Power System:

A Power system consists of Generation Transmission and Distribution. An Modern power system is divided into four major parts They are Generation, Transmission Sub Transmission and Distribution.

Power System Analysis:

The Evolution of power system is called as Power system Analysis.

Functions of Power System Analysis:

1. To monitor the voltage at various buses.
2. Real & Reactive power flow between the buses.
3. To Design the circuit brakers.
4. To Plan future expansion of the existing system.
5. To Analise the system under different fault conditions.
6. To study the ability of the system for small and large disturbances. (steady state & Transience stability)
7. Co

Components of Power System:

The components of power system includes Generators, Power Transformer, Transmission lines, distribution lines, Loads, & Compensation devices like shunt, series, static VAR compensators.

Representation of Power system Components :

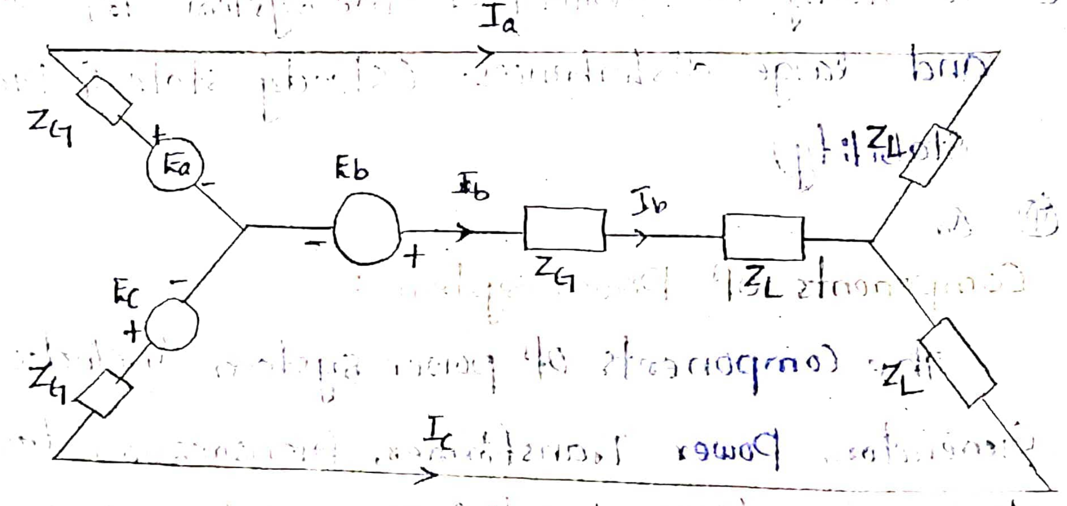
A Complete diagram of a power system representing all 3 phases becomes too complicated for a system of practical size. So ~~that~~ much so that, It may no longer convey the information it is intended to convey..

It is much more practical ~~to~~ represent a power system by means of simple ~~single~~ symbols for each component resulting in what is called One Line (or) Single Line Diagram.

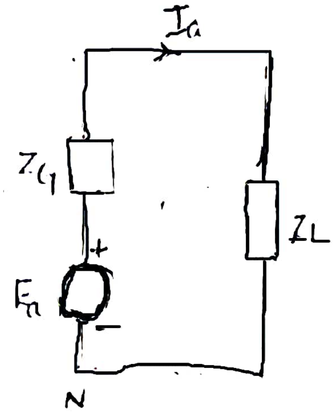
Per Unit System leads to great simplification of 3-ϕ network involving Transformer. An Impedance diagram drawn on a per unit basis doesn't requires ideal Transformers to be included in it.

An Important Element of the Power System is the Synchronous machine which is greatly influenced system during both Steady state and Transient Condition

The synchronous machine model in steady state is represented as



The 1-ϕ equivalent of balanced 3-ϕ [network] is as shown in figure

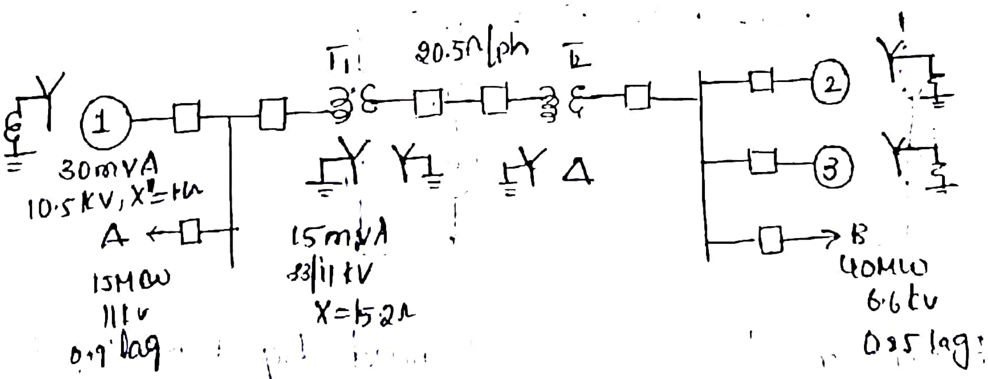


Impedence (or) Reactance Diagram :

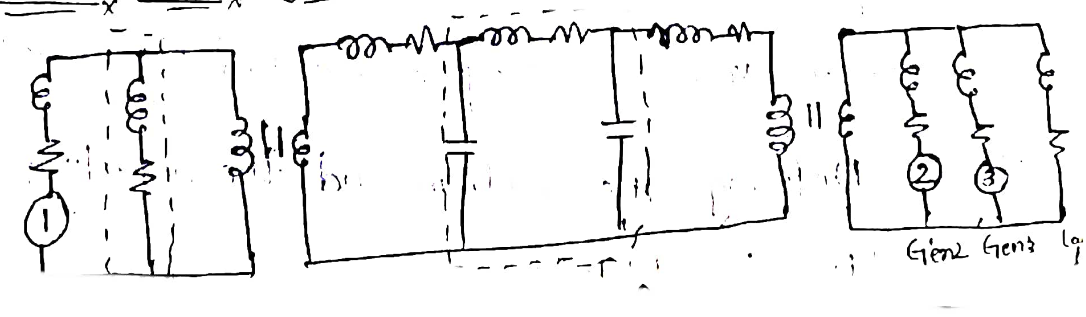
The Circuit Brakers need not be shown in a Load flow study but are must for protection. Steady represented by rectangular blocks.

Generator & Transformer connection Star, Delta or the nutral ground are indicated by the symbol drawn by the sign of the representation of these elements.

Single Line diagram of Simple Power system :

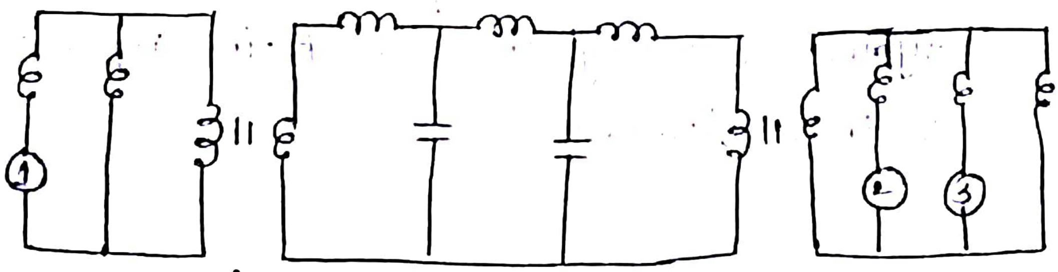


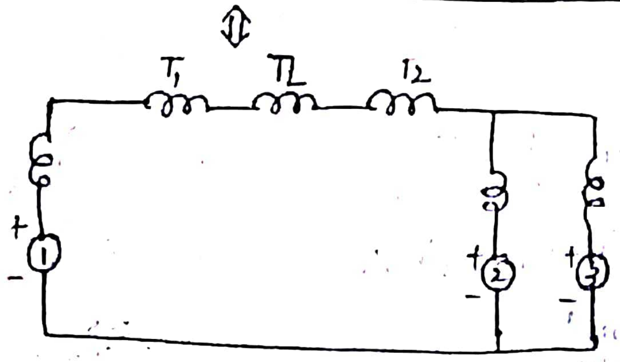
Impedence diagram :



* Magnetising reactance of the Transformers have been neglected.
* Generators are represented as Voltage sources with series resistance and inductive reactance.
* Transmission line is represented by π model
  Loads are assumed to be passive.
* Not involving rotating machines and are represented by resistance & inductive reactance in series.
* Nutral grounding Impedences don't appears in the diagram.
* As Balanced conditions are assumed

Reactance Diagram :-





Reactance diagrame is obtained by removing resistance in Impedence diagram.

* Steps Involved in Solving a Problem Using digital Computer

Step 1: Defining the problem and the objective of the Analysis are studied.

* Assesment of data available and preparation of the input data in proper format.
* This step also involves methods to synthesize are estimate the missing data.
* Development of suitable mathematical models to represent the power system and various components.
* This step also involves the integrating the models of various components and formulating the mathematical equations to be formed.
* Selection of suitable techinque to solve the mathematical equations.
* In most of the Analysis the equations are non linear and run suitable numerical methods has to be choosen which depends on computation time involves, Memory required, and accuracy, reliability of method and convergence characteristics
* Modern digital computers have very high computation Speed and large memory.
* For real time Applications the computation time is important.
* Development of flow chart or Algorithm for the solution of the equations.
* The Algorithm should have the All operational limitations of the power system.
* The Last step involves the Actual development of computer programme in a high level languages those are FORTRON, C++, MATLab are extensively used for the programming.

* The programme developed should be flexible & reliable and capable of handling diverse data.

Graph Theory:

Graph:

* The Geometrical interconnection of the elements in a network.
* Which describes to replace the network components by single ligne segments irrespective of the characteristics of a components.
* The Line segments are called elements and their terminals are called as nodes.
* A Node of an element are incident if the node is a terminal of the element
* Nodes can be incident to one or more elements.

Sub Graph:

* Sub Graph is any subset of elements of graph.

Path:

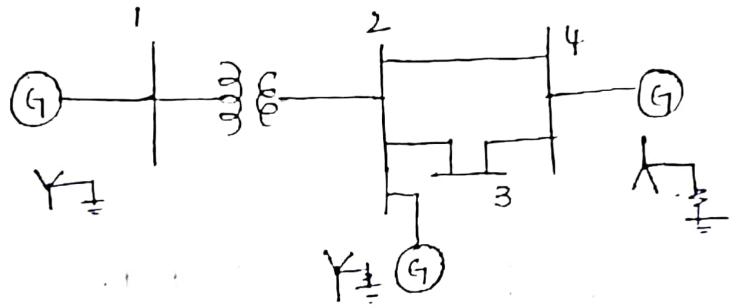
* It is a Sub Graph of connected elements with not more that two elements connected to any one node

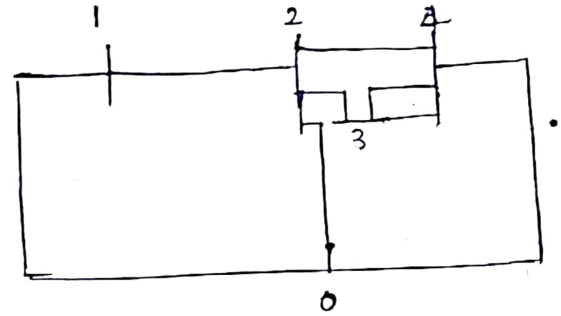
Connected Graph:

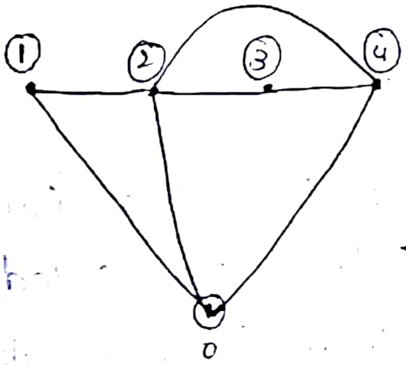
* A Graph is connected if and only if there is a path between every pair of nodes.

Oriented (or) Directed graph:

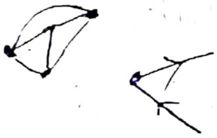
* If each element of the connected Graph is asign a direction is called Oriented (or) Directed Graph

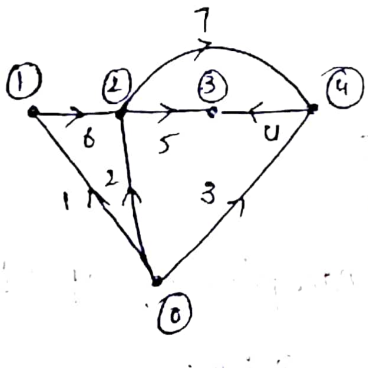


⇒ 

⇒ 

Graph



⇒ 

Directed Graph

Tree :

* A connected Sub Graph containing all nodes of the Graph but no loop is called a Tree.
* Elements of ~~Branch~~ Tree are called Branches
* No. of branches required $b = n-1$

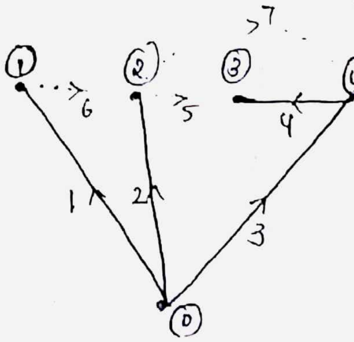
b – branches

n – no. of nodes in Graph

Number of possible trees for a Graph is

$\Rightarrow \left| [A][A]^T \right|$

where A is reduced incidence Matrix.

$\Rightarrow$ 

Tree for a given Graph

Co-Tree :-

* The Elements of the connected Graph that are not included in the tree are called Links and form a sub Graph not necessarily connected called Co-Tree.

$\Rightarrow$ 

* The Co-Tree is a compliment of the tree.
* The no.of links in a Co.Tree.

$l = e - b$ $\qquad$ $e \rightarrow$ Total no.of elements of graph

$= e - (n-1)$ $\qquad$ $b \rightarrow$ no.of branches in a tree

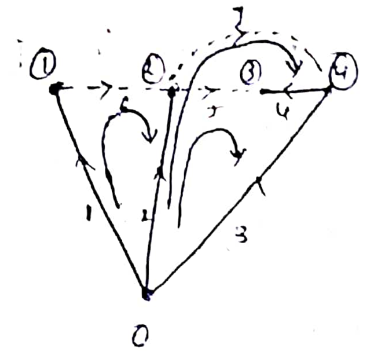
$= e - n + 1$ $\qquad$ $n \rightarrow$ Total no.of nodes

Loop :-

* If a link is added to the Tree the resulting Graph contains one closed path called a loop.
* The addition of each subsequent link forms one (or) more additional loops

* Loops which contains only one link are independent and are called Basic Loop (or) Indipendent Loop.

* Orientation of the Basic Loop is choosen to be same that of Links.



Cut Set :-

* A Set of elements that if removed divides a connected graph into two connected sub Graphs.

* A Unique indipendent Group of cut sets may be choosen if each cutset contains only one branch

* Indipendent Cut-set are called as Basic Cutsets

No.of cutsets = No.of Branches

* Orientation of the Basic Cutset is Choosen to be same as that of its Branch.

Incedence Matrices :-

* Any Oriented Graph can be described completely in the compact complex Matrix form.

* where it specified the orientation of the each branch in the Graph and the nodes at which this Branch is incident. This matrix is called as Incidence Matrix.

