∑ DANEMATHICS
FREE CAPS MATHS RESOURCES

GRADE 11 · TRIGONOMETRY
Sine, Cosine & Area Rules (Grade 11)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
50	1 hour 15 minutes	4

Name: ..... Class: .....

Date: .....

MARK ____ / 50

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The SINE rule is the one to use when you know ... (1)

A) all three sides
B) two angles and a side ✓
C) all three angles
D) two sides and the angle BETWEEN them

Answer: B — The sine rule pairs each side with the angle OPPOSITE it, so it needs one complete pair.

- **A** — that is the cosine rule's case
- **C** — three angles fix the shape but no side length
- **D** — that is the cosine rule's case

1.2 The COSINE rule $a^2 = b^2 + c^2 - 2bc$ cos A is used when ... (1)

A) only one side is known
B) the triangle is right-angled
C) two sides and the INCLUDED angle are known ✓
D) two angles and a side are known

Answer: C — The cosine rule needs the angle BETWEEN the two known sides, or else all three sides.

- **A** — one side is never enough
- **B** — for a right-angled triangle, Pythagoras and SOH-CAH-TOA are simpler
- **D** — that is the sine rule's case

1.3 In $\triangle ABC$, $A = 40°$, $B = 60°$ and $a = 10$. Then $b =$ ... (1)

A) 8,66 **B)** 15,32

C) 13,47 ✓ **D)** 7,42

> **Answer: C** — $b \div \sin B = a \div \sin A$, so $b = 10 \sin 60° \div \sin 40° = 13{,}47$.
> - **A** — used $10 \sin 60°$ without dividing by $\sin 40°$
> - **B** — used the third angle, $80°$, in place of $60°$
> - **D** — inverted the ratio: that is $10 \sin 40° \div \sin 60°$

1.4 In $\triangle PQR$, $p = 5$, $q = 7$ and $R = 60°$. Then $r =$ ... (1)

A) 6,24 ✓ **B)** 39

C) 10,44 **D)** 2

> **Answer: A** — $r^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 74 - 35 = 39$, so $r = \sqrt{39} = 6{,}24$.
> - **B** — forgot to take the square root of 39
> - **C** — ADDED the 35 instead of subtracting it
> - **D** — used $7 - 5$ and ignored the cosine rule altogether

1.5 The AREA rule states that the area of a triangle is ... (1)

A) $\frac{1}{2}ab \sin C$ **✓** **B)** $\frac{1}{2}$ × base × height only

C) $ab \sin C$ **D)** $\frac{1}{2}ab \cos C$

> **Answer: A** — The area rule uses two sides and the SINE of the angle between them.
> - **B** — that formula needs a perpendicular height, which a general triangle rarely gives
> - **C** — the ½ has been left off
> - **D** — the area rule uses the SINE of the included angle, not its cosine

1.6 Find the area of $\triangle ABC$ if $b = 8$, $c = 6$ and $A = 30°$. (1)

A) 24 **B) 12 ✓**

C) 48 **D)** 20,78

> **Answer: B** — Area $= \frac{1}{2}(8)(6)\sin 30° = 24 \times 0{,}5 = 12$.
> - **A** — left off the $\sin 30°$ factor
> - **C** — left off both the ½ and the sine
> - **D** — used $\cos 30°$ instead of $\sin 30°$

1.7 In $\triangle ABC$, $a = 9$, $b = 7$ and $c = 5$. Then $\cos A =$ ... (1)

A) -7 **B)** 0,1

C) $-0{,}1$ ✓ **D)** 0,83

> **Answer: C** — $\cos A = (7^2 + 5^2 - 9^2) \div (2 \times 7 \times 5) = -7 \div 70 = -0{,}1$.
> - **A** — forgot to divide by $2bc$
> - **B** — dropped the minus — $49 + 25$ is LESS than 81
> - **D** — used $a^2 + b^2 - c^2$, which finds $\cos C$ instead

