MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
48	1 hour 15 minutes	4

Name: ..... Class: .....

Date: .....

MARK ____ / 48

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 On a tree diagram, the probabilities on the branches leaving any one point add up to ... (1)

A) 0 **B)** 1 ✓

C) 0,5 **D)** the number of branches

Answer: B — One of the available outcomes must happen.

- **A** — the probabilities of the possible outcomes cannot total zero
- **C** — 0,5 applies only when there are exactly two equally likely branches
- **D** — the COUNT of branches is not a probability

1.2 To find the probability of one complete PATH through a tree diagram, you ... (1)

A) multiply and then subtract from 1 **B)** take the largest branch probability

C) add along the branches **D)** **multiply along the branches** ✓

Answer: D — Each step is conditional on the one before it, so the probabilities multiply.

- **A** — there is nothing to subtract
- **B** — every step along the path matters, not just the largest
- **C** — adding is for combining SEPARATE paths, not the steps within one path

1.3 Two coins are tossed. P(exactly one head) = ... (1)

A) $\frac{1}{4}$ **B)** $\frac{3}{4}$

C) $\frac{1}{2}$ ✓ **D)** $\frac{1}{3}$

> **Answer: C** — HT and TH are two of the four equally likely outcomes.
> - **A** — that counts only ONE of the two orders
> - **B** — that is P(at least one head)
> - **D** — there are FOUR equally likely outcomes, not three

1.4 A bag holds 3 red and 2 green sweets. Two are taken WITHOUT replacement. P(red then green) = ... (1)

A) $\frac{3}{5}$ **B)** $\frac{3}{10}$ ✓

C) $\frac{1}{2}$ **D)** $\frac{6}{25}$

> **Answer: B** — $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.
> - **A** — used the first draw's probability alone
> - **C** — used the second draw's probability alone
> - **D** — treated the draws as WITH replacement: $\frac{3}{5} \times \frac{2}{5}$

1.5 In a two-way table, the row totals added together give ... (1)

A) the number of rows **B)** the largest single cell

C) 1 **D) the grand total** ✓

> **Answer: D** — Every observation falls into exactly one row and exactly one column.
> - **A** — counting rows counts categories, not observations
> - **B** — one cell is only part of the total
> - **C** — a table holds counts; only probabilities total 1

1.6 Of 200 people, 120 own a car, 90 own a bicycle and 40 own both. P(owns a car OR a bicycle) = ... (1)

A) 0,85 ✓ **B)** 1,05

C) 0,6 **D)** 0,2

> **Answer: A** — $120 + 90 - 40 = 170$, and $170 \div 200 = 0{,}85$.
> - **B** — forgot to subtract the 40 who were counted twice
> - **C** — used the car owners only
> - **D** — used the 40 who own both only

1.7 For that survey, are 'owns a car' and 'owns a bicycle' INDEPENDENT? (1)

A) no — 0,2 does not equal $0{,}6 \times 0{,}45$ ✓ **B)** yes — both probabilities are less than 1

C) yes — 40 divides into 200 exactly **D)** it cannot be decided from a two-way table

> **Answer: A** — Independence needs P(both) = P(car) × P(bicycle), and $0{,}6 \times 0{,}45 = 0{,}27$, not 0,2.
> - **B** — every probability is at most 1, which says nothing about independence
> - **C** — whether the counts divide evenly is irrelevant
> - **D** — a two-way table supplies exactly the numbers this test needs

1.8 A tree diagram is more useful than a Venn diagram when ... (1)

A) the events are mutually exclusive

B) the events happen in stages, one after another ✓

C) all the probabilities are equal

D) there are exactly two events

> **Answer: B** — A tree shows the SEQUENCE, and lets conditional probabilities be written on the branches.
> - **A** — a Venn diagram shows mutual exclusivity most clearly of all
> - **C** — equal probabilities suit either diagram
> - **D** — a Venn diagram handles two events perfectly well

1.9 Events A and B are independent, with P(A) = 0,4 and P(B) = 0,5. Then P(A or B) equals ... (1)

A) 0,9

B) 0,2

C) 0,7 ✓

D) 0,1

> **Answer: C** — P(A and B) = $0,4 \times 0,5 = 0,2$ for independent events, so P(A or B) = $0,4 + 0,5 - 0,2 = 0,7$.
> - **A** — ADDED without subtracting the overlap P(A and B)
> - **B** — that is P(A and B), not P(A or B)
> - **D** — subtracted instead of adding the two probabilities

