

Short-Circuit Study

Perunit Representation

\* In the process of computation of power system problems some times it is more convenient to express impedences, currents, voltages, and powers, in terms of per unit values rather than ohms, Amps, Volts, Watts, and VAR's.

\* The Perunit Value can be defined as the ratio of Actual value to the base value (or) reference value.

\* The ratio in percent is 100 times the perunit

\* The perunit method has the advantage over the percentage method because the product of the two quantities expressed in perunit itself but the product of two quantities expressed in percentage must be divided by 100 to obtain the result in percent. The perunit values are dimensionless.

\* % Value = PU value  $\times$  100

\* Let  $I_a$  be the actual value of current in Amps  
 $I_b$  be the base value of current in Amps

$$\therefore I_{pu} = \frac{I_a}{I_b}$$

$$\text{ii) } V_{pu} = \frac{V_a}{V_b}$$

$$Z_{pu} = \frac{Z_a}{Z_b} ; Z_a = R_a + jX_a$$
$$Z_b = R_b + jX_b$$

$$= \frac{Z_a}{Z_b} = \frac{R_a + jX_a}{Z_b}$$

$$Z_{pu} = \frac{R_a}{Z_b} + j \frac{X_a}{Z_b} = R_{pu} + jX_{pu}$$

\* Let ' $S_a$ ' be the actual value of apparent power given by

$$S_a = VI^* = P + jQ$$

Let ' $S_b$ ' be the base value of apparent power

$$S_{pu} = \frac{S_a}{S_b}$$

$$= \frac{P + jQ}{S_b}$$

$$S_{pu} = P_{pu} + jQ_{pu} \quad ; \quad P_{pu} = \frac{P}{S_b} \quad ; \quad Q_{pu} = \frac{Q}{S_b}$$

$$S_{pu} = P_{pu} + jQ_{pu}$$

\* For a 1- $\phi$  system  $Z_b = \frac{V_b}{I_b}$  ;  $S_b = V_b I_b$

$$I_b = \frac{S_b}{V_b}$$

$$\Rightarrow Z_b = \frac{V_b}{S_b / V_b}$$

$$Z_b = \frac{V_b^2}{S_b} \Omega$$

$$Y_b = \frac{S_b}{V_b^2}$$

$$\Rightarrow Z_{pu} = \frac{Z_a}{Z_b} = \frac{Z_a}{(V_b^2 / S_b)} = Z_a \frac{S_b}{V_b^2}$$

$$\Rightarrow Y_{pu} = \frac{Y_a}{Y_b} = \frac{Y_a}{(S_b / V_b^2)} = Y_a \left( \frac{V_b^2}{S_b} \right)$$

Change of Base

\* It is sometimes necessary to convert the per unit quantities of one base to another.

\* Let the Base power and Base voltage be  $S_{b1}$  &  $V_{b1}$  and the corresponding values of base 2 is  $S_{b2}$  &  $V_{b2}$

$$S_{b1} = V_{b1} I_{b1} \Rightarrow I_{b1} = \frac{S_{b1}}{V_{b1}}$$

$$\text{|| } I_{b2} = \frac{S_{b2}}{V_{b2}}$$

$$I_{pu1} = \frac{I_a}{I_{b1}}$$

$$\text{ii) } I_{pu2} = \frac{I_a}{I_{b2}}$$

$$\frac{I_{pu2}}{I_{pu1}} = \frac{I_{b1}}{I_{b2}}$$

$$\Rightarrow I_{pu2} = \frac{I_{b1}}{I_{b2}} \times I_{pu1}$$

$$= \frac{(S_{b1}/V_{b1})}{(S_{b2}/V_{b2})} \times I_{pu1}$$

$$= \frac{S_{b1}}{V_{b1}} \times \frac{V_{b2}}{S_{b2}} \times I_{pu1}$$

$$I_{pu2} = I_{pu1} \times \frac{S_{b1}}{S_{b2}} \times \frac{V_{b2}}{V_{b1}}$$

$$* \therefore \text{Now } Z_{pu1} = Z_a \times \frac{S_{b1}}{V_{b1}^2}$$

$$\text{ii) } Z_{pu2} = Z_a \times \frac{S_{b2}}{V_{b2}^2}$$

$$\frac{Z_{pu2}}{Z_{pu1}} = \frac{S_{b2}/V_{b2}^2}{V_{b1}^2}$$

$$= \frac{S_{b2}}{S_{b1}} \times \frac{V_{b1}^2}{V_{b2}^2}$$

$$\Rightarrow Z_{pu2} = Z_{pu1} \times \frac{S_{b2}}{S_{b1}} \times \left(\frac{V_{b1}}{V_{b2}}\right)^2$$

$$Z_{pu(\text{new})} = Z_{pu(\text{old})} \times \frac{S_{b(\text{new})}}{S_{b(\text{old})}} \times \left(\frac{V_{b(\text{old})}}{V_{b(\text{new})}}\right)^2$$

$$\therefore Z_{pu(\text{new})} = Z_{pu(\text{old})} \times \frac{\text{MVA}(\text{new})}{\text{MVA}(\text{old})} \times \left[\frac{\text{kV}(\text{old})}{\text{kV}(\text{new})}\right]^2$$

$$\therefore Z_{pu(\text{new})} = Z_{pu(\text{old})} \times \frac{\text{MVA}(\text{new})}{\text{MVA}(\text{old})} \times \left[\frac{\text{kV}(\text{old})}{\text{kV}(\text{new})}\right]^2$$

Per Unit Impedance of a Transformer

\* Let  $V_1$  is the base voltage of the transformer in the primary.

$I_1$  is the base current of the TF in primary

$Z_{b1}$  is base impedances of TF on primary & secondary

$V_2, I_2$  are base voltage & current of TF at secondary

$$Z_{b1} = \frac{V_1}{I_1} \Omega$$

$$Z_{b2} = \frac{V_2}{I_2} \Omega$$

\* Equivalent impedance of TF is referred to primary

$$\begin{aligned} Z_{01}(\text{pu}) &= \frac{Z_{01}}{Z_{b1}} \\ &= \frac{Z_{01}}{(V_1/I_1)} \end{aligned}$$

$$Z_{01}(\text{pu}) = \frac{I_1^2 Z_{01}}{V_1}$$

$$\begin{aligned} \text{Ily } Z_{02}(\text{pu}) &= \frac{Z_{02}}{Z_{b2}} \\ &= \frac{Z_{02}}{(V_2/I_2)} \end{aligned}$$

$$Z_{02}(\text{pu}) = \frac{I_2^2 Z_{02}}{V_2} \quad \text{--- (1)}$$

$$\Rightarrow Z_{02} = Z_{01} \left( \frac{N_2}{N_1} \right)^2$$

$$I_2 = I_1 \left( \frac{N_1}{N_2} \right)$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_2 = V_1 \left( \frac{N_2}{N_1} \right)$$

$$\therefore Z_{02}(\text{pu}) = Z_{01} \left( \frac{N_2}{N_1} \right)^2 \left[ \frac{I_1^2 \left( \frac{N_1}{N_2} \right)}{V_1 \left( \frac{N_2}{N_1} \right)} \right] \leftarrow \text{by substitution in eq (1)}$$

$$Z_{02}(\text{pu}) = \frac{Z_{01}}{V_1/I_1} = Z_{01}(\text{pu})$$

$$Z_{02}(\text{pu}) = Z_{01}(\text{pu})$$

\* Hence the pu representation of 2 winding T/F is referred to same. In either side.

\* In other words the pu representation of 2 winding T/F is same whether the calculations is made from HV (or) LV side.

\* Base MVA

$$\frac{\text{Base MVA}}{\underline{\quad} \times \underline{\quad} \times \underline{\quad}}$$

\* If numbers of equipment such as generators, T/F's, T/m lines etc., and their percentage resistance and reactance also refer to their own kVA ratings it is difficult to compare these percentage resistance & reactances and their combined effect until and unless they are all referred to a common (Base MVA).

\* This common Base MVA is an arbitrary one. A Base MVA may be chosen in the following manners

- ① Equal to MVA rating of the largest unit connected in the network.
- ② Equal to the sum of the MVA ratings of all the units connected in the network.
- ③ Any arbitrary value.

\* The value of Base MVA has no bearing what so ever on the results since in the ultimate formula for calculation of short circuit current the Base MVA is to be taken into consideration.

$$\frac{\text{Base kVA}}{\underline{\quad} \times \underline{\quad} \times \underline{\quad}}$$

\* In some cases it is convenient to work in ohmic values of the various reactances rather than the percentage value. The method would become simple, if all the reactances related to same voltage, but, if step-up (or) step-down transformers operating at different voltages are

also included all the ohmic values has to be reduced to a common base voltage.

