MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
34	55 minutes	3

Name: _____ Class: _____

Date: _____

MARK _____ / 34

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The general solution of $\sin\theta = 0{,}5$ is ... (1)

A) $\theta = 30° + k(180°)$

B) $\theta = 30° + k(360°)$ only

C) $\theta = 30° + k(360°)$ **or** $\theta = 150° + k(360°)$ ✓

D) $\theta = 30° + k(360°)$ or $\theta = 330° + k(360°)$

Answer: C — The reference angle is $30°$, and sine is positive in the FIRST and SECOND quadrants, so $180° - 30° = 150°$ is the second family.

- **A** — the $180°$ period belongs to the TANGENT function
- **B** — sine is positive in TWO quadrants, so a second family of solutions is missing
- **D** — $330°$ is where sine is NEGATIVE

1.2 The general solution of $\cos\theta = 0{,}5$ is ... (1)

A) $\theta = 60° + k(360°)$ or $\theta = 120° + k(360°)$

B) $\theta = 60° + k(360°)$ **or** $\theta = -60° + k(360°)$ ✓

C) $\theta = 60° + k(180°)$

D) $\theta = 60° + k(360°)$ only

Answer: B — Cosine is positive in the FIRST and FOURTH quadrants, and the fourth-quadrant angle is $-60°$, that is $300°$.

- **A** — $180° - 60° = 120°$ is where cosine is NEGATIVE — that rule is for sine
- **C** — the $180°$ period belongs to the TANGENT function
- **D** — cosine is positive in TWO quadrants, so a second family is missing

1.3 The general solution of $\tan\theta = 1$ is ... (1)

A) $\theta = 45° $ only
B) $\theta = 45° + k(360°)$
C) **$\theta = 45° + k(180°)$ ✓**
D) $\theta = 45°$ or $\theta = 135° + k(360°)$

> **Answer: C** — Tangent repeats every $180°$, so one family covers every solution.
> - **A** — there are infinitely many solutions, one every $180°$
> - **B** — tangent's period is $180°$, not $360°$
> - **D** — $\tan 135° = -1$, not $+1$

1.4 Solve $\sin\theta = -0{,}5$ for $0° \leq \theta \leq 360°$. (1)

A) **$210°$ and $330°$ ✓**
B) $-30°$ only
C) $30°$ and $150°$
D) $150°$ and $210°$

> **Answer: A** — The reference angle is $30°$, and sine is negative in the third and fourth quadrants: $180° + 30°$ and $360° - 30°$.
> - **B** — $-30°$ lies outside the given interval
> - **C** — those are the solutions of $\sin\theta = +0{,}5$
> - **D** — $150°$ is a POSITIVE-sine angle

1.5 Solve $\cos\theta = -0{,}5$ for $0° \leq \theta \leq 360°$. (1)

A) **$120°$ and $240°$ ✓**
B) $240°$ only
C) $60°$ and $120°$
D) $60°$ and $300°$

> **Answer: A** — The reference angle is $60°$, and cosine is negative in the second and third quadrants.
> - **B** — there are TWO solutions inside this interval
> - **C** — $60°$ is a POSITIVE-cosine angle
> - **D** — those are the solutions of $\cos\theta = +0{,}5$

1.6 In which quadrants is $\tan\theta$ POSITIVE? (1)

A) the second and fourth
B) **the first and third ✓**
C) the first and fourth
D) the first and second

> **Answer: B** — Tangent is sine ÷ cosine, so it is positive wherever the two have the SAME sign.
> - **A** — that is where tangent is NEGATIVE
> - **C** — that is where COSINE is positive
> - **D** — that is where SINE is positive

1.7 Solve $2\sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$. (1)

A) $30°$ and $330°$
B) $30°$ only
C) **$30°$ and $150°$ ✓**
D) $60°$ and $120°$

> **Answer: C** — $\sin\theta = 0{,}5$, and sine is positive in the first and second quadrants.
> - **A** — $330°$ is where sine is NEGATIVE
> - **B** — sine is positive in TWO quadrants inside this interval
> - **D** — those are the solutions of $\cos\theta = 0{,}5$