Types of Incidence Matrix:

1. Element Node Incidence Matrix $[\hat{A}]$ $e \times n$
2. Bus Incidence Matrix $[A]$ $e \times n-1$
3. Branch path Incidence Matrix $[K]$ $b \times$ nodes
4. Basic cutset Incidence Matrix $[B]$ $e \times n-1$
5. Augmented cutset Matrix $[\hat{B}]$ $e \times e$
6. Basic Loop Incidence Matrix $[C]$ $e \times l$
7. Augmented Loop Incidence Matrix $[\hat{C}]$ $e \times e$

1. Element Node Incidence Matrix $[\hat{A}]$:

* The Incidence of elements to nodes in a connected graph is shown by the Element node Incidence Matrix $[\hat{A}]$

$a_{ij} = +1$

If the ith element is incident to and oriented away from jth node.

$a_{ij} = -1$

If the ith element is incident to and oriented towards jth node.

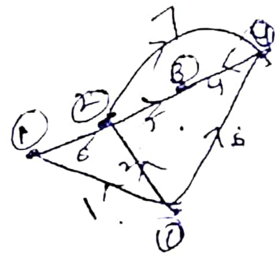
$a_{ij} = 0$

If the ith element is not incident to the jth node.

The dimensions of $\hat{A}$ matrix is $e \times n$, where e is no. of elements and n is no. of nodes.

e \ n	0	1	2	3	4
1	+1	-1	0	0	0
2	+1	0	-1	0	0
3	+1	0	0	0	-1
4	0	0	0	-1	+1
5	0	0	+1	-1	0
6	0	+1	-1	0	0
7	0	0	+1	0	-1

$\hat{A} \Rightarrow$ $e \times n$



Reduce (or)

② Bus Incidence Matrixes [A] :-

* Any node of a connected graph can be selected as the reference node. The variables of the other nodes referred to as buses can be measures with respect to the unsigned reference.

* Delete the column corresponding to the reference node is bus incidence matrix (or) Element Bus Incidence Matrix.

The order is $A = (e \times n-1)$

$A \Rightarrow$

e \ bus	1	2	3	4
1	-1	0	0	0
2	0	-1	0	0
3	0	0	0	-1
4	0	0	-1	+1
5	0	+1	-1	0
6	+1	-1	0	0
7	0	+1	0	-1

e \ bus	Buses
Branches	A_b
Links.	A_l

$A_b = b \times (n-1)$ = branches × Buses

$A_l = l \times (n-1)$

* Where A_b is a non singular matrix with rank $(n-1)$

③ Branch Path Incidence Matrix [K] :-

The incidence of branches to paths in a tree is shown by the branch path incidence matrix, where a path is oriented from a bus to the reference node.

$K_{ij} = 1$

If the ith branch is in the path from jth bus to reference and is oriented in the same direction.

$K_{ij} = -1$

If the ith branch is in the path from jth bus to reference and is oriented in the opposite direction.

$K_{ij} = 0$

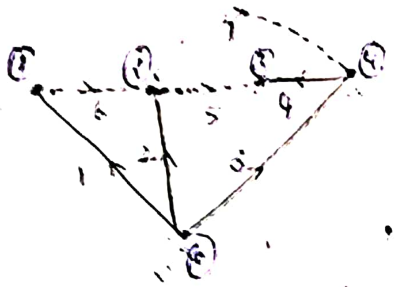
If the ith branch is not in the path from jth bus to reference.

With the node zero as reference the branch path incidence matrix associated with the tree is given as follows.

Path / Br	(1)	(2)	(3)	(4)
1	-1	0	0	0
2	0	-1	0	0
3	0	0	-1	0
4	0	0	-1	-1

K = branches × paths

K^T = paths × branches



$K^T . A_b$ = (paths × branches)(branches × Buses)

= paths × Buses.

$K^T = U A_b^{-1} = A_b^{-1}$

This is a non singular square matrix with rank $(n-1)$.

The Branch path incidences to p matrix K and a sub branch A_b relate the branches to path and branches to buses respectively.

Since there is a one-to-one correspondence between paths and buses.

$$A_b K^t = U \Rightarrow K^t = A_b^{-1}$$

④ Basic Cutset Matrix [B]:

The Incidence of elements to basic cutsets of a connected graph is shown by the basic cutset incidence matrix [B].

$b_{ij} = +1$

If ith element is incident to and oriented in the same direction as jth basic cutset.

$b_{ij} = -1$

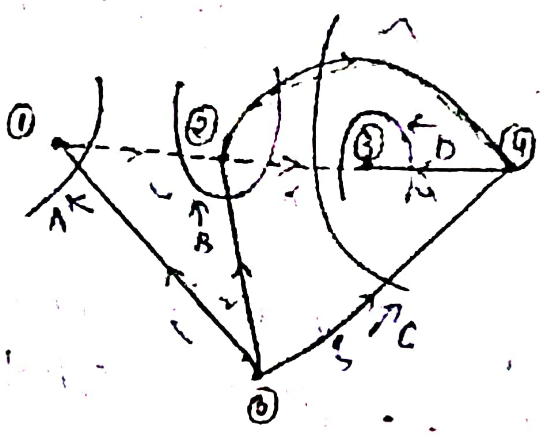
If the ith element is incident to and oriented in the opposite direction to the jth basic cutset.

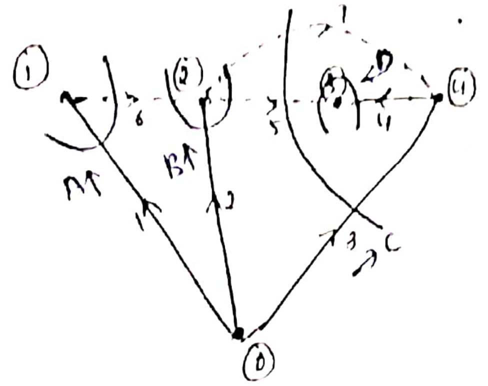
$b_{ij} = 0$

If the ith element is not incident to the jth basic cutset.

e \ b	A	B	C	D
1	+1	0	0	0
2	0	+1	0	0
3	0	0	+1	0
4	0	0	0	+1
5	0	-1	+1	+1
6	-1	+1	0	0
7	0	-1	+1	0

e \ b	
Branches	U_b
Links	B_l





* The Identity Matrix U_b will shows one to one Corespondence of Branches & Basic cutsets.

* The Sub Matrix B_l can be obtained from Bus Incidence Matrix A

* Since there is a One to One Correspondence of the branches and cutsets $B_l A_b$ shows the incidenc of the links to Buses.

$$B_l A_b = A_l$$

$$B_l = A_l A_b^{-1}$$

We know that $A_b^{-1} = K^t$

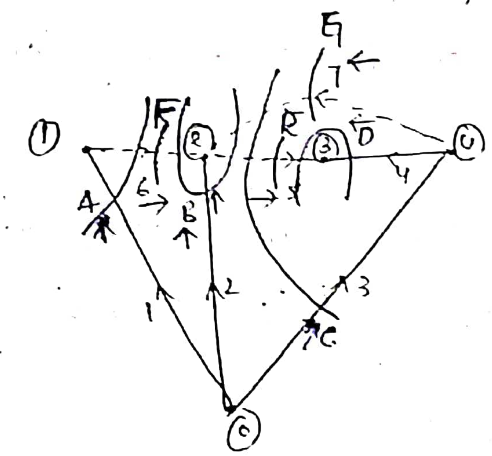
$$\boxed{B_l = A_l K^t}$$

⑤ Augmented Cutset Matrix :- $(\hat{B})$

* Ficticious Cutset called Tie Cutsets

* An Augmented Cutset Incidence Matrix is formed by adjoining to the Basic Cutset Incidence Matrix ... corresponding to these Tie cutsets

	e	A	B	C	D	E	F	G
		Basic Cutset				Tie cutset		
branches	1	1						
	2		1					
	3			1				
	4				1			
links	5		−1	1	1	1		
	6	−1	1				1	
	7		−1	1				1

⇒ 

* A Tie cutset is oriented in the same direction as the assosiated Links.

⇒

e	Basic Cutset	Tie Cutsets
Branchs	U_b	0
links	B_l	U_l

* This is the square matrix of order (e×e) and e is non singular.

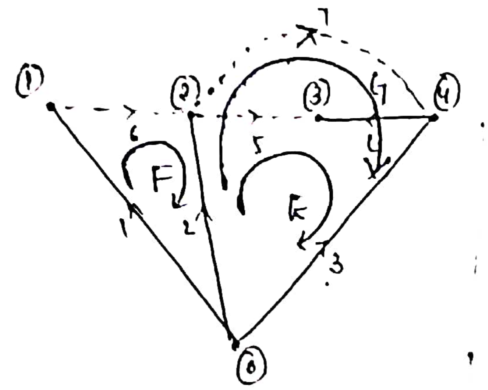
⑥ Basic Loop Incidence Matrix :- (C)

* The Incidence of elements to Basic loops of a connected graph is shown by Basic Loop Incidence Matrix [C].

⇒

e		E	F	G
1		0	+1	0
2		+1	-1	+1
3		-1	0	-1
4		-1	0	0
5		+1	0	0
6		0	+1	0
7		0	0	+1

⇒

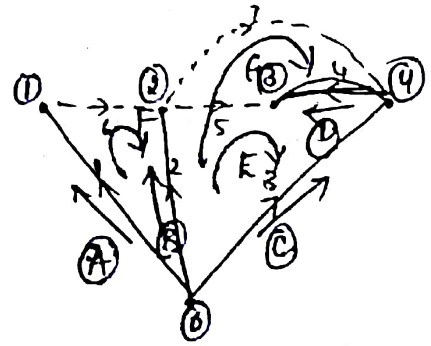


⇒

$$e \backslash l \quad \begin{bmatrix} \hat{C}_b \\ U_l \end{bmatrix}$$

7) Augmented Loop Incidence Matrix :- $[\hat{C}]$

⇒



⇒ A, B, C, D are the imaginary loops

	e\e	Open loops: A	B	C	D	Basic loop: E	F	G
Branches	1	+1				0	+1	0
	2		+1			+1	-1	+1
	3			+1		-1	0	-1
	4				+1	-1	0	0
Links	5	0	0	0	0	+1	0	0
	6	0	0	0	0	0	+1	0
	7	0	0	0	0	0	0	+1

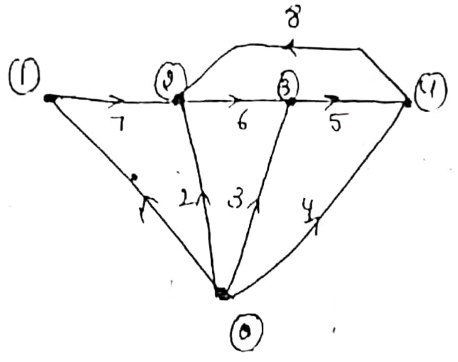
e\e	Open loops	Basic loops
Branches	U_b	C_b
Links	0	U_l

* An Open loop is defined as a path between adjacent loads conected by a branch.

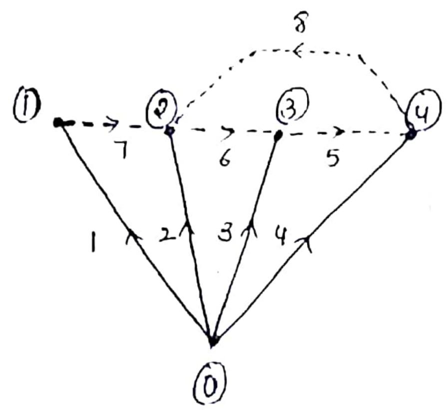
* The orientation of an open loop is same as that of the associated branch.

Problems

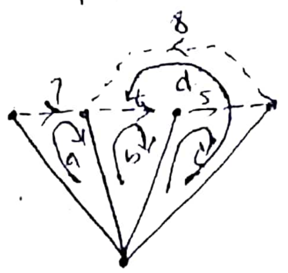
① Show the Basic loops and Basic cutsets for the Graph shown below. and verify any relation between them.



⇒ Consider the tree having branches 1, 2, 3, 4



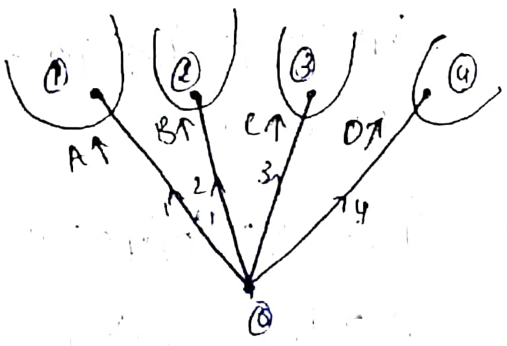
Basic loops $= l = 4$



$b = n-1 = 5-1 = 4$

$l = e-b = 8-4 = 4$

⇒ Basic cutsets $= b = 4$



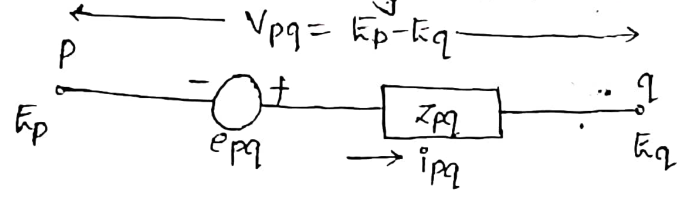
Basic cutsets = branches = no. of links = 4

Primitive Network :-

* A set of un connected elements is "Primitive Network". (or) A set of un coupled elements is called "Primitive Network".

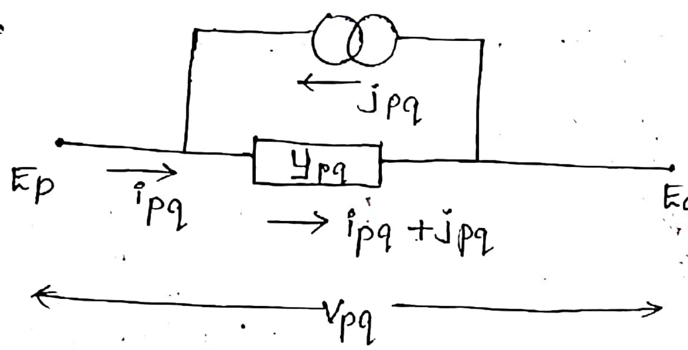
* The Network components represented both in impedance form and a admitance form.

⇒



:- Impedence form :-

⇒



:- Admitance form :-

* In Impedence form $V_{pq} + e_{pq} - i_{pq} Z_{pq} = 0$

$$V_{pq} + e_{pq} = i_{pq} Z_{pq} \quad \text{--- (1)}$$

$$\bar{v} + \bar{e} = [z]\, \bar{i}$$

* In Admitance form $i_{pq} + j_{pq} - y_{pq} V_{pq} = 0$

$$i_{pq} + j_{pq} = y_{pq} V_{pq} \quad \text{--- (2)}$$

$$\bar{i} + \bar{j} = [y]\, \bar{v}$$

* Where V_{pq} is voltage across the elements p,q
  e_{pq} is source voltage in series with element p&q

i_{pq} is current through element pq

j_{pq} is source & current in parallel with element pq

z_{pq} is self Impedence

y_{pq} is self Admittance.

* The performance equation of premitive network Can be derived from eq ① & ② by expressing the Variables and as vectors and parameters as matrices.

* In Impedence form

$$\bar{v}+\bar{e}=[z]\,\bar{i} \quad \text{—①}$$

* In Admittance form

$$\bar{i}+\bar{j}=[y]\,\bar{v} \quad \text{—②}$$

* These network equations can formed in the reference of i) Bus frame of reference

ii) Loop frame of reference &

iii) Branch fram of reference.

* In the above equations the diagnol elements of the matrices [z] & [y] is selef impendence i.e. z_{pqpq} & Self Admittance i.e. y_{pqpq}

* the half diagnal elements are z_{pqrs} (or) y_{pqrs}.

