A simple model with liquidity

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Abstract

We introduce liquidity motives in an otherwise standard monetary model. The Central Bank's policy rule is adapted to target the interest rate on liquid bonds. These deviations are sufficient to relax the requirement for active monetary policy and warrant determinacy in both passive and active policy regimes. We compare this model of liquidity with workhorse models and find that it can substantially replicate usual dynamics. By means of stochastic simulations, we also study how monetary policy stance affects inflation dynamics and find evidence of increased persistence for passive monetary policy.
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1 Introduction

In the wake of the 2008 Global Financial Crisis liquidity and finance have risen to the central stage of macroeconomics. As financial and interbank markets froze and policy interest rates fell to zero, major central banks injected unprecedented amounts of liquidity (Quantitative Easing measures, QE) to revive financial markets. Fed's Chairman Ben Bernanke famously commented on the ex-ante effectiveness of Quantitative Easing policies stating that "The problem with QE is [that] it works in practice, but it doesn't work in theory." Indeed, monetary policy frameworks at the time were ill-equipped to deal with such unconventional interventions. Besides lacking a thorough characterisation of the effects of financial turmoil, workhorse models implied a number of puzzles at the Zero Lower Bound

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(ZLB): implausibly large current effects of small (announced) policy interventions in the future, counter-intuitive effects of increased price flexibility, and sunspot equilibria for output and inflation as the policy rate becomes irresponsive to expectations. The last implication became salient when the ZLB became binding and central banks switched to liquidity management.

This paper contributes to the study of the role of liquidity in monetary policy in general, and how it affects the restrictions on interest rate rules in particular. We introduce liquid bonds and total liquidity in an otherwise standard New Keynesian monetary model. We equip the monetary authority with a simple rule that fixes total liquidity and sets the interest rate on liquid assets. Expanding the set of bonds, differentiated by their degree of liquidity, permits a more realistic characterisation of interest rate policy, with the policy rate associated to a liquid bond, rather than the risk-free non-liquid bond typical of standard models. Moreover, with liquid and illiquid assets, the monetary instruments of the central bank are both an interest rate and a quantitative measure of total liquidity.

The first result is that, in contrast with the traditional Taylor principle, this framework yields a stable equilibrium as long as the central bank responds positively to expected inflation, regardless of the intensity of the interest rate adjustment. Secondly, we study how inflation dynamics are affected by monetary policy stances. We find that less reactive central banks impart a higher degree of persistence to realised inflation, while active policy rules reduce volatility and persistence in simulated inflation.

This model thus provides a simple framework to analyse how the introduction of liquidity affects the response of the economy under a supposedly indeterminate regime. We compare this liquidity model to the baseline standard model that is at the core of the New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) class, with nominal frictions, technological shocks, and rational expectations. The inclusion of liquidity and a simpler monetary policy rule are sufficient to broadly match and reproduce the behaviour of the NKDSGE baseline model. This approach has several attractive features. First, it allows us to move in known territory, making thus possible to compare the properties of the model with the standard NKDSGE model. Second, it can be easily implemented in existing theoretical structures with negligible adjustments, making possible a direct test against other policy-relevant models. Moreover, such a common ground allows further extensions to the analysis of fiscal policy and banking.

Through stochastic simulations, we analyse how the degree of policy accommodation (summarised by the coefficient on inflation in the interest rate rule) directly affects inflation dynamics. We find that an accommodative monetary policy induces more persistence in the inflation process, making it harder to reach the inflation target.

Finally, in close connection to the 2008 crisis, we experiment with a large drop in liquidity and analyse how different policy regimes affect economic responses.

Related literature

This paper studies the role and consequences of liquidity in monetary policy and formulates a setup encompassing both aggressive and passive monetary policy interest setting rules. This theoretical model builds on Calvo (2016) and also relates in spirit with Michaillat and Saez (2015, 2021), Benhabib, Schmitt-Grohe, and Uribe (2001), and Canzoneri, Cumby, et al. (2008a,b) and Canzoneri and Diba (2005).

These latter contributions in particular extensively study the role of liquid (government) bonds in providing transaction services. As shown in those works, this specification is equivalent to one in which rather than liquid government bonds, the distinction is made between base money and bank deposits, which pay an interest rate, which in turn represents the policy rate. In this different specification, total liquidity is given by broad money (base money plus deposits, usually defined as M1). Canzoneri, Cumby, et al. (2011) take a similar approach in showing that money, liquid bonds, and transaction frictions help in ruling out controversial sunspot equilibria even with passive monetary policy. A similar framework, developed in Canzoneri and Diba, 2005, draws comparable conclusions with a more detailed role for fiscal policy. Our results are similar to those achieved in Canzoneri, Cumby, et al. (2011) and Canzoneri and Diba (2005), but arise from a simpler framework that abstracts from the presence of transaction frictions and fiscal policy. A series of contributions (Benhabib, Schmitt-Grohe, and Uribe, 2001; Benhabib, Schmitt-Grohé, and Uribe, 2002; Schmitt-Grohe, Benhabib, and Uribe, 2001; Schmitt-Grohe and Uribe, 2000, 2007) investigates the determinacy properties of monetary policy rules with a framework similar to ours. A common feature of those works is to assume that also firms use money for transactions, and that the central bank reacts only to inflation. We adopt a similar simplified feedback rule for the central bank but we restrict liquidity only to the consumer block of the model. Schmitt-Grohe and Uribe (2007), moreover, show how indeed feedback rules that include output are dominated by simpler rules in terms of welfare.

We elaborate the Calvo (2016) model by fully specifying the supply side of the economy and studying its behaviour in discrete time. This model is a stripped-down version of the framework presented in Calvo and Végh (1990a,b) and Calvo and Vegh (1996), which instead are applications to exchange rates in a small open economy. Throughout the paper, our aim is to focus on a small deviation from the standard NKDSGE framework that constitutes the core of modern fluctuations theory in macroeconomics (Gali, 2015; Walsh, 2003; Woodford, 2003). Diba and Loisel (2021) also propose a simple departure from the baseline NK model, featuring a money-in-utility extension and a pegged policy rate, that solves part of the puzzles.

A foundational approach to the role of money in a search setup is developed and discussed in a series of contributions such as Aiyagari and Wallace (1992), Aiyagari, Wallace, and Wright (1996), and Kiyotaki and Wright (1991). The main focus there is to provide a role for fiat money as medium of exchange. Our take is simpler and assumes away the utility provided by liquid assets and cash, in the interest of studying how they affect monetary policy. Our take is simpler, as we concentrate on reduced-form models in which money or liquidity entered directly the utility function or it enters through liquidity in advance constraints. A general overview of the salient approaches to model liquidity is provided in Lagos, Rocheteau, and Wright (2017), with a focus on money liquidity.

This paper is also related to studies on conditions for determinacy in several vintages of New Keynesian models. Benhabib, Schmitt-Grohe, and Uribe (2001) include money in the production function and studies under which fiscal and monetary policy regimes the model displays indeterminacy. Relatedly, Leeper (1991) studies how fiscal and monetary policy interact and establishes parametric regions for determinacy: fiscal dominance can ensure determinacy over a passive monetary policy. Along the same lines of fiscal interaction, Cochrane (2020) further develops a price level theory grounded on debt and taxes, working around the limitations and shortcomings of the monetary NKDSGEs. In this model, we abstract from fiscal considerations and focus exclusively on the liquidity factor. Likewise, we also avoid considerations on market frictions and distributional consequences of liquidity shocks, which are discussed more broadly in Iacopetta and Minetti (2019): we stick to a representative agent framework and focus on the consequences of liquidity for determinacy.

Similarly, our simplified model abstracts from a refined banking sector to emphasise the role of liquidity. Recent works find that the credit and bank lending channel affect the transmission of monetary policy, especially when credit is directed to the housing sector (Iacoviello and Minetti, 2003, 2008): the inclusion of such channel, though, constitutes a promising avenue for future research.

The remainder of the paper is structured as follows: Section 2 introduces liquidity in the NKDSGE framework; Section 3 compares the liquidity model with a baseline one; Section 4 analyses how monetary rules influence inflation dynamics; Section 5 studies the effects of a severe liquidity shock. Finally, Section 6 concludes.

2 A model with liquidity

We introduce liquid bonds in an otherwise standard, small scale NKDSGE model. This extension is motivated by the utility they provide in being liquid assets, which provide utility services. Introducing liquidity services directly in the agent's utility function is a reduced-form approach, but is an informative exercise to explore the implications of such extensions. Reasonable micro-foundations call for more complete modelling of the financial block of our stylised economy. The main idea is to model agents' liquidity portfolio allocation between plain money and liquid assets, interchangeably bonds or bank deposits. Including banks, firms' financial position, financial frictions is typically carried out in a structural way following the footsteps of Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), and Kiyotaki and Moore (1997). Since our interest lays in understanding the consequences of liquidity rather than its deeper motives, at present we opt for the simplicity of a transparent reduced form.

In the same vein, we approach the policy modelling. While Gertler and Karadi (2011) set out to directly model unconventional monetary policies, we restrict the scope of monetary policy with a simplified rule and the assumption of a fixed amount of nominal circulating assets. We thus postulate a simple feedback rule that relates the interest rate paid on liquid assets to expected inflation. Such simplified rules have been extensively explored for their debated determinacy properties, as is shown in Benhabib, Schmitt-Grohe, and Uribe (2001), Benhabib, Schmitt-Grohé, and Uribe (2002), Schmitt-Grohe, Benhabib, and Uribe (2001), and Schmitt-Grohe and Uribe (2000, 2007).

2.1 Consumer

The major variation from workhorse models is limited to the structure of flow-utility and overall assets portfolio of the consumer. We postulate an economy with an infinitely-lived representative agent, who works, consumes, and holds money alongside two types of assets. Therefore, total wealth is allotted between a liquid bond B, cash M, and an illiquid bond X. This latter serves the only purpose of smoothing consumption over time and allocate wealth intertemporally. All assets mature after one period. We assume the consumer is willing to hold B and M because they provide transaction services, and therefore utility. In addition, B bonds pay a nominal interest rate s, X ones pay nominal interest rate i. Cash holdings pay no interest and are carried on to the next period, suffering inflation erosion.

While inserting money balances in the utility function is not new (Sidrauski, 1969), adding bond holdings in the utility function is less common. In this respect, our model is close to Michaillat and Saez (2021), although the resulting Euler equation at the steady-state is not modified by bonds but rather by money holdings. In fact, it is completely homomorphic to introduce a cash-in-advance (or rather, liquidity-in-advance) constraint, as in Calvo and Vegh (1996).[1]

Under these assumptions, the utility maximization problem of the consumer takes the following form:

$$\max_{\{c_s, M_s, b_s, N_s\}_{s=0}^{\infty}} E_t \left[\sum_{s=0}^{\infty} \beta^s \left(u(c_{t+s}) + h\left(b_{t+s}\right) + v\left(M_{t+s}/P_{t+s}\right) - g\left(N_{t+s}\right) \right) \right] \tag{1}$$

$$s.t. \quad C_t + M_t + X_t + B_t = W_t N_t + \left(1 + s_{t-1}\right) B_{t-1} + \left(1 + i_{t-1}\right) X_{t-1} + M_{t-1}$$

Where we assume additively separable (dis)utilities for consumption c, cash m, liquid bonds b, and hours worked N. Moreover, c is the result of aggregating a measure one of differentiated goods via a Dixit-Stiglitz aggregator with constant elasticity of substitution θ. This also implies that P is the price index of the underlying goods.

The intertemporal budget constraint summarises expenditures, allocations and income sources: interests promised at $t-1$, carried-on money, and labour income.

It is useful to reformulate the budget constraint in terms of total wealth and real quantities before deriving the system of first-order conditions. Therefore, let $D = X + M + B$ be the total wealth held by the consumer. Replacing $X = D - M - B$ in the budget constraint and appropriately dividing through P gives the following real budget constraint.

$$c_t + d_t = w_t N_t - \frac{i_{t-1} - s_{t-1}}{1 + \pi_t} b_{t-1} + \left(1 + r_{t-1}\right) d_{t-1} - \frac{i_{t-1}}{1 + \pi_t} m_{t-1} \tag{2}$$

Where $\pi = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate and lower-case indicates real quantities. Rewriting the constraint in such forms highlights the opportunity costs of holding bonds b and cash m: assuming a positive spread $i > s$, the consumer gives up the additional interest paid by illiquid bonds for every additional unit of b the consumer holds.

Similarly, holding cash entails giving up entirely on the nominal interest rate i. These two opportunity costs are offset by the marginal utilities provided by holding such assets and will determine holdings and allocations at the equilibrium.[2]

[1] Appendix C proposes an alternative setup with a Liquidity-In-Advance which yields very similar quantitative results.

[2] This reduced form specification is presented to highlight the effects of liquidity on monetary policy. A Liquidity-in-Advance version is deferred to Appendix (C).

With this reformulation, the FOCs system implies the equilibrium conditions in (3):

$$\begin{aligned} u'(c_t) &= E_t\left[\beta(1+r_t)u'(c_{t+1})\right] & h'(b_t) &= E_t\left[\beta\lambda_{t+1}\frac{i_t - s_t}{1+\pi_{t+1}}\right] \\ \frac{g'(N_t)}{u'(c_t)} &= w_t & v'(m_t) &= E_t\left[\beta\lambda_{t+1}\frac{i_t}{1+\pi_{t+1}}\right] \end{aligned} \tag{3}$$

where λ_t is the Lagrangian multiplier, the first equation is the usual Euler equation for intertemporal consumption, the second equates marginal cost and benefits of work, and the last two equations govern the allocation decision between liquid bonds b and real money balances m. In particular, the latter condition discounts money holdings by the real interest rate, that is the loss from inflation; the equilibrium condition on bonds, instead, regulates liquid asset holdings on the spread between nominal interest rate i and yield of such bonds, both in real terms. For the purpose of this paper, we will assume a non-negative spread between i and s, such that the consumer holds simultaneously cash and bonds at every period.

