Exponent Laws & Exponential Equations (Grade 11)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
36	55 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 36

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 $\frac{2^{x+1}}{2^{x-1}} = \ldots$ (1)

A) 4 ✓ **B)** 2

C) 2^{2x} **D)** 1

Answer: A — Dividing SUBTRACTS the exponents: $(x+1)-(x-1)=2$, so the answer is 2^2.

- **B** — subtracted to an exponent of 1 instead of 2
- **C** — ADDED the exponents instead of subtracting them
- **D** — assumed the x's cancel to leave 2^0

1.2 If $3^{2x} = 27$, then $x = \ldots$ (1)

A) 3 **B) 1,5 ✓**

C) 0,5 **D)** 9

Answer: B — $27 = 3^3$, so $2x = 3$ and $x = 1{,}5$.

- **A** — solved $2x = 3$ but never divided by the 2
- **C** — wrote $2x = 1$ instead of $2x = 3$
- **D** — divided 27 by 3 instead of matching the bases

1.3 $\frac{9^x}{3^{x-1}} = \ldots$ (1)

A) 3^{2x} **B)** 3^{x+1} ✓

C) 3^{x-1} **D)** 6^{x+1}

Answer: B — Write 9^x as 3^{2x}; then $2x - (x-1) = x + 1$.

- **A** — never divided by the denominator at all
- **C** — dropped the sign on the -1 when subtracting the exponents
- **D** — changed the BASE instead of the exponent

1.4 Solve for x: $2^{2x} - 5(2^x) + 4 = 0$ (1)

A) $x = 0$ or $x = 2$ ✓ **B)** $x = 2$ only

C) $x = 5$ or $x = 4$ **D)** $x = 1$ or $x = 4$

Answer: A — Let $k = 2^x$; then $k^2 - 5k + 4 = 0$ gives $k = 1$ or 4, so $2^x = 1$ or $2^x = 4$.

- **B** — found only one of the two solutions
- **C** — read off the coefficients instead of solving
- **D** — gave the values of $k = 2^x$ instead of the values of x

1.5 $16^{\frac{3}{4}} = \ldots$ (1)

A) 2 **B)** 64

C) 8 ✓ **D)** 12

Answer: C — The fourth root of 16 is 2, and $2^3 = 8$.

- **A** — took the fourth root but never cubed it
- **B** — cubed 4 instead of cubing the fourth root
- **D** — multiplied 16 by $\frac{3}{4}$ — an exponent is not a multiplier

1.6 $\frac{5^{x+2}}{5^x} = \ldots$ (1)

A) 5 **B)** 5^{2x+2}

C) 10 **D)** 25 ✓

Answer: D — $(x + 2) - x = 2$, so the answer is 5^2.

- **A** — subtracted to an exponent of 1 instead of 2
- **B** — ADDED the exponents instead of subtracting them
- **C** — multiplied the base by 2 instead of squaring it

1.7 Solve for x: $3^{x+1} = 9^{x-1}$ (1)

A) $x = 2$ **B)** $x = 1$

C) $x = 3$ ✓ **D)** $x = -3$

Answer: C — $9^{x-1} = 3^{2x-2}$, so $x + 1 = 2x - 2$ and $x = 3$.

- **A** — doubled the x but not the -1: solved $x + 1 = 2x - 1$
- **B** — dropped the -2: solved $x + 1 = 2x$
- **D** — moved the terms across without changing their signs

1.8 $\dfrac{2^x \times 4^x}{8^x} = \ldots$ (1)

A) 1 ✓ **B)** 0

C) 2^{3x} **D)** 2^x

> **Answer: A** — In base 2 the exponents give $x + 2x - 3x = 0$, and $2^0 = 1$.
> - **B** — took 2^0 as 0 instead of 1
> - **C** — added all three exponents instead of subtracting the denominator's
> - **D** — left one x over after subtracting

1.9 If $2^x = 5$, then $2^{x+3} = \ldots$ (1)

A) 15 **B)** 125

C) 8 **D)** 40 ✓

> **Answer: D** — $2^{x+3} = 2^x \times 2^3 = 5 \times 8 = 40$.
> - **A** — multiplied 5 by 3 instead of by 2^3
> - **B** — cubed the 5 instead of multiplying it by 2^3
> - **C** — gave 2^3 on its own and forgot the factor of 5

1.10 For $x > 0$, $\sqrt{x} \times x^{½} = \ldots$ (1)

A) x^2 **B)** $x^{\frac{1}{4}}$

C) $\sqrt{x}$ **D)** x ✓

> **Answer: D** — $\sqrt{x} = x^{½}$, and $½ + ½ = 1$.
> - **A** — added the exponents as though each were 1
> - **B** — MULTIPLIED the exponents instead of adding them
> - **C** — gave one of the two factors instead of their product

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	(B)	C	D
1.3	A	(B)	C	D
1.4	(A)	B	C	D
1.5	A	B	(C)	D
1.6	A	B	C	(D)
1.7	A	B	(C)	D
1.8	(A)	B	C	D
1.9	A	B	C	(D)
1.10	A	B	C	(D)

QUESTION 2 [12 MARKS]

Simplify the following WITHOUT the use of a calculator.

2.1 $(\frac{8}{27})^{-\frac{2}{3}}$ **(4)**

The negative exponent flips the fraction *(1)*
$(\frac{27}{8})^{\frac{2}{3}}$ *(1)*
$= (\frac{3}{2})^2$ *(1)* $= \mathbf{\frac{9}{4}}$ *(1)*

2.2 $\frac{3^{x+1} - 3^x}{3^x}$ **(4)**

Factorise the numerator: $3^x(3 - 1)$ *(2)*
$= \frac{3^x \times 2}{3^x}$ *(1)* $= \mathbf{2}$ *(1)*

2.3 $\frac{2^{x+2} + 2^x}{5 \times 2^x}$ **(4)**

Numerator $= 2^x(2^2 + 1) = 5(2^x)$ *(3)*
$= \mathbf{1}$ *(1)*

QUESTION 3 [14 MARKS]

Solve for x.

3.1 $3^{2x} - 4(3^x) + 3 = 0$ **(5)**

Let $k = 3^x$ *(1)*
$k^2 - 4k + 3 = 0$ *(1)*
$(k - 1)(k - 3) = 0$ *(1)*
$3^x = 1$ gives $x = 0$ *(1)*
$3^x = 3$ gives $x = 1$ *(1)*

3.2 $5^{x+2} - 5^x = 600$ **(4)**

$5^x(5^2 - 1) = 600$ *(2)*
$24 \times 5^x = 600$, so $5^x = 25$ *(1)*
$\boldsymbol{x = 2}$ *(1)*

3.3 Explain why $2^x = -4$ has no solution. **(2)**

2^x is positive for EVERY real x *(1)*, so it can never equal a negative number *(1)*

3.4 Show, without a calculator, that $2^{10} \times 5^{10} = 10^{10}$. **(3)**

$2^{10} \times 5^{10} = (2 \times 5)^{10}$ *(2)* — the power of a product law
$= \mathbf{10^{10}}$ *(1)*

TOTAL: 36 marks

This question paper consists of 3 questions.

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