Quadratic Number Patterns (Grade 11 & 12)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
34	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK ____ / 34

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 In a QUADRATIC number pattern, the second difference is ... (1)

A) equal to the first difference
B) constant ✓
C) zero
D) increasing

Answer: B — A constant second difference is exactly what makes a pattern quadratic.

- **A** — the first differences change; the second ones do not
- **C** — a zero second difference makes the pattern LINEAR
- **D** — an increasing second difference means a cubic pattern or higher

1.2 For the pattern 204 ; 176 ; 150 ; 126 ; ..., the SECOND difference is ... (1)

A) -26
B) -2
C) 2 ✓
D) -28

Answer: C — The first differences are -28, -26 and -24, and each is 2 more than the one before.

- **A** — that is the second of the first differences
- **B** — the first differences are INCREASING, so the second difference is positive
- **D** — that is the FIRST difference

1.3 The next TWO terms of 204 ; 176 ; 150 ; 126 ; ... are ... (1)

A) 104 and 82
B) 104 and 84 ✓
C) 102 and 78
D) 100 and 74

Answer: B — The first differences continue -22 and then -20.

- **A** — used -22 twice instead of -22 then -20
- **C** — carried on subtracting 24 each time, ignoring the second difference
- **D** — used -26 twice, going backwards through the differences

1.4 For $T_n = an^2 + bn + c$, the value of a is ... (1)

A) half the second difference ✓
B) the first term
C) the second difference itself
D) the first difference

> **Answer: A** — The second difference always equals $2a$.
> - **B** — the first term is $a + b + c$, not a
> - **C** — the second difference is $2a$, so a is half of it
> - **D** — the first difference changes from term to term

1.5 The general term of $204\ ;\ 176\ ;\ 150\ ;\ 126\ ;\ \ldots$ is ... (1)

A) $T_n = n^2 - 31n + 234$ ✓
B) $T_n = 2n^2 - 31n + 234$
C) $T_n = n^2 + 31n + 234$
D) $T_n = n^2 - 28n + 231$

> **Answer: A** — $2a = 2$ gives $a = 1$; $3a + b = -28$ gives $b = -31$; and $a + b + c = 204$ gives $c = 234$.
> - **B** — used $a = 2$, the second difference itself, instead of half of it
> - **C** — b must be negative, since the pattern falls at first
> - **D** — used the first difference -28 as b directly

1.6 A quadratic pattern with $a > 0$ has ... (1)

A) a constant first difference
B) no turning term
C) a minimum term ✓
D) a maximum term

> **Answer: C** — The terms lie on an upward parabola, so they fall and then rise.
> - **A** — a constant FIRST difference would make the pattern linear
> - **B** — the parabola always turns somewhere
> - **D** — a maximum term needs $a < 0$

1.7 The general term of $3\ ;\ 8\ ;\ 17\ ;\ 30\ ;\ \ldots$ is ... (1)

A) $T_n = 2n^2 - n$
B) $T_n = 2n^2 + n + 2$
C) $T_n = 4n^2 - n + 2$
D) $T_n = 2n^2 - n + 2$ ✓

> **Answer: D** — The second difference is 4, so $a = 2$; then $3a + b = 5$ gives $b = -1$, and $c = 2$.
> - **A** — left out c, which would make T_1 equal 1, not 3
> - **B** — $3a + b = 5$ with $a = 2$ gives $b = -1$, not $+1$
> - **C** — used the second difference 4 as a instead of half of it

1.8 For $T_n = n^2 - 31n + 234$, the SMALLEST term occurs at ... (1)

A) $n = 15{,}5$
B) $n = 1$
C) $n = 31$
D) $n = 15$ and $n = 16$, which are equal ✓

> **Answer: D** — The vertex is at $n = 15{,}5$, and n must be a whole number, so the two neighbouring terms tie at -6.
> - **A** — n counts terms, so it must be a whole number
> - **B** — the pattern falls at first, so the first term is not the smallest
> - **C** — 31 is the value of $-b$, before dividing by $2a$

