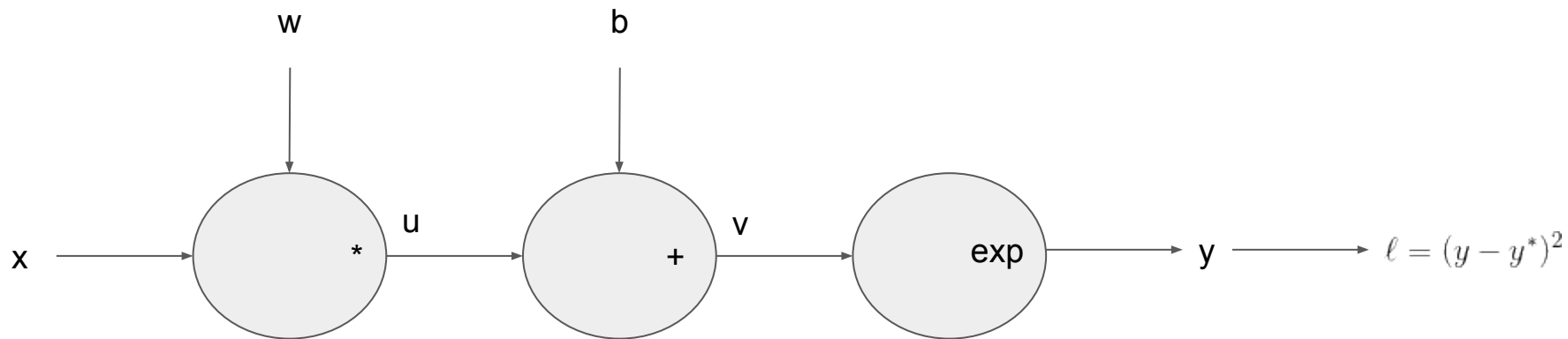


How automatic differentiation works

on an example

FORWARD PASS

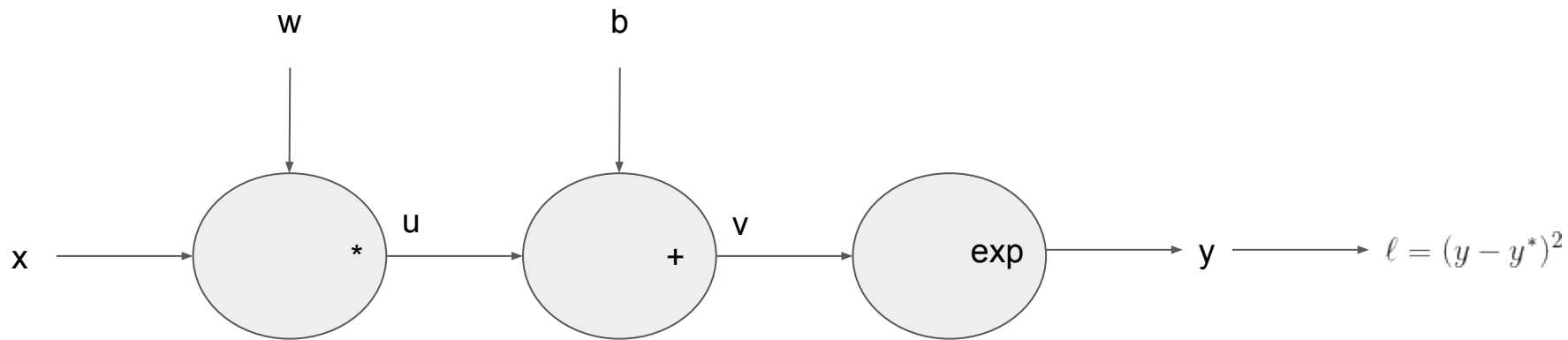


$$u = wx$$

$$v = u + b$$

$$y = e^v$$

$$\ell = (y - y^*)^2$$



$$u = wx$$

$$v = u + b$$

$$y = e^v$$

$$\ell = (y - y^*)^2$$

$$\frac{\partial u}{\partial w} = x$$

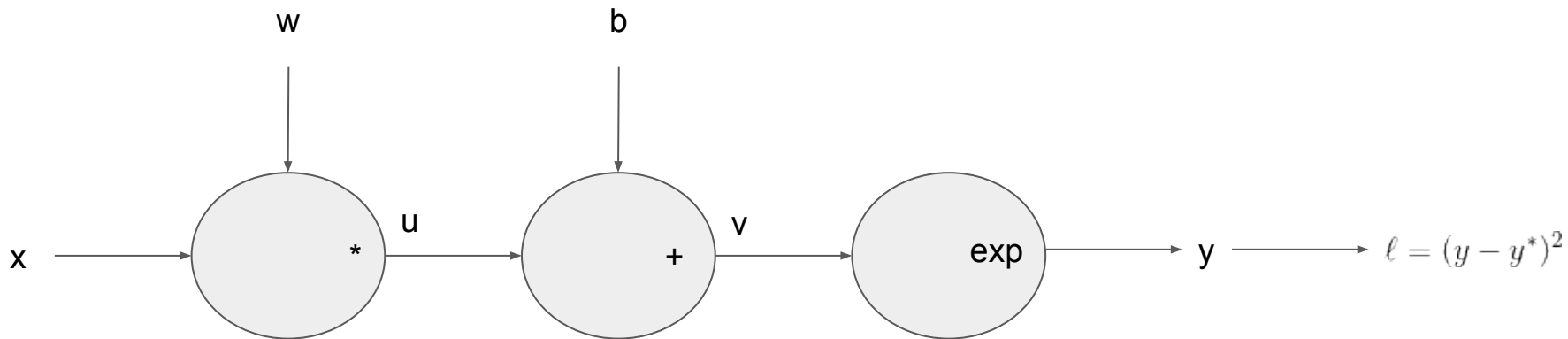
$$\frac{\partial v}{\partial b} = 1, \quad \frac{\partial v}{\partial u} = 1$$

$$\frac{\partial y}{\partial v} = e^v = y.$$

$$\frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w}$$

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}$$

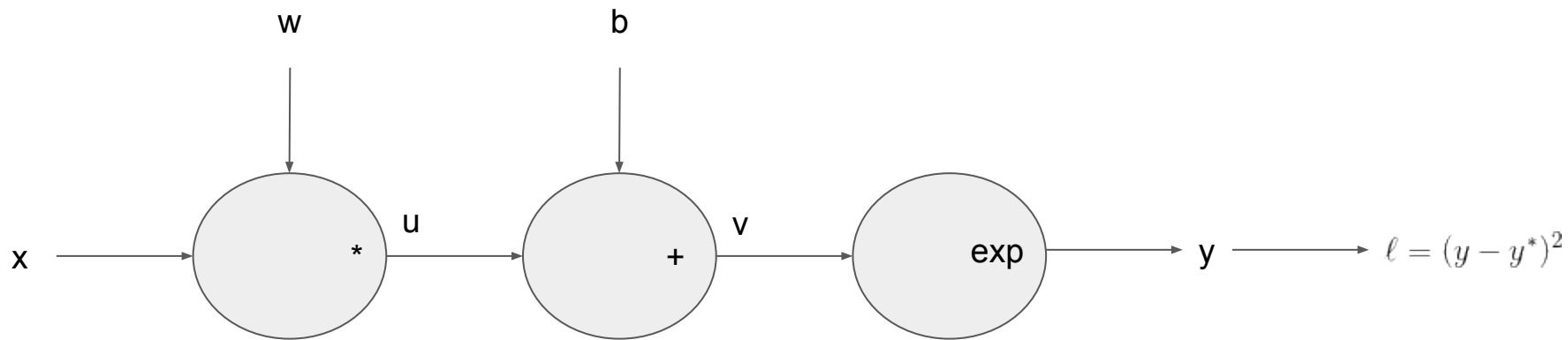
CHAIN RULE



$$\begin{aligned}u &= wx \\v &= u + b \\y &= e^v \\ \ell &= (y - y^*)^2\end{aligned}$$

$$\begin{aligned}\frac{\partial u}{\partial w} &= x \\ \frac{\partial v}{\partial b} &= 1, \quad \frac{\partial v}{\partial u} = 1 \\ \frac{\partial y}{\partial v} &= e^v = y.\end{aligned}$$

$$\begin{aligned}\frac{\partial y}{\partial w} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial u} \frac{\partial u}{\partial w} & \frac{\partial \ell}{\partial w} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w} \\ \frac{\partial y}{\partial b} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial b} & \frac{\partial \ell}{\partial b} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}\end{aligned}$$



