Sketching the Parabola $y = ax^2 + q$ (Grade 10)

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The turning point of $y = x^2 - 4$ is ... (1)

A) $(0\ ;\ 4)$ **B)** $(0\ ;\ -4)$ ✓

C) $(-4\ ;\ 0)$ **D)** $(2\ ;\ 0)$

Answer: B — For $y = ax^2 + q$ the turning point is $(0\ ;\ q)$, and here $q = -4$.

- **A** — the sign of q has been changed
- **C** — the coordinates are swapped: x comes first
- **D** — $(2\ ;\ 0)$ is an x-INTERCEPT, not the turning point

1.2 The range of $y = x^2 + 2$ is ... (1)

A) $y \leq 2$ **B)** $y \in \mathbb{R}$

C) $y > 2$ **D)** $y \geq 2$ ✓

Answer: D — It opens upward, so the lowest y-value is the turning point's, 2 — and 2 itself is reached.

- **A** — that is the range of a parabola that opens DOWNWARDS
- **B** — a parabola never covers every y-value
- **C** — the turning point $(0\ ;\ 2)$ is ON the graph, so 2 is included

1.3 The x-intercepts of $y = x^2 - 9$ are ... (1)

A) $x = -3$ and $x = 3$ ✓ **B)** $x = 9$ only

C) $x = 3$ only **D)** $x = -9$ and $x = 9$

Answer: A — $x^2 = 9$ has two solutions, -3 and 3.

- **B** — $x^2 = 9$ has TWO solutions, and neither is 9
- **C** — $x^2 = 9$ also has the negative solution -3
- **D** — the square ROOT of 9 is 3, not 9

1.4 The graph of $y = -2x^2 + 5$ ... (1)

A) has a minimum turning point

B) does not cut the y-axis

C) has a maximum turning point ✓

D) is symmetric about the x-axis

> **Answer: C** — $a = -2$ is negative, so the graph opens downward and $(0\ ;\ 5)$ is a maximum.
>
> - **A** — a is NEGATIVE, so it opens downwards and has a maximum
> - **B** — every parabola $y = ax^2 + q$ cuts the y-axis, at $(0\ ;\ q)$
> - **D** — its axis of symmetry is the y-AXIS, $x = 0$

1.5 The axis of symmetry of $y = 3x^2 - 7$ is ... (1)

A) $y = -7$

B) $x = 0$ ✓

C) $x = 3$

D) $x = -7$

> **Answer: B** — Every graph $y = ax^2 + q$ is symmetric about the y-axis, $x = 0$.
>
> - **A** — that is the y-value of the turning point, not the axis
> - **C** — 3 is a, which changes the shape, not the axis
> - **D** — -7 is q, which moves the graph up or down, not sideways

1.6 How many times does $y = x^2 + 4$ cut the x-axis? (1)

A) none ✓

B) once

C) twice

D) three times

> **Answer: A** — $x^2 + 4 = 0$ would need $x^2 = -4$, which has no real solution.
>
> - **B** — $x^2 + 4 = 0$ has no real solution, so there is no point at all
> - **C** — $x^2 = -4$ has no real solution
> - **D** — a parabola can cut the x-axis at most twice

1.7 Compared with $y = x^2$, the graph of $y = x^2 - 5$ is ... (1)

A) 5 units to the right

B) 5 units to the left

C) 5 units up

D) 5 units down ✓

> **Answer: D** — $q = -5$ moves every point 5 units down.
>
> - **A** — q moves the graph UP or DOWN, never sideways
> - **B** — q moves the graph UP or DOWN, never sideways
> - **C** — $q = -5$ is negative, so the graph moves DOWN

1.8 In $y = ax^2 + q$, the value of q is ... (1)

A) the gradient of the graph

B) one of the x-intercepts

C) the y-value of the turning point ✓

D) the axis of symmetry

> **Answer: C** — The turning point is $(0\ ;\ q)$.
>
> - **A** — a parabola has no single gradient
> - **B** — the x-intercepts come from solving $ax^2 + q = 0$
> - **D** — the axis of symmetry is $x = 0$ for every graph of this form

