MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
52	1 hour 20 minutes	4

Name: _____ Class: _____

Date: _____

MARK ____ / 52

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The PERIOD of $y = \sin 2x$ is ... (1)

A) $180°$ ✓ **B)** $720°$

C) $360°$ **D)** $90°$

Answer: A — The period of $\sin kx$ is $360° \div k$.

- **B** — MULTIPLIED by 2 instead of dividing
- **C** — that is the period of $y = \sin x$
- **D** — divided by 4 instead of by 2

1.2 The PERIOD of $y = \tan 3x$ is ... (1)

A) $540°$ **B)** $180°$

C) $60°$ ✓ **D)** $120°$

Answer: C — Tangent's basic period is $180°$, so it becomes $180° \div 3 = 60°$.

- **A** — MULTIPLIED by 3 instead of dividing
- **B** — that is the period of $y = \tan x$
- **D** — used $360° \div 3$, but tangent's period is $180°$, not $360°$

1.3 The AMPLITUDE of $y = -4 \cos x$ is ... (1)

A) 8 **B)** 4 ✓

C) -4 **D)** 1

Answer: B — Amplitude is the SIZE of a, and is always positive.

- **A** — that is the distance from minimum to maximum, twice the amplitude
- **C** — amplitude is never negative — the minus only flips the graph over
- **D** — that is the amplitude of $y = \cos x$

1.4 The RANGE of $y = 3 \sin x - 1$ is ... (1)

A) $-4 \leq y \leq 2$ ✓ **B)** $-2 \leq y \leq 4$

C) $-3 \leq y \leq 3$ **D)** $-1 \leq y \leq 3$

> **Answer: A** — $3 \sin x$ runs from -3 to 3, and the -1 lowers both ends.
>
> - **B** — ADDED the 1 instead of subtracting it
> - **C** — forgot the vertical shift of -1
> - **D** — used the shift as though it were the amplitude

1.5 Compared with $y = \sin x$, the graph of $y = \sin(x - 30°)$ is ... (1)

A) reflected in the x-axis **B)** shifted $30°$ to the left

C) shifted 30 units down **D)** **shifted $30°$ to the right** ✓

> **Answer: D** — A minus INSIDE the bracket shifts the graph in the POSITIVE x-direction.
>
> - **A** — a reflection needs a minus in FRONT of the function
> - **B** — a minus inside the bracket shifts to the RIGHT, not to the left
> - **C** — the $30°$ sits inside the function, so the shift is horizontal

1.6 The graph of $y = \cos(x + 60°)$ has a maximum at $x = ...$ (1)

A) $180°$ **B)** $0°$

C) $-60°$ ✓ **D)** $60°$

> **Answer: C** — Cosine peaks when its argument is $0°$, so $x + 60° = 0°$.
>
> - **A** — $180°$ is where cosine is at its MINIMUM
> - **B** — $0°$ is where $y = \cos x$ peaks, before the shift
> - **D** — the shift is to the LEFT, so the peak moves to $-60°$

1.7 $y = 2 \sin x$ and $y = \sin 2x$ differ because ... (1)

A) **the outside 2 changes the AMPLITUDE; the inside 2 changes the PERIOD** ✓ **B)** both change the period

C) they are in fact the same graph **D)** both change the amplitude

> **Answer: A** — A factor in front stretches the graph vertically; a factor on x squeezes it horizontally.
>
> - **B** — a factor in FRONT affects the amplitude, not the period
> - **C** — their maxima and their periods both differ
> - **D** — a factor ON x affects the period, not the amplitude

1.8 On $[0° ; 360°]$, how many complete cycles does $y = \cos 3x$ complete? (1)

A) 6 **B)** 1

C) $\frac{1}{3}$ **D)** 3 ✓

> **Answer: D** — The period is $120°$, and $360° \div 120° = 3$.
>
> - **A** — doubled the correct count
> - **B** — that is the number of cycles of $y = \cos x$
> - **C** — inverted the relationship

