Unit-2

Power flow Studies

* Power flow studies are conducted at the stage of planning operation and control and it is done at the most important mode of operation. i.e., symmetric mode of operation.

* They are used to determine the magnitude and phase angle of new buses and active and reactive powers injected at the buses and also over the T/m lines.

* This information is necessary for the following functions.

1) To keep the voltage level of certain buses with in the closed tolerance by proper reactive power sheduling.

2) The reactive power generation must be equal to the load demand plus losses.

3) This should be devided amongest the generators in the unique ratio for optimum economic operation.

4) The effects of disturbances which may results in the system failure during pre fault, at the falut incide and post fault condition.

5) The effectiveness of alternative plant for future system expansions to meet the increased load demand or to design a new system.

6) To analise and determine the best location for capacitor (or) voltage reggulator for improvement of voltage regguleton.

7) To determine the system conditions at various stages of Steady state, Transient & Dynamic state stabilities

Representation of Power System:-

* A Single-phase representation is adiquate since the power systems are usually balanced.

Load flow problem:-

* The performance equations of the network are

$$V_{BUS} = Z_{BUS}\, I_{BUS}$$
$$I_{BUS} = Y_{BUS}\, V_{BUS}$$
$\Big\} \rightarrow (1)$

* Nodal equations are expensively employed in Y_{BUS} because of its symmetry and sparcity we can save computer storage requirement and operation time.

* $I_i = \sum_{k=1}^{n} Y_{ik} V_k$

* Therefore the complex power injected by the source into the ith bus of the power system is

$$S_i = S_{Gi} - S_{Li}$$
$$= P_i + jQ_i$$
$$= (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

$\therefore S_i = V_i^* I_i^*$; $i = 1, 2 \ldots n$

* Now substitute $I_i = \sum_{k=1}^{n} Y_{ik} V_k$; $i = 1, 2, 3, \ldots n$

$$S_i = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \; ; \; i = 1, 2, 3 \ldots n$$

$P_i = \mathrm{Re}\{V_i^* \sum_{K=1}^{n} y_{iK} V_K\}$

$Q_i = -\mathrm{Img}\{V_i^* \sum_{K=1}^{n} y_{iK} V_K\}$

* Let the bus voltages and admittance may be in the form of rectangular & polar forms as follows

$V_i = e_i + j f_i = |V_i| \angle \delta_i$ → for ith bus

$V_K = e_K + j f_K = |V_K| \angle \delta_K$ → for Kth bus

$y_{iK} = G_{iK} + jB_{iK} = |y_{iK}| \angle \theta_{iK}$

Magnitude & angle of voltage

$|V_i| = \sqrt{e_i^2 + f_i^2}$

$\delta_i = \tan^{-1}\left(\frac{f_i}{e_i}\right)$

Magnitude & Angle of Admittance

$|y_{iK}| = \sqrt{G_{iK}^2 + B_{iK}^2}$

$\theta_{iK} = \tan^{-1}\left(\frac{B_{iK}}{G_{iK}}\right)$

* ∴ $P_i - jQ_i = \sum_{K=1}^{n} (G_{iK} + jB_{iK})(e_i - jf_i)(e_K + jf_K)$

Real power

$P_i = e_i \sum_{K=1}^{n} (G_{iK} e_K - B_{iK} f_K) + f_i \sum_{K=1}^{n} (G_{iK} f_K + B_{iK} e_K)$

$i = 1, 2, 3 \cdots n$

$Q_i = e_i \sum_{K=1}^{n} (G_{iK} f_K + B_{iK} e_K) - f_i \sum_{K=1}^{n} (G_{iK} e_K - B_{iK} f_K)$

$i = 1, 2, 3, \cdots n$

* $P_i - jQ_i = |V_i| e^{-j\delta_i} \sum_{K=1}^{n} |y_{iK}| e^{j\theta_{iK}} |V_K| e^{j\delta_K}$

$$\boxed{\begin{aligned} P_i &= |V_i| \sum_{K=1}^{n} |V_K| |y_{iK}| \cos(\theta_{iK} + \delta_K - \delta_i) \; ; \; i = 1, 2 \cdots n \\ Q_i &= |V_i| \sum_{K=1}^{n} |V_K| |y_{iK}| \sin(\theta_{iK} + \delta_K - \delta_i) \; ; \; i = 1, 2 \cdots n \end{aligned}} \rightarrow \text{I \& II}$$

09/01/2011

* Each Bus is characterised by 4 variables $P_i, Q_i, |V_i|, \delta_i$
* Thus, For 'n' Bus system there are total of 4n variables.
* The above equations are solved for 2n variables provided the other 2n variables are specified as input data.
* Solution can only be obtained by iterative numerical techniques.
* Depending upon which two variables are specified in the priori. the buses are classified into 3 categories.
  1. Load Bus (or) PQ Bus
  2. Voltage controlled Bus (or) Generated Bus (or). PV Bus
  3. Slack Bus (or) Swing Bus (or) Reference Bus

① Load Bus (or) PQ Bus :-

* At this Bus the total injected complex power is specified (P_{Di}, Q_{Di} & P_{Gi}, Q_{Gi} are specified).
* The magnitude V_i and the phase angle δ_i of such a Bus 'i' are unknown & this bus is called as "PQ Bus".
* A Pure Load Bus is PQ Bus where $P_{Gi} = Q_{Gi} = 0$.

② Voltage Controlled Bus (or) Generator Bus (or) PV Bus :-

* Active and reactive power load demand are ~~(or) load~~ is known in prior and P_{Gi}, magnitude of V_i are specified. The unknowns are Q_{Gi} & δ_i to be calculated.

③ Slack Bus (or) Swing Bus (or) Reference Bus :-

* Here $|V_i|$ & δ are specified, Real & reactive powers are not specified.
* There is only one bus of this type in a given power system.

* The Bus connected to the largest generating station is normally selected as slack Bus

* The Equations I & II are called static load flow Equations (SLFE). SLFE solution to have practical significance, the variables must be lies with in the satisfying limits.

* i) $\Rightarrow |V_i|_{min} \leq |V_i| \leq |V_i|_{max}$ (i.e., ±5–10%)

ii) $\Rightarrow |\delta_i - \delta_k| \leq |\delta_i - \delta_k|_{max}$

iii) $\Rightarrow P_{Gi\,min} \leq P_{Gi} \leq P_{Gi\,max}$

$Q_{Gi\,min} \leq Q_{Gi} \leq Q_{Gi\,max}$

iv) $\Rightarrow \sum_{i=1}^{n} P_{Gi} = \sum_{i=1}^{n} P_{Di} + P_L$

$$\sum_{i=1}^{n} Q_{Gi} = \sum_{i=1}^{n} Q_{Di} + Q_L$$

Types of Buses at Glance:

Types of Buses	Quantity specified	Quantity Unspecified
Slack Bus	$\lvert V_i \rvert$ δ_i	P_i, Q_i
PV Bus	P_i $\lvert V_i \rvert$	Q_i, δ_i
PQ Bus	P_i Q_i	$\lvert V_i \rvert$, δ_i

* A Complex power flow solution should have the following properties:

① High Computational speed
② Simplicity of the program
③ flexibility of the program
④ Low computer storage
⑤ Reliability of the solution.

Need of Slack Bus:-

① The Voltage angle of slack Bus serves as reference for the Angle of all other Bus Voltages

② To understand why P & Q are not specified (or) sheduled at the slack Bus consider that at each of the 'n' bus system.

$$* P_L = \sum_{i=1}^{n} P_i$$

P_L = Real power loss

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} P_{Gi} - \sum_{i=1}^{n} P_{Di}$$

③ The Equation is evidently the total I^2R losses in the transmission line and transformer of the network.

④ The individual current in the various T/m lines of the network can't be calculated untill ~~the~~ and after the voltage magnitude & angle are known at any bus of the system.

⑤ Therefore P_L is initially unknown & it is not possible to prespecify all the quantities in the above equation.

⑥ In the formulation of power flow problem we choose one bus i.e., slack bus at which P_{Gi} is not sheduled (or) not specified.

⑦ So At the end of power flow solution total power generated is equal to the sum of power demand & losses at the slack Bus.

Approximate Load Flow Bus Solution :-

* The following assumptions are made for the Load flow analysis for Approximate Solution

① Line resistances are neglected which means that active power losses in the line is zero, which reduces the complexity of equations because the total.

⇒ $P_{Gi} = P_D$ & the effect is $\theta_{ik} \approx 90°$ & $\theta_{ii} = -90°$

② The Angle δ_i is so small, so that $\sin\delta_i \approx \delta_i$.
This approximately converts the non linear load flow solutions into linear, so a that analytical solution is possible.

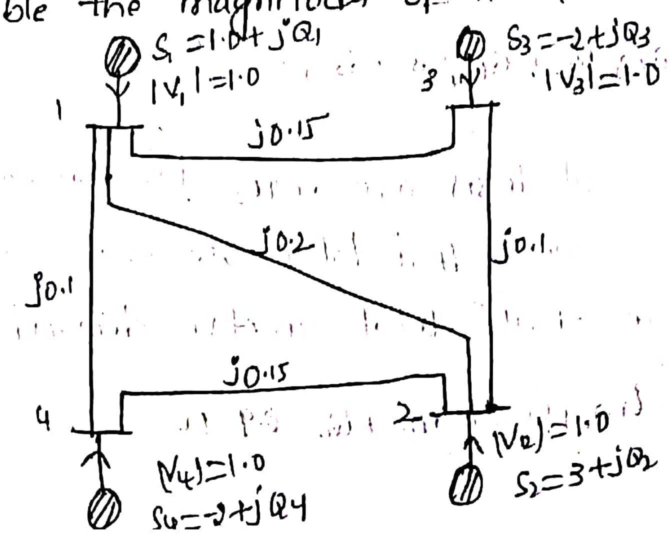
③ All Buses except slack Bus are Voltage controlled bus i.e., the Voltages at all the buses are specified.

$$P_i = V_i \sum_{k=1}^{n} y_{ik} V_k (\delta_i - \delta_k)$$

$$Q_i = -V_i \sum_{\substack{k=1 \\ k \neq i}}^{n} V_k y_{ik} \cos(\delta_i - \delta_k) + V_i^2 y_{ii}$$

Problem

Consider the 4 Bus Sample system where line reactances are indicated per unit line resistances are considered negligible the magnitudes of All 4 bus voltages are



specified to be 1 pu the bus [illegible] powers [illegible] [illegible] given in table given below.

Bus	Real demand	Reactive demand	Real Generation	Reactive Generation
1	$P_{D_1} = 1.0$	$Q_{D_1} = 0.5$	$P_{G_1} = ?$	Q_{G_1} (unspecified)
2	$P_{D_2} = 1.0$	$Q_{D_2} = 0.4$	$P_{G_2} = 4.0$	Q_{G_2} (unspecified)
3	$P_{D_3} = 2.0$	$Q_{D_3} = 1.0$	$P_{G_3} = 0.0$	Q_{G_3} (unspecified)
4	$P_{D_4} = 2.0$	$Q_{D_4} = 1.0$	$P_{G_4} = 0.0$	Q_{G_4} (unspecified)

Solution   1 Bus- Slack Bus

* As the Bus Voltages are specified at all the Buses must have controllable Q sources. & Buses 3 & 4 has only Q sources.

* Since system is assumed to be lossless the real power generation is equal to be sum of demands.

$$P_{G_1} = P_{D_1} + P_{D_2} + P_{D_3} + P_{D_4} - P_{G_2}$$

$$= 1 + 1 + 2 + 2 - 4$$

$$P_{G_1} = 2$$

∴ We have 7 unknowns instead of 8.

* In the present problem the unknowns are

$$Q_{G_1}, Q_{G_2}, Q_{G_3}, Q_{G_4}, \delta_2, \delta_3, \delta_4.$$

* Though the real losses are zero, the presence of the reactive losses requires that total reactive generation must be more that total reactive demand.

$$Q_{DT} = Q_{D_1} + Q_{D_2} + Q_{D_3} + Q_{4} = 2.9 \text{ pu}$$

From the given data Y_{BUS} can be written as

Y_{BUS} Calculation:

$$
Y_{BUS} = \begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & y_{11} & y_{12} & y_{13} & y_{14} \\
2 & y_{21} & y_{22} & y_{23} & y_{24} \\
3 & y_{31} & y_{32} & y_{33} & y_{34} \\
4 & y_{41} & y_{42} & y_{43} & y_{44}
\end{array}
$$

$$y_{11} = y_{12} + y_{13} + y_{14}$$
$$= \frac{1}{j0.15} + \frac{1}{j0.2} + \frac{1}{j0.1}$$
$$= -j21.67$$

$$y_{12} = y_{21} = \frac{-1}{0.2j} = j5$$
$$y_{13} = y_{31} = \frac{-1}{0.15j} = j6.667$$
$$y_{14} = y_{41} = \frac{-1}{0.1j} = j10$$

$$\cancel{y_{12} = -j0.2 = y_{21}}$$
$$\cancel{y_{13} = -j0.15 =}$$
$$\cancel{y_{14} = j0.1}$$

$$y_{22} = y_{20} + y_{23} + y_{24}$$
$$= \frac{1}{j0.2} + \frac{1}{j0.1} + \frac{1}{j0.15}$$
$$= -j21.67$$

$$y_{23} = y_{32} = -y_{23} = \frac{-1}{j0.1} = j10$$
$$y_{24} = y_{42} = -y_{24} = \frac{-1}{j0.15} = j6.667$$

$$y_{33} = y_{31} + y_{32}$$
$$= \frac{1}{j0.15} + \frac{1}{j0.1} = -j16.667$$

$$y_{34} = y_{43} = -y_{34} = 0$$

$$y_{44} = -j16.667$$

$$
Y_{BUS} = \begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & -j21.67 & j5 & j6.667 & j10 \\
2 & j5 & -j21.667 & j10 & j6.667 \\
3 & j6.667 & j10 & -j16.667 & j0.00 \\
4 & j10 & j6.667 & j0.00 & -j16.667
\end{array}
$$

By using this Y_{BUS} approximate load flow equation is

$$\Rightarrow P_i = |V_i| \sum_{K=1}^{n} |Y_{ik}| \, |V_k| \cos(\delta_i - \delta_k)$$

$$i = 1, 2 \ldots n$$

1st Bus is slack bus $\therefore \delta_1 = 0$

For $i=2$, $n=4$

$$P_2 = P_{G_2} - P_{D_2} = 4 - 1 = 3$$

$$P_2 = 3 = |V_2| \, |V_1| \, |Y_{21}| (\delta_2 - \delta_1) + |V_2| \, |V_2| \, |Y_{22}| (\underbrace{\delta_2 - \delta_2}_{0})$$

$$+ |V_2| \, |V_3| \, |Y_{23}| (\delta_2 - \delta_3) + |V_2| \, |V_4| \, |Y_{24}| (\delta_2 - \delta_4)$$

$$3 = 1 \cdot 1 \cdot (5) (\delta_2 - \delta_1) + 1 \cdot 1 \cdot (10) (\delta_2 - \delta_3) + 1 \cdot 1 \cdot (6.667)(\delta_2 - \delta_4)$$

$$P_2 = 3 = 5(\delta_2 - \delta_1) + 10(\delta_2 - \delta_3) + 6.667(\delta_2 - \delta_4)$$