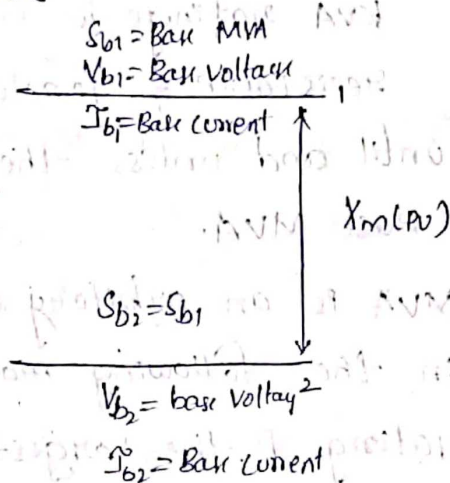
\* Let  $X_1$  is reactance at  $E_1$

$X_2$  is reactance at  $E_2$

$$\text{then } X_2 = \left(\frac{E_2}{E_1}\right)^2 X_1$$

\* The resistances, reactances and voltages refer to the phase value and not the line value.

\* Mutual impedance in per unit between line of different voltage levels.



$$X_m(\text{PU}) = \frac{X_m(\text{actual}) \times S_{b1} \times S_{b2}}{V_{b1} \times V_{b2}}$$

$$= X_m(\text{actual}) \times \frac{\text{MVA}_b}{\text{KV}_{b1} \times \text{KV}_{b2}}$$

Problems :-

① Base Voltage = 1100V, Base KVA is  $10^6$ , what is the Base impedance.

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$$(Z_b = \frac{V_b^2}{S_b} = \frac{1100^2}{10^6} = 1.21) \times$$

$$Z_b = \frac{\text{KV}_b^2}{\text{MVA}_b}$$

$$\text{KV}_b = 1100 \times 10^{-3} = 1.1 \text{ KV}$$

$$\text{MVA}_b = 10^6 \times 10^{-3} = 10^3$$

$$Z_b = \frac{1.1^2}{10^3} = 1.21 \times 10^{-3} \Omega$$

② If the resistance is  $5\Omega$  find the pu value. Given base kVA is 10 & kv is 11.

$$R_a = 5\Omega \quad R_{pu} = ?$$

$$R_{pu} = \frac{R_a}{R_b} = \frac{5}{R_b}$$

$$R_b = \frac{V}{I} = \frac{V_b^2}{S_b} = \frac{11^2}{10 \times 10^3} = 12100$$

$$\frac{V}{I} = \frac{V^2}{S_b}$$

$$R_{pu} = \frac{5}{12100}$$

$$= 4.13 \times 10^{-4} \text{ pu}$$

③ A single phase T/F is rated as 2.5 kVA, 11/0.4 kv if the leakage reactance is  $0.96\Omega$  when referred to kv side. Determine the leakage reactance per unit.

$$S_b = 2.5 \text{ kVA} = 2.5 \times 10^3 \text{ MVA}; \quad KV_b = 0.4$$

$$X_a = 0.96\Omega$$

$$X_{pu} = \frac{(KV_b)^2}{\text{MVA}_b}$$

$$= \frac{(0.4)^2}{2.5 \times 10^3}$$

$$= 64 \text{ pu}\Omega$$

$$X_{pu} = \frac{0.96}{64}$$

$$= 0.015 \text{ pu}$$

Another method:

$$K = \frac{V_2}{V_1} = \frac{0.4}{11} = 0.036$$

$$X_{02} = 0.96$$

$$X_{01} = \frac{X_{02}}{K^2}$$

$$= \frac{0.96}{0.036^2}$$

$$= 740.74$$

$$X_{01}(\text{pu}) = 740.74 \times \frac{2.5 \times 10^3}{11^2}$$

$$= 0.015 \text{ pu}$$

④ For a 110/440V, 25 kVA, single phase transformer primary and secondary leakage reactances are  $0.04\Omega$  &  $0.1\Omega$  respectively show that the net pu leakage reactance of the transformer referred to LV side is same as that referred to HV side

$$S_b = 25 \times 10^3 \text{ MVA} \quad V_1 = 110 \times 10^3 \text{ KV} \quad V_2 = 440 \times 10^3 \text{ KV} \quad X_{01} = 0.04\Omega$$

$$X_{02} = 0.1\Omega$$

$$X_{01b} = \frac{KV_b^2}{\text{MVA}_b} = \frac{(110 \times 10^3)^2}{25 \times 10^3}$$

$$= 0.484\Omega$$

$$X_{pu} = \frac{X_{01}}{X_{01b}} = \frac{0.04}{0.484} = 0.0826$$

$$X_{02b} = \frac{(440 \times 10^3)^2}{25 \times 10^3}$$

$$= 7.744$$

$$X_{pu} = \frac{X_{02}}{X_{02b}} = \frac{0.1}{7.744} = 0.0129$$

$$X_{01} = \frac{X_0}{K^2}$$

$$\Rightarrow X_{01 \text{ actual}} = 0.04 + \frac{X_0}{K^2}$$

$$= 0.04 + \frac{0.1}{4^2}$$

$$= 0.04625$$

$$\therefore [K = \frac{110}{110} = 1]$$

$$X_{01 b} = 0.484$$

$$X_{01(pu)} = \frac{0.04625}{0.484}$$

$$= 0.095 \text{ pu}$$

$$\Rightarrow X_{02} = X_{01} K^2$$

$$X_{02(\text{actual})} = 0.1 + X_{01} K^2$$

$$= 0.1 + 0.04 \times 4^2$$

$$= 0.74$$

$$(or) X_{02} = X_{01 \text{ act}} \times K^2$$

$$= 0.04625 \times 4^2$$

$$= 0.74$$

$$X_{02 b} = 7.744$$

$$X_{02(pu)} = \frac{0.74}{7.744}$$

$$= 0.095 \text{ pu}$$

\* Hence it is proved that either side of the transformer pu reactances are same

(5) An 11/0.4 KV 200 KVA T/F has an equivalent impedance of  $(2.4 + j12.4)\Omega$  referred to H.V. sided. Determine the base values for the per unit system & the equivalent impedance drop at one ~~half~~  $\frac{1}{2}$  rated current.

Sol

$$V_1 = 11 \text{ KV} \quad V_2 = 0.4 \text{ KV}$$

$$S_b = 200 \times 10^3 \text{ MVA}$$

$$X_{01} = 2.4 \Omega \quad R_{01} = 12.4 \Omega$$

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{11^2}{200 \times 10^3} = 605 \Omega$$

$$Z_{01(pu)} = \frac{Z_{01(\text{act})}}{Z_b} = \frac{2.4 + j12.4}{605} = 3.96 \times 10^{-3} + j0.0204$$

$$= (0.0039 + j0.0204) \text{ pu} = Z_{0e(pu)}$$



$$S = V_1 I_1$$

$$I_1 = \frac{S}{V_1}$$

$$= \frac{200}{11}$$

$$= 18.18 \text{ A} = I_b$$

$$I_1 \text{ pu} = \frac{18.18}{18.18} = 1 \text{ pu}$$

$$I_1 Z_{01} = \frac{1}{2} \times (0.0039 \times j0.0204) \left[ (I_b Z_{01})_b \text{ impedance drop at half of rated current is} \right]$$

$$= 0.00195 + j0.0102$$

$$I_1 Z_{01} (\text{actual}) = \text{pu} \times \text{Base}$$

$$=$$

$$b = \frac{\text{kV}_b}{\text{MVA}_b}$$

Per unit Quantities for 3-φ system

\* The per unit value of a line to neutral  $V_{LN}$  on the line to neutral base  $V_{bLN}$  is equal to the per unit value of the line to line voltage  $V_{LL}$  on the line to line voltage base  $V_{bLL}$  if the system is balanced.

$$\text{i.e., } \frac{V_{LN}}{V_{bLN}} = \frac{V_{LL}}{V_{bLL}}$$

$$* \frac{3\text{-}\phi \text{ kVA}}{3\text{-}\phi \text{ base kVA}} = \frac{\text{Per phase kVA}}{\text{Per phase base kVA}}$$

\* The per unit value of a 3-φ kVA on the 3-φ base kVA is identical to the per unit value of the per phase kVA on the per phase base kVA.

\* In a 3- $\phi$  system the line value of voltage & 3- $\phi$  KVA are directly used for per unit calculations

\* Base impedance and Base current are calculated using line value of voltage and 3- $\phi$  KVA.

\* Let  $KV_b$  = line to line base KV

$I_b$  = line value of base current

Then  $KVA_b = \sqrt{3} \times KV_b \times I_b$

$$I_b = \frac{KVA_b}{\sqrt{3} \times KV_b}$$

\* Base impedance per phase  $Z_b = \frac{(KV_b)^2}{MVA_b}$

\* This equation is similar to the 1- $\phi$  system but in 3- $\phi$  system the  $KV_b$  is a line voltage, and  $MVA_b$  is a 3- $\phi$  MVA.

