Analytical Geometry: Distance, Midpoint and Gradient

MEMORANDUM

Name: ..... Class: .....

Date: .....

MARK ____ / 37

Answer all questions. Show all your working — marks are awarded for method as well as the final answer. **Teacher copy: accept any correct equivalent method.**

1 Distance between $(1\ ;\ 1)$ and $(4\ ;\ 5)$. *(distance)* (2)

$\sqrt{9+16} = \sqrt{25} = \mathbf{5}$ ✓✓ (2)

2 Distance between $(-2\ ;\ 1)$ and $(1\ ;\ 5)$. *(distance)* (2)

$\sqrt{9+16} = \mathbf{5}$ ✓✓ (2)

3 Midpoint of $(2\ ;\ 4)$ and $(6\ ;\ 10)$. *(midpoint)* (2)

$\frac{2+6}{2}, \frac{4+10}{2} = \mathbf{(4\ ;\ 7)}$ ✓✓ (2)

4 Gradient of the line through $(1\ ;\ 2)$ and $(5\ ;\ 10)$. *(gradient)* (2)

$\frac{10-2}{5-1} = \mathbf{2}$ ✓✓ (2)

5 Are gradients 3 and 3 parallel or perpendicular? *(parallel)* (1)

Equal → **parallel** ✓ (1)

6 What gradient is perpendicular to gradient 2? *(perpendicular)* (1)

$-\frac{1}{2}$ (negative reciprocal) ✓ (1)

7 What gradient is perpendicular to gradient $-\frac{3}{4}$? *(perpendicular)* (1)

$\frac{4}{3}$ ✓ (1)

8 Are $(0\ ;\ 0)$, $(2\ ;\ 4)$, $(3\ ;\ 6)$ collinear? *(collinear)* (3)

Gradients 2 and 2 → **Yes** ✓✓✓ (3)

9 Midpoint of $(1\ ;\ 2)$ and $(x\ ;\ 8)$ is $(4\ ;\ 5)$. Find x. *(unknown)* **(2)**

> $\frac{1+x}{2} = 4 \rightarrow x = \mathbf{7}$ ✓✓ (2)

10 Distance from $(0\ ;\ 0)$ to $(x\ ;\ 12)$ is 13 $(x > 0)$. Find x. *(unknown)* **(3)**

> $x^2 + 144 = 169 \rightarrow x = \mathbf{5}$ ✓✓✓ (3)

11 P$(-1\ ;\ 1)$, Q$(3\ ;\ 3)$, R$(5\ ;\ 7)$ and S$(1\ ;\ 5)$ are the vertices of a quadrilateral. Prove that PQRS is a parallelogram. **(4)**

> $m(\text{PQ}) = \frac{2}{4} = ½$ ✓; $m(\text{SR}) = \frac{2}{4} = ½$ ✓, so PQ ∥ SR; PQ $= \sqrt{20}$ and SR $= \sqrt{20}$ ✓; ∴ parallelogram (one pair opp sides equal and parallel) ✓ (4)

12 A$(0\ ;\ 0)$, B$(4\ ;\ 2)$ and C$(2\ ;\ 6)$. Show that AB ⊥ BC. **(3)**

> $m(\text{AB}) = \frac{2}{4} = ½$ ✓; $m(\text{BC}) = \frac{6-2}{2-4} = -2$ ✓; $½ \times (-2) = -1$, so AB ⊥ BC ✓ (3)

13 The diagonals of quadrilateral KLMN are KM and LN. K$(1\ ;\ 1)$, L$(6\ ;\ 2)$, M$(7\ ;\ 6)$, N$(2\ ;\ 5)$. Show that the diagonals bisect each other. **(3)**

> Midpoint KM = $(4\ ;\ 3{,}5)$ ✓; midpoint LN = $(4\ ;\ 3{,}5)$ ✓; the midpoints are the same, so the diagonals bisect each other ✓ (3)

14 P$(-1\ ;\ 2)$, Q$(3\ ;\ 5)$ and R$(6\ ;\ 1)$. Show that triangle PQR is right-angled at Q. **(4)**

> $m_{\text{PQ}} = \frac{3}{4}$, $m_{\text{QR}} = \frac{-4}{3} = -\frac{4}{3}$ *(2)*
>
> $\frac{3}{4} \times (-\frac{4}{3}) = -1$ *(1)*
>
> so PQ ⊥ QR and $\hat{\text{Q}} = \mathbf{90°}$ *(1)* ✓✓✓✓ (4)

15 ABCD is a parallelogram with A$(-2\ ;\ 0)$, B$(3\ ;\ 1)$ and C$(5\ ;\ 5)$. Find the coordinates of D. **(4)**

> Midpoint of AC = $(1{,}5\ ;\ 2{,}5)$ *(1)*
>
> Midpoint of BD: $(\frac{3+x}{2}\ ;\ \frac{1+y}{2}) = (1{,}5\ ;\ 2{,}5)$ *(1)*
>
> $x = 0$ and $y = 4$, so **D(0 ; 4)** *(2)* ✓✓✓✓ (4)

TOTAL: 37 marks

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