∑ DANEMATHICS
FREE CAPS MATHS RESOURCES

GRADE 12 · CALCULUS
Sketching Cubic Graphs (Grade 12)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
32	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 32

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 At a STATIONARY point of a cubic ... (1)

A) $f(x) = 0$ **B)** $x = 0$

C) $\boldsymbol{f'(x) = 0}$ ✓ **D)** $f''(x) = 0$

Answer: C — A stationary point is where the tangent is horizontal.

- **A** — that is an x-INTERCEPT
- **B** — $x = 0$ is on the y-axis, which has nothing to do with it
- **D** — that is a point of INFLECTION

1.2 $h(x) = ax^3 + bx^2$ has turning points at $(0\ ;\ 0)$ and $(4\ ;\ 32)$. Then ... (1)

A) $\boldsymbol{a = -1}$ **and** $\boldsymbol{b = 6}$ ✓ **B)** $a = -1$ and $b = -6$

C) $a = 6$ and $b = -1$ **D)** $a = 1$ and $b = -6$

Answer: A — $h'(x) = 3ax^2 + 2bx$ is zero at $x = 4$, and $h(4) = 32$; solving both gives $a = -1$ and $b = 6$.

- **B** — b must be positive for $(4\ ;\ 32)$ to be a maximum
- **C** — the values of a and b have been swapped
- **D** — every sign has been reversed, which would put the turning point below the axis

1.3 For $h(x) = -x^3 + 6x^2$, the x-intercept other than the origin is at ... (1)

A) $x = 4$ **B)** $\boldsymbol{x = 6}$ ✓

C) $x = 32$ **D)** $x = -6$

Answer: B — $-x^3 + 6x^2 = x^2(6 - x)$, which is zero at $x = 0$ and at $x = 6$.

- **A** — $x = 4$ is a TURNING point, not an x-intercept
- **C** — 32 is the y-value at the turning point, not an x-intercept
- **D** — $6 - x = 0$ gives $x = +6$

1.4 For $h(x) = -x^3 + 6x^2$, h is INCREASING for ... (1)

A) $x < 0$ or $x > 4$ **B)** $x < 0$

C) $\mathbf{0 < x < 4}$ ✓ **D)** $x > 4$

> **Answer: C** — $h'(x) = 3x(4 - x)$, which is positive between its roots 0 and 4.
> - **A** — that is where h is DECREASING
> - **B** — below $x = 0$ the graph falls
> - **D** — beyond $x = 4$ the graph falls

1.5 For $h(x) = -x^3 + 6x^2$, h is CONCAVE DOWN for ... (1)

A) $x > 4$ **B)** $x < 2$

C) $0 < x < 4$ **D)** $\mathbf{x > 2}$ ✓

> **Answer: D** — $h''(x) = -6x + 12$, which is negative when $x > 2$.
> - **A** — 4 is a turning point, not the point of inflection
> - **B** — that is where h is concave UP
> - **C** — that is where h is INCREASING, which is a different question

1.6 The POINT OF INFLECTION of a cubic is where ... (1)

A) $f(x) = 0$ **B)** the graph cuts the y-axis

C) $\mathbf{f''(x) = 0}$ ✓ **D)** $f'(x) = 0$

> **Answer: C** — The concavity changes there, so the second derivative is zero.
> - **A** — that is an x-intercept
> - **B** — the y-intercept has nothing to do with concavity
> - **D** — that is a STATIONARY point

1.7 A cubic whose coefficient of x^3 is POSITIVE ... (1)

A) rises to the right and falls to the left ✓ **B)** has no turning points

C) is increasing everywhere **D)** falls to the right and rises to the left

> **Answer: A** — With $a > 0$ the graph comes up from below on the left and heads upward on the right.
> - **B** — a cubic may have two turning points, or none
> - **C** — a cubic with two turning points decreases between them
> - **D** — that describes a cubic with a NEGATIVE coefficient

1.8 How many turning points can a cubic graph have? (1)

A) at most three **B)** exactly one

C) exactly two **D) at most two** ✓

> **Answer: D** — $f'(x)$ is a quadratic, so it has at most two roots.
> - **A** — f' is a quadratic and cannot have three roots
> - **B** — exactly one would need f' to have a single root, which is the inflection case
> - **C** — $y = x^3$ has none at all

1.9 For $f(x) = x^3 - 3x^2$, the LOCAL MINIMUM is at ... (1)

A) $(2\ ;\ 4)$ **B) $(2\ ;\ -4)$ ✓**

C) $(0\ ;\ 0)$ **D)** $(-2\ ;\ -20)$

> **Answer: B** — $f'(x) = 3x(x-2)$ is zero at $x = 0$ and $x = 2$, and $f''(2) = 6 > 0$ makes $x = 2$ the minimum.
>
> - **A** — a sign slip: $8 - 12 = -4$, not $+4$
> - **C** — that is the local MAXIMUM, where $f''(0) = -6 < 0$
> - **D** — $x = -2$ is not a stationary point at all

1.10 To decide whether a stationary point is a maximum or a minimum, you can ... (1)

A) test the sign of $f(x)$ at that point **B)** count the x-intercepts

C) check whether x is positive **D) test the sign of $f'(x)$ at that point ✓**

> **Answer: D** — A positive second derivative means concave up, and hence a minimum.
>
> - **A** — the VALUE of f says nothing about the shape there
> - **B** — the number of intercepts does not identify a turning point's nature
> - **C** — the sign of x is irrelevant

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	**A**	B	C	D
1.3	A	**B**	C	D
1.4	A	B	**C**	D
1.5	A	B	C	**D**
1.6	A	B	**C**	D
1.7	**A**	B	C	D
1.8	A	B	C	**D**
1.9	A	**B**	C	D
1.10	A	B	C	**D**

QUESTION 2 [12 MARKS]

Given $f(x) = -x^3 + 6x^2$.

2.1 Calculate the coordinates of the x-intercepts of f. (4)

> $x^2(6 - x) = 0$ *(2)*
> $x = 0$ (a double root) or $x = 6$ *(1)*
> **(0 ; 0) and (6 ; 0)** *(1)*

2.2 Determine the coordinates of the stationary points of f. (5)

> $f'(x) = -3x^2 + 12x$ *(1)*
> $-3x(x - 4) = 0$ *(2)*
> $x = 0$ gives $(0\ ;\ 0)$ *(1)*; $x = 4$ gives **(4 ; 32)** *(1)*

2.3 Determine the coordinates of the point of inflection. (3)

> $f''(x) = -6x + 12 = 0$ *(1)*
> $x = 2$ *(1)*
> $f(2) = 16$, so **(2 ; 16)** *(1)*

QUESTION 3 [10 MARKS]

Answer the questions below about the same graph.

3.1 Write down the values of x for which f is concave up. **(2)**

> $f''(x) = -6x + 12 > 0$ *(1)*
> $\boldsymbol{x < 2}$ *(1)*

3.2 Determine the equation of the tangent to f at $x = 1$. **(4)**

> $f'(1) = -3 + 12 = 9$ *(1)*
> $f(1) = 5$, so the point is $(1\ ;\ 5)$ *(1)*
> $5 = 9(1) + c$ gives $c = -4$ *(1)*
> $\boldsymbol{y = 9x - 4}$ *(1)*

3.3 For which values of k will $-x^3 + 6x^2 = k$ have THREE distinct real roots? Justify your answer. **(4)**

> The horizontal line $y = k$ must cut the graph three times *(1)*
> That happens strictly between the two turning-point values *(1)*
> $\boldsymbol{0 < k < 32}$ *(2)*

TOTAL: 32 marks

This question paper consists of 3 questions.

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