Note :-

\* Impedance is always expressed as phase value.

Percentage Resistance & Reactance :-

\* Resistance & Reactance for generators, Transformers and motors is always represented in percentage terms.

\* For cables and Transmission lines ohmic values are specified.

\* For short circuit calculations percentage values are employed.

Percentage Resistance :-

$$* \% R = R_{pu} \times 100$$

$$= \left[ \frac{IR}{V} \right] \times 100$$

$$\% R = R_{pu} \times 100 \\ = \left( \frac{IR}{V} \right) \times 100$$

\* It is the voltage drop across the resistance expressed as % of normal voltage when carrying full load current.

$$R_{pu} = \frac{R}{Z_b}; \quad X_{pu} = \frac{X}{Z_b}$$

### Percentage Reactance

\* It is the voltage drop across the reactance expressed as the percentage of normal value when carrying full load current.

$$\text{i.e. } \%X = \left( \frac{IX}{V} \right) \times 100$$

$$X = \frac{\%X \cdot V}{I} \times 100$$

$$* X_{pu} = \frac{(kV_b)^2}{MVA_b}$$

$$X_{pu} = \frac{X \cdot MVA_b}{(kV_b)^2}$$

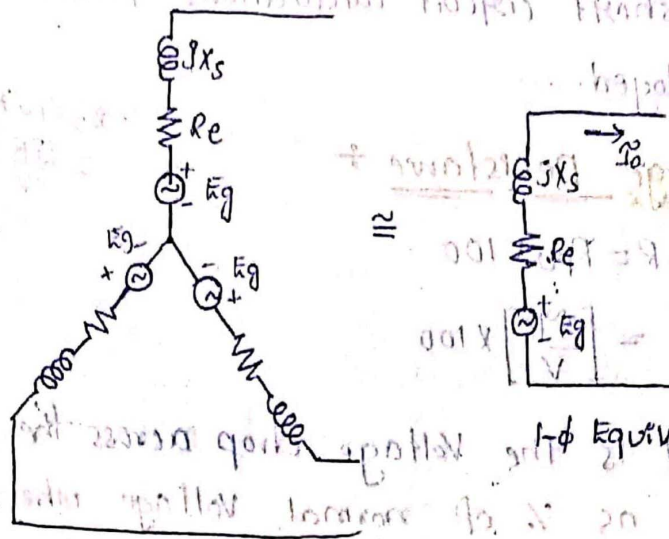
$$\%X = X_{pu} \times 100$$

$$= \frac{X \cdot MVA_b}{(kV_b)^2} \times 100$$

$$X = \frac{(\%X) (kV_b)^2}{(MVA_b) \times 100}$$

### Representation of Power System Components

#### ① Alternator (or) Synchronous generator:



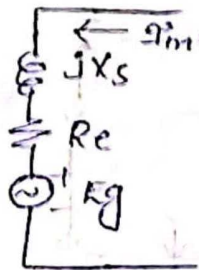
1- $\phi$  Equivalent circuit

\* It can be represented by induced EMF per phase  
 A series reactance representing the armature leakage reactance, reactance due to armature reaction and a series resistance representing the armature resistance

\*  $X_s = X_l + X_{ar}$

② Synchronous Motor :-

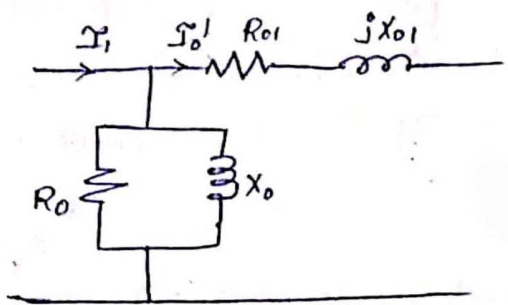
\* The equivalent circuit of synchronous motor is similar to that of Alternator but it performs the reverse action of generator.



1- $\phi$  Equivalent circuit.

③ Transformer :-

\* The equivalent circuit of 1- $\phi$  two winding transformer referred to primary is as shown in figure

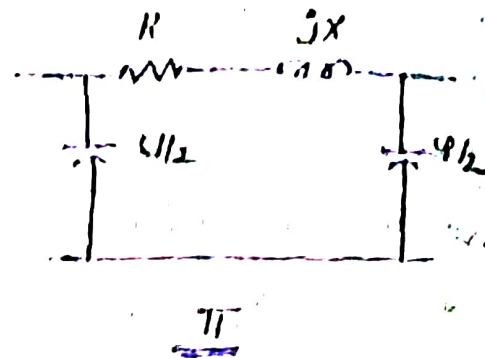
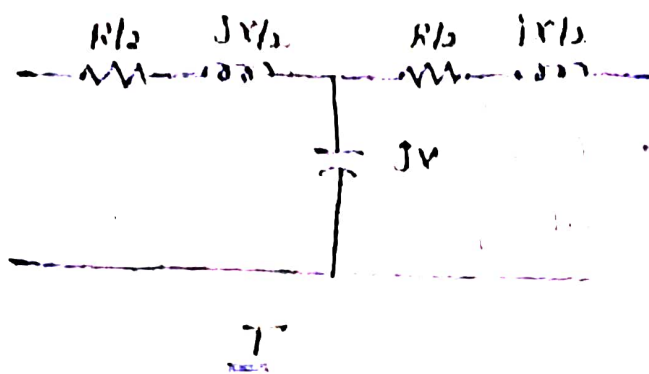


\* A 3- $\phi$  transformer can be represented by its 1- $\phi$  equivalent circuit

\* In a 3- $\phi$  transformer the transformation ratio 'k' is taken as the ratio of line voltages.

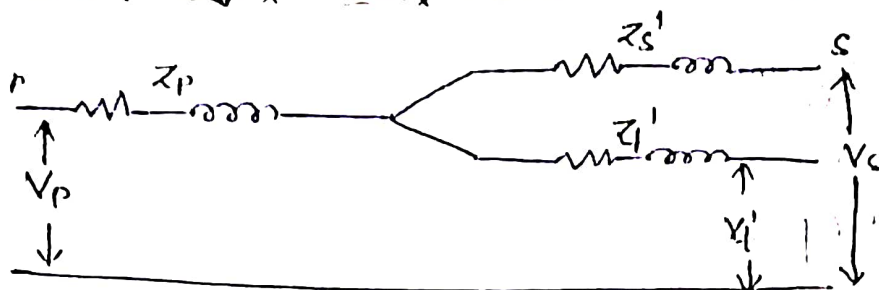
④ Transmission line :-

\* Transmission line can be represented by its resistance, inductance and capacitance



\* The 1-φ equivalent circuit of nominal T & π are as shown in above

⑤ 3-Winding Transformer



\* The leakage impedances are given by as follows.

$$Z_{ps} = Z_p + Z_s$$

$$Z_{pl} = Z_p + Z_l'$$

$$Z_{st}' = Z_s' + Z_l'$$

$$Z_p = \frac{1}{2} [Z_{ps} + Z_{pl} - Z_{st}']$$

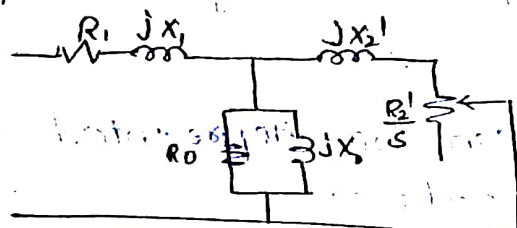
$$Z_l' = \frac{1}{2} [Z_{pl} + Z_{st}' - Z_{ps}]$$

$$Z_s' = \frac{1}{2} [Z_{ps} + Z_{st}' - Z_{pl}]$$

10-02-2017

⑥ Induction Motor

\* Induction motors are neglected in computing fault current a few cycles after the fault occurred, because the current contributed by an induction motor is short circuited.



## Impedance Diagram

\* Impedance Diagram of power system is the equivalent circuit of power system in which the various components are represented by simplified equivalent circuit.

\* This simplified diagram is used to analyse the performance of the system under load condition (or) to analyse the system condition under fault conditions.

\* The impedance diagram can be obtained from the single line diagram by replacing all the components of the system by their equivalent single phase circuit.

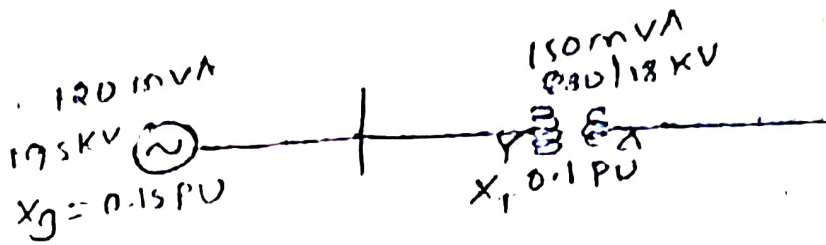
### Assumptions :-

① The current limiting impedance connected between the generator neutral and ground is neglected. Because under balanced condition no current is flowing through neutral of a generator.

