Analytical Geometry: Distance, Midpoint and Gradient

MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
33	50 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 33

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 The distance between A$(1\ ;\ 2)$ and B$(4\ ;\ 6)$ is ... (1)

A) 2,65 **B) 5 ✓**

C) 25 **D)** 7

Answer: B — $d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{25} = 5$.

- **A** — SUBTRACTED the squares instead of adding them
- **C** — forgot to take the square root
- **D** — ADDED the two differences instead of squaring and rooting

1.2 The midpoint of P$(-2\ ;\ 3)$ and Q$(6\ ;\ -1)$ is ... (1)

A) $(4\ ;\ 2)$ **B)** $(-4\ ;\ 1)$

C) $(2\ ;\ 1)$ ✓ **D)** $(8\ ;\ -4)$

Answer: C — Average each coordinate: $(-2 + 6) \div 2 = 2$ and $(3 + (-1)) \div 2 = 1$.

- **A** — added the coordinates but never halved them
- **B** — SUBTRACTED the x-values instead of adding them before halving
- **D** — SUBTRACTED the coordinates instead of averaging them

1.3 The gradient of the line through $(2\ ;\ 5)$ and $(6\ ;\ 13)$ is ... (1)

A) -2 **B)** $\frac{1}{2}$

C) 4 **D)** $\mathbf{2}$ ✓

> **Answer: D** — $m = (13 - 5) \div (6 - 2) = 8 \div 4 = 2$.
>
> - **A** — subtracted in opposite orders on the top and the bottom
> - **B** — inverted the formula — it is the change in y OVER the change in x
> - **C** — that is the change in x, not the gradient

1.4 If both gradients are defined, two lines are PERPENDICULAR when ... (1)

A) $\boldsymbol{m_1 \times m_2 = -1}$ ✓ **B)** $m_1 + m_2 = 0$

C) $m_1 = m_2$ **D)** $m_1 \times m_2 = 1$

> **Answer: A** — Perpendicular gradients are negative reciprocals of each other.
>
> - **B** — that makes them negatives of each other, not negative RECIPROCALS
> - **C** — equal gradients means PARALLEL
> - **D** — the product must be -1, not $+1$

1.5 A line has gradient 3. A line perpendicular to it has gradient ... (1)

A) $\frac{-1}{3}$ ✓ **B)** -3

C) $\frac{1}{3}$ **D)** 3

> **Answer: A** — The perpendicular gradient is the negative reciprocal, $\frac{-1}{3}$.
>
> - **B** — changed the sign but never took the reciprocal
> - **C** — took the reciprocal but left off the minus
> - **D** — that is the gradient of a PARALLEL line

1.6 A$(1\ ;\ 2)$, B$(3\ ;\ 6)$ and C$(5\ ;\ 10)$ are collinear because ... (1)

A) they form a triangle **B)** $\boldsymbol{m_{AB} = m_{BC} = 2}$ ✓

C) AC is the longest of the three lengths **D)** AB = BC

> **Answer: B** — Collinear points lie on ONE straight line, so every pair of them has the same gradient.
>
> - **A** — collinear points do NOT form a triangle
> - **C** — a longest side describes a triangle, not a straight line
> - **D** — equal lengths do not put three points on one line

1.7 The line through $(0\ ;\ -3)$ with gradient 2 has equation ... (1)

A) $y = -3x + 2$ **B)** $y = 2x + 3$

C) $\boldsymbol{y = 2x - 3}$ ✓ **D)** $y = 3x - 2$

> **Answer: C** — $y = mx + c$ with $m = 2$ and $c = -3$.
>
> - **A** — swapped the gradient and the y-intercept
> - **B** — the y-intercept is -3, not $+3$
> - **D** — swapped them and changed the sign as well