2.2 Firms

The production side of this economy is straightforward and assumes a measure one of infinitesimal firms, indexed by $j \in [0,1]$. Each firm is embedded with a technology that employs only labour, so that the production function is

$$Y_{jt} = A_t N_{jt}^a. \tag{4}$$

The term A captures the stochastic productivity of the economy and follows a simple AR(1) process, $a \in (0,1)$ represents the decreasing returns to scale, and N_{jt} the individual employment of each firm. As the consumption good results from the CES aggregator, every firm j faces a demand schedule (5), relative to aggregate production, with θ being the elasticity of substitution and P_{jt} the firm's price.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t \tag{5}$$

We assume nominal rigidity à la Calvo (1983):[3] every period, a $(1-\alpha)$ portion of firms is given the chance to update their price, while the remaining share will stick to previously set prices. This entices a forward-looking behaviour in firms when they optimise their expected discounted profits, as they take into account

[3] The precise modelling of the nominal rigidity is inconsequential, as the main novelties are in the consumers' side of the model. Hence, quadratic adjustment costs as in Rotemberg (1982) might be added without loss of fundamental insights.

the duration of their price. Firm j's marginal cost is $\mathcal{MC}_{jt} = \frac{W_t/P_t}{aY_{jt}/N_{jt}}$, whence the expected discounted profits in eq.(6).

$$\begin{aligned} &\max_{P_j^*} \quad E_t\left[\sum_{s=0}^{\infty}\alpha^s Q_{t,t+s}\left(P_j^* Y_{jt+s} - \mathcal{MC}_{jt+s}Y_{jt+s}\right)\right] \\ &s.t. \quad Y_{jt+s} = \left(\frac{P_{jt+s}}{P_{t+s}}\right)^{-\theta} Y_{t+s} \end{aligned} \tag{6}$$

Where α is the Calvo pricing parameter, $Q_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ the consumer, who owns all firms, uses to weight future profits, and P_j^* is the optimal price chosen by the firm. Factoring in the constraint and solving the program with a symmetry argument gives two results. First, firms price with a constant markup over the marginal cost, and second that the optimal price is specified as a function of expected future marginal costs, price index levels and economic activity, as shown in eq.(7). This same equation also presents the inflation dynamic as an autoregressive process of order one, depending on past prevailing prices and current updated prices.

$$\frac{P^*}{P_t} = \frac{\theta}{\theta-1}\frac{E_t\sum_{s=0}^{\infty}(\beta\alpha)^s Y_{t+s}\left(\frac{P_{t+s}}{P_t}\right)^{\theta}\mathcal{MC}_{t+s}}{E_t\sum_{s=0}^{\infty}(\beta\alpha)^s Y_{t+s}\left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}} \qquad P_t^{1-\theta} = (1-\alpha)\,P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{7}$$

2.3 Monetary authority and market clearing

We also depart from the standard setup in characterising the central bank, its policy, and total liquidity management. We assume the existence of a monetary authority that operates in two ways in the economy. First, it sets the total amount of liquidity in circulation, namely eq.(8). Importantly, the central bank does not directly determine the allocation between cash and liquid bonds, but only the sum of the two, irrespectively of the portfolio composition. This setup provides the baseline building block for further extensions of this framework. Equipping monetary policy with quantitative easing tools equates to provide the central bank with a balance sheet and the channels to adjust the total nominal liquidity Z_t. For the sake of simplicity, we will also assume a fixed nominal amount of circulating liquidity, so that

$$Z_t = B_t + M_t = \bar{Z} \quad \forall t \tag{8}$$

A fixed amount of liquidity implies no issuance of liquid bonds: ideally, to capture the effects of the central bank's quantitative easing and balance sheet policies, B would be issued by either a government (Cochrane, 2020), banks (Gertler and Karadi, 2011), firms (Ravenna and Walsh, 2006; Surico, 2008), or combinations thereof. The same principle applies to money, which is injected or withdrawn to seamlessly counteract the movements in liquid bonds trade. In this light, B closely resembles bank deposits, for both can be readily exchanged for cash. One can think of Z as a fixed money supply (M2), although a full characterisation would have B issued by the government in the form of liquid, risk-free treasury bonds. Along the same lines, the central bank would influence the circulation of these bonds to manage total liquidity and, more importantly, to coordinate with fiscal policy.[4]

The central bank also sets the policy interest rate s_t paid on liquid bonds following a constant rule. It is useful to think in this context to the link between the Federal fund rate and the interest rate on the shortest-maturity Treasury Bill. In detail, the rule responds solely to inflation expectations and does ignore any level of economic slack contrary to more usual Taylor feedback rules:

$$\exp(s_t) = \exp(E_t \gamma \pi_{t+1}). \tag{9}$$

This skeletal structure is clearly a simplified Taylor rule, but the values for γ are crucial. An accommodative central bank, with $0 < \gamma \leq 1$, does not necessarily trigger unstable sunspot equilibria in our model, but rather stabilises the economy despite the accommodative stance.[5] Such a rule can easily be extended to a full-fledged Taylor Rule including real slack without impairing or modifying the final results. As an actual example, this specification of the Taylor Rule relates to that of the European central bank, which in its mandate contemplates explicitly only price stability and not employment or economic slack.

The rationale behind this specification is that, keeping under control the liquidity in circulation, the Central bank assures that inflation follows a specified path. This particular specification of the monetary policy links the inflation rate and the return yield on liquid bonds. Therefore, the central bank in our model levers the liquidity allocation via interest rate setting over a fixed amount of Z_t.

Last, taking eq.(8) and dividing through by the prices level, one can obtain the values for liquidity allocation in real terms, as well as a backwards-looking expression for real liquidity depending on current inflation.

[4]Notable examples are the CARES Act in the US, or the combined SURE and Next Generation EU in Europe, both in response to Covid-19 crisis. These were preceded by ECB's APP-PSPP and Fed's Open Market Operations as measures for the 2008 crisis.

[5]Although it is worth recalling that a central bank complying with the Taylor principle actually *destabilizes* the economy (Benhabib, Schmitt-Grohe, and Uribe, 2001).

$$z_t = m_t + b_t \Longleftrightarrow z_t = \frac{z_{t-1}}{1+\pi_t} \tag{10}$$

This last equation, with the market clearing condition $C_t = Y_t$, closes the model.

2.4 Linearised model

In this section we briefly present the system of equations resulting from loglinearising eqs. (3), (7), (9), and (10) around a zero-inflation steady state. To this end, we assume precise functional forms for the utility functions, namely CRRA

$$\begin{aligned} u(c) &= \frac{c^{1-\sigma}}{1-\sigma} & v(m) &= \frac{m^{\psi}}{\psi} \\ h(b) &= \frac{b^{\phi}}{\phi} & g(N) &= \chi\frac{N^{1+\eta}}{1+\eta} \end{aligned} \tag{11}$$

With $1 \geq \phi > \psi > 0$, which implies that the consumer is more sensitive to bonds rather than money, in line with everyday financial decisions. The other functional forms assumed are consistent with more traditional exercises and well settled in the NKDSGE literature. This shared ground highlights how consequential liquid assets can be, once modelled as a complement to cash, especially on the policy rule.

The loglinearised model consists of a system of linear equations whose properties can be easily and extensively studied. We perform a comparison with the simple 3-equation model presented in Gali (2015), for example.[6]

$$\hat{y}_t = \frac{1-\psi}{\sigma}\hat{m}_t + E_t\hat{y}_{t+1} + \frac{1}{\sigma}E_t\pi_{t+1} \tag{12}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\left(\hat{y}_t - \hat{y}_t^f\right) \tag{13}$$

$$(1-\phi)\left(1+m^{\frac{\phi-\psi}{1-\psi}}\right)\hat{z}_t = \left[\frac{1-\psi}{m^2} + m^{\frac{\phi-\psi}{1-\psi}}(1-\psi)\right]\hat{m}_t + \left(\beta y^{-\sigma}m^{1-\psi}\right)s_t \tag{14}$$

$$\hat{z}_t = \hat{z}_{t-1} - \pi_t \tag{15}$$

[6]A more detailed walk-through for obtaining this system is provided in the Appendix (A.1)

$$s_t = \gamma E_t \pi_{t+1} \tag{16}$$

Eq.(13) is the Phillips curve of this economy, in line with more classical models. Eq.(12) is the Euler equation augmented with the real money balances, which affect positively the contemporaneous output gap. This wedge subsumes the transaction utility of liquidity: positive values for $\hat{m}$ entail higher output levels. As the Phillips Curve (13) relates economic slack to inflation, the wedge also affects this latter, and eventually its expected value.

Eq.(14) summarises the money demand function (depending on liquid bonds interest rate s, total real liquidity z). This relation shows how, indeed, the central bank affects asset allocation for a given level of total liquidity, levering the policy rate s_t.

Eq.(15) captures intertemporal changes in total real liquidity, and finally eq.(16) represents the monetary policy rule.

We adopt the convention that $\hat{x}$ is the percentage deviation of x from its steady-state, while all lower-case, unhatted, and time-independent variables are steady-state values. Moreover, we use y^f for frictionless output, so that $(\hat{y}_t - \hat{y}_t^f)$ is the output gap.

Finally, some exogenous variables and parameters are grouped as follows:

$$\begin{aligned} \kappa &= \left[\frac{(1-\alpha)(1-\alpha\beta)\,a}{\alpha\,(a+\theta\,(1-a))}\right]\left[\frac{1+\eta+a\,(\sigma-1)}{a}\right] & m &= \left(\frac{y^{\sigma}}{1-\beta}\right)^{\frac{1}{1-a}} \\ \hat{y}_t^f &= \frac{\eta+1}{1+\eta+a\,(\sigma-1)}\hat{A}_t & y &= \frac{\eta+1}{1+\eta+a\,(\sigma-1)} \end{aligned} \tag{17}$$

As this approximation of the full model can be easily simulated, we exploit it to check for which calibration sets the model generates a unique and stable equilibrium.

To fully specify the model process, we introduce two standard shocks to perturb the model around its steady state to display convergence dynamics. Both shocks follow a stationary autoregressive process of order one:

$$\begin{aligned} \hat{A}_t &= (1-\rho_A)\,\bar{A} + \rho_A \hat{A}_{t-1} + \epsilon_t^A & \epsilon_t^A &\sim N(0, \sigma_A) \\ \nu_t &= \rho_\nu \nu_{t-1} + \epsilon_t^\nu & \epsilon_t^\nu &\sim N(0, \sigma_\nu) \end{aligned}$$

In addition to the real, technological disturbance, we add a policy shock impulsed by the central bank. The former induces a change in the total factor productivity A from its steady-state value, set to 1. The latter is a shock to the monetary rule detailed in eq.(9). These two shocks allow the comparison with the aforementioned standard models, so to perform a horse race and check the consistency of our augmented model. Assessing whether our setup replicates the classic reactions of well-known models is a first paramount consistency check to validate its structure and internal workings before further analysing policy stance implications.

Both shocks are simply added to their respective equations, the production function and the monetary policy rule, and throughout the whole simulations, we calibrate their persistence parameters ρ to the same value.

3 Calibration and IRFs

Calibration and simulation is a usual exercise and there is a large literature to inform our choice of parameters' values.

We calibrate the model to the values presented in Table (1), taking the most common values used in the literature. The novel restriction involved in the model concerns exponents of bonds and real balances utility functions. As of the values chosen for semi-elasticities ψ and ϕ, values from the literature are scarce, thus we conservatively underweight liquid bonds with respect to money holdings. For the "Taylor Principle" parameter of our policy rule, we explore two candidate values. The first is supposed to violate the Blanchard and Kahn (1980) condition and thus produce an unstable solution, whilst the second is the one most commonly found in both empirical studies and theoretical exercises.[7] These two values are also what we find in **??** for a variety of regimes that include or exclude financial liquidity.

The remaining calibrated parameters are fairly standard. Price duration is chosen to obtain four quarters on average Woodford (2003). The parameter governing returns to scale matches the labour share usually observed in the data. Intratemporal elasticity offers several values and affects firms' markup, we conservatively pick a low value from Bilbiie, Ghironi, and Melitz (2019), used to target micro evidence. For intertemporal elasticity of substitution, we pick a fairly high value in comparison to what Havranek et al. (2015) find in a meta-analysis of roughly 170 studies. If anything, this high value penalises the wedge present in the Euler equation (12), but equally affects the comparison with NKDSGE models.

Two facts emerge from this calibration exercise. First, our calibrated model generates a unique, stable equilibrium for both values of γ. Thus, sunspot equilib-

[7] Appendix D offers some values for semi-elasticities ψ and ϕ. These changes mainly affect persistence and do not dramatically affect the quantitative results, nor do they impart the qualitative profiles of IRFs. Similarly, Appendix E explores the implications of varying values of γ on inflation persistence.

Table 1: Calibration for model simulations

Parameter	Descr.	Value
a	ret. to scale	.6
β	discount rate	.975
σ	intertemp. el. of subst.	5
θ	intratemp. el. of subst.	3.8
α	price duration	.75
η	Frisch elast.	1
χ	labour disutility	1
ρ_A	persistence, TFP shock	.65
ρ_ν	persistence, MP shock	.65
ψ	bond el.	.02
ϕ	money el.	.65
γ	Passive Rule	.5
	Active Rule	1.8

ria are ruled out even if the central bank reacts passively to the inflation expectations of the economy.

Second, our model behaves as expected once compared with the 3-equation NKDSGE counterpart.[8]

To illustrate this point, we first compare side-by-side the effects of a technological shock (Fig.(1)), then compare the effects of a monetary policy shock under two regimes for our model (Fig.(2) and Fig.(4)).

Both shocks are present in the baseline versions of the models in exam, which allow for a meaningful comparison. In order to carry out the comparison on equal ground, the two models are calibrated with the very same values for the common parameters. In fact, under the same calibration, the two models show the same behaviour in terms of reactions to shock, as it is possible to see in the next figures.

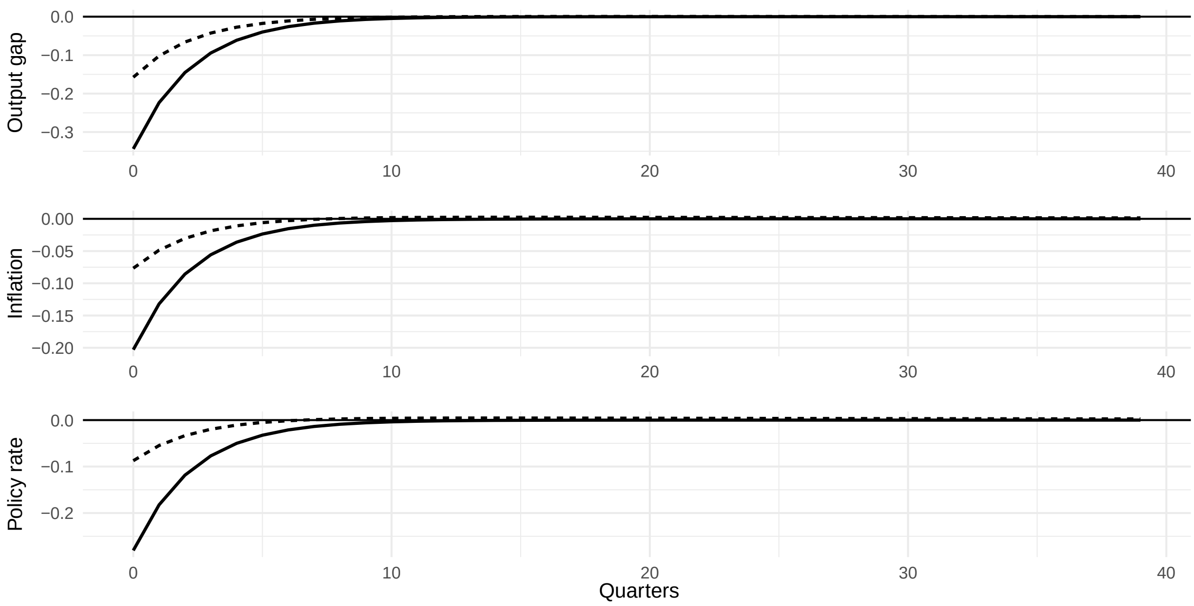
Technological shock

Fig. (1) depicts the reactions to a one-standard-deviation, positive shock to total factor productivity, $\hat{A}_t$. It produces the same response in our model and in the standard NKDSGE one. This first result is not surprising, because of the very same structure of the production side of the two stylised economies.

