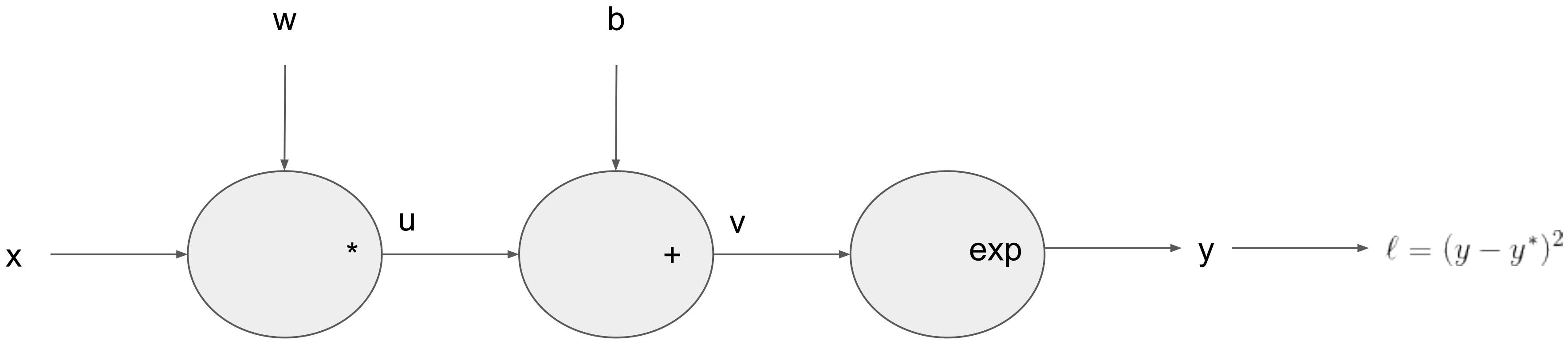
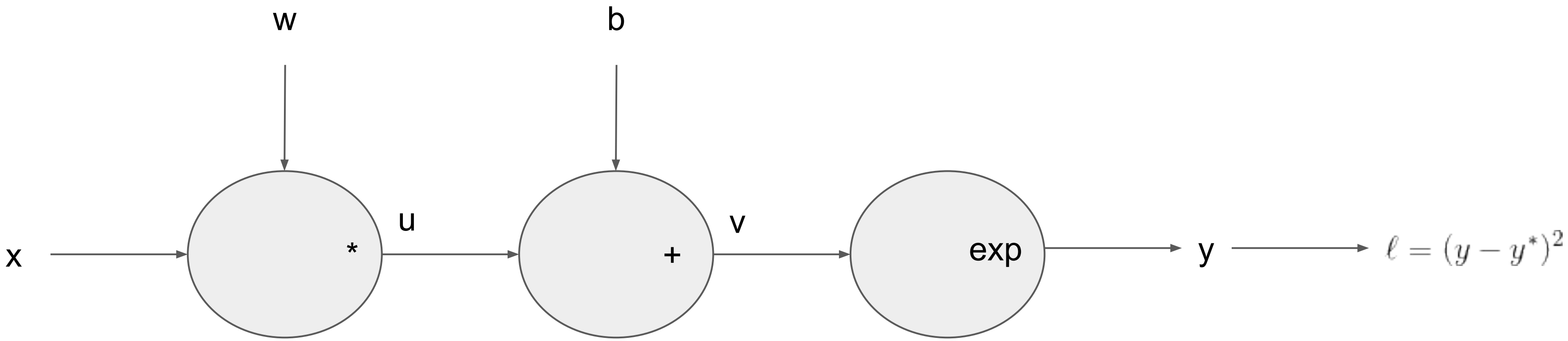
How automatic differentiation works

on an example

FORWARD PASS



$$u = wx$$
$$v = u + b$$
$$y = e^v$$
$$\ell = (y - y^*)^2$$



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\ell &= (y - y^*)^2
\end{aligned}
$$

$$\frac{\partial u}{\partial w} = x$$

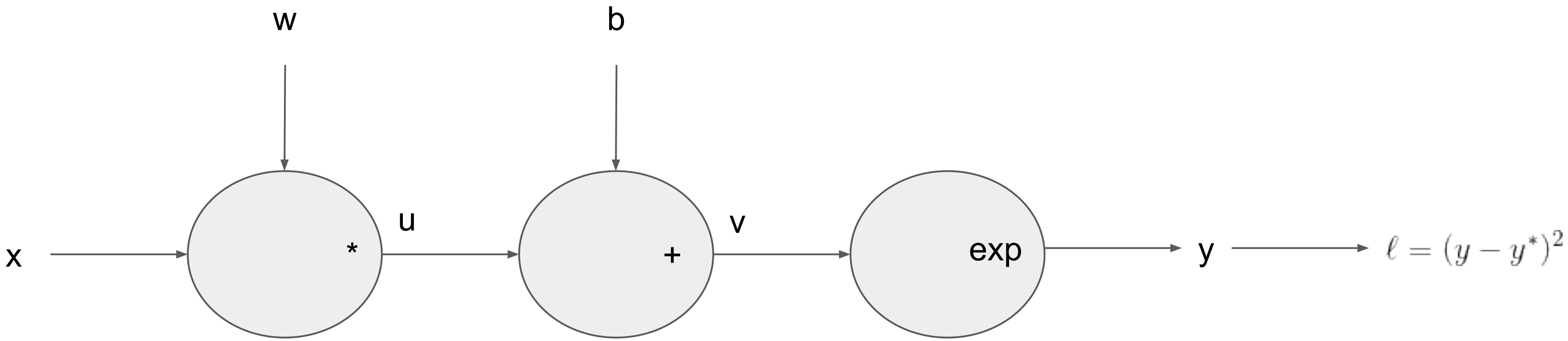
$$\frac{\partial v}{\partial b} = 1, \quad \frac{\partial v}{\partial u} = 1$$

