# MARKING GUIDELINE

| MARKS | DURATION | QUESTIONS |
|---|---|---|
| 31 | 50 minutes | 3 |

Name: _____ Class: _____

Date: _____

MARK ____ / 31

## INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

## QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

**1.1** To MAXIMISE or MINIMISE a quantity, you ... (1)

**A)** set the expression itself equal to 0

**B) differentiate the expression and set the derivative equal to 0** ✓

**C)** substitute the largest allowed value

**D)** set the second derivative equal to 0

**Answer: B** — A maximum or minimum occurs where the rate of change is zero.

- **A** — that finds where the quantity is zero, not where it is largest
- **C** — the optimum need not lie at an endpoint
- **D** — setting $f''$ to zero finds the point of INFLECTION

**1.2** A rectangle has a perimeter of 40 m. Its greatest possible AREA is ... (1)

**A)** 400 m$^2$

**B) 100 m$^2$** ✓

**C)** 40 m$^2$

**D)** 200 m$^2$

**Answer: B** — A $= l(20 - l)$, and $\frac{dA}{dl} = 20 - 2l = 0$ gives $l = 10$, so A $= 100$ m$^2$.

- **A** — squared half the perimeter instead of optimising
- **C** — used the perimeter as though it were the area
- **D** — halved the perimeter and multiplied by 10 without optimising

**1.3** A closed box has a square base of side $x$ and a volume of $500$ cm$^3$. Its height is ... (1)

**A)** $\frac{500}{x}$

**B)** $500x^2$

**C)** $\frac{x^2}{500}$

**D)** $\frac{500}{x^2}$ ✓

**Answer: D** — $V = x^2h = 500$, so $h = 500 \div x^2$.

- **A** — the base is a SQUARE, so its area is $x^2$, not $x$
- **B** — multiplied instead of dividing
- **C** — inverted the expression

**1.4** If $s(t) = 5t^2 + 2t$ metres, the VELOCITY at $t = 3$ seconds is ... (1)

**A)** $30$ m per second

**B)** $51$ m per second

**C)** $10$ m per second

**D)** **$32$ m per second** ✓

**Answer: D** — $v = s'(t) = 10t + 2$, so $v(3) = 32$.

- **A** — forgot to add the $2$
- **B** — substituted into $s(t)$ instead of $s'(t)$: $45 + 6 = 51$
- **C** — gave the coefficient without substituting $t = 3$

**1.5** ACCELERATION is the derivative of ... (1)

**A)** **velocity with respect to time** ✓

**B)** velocity with respect to distance

**C)** time with respect to velocity

**D)** distance with respect to time

**Answer: A** — $a = \frac{dv}{dt}$, which is the second derivative of displacement.

- **B** — acceleration is measured against TIME
- **C** — that inverts the relationship
- **D** — that is VELOCITY

**1.6** The sum of two positive numbers is $30$. Their product is largest when the numbers are ... (1)

**A)** **$15$ and $15$** ✓

**B)** $1$ and $29$

**C)** $0$ and $30$

**D)** $10$ and $20$

**Answer: A** — $P = x(30 - x)$, and $\frac{dP}{dx} = 30 - 2x = 0$ gives $x = 15$, so the product is $225$.

- **B** — $1 \times 29 = 29$, far below $225$
- **C** — $0 \times 30 = 0$, the smallest product possible
- **D** — $10 \times 20 = 200$, still below $225$

**1.7** A cylinder of volume $1\,000$ cm$^3$ has radius $r$. Its height is ... (1)

**A)** $\frac{1\,000}{\pi r}$

**B)** $1\,000\pi r^2$

**C)** $\frac{1\,000}{\pi r^2}$ ✓

**D)** $\frac{1\,000}{r^2}$

**Answer: C** — $V = \pi r^2 h = 1\,000$, so $h = 1\,000 \div (\pi r^2)$.

- **A** — the base area is $\pi r^2$, not $\pi r$
- **B** — multiplied instead of dividing
- **D** — left out the $\pi$

**1.8** At a MAXIMUM turning point, $f''(x)$ is ... (1)

**A)** undefined **B)** positive

**C) negative ✓** **D)** zero

> **Answer: C** — A maximum is concave down, so the second derivative is negative there.
>
> - **A** — the second derivative of a polynomial exists everywhere
> - **B** — a positive second derivative means a MINIMUM
> - **D** — at a maximum the curve is concave DOWN, so $f''$ is negative; $f'' = 0$ is where a cubic changes concavity

