MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
39	1 hour	3

Name: ..... Class: .....

Date: .....

MARK ____ / 39

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The asymptotes of $y = \frac{2}{x-1} + 3$ are ... (1)

A) $x = 3$ and $y = 1$
B) $x = 1$ and $y = 0$
C) $x = 1$ and $y = 3$ ✓
D) $x = -1$ and $y = 3$

> **Answer: C** — The vertical asymptote is where the denominator is zero; the horizontal one is the shift q.
> - **A** — the two asymptotes have been swapped
> - **B** — the $+3$ lifts the horizontal asymptote off the x-axis
> - **D** — $x - 1 = 0$ gives $x = +1$

1.2 The y-intercept of $y = \frac{-2}{x-1} + 3$ is ... (1)

A) 5 ✓
B) -2
C) 1
D) 3

> **Answer: A** — At $x = 0$, $y = -2 \div (-1) + 3 = 2 + 3 = 5$.
> - **B** — gave the value of a instead of evaluating at $x = 0$
> - **C** — took $-2 \div (-1)$ as -2 instead of $+2$
> - **D** — gave only the shift and left out the fraction

1.3 The domain of $y = \frac{3}{x+2}$ is ... (1)

A) $x \in \mathbb{R}, x \neq -2$ ✓ **B)** $x \in \mathbb{R}, x \neq 2$

C) $x \in \mathbb{R}$ **D)** $x > -2$

> **Answer: A** — $x + 2 = 0$ at $x = -2$, where the expression is undefined.
> - **B** — $x + 2 = 0$ gives $x = -2$, not $+2$
> - **C** — $x = -2$ must be excluded
> - **D** — values below -2 are perfectly allowed

1.4 The range of $y = \frac{4}{x-1} - 2$ is ... (1)

A) $y \in \mathbb{R}, y \neq 0$ **B)** $y > -2$

C) $y \in \mathbb{R}, y \neq 1$ **D)** $y \in \mathbb{R}, y \neq -2$ ✓

> **Answer: D** — The curve never reaches its horizontal asymptote, $y = -2$.
> - **A** — the vertical shift moves the excluded value away from 0
> - **B** — the curve takes values both above and below -2
> - **C** — 1 is the VERTICAL asymptote's x-value

1.5 The axes of symmetry of $y = \frac{a}{x}$ are ... (1)

A) $y = x$ only **B)** $y = x$ and $y = -x$ ✓

C) it has none **D)** the x-axis and the y-axis

> **Answer: B** — A hyperbola centred at the origin is symmetrical about both diagonals.
> - **A** — there are TWO axes of symmetry, one of each gradient
> - **C** — it has two
> - **D** — the axes are the ASYMPTOTES, which the curve never touches

1.6 For $y = \frac{a}{x-1} + 3$, the axis of symmetry with a POSITIVE gradient is ... (1)

A) $y = x - 2$ **B)** $y = -x + 4$

C) $y = x$ **D)** $y = x + 2$ ✓

> **Answer: D** — It runs through the centre $(1\ ;\ 3)$ with gradient 1, and $3 = 1 + c$ gives $c = 2$.
> - **A** — substituting the centre gives $3 = 1 + c$, so c is $+2$, not -2
> - **B** — that is the axis with a NEGATIVE gradient
> - **C** — $y = x$ passes through the origin, not through $(1\ ;\ 3)$

1.7 The asymptote of $y = 3(2^x) - 6$ is ... (1)

A) $y = 3$ **B)** $y = 0$

C) $y = -6$ ✓ **D)** $x = 0$

> **Answer: C** — The exponential part shrinks towards 0, leaving $y = -6$.
> - **A** — 3 is the value of a, not the shift
> - **B** — that is the asymptote before the vertical shift
> - **D** — an exponential graph has no VERTICAL asymptote

1.8 The x-intercept of $y = 3(2^x) - 6$ is ... (1)

A) $x = 2$ **B)** $x = 1$ ✓

C) $x = 6$ **D)** $x = 0$

> **Answer: B** — $3(2^x) = 6$ gives $2^x = 2$, so $x = 1$.
> - **A** — solved $2^x = 2$ and then wrote down the base
> - **C** — divided 6 by 1 instead of by 3
> - **D** — $x = 0$ gives $y = 3 - 6 = -3$, not 0