* Matrix of Admittance [y] can be obtained by inverting the matrix of impedence [z].

$$[y]=[z]^{-1}$$

* [z],[y] are diagnal matrices if no mutual Coupling between the elements.

⇒ from eq ①

$$V_{pq} + e_{pq} = i_{pq} z_{pq}$$

$$y_{pq} V_{pq} + y_{pq} e_{pq} = i_{pq}$$

Sub i_{pq} in eq ②

$$i_{pq} + j_{pq} = y_{pq} V_{pq}$$

$$\cancel{y_{pq} V_{pq}} + y_{pq} e_{pq} + j_{pq} = \cancel{y_{pq} V_{pq}}$$

$$\boxed{j_{pq} = - y_{pq} e_{pq}}$$

* The parallel source current in Admittance form is related to the series source voltage in impedence form by using equation $j_{pq} = -y_{pq} e_{pq}$

Bus

Formation of Network by Bus frame of Reference:-

* $\bar{E}_{BUS} = \bar{Z}_{BUS} \cdot \bar{I}_{BUS}$

* $\bar{I}_{BUS} = Y_{BUS} \bar{E}_{BUS}$

where * E_{BUS} is vector of Bus Voltages measured with respect to the reference voltages.

* I_{BUS} is vector of impressed Bus currents

* Z_{BUS} is Bus Impedence Matrix whose elements are open circuit driving point & transfer impedence.

* Y_{BUS} is Bus Admittance Matrix whose elements are short circuit driving point & transfer Admittance

* There are $(n-1)$ no.of nodal equations
  where n = no.of nodes.

Branch frame of Reference :-

* 'b' independent branch equations

$$\bar{E}_{BR} = Z_{BR} \bar{I}_{BR}$$

$$\bar{I}_{BR} = Y_{BR} \bar{E}_{BR}$$

* where

E_{BR} = vector of voltages across branches
I_{BR} = Vector of currents through branches.
Z_{BR} = Branch Impedence Matrix
Y_{BR} = Branch Admittance Matrix

Loop frame of Reference :-

* 'l' independent Loop equations

$$\bar{E}_{loop} = Z_{loop} \bar{I}_{loop}$$

$$\bar{I}_{loop} = Y_{loop} \bar{E}_{loop}$$

* where

$\bar{E}_{loop}$ = Vector of Voltages of basic loops
$\bar{I}_{loop}$ = Vector of Basic loop current
Z_{loop} = Loop impedence Matrix
Y_{loop} = Loop Admittance matrix.

Bus Admittance & Bus Impedence Matrix :-

* The Bus Admittance Matrix $[Y_{kk}]$ Y_{Bus} can be obtained by using Bus Incidence Matrix $[A]$.

* The performance equation of premelive network

$$\bar{i} + \bar{j} = [y]\bar{v} \quad \text{—} ①$$

* Premultiply by A^t.

$$A^t\bar{i} + A^t\bar{j} = A^t[y]\bar{v} \quad \text{—} ①$$

* Since the Matrix A shows the incidence of elements to Buses.

* Where $A^t\bar{i}$ is a vector in which each element is Algebroic sum of the current through the network element terminating at Bus.

* With KCL The Algebroic sum of the currents at the node is zero.

$$\therefore A^t\bar{i} = 0 \quad \text{—} ②$$

* Simillary $A^t\bar{j}$ gives the Algebroic sum of the source currents at each bus and equals the vector Impressed Bus currents.

$$\bar{I}_{Bus} = A^t.\bar{j} \quad \text{—} ③$$

* Now $I_{Bus} = A^t[y]\bar{v}$ — ④ By substituting eq ② & eq ③ in eq①.

* Powere into the network is

$(I_{Bus}^*)^t E_{Bus}$ and the sum of the power in the premitive network is

$(j^*)^t\bar{v}$.

* The power in the premetive and interconnect-ed network must be equal i.e., the transform-ation of variable must be power invarient.

* Hence

$$(I_{BUS}^{*})^{t} E_{BUS} = (j^{*})^{t} \bar{v} \quad — (5)$$

* From Eq (3)

$$(I_{BUS}^{*})^{t} = (j^{*})^{t} A^{*}$$

where $A^{*} = A$

* Now From Eq (5)

$$(j^{*})^{t} A \; E_{BUS} = (j^{*})^{t} \bar{v} \quad \forall j$$

$$\bar{v} = A \, E_{BUS} \quad — (6)$$

* Substitute in eq (4)

$$I_{BUS} = A^{t} [y] A \, E_{BUS}$$

$$I_{BUS} = y_{BUS} \, E_{BUS}$$

$$\left[\text{where } y_{BUS} = A^{t} [y] A\right]$$

* Bus Incidence Matrix is a singular matrix.

$A^{t} [y] A$ is a singular transfer matrix is y_{BUS}.

$$\therefore \; Z_{BUS} = y_{BUS}^{-1} = [A^{t} [y] A]^{-1}$$

$$\therefore \; \boxed{\begin{aligned} y_{BUS} &= A^{t} [y] A \\ Z_{BUS} &= [A^{t} [y] A]^{-1} \end{aligned}}$$

Branch Admittance & Branch Impedence Matrices

* The Branch Admittance Matrix Y_{BR} can be obtained by Basic Cutset Incidence Matrix.
* The performance equation is

$$\bar{i}+\bar{j} = [y]\bar{v}$$

* Pre multiply with the above B^t

$$B^t\bar{i} + B^t\bar{j} = B^t[y]\bar{v} \quad \text{—(1)}$$

* By using KCL $B^t\bar{i}=0$ since $B^t\bar{i}$ is the Algebraic sum entering into the Sub network.

$$I_{BR} = B^t\bar{j} \quad \text{—(2)}$$

* Substitute in above equation.

$$I_{BR} = B^t[y]\bar{v} \quad \text{—(3)}$$

* The power into the network = sum of power in premitive nlw

$$(I_{BR}^*)^t E_{BR} = (\bar{j}^*)^t \bar{v}$$

$$(\bar{j}^*)^t B^* E_{BR} = (\bar{j}^*)^t \bar{v}$$

$$(B^* = B)$$

$$\bar{v} = B E_{BR}$$

$$I_{BR} = B^t[y] B E_{BR}$$

* let $Y_{BR} = B^t[y] B$

$$I_{BR} = Y_{BR} E_{BR}$$

* $Z_{BR} = Y_{BR}^{-1} = [B^t[y]B]^{-1}$

$$\therefore \boxed{\begin{aligned} Y_{BR} &= B^t[y]B \\ Z_{BR} &= [B^t[y]B]^{-1} \end{aligned}}$$

Loop Admittance & Loop Impedance Matrix :-

* The $[Z_{loop}]$ & $[Y_{loop}]$ can be obtained by Basic Loop Incidence Matrix.

$$\bar{v} + \bar{e} = [Z]\bar{i} \quad -① $$

* pre multiply with c^t

$$c^t\bar{v} + c^t\bar{e} = c^t[Z]\bar{i} \quad -②$$

* By using KVL $c^t\bar{v} = 0$ since $c^t\bar{v}$ is the Algebraic sum of voltages in the loop.

$$c^t\bar{e} = E_{loop} \quad -③$$

* Substitute in eq ②

$$E_{loop} = c^t[Z]\bar{i} \quad -④$$

* The power in the network is equal to sum of powers in the loop network

$$(I_{loop}^*)^t E_{loop} = (i^*)^t \bar{e}$$

$$(I_{loop}^*)^t c^t \bar{e} = (i^*)^t \bar{e} \qquad \text{from ③}$$

$$(i^*)^t = (I_{loop}^*)^t c^t$$

$$\bar{i} = c^* \bar{I}_{loop}$$

$$i = c\, I_{loop} \quad (\because c^* = c)$$

Substitute in eq ④

$$E_{loop} = c^t[Z]c\, I_{loop}.$$

$$Z_{loop} = c^t[Z]c$$

$$E_{loop} = Z_{loop} I_{loop}$$

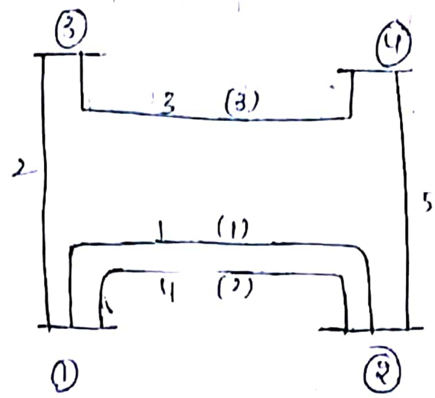
$$Y_{loop} = Z_{loop}^{-1} = [c^t[Z]c]^{-1}$$

$$\boxed{\begin{aligned} Z_{loop} &= c^t[Z]c \\ Y_{loop} &= [c^t[Z]c]^{-1} \end{aligned}}$$

*

	Loop	Bus	Branch
Current	$\bar{i} = C \bar{I}_{loop}$	$\bar{I}_{BUS} = A^t \bar{j}$	$\bar{I}_{BR} = B^t \bar{j}$
Voltage	$\bar{E}_{loop} = C^t \bar{e}$	$\bar{v} = A \bar{E}_{BUS}$	$\bar{v} = B \bar{E}_{BR}$

① Form the Incidence Matrices $[\hat{A}]$, $[A]$, $[B]$, $[\hat{B}]$, $[C]$, $[\hat{C}]$ for the network shown in figure.



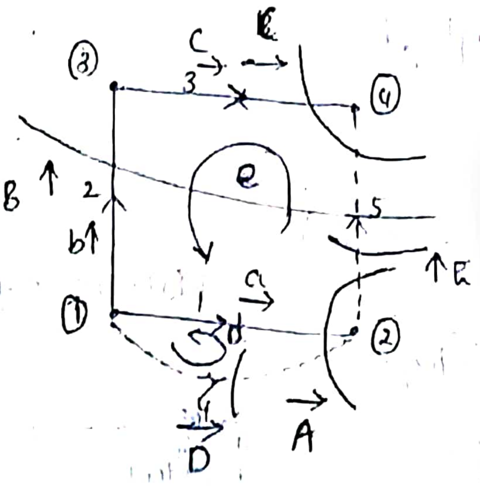
(n) Nodes = 4.

(e) Elements = 5

(b) Branches = b = (n-1) = 4-1 = 3

(l) Links = e-b = 5-3 = 2

Tree : Assign directions



⇒ Element node incidence matrix $[\hat{A}]$:

$[\hat{A}]$ ⇒

e \ n	(1)	(2)	(3)	(4)
1	+1	-1	0	0
2	+1	0	-1	0
3	0	0	+1	-1
4	+1	-1	0	0
5	0	+1	0	-1

⇒ Bus Incidence Matrix [A]:

⇒ node 1 is reference

[A] ⇒

e \ buses	(2)	(3)	(4)
1	-1	0	0
2	0	-1	0
3	0	+1	-1
4	-1	0	0
5	+1	0	1

⇒ Branch path Incidence matrix [K]:

⇒ node 1 is reference

[K] ⇒

b \ path	(2)	(3)	(4)
1	-1	0	0
2	0	-1	-1
3	0	0	-1

⇒ Basic Cutset matrix [B]:

no. of cutsets = no. of branches = 3

[B] ⇒

e \ basic cut-set	A	B	C
1	1	0	0
2	0	1	0
3	0	0	1
4	1	0	0
3	-1	+1	+1

⇒ Augmented Cutset Incidence matrix $[\hat{B}]$

e \ tr	A	B	C	D	E
1	1				
2		1			
3			1		
4	+1	0	0	1	0
5	-1	+1	+1	0	1

$\Rightarrow [\hat{B}]$

⇒ Basic Loop Incidence Matrix $[C]$

e \ loop	d	e
1	-1	+1
2	0	-1
3	0	-1
4	+1	0
5	0	[illegible]

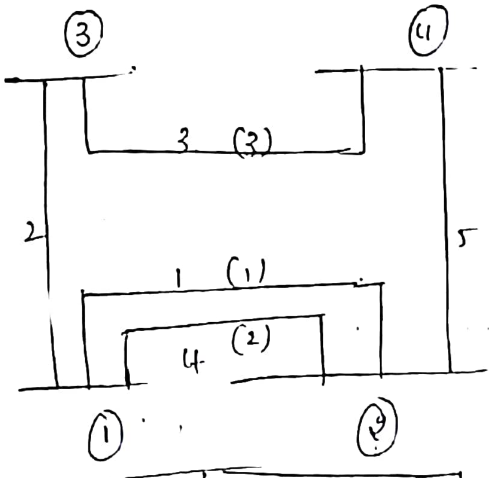
$\Rightarrow [C]$

⇒ Augment Loop Incidence matrix $[\hat{C}]$

e \ loops	a	b	c	d	e
1	1	0	[illegible]	1	+1
2	0	1	0	0	-1
3	0		1	0	-1
4	0	0	0	+1	+1
5	0	0	0	0	0

$\Rightarrow [\hat{C}]$

(2) Form the N/w matrix Y_{BUS}, Y_{BR}, Z_{loop} by Singular Transformation



Element	Bus code (p-q)	Impedence $Z_{pq\,pq}$
1	1-2 (1)	0.6
2	1-3	0.5
3	3-4	0.5
4	1-2 (2)	0.4
5	2-4	0.2

[Z] ⇒

e	1	2	3	4	5
1	0.6				
2		0.5			
3			0.5		
4				0.4	
5					0.2

$$[Y] = [Z]^{-1} = \begin{bmatrix} 1/0.6 & 0 & 0 & 0 & 0 \\ 0 & 1/0.5 & 0 & 0 & 0 \\ 0 & 0 & 1/0.5 & 0 & 0 \\ 0 & 0 & 0 & 1/0.4 & 0 \\ 0 & 0 & 0 & 0 & 1/0.2 \end{bmatrix}$$

$$[y] = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1.67 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \end{array}$$

$$\Rightarrow Y_{BUS} = A^t \, [y] \, A$$

$$= \underset{3\times 5}{\begin{bmatrix} -1 & 0 & 0 & -1 & +1 \\ 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}} \times \underset{5\times 5}{\begin{bmatrix} 1.67 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}} \times \underset{5\times 3}{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & +1 & -1 \\ -1 & 0 & 0 \\ +1 & 0 & -1 \end{bmatrix}}$$

$$= \underset{3\times 5}{\begin{bmatrix} -1.67 & 0 & 0 & -2.5 & 5 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 0 & -5 \end{bmatrix}} \underset{5\times 3}{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & +1 & -1 \\ -1 & 0 & 0 \\ +1 & 0 & -1 \end{bmatrix}}$$

$$Y_{Bus} = \underset{3\times 3}{\begin{bmatrix} 9.17 & 0 & -5 \\ 0 & 4 & -2 \\ -5 & -2 & 7 \end{bmatrix}}$$

1.67
2.5
5
9.17

$$\Rightarrow Y_{BUS} = B^t \, [y] \, B$$

$$= \underset{3\times 5}{\begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}} \underset{5\times 5}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}} \underset{5\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}}$$

$$= \underset{3\times 5}{\begin{bmatrix} 1.67 & 0 & 0 & 2.5 & -5 \\ 0 & 2 & 0 & 0 & 5 \\ 0 & 0 & 2 & 0 & 5 \end{bmatrix}} \underset{5\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}}$$

$$Y_{BUS} = \begin{bmatrix} 7.11 & -5 & -5 \\ -5 & 7 & 5 \\ -5 & 5 & 7 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow Z_{loop} = C^{t} [Z] C$$

$$= \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 1 \end{bmatrix}_{2\times 5} \begin{bmatrix} 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}_{5\times 5} \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5\times 2}$$

$$= \begin{bmatrix} -0.6 & 0 & 0 & 0.4 & 0 \\ 0.6 & -0.5 & -0.5 & 0.4 & 0.2 \end{bmatrix}_{2\times 5} \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5\times 2}$$