1.8 Hence, in that triangle, $\hat{A} = \ldots$ (1)

A) $1{,}67°$ **B)** $84{,}26°$

C) $5{,}74°$ **D)** **$95{,}74°$ ✓**

> **Answer: D** — $A = \cos^{-1}(-0{,}1) = 95{,}74°$, which is obtuse because the cosine is negative.
> - **A** — the calculator was in RADIAN mode
> - **B** — that is $\cos^{-1}(+0{,}1)$ — the minus was dropped
> - **C** — subtracted $90°$ from the correct answer

1.9 The AMBIGUOUS case of the sine rule can arise when ... (1)

A) two sides and a NON-included angle are given ✓ **B)** one of the angles is a right angle

C) all three sides are given **D)** two angles and a side are given

> **Answer: A** — The unknown angle may be acute or obtuse, and both can fit the given lengths.
> - **B** — a right angle leaves no ambiguity
> - **C** — three sides fix the triangle completely
> - **D** — two angles fix the third, so the triangle is determined

1.10 In $\triangle ABC$, if $C = 90°$ the cosine rule becomes ... (1)

A) $c = a + b$ **B)** $c^2 = a^2 + b^2 - 2ab$

C) $c^2 = a^2 - b^2$ **D)** **$c^2 = a^2 + b^2$ ✓**

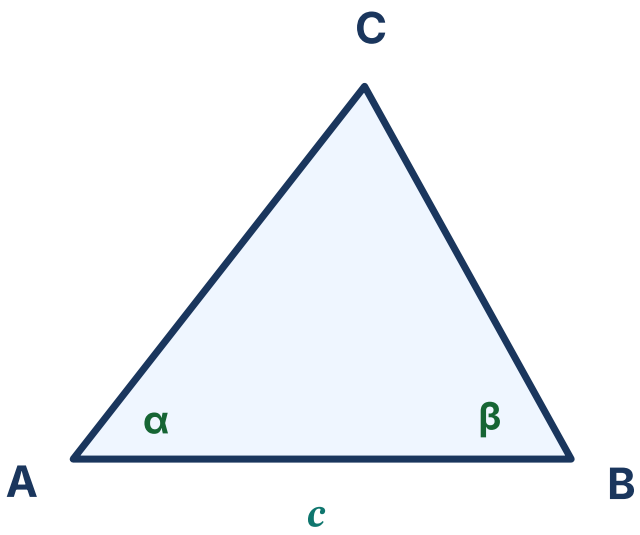
> **Answer: D** — $\cos 90° = 0$, so the $-2ab \cos C$ term vanishes and Pythagoras is what is left.
> - **A** — the theorem relates the SQUARES of the sides, not the sides
> - **B** — $\cos 90° = 0$, so the whole $2ab$ term goes, not just the cosine
> - **C** — the term that vanishes is $-2ab \cos C$, not the $+ b^2$

ANSWER GRID — MARKING GUIDELINE

1.1	A	**(B)**	C	D
1.2	A	B	**(C)**	D
1.3	A	B	**(C)**	D
1.4	**(A)**	B	C	D
1.5	**(A)**	B	C	D
1.6	A	**(B)**	C	D
1.7	A	B	**(C)**	D
1.8	A	B	C	**(D)**
1.9	**(A)**	B	C	D
1.10	A	B	C	**(D)**

QUESTION 2 **[12 MARKS]**

In the diagram below, $\triangle ABC$ is drawn with $B\hat{A}C = \alpha$, $A\hat{B}C = \beta$ and $AB = c$.



2.1 Write down the size of $A\hat{C}B$ in terms of α and β. **(1)**

$A\hat{C}B = \mathbf{180° - \alpha - \beta}$ *(1)*

2.2 Show that $BC = \dfrac{c \cdot \sin \alpha}{\sin(\alpha + \beta)}$. **(4)**

By the sine rule: $\dfrac{BC}{\sin \hat{A}} = \dfrac{AB}{\sin \hat{C}}$ *(1)*

$BC = \dfrac{c \cdot \sin \alpha}{\sin(180° - \alpha - \beta)}$ *(1)*

$\sin(180° - \theta) = \sin \theta$ *(1)*

$\therefore BC = \dfrac{c \cdot \sin \alpha}{\sin(\alpha + \beta)}$ *(1)*

2.3 Given that $\alpha = 52°$, $\beta = 61°$ and $c = 48$ m, calculate the length of BC, correct to TWO decimal places. **(3)**

