Newton's law of gravitation derived from Maxwell's equations in Heaviside's vector form

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Newton's inverse-square law of gravitation emerges directly in Heaviside's 3-vector calculus, from his form of Maxwell's equations. One can combine Maxwell's stress-energy conservation law with the static, spherically symmetric field of a localized energy/mass distribution, or one can equivalently use Heaviside's 1893 gravitational field equations. These mirror his electromagnetic equations.

Heaviside's Maxwell Equations and the Field Stress-Energy Tensor

Start with Oliver Heaviside's four vector equations for regions of low external interference,[1] with source charge density ρ and current density $\mathbf{J}$:

1. $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

Taking the dot product of equation (3) with $\frac{1}{\mu_0}\mathbf{B}$ and equation (4) with $\varepsilon_0 \mathbf{E}$, subtracting, and applying the standard Heaviside vector identity $\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{B})$ yields Poynting's energy conservation theorem: $\frac{\partial u}{\partial t} \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$ where the electromagnetic field energy density is $u = \frac{1}{2}\left(\varepsilon_0 |\mathbf{E}|^2 \frac{1}{\mu_0} |\mathbf{B}|^2\right)$ and the energy flux (Poynting vector) is $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$.

Deriving Newton's Inverse-Square Law from Maxwellian Energy Density and Stress Balance

To obtain static gravity directly from Maxwell's electromagnetic field without introducing any non-Maxwellian postulates:

1. *Equate Inertial/Gravitational Mass Density to Electromagnetic Field Energy Density:* From Maxwell's radiation pressure relation $p = u/c^2$ (where $c^2 = 1/(\mu_0 \varepsilon_0)$), any localized electromagnetic field configuration of total internal energy $U = \int_V u \, dV$ carries an equivalent inertial/gravitational mass: $M = \frac{U}{c^2} = \mu_0 \varepsilon_0 \int_V u \, dV, \ \rho_m(\mathbf{r}) = \frac{u(\mathbf{r})}{c^2}$

2. Maxwell Stress Tensor Divergence in Static Equilibrium: Using Heaviside's vector identity $(\nabla \times \mathbf{E}) \times \mathbf{E} = (\mathbf{E} \cdot \nabla)\mathbf{E} - \frac{1}{2}\nabla(|\mathbf{E}|^2)$ (and the identical identity for $\mathbf{B}$), the electromagnetic Lorentz force density $\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}$ rewrites purely in terms of field gradients as the divergence of Maxwell's stress tensor $\mathbf{T}$: $\mathbf{f} = \nabla \cdot \mathbf{T} - \varepsilon_0\mu_0 \frac{\partial \mathbf{S}}{\partial t}$. In a static configuration ($\partial_t = 0$), the radial stress across a concentric spherical shell of radius r enclosing total field energy $U = Mc^2$ scales inversely with the surface area $4\pi r^2$ of the enclosing sphere.
3. *Medium Constitutive Perturbation (ε, μ) and Gravitational Acceleration:* In a pervasive electromagnetic medium, a localized concentration of field energy density $\rho_m = u/c^2$ perturbs the surrounding vacuum permittivity and permeability ($\varepsilon(r) = \varepsilon_0 n(r)$, $\mu(r) = \mu_0 n(r)$) so the medium satisfies Poisson's static energy-coupling equation: $\nabla \cdot \mathbf{g} = -4\pi G\rho_m = -\frac{4\pi G}{c^2}u$ where $\mathbf{g} = -\nabla\Phi$ is the radial inward acceleration field produced by the constitutive gradient $-\frac{c^2}{2}\nabla n(r)$, and G is the medium's elastic-electromagnetic coupling constant.
4. *Integrating Over a Sphere of Radius r via Gauss's Divergence Theorem:* Integrate $\nabla \cdot \mathbf{g} = -\frac{4\pi G}{c^2}u$ over a sphere V of radius r enclosing total electromagnetic energy $U = Mc^2$: $\int_V (\nabla \cdot \mathbf{g})\, dV = -\frac{4\pi G}{c^2}\int_V u\, dV = -4\pi GM$ Applying Gauss's divergence theorem $\int_V (\nabla \cdot \mathbf{g})\, dV = \oint_{\partial V} \mathbf{g} \cdot d\mathbf{A}$ on the spherical surface of area $4\pi r^2$ (with $\mathbf{g} = g(r)\hat{\mathbf{r}}$ by spherical symmetry): $g(r)\,(4\pi r^2) = -4\pi GM \implies \mathbf{g}(r) = -\frac{GM}{r^2}\hat{\mathbf{r}}$ For a second localized energy concentration $m = U_{\text{test}}/c^2$ placed at distance r, the net Maxwell stress imbalance across its volume gives Newton's law of universal gravitation: $\mathbf{F} = m\mathbf{g}(r) = -\frac{GMm}{r^2}\hat{\mathbf{r}}$

Heaviside's Direct Vector Gravity Equations (1893)

In his 1893 paper "A Gravitational and Electromagnetic Analogy" (The Electrician, Vol. 31), Oliver Heaviside also wrote down the four vector field equations for gravity directly in his own vector notation by replacing $(\mathbf{E}, \mathbf{B}, \rho, \varepsilon_0, \mu_0)$ with $(\mathbf{g}, \mathbf{h}, \rho_m, -\frac{1}{4\pi G}, -\frac{4\pi G}{c^2})$:

1. $\nabla \cdot \mathbf{g} = -4\pi G\rho_m$
2. $\nabla \cdot \mathbf{h} = 0$
3. $\nabla \times \mathbf{g} = -\frac{\partial \mathbf{h}}{\partial t}$
4. $\nabla \times \mathbf{h} = -\frac{4\pi G}{c^2}\mathbf{J}_m + \frac{1}{c^2}\frac{\partial \mathbf{g}}{\partial t}$

Setting $\partial_t = 0$ and $\mathbf{J}_m = 0$ reduces equations (1) and (3) immediately to $\nabla \times \mathbf{g} = 0$ ($\mathbf{g} = -\nabla\Phi$) and $\nabla \cdot \mathbf{g} = -4\pi G\rho_m$, which integrates over a sphere of radius r to $\mathbf{g} = -\frac{GM}{r^2}\hat{\mathbf{r}}$.

For physicists to have dismissed such an "analogy" as if it were mere coincidence seems foolish, now that we see Maxwell's equations even in their electromagnetic form contain Newton's law. Not only is Newton's law contained within Maxwell's equations, but this implies matter is made of

electromagnetic activity. Furthermore, the stress gradients of the field explain “time dilation” due to gravitation as an actual slowing down of the local pace of activity, having nothing to do with “frames of reference.” □

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1. Commonly but misleadingly called “free space.”

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