We focus on three common endogenous aggregates to evaluate the matching between the models. Following a TFP shock, in both cases the output gap turns

[8]Gali (2015), paper III, version with interest rate rule.

Figure 1: TFP Shock – IRFs



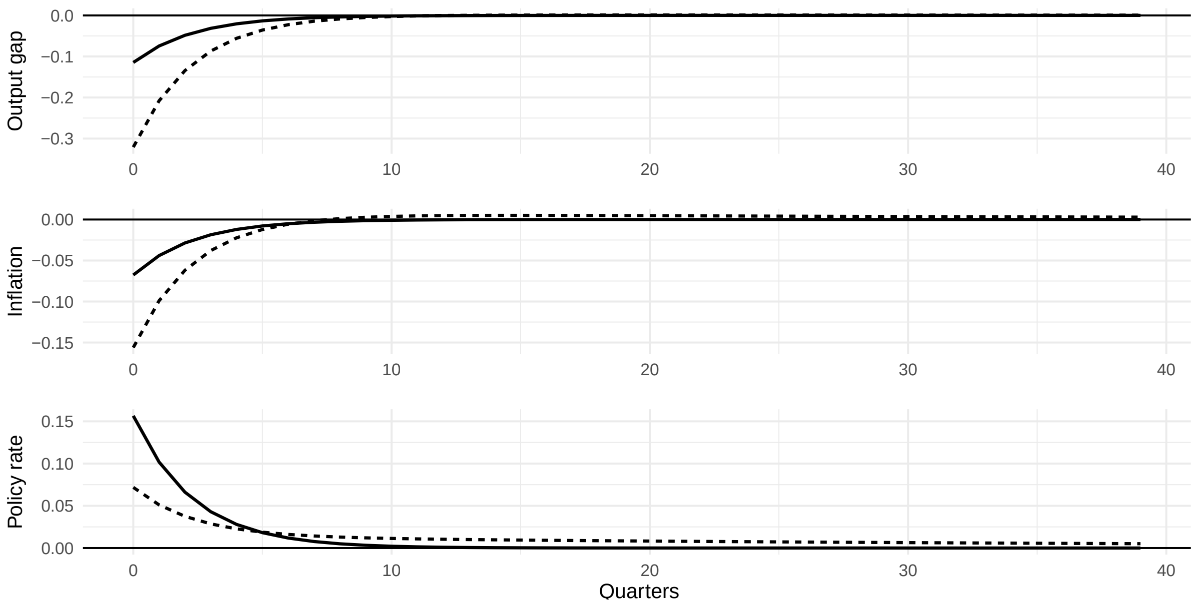
Liquidity model (dashed, $\gamma = 1.8$) and Gali (2015) (solid): impulse response functions following a one standard-deviation technology shock. Values in percentages.

negative, inflation falls, and the reference interest rate falls in response. While this reaction is common across models, magnitudes mark little differences: output and inflation fall more in the NKDSGE model, whereas our proposal implies less movement in these aggregates: for inflation and output gap, the response in our model is broadly half on impact. This milder response in our model is driven by the stripped-down version of our monetary policy rule, which prevents output gap movements to percolate into the variations in s. An alternative, concurrent explanation for the dampened propagation works through the two complete financial markets included in our model, where the TFP shock is dissipated smoothly through two complete financial markets.

Monetary policy shock

When the modelled economies are hit by a monetary policy shock and the liquidity model complies with the Taylor Principle, they generate the IRFs pictured in Fig.(2). The IRFs produce the same profiles in both cases, but the liquidity models report higher volatility and greater impact on output gap and inflation. In both cases, the magnitude is roughly thrice that of the baseline NKDSGE, while direction and adjustment correspond closely. The skeletal policy rule engrained in the liquidity model abstracts from the economic activity level, thus the central bank is not facing trade-offs between economic activity and inflation and focuses solely on the latter. This monetary policy rule, moreover, impedes the feedback loops between policy rate, inflation and economic activity, designing a different propa-

Figure 2: Interest Rate Shock – IRFs



Liquidity model (dashed, $\gamma = 1.8$) and Gali (2015) (solid): impulse response functions following a 1% key rate hike, annualised. Values in percentages.

gation mechanism: adjustments in the market for money and liquid bonds affect consumption and, then, the supply side of the economy.

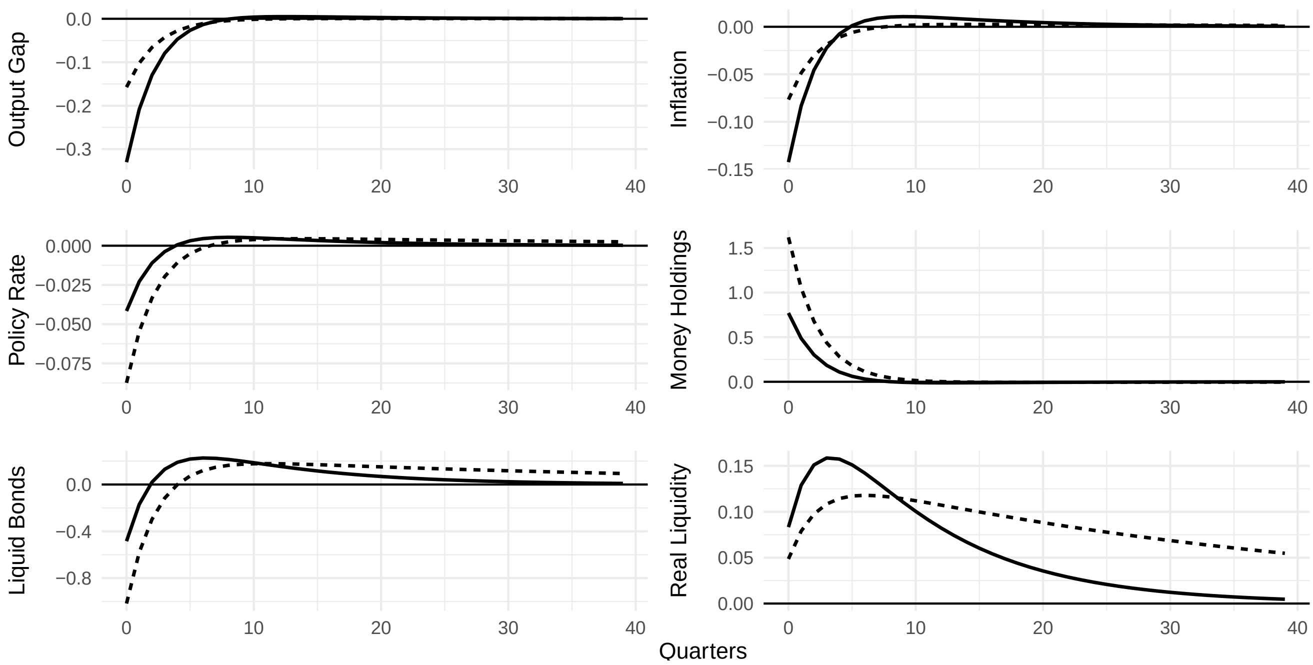
Focusing on the policy rate, our model responds roughly twice less than the NKDSGE model to the very same shock: the propagation mechanism works mainly through the consumer's financial position in the model with liquidity. In addition, our monetary shock is well more persistent in the interest rate, converging back to zero only when inflation levels off, too. The absence of economic activity in the rule makes that the policy rate essentially mirrors the inflation profile. In comparison to the baseline model, the liquidity one presents a slower convergence path to the steady-state. This model, therefore, entails a higher degree of persistence in its design, even with an active central bank.

A more thorough analysis of the effects of monetary policy stance on inertia will be the focus of Section 4.

3.1 Liquidity with an accommodative central bank

After checking that our model consistently matches the response of the workhorse model, we turn now to showing how the liquidity model reacts to the previous shocks under different policy stances. The focus of this exercise is on the passive regime, where we set $\gamma = .5$ and analyse the behaviour of all endogenous aggregates in comparison to the active policy stance. We also include the dynamics of liquid bond holdings, recovered from the FOCs, to complete the picture.

Figure 3: TFP Shock IRFs – Both Regimes



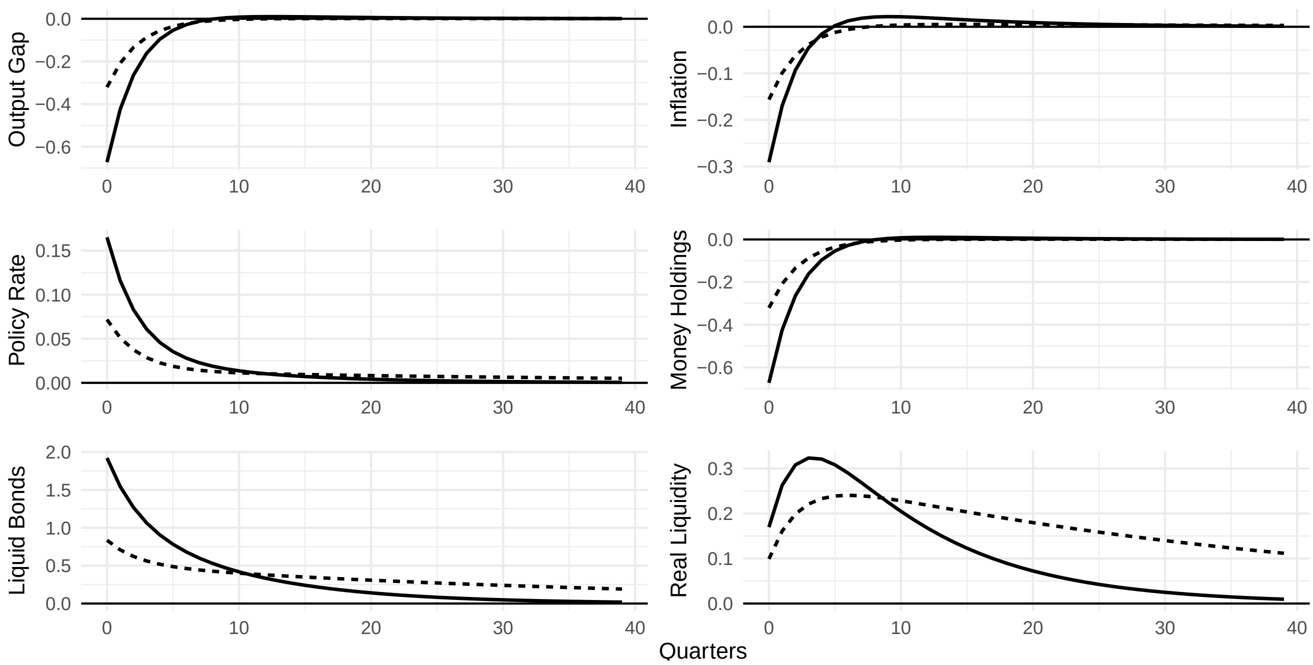
Impulse Response functions for our model: with accommodative (solid) and aggressive (dashed) reaction to expected inflation after a productivity shock (one standard deviation) – all endogenous variables, values in percentages.

We start with the IRFs generated by a positive TFP shock, collected in Fig.(3). In the aggressive regime, output, inflation, and the policy rate s fall. Interestingly, s and π slightly overshoot before converging to zero from above after about five quarters. Inflation and policy rate dynamics map almost one to one to the asset allocation and real liquidity. This behaviour is reflected in real liquidity and its components: in comparison to the active stance, the technological shock doubles the impact on total real liquidity, the interest rate drop triggers a reallocation away from bonds towards cash m due to deflation, until s overshoots and subsequently offsets the overall effect on liquidity reshuffling.

All in all, the main discrepancy between the two regimes is in the asset block: the inflation path influences dramatically and on a shorter period the amount of real liquidity when the central bank is passive. Conversely, the path back to the steady-state is significantly quicker in the passive regime.

Figure 4 summarises the response to a monetary policy shock that raises the interest rate s on bonds of 1%. First off, in both policy regimes output moves in the same direction and is affected similarly. Secondly, inflation falls as expected and after 15 quarters the shock is fully absorbed, without unexpected evolutions of the prices. This last result is particularly relevant as one would expect erratic

Figure 4: Interest Rate Shock IRFs – Both Regimes



Impulse Response functions for our model: with accommodative (solid) and aggressive (dashed) reaction to expected inflation after a monetary policy shock (annualised 1% policy rate hike) – all endogenous variables, values in percentages.

paths instead of a well-determined dynamic for prices under a passive monetary policy regime.

Accounting for the effects of financial liquidity relaxes the stringency of the Taylor Principle. The relevance of financial markets is also shown by Bilbiie and Straub (2013), which provide a theoretical framework to analyse how financial market participation impacts the policy requirements for determinacy.

On the total liquidity side, two forces are at work: the inflation effect and arbitrage towards the more remunerative asset. On impact, inflation decreases: this path influences real liquidity as future inflation will be higher than today, increasing current liquidity until inflation overshoots its steady-state level and turns slightly positive. This happens over a relatively short period. When inflation turns back positive (although extremely close to zero), real liquidity peaks and decreases smoothly.

In this process, the agent adjusts its portfolio of assets profiting from the increased return on the liquid asset, b. In this respect, the IRF for m mirrors that of b, with a reallocation away from money holdings to liquid bonds. The asset flows following a positive productivity shock are thus flipped when the monetary authority raises the policy rate.