0.6
0.2
0.5
0.1
1.8

$$= \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1.8 \end{bmatrix}$$

$$Z_{loop} = \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1.8 \end{bmatrix}_{2\times 2}$$

$$\Rightarrow Y_{loop} = [Z_{loop}]^{-1}$$

$$= \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1.8 \end{bmatrix}^{-1}$$

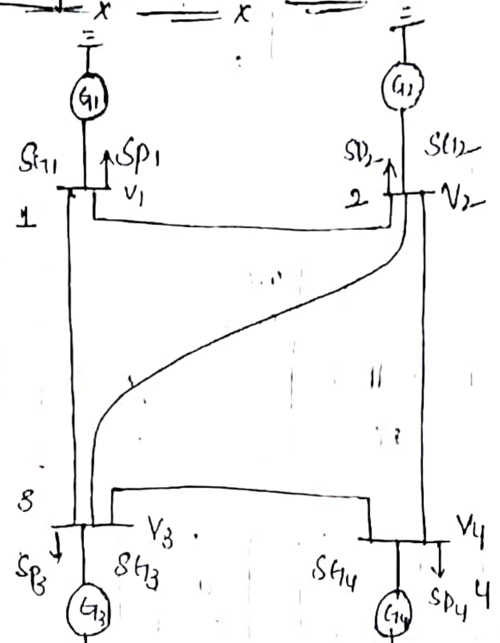
$$= \frac{1}{1.8 - 0.36} \begin{bmatrix} 1.8 & 0.6 \\ 0.6 & 1 \end{bmatrix}$$

2.6
1.8
1.0

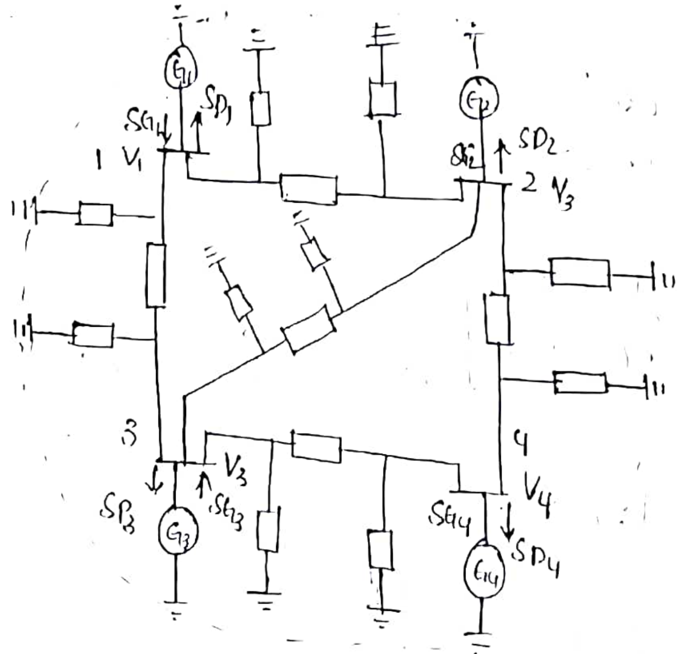
1.8
0.36
1.44

$$Y_{loop} = \begin{bmatrix} 1.25 & 0.416 \\ 0.416 & 0.69 \end{bmatrix}_{2\times 2}$$

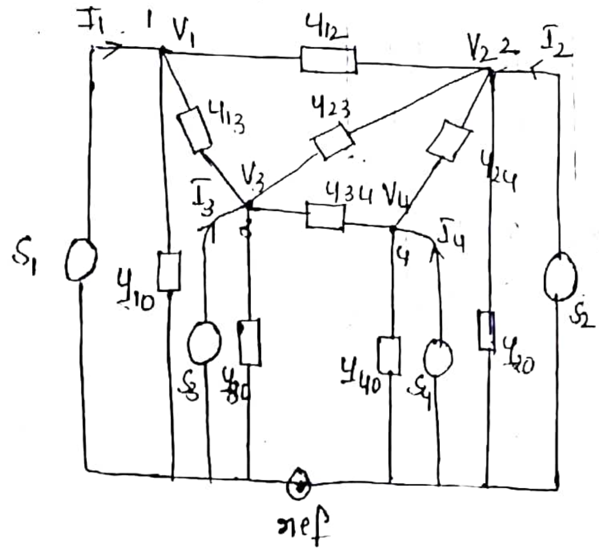
Direct Inspection Method :-



⇒ Equivalent Circuit :- [with references & grounds]



⇒ Combining all the references



$S_i = P_i + jQ_i$

At ith Bus the net complex power is injected into the bus is given by

$$S_i = P_i + jQ_i$$

$$= (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

Applying KCL at nodes 1, 2, 3 & 4

Node 1: $I_1 = V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$

$$I_1 = V_1 (y_{10} + y_{12} + y_{13}) + V_2 (y_{12}) - V_3 (y_{13})$$

Node 2: $I_2 = V_2 y_{20} + (V_2 - V_1) y_{12} + (V_2 - V_3) y_{23} + (V_2 - V_4) y_{24}$

$$I_2 = V_2 (y_{20} + y_{12} + y_{23} + y_{24}) - V_1 y_{12} - V_3 y_{23} - V_4 y_{24}$$

Node 3: $I_3 = V_3 y_{30} + (V_3 - V_1) y_{13} + (V_3 - V_2) y_{23} + (V_3 - V_4) y_{34}$

$$I_3 = -V_1 y_{13} + V_3 (y_{30} + y_{13} + y_{23} + y_{34}) - V_2 y_{23} - V_4 y_{34}$$

Node 4: $I_4 = V_4 y_{40} + (V_4 - V_2) y_{24} + (V_4 - V_3) y_{34}$

$$I_4 = -V_2 y_{24} - V_3 y_{34} + V_4 [y_{40} + y_{24} + y_{34}]$$

Write these equations in Matrix form.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{12} & y_{20}+y_{12}+y_{23}+y_{34} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{30}+y_{13}+y_{23}+y_{34} & -y_{34} \\ 0 & -y_{24} & -y_{34} & y_{40}+y_{24}+y_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

It can write as follows.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$\Rightarrow \therefore Y_{11} = y_{10} + y_{12} + y_{13}$; $Y_{22} = y_{20} + y_{12} + y_{23} + y_{24}$

$Y_{12} = Y_{21} = -y_{12}$ $\qquad$ $Y_{33} = y_{30} + y_{13} + y_{23} + y_{34}$

$Y_{23} = Y_{32} = -y_{23}$ $\qquad$ $Y_{44} = y_{40} + y_{24} + y_{34}$

$Y_{31} = Y_{13} = -y_{13}$

$Y_{14} = Y_{41} = -y_{14} = 0$

$Y_{24} = Y_{42} = -y_{24}$

$Y_{34} = Y_{43} = -y_{34}$

$\Rightarrow$ Where Y_{ii} = Self Admittance

Y_{ik} = Mutual Admittance

$$\Rightarrow \quad I_i = \sum_{i=1}^{n} Y_{ik} V_k$$

$i = 1, 2, 3 \cdots n$

$$\Rightarrow \quad I_{BUS} = Y_{BUS} V_{BUS}$$

* Where Y_{BUS} is $n \times n$ matrix where n is no. of buses.

$m = n + 1$ i.e., no. of node.

* Y_{BUS} is the symmetrical matrix so only $\frac{n(n+1)}{2}$ terms are to be stored in Y_{BUS} system.

* As i & k are not connected. Y_{BUS} of large network is very sparse. [A matrix containing more no. of

zero's] i.e., It has a large no. of zero elements.

* The sparsity may be as high as 90% which reduced numerical calculation and less memory required. $V_{BUS} = Z_{BUS} I_{BUS}$

$\Rightarrow Z_{BUS} = Y_{BUS}^{-1}$

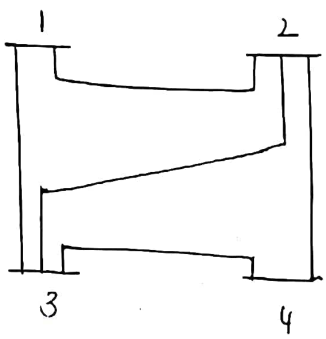
* While Y_{BUS} is a sparse matrix, Z_{BUS} is a full Matrix i.e, zero elements in the Y_{BUS} Matrix becomes non zero's in Z_{BUS} Matrix.

* Y_{BUS} is used in Load flow problems and Z_{BUS} is used in short circuit study.

26/12/2016

Problem

① The fig. shows Single Line Diagram of simple four bus system. Find Y_{BUS} by direct inspection method.



Line	R_{pu}	X_{pu}	Z_{pu}	$y_{pu} = \frac{1}{Z_{pu}}$
1-2	0.05	0.15	→ $0.05 + j0.15 - Z_{12}$	→ $2 - j6 = y_{12}$
1-3	0.10	0.30	→ $0.10 + j0.30 - Z_{13}$	→ $1 - j3 = y_{13}$
2-3	0.15	0.45	→ $0.15 + j0.45 - Z_{23}$	→ $0.667 - j2 - y_{23}$
2-4	0.10	0.30	→ $0.10 + j0.30 - Z_{24}$	→ $1 - j3 - y_{24}$
3-4	0.05	0.15	→ $0.05 + j0.15 - Z_{34}$	→ $2 - j6 - y_{34}$

Sol: $[Y_{BUS}]$ order is [no. of Buses × no. of Buses]

$$[Y_{BUS}]_{4\times4} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix}_{4\times4}$$

$y_{11} = y_{12} + y_{13}$

$= 2-j6+1-j3$

$= 3-j9$

$y_{12} = (2-j6) = -2+j6 = y_{21}$

$y_{13} = (1-j3) = -1+j3 = y_{31}$

$y_{14} = y_{41} = 0$

$y_{22} = y_{21} + y_{23} + y_{24}$

$= +2+j6 + 0.667-j2 + 1-j3$

$= \cancel{-0.33+j1} \quad = 3.667-j11$

$y_{23} = y_{32} = -(y_{23}) = -(0.667-j2) = -0.667+j2$

$y_{24} = y_{42} = -(y_{24}) = -(1-j3) = -1+j3$

$y_{32} = -0.667+j2$

$y_{33} = y_{31} + y_{32} + y_{34}$

$= 1-j3 + 0.667-j2+2-j6$

$= 3.667-j11$

$y_{34} = -(2-j6) = 2+j6$

$y_{41} = 0$

$y_{42} = -1+j3$

$y_{43} = -2+j6$

$y_{44} = y_{42} + y_{43}$

$= -1+j3 + 2+j6$

$= 3+j9$

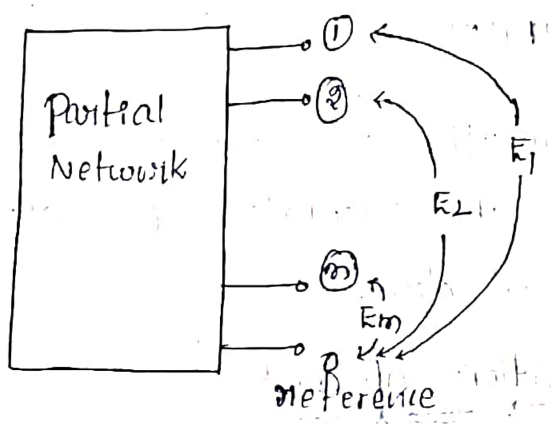
$$y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ +2+j6 & 3.66-j11 & -0.667+j2 & -1+j3 \\ -1+j3 & +0.66+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}_{4\times4}$$

The procedure for obtaining Y (or) Z matrix

* The procedure for obtaining y (or) z in any frame of reference requires involving inversions and multiplication which is laborious and time consuming processes.

* For large systems involving more number of nodes. If the mutual coupling exist between the elements direct inspection method is not aplicable so it is not aplicable to built Z_{Bus} by using an algorithm where symmetrically consider element by element for addition and build the complete network derectly from element parameters and such an algorithm can be very convinent to various manipulation that may be needed when system is in operation such as addition of lines, removal of lines and change in parameters

Algorithm for Bus Impedence Matrix :-



* Assume the partial network having 'm' buses and reference node '0'.

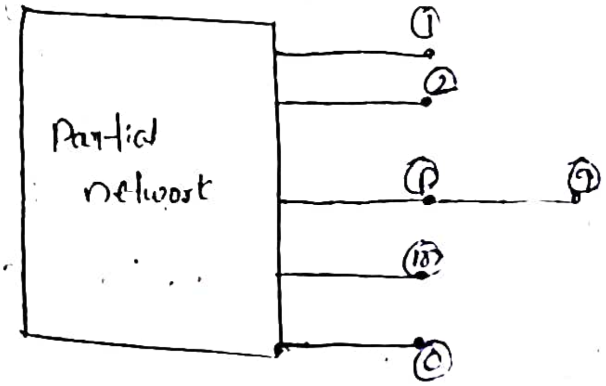
* $\bar{E}_{BUS} = Z_{BUS} \times \bar{I}_{BUS}$

$\bar{E}_{BUS}$ = m×1 vector of bus voltages measure with respect to all nodes with reference

$\bar{I}_{BUS}$ = m×1 vector of impressed bus currents

* When an element pq is added to the partial network it may be a branch (or) link.

Case: 1 ⇒ Addition of branch



* If pq is a branch a new bus 'q' is added to the partial network & the resultant Z_{BUS} is of order $(m+1)\times(m+1)$.

* The new voltage & current vectors of dimensions

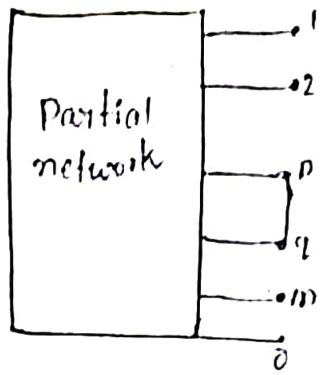
$\bar{E}_{BUS} = (m+1)\times 1$

$\bar{I}_{BUS} = (m+1)\times 1$

* To determine the new bus impedance matrix requires only the calculation of elements required is new row and column.

Case 2: ⇒ Addition of link

* If pq is a link no new bus is added to the partial network. The dimensions are unchanged



* All the Elements of the Bus Impedance Matrix must be recalculated to include the effect of the added link.

Case 1 :- Addition of Branch

* The performance equation is given as

$$\bar{E}_{BUS} = Z_{BUS}\ \bar{I}_{BUS}$$

$$\Rightarrow \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \quad \text{—(1)}$$

* The network consist of bilateral passive element

Here $Z_{qi} = Z_{iq}$ $\quad i = 1, 2, 3 \cdots m$

* And referes to the buses not including new bus 'q'.

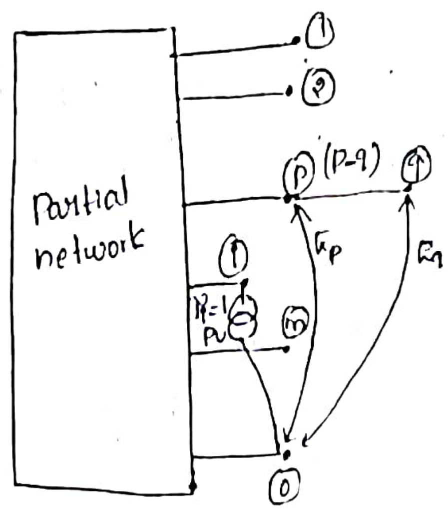
* The Added branch pq is assumed to be mutually coupled with one (or) more elements of partial network.

