MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
33	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 33

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 $T_n = a + (n-1)d$ is the general term of ... (1)

A) a quadratic pattern
B) a geometric sequence
C) an arithmetic sequence ✓
D) a convergent series

> **Answer: C** — A common DIFFERENCE d is what makes a sequence arithmetic.
> - **A** — a quadratic pattern has an n^2 term
> - **B** — a geometric sequence uses $T_n = ar^{n-1}$
> - **D** — a series is a SUM, not a sequence

1.2 The 20th term of $7\ ;\ 11\ ;\ 15\ ;\ \ldots$ is ... (1)

A) 87
B) 83 ✓
C) 80
D) 76

> **Answer: B** — $T_{20} = 7 + 19(4) = 83$.
> - **A** — used $20d$ instead of $19d$
> - **C** — used $20d$ and then dropped the first term
> - **D** — used $19d$ without adding the first term

1.3 The common ratio of $3\ ;\ 6\ ;\ 12\ ;\ \ldots$ is ... (1)

A) 3 **B)** 6

C) $\frac{1}{2}$ **D) 2 ✓**

> **Answer: D** — Each term is DIVIDED by the one before: $6 \div 3 = 2$.
> - **A** — that is the first term
> - **B** — SUBTRACTED consecutive terms instead of dividing them
> - **C** — inverted the ratio

1.4 The 8th term of $3\ ;\ 6\ ;\ 12\ ;\ \ldots$ is ... (1)

A) 768 **B)** 192

C) 384 ✓ **D)** 24

> **Answer: C** — $T_8 = 3(2)^7 = 384$.
> - **A** — used r^8 instead of r^7
> - **B** — used r^6 instead of r^7
> - **D** — worked out 3×8 instead of 3×2^7

1.5 The sum of the first 20 terms of $7\ ;\ 11\ ;\ 15\ ;\ \ldots$ is ... (1)

A) $1\ 800$ **B)** 830

C) 90 **D) 900 ✓**

> **Answer: D** — $S_n = \frac{n}{2}[2a + (n-1)d] = 10(14 + 76) = 900$.
> - **A** — left out the factor of a half
> - **B** — used $n/2 \times T_{20}$, leaving out the first term
> - **C** — left off the factor of 10 altogether

1.6 In an arithmetic sequence $T_5 = 17$ and $T_9 = 33$. Then $d =$ ... (1)

A) 8 **B) 4 ✓**

C) 16 **D)** 2

> **Answer: B** — Four steps separate T_5 from T_9, so $4d = 16$ and $d = 4$.
> - **A** — divided by 2 instead of by 4
> - **C** — that is $4d$, before dividing by the four steps
> - **D** — divided by 8 instead of by 4

1.7 A geometric SERIES converges when ... (1)

A) $-1 < r < 1$ ✓ **B)** $r > 1$

C) $r < 0$ **D)** $r = 1$

> **Answer: A** — Only then do the terms shrink fast enough for the sum to settle on a value.
> - **B** — the terms grow without limit, so the sum diverges
> - **C** — $r = -2$ is negative, but that series still diverges
> - **D** — every term is then the same, so the sum grows without limit

1.8 The sum to infinity of $8 + 4 + 2 + \ldots$ is ... (1)

A) 8 **B)** 14

C) 16 ✓ **D)** 32

> **Answer: C** — $S_\infty = a \div (1 - r) = 8 \div 0{,}5 = 16$.
> - **A** — that is the first term alone
> - **B** — added the first three terms and stopped
> - **D** — used $1 - r$ as $0{,}25$ instead of $0{,}5$

1.9 In a geometric sequence $T_3 = 20$ and $T_5 = 80$. The POSITIVE value of r is ... (1)

A) 4 **B) 2 ✓**

C) 60 **D)** $\sqrt{60}$

> **Answer: B** — $T_5 = T_3 r^2$, so $r^2 = 80 \div 20 = 4$ and $r = 2$.
> - **A** — that is r^2, before taking the square root
> - **C** — SUBTRACTED the two terms instead of dividing
> - **D** — square-rooted the difference instead of the ratio

1.10 The sum of the first n terms of a GEOMETRIC series ($r - 1$ is not 0) is ... (1)

A) $\dfrac{a(r^n - 1)}{r - 1}$ ✓ **B)** $\dfrac{a}{1 - r}$

C) $\dfrac{a(r^n + 1)}{r - 1}$ **D)** $\dfrac{n}{2}[2a + (n - 1)d]$

> **Answer: A** — That is the geometric sum formula from the information sheet.
> - **B** — that is the sum to INFINITY, not the sum of n terms
> - **C** — the numerator carries a MINUS, not a plus
> - **D** — that is the ARITHMETIC sum formula

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	A	**B**	C	D
1.3	A	B	C	**D**
1.4	A	B	**C**	D
1.5	A	B	C	**D**
1.6	A	**B**	C	D
1.7	**A**	B	C	D
1.8	A	B	**C**	D
1.9	A	**B**	C	D
1.10	**A**	B	C	D

QUESTION 2 [11 MARKS]

In an ARITHMETIC sequence the third term is 11 and the seventh term is 27.

2.1 Determine the common difference d. (3)

$T_7 - T_3 = 4d$ *(1)*
$27 - 11 = 4d$ *(1)*
$\boldsymbol{d = 4}$ *(1)*

2.2 Determine the first term a. **(2)**

$T_3 = a + 2d$: $11 = a + 8$ *(1)*
$\boldsymbol{a = 3}$ *(1)*

2.3 Calculate the sum of the first 20 terms. **(3)**

$S_{20} = \frac{20}{2}[2(3) + 19(4)]$ *(2)* $= \mathbf{820}$ *(1)*

2.4 Determine which term of the sequence is equal to 99. **(3)**

$3 + (n - 1)4 = 99$ *(1)*
$4n - 1 = 99$ *(1)*
$\boldsymbol{n = 25}$ *(1)*

QUESTION 3 [12 MARKS]

In a GEOMETRIC sequence the second term is 6 and the fifth term is 48.

3.1 Determine the constant ratio r. **(4)**

$\frac{T_5}{T_2} = r^3$ *(1)*
$\frac{48}{6} = 8$ *(1)*
$r^3 = 8$ *(1)*
$\boldsymbol{r = 2}$ *(1)*

3.2 Determine the first term. **(2)**

$T_2 = ar$: $6 = 2a$ *(1)*
$\boldsymbol{a = 3}$ *(1)*

3.3 Calculate the sum of the first 8 terms. **(3)**

$S_8 = \frac{3(2^8 - 1)}{2 - 1}$ *(2)* $= \mathbf{765}$ *(1)*

3.4 Explain why this series does NOT have a sum to infinity, and give an example of a geometric series that does. **(3)**

S_∞ exists only when $|r| < 1$ *(1)*, and here $r = 2$ *(1)*
For example $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, where $r = \frac{1}{2}$ and $S_\infty = 1$ *(1)*

TOTAL: 33 marks

This question paper consists of 3 questions.

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