1.9 The MAXIMUM value of $y = -2\cos x + 3$ is ... (1)

A) 3 **B)** 5 ✓

C) 1 **D)** -2

> **Answer: B** — With a negative a, the maximum happens where $\cos x = -1$, giving $2 + 3 = 5$.
>
> - **A** — that is the vertical shift on its own
> - **C** — that is the MINIMUM, where $\cos x = +1$
> - **D** — that is the value of a

1.10 The graph of $y = \tan x$ has asymptotes wherever ... (1)

A) $\sin x = 0$ **B)** $\cos x = 0$ ✓

C) $x = 0$ **D)** $\tan x = 0$

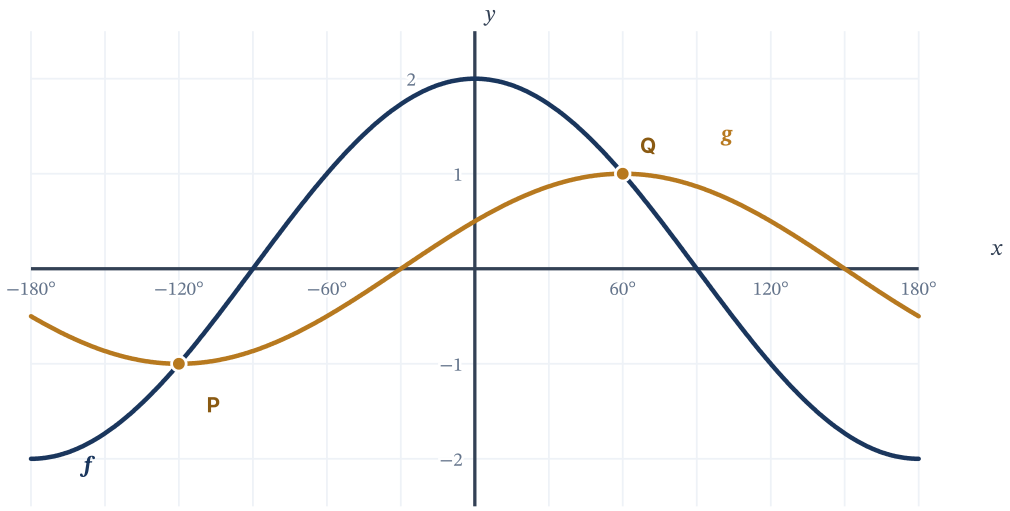
> **Answer: B** — $\tan x = \sin x \div \cos x$, so it is undefined where the denominator is zero.
>
> - **A** — $\sin x = 0$ gives $\tan x = 0$, which is an x-INTERCEPT
> - **C** — $\tan 0° = 0$, so $x = 0$ is an intercept, not an asymptote
> - **D** — $\tan x = 0$ is where the graph crosses the axis

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	B	(C)	D
1.3	A	(B)	C	D
1.4	(A)	B	C	D
1.5	A	B	C	(D)
1.6	A	B	(C)	D
1.7	(A)	B	C	D
1.8	A	B	C	(D)
1.9	A	(B)	C	D
1.10	A	(B)	C	D

QUESTION 2 [17 MARKS]

The graphs of $f(x) = 2\cos x$ and $g(x) = \sin(x + 30°)$ are drawn below on the SAME set of axes for $x \in [-180° ; 180°]$. P and Q are the points of intersection of f and g.



2.1 Write down the range of f and the period of g, and state how many complete cycles f completes in the interval $[-180° ; 180°]$. **(3)**

Range of f: $\mathbf{y \in [-2 ; 2]}$ *(1)*
Period of g: **360°** *(1)*
The interval is $360°$ wide and f also has a period of $360°$, so f completes **ONE** complete cycle *(1)*

2.2 Write down the x-coordinates of the points at which g cuts the x-axis in the given interval, and show how you obtained them. **(3)**

$g(x) = 0$ when $\sin(x + 30°) = 0$ *(1)*
$x + 30° = 0°$ or $x + 30° = 180°$ *(1)*
$x = \mathbf{-30°}$ or $x = \mathbf{150°}$ *(1)*