Under the passive monetary policy regime the economy experiences different magnitudes in the aggregates, but qualitatively similar profiles. When the central bank is accommodative, output, inflation, and the key policy rate double their impact change, and the latter becomes less persistent, overall. This last outcome falls in line with Primiceri (2005), relating to a passive pre-Volcker Fed, and sheds light on how monetary policy stances can affect aggregate dynamics in general, and inflation dynamics in particular. Taking this implication to the data would imply that prior to the Great Moderation period inflation was reporting higher degrees of inertia, whilst it would decrease afterwards. paper **??** reports indeed decreasing levels of inertia, although the timing does not univocally point to policy switches.

We analyse the effects of different monetary policy regimes on inflation dynamics in Section (4).

4 Dissecting simulated inflation dynamics

As discussed, the monetary policy stance could influence the dynamics of some endogenous aggregates. We are particularly interested in inflation, as typically a passive central bank enables sunspot, degenerate paths for prices. While this inquiry relies on stylised economies, paper **??** delves into an empirical analysis of inflation persistence in the US.

The motivating question we ask is, thus, whether a passive central bank produces unstable inflation dynamics. In this framework, it is straightforward to generate abundant time series and hence conduct some ex-post econometric exploration. To offer a more comprehensive comparison, we simulate and analyse two other DSGE models, namely Ascari and Sbordone (2014) and Smets and Wouters (2007). The former study how trend inflation affects aggregate dynamics and policy in a generalised New Keynesian model, while the latter builds a rich environment with nominal frictions and indexation for wages and prices, investment adjustment cost, and a large number of shocks and outperform VARs in short term forecasting.

We generate for each model 500 thousand observations, or 125 thousand years of simulated history: this should assure convergence of the estimators and tight confidence intervals. By the very structure of the models, the Data Generating Processes of these series is a linear system shocked by AR(1) normal innovations: one should not be surprised that the data generated are also Gaussian. Comparability across models is meaningful because of the close parametrisation: as in the previous Sections, all common blocks are calibrated to the same values. All remaining parameters are either those chosen in the original paper or those estimated. Crucially, every model is also perturbed with the very same sequence of monetary and technological shocks, so to ideally elicit differences in the transmission and propagation mechanisms.

For our interest is chiefly on inflation dynamics, we limit our interest to the global autoregressive properties of the inflation series, which is generated by a single, stable, and well behaved DGP for each case. The statistical framework of reference is an autoregressive one, with varying orders. As we calibrate the persistence of all shocks to $\rho = .65$, we expect to find values in this neighbourhood, the more so since all shocks are identical and other exogenous disturbances are muted. Any difference between the simulated series is due to the propagation mechanism and, most importantly, the monetary response function. Moreover, when comparing our model in its two policy regimes, we will be able to pick up the different inflation dynamics enticed by the passive monetary policy stance.

Our analysis starts with an $AR\,(5)$: our model complying to the Taylor Principle, the same violating it, and the workhorse NKDSGEs. Secondly, we set an upper bound on the lags to 120, and pick the optimal lag number minimising the Bayesian Information Criterion. Table (2) presents the results for an $AR\,(5)$.

Looking at the coefficients on the first lag, we see our expectations confirmed, as all coefficients are tightly close to the calibrated parameter, no constant is statistically different from zero, and significance decreases after the first lag for classic NKDSGE models. A notable exception is the Smets and Wouters (2007) model, which reports five significant lags with relatively large coefficients. The most remarkable feature of such model, though, is that it displays a quasi unit-root in inflation, as the first lag is statistically close, but different, than one.

Interestingly, as we depart from the NKDSGE models to analyse an accommodating central bank, the coefficient on the first lag moves away from the calibrated value, downwards (column (2) in Table 2). This result corroborates the consensus that a passive monetary authority has less command over the inflation path and fails at taming its dynamics back on target. The flip side of this latter aspect is that, when the first lag becomes less relevant, previous ones acquire more weight. Overall, hence, inflation seems to become more persistent when central banks do not follow an aggressive Taylor rule.

On the other hand, the magnitude of the significant coefficients of the two calibrations of the liquidity model is comparatively small. While for an active central bank (model (3)) the autoregressive coefficients quickly approach zero, for a passive one these still become smaller but remain roughly ten times bigger than those of the other models, and still significant.

These results point to a role for monetary policy stance in substantially influencing the inflation dynamics, along the same lines traced by Cogley, Primiceri, and Sargent (2008), Cogley and Sargent (2002, 2005), and Primiceri (2005).

Table 2: Estimates on simulated data

	Simulated Inflation				
	(1)	(2)	(3)	(4)	(5)
Const.	$-.0001$	$-.0002$	$-.0001$	$-.0002$	$-.0004$
	(.0003)	(.0005)	(.0002)	(.0003)	(.0005)
1^{st} lag	$.648^{***}$	$.597^{***}$	$.640^{***}$	$.652^{***}$	$.966^{***}$
	(.001)	(.001)	(.001)	(.001)	(.001)
2^{nd} lag	.001	$-.017^{***}$	$-.005^{***}$	$-.001$	$-.004^{**}$
	(.002)	(.002)	(.002)	(.002)	(.002)
3^{rd} lag	.00003	$-.014^{***}$	$-.003^{**}$	.0004	$-.004^{**}$
	(.002)	(.002)	(.002)	(.002)	(.002)
4^{th} lag	$-.001$	$-.013^{***}$	$-.003^{**}$	.0001	$-.005^{**}$
	(.002)	(.002)	(.002)	(.002)	(.002)
5^{th} lag	.002	$-.029^{***}$	$-.009^{***}$	.001	$-.025^{***}$
	(.001)	(.001)	(.001)	(.001)	(.001)
Adjusted R^2	.421	.338	.402	.424	.875

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

AR (5) estimates on simulated data from five models: (1) Gali (2015), (2) liquidity model $\gamma = .5$, (3) liquidity model $\gamma = 1.8$, (4) Ascari and Sbordone (2014), (5) Smets and Wouters (2007). Only technological and monetary policy shocks are allowed, each model is simulated for 500000 periods, after discarding the first 100000 iterations. All shocks are set to have zero mean, equal variance, and are iid. Second and third columns present estimates for our model with liquidity, complying to the Taylor Principle and violating it, respectively.

To carry out a deeper analysis into inflation dynamics, we analyse a crude measure of persistence, that is to compare the number of optimal lags for an $AR(k)$ process. This procedure finds that the optimal number of lags for our model with liquidity and Taylor principle compliance (model (3) in Tab.(3)) is around 70, whilst for the version parametrised in accordance to the Taylor Principle (model (2), *ibid*) it is around 50, for the standard NKDSGEs it is merely 2. Table (3) offers more details on this result.

Table 3: Optimal lags

Models	Opt. lags	Sign. lags	adj. R^2	BIC
(1) Gali (2015)	2	50%	.421	−280676.9
(2) Liq. $\gamma = .5$	51	80%	.348	−413028.9
(3) Liq. $\gamma = 1.8$	71	32%	.401	−1033066
(4) Ascari and Sbordone (2014)	2	50%	.421	−422629.6
(5) Smets and Wouters (2007)	13	38%	.719	−486558.7

Optimal lags for AR process of inflation for (1) baseline NKDSGE (Gali, 2015), (2) liquidity model violating the Taylor Principle ($\gamma = 0.5$), (3) liquidity model complying to it ($\gamma = 1.8$), a model with time-varying trend inflation (Ascari and Sbordone, 2014), and (5) a medium scale workhorse DSGE (Smets and Wouters, 2007). Optimal lags are those minimising the BIC. All models are fed the same sequence of shocks of the same variance, generating 500000 quarterly observations.

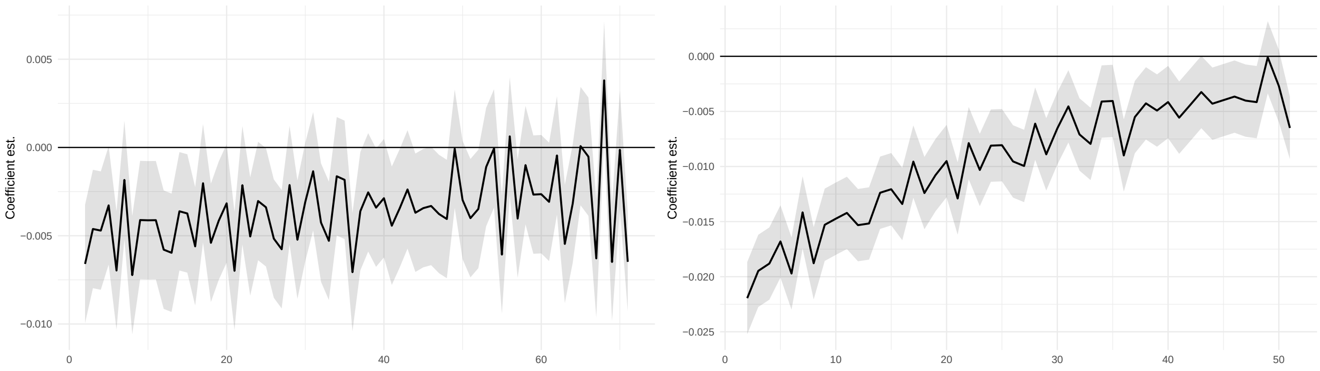
An optimal lag number does not imply that *all* regressor lags are significant. It only implies that all significant lags are part of the regression, likely a small subset of lags explains the largest share of variation. With this in mind, our model of liquidity with an accommodative monetary authority shows that today's inflation depends on a long sequence of lags. The number of optimal lags increases when we let the central bank respond aggressively to expected inflation.

On the other hand, the baseline NKDSGE model produces a process with extremely short memory, likewise for the model of trend inflation. In line with the result from Tab.(2), the medium-scale Smets and Wouters (2007) model displays longer endogenous lags, due to numerous frictions and feedback engrained in this rich model.

Panel 6a plots the point estimates and confidence bands[9] for the 71 optimal lags of the liquidity model with an active policy rule. Of all lags included, only 21 are strongly significant (±30%); interestingly, these are largely negative in sign and relatively small in magnitude. This last evidence might suggest that in this

[9] For the sake of readability, we do not plot the first lag. All-inclusive plots are in Appendix F.

Figure 5: $AR(k^*)$ Lags



(a) $\gamma = 1.8$ **(b)** $\gamma = 0.5$

Autoregressive estimates on optimally selected lags. Left panel 6a liquidity model with aggressive central bank ($\gamma = 1.8$). Right panel 6b liquidity model with accommodative policy ($\gamma = 0.5$). Estimated coefficients in solid, bands are twice the estimated standard errors. Note: first lag excluded from the plot for scale readability.

policy regime inflation depends negligibly on past realisations, being constantly nudged into a well-determined path.

Panel 6b plots the same information for an accommodative policy rule. What is striking is the length of lags deemed relevant and the share of significant ones (±80%), as opposed to the aggressive policy rule. As remarked in Table 2, coefficients are greater and all significant at 1% up to the 33$^{\text{rd}}$. These features point toward a higher persistence in inflation, something compatible with a central bank that, for instance, targets monetary aggregates (prior to Volcker's chairmanship) or finds itself short of conventional monetary tools (QE at the ZLB), as it has been for long periods recently. Nonetheless, this setting does not imply necessarily sunspot equilibria or spiralling aggregates, the system eventually converges back to the steady-state even when the monetary policy stance is accommodative.

This result offers a framework to analyse inflation dynamics in light of diverse monetary policy regimes. Higher levels of persistence – as long as it is measured as the number of significant lags – are expected in periods like the US high inflation of the '70s, when indeed prices skyrocketed out of control. Conversely, tranquil periods results from aggressive monetary stances, like the Great Moderation. The inclusion of liquidity is extremely helpful to systematise the 2008 crisis, while also accounting for the inactive interest rate policy, as shown in **??**. For this latter case, though, other factors need to be considered, like the level of liquidity and the wide range of unconventional policies put in place, from which we abstracted in this paper.

5 The 2008 crisis: severe liquidity shortage

This model lends itself to an interesting experiment: although nominal liquidity is assumed to be constant at $\bar{Z}$ and real liquidity z moves with the inflation rate, we hit z with a negative shock and study the behaviour of our model, as in eq.(18). Although not orthodox, this is a practical short-cut: neglecting where that missing liquidity goes *physically* – and in which proportion money and bonds are affected – lets us focus on the dynamics of convergence to the steady-state. This negative shock affects solely the current level of real liquidity $\hat{z}_t$, leaving inflation untouched. The adjustment then has to run primarily through the remaining contemporaneous relations (12) and (14). In eq. (14), for given inflation expectations $E_t\pi_{t+1}$, the policy interest rate does not yet move, so that the sudden drop in real liquidity has to be offset by money holdings $\hat{m}_t$. This adjustment then passes on to the Euler equation eq. (12) through the wedge: current output gap moves accordingly and then transmits to current inflation via the Phillips Curve (13).

One could think of this experiment as a sudden drop in the combined liquidity of bonds and money, closely related to the liquidity dry up triggering the 2008 Global Financial Crisis. In the aftermath of such recession, major central banks rapidly hit the zero lower bound on policy interest rates, thus in effect adopting an accommodative monetary policy stance, equivalent to that admitted by the liquidity model. In this respect, analysing how aggregates react to a sudden liquidity dry up sheds light on how different interest rate rules interact and affect the overall dynamics.

We produce IRFs and discuss their economic interpretations under the two regimes of monetary policy.

