Compound & Double Angle Identities (Grade 12)

MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 26

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 Expand cos(A − B). **(1)**

cos A cos B + sin A sin B ✓ (1)

2 Expand sin(A + B). **(1)**

sin A cos B + cos A sin B ✓ (1)

3 Determine the exact value of cos 75° without a calculator. **(3)**

$\cos(45°+30°) = \frac{\sqrt{6}-\sqrt{2}}{4}$ ✓✓✓ (3)

4 Determine the exact value of sin 15° without a calculator. **(3)**

$\sin(45°-30°) = \frac{\sqrt{6}-\sqrt{2}}{4}$ ✓✓✓ (3)

5 Write down the three forms of cos 2A. **(3)**

$\cos^2 A - \sin^2 A$, $2\cos^2 A - 1$, $1 - 2\sin^2 A$ ✓✓✓ (3)

6 If $\sin\theta = \frac{3}{5}$ and θ is acute, determine cos 2θ. **(3)**

$1 - 2(\frac{9}{25}) = \frac{7}{25}$ ✓✓✓ (3)

7 If $\sin\theta = \frac{3}{5}$ and θ is acute, determine sin 2θ. **(3)**

$\cos\theta = \frac{4}{5}$, so $2(\frac{3}{5})(\frac{4}{5}) = \frac{24}{25}$ ✓✓✓ (3)

8 Prove that $\frac{\sin 2A}{1+\cos 2A} = \tan A$, and state the restriction on A. **(5)**

> $\frac{2\sin A\cos A}{1+(2\cos^2 A-1)}$ ✓ $= \frac{2\sin A\cos A}{2\cos^2 A}$ ✓ $= \frac{\sin A}{\cos A}$ ✓ $=$ **tan A** ✓
>
> The original denominator $1+\cos 2A = 2\cos^2 A$ is zero when $\cos A = 0$, so **$\cos A \neq 0$**, i.e. $A \neq 90° + k\cdot 180°$, $k \in \mathbb{Z}$ ✓ **(5)**

9 Simplify $2\sin 15°\cos 15°$. **(2)**

> $= \sin 30° = \frac{1}{2}$ ✓✓ **(2)**

10 Why is $\sin(A+B) \neq \sin A + \sin B$? **(2)**

> Test $A = 30°$, $B = 60°$: $\sin 90° = 1$ but $\sin 30° + \sin 60° \approx 1{,}37$, so **they are not equal.** ✓✓ **(2)**

TOTAL: 26 marks

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