Average Gradient on a Curve (Grade 11)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
33	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 33

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The average gradient of f between $x = a$ and $x = b$ is ... (1)

A) $\dfrac{f(b) - f(a)}{b - a}$ ✓

B) $\dfrac{f(b) + f(a)}{b - a}$

C) $\dfrac{b - a}{f(b) - f(a)}$

D) $f(b) - f(a)$

Answer: A — Change in y over change in x — the gradient of the chord.

- **B** — the change in height needs a MINUS
- **C** — upside down: it must be the change in y over the change in x
- **D** — never divided by the change in x

1.2 For $f(x) = x^2$, the average gradient between $x = 1$ and $x = 4$ is ... (1)

A) 15

B) 3

C) 8

D) 5 ✓

Answer: D — $\dfrac{16 - 1}{4 - 1} = \dfrac{15}{3} = 5$.

- **A** — forgot to divide by $4 - 1 = 3$
- **B** — divided the change in x by itself
- **C** — that is the gradient AT $x = 4$, not the average

1.3 For $f(x) = x^2 - 2x$, the average gradient between $x = 0$ and $x = 2$ is ... (1)

A) 2 **B)** -2

C) 0 ✓ **D)** 4

> **Answer: C** — $f(0) = 0$ and $f(2) = 0$, so $\frac{0-0}{2-0} = 0$.
>
> - **A** — used the change in x as the answer
> - **B** — that is $f(1) - f(0)$, a different interval
> - **D** — that is only the x^2 part

1.4 For $g(x) = \frac{6}{x}$, the average gradient between $x = 1$ and $x = 3$ is ... (1)

A) 2 **B)** -2 ✓

C) -4 **D)** 4

> **Answer: B** — $g(1) = 6$ and $g(3) = 2$, so $\frac{2-6}{3-1} = -2$.
>
> - **A** — the curve is LOWER at $x = 3$, so the answer is negative
> - **C** — forgot to divide by $3 - 1 = 2$
> - **D** — the wrong sign and no division

1.5 A NEGATIVE average gradient over an interval means that the curve ... (1)

A) ends lower than it started ✓ **B)** falls all the way

C) is a straight line **D)** has no turning point in between

> **Answer: A** — It compares only the two end points.
>
> - **B** — it may rise and fall in between
> - **C** — a curve has average gradients too
> - **D** — a turning point in between is possible

1.6 As the two points slide together, the chord becomes the ... (1)

A) asymptote **B)** axis of symmetry

C) diameter **D)** tangent at that point ✓

> **Answer: D** — The chord through two ever-closer points becomes the tangent.
>
> - **A** — an asymptote is a line the graph approaches but never reaches
> - **B** — that is a different line altogether
> - **C** — a diameter belongs to a circle

1.7 For $f(x) = x^2$, the average gradients from $x = 3$ to $x = 3{,}1$; $3{,}01$; $3{,}001$ get closer and closer to ... (1)

A) 9 **B)** 3

C) 6 ✓ **D)** 0

> **Answer: C** — $6{,}1$; $6{,}01$; $6{,}001$ ... settle on 6, the gradient at $x = 3$.
>
> - **A** — that is $f(3)$, a height, not a gradient
> - **B** — that is the x-value
> - **D** — the curve is not flat at $x = 3$

1.8 An object's distance is $s(t) = 2t^2$ metres after t seconds. Its average speed from $t = 1$ to $t = 3$ is ... (1)

A) 16 m/s

B) **8 m/s** ✓

C) 18 m/s

D) 4 m/s

> **Answer: B** — $\frac{18 - 2}{3 - 1} = 8$ m/s.
>
> - **A** — forgot to divide by the 2 seconds
> - **C** — that is the distance at $t = 3$
> - **D** — divided by 4 instead of by 2