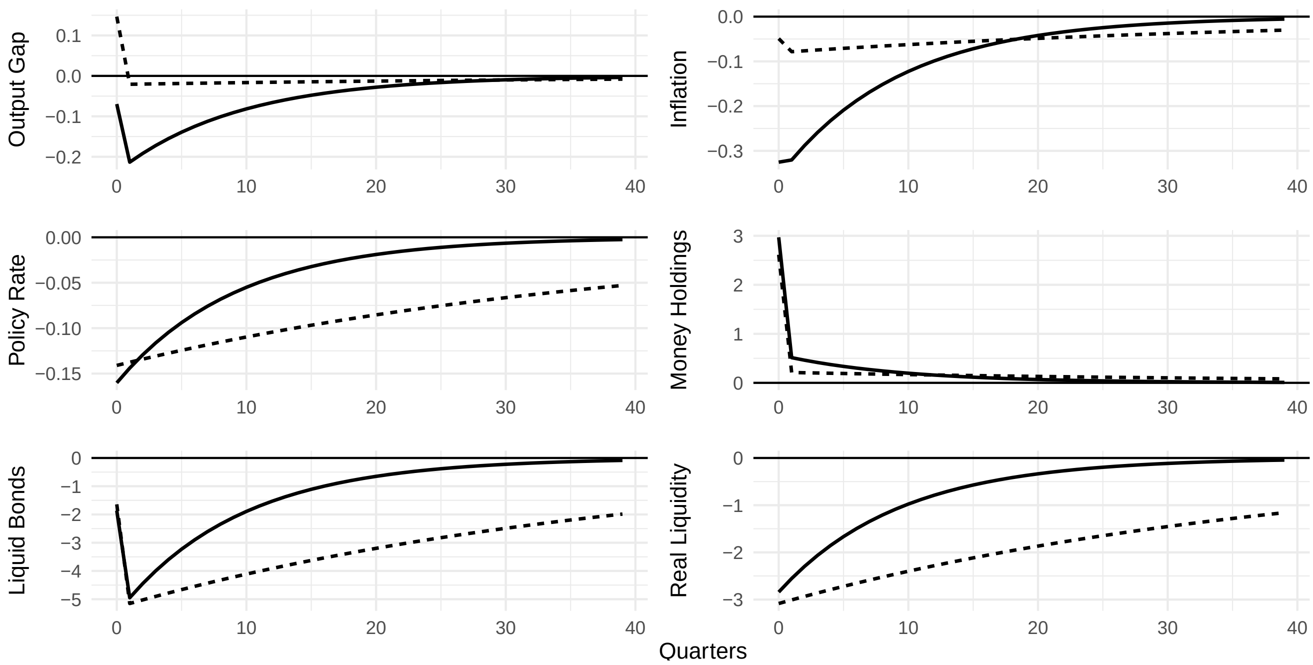
$$\hat{z}_t - \epsilon_t^z = \hat{z}_{t-1} - \pi_t \tag{18}$$

Following an abrupt and violent liquidity dry-up the model shows a general reaction broadly independent from the behaviour of the monetary authority, but substantial differences emerge in magnitude and reversion to steady-state levels.

The shock impacts at first money demand $\hat{m}$, for a given policy rate s, which spikes up. Conversely, the representative agent disinvests from liquid bonds, proportionally more than the missing liquidity. This results from the preference for money with respect to bonds that were assumed in the calibration.

At this stage, the Phillips curve (13) and the IS equation (12) propagate the shock to the rest of the economy. The money wedge in the Euler equation (12) transmits the shock to current output that spikes as well on impact. Under a passive monetary policy, this translates into a limited effect of the shock on impact, and a degradation in the following quarters. The Phillips curve then squares the

Figure 6: Liquidity Shock



Real liquidity shock: aggressive ($\gamma = 1.8$, dashed) vs accommodative ($\gamma = .5$, solid) policy rules, values in percentages.

expected inflation with current π and a widened output gap. Inflation expectations subsequently drive the monetary policy decisions which translate into two distinct paths for realised inflation.

The money term in the IS curve is the telling point between the two regimes, together with the path for liquid bonds, b.

Under both regimes, money converges back to the steady-state relatively quickly, showing that real liquidity is deeply intertwined with liquid bonds. Most notably, under an accommodative central bank real liquidity recovers rapidly, thanks to a deflation that accelerates the recovery of z but impedes a quick rebound in output. Interestingly, when the money authority conducts an active policy, the inflation path – contained deflation with slow recovery – turns into persistently low rates, well beyond the case of a passive central bank.

The sharp difference in the reaction of the two regimes lies in the severity of the impact and the duration of the recovery. Inflation and output, in particular, show starkly different behaviours: when the central bank has a passive stance, a liquidity dry-up triggers a deep recession with a painfully long recovery (more than thirty quarters); the same applies to inflation, too. An active monetary stance against a liquidity shock tames the damage and facilitates the recovery, somewhat containing the effects within the financial sector of the economy.

To sum up, and combining these results with previous information, this exercise suggests that central banks complying with the Taylor Principle have a firmer control on contagion when a liquidity crisis hits. According to our stylised model, in fact, active monetary policy helps to contain and limit the damage to the sole financial sector of the economy, with reduced impact and consequence on real activity. The stark difference in the set of IRFs, clearly, lies in the fall of the output gap: it widens under passive monetary policy while it marks a mild recession under active policy, although the duration is similar. As we already remarked, recovery speeds are substantially different.

Stretching our model to draw policy implications, one can think back to the Zero Interest Rate Policies (ZIRP) that major central banks deployed both in the 2008 GFC and in the current Covid19 crisis. Although hazardous, such an aggressive reaction could be strong enough to facilitate a speedy recovery, as it has been the case for the US after the GFC.

6 Conclusion

Since the Global Financial Crisis, liquidity has gained a central role in the general macroeconomic discussion, and in the debate on monetary policy in particular. It has been the main concern for major central banks in engaging in unconventional policies.

In this paper, we present a parsimonious framework with minor departures from the core new Keynesian model of monetary policy and derive relevant results for central banks' mandate of price stabilisation.

First and foremost, we show that the simple addition of a liquid asset – and the consequent modification of the intertemporal Euler equation – pins down a solution with an accommodative, stripped-down policy rate rule. The latter needs only to *positively* correlate the policy rate and expected inflation to rule out degenerate, multiple equilibria. These latter sunspot equilibria would arise in the baseline New Keynesian Dynamic Stochastic General Equilibrium model when the central bank does not react to inflation aggressively.

We compare the responses of our liquidity model with those of the baseline NKDSGE and confirm that they broadly match for technological and monetary policy shocks, under identical, common calibration. We then study how our model responds to such shocks when the central bank under-reacts to expected inflation. We find no evidence of degenerate behaviour for the model aggregates, contrary to the predictions of the baseline NKDSGE.

In fact, all aggregates, and inflation especially, broadly react in the same way under the two regimes: we signal, though, a change in the dynamics, rather than direction. We find that an accommodative central bank generates more persistent inflation. We test such hypothesis on simulated data, including two other workhorse models from the monetary field.

We find that the inclusion of liquidity and liquid bonds generates overall more persistence in inflation. Within the sets of calibration parameters, inflation displays more persistence when the monetary policy stance is passive, in line with evidence from the US high inflation period.

Going forward, more sophisticated versions of this model may be easily developed, as it accommodates additional layers of complexity. For example, one might want to include sticky wages, capital stock, financial blocks in the spirit of the financial accelerator, or occasionally binding constraints like a properly modelled Zero Lower Bound. These theoretical devices have been developed as modules for the basic NKDSGE model, with which our model shares the core features. The first, natural extension for this model would be the relaxation of the fixed total liquidity in nominal terms, $\bar{Z}$. This would provide the monetary authority an additional tool to carry out its mandate and, most importantly, a framework to study liquidity management.

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A Model appendix

A.1 Obtaining the system of equations

We combine the equations presented in the body of the paper so to obtain the system of equations that will be later loglinearised and fed to `Dynare` for simulations.

The first target is the augmented Euler equation. The version proposed in the paper includes real money balances on top of the usual terms. Take the first intertemporal FOC from the consumer utility maximisation program as the starting point:

$$u'\left(c_t\right) = E_t\left[\beta\left(1+r_t\right)u'\left(c_{t+1}\right)\right]$$

recall the Fisher equation $\left(1+r_t\right)\left(1+\pi_{t+1}\right) = \left(1+i_t\right)$ and replace r_t

$$\begin{aligned} u'\left(c_t\right) &= E_t\left[\beta u'\left(c_{t+1}\right)\frac{1+i_t}{1+\pi_{t+1}}\right] \\ &= E_t\left[\frac{\beta u'\left(c_{t+1}\right)}{1+\pi_{t+1}} + \frac{\beta u'\left(c_{t+1}\right)i_t}{1+\pi_{t+1}}\right] \end{aligned}$$

Now recall the condition on marginal utility of real money balances, $v'\left(m_t\right)$, and employ it to define the nominal interest rate, i_t:

$$\begin{aligned} v'\left(m_t\right) &= E_t\left[\beta\lambda_{t+1}\frac{i_t}{1+\pi_{t+1}}\right] \\ i_t &= v'\left(m_t\right)E_t\left[\frac{1+\pi_{t+1}}{\beta\lambda_{t+1}}\right] \\ \text{moreover } \lambda_{t+1} &= u'\left(c_{t+1}\right) \end{aligned}$$

Turning back to the Euler equation, plug the nominal interest rate in the relation just recovered:

$$u'\left(c_t\right) = E_t\left[\frac{\beta u'\left(c_{t+1}\right)}{1+\pi_{t+1}} + \frac{\beta u'\left(\cancel{c_{t+1}}\right)}{\cancel{1+}\pi_{t+1}}v'\left(m_t\right)\frac{1+\cancel{\pi_{t+1}}}{\cancel{\beta u'}\left(c_{t+1}\right)}\right]$$

which rearranges in

$$= E_t\left[\beta u'\left(c_{t+1}\right)\left(\frac{1}{1+\pi_{t+1}} + \frac{v'\left(m_t\right)}{\beta u'\left(c_{t+1}\right)}\right)\right].$$

This last expression is then loglinearised to obtain Equation 12.

To condense the money equation, start with the last two relations in (3):

$$v'(m_t) = E_t\left[\beta\lambda_{t+1}\frac{i_t}{1+\pi_{t+1}}\right]$$

$$i_t = v'(m_t)\, E_t\left[\frac{1+\pi_{t+1}}{\beta\lambda_{t+1}}\right]$$

plug this result into $h'(b_t)$

$$h'(b_t) = E_t\left[\beta\lambda_{t+1}\frac{i_t - s_t}{1+\pi_{t+1}}\right]$$

$$\Rightarrow h'(b_t) = E_t\left[\frac{\cancel{\beta\lambda_{t+1}}}{\cancel{1+\pi_{t+1}}}v'(m_t)\frac{\cancel{1+\pi_{t+1}}}{\cancel{\beta\lambda_{t+1}}} - \frac{\beta\lambda_{t+1}s_t}{1+\pi_{t+1}}\right]$$

exploit the fact that $\lambda_{t+1} = u'(c_{t+1})$ and $b_t = z_t - m_t$ to obtain

$$h'(z_t - m_t) - v'(m_t) = E_t\left[-\frac{\beta s_t u'(c_{t+1})}{1+\pi_{t+1}}\right]$$

Setting this equation to its steady-state and using a first Taylor approximation generates equation (14) in the text.

The backward dependence of real liquidity (10) results as follows:

$$Z_t = \bar{Z} = M_t + B_t$$

$$\frac{\bar{Z}}{P_{t-1}} = \frac{(m_t + b_t)\, P_t}{P_{t-1}}$$

$$\Rightarrow z_t = \frac{z_{t-1}}{1+\pi_t}$$

Concerning the Phillips curve, it remains unchanged from traditional New Keynesians models and derives from the use of equations (7). The output gap it includes results from the comparison to the flexible prices version of the model. Other relations do not need further manipulation.

B Loglinearisation

The model is loglinearised around a zero-inflation steady state as in the early New Keynesian models. This assumption yields the remaining variable values in the long run and without shocks. We employ directly the functional forms from (11).

Steady state values

Euler equation / IS curve at the steady state:

$$
\begin{aligned}
u'(c_t) &= E_t\left[\beta\left(1+r_t\right)u'\left(c_{t+1}\right)\right] \\
c^{-\sigma} &= m^{\psi-1} + \beta\frac{c^{-\sigma}}{1+\pi} \\
m^{1-\psi} &= \frac{c^{\sigma}}{1-\beta}
\end{aligned}
$$

Output level: $y = y^f$.

Money demand and policy rate:

$$
\begin{aligned}
s &= \gamma\pi = 0 \\
h'(z-m) - v'(m) &= \left[-\frac{\beta s u'(c)}{1+\pi}\right] \\
(z-m)^{\phi-1} - m^{\psi-1} &= -\frac{\beta \overbrace{s}^{=0} c^{-\sigma}}{1+\underbrace{\pi}_{=0}} = 0 \\
(z-m)^{\phi-1} &= m^{\psi-1} \\
z - m &= m^{\frac{\psi-1}{\phi-1}} \\
z &= m + m^{\frac{1-\psi}{1-\phi}} = m\left(1 + m^{\frac{\phi-\psi}{1-\phi}}\right)
\end{aligned}
$$

Remarkably, if $\psi = \phi$, so that the agent is indifferent between money and liquid bonds, $z = 2m$ as in the log preferences case.

Linearised system

Linearised liquidity law of motion:

$$
\begin{aligned}
z_t &= \frac{z_{t-1}}{1+\pi_t} \\
\Rightarrow \hat{z}_t &= \hat{z}_{t-1} - \pi_t
\end{aligned}
$$

This equation does not pin down the steady-state value for z, since it results from the sum of real money balances, m, and liquid bonds, b.

$$
\hat{b}_t = \frac{z}{z-m}\hat{z}_t - \frac{m}{z-m}\hat{m}_t
$$

Linearised Euler equation:

$$
c_t^{-\sigma} = E_t\left[\beta c_{t+1}^{-\sigma}\left(\frac{1}{1+\pi_{t+1}} + \frac{m_t^{\psi-1}}{\beta c_{t+1}^{-\sigma}}\right)\right]
$$

taking logs and first derivatives - drop E_t for convenience - and rearrange:

$$-\sigma \hat{c}_t = (\psi - 1)\,\hat{m}_t + (-\sigma \hat{c}_{t+1} - \pi_{t+1})$$
$$\hat{c}_t = \frac{1-\psi}{\sigma}\hat{m}_t + \hat{c}_{t+1} + \frac{1}{\sigma}\pi_{t+1}$$

Linearised money demand:

$$h'\left(z_t - m_t\right) - v'\left(m_t\right) = E_t\left[-\frac{\beta s_t u'\left(c_{t+1}\right)}{1+\pi_{t+1}}\right]$$
$$\left(z_t - m_t\right)^{\phi-1} - m_t^{\psi-1} = E_t\left[-\frac{\beta s_t c_{t+1}^{-\sigma}}{1+\pi_{t+1}}\right]$$
$$\left(1+\pi_{t+1}\right)\left(z_t - m_t\right)^{\phi-1} = \left(1+\pi_{t+1}\right) m_t^{\psi-1} - \beta c_{t+1}^{-\sigma} s_t$$

Focus first on the left-hand side of the above equation and drop the steady state terms – which will cancel out eventually:

$$\ln(1+\pi_{t+1}) + (\psi-1)\ln\left(z_t - m_t\right) \;\Rightarrow \pi_{t+1} + (\psi-1)\left[\frac{z}{z-m}\hat{z}_t - \frac{m}{z-m}\hat{m}_t\right]$$

Now replace the steady state value for z found in the money demand above and plug in the previous equation, factoring out the common terms, we obtain

$$\pi_{t+1} + (\psi-1)\left[m^{\frac{1-\psi}{1-\phi}}\left(z\hat{z}_t - m\hat{m}_t\right)\right]$$
$$\pi_{t+1} + (\psi-1)\left[\hat{z}_t + \left(\hat{z}_t - \hat{m}_t\right) m^{\frac{\psi-\phi}{1-\phi}}\right]$$

Turning to the right-hand side of the previous equation, we know that at the steady-state and in logs it equals $\ln\left(m^{\psi-1}\right)$, as the term involving s collapses to 0. Furthermore, we can break down the linearisation into two chunks, the first yielding simply

$$\pi_{t+1} + \frac{\psi-1}{m^2}\hat{m}_t$$

and the second one not more difficult. Namely, linearising with respect to c yields 0, as it contains $s = \gamma\pi = 0$ at the steady-state; approximation for s gives

$$-\beta\frac{c^{-\sigma}}{m^{\psi-1}}s_t$$

Gathering all pieces together gives, after slight rearrangements:

$$\left(\frac{1-\psi}{m^2}\right)\hat{m}_t + \beta c^{-\sigma} m^{1-\psi} s_t - (1-\phi)\left[\hat{z}_t + \left(\hat{z}_t - \hat{m}_t\right) m^{\frac{\psi-\phi}{1-\phi}}\right] = 0$$

From which we can recover eq.(14) in the text.