$BC = \dfrac{48 \sin 52°}{\sin 113°}$ *(2)*

$= \mathbf{41{,}09\ m}$ *(1)*

2.4 Hence calculate the area of $\triangle ABC$, correct to TWO decimal places. **(4)**

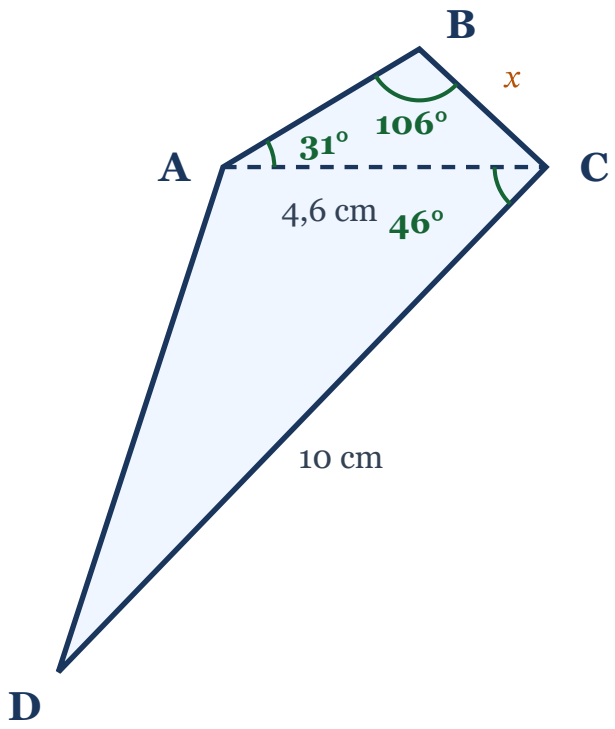
Area $= \frac{1}{2} \cdot AB \cdot BC \cdot \sin \hat{B}$ *(the area rule, with the INCLUDED angle)* *(1)*

$= \frac{1}{2} \times 48 \times 41{,}09 \times \sin 61°$ *(2)*

$= \mathbf{862{,}54\ m^2}$ *(1)*

QUESTION 3 **[15 MARKS]**

In the diagram below, ABCD is a quadrilateral with the diagonal AC drawn. $A\hat{B}C = 106°$, $B\hat{A}C = 31°$, $A\hat{C}D = 46°$, AC = 4,6 cm and CD = 10 cm. BC = x. *The diagram is not drawn to scale.*