For $i = 3$, $n = 4$

$$P_3 = P_{G_3} - P_{D_3} = 0 - 2 = -2$$

$$P_3 = -2 = |V_3| \Big[|V_1| \, |Y_{31}| (\delta_3 - \delta_1) + |V_2| \, |Y_{32}| (\delta_3 - \delta_2)$$

$$+ |V_3| \, \underbrace{|Y_{33}| (\delta_3 - \delta_3)}_{0} + |V_4| \, |Y_{34}| (\delta_3 - \delta_4) \Big]$$

$$= 1 \Big[1 \cdot 6.667 (\delta_3 - \delta_1) + 1 \cdot 10 (\delta_3 - \delta_2) + \underbrace{1 \cdot 0 (\delta_3 - \delta_4)}_{0}$$

$$P_3 = -2 = 6.667(\delta_3 - \delta_1) + 10(\delta_3 - \delta_2)$$

For $i = 4$, $n = 4$

$$P_4 = P_{G_4} - P_{D_4} = 0 - 2 = -2$$

$$P_4 = -2 = |V_4| \Big[|V_1| \, |Y_{41}| (\delta_4 - \delta_1) + |V_2| \, |Y_{42}| (\delta_4 - \delta_2)$$

$$+ |V_3| \underbrace{|Y_{43}|}_{0} (\delta_4 - \delta_3) + |V_4| \underbrace{|Y_{44}|}_{0} (\underbrace{\delta_4 - \delta_4}_{0}) \Big]$$

$P_4 = 10(\delta_4 - \delta_1) + 6.667(\delta_4 - \delta_2) = -3$

From P_2, P_3, P_4 Equations

$\Rightarrow$ ~~$6.667 + 16.\delta_2$~~

$-15\delta_1 - 31.667\,\delta_2 - 10\,\delta_3 - 6.667\,\delta_4 = 3$ — (1)

$-6.667\,\delta_1 - 10\,\delta_2 + 16.667\,\delta_3 = -2$ — (2)

$-10\,\delta_1 - 6.667\,\delta_2 + 16.667\,\delta_4 = -3$ — (3)

From (2) & (3)

$-66.667\,\delta_1 - 100\,\delta_2 + 166.67\,\delta_3 = -20$

$-66.667\,\delta_1 - 44.44\,\delta_2 + 111.11\,\delta_4 = -13.334$

$+ \quad + \quad - \quad +$

$\delta_1 = 0$

$\therefore \quad -31.667\,\delta_2 - 10\,\delta_3 - 6.667\,\delta_4 = 3$

$-10\,\delta_2 + 16.667\,\delta_3 = -2$

$-6.667\,\delta_2 + 16.667\,\delta_4 = -2$

$\delta_2 = -0.0377$

$\delta_3 = -0.137$

$\delta_4 = -0.1331$

By solving $\quad \delta_2 = 0.077$ rad

$\delta_3 = -0.074$ rad

$\delta_4 = -0.609$ rad

$$Q_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|\,|Y_{ik}| \cos(\delta_i - \delta_k) + |V_i|\,|Y_{ii}|$$

$i = 1$

$$Q_{G_1} = -\Big[|V_1|\,|V_1|\,|Y_{11}| \cos(\delta_1 - \delta_1) + |V_1|^2\,|Y_{11}|$$
$$+ |V_1|\,|V_2|\,|Y_{12}| \cos(\delta_1 - \delta_2) + |V_1|^2\,|Y_{11}|$$
$$+ |V_1|\,|V_3|\,|Y_{13}| \cos(\delta_1 - \delta_3) + |V_1|^2\,|Y_{11}|$$

$$+ |V_1| \, |V_4| \, |Y_{14}| \cos(\delta_1 - \delta_4) + |V_1|^2 |Y_{11}|$$

$$Q_{G_1} = -\Big[1 \cdot 1 \, (21.67) \cos(0 - 0) + 1^2 (21.67) + 1 \cdot 1 \, (5) \cos(0 - 0.077) + 1^2 (5) + 1 \cdot 1 \, (10) \cos(0 + 0.089) + 10$$

$$+ 1 \cdot 1 \, (6.667) \cos(0 + 0.077) + 1^2 (6.667) + 1 \cdot 1 \, (10) \cos(0 + 0.089)$$

$$= -\Big[21.67 + 21.67 + 5 \cos(-0.077) + 5 + 6.667 \cos(0.077) + 6.667 +$$

$$10 \cos(0.089) + 10 \Big]$$

$$Q_{G_1} = -86.67$$

$$P = \frac{|V_s|^2}{Z} \cos\theta - \frac{|V_s||V_R|}{Z} \cos(\theta + \delta)$$

$$|Z| = |X| \Rightarrow \theta = 90^\circ$$

$$P_{PK} = -P_{KP} = \frac{|V_P| \, |V_K|}{X_{PK}} \sin(\delta_P - \delta_K)$$

$$P_{13} = -P_{31} = \frac{|V_1| \, |V_3|}{X_{13}} \sin(\delta_1 - \delta_3)$$

$$= \frac{1}{X_{13}} \sin(-0.1331) = \frac{\sin(\cancel{-0.1331}\ 4.93)}{0.15} = 0.492 \text{ PU}$$

$$P_{12} = -P_{21} = \frac{|V_1| \, |V_2|}{X_{12}} \sin(\delta_1 - \delta_2)$$

$$P_{23} = 1.5 \text{ PU} = -P_{32}$$

$$P_{24} = -P_{42} = 1.10 \text{ PU}$$

$$= \frac{1}{0.2} \sin(044.3)$$

$$= -0.87$$

$$P_{14} = -P_{41} = \frac{|V_1| \, |V_4|}{X_{14}} \sin(\delta_1 - \delta_4) = \frac{1}{0.1} \sin(5.1)$$

$$= 0.89 \text{ PU}$$

$$Q_{ik} = \frac{|V_i|^2}{X_{ik}} - \frac{|V_i||V_k|}{X_{ik}} \cos(\delta_i - \delta_k)$$

$$Q_{12} = \frac{|V_1|^2}{X_{12}} - \frac{|V_1||V_2|}{X_{12}} \cos(\delta_1 - \delta_2)$$

$$= \frac{1}{0.2} - \frac{1}{0.2} \cos(0+4.3)$$

$$= 0.0145 = Q_{21}$$

$Q_{13} = -Q_{31} = 0.018$

$Q_{41} = -Q_{14} = 0.04$

$Q_{23} = -Q_{32} = 0.1129$

$Q_{24} = -Q_{42} = 0.0916$

$\frac{|V_i||V_k|}{X} \cos(\delta_i - \delta_k)$

$\frac{|V_i|^2}{X_{ik}} - V_iV_k$

$Q_i = Q_G - Q_D$

$Q_G = Q_i + Q_D$

In Power flow calculations usually employed such as 1) Gauss-Seidal method [GS]
2) Newton-Raphson method [NR]
3) Fast Decoupled Load Flow [FDLF]

① GS Method [Gauss Seidal Method]:

* Iterative Algorithm for solving set of non-linear Algebra equations. The nodal current equation for n-bus system is given by

$$I_i = \sum_{\substack{K=1 \\ K \neq i}}^{n} y_{iK} V_K \quad \forall i = 1,2,\dots n$$

$$I_i = y_{ii} V_i + \sum_{\substack{K=1 \\ K \neq i}}^{n} y_{iK} V_K$$

$$\Rightarrow y_{ii} V_i = I_i - \sum_{\substack{K=1 \\ K \neq i}}^{n} y_{iK} V_K$$

$$V_i = \frac{1}{y_{ii}} \left[I_i - \sum_{\substack{K=1 \\ K \neq i}}^{n} y_{iK} V_K \right]$$

*Conjugate power

$$V_i^* I_i = P_i - jQ_i$$

$$I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{K=1 \\ K \neq i}}^{n} y_{iK} V_K \right] \quad \forall i = 1,2, \dots n$$

* Let it be assumed that all the buses other than the slack bus are PQ buses.

* The slack buses voltages being specified there are n-1 bus voltages and starting values of whose magnitude and angles are assumed. and they are assumed as 1+j0

At every step of iteration the updated values of bus are used to compute with the new values of bus voltages

∴ expression of new voltage.

$$V_i^{r+1} = \frac{1}{y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{r})^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} y_{ik} V_k^{r}\right] \quad i = 1, 2, \ldots n$$

* Iterations are repeated till no bus voltage magnitude changes by more than a prescribed value during an iteration. The computation process then said to converge to a solution.

Advantages of GS Method:-

① It is one of the simplest iterative method: suitable for

② suitable for small power system.

⇒ Case 1 :-

* When PQ buses are considered (or) PV buses are not considered :-

* Let us consider the case where All the buses other than the slack bus are PQ Buses.

① With load profile known at each poles i.e., P_{Di} & Q_{Di} are known. Allocate P_{Gi} & Q_{Gi} to All the generating station. With this step Bus injections $(P_i + jQ_i)$ are known at All the buses except slack bus.

② Assembly of Bus Admittance matrix Using

$$Y_{Bus} = A^T Y A \quad (\text{Singular Transformation Method})$$

③ Iterative computation of Bus Voltages

$(V_i = 2, 3 \ldots n)$

To start the iterations a set of Initial voltage values are assumed i.e., initially All the voltages are set equal to $(1+j0)$, except the voltage of slack bus which is fixed.

The $(n-1)$ equations in Complex numbers are to be solved iteratively for finding $(n-1)$ complex voltages, if complex number operations are not available in a computer then it can be converted into 2(n-1) equations in real unknowns.

i.e., $(e_i, \delta_i$ (or) $|V_i|, \delta_i)$

$V_i = e_i + j\delta_i$

$\Rightarrow |V_i| e^{j\delta_i}$

$$A_i = \frac{P_i - jQ_i}{y_{ii}} \; ; \; i = 2,3 \ldots n$$

$$B_{ik} = \frac{y_{ik}}{y_{ii}} \; ; \; i = 2 \ldots n$$

$k = 1,2 \ldots n$

$i \neq k$

* Now for $(r+1)^{th}$ iteration

$$V_i^{(r+1)} = \frac{A_i}{(V_i)^{r*}} - \sum_{k=1}^{i-1} B_{ik} V_k^{r+1} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \; ; \; i = 2 \ldots n$$

* The iterative process is continued till the change in magnitude of bus voltage i.e., Modulus of $|\Delta V_i|^{r+1}$ between 2 consequative iterations is less than the certain toloarance for this

i.e., $|\Delta V_i^{r+1}| = |V_i^{r+1} - V_i^{r}| < \varepsilon \; ; \; i = 2,3 \ldots n$

④ Compitation of slack bus power

Substitution of all bus voltages computed in step③ along with V_1 in $P_i - jQ_i$ results

$$\boxed{S_i^* = P_i - jQ_i}$$

⑤ Computation of line flows

power flows on the various lines of the network are computed slack bus power can also be found by suming the flows on the lines terminating at the slack bus.

Acceleration of convergence

* Convergence in the Gauss-Seidal method sometimes be speeded up by the use of accelerating factor. By the fact that the accelerated value of voltage at r+1 iteration is given by

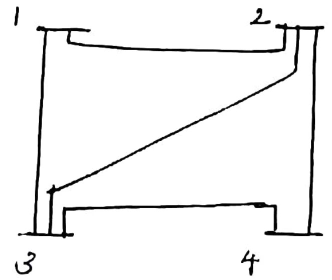
$$V_i^{(r+1)}(\text{acceleration}) = V_i^{(r)} + \alpha\left(V_i^{(r+1)} - V_i^{(r)}\right)$$

⇒ where 'α' is the real number called acceleration factor. A suitable value of 'α' for any system can be obtained by Trail Load flow studies and the recommened value of 'α' is 1.6.

$\alpha = 1.6$

Problem

① For the system shown in figure the generator are connected at all the four buses while loads are at buses 2 & 3. All buses other than slack are PQ type and assuming a flat voltage slack. find the voltages & bus angles at the three buses, at the end of first iteration



Line data ÷

Bus-Bus	Rpu	Xpu
1-2	0.05	0.15
1-3	0.10	0.30
2-3	0.15	0.45
2-4	0.10	0.30
3-4	0.05	0.15

Bus data ÷

Bus	Ppu	Qpu	Vpu	Remarks
1	–	–	$1.04\angle 0°$	Slack Bus
2	0.5	−0.2	–	PQ Bus
3	−1.0	0.5	–	PQ Bus
4	0.3	−0.1	–	PQ Bus

Step 1:

$$Y_{BUS} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix} \end{array}$$

$$y_{11} = y_{12} + y_{13}$$

$$= \frac{1}{0.05+j0.15} + \frac{1}{0.10+j0.30}$$

$$= 3-j9$$

$$y_{12} = y_{21} = -y_{12}$$

$$= \frac{-1}{0.05+j0.15}$$

$$= -2+j6$$

$$y_{13} = y_{31} = -y_{13}$$

$$= \frac{-1}{0.10+j0.30}$$

$$= -1+j3$$

$$y_{14} = 0$$

$$y_{22} = y_{21} + y_{23} + y_{24}$$

$$= \frac{1}{0.05+j0.15} + \frac{1}{0.15+j0.45} + \frac{1}{0.10+j0.30}$$

$$= 3.667 - j11$$

$$y_{23} = y_{32} = -y_{23}$$

$$= \frac{-1}{0.15+j0.45}$$

$$= -0.667+j2$$

$$y_{24} = y_{42} = -y_{24}$$

$$= \frac{-1}{0.10+j0.30}$$

$$= -1+j3$$

$$y_{33} = y_{31} + y_{32} + y_{34}$$

$$= \frac{1}{0.10+j0.30} + \frac{1}{0.15+j0.45} + \frac{1}{0.05+j0.15}$$

$$= 3.66 - j11$$

$$y_{34} = \frac{-1}{0.05+j0.15} = y_{43} = -y_{34}$$

$$= -2+j6$$

$$y_{44} = y_{42} + y_{43}$$

$$= \frac{1}{0.10+j0.30} + \frac{1}{0.05+j0.15}$$

$$= 3-j9$$

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.667-j11 & -0.667+j2 & -1+j3 \\ -1+j3 & -0.667+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Step 2:

$$V_i^{(r+1)} = \frac{1}{y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{r})^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} y_{ik} V_k^{r}\right]$$

when $n=4$ $i=2$

$r=0$ (First Iteration)

Assume $V_2 = 1+j0$
$V_3 = 1+j0$
$V_4 = 1+j0$

$$V_2^{1} = \frac{1}{y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^{0})^*} - \sum_{\substack{k=1 \\ k\neq 2}}^{4} y_{21} V_1^{0}\right]$$

$$V_2^{1} = \frac{1}{3.667-j11}\left[\frac{P_2 - jQ_2}{V_2^*} - y_{21}V_1 - y_{23}V_3 - y_{24}V_4\right]$$

$$= \frac{1}{3.667-j11}\left[\frac{0.5+j0.2}{1-j0} - (-2+j6)(1.04\angle 0^\circ) - (-0.667+j2)(1+j0) - (-1+j3)(1+j0)\right]$$

$$= 0.0272 + j0.081\left[(0.5+j0.2) - (-3.747+j11.24)\right]$$

$$V_2^{1} = 1.019 + j0.046 \ \text{Vpu}$$

when $n=4$ $i=3$

$r=0$

$V_3 = 1+j0$
$V_4 = 1+j0$

$$V_3^{1} = \frac{1}{y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - y_{31}V_1 - y_{32}V_2^{1} - y_{34}V_4\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{-1-j0.5}{1-j0} - (-1+j3)(1.04\angle 0^\circ) - (-0.667+j2)(1.019+j0.046) - (-2+j6)(1+j0)\right]$$

$$V_3^{1} = 1.0282 - j0.086$$

when $n=4$ $i=4$ $V_4 = 1+j0$

$r=0$

$$V_4^{1} = \frac{1}{y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - y_{41}V_1 - y_{42}V_2^{1} - y_{43}V_3^{1}\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1-j0} - 0(1.04\angle 0^\circ) - (-1+j3)(1.019+j0.046) - (-2+j6)(1.0282-j0.086)\right]$$

$= 1.025 -j$

If acceleration factor is 1.6

$$V_2^1 = V_2^0 + \alpha (V_2^1 - V_2^0)$$

$$= (1+j0) + 1.6(1.019 + j0.046 - 1 + j0)$$

$$V_2^1 = (1.0304 + j0.013)\ p.u$$

(acceleration factor)

Case 2:

Algorithm for GS Method when PV Bus is also present:

* At PV Buses P and magnitude of voltage are specified Q and δ are unknowns to be determined.
* The following steps for ith PV Bus.

