中山大学软件学院 **2009** 级软件工程专业 (2011 秋季学期)

《数字图像处理》期末试题 (A 卷)

(考试形式：闭（45 分钟）/开（75 分钟）　考试时间：共 2 小时)

警示

《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：_____　姓名：_____　学号：_____

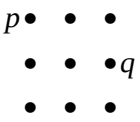
注意：答案一定要写在答卷中，写在本试题卷中不给分。本试卷要和答卷一起交回。

Part I (Close book exam 40 pts) 45min

1. (4 pts) Given an image with size 3×3 as following. Reduce its gray level resolution in half, generate a result image and describe your methods.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 5 & 4 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

2. (6 pts) Consider two pixels p and q in the following figure. Calculate the (a) Euclid distance, (b) city-block distance and (c) chessboard distance between p and q.

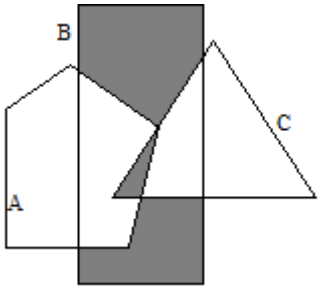


(a) $\sqrt{1^2+2^2}=\sqrt{5}$

(b) 1+2 = 3

(c) Max(1, 2) = 2

3. (6 pts) Give the expression for the set shown shaded in the following figure in terms of sets A, B, and C.



$(B-(A\cup B))\cup(A\cap B)$

4. (5 pts) Can a typical image display device show any natural color? Why?

通常的彩色显示设备是不能显示自然界的所有颜色的。显示器是所能显示的颜色范围仅限于该显示器的三原色所构成的三角形内。而在色度图中任意三角形都是不能完全覆盖整个色度图的。

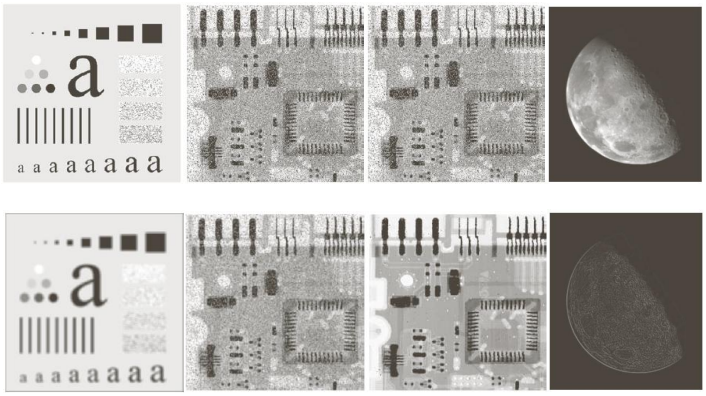
5. (6 pts) Consider an image pixel with~~one pixel have color~~ [10, 20, 30] in RGB color space. (Assume that the intensity level range of each color channel is [0, 255].) Please answer the following questions:
   a) What is the intensity of this pixel?
   b) What is the saturation of this pixel?
   c) What is the complement of this pixel?

   a) $\frac{10+20+30}{3}=20$

   b) $S=1-\frac{3}{10+20+30}\min(10,20,30)=0.5$

   c) [245, 235, 225]

6. (4 pts) As shown in the following figure. The upper row is the input images, and the bottom row is the output images after a spatial filtering. Choose the corresponding filter for each image pair.



(a) (b) (c) (d)

A. $\begin{pmatrix}0&1&0\\1&-4&1\\0&1&0\end{pmatrix}$ B. $\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ C. median filter D. $\begin{pmatrix}0&-1&0\\-1&5&-1\\0&-1&0\end{pmatrix}$

(a) B (b) B (c) C (d) A

7. (4 pts) Consider the continuous function $f(t) = e^{j2\pi nt}$. Please answer the following questions:
   a) What is the period of $f(t)$?
   b) What is the frequency of $f(t)$?

   a) $T=\frac{2\pi}{2\pi n}=\frac{1}{n}$

   b) $F=\frac{1}{T}=n$

8. (5 pts) Give both the 1D Fourier transformation and inverse Fourier transformation pair.

