MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 $\sin(A + B) = \ldots$ (1)

A) $\sin A \cos B + \cos A \sin B$ ✓ **B)** $\cos A \cos B - \sin A \sin B$

C) $\sin A + \sin B$ **D)** $\sin A \cos B - \cos A \sin B$

Answer: A — That is the compound-angle formula on the information sheet.

- **B** — that is $\cos(A + B)$
- **C** — sine does not distribute over addition
- **D** — that is $\sin(A - B)$

1.2 $\cos(A + B) = \ldots$ (1)

A) $\cos A \cos B + \sin A \sin B$ **B)** $\cos A + \cos B$

C) $\cos A \cos B - \sin A \sin B$ ✓ **D)** $\sin A \cos B + \cos A \sin B$

Answer: C — The cosine compound formula carries the OPPOSITE sign to the one inside the bracket.

- **A** — that is $\cos(A - B)$
- **B** — cosine does not distribute over addition
- **D** — that is $\sin(A + B)$

1.3 $\sin 2\theta = \ldots$ (1)

A) $\sin\theta + \cos\theta$ **B)** $\sin^2\theta + \cos^2\theta$

C) $2\sin\theta\cos\theta$ ✓ **D)** $2\sin\theta$

Answer: C — Put $A = B = \theta$ into $\sin(A + B)$.

- **A** — the two ratios are multiplied, not added
- **B** — that expression equals 1, not $\sin 2\theta$
- **D** — doubling the ANGLE is not the same as doubling the ratio

1.4 Which is NOT one of the three forms of $\cos 2\theta$? (1)

A) $2\sin^2\theta - 1$ ✓ **B)** $\cos^2\theta - \sin^2\theta$

C) $1 - 2\sin^2\theta$ **D)** $2\cos^2\theta - 1$

> **Answer: A** — The sine form is $1 - 2\sin^2\theta$; reversing it changes the sign of the whole expression.
>
> - **B** — that is the basic form
> - **C** — that is the sine form
> - **D** — that is the cosine form

1.5 Using the double-angle formula on $30°$, $\sin 60° = \ldots$ (1)

A) $\frac{\sqrt{3}}{4}$ **B)** 1

C) $\frac{1}{2}$ **D)** $\frac{\sqrt{3}}{2}$ ✓

> **Answer: D** — $\sin 60° = 2\sin 30°\cos 30° = 2(½)\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$.
>
> - **A** — forgot the factor of 2
> - **B** — $\sin 60°$ is less than 1
> - **C** — that is $\sin 30°$, not $\sin 60°$

1.6 $\cos 2\theta$ written in terms of $\cos\theta$ only is ... (1)

A) $1 - 2\cos^2\theta$ **B)** $2\cos^2\theta + 1$

C) $\cos^2\theta - 1$ **D)** $2\cos^2\theta - 1$ ✓

> **Answer: D** — Substitute $\sin^2\theta = 1 - \cos^2\theta$ into $\cos^2\theta - \sin^2\theta$.
>
> - **A** — the signs have been reversed — that is $-\cos 2\theta$
> - **B** — the constant is -1, not $+1$
> - **C** — the coefficient of $\cos^2\theta$ is 2, not 1

1.7 If $\sin\theta = \frac{3}{5}$ and θ is acute, then $\sin 2\theta = \ldots$ (1)

A) $\frac{6}{5}$ **B)** $\frac{24}{25}$ ✓

C) $\frac{12}{25}$ **D)** $\frac{7}{25}$

> **Answer: B** — $\cos\theta = \frac{4}{5}$, so $\sin 2\theta = 2\frac{3}{5}\frac{4}{5} = \frac{24}{25}$.
>
> - **A** — doubled $\sin\theta$ instead of using the formula
> - **C** — forgot the factor of 2
> - **D** — that is $\cos 2\theta$, not $\sin 2\theta$

1.8 For that same θ, $\cos 2\theta = \ldots$ (1)

A) $\frac{7}{25}$ ✓ **B)** $\frac{16}{25}$

C) $\frac{24}{25}$ **D)** $\frac{-7}{25}$

> **Answer: A** — $\cos 2\theta = 1 - 2\sin^2\theta = 1 - \frac{18}{25} = \frac{7}{25}$.
>
> - **B** — that is $\cos^2\theta$, not $\cos 2\theta$
> - **C** — that is $\sin 2\theta$
> - **D** — $\sin\theta = 0{,}6$ is less than $\sin 45°$, so $\theta < 45°$, $2\theta < 90°$ and $\cos 2\theta$ must be POSITIVE