② Since the magnetising current of a transformer is very low when compared to load current the shunt branches in the equivalent circuit of a transformer can be neglected.

### Problem

- ① A 120 MVA, 19.5 kV generator has a synchronous reactance of 0.15 pu and is connected to a transmission line through a transformer rated 150 MVA, 230/118 kV star/delta with  $x_T = 0.1$  pu
- calculate the pu reactances, by taking transformer ratings as base value
  - calculate the pu reactances, by taking transformer generating ratings as base values
  - calculate the pu reactance, for a base value of 100 MVA and 220 kV on HV side of TF.



(i) Base MVA = 120

Base KV = 19.5

$X_G = 0.15 \text{ PU}$

$$X_T(\text{PU}) = X_{T(\text{PU})}(\text{old}) \times \frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}} \times \left( \frac{\text{KV}_{\text{old}}}{\text{KV}_{\text{new}}} \right)^2$$

$$= 0.1 \times \frac{120}{150} \times \left( \frac{19.5}{18} \right)^2$$

$$= 0.068 \text{ PU}$$

(ii) Base MVA = 150

Base KV = 18

$X_T = 0.1 \text{ PU}$

$$X_G(\text{PU}) = X_{G(\text{PU})}(\text{old}) \times \frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}} \times \left( \frac{\text{KV}_{\text{old}}}{\text{KV}_{\text{new}}} \right)^2$$

$$= 0.15 \times \frac{150}{120} \times \left( \frac{19.5}{18} \right)^2$$

$$= 0.22 \text{ PU}$$

(iii) Base MVA = 100

Base KV = 220

$$X_T(\text{PU}) = 0.1 \times \frac{100}{150} \times \left( \frac{230}{220} \right)^2$$

$$= 0.0728 \text{ PU}$$

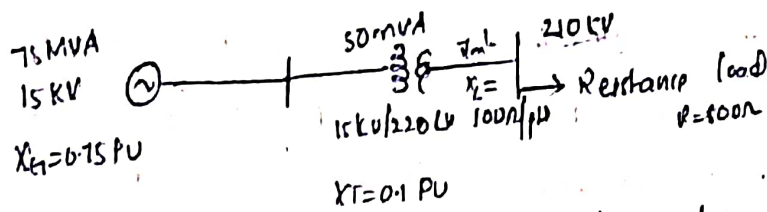
$$X_G(\text{PU}) = 0.15 \times \frac{100}{120} \times \left( \frac{19.5}{220} \right)^2$$

$$\text{KV}(\text{new}) = \frac{220}{230} \times 18 = 17.217$$

$$= 0.15 \times \frac{100}{120} \times \left( \frac{220}{17.21} \right)^2$$

$$= 19.01604 \text{ PU}$$

② For the system shown in figure determine the generator voltage.



Base MVA = 100 MVA (Assumption) arbitrary value  
 Base kV = 15 kV

$$\Rightarrow X_G(\text{pu}) = X_G(\text{pu})_{\text{old}} \times \frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}} \times \left( \frac{\text{kV}_{\text{old}}}{\text{kV}_{\text{new}}} \right)^2$$

$$= 0.75 \times \frac{100}{75} \times \left( \frac{15}{15} \right)^2$$

$$= 1 \text{ pu}$$

$$\Rightarrow X_T(\text{pu}) = 0.1 \times \frac{100}{50} \times \left( \frac{15}{15} \right)^2$$

$$= 0.2 \text{ pu}$$

$$X_L(\text{pu}) = X_L \times \frac{\text{MVA}_b}{(\text{kV}_b)^2}$$

$$\text{kV}_b(\text{new}) = \frac{220}{15} \times 15$$

$$= 220 \text{ kV}$$

$$= 100 \times \frac{100}{(220)^2}$$

$$\Rightarrow X_L(\text{pu}) = 0.206$$

$$R_{\text{pu}} = R \times \frac{\text{MVA}_b}{\text{kV}_b^2} = 500 \times \frac{100}{220^2} = 1.03 \text{ pu}$$

$$V_{\text{pu}} = \frac{210}{220} = 0.95 \text{ pu}$$

$$I_{\text{pu}} = \frac{V_{\text{pu}}}{R_{\text{pu}}} = 0.926 \text{ pu}$$

$$\text{Voltage drop} = I_{\text{pu}} (R_{\text{pu}} + j(X_G + X_T + X_L))$$

$$= 0.926 (1.03 + j(1 + 0.206 + 0.2))$$

$$= 1.614 \text{ pu}$$

$$\text{Generator voltage} = 1.614 \times V_b$$

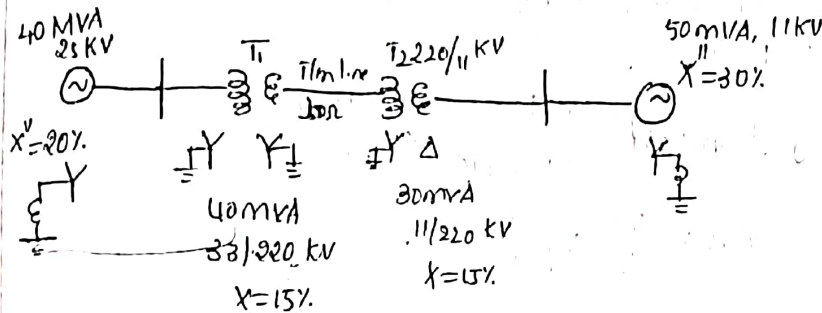
$$= 1.614 \times 15$$

$$= 24.15 \text{ kV}$$

$$\text{Generator phase voltage} = 13.94 \text{ kV}$$



3. Draw the reactance diagram for the power system shown in figure neglect resistance and use the base of 100 MVA, 220 kV in 500 line. The rating of the generator motor and transformer are as shown in figure.



Sol

Base MVA is  $MVA_b = 100 \text{ MVA}$

Base KV is  $KV_b = 220 \text{ KV}$

$$X_{G(\text{new})} = 0.2 \times 100$$

$$= X_{G(\text{old})} \times \frac{MVA_b}{MVA_{\text{old}}} \times \left( \frac{KV_{\text{old}}}{KV_b} \right)^2$$

$$= 0.2 \times \frac{100}{40} \times \left( \frac{25}{33} \right)^2$$

$$= 0.2869$$

$$X_{G(\text{new})} = 0.287 \text{ PU}$$

$$X_{T_1(\text{new})} = X_{T_1(\text{old})} \times \frac{MVA_b}{MVA_{\text{old}}} \times \left( \frac{KV_{\text{old}}}{KV_b} \right)^2$$

$$= 0.15 \times \frac{100}{40} \times \left( \frac{220}{220} \right)^2$$

$$= 0.375$$

$$X_{T_1(\text{new})} = 0.375 \text{ PU}$$

$$X_{T_2(\text{new})} = X_{T_2(\text{old})} \times \frac{MVA_b}{MVA_{\text{old}}} \times \left( \frac{KV_{\text{old}}}{KV_b} \right)^2$$

$$= 0.5 \times \frac{100}{30} \times \left( \frac{220}{220} \right)^2$$

$$= 0.5 \text{ PU}$$

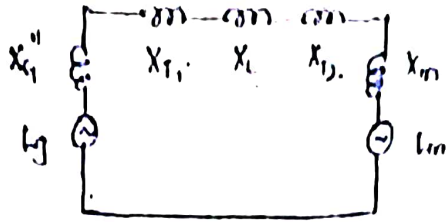
$$X_{M(\text{new})} = X_{M(\text{old})} \times \frac{MVA_b}{MVA_{\text{old}}} \times \left( \frac{KV_{\text{old}}}{KV_b} \right)^2$$