$$F(t)=\int f(x)e^{-j2\pi tx}dx \quad \text{或者} \quad F(t)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi tx}$$
$$f(x)=\int F(t)e^{j2\pi xt}dt \qquad f(x)=\frac{1}{M}\sum_{t=0}^{M-1} F(t)e^{j2\pi xt}$$

Part II (Open book exam 60 pts) 75min

1. (6 pts) Two images, $f(x, y)$ and $g(x, y)$, have histograms as following. Please determine the histograms of image $f(x, y) + g(x, y)$.

r_k	0	1	2	3	4	5	6	7
$p_f(r_k)$	0.3	0.2	0.2	0.1	0.05	0.15	0	0
$p_g(r_k)$	0	0	1.0	0	0	0	0	0

r_k	0	1	2	3	4	5	6	7
$p(r_k)$	0	0	0.3	0.2	0.2	0.1	0.05	0.15

2. (8 pts) Give the expression of contraharmonic mean filter,

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Please explain that if the parameter Q>0, it can eliminates pepper noise, but invalid for salt noise.

该题允许学生举例说明，要点是说明椒噪声是灰度值很小的点，周围应该是灰度值较大的点，因此在滤波公式中椒噪声点的灰度值不起作用。下面是参考答案：

逆谐波均值滤波器即为一个加权均值滤波器，像素 (s,t) 的权值为

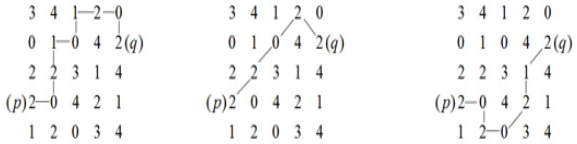
$$g(s,t)^{Q} / \sum_{(s,t)\in S_{xy}} g(s,t)^{Q}$$

由于对(x, y)的邻域 S_{xy}, $\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}$ 是常数。因此，像素 (s,t) 的权值的大小取决于 $g(s,t)^{Q}$：

当 Q>0 时，滤波器掩模下各像素若灰度值 $g(s,t)$ 越大权值 $g(s,t)^{Q}$ 越大，对滤波结果的贡献越大；相反若灰度值 $g(s,t)$ 越小权值 $g(s,t)^{Q}$ 则越小，对滤波结果的贡献越小；特别地当 $g(s,t)=0$ 时，其贡献也为零，滤波的结果将由其他像素的加权平均决定。所以当 Q>0 时，对胡椒噪声有效。

3. (9 pts) Let $V=\{0,1,2\}$ and compute the lengths of the shortest 4-, 8-, and *m*-path between *p* and *q*. If a particular path does not exist between these points, explain why.

$$\begin{array}{ccccc} 3 & 4 & 1 & 2 & 0 \\ 0 & 1 & 0 & 4 & 2(q) \\ 2 & 2 & 3 & 1 & 4 \\ (p)2 & 0 & 4 & 2 & 1 \\ 1 & 2 & 0 & 3 & 4 \end{array}$$



最短 4 通路长度为 8，最短 8 通路长度为 4， 最短 m 通路为 6.

4. (9 pts) Based on the relationship between the original image and its filtered image, determine the linear filter coefficients. (Ignore the border effects.)

$$\begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{0} & \boxed{\frac{1}{4}} & \boxed{0} \\ \boxed{1/4} & \boxed{1} & \boxed{1/4} \\ \boxed{0} & \boxed{\frac{1}{4}} & \boxed{0} \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{pmatrix}$$

5. (8 pts) Design a morphological operation to extract the edge of the image ~~bellow~~as following.

$$\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

用结构 $\begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{1} & 1 \\ 1 & 1 & 1 \end{pmatrix}$ 对原图进行腐蚀，得到结果 S $\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$，再用原图减去

腐蚀结果 S 即得到边缘
$$\begin{matrix}0&0&0&0&0&0&0&0\\0&0&1&1&1&1&0&0\\0&1&1&0&0&1&1&0\\0&1&0&0&0&0&1&0\\0&1&1&1&1&1&1&0\\0&0&0&0&0&0&0&0\end{matrix}$$
。

6. (10 pts) In general, spatial filtering of an image of size $M\times N$ with a filter of size $m\times n$ is given by the expression:

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)\,.$$

Show that this is a linear operator.

