MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
33	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 33

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The number of ways to arrange 5 different books in a row is ... (1)

A) 120 ✓ **B)** 10

C) 5 **D)** 25

Answer: A — $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

- **B** — 5×2 is not a factorial
- **C** — gave the number of books, not the number of arrangements
- **D** — squared the 5 instead of taking its factorial

1.2 The FUNDAMENTAL COUNTING PRINCIPLE says the total number of ways is found by ... (1)

A) taking the factorial of the number of stages **B)** **multiplying the number of choices at each stage** ✓

C) adding the number of choices at each stage **D)** taking the largest number of choices

Answer: B — Every choice at one stage can pair with every choice at the next.

- **A** — the factorial applies only when ALL the items are being arranged
- **C** — adding counts alternatives, not sequences
- **D** — every stage contributes, not only the largest

1.3 A PIN has 4 digits, each from 0 to 9, and repeats ARE allowed. The number of possible PINs is ... (1)

A) 24 **B)** **10 000** ✓

C) 5 040 **D)** 40

> **Answer: B** — There are 10 choices for each of the 4 positions, giving 10^4.
>
> - **A** — that is 4!, the arrangements of four fixed digits
> - **C** — that is $10 \times 9 \times 8 \times 7$, which assumes NO repeats
> - **D** — multiplied 10 by 4 instead of raising to the power

1.4 If repeats are NOT allowed, the number of 4-digit PINs is ... (1)

A) **5 040** ✓ **B)** 40

C) 24 **D)** 10 000

> **Answer: A** — Each digit used removes one choice from the next position: $10 \times 9 \times 8 \times 7$.
>
> - **B** — multiplied 10 by 4
> - **C** — that is 4!, ignoring the ten available digits
> - **D** — that allows repeats

1.5 The number of ways to arrange the letters of the word LEVEL is ... (1)

A) 60 **B)** 120

C) 20 **D)** **30** ✓

> **Answer: D** — Five letters with L repeated twice and E repeated twice: $5! \div (2! \times 2!) = 30$.
>
> - **A** — divided by only one of the two repeats
> - **B** — treated all five letters as different
> - **C** — divided by 3! instead of by $2! \times 2!$

1.6 Five people sit in a row, but two particular people must sit TOGETHER. The number of arrangements is ... (1)

A) 240 **B)** 120

C) **48** ✓ **D)** 24

> **Answer: C** — Treat the pair as one block: 4! ways to arrange the blocks, times 2! within the pair.
>
> - **A** — multiplied by 10 instead of by 2
> - **B** — that is the number with no restriction at all
> - **D** — forgot to let the pair swap places

1.7 Four DIFFERENT letters are arranged at random. The probability of one particular arrangement is ... (1)

A) $\frac{1}{4}$ **B)** $\frac{1}{16}$

C) $\mathbf{\frac{1}{24}}$ ✓ **D)** $\frac{1}{12}$

> **Answer: C** — There are $4! = 24$ equally likely arrangements.
>
> - **A** — that is $1 \div$ the number of letters, not of arrangements
> - **B** — that is $1 \div 4^2$, which would allow repeats
> - **D** — that halves the correct count

1.8 Three coins are tossed and a die is rolled. The sample space has ... (1)

A) 36 outcomes
B) 12 outcomes
C) 18 outcomes
D) 48 outcomes ✓

> **Answer: D** — $2 \times 2 \times 2 \times 6 = 48$.
> - **A** — used 6×6 instead of $2^3 \times 6$
> - **B** — used 2×6 and ignored two of the coins
> - **C** — ADDED the three coins' outcomes to get 3, then multiplied by 6

1.9 In how many ways can a chairperson and a secretary be chosen from 8 people? (1)

A) 40 320
B) 28
C) 64
D) 56 ✓

> **Answer: D** — There are 8 choices for the chair and then 7 for the secretary.
> - **A** — that is $8!$, which arranges all eight people
> - **B** — that is $8 \times 7 \div 2$, which would apply if the two posts were identical
> - **C** — allowed the same person to hold both posts

1.10 Two events are INDEPENDENT when ... (1)

A) P(A) = P(B)
B) P(A and B) = P(A) × P(B) ✓
C) P(A or B) = P(A) + P(B)
D) P(A and B) = 0

> **Answer: B** — One event happening leaves the other's probability unchanged.
> - **A** — equal probabilities say nothing about independence
> - **C** — that also describes mutually exclusive events
> - **D** — that is MUTUALLY EXCLUSIVE

ANSWER GRID — MARKING GUIDELINE

1.1	**(A)**	B	C	D
1.2	A	**(B)**	C	D
1.3	A	**(B)**	C	D
1.4	**(A)**	B	C	D
1.5	A	B	C	**(D)**
1.6	A	B	**(C)**	D
1.7	A	B	**(C)**	D
1.8	A	B	C	**(D)**
1.9	A	B	C	**(D)**
1.10	A	**(B)**	C	D

QUESTION 2 [11 MARKS]

The letters of the word **NUMBERS** are arranged in a row. The word has 7 different letters.

2.1 Determine the number of different arrangements possible, and how many of them begin with the letter N. (3)

> $7!$ *(1)*
> $= \mathbf{5\,040}$ *(1)*
> Fixing N in the first position leaves 6 letters to arrange: $6! = \mathbf{720}$ *(1)*

2.2 Determine the number of arrangements in which the letters N and U are next to each other. **(4)**

> Treat NU as one item, leaving 6 items: $6!$ *(2)*
> N and U can swap: $\times 2$ *(1)*
> $= \mathbf{1\,440}$ *(1)*

2.3 Hence determine the probability that in a random arrangement N and U are NOT next to each other. **(4)**

> Not together $= 5\,040 - 1\,440 = 3\,600$ *(2)*
> $P = \frac{3\,600}{5\,040}$ *(1)* $= \mathbf{\frac{5}{7}}$ *(1)*

QUESTION 3 [12 MARKS]

A code consists of 4 digits chosen from 0 to 9.

3.1 Determine how many codes are possible if digits may be repeated, and how many of those begin with an EVEN digit. **(3)**

> 10^4 *(1)*
> $= \mathbf{10\,000}$ *(1)*
> There are 5 even digits $(0; 2; 4; 6; 8)$ for the first position, so $5 \times 10^3 = \mathbf{5\,000}$ *(1)*

3.2 Determine how many codes are possible if no digit may be repeated. **(3)**

> $10 \times 9 \times 8 \times 7$ *(2)* $= \mathbf{5\,040}$ *(1)*

3.3 Determine the probability that a randomly chosen 4-digit code has no repeated digit. **(3)**

> $\frac{5\,040}{10\,000}$ *(2)* $= \mathbf{0{,}504}$ *(1)*

3.4 Explain why the answer to the previous part would be smaller for a 5-digit code, and calculate that probability. **(3)**

> Each extra position has FEWER digits still available, so the fraction of codes with no repeat keeps shrinking *(1)*
> $P = \frac{10 \times 9 \times 8 \times 7 \times 6}{10^5} = \frac{30\,240}{100\,000}$ *(1)*
> $= \mathbf{0{,}3024}$, which is indeed less than $0{,}504$ *(1)*

TOTAL: 33 marks

This question paper consists of 3 questions.

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