Average Gradient on a Curve (Grade 11)

MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 25

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 Determine the average gradient of $f(x) = x^2$ between $x = 2$ and $x = 5$. (2)

$f(2) = 4$, $f(5) = 25$ ✓; $(25 - 4) \div (5 - 2) = 21 \div 3 = 7$ ✓ (2)

2 Determine the average gradient of $f(x) = x^2 + 1$ between $x = -1$ and $x = 2$. (2)

$f(-1) = 2$, $f(2) = 5$ ✓; $(5 - 2) \div (2 - (-1)) = 3 \div 3 = 1$ ✓ (2)

3 Write down the formula for the average gradient of f between $x = a$ and $x = b$. (2)

$\frac{f(b) - f(a)}{b - a}$ ✓✓ (2)

4 For $f(x) = x^2$, calculate the average gradient between $x = 2$ and $x = 2{,}1$ (to two decimal places). (3)

$f(2) = 4$; $f(2{,}1) = 4{,}41$ ✓; $(4{,}41 - 4) \div 0{,}1$ ✓ $= 4{,}10$ ✓ (3)

5 Using the answer above, estimate the gradient of $y = x^2$ AT the point where $x = 2$. (2)

Approximately 4 ✓✓ (the average gradients close in on 4 as the interval shrinks) (2)

6 Explain why the gradient of a curve cannot be given as a single number, but the gradient of a straight line can. (3)

A straight line has the same steepness everywhere ✓; a curve changes steepness from point to point ✓, so a gradient can only be quoted for a stated interval or at a stated point ✓ (3)

7 The average gradient of f between $x = 0$ and $x = 4$ is 0. Does this mean the graph is a horizontal line? Explain. (3)

No ✓. It means $f(4) = f(0)$ ✓ — the graph returns to the same height, but it may have risen and fallen in between ✓ (3)

8 A ball's height is $h(t) = 30t - 5t^2$ metres. Determine its average velocity between $t = 1$ and $t = 4$ seconds. **(3)**

> $h(1) = 25$, $h(4) = 120 - 80 = 40$ ✓; $(40 - 25) \div 3$ ✓ $= 5$ m/s ✓ **(3)**

9 What does the chord between two points on a curve become as the two points slide together? **(2)**

> The tangent to the curve at that point ✓✓ **(2)**

10 For $f(x) = 2x^2$, determine the average gradient between $x = 1$ and $x = 1 + h$, and simplify your answer. **(3)**

> $f(1) = 2$; $f(1 + h) = 2(1 + h)^2 = 2 + 4h + 2h^2$ ✓; average gradient $= \frac{4h + 2h^2}{h}$ ✓ $= 4 + 2h$, $h \neq 0$ ✓ **(3)**

TOTAL: 25 marks

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