软件学院　2010　级软件工程专业(2011-1)

《线性代数》期末试题试卷

(考试形式：　闭　卷　　考试时间：120 分钟)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：__________　姓名：__________　学号：__________成绩：__________

1. Fill the blank (4 titles * 4 points/title = 16 points)

(1) The determinant of $\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$ is ________.

(2) Assume that $A = \begin{pmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{pmatrix}$ is row equivalent to $\begin{pmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Bases for Col A are ________.

(3) The characteristic polynomial of $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 0 & 6 & 0 \end{pmatrix}$ is ________.

(4) The orthogonal projection of $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$ onto the line through $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and the origin is ________.

2. Mark each statement True or False, and descript your reasons (5titles * 4 points/title = 20 points)

(1) If A is m × n and rank A = m, then the linear transformation $\vec{x} \mapsto A\vec{x}$ is one-to-one.

(2) If matrices A and B are row equivalent, they have the same echelon form.

(3) If matrices A and B are both n × n invertible matrices, then (A+B) is also invertible.

(4) The matrices A and A^T have the same eigenvalues, counting multiplicities.

(5) An n × n symmetric matrix has n distinct real eigenvalues.

3. Problem issues (7 + 9 + 10 + 11 = 37 points)

(1) Let S be the parallelogram determined by the vectors $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$, and let $A = \begin{pmatrix} 1 & -0.1 \\ 0 & 2 \end{pmatrix}$. Computer the area of the image of S under the mapping $\vec{x} \mapsto A\vec{x}$ (7 points)

(2) Assume the mapping $T: P_2 \to P_2$ defined by

$$T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$$

is linear. Find the matrix representation of T relative to the basis $B = \{1, t, t^2\}$ (9 points).

(3) Computer A^k, where $A = \begin{pmatrix} a & 0 \\ 3(a-b) & b \end{pmatrix}$ and k represents an arbitrary positive integer (10 points).

(4) Find a QR factorization of the matrix $\begin{pmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{pmatrix}$ (11 points).

4. Prove issues (10 + 9 = 19 points)

(1) Prove that a linear transformation T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ if and only if T^{-1} exists and also maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ (10 points).

(2) Suppose the solutions of a homogeneous system of five linear equations in six unknowns are all multiples of one nonzero solution. Will the system necessarily be consistent for every possible choice of constants on the right sides of the equations? Explain.(9 points)

5. Synthesis (8 points)

If $p(t) = c_0 + c_1 t + \cdots + c_n t^n$, define $p(A)$ to be $p(A) = c_0 + c_1 A + \cdots + c_n A^n$. Show that if λ is an eigenvalue of A, then one eigenvalue of $p(A)$ is $p(\lambda)$ (8 points).