Average Gradient & Average Rate of Change (Grade 12)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The AVERAGE GRADIENT between two points on a curve is ... (1)

A) the gradient of the tangent at the first point
B) the average of the two y-values
C) the gradient of the straight line joining them ✓
D) the difference between the two y-values

> **Answer: C** — Average gradient is the change in y divided by the change in x — the chord's gradient.
> - **A** — the tangent gives the INSTANTANEOUS gradient at one point
> - **B** — the y-values are subtracted and then divided, not averaged
> - **D** — the difference in y must still be divided by the difference in x

1.2 For $f(x) = x^2$, the average gradient between $x = 1$ and $x = 3$ is ... (1)

A) 2
B) 8
C) 4 ✓
D) 5

> **Answer: C** — $(9 - 1) \div (3 - 1) = 8 \div 2 = 4$.
> - **A** — gave the change in x instead of the gradient
> - **B** — gave the change in y without dividing by the change in x
> - **D** — averaged the two y-values instead

1.3 For $f(x) = x^2$, the average gradient between $x = 1$ and $x = 1 + h$ is ... (1)

A) h
B) $2 + h$ ✓
C) 2
D) $2h$

> **Answer: B** — $((1 + h)^2 - 1^2) \div h = (2h + h^2) \div h = 2 + h$.
> - **A** — dropped the 2 from $2h + h^2$
> - **C** — that is the LIMIT as h approaches 0, not the average gradient
> - **D** — forgot to divide by the h

1.4 As h approaches 0, the average gradient becomes ... (1)

A) the gradient of the chord

B) zero

C) undefined

D) the gradient of the tangent at that point ✓

> **Answer: D** — That limit is the definition of the derivative.
>
> - **A** — the chord BECOMES the tangent in the limit
> - **B** — the average gradient tends to a finite value, not necessarily zero
> - **C** — the limit exists wherever the function is differentiable

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1.5 The derivative from FIRST PRINCIPLES is defined as the limit of ... (1)

A) $\frac{f(x)-f(x+h)}{h}$, as x approaches 0

B) $\frac{f(x+h)+f(x)}{h}$, as h approaches 0

C) $\frac{f(x+h)-f(x)}{x}$, as h approaches 0

D) $\frac{f(x+h)-f(x)}{h}$**, as h approaches 0** ✓

> **Answer: D** — That is the definition printed on the information sheet.
>
> - **A** — the limit is taken as h approaches 0, and the numerator is the other way round
> - **B** — the two function values are SUBTRACTED, not added
> - **C** — the denominator is h, the change in x, not x itself

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1.6 Using first principles, $f'(x)$ for $f(x) = 2x^2 - 3x$ is ... (1)

A) $4x - 3$ ✓

B) $4x - 3 + 2h$

C) $2x - 3$

D) $4x$

> **Answer: A** — The difference quotient simplifies to $4x - 3 + 2h$, and letting h approach 0 leaves $4x - 3$.
>
> - **B** — the limit has not been taken; h must go to 0
> - **C** — halved the $4x$ — the derivative of $2x^2$ is $4x$
> - **D** — dropped the derivative of the $-3x$ term

1.7 The AVERAGE RATE OF CHANGE of a quantity over an interval is ... (1)

A) the total change on its own

B) the total change divided by the length of the interval ✓

C) the change at a single instant

D) the largest value in the interval

> **Answer: B** — It is the same calculation as an average gradient, expressed in context.
>
> - **A** — the change must still be divided by the interval
> - **C** — that is the INSTANTANEOUS rate, which needs a derivative
> - **D** — a maximum value is not a rate

1.8 A tank holds $V = 100 - 4t$ litres after t minutes. Its rate of change is ... (1)

A) -4 litres per minute ✓

B) 4 litres per minute

C) 96 litres per minute

D) 100 litres per minute

> **Answer: A** — The coefficient of t is the rate, and the minus sign shows the tank is EMPTYING.
>
> - **B** — the tank is emptying, so the rate is negative
> - **C** — 96 is the volume after one minute, not the rate
> - **D** — 100 is the STARTING volume, not the rate

1.9 For $f(x) = x^2$, the average gradient between $x = -2$ and $x = 2$ is ... (1)

A) 4 **B)** 2

C) 8 **D) 0 ✓**

> **Answer: D** — Both points have $y = 4$, so the chord joining them is horizontal.
> - **A** — that is the change in x, not the gradient
> - **B** — that is half the change in x
> - **C** — that is the sum of the two y-values

1.10 The average gradient of a STRAIGHT line between any two of its points is ... (1)

A) different for every pair of points **B) always the same, and equal to the line's gradient** ✓

C) undefined **D)** always zero

> **Answer: B** — A straight line has a constant gradient, so every chord of it has that same gradient.
> - **A** — that is true of a CURVE, not of a straight line
> - **C** — only a vertical line has an undefined gradient
> - **D** — only a horizontal line has gradient zero

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	A	B	**C**	D
1.3	A	**B**	C	D
1.4	A	B	C	**D**
1.5	A	B	C	**D**
1.6	**A**	B	C	D
1.7	A	**B**	C	D
1.8	**A**	B	C	D
1.9	A	B	C	**D**
1.10	A	**B**	C	D

QUESTION 2 [13 MARKS]

Given $f(x) = x^2 - 4x + 3$.

2.1 Calculate the average gradient of f between $x = 1$ and $x = 5$. (4)

> $f(1) = 0$ and $f(5) = 8$ *(2)*
> Average gradient $= \frac{8 - 0}{5 - 1}$ *(1)* $= \mathbf{2}$ *(1)*

2.2 Determine $f'(x)$, and calculate the gradient of the tangent to f at $x = 3$. (3)

> $f'(x) = 2x - 4$ *(2)*
> $f'(3) = \mathbf{2}$ *(1)*

2.3 The two answers above are equal. Explain what that means about the graph. (3)

> The tangent at $x = 3$ is PARALLEL to the chord joining $(1\ ;\ 0)$ and $(5\ ;\ 8)$ *(2)*
> $x = 3$ is the midpoint of the interval, which is where a parabola's tangent matches its average gradient *(1)*

2.4 Determine the value of x at which the gradient of f is zero, and state what this point is called. **(3)**

> $2x - 4 = 0$ *(1)*
> $x = 2$ *(1)*
> It is the **turning point** (a minimum here) *(1)*

QUESTION 3 [12 MARKS]

The height, in metres, of a ball t seconds after it is thrown is given by $h(t) = 20t - 5t^2$.

3.1 Calculate the average speed of the ball over the first 2 seconds. **(3)**

> $h(0) = 0$ and $h(2) = 20$ *(1)*
> $\frac{20}{2}$ *(1)* $= \mathbf{10\ m/s}$ *(1)*

3.2 Determine the speed of the ball at $t = 1$ second. **(3)**

> $h'(t) = 20 - 10t$ *(2)*
> $h'(1) = \mathbf{10\ m/s}$ *(1)*

3.3 Determine when the ball reaches its maximum height, and what that height is. **(4)**

> The ball is at its highest when $h'(t) = 0$ *(1)*
> $20 - 10t = 0$, so $t = 2$ s *(1)*
> $h(2) = 40 - 20$ *(1)* $= \mathbf{20\ m}$ *(1)*

3.4 Explain what a NEGATIVE value of $h'(t)$ would tell you about the ball. **(2)**

> The height is decreasing *(1)* — the ball is falling *(1)*

TOTAL: 35 marks

This question paper consists of 3 questions.

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