1.10 Three coins are tossed. P(all three the same) = ... (1)

A) $\frac{1}{8}$

B) $\frac{1}{2}$

C) $\frac{3}{8}$

D) $\frac{1}{4}$ ✓

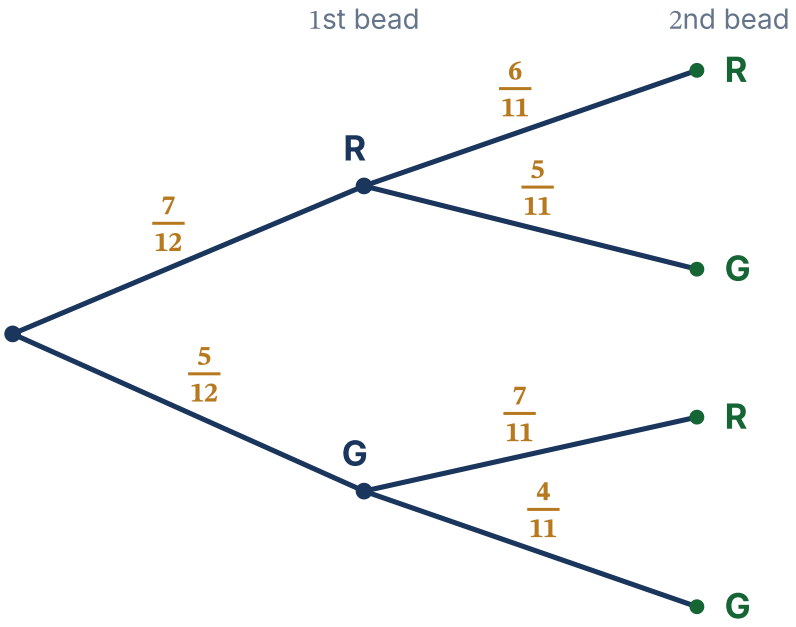
> **Answer: D** — HHH and TTT are two of the eight equally likely outcomes.
> - **A** — that counts only HHH and leaves out TTT
> - **B** — that is the chance of TWO coins matching, not three
> - **C** — $\frac{3}{8}$ is the chance of exactly two heads, not of all three matching

ANSWER GRID — MARKING GUIDELINE

1.1	A	**(B)**	C	D
1.2	A	B	C	**(D)**
1.3	A	B	**(C)**	D
1.4	A	**(B)**	C	D
1.5	A	B	C	**(D)**
1.6	**(A)**	B	C	D
1.7	**(A)**	B	C	D
1.8	A	**(B)**	C	D
1.9	A	B	**(C)**	D
1.10	A	B	C	**(D)**

QUESTION 2 [15 MARKS]

A bag contains 7 red beads and 5 green beads. Two beads are drawn from the bag, one after the other, WITHOUT replacement. The tree diagram below shows the possible outcomes, where R is a red bead and G is a green bead.



2.1 Calculate the probability that both beads drawn are red. Leave your answer as a common fraction in its simplest form. **(3)**

Multiply along the top path: P(R and R) $= \frac{7}{12} \times \frac{6}{11}$ *(1)*

$= \frac{42}{132}$ *(1)*

$= \mathbf{\frac{7}{22}}$ *(1)*

2.2 Calculate the probability that the two beads are of DIFFERENT colours. **(4)**

There are TWO paths that give one of each — red then green, and green then red *(1)*

$P = \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$ *(1)*

$= \frac{35}{132} + \frac{35}{132} = \frac{70}{132}$ *(1)*

$= \mathbf{\frac{35}{66}} \approx 0{,}53$ *(1)*

2.3 Calculate the probability that AT LEAST ONE of the beads is green. **(4)**

"At least one green" is the complement of "no green at all" *(1)*
and "no green" means both beads are red *(1)*

$P = 1 - \frac{7}{22}$ *(1)*

$= \mathbf{\frac{15}{22}} \approx 0{,}68$ *(1)*

2.4 Suppose instead that the first bead is REPLACED before the second is drawn. Calculate the probability that both beads are red, and explain why this answer differs from your answer above. **(4)**

> With replacement the bag is unchanged, so the second denominator stays at 12 *(1)*
> $P = \frac{7}{12} \times \frac{7}{12} = \frac{49}{144} \approx 0{,}34$ *(1)*
> This is LARGER than $0{,}32$ *(1)*
> because without replacement one red bead has already been removed, so only 6 of the remaining 11 beads are red *(1)*

QUESTION 3 [13 MARKS]

The two-way table below records how 200 learners at a school travel to school each morning.