2.3 Write down the coordinates of P and Q, and hence use the graphs to determine the values of x for which $f(x) > g(x)$. **(4)**

P$(-120° ; -1)$ *(1)*
Q$(60° ; 1)$ *(1)*
f lies ABOVE g strictly between P and Q *(1)*
$\therefore \mathbf{-120° < x < 60°}$ *(1)*

2.4 Determine, from the graphs, the values of $x \in [-180° ; 180°]$ for which $f(x) \cdot g(x) \leq 0$. **(4)**

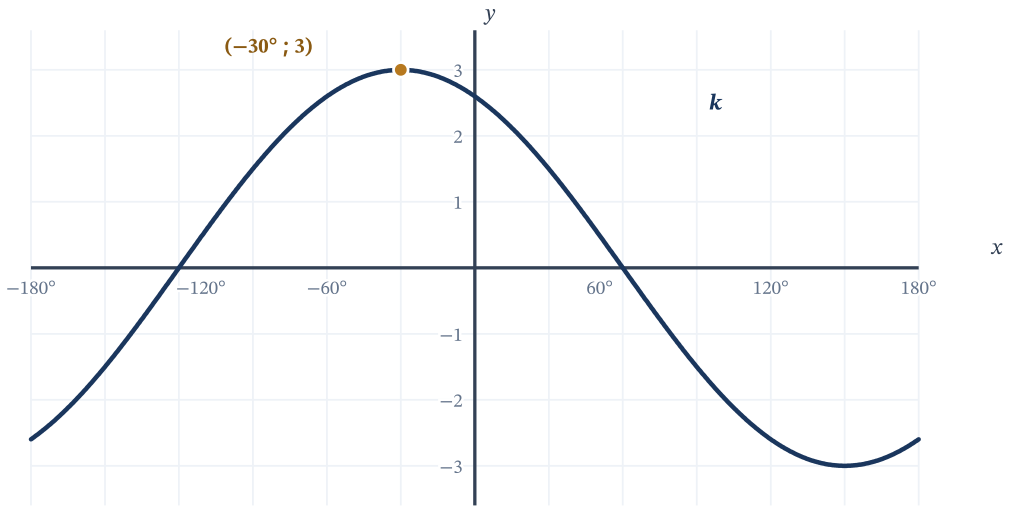
A product is negative or zero where the two graphs have OPPOSITE signs, or where either one is zero *(1)*
f is zero at $-90°$ and $90°$; g is zero at $-30°$ and $150°$ *(1)*
Between $-90°$ and $-30°$: f is above the axis, g is below it *(1)*
$\therefore \mathbf{x \in [-90° ; -30°]}$ **or** $\mathbf{x \in [90° ; 150°]}$ *(1)*

2.5 The graph of f is shifted 3 units DOWN to give a new graph m. Write down the range of m, and state, with a reason, whether m still cuts the x-axis. **(3)**

> $m(x) = 2\cos x - 3$, so the range is $\boldsymbol{y \in [-5\,;\,-1]}$ *(2)*
> m does NOT cut the x-axis, because 0 does not lie in that range — the maximum value of m is -1 *(1)*

QUESTION 3 [15 MARKS]

The graph of $k(x) = a\sin(x + p)$ is sketched below for $x \in [-180° ; 180°]$. The maximum turning point of k in this interval is $(-30° ; 3)$, and $0° < p < 180°$.



3.1 Show that $a = 3$ and $p = 120°$. **(4)**

> The maximum value of $a\sin(\theta)$ is a, and the sketch gives a maximum of 3 *(1)*
> $\therefore a = \mathbf{3}$ *(1)*
> A sine graph reaches its maximum when the angle equals $90°$: $-30° + p = 90°$ *(1)*
> $\therefore p = \mathbf{120°}$ *(1)*

3.2 Write down the range of k, and determine the coordinates of the minimum turning point of k in the given interval. **(4)**

> Range: $\boldsymbol{y \in [-3\,;\,3]}$ *(1)*
> The minimum occurs when $x + 120° = 270°$ *(1)*
> $x = 150°$ *(1)*
> $\therefore$ the minimum turning point is $\mathbf{(150° ; -3)}$ *(1)*

3.3 Determine the x-intercepts of k in the interval $[-180° ; 180°]$. **(4)**

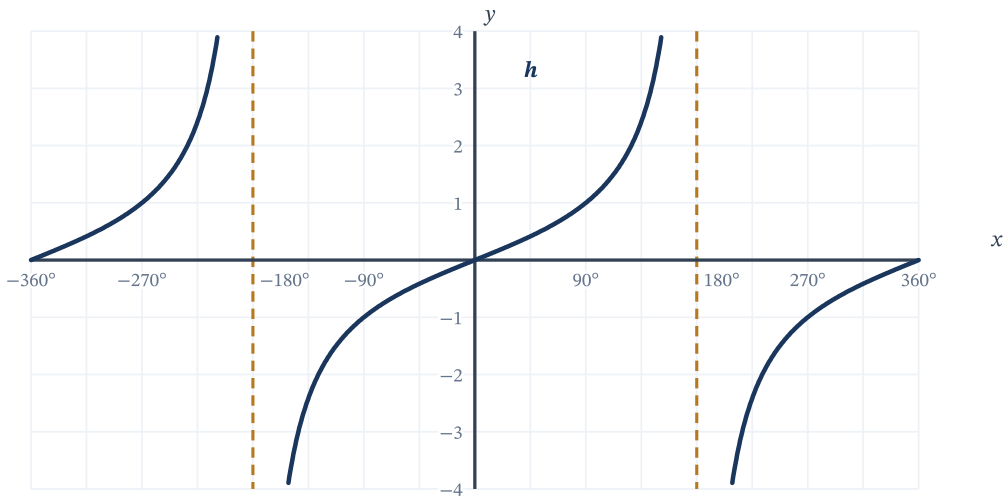
> $3\sin(x + 120°) = 0$, so $\sin(x + 120°) = 0$ *(1)*
> $x + 120° = 0° ; 180° ; 360° ; \ldots$ *(1)*
> $x = -120° ; 60° ; 240°$ *(1)*
> Only $\boldsymbol{x = -120°}$ **and** $\boldsymbol{x = 60°}$ lie in the interval *(1)*

3.4 Describe fully the transformation that maps $y = 3\sin x$ onto k, and write down the period of $y = 3\sin 2(x + 120°)$. **(3)**

> k is $y = 3\sin x$ shifted $120°$ to the LEFT *(2)*
> Doubling the value of the constant inside HALVES the period: $360° \div 2 = \mathbf{180°}$ *(1)*

QUESTION 4 [10 MARKS]

The graph of $h(x) = \tan \frac{1}{2}x$ is sketched below for $x \in [-360° ; 360°]$. The dashed lines are the asymptotes of h.



4.1 Write down the period of h, and the equations of the two asymptotes shown in the sketch. **(4)**

Period of tan kx is $180° \div k$, and here $k = \frac{1}{2}$ *(1)*
$\therefore$ period $= \mathbf{360°}$ *(1)*
tan is undefined when $\frac{1}{2}x = 90° + 180°n$ *(1)*
$\therefore$ $\boldsymbol{x = -180°}$ **and** $\boldsymbol{x = 180°}$ *(1)*

4.2 Determine the values of $x \in [-360° ; 360°]$ for which $h(x) = 1$. **(3)**

$\tan \frac{1}{2}x = 1$, so $\frac{1}{2}x = 45° + 180°n$ *(1)*
$x = 90° + 360°n$ *(1)*
$\therefore$ $\boldsymbol{x = -270°}$ **or** $\boldsymbol{x = 90°}$ *(1)*

4.3 Use the sketch to write down the values of x, where $-180° < x < 180°$, for which $h(x) < 0$. **(3)**

Between the two asymptotes the curve lies BELOW the x-axis to the left of the origin *(1)*
and $h(0°) = 0$, which is not less than zero *(1)*
$\therefore$ $\boldsymbol{-180° < x < 0°}$ *(1)*

TOTAL: 52 marks

This question paper consists of 4 questions.

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