MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The inverse of a function is found by ... (1)

A) taking the reciprocal of the function
B) reflecting in the y-axis
C) swapping x and y, then making y the subject ✓
D) changing the sign of y

> **Answer: C** — The inverse reverses the roles of input and output.
> - **A** — f^{-1} means the INVERSE, not $1 \div f$
> - **B** — the inverse is a reflection in $y = x$, not in the y-axis
> - **D** — that reflects the graph in the x-axis, a different transformation

1.2 The graph of f^{-1} is the reflection of f in the line ... (1)

A) $y = x$ ✓
B) the y-axis
C) $y = -x$
D) the x-axis

> **Answer: A** — Swapping x and y is exactly a reflection in the line $y = x$.
> - **B** — reflecting in the y-axis changes the sign of x only
> - **C** — that is the other diagonal
> - **D** — reflecting in the x-axis changes the sign of y only

1.3 The inverse of $f(x) = 3x - 6$ is ... (1)

A) $3x + 6$

B) $\frac{1}{3x - 6}$

C) $\frac{x + 6}{3}$ ✓

D) $\frac{x - 6}{3}$

> **Answer: C** — Swap and solve: $x = 3y - 6$ gives $y = (x + 6) \div 3$.
> - **A** — changed the sign of the constant but never divided by the 3
> - **B** — that is $1 \div f(x)$, not the inverse
> - **D** — the -6 becomes $+6$ when it is moved across

1.4 The inverse of $f(x) = x^2$ for $x \geq 0$ is ... (1)

A) x^2

B) $\sqrt{x}$ ✓

C) $\frac{1}{x^2}$

D) $-\sqrt{x}$

> **Answer: B** — Restricting to $x \geq 0$ makes f one-to-one, and its inverse is the POSITIVE square root.
> - **A** — that is f itself
> - **C** — that is $1 \div f(x)$, not the inverse
> - **D** — the restriction $x \geq 0$ keeps only the positive branch

1.5 Why must the domain of $f(x) = x^2$ be restricted before it has an inverse function? (1)

A) the inverse would otherwise be a straight line

B) x^2 is undefined for negative x

C) x^2 has no y-intercept

D) **without the restriction the inverse fails the vertical line test** ✓

> **Answer: D** — The full parabola is many-to-one, so its reflection is one-to-many and is not a function.
> - **A** — the inverse is a sideways parabola, not a line
> - **B** — x^2 is perfectly well defined for negative x
> - **C** — x^2 passes through $(0\ ;\ 0)$, so it does have a y-intercept

1.6 If $f(x) = 2^x$, then $f^{-1}(x) = ...$ (1)

A) x^2

B) $\log_2 x$ ✓

C) 2^{-x}

D) $\frac{1}{2^x}$

> **Answer: B** — A logarithm is the inverse of an exponential with the same base.
> - **A** — that swaps the base and the exponent
> - **C** — that reflects the graph in the y-axis, not in $y = x$
> - **D** — that is $1 \div f(x)$, not the inverse

1.7 The DOMAIN of f^{-1} is ... (1)

A) **the range of f** ✓

B) always $x > 0$

C) the domain of f

D) always all real x

> **Answer: A** — The inverse swaps inputs and outputs, so it swaps domain and range too.
> - **B** — that holds for a logarithm, but not for every inverse
> - **C** — the domain of f becomes the RANGE of f^{-1}
> - **D** — it is whatever f's range happened to be, which need not be all of $\mathbb{R}$

1.8 If $(3\ ;\ 7)$ lies on f, then f^{-1} passes through ... (1)

A) $(7\ ;\ 3)$ ✓ **B)** $(-7\ ;\ 3)$

C) $(3\ ;\ 7)$ **D)** $(-3\ ;\ -7)$

> **Answer: A** — The inverse swaps the coordinates of every point.
>
> - **B** — the inverse swaps them without changing any sign
> - **C** — that is the point on f itself
> - **D** — the inverse swaps the coordinates; it does not negate them

1.9 For $a \neq 0$, the inverse of $y = ax$ is ... (1)

A) $\frac{a}{x}$ **B)** ax

C) $-ax$ **D)** $\frac{x}{a}$ ✓

> **Answer: D** — Swap and solve: $x = ay$ gives $y = x \div a$, which needs $a \neq 0$ (if $a = 0$, $y = 0$ is a constant and has no inverse function).
>
> - **A** — that is a hyperbola, not the inverse of a straight line
> - **B** — that is the function itself
> - **C** — changing the sign is a reflection, not an inverse

