MARKING GUIDELINE

MARKS	DURATION	QUESTIONS
36	55 minutes	3

Name: ..... Class: .....

Date: .....

MARK ____ / 36

INSTRUCTIONS AND INFORMATION

1. Answer **ALL** the questions in this question paper.
2. Answer QUESTION 1 by **circling** the letter (A–D) in the answer grid at the end of that section.
3. Show **ALL** calculations clearly.
4. Show all units where applicable.
5. Number the answers correctly according to the numbering system used in this question paper.
6. A non-programmable calculator may be used, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1 [10 MARKS]

Four options are given as possible answers to the following questions. Choose the correct answer and circle the letter (A–D) in the grid at the end of this section. If you want to change your choice, put a cross through the wrong letter and circle your new choice.

1.1 $\log_2 8 = \ldots$ (1)

A) 3 ✓ **B)** 2

C) 4 **D)** 16

Answer: A — $2^3 = 8$.

- **B** — read off the base instead of the exponent
- **C** — $2^4 = 16$, not 8
- **D** — multiplied 2 by 8

1.2 $\log 100 = \ldots$ (1)

A) 100 **B)** 2 ✓

C) 1 000 **D)** 10

Answer: B — The base is 10, and $10^2 = 100$.

- **A** — read off the argument instead of the exponent
- **C** — $10^3 = 1\,000$, not 100
- **D** — read off the base instead of the exponent

1.3 The inverse of $y = 2^x$ is ... (1)

A) $y = \log_2 x$ ✓ **B)** $y = 2^{-x}$

C) $y = x^2$ **D)** $y = \log_x 2$

Answer: A — Swap x and y: $x = 2^y$, which in log form is $y = \log_2 x$.

- **B** — that is the reflection in the y-axis, not the inverse
- **C** — that is a parabola — the base and the exponent have swapped roles
- **D** — the base stays 2; only x and y swap

1.4 The graph of $y = \log_2 x$ passes through ... (1)

A) $(0\ ;\ 1)$ **B) $(1\ ;\ 0)$ ✓**

C) $(2\ ;\ 0)$ **D)** $(0\ ;\ 2)$

> **Answer: B** — $\log_2 1 = 0$ because $2^0 = 1$, so the graph cuts the x-axis at $(1\ ;\ 0)$.
>
> - **A** — that point lies on $y = 2^x$, the inverse
> - **C** — at $x = 2$ the graph is at $y = 1$, not 0
> - **D** — a log graph never meets the y-axis

1.5 The graph of $y = \log_{\frac{1}{2}} x$ ... (1)

A) increases from left to right **B) decreases from left to right ✓**

C) is a straight line **D)** is defined for $x < 0$

> **Answer: B** — With a base between 0 and 1 the log graph DECREASES; it still passes through $(1\ ;\ 0)$ and exists only for $x > 0$.
>
> - **A** — that happens when the base is greater than 1
> - **C** — a log graph is a curve
> - **D** — no power of $\frac{1}{2}$ is zero or negative, so $x > 0$

1.6 If $\log_3 x = 4$, then $x =$ ... (1)

A) $1{,}32$ **B)** 12

C) 81 ✓ **D)** 64

> **Answer: C** — $\log_3 x = 4$ means $3^4 = x$.
>
> - **A** — took the fourth root of 3 instead of the fourth power
> - **B** — multiplied 3 by 4
> - **D** — used 4^3 instead of 3^4

1.7 $\log 1 =$ ... (1)

A) 10 **B)** undefined

C) 0 ✓ **D)** 1

> **Answer: C** — Any base raised to the power 0 gives 1.
>
> - **A** — that is the base, not the answer
> - **B** — $\log 1$ is perfectly well defined; it is $\log 0$ that is not
> - **D** — $\log 10 = 1$; it is $\log 1$ that is 0

1.8 $\log(-5)$ is ... (1)