$$= 0.3 \times \frac{100}{50} \times \left( \frac{11}{220} \right)^2$$

$$= 0.407$$

*Handwritten note:*  $X_{\text{new}} = 25 \times \frac{220}{33}$

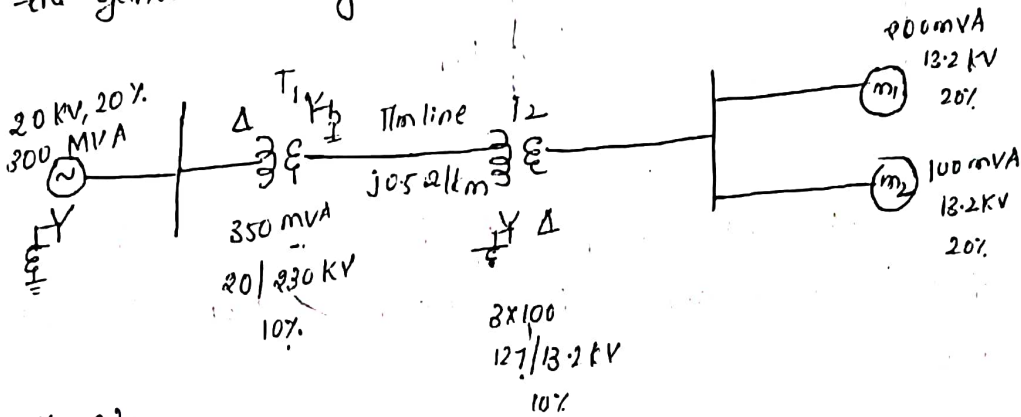
$$= 25 \times \frac{220}{33}$$



$$X_L = j 50 \times \frac{100}{230^2} \rightarrow (X_{100}) \times \frac{\text{MVA}}{\text{kV}^2}$$

$$= j 0.1033$$

Q) A 300 MVA, 20 kV, 3- $\phi$  Generator has sub-transient reactance of 20%, the generator supplied two synchronous motors through a 64 km TL line. Having TLF's at both ends as shown in fig. In this  $T_1$  is 3- $\phi$  transformer &  $T_2$  is made up of three 1- $\phi$  TLF's of rating 100 MVA, 127/13.2 kV, 10% reactance, series reactance of the TL line is  $j 0.5 \text{ ohm/km}$ . Draw the reactance diagram and select the generator rating as base values.



$$X_g = 0.2$$

Sol

$$X_{T1}(\text{pu}) = 0.1 \times \frac{300}{350} \times \left(\frac{20}{20}\right)^2 = 0.0857 \text{ pu}$$

$$X_{L(\text{pu})} = j 0.5 \times \frac{200}{(230)^2} = j 0.00283 \times 64 \quad [\text{for } 64 \text{ km length of TL line}]$$

$$= j 0.1814 \text{ pu}$$

$$X_{T2}(\text{pu}) = 0.1 \times$$

$$V_{ph} = 127 \text{ kV}$$

$$V_L = \sqrt{3} \times 127 = 219.97 \text{ kV}$$

$$X_{T2}(\text{pu}) = 0.1 \times \frac{\text{MVA}_b}{\text{MVA}_{TL}} \times \left(\frac{\text{kV}_{old}}{\text{kV}_b}\right)^2$$

$$= 0.1 \times \frac{300}{300} \times \left(\frac{230}{230}\right)^2$$

$$= 0.0714 \text{ pu}$$

$$0.2 \times \frac{300}{200} \times \left(\frac{13.2}{13.2}\right)^2$$

$$\frac{230}{127} = \frac{127}{13.2}$$

$$n = \frac{230 \times 13.2}{127}$$

$$x_1(\text{pu}) = 0.2 \times \frac{200}{200} \times \left(\frac{13.2}{13.8}\right)^2$$

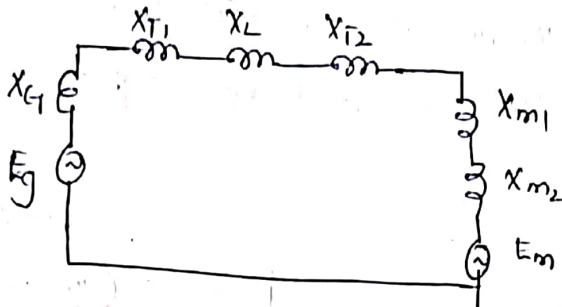
$$\left[ \begin{array}{l} 200 \text{ KV} \rightarrow 13.2 \text{ KV} \\ 200 \text{ KV} \rightarrow \frac{200 \times 13.2}{13.8} \\ \quad \quad \quad = 13.8 \end{array} \right]$$

$$= 0.2 \times \frac{200}{200} \times \left(\frac{13.2}{13.8}\right)^2$$

$$= 0.47 \text{ pu}$$

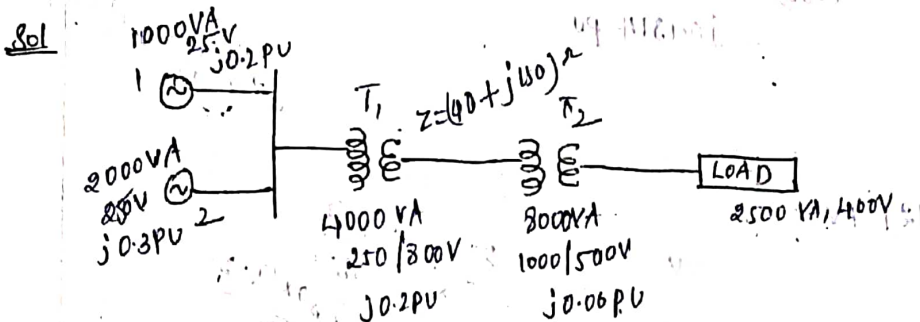
$$x_{m2}(\text{pu}) = 0.2 \times \frac{300}{100} \times \left(\frac{13.2}{13.8}\right)^2$$

$$= 0.548 \text{ pu}$$



- ⑤ A 100 MVA, 33 KV, 3-φ Generator has a sub transient reactance of 15%, the Generator is connected to motor through a line.

- ⑤ A simple power system draw where the simple powersystem represent as common 5000VA and common voltage is 250 KV



Sol

$$\text{Base KVA} = \frac{5000}{1000} = 5 \text{ KVA}$$

$$\text{Base KV} = \frac{250}{1000} = 0.25 \text{ KV}$$

$$X_{PU} m_1 = j0.0x \left(\frac{5}{1}\right) \times \left(\frac{0.25}{0.25}\right)^2$$

$$= j1.0 \text{ pu}$$

$$X_{PU} m_2 = j0.3x \left(\frac{5}{3}\right) \times \left(\frac{0.25}{0.25}\right)^2$$

$$= j0.75 \text{ pu}$$

$$X_{PU} T_1 = j0.2x \left(\frac{5}{4}\right) \times \left(\frac{0.25}{0.25}\right)^2$$

$$= j0.25 \text{ pu}$$

$$X_{(PU)} T_2 \Delta$$

$$\Rightarrow V_B (\text{line}) = 0.25 \times \frac{500}{250} = 0.5 \text{ kV}$$

$$X_{(PU)} L = (40 + j150) \times \frac{5}{0.3^2 \times 1000}$$

$$= (0.3125 + j1.17) \text{ pu}$$

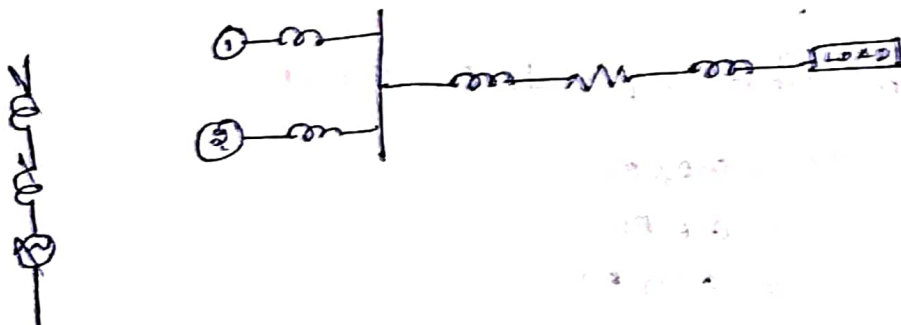
$$\left[ X_{PU(L)} = X_L \times \frac{1.0 \text{ pu}}{(V_B)^2} \right]$$

$$X_{(PU)} T_3 = j0.06 \times \frac{5}{8} \times \left(\frac{0.25}{0.25}\right)^2 = j0.0535 \text{ pu}$$

$$= j0.0535 \text{ pu}$$

$$KVA_{PU} = \frac{2.5}{5} = 0.5 \text{ pu}$$

Active KVA  
Base KVA



- ⑥ The 3-d ratings of a 3 winding transformer are  
 Primary - Y connected, 66 kV, 15 MVA  
 Secondary - Y connected, 13.2 kV, 10 MVA  
 Tertiary - Δ connected, 2.3 kV, 5 MVA  
 Neglecting resistance & leakage impedances of

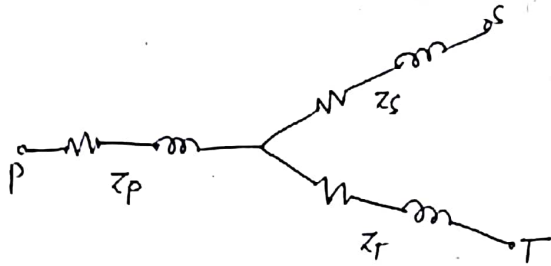
$$Z_{PS} = 0.08 \text{ pu on } 15 \text{ MVA, } 66 \text{ KV base}$$

$$Z_{PT} = 0.10 \text{ pu on } 15 \text{ MVA, } 66 \text{ KV base}$$

$$Z_{ST} = 0.09 \text{ pu on } 10 \text{ MVA, } 13.2 \text{ KV base}$$

Find the pu impedances of the Y connected equivalent circuit for a base of 15 MVA, 66 KV base in the primary circuit.