$$\frac{\partial y}{\partial v} = e^v = y.$$

$$\frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w}$$

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial b}$$

CHAIN RULE



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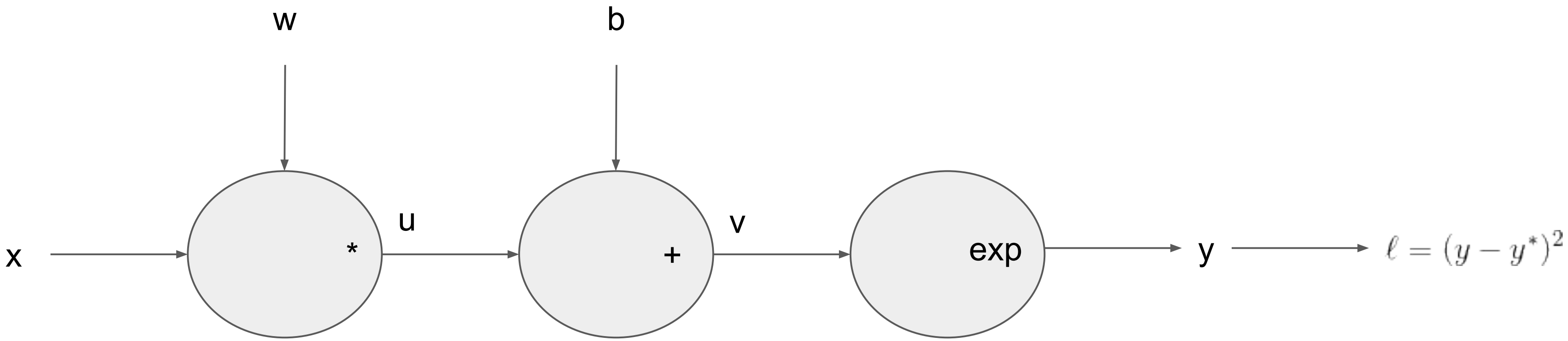
$$\frac{\partial y}{\partial w} = \frac{\partial y}{\partial v}\frac{\partial v}{\partial u}\frac{\partial u}{\partial w} \qquad \frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w}$$
$$\frac{\partial y}{\partial b} = \frac{\partial y}{\partial v}\frac{\partial v}{\partial b} \qquad \frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial b}$$



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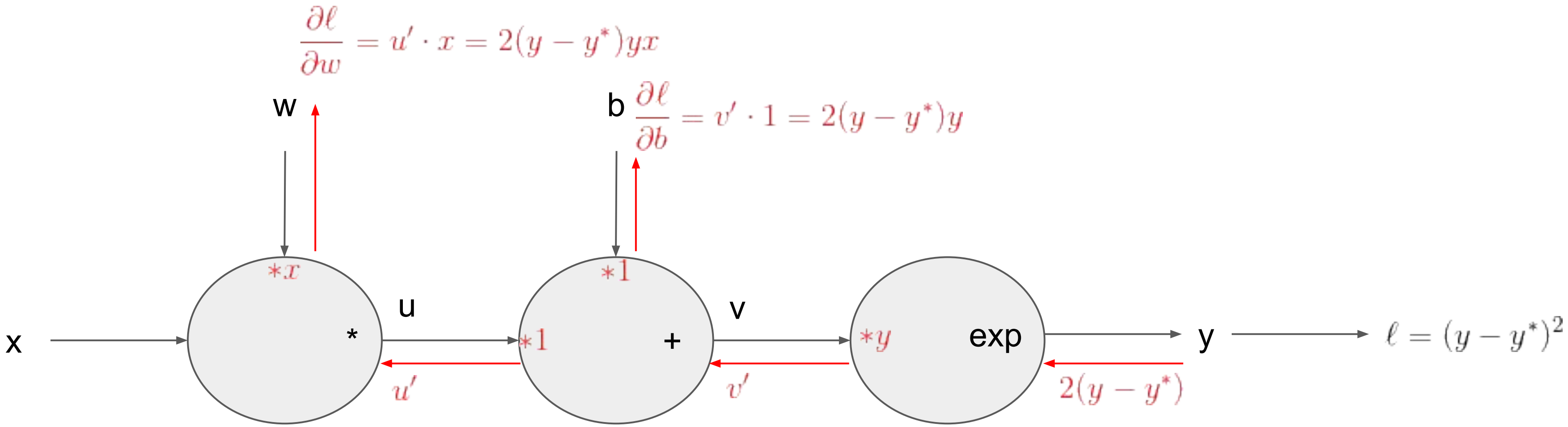
$$\frac{\partial \ell}{\partial w} = 2(y - y^*) \cdot y \cdot 1 \cdot x$$

$$\frac{\partial \ell}{\partial b} = 2(y - y^*) \cdot y \cdot 1$$

$$\frac{\partial y}{\partial w} = \frac{\partial y}{\partial v}\frac{\partial v}{\partial u}\frac{\partial u}{\partial w} \qquad \frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w}$$

$$\frac{\partial y}{\partial b} = \frac{\partial y}{\partial v}\frac{\partial v}{\partial b} \qquad \frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial b}$$

BACKWARD PASS



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$$\frac{\partial \ell}{\partial b} = 2(y - y^*) \cdot y \cdot 1$$

$$v' = 2(y - y^*) \cdot y)$$
$$u' = v' \cdot 1$$