1.9 For $f(x) = 2^x$, the average gradient between $x = 0$ and $x = 3$ is ... (1)

A) $\frac{7}{3}$ ✓

B) $\frac{8}{3}$

C) 7

D) 2

> **Answer: A** — $\frac{8 - 1}{3 - 0} = \frac{7}{3}$.
>
> - **B** — used $f(3)$ and forgot to subtract $f(0) = 1$
> - **C** — forgot to divide by 3
> - **D** — that is the base, not a gradient

1.10 For $f(x) = x^2$, which statement is TRUE? (1)

A) The average gradient is the same over every interval

B) The average gradient is always 2

C) The average gradient from $x = -1$ to $x = 1$ is 2

D) **The average gradient from $x = 1$ to $x = 2$ is greater than from $x = 0$ to $x = 1$** ✓

> **Answer: D** — From 1 to 2 it is 3; from 0 to 1 it is 1. The parabola gets steeper.
>
> - **A** — only a straight line has that
> - **B** — it changes from interval to interval
> - **C** — the ends are at the same height, so it is 0

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	C	(D)
1.3	A	B	(C)	D
1.4	A	(B)	C	D
1.5	(A)	B	C	D
1.6	A	B	C	(D)
1.7	A	B	(C)	D
1.8	A	(B)	C	D
1.9	(A)	B	C	D
1.10	A	B	C	(D)

QUESTION 2 [12 MARKS]

Given $f(x) = -x^2 + 4x$.

2.1 Calculate $f(1)$ and $f(3)$. **(2)**

$f(1) = -1 + 4 = \mathbf{3}$ *(1)*
$f(3) = -9 + 12 = \mathbf{3}$ *(1)*

2.2 Determine the average gradient of f between $x = 1$ and $x = 3$, and explain what your answer tells you about the curve. **(3)**

$\frac{3-3}{3-1} = \mathbf{0}$ *(2)*
the curve is at the same height at both ends — it rises and then falls in between *(1)*

2.3 Determine the average gradient of f between $x = 0$ and $x = 2$. **(3)**

$f(0) = 0$ and $f(2) = -4 + 8 = 4$ *(1)*
$\frac{4-0}{2-0}$ *(1)* $= \mathbf{2}$ *(1)*

2.4 Determine the average gradient of f between $x = 1$ and $x = 1 + h$, where $h \neq 0$. Simplify your answer. **(4)**

$f(1+h) = -(1+h)^2 + 4(1+h) = 3 + 2h - h^2$ *(2)*
$\frac{(3 + 2h - h^2) - 3}{h} = \frac{2h - h^2}{h}$ *(1)*
$= \mathbf{2 - h}$ *(1)*

QUESTION 3 [11 MARKS]

A stone is thrown upwards. Its height after t seconds is $h(t) = 20t - 5t^2$ metres.

3.1 Calculate the average rate of change of the height between $t = 0$ and $t = 1$, and give its units. **(3)**

$h(0) = 0$ and $h(1) = 15$ *(1)*
$\frac{15-0}{1-0} = 15$ *(1)*
15 m/s *(1)*

3.2 Calculate the average rate of change between $t = 2$ and $t = 4$, and explain the sign of your answer. **(4)**

$h(2) = 20$ *(1)* and $h(4) = 0$ *(1)*
$\frac{0-20}{4-2} = \mathbf{-10\ m/s}$ *(1)*
negative: the stone is lower at $t = 4$ than at $t = 2$ — it is coming down *(1)*

3.3 Use the average rates of change from $t = 2$ to $t = 2{,}1$ and from $t = 2$ to $t = 2{,}01$ to estimate the rate of change at $t = 2$, and explain your estimate. **(4)**

> $h(2{,}1) = 19{,}95$, so $\frac{19{,}95 - 20}{0{,}1} = -0{,}5$ *(1)*
>
> $h(2{,}01) = 19{,}9995$, so $\frac{19{,}9995 - 20}{0{,}01} = -0{,}05$ *(1)*
>
> the values approach $\mathbf{0}$ *(1)*
>
> at $t = 2$ the stone is at the top, momentarily at rest *(1)*

TOTAL: 33 marks

This question paper consists of 3 questions.

© Danemathics — free to print and share for classroom use.