6 May, 2018

Dear Carmona,

I had forgotten this letter ([1]), and started reading it.

I understand the triangle (T) on page 1 but I expect the "universal argument" on page 3 to be flawed, and that so is the "0" at the end of $(*)$ page 2.

A suitable universal case is the topos of presheaves on the category opposite to that of (finite dimensional) vector spaces over $\mathbf{F}_2$, i.e the topos of covariant functors

$$\text{Such vector spaces} \longrightarrow \text{Sets}.$$

If in this topos one takes as abelian sheaves M and N the tautological send V to V and if in $\mathrm{Hom}(M.N)$ one takes the identity, surjectivity of $(*)$ implies that to V one can functorially attach a commutative Picard category $P(V)$ having as abelian group of isomorphism classes of objects V, as abelian group of automorphisms of the unit object 0 (or any object) again V, and for which the commutativity $XY \longrightarrow YX$ is, for $X = Y$ given by the class of X in V. Functorially means :

For $V \longrightarrow W$, a functor $P(V) \longrightarrow P(W)$ with a compatibitity with product (respecting the commutativity and associativity data) ; for $U \longrightarrow V \longrightarrow W$, an isomorphism of functors (from $P(U)$ to $P(W)$, respecting the compat-

[1] Letter of A. Grothendieck to P. Deligne on Picard stacks (7.8.74)

ibilities with products); for $T \longrightarrow U \longrightarrow V \longrightarrow W$, a compatibility between those isomorphisms.

I convinced myself that this does not exist, even when one considers just the topos of presheaves on the category opposite to that of vector spaces over $\mathbf{F}_2$ of dimension at most 2, the rest being the same.

Best,

Pierre Deligne