1.9 The y-intercept of $y = 3(2^x) - 6$ is ... (1)

A) 0 **B)** -6

C) 3 **D)** -3 ✓

> **Answer: D** — At $x = 0$, $2^0 = 1$, so $y = 3 - 6 = -3$.
> - **A** — that is the y-value at the X-intercept, not the y-intercept
> - **B** — took 2^0 as 0 instead of 1
> - **C** — forgot to subtract the 6

1.10 As x becomes very large and NEGATIVE, the graph of $y = 3(2^x) - 6$ approaches ... (1)

A) $y = 0$ **B)** $y = -6$ from above ✓

C) no limiting value at all **D)** $y = -6$ from below

> **Answer: B** — $3(2^x)$ stays POSITIVE however small it becomes, so the curve sits just above -6.
> - **A** — the -6 shift moves the asymptote off the x-axis
> - **C** — the curve levels off at -6 rather than falling forever
> - **D** — $3(2^x)$ is always positive, so the curve stays ABOVE the asymptote

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	**A**	B	C	D
1.3	**A**	B	C	D
1.4	A	B	C	**D**
1.5	A	**B**	C	D
1.6	A	B	C	**D**
1.7	A	B	**C**	D
1.8	A	**B**	C	D
1.9	A	B	C	**D**
1.10	A	**B**	C	D

QUESTION 2 [15 MARKS]

Given the hyperbola $g(x) = \dfrac{4}{x - 2} + 1$.

2.1 Write down the equations of the asymptotes, and explain how each one is read off the equation. (3)

> Vertical: $x = 2$ *(1)*
> Horizontal: $y = 1$ *(1)*
> $x = 2$ makes the denominator zero, and the $+1$ lifts the horizontal asymptote from $y = 0$ to $y = 1$ *(1)*

2.2 Calculate the coordinates of the intercepts with the axes. **(5)**

> y-intercept: $g(0) = \frac{4}{-2} + 1 = -1$, so $\mathbf{(0\,;\,-1)}$ *(2)*
>
> x-intercept: $\frac{4}{x-2} = -1$ gives $x - 2 = -4$ *(2)*
>
> so $\mathbf{(-2\,;\,0)}$ *(1)*

2.3 Write down the equations of the two axes of symmetry of g. **(4)**

> Both pass through the centre $(2\,;\,1)$ with gradients 1 and -1 *(2)*
>
> $\boldsymbol{y = x - 1}$ *(1)* and $\boldsymbol{y = -x + 3}$ *(1)*

2.4 Write down the domain and the range of g, and explain why each one excludes exactly ONE value. **(3)**

> Domain: $\boldsymbol{x \in \mathbb{R}, x \neq 2}$ *(1)*
>
> Range: $\boldsymbol{y \in \mathbb{R}, y \neq 1}$ *(1)*
>
> The two excluded values are the asymptotes — the curve approaches each one without ever reaching it *(1)*

QUESTION 3 **[14 MARKS]**

Given the exponential function $h(x) = 2(3)^x - 6$.

3.1 Write down the equation of the asymptote of h, and explain why the graph never reaches it. **(3)**

> The asymptote is $\boldsymbol{y = -6}$ *(1)*
>
> 3^x is positive for every x *(1)*
>
> so $2(3)^x - 6$ is always greater than -6 *(1)*

3.2 Calculate the coordinates of the intercepts with the axes. **(5)**

> y-intercept: $h(0) = 2 - 6 = -4$, so $\mathbf{(0\,;\,-4)}$ *(2)*
>
> x-intercept: $2(3)^x = 6$, so $3^x = 3$ *(2)*
>
> so $\mathbf{(1\,;\,0)}$ *(1)*

3.3 Write down the range of h, and explain why h has exactly ONE x-intercept. **(3)**

> Range: $\boldsymbol{y > -6}$ *(1)*
>
> $2(3)^x$ is positive for every x, so h never falls to -6, but it grows without bound as x increases *(1)*
>
> An increasing graph that starts below the x-axis and rises without bound crosses it **exactly once** *(1)*

3.4 Determine the equation of the graph obtained when h is shifted 2 units to the LEFT. **(3)**

> Replace x by $(x + 2)$ *(2)*
>
> $\boldsymbol{y = 2(3)^{x+2} - 6}$ *(1)*

TOTAL: 39 marks

This question paper consists of 3 questions.

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