MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 If $y = 4x^5 - 6x^4 + 3x$, then $\frac{dy}{dx} = \ldots$ (1)

A) $20x^4 - 24x^3 + 3x$

B) $20x^4 - 24x^3 + 3$ ✓

C) $20x^5 - 24x^4 + 3$

D) $4x^4 - 6x^3 + 3$

Answer: B — Multiply by the power, then reduce the power by one.

- **A** — the derivative of $3x$ is 3, not $3x$
- **C** — the powers must DROP by one, not stay the same
- **D** — the coefficients must be multiplied by the old power

1.2 The derivative of a CONSTANT is ... (1)

A) undefined

B) 0 ✓

C) the constant itself

D) 1

Answer: B — A constant function has a horizontal graph, whose gradient is zero.

- **A** — constants are perfectly differentiable
- **C** — a constant does not change, so its rate of change is zero
- **D** — that is the derivative of x

1.3 If $f(x) = \sqrt{x}$, then $f'(x) = \ldots$ (1)

A) $\frac{1}{\sqrt{x}}$ **B)** $2\sqrt{x}$

C) $\frac{1}{2x}$ **D)** $\frac{1}{2\sqrt{x}}$ ✓

> **Answer: D** — Write $\sqrt{x}$ as $x^{½}$; its derivative is $½x^{-½} = 1 \div (2\sqrt{x})$.
> - **A** — forgot the factor of ½ that the power brings down
> - **B** — differentiated as though the power were increasing
> - **C** — reduced the power by one but dropped the square root

1.4 If $y = \frac{3}{x}$, then $\frac{dy}{dx} = \ldots$ (1)

A) $\frac{-3}{x^2}$ ✓ **B)** $\frac{-3}{x}$

C) $-3x$ **D)** $\frac{3}{x^2}$

> **Answer: A** — Write it as $3x^{-1}$; the derivative is $-3x^{-2}$.
> - **B** — the power must drop from -1 to -2
> - **C** — the power drops; it does not become $+1$
> - **D** — the power of -1 brings down a MINUS

1.5 Before differentiating $(3x + 2)(x - 1)$, you should ... (1)

A) differentiate each bracket and multiply the results **B) expand the brackets first** ✓

C) differentiate the first bracket only **D)** take logarithms

> **Answer: B** — The Grade 12 syllabus has no product rule, so the expression is expanded into a sum of powers first.
> - **A** — differentiating each factor and multiplying does NOT give the derivative of a product
> - **C** — both factors depend on x, so neither may be left alone
> - **D** — logarithms are for exponential equations, not for this

1.6 If $f(x) = x^3 - 3x$, then $f'(2) = \ldots$ (1)

A) 9 ✓ **B)** 2

C) 12 **D)** 6

> **Answer: A** — $f'(x) = 3x^2 - 3$, so $f'(2) = 12 - 3 = 9$.
> - **B** — substituted into $f(x)$ instead of $f'(x)$: $8 - 6 = 2$
> - **C** — forgot to subtract the 3
> - **D** — differentiated $3x$ to $3x^2$ instead of to 3

1.7 D_x notation means ... (1)

A) the value of the function at x **B)** the domain of the function

C) differentiate with respect to x ✓ **D)** the discriminant

> **Answer: C** — D_x, $\frac{dy}{dx}$ and $f'(x)$ all mean the same operation.
> - **A** — that is $f(x)$
> - **B** — the domain is the set of allowed inputs, not an operation
> - **D** — the discriminant belongs to a quadratic equation

1.8 The gradient of the TANGENT to $y = x^2$ at $x = 3$ is ... (1)

A) 3 **B)** 9

C) 2 **D) 6 ✓**

> **Answer: D** — $\frac{dy}{dx} = 2x$, so at $x = 3$ the gradient is 6.
>
> - **A** — that is the x-value itself
> - **B** — that is the y-VALUE at $x = 3$, not the gradient
> - **C** — that is the coefficient in the derivative, before substituting

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1.9 If $f'(a) = 0$, the graph of f has ... (1)

A) a point of inflection at $x = a$ **B)** an asymptote at $x = a$

C) an x-intercept at $x = a$ **D) a stationary point at $x = a$ ✓**

> **Answer: D** — A zero gradient means the tangent there is horizontal.
>
> - **A** — $f'(a) = 0$ does not by itself make $x = a$ a point of inflection — that is where the concavity CHANGES, which you check with the second derivative
> - **B** — an asymptote is where the function is undefined, not where its gradient is zero
> - **C** — an x-intercept needs $f(a) = 0$, not $f'(a) = 0$

1.10 If $y = (2x)^3$, then $\frac{dy}{dx} = ...$ (1)

A) $3(2x)^2$ **B)** $8x^2$

C) $24x^2$ ✓ **D)** $6x^2$

> **Answer: C** — $(2x)^3 = 8x^3$, whose derivative is $24x^2$.
>
> - **A** — left the bracket unexpanded, which the Grade 12 rules do not allow
> - **B** — differentiated the 8 but not the x^3
> - **D** — cubed only the x, leaving the 2 outside

ANSWER GRID — MARKING GUIDELINE

1.1	A	(B)	C	D
1.2	A	(B)	C	D
1.3	A	B	C	(D)
1.4	(A)	B	C	D
1.5	A	(B)	C	D
1.6	(A)	B	C	D
1.7	A	B	(C)	D
1.8	A	B	C	(D)
1.9	A	B	C	(D)
1.10	A	B	(C)	D

QUESTION 2 [13 MARKS]

Answer the questions below. Show ALL your working.