1.8 P$(x\ ;\ 4)$ is 5 units from Q$(1\ ;\ 0)$. Then $x =$ ... (1)

A) 3 or -3 **B)** 6 or -4

C) 4 only **D) 4 or -2 ✓**

> **Answer: D** — $(x-1)^2 + 4^2 = 5^2$ gives $(x-1)^2 = 9$, so $x - 1 = \pm 3$.
> - **A** — solved $x^2 = 9$ and never moved the -1 across
> - **B** — solved $(x-1)^2 = 25$ by forgetting to subtract the 16
> - **C** — a square root has TWO values, so there is a second answer

1.9 The distance from the origin to $(-6\ ;\ 8)$ is ... (1)

A) 14 **B)** 100

C) 2 **D) 10 ✓**

> **Answer: D** — $\sqrt{(-6)^2 + 8^2} = \sqrt{100} = 10$.
> - **A** — added the two coordinates' sizes instead of squaring and rooting
> - **B** — forgot to take the square root
> - **C** — added -6 and 8 as they stand

1.10 Line AB has gradient 4. A line PARALLEL to AB has gradient ... (1)

A) $\frac{-1}{4}$ **B) 4 ✓**

C) -4 **D)** $\frac{1}{4}$

> **Answer: B** — Parallel lines have EQUAL gradients.
> - **A** — that is the PERPENDICULAR gradient
> - **C** — changing the sign does not make a line parallel
> - **D** — taking the reciprocal does not make a line parallel

ANSWER GRID — MARKING GUIDELINE

1.1	A	**(B)**	C	D
1.2	A	B	**(C)**	D
1.3	A	B	C	**(D)**
1.4	**(A)**	B	C	D
1.5	**(A)**	B	C	D
1.6	A	**(B)**	C	D
1.7	A	B	**(C)**	D
1.8	A	B	C	**(D)**
1.9	A	B	C	**(D)**
1.10	A	**(B)**	C	D

QUESTION 2 [11 MARKS]

A$(2\ ;\ 10)$ and B$(6\ ;\ -2)$ are two points on the Cartesian plane.

2.1 Calculate the length of AB. Leave your answer in simplest surd form. (4)

> AB $= \sqrt{(6-2)^2 + (-2-10)^2}$ *(2)*
> $= \sqrt{160}$ *(1)* $= \mathbf{4\sqrt{10}}$ *(1)*

2.2 Calculate the coordinates of M, the midpoint of AB. **(3)**

> M $= \left(\frac{2+6}{2}\ ;\ \frac{10+(-2)}{2}\right)$ *(2)* $=$ **(4 ; 4)** *(1)*

2.3 Calculate the gradient of AB, and hence write down the gradient of any line perpendicular to AB. **(4)**

> $m_{AB} = \frac{-2-10}{6-2} = -3$ *(2)*
>
> Perpendicular: $m = \mathbf{\frac{1}{3}}$ *(1)* — the product must be -1 *(1)*

QUESTION 3 [12 MARKS]

Answer the questions below.

3.1 Determine the equation of the line through A$(2\ ;\ 10)$ and B$(6\ ;\ -2)$ in the form $y = mx + c$. **(4)**

> $m = -3$ *(1)*
> $10 = -3(2) + c$ *(2)*
> $c = 16$, so $\boldsymbol{y = -3x + 16}$ *(1)*

3.2 Determine whether the point C$(4\ ;\ 4)$ lies on the line AB. Show your working. **(3)**

> $-3(4) + 16 = 4$ *(2)*
> The y-value matches, so **C lies on AB** *(1)*

3.3 Points P$(-8\ ;\ 0)$, Q$(-5\ ;\ -8)$ and R$(x\ ;\ -14)$ are collinear. Calculate the value of x. **(5)**

> Collinear means $m_{PQ} = m_{QR}$ *(1)*
> $m_{PQ} = \frac{-8-0}{-5+8} = -\frac{8}{3}$ *(1)*
> $\frac{-14+8}{x+5} = -\frac{8}{3}$ *(1)*
> $-18 = -8(x+5)$ *(1)*
> $\boldsymbol{x = -2{,}75}$ *(1)*

TOTAL: 33 marks

This question paper consists of 3 questions.

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