中 山 大 学 软 件 学 院 **2011** 级 软 件 工 程 专 业 (2011 学年秋季学期)

《SE-103+线 性 代 数》 期 末 试 题 (B 卷)

(考 试 形 式： 闭 卷 考 试 时 间：2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：________________ 姓名：________ 学号：__________

出卷：<u>伍丽华</u> 复核：<u>高成英</u>

1. Fill in the blank（5×4=20 Pts）

(1) If T **is the linear transformation from** P_2 **to** P_2 **whose matrix relative to** $B=\{1,\ t,\ t^2\}$ **is**

$$[T]_B=\begin{bmatrix}2 & 1 & 0\\ 0 & 3 & -1\\ 1 & -2 & 4\end{bmatrix}$$**, then** $T(a_0+a_1t+a_2t^2)=$____________________________.

(2) If the row space of a 4×7 matrix A **is 4-dimentional, then the dimension of the null space of** A **is** ______________. **Is** $\text{Col }A=R^4$ **?**________________ **(Yes or No).**

(3) Let $v_1=\begin{bmatrix}2\\2\\0\end{bmatrix}$, $v_2=\begin{bmatrix}2\\2\\2\end{bmatrix}$, **and** $v_3=\begin{bmatrix}0\\2\\2\end{bmatrix}$ **be eigenvectors of a 3×3 matrix** A **, with corresponding eigenvalues 3, 2, and 1. Compute** A **.** A **=**______________________.

(4) Determine the value(s) of a **such that the system** $\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & a+2\\ 1 & a & -2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\3\\0\end{bmatrix}$ **is inconsistent.** a **=**___________________________________.

(5) For x **in** R^3**, Let** $Q(x)=3x_1^2+5x_2^2-x_1x_2+8x_2x_3$**, this quadratic form as** x^TAx **is** ____.

2．Make each statement True or False, and descript your reasons.（5×4=20 Pts）

(1) Whenever a system has free variables, the solution set contains many solutions.

(2) If $v_1, v_2, \cdots, v_k$ **are vectors in a vector space** V **and**

$\text{Span}\{v_1, v_2, \cdots, v_k\} = \text{Span}\{v_1, v_2, \cdots, v_{k-1}\}$**, then** $v_1, v_2, \cdots, v_k$ **are linearly dependent.**

(3) Let $T: R^n \to R^n$ **be a linear transformation. If** A **is the standard matrix representation of** T**, then an** $n\times n$ **matrix** B **will also be a matrix representation of** T **if and only if** B **is similar to** A**.**

(4) If A **is an** $n\times n$ **matrix, then** A **and** A^T **have the same eigenvectors.**

(5) If A **is symmetric and det(**A**)>0, then** A **is positive definite.**

3. Calculation (5×8=40 Pts）

(1) let $A = [b_1 \ \ b_2 \ \ b_3]$ **and** $B = [b_1 + b_2 + b_3 \ \ \ b_1 + 2b_2 + 4b_3 \ \ \ b_1 + 3b_2 + 9b_3]$**, where** b_1, b_2 **and** b_3 **are vectors in** R^3**. Suppose** $\det A = 1$**, find** $\det B$**.**

(2) Computer A^6**, where** $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$**.**

(3) Let $H = \left\{ \begin{bmatrix} 3a + 7b - c \\ -5b + 8c - 2d \\ 3d - 4e \\ 5b - 8c - d + 4e \end{bmatrix} a, b, c, d, e \text{ any real numbers} \right\}$,

a. **Show that** H **is a subspace of** R^4

b. **Find a basis for** H**.**

(4) Let $A = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$**.**

a. **Find the orthogonal projection of** b **onto Col** A**.**

b. **Find a least-squares solution of** $Ax = b$**.**

c. **Determine the associated least-squares error.**

(5) Let W =Span$\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$, and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, Construct an orthonormal basis for W.

4. Prove issues（2×6=12 Pts）

(1) Let W be a subspace of R^n such that $\dim W = p$, and let $S = \{w_1, w_2, \cdots, w_p\}$ be an orthonomal basis for W. Define $T : R^n \to W$ by

$$T(v) = (v \cdot w_1)w_1 + (v \cdot w_2)w_2 + \cdots + (v \cdot w_p)w_p$$

Prove that T is a linear transformation.

(2) Let A and B be similar matrices. Show that if A satisfies the equation $A^3 - 3A + I = 0$, then B also satisfies a similar equation $B^3 - 3B + I = 0$.

5. Synthesis (8 points)

Let x be a vector in R^n with $x^T x = 1$, Show that if $A = I - xx^T$, then $rank(A) < n$.