3.1 Calculate the length of x, correct to TWO decimal places. **(3)**

In △ABC: $\frac{x}{\sin 31°} = \frac{4,6}{\sin 106°}$ *(the sine rule) (1)*

$x = \frac{4,6 \sin 31°}{\sin 106°}$ *(1)*

$= \mathbf{2,46\ cm}$ *(1)*

3.2 Calculate the area of △ABC, correct to TWO decimal places. **(4)**

$A\hat{C}B = 180° - 106° - 31° = 43°$ *(1)*

Area = ½ × AC × x × sin($A\hat{C}B$) *(1)*

= ½ × 4,6 × 2,4646... × sin 43° *(1)* — use the unrounded x

$= \mathbf{3,87\ cm^2}$ *(1)*

3.3 Calculate the length of AD, correct to TWO decimal places. **(4)**

In △ACD: $AD^2 = AC^2 + CD^2 - 2(AC)(CD)\cos(A\hat{C}D)$ *(the cosine rule) (1)*

$= 4,6^2 + 10^2 - 2(4,6)(10)\cos 46°$ *(1)*

$= 57,25$ *(1)*

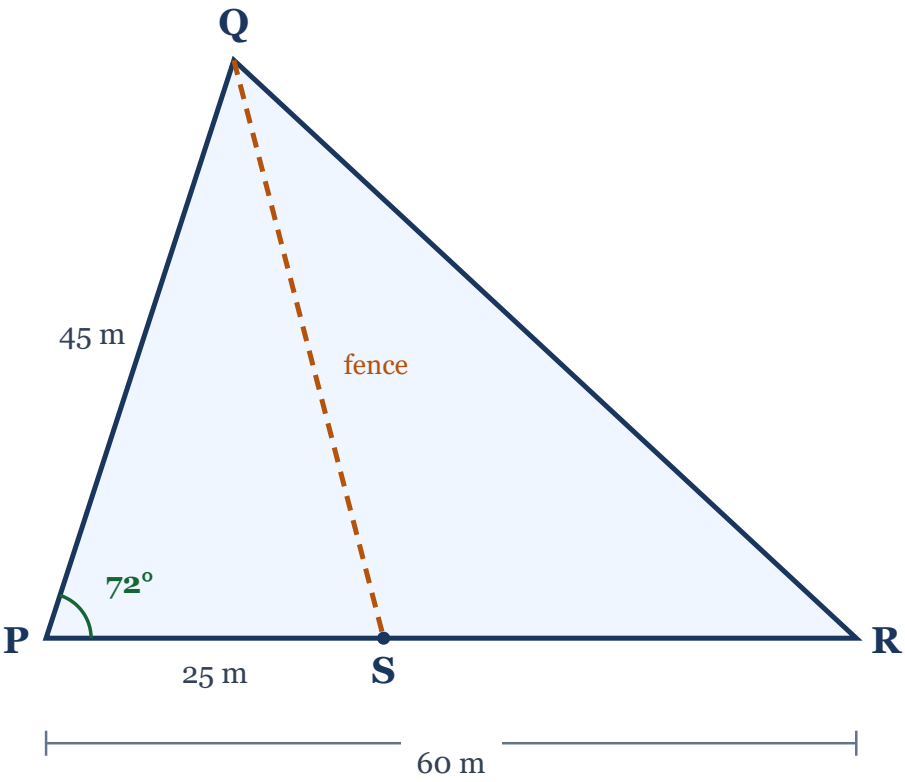
AD = **7,57 cm** *(1)*

3.4 Hence calculate the area of quadrilateral ABCD, correct to TWO decimal places. **(4)**

Area $\triangle ACD = \frac{1}{2} \times 4{,}6 \times 10 \times \sin 46°$ *(1)*
$= 16{,}54 \text{ cm}^2$ *(1)*
Area ABCD $= 3{,}87 + 16{,}54$ *(1)*
$= \mathbf{20{,}41 \text{ cm}^2}$ *(1)*

QUESTION 4 [13 MARKS]

The diagram below shows a triangular plot of land PQR with PQ = 45 m, PR = 60 m and $Q\hat{P}R = 72°$. A straight fence QS is built from Q to the point S on PR, with PS = 25 m.



4.1 Calculate the length of QR, correct to TWO decimal places. **(3)**

$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos \hat{P}$ *(the cosine rule) (1)*
$= 45^2 + 60^2 - 2(45)(60)\cos 72°$ *(1)*
$QR = \mathbf{62{,}90 \text{ m}}$ *(1)*

4.2 Calculate the size of $P\hat{R}Q$, correct to ONE decimal place. **(3)**

$\frac{\sin P\hat{R}Q}{45} = \frac{\sin 72°}{62{,}90}$ *(the sine rule) (1)*
$\sin P\hat{R}Q = \frac{45 \sin 72°}{62{,}90} = 0{,}6804$ *(1)*
$P\hat{R}Q = \mathbf{42{,}9°}$ *(1)*

4.3 Calculate the area of $\triangle PQR$, correct to TWO decimal places. **(3)**

Area $= \frac{1}{2} \times PQ \times PR \times \sin \hat{P}$ *(the area rule) (1)*
$= \frac{1}{2} \times 45 \times 60 \times \sin 72°$ *(1)*
$= \mathbf{1\,283{,}93 \text{ m}^2}$ *(1)*

4.4 Calculate the length of the fence QS, correct to TWO decimal places. **(4)**

In $\triangle PQS$: $QS^2 = PQ^2 + PS^2 - 2(PQ)(PS)\cos \hat{P}$ *(the cosine rule)* *(1)*
$= 45^2 + 25^2 - 2(45)(25)\cos 72°$ *(1)*
$= 1\,954{,}71$ *(1)*
$QS =$ **44,21 m** *(1)*

TOTAL: 50 marks

This question paper consists of 4 questions.

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