* The elements Z_{qi} can be determined by

Injecting a current at ith bus and calculating the voltage at qth bus with respect to the

$$Z_{qi} = \frac{V_q}{I_i}$$

reference node as shown in the figure.



* Since All other bus currents are zero

$$\Rightarrow \quad \left.\begin{aligned} E_1 &= Z_{1i} I_i \\ E_2 &= Z_{2i} I_i \\ E_3 &= Z_{3i} I_i \\ &\vdots \\ E_p &= Z_{pi} I_i \\ E_m &= Z_{mi} I_i \\ &\vdots \\ E_q &= Z_{qi} I_i \end{aligned}\right\} \longrightarrow (2)$$

* Z_{qi} can be obtained directly by calculating E_q by letting the current at ith bus as 1PU in above equations

$$I_i = 1 \text{ PU}$$

* The Bus Voltages associated with added element and the voltage across the element is related as

* However v_{pq} is not equal to zero. Since the added branch is mutually coupled to one (or) more elements of partial network.

$$v_{\rho\sigma} = E_\rho - E_\sigma \quad -⑥$$

where

E_ρ & E_σ are voltages at the buses in partial network.

Now from equations ④ & ⑤

$$i_{pq} = y_{pq\,pq}\, v_{pq} + y_{pq\,\rho\sigma}\, v_{\rho\sigma} = 0.$$

$$v_{pq} = \frac{-y_{pq\,\rho\sigma}\, v_{\rho\sigma}}{y_{pq\,pq}}$$

from eq ⑥ $v_{\rho\sigma} = E_\rho - E_\sigma$

Substituting eq ⑥ in it

$$v_{pq} = \frac{-y_{pq\,\rho\sigma}\,(E_\rho - E_\sigma)}{y_{pq\,pq}}$$

Substituting this in eq ③

from eq ③

$$E_q = E_p - v_{pq}$$

$$= \frac{E_p + y_{pq\,\rho\sigma}\,(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq\,pq}}$$

$$E_q = E_p + \frac{y_{pq\,\rho\sigma}\,(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq\,pq}}$$

* Substitute for E_q, E_p, E_ρ & E_σ from eq (6) with $I_i = 1$

$$\Rightarrow Z_{qi} = Z_{pi} + \frac{y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pqpq}} \qquad i = 1,2,3 \dots m, \; i \neq q$$

$$\boxed{Z_{qi} = Z_{pi} + \frac{y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pqpq}}} \quad -(7)$$

$i = 1,2,3 \dots m$, $i \neq q$

(if mutual imped is absent $y_{pq\rho\sigma} = 0$ $\therefore Z_{qi} = Z_{pi} + Z_{pq\,pq}$)

* The Element Z_{qq} can be calculated by injecting current at qth bus and calculating voltage at that bus since all the other bus currents are zero.

$$E_1 = Z_{1q} I_q$$

$$E_2 = Z_{2q} I_q$$

$$\vdots$$

$$E_p = Z_{pq} I_q$$

$$\vdots$$

$$E_m = Z_{mq} I_q$$

$$E_q = Z_{qq} I_q$$

* Letting $I_q = 1$ pu in above equations
* Z_{qq} can be obtained directly by calculating E_q

the voltages at buses p & q are related as

$$E_q = E_p - V_{pq} \quad -(9)$$

$$\Rightarrow i_{pq} = -I_q = -1 \text{ pu} \quad -(8)$$

From eq (4) & eq (8)

$$i_{pq} = y_{pqpq} V_{pq} + y_{pq\rho\sigma} V_{\rho\sigma} = -1$$

$$V_{pq} = -\left[\frac{1 + y_{pq\rho\sigma} V_{\rho\sigma}}{y_{pqpq}}\right]$$

$$\Rightarrow \quad E_q = E_p - V_{pq} \quad \text{---} (3)$$

* The currents in the elements of the network are expressed in terms of the primitive admittances and the voltages across the element.

$$\Rightarrow \begin{bmatrix} i_{pq} \\ i_{e\sigma} \end{bmatrix} = \begin{bmatrix} y_{pqpq} & y_{pqe\sigma} \\ y_{e\sigma pq} & y_{e\sigma e\sigma} \end{bmatrix} \cdot \begin{bmatrix} v_{pq} \\ v_{e\sigma} \end{bmatrix} \quad \text{---} (4)$$

* where pq is the fixed element subscript and refers to the added element &
* $e\sigma$ is the variable subscript and refers to all other elements.
* i_{pq} & v_{pq} are current through and voltage across the added elements.
* $i_{e\sigma}$ & $v_{e\sigma}$ are current and voltage vectors of elements of the partial network.
* y_{pqpq} self admittance of the added element.
* $y_{pqe\sigma}$ vector of mutual admittances between the added element pq and elements $e\sigma$ of the partial network.
* $y_{e\sigma pq}$ is the transpose of vector $y_{pqe\sigma}$
* $y_{e\sigma e\sigma}$ is the primitive admittance matrix of partial elements.

* The current in the added branch, $i_{pq} = 0$ since open circuit.

$$\Rightarrow \quad i_{pq} = 0 \quad \text{---} (5)$$

$$V_{pq} = -\left[\frac{1 + y_{pq\rho\sigma}(E_\rho - E_\sigma)}{y_{pqpq}}\right]$$

* from eq (9)

$$E_q = E_p - V_{pq}$$

$$E_q = E_p + \frac{1 + y_{pq\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pqpq}}$$

* By substituting E_q, E_p and E_ρ & E_σ we get

$$\boxed{Z_{qq} = Z_{pq} + \frac{1 + y_{pq\rho\sigma}(Z_{\rho q} - Z_{\sigma q})}{y_{pqpq}}}$$

* If there is no mutual coupling between added & other elements of partial network then the elements of $y_{pq\sigma\rho} = 0$

$$\Rightarrow \therefore \frac{1}{y_{pqpq}} = z_{pq\rho\sigma}$$

$$Z_{qi} = Z_{pi} \quad i = 1, 2, \cdots m \quad i \neq q$$

$$\boxed{Z_{qq} = Z_{pq} + z_{pqpq}} \quad \text{— (10)}$$

* If there is no mutual coupling and p is the reference bus.

$\Rightarrow Z_{pi} = 0$ for all the values of $i = 1, 2, \cdots m$, $i \neq q$

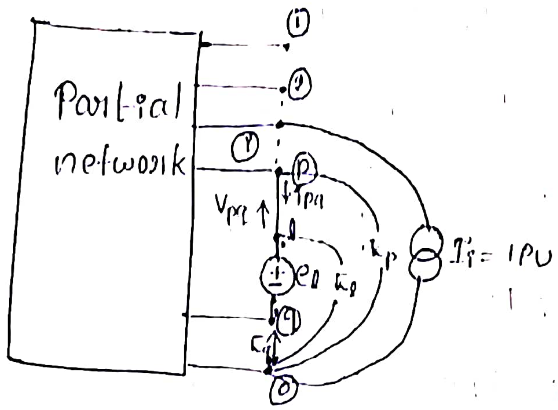
$\Rightarrow Z_{qi} = 0 \quad i = 1, 2, \cdots m, \quad i \neq q$

$\Rightarrow Z_{pq} = 0$

from (10)

$\Rightarrow Z_{qq} = z_{pqpq}$

Case 2: Addition of a link:



* If the added element pq is the link the procedure for recalculating the elements of the bus impedence matrix is to connect in series with the added element.

* A voltage source e_l as shown in the figure. This creates a ficticious node 'l' which will be eliminated later. The voltage source e_l is selected such that the current through the added link is zero.

* The performance equation for partial network with the added element & series with voltage source.

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ \hline e_l \end{bmatrix} = \left[\begin{array}{cccccccc|c} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} \\ Z_{31} & Z_{32} & \cdots & Z_{3p} & \cdots & Z_{3m} & Z_{3l} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} \\ \hline Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \hline I_l \end{bmatrix}$$

* Now from figure $e_l = E_l - E_q$

* The element Z_{li} can be determined by injecting current at ith bus and calculating voltage at lth node with respect to bus q.

* Since all other bus current is equal to zero

$$E_k = Z_{ki} I_i \quad k = 1,2,3 \ldots m$$

$$e_l = Z_{li} I_i$$

* Letting $I_i = 1\,pu$, Z_{li} can be obtained directly by calculating e_l.

* The series voltage source e_l is

$$e_l = E_p - E_q - v_{pl} \quad [\text{Here } v_{pl} = v_{pq}]$$

* Since $i_{pq} = 0$, the element pl can be treated as a branch.

$$\therefore i_{pl} = y_{pl,pl} \bar{v}_{pl} + y_{pl\,\rho\sigma} \bar{v}_{\rho\sigma}$$

$$[i_{pl} = i_{pq} = 0]$$

$$\therefore v_{pl} = \frac{-y_{pl\,\rho\sigma} v_{\rho\sigma}}{y_{pl\,pl}}$$

We can write $\quad \because [pl = pq]$

$$y_{pl\,\rho\sigma} = y_{pq\,\rho\sigma}$$

$$y_{pl\,pl} = y_{pq\,pq}$$

$$\Rightarrow v_{pl} = \frac{-y_{pq\,\rho\sigma} v_{\rho\sigma}}{y_{pq\,pq}}$$

* $$Z_{li} = Z_{pi} - Z_{qi} + \frac{y_{pq\,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq\,pq}} \quad i = 1,2,3 \ldots m, \; i \neq l$$

* The element $z_{\ell\ell}$ can be calculated by injecting the current at ℓth bus with bus 'p' as reference and calculating the voltage at ℓth bus w.r.t to bus 'q'.

* Since all the bus current are zero

$$E_K = Z_{K\ell} I_\ell$$

$$e_\ell = z_{\ell\ell} I_\ell$$

Now $i_{p\ell} = -I_\ell = -1$

* The current in terms of primitive admittances and voltage in terms of

$$i_{p\ell} = y_{p\ell p\ell} v_{p\ell} + y_{p\ell\rho\sigma} v_{\rho\sigma} = -1$$

$$v_{p\ell} = -\left[\frac{1 + y_{p\ell\rho\sigma} v_{\rho\sigma}}{y_{p\ell p\ell}}\right]$$

$\because$ $y_{p\ell\rho\sigma} = y_{pq\rho\sigma}$

$y_{p\ell p\ell} = y_{pqpq}$

$$\therefore v_{p\ell} = -\left[\frac{1 + y_{pq\rho\sigma} v_{\rho\sigma}}{y_{pqpq}}\right]$$

* $$z_{\ell\ell} = z_{p\ell} - z_{q\ell} + \frac{1 + y_{pq\rho\sigma}(z_{\rho\ell} - z_{\sigma\ell})}{y_{pqpq}}$$

* If there is no mutual coupling then the element

$$y_{pq\rho\sigma} = 0$$

$$\frac{1}{y_{pqpq}} = z_{pqpq}$$

* $$z_{\ell i} = z_{pi} - z_{qi} \quad i = 1, 2, 3 \ldots m$$

$$z_{\ell\ell} = z_{p\ell} - z_{q\ell} + z_{pqpq}$$

* If there is no mutual coupling & p as reference

$$Z_{pi} = 0$$

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$= -Z_{qi}$$

$$Z_{pl} = 0$$

$$Z_{ll} = -Z_{ql} + z_{pq\,pq}$$

* Now it remains to calculate the required Bus Impedence Matrix to include the effect of Added link.

* The ficticious node l is eliminated by short-circuiting the series voltage source e_l.

$$E_{BUS} = Z_{BUS}\, I_{BUS} + Z_{li}\, I_l \quad \text{— (i)}$$

Now $e_l - Z_{lj}\, I_{BUS} + Z_{ll}\, I_l = 0$ — (ii)

$i, j = 1, 2 \ldots m$

* By solving (i) & (ii)

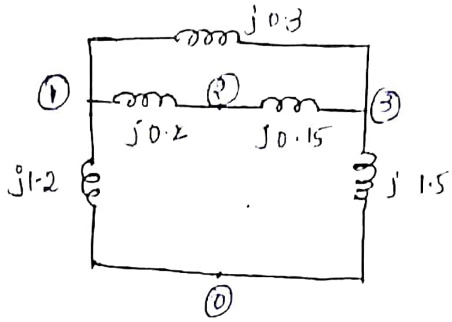
$$E_{BUS} = \left(Z_{BUS} - \frac{Z_{il}\, Z_{lj}}{Z_{ll}} \right) I_{BUS} = \text{Upd}$$

* Z_{BUS} (modified) = Z_{BUS} (before modified) $- \dfrac{Z_{il} Z_{lj}}{Z_{ll}}$

* Z_{ij} (new) = Z_{ij} (old) $- \dfrac{Z_{il}\, Z_{lj}}{Z_{ll}}$

Problem:

① Form the Z_{BUS} for the following network.



Sol

Step ①:

[We are adding element in reference & 1st bus]

* Add Element(1) between 1st node and reference.

$P=0, \; q=1$

Which is a branch & no. mutual coupling.

$$Z_{qi} = Z_{pi} + \frac{y_{pq\,\sigma\rho}(Z_{\rho i} - Z_{\sigma i})}{y_{pq\,pq}}$$

As no mutual coupling $y_{pq\,\sigma\rho} = 0$

$Z_{qi} = Z_{pi}$

$Z_{qi} = Z_{pi} = 0$ (As reference is P)

$Z_{1i} = Z_{0i} = 0$

$\Rightarrow i = 0$

$Z_{10} = Z_{01} = 0$

$\Rightarrow q = 1$

$Z_{11} = Z_{qq} = Z_{pq} + Z_{pq\,pq}$

$= Z_{01} + j1.2$

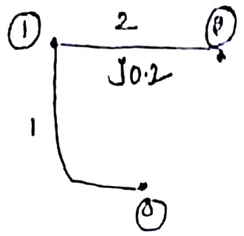
$= j1.2$

$$\therefore Z_{BUS} = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & j1.2 \end{bmatrix} \end{array}$$

Step 2: Adding element between 1st & 2nd bus

$P=1, q=2$

Added element is branch



$y_{pq\sigma}=0$ [The branch has no mutual coupling]

$z_{pqpq} = z_{1212} = j0.2$

$$z_{BUS} = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} j1.2 & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \end{array}$$

We know $z_{11} = j1.2$

Now $z_{qi} = z_{pi}$

$z_{2i} = z_{1i}$

$i=1 \Rightarrow z_{21} = z_{11}$

$z_{12} = z_{21} = z_{11}$

$z_{12} = z_{21} = z_{11} = j1.2$

Now $z_{qq} = z_{pq} + 1/y_{pqpq}$

$$z_{qq} = z_{pq} + \frac{1 + y_{pq\rho\sigma}(z_{\rho q} - z_{\sigma q})}{y_{pqpq}}$$

$$= z_{pq} + \frac{1}{y_{pqpq}}$$

$$z_{22} = z_{12} + \frac{1}{y_{1212}}$$

$= z_{12} + z_{1212}$

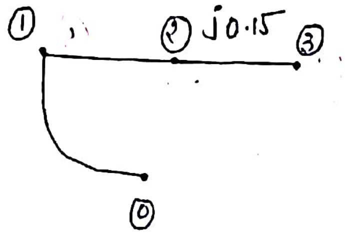
$= z_{11} + j0.2$

$= j1.2 + j0.2$

$= j1.4$

$$Z_{BUS} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & j1.2 & j1.2 \\ 2 & j1.2 & j1.4 \end{array}$$