1.10 A function's inverse is itself a function only when the original is ... (1)

A) always increasing **B)** many-to-one

C) **one-to-one** ✓ **D)** continuous

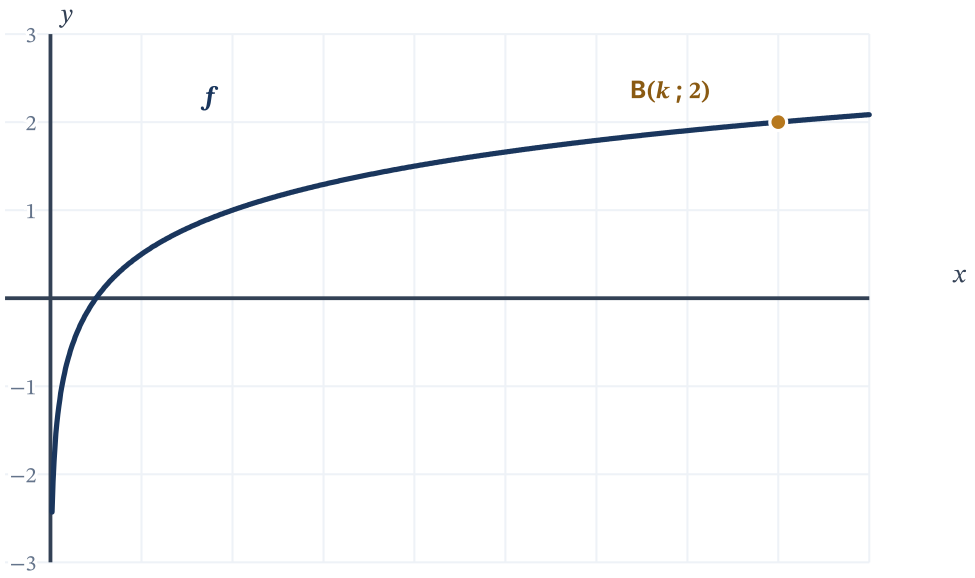
> **Answer: C** — Only a one-to-one function reflects into something that passes the vertical line test.
>
> - **A** — a decreasing function such as $y = -x$ has a perfectly good inverse
> - **B** — a many-to-one function reflects into a one-to-many relation
> - **D** — continuity is not what decides this

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	(C)	D
1.2	(A)	B	C	D
1.3	A	B	(C)	D
1.4	A	(B)	C	D
1.5	A	B	C	(D)
1.6	A	(B)	C	D
1.7	(A)	B	C	D
1.8	(A)	B	C	D
1.9	A	B	C	(D)
1.10	A	B	(C)	D

QUESTION 2 [12 MARKS]

The graph of $f(x) = \log_4 x$ is drawn. B$(k\ ;\ 2)$ is a point on f.



2.1 Calculate the value of k, and hence write down the coordinates of the point at which f cuts the x-axis. **(3)**

$\log_4 k = 2$ *(1)*
$k = 4^2 = \mathbf{16}$ *(1)*
$f(x) = 0$ when $x = 1$, so f cuts the x-axis at **(1 ; 0)** *(1)*

2.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. **(3)**

$4^{-1} \leq x \leq 4^2$ *(2)*
$\mathbf{\frac{1}{4} \leq x \leq 16}$ *(1)*

2.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$, and state its domain. **(3)**

Swap x and y: $x = \log_4 y$ *(1)*
$y = \mathbf{4^x}$ *(1)*
The domain of f^{-1} is the RANGE of f, so $\boldsymbol{x \in \mathbb{R}}$ *(1)*

2.4 For which values of x will $x \cdot f^{-1}(x) \leq 0$? **(3)**

$f^{-1}(x) = 4^x$ is ALWAYS positive *(1)*
So the product is ≤ 0 only when $x \leq 0$ *(1)*
$\mathbf{x \leq 0}$ *(1)*

QUESTION 3 [13 MARKS]

Given $g(x) = 2x^2$ for $x \geq 0$.

3.1 Determine the equation of g^{-1} in the form $y = \ldots$ **(4)**

$x = 2y^2$ *(1)*

$y^2 = \frac{x}{2}$ *(1)*

$y = \pm\sqrt{\frac{x}{2}}$ *(1)*

The domain of g is $x \geq 0$, so $\boldsymbol{g^{-1}(x) = \sqrt{\frac{x}{2}}}$ *(1)*

3.2 Write down the domain and the range of g^{-1}, and say where each one comes from. **(3)**

Domain of g^{-1}: $\boldsymbol{x \geq 0}$ *(1)*
Range of g^{-1}: $\boldsymbol{y \geq 0}$ *(1)*
The domain of an inverse is the RANGE of the original, and its range is the original's DOMAIN *(1)*

3.3 Explain why the inverse of g would NOT be a function if the domain of g were not restricted. **(3)**

Every positive value of g comes from two x-values, $+x$ and $-x$ *(1)*
The inverse would then give two outputs for one input *(1)*
A function may give only one *(1)*

3.4 Describe the geometric relationship between the graphs of g and g^{-1}, and hence determine the coordinates of the point, other than the origin, at which they intersect. **(3)**

They are reflections of each other in the line $\boldsymbol{y = x}$ *(1)*
So they meet ON that line: $2x^2 = x$, giving $x(2x - 1) = 0$ *(1)*
$x = ½$, so the point is **(½ ; ½)** *(1)*

TOTAL: 35 marks

This question paper consists of 3 questions.

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