MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 29

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 The general term of an arithmetic sequence is $T_n = 2n + 4$. Determine T_{10}. **(1)**

$2(10) + 4 = \mathbf{24}$ ✓ (1)

2 Determine the sum of the first 19 terms of $T_n = 2n + 4$. **(3)**

$a = 6,\ d = 2.\ S_{19} = \frac{19}{2}[12 + 36] = \mathbf{456}$ ✓✓✓ (3)

3 Evaluate $\sum_{n=1}^{20} (3n - 1)$ **(3)**

$a = 2,\ d = 3,\ n = 20.\ S = 10[4 + 57] = \mathbf{610}$ ✓✓✓ (3)

4 How many terms are there in $\sum_{n=3}^{10} T_n$? **(1)**

$10 - 3 + 1 = \mathbf{8\ terms}$ ✓ (1)

5 Write $2 + 5 + 8 + \ldots + 59$ in sigma notation. **(3)**

$T_n = 3n - 1$ and $3n - 1 = 59$ gives $n = 20$, so $\mathbf{\sum_{n=1}^{20} (3n - 1)}$ ✓✓✓ (3)

6 Determine S_∞ for $8 + 4 + 2 + 1 + \ldots$ **(2)**

$r = \frac{1}{2}$, so $S_\infty = \frac{8}{1 - \frac{1}{2}} = \mathbf{16}$ ✓✓ (2)

7 Determine the values of x for which $\sum 3(2x)^{n-1}$ converges. **(3)**

$-1 < 2x < 1$, so $\mathbf{-\frac{1}{2} < x < \frac{1}{2}}$ ✓✓✓ (3)

8 Calculate S_{10} for the geometric series $2 + 6 + 18 + \ldots$ **(3)**

$a = 2,\ r = 3.\ S_{10} = \frac{2(3^{10} - 1)}{2} = \mathbf{59\ 048}$ ✓✓✓ (3)

9 Derive the formula $S_n = \frac{n}{2}(a + l)$ for an arithmetic series. **(4)**

> Write S_n forwards and backwards ✓; adding gives $2S_n = n(a + l)$ because each pair sums to $a + l$ ✓✓; $\therefore S_n = \frac{n}{2}(a + l)$ ✓ **(4)**

10 Derive the formula for the sum of the first n terms of a geometric series. **(4)**

> $S_n = a + ar + \ldots + ar^{n-1}$; multiply by r ✓; subtract to get $rS_n - S_n = ar^n - a$ ✓; factorise: $S_n(r - 1) = a(r^n - 1)$ ✓; $S_n = a\frac{r^n - 1}{r - 1}$, $r \neq 1$ ✓ **(4)**

11 Explain why the sum to infinity only exists when $-1 < r < 1$. **(2)**

> Only then does r^n approach 0 as n grows ✓, so the terms shrink and the total settles on a finite value ✓ **(2)**

TOTAL: 29 marks

© Danemathics — free to print and share for classroom use.