Step 1: From load flow equation

$$Q_i = -\text{img}\left[V_i^* \sum_{K=1}^{n} Y_{ik} V_K\right]$$

* The revised value of Q_i is obtained from the above equation by substituting most updated values of voltages on the right hand side

* For $(r+1)$th iteration

$$Q_i^{(r+1)} = -\text{Img}\left[(V_i^{r})^* \sum_{K=1}^{i-1} Y_{ik} V_K^{(r+1)} + (V_i^{r})^* \sum_{K=i}^{n} Y_{ik} V_K^{r}\right]$$

Step 2: The revised value of δ_i is obtained from V_i immediatly following step-1 thus

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= \text{Angle of}\left[\frac{A_i^{(r+1)}}{(V_i^{r})^*} - \sum_{K=1}^{i-1} B_{ik} V_K^{r+1} - \sum_{K=i+1}^{n} B_{ik} V_K^{r}\right]$$

where $A_i^{(r+1)} = \dfrac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$

* The Algorithm for PQ Buses remains (Constant) unchang

* Q Generation requires that Q demand at any bus must be in a range Q_{min} to Q_{max}.

* If at any stage during the computation Q at any bus goes outside these limits it is fixed at Q_{min} (or) Q_{max} and the bus is treated like a PQ Bus.

* Thus, Step-1 above branches out to Step-3 below.

<u>Step-3</u>: If $Q_i^{(r+1)} < Q_{imin}$ then set $Q_i^{(r+1)} = Q_{imin}$

If $Q_i^{(r+1)} > Q_{imax}$ then set $Q_i^{(r+1)} = Q_{imax}$

* Treat Bus 'i' as PQ Bus and compute $\angle A_i^{(r+1)}$ & $V_i^{(r+1)}$ respectively.

<u>Problem</u>:

If in the previous problem, let Bus '2' be a PV Bus now with the magnitude of $|V_2| = 1.04$ PU. Once again assuming a flat Voltage start $1+j0$. Find Q_2, δ_2, V_3, V_4 at the end of first GS method. $[0.2 \le Q_2 \le 1]$

$$Q_i^{(r+1)} = -\text{Img}\left[(V_i^{r})^* \sum_{K=1}^{i-1} Y_{iK} V_K^{(r+1)} + (V_i^{r})^* \sum_{K=i}^{n} Y_{iK} V_K^{r}\right]$$

$i = 2, r = 0$

$$Q_2^1 = -\text{Img}\left[V_2^* \sum_{K=1}^{1} Y_{21} V_1 + V_2^* \sum_{K=2}^{4} Y_{2K} V_K^{r}\right]$$

$$= -\text{Img}\Big[(1.04 - j0)(-2+j6)(1.04+j0) + (1.04 - j0)\big[(3.66 - j11)(1.04) + (-0.66+j2)(1+j0) + (-1+j3)(1+j0)\big]\Big]$$

$= -0.37 + j0.65$

$= -0.15 + j0.45$

$= -\text{Img}\{0.128 - j0.208\}$

$|Q_2| = 0.208$

It is clear that the reactive power is within the specified limits

$$\therefore \delta_i^{(r+1)} = \text{Angle of} \left\{ \frac{A_i^{(r+1)}}{(V_i^r)^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{r+1} - \sum_{k=i+1}^{n} B_{ik} V_k^r \right\}$$

$i=2$

$$A_i^{(r+1)} = \frac{P_i - jQ_i^{r+1}}{y_{ii}} = \frac{0.5 - j0.203}{y_{22}} = \frac{0.5-j0.203}{3.261-j11} = 0.366+j0.035$$

$$\delta_2' = \text{Angle of} \left\{ \frac{0.366+j0.035}{V_2^*} - (B_{21} V_1) - (B_{23} V_3 + B_{24} V_4) \right]$$

$$= \text{Angle of} \left\{ \frac{0.366+j0.035}{1.04-j0} - (-j6.667 \times 1.04) - (-j2.22(1.00) + (-j3.33)(1+j0)) \right]$$

$$= \text{Angle of} \{ 0.357 + j12.51 \}$$

$$= \text{Angle of} \{ 12.52 \angle 88.389 \}$$

$$= (88.389^\circ$$

$$1.54^\circ)$$

$$= \cancel{0.0827 rad} = 1.874$$

$$V_2' = 1.04 \angle \delta_2'$$

$$= 1.04 \angle 1.874^\circ$$

$$V_3' = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} y_{ik} V_k^r \right]$$

$i=3,\ r=0,\ n=4$

$$= \frac{1}{y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*} - \sum_{\substack{k=1 \\ k \neq 3}}^{4} y_{3k} V_k \right]$$

$$= \frac{1}{y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*} - (y_{31} V_1 + y_{32} V_2 + y_{34} V_4) \right]$$

$$= \frac{1}{3.66-j11} \left[\frac{-1.0-j0.5}{1-j0} - ((-1+j3)(1.04) + (-0.667+j2)(1.04 \angle 1.874) + (-2+j6)(1+j0)) \right]$$

$$= (1.031 - j0.089) \text{ pu}$$

$$\frac{1}{y_{44}}\left[\frac{P_4-jQ_4}{V_4}-y_{41}V_1'-y_{42}V_2'-y_{43}V_3'\right]$$

$$V_4'=\frac{1}{3-j9}\left[\frac{0.3+j0.1}{1}-0-(-1+j3)(1.04(1.874)-(2+j6)(1.0137-j0.0\ldots\right.$$

$$=(1.034-j0.0146)\ PU$$

ⅰ) suppose permissible limits of Q_2 is revised as $0.25\le Q_2\le 1\,PU$.

But we have $Q_2=0.208$

$Q_{calculated}\le Q_{min}$

$0.208\le 0.25$

As reactive power violets the minimum power, Hence Q is fixed to 0.25PU. and Bus 2 is treated as PQ Bus.

$$V_i^{r+1}=\frac{1}{y_{ii}}\left[\frac{P_i-jQ_i}{(V_i^r)^*}-\sum_{\substack{k=1\\k\neq i}}^{n}y_{ik}V_k^*\right]$$

$i=2\quad r=0\quad n=4$

$$V_2'=\frac{1}{y_{22}}\left[\frac{P_2-jQ_2}{(V_2^0)^*}-y_{21}V_1^0+y_{23}V_3^0+y_{24}V_4^0\right]$$

$$=\frac{1}{3.617-j11}\left[\frac{0.5+j0.2}{1-j0}-(-2+j6)(1.04+j0)-(-0.617+j2)^{(1+j0)}-(-1+j3)^{(1+j0)}\right]$$

$$=(1.019+j0.046)\ PU$$

$$=1.0559+j0.0341\ pu$$

$i=3\quad r=0\quad n=4$

$$V_3'=\frac{1}{y_{33}}\left[\frac{P_3-jQ_3}{(V_3^0)^*}-y_{31}V_1^0-y_{32}V_2^1-y_{34}V_4^0\right]$$

$$=\frac{1}{8.66-j11}\left[\frac{-1-j0.5}{1-j0}-(-1+j3)(1.04+j0)-(-0.662+j2)(1.019+j0.046)-(-2+j6)^{(1+j0)}\right]$$

$$=0.8159-j0.0048$$

$$V_3'=(1.034-j0.0887)\ PU$$

$$=1.0347-j0.0893\ pu$$

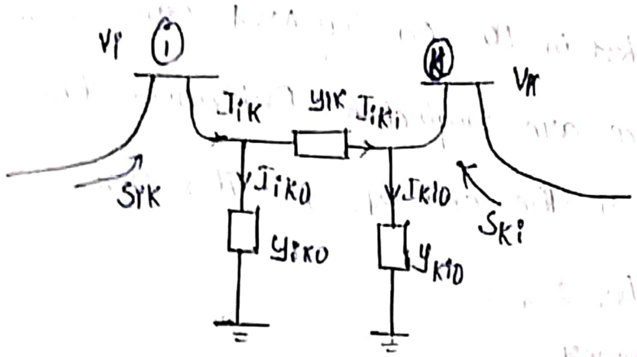
$$V_4' = \frac{1}{y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - y_{41}V_1^* - y_{42}V_2 - y_{43}V_3\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1-j0} - 0 - (-1+j3)(1.019+j0.046) - (-2+j6)(1.034-j0.0883)\right]$$

$$= 1.0775 + j0.0923$$

$$V_4' = 1.029 + j0.1078$$

Computation of Line flows :-



$$I_{ik} = I_{ik0} + I_{ik1}$$

$$= V_i y_{ik0} + y_{ik}[V_i - V_k]$$

$S_{ik} = V_i(V_i^* - V_i^*)\, y_{ik}$; $(V_i V_i^* y_{ik}$

$$S_{ik} = P_{ik} + jQ_{ik}$$

$$= V_i I_{ik}^*$$

$$= V_i\left[V_i y_{ik0} + y_{ik}[V_i - V_k]\right]^*$$

$$= V_i\left[V_i^* y_{ik0}^* + y_{ik}^*(V_i^* - V_k^*)\right]$$

$$S_{ik} = V_i(V_i^* - V_k^*)\, y_{ik}^* + V_i V_i^* y_{ik0}^* \quad \text{—(1)}$$

$$S_{ki} = V_k(V_k^* - V_i^*)\, y_{ki}^* + V_k V_k^* y_{ki0}^* \quad \text{—(2)}$$

* The power loss in i^{th} to k^{th} line is the sum of the power flows determine in eq(1) & eq(2)

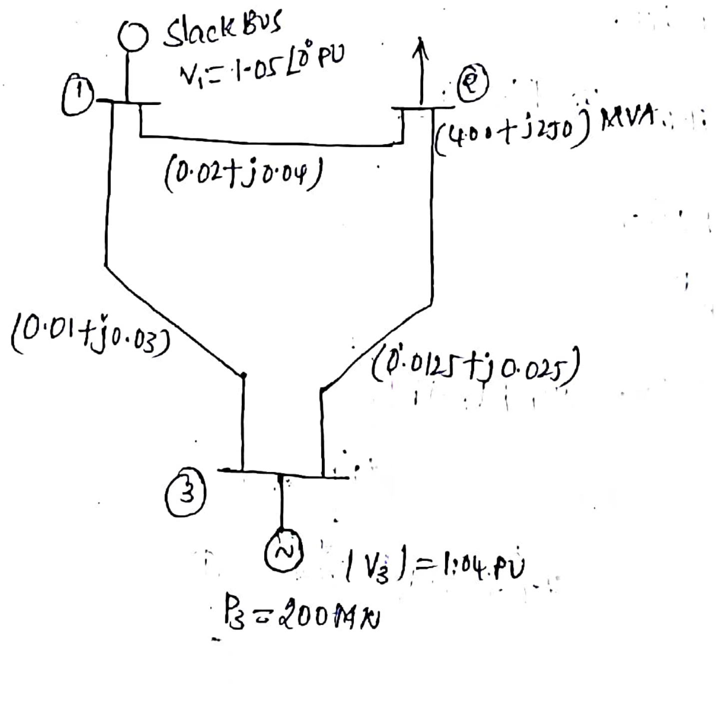
* Total transmission loss can be computed by all the load flows $S_{ik} + S_{ki}$ for every i & k

Problem :-

=x=

① The single line diagram of simple power system with generation at bus 1 & 3 as shown in figure. The magnitude of voltage at Bus '1' is 1.05 PU. The Voltage magnitude at Bus '3' is fixed at 1.04 PU with active power generation of 200 MW. A load consisting of 400 MW and 250 MVAR is taken from BUS 2. The line impedences are marked in PU. On a 100 MVA base & the line charging suceptance are neglected. Determine the following using GS Method at the end of 1st iteration.

1) Voltage at bus 2 & 3
2) Slack bus power
3) Line flows and Line losses



Sol

① $Z_{12} = 0.02 + j0.04$

$\Rightarrow y_{12} = (10 - j20)$ PU

$Z_{13} = 0.01 + j0.03$

$\Rightarrow y_{13} = 10 - j30$

$Z_{23} = 0.0125 + j0.025$

$\Rightarrow y_{23} = 16 - j32$

$$Y_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & y_{11} & y_{12} & y_{13} \\ 2 & y_{21} & y_{22} & y_{23} \\ 3 & y_{31} & y_{32} & y_{33} \end{array}$$

$y_{11} = y_{12} + y_{13}$

$= 10 - j20 + 10 - j30$

$= 20 - j50$

$y_{12} = y_{21} = -10 + j20$

$y_{13} = y_{31} = -10 + j30$

$y_{23} = y_{32} = -16 + j32$

$y_{22} = y_{21} + y_{23}$

$= 10 - j20 + 16 - j32$

$= 26 - j52$

$y_{33} = y_{31} + y_{32}$

$= 10 - j30 + 16 - j32$

$= 26 - j62$

$$Y_{BUS} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j3 & -16+j32 & 26-j62 \end{bmatrix}$$

$$V_P^{r+1} = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{r})^*} - \sum_{\substack{K=1 \\ K\neq i}}^{i-1} y_{ik} V_k^{r+1} - \sum_{K=i+1}^{n} y_{ik} V_K^{r} \right]$$

$i = 2 \quad r = 0 \qquad V_2^0 = 1 + j0 \quad V_3^0 = (1.04 + j0) \quad V_1^0 = 1.05 \angle 0 \text{ PU}$

$$\Rightarrow V_2^1 = \frac{1}{y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - \underbrace{\sum_{K=2}^{1} y_{2K} V_K}_{y_{21} V_1^1} - \underbrace{\sum_{K=3}^{3} y_{2K} V_2^{r}}_{y_{23} V_3^0} \right]$$

$P_{3_{PU}} = \frac{200}{100} = 2 \text{ PU}$

$P_{D} = \frac{400}{100} = 4 \text{ PU} \Rightarrow P_{2PU} = P_g - P_d$

$= 0 - 4$

$= -4 \text{ PU}$

$Q_{2PU} = \frac{250}{100} = 2.5 \text{PU}$

$\text{Illy } Q_2 = -2.5 \text{PU}$

$$\Rightarrow V_2^1 = \frac{1}{26-j52} \left[\frac{4 - j2.5}{1 - j0} - (-10+j20)(1.05 \angle 0) - (-16+j32)(1.04+j0) \right]$$

$V_2^1 = (0.97 - j\,0.042307) \text{ PU}$

$$Q_i^{r+1} = -\text{Img}\left\{ (V_i^{r})^* \sum_{K=1}^{i-1} y_{iK} V_K^{r+1} + (V_i^{r})^* \sum_{K=i}^{n} y_{iK} V_K^{r} \right\}$$

$i=3 \quad r=0$

$$Q_3^1 = -\text{Img}\left\{ (V_3^0)^* \sum_{K=1}^{2} y_{3K} V_K^1 + (V_3^0)^* \sum_{K=3}^{3} y_{3K} V_K^{r} \right\}$$