**1.9** A stone's height is $h = 30t - 5t^2$ metres after $t$ seconds. Its greatest height is ... (1)

**A)** 30 m **B)** 3 m

**C)** 90 m **D) 45 m ✓**

> **Answer: D** — $h'(t) = 30 - 10t = 0$ gives $t = 3$, and $h(3) = 90 - 45 = 45$ m.
>
> - **A** — gave the coefficient 30 instead of a height
> - **B** — that is the TIME at the maximum, not the height
> - **C** — used $30t$ alone and forgot to subtract the $5t^2$

**1.10** In an optimisation problem, the CONSTRAINT is used to ... (1)

**A) write the quantity in terms of a single variable** ✓ **B)** avoid using calculus at all

**C)** find the derivative directly **D)** check the answer at the very end only

> **Answer: A** — In Grade 12 optimisation the quantity is first written in terms of ONE variable, and the constraint is what removes the other.
>
> - **B** — calculus is still what finds the optimum
> - **C** — the derivative can only be taken once one variable is left
> - **D** — the constraint is needed BEFORE differentiating, not only afterwards

**ANSWER GRID — MARKING GUIDELINE**

| | | | | |
|---|---|---|---|---|
| **1.1** | A | **(B)** | C | D |
| **1.2** | A | **(B)** | C | D |
| **1.3** | A | B | C | **(D)** |
| **1.4** | A | B | C | **(D)** |
| **1.5** | **(A)** | B | C | D |
| **1.6** | **(A)** | B | C | D |
| **1.7** | A | B | **(C)** | D |
| **1.8** | A | B | **(C)** | D |
| **1.9** | A | B | C | **(D)** |
| **1.10** | **(A)** | B | C | D |

## QUESTION 2 [13 MARKS]

Given $f(x) = x^2$. Determine the MINIMUM distance between the point $(10\ ;\ 2)$ and a point on $f$.

**2.1** Write down an expression for the square of the distance $d$ between $(10\ ;\ 2)$ and the point $(x\ ;\ x^2)$ on $f$. (3)

> $d^2 = (x-10)^2 + (y-2)^2$ *(1)*
> $y = x^2$ on $f$ *(1)*
> **$d^2 = (x-10)^2 + (x^2-2)^2$** *(1)*

**2.2** Explain why minimising $d^2$ gives the same answer as minimising $d$. (2)

> $d$ is never negative *(1)*, and squaring preserves the order of non-negative numbers, so both are smallest at the same $x$ *(1)*

**2.3** Determine the value of $x$ for which the distance is a minimum. (5)

> $d^2 = x^4 - 3x^2 - 20x + 104$ *(1)*
> $\frac{d(d^2)}{dx} = 4x^3 - 6x - 20$ *(2)*
> Set it equal to $0$ and test factors: $x = 2$ gives $32 - 12 - 20 = 0$ *(1)*
> **$x = 2$** *(1)*

**2.4** Hence calculate the minimum distance, in simplest surd form. (3)

> At $x = 2$ the point on $f$ is $(2\ ;\ 4)$ *(1)*
> $d^2 = (-8)^2 + 2^2 = 68$ *(1)*
> $d = \mathbf{2\sqrt{17}}$ units *(1)*

## QUESTION 3 [8 MARKS]

A container is being filled with water. After $t$ seconds the volume, in litres, is given by $V(t) = 3t^2 + 2t$.

**3.1** Determine the AVERAGE rate at which the volume increases over the first $5$ seconds. (3)

> $V(0) = 0$ and $V(5) = 85$ *(1)*
> $\frac{85-0}{5-0}$ *(1)* = **17 litres per second** *(1)*

**3.2** Determine the rate at which the volume is increasing AT $t = 5$ seconds. (3)

> $V'(t) = 6t + 2$ *(2)*
> $V'(5) =$ **32 litres per second** *(1)*

**3.3** Explain why the two answers differ. (2)

> The first is the average over the whole interval *(1)*; the second is the instantaneous rate at one moment, and the flow is speeding up *(1)*

**TOTAL: 31 marks**

*This question paper consists of 3 questions.*

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