Analytic Tableau Rules

1.	$T\ \varphi_1$	Premise
2.	$T\ \varphi_2$	Premise
⋮	⋮	⋮
n.	$T\ \varphi_n$	Premise
n+1.	$F\ \psi$	Conclusion
⋮	⋮	⋮

(a) Initial Tableau

⋮	⋮	⋮
$m.$	T φ	
⋮	⋮	⋮
$n.$	F φ	
⋮	⋮	⋮
$p.$	$\bot$	m,n

(b) Closed Branch

⋮	⋮	⋮
m.	T $\neg\varphi$	
⋮	⋮	⋮
n.	F φ	m

(c) Negation-true ($\neg T$)

⋮	⋮	⋮
m.	F $\neg\varphi$	
⋮	⋮	⋮
n.	T φ	m

(d) Negation-false ($\neg F$)

⋮	⋮	⋮
m.	T $\varphi \wedge \psi$	
⋮	⋮	⋮
n.	T φ	m
n+1.	T ψ	m

(e) And-true ($\wedge T$)

⋮	⋮		⋮
m.	F $\varphi \wedge \psi$		
⋮	⋮		⋮
n.	{	F φ	m
⋮		⋮	⋮
	}		
p.	{	F ψ	m
⋮		⋮	⋮
	}		

(f) And-false ($\wedge F$)

⋮	⋮		⋮
m.	T $\varphi \vee \psi$		
⋮	⋮		⋮
n.	{	T φ	m
⋮		⋮	⋮
	}		
p.	{	T ψ	m
⋮		⋮	⋮
	}		

(g) Or-true ($\vee T$)

⋮	⋮	⋮
m.	F $\varphi \vee \psi$	
⋮	⋮	⋮
n.	F φ	m
n+1.	F ψ	m

(h) Or-false ($\vee F$)

⋮	⋮		⋮
m.	T $\varphi \rightarrow \psi$		
⋮	⋮		⋮
n.	{	F φ	m
⋮		⋮	⋮
	}		
p.	{	T ψ	m
⋮		⋮	⋮
	}		

(i) Implication-true ($\rightarrow T$)

⋮	⋮	⋮
m.	F $\varphi \rightarrow \psi$	
⋮	⋮	⋮
n.	T φ	m
n+1.	F ψ	m

(j) Implication-false ($\rightarrow F$)

Analytic Tableau First-Order Rules

$$\begin{array}{lll} \vdots & \vdots & \vdots \\ \text{m.} & \text{T } \forall x\varphi(x) & \\ \vdots & \vdots & \vdots \\ \text{n.} & \text{T } \varphi(t) & m \end{array}$$

x is substitutable for t in φ

(a) Universal-true ($\forall T$)

$$\begin{array}{lll} \vdots & \vdots & \vdots \\ \text{m.} & \text{F } \forall x\varphi(x) & \\ \vdots & \vdots & \vdots \\ \text{n.} & \text{F } \varphi(a) & m \end{array}$$

a is a new variable

(b) Universal-false ($\forall F$)

$$\begin{array}{lll} \vdots & \vdots & \vdots \\ \text{m.} & \text{T } \exists x\varphi(x) & \\ \vdots & \vdots & \vdots \\ \text{n.} & \text{T } \varphi(a) & m \end{array}$$

a is a new variable

(c) Existential-true ($\exists T$)

$$\begin{array}{lll} \vdots & \vdots & \vdots \\ \text{m.} & \text{F } \exists x\varphi(x) & \\ \vdots & \vdots & \vdots \\ \text{n.} & \text{F } \varphi(t) & m \end{array}$$

x is substitutable for t in φ

(d) Existential-false ($\exists F$)