1.8 The general solution of $\sin 2\theta = 0{,}5$ is ... (1)

A) $\theta = 15° + k(360°)$ or $\theta = 75° + k(360°)$

B) $\theta = 15° + k(180°)$ **or** $\theta = 75° + k(180°)$ ✓

C) $\theta = 30° + k(360°)$ or $\theta = 150° + k(360°)$

D) $\theta = 60° + k(180°)$

> **Answer: B** — Solve for 2θ first, then divide EVERYTHING by 2 — the angles and the $k(360°)$ alike.
>
> - **A** — halved the angles but left the period at $k(360°)$
> - **C** — never divided by the 2 — those are the values of 2θ, not of θ
> - **D** — doubled the reference angle instead of halving it

1.9 Why is a GENERAL solution needed? (1)

A) the reference angle is always negative

B) there is never more than one solution

C) the calculator gives a wrong answer

D) **the trigonometric functions repeat, so there are infinitely many solutions** ✓

> **Answer: D** — The calculator returns ONE angle; the general solution accounts for every repeat.
>
> - **A** — a reference angle is always taken as positive and acute
> - **B** — there are infinitely many, which is precisely why the k-form exists
> - **C** — the calculator is right — it just returns only one of the solutions

1.10 For $\sin\theta = 0{,}8$ the calculator gives $53{,}13°$. The SECOND solution in $[0° ; 360°]$ is ... (1)

A) $36{,}87°$

B) $233{,}13°$

C) $306{,}87°$

D) **$126{,}87°$** ✓

> **Answer: D** — Sine is positive in the second quadrant too, at $180° - 53{,}13° = 126{,}87°$.
>
> - **A** — that is the complement, whose COSINE is $0{,}8$
> - **B** — that is $180° + 53{,}13°$, where sine is NEGATIVE
> - **C** — that is $360° - 53{,}13°$, where sine is NEGATIVE

ANSWER GRID — MARKING GUIDELINE

1.1	A	B	**C**	D
1.2	A	**B**	C	D
1.3	A	B	**C**	D
1.4	**A**	B	C	D
1.5	**A**	B	C	D
1.6	A	**B**	C	D
1.7	A	B	**C**	D
1.8	A	**B**	C	D
1.9	A	B	C	**D**
1.10	A	B	C	**D**

QUESTION 2 [13 MARKS]

Determine the GENERAL SOLUTION of each equation, with $k \in \mathbb{Z}$.

2.1 $3 \sin x - 1 = 0$, correct to TWO decimal places (5)

$\sin x = \frac{1}{3}$ *(1)*

Reference angle $= 19{,}47°$ *(1)*

Sine is positive in the first and second quadrants *(1)*

$\boldsymbol{x = 19{,}47° + k \cdot 360°}$ *(1)* **or** $\boldsymbol{x = 160{,}53° + k \cdot 360°}$ *(1)*

2.2 $\cos 2x = 0{,}5$ (4)

Reference angle $= 60°$ *(1)*

$2x = \pm 60° + k \cdot 360°$ *(2)*

$\boldsymbol{x = \pm 30° + k \cdot 180°}$ *(1)*

2.3 $\tan(x - 30°) = 1$ (4)

Reference angle $= 45°$ *(1)*

$x - 30° = 45° + k \cdot 180°$ *(2)*

$\boldsymbol{x = 75° + k \cdot 180°}$ *(1)*

QUESTION 3 [11 MARKS]

Answer the questions below.

3.1 Solve for $x \in [0° ; 360°]$: $2\cos^2 x - \cos x - 1 = 0$ (6)

Let $p = \cos x$ *(1)* — k is already in use for the general solution

$2p^2 - p - 1 = 0$ *(1)*

$(2p + 1)(p - 1) = 0$ *(1)*

$\cos x = 1$ gives $x = 0°$ or $360°$ *(1)*

$\cos x = -\frac{1}{2}$ gives $x = 120°$ *(1)* or $x = 240°$ *(1)*

3.2 Explain why a trigonometric equation has infinitely many solutions, but only a few in a given interval. (3)

The trig graphs repeat every period *(1)*, so the same ratio value recurs forever *(1)*; restricting x to an interval keeps only the solutions inside it *(1)*

3.3 Explain why $\sin x = 1{,}5$ has no solution. (2)

$\sin x$ lies between -1 and 1 for every x *(1)*, and $1{,}5$ is outside that range *(1)*

TOTAL: 34 marks

This question paper consists of 3 questions.

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