1.9 $\sin 75°$ written as a compound angle is ... (1)

A) $\sin 45° \cos 30° - \cos 45° \sin 30°$

B) $\sin 45° \cos 30° + \cos 45° \sin 30°$ ✓

C) $\sin 45° + \sin 30°$

D) $\cos 45° \cos 30° - \sin 45° \sin 30°$

> **Answer: B** — $75° = 45° + 30°$, and $\sin(A + B) = \sin A \cos B + \cos A \sin B$.
>
> - **A** — that is $\sin(45° - 30°) = \sin 15°$
> - **C** — sine does not distribute over addition
> - **D** — that is $\cos 75°$, not $\sin 75°$

1.10 $\cos 15° \cos 45° + \sin 15° \sin 45° = \ldots$ (1)

A) $\cos 75°$

B) $\frac{1}{2}$

C) $\frac{\sqrt{3}}{2}$ ✓

D) $\frac{\sqrt{2}}{2}$

> **Answer: C** — This is $\cos(45° - 15°) = \cos 30° = \frac{\sqrt{3}}{2}$.
>
> - **A** — the PLUS in the expansion means a MINUS inside the bracket
> - **B** — that is $\cos 60°$, which would come from $45° + 15°$
> - **D** — that is $\cos 45°$

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	(C)	D
1.3	A	B	(C)	D
1.4	(A)	B	C	D
1.5	A	B	C	(D)
1.6	A	B	C	(D)
1.7	A	(B)	C	D
1.8	(A)	B	C	D
1.9	A	(B)	C	D
1.10	A	B	(C)	D

QUESTION 2 [13 MARKS]

Answer the questions below WITHOUT the use of a calculator.

2.1 Determine the exact value of $\cos 15°$ using a compound-angle formula. (5)

$\cos 15° = \cos(45° - 30°)$ *(1)*

$= \cos 45° \cos 30° + \sin 45° \sin 30°$ *(2)*

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ *(1)*

$= \frac{\sqrt{6} + \sqrt{2}}{4}$ *(1)*

2.2 Simplify fully: $\dfrac{\sin 2A}{1+\cos 2A}$ **(4)**

> $\sin 2A = 2\sin A\cos A$ *(1)*
> $1+\cos 2A = 2\cos^2 A$ *(2)*
> $= \mathbf{\tan A}$ *(1)*
> (Valid only where $\cos A \neq 0$ — the original denominator $2\cos^2 A$ is zero there.)

2.3 Prove the identity: $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$ **(4)**

> LHS $= (\sin A\cos B + \cos A\sin B) - (\sin A\cos B - \cos A\sin B)$ *(2)*
> $= 2\cos A\sin B$ *(1)* $=$ RHS *(1)*

QUESTION 3 [12 MARKS]

Answer the questions below.

3.1 Write down the THREE forms of the identity for $\cos 2A$. **(3)**

> $\cos^2 A - \sin^2 A$ *(1)*; $2\cos^2 A - 1$ *(1)*; $1 - 2\sin^2 A$ *(1)*

3.2 If $\sin 25° = p$, express $\sin 50°$ in terms of p. **(4)**

> $\sin 50° = 2\sin 25°\cos 25°$ *(2)*
> $\cos 25° = \sqrt{1-p^2}$ *(1)*
> $= \mathbf{2p\sqrt{1-p^2}}$ *(1)*

3.3 Solve for $x \in [0° ; 360°]$: $\sin 2x = \cos x$ **(5)**

> $2\sin x\cos x - \cos x = 0$ *(1)*
> $\cos x(2\sin x - 1) = 0$ *(1)*
> $\cos x = 0$ gives $x = 90°$ or $270°$ *(1)*
> $\sin x = \dfrac{1}{2}$ gives $x = 30°$ or $150°$ *(2)*

TOTAL: 35 marks

This question paper consists of 3 questions.

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