$y_{31}V_1^1 + y_{32}V_2^1 \qquad y_{33} \; V_3^0$

$= -\text{Img}\{ (1.04 - j0) [(-10+j30)(1.05) + (-16+j32)(0.97 - j0.0423)] +$

$(1.04-j0)(26-j62)(1.04+j0) \}$

$= -\text{Img}\{ 2.468 - \underline{j1.3137} \}$

$= +1.16 \text{ PU}$

$$\delta_i^{r+1} = \text{Angle of}\left\{ \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{r})^*} - \sum_{K=1}^{i-1} y_{iK} V_K^{r+1} - \sum_{K=i+1}^{n} y_{iK} V_K^{r} \right] \right\}$$

$i=3, \; r=0$

$= \text{Angle of}\left\{ \frac{1}{y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - y_{31}V_1^1 - y_{32}V_2^1 \right] \right\}$

$= \text{Angle of}\left\{ \frac{1}{26-j62} \left[\frac{2-j1.16}{(1.04-j0)} - (-10+j30)(1.05\angle 0) - (-16+j32)(0.97 - j0.04230) \right] \right\}$

$= \text{Angle of}\{ 1.035 - j5.333 \}$

$= \text{Angle of}\{ 1.035 \angle -0.295 \}$

$= -0.29$

$= -0.2854^\circ$

$\left(\times \frac{180}{\pi} = 5.10\times 10^{-3}\right)$

$-10+j20$

$-10-j20$

$V_3^0 = 1.04 + j0$

$V_3^1 = 1.04 \angle -0.2854$

$= 1.039 - j0.00518$

(2) Slack bus power

$S_i = P_i - jQ_i$

$$= V_i^{0*} \sum_{K=1}^{n} y_{iK} V_K$$

$i=1 \qquad , n=3$

$S_1 = V_1^* \left[y_{11}V_1^1 + y_{12}V_2^1 + y_{13}V_3 \right]$

$$S_1 = (1.05 - j0)\left[(20-j50)(1.05+j0) + (-10+j20)(0.97-j0.042307) + (-10+j30)(1.039-j0.00518)\right]$$

$$S_1 = (1.948 - j1.399)\ PU = \left(\frac{\text{Actual}}{\text{base}}\right)$$

$$S_1 = (1.948 - j1.399) \times 100\ MVA \quad \text{Actual} = \text{Ans} \times \text{base}$$

$$S_1 = (194.8 - j139.97)\ MVA$$

(3) Line flows & Line losses

$$S_{ik} = P_{ik} + jQ_{ik}$$

$$= V_i I_{ik}^*$$

$$= V_i\left[V_i^* - V_k^*\right] y_{ik}^* + V_i V_i^* y_{ik0}^*$$

Here $y_{ik0} = 0$ bcz susceptances are neglected

$$S_{12} = V_1\left[V_1^* - V_2^*\right] y_{12}^*$$

$$= 1.05\left[1.05 - 0.97 - j0.042307\right](-10+j20)$$

$$= (1.67 + j1.230)\ PU$$

$$= (1.7 + j1.23)\ PU$$

$$S_{21} = V_2\left[V_2^* - V_1^*\right] y_{21}^*$$

$$= (0.97 - j0.042307)\left[0.97 + j0.042307 - 1.05\right] \times (-10 - j20)$$

$$= +1.64 + j1.071$$

&

Line loss $\Rightarrow S_{12} + S_{21}$

$$= (0.06 + j0.159)$$

$$S_{13} = V_1\left[V_1^* - V_3^*\right] y_{13}^*$$

$$= 1.05\left[1.05 - \left[1.039 + j0.00518\right]\right](-10-j30)$$

$$= -0.278 - j0.292$$

$S_{31} = V_3 \left[V_3^* - V_1^* \right] y_{31}^*$

$= (1.039 - j0.00518) \left[1.039 + j0.00518 - 1.05 \right] (-10 - j30)$

$= 0.27719 + j0.2876$

Total Line loss $\Rightarrow S_{13} + S_{31}$

$\Rightarrow -0.00081 - j\,0.0044$

Newton Rapson Method : (N-R) Method

① It is a powerful method of solving non linear algebraic equations.

② It works faster and is assured to convergence in most cases as compared to GS Method.

③ It is in deep the practical method of load flow solution of large power networks

④ Convergence can be considerable and speeded up by performing the first iteration through the GS Method. and the values so obtained is used for starting NR iteration

⑤ NR Method is an iterative procedure based on an initial estimate of the unknown variable and the use of Tailor series expansion and partial derivatives.

Case 1: NR Method for Single Dimension Case

* Let the Single Dimensional non-linear equation is expressed as

$$f(x) = y$$

Where x is unknown variable
y is specified quantity

* Let the intial guess be x^0 and Δx^0 is small deviation from correct solution

$$\therefore f(x^0+\Delta x^0) = y$$

* By Tailor Series expansion, around the operating point x^0 gives

$$f(x^0) + \frac{\Delta x^0}{1!} f'(x^0) + \frac{(\Delta x^0)^2}{2!} f''(x^0) + \cdots = y$$

Where f' & f'' are the first and second order derivatives of 'f' with respect to x.

As the error Δx^0 is very small the higher order terms can be neglected, retaining only the linear terms and is given as

$$f(x^0) + \Delta x^0 f'(x^0) = y$$

$$\Delta x^0 f'(x^0) = y - f(x^0) = \Delta y^0$$

Let $\Delta y^0 \simeq y - f(x^0)$

$$\therefore \Delta x^0 = \frac{\Delta y^0}{f'(x^0)}$$

$$\Delta x^0 = \frac{y - f(x^0)}{f'(x^0)}$$

* Then an improved estimate (which is considered as x') is obtained by ~~obtained~~ adding Δx^0 to the initial estimate x^0.

Thus $x' = x^0 + \Delta x^0$

$$= x^0 + \frac{y - f(x^0)}{f'(x^0)}$$

* f(x) is expanded around x' and an improved estimate is obtained in similar manner and in general for k^{th} iteration

$$\Delta y^k = y - f(x^k)$$

$$\Delta x^k = \frac{y - f(x^k)}{f'(x^k)}$$

$\therefore$ Now $x^{K+1} = x^K + \Delta x^K$

$$\boxed{x^{K+1} = x^K + \frac{y - f(x^K)}{f'(x^K)}}$$

* The iterative process is continued till the function $f(x)$ converges with in specified tolarence.

Case 2:- NR Method for n dimension case

* Let the non-linear ~~exp~~ equation can be expressed in matrix form as

$$F(X) = Y$$

$$f_i(x_1, x_2 \cdots x_n) = y_i; \quad i = 1, 2, \cdots n$$

* Let the initial estimate of solution vector is $x_1^0, x_2^0 \cdots x_n^0$.

* Let $\Delta x_1^0, \Delta x_2^0 \cdots \Delta x_n^0$ be the corrections which on being added to the initial guess gives the actual solution.

$\therefore$ Now

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \cdots x_n^0 + \Delta x_n^0) = y_i \quad ; i = 1, 2, 3, \cdots n$$

* By Tailor series expansion

$$f_i(x_1^0, x_2^0, \cdots x_n^0) + \left[\left(\frac{df_i}{dx_1}\right)^0 \Delta x_1^0 + \left(\frac{df_i}{dx_2}\right)^0 \Delta x_2^0 + \cdots + \left(\frac{df_i}{dx_n}\right)^0 \Delta x_n^0\right] = y_i$$

$\forall i = 1, 2 \cdots n$

Since we are neglecting higher order terms as Δx_0 are very small & In Matrix form the equation can be written as.

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0 \dots x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0 \dots x_n^0) \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \dots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \vdots & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \dots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$$

$\Rightarrow$ $B = JC$

$\Delta y^0 = J^0 \Delta x^0$

where J^0 = Jacobian for the function f_i

Δx^0 = change vector Δx_i and are obtained from $\Delta x^0 = [J^0]^{-1} \Delta y^0$

The new values of x_i are computed from

$x_i^1 = x_i^0 + \Delta x_i^0$

* These being a set of linear - Algebraic equations can be solved by Triangularisation and back substit-ution.

In general for $k+1$ iterations

$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$

$x^{(k+1)} = x^{(k)} + [J^k]^{-1} \Delta y^{(r)}$

* The newton raphson Method has unique feature in possesing quadratic convergence characteristics.

Power flow solution by NR Method :-

* P_i and Q_i are either in polar quardinates (or) in rectangular quardinates.

① Polar form:

P_i and Q_i at the bus in polar form can be expressed as.

$$P_i = |V_i| \sum_{K=1}^{n} |y_{iK}||V_K| \cos(\theta_{iK} - \delta_i + \delta_K) \quad ; i = 1,2 \ldots n$$

$$Q_i = -|V_i| \sum_{K=1}^{n} |y_{iK}||V_K| \sin(\theta_{iK} - \delta_i + \delta_K) \quad ; i = 1,2 \ldots n$$

Now

$$P_i - |V_i| \sum_{K=1}^{n} |y_{iK}||V_K| \cos(\theta_{iK} - \delta_i + \delta_K) = 0$$

$$Q_i - \left[-|V_i| \sum_{K=1}^{n} |y_{iK}||V_K| \sin(\theta_{iK} - \delta_i + \delta_K)\right] = 0$$

* Let the number of PQ buses be m_1 and PV buses be m_2 among the n buses.

$$n = m_1 + m_2 + 1$$

where 1 is slack bus

Here to find m_1 unknown Bus voltages $|V|$ at PQ buses & $m_1 + m_2$ of unknown Bus voltage angles δ at PQ & PV buses & m_2 unknown Q of PV buses.

Let 'x' be the vector of all unknown voltage δ and 'y' be the vector of all specified variables

$$X = \begin{bmatrix} \left.\begin{matrix} \delta \\ |V| \end{matrix}\right\} \text{ on each PQ bus} \\ \\ \left.\begin{matrix} \delta \\ Q \end{matrix}\right\} \text{ on each PV bus} \end{bmatrix}$$

$$y = \begin{bmatrix} \left.\begin{matrix} V_1 \\ \delta_1 \end{matrix}\right\} \text{ on slack bus} \\ \left.\begin{matrix} P_i^{sp} \\ Q_i^{sp} \end{matrix}\right\} \text{ On PQ bus} \\ \left.\begin{matrix} P_i^{sp} \\ |V_i|^{sp} \end{matrix}\right\} \text{ on PV bus} \end{bmatrix}$$

$\Rightarrow F(X, y) = 0$

$$\Rightarrow F(X,y) = \begin{bmatrix} P_i - \left[|V_i| \sum_{k=1}^{n} |y_{ik}| |V_k| \cos(\theta_{ik} - \delta_i + \delta_k)\right] \text{ for each PQ \& PV with } P_i = P_i^{sp} \\ Q_i - \left[-|V_i| \sum_{k=1}^{n} |y_{ik}| |V_k| \sin(\theta_{ik} - \delta_i + \delta_k)\right] \text{ for each PQ bus } Q_i = Q_i^{sp} \end{bmatrix} = 0$$

* The Above equation can be written in the form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = 0$$

$$\Delta P_i = P_i^{Spec} - P_i^{Calcu}$$

$$\Delta Q_i = Q_i^{Spec} - Q_i^{Calcu}$$

* For 'n' dimensional case the NR iterates for the power flow studies takes the form as

$$\begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix} = \begin{bmatrix} J_1^k & J_2^k \\ J_3^k & J_4^k \end{bmatrix} \begin{bmatrix} \Delta \delta^k \\ \Delta |V|^k \end{bmatrix}$$

$$= [J] \begin{bmatrix} \Delta \delta^k \\ \Delta |V|^k \end{bmatrix}$$

* where $\Delta\delta$ is the subvector of incremental angles at PQ buses and PV buses

$\Delta |V|$ is the subvector of incremental voltage magnitudes at PQ bus.

J is Jacobian matrix of partial derivates given as

$$H = J_1 = \frac{\partial P}{\partial \delta}$$

$$N = J_2 = \frac{\partial P}{\partial |V|}$$

$$\mathcal{J} = J_3 = \frac{\partial Q}{\partial \delta}$$

$$L = J_4 = \frac{\partial Q}{\partial |V|}$$

* System having only PQ buses

* If Bus 1 is specified as slag bus and all other $n-1$ buses are PQ buses

$$\therefore \begin{bmatrix} \Delta P_2^k \\ \vdots \\ \Delta P_n^k \\ \Delta Q_2^k \\ \vdots \\ \Delta Q_n^k \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^k & \cdots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^k & \left(\frac{\partial P_2}{\partial |V_2|}\right)^k & \cdots & \left(\frac{\partial P_2}{\partial |V_n|}\right)^k \\ \vdots & & & \vdots & & \\ \left(\frac{\partial P_n}{\partial \delta_2}\right)^k & \cdots & \left(\frac{\partial P_n}{\partial \delta_n}\right)^k & \left(\frac{\partial P_n}{\partial |V_2|}\right)^k & \cdots & \left(\frac{\partial P_n}{\partial |V_n|}\right)^k \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^k & \cdots & \left(\frac{\partial Q_2}{\partial \delta_n}\right)^k & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^k & \cdots & \left(\frac{\partial Q_2}{\partial |V_n|}\right)^k \\ \vdots & & & \vdots & & \\ \left(\frac{\partial Q_n}{\partial \delta_2}\right)^k & \cdots & \left(\frac{\partial Q_n}{\partial \delta_n}\right)^k & \left(\frac{\partial Q_n}{\partial |V_2|}\right)^k & \cdots & \left(\frac{\partial Q_n}{\partial |V_n|}\right)^k \end{array}\right] \begin{bmatrix} \Delta \delta_2^k \\ \vdots \\ \Delta \delta_n^k \\ \Delta |V_2|^k \\ \vdots \\ \Delta |V_n|^k \end{bmatrix}$$

* The terms ΔP_i^k & ΔQ_i^k are known as the power residues.

$$P_i = \sum_{k=1}^{n} |V_i||Y_{ik}||V_k| \cos(\theta_{ik} + \delta_i - \delta_k)$$

$$\Delta P_i^k = P_i^{spe} - P_i^k$$

$$\Delta Q_i^k = Q_i^{spec} - Q_i^k \quad \forall i = 2,3,\ldots n$$

* In order to compute δ & $|V|$ the inverse of the matrix J has to be computed that is given as follows

$$\begin{bmatrix} \Delta \delta^k \\ \Delta |V|^k \end{bmatrix} = \begin{bmatrix} H^k & N^k \\ J^k & L^k \end{bmatrix}^{-1} \begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k||Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k)$$

$$\begin{bmatrix} \delta^{k+1} \\ |V|^{k+1} \end{bmatrix} = \begin{bmatrix} \delta^k \\ |V|^k \end{bmatrix} + \begin{bmatrix} \Delta \delta^k \\ \Delta |V|^k \end{bmatrix}$$

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i||V_k||Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad k \neq i$$

* The diagnol and half-diagnol elements of the Jacobian matrix are as follows.

$$J_1 \text{ or } H : \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{m \neq i \\ m=1}}^{n} |V_i||V_m||Y_{im}| \sin(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial P_i}{\partial \delta_m} = -|V_i||V_m||Y_{im}| \sin(\theta_{im} - \delta_i + \delta_m) \text{ for } m \neq i$$

* The diagnal & half diagnal elements of J_2 or N Jacobian elements J_2 or N:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{im}|\cos\theta_{ii} + \sum_{m \neq i} |V_m||Y_{im}|\cos(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial P_i}{\partial |V_m|} = |V_i||Y_{im}|\cos(\theta_{im} - \delta_i + \delta_m) \quad \text{for } m \neq i$$

* The diagnal & half diagnal elements of J_3 or J

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{m \neq i} |V_i||V_m||Y_{im}|\cos(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial Q_i}{\partial \delta_m} = -|V_i||V_m||Y_{im}|\cos(\theta_{im} - \delta_i + \delta_m) \quad \text{for } m \neq i$$

* The diagnal & half diagnal elements of J_4 or L

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{im}|\sin\theta_{ii} + \sum_{m \neq i} |V_m||Y_{im}|\sin(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial Q_i}{\partial |V_m|} = -|V_i||Y_{im}|\sin(\theta_{im} - \delta_i + \delta_m) \quad \text{for } m \neq i$$

* When both PQ & PV buses are present in the system if ith bus is PV bus Q_i is unspecified, V_i is specified i.e., $\Delta V_i = 0$

* ∴ The Equations involving $\Delta|V|$ & ΔQ and the corresponding elements of Jacobian matrix are elementated.