Step 3:



The Element is added in between 2 & 3, as a branch

Now $p=2,\ q=3$

$$Z_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j1.2 & j1.2 & Z_{13} \\ 2 & j1.2 & j1.4 & Z_{32} \\ 3 & Z_{31} & Z_{32} & Z_{33} \end{array}$$

Now $Z_{qi} = Z_{pi} \Rightarrow Z_{3i} = Z_{2i}$

$i = 1,2$

$i=1 \quad Z_{31} = Z_{21} = j1.2$

$Z_{31} = Z_{13} = j1.2$

$i=2 \quad Z_{32} = Z_{22} = j1.4$

$Z_{32} = Z_{23} = j1.4$

Now $Z_{qq} = Z_{pq} + \dfrac{1}{y_{pq\,pq}}$

$Z_{33} = Z_{23} + \dfrac{1}{y_{23\,23}}$

$= j1.4 + z_{2323}$

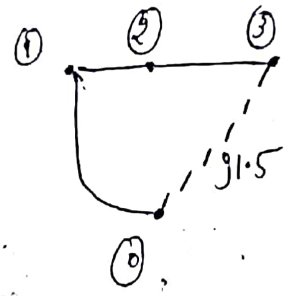
$= j1.4 + j0.15$

$= j1.55$

$$Z_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j1.2 & j1.2 & j1.2 \\ 2 & j1.2 & j1.4 & j1.4 \\ 3 & j1.2 & j1.4 & j1.55 \end{array}$$

Step 4:

Adding element in between reference & 3rd bus



Now $P=0$ & $q=3$

Added element is link

W.K.T

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pqpq}}$$

$$Z_{li} = Z_{pi} - Z_{qi}$$

$i = 1,2,3$

$Z_{BUS} = \left[\quad \right.$

$$Z_{l1} = Z_{01} - Z_{31}$$

$$= 0 - j1.2$$

$$= -j1.2 = Z_{1l}$$

$$Z_{l2} = Z_{02} - Z_{32}$$

$$= 0 - j1.4$$

$$= -j1.4 = Z_{2l}$$

$$Z_{l3} = Z_{03} - Z_{33}$$

$$= 0 - j1.55$$

$$= -j1.55 = Z_{3l}$$

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1}{y_{pqpq}}$$

$$= Z_{0l} - Z_{3l} + Z_{0303}$$

$$= 0 + j1.55 + j1.5$$

$$= j3.05$$

$\underline{\underline{\text{Step 4}}}$ ÷

$$Z_{BUS}(new) = Z_{BUS}(old) - \frac{Z_{il}\,Z_{lj}}{Z_{ll}}$$

$$\frac{[Z_{il}][Z_{lj}]}{Z_{ll}} = \frac{1}{Z_{ll}} \begin{bmatrix} Z_{1l} \\ Z_{2l} \\ Z_{3l} \end{bmatrix}_{3\times1} \begin{bmatrix} Z_{l1} & Z_{l2} & Z_{l3} \end{bmatrix}_{1\times3}$$

$$= \frac{1}{j3.05} \begin{bmatrix} -j1.2 \\ -j1.4 \\ -j1.55 \end{bmatrix} \begin{bmatrix} -j1.2 & -j1.4 & -j1.55 \end{bmatrix}$$

$$= \frac{1}{j3.05} \begin{bmatrix} \cancel{-5}.8025 \end{bmatrix}$$

$$= \frac{1}{j3.05} \begin{bmatrix} -1.44 & -1.68 & -1.86 \\ -1.68 & -1.96 & -2.17 \\ -1.86 & -2.17 & -2.40 \end{bmatrix}$$

$$= \begin{bmatrix} j0.472 & j0.5508 & j0.609 \\ j0.5508 & j0.642 & j0.7114 \\ j0.609 & j0.7114 & j0.786 \end{bmatrix}$$

$$Z_{BUS}(new) = \begin{bmatrix} j1.2 & j1.2 & j1.4 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix} - \begin{bmatrix} j0.472 & j0.5508 & j0.609 \\ j0.5508 & j0.642 & j0.714 \\ j0.609 & j0.714 & j0.781 \end{bmatrix}$$

$$Z_{BUS}(new) = \begin{bmatrix} j0.728 & j0.692 & j0.591 \\ j0.692 & j0.758 & j0.659 \\ j0.591 & j0.659 & j0.763 \end{bmatrix}$$

$\underline{\underline{\text{Step 5}}}$ ÷ Add Element in between 1 & 3

$p=1,\ q=3$

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{y_{pq\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pqpq}}$$

$= Z_{pi} - Z_{qi}$

$i = 1, 2, 3$

$Z_{1l} = Z_{11} - Z_{21}$

$= j0.718 - j0.591$

$= j0.127 = Z_{1l}$

$Z_{2l} = Z_{12} - Z_{22}$

$= j0.629 - j0.659$

$= -j0.03 = Z_{2l}$

$Z_{3l} = Z_{13} - Z_{23}$

$= j0.591 - j0.763$

$= -j0.172 = Z_{3l}$

$$Z_{ll} = Z_{1l} - Z_{2l} + \frac{1}{y_{mn}}$$

$= Z_{1l} - Z_{3l} + j1.3$

$= j0.127 + j0.172 + j0.3$

$= \cancel{j0.455}$

$= j0.599$

$$Z_{BUS}(new) = Z_{BUS}(old) - \frac{Z_{il} Z_{lj}}{Z_{ll}}$$

$$\frac{Z_{il} Z_{lj}}{Z_{ll}} = \frac{1}{Z_{ll}} \begin{bmatrix} Z_{1l} \\ Z_{2l} \\ Z_{3l} \end{bmatrix} \begin{bmatrix} Z_{1l} & Z_{2l} & Z_{3l} \end{bmatrix}$$

$$= \frac{1}{j0.599} \begin{bmatrix} j0.127 \\ -j0.03 \\ -j0.172 \end{bmatrix} \begin{bmatrix} j0.127 & -j0.03 & -j0.172 \end{bmatrix}$$

$$= \frac{1}{j0.599} \begin{bmatrix} -0.0161 & 0.00381 & 0.0218 \\ 0.00381 & -0.0009 & -0.00516 \\ 0.0218 & -0.00516 & -0.029 \end{bmatrix}$$

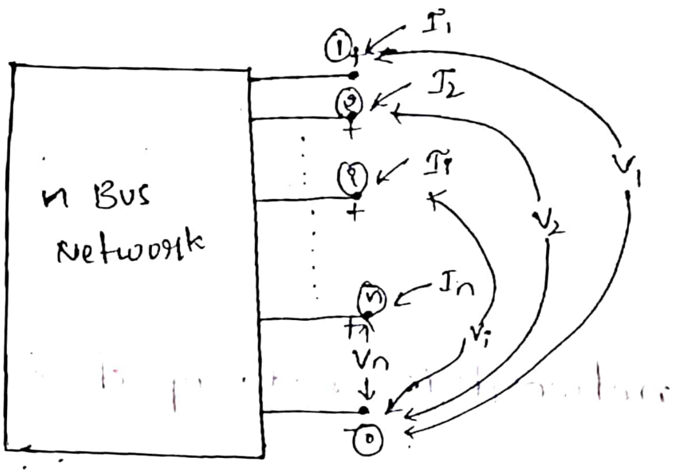
$$= \begin{bmatrix} j0.0268 & -j0.00636 & -j0.0363 \\ -j0.00636 & -j0.0015 & j0.00861 \\ -j0.0363 & j0.00861 & j0.0484 \end{bmatrix}$$

$$Z_{BUS}(new) = Z_{BUS}(old) - \frac{Z_{01}\, Z_{01}}{Z_{11}}$$

$$= \begin{bmatrix} j0.728 & j0.691 & j0.591 \\ j0.692 & j.0.738 & j0.639 \\ j0.59 & j0.659 & j0.735 \end{bmatrix} - \begin{bmatrix} j0.0268 & -j0.00631 & - \\ -j0.00636 & -j0 & \\ & & \end{bmatrix}$$

31/12/2016

Formulation of Z_{BUS} Matrix :- [Type Modification Method]



* Figure shows a general n-Bus linear passive network the general form of the network performance equation.

* $V_{BUS} = Z_{BUS}\, I_{BUS}$

  Where V_{BUS} is $n \times 1$ vector of Bus voltages with respect to reference.

  I_{BUS} is $n \times 1$ vector of injected bus currents.

  Z_{BUS} is $n \times n$ vector of impedence of

* The known Voltage equation in expanded form can be written as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1i} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2n} \\ \vdots & & & & & \\ Z_{i1} & Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{in} \\ Z_{n1} & Z_{n2} & \cdots & Z_{ni} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix}$$

* If a unit current is injected into the i^{th} Bus and All the other Bus currents as zero. and the open circuit bus currents at each bus is measured.

$$Z_{i1} = \frac{V_1}{I_i}$$

$$Z_{i2} = \frac{V_2}{I_i}$$

$$\vdots$$

$$Z_{i3} = \frac{V_3}{I_i}$$

$$Z_{in} = \frac{V_n}{I_i}$$

* Since the network is made up of linear passive network

$$Z_{ij} = Z_{ji}$$

* By Adding impedences one at a time till all the Impedences have been included with this a new matrix is produced and the order of the matrix may or may not increase depending upon whether the addition of the impedence creates a new bus (or) not.

* Assume an n bus partial network and there is no mutual coupling between the components of the netw

* When an impedence Z_b is added to the network. the 4 types of modifications are possible.

Z_b = Branch impedence.

$$Z_{BUS}(old) \longrightarrow Z_{BUS}(new)$$

① Z_b is added from a new bus to the reference bus i.e., a new branch is added and the dimension of Z_{BUS} goes up by 1. This is Type-1 modification.

② Z_b is added from a new bus to the old bus and this is Type-2 Modification.

③ Z_b is added from an old bus to the reference bus i.e., a new link is added and the dimension of Z_{BUS} does not change and This is Type-3 Modification.

④ Z_b is added in between two old buses and this is Type-4 Modification.

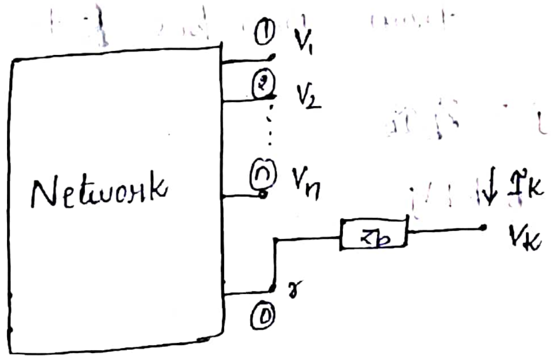
* In the following discussion

i & j are old buses

r is reference buses &

k is new bus.

* Type 1 - Modification:



* For this new branch k

$V_k = Z_b I_k$

$Z_{ki} = Z_{ik} = 0$ [Since there is no mutual coupling]

$i = 1, 2 \dots n$

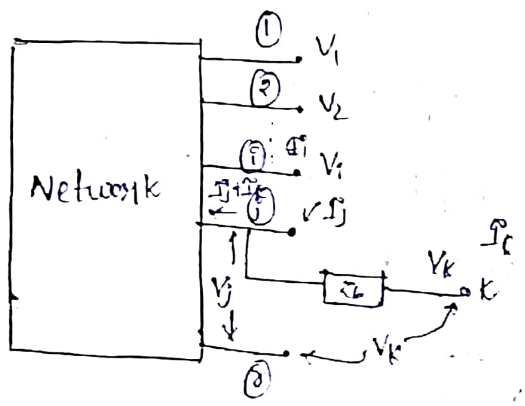
$Z_{kk} = Z_b$

* ∴ The new Bus Impedence matrix is given as

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ V_k \end{bmatrix}_{(n+1)\times 1} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} & 0 \\ Z_{21} & \cdots & Z_{2n} & \vdots \\ \vdots & & & \\ Z_{n1} & \cdots & Z_{nn} & 0 \\ 0 & \cdots & 0 & Z_{bb} \end{bmatrix}_{(n+1)\times(n+1)} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix}_{(n+1)\times 1}$$

$$Z_{BUS(new)} = \begin{bmatrix} \begin{bmatrix} Z_{BUS}(old) \end{bmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \cdots\cdots 0 & Z_{bb} \end{bmatrix} = [Z_{bb}]$$

* Type-2 Modification ÷



* Z_b is added from new bus k to the old bus j

$V_k - V_j = Z_b I_k$

$V_k = Z_b I_k + V_j$

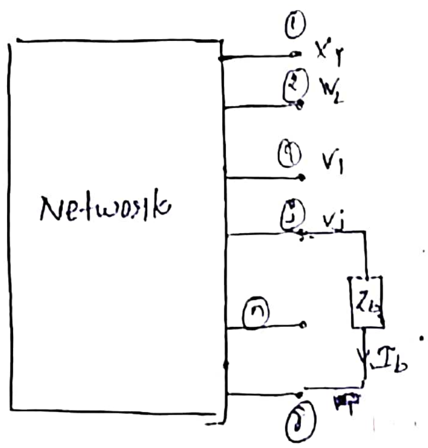
$$= Z_b I_K + Z_{j1} I_1 + Z_{j2} I_2 + \cdots Z_{jj}(I_j + I_K) + \cdots + Z_{jn} I_n$$

$$V_K = Z_{j1} I_1 + \cdots + Z_{jj} I_j + \cdots + Z_{jn} I_n + (Z_{jj} + Z_b) I_K .$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \vdots \\ V_K \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} & Z_{1j} \\ \vdots & & & \vdots \\ & & & Z_{jj} \\ Z_{n1} & \cdots & Z_{nn} & Z_{nj} \\ Z_{j1} & \cdots Z_{jj} & Z_{jn} & Z_{jj}+Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \vdots \\ I_L \end{bmatrix}$$

$$Z_{BUS}(\text{new}) = \left[\begin{array}{c|c} Z_{BUS}(\text{old}) & \begin{matrix} Z_{1j} \\ Z_{2j} \\ Z_{jj} \\ Z_{nj} \end{matrix} \\ \hline Z_{j1} \cdots Z_{jj} \; Z_{jn} & Z_{jj}+Z_b \end{array}\right]$$

* Type-3 Modification :-



* Z_b is connected to old bus from reference bus.
* By connecting Bus k to the reference (or) By setting $V_K = 0$.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|c} Z_{11} & \cdots & Z_{1n} & Z_{1j} \\ \vdots & & & \vdots \\ & & & Z_{jj} \\ Z_{n1} & \cdots & Z_{nn} & Z_{nj} \\ \hline Z_{j1} & \cdots Z_{jj} & Z_{jn} & Z_{jj}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ \vdots \\ I_K \end{bmatrix}$$

* Eliminate I_k in the set of equations

$$0 = Z_{j1} I_1 + Z_{j2} I_2 + \dots Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

$$I_k = \frac{-1}{Z_{jj}+Z_b}\left(Z_{j1} I_1 + \dots + Z_{jn} I_n\right)$$

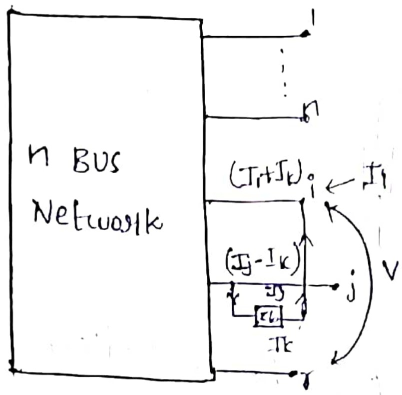
* Now $V_i = Z_{i1} I_1 + Z_{i2} I_2 + \dots Z_{in} I_n + Z_{ij} I_k$

$$V_i = \left[Z_{i1} - \frac{1}{Z_{jj}+Z_b}\left(Z_{ij} \dots Z_{j1}\right)\right] I_1 +$$

$$\left[Z_{i2} - \frac{1}{Z_{jj}+Z_b}\left(Z_{ij} \dots Z_{j2}\right)\right] I_2 + \dots$$