C CIA setup

The model presented in the main body follows the reduced-form strategy that was introduced by Sidrauski (1969), which is usually referred to as Money in the Utility function, MIU. It is a straightforward way to have agents hold cash balances or other assets, but it lacks a proper microfoundation. Cash in Advance (CIA) models partly overcome this lack of microfoundation: instead of providing direct utility, money holdings are used to purchase the desired amount of consumption. The CIA approach was proposed by Clower (1967) and further developed by Grandmont and Younes (1972) and Lucas (1980).

Compared to MIU models, therefore, CIA models add one more inequality constraint: agents allocate money to purchase the desired level of consumption, either within the same period or with a lag. This timing difference is key. Lucas (1982) develops a deterministic CIA model where assets are allocated at the beginning of the period, in line with the consumption decision. Svensson (1985), conversely, constrains agents to allocate assets in advance, before shocks and adjustments take place. This latter approach yields more informative dynamics, since agents might over (under) accumulate money balances with respect to their future desired level of consumption. Finally, Cooley and Hansen (1989, 1991) build on Lucas and Stokey (1987) and propose a RBC model with CIA and uncertainty. The setup proposed here is akin to this latter, as we embed in the NKDSGE structure a Liquidity in Advance (LIA) constraint. In this exercise, we draw from Calvo and Végh (1990a,b), who study a similar economy under a different setup.

The LIA constraint hinges on a liquidity production function $Q\left(M, B\right)$ that combines enough bonds and cash to purchase the desired level of consumption. Liquidity in-house production function Q might take several forms (Calvo and Végh (1990a,b) lay out some restrictions on these forms): to grant flexibility and generality, assume Q is increasing and concave in both assets. For the sake of generality, timing will be detailed momentarily. Under these assumptions, the LIA constraint takes the following form:

$$C_t \leq Q_t(M_{t-j}, B_{t-j}) \tag{19}$$

With this additional constraint, the representative agent now solves the following program:

$$\begin{aligned}
&\max_{c,N} E_t \sum \beta^t \left(u\left(c_t\right) - g\left(N_t\right)\right) \\
&st{:}\ C_t + B_t + M_t + X_t = W_t N_t + \left(1 + i_{t-1}\right) X_{t-1} + \left(1 + s_{t-1}\right) B_{t-1} + M_{t-1} \\
&\quad C_t \leq Q\left(M_{t-j}, B_{t-j}\right)
\end{aligned} \tag{20}$$

Notice that rewriting the budget constraint in terms of total (real) assets highlights the nominal (real) opportunity costs incurred when holding positive quantities of money and bonds. Let $D_t = X_t + B_t + M_t$, where D is total assets. One can replace X in the flow constraint and turn to real quantities:

$$\begin{aligned}
C_t + B_t + M_t + X_t &= W_t N_t + \left(1 + i_{t-1}\right) X_{t-1} + \left(1 + s_{t-1}\right) B_{t-1} + M_{t-1} \\
C_t + D_t &= W_t N_t + \left(1 + i_{t-1}\right) D_{t-1} + \left(i_{t-1} - s_{t-1}\right) B_{t-1} - i_{t-1} M_{t-1} \\
c_t + d_t &= w_t N_t + \left(1 + r_{t-1}\right) d_{t-1} - \frac{i_{t-1} - s_{t-1}}{1 + \pi_t} b_{t-1} - \frac{i_{t-1}}{1 + \pi_t} m_{t-1}
\end{aligned} \tag{21}$$

Where $\left(1 + i_t\right) / \left(1 + E_t \pi_{t+1}\right) = \left(1 + r_t\right)$ follows from Fischer equation and is the real interest rate. As of the liquidity in advance constraint in real terms, timing is relevant:

$$c_t = \begin{cases} Q\left(m_t, b_t\right) & \text{for } j = 0 \\ \frac{Q\left(m_{t-1}, b_{t-1}\right)}{1 + \pi_t} & \text{for } j = 1 \end{cases} \tag{22}$$

When allocating one additional unit of income to liquid bonds, the consumer will receive interest s next period, but gives up i on the purely illiquid bond. Therefore the opportunity cost of holding such bonds is $i - s$. Similarly, money yields no interest so that the agent gives up entirely to i. The elasticity of substitution q, together with the liquidity-in-advance constraint will force agents to hold nonnegative amounts of m and b.

Lagrangian reads

$$\mathcal{L} = \sum \beta^t \begin{bmatrix} \left[u\left(c_t\right) - g\left(N_t\right)\right] + \\ \lambda_t \left(w_t N_t + \left(1 + r_{t-1}\right) d_{t-1} - \frac{i_{t-1} - s_{t-1}}{1 + \pi_t} b_{t-1} - \frac{i_{t-1}}{1 + \pi_t} m_{t-1} - c_t - d_t\right) + \\ \mu_t \left(\frac{Q\left(m_{t-1}, b_{t-1}\right)}{1 + \pi_t} - c_t\right) \end{bmatrix} \tag{23}$$

FOCs for consumption, labour supply, and intertemporal allocation are independent of the timing of the liquidity constraint and fairly standard:

$$
\begin{aligned}
u_t' - \lambda_t - \mu_t &= 0 \\
-g_t' + \lambda_t W_t &= 0 \\
-\lambda_t + \beta\lambda_{t+1}\left(1 + r_t\right) &= 0
\end{aligned}
\tag{24}
$$

FOCs for money holdings and bonds, instead, slightly differ depending on the timing of assets allocation. The $j = 1$ is the most interesting case: with uncertain productivity and monetary policy shock, agents will rationally choose their portfolio composition, level of consumption, labour supply. For instance, for a productivity shock, they wish to increase consumption but face a binding liquidity in advance constraint and therefore do not adjust as they wish.

For $j = 1$, FOCs for cash and bonds read:

$$
\frac{\mu_{t+1} Q_t^M}{1+\pi_{t+1}} = \left(\frac{i_t}{1+\pi_{t+1}}\right)\lambda_{t+1} \qquad \frac{\mu_{t+1} Q_t^B}{1+\pi_{t+1}} = \left(\frac{i_t - s_t}{1+\pi_{t+1}}\right)\lambda_{t+1} \tag{25}
$$

Where λ and μ are the multipliers associated with the budget and liquidity in advance constraint, respectively. Taking the ratio of the two equations provides the relative allocation between cash and bonds. The functional form of Q is now required, and we assume a Cobb-Douglas one with elasticity α:

$$
\begin{aligned}
\frac{Q_t^B}{Q_t^M} &= \frac{i_t - s_t}{i_t} \\
\frac{m_t}{b_t} &= \frac{\alpha}{1-\alpha}\frac{i_t - s_t}{i_t} \qquad \forall t
\end{aligned}
\tag{26}
$$

The last equation allows for recovering demand for bonds b (money m) as a function of interest rates and money holdings m (bonds b) for each time t. Indeed, since the one-period-ahead LIA constraint in real terms reads $c_t = \left(1+\pi_t\right)^{-1} m_{t-1}^{\alpha} b_{t-1}^{1-\alpha}$ it obtains that, for b and m given from $t-1$:

$$
\begin{aligned}
b_{t-1} &= \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{i_{t-1}}{i_{t-1} - s_{t-1}}\right) m_{t-1} & m_{t-1} &= \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{i_{t-1} - s_{t-1}}{i_{t-1}}\right) b_{t-1} \\
c_t\left(m_{t-1}\right) &= m_{t-1}^{\alpha}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{i_{t-1}}{i_{t-1} - s_{t-1}}\right) m_{t-1}\right]^{1-\alpha} & c_t\left(b_{t-1}\right) &= b_{t-1}^{1-\alpha}\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{i_{t-1} - s_{t-1}}{i_{t-1}}\right) b_{t-1}\right]^{\alpha} \\
&= \left[\frac{1-\alpha}{\alpha}\frac{i_{t-1}}{i_{t-1} - s_{t-1}}\right]^{1-\alpha} m_{t-1} & &= \left[\frac{\alpha}{1-\alpha}\frac{i_{t-1} - s_{t-1}}{i_{t-1}}\right]^{\alpha} b_{t-1} \\
m_{t-1}\left(c_t\right) &= \left[\frac{\alpha}{1-\alpha}\frac{i_{t-1} - s_{t-1}}{i_{t-1}}\right]^{1-\alpha} c_t & b_{t-1}\left(c_t\right) &= \left[\frac{\alpha}{1-\alpha}\frac{i_{t-1} - s_{t-1}}{i_{t-1}}\right]^{-\alpha} c_t
\end{aligned}
\tag{27}
$$

Whence it follows that:

$$b_{t-1} = \left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{-\alpha} c_t\left(1+\pi_t\right); \quad m_{t-1} = \left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{1-\alpha} c_t\left(1+\pi_t\right); \tag{28}$$

Taking $z_t = m_t + b_t, \forall t$ in combination with $z_t = z_{t-1}/\left(1+\pi_t\right)$, it finally obtains:

$$\begin{aligned} z_{t-1} &= \left[\left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{\alpha}\right] c_t\left(1+\pi_t\right) \\ \frac{z_t}{c_t} &= \left[\left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{\alpha}\right] \\ \frac{c_t}{z_t} &= \frac{\left(\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)^{\alpha}}{\left(1+\frac{\alpha}{1-\alpha}\frac{i_{t-1}-s_{t-1}}{i_{t-1}}\right)} = h\left(i_{t-1}, s_{t-1}\right) \end{aligned} \tag{29}$$

Which determines the level of real consumption relative to real liquidity as a function of past interest rates i and s.

The zero-inflation steady state, which implies that $s = 0$, pins down the ratio:

$$\frac{c_{ss}}{z_{ss}} = \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}{\left(1+\frac{\alpha}{1-\alpha}\right)} = \alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} \tag{30}$$

which pins down the amount of real consumption over total real liquidity.

The LIA constraint also affects the wedge between marginal utility and λ_t, wealth shadow price. In fact, marginal consumption from the first line of Equation 24 equates the sum of the two Lagrangian multipliers. To factor out μ, we exploit the FOCs for the LIA constraint. Indeed, from the first order condition on cash

$$\mu_{t+1} = \frac{i_t \lambda_{t+1}}{Q^M\left(m_t, b_t\right)} \qquad \forall t \tag{31}$$

Where indeed $\frac{i_t}{Q^M(m_t, b_t)}$ is a wedge introduced by the LIA constraint that increases the marginal cost of consumption above unity. As noted in Calvo and Végh (1990a, footnote 18), this wedge appears because one extra unit of consumption requires one extra unit of liquidity $\frac{1}{Q_t^M}$, which in cash terms costs i_t: giving up on nominal interest.

Using this in the marginal utility FOC, with appropriate timing, one obtains

$$
\begin{aligned}
u'_t &= \left(1 + \overbrace{\frac{i_{t-1}}{Q^M\left(m_{t-1}, b_{t-1}\right)}}^{=v_{t-1}}\right)\lambda_t \\
\Rightarrow u'_t &= \left(1 + i_t\right)\beta\frac{1 + v_{t-1}}{1 + v_t}u'_{t+1}
\end{aligned}
\tag{32}
$$

Simpler liquidity production

The Cobb-Douglas functional form for liquidity production determines c/z as $h\left(i_{t-1}, s_{t-1}\right)$ but does not offer an analytical expression for h^{-1}. To further simplify the analysis, let us assume that Q is a linear liquidity production function. In the most general case one has:

$$Q = \phi m + \psi b \tag{33}$$

With (ϕ, ψ) strictly positive. Then Equation 26 boils down to a simpler linear relation

$$\frac{\psi}{\phi} = \frac{i - s}{i} \Rightarrow i = \frac{\phi}{\phi - \psi}s \tag{34}$$

In such a setting, ensuring a positive spread between nominal interest rate i and the policy rate on liquid bonds s equates to assuming $\phi > \psi > 0$, mirroring the restriction in the body of the chapter. This assumption entails a higher, constant marginal liquidity productivity of cash with respect to liquid bonds (or bank deposits). This relation pins down the nominal interest rate as a proportional function of the policy interest rate. Transmission of policy shocks is therefore immediate.

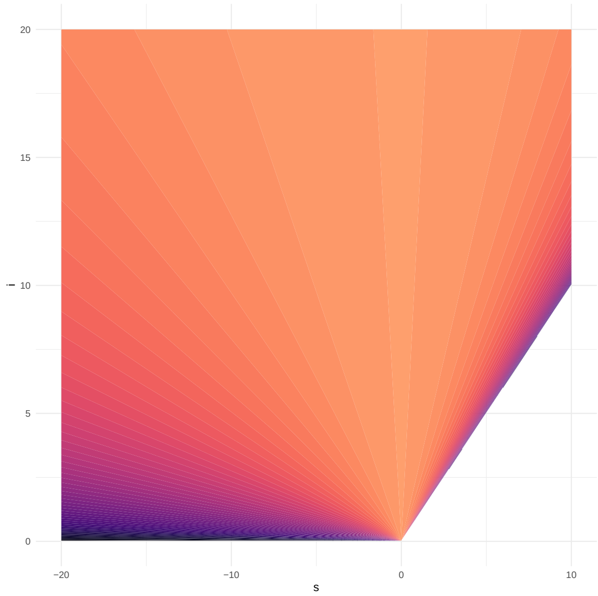
To further simplify the setup, one could assume that liquidity is an affine combination of cash and bonds so that $\phi = \alpha$ and $\psi = 1 - \alpha$. This implies that

$$i = \alpha s \tag{35}$$

which can be easily plugged into other expressions – s is set by the forward-looking central bank. Thus

$$
\begin{aligned}
v_t &= \frac{i_t}{Q^M} = \frac{\alpha s_t}{\alpha} = s_t \\
u'_t &= \left(1 + i_t\right)\beta\frac{1 + s_{t-1}}{1 + s_t}u'_{t+1}
\end{aligned}
\tag{36}
$$

Figure 7: h with Cobb-Douglas liquidity function



Numerical surface for $c/z = h(i,s)$ as defined in Equation 29, with $i > s$ and $\alpha = 1/3$. Lighter area, higher values of h.

