MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 23

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 Simplify $\sin\theta \cdot \tan\theta \cdot \cos\theta$, and state the restriction on θ. *(simplify)* (3)

$\sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \cos\theta = \mathbf{\sin^2\theta}$ ✓✓

The original expression contains $\tan\theta$, so $\mathbf{\cos\theta \neq 0}$, i.e. $\theta \neq 90° + k \cdot 180°$ ✓ (3)

2 Simplify $(1 - \sin^2\theta)$. *(identity)* (1)

$\mathbf{\cos^2\theta}$ ✓ (1)

3 Simplify $\frac{\cos^2\theta}{1 - \sin^2\theta}$. *(identity)* (2)

$\frac{\cos^2\theta}{\cos^2\theta} = \mathbf{1}$ ✓✓ (2)

4 Prove $\tan\theta \cdot \cos\theta = \sin\theta$, and say where the identity does not hold. *(prove)* (3)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓, so $\frac{\sin\theta}{\cos\theta} \cdot \cos\theta = \mathbf{\sin\theta}$ ✓

$\tan\theta$ is undefined where $\cos\theta = 0$, so the identity does not hold at $\theta = 90° + k \cdot 180°$ ✓ (3)

5 In which quadrant are only sine positive? *(CAST)* (1)

2nd quadrant ✓ (1)

6 $\sin\theta = 0{,}5$, θ in Q2. Is $\cos\theta$ + or −? *(CAST)* (1)

Q2 → cosine **negative** ✓ (1)

7 Simplify $\cos(180° - \theta)$. *(reduction)* (1)

$\mathbf{-\cos\theta}$ ✓ (1)

8 Simplify $\sin(360° - \theta)$. *(reduction)* (1)

$\mathbf{-\sin\theta}$ ✓ (1)

9 Exact value of $\sin 150°$. *(exact)* (2)

> $150° = 180° - 30°$, Q2 sine $+ \rightarrow \sin 30° = \frac{1}{2}$ ✓✓ (2)

10 Exact value of $\cos 210°$. *(exact)* (2)

> $210° = 180° + 30°$, Q3 cosine $- \rightarrow -\cos 30° = \frac{-\sqrt{3}}{2}$ ✓✓ (2)

11 For which values of θ is $\tan\theta$ undefined? Give the general answer. (2)

> $\theta = 90° + k \cdot 180°$, $k \in \mathbb{Z}$ ✓✓ ($\cos\theta = 0$ there, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$) (2)

12 For $\theta \in [0° ; 360°]$, state the values for which $\frac{\sin\theta \times \tan\theta}{\cos\theta}$ is undefined. (2)

> $\cos\theta = 0$, so $\theta = 90°$ and $\theta = 270°$ ✓✓ (2)

13 Explain why the restriction must be read off the ORIGINAL expression and not the simplified one. (2)

> Simplifying can cancel the factor that caused the division by zero ✓; the original expression is still undefined there, so the restriction stands ✓. (2)

TOTAL: 23 marks

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