GRADE 11 · EQUATIONS AND INEQUALITIES

Simultaneous Equations: One Linear, One Quadratic (Grade 11)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
32	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 32

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 Solve simultaneously: $y = x + 1$ and $y = x^2 - 1$ (1)

A) (2 ; 3) and (−1 ; 0) ✓
B) (−1 ; 0) only
C) (2 ; 3) and (1 ; 2)
D) (2 ; 3) only

Answer: A — $x + 1 = x^2 - 1$ gives $x^2 - x - 2 = (x - 2)(x + 1) = 0$.

- **B** — only one of the two solutions was given
- **C** — $x = 1$ does not satisfy $x^2 - x - 2 = 0$
- **D** — a line can cut a parabola TWICE — the second point was dropped

1.2 For one LINEAR and one QUADRATIC equation, the standard method is to ... (1)

A) subtract the two equations
B) add the two equations
C) make a variable the subject of the QUADRATIC equation, then substitute
D) make a variable the subject of the LINEAR equation, then substitute ✓

Answer: D — The linear equation rearranges cleanly; the quadratic one does not.

- **A** — subtracting works only for two LINEAR equations
- **B** — adding works only for two LINEAR equations
- **C** — rearranging the quadratic brings in a square root and both of its signs

1.3 Solve simultaneously: $y = 2x$ and $x^2 + y^2 = 20$ (1)

A) $(4\ ;\ 2)$ and $(-4\ ;\ -2)$ **B)** $(2\ ;\ 4)$ and $(-2\ ;\ 4)$

C) $(2\ ;\ 4)$ and $(-2\ ;\ -4)$ ✓ **D)** $(2\ ;\ 4)$ only

> **Answer: C** — Substituting gives $5x^2 = 20$, so $x = \pm 2$ and $y = 2x$ follows.
> - **A** — the coordinates are the wrong way round
> - **B** — $y = 2x$, so a negative x gives a NEGATIVE y
> - **D** — $x^2 = 4$ has TWO solutions, so there are two points

1.4 How many solutions can a straight line and a parabola have? (1)

A) always 1 **B) 0, 1 or 2 ✓**

C) 0 or 1 only **D)** always 2

> **Answer: B** — The line may miss the parabola, touch it as a tangent, or cut it twice.
> - **A** — a line usually cuts a parabola twice
> - **C** — a line can cut a parabola twice
> - **D** — a line that misses the parabola gives no solution at all

1.5 Solve simultaneously: $x + y = 5$ and $xy = 6$ (1)

A) $(2\ ;\ 3)$ only **B)** $(-2\ ;\ -3)$ and $(-3\ ;\ -2)$

C) $(2\ ;\ 3)$ and $(3\ ;\ 2)$ ✓ **D)** $(1\ ;\ 6)$ and $(6\ ;\ 1)$

> **Answer: C** — $y = 5 - x$ gives $x^2 - 5x + 6 = 0$, so $x = 2$ or $x = 3$.
> - **A** — both roots give a valid point, so there are TWO solutions
> - **B** — $(-2)(-3) = 6$, but $-2 + (-3) = -5$, not $+5$
> - **D** — $1 \times 6 = 6$, but $1 + 6 = 7$, not 5

1.6 If substituting produces a quadratic with a NEGATIVE discriminant, then ... (1)

A) the line and the curve do not meet ✓ **B)** there are infinitely many solutions

C) there are two points of intersection **D)** the line is a tangent

> **Answer: A** — No real roots means no real point of intersection.
> - **B** — a line and a curve never share infinitely many points
> - **C** — two points need $\Delta > 0$
> - **D** — a tangent gives $\Delta = 0$

1.7 If substituting produces a quadratic with $\Delta = 0$, the line is ... (1)

A) parallel to the curve **B) a tangent to the curve ✓**

C) a secant cutting it twice **D)** missing the curve entirely

> **Answer: B** — One repeated root means the line touches the curve at exactly one point.
> - **A** — 'parallel' has no meaning for a line and a parabola
> - **C** — cutting twice needs $\Delta > 0$
> - **D** — missing it entirely needs $\Delta < 0$