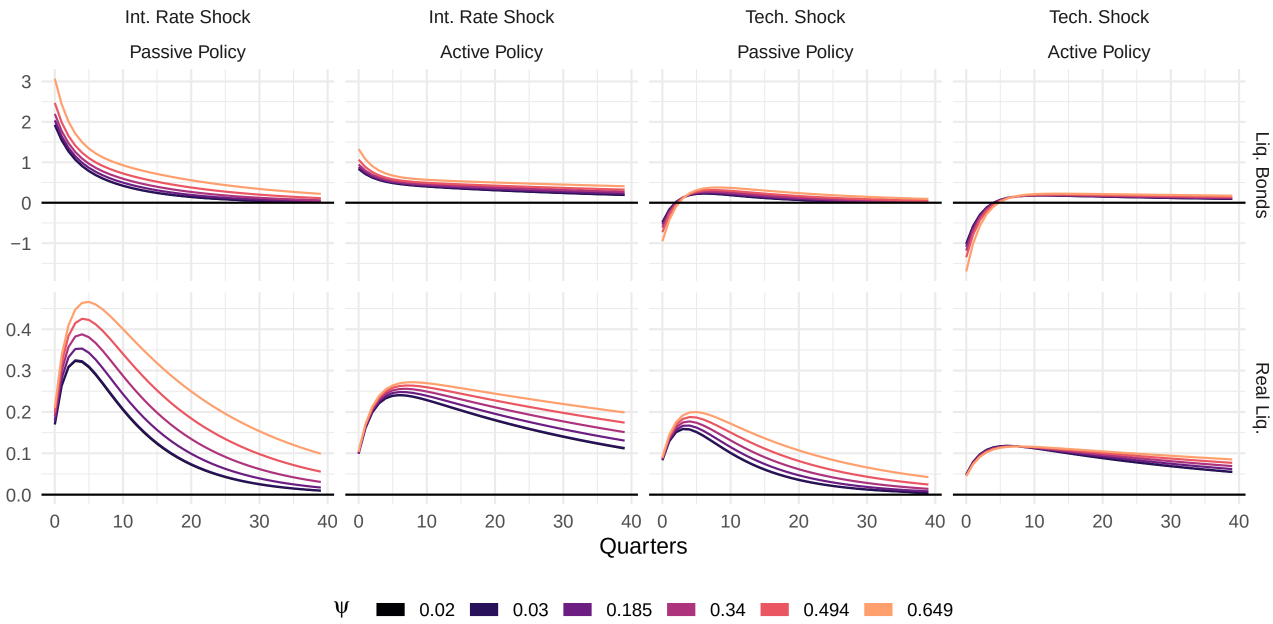
The resulting Euler/IS equation is therefore modified: in this setting, past and current policy rates affect the intertemporal allocation of consumption. Akin to the money wedge in the main model, the wedge hinges on liquidity, although through a different channel. In the MIU form, money holdings are indirectly affected by changes in the liquid bonds' interest rate: the higher such rate, the lower the money holdings, the lower current consumption for given values of other aggregates. In the LIA for, by contrast, the effect is direct: all else equal, higher current s (hence, higher allocation towards liquid bonds) depresses current consumption.

D Sensitivity analyses

D.1 Semi-elasticities for bonds and cash

To assess how our results depend on some particular values in the calibration of the parameters, this section briefly explores a sensitivity exercise for some values of ψ and ϕ, which are the most exotic plug-in from the main model. These parameters are also particularly hard to estimate in an empirical setting, with very scattered references. We select the more affected variables for each parameter, liquid bond holdings b, money holdings m, and real liquidity z. Other variables report negligible variations in responses to shocks in magnitude, persistence, and overall profile, but are available in the companion online repository.

Figure 8: Sensitivity on ψ – IRFs



Liquidity model, IRFs for bonds b (top row) and real liquidity z (bottom row), under several values for bond holdings semi-elasticity, $0 < \psi < \phi = .65$. Monetary shock is a 1% key rate hike, annualised; technology shock is one standard-deviation. All values in percentages.

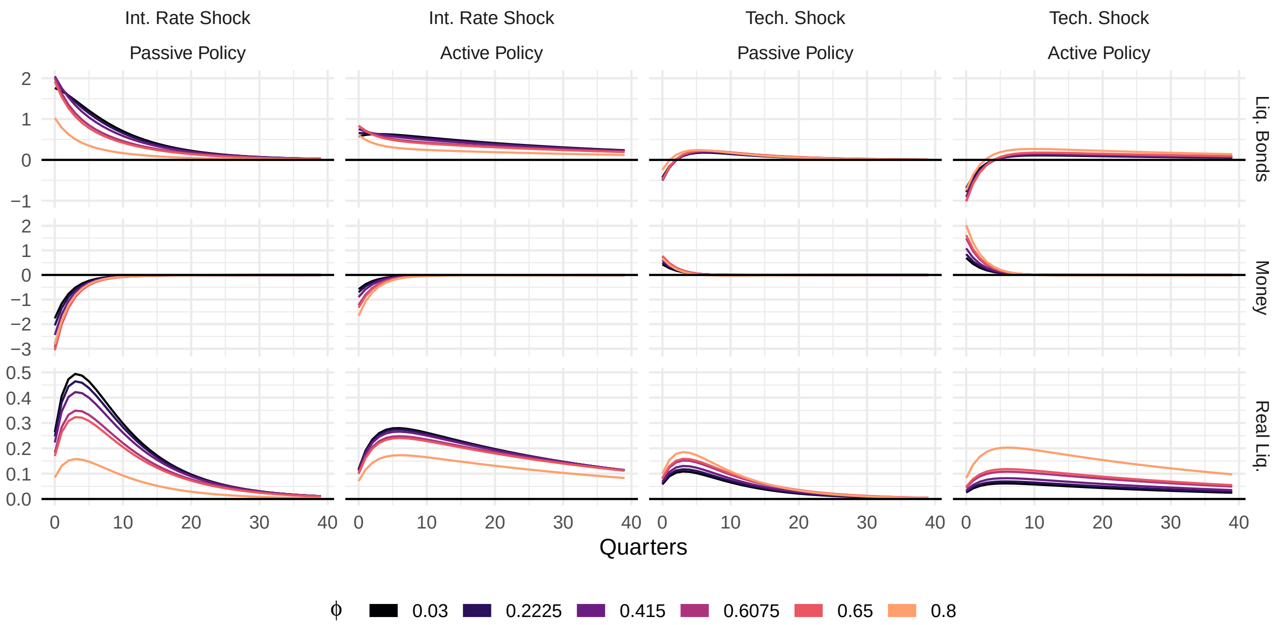
While the overall IRFs profile does not vary much over the values, higher semi-elasticity in bond holdings mainly affects the magnitude and persistence of the shock. Case in point is the monetary policy shock under a passive regime, where a higher value of ψ monotonically amplifies adjustments in real liquidity, driven by increased action in bond holdings. The same line of reasoning applies, to a reduced scale, to other combinations of shocks and regimes.

In turn, technological shocks display for both regimes a slight variation in reaction: impact variation inversely depends on ψ, but convergence to steady-state values takes longer. These changes are nevertheless limited in magnitude, so their effect is eventually negligible with respect to overall dynamics.

All in all, the especially low value used in the body of the paper is conservative and reduces considerably the persistence properties ingrained in our model of liquidity. Higher values would further increase the persistence that our model generates in comparison to other setups.

Turning to the semi-elasticity for money holdings, ϕ, Figure 9 plots an ensemble of IRFs for several candidate values. In such case, ψ is fixed at 0.02 as in the main analysis and ϕ takes increasing values. As ϕ affects mainly m and z, less so b, we restrict our analysis to these variables.

Figure 9: Sensitivity on ϕ – IRFs

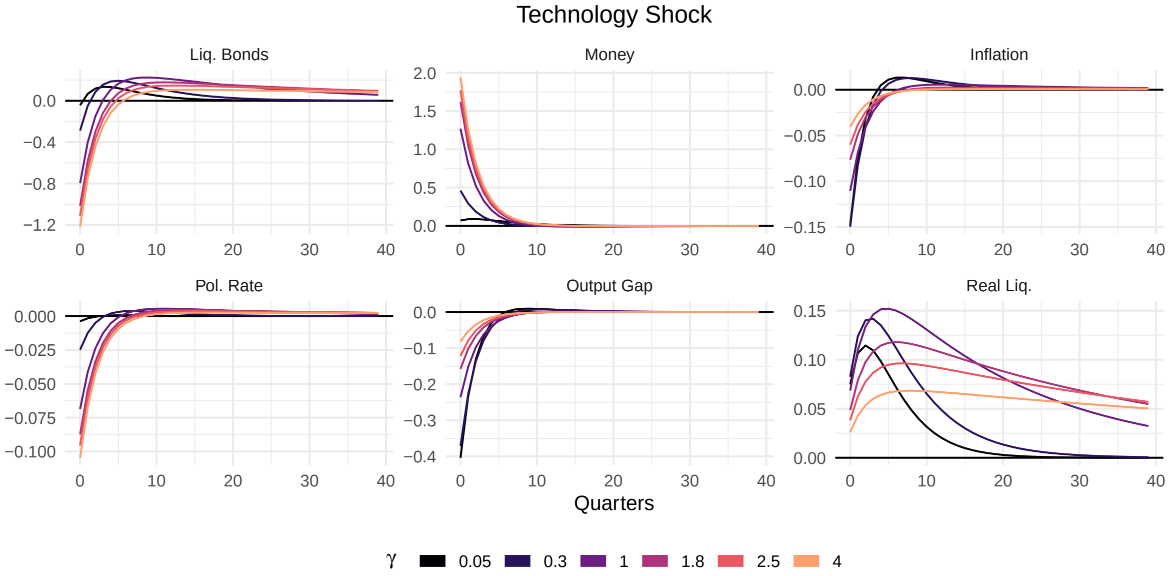


Liquidity model, IRFs for bonds b (top row), money holdings m (middle row), and real liquidity z (bottom row), under several values for money holdings semi-elasticity, ϕ, with $\psi = .02$. Monetary shock is a 1% key rate hike, annualised; technology shock is one standard-deviation. All values in percentages.

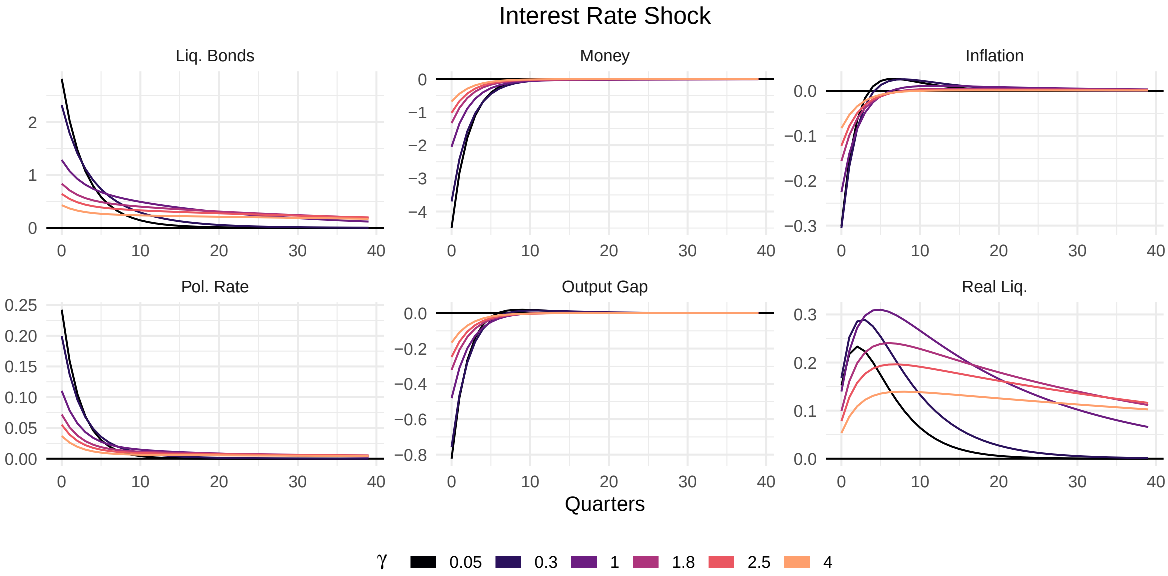
For both technology and interest rate shocks, higher semi-elasticity ϕ monotonically translates in higher responses on impact and little to no effect in terms of convergence speed. On the other hand, for liquid bonds, ϕ affects the response profile, especially for the monetary shock. In both regimes when ϕ is close to ψ IRFs take a smooth declining profile, whereas as ϕ grows away the reaction on impact decreases and the overall profile becomes more convex, speeding up convergence to the steady-state.

Turning to the effects on real liquidity z, the shock's nature determines the changes. Following a monetary shock, irrespective of the policy regime, higher values of ϕ imply less volatile movements in z and a quick convergence. The opposite holds following a technological shock, again for both policy regimes, although an active monetary policy seems to impart higher persistence in real liquidity after a TFP shock.