A) 5 **B) undefined ✓**

C) 0 **D)** $-\log 5$

> **Answer: B** — No power of a positive base gives a negative result, so the logarithm of a negative number does not exist.
>
> - **A** — that is the argument itself
> - **C** — $\log 1 = 0$, but -5 is not 1
> - **D** — a minus cannot be pulled out of a logarithm's argument

1.9 R5 000 grows to R10 000 at 8% p.a. compounded annually. The number of years n is given by ... (1)

A) $n = \log_{1,08} 2$ ✓ **B)** $n = \log_2 1{,}08$

C) $n = 2 \div 1{,}08$ **D)** $n = 1{,}08^2$

> **Answer: A** — $5\,000(1{,}08)^n = 10\,000$ gives $1{,}08^n = 2$, so $n = \log_{1,08} 2 \approx 9{,}01$ years.
> - **B** — the base is the growth factor 1,08, not 2
> - **C** — an exponent is not found by dividing
> - **D** — that is two years of growth, not the number of years

1.10 Solve for x, correct to TWO decimal places: $2^x = 20$ (1)

A) 10 **B)** 0,30

C) 1,30 **D)** **4,32** ✓

> **Answer: D** — $x = \log 20 \div \log 2 = 4{,}32$.
> - **A** — halved 20 instead of solving with logarithms
> - **B** — that is log 2 on its own
> - **C** — that is log 20 on its own

ANSWER GRID — MARKING GUIDELINE

1.1	(A)	B	C	D
1.2	A	(B)	C	D
1.3	(A)	B	C	D
1.4	A	(B)	C	D
1.5	A	(B)	C	D
1.6	A	B	(C)	D
1.7	A	B	(C)	D
1.8	A	(B)	C	D
1.9	(A)	B	C	D
1.10	A	B	C	(D)

QUESTION 2 [13 MARKS]

Solve for x. Show ALL your working.

2.1 R5 000 is invested at 9% p.a. compounded annually. Solve $5\,000(1{,}09)^n = 12\,000$ to find how long it takes to grow to R12 000, correct to TWO decimal places, and interpret your answer. (6)

$(1{,}09)^n = \frac{12\,000}{5\,000} = 2{,}4$ *(2)*

$n = \log_{1,09} 2{,}4$ *(2)*

$n =$ **10,16 years** *(1)*

Interest is added once a year, so the investment first exceeds R12 000 after **11 years** *(1)*

2.2 $3^x = 20$, correct to TWO decimal places (4)

Write it in log form: $x = \log_3 20$ *(2)*

$= 2{,}7268\ldots$ *(1)*

$x =$ **2,73** *(1)*

2.3 $\log_x 81 = 4$ **(3)**

> $x^4 = 81$ *(2)*
> $\boldsymbol{x = 3}$ *(1)* — a log base must be positive

QUESTION 3 [13 MARKS]

Answer the questions below WITHOUT the use of a calculator.

3.1 Write $\log_3 81 = 4$ in exponential form, and hence determine $\log_3 \frac{1}{9}$. **(4)**

> $3^4 = 81$ *(2)*
> $\frac{1}{9} = 3^{-2}$ *(1)*, so $\log_3 \frac{1}{9} = \mathbf{-2}$ *(1)*

3.2 Determine $\log_2 32 + \log_5 1$. **(3)**

> $\log_2 32 = 5$, because $2^5 = 32$ *(1)*
> $\log_5 1 = 0$, because $5^0 = 1$ *(1)*
> Total $= \mathbf{5}$ *(1)*

3.3 Write down the equation of the inverse of $f(x) = 2^x$ in the form $y = \ldots$ **(3)**

> Swap x and y: $x = 2^y$ *(2)*
> $\boldsymbol{y = \log_2 x}$ *(1)*

3.4 Write down the domain of $y = \log_2 x$, and explain why. **(3)**

> $\boldsymbol{x > 0}$ *(2)*
> No power of 2 gives zero or a negative number, so the log of such a number does not exist *(1)*

TOTAL: 36 marks

This question paper consists of 3 questions.

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