Sol



\* The KVA and KV ratings of the 3 windings may be different, however the pu impedances in the impedance diagram are expressed on the same base KV.

\* The leakage impedance measured in primary with secondary short circuited and tertiary open is  $Z_{PS} = 0.08$  15 MVA & 66 KV.

\* The leakage impedance measured in secondary with tertiary short circuited and primary open.

$$Z_{ST} = 0.09 \times \frac{15}{10} \times \left(\frac{13.2}{13.2}\right)^2$$

$$= 0.135 \text{ pu}$$

It is seen from equivalent circuit

$$Z_{PS} = Z_p + Z_s = 0.08 \text{ pu}$$

$$Z_{PT} = Z_p + Z_t = 0.1 \text{ pu}$$

$$Z_{ST} = Z_s + Z_t = 0.135 \text{ pu}$$

$$Z_p = \frac{1}{2} [Z_{PS} + Z_{PT} - Z_{ST}]$$

$$= \frac{1}{2} [0.08 + 0.1 - 0.135]$$

$$= 0.0225 \text{ pu}$$

$$z_S = \frac{1}{2} [z_{PS} + z_{ST} - z_{PT}]$$

$$= \frac{1}{2} [0.08 + 0.135 - 0.1]$$

$$= 0.0575 \text{ PU}$$

$$z_T = \frac{1}{2} [z_{PT} + z_{ST} - z_{PS}]$$

$$= \frac{1}{2} [0.1 + 0.135 - 0.08]$$

$$= 0.0775 \text{ PU}$$

## Nature and Causes of faults

\* Faults are caused either by insulation failure (or) by conducting path failure. Failure of insulation results in short circuits which are very harmful as they may damage some equipment of the power system.

\* Most of the faults on transmission & distribution system are caused by over voltages due to lightning and switching surges (or) by external conducting objects falling on overhead lines.

\* Over voltages due to lightning or switching surges cause flash over on the surface of insulators resulting in short circuit which some times causes insulators get punctured.

\* Due to dust or soot in industrial areas, salt in coastal areas, dirt accumulates on the surface of insulators.

\* This reduces the insulation strength and causes flash over.

\* Short circuits are also caused by falling of tree branches (or) other conducting objects falling on overhead transmission lines.

\* Birds also cause faults on overhead lines if their body touches one of the phase and earth (or) neutral.

\* The faults are also caused by snakes, storms, earthquakes etc.

\* If the conductors are broken there is a failure of conducting path and the conductor becomes open circuited

\* If the open conductor falls on the ground it results in short-circuit

\* Opening of one (or) two of three phases makes the system unbalance which sets up the Harmonics in the rotating machines and hence heating of the machines will result. Therefore unbalance of line is not allowed in the normal operation of power system.

\* In case of cables, transformers, generators and other equipment the causes of faults are failure of the insulation due to aging, heat, moisture, over voltage, mechanical damage, accidental contact with ground etc.

### Statistics of Fault Occurrence :-

Equipment	% of total faults
Over head lines	50%
Switch gears	12%
CT's & PT's & control equipment	12%
TLE's	10%
Under ground cables	9%
Generators	7%

### Types of faults :-

Faults are broadly classified into

1) Short circuit faults

2) Open conductor faults



## ① Short Circuit Faults =

Short circuit faults are mainly due to insulation failure which are classified as

(i) Symmetrical faults

(ii) Unsymmetrical faults

### (i) Symmetrical faults =

\* A 3- $\phi$  fault is called as symmetrical fault in which all the three phases are short circuited.

\* All the 3- $\phi$ 's may be short circuited to the ground.

\* They may be short circuited without involving the ground.

A 3- $\phi$  fault is treated as the standard fault to determine the fault level.

### (ii) Unsymmetrical faults =

a) 1- $\phi$  to ground (LG)

b) 2- $\phi$  to ground (LLG)

c) 1- $\phi$  to 1- $\phi$  (LL)

d) 1- $\phi$  open circuit

e) 2- $\phi$  open circuit

\* These are the faults under the category of unsymmetric faults

#### a) LG fault =

\* A short circuit between any one of the phase conductors and earth is called single-phase to ground fault.

\* LG fault may be due to failure of insulation b/n  $\phi$ -phase conductor & earth (or) due to a phase

conductor breaking & falling on ground.

(b) 2- $\phi$  to Ground (or) LLG Fault

\* A short circuit between any two phases and the earth is called double line to ground (or) LLG (or) 2- $\phi$  to ground fault.

(c) phase to phase (or) LL Fault

\* A short circuit b/w any two phases is called LL (or) phase to phase (or) line to line fault.

(d) Open conductor fault

\* This type of faults are caused by break in the conducting path such faults occurred when one or more phase conductors break on a cable joint (or) a joint on the over head line phase.

Due to the opening of one or more phases the unbalanced currents flow in the system there by meeting the rotating machine.

Fault Statistics:  
For the design & application of protective scheme it is very useful to have an idea of occurrence of faults on various elements of the power system.

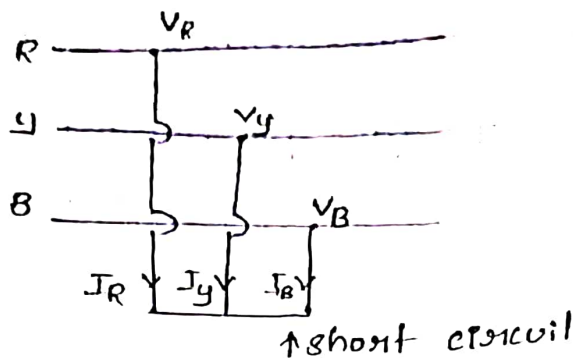
Fault % of Total Fault

LLG (most frequent)	85%	70%
LL	8%	15%
LLG	5%	10%
3- $\phi$ (most severe)	2%	5%

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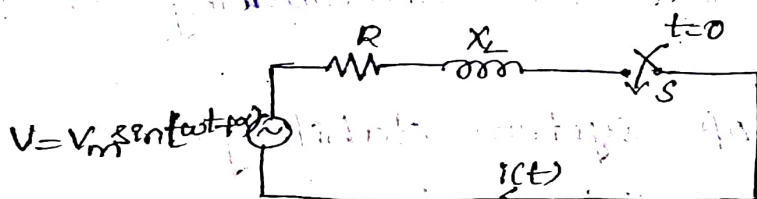
- to minimise the effect of faults and other abnormalities.

### Symmetrical faults on 3- $\phi$ system



- \* It gives rise to symmetrical currents which are equal fault currents in the lines with  $120^\circ$  displacement.
- \*  $I_R, I_Y, I_B$  are equal in magnitude, with  $120^\circ$  phase displacement between them.
- \* Because of balanced nature of fault only one phase should be considered in calculation since the other two phases are also similar.
- \* Symmetrical fault rarely occurs as majority of faults are unsymmetrical in nature.
- \* The symmetrical fault is most severe & imposes more heavy duty on circuit breaker.

### Transients due to short circuit in Transmission Line



- \* Consider the line is unloaded under short condition the line capacitance is neglected & the lines can be

represented by lumped RLC.

\* Let  $i(t)$  is the current in the transmission line under the short circuit (or) fault condition.

\* Assuming that under fault condition the switch is closed at  $t=0$ , At this instant the fault current flows in the circuit.

\* The circuit equation can be written by KVL

$$Ri(t) + L \frac{di(t)}{dt} = V_m \sin(\omega t + \alpha) \quad \text{--- (1)}$$

\* Rearranging the above equation and putting in the form as below i.e., differential form

$$\left(D + \frac{R}{L}\right) i(t) = \frac{V_m}{L} \sin(\omega t + \alpha) \quad \left[ \because D = \frac{d}{dt} \right] \quad \text{--- (2)}$$

\* By Mathematical differential equation method the complementary function of eq (2) is

$$i_c = C e^{-\left(\frac{R}{L}\right)t}$$

\* The particular integral is

$$i_p = \frac{V_m}{\sqrt{R^2 + X_L^2}} \sin\left[\omega t + \alpha - \tan^{-1}\left(\frac{X_L}{R}\right)\right] \quad \text{--- (4)}$$

\* The total solution is

$$i_T = i_c + i_p \quad \text{--- (5)}$$

$$i(t) = C e^{-\left(\frac{R}{L}\right)t} + \frac{V_m}{\sqrt{R^2 + X_L^2}} \sin\left[\omega t + \alpha - \tan^{-1}\left(\frac{X_L}{R}\right)\right] \quad \text{--- (6)}$$

\* Applying the initial conditions, At  $t=0$

$$i(t) = 0$$

$$\text{then } C = \frac{V_m}{|Z|} \sin(\theta)$$

\*  $\therefore$  eq (6) becomes

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \alpha - \theta) + \frac{V_m}{|Z|} e^{-\left(\frac{R}{L}\right)t} \sin(\theta - \alpha)$$

\* In this 1st term is called symmetrical s/c current which is alternating quantity at the instant of fault i.e.,  $t=0$  and its fault current is max. & based on circuit current operation. This current will reduce & fault is cleared.