* $$Z_{Bus}(new) = Z_{Bus}(old) + \frac{1}{Z_{jj}+Z_b}\begin{bmatrix} Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix}\begin{bmatrix} Z_{j1} & \dots & Z_{jn} \end{bmatrix}$$

Type-4 :



* Z_b is connected in between 2 old buses i & j. The Voltage at ith bus is given as

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + Z_{i3} I_3 + \dots + Z_{ii}(I_i + I_k) + Z_{ij}(I_j - I_k) + \dots Z_{in} I_n$$

The voltage at jth bus is given as.

$$V_j = Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{ji}(I_i + I_k) + Z_{jj}(I_j - I_k) + \dots + Z_{jn} I_n$$

$$V_j = Z_b I_k + V_i$$

* Substituting & rearranging the equations

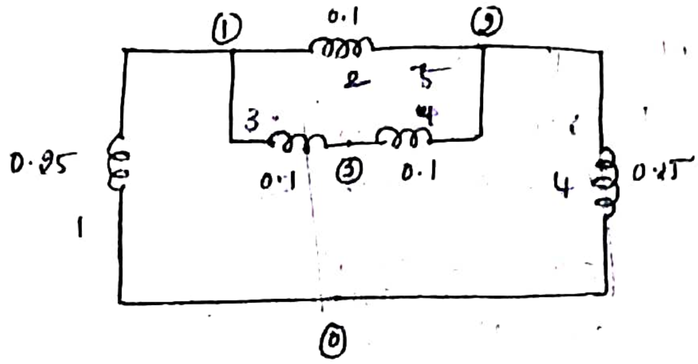
$$\Rightarrow 0 = (Z_{i1} - Z_{j1}) + \dots + (Z_{ii} - Z_{ji}) I_i + (Z_{ij} - Z_{ji}) I_j + \dots + (Z_{in} - Z_{jn}) I_n + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji}) I_k$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} Z_{Bus} & \begin{matrix} Z_{1i} - Z_{1j} \\ \vdots \\ Z_{ni} - Z_{nj} \end{matrix} \\ \hline (Z_{i1} - Z_{j1}) \dots (Z_{in} - Z_{jn}) & Z_b + Z_{ii} + Z_{jj} - 2Z_{ij} \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_k \end{bmatrix}$$

$$\therefore \Rightarrow Z_{Bus}(new) = Z_{Bus}(old) \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} - Z_{1j} \\ \vdots \\ Z_{ni} - Z_{nj} \end{bmatrix} \begin{bmatrix} Z_{i1} - Z_{j1} & \dots & Z_{in} - Z_{jn} \end{bmatrix}$$

Problem:

For the three Bus network shown in figure built Z_{BUS} by using Type Modification.



Sol

Step 1: Add Element b/n reference & 1st Bus

Type-1 Modification : Added element is Branch

$$Z_{bb} = 0.25\,\Omega$$

$$Z_{BUS} = \begin{bmatrix} 0.25 \end{bmatrix}$$

Step 2:

Add element b/n 1st & 2nd Buses

Type-2 Modification:

$Z_b = 0.1\,\Omega$

$$Z_{BUS}(new) = \begin{bmatrix} Z_{BUS}(old) \rightarrow & \\ \downarrow & Z_{jj} + Z_b \end{bmatrix}$$

$$= \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0.25 \rightarrow & 0.25 \\ 2 & 0.25 & 0.35 \end{array}$$

Step 3:

Add element b/n 1st & 3rd where $i=1$ $k=3$

Type-2 Modification

$Z_b = 0.1$

$$Z_{BUS}(new) = \begin{array}{c|ccc|c} & 1 & 2 & 3 & \\ \hline 1 & 0.25 & 0.25 & 0.25 & Z_{i1} \\ 2 & 0.25 & 0.35 & 0.25 & Z_{i2} \\ 3 & 0.25 & 0.25 & (0.25+0.1) = 0.35 & \\ \hline & Z_{1i} & Z_{2i} & Z_{22} & \end{array}$$

Where $i=1$ & $k=3$

$Z_{11} = Z_{1i} = 0.25$

$Z_{2i} = Z_{i2} = 0.25$

$Z_{ii} + Z_b = Z_{11} + Z_b = 0.25 + 0.1 = 0.35\,\Omega$

$Z_{ai} = Z_{pi} + 0$

$Z_{31} = Z_{11} = 0.25$

$Z_{32} = \ldots$

Step 4:

Add Element between ② & ref

Type-3 Modification

$Z_b = 0.25\Omega$

$$Z_{BUS(new)} = Z_{BUS(old)} - \frac{1}{Z_{jj}+Z_b}\begin{bmatrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{bmatrix}\begin{bmatrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.35+0.25}\begin{bmatrix} Z_{12} \\ Z_{22} \\ \vdots \\ Z_{32} \end{bmatrix}\begin{bmatrix} Z_{21} & \cdots & Z_{23} \end{bmatrix}$$

where $j=2$, $r=0$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.6}\begin{bmatrix} 0.25 \\ 0.35 \\ 0.25 \end{bmatrix}\begin{bmatrix} 0.25 & 0.35 & 0.25 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.6}\begin{bmatrix} 0.0625 & 0.0875 & 0.0625 \\ 0.0875 & 0.1225 & 0.0875 \\ 0.0625 & 0.0875 & 0.0625 \end{bmatrix}$$

$$Z_{BUS} = \begin{bmatrix} 0.1458 & 0.1041 & 0.1458 \\ 0.1041 & 0.1458 & 0.1041 \\ 0.1458 & 0.1041 & 0.2458 \end{bmatrix}$$

Step 5:

Add Element b/n ② & ③

Type-4 Modification

wher $i=2$ $j=3$

$Z_b = 0.1$

$$Z_{BUS(new)} = Z_{BUS(old)} - \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} - Z_{1j} \\ Z_{2i} - Z_{2j} \\ Z_{3i} - Z_{3j} \end{bmatrix} \begin{bmatrix} Z_{i1} - Z_{j1} & Z_{i2} - Z_{j2} & Z_{i3} - Z_{j3} \end{bmatrix}$$

$$= Z_{BUS(old)} - \frac{1}{0.1 + Z_{22} + Z_{33} - 2Z_{23}} \begin{bmatrix} Z_{12} - Z_{13} \\ Z_{22} - Z_{23} \\ Z_{32} - Z_{33} \end{bmatrix} \begin{bmatrix} Z_{21} - Z_{31} & Z_{22} - Z_{32} & Z_{23} - Z_{33} \end{bmatrix}$$

$$\Rightarrow \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.1458 & 0.1041 & 0.1458 \\ 2 & 0.1041 & 0.1458 & 0.1041 \\ 3 & 0.1458 & 0.1041 & 0.2458 \end{array} - \frac{1}{0.1 + 0.1458 + 0.2458 - 2(0.1041)} \begin{bmatrix} 0.1041 - 0.1458 \\ 0.1458 - 0.1041 \\ 0.1041 - 0.2458 \end{bmatrix} \begin{bmatrix} 0.1041 - 0.1458 & 0.1458 - 0.1041 & 0.1041 - 0.2458 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.1458 & 0.1041 & 0.1458 \\ 0.1041 & 0.1458 & 0.1041 \\ 0.1458 & 0.1041 & +0.2458 \end{bmatrix} - \frac{1}{0.2834} \begin{bmatrix} -0.0417 \\ 0.0417 \\ -0.1417 \end{bmatrix} \cancel{0.0417} \begin{bmatrix} -0.0417 & 0.0417 & -0.1417 \end{bmatrix}$$

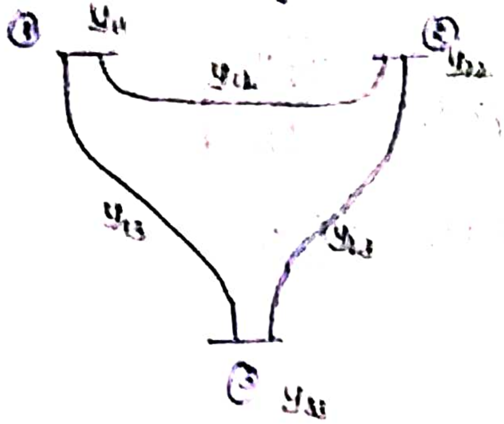
$$\Rightarrow \begin{bmatrix} 0.1458 & 0.1041 & 0.1458 \\ 0.1041 & +0.1458 & 0.1041 \\ 0.1458 & 0.1041 & 0.2458 \end{bmatrix} - \frac{1}{0.2834} \begin{bmatrix} 0.00173 & -0.00173 & 0.0059 \\ -0.00173 & 0.00173 & -0.0059 \\ 0.00590 & -0.00590 & 0.020 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.1396 & 0.1102 & 0.1250 \\ 0.1102 & 0.1396 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

$$Z_{BUS} = \begin{bmatrix} 0.1396 & 0.1102 & 0.1250 \\ 0.1102 & 0.1396 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

Elimination of node

Consider a 3 Bus system as shown in the figure



The performance equations is

$$I_1 = y_{11}V_1 + y_{12}V_2 + y_{13}V_3$$

Ily I_2, I_3

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Ily $I_2 = y_{21}V_1 + y_{22}V_2 + y_{23}V_3$

$$I_3 = y_{31}V_1 + y_{32}V_2 + y_{33}V_3$$

$I_3 = 0$ if node 3 is eliminated

$$0 = y_{31}V_1 + y_{32}V_2 + y_{33}V_3$$

$$V_3 = \frac{-(y_{31}V_1 + y_{32}V_2)}{y_{33}}$$

Substitute V_3 in equation I_1

$$I_1 = y_{11}V_1 + y_{12}V_2 + y_{13}\left[\frac{-y_{31}V_1 - y_{32}V_2}{y_{33}}\right]$$

$$= V_1\left[y_{11} - \frac{y_{13}y_{31}}{y_{33}}\right] + V_2\left[y_{12} - \frac{y_{13}y_{32}}{y_{33}}\right]$$

Assume $y_{11}' = y_{11} - \frac{y_{13}y_{31}}{y_{3}}$ $\quad y_{12}' = y_{12} - \frac{y_{13}y_{32}}{y_{33}}$

$$y_{12} = y_{12}(old) - \frac{y_{13}\, y_{32}}{y_{33}}$$

$$y_{21} = y_{21}(old) - \frac{y_{23}\, y_{31}}{y_{33}}$$

$$\therefore \boxed{y_{ij}(new) = y_{ij}(old) - \frac{y_{il}\, y_{lj}}{y_{ll}}}$$

$$\text{Ily } z_{ij}(new) = z_{ij}(old) - \frac{z_{il}\, z_{lj}}{z_{ll}}$$

Problem :

① Eliminate 3rd bus for the given Bus Admittance Matrix. (without mutual coupling)

$$y_{bus} = \begin{bmatrix} -j2.86 & 0 & j2.86 \\ 0 & -j4 & j2 \\ j2.86 & j2 & -j8.86 \end{bmatrix}$$

Sol

$$y_{ij}(new) = y_{ij}(old) - \frac{y_{il}\, y_{lj}}{y_{ll}}$$

where $l=3$

~~i=1, j=1~~ j=1,2

$\Rightarrow$ $i=1,\ j=1$

$$y_{11}(new) = y_{11}(old) - \frac{y_{13}\, y_{31}}{y_{33}}$$

$$= -j8.86 - \frac{j2.86 \cdot j2.86}{-j8.86}$$

$$= -j1.9$$

$$y_{12}(new) = y_{12}(old) - \frac{y_{13}\, y_{32}}{y_{33}}$$

$$= 0 - \frac{j2.86 \cdot j2}{-j8.86}$$

$$= j0.64$$

$$y_{21}(new) = y_{21}\,old - \frac{y_{23}\,y_{31}}{y_{33}}$$

$$= 0 - \frac{j2 \cdot j2.86}{-j8.86}$$

$$= j0.64$$

$$y_{22}(new) = y_{22}\,old - \frac{y_{23}\,y_{32}}{y_{33}}$$

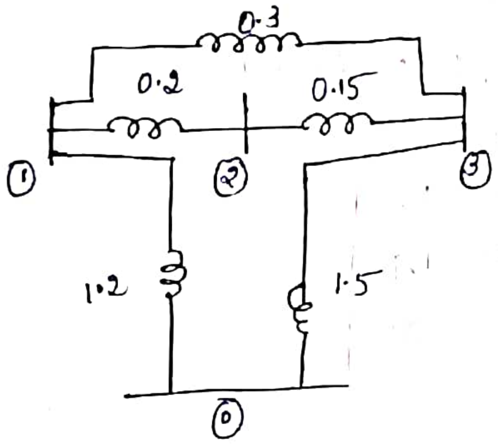
$$= -j6 - \frac{j2\,j2}{-j8.86}$$

$$= -j5.54$$

$$Y_{BUS}(new) = \begin{bmatrix} -j1.9 & j0.64 \\ j0.64 & -j5.54 \end{bmatrix}$$

(2) Determine Z_{BUS} for the given network

(without mutual coupling)



Step 1:

Adding element b/n ref & 1st bus

$j=0, \; K=1$

Type-1 Modification

Added element is Branch

$$Z_{BUS} = \begin{bmatrix} 1.2 \end{bmatrix}_{1\times1}$$

Step2: Adding Element b/n 1 & 2

Type-2 Modification

old - 1   New K=2

Added element is a branch

$$Z_{BUS} = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1.2 & 1.2 \\ 1.2 & 1.4 \end{bmatrix} \end{array}$$

$$Z_{22} = 1.2 + 0.2 = 1.4$$

Step 3: Adding Element b/n 2 & 3

Old i=2   New-K-3

Type 2 Modification

Added element is branch

$$Z_{BUS} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.2 & 1.2 & 1.2 \\ 1.2 & 1.4 & 1.4 \\ 1.2 & 1.4 & 1.55 \end{bmatrix} \end{array}$$

Z

Step4: Adding element b/n 3 & ref

$$Z_{BUS} = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1.2 & 1.2 & 1.2 & 1.2 \\ 1.2 & 1.4 & 1.4 & 1.4 \\ 1.2 & 1.4 & 1.55 & 1.55 \\ 1.2 & 1.4 & 1.55 & 3.05 \end{bmatrix} \end{array}$$

$l=4$

$$Z_{11}(new) = Z_{11}(old) - \frac{Z_{14} Z_{41}}{Z_{44}}$$

$$= 1.2 - \frac{1.2 \times 1.2}{3.05}$$

$$= 0.728$$

$$Z_{12}(new) = Z_{12}(old) - \frac{Z_{14} Z_{41}}{Z_{44}}$$

$$= 1.2 - \frac{1.4 \times 1.2}{3.05}$$

$$= 0.649 = Z_{21}$$

$$Z_{13}(new) = Z_{13}(old) - \frac{Z_{34} Z_{41}}{Z_{44}}$$

$$= 1.2 - \frac{1.55 \times 1.2}{3.05}$$

$$= 0.591 = Z_{31}$$

$$Z_{22}(new) = Z_{22}(old) - \frac{Z_{24} Z_{42}}{Z_{44}}$$

$$= 1.4 - \frac{1.4 \times 1.4}{3.05}$$

$$= 0.757$$

$$Z_{23} = Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}}$$

$$= 1.4 - \frac{1.4 \times 1.55}{3.05}$$

$$= 0.68 = Z_{32}$$

$$Z_{33} = 0.76$$

$$Z_{BUS} = \begin{bmatrix} 0.723 & 0.649 & 0.59 \\ 0.649 & 0.591 & 0.68 \\ 0.591 & 0.68 & 0.76 \end{bmatrix}$$

Steps :-

Adding element b/w 1 & 3 as a link
$i=1$ $j=3$

~~ii~~ ~~1~~ 4

Type 4 Modification

$$Z_{BUS} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.728 & 0.649 & 0.59 & 0.59 \\ 0.649 & 0.757 & 0.68 & 0.68 \\ 0.59 & 0.68 & 0.76 & 0.76 \\ 0.59 & 0.68 & 0.76 & 1.06 \end{bmatrix} \end{array}$$

$$Z_{44} = Z_b + Z_{ii} + Z_{jj} - 2Z_{ji}$$

$$= 0.3 + 0.728 + 0.76 - 2(0.59)$$

$$= 0.608$$

$$Z_{11}(new) = Z_{11}(old) - \frac{Z_{14} Z_{41}}{Z_{44}}$$

$$= 0.728 - \frac{0.59 \; 0.59}{1.06}$$

$$= 0.399$$

$$Z_{12}(new) = 0.649 - \frac{0.59 \times 0.68}{1.06}$$

$$= 0.270 = Z_{21}$$

$$Z_{13}(new) = 0.59 - \frac{0.59 \times 0.76}{1.06}$$

$$= 0.1669 = Z_{31}$$

$$Z_{22} = 0.649 - \frac{0.68 \times 0.68}{1.06}$$

$$= 0.212$$

$$Z_{33} = 0.76 - \frac{0.76 \times 0.76}{1.06}$$

$$= \cancel{0.0450} \; 0.215$$

$$Z_{23} = 0.68 - \frac{0.68 \times 0.76}{1.06}$$

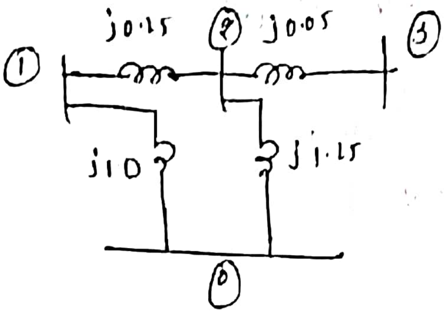
$$= 0.192 = Z_{32}$$

Ans:

$$\begin{bmatrix} 0.697 & 0.658 & 0.629 \\ 0.658 & 0.754 & 0.678 \\ 0.629 & 0.678 & 0.714 \end{bmatrix}$$

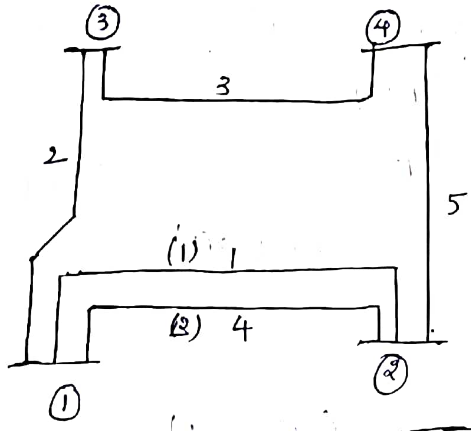
Assignment

Find: Z_{BUS}



(With Mutual Coupling)

(3) Form Z_{BUS}



Element number	Self		Mutual	
	Bus code p-q	impedance	Bus code r-s	impedence
1	1-2(1)	0.6		
4	1-2(2)	0.4	1-2(1)	0.2
2	1-3	0.5	1-2(1)	0.1
3	3-4	0.5		
5	2-4	0.2		

Sol

Step 1: Element-1 b/n $p=1$ & $q=2$

$p=1=$ reference.

$$Z_{BUS(new)} = \begin{array}{c|cc} & p & 2 \\ \hline p & 0 & 0 \\ 2 & 0 & 0.6 \end{array} = \begin{array}{c|c} & 2 \\ \hline 2 & 0.6 \end{array}$$

Step 2:

Element 4 connected b/n ① & ②

$p = 1 = \text{ref}$ $q = 2$ and $i = 2$

$\rho = 1; \sigma = 2$

Mutually coupled element-1

$$Z_{BUS} = \begin{array}{c|cc} & 2 & l \\ \hline 2 & 0.6 & Z_{2l} \\ l & Z_{l2} & Z_{ll} \end{array}$$

$Z_{2l} = Z_{l2} =$

$$\Rightarrow Z_{li} = Z_{pi} - Z_{qi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq\,pq}}$$

$$\Rightarrow Z_{2l} = Z_{l2} = \underset{0}{Z_{12}} - Z_{22} + \frac{y_{12(2),12(1)}(Z_{12} - Z_{22})}{y_{12(2),12(2)}} \quad \text{—(a)}$$

$$\Rightarrow Z_{ll} = -Z_{2l} + \frac{1 + y_{12(2),12(1)}(Z_{1l} - Z_{2l})}{y_{12(2)12(2)}} \quad \text{—(b)}$$

Here $Z_{12} = Z_{1l} = 0$

Now we have to consider premitive impedence matrix for mutually coupled element

$$Z_{e\sigma} \Rightarrow \begin{array}{c|cc} & 1\text{-}2(1) & 1\text{-}2(2) \\ \hline 1\text{-}2(1) & 0.6 & 0.2 \\ 1\text{-}2(2) & 0.2 & 0.4 \end{array}$$

$$y_{e\sigma} = (Z_{e\sigma})^{-1} = \frac{1}{0.24 - 0.08}\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$= \frac{0.4}{6.25} \quad \frac{-0.2}{6.25}$$

$$= 5\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \Rightarrow \begin{array}{c|cc} & 12(1) & 12(2) \\ \hline 12(1) & 2 & -1 \\ 12(2) & -1 & 3 \end{array}$$

$=$

$y_{12(1)12(2)} = -1 = y_{12(2)12(1)}$

$\therefore$ from (a)

$$Z_{21} = Z_{12} = -0.6 + \frac{(-1)(-0.1)}{3}$$

$$= -0.4$$

$$Z_{11} = +0.4 + \frac{1 \div (-1)\ (0.4)}{3}$$

$$= 0.6$$

$$Z_{BUS} = \begin{array}{c c} & 2 \\ \begin{array}{c} 2 \\ 1 \end{array} & \begin{bmatrix} 0.6 & -0.4 \\ -0.4 & 0.6 \end{bmatrix} \end{array}$$

$$Z_{22}(\text{new}) = Z_{22}(\text{old}) - \frac{Z_{21} Z_{12}}{Z_{11}}$$

$$= 0.6 - \frac{-0.4 \times -0.4}{0.6}$$

$$= 0.33$$

$$Z_{BUS}(\text{new}) = [0.33]$$

Step3

Add Element-2 b/n ① & ③

$p = 1$ (ref), $q = 3$ (new) $\quad i = 2$ $\quad p = 1\ \sigma = 2$

Mutually coupled element $-$ 1

$$Z_{BUS} = \begin{array}{c c} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} 0.33 & Z_{23} \\ Z_{32} & Z_{33} \end{bmatrix} \end{array}$$

$$Z_{23} = Z_{32} = Z_{13} - Z_{23} + \frac{y_{13,12(1)}\,[Z_{13} - Z_{23}]}{y_{13,12(2)}}$$

$$Z_{23} = Z_{32} = \frac{\left[y_{13,12(1)} \quad y_{13,12(2)} \right] \begin{bmatrix} Z_{12} - Z_{22(1)} \\ Z_{12} - Z_{22(2)} \end{bmatrix}}{y_{1313}}$$

$p=1 \quad i=2$

$\sigma=2$

$q=3 \quad i=$

Z_{23}

$$Z_{33} = \frac{1 + \left[y_{13,12(1)} \quad y_{13,12(2)} \right] \begin{bmatrix} Z_{12} - Z_{22} \\ Z_{13} - Z_{23} \end{bmatrix}}{y_{1313}}$$

Now Premitive impedence matrix for mutual coupling

$$Z_{pq\rho\sigma} = \begin{array}{c|ccc} & 1-2(1) & 1-2(2) & 1-3 \\ 1-2(1) & 0.6 & 0.2 & 0.1 \\ 1-2(2) & 0.2 & 0.4 & 0 \\ 1-3 & 0.1 & 0 & 0.5 \end{array}$$

$$= \frac{\{Adj\}}{|Z_{pq\rho\sigma}|}$$

$$= |Z_{pq\rho\sigma}| = 0.6(0.4 \times 0.5 - 0) - 0.2(0.2 \times 0.5 - 0) + 0.1(0 - 0.1 \times 0.4)$$

$$= 0.096$$

Adj:

$$\text{Co-factor} = \begin{bmatrix} 0.4 & 0 & 0.2 & 0.4 \\ 0 & 0.5 & 0.1 & 0 \\ 0.2 & 0.1 & 0.6 & 0.2 \\ 0.4 & 0 & 0.2 & 0.4 \end{bmatrix}$$

$$Adj = \begin{bmatrix} 0.2 & -0.1 & -0.04 \\ -0.1 & 0.29 & 0.02 \\ -0.04 & 0.02 & 0.2 \end{bmatrix}$$

$$(Z_{pq\rho\sigma})^{-1} = y_{pq\rho\sigma} = \begin{array}{c|ccc} & 1-2(1) & 1-2(2) & 1-3 \\ 1-2(1) & 2.083 & -1.0417 & -0.416 \\ 1-2(2) & -1.0417 & 3.0208 & 0.208 \\ 1-3 & -0.4167 & 0.2083 & 2.0833 \end{array}$$

$$Z_{32} = Z_{23} = \frac{\begin{bmatrix} -0.4117 & 0.2083 \end{bmatrix} \begin{bmatrix} -0.33 \\ -0.33 \end{bmatrix}}{2.083}$$

$$= \frac{\begin{bmatrix} 0.13751 & -0.068 \\ 0.13751 & -0.068 \end{bmatrix}}{2.083}$$

$$= \frac{0.0687}{2.083}$$

$$= 0.033$$

$$Z_{33} = \frac{1 + \begin{bmatrix} -0.4167 & 0.2083 \end{bmatrix} \begin{bmatrix} -0.33 \\ -0.53 \end{bmatrix}}{2.083}$$

$$= \frac{1 + 0.0687}{2.083}$$

$$= \cancel{0.513}$$

$$= 0.4833$$

$$Z_{BUS\,(new)} = \begin{array}{c|cc} & 2 & 3 \\ \hline 2 & 0.033 & 0.033 \\ 3 & 0.033 & 0.4833 \end{array}$$

Step 4:-

Add. elements b/n ③ & ④

$P = 3 \quad q = 4$

$$Z_{BUS\,(new)} = \begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 & 0.033 & 0.033 & \rightarrow 0.033 \\ 3 & 0.033 & 0.4833 & \rightarrow 0.4833 \\ 4 & 0.033 & 0.4833 & 0.9833 \end{array}$$

0.25

Step 5:

Add Element 5 b/n p=2, q=4

l=4

Added element is link

$$Z_{BUS(new)} = \begin{array}{c|cccc} & 2 & 3 & 4 & l \\ \hline 2 & 0.033 & 0.033 & 0.033 & 0.333 \\ 3 & 0.033 & 0.4833 & 0.4833 & 0.4833 \\ 4 & 0.033 & 0.4833 & 0.9833 & 0.9833 \\ l & 0.033 & 0.4833 & 0.9833 & 1.1833 \end{array}$$

$0.333 - 0.033 = 0.3$

$\rightarrow -0.45$

$\rightarrow -0.95$

$\rightarrow 1.45$

$0.3 \quad -0.45 \quad -0.95$

$$Z_{22(new)} = 0.33 - \frac{0.033 \times 0.033}{1.1833}$$

$= 0.3290$

$$Z_{23(new)} = 0.033 - \frac{0.033 \times 0.9833}{1.1833}$$

$= 0.3025$

$$Z_{BUS(new)} = \begin{array}{c|cccc} & 2 & 3 & 4 & l \\ \hline 2 & 0.333 & 0.033 & 0.033 & 0.3 \\ 3 & 0.033 & 0.4833 & 0.4833 & -0.45 \\ 4 & 0.033 & 0.4833 & 0.9833 & -0.95 \\ l & 0.3 & -0.45 & -0.95 & 1.45 \end{array}$$

$$Z_{22(new)} = 0.33 - \frac{0.3 \times 0.3}{1.45}$$

$= 0.267$

$$Z_{23(new)} = 0.033 - \frac{0.3 \times -0.45}{1.45}$$

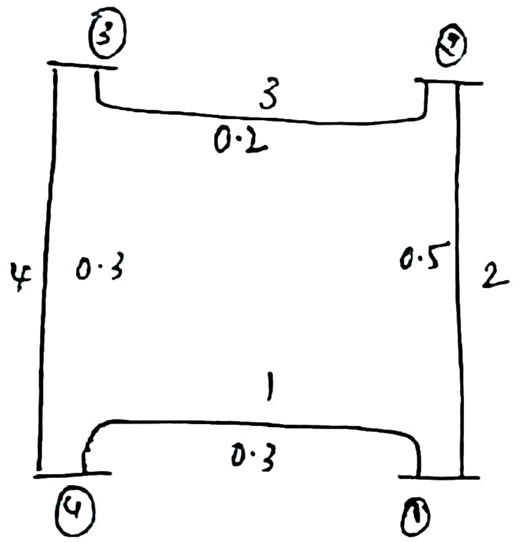
$= 0.4231 = Z_{32}$

$$Z_{24} = 0.033 - \frac{0.3 \times -0.95}{1.45}$$

$= 0.526 = Z_{42} = Z_{42}$

$$Z_{BUS}(new) = \begin{bmatrix} 0.271 & 0.126 & \\ 0.126 & 0.34336 & 0.1585 \\ & 0.1585 & 0.3609 \end{bmatrix}$$

Form Z_{BUS}



Elimination of Node [Matrix Algebra]

* $I = YV \qquad [I_{BUS} = Y_{BUS} V_{BUS}]$

$$\begin{bmatrix} I_A \\ I_x \end{bmatrix} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_x \end{bmatrix}$$

⇒ where V_x & I_x are sub matrix of voltage & currents of the nodes to be eliminated.

$$I_x = L^T V_A + M V_x = 0$$

$$V_x = M^{-1} L^T V_A$$

$$I_A = V_A (K - L M^{-1} L^T)$$

$$\therefore Y_{BUS} = K - L M^{-1} L^T$$

$$Y_{BUS} = \begin{array}{c} \begin{array}{cc} K & L \end{array} \\ \left[\begin{array}{cc|c} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ \hline y_{31} & y_{32} & y_{33} \end{array}\right] \\ \begin{array}{cc} L^T & M \end{array} \end{array}$$

$$\therefore Y_{BUS} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} - \begin{bmatrix} y_{13} \\ y_{23} \end{bmatrix} \frac{1}{y_{33}} \begin{bmatrix} y_{31} & y_{32} \end{bmatrix}$$

$$= \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} - \frac{1}{y_{33}} \begin{bmatrix} y_{13} \\ y_{23} \end{bmatrix} \begin{bmatrix} y_{31} & y_{32} \end{bmatrix}$$

$$\boxed{y_{ij}(\text{new}) = y_{ij}(\text{old}) - \frac{y_{in}\, y_{nj}}{y_{nn}} \qquad i = j = 1, 2 \cdots (n-1)}$$