	Bus	Taxi	Walks	TOTAL
Male	48	36	26	a
Female	42	30	18	b
TOTAL	90	66	c	200

3.1 Write down the values of a, b and c. **(3)**

> $a = 48 + 36 + 26 = \mathbf{110}$ *(1)*
> $b = 42 + 30 + 18 = \mathbf{90}$ *(1)*
> $c = 26 + 18 = \mathbf{44}$ *(1)*

3.2 Calculate the probability that a learner chosen at random is female AND travels by taxi. **(3)**

> Only the one cell counts: 30 learners *(1)*
> $P = \frac{30}{200}$ *(1)*
> $= \mathbf{0{,}15}$ *(1)*

3.3 Calculate the probability that a learner chosen at random is male OR walks to school. **(3)**

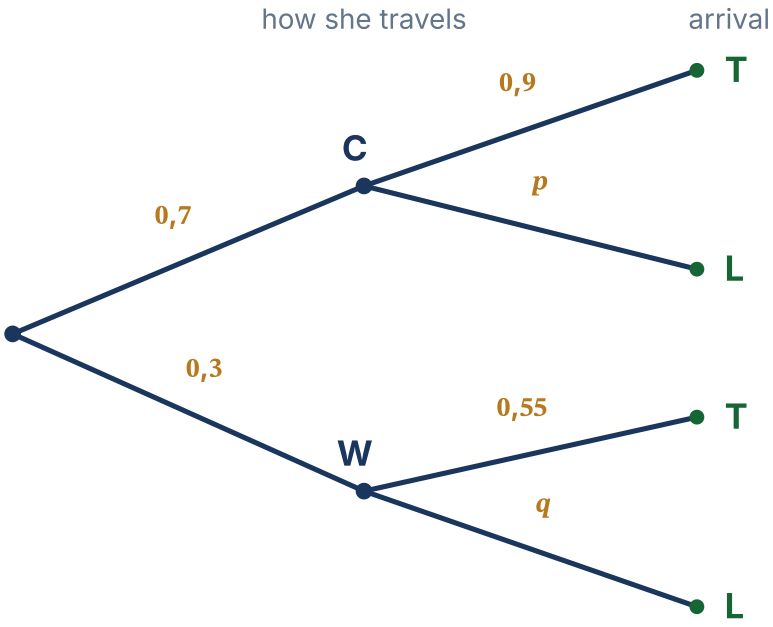
> P(male or walks) = P(male) + P(walks) − P(male and walks) *(1)*
> $= \frac{110}{200} + \frac{44}{200} - \frac{26}{200} = \frac{128}{200}$ *(1)*
> $= \mathbf{0{,}64}$ *(1)*

3.4 Determine, showing ALL calculations, whether the events "the learner is male" and "the learner travels by bus" are independent. **(4)**

> $P(\text{male}) = \frac{110}{200} = 0{,}55$ and $P(\text{bus}) = \frac{90}{200} = 0{,}45$ *(1)*
> $P(\text{male}) \times P(\text{bus}) = 0{,}2475$ *(1)*
> $P(\text{male and bus}) = \frac{48}{200} = 0{,}24$ *(1)*
> $0{,}24 \neq 0{,}2475$, so the events are **NOT independent** *(1)*

QUESTION 4 [10 MARKS]

Lerato either cycles (C) to school or walks (W). The probability that she cycles on any given day is $0{,}7$. If she cycles, the probability that she arrives on time (T) is $0{,}9$; if she walks, the probability that she arrives on time is only $0{,}55$. L means she is late. The tree diagram below represents this information.



4.1 Write down the values of p and q on the tree diagram, and hence calculate the probability that Lerato cycles AND is late on a given day. (3)

The branches leaving any one point must add to 1 *(1)*
$p = 1 - 0{,}9 =$ **0,1** and $q = 1 - 0{,}55 =$ **0,45** *(1)*
P(C and L) $= 0{,}7 \times 0{,}1 =$ **0,07** *(1)*

4.2 Calculate the probability that Lerato arrives at school on time on a given day. (4)

TWO paths end in T, so multiply along each and add them *(1)*
P(T) $= (0{,}7)(0{,}9) + (0{,}3)(0{,}55)$ *(1)*
$= 0{,}63 + 0{,}165$ *(1)*
$=$ **0,795** *(1)*

4.3 A school term is 60 school days long. Determine how many of those days Lerato can be expected to arrive LATE. (3)

P(L) $= 1 - 0{,}795 = 0{,}205$ *(1)*
expected number of late days $= 0{,}205 \times 60 = 12{,}3$ *(1)*
≈ 12 days *(1)*

TOTAL: 48 marks

This question paper consists of 4 questions.

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