Figure 10: Sensitivity on γ – IRFs



(a)

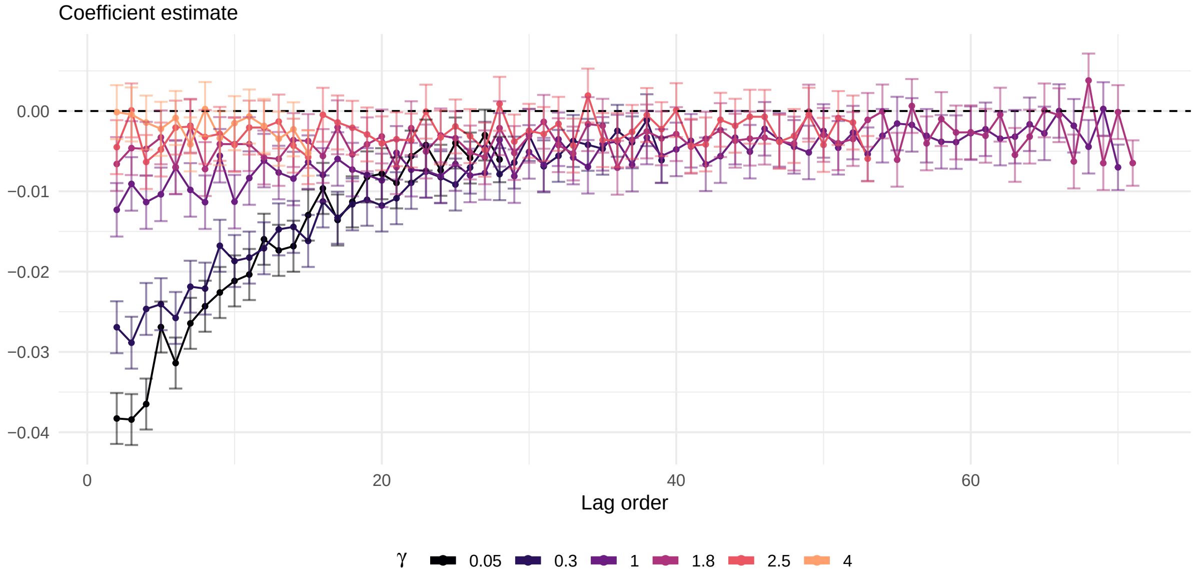


(b)

Full set of IRFs for several values of γ, subsuming the monetary policy stance.

E How γ affects inflation dynamics

Figure 11: Sensitivity on $\gamma - AR(k^*)$ on Inflation



Coefficient estimates for autoregressive model on simulated inflation, optimal lag order. Error bars are twice estimated standard deviations around the point value. The first lag is excluded in the figure for scale: it is overall in the neighbourhood of the shock persistence, .65.

In the main body of the paper, we restrict our attention to two values for γ, while this Section experiments with a larger set of values to assess how it affects the model dynamics. As shown in the paper, the model we propose allows for passive monetary policy, which indeed influences inflation dynamics.

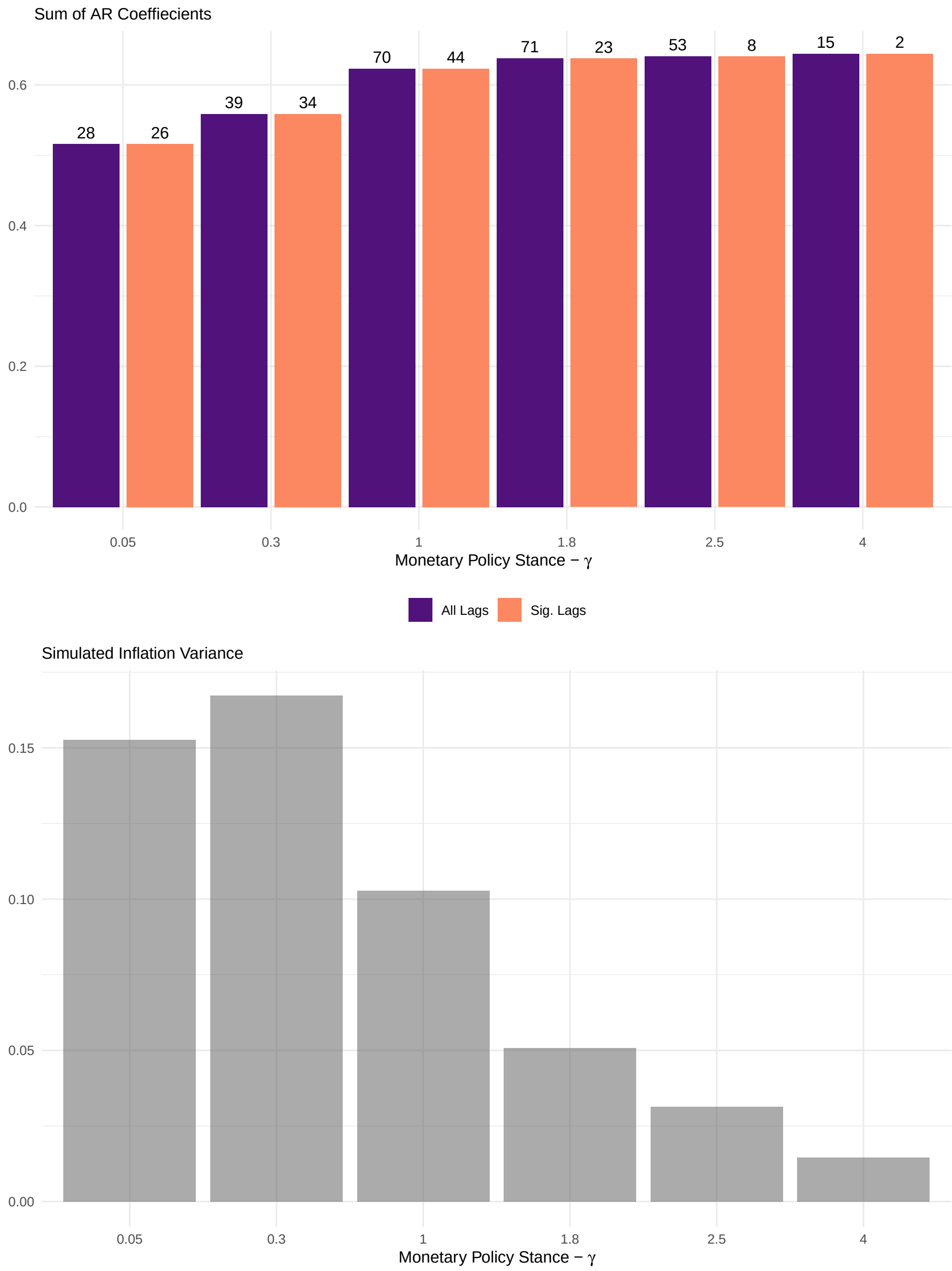
Consistently with the analysis in the main body, we compute optimal lags for the simulated series: as shown in Figure 11, when the central bank under-reacts to expected inflation in its rate levering, actual inflation depends more heavily on past values and readjusts more slowly. The number of significant lags and their magnitude is sensibly higher than the case of an extremely aggressive response to inflation (ie, $\gamma = 4$).

Figure 12 shows that inflation in itself exhibits lower persistence, but most importantly a high variance – which drives the results on persistence as measured with the sum of an AR coefficients. The tension originates from the effectiveness (or lack thereof) of the central bank to steer inflation back to its zero steady-state after a shock.[10]

The mechanism is the following: current high inflation π_t drives down real liquidity $\hat{z}_t$, which relates to liquidity allocation (eq. 14 in the main text). As the

[10] The same logic holds for the baseline NKDSGE model, see below Section G.

Figure 12: Sensitivity on γ – Inflation Persistence and Volatility



Results based on simulations. Values for π persistence. Top: Sum of optimal autoregressive coefficients; Bottom: variance for simulated π.

central bank does not keep sufficiently up with a raise in s_t, the liquid bond interest rate, money holdings $\hat{m}_t$ need to compensate; reallocating away from cash pushes down s_t to clear the asset market. In turn, degradation of money holdings and liquid bonds' yield fosters demand, triggering inflationary pressure in expectations. As expected inflation $E_t\pi_{t+1}$ increases, the central bank raises the interest rate s_t

by γ, ie less than proportionally, thus not enough to absorb into liquid bonds the asset reshuffling, which feeds again back into demand.

E.1 Misspecification

From a statistical standpoint, we can illustrate the relation between measured persistence, observed variance, and model misspecification. Assuming an econometrician who observes the series and wants to measure persistence, she fits a simple first-order autoregressive model $\pi_t = \rho\pi_{t-1} + \varepsilon_t$. The error in such specification is correlated with observed inflation via the transmission mechanism that is imparted by the model, namely the Phillips curve and the feedback through the central bank reaction. While the structural shocks impulsing the dynamics are the same sequence across simulation rounds, residuals also capture the channels of asset holding and total liquidity.

To see formally the link between measured persistence, observed variance, and model misspecification, recall that the variance of a first order autoregressive process is $V(\pi) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$, so that expressing ρ leads to

$$\rho = \sqrt{1 - \frac{\sigma_\varepsilon^2}{V(\pi)}}. \tag{37}$$

Provided the fundamental shocks are the same across simulations and have thus identical variance σ_ε^2, a more volatile series influenced by varying transmissions mechanisms will report a higher estimated persistence, ρ.

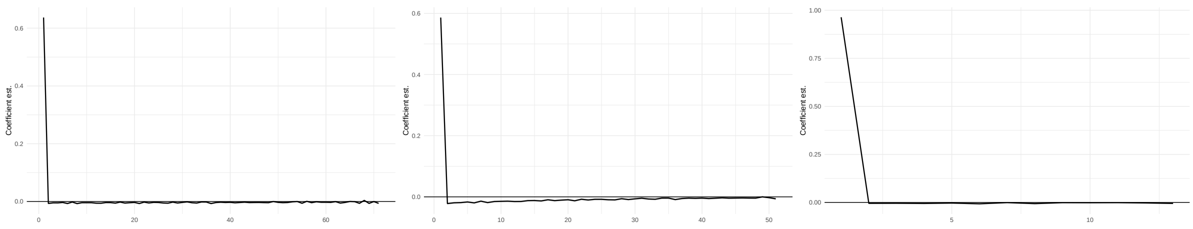
This relation is less transparent when the model to fit comprises more lags and the econometrician measures persistence summing over the autoregressive coefficients. Then, the estimated model imparts the following relationship, where L is the usual lag operator and $\Psi(L)$ is the associated characteristic polynomial:

$$\begin{aligned} \pi_t &= \mu + \sum_{i=1}^{p} \rho_i \pi_{t-i} + \varepsilon_t \\ \left(1 - \sum_{1}^{p} \rho_i L^i\right) \pi_t &= \Psi(L)\, \pi_t = \varepsilon_t \end{aligned} \tag{38}$$

Switching over to the roots of Ψ while keeping the relation with ρ coefficients is algebraically cumbersome for $p > 2$, the more so when the econometrician measures persistence summing over the estimated ρ's.

F Full lags structure

Figure 13: Autoregressive process lags



(a) $\gamma = 1.8$ **(b)** $\gamma = .5$ **(c)** Smets and Wout

$AR\,(k^*)$ model, with k^* optimally selected lags. Estimated coefficients in solid, bands are twice the estimated standard errors. Full lags, only the intercept is omitted.

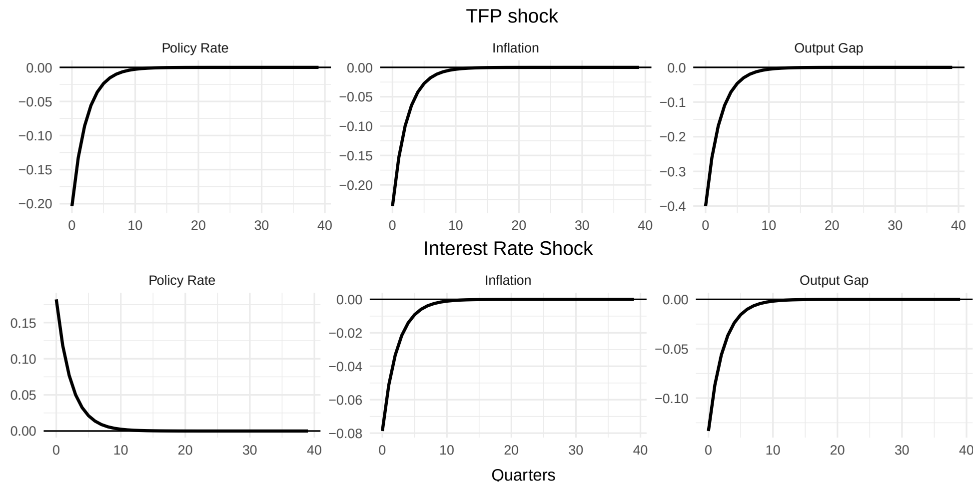
G Extreme policies

This section investigates what are the consequences of two extreme calibrations for the textbook NKDSGE models.

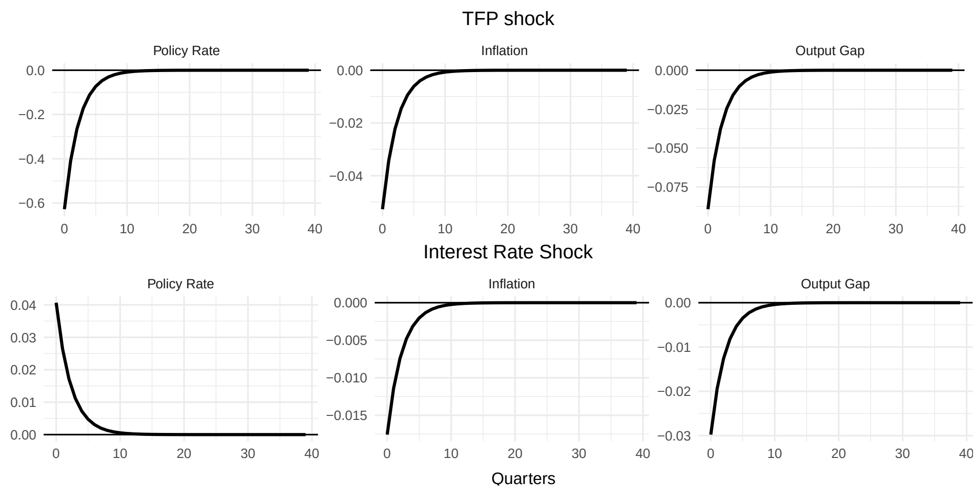
We let the reaction to expected inflation be negligibly close to its determinacy limit, such that $\gamma \to 1^+$, and then test for extremely high levels of reaction. The IRFs for these two cases are presented in the Figures below.

Interestingly, while calibrations close to indeterminacy do not display dramatic changes but only a higher effect on impact, an extremely reactive central bank can substantially suppress a sizeable share of volatility in the economy. Indeed, it appears to curb inflation and economic gap in a much more effective way when a fundamental productivity shock hits the economy – at the cost of considerable volatility in the policy rate, though. On the monetary policy shock, the decrease in volatility is even sharper.

Figure 14: Extreme Policies in baseline NKDSGE



(a) $\gamma \to 1^+$



(b) $\gamma \to +\infty$

Baseline NKDSGE (Gali, 2015) with extreme monetary policies. Top panel: very low reaction to expected inflation, $\gamma \to 1^+$. Bottom panel: extremely aggressive reaction to expected inflation. Values in percentages.