Differentiation: The Rules and How Exams Use Them

MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 37

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 Differentiate $y = x^4$. *(power rule)* (1)

$\mathbf{4x^3}$ ✓ (1)

2 Differentiate $y = 5x^3 - 2x^2 + 7$. *(polynomial)* (2)

$\mathbf{15x^2 - 4x}$ ✓✓ (2)

3 Differentiate $y = 2x^2 - 3x + 9$. *(polynomial)* (2)

$\mathbf{4x - 3}$ ✓✓ (2)

4 Differentiate $y = \frac{3}{x}$ (write as $3x^{-1}$). *(rewrite)* (2)

$-3x^{-2} = \mathbf{\frac{-3}{x^2}}$ ✓✓ (2)

5 Differentiate $y = \sqrt{x}$ (write as $x^\wedge\frac{1}{2}$). *(rewrite)* (2)

$\frac{1}{2}x^\wedge(-\frac{1}{2}) = \mathbf{\frac{1}{2\sqrt{x}}}$ ✓✓ (2)

6 From first principles, differentiate $f(x) = 3x$. *(first principles)* (4)

$f'(x) = \lim_{h\to 0}\frac{3(x+h)-3x}{h} = \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = \mathbf{3}$ ✓✓✓✓ (4)

7 Gradient of $y = x^2 + 2x$ at $x = 1$. *(gradient)* (2)

$2x + 2$ at $x{=}1 = \mathbf{4}$ ✓✓ (2)

8 Gradient of $y = x^3$ at $x = 2$. *(gradient)* (2)

$3x^2$ at $x{=}2 = \mathbf{12}$ ✓✓ (2)

9 Find x where $y = x^2 - 6x$ has a turning point. *(turning point)* (2)

$2x - 6 = 0 \rightarrow x = \mathbf{3}$ ✓✓ (2)

10 Find the turning points of $y = x^3 - 12x$. *(turning point)* (4)

$3x^2 - 12 = 0 \rightarrow x = \pm 2 \rightarrow$ **(2 ; −16) and (−2 ; 16)** ✓✓✓✓ (4)

11 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 3x$. (5)

$f(x+h) = 2(x+h)^2 - 3(x+h) = 2x^2 + 4xh + 2h^2 - 3x - 3h$ ✓; $f(x+h) - f(x) = 4xh + 2h^2 - 3h$ ✓; $\div h$: $4x + 2h - 3$ ✓; lim as $h \rightarrow 0$ ✓; $f'(x) = 4x - 3$ ✓ (5)

12 Determine $f'(x)$ from first principles if $f(x) = \frac{3}{x}$. (5)

$$f'(x) = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \checkmark$$

$$= \lim_{h\to 0} \frac{3x - 3(x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-3h}{hx(x+h)} \checkmark\checkmark$$

$$= \lim_{h\to 0} \frac{-3}{x(x+h)} \checkmark$$

$$= -\frac{3}{x^2} \checkmark \text{ (5)}$$

13 Use first principles to show that the derivative of $f(x) = 7$ is 0. (2)

$f(x+h) - f(x) = 7 - 7 = 0$ ✓; $0 \div h = 0$, so $f'(x) = 0$ ✓ (2)

14 Explain why you may NOT substitute $h = 0$ at the start of a first-principles question. (2)

It gives $0 \div 0$, which is undefined ✓; the h in the denominator must be cancelled by simplifying first ✓ (2)

TOTAL: 37 marks

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