MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
32	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 32

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 A circle with centre $(a\ ;\ b)$ and radius r has equation ... (1)

A) $(x-a)^2+(y-b)^2=r^2$ ✓ **B)** $(x+a)^2+(y+b)^2=r^2$

C) $(x-a)^2+(y-b)^2=r$ **D)** $(x-a)^2-(y-b)^2=r^2$

Answer: A — That is the formula on the information sheet.

- **B** — the signs inside the brackets are the OPPOSITE of the centre's coordinates
- **C** — the right-hand side is r SQUARED
- **D** — the two squares are ADDED, not subtracted

1.2 The centre of $(x-3)^2+(y+2)^2=25$ is ... (1)

A) $(-3\ ;\ 2)$ **B)** $(3\ ;\ 2)$

C) $(-3\ ;\ -2)$ **D)** $(3\ ;\ -2)$ ✓

Answer: D — The centre's coordinates are the OPPOSITE of the signs inside the brackets.

- **A** — both signs have been read the wrong way round
- **B** — the bracket reads $(y+2)$, so $b=-2$
- **C** — the bracket reads $(x-3)$, so $a=+3$

1.3 The RADIUS of $(x-3)^2+(y+2)^2=25$ is ... (1)

A) 25 **B)** $\sqrt{5}$

C) 5 ✓ **D)** $12{,}5$

Answer: C — $r^2=25$, so $r=5$.

- **A** — that is r^2, before the square root
- **B** — square-rooted 5 instead of 25
- **D** — halved 25 instead of square-rooting it

1.4 $x^2 + y^2 - 6x + 4y - 12 = 0$ has centre ... (1)

A) $(6\ ;\ -4)$ **B)** $(3\ ;\ -2)$ ✓

C) $(-3\ ;\ 2)$ **D)** $(-6\ ;\ 4)$

> **Answer: B** — Completing the square gives $(x-3)^2 + (y+2)^2 = 25$.
> - **A** — reversed the signs but did not halve the coefficients
> - **C** — halved the coefficients but did not reverse their signs
> - **D** — read the coefficients straight off instead of halving them and reversing the signs

1.5 The radius of $x^2 + y^2 - 6x + 4y - 12 = 0$ is ... (1)

A) 5 ✓ **B)** $\sqrt{12}$

C) 25 **D)** 12

> **Answer: A** — $r^2 = 9 + 4 + 12 = 25$, so $r = 5$.
> - **B** — square-rooted the constant term only
> - **C** — that is r^2, before the square root
> - **D** — used the constant 12 without adding the completed squares

1.6 A tangent to a circle at a point P is PERPENDICULAR to ... (1)

A) any chord through P **B)** the x-axis

C) **the radius drawn to P** ✓ **D)** every diameter of the circle

> **Answer: C** — That is what makes the tangent's gradient the negative reciprocal of the radius's gradient.
> - **A** — a chord through P is not generally perpendicular to the tangent
> - **B** — the tangent's direction depends on where P lies
> - **D** — only the diameter THROUGH P qualifies, and that is the radius extended

1.7 A circle has centre C$(2\ ;\ 1)$ and P$(5\ ;\ 5)$ lies on it. Its radius is ... (1)

A) 25 **B)** 7

C) 5 ✓ **D)** $\sqrt{7}$

> **Answer: C** — $r = \sqrt{(5-2)^2 + (5-1)^2} = \sqrt{25} = 5$.
> - **A** — forgot to take the square root
> - **B** — ADDED the two differences instead of squaring and rooting
> - **D** — subtracted the squares instead of adding them

1.8 The gradient of the radius from C$(2\ ;\ 1)$ to P$(5\ ;\ 5)$ is ... (1)

A) $\frac{4}{3}$ ✓ **B)** $\frac{3}{4}$

C) $\frac{-4}{3}$ **D)** $\frac{-3}{4}$

> **Answer: A** — $m = (5-1) \div (5-2) = \frac{4}{3}$.
> - **B** — inverted the formula
> - **C** — changed the sign as well as inverting
> - **D** — that is the TANGENT's gradient

1.9 So the gradient of the TANGENT at P(5 ; 5) is ... (1)

A) $\frac{4}{3}$ **B)** $\frac{-4}{3}$

C) $\frac{3}{4}$ **D)** $\frac{-3}{4}$ ✓

Answer: D — The tangent is perpendicular to the radius, so its gradient is the negative reciprocal of $\frac{4}{3}$.

- **A** — that is the RADIUS's gradient
- **B** — changed the sign but did not take the reciprocal
- **C** — took the reciprocal but left off the minus

1.10 The equation of that tangent at P(5 ; 5) is ... (1)

A) $y = -3x + 5$ **B)** $4y = -3x + 35$ ✓

C) $3y = -4x + 35$ **D)** $4y = 3x + 5$

Answer: B — $y - 5 = -¾(x - 5)$ gives $4y - 20 = -3x + 15$, so $4y = -3x + 35$.

- **A** — used -3 instead of $-\frac{3}{4}$ for the gradient
- **C** — used $-\frac{4}{3}$ instead of $-\frac{3}{4}$
- **D** — used the radius's gradient instead of the tangent's

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	C	(D)
1.3	A	B	(C)	D
1.4	A	(B)	C	D
1.5	(A)	B	C	D
1.6	A	B	(C)	D
1.7	A	B	(C)	D
1.8	(A)	B	C	D
1.9	A	B	C	(D)
1.10	A	(B)	C	D

QUESTION 2 [12 MARKS]

A circle has the equation $x^2 + y^2 - 4x + 6y - 12 = 0$.

2.1 Determine the coordinates of the centre and the length of the radius by completing the square. (5)

$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$ *(3)*
$(x - 2)^2 + (y + 3)^2 = 25$ *(1)*
Centre **(2 ; −3)**, radius **5** *(1)*

2.2 Show that the point (5 ; 1) lies on the circle. (2)

$(5 - 2)^2 + (1 + 3)^2 = 9 + 16$ *(1)* $= 25$ ✓ *(1)*

2.3 Determine the equation of the tangent to the circle at $(5\ ;\ 1)$. **(5)**

$m_{\text{radius}} = \frac{1-(-3)}{5-2} = \frac{4}{3}$ *(2)*

The tangent is perpendicular: $m = -\frac{3}{4}$ *(1)*

$1 = -\frac{3}{4}(5) + c$ gives $c = \frac{19}{4}$ *(1)*

$\boldsymbol{y = -\frac{3}{4}x + \frac{19}{4}}$ *(1)*

QUESTION 3 [10 MARKS]

Answer the questions below about the same circle.

3.1 Determine the coordinates of the point on the circle DIAMETRICALLY opposite $(5\ ;\ 1)$. **(3)**

The centre is the midpoint of the diameter *(1)*

$(2\ ;\ -3) = (\frac{5+x}{2}\ ;\ \frac{1+y}{2})$ *(1)*

$\boldsymbol{(-1\ ;\ -7)}$ *(1)*

3.2 Determine whether the point $(6\ ;\ 1)$ lies inside, on, or outside the circle. Justify with a calculation. **(4)**

$(6-2)^2 + (1+3)^2 = 16 + 16 = 32$ *(2)*

$32 > 25 = r^2$ *(1)*

The point lies **OUTSIDE** the circle *(1)*

3.3 Write down the equation of the circle with the same centre but twice the radius. **(3)**

$r = 10$, so $r^2 = 100$ *(1)*

$\boldsymbol{(x-2)^2 + (y+3)^2 = 100}$ *(2)*

TOTAL: 32 marks

This question paper consists of 3 questions.

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