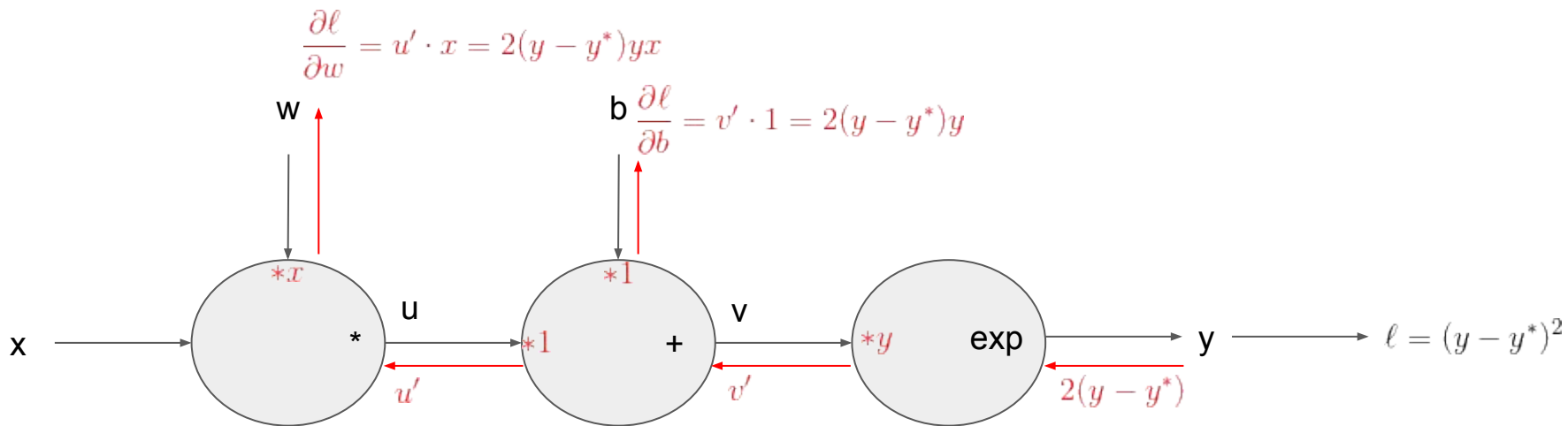
$$\begin{aligned}
 u &= wx \\
 v &= u + b \\
 y &= e^v \\
 \ell &= (y - y^*)^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial u}{\partial w} &= x \\
 \frac{\partial v}{\partial b} &= 1, \quad \frac{\partial v}{\partial u} = 1 \\
 \frac{\partial y}{\partial v} &= e^v = y.
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \ell}{\partial w} &= 2(y - y^*) \cdot y \cdot 1 \cdot x \\
 \frac{\partial \ell}{\partial b} &= 2(y - y^*) \cdot y \cdot 1
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial y}{\partial w} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial u} \frac{\partial u}{\partial w} & \frac{\partial \ell}{\partial w} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w} \\
 \frac{\partial y}{\partial b} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial b} & \frac{\partial \ell}{\partial b} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}
 \end{aligned}$$

BACKWARD PASS



$$\begin{aligned}
 u &= wx \\
 v &= u + b \\
 y &= e^v \\
 \ell &= (y - y^*)^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial u}{\partial w} &= x \\
 \frac{\partial v}{\partial b} &= 1, \quad \frac{\partial v}{\partial u} = 1 \\
 \frac{\partial y}{\partial v} &= e^v = y.
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \ell}{\partial w} &= 2(y - y^*) \cdot y \cdot 1 \cdot x \\
 \frac{\partial \ell}{\partial b} &= 2(y - y^*) \cdot y \cdot 1
 \end{aligned}$$

$$\begin{aligned}
 v' &= 2(y - y^*) \cdot y \\
 u' &= v' \cdot 1
 \end{aligned}$$