1.9 The domain of $y = 2x^2 + 1$ is ... (1)

A) $x \in \mathbb{R}$ ✓ **B)** $x \geq 1$

C) $x \geq 0$ **D)** $y \geq 1$

> **Answer: A** — Any real number can be squared, so every x is allowed.
> - **B** — 1 is where the RANGE starts, and the range is about y
> - **C** — negative x-values can be squared too
> - **D** — that is the RANGE, not the domain

1.10 $y = ax^2 + q$ has turning point $(0\ ;\ -3)$ and passes through $(1\ ;\ -1)$. The value of a is ... (1)

A) -2 **B)** 2 ✓

C) -1 **D)** 4

> **Answer: B** — $q = -3$, and $-1 = a(1)^2 - 3$ gives $a = 2$.
> - **A** — the sign is wrong: $a - 3 = -1$ gives $a = 2$
> - **C** — that is the y-value of the point, not a
> - **D** — added 3 and 1 instead of solving $a - 3 = -1$

ANSWER GRID — MARKING GUIDELINE

1.1	A	**(B)**	C	D
1.2	A	B	C	**(D)**
1.3	**(A)**	B	C	D
1.4	A	B	**(C)**	D
1.5	A	**(B)**	C	D
1.6	**(A)**	B	C	D
1.7	A	B	C	**(D)**
1.8	A	B	**(C)**	D
1.9	**(A)**	B	C	D
1.10	A	**(B)**	C	D

QUESTION 2 [12 MARKS]

Given $f(x) = x^2 - 4$.

2.1 Write down whether the parabola opens upwards or downwards, and give a reason. (2)

> It opens **UPWARDS** *(1)*
> $a = 1$, which is positive *(1)*

2.2 Calculate the coordinates of the x-intercepts and the y-intercept. (4)

> $y = 0$: $x^2 - 4 = 0$ *(1)*
> $x^2 = 4$, so **$(-2\ ;\ 0)$ and $(2\ ;\ 0)$** *(2)*
> $x = 0$: **$(0\ ;\ -4)$** *(1)*

2.3 Write down the coordinates of the turning point and the equation of the axis of symmetry. (3)

> Turning point **$(0\ ;\ -4)$** *(2)* — for $y = ax^2 + q$ it is always $(0\ ;\ q)$
> Axis of symmetry **$x = 0$** *(1)*

2.4 Write down the domain and the range of f. **(3)**

> Domain: $x \in \mathbb{R}$ *(1)*
> Range: $y \geq -4$ *(2)* — the turning point is a minimum

QUESTION 3 [13 MARKS]

$g(x) = ax^2 + q$ has a turning point at $(0\ ;\ 8)$ and passes through the point $(2\ ;\ 0)$.

3.1 Write down the value of q, and show that $a = -2$. **(3)**

> $q = 8$ *(1)* — the turning point is $(0\ ;\ q)$
> $0 = a(2)^2 + 8$ *(1)*
> $4a = -8$, so $a = -2$ *(1)*

3.2 Write down the coordinates of the other x-intercept, and give a reason. **(3)**

> $(-2\ ;\ 0)$ *(2)*
> The parabola is symmetric about the y-axis *(1)*

3.3 Write down the range of g, and explain your answer. **(3)**

> $y \leq 8$ *(2)*
> $a < 0$, so $(0\ ;\ 8)$ is a maximum *(1)*

3.4 Describe, in words, how $y = x^2$ is changed to give $g(x) = -2x^2 + 8$. **(4)**

> It is reflected in the x-axis *(1)*
> and made narrower, stretched by a factor of 2 *(1)*
> then moved 8 units UP *(2)*

TOTAL: 35 marks

This question paper consists of 3 questions.

© Danemathics — free to print and share for classroom use.