\* The 2nd term is called unidirectional transient component or offset <sup>current</sup> total s/c current to be unsymmetrical till the transient leakage.

01-03-2017

\* Let us assume the resistance is small i.e.,  $\theta \approx 90^\circ$  & At  $t=0$  the total current  $i_{mm}$  is

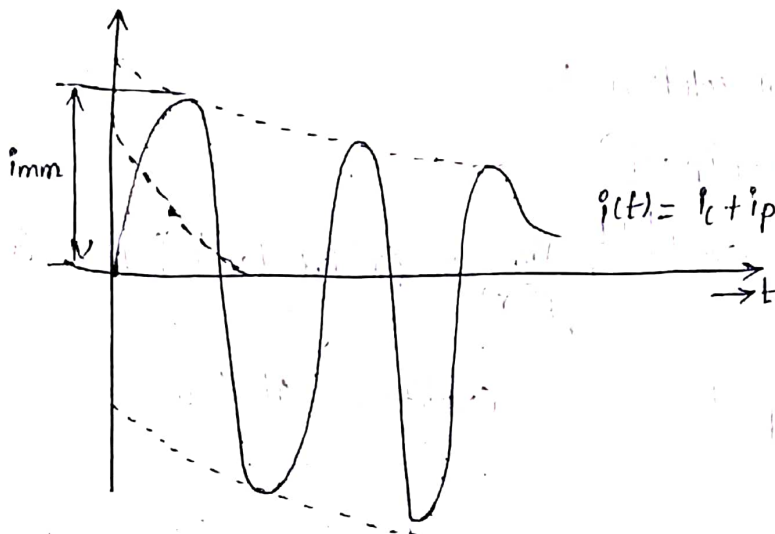
$$i_{mm} = \frac{V_m}{|Z|} \cos \alpha + \frac{V}{|Z|}$$

\* where  $i_{mm}$  is maximum momentary current &

if  $\alpha=0$

$$i_{mm} = 2 \frac{V_m}{|Z|}$$

\* i.e.,  $i_{mm}$  is twice the maximum of symmetrical short circuit current known as doubling effect.

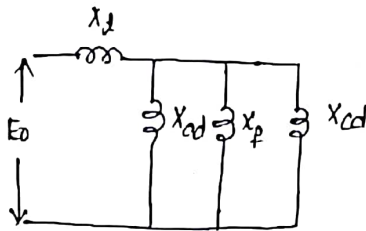


## Transients due to short circuit in 3- $\phi$ Alternators

\* A synchronous generator generates the Alternating voltage. A sudden short circuit occurs on the terminals of it, the following assumptions are followed.

- 1) Armature resistance is neglected
- 2) All the 3- $\phi$ s are short circuited simultaneously
- 3) The machine is under no-load condition before short circuited.

\* The Approximate equivalent circuit of a synchronous generator immediately after short circuit is as shown below figure

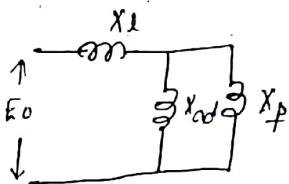


- \* Where  $X_f$  = field winding leakage reactance.  
 $X_{cd}$  = damper winding leakage reactance  
 $X_{ad}$  = Mutual (or) magnetising reactance

$\therefore$  sub transient reactance of the circuit is

$$X_d'' = X_2 + \frac{1}{\frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_{cd}}}$$

\* After a few cycles of short circuit the equivalent circuit of the machine is



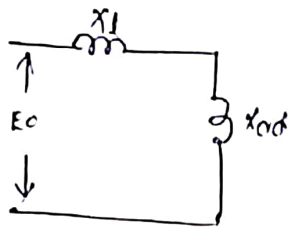
\* The effect of the damper winding and eddy currents in other metallic parts disappear after few cycles of short circuit current, because of large resistances associated with these currents resulting in large damping current.

∴ Direct axis transient reactance is

$$X_d' = X_d + \frac{1}{\frac{1}{X_{ad}} + \frac{1}{X_f}}$$

\* After some time the fault will be cleared then the current will come to steady state condition.

The equivalent circuit of machine is



$$\therefore X_d = X_d + X_{ad}$$

Percentage reactance ÷

\* The reactance of generators, transformers, reactors etc is generally expressed in percentage reactance to permit rapid short circuit calculations which is defined as the percentage of total phase voltage drop of circuit when full load current is flowing

$$\%X = \frac{I_f X}{V} \times 100$$

Where,  $I_f$  - full load current  
 $X$  - reactance per phase  
 $V$  - phase voltage

$$\Rightarrow X = \frac{(\%X) V}{I \times 100}$$

$$= \frac{(\%X) V^2}{100 (VA)}$$

$$= \frac{(\%X) \left(\frac{V}{1000}\right) \left(\frac{V}{1000}\right) \times 10000}{100 \times \left(\frac{VA}{1000}\right)}$$

$$= \frac{(\%X) (KV)^2 \times 10}{(KVA)}$$

$$\%X = \frac{(KVA) X}{10 (KV)^2}$$

$$\therefore \%X = \frac{(KVA) X}{10 (KV)^2}$$

\* If  $X$  is the only reactance element present in the circuit then short circuit current in the circuit is

$$I_{sc} = \frac{V}{X}$$

$$= \frac{V}{\left(\frac{(\%X) V}{100 VA}\right)}$$

$$I_{sc} = I \left[ \frac{100}{\%X} \right]$$

Ex: If percentage reactance of element is 20% and

full load current is 50A then  $I_{sc} = \frac{250A}{1}$

$$I_{sc} = 50 \times \left[ \frac{100}{20} \right]$$

$$= 250A$$

\* % reactance at base KVA is

$$(\%X)_{\text{Base}} = \frac{\text{Base KVA}}{\text{Base stated KVA}} \times \%X \text{ at stated KVA}$$



## Problems

- ① A 1000 KVA T/F with 5% reactance will have reactance of 10% at 2000 KVA.

$$\begin{aligned} \%X \text{ at } 2000 \text{ KVA} &= \frac{2000}{1000} \times 5 \\ &= 10\% \end{aligned}$$

### Short circuit KVA

\* Although the potential at the point of fault is zero, it is normal practice to express the short circuit current in terms of short circuit KVA base on the normal system voltage at the point of fault.

\* It is the product of normal system voltage and short current at the point of fault expressed in MVA or MVA known as short circuit KVA.

$$* \text{ Let } I_{sc} = I \left( \frac{100}{\%X} \right)$$

then s/c KVA of fault is

$$\text{s/c KVA} = \frac{3V I_{sc}}{1000}$$

$$= \frac{3V}{1000} \times I \left( \frac{100}{\%X} \right)$$

$$= \frac{3V I}{10(\%X)}$$

\* So, short circuit KVA is obtained by multiplying KVA rating by  $\frac{100}{(\%X)}$

$$\text{s/c KVA} = \frac{3V I}{1000} \times \frac{100}{\%X}$$

$$= (\text{Base KVA}) \times \frac{100}{\%X}$$

$$\begin{aligned} I_{sc} &= I \left( \frac{100}{\%X} \right) \\ \text{s/c KVA} &= \frac{3V I_{sc}}{1000} \\ &= \frac{3V I}{1000} \times \frac{100}{\%X} \\ &= \frac{3V I}{10(\%X)} \end{aligned}$$

