MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
39	1 hour	3

Name: _____ Class: _____

Date: _____

MARK ____ / 39

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The asymptotes of $y = \frac{3}{x}$ are ... (1)

A) $x = 0$ and $y = 0$ ✓ **B)** $x = 3$ and $y = 3$

C) $y = 3$ only **D)** $x = 0$ only

Answer: A — The curve approaches both axes but never reaches either.

- **B** — 3 is the value of a, which controls the shape, not the asymptotes
- **C** — $y = 3$ would be the asymptote of $y = \frac{3}{x} + 3$
- **D** — there are TWO asymptotes, one vertical and one horizontal

1.2 The horizontal asymptote of $y = \frac{2}{x} + 3$ is ... (1)

A) $y = 2$ **B)** $x = 3$

C) $y = 3$ ✓ **D)** $y = 0$

Answer: C — The $+ 3$ shifts the whole graph up by 3, and the asymptote moves with it.

- **A** — 2 is the value of a, which controls the shape, not the shift
- **B** — the VERTICAL asymptote is still $x = 0$
- **D** — that is the asymptote BEFORE the vertical shift

1.3 The graph of $y = \frac{a}{x}$ with $a < 0$ lies in quadrants ... (1)

A) 3 and 4 **B)** 1 and 3

C) 2 and 4 ✓ **D)** 1 and 2

> **Answer: C** — With a negative, x and y always have OPPOSITE signs.
>
> - **A** — quadrants 3 and 4 are separated by the y-axis, which the hyperbola never crosses
> - **B** — that is where $y = \frac{a}{x}$ lies when a is POSITIVE
> - **D** — quadrants 1 and 2 are separated by the y-axis, which the hyperbola never crosses

1.4 The y-intercept of $y = 3 \times 2^x$ is ... (1)

A) 1 **B) 3 ✓**

C) 0 **D)** 2

> **Answer: B** — At $x = 0$, $2^0 = 1$, so $y = 3 \times 1 = 3$.
>
> - **A** — took 2^0 as 1 but then forgot to multiply by the 3
> - **C** — took 2^0 as 0 instead of 1
> - **D** — read off the base instead of working the value out

1.5 The asymptote of $y = 2^x + 1$ is ... (1)

A) $y = 2$ **B)** $x = 0$

C) $y = 0$ **D) $y = 1$ ✓**

> **Answer: D** — The $+1$ lifts the whole curve, and the asymptote lifts with it.
>
> - **A** — 2 is the base, not the shift
> - **B** — an exponential graph has no VERTICAL asymptote
> - **C** — that is the asymptote before the shift

1.6 Does $y = 2^x$ have an x-intercept? (1)

A) no — 2^x is never 0 ✓ **B)** yes, at $x = 2$

C) yes, at $x = 0$ **D)** yes, at $x = 1$

> **Answer: A** — A positive base raised to any power stays positive, so the curve never reaches the x-axis.
>
> - **B** — at $x = 2$ the graph is at $y = 4$
> - **C** — at $x = 0$ the graph is at $y = 1$, which is the Y-intercept
> - **D** — at $x = 1$ the graph is at $y = 2$

1.7 $y = 3^x$ is increasing. The graph of $y = (\frac{1}{3})^x$ is ... (1)

A) undefined for negative x **B) decreasing ✓**

C) a straight line **D)** also increasing

> **Answer: B** — A base between 0 and 1 gives a decreasing exponential graph.
>
> - **A** — $(\frac{1}{3})^x$ is defined for every real value of x
> - **C** — an exponential graph is a curve, never a straight line
> - **D** — a base smaller than 1 makes each step SMALLER, not bigger

1.8 The domain of $y = \frac{4}{x}$ is ... (1)

A) $x > 0$ **B)** $x \in \mathbb{R}, x \neq 0$ ✓

C) $y \in \mathbb{R}, y \neq 0$ **D)** $x \in \mathbb{R}$

> **Answer: B** — Division by zero is undefined, so $x = 0$ must be left out.
> - **A** — negative values of x are perfectly allowed
> - **C** — that is the RANGE, not the domain
> - **D** — $x = 0$ must be excluded — it would divide by zero

1.9 The point $(2\ ;\ 6)$ lies on $y = \frac{a}{x}$. Then $a = ...$ (1)

A) $\frac{1}{3}$ **B)** 3

C) 8 **D)** 12 ✓

> **Answer: D** — $6 = a \div 2$, so $a = 12$.
> - **A** — inverted the ratio and worked out $2 \div 6$
> - **B** — DIVIDED 6 by 2 instead of multiplying
> - **C** — ADDED 6 and 2 instead of multiplying them

1.10 For $y = 5 \times 3^x$, each time x increases by 1 the value of y is ... (1)

A) multiplied by 3 ✓ **B)** increased by 5

C) multiplied by 5 **D)** increased by 3

> **Answer: A** — Each unit step in x multiplies y by the BASE, which is 3.
> - **B** — 5 is the starting value, and the change is a multiplication, not an addition
> - **C** — 5 is the starting value, not the growth factor
> - **D** — an exponential graph MULTIPLIES; only a straight line adds a fixed amount

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	(C)	D
1.3	A	B	(C)	D
1.4	A	(B)	C	D
1.5	A	B	C	(D)
1.6	(A)	B	C	D
1.7	A	(B)	C	D
1.8	A	(B)	C	D
1.9	A	B	C	(D)
1.10	(A)	B	C	D

QUESTION 2 [13 MARKS]

Given the function $f(x) = \frac{6}{x} - 2$.

2.1 Write down the equations of the asymptotes of f, and explain how each one is read off the equation. (3)

> Vertical: $\boldsymbol{x = 0}$ *(1)*
> Horizontal: $\boldsymbol{y = -2}$ *(1)*
> $x = 0$ is the value that makes the denominator zero, and the -2 drops the horizontal asymptote from $y = 0$ to $y = -2$ *(1)*

2.2 Explain why f has no y-intercept. (3)

> A y-intercept needs $x = 0$ *(1)*
> but $\frac{6}{x}$ is undefined at $x = 0$ *(1)*
> so the graph never crosses the y-axis — that axis is its vertical asymptote *(1)*

2.3 Calculate the coordinates of the x-intercept of f. (4)

> Let $y = 0$: $\frac{6}{x} - 2 = 0$ *(1)*
> $\frac{6}{x} = 2$ *(1)*
> $6 = 2x$, so $x = 3$ *(1)*
> **(3 ; 0)** *(1)*

2.4 Write down the domain and the range of f, and explain why each one excludes exactly ONE value. (3)

> Domain: $\boldsymbol{x \in \mathbb{R}, x \neq 0}$ *(1)*
> Range: $\boldsymbol{y \in \mathbb{R}, y \neq -2}$ *(1)*
> The excluded values are precisely the asymptotes — the graph approaches each one but never reaches it *(1)*

QUESTION 3 [16 MARKS]

Given the function $g(x) = 2(3)^x - 6$.

3.1 Write down the equation of the asymptote of g, and explain why the graph never reaches it. (3)

> The asymptote is $\boldsymbol{y = -6}$ *(1)*
> 3^x is always POSITIVE, however negative x becomes *(1)*
> so $2(3)^x - 6$ is always a little more than -6, and the graph approaches the line without touching it *(1)*

3.2 Calculate the coordinates of the y-intercept of g. (3)

> $g(0) = 2(1) - 6$ *(2)* = **(0 ; −4)** *(1)*

3.3 Calculate the coordinates of the x-intercept of g. (4)

> $2(3)^x = 6$ *(1)*
> $3^x = 3$ *(2)*
> **(1 ; 0)** *(1)*

3.4 Describe what happens to g as x becomes very small (very negative), and write down the range of g. **(3)**

> 3^x approaches 0 *(1)*, so g approaches -6 without ever reaching it *(1)*
> Range: $\boldsymbol{y > -6}$ *(1)*

3.5 Determine the value of $g(2)$, and hence state whether the point $(2\ ;\ 10)$ lies on g. **(3)**

> $g(2) = 2(9) - 6 = 12$ *(2)*
> $12 \neq 10$, so **$(2\ ;\ 10)$ does NOT lie on g** *(1)*

TOTAL: 39 marks

This question paper consists of 3 questions.

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