1.8 Solve simultaneously: $y = x - 1$ and $x^2 + y^2 = 25$ (1)

A) $(4\ ;\ 3)$ and $(3\ ;\ 4)$ **B) $(4\ ;\ 3)$ and $(-3\ ;\ -4)$ ✓**

C) $(3\ ;\ 4)$ and $(-4\ ;\ -3)$ **D)** $(4\ ;\ 3)$ only

> **Answer: B** — Substituting gives $x^2 - x - 12 = 0$, so $x = 4$ or $x = -3$, and $y = x - 1$.
> - **A** — $(3\ ;\ 4)$ does not satisfy $y = x - 1$
> - **C** — the coordinates are the wrong way round
> - **D** — the quadratic has TWO roots, so there are two points

1.9 Two numbers have a product of 24 and a sum of 11. The numbers are ... (1)

A) 3 and 8 ✓ **B)** 2 and 12

C) 1 and 24 **D)** 4 and 6

> **Answer: A** — $x^2 - 11x + 24 = 0$ gives $x = 3$ or $x = 8$, and $3 + 8 = 11$.
> - **B** — $2 \times 12 = 24$, but $2 + 12 = 14$, not 11
> - **C** — $1 \times 24 = 24$, but $1 + 24 = 25$, not 11
> - **D** — $4 \times 6 = 24$, but $4 + 6 = 10$, not 11

1.10 A rectangle has perimeter 26 cm and area 40 cm². Its dimensions are ... (1)

A) 6 cm by 7 cm **B)** 4 cm by 10 cm

C) 2 cm by 20 cm **D) 5 cm by 8 cm ✓**

> **Answer: D** — $l + b = 13$ and $lb = 40$ give $x^2 - 13x + 40 = 0$, so the sides are 5 and 8.
> - **A** — $6 + 7 = 13$ gives the right perimeter, but $6 \times 7 = 42$, not 40
> - **B** — $4 \times 10 = 40$, but that perimeter is 28, not 26
> - **C** — $2 \times 20 = 40$, but that perimeter is 44, not 26

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	C	(D)
1.3	A	B	(C)	D
1.4	A	(B)	C	D
1.5	A	B	(C)	D
1.6	(A)	B	C	D
1.7	A	(B)	C	D
1.8	A	(B)	C	D
1.9	(A)	B	C	D
1.10	A	B	C	(D)

QUESTION 2 [10 MARKS]

Solve the following simultaneously. Show ALL your working.

2.1 $y = x + 1$ and $y = x^2 - 3x + 4$ (5)

> $x + 1 = x^2 - 3x + 4$ *(1)*
> $x^2 - 4x + 3 = 0$ *(1)*
> $(x - 1)(x - 3) = 0$ *(1)*
> $x = 1$ gives $y = 2$ *(1)*; $x = 3$ gives $y = 4$ *(1)*

2.2 $x + y = 5$ and $x^2 + y^2 = 17$ **(5)**

> $y = 5 - x$ *(1)*
> $x^2 + (5 - x)^2 = 17$ *(1)*
> $2x^2 - 10x + 8 = 0$ *(1)*
> $(x - 1)(x - 4) = 0$ *(1)*
> **(1 ; 4) and (4 ; 1)** *(1)*

QUESTION 3 [12 MARKS]

Answer the questions below.

3.1 Solve simultaneously: $y - x = 2$ and $xy = 8$ **(5)**

> $y = x + 2$ *(1)*
> $x(x + 2) = 8$ *(1)*
> $x^2 + 2x - 8 = 0$ *(1)*
> $(x + 4)(x - 2) = 0$ *(1)*
> **(2 ; 4) and (−4 ; −2)** *(1)*

3.2 Explain which of the two equations should always be rearranged first, and why. **(2)**

> The LINEAR one *(1)* — making a variable its subject introduces no squares, so the substitution stays simple *(1)*

3.3 The line $y = x + c$ cuts the parabola $y = x^2$ at two points. Determine the values of c for which this happens. **(5)**

> $x^2 = x + c$ gives $x^2 - x - c = 0$ *(2)*
> Two points means $\Delta > 0$ *(1)*
> $1 + 4c > 0$ *(1)*
> $c > -\frac{1}{4}$ *(1)*

TOTAL: 32 marks

This question paper consists of 3 questions.

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