2.1 Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = 2x^2 - 3x$. **(5)**

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ *(1)*

$= \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$ *(1)*

$= \lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h}$ *(1)*

$= \lim_{h\to 0} (4x + 2h - 3)$ *(1)*

$= \mathbf{4x - 3}$ *(1)*

2.2 Determine $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$. **(3)**

$\mathbf{\frac{dy}{dx} = 20x^4 - 24x^3 + 3}$ *(3)*

2.3 Determine $D_x[\,(\frac{3x}{2} - \frac{1}{3x})^2\,]$. **(5)**

Expand FIRST — you may not differentiate a bracket term by term *(1)*

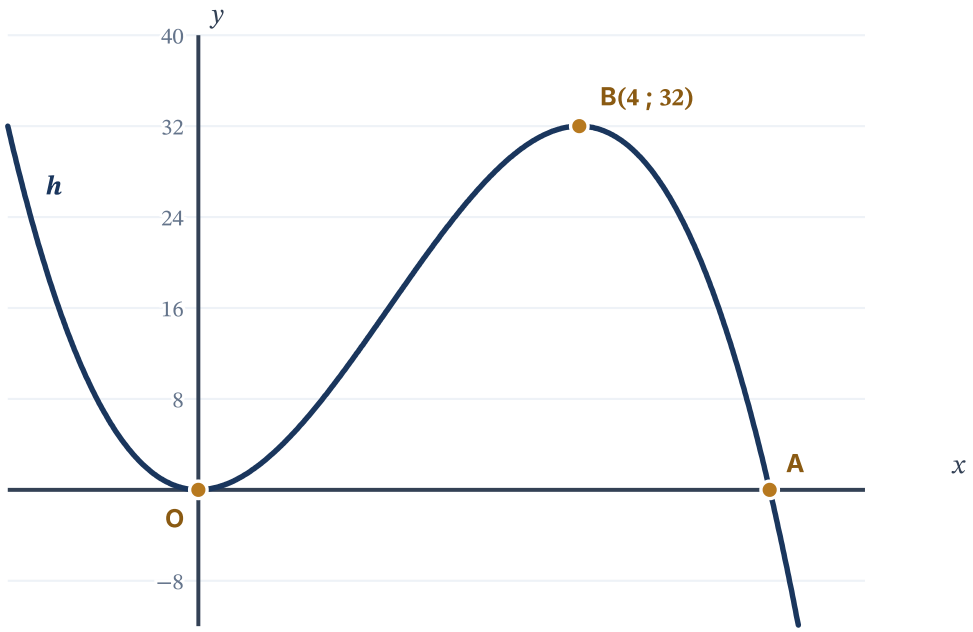
$= \frac{9x^2}{4} - 1 + \frac{1}{9x^2}$ *(2)*

Write the last term as $\frac{1}{9}x^{-2}$ *(1)*

$= \mathbf{\frac{9x}{2} - \frac{2}{9x^3}}$ *(1)*

QUESTION 3 [12 MARKS]

The graph of $h(x) = ax^3 + bx^2$ has turning points at the origin O(0 ; 0) and at B(4 ; 32). A is the other x-intercept of h.



3.1 Show that $a = -1$ and $b = 6$. **(5)**

$h'(x) = 3ax^2 + 2bx$ *(1)*
$h'(4) = 0$: $48a + 8b = 0$ *(1)*
$h(4) = 32$: $64a + 16b = 32$ *(1)*
From the first, $b = -6a$ *(1)*
$64a - 96a = 32$ gives $a = -1$, so $b = 6$ *(1)*

3.2 Calculate the coordinates of A. **(3)**

$-x^3 + 6x^2 = 0$ *(1)*
$x^2(6 - x) = 0$ *(1)*
A(6 ; 0) *(1)*

3.3 Write down the values of x for which h is INCREASING. **(2)**

h rises between its turning points *(1)*
$\mathbf{0 < x < 4}$ *(1)*

3.4 Write down the values of x for which h is CONCAVE DOWN. **(2)**

$h''(x) = -6x + 12 < 0$ *(1)*
$\mathbf{x > 2}$ *(1)*

TOTAL: 35 marks

This question paper consists of 3 questions.

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