MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
36	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK _____ / 36

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The turning point of $y = 2(x - 3)^2 + 5$ is ... (1)

A) $(3\ ;\ -5)$ **B)** $(2\ ;\ 3)$

C) $(3\ ;\ 5)$ ✓ **D)** $(-3\ ;\ 5)$

Answer: C — The bracket is zero when $x = 3$, and then $y = 5$.

- **A** — q is added, so it keeps its sign
- **B** — 2 is the value of a, not a coordinate
- **D** — the bracket reads $(x - 3)$, so the turning point is at $+3$

1.2 The turning point of $y = (x + 4)^2 - 1$ is ... (1)

A) $(-4\ ;\ -1)$ ✓ **B)** $(4\ ;\ 1)$

C) $(4\ ;\ -1)$ **D)** $(-4\ ;\ 1)$

Answer: A — $(x + 4)$ is zero at $x = -4$, and then $y = -1$.

- **B** — both signs have been read the wrong way round
- **C** — the bracket reads $(x + 4)$, so the turning point is at -4
- **D** — the -1 keeps its sign

1.3 The range of $y = -3(x - 1)^2 + 7$ is ... (1)

A) $y \leq -3$ **B)** $y \geq 7$

C) $y \geq 1$ **D) $y \leq 7$ ✓**

Answer: D — $a = -3$ is negative, so the graph opens downward and 7 is a MAXIMUM.

- **A** — -3 is the value of a, not a y-value
- **B** — the graph opens downward, so 7 is the maximum, not the minimum
- **C** — 1 is the x-coordinate of the turning point

1.4 The axis of symmetry of $y = 2(x + 5)^2 - 3$ is ... (1)

A) $x = 2$ **B)** $x = -5$ ✓

C) $y = -3$ **D)** $x = 5$

> **Answer: B** — The axis of symmetry runs through the turning point, at $x = -5$.
>
> - **A** — 2 is the value of a
> - **C** — a parabola's axis of symmetry is a VERTICAL line, written $x = \ldots$
> - **D** — the bracket reads $(x + 5)$, so the axis is at -5

1.5 $y = x^2 - 4x + 1$ in the form $a(x + p)^2 + q$ is ... (1)

A) $y = (x + 2)^2 - 3$ **B)** $y = (x - 2)^2 + 1$

C) $y = (x - 4)^2 - 15$ **D)** $y = (x - 2)^2 - 3$ ✓

> **Answer: D** — Half of -4 is -2, and $(x - 2)^2 = x^2 - 4x + 4$, so $1 - 4 = -3$.
>
> - **A** — the sign inside the bracket follows b's, so it is -2
> - **B** — forgot to subtract the 4 that completing the square adds in
> - **C** — used -4 inside the bracket instead of half of it

1.6 A parabola has turning point $(2\ ;\ -5)$ and passes through $(0\ ;\ 3)$. Its equation is ... (1)

A) $y = (x - 2)^2 - 5$ **B)** $y = 2(x - 2)^2 - 5$ ✓

C) $y = 2(x + 2)^2 - 5$ **D)** $y = 8(x - 2)^2 - 5$

> **Answer: B** — $3 = a(0 - 2)^2 - 5$ gives $4a = 8$, so $a = 2$.
>
> - **A** — assumed $a = 1$ without using the given point
> - **C** — the turning point is at $+2$, so the bracket is $(x - 2)$
> - **D** — took $4a = 8$ as $a = 8$, without dividing by the 4

1.7 If $a > 0$, the graph of $y = a(x + p)^2 + q$ has ... (1)

A) a minimum value of p **B)** no turning point

C) a maximum value of q **D)** a minimum value of q ✓

> **Answer: D** — The squared bracket is never negative, so the smallest value of y is q.
>
> - **A** — p fixes the x-position, not the y-value
> - **B** — every parabola has exactly one turning point
> - **C** — a maximum needs $a < 0$

1.8 The y-intercept of $y = 2(x - 3)^2 + 5$ is ... (1)

A) 5 **B)** 18

C) 23 ✓ **D)** 9

> **Answer: C** — At $x = 0$, $y = 2(-3)^2 + 5 = 18 + 5 = 23$.
>
> - **A** — that is the y-coordinate of the TURNING POINT
> - **B** — forgot to add the 5
> - **D** — squared the 3 but neither multiplied by 2 nor added 5

1.9 How many x-intercepts does $y = 2(x-3)^2 + 5$ have? (1)

A) one **B)** two

C) none ✓ **D)** three

> **Answer: C** — The graph opens upward with a minimum of 5, so it never reaches $y = 0$.
> - **A** — one x-intercept needs the turning point to sit ON the axis
> - **B** — two needs the turning point BELOW the axis
> - **D** — a parabola never cuts the x-axis three times

1.10 Reflecting $y = (x-1)^2 + 2$ in the x-axis gives ... (1)

A) $y = -(x-1)^2 - 2$ ✓ **B)** $y = (x+1)^2 + 2$

C) $y = (x-1)^2 - 2$ **D)** $y = -(x-1)^2 + 2$

> **Answer: A** — A reflection in the x-axis replaces y by $-y$, so EVERY term on the right changes sign.
> - **B** — that is a reflection in the Y-axis
> - **C** — that shifts the graph down instead of reflecting it
> - **D** — the $+2$ must change sign as well

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	**A**	B	C	D
1.3	A	B	C	**D**
1.4	A	**B**	C	D
1.5	A	B	C	**D**
1.6	A	**B**	C	D
1.7	A	B	C	**D**
1.8	A	B	**C**	D
1.9	A	B	**C**	D
1.10	**A**	B	C	D

QUESTION 2 [13 MARKS]

The parabola $f(x) = -2(x-1)^2 + 8$ is given.

2.1 Write down the coordinates of the turning point and the equation of the axis of symmetry, and state whether the turning point is a maximum or a minimum, with a reason. (3)

> Turning point **(1 ; 8)** *(1)*
> Axis of symmetry $x = 1$ *(1)*
> It is a **MAXIMUM**, because $a = -2$ is negative, so the parabola opens downwards *(1)*

2.2 Calculate the x-intercepts of f. (4)

> $-2(x-1)^2 + 8 = 0$ *(1)*
> $(x-1)^2 = 4$ *(1)*
> $x - 1 = \pm 2$ *(1)*
> $x = 3$ **or** $x = -1$ *(1)*

2.3 Write f in the form $y = ax^2 + bx + c$. **(3)**

> $-2(x^2 - 2x + 1) + 8$ *(2)*
> $= \mathbf{-2x^2 + 4x + 6}$ *(1)*

2.4 Write down the range of f, and give a reason. **(3)**

> $\mathbf{y \leq 8}$ *(2)* — $a < 0$, so the graph opens downwards and 8 is the MAXIMUM *(1)*

QUESTION 3 [13 MARKS]

Answer the questions below about the same parabola.

3.1 For which values of x is $f(x) \geq 0$? Give your answer in interval notation. **(3)**

> The graph is on or above the x-axis BETWEEN its roots *(1)*
> **$[-1\ ;\ 3]$** *(2)*

3.2 For which values of x is f increasing? Hence, using your answer to the previous part, for which values of x is f increasing AND $f(x) > 0$? **(4)**

> f increases to the LEFT of the turning point: $\mathbf{x < 1}$ *(1)*
> From the previous part, $f(x) > 0$ for $-1 < x < 3$ *(1)*
> Both hold together: $\mathbf{-1 < x < 1}$ *(2)*

3.3 Determine the value(s) of k for which $f(x) = k$ has NO real roots. **(3)**

> The maximum value of f is 8 *(1)*
> A horizontal line above the turning point never meets the graph *(1)*
> $\mathbf{k > 8}$ *(1)*

3.4 Describe the transformation that maps f onto $g(x) = -2(x - 1)^2 + 3$, and write down the turning point of g. **(3)**

> Only the constant has changed, from 8 to 3 *(1)*
> So f is shifted **5 units DOWN** *(1)*
> The turning point of g is **$(1\ ;\ 3)$** *(1)*

TOTAL: 36 marks

This question paper consists of 3 questions.

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