MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
32	50 minutes	3

Name: _____ Class: _____

Date: _____

MARK ____ / 32

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The value of the sum of $2k$, for k running from 1 to 5, is ... (1)

A) 10 **B)** 20

C) 15 **D) 30 ✓**

> **Answer: D** — $2 + 4 + 6 + 8 + 10 = 30$.
>
> - **A** — gave only the LAST term
> - **B** — left out the last term, 10
> - **C** — added k from 1 to 5 without the factor of 2

1.2 In sigma notation, the number written BELOW the sigma is ... (1)

A) the last value of the counter **B)** the common difference

C) the first value of the counter ✓ **D)** the number of terms

> **Answer: C** — The lower limit says where the counting starts; the upper limit says where it stops.
>
> - **A** — that is the number written ABOVE the sigma
> - **B** — the difference lives inside the expression, not on the limits
> - **D** — the number of terms is upper − lower + 1

1.3 A sum runs with its counter from $k = 3$ to $k = 20$. The number of terms is ... (1)

A) 23 **B) 18 ✓**

C) 20 **D)** 17

> **Answer: B** — upper − lower + 1 = 20 − 3 + 1 = 18.
>
> - **A** — ADDED the two limits
> - **C** — used the upper limit on its own
> - **D** — forgot the + 1, which counts the starting term itself

1.4 A geometric series CONVERGES when ... (1)

A) $r \leq -1$ **B)** $r > 1$

C) $r \geq 1$ **D)** $-1 < r < 1$ ✓

> **Answer: D** — Only then do the terms shrink towards zero.
>
> - **A** — the terms grow in size and alternate in sign
> - **B** — the terms grow, so the sum has no limit
> - **C** — $r = 1$ makes every term the same, so the sum grows without limit

1.5 The sum to infinity of $12 + 6 + 3 + \ldots$ is ... (1)

A) 24 ✓ **B)** 21

C) 48 **D)** 12

> **Answer: A** — $S_\infty = 12 \div (1 - 0{,}5) = 24$.
>
> - **B** — added the first three terms and stopped
> - **C** — used $1 - r$ as $0{,}25$ instead of $0{,}5$
> - **D** — that is the first term

1.6 For which values of x does $1 + x + x^2 + x^3 + \ldots$ converge? (1)

A) $x > 1$ **B)** all real x

C) $-1 < x < 1$ ✓ **D)** x is not 0

> **Answer: C** — The common ratio is x, so convergence needs x to lie strictly between -1 and 1.
>
> - **A** — for $x > 1$ the terms grow and the sum diverges
> - **B** — for x of size 1 or more the series diverges
> - **D** — $x = 0$ is perfectly fine; it is LARGE values of x that break convergence

1.7 The value of the sum of $3k$, for k running from 1 to 10, is ... (1)

A) 150 **B)** 30

C) 165 ✓ **D)** 55

> **Answer: C** — $1 + 2 + \ldots + 10 = 55$, and $3 \times 55 = 165$.
>
> - **A** — used $1 + 2 + \ldots + 10 = 50$ instead of 55
> - **B** — gave only the last term, 3×10
> - **D** — left out the factor of 3

1.8 The value of the sum of the CONSTANT 4, for k running from 1 to 7, is ... (1)

A) 11 **B)** 28 ✓

C) 7 **D)** 4

> **Answer: B** — The constant is added seven times.
>
> - **A** — ADDED the constant to the number of terms
> - **C** — that is the number of terms
> - **D** — that is the constant itself, added once

1.9 A SERIES is the ... of a sequence. (1)

A) sum ✓ **B)** ratio

C) difference **D)** list

> **Answer: A** — A sequence lists the terms; a series adds them up.
> - **B** — the ratio belongs to a geometric SEQUENCE
> - **C** — the difference belongs to an arithmetic SEQUENCE
> - **D** — the list IS the sequence

1.10 If $S_\infty = 40$ and $a = 10$, then $r =$ ... (1)

A) 0,75 ✓ **B)** 4

C) 0,25 **D)** 30

> **Answer: A** — $10 \div (1 - r) = 40$ gives $1 - r = 0{,}25$, so $r = 0{,}75$.
> - **B** — that is $40 \div 10$, which equals $1 \div (1 - r)$
> - **C** — that is $1 - r$, not r
> - **D** — SUBTRACTED a from the sum instead of solving for r

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	C	**(D)**
1.2	A	B	**(C)**	D
1.3	A	**(B)**	C	D
1.4	A	B	C	**(D)**
1.5	**(A)**	B	C	D
1.6	A	B	**(C)**	D
1.7	A	B	**(C)**	D
1.8	A	**(B)**	C	D
1.9	**(A)**	B	C	D
1.10	**(A)**	B	C	D

QUESTION 2 [9 MARKS]

Consider the series $3 + 9 + 15 + \ldots + 273$.

2.1 Determine how many terms there are in the series. (3)

> $a = 3$ and $d = 6$ *(1)*
> $3 + (n - 1)6 = 273$ *(1)*
> $\boldsymbol{n = 46}$ *(1)*

2.2 Determine the sum of the series. (3)

> $S_{46} = \frac{46}{2}(3 + 273)$ *(2)* $= \mathbf{6\,348}$ *(1)*

2.3 Write the series in sigma notation. (3)

> $T_n = 6n - 3$ *(2)*
> $\boldsymbol{\Sigma_{n=1}^{46}(6n - 3)}$ *(1)*

QUESTION 3 [13 MARKS]

Answer the questions below on geometric series.

3.1 Determine the value of n if $\Sigma_{k=3}^{n}\ 8(2)^{k-1} = 131\,040$. **(5)**

The full sum from $k = 1$ is $8(2^n - 1)$ *(1)*
The first two terms are $8 + 16 = 24$ *(1)*
$8(2^n - 1) - 24 = 131\,040$ *(1)*
$2^n = 16\,384$ *(1)*
$\boldsymbol{n = 14}$ *(1)*

3.2 Gold is extracted from an old mine heap. Processing $1\,000$ tons of gravel yields 30 kg of gold the first time, 24 kg the second, $19{,}2$ kg the third and so on. Determine the MAXIMUM amount of gold that can be recovered. **(4)**

$r = \frac{24}{30} = 0{,}8$, and $|r| < 1$ so the series converges *(2)*

$S_\infty = \frac{30}{1 - 0{,}8}$ *(1)* $= \mathbf{150\ kg}$ *(1)*

3.3 Determine the values of x for which the series $\Sigma 3(2x)^{n-1}$ converges. **(4)**

$r = 2x$ *(1)*
$|2x| < 1$ *(1)*
$-1 < 2x < 1$ *(1)*
$\boldsymbol{-\frac{1}{2} < x < \frac{1}{2}}$ *(1)*

TOTAL: 32 marks

This question paper consists of 3 questions.

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