1.9 For $T_n = n^2 - 31n + 234$, the number of NEGATIVE terms is ... (1)

A) 5 **B) 4 ✓**

C) 2 **D)** 6

> **Answer: B** — $n^2 - 31n + 234 = (n - 13)(n - 18)$, which is negative for $13 < n < 18$, so $n = 14, 15, 16$ and 17.
>
> - **A** — included $n = 13$ or $n = 18$, where the term is exactly 0
> - **C** — counted only the two smallest terms
> - **D** — included both endpoints, but a zero term is not a negative one

1.10 The FIRST differences of a quadratic pattern themselves form ... (1)

A) a linear pattern ✓ **B)** a constant pattern

C) a quadratic pattern **D)** a geometric pattern

> **Answer: A** — They change by the constant second difference each time, which is what a linear pattern does.
>
> - **B** — constant first differences would make the ORIGINAL pattern linear
> - **C** — the SECOND differences would then be changing, not constant
> - **D** — a geometric pattern multiplies by a ratio, it does not add

ANSWER GRID — MARKING GUIDELINE

1.1	A	(B)	C	D
1.2	A	B	(C)	D
1.3	A	(B)	C	D
1.4	(A)	B	C	D
1.5	(A)	B	C	D
1.6	A	B	(C)	D
1.7	A	B	C	(D)
1.8	A	B	C	(D)
1.9	A	(B)	C	D
1.10	(A)	B	C	D

QUESTION 2 [12 MARKS]

A quadratic number pattern $T_n = an^2 + bn + c$ has these properties:

- $T_1 = -2$
- the n^{th} term of the FIRST differences is $-6n - 2$

2.1 Show that $a = -3$ and $b = 1$. (4)

> The first differences of $an^2 + bn + c$ are $a(2n + 1) + b$ *(1)*
> $2an + a + b = -6n - 2$ *(1)*
> $2a = -6$, so $\boldsymbol{a = -3}$ *(1)*
> $a + b = -2$ gives $\boldsymbol{b = 1}$ *(1)*

2.2 Hence determine the general term T_n. (3)

> $T_1 = a + b + c = -2$ *(1)*
> $-3 + 1 + c = -2$, so $c = 0$ *(1)*
> $\boldsymbol{T_n = -3n^2 + n}$ *(1)*

2.3 Determine whether $-19\,122$ is a term of the sequence. Justify your answer with calculations. **(5)**

$-3n^2 + n = -19\,122$ *(1)*
$3n^2 - n - 19\,122 = 0$ *(1)*
$\Delta = 1 + 4(3)(19\,122) = 229\,465$ *(1)*
$\sqrt{229\,465} \approx 479{,}03$, which is not a whole number *(1)*
n is not a natural number, so **$-19\,122$ is NOT a term** *(1)*

QUESTION 3 [12 MARKS]

Consider the quadratic pattern 3 ; 8 ; 17 ; 30 ; 47 ; ...

3.1 Show that the pattern is quadratic and determine its general term. **(5)**

First differences 5; 9; 13; 17 *(1)*, second differences 4; 4; 4 — constant *(1)*
$2a = 4$ so $a = 2$ *(1)*; $3a + b = 5$ so $b = -1$ *(1)*; $a + b + c = 3$ so $c = 2$
$\mathbf{T_n = 2n^2 - n + 2}$ *(1)*

3.2 Determine which term of the pattern is equal to 122. **(4)**

$2n^2 - n - 120 = 0$ *(2)*
$(n - 8)(2n + 15) = 0$ *(1)*
$n = -7{,}5$ is rejected, so it is the **8th term** *(1)*

3.3 Determine the value of the FIRST difference between T_{20} and T_{21}. **(3)**

The first differences have the rule $a(2n + 1) + b = 4n + 1$ *(2)*
At $n = 20$: **81** *(1)*

TOTAL: 34 marks

This question paper consists of 3 questions.

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