## Short circuit MVA

Consider a supply system of impedance  $Z \Omega$  supplying a load. If the load is suddenly short circuited then

$$I_{sc} (A) = \frac{E_L (V)}{Z (\Omega)} \rightarrow (1)$$

Where  $E_L$  is stated line voltage

If  $E_L$  is assumed to be base voltage and  $I$  is base current then base impedance is

$$Z_b = \frac{E_L}{I} \rightarrow (2)$$

$$Z_{pu} = \frac{Z (\Omega)}{Z_b (\Omega)}$$

$$Z_{pu} = \frac{Z}{\left(\frac{E_L}{I}\right)}$$

$$Z_{pu} = \frac{Z}{E_L} \times I \rightarrow (3)$$

∴ Eq (1) will be written as

$$Z = \frac{E_L}{I_{sc}} \rightarrow (4)$$

$$\therefore Z_{pu} = \left(\frac{E_L}{I_{sc}}\right) \frac{1}{E_L} \times I$$

$$Z_{pu} = \frac{I}{I_{sc}} \rightarrow (5)$$

$$\therefore Z_{pu} = \frac{\text{Base KVA}}{\text{slc KVA}}$$

\* Short circuit KVA for 1- $\phi$  system

$$= \frac{\text{Base KVA (1-}\phi\text{)}}{Z_{pu}}$$

$$* \text{ Short circuit MVA for 1-}\phi \text{ system} = \frac{\text{Base MVA (1-}\phi\text{)}}{Z_{pu}} \times 10^3$$

$$\text{S/c KVA for } 3-\phi = \frac{\sqrt{3} \times \text{Base KVA} \times 10^4}{I_{sc}} \times \frac{10^4}{10^3}$$

$$\text{S/c MVA for } 3-\phi = \frac{\sqrt{3} \times \text{Base MVA} \times 10^3}{I_{sc}}$$

$$= \frac{\sqrt{3} \times (\text{Base MVA}) \times 10^3}{I_{sc}}$$

$$= \frac{\sqrt{3} \times (\text{Base MVA}) \times 10^3}{I_{sc}}$$

$$= \frac{\sqrt{3} \times (E_f \times I_{sc})}{I_{sc}} \times 10^3$$

$$= \sqrt{3} \times E_f \times I_{sc} \times 10^3$$

$$= \sqrt{3} \times \text{rated line voltage in KV} \times$$

Short circuit current  $\times 10^3$  in Amps

03-03-2017

### Fault in levels +

Fault level of a bus is known as short-circuit capacity of that bus. The main objectives of finding short circuit capacity (or) fault levels of bus are

- 1) To determine the size of bus bar
- 2) To determine the interrupting capacity of the circuit breaker connected to the bus
- 3) To determine the strength of the bus  
Strength of the network is nothing but the ability of the bus to maintain the voltage when fault occurs on bus
- 4) To know severity of the bus short circuit stresses

\* The short circuit capacity is defined as the product of magnitude of pre-fault voltage and post-fault current (MVA)

\* It can be defined as the product of rated voltage and short circuit current.

\* Short circuit capacity of 1- $\phi$

$$\Rightarrow E_s \times I_{SC} \times 10^3 \text{ (for 1-}\phi\text{)}$$

$$\Rightarrow \sqrt{3} \times E_s \text{ (kV)} \times I_{SC} \text{ (A)} \times 10^3 \text{ (for 3-}\phi\text{)}$$

[dead short circuit]

Note:

\* Always the voltage at a bus prior to any fault will be 1 pu. which is given as

$$E_{PD} = (1 + j0.0) \text{ pu}$$

Short circuit capacity in (pu) would be numerically equal to short circuit current per unit

$$SCC \text{ (pu)} = I_{SC} \text{ (pu)}$$

\* The following are the main consequences results due to higher short circuit capacity of bus

(i) High current flows in that bus

(ii) Voltage at faulted bus reduced to zero

(iii) The net (or) equivalent impedance as found in b/n faulted bus and reference bus is reduce zero potential bus at i.e., reference bus is zero reduced.

\* Thus, a Bus with an infinite s.c. i.e., zero equivalent impedance which is called as "Infinite Bus".

A 3- $\phi$  10,000 KVA

Problems:-

① A 3- $\phi$ , 10,000 MVA, 11 KV  $\Delta$  Alternator has sub transient reactance of 8%. A 3- $\phi$  short circuit occurs at its terminals. Determine the fault current and fault MVA.

Sol

10,000 MVA

11 KV

%X = 8%

$\Rightarrow$  Base current  $I_b = \frac{\text{MVA Base}}{\text{KV Base}}$

$$I_b = \frac{10000 \text{ MVA}}{\sqrt{3} \times 11} = 524.86 \text{ A}$$

$$= 0.524$$

$$I_{pu} = \frac{V_{pu}}{X_{pu}} = \frac{1}{0.08} = 12.5 \text{ pu}$$

$$I_{pu} = \frac{I_{\text{actual}}}{I_b}$$

$$I_{\text{actual}} = I_b \times I_{pu} = 524.86 \times 12.5$$

$$= 6560.7 \text{ A}$$

$$\Rightarrow \text{Short circuit MVA} = \sqrt{3} \times E_L \times I_{sc} \times 10^{-3}$$

$$= \sqrt{3} \times 11 \times 6560.7 \times 10^{-3}$$

$$= 125 \text{ MVA}$$

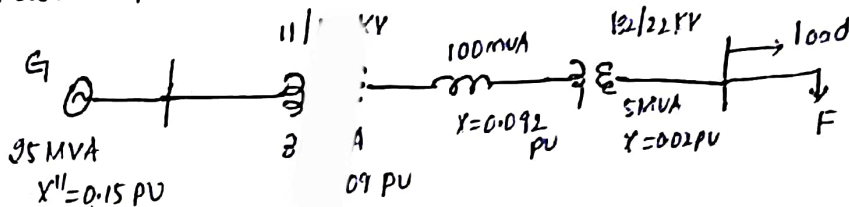
(02)

$$S/c \text{ MVA} = \frac{P_{base} \text{ MVA}}{X_{pu}}$$

$$= \frac{10000}{0.07} = 142.857 \text{ MVA}$$

$$I_{sc} = \frac{S/c \text{ MVA}}{\sqrt{3} \times V_L} = \frac{142.857}{\sqrt{3} \times 11} = 6.5$$

Q) A symmetrical 3- $\phi$  s/c occurs on the 22 kV bus of the circuit shown. Calculate the fault current & fault apparent power.



Sol

Let the base MVA for complete system be 100 MVA of TL line

Base kV on generator side is 11 kV

Base kV on line side is 132 kV

Base kV on load side is 22 kV

$$X_G = j0.15 \times \frac{100}{25} \times \left(\frac{11}{11}\right)^2$$

$$= j0.6 \text{ pu}$$

$$X_{T1} = j0.092 \times \frac{100}{30} \times \left(\frac{11}{11}\right)^2$$

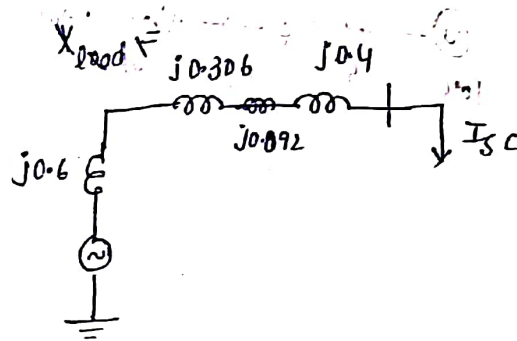
$$= j0.306 \text{ pu}$$

$$X_{L1} = j0.092 \times \frac{100}{132} \times \left(\frac{11}{132}\right)^2$$

$$= j5.28 \times 10^{-4} \text{ pu}$$

$$X_{T2} = j0.02 \times \frac{100}{5} \times \left(\frac{132}{132}\right)^2$$

$$= j0.4 \text{ pu}$$



$$\Rightarrow I_{sc} (\text{pu}) = \frac{1.0 \text{ pu}}{j0.6 + j0.306 + j0.000528 + j0.4}$$

$$= 0.718 \text{ pu}$$

$$I_L = \frac{S_L}{\sqrt{3} V_L}$$

$$= \frac{100}{\sqrt{3} \times 20}$$

$$= 2.624 \text{ A}$$

$$I_{sc} = I_a = I_L \times h$$

$$= 0.1183 \times 2.624$$

$$= 1.885 \text{ A}$$

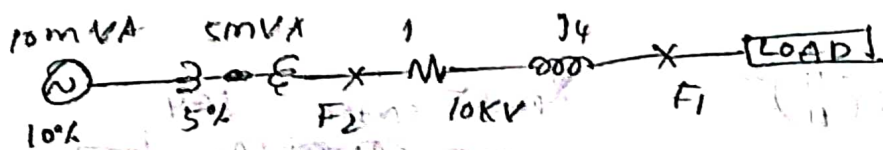
$$\Rightarrow S_{sc} \text{ MVA} = I_{sc} (\text{pu}) \times V (\text{pu})$$

$$= 1.885 \text{ MW}$$

$$= 0.1183 \times 1$$

$$\therefore = 0.1183 \text{ (pu)}$$

③ A 3- $\phi$  Tlm line operating at 10KV and having a resistance of 1 $\Omega$  and reactance of 4 $\Omega$  is connected to generating station busbar through a 5MVA transformer having reactance of 5%. The bus bars are <sup>supplied</sup> separated by 10MVA alternator, having 10% reactance. Calculate the SLC kVA, fed to symmetrical faults between phases, if it occurs i) At load end of Tlm line ii) At HