MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
35	55 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 35

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 $\sin^2\theta + \cos^2\theta = \ldots$ (1)

A) 0 **B)** 1 ✓

C) 2 **D)** $\sin\theta + \cos\theta$

Answer: B — That is the square identity, and it comes straight from Pythagoras.

- **A** — the two squares ADD to 1; they do not cancel
- **C** — each is at most 1, and together they total exactly 1
- **D** — squaring first is what makes the total 1 — $\sin\theta + \cos\theta$ is not constant

1.2 $\tan\theta = \ldots$ (1)

A) $\frac{1}{\sin\theta}$ **B)** $\frac{\cos\theta}{\sin\theta}$

C) $\sin\theta \times \cos\theta$ **D)** $\frac{\sin\theta}{\cos\theta}$ ✓

Answer: D — That is the quotient identity.

- **A** — that is cosec θ
- **B** — that is cot θ
- **C** — that is a PRODUCT, not the quotient $\tan\theta = \frac{\sin\theta}{\cos\theta}$

1.3 $\sin(180° - \theta) = \ldots$ (1)

A) **sin θ ✓** **B)** $-\cos\theta$

C) $-\sin\theta$ **D)** $\cos\theta$

> **Answer: A** — $180° - \theta$ lies in the second quadrant, where sine is POSITIVE.
> - **B** — both the sign and the ratio are wrong
> - **C** — sine is positive in the second quadrant
> - **D** — the ratio changes only for the $90°$ and $270°$ reductions

1.4 $\cos(180° + \theta) = \ldots$ (1)

A) $\cos\theta$ **B)** $-\sin\theta$

C) **$-\cos\theta$ ✓** **D)** $\sin\theta$

> **Answer: C** — $180° + \theta$ lies in the third quadrant, where cosine is NEGATIVE.
> - **A** — cosine is negative in the third quadrant
> - **B** — the ratio changes only for the $90°$ and $270°$ reductions
> - **D** — both the sign and the ratio are wrong

1.5 $\tan(360° - \theta) = \ldots$ (1)

A) $\cot\theta$ **B)** $-\cot\theta$

C) **$-\tan\theta$ ✓** **D)** $\tan\theta$

> **Answer: C** — $360° - \theta$ lies in the fourth quadrant, where tangent is NEGATIVE.
> - **A** — both the sign and the ratio are wrong
> - **B** — the ratio changes only for the $90°$ and $270°$ reductions
> - **D** — tangent is negative in the fourth quadrant

1.6 $\sin(90° - \theta) = \ldots$ (1)

A) $-\sin\theta$ **B)** $\sin\theta$

C) $-\cos\theta$ **D)** **$\cos\theta$ ✓**

> **Answer: D** — A $90°$ reduction is a CO-RATIO reduction, so sine becomes cosine.
> - **A** — the ratio must change to a cosine
> - **B** — the $90°$ reductions DO change the ratio, unlike the $180°$ ones
> - **C** — $90° - \theta$ lies in the first quadrant, where cosine is positive

1.7 $\cos(-\theta) = \ldots$ (1)

A) **$\cos\theta$ ✓** **B)** $\sin\theta$

C) $-\sin\theta$ **D)** $-\cos\theta$

> **Answer: A** — $-\theta$ lies in the fourth quadrant, where cosine is positive.
> - **B** — the ratio does not change for $a - \theta$ reduction
> - **C** — both the sign and the ratio are wrong
> - **D** — cosine is POSITIVE in the fourth quadrant

1.8 Simplify $\frac{1 - \cos^2\theta}{\sin\theta}$, given $\sin\theta \neq 0$. (1)

A) 1 **B)** $\tan\theta$

C) $\cos\theta$ **D) $\sin\theta$ ✓**

> **Answer: D** — $1 - \cos^2\theta = \sin^2\theta$, and $\sin^2\theta \div \sin\theta = \sin\theta$.
> - **A** — cancelled both powers of sine instead of one
> - **B** — that would need a cosine in the denominator
> - **C** — $1 - \cos^2\theta$ is $\sin^2\theta$, not $\cos^2\theta$

1.9 $\sin 150°$ in exact form is ... (1)

A) $\frac{-1}{2}$ **B)** $\frac{1}{2}$ ✓

C) $\frac{-\sqrt{3}}{2}$ **D)** $\frac{\sqrt{3}}{2}$

> **Answer: B** — $150° = 180° - 30°$, so $\sin 150° = \sin 30° = ½$.
> - **A** — sine is POSITIVE in the second quadrant
> - **C** — that is $\cos 150°$
> - **D** — that is $\cos 30°$, not $\sin 30°$

1.10 Simplify $\frac{\sin\theta \times \cos\theta}{\tan\theta}$. (1)

A) $\sin^2\theta$ **B) $\cos^2\theta$ ✓**

C) 1 **D)** $\cos\theta$

> **Answer: B** — Replace $\tan\theta$ with $\sin\theta \div \cos\theta$: the sines cancel and a second cosine appears.
> - **A** — the SINES cancel, not the cosines
> - **C** — nothing here cancels all the way down to 1
> - **D** — dividing by $\tan\theta$ brings in a second cosine, so the power is 2

ANSWER GRID — MARKING GUIDELINE

1.1	A	**(B)**	C	D
1.2	A	B	C	**(D)**
1.3	**(A)**	B	C	D
1.4	A	B	**(C)**	D
1.5	A	B	**(C)**	D
1.6	A	B	C	**(D)**
1.7	**(A)**	B	C	D
1.8	A	B	C	**(D)**
1.9	A	**(B)**	C	D
1.10	A	**(B)**	C	D

QUESTION 2 [11 MARKS]

Answer the questions below WITHOUT the use of a calculator.

2.1 Simplify fully: $\sin(90° - x)\cdot\cos(180° + x) + \tan x\cdot\cos x\cdot\sin(x - 180°)$ **(6)**

$\sin(90° - x) = \cos x$ *(1)*
$\cos(180° + x) = -\cos x$ *(1)*
$\sin(x - 180°) = -\sin x$ *(1)*
$= -\cos^2 x + \frac{\sin x}{\cos x}\cdot\cos x\cdot(-\sin x)$ *(1)*
$= -\cos^2 x - \sin^2 x$ *(1)*
$= \mathbf{-1}$ *(1)*
(Valid only where $\cos x \neq 0$ — the original expression contains $\tan x$.)

2.2 If $-3\sin\theta - 2 = 0$ and $\theta \in [0° ; 270°]$, use a SKETCH in the correct quadrant to determine the value of $1 + \tan^2\theta$. **(5)**

$\sin\theta = -\frac{2}{3}$ *(1)*
Sine is negative and $\theta \leq 270°$, so θ is in the THIRD quadrant *(1)*
$y = -2$, $r = 3$, so $x = -\sqrt{5}$ *(1)*
$\tan\theta = \frac{2}{\sqrt{5}}$ *(1)*
$1 + \tan^2\theta = 1 + \frac{4}{5} = \mathbf{\frac{9}{5}}$ *(1)*

QUESTION 3 [14 MARKS]

If $\cos 75° = k$, express each of the following in terms of k. Show ALL your working.

3.1 $\cos 105°$ **(3)**

$\cos 105° = \cos(180° - 75°)$ *(1)*
$= -\cos 75°$ *(1)* $= \mathbf{-k}$ *(1)*

3.2 $\sin 15°$ **(3)**

$\sin 15° = \sin(90° - 75°)$ *(1)*
$= \cos 75°$ *(1)* $= \mathbf{k}$ *(1)*

3.3 $\tan 15°$ **(4)**

$\cos 15° = \sqrt{1 - k^2}$ *(2)*
$\tan 15° = \frac{\sin 15°}{\cos 15°}$ *(1)* $= \mathbf{\frac{k}{\sqrt{1 - k^2}}}$ *(1)*

3.4 Prove the identity: $\frac{1 - \cos^2\theta}{\sin\theta\cdot\cos\theta} = \tan\theta$ **(4)**

LHS $= \frac{\sin^2\theta}{\sin\theta\cdot\cos\theta}$ *(2)*
$= \frac{\sin\theta}{\cos\theta}$ *(1)* $= \tan\theta =$ RHS *(1)*
(Valid only where $\sin\theta \neq 0$ and $\cos\theta \neq 0$ — the original denominator is zero otherwise.)

TOTAL: 35 marks

This question paper consists of 3 questions.

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