证：令

$$g(x,y)=H[f(x,y)]=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$

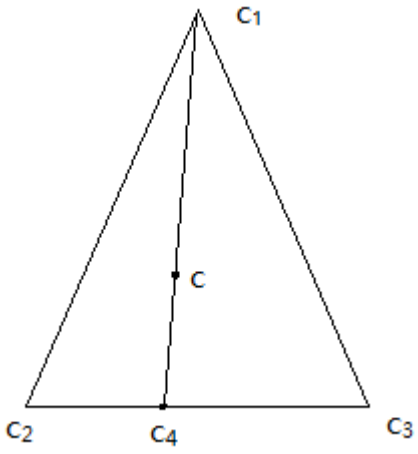
则

$$\begin{aligned}H[af_a(x,y)+bf_b(x,y)]&=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)[af_a(x+s,y+t)+bf_b(x+s,y+t)]\\&=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)af_a(x+s,y+t)+\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)bf_b(x+s,y+t)\\&=a\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f_a(x+s,y+t)+b\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f_b(x+s,y+t)\\&=aH[f_a(x,y)]+bH[f_b(x,y)]\end{aligned}$$

得证。

7. (10 pts) Consider ~~any~~ three valid colors c_1, c_2, and c_3 with coordinates (0, 0), (10, 0) and (6, 8) in the chromaticity diagram. A color c is lie within the triangle with coordinate (5, 4). Computing the relative percentages of c_1, c_2, and c_3 composing the given color c.

A：如右图所示，彩色 c 可以由 c_1，c_2 和 c_3 线性组合而成。要求其中各个颜色的通用比例。可以这么考虑：彩色 c 可以由 c_1 和 c_4 组成，而 c_4 可以由 c_2 和 c_3 组成。因此



$$\lambda(c_1)=\frac{d(c,c_4)}{d(c_1,c_4)}$$

$$\lambda(c_2)=\frac{d(c,c_1)}{d(c_1,c_4)}\cdot\frac{d(c_3,c_4)}{d(c_2,c_3)}$$

$$\lambda(c_3)=\frac{d(c,c_1)}{d(c_1,c_4)}\cdot\frac{d(c_2,c_3)}{d(c_2,c_3)}$$

或者

$$\Leftrightarrow \begin{cases} \lambda(c_1)\vec{c}_1 + \lambda(c_2)\vec{c}_2 + \lambda(c_3)\vec{c}_3 = \vec{c} \\ \lambda(c_1) + \lambda(c_2) + \lambda(c_3) = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \lambda(c_1)x_1 + \lambda(c_2)x_2 + \lambda(c_3)x_3 = x \\ \lambda(c_1)y_1 + \lambda(c_2)y_2 + \lambda(c_3)y_3 = y \\ \lambda(c_1) + \lambda(c_2) + \lambda(c_3) = 1 \end{cases}$$

解得：

$$\begin{cases} \lambda(c_1) = \dfrac{(x-x_3)(y_2-y_3)-(y-y_3)(x_2-x_3)}{(x_1-x_3)(y_2-y_3)-(y_1-y_3)(x_2-x_3)} \\ \lambda(c_2) = \dfrac{(x-x_3)(y_1-y_3)-(y-y_3)(x_1-x_3)}{(x_2-x_3)(y_1-y_3)-(y_2-y_3)(x_1-x_3)} \\ \lambda(c_3) = \dfrac{(x-x_1)(y_2-y_1)-(y-y_1)(x_2-x_1)}{(x_3-x_1)(y_2-y_1)-(y_3-y_1)(x_2-x_1)} \end{cases}$$

由通用表达式可以计算出

$$\begin{cases} \lambda(c_1) = 0.3 \\ \lambda(c_2) = 0.2 \\ \lambda(c_3) = 0.5 \end{cases}$$