Algorithm for Power flow solution by NR Method in Polar form:

Step 1: Initialise NR iterative process by setting the iteration count k=0 and set the voltage magnitude $|V_i|^0$ equal to slack bus voltage or equal to 1.

* Set the bus voltage angles $\delta_i^0 = 0$.

- For PQ (or) load buses, set $\delta_i = 0$ and for PV buses also $\delta_i = 0$

Step 2: For load buses compute the real and reactive power and for PV buses compute real powers and then compute ΔP_i is for PV and PQ buses. ΔQ_i is for all PQ buses.

Step 3: Compute the elements of Jacobian matrix by computing the sub matrices.

Step 4: Solve the equations for computing the inverse of Jacobian matrix to compute $\Delta\delta^k$ & $\Delta|V_i|^k$.

Step 5: Compute the new estimates of Bus voltage magnitudes and the angle.

Step 6: Apply the following test for convergence

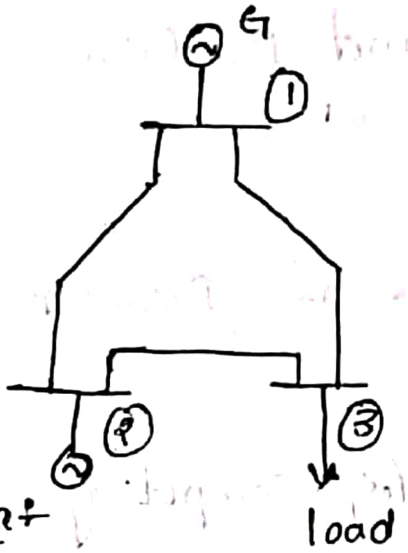
$$|\Delta P_i^k| = |P_i^{spec} - P_i^k| \leq \epsilon$$
$$|\Delta Q_i^k| = |Q_i^{spec} - Q_i^k| \leq \epsilon$$

Step 7: The power mismatch at each bus is used to specify the tolerance which is usually of the order of 0.01 PU for real and reactive powers.

Step 8: If the tolerance condition is satisfied, the solution of power flow solution is obtained. Otherwise repeat the steps from step 2.

Problem :-

① For the system shown in figure with Bus 1 as slack bus obtain the power flow solution after 1st iteration using polar co-ordinate form of NR Method.



Line data :-

Bus code P	Bus code Q	Line impedance (PU)	Half line charging admittance
1	2	j0.1	0
2	3	j0.2	0
3	1	j0.2	0

Bus data :-

Bus code	Generation	Load	|V|	Reactive power limits	Type of Bus

$$y_{11} = y_{12} + y_{13}$$
$$= \frac{1}{j0.1} + \frac{1}{j0.2}$$
$$= -j15$$
$$= 15\angle -90^\circ$$

$$y_{12} = y_{21} = \frac{-1}{j0.1}$$
$$= 10\angle 90$$

$$y_{13} = y_{31} = \frac{-1}{j0.2}$$
$$= 5\angle 90^\circ$$

$$y_{22} = y_{21} + y_{23}$$
$$= \frac{1}{j0.1} + \frac{1}{j0.2}$$
$$= 15\angle -90^\circ$$

$$y_{23} = y_{32} = \frac{-1}{j0.2}$$
$$= 5\angle 90^\circ$$

$$y_{33} = y_{31} + y_{32}$$
$$= \frac{1}{j0.2} + \frac{1}{j0.2}$$
$$= 10\angle -90^\circ$$

$$Y_{BUS} = \begin{bmatrix} 15\angle -90^\circ & 10\angle 90^\circ & 5\angle 90^\circ \\ 10\angle 90^\circ & 15\angle -90^\circ & 5\angle 90^\circ \\ 5\angle 90^\circ & 5\angle 90^\circ & 10\angle -90^\circ \end{bmatrix}$$

* Given

$V_1 = 1\angle 0^\circ$ pu

$V_2 = 1.1\angle 0^\circ$ pu

$V_3 = 1\angle 0^\circ$ pu [Assume is flat voltage (or) slack bus voltage]

* Since Bus (2) is PV bus

P_2, P_3, Q_3 are to be calculated. at 2nd & 3rd buses

$$P_i = \sum_{k=1}^{n} |V_i||y_{ik}||V_k| \cos(\theta_{ik} - \delta_i + \delta_k)$$

$$Q_i = -\sum_{k=1}^{n} |V_i||y_{ik}||V_k| \sin(\theta_{ik} - \delta_i + \delta_k)$$

∴ now $i = 2$

$$P_2 = \sum_{k=1}^{n=3} |V_2||y_{2k}||V_k| \cos(\theta_{2k} - \delta_2 + \delta_k)$$

$$P_2 = |V_2||y_{21}||V_1| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |y_{22}| \cos(\theta_{22})$$
$$+ |V_2||y_{23}||V_3| \cos(\theta_{23} + \delta_2 - \delta_3)$$

δ_2^0 & $\delta_3^0 = 0$

$P_2^0 = 1.1$

$$P_3 = |V_3||y_{31}||V_1| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||y_{32}||V_2| \cos(\theta_{32} - \delta_3 + \delta_2)$$
$$+ |V_3|^2 |y_{33}| \cos(\theta_{33})$$

$$\Rightarrow P_2^0 = 1.1 \times 10 \times 1 \times \cos(90 - 0 + 0) + 1.1^2 \times 15 \times \cos(90) + 1.1 \times 5 \times 1 \times \cos(90)$$
$$= 0$$

$\Rightarrow P_3^0 = 0$

$i = 3$

$$\Rightarrow Q_3^0 = -|V_3||y_{31}||V_1| \sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||y_{32}||V_2| \sin(\theta_{32} - \delta_3 + \delta_2)$$
$$- |V_3|^2 |y_{33}| \sin(\theta_{33})$$

$$= -1 \times 5 \times 1 \times \sin(90) - 1 \times 5 \times 1.1 \times \sin(90) - 1^2 \times 10 \times \sin(90)$$

$$Q_3^0 = -0.5 \text{ PU}$$

Now $\Delta P_2 = P_2^{spec} - P_2^0 = 5 - 0 = 5$ PU

$\Delta P_3 = P_3^{spec} - P_3^0 = -3.5 - 0 = -3.5$ PU

$$\left[\begin{aligned} P_3^{spec} &= P_{3gen} - P_{3demand} \\ &= 0 - 3.5 \\ &= -3.5 \text{ pu} \end{aligned} \right]$$

$\Delta Q_3 = Q_3^{spec} - Q_3^0 = -0.5 + 0.5 = 0$ PU

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_3| \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = \sum_{\substack{m \neq 2 \\ m=1}}^{3} \Big[|V_2| |V_1| |y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2| |V_2| |y_{22}| \sin(\theta_{22}) + |V_2| |y_{23}| |V_3| \sin(\theta_{23} - \delta_2 + \delta_3) \Big]$$

$= 1.1 \times 1 \times 10 \times \sin(+90) + 1.1^2 \times \sin(-90) + 1.1 \times 5 \times 1 \sin(90)$

$= \cancel{15.29} = 16.5$ PU

$$\frac{\partial P_2}{\partial \delta_3} = -|V_i| |V_m| |y_{im}| \sin(\theta_{ik} - \delta_i + \delta_m)$$

$= -|V_2| |V_3| |y_{12}| \sin(\theta_{12} - \delta_2 + \delta_3)$

$= -1.1 \times \cancel{1} \times 15 \times \sin(-90 - 0 + 0)$

$= \cancel{18.15}\ -5.5$

$$\frac{\partial P_2}{\partial |V_3|} = 0$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3| |V_2| |y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$= -1 \times 1.1 \times 5 \sin(90)$

$= -5.5$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3| |V_1| |y_{31}| \sin(\theta_{31}) + |V_3| |V_2| |y_{32}| \sin(\theta_{32}) + |V_3|^2 y_{33} \sin(\theta_{33})$$

$= 1 \times 1 \times 5 \sin(90°) + 1 \times 1.1 \times 5 \sin(90°) + 1^2 \times \sin(-90°) \times 10$

$= 10.5$ PU

$$\frac{\partial P_3}{\partial |V_3|} = 0$$

$$\frac{\partial Q_3}{\partial \delta_2} = 0 \qquad \frac{\partial Q_3}{\partial V_3} = 9.5$$

$$\frac{\partial Q_3}{\partial \delta_3} = 0$$

$$\begin{bmatrix} 5 \\ -3.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 16.5 & -5.5 & 0 \\ -5.5 & 10.5 & 0 \\ 0 & 0 & 9.5 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ |V_3| \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ |V_3| \end{bmatrix} = \begin{bmatrix} 16.5 & -5.5 & 0 \\ -5.5 & 10.5 & 0 \\ 0 & 0 & 9.5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ -3.5 \\ 0 \end{bmatrix}$$

$|A| = 1358.5$

$$\begin{bmatrix} 10.5 & 0 & -5.5 & 16.5 \\ 0 & 9.5 & 0 & 0 \\ -5.5 & 0 & 16.5 & -5.5 \\ 10.5 & 0 & -5.5 & 10.5 \end{bmatrix} = \begin{bmatrix} 99.75 & 52.25 & 0 \\ 52.25 & 156.75 & 0 \\ 0 & 0 & 143 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0734 & 0.0384 & 0 \\ 0.0384 & 0.1153 & 0 \\ 0 & 0 & 0.105 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ |V_3| \end{bmatrix} = \begin{bmatrix} 0.2326 \\ -0.2154 \\ 0 \end{bmatrix}$$

21.28

$\delta_2' = \delta_2^0 + \Delta\delta_2^0 = 13.32^\circ$

$\delta_3' = \delta_3^0 + \Delta\delta_3 = -12.12^\circ$

$V_3' = V_3^0 + \Delta|V_3| = 1.0\,pu$

$\theta_{21} = \frac{1}{90} = 90$

$$Q_2 = -|V_2||y_{21}||V_1|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2|y_{22}|\sin(\theta_{22})$$
$$- |V_2||y_{23}||V_3|\sin(\theta_{23} - \delta_2 + \delta_3)$$
$$= -1.1 \times 40 \times 1 \times \sin(90 - 13.32 + 0) - 1.1^2 \times 15 \sin(90^\circ)$$
$$- 1.1 \times 5 \times 1 \times \sin(90^\circ - 13.32 + 12.12)$$

$= \cancel{9.479}$

$= -33.81$

General Expression

N-R Rectangular co-ordinates

* General expression for power

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{k=1}^{n} y_{ik} V_k \rightarrow (1)$$

* let $V_i = e_i + jf_i$

where e_i & f_i are real and imaginary components of the bus voltage V_i and ∴ V_i^* is

$$V_i^* = e_i - jf_i \quad -(2)$$

* let $V_k = e_k + jf_k \quad -(3)$

* $y_{ik} = G_{ik} + jB_{ik} \quad -(4)$

where G_{ik} & B_{ik} are the conductance and suseptance.

* substitute (2), (3) (4) in eq(1)

$$P_i - jQ_i = (e_i - jf_i) \sum_{k=1}^{n} (G_{ik} - jB_{ik})(e_k + jf_k) \quad -(5)$$

* Separating real & imaginary parts we get

$$P_i = \sum_{k=1}^{n} e_i(e_k G_{ik} + f_k B_{ik}) + f_i(f_k G_{ik} - e_k B_{ik}) \quad -(6)$$

$$Q_i = \sum_{k=1}^{n} f_i(e_k G_{ik} + f_k B_{ik}) - e_i(f_k G_{ik} - e_k B_{ik}) \quad -(7)$$

Also $V_i^2 = e_i^2 + f_i^2 \quad -(8)$

* Separating for ith bus the power equations (6) & (7) becomes

$$P_i = e_i(e_i G_{ii} + f_i B_{ii}) + f_i(f_i G_{ii} - e_i B_{ii}) + \sum_{\substack{k=1 \\ k \neq i}}^{n} e_i(e_k G_{ik} + f_k B_{ik}) + f_i(f_k G_{ik} - e_i B_{ik}) \quad -(9)$$

$$Q_i = f_i(e_i G_{ii} + f_i B_{ii}) - e_i(f_i G_{ii} - e_i B_{ii}) +$$

* The above equations results in a system of non-linear Algebraic equations 1 for P_i and other for Q_i at each bus excluding slack bus i.e., Bus 1 where V and δ are specified and remains fixed through out the total no.of equations to be solved for n-Bus system will be (2n-1) equations

* With the help of Newton Raphson method the above non-linear algebraic equations of power is transfered into a set of linear algebraic equations inter related to error in power with the change in real and reactive powers of components of Bus Voltages with the help of Jacobian matrix.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \frac{\partial P_2}{\partial f_n} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \frac{\partial P_n}{\partial f_n} \\ \hline \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \cdots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \cdots & \frac{\partial Q_n}{\partial f_n} \end{array}\right] \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix} \quad — (11)$$

* In short form we can write eq (11) as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad — (12)$$

* Incase the system contains all type of buses [PQ & PV] the set of equations can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V_i^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad — (13)$$

* The elements of Jacobian matrix ~~are~~ can be derived from three power flow equations (8) & (9) & (10)

$i = 2, 3 \cdots n$

J_1: Half diagonal elements of J_1

$$\frac{dP_i}{de_K} = e_i G_{iK} - f_i B_{iK} \quad \forall K \neq i \quad -(14)$$

Diagonal element of J

$$\frac{dP_i}{de_i} = 2e_i G_{ii} + \cancel{f_i B_{ii}} - \cancel{f_i B_{ii}} + \sum_{\substack{K=1 \\ K\neq i}}^{n} (e_K G_{iK} + f_K B_{iK}) \quad -(15)$$

$$\frac{dP_i}{de_i} = 2e_i G_{ii} + \sum_{\substack{K=1 \\ K\neq i}}^{n} (e_K G_{iK} + f_K B_{iK}) \quad -(15)$$

J_2:

Half diagonal elements

$$\frac{dP_i}{df_K} = e_i B_{iK} + f_i G_{iK} \quad \forall K \neq i \quad -(16)$$

Diagonal elements

$$\frac{dP_i}{df_i} = 2f_i G_{ii} + \sum_{\substack{K=1 \\ K\neq i}}^{n} (f_K G_{iK} - e_K B_{iK}) \quad -(17)$$

J_3:

Half diagonal elements

$$\frac{dQ_i}{de_K} = e_i B_{iK} + f_i G_{iK} \quad \forall K \neq i \quad -(18)$$

Diagonal elements.

$$\frac{dQ_i}{de_i} = 2e_i B_{ii} + \sum_{\substack{K=1 \\ K\neq i}}^{n} (f_K G_{iK} - e_K B_{iK}) \quad -(19)$$

J_4:

Half diagonal element

$$\frac{dQ_i}{df_K} = -e_i G_{iK} + f_i B_{ii} \quad \forall K \neq i \quad -(20)$$

Diagonal elements

$$\frac{dQ_i}{df_i} = 2f_i B_{ii} + \sum_{\substack{K=1 \\ K\neq i}}^{n} (e_K G_{iK} + f_K B_{iK}) \quad -(21)$$

jstem of non-

J5 :

Half diagonal elements

$$\frac{dV_i^2}{de_K} = 0 \quad \forall\, K \neq i \quad — (22)$$

Diagonal elements

$$\frac{dV_i^2}{de_i} = 2e_i \quad — (23)$$

J6 :

Half diagonal elements

$$\frac{dV_i^2}{df_K} = 0 \quad , K \neq i \quad — (24)$$

Diagonal elements

$$\frac{dV_i^2}{df_K} = 2f_i \quad — (25)$$

Algorithm for NR Method : [Rectangular form]

Step-1 :

* For the load buses where P and Q are given we assume the bus voltage magnitudes and phase angles for all the buses except the slack bus where V and δ are specified. Normally we have the flat voltage start i.e., we set the assumed bus voltage magnitude and its phase angles i.e., the real and imaginary components of bus voltage (e and f) equal to to slack bus quantities.

Step-2 :

* Substituting this assumed bus voltages e and f in P_i and Q_i we calculate the real and reactive components of power i.e., P_i and Q_i for all the buses

$i = 2, 3 \cdots n$ because first bus is slack bus

Step 3 :-

* Since P_i & Q_i for any bus 'i' i.e., specified, the error in power will be ΔP_i^r is

$$\Delta P_i^r = P_i^{spec} - P_i^{calculated}$$

$$\Delta P_i^r = P_i^{spec} - P_i^r$$

$$\Delta Q_i^r = Q_i^{spec} - Q_i^r$$

* P_i^r & Q_i^r are power calculated with the latest value of bus voltages at any iteration 'r'.

Step 4 :-

* Then the elements of Jacobian matrix J_1, J_2, J_3 & J_4 are determined with the latest bus voltage and calculated power equation P_i and Q_i

Step 5 :-

* After this, the linear set of equations is solved by the iterative techinque to determine the voltage correction i.e., Δe_i & Δf_i at any bus 'i'.

Step 6 :-

* This value of voltage correction is used to determine the new estimated of bus voltages as follows.

$$e_i^{r+1} = e_i^r + \Delta e_i^r$$

$$f_i^{r+1} = f_i^r + \Delta f_i^r$$

* Now ~~the~~ this new estimate of bus voltage i.e., e_i^{r+1} & f_i^{r+1} is used in P_i and Q_i equations to ... error in power and thus entire algo

- When starting from step(3) or hybrid above is repeated.

Step 11:

* In each iteration the elements of Jacobian are computed as these depends upon latest voltage estimate and calculated power.
* The process is continued till the error in power becomes very small $\Delta P < E$ & $\Delta Q < E$ where E is very small number.

Decoupled & Fast Decoupled Load flow Studies:-

* The characteristics of any practical power transmission system operating in steady state is that the change in real power from the specified value at a bus is more dependent on the changes in voltage angles at various buses than the changes in v/g magnitudes. And change in reactive power from the specified value at a bus is more dependent on the changes in voltage magnitude at the various buses than the changes in voltage angle.
* Fast Decoupled method is a very fast and efficient method of obtaining power flow solution problem.
* This is actually an extension of N-R Method formulated in polar co-ordinates with certain approximations which results into a fast algorithm for power flow solution.
* This method exploits the property of the power system where in megawatt flow voltage angle and MVAR flow voltage magnitude are loosely coupled.

* In other words a small change in the magnitude of bus voltage doesn't affect the real power flow at the bus & similarly change in phase angle of bus voltage doesn't affect reactive power flow because of this loose physical interaction between MW & MVAR flows in power system the MW-δ & MVAR - voltage magnitude.

* Interactions (or) calculation can be decoupled which results in very simple, fast & reliable algorithm.
* Power flow equation in N-R method in polar co-ordinates is given by

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta V}{V} \end{bmatrix} \rightarrow (1)$$

* $\therefore$ changes in real power ΔP are less sensitive to voltage magnitude ΔV & changes in the reactive power ΔQ are less sensitive to power angle $\Delta\delta$ the equation (1) can be reduced as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta V}{V} \end{bmatrix} \rightarrow (2)$$

* The decoupled equation can be represented as.

$$\Delta P = H\,\Delta\delta \rightarrow (3)$$
$$\Delta Q = L\,\frac{\Delta V}{V} \rightarrow (4)$$

* The next step in deriving the algorithm is to make suitable assumptions in deriving the expression for H & L.

~~HRA~~ i≠k

* Half diagonal elements

$$H_{ik} = \frac{dP_i}{d\delta_k} = V_i V_k\, y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k)$$
$$= V_i V_k\, y_{ik} \left(\sin\theta_{ik} \cos(\delta_i - \delta_k) + \cos\theta_{ik} \sin(\delta_i - \delta_k) \right)$$

$$= V_i V_K (Y_{iK} \sin \theta_{iK} \cos(\delta_i - \delta_K) + Y_{iK} \cos \theta_{iK} \sin(\delta_i - \delta_K))$$
$$= V_i V_K [-B_{iK} \cos(\delta_i - \delta_K) + G_{iK} \sin(\delta_i - \delta_K)] \rightarrow (5)$$

* $L_{iK} = \frac{\partial Q_i}{\partial V_K} V_K = V_i V_K Y_{iK} \sin(\theta_{iK} + \delta_i - \delta_K)$
$$= V_i V_K [G_{iK} \sin(\delta_i - \delta_K) - B_{iK} \cos(\delta_i - \delta_K)] \rightarrow (6)$$

* $H_{iK} = L_{iK} = V_i V_K [-G_{iK} (\sin(\delta_i - \delta_K)) - B_{iK} \cos(\delta_i - \delta_K)]$

$$H_{ii} = \frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{K=1 \\ K \neq i}}^{n} V_i V_K Y_{iK} \sin(\theta_{iK} + \delta_i - \delta_K) + V_i V_i Y_{ii} \sin(\theta_{ii} + \delta_i - \delta_i)$$

$$= -Q_i + V_i V_i Y_{ii} \sin$$

$$= -Q_i - V_i^2 B_{ii} \rightarrow (7)$$

* $L_{ii} = \frac{\partial Q_i V_i}{\partial V_i} = 2V_i^2 Y_{ii} \sin \theta_{ii} + \sum_{\substack{K=1 \\ K \neq i}}^{n} V_i V_K Y_{iK} \sin(\theta_{iK} + \delta_i - \delta_K)$

$$= 2V_i^2 Y_{ii} \sin \theta_{ii} + Q_{ii} - V_i^2 Y_{ii} \sin \theta_i$$

$$= Q_i + V_i^2 Y_{ii} \sin \theta_i$$

$$= Q_i - V_i^2 B_{ii}$$

* In the case of FDL method the following approxim-ations are made for evaluting jacobian elements

1. $\cos(\delta_i - \delta_K) \simeq 1$
2. $G_{iK} (\sin(\delta_i - \delta_K)) \leq B_{iK}$
3. $Q_i << B_{ii} V_i^2$

* With the above assumptions the jacobian elements become

$H_{iK} = L_{iK} = -V_i V_K B_{iK}$ for $K \neq i$

$H_{ii} = L_{ii} = -V_i^2 B_{ii}$

With these jacobian elements equations of ΔP & ΔQ becomes.

$$\Delta P_i = H\Delta\delta = V_i V_K B_{ik}' \Delta\delta_K$$

$$\Delta Q_i = L\frac{\Delta V}{V} = V_i V_K B_{ik}'' \frac{\Delta V}{V}$$

Where B_{ik}', B_{ik}'' are elements of B_{ik} matrix.

* Further decoupling & the final algorithm for fast decoupled power flow studies are obtained from

(i) Omitting from V' the representation of those network elements that affect MVAR flows i.e., shunt reactances, off nominal in phase transformer tapps.

(ii) Omitting from V'' the angles shifting affects of phase shifters.

(iii) diving ΔP_i; ΔQ_i & Assuming $V_K = 1.0$ PU and also negleting series resistance in calculating the elements of B'.

* Now with the above assumptions ΔP_i & ΔQ_i for power flow studies becomes.

$$\frac{\Delta P_i}{V_i} = B'\Delta\delta$$

$$\frac{\Delta Q_i}{V_i} = B''\Delta V$$

*

Derivation of D.c power flow:-

* The power flow equations are given as

$$P_p = \sum_{q=1}^{n} |V_p||V_q|\left[G_{pq}\cos(\delta_p-\delta_q) + B_{pq}\sin(\delta_p-\delta_p)\right]$$

$$Q_p = \sum_{q=1}^{n} |V_p||V_q|\left[G_{pq}\cos(\delta_p-\delta_q) - B_{pq}\cos(\delta_p-\delta_q)\right]$$

* We can derive the equations for d.c power flow using following simplifing approximations.

(i) Approximate the T/m resistance to zero. The resistance of T/m circuits is significantly less than reactance where the x/R ratio is b/n

5 & 10

* For a T/m circuit with impedance $Z = r + jx$.

conductance $g = \frac{r}{r^2+x^2}$ & $b = \frac{-x}{r^2+x^2}$

* If r is very small hence $g=0$, $b = \frac{-1}{x}$ Now if $g=0$, the real part of all of the Ybus elements are also zero, Hence now

$$P_p = \sum_{q=1}^{n} |V_p||V_q| [B_{pq} \sin(\delta_p - \delta_q)]$$

$$Q_p = \sum_{q=1}^{n} |V_p||V_q| [-B_{pq} \cos(\delta_p - \delta_q)]$$

(ii) Approximate the cosine term to zero & sine terms to the radian angle the difference in the angle of voltage phasor at 2 buses p & Q i.e., ΔP & ΔQ is less than 10-15°.

*And it is very rare to see where the angular separation exceeds 30°.

* ∴ we can say that the angular separation across T/m circuit is small

* ∴ $\delta = (\delta_p - \delta_q)$ which is small

* ∴ Cosine function approaches to 1 & sine of the angle is approximately equal to angle itself.

* ∴ $P_p = \sum_{q=1}^{n} |V_p||V_q| \cdot B_{pq} (\delta_p - \delta_p)$

$$Q_p = \sum_{q=1}^{n} |V_p||V_q| (-B_{pq})$$

ii) Approximate the product of voltages of to 1

$$P_p = \sum_{q=1}^{n} B_{pq} (\delta_p - \delta_q)$$

$$Q_p = \sum_{q=1}^{n} (-B_{pq})$$

DC power flow is commonly used in optimal power flow & economic despatch problem.

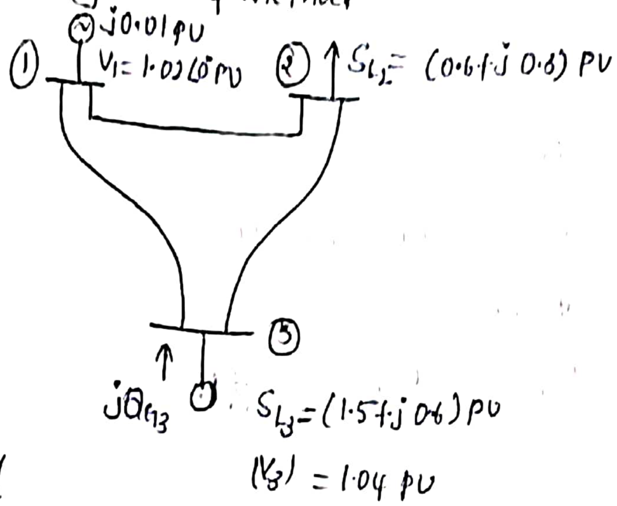
Comparision of Load flow studies :-

S.no	Parameter for Comparision	GS Method	NR Method	FDLF Method
1	Co-ordinates	Works well with the rectangular co-ordinates.	The Polar co-ordinates is prefer as rectangular coordinates occupies more memory	It works with polar co-ordinates.
2	Arithmateic operations	Least in number to complete in one iteration.	The elements of Jacobian is to be calculated in each iteration	Less than NR method.
3	Time	requires less time for iteration but increases with number of buses increasing.	Time per iteration is seven times of Gauss seidel and increases in no. of buses	Less time when compared to NR and GS Methods.
4	Convergence	It is Linear	It is quadratic	It is Geometric
5	No. of Iterations	Larger number increases with increase in buses	very less [3 to 5 only] for large system and is practically constant	only [2 to 5] iterations for practical accuracy
6	slack bus selection	Choice of slack bus affects convergence adversly	Sensitivity to this is minimise	Moderate

7	Accelaration factor	Less Accurate	More Accurate	Moderate
8	Memory	Less memory because of sparcity of matrix (containing more zero's)	Large memory even with compact storage scheme	Only 60% of memory when compared with NR
9	Usage / Application	Small size systems	Large systems Ill conditioned systems optimal load flow studies	Optimization studies Multiple load flow studies Contingency evoluation for security assesment & enhancement
10	Programming Logic	GS is easy	It is very difficult	It is moderate
11	Reliability	Reliable only for small systems.	Reliable even for large systems	More reliable than NR method.

problem 8

Q For the power system shown in figure each line has series impedence of (0.03 + j 0.07) PU & shunt admittance of j0.01 PU. The specified values of buses are shown in figure. Determine the elements of Jacobian matrix by rectangular coordinates formation by N-R method.



Rectangular solution

① bus is slack bus $\Rightarrow V_1 = 1.02\angle 0°$

② bus is PQ bus $\Rightarrow P_2 - jQ_2 = -0.6 + j0.3$

③ bus is PV bus $\Rightarrow |V_3| = 1.04$ PU

$P_3 = -1.5$ PU

Step 1: Formation of Y_{BUS}

Self admittance $Y_{PP} = 2\left[\frac{1}{0.03 + j0.07} + j\frac{0.01}{2}\right]$

$= (10.344 - j24.128)$ PU

Mutual admittance $Y_{PQ} = \frac{-1}{0.03 + j0.07}$

$= (-5.17 + j12.06)$ PU

$Y_{BUS} = G - jB = \begin{bmatrix} Y_{PP} & Y_{PQ} & Y_{PQ} \\ Y_{PQ} & Y_{PP} & Y_{PQ} \\ Y_{PQ} & Y_{PQ} & Y_{PP} \end{bmatrix} = \begin{bmatrix} 10.34 - j24.12 & & \\ & & \\ & & \end{bmatrix}$

$$
Y_{BUS} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 10.34-j24.12 & -5.17+j12.069 & -5.17+j12.069 \\ -5.17+j12.069 & 10.34-j24.12 & -5.17+j12.069 \\ -5.17+j12.069 & -5.17+j12.069 & 10.34-j24.12 \end{bmatrix}
$$

$G_{11} = G_{22} = G_{33} = 10.34$

$G_{12} = G_{21} = G_{13} = G_{31} = -5.17 = G_{23} = G_{32}$

$B_{11} = B_{22} = B_{33} = 24.12$

$B_{12} = B_{21} = B_{13} = B_{31} = -12.069 = B_{23} = B_{32}$

Step 2:

Assume $V_2 = 1+j0 = e_2 + jf_2$; $\delta_2 = 0$

Specified $V_1 = 1.02 + j0$

$V_3 = 1.04 + j0.0$ pu $= e_3 + jf_3$

Step 3:

Performance (or) Jacobian Matrix

$$
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} \\ \frac{\partial V_3}{\partial e_2} & \frac{\partial V_3}{\partial e_3} & \frac{\partial V_3}{\partial f_2} & \frac{\partial V_3}{\partial f_3} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}
$$

Half diagonal elements of J_1

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_q G_{pq}, \quad q \neq p$$

$$\frac{\partial P_2}{\partial e_3} = e_2 G_{23} - f_3 G_{23}$$

$$= 1\times -5.17 - 0\times -5.17$$

$$= -5.17 \text{ pu}$$

$$\frac{\partial P_3}{\partial e_2} = e_3 G_{32} - \underbrace{f_2 G_{32}}_{0}$$

$$= 1.04\times -5.17$$

$$= -5.379 \text{ pu}$$

diagonal elements of J_1

$$\frac{\partial P_i}{\partial e_i} = 2e_i G_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} (e_k G_{ik} + f_k B_{ik})$$

$$\frac{\partial P_2}{\partial e_2} = 2e_2 G_{22} + \sum_{\substack{k\neq 1 \\ k\neq 2}}^{3} (e_1 G_{21} + \underbrace{f_1 B_{21}}_{0} + e_3 G_{23} + \underbrace{f_3 B_{23}}_{0})$$

$$= 2\times 1\times 10.34 + (1.02\times -5.17 + \cancel{0\times}\ 1.04\times -5.17)$$

$$= 10.0298$$

$$\frac{\partial P_3}{\partial e_3} = 2e_3 G_{33} + (e_1 G_{31} + \underbrace{f_1 B_{31}}_{0} + e_2 G_{32} + \underbrace{f_2 B_{32}}_{0})$$

$$= 2\times 1.04\times 10.34 + (1.02\times -5.17 + 1\times -5.17)$$

$$= 11.0633$$

Half diagonal elements of J_2

$$\frac{\partial P_i}{\partial f_k} = e_i B_{ik} + f_i G_{ik} \quad \forall k \neq i$$

$$\frac{\partial P_2}{\partial f_3} = e_2 B_{23} + f_2 G_{23}$$

$$= 1\times -12.069 + \underbrace{0\times -5.17}_{0}$$

$$= \cancel{-12.31}\ -12.069$$

$$\frac{dP_3}{df_2} = e_3 B_{32} + \underbrace{f_3 G_{32}}_{0}$$

$$= 1.04 \times -12.069$$

$$= -12.551$$

Diagonal elements of J_2

$$\frac{dP_i}{df_i} = 2 f_i G_{ii} + \sum_{\substack{K=1 \\ K \neq i}}^{n} (f_k G_{ik} - e_k B_{ik})$$

$$\frac{dP_2}{df_2} = 2 \underbrace{f_2 G_{22}}_{0} + (\underbrace{f_1 G_{21}}_{0} - e_1 B_{21} + \underbrace{f_3 G_{23}}_{0} - e_3 B_{23})$$

$$= \cancel{2} \; -(1.02 \times -12.069) + (1.04 \times -12.069)$$

$$= 24.862$$

$$\frac{dP_3}{df_3} = 2 \underbrace{f_3 G_{33}}_{0} + (\underbrace{f_1 G_{31}}_{0} - e_1 B_{31} + \underbrace{f_2 G_{32}}_{0} - e_2 B_{32})$$

$$= (-(1.02 \times -12.069) - (1 \times -12.069)$$

$$= 24.379$$

Half Diagonal elements of J_3..

$$\frac{dQ_i}{de_k} = e_i B_{ik} + \underline{f_i G_{ik}} \quad \forall \; k \neq i$$

$$\frac{dQ_2}{de_3} = e_2 B_{23} + \underbrace{f_2 G_{23}}_{0}$$

$$= 1 \times -12.069$$

$$= -12.069$$

$$\cancel{\frac{dQ_3}{de_1}} =$$

Diagonal elemts of J_3

$$\frac{dQ_i}{de_i} = 2 e_i B_{ii} - \sum_{\substack{K=1 \\ K \neq i}}^{n} (f_k G_{ik} - e_k B_{ik})$$

$$\frac{dQ_2}{de_2} = 2 e_2 B_{22} - [\underbrace{f_1 G_{21}}_{0} - e_1 B_{21} + \underbrace{f_3 G_{23}}_{0} - e_3 B_{23}]$$

$$= \underbrace{2 \times 1 \times \cancel{12.069}}_{24.12} - [-(1.02 \times -12.069) - (1.04 \times -12.069)]$$

$$= 23.374$$

Half diagonal elemts of J_4.

$$\frac{\partial Q_i}{\partial f_k} = -e_i G_{ik} + f_i B_{ii}$$

$$\frac{\partial Q_2}{\partial f_3} = -e_2 G_{23} + \underset{0}{\underline{f_2}} B_{22}$$

$$= -1 \times -5.17$$

$$= 5.17$$

Diagonal elemts of J_4

$$\frac{\partial Q_i}{\partial f_i} = -2 f_i B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} (e_k G_{ik} + f_k B_{ik})$$

$$\frac{\partial Q_2}{\partial f_2} = -2 \underset{0}{\underline{f_2}} B_{22} + \left[e_1 G_{21} + \underset{0}{\underline{f_1 B_{21}}} + e_3 G_{23} + \underset{0}{\underline{f_3 B_{23}}} \right]$$

$$= \left[1.02 \times -5.17 + 1.04 \times -5.17 \right]$$

$$= -10.65$$

Half diagonal elemts of J_5

$$\frac{\partial V_i^2}{\partial e_k} = 0 \quad \forall k = i$$

$$\frac{\partial V_3^2}{\partial e_2} = 0$$

Diagonal elements of J_5

$$\frac{\partial V_i^2}{\partial e_i} = 2e_i$$

$$\frac{\partial V_3^2}{\partial e_3} = 2e_3$$

$$= 2 \times 1.04 = 2.08$$

Half diagonal elemts of J_6

$$\frac{\partial V_i^2}{\partial f_k} = 0 \quad , k \neq i$$

$$\frac{\partial V_3^2}{\partial f_2} = 0$$

Diagonal elements of J_6

$$\frac{\partial V_i^2}{\partial f_i} = 2 f_i \Rightarrow \frac{\partial V_3^2}{\partial f_3} = 2 \times f_3 = 0$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta V \end{bmatrix} = \begin{bmatrix} 10.041 & -5.172 & 24.86 & -12.06 \\ -5.379 & 11.07016 & -12.5517 & 24.379 \\ 23.374 & -12.069 & -10.487 & 5.172 \\ 2.008 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

Polar:

Step 1:

$$Y_{BUS} = \begin{bmatrix} 10.34-j24.12 & -5.17+j12.069 & -5.17+j12.069 \\ -5.17+j12.069 & 10.34-j24.12 & -5.17+j12.069 \\ -5.17+j12.069 & -5.17+j12.069 & 10.34-j24.12 \end{bmatrix} = G-jB$$

$$= \begin{bmatrix} 26.242\angle -66.79 & 13.129\angle 113.18 & 13.129 \\ 13.129\angle 113.18 & 26.24\angle -66.79 & 13.129 \\ 13.129\angle 113.18 & 13.129\angle 113.18 & 26.24 \end{bmatrix}$$

$G_{11} = G_{22} = G_{33} = 10.34$

$B_{11} = B_{22} = B_{33} = 24.12$

$G_{12} = G_{21} = G_{31} = G_{13} = G_{23} = G_{32} = -5.17$

$B_{12} = B_{21} = B_{13} = B_{31} = B_{23} = B_{32} = -12.069$

$|Y_{11}| = |Y_{22}| = |Y_{33}| = 26.242$

$|Y_{12}| = |Y_{21}| = |Y_{31}| = |Y_{13}| = |Y_{23}| = |Y_{32}|$

$= 13.127$

$\theta_{11} = \theta_{22} = \theta_{33} = -66.79$

$\theta_{12} = \theta_{21} = \theta_{13} = \theta_{31} = \theta_{23} = \theta_{32} = 113.18$

Step 2:

Given $V_1 = 1.02\angle 0^\circ$

$V_2 = 1\angle 0^\circ$ (Assumed)

$V_3 = 1.04\angle 0^\circ$

Step 3:

Performance (or) Jacobian Matrix

$$\begin{bmatrix} \Delta P_2 \\ \cancel{\Delta P_3} \\ \Delta Q_2 \\ \cancel{\Delta V_3^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial} \end{bmatrix}$$

Since Bus (2) is PQ Bus & (3) is PV bus
we have to determine P_2, P_3, Q_3

$$P_i = \sum_{k=1}^{n} |V_i||y_{ik}||V_k| \cos(\theta_{ik} - \delta_i + \delta_k)$$

$$Q_i = -\sum_{k=1}^{n} |V_i||y_{ik}||V_k| \sin(\theta_{ik} - \delta_i + \delta_k)$$

$\therefore i = 2$

$$P_2 = \sum_{k=1}^{3} |V_2||y_{2k}||V_k| \cos(\theta_{2k} - \delta_2 + \delta_k)$$

As we have $V_1 = 1.02\angle 0^\circ$ $V_2 = 1\angle\delta$ $V_3 = 1.04\angle\delta$
$\delta_1^0 = 0$ $\delta_2^0 = 0$ $\delta_3^0 = 0$

$$P_2^0 = |V_2||y_{21}||V_1| \cos(\theta_{21} - \delta_2^0 + \delta_1^0) + |V_2||y_{22}||V_2| \cos(\theta_{22} - \delta_2^0 + \delta_2^0)$$
$$+ |V_2||y_{23}||V_3| \cos(\theta_{23} - \delta_2^0 + \delta_3^0)$$

$= 1 \times 13.129 \times 1.02 \times \cos(113.18 - 0 + 0) + 1 \times 26.042 \times 1 \times \cos(-66.79)$
$+ 1 \times 13.129 \times 1.04 \times \cos(113.18 - 0 + 0)$

$= -0.3037$

$P_2^0 = -0.3031 \simeq -0.31$

$$P_3^0 = |V_3||y_{31}||V_1| \cos(\theta_{31} - \delta_3^0 + \delta_1^0) + |V_3||y_{32}||V_2| \cos(\theta_{32} - \delta_3 + \delta_2)$$
$$+ |V_3|^2 |y_{33}| \cos(\theta_{33})$$

$= 1.04 \times 13.129 \times 1.02 \times \cos(113.18 - 0 + 0) + 1.04 \times 13.129 \times 1 \times \cos(113.18)$
$+ (1.04)^2 \times 26.042 \times \cos(-66.79)$

$P_3^0 = 0.3293$

$i = 3$

$$Q_3^0 = -\left[|V_3||y_{31}||V_1| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||y_{32}||V_2| \sin(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2 |y_{33}| \sin(\theta_{33})\right]$$

$= -[1.04 \times 13.129 \times 1.02 \times \sin(113.18) + 1.04 \times 13.129 \times 1 \sin(113.18)$
$+ 1.04^2 \times 26.242 \times \sin(-66.79)]$

$Q_3^0 = 0.73135$

$$Q_2^0 = -\left[|V_2||y_{21}||V_1| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |y_{22}| \sin(\theta_{22}) + |V_2||y_{23}||V_3| \sin(\theta_{23} - \delta_2 + \delta_3)\right]$$
$= 8.694$

Now given

$$P_2^{spec} = 0.6 \quad Q_2^{spec} = 0.3 \quad P_3^{spec} = 1.5$$

$\Delta P_2 = P_2^{spec} - P_2^0$

$= 0.6 - (-0.3037)$

$= 0.9037$

$\Delta Q_2 = Q_2^{spec} - Q_2^0$

$= \cancel{0.3} - 0.7313$

$= \cancel{-0.4313}$

$= 0.3 - 8.694$

$= -8.394$

$\Delta P_3 = P_3^{spec} - P_3^0$

$= 1.5 - 0.3293$

$= 1.1707$

Step 8:

Computation for Jacobian elements

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

Half diagonal elements of J_1

$$\frac{\partial P_i}{\partial \delta_m} = -|V_i||V_m||Y_{im}| \sin(\theta_{im} - \delta_i + \delta_m) \quad \forall m \neq i$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$= -1 \times 1.04 \times 13.129 \sin(113.18 - 0 + 0)$

$= -12.5518$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$= -1.04 \times 1 \times 13.129 \sin(113.18)$

$= -12.5518$

diagonal elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{m \neq i \\ m=1}}^{n} |V_i||V_m||Y_{im}| \sin(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23})$$

$= 1 \times 1.02 \times 13.129 \sin(113.18) + 1 \times 1.04 \times 13.129 \times \sin(113.18)$

$= 24.86$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} \ldots)$$

$$= 1.04 \times 1.02 \times 13.129 \sin(113.18) + 1.04 \times 1 \times 13.129 \times \sin(113.18)$$

$$= 25.354$$

Half diagonal elements of J_2

$$\frac{\partial P_i}{\partial |V_m|} = |V_i| |Y_{im}| \cos(\theta_{im} - \delta_i + \delta_m) \quad \forall m \neq i$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3| |Y_{32}| \cos\theta_{32}$$

$$= 1.04 \times 13.129 \times \cos(113.18)$$

$$= -5.374$$

diagonal element of J_2

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i| |Y_{ii}| \cos\theta_{ii} + \sum_{m \neq i} |V_m| |Y_{im}| \cos(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}| \cos\theta_{22} + |V_1||Y_{21}| \cos(\theta_{21}) + |V_3| |Y_{23}| \cos(\theta_{23})$$

$$= 2 \times 1 \times 26.242 \times \cos(-66.79) + 1.02 \times 13.129 \times \cos(113.18) + 1.04 \times 13.129 \times \cos(113.18)$$

$$= 28.707$$

Half diagonal elements of J_3

$$\frac{\partial Q_i}{\partial \delta_m} = -|V_i| |V_m| |Y_{im}| \cos(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2| |V_3| |Y_{23}| \cos(\theta_{23})$$

$$= -1 \times 1.04 \times 13.129 \times \cos(113.18)$$

$$= 5.374$$

diagonal element of J3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{m \neq i} |V_i| |V_m| |Y_{im}| \cos(\theta_{im} - \delta_i + \delta_m)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2| |V_1| |Y_{21}| \cos(\theta_{21}) + |V_2| |V_3| |Y_{23}| \cos(\theta_{23})$$

$$= 1 \times 1.02 \times 13.129 \times \cos(113.18) + 1 \times 1.04 \times 13.129 \times \cos(113.18)$$

$$= -10.645$$

diagonal element of J_4

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{m\neq i} |V_m||Y_{im}|\sin(\theta_{im}-\delta_i+\delta_m)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2||Y_{22}|\sin\theta_{22} - |V_1||Y_{21}|\sin(\theta_{21}) - |V_3||Y_{23}|\sin(\theta_{23})$$

$$= -2\times1\times26.24\times\sin(-66.79) - 1.02\times13.129\times\sin(113.18)$$

$$- 1.04\times13.129\times\sin(113.18)$$

$$= 32.627$$

∴ Jacobian Matrix is

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\\ \Delta Q_2\end{bmatrix} = \begin{bmatrix}24.86 & -12.55 & 28.707\\ -12.55 & 25.35 & -5.374\\ -10.645 & 5.374 & 32.627\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta|V_2|\end{bmatrix}$$

$$\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta|V_2|\end{bmatrix} = \begin{bmatrix}24.86 & -12.55 & 28.707\\ -12.55 & 25.35 & -5.374\\ -10.645 & 5.374 & 32.627\end{bmatrix}^{-1}\begin{bmatrix}0.903\\ \cancel{8.394}\ 1.1707\\ -8.394\end{bmatrix}$$

$$= \begin{bmatrix}0.0403 & 0.0265 & -0.0310\\ 0.0219 & 0.0525 & -0.0106\\ 9.53\times10^{-3} & -1.321\times10^{-7} & 0.0222\end{bmatrix}\begin{bmatrix}0.903\\ 1.1707\\ -8.394\end{bmatrix}$$

-8994

$$\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta|V_2|\end{bmatrix} = \begin{bmatrix}0.0444\\ 0.3481\\ -0.172\end{bmatrix}$$

$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0.044^{rad} = 2.52^0$

$\delta_3^1 = \delta_3^0 + \Delta\delta_3 = 0.3481 = 19.94^0$

$V_2^1 = V_2^0 + \Delta|V_2| = 1\angle0 + [0.172\angle 19.948] = 0.8367\angle{-4.113}$

Polar Method

⇒ 1^{st} bus is slack bus $V_1 = 1.02\angle 0^\circ$

2^{nd} bus is PQ bus $P_2 - jQ_2 = (-0.6 + j0.3)$ pu

3^{rd} bus is PV bus $P_3 = -1.5$ pu, $|V_3| = 1.04\angle 0^\circ$ pu

⇒ We have Y_{BUS} from that

$y_{11} = y_{22} = y_{33} = 26.252\angle -66.8$, $\theta_{pp} = 66.8^\circ$

$y_{12} = y_{13} = y_{23} = 13.13\angle 113.2^\circ$ pu , $\theta_{pq} = -113.2^\circ$

⇒ Unspecified values has to be assume as

$|V_2| = 1\angle 0^\circ$ pu

$\delta_2^0 = 0$

$\delta_3^0 = 0$

⇒ Now

$$P_2 = |V_2||y_{21}||V_1|\cos(\theta_{21} - \delta_2^0 + \delta_1^0) + |V_2||y_{22}||V_2|\cos(\theta_{22} - \delta_2 + \delta_2)$$
$$+ |V_2||y_{23}||V_3|\cos(\theta_{23} - \delta_2^0 + \delta_3^0)$$

$$= 1 \times 13.13 \times 1.02 \times \cos(-113.2^\circ) + 1^2 \times 26.25 \times \cos(66.8)$$
$$+ 1 \times 13.13 \times 1.04 \times \cos(-113.2)$$

$$= -0.314 \text{ pu}$$

$$P_3 = |V_3||y_{31}||V_1|\cos(\theta_{31} - \delta_3^0 + \delta_1^0) + |V_3||y_{32}||V_2|\cos(\theta_{32} - \delta_3 + \delta_2)$$
$$+ |V_3|^2|y_{33}|\cos(\theta_{33})$$

$$= 1.04 \times 13.13 \times 1.02 \times \cos(-113.2) + 1.04 \times 13.13 \times 1 \times \cos(-113.2)$$
$$+ 1.04^2 \times 26.252 \times \cos(66.8)$$

$$= 0.319 \text{ PU} \simeq 0.323 \text{ PU}$$

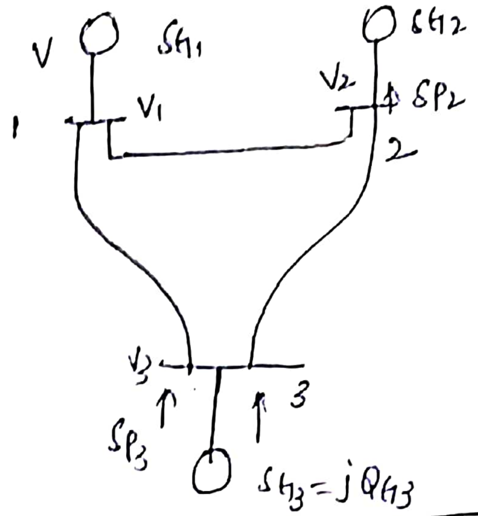
$$Q_2 = -\Big[|V_2||y_{21}||V_1|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2|y_{22}|\sin(\theta_{22}) +$$
$$|V_2||y_{23}||V_3|\sin(\theta_{23} - \delta_2 + \delta_3)\Big]$$

$$= -\Big[1 \times 13.13 \times 1.02 \times \sin(-113.2) + 1^2 \times 26.252 \times \sin(66.8) +$$
$$1 \times 13.13 \times 1.04 \times \sin(-113.2)\Big]$$

$$= 0.731$$

Problems:

(5) Consider a 3 bus system as shown in figure. Each of the three lines has series impedance of $(0.02+j0.08)$ & total shunt admittance of $j0.02$ PU. The specified quantities at the buses are as following



Bus	Real load demand P_D	Reactive load demand Q_D	Real power Generation P_G	Reactive power Generation Q_G	Voltage Specified
1	2.0	1.0	Unspecified	Unspeci	$V_1 = 1.04 +$
2	0.0	0.0	0.5	1.0	$V_2 =$ Unspec
3	1.5	0.6	0.0	$Q_{G_3} = ?$	$V_3 = 1.04$

Use Decoupled-NR and Fast Decoupled Load flow method to obtain one iteration of load flow solutions.

Solution

Given 1st bus is slack bus
2nd bus is PQ bus
3rd bus is PV bus

Given $V_1 = 1.04 \angle 0^\circ$
$V_3 = 1.04 \angle 0^\circ$
(Assume) $V_2 = 1 \angle 0^\circ$

series impedence = (0.02 + j0.08) pu
shunt admittance = j0.02 pu
series admittance = $\frac{1}{0.02+j0.08}$ = (2.94 − j11.76) pu
= −2.94 + j11.76

Calculating Ybus,

$$Y_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 24.23\angle{-75.95} & 12.126\angle{104.03} & 12.126\angle{104.03} \\ 2 & 12.126\angle{104.03} & 24.23\angle{-75.95} & 12.126\angle{104.03} \\ 3 & 12.126\angle{104.03} & 12.126\angle{104.03} & 24.23\angle{-75.95} \end{array}$$

$y_{11} = y_{22} = y_{33}$ = self admittance

$$= 2\left[\frac{1}{0.02+j0.08} + \frac{j0.02}{2}\right]$$

$= 24.23\angle{-75.95} = 5.882 - j23.505$

$B_{22} = 23.505$, $G_{22} = 5.882$

$y_{12} = y_{13} = y_{23} = \frac{-1}{0.02+j0.08} = 12.126\angle{104.03} = y_{21} = y_{31} = y_{32}$

$(-y_{12}) = -2.93 + j11.76$ $\quad \theta_{pp} = -75.95 \quad \theta_{pq} = +104.03$

Now calculating P_i & Q_i $\quad \delta_2^0 = 0 \quad \delta_3^0 = 0$

$$P_i = \sum_{K=1}^{n} |V_i| \, |Y_{ik}| \, |V_k| \cos(\theta_{ik} - \delta_i + \delta_k)$$

i = 2

$$P_2^0 = |V_2| \, |V_1| \, |y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2) + |V_3| \, |V_2| \, |y_{32}| \cos(\theta_{32} - \delta_2 + \delta_3)$$

$= 1 \times 1.04 \times 12.126 \times \cos(+104.03) + 1^2 \times 24.23 \cos(-75.95) +$
$1.04 \times 1 \times 12.126 \cos(+104.03)$

$P_2^0 = -0.23$ pu

i=3

$$P_3^0 = \left[|V_3| \, |V_1| \, |y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3| \, |V_2| \, |y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2 |y_{33}| \cos(\theta_{33})\right]$$

$= 1.04 \times 1.04 \times 12.13 \times \cos(104.03) + 1.04 \times 1 \times 12.13 \times \cos(104.03)$
$+ 1.04^2 \times 24.23 \times \cos(-75.95)]$

$= 0.4495$ $\quad$ 0.123 pu

$P_3^0 = 0.123$ PU

$= 0.12$ PU

$$Q_i^0 = -\left[\sum_{k=1}^{n} |V_i||V_k||Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k)\right]$$

$i=2$

$$Q_2^1 = -\left[|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |Y_{22}| \sin(\theta_{22}) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)\right]$$

$$= -\left[1 \times 1.04 \times 12.13 \times \sin(104.03) + 1^2 \times 24.23 \times \sin(-75.95) + 1 \times 1.04 \times 12.13 \times \sin(104.03)\right]$$

$= -0.972$

$Q_2^0 \simeq -0.96$

$\Delta P_2 = P_2^{spec} - P_2^0 = 0.5 + 0.23 = 0.73$ PU

$\Delta P_3 = P_3^{spec} - P_3^0 = -1.5 - 0.12 = -1.62$ PU

$\Delta Q_2 = Q_2^{spec} - Q_2^0 = 1.0 + 0.96 = 1.96$ PU

Calculate Jacobian Matrix elements.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \frac{\Delta V_2}{V_2} \end{bmatrix}$$

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - V_i^2 B_{ii}$$

$i=2$

$$\frac{\partial P_2}{\partial \delta_2} = -Q_2 - V_2^2 B_{22}$$

$= +0.96 + 1 \times 23.55$

$= 24.4$

$$\frac{\partial P_i}{\partial \delta_k} = V_i V_k\left(-B_{ik}\cos(\delta_i-\delta_k) + G_{ik}\sin(\delta_i-\delta_k)\right]$$

$$\frac{\partial P_2}{\partial \delta_3} = V_2 V_3\left(-B_{23}\cos(\delta_2-\delta_3) + G_{23}\sin(\delta_2-\delta_3)\right]$$

$$= 1 \times 1.04\left[11.76\cos(0) + (-2.93) \times \sin(0)\right]$$

$$= -12.23$$

$$\frac{\partial P_3}{\partial \delta_2} = -12.23$$

$$\frac{\partial P_3}{\partial \delta_3} = -Q_3 - V_3^2 B_{33}$$

$$= -0.6 - 1.04^2 \times -23.505$$

$$= 24.8$$

$$\frac{\partial Q_i}{\partial \delta_k} = V_i V_k\left[G_{ik}\sin(\delta_i-\delta_k) - B_{ik}\cos(\delta_i-\delta_k)\right]$$

$$\frac{\partial Q_i}{\partial V_i} = Q_i - V_i^2 B_{ii}$$

$$\frac{\partial Q_2}{\partial |V_2|} = Q_2 - V_2^2 \times B_{22}$$

$$= -0.96 - 1^2 \times -23.55$$

$$= 22.54$$

$$\frac{\partial P_2}{\partial |V_2|} = 5.64$$

$$\frac{\partial P_3}{\partial |V_2|} = -3.05$$

$$\frac{\partial Q_i}{\partial \delta_i} = \frac{\partial Q_2}{\partial \delta_2} = -6.11$$

$$\frac{\partial Q_2}{\partial \delta_3} = 3.05$$

$$\begin{bmatrix} 0.73 \\ -1.62 \\ 1.93 \end{bmatrix} = \begin{bmatrix} 24.4 & -12.23 & 5.64 \\ -12.23 & 24.8 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix} = \begin{bmatrix} 24.4 & -12.23 & 5.64 \\ -12.23 & 24.8 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0520 & 0.0268 & -9.40\times10^{-3} \\ 0.0259 & 0.0535 & 4.98\times10^{-4} \\ 0.0104 & 2.796\times10^{-5} & 0.0417 \end{bmatrix} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix} = \begin{bmatrix} -0.07 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2' \\ \Delta\delta_3' \\ \Delta|V_3|' \end{bmatrix} = \begin{bmatrix} -0.0231 \\ -0.0654 \\ 1.0891 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \\ |V_3| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} -0.0231 \\ -0.0654 \\ 0.0891 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0231 \\ -0.0654 \\ 1.0891 \end{bmatrix} = \begin{bmatrix} -1.32^\circ \\ -3.74^\circ \\ 1.087 \end{bmatrix}$$

$$Q_3^0 = -\Big[|V_3||V_1||Y_{31}|\sin(\theta_{31}-\delta_3+\delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32}-\delta_3+\delta_2) + |V_3|^2 |Y_{33}| \sin(\theta_{33})\Big]$$

$$= -\Big[1.04 \times 1.04 \times 12.12 \times \sin(104.03) + 1.04 \times 1.08 \times 12.12 \sin(104.03) + 1.04^2 \times 24.23 \times \sin(-75.95)\Big]$$

$+0.0654-0.0231$

$Q_3^0 = 0.467$

$-[12.64+12.57-25.42]$

$=0.26$

$$Q_{G3} = Q_3' + Q_{P3}$$

$$= 0.467 + 0.6$$

$$= (1.067)\ \text{PU}$$

the equations to solve

$[\Delta P] = [H][\Delta\delta]$

$[\Delta Q] = [L]\left[\frac{\Delta|V|}{|V|}\right]$

$H_{ii} = -B_{ii}\left[|V_i|^2 + \sum_{k=1}^{n} |V_i||V_k|(-G_{ik}\sin(\delta_i-\delta_k) + B_{ik}\cos(\delta_i-\delta_k))\right]$

$L_{ii} = -B_{ii}\left[|V_i|^2 + \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin(\delta_i-\delta_k) - B_{ik}\cos(\delta_i-\delta_k))\right]$

$i=2$

$H_{22} = -B_{22}\left[|V_2|^2 + \sum_{k=1}^{n} |V_2||V_k|(-G_{2k}\sin(\delta_2-\delta_k) + B_{2k}\cos(\delta_2-\delta_k))\right]$

$H_{22} = 24.47$

$H_{23} = H_{32} = -12.23$

$H_{33} = 25.89$

$L_{22} = 24.508$

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\end{bmatrix} = \begin{bmatrix}H_{22} & H_{23}\\ H_{32} & H_{33}\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\end{bmatrix}$$

$\Delta\delta_2^{\circ} = -0.115$

$\Delta\delta_3 = 3.55$

$[\Delta Q_2]^{(1)} = [L_{22}]\left[\frac{\Delta|V_2|}{V_2}\right]$

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\end{bmatrix} = \begin{bmatrix}24.47 & -12.23\\ -12.23 & 25.89\end{bmatrix}\begin{bmatrix}-1.32\\ -3.76\end{bmatrix} = \begin{bmatrix}0.73\\ -1.62\end{bmatrix}$$

$\delta_2^1 = 0 - 0.115^{\circ} = -0.115$

$\delta_3^1 = 0 - 3.55 = \cancel{3.05} \quad -3.55^{\circ}$

$[\Delta Q_2^1] = [24.508][1.08]$

$\Rightarrow Q_2^0 = -|V_2||V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2^1) - |V_2|^2|Y_{22}|\sin(\theta_{22})$

$\quad - |V_2||V_3||Y_{23}|\sin(\theta_{23}+\delta_2^1-\delta_3^1)$

$= -1\times1.04\times12.12\times\sin(\overset{104.03}{\cancel{75.95+0.115}}) - 1\times1.04^2\times24.23\times\sin(\overset{-75.95}{\cancel{104.03}})$

$\quad - 1\times1.04\times12.12\times\sin(104.03 - 0.115 + 3.55)$

$= -1.125$

$$\Delta|V_2| = 0.086$$

$$|V_2|' = |V_2|^0 + \Delta V_2 = 1 + 0.086$$

$$= 1.086$$

$$\left[\frac{\Delta P}{|V|}\right] = [B'][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''][\Delta|V|]$$

$$\left[\frac{\Delta P_2^0}{|V_2|^0}\right] = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix}$$

$$\left[\frac{\Delta Q_2^0}{|V_2|}\right] = [-B_{22}]\,[\Delta|V_2|^0]$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} 23.508 & -11.76 \\ -11.76 & 23.508 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.0567 & 0.0283 \\ 0.0283 & 0.0567 \end{bmatrix} \begin{bmatrix} 0.73 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} -0.003 \text{ rad} \\ -0.068 \text{ rad} \end{bmatrix}$$

$$\Rightarrow \quad \delta_2' = -0.003 \text{ rad}$$
$$\delta_3' = -0.068 \text{ rad}$$

$$\left[\frac{\Delta Q_2^0}{|V_2|}\right] = [23.507]\,[\Delta|V_2|']$$

$$\Rightarrow \quad V_2' = (V_2)^0 + \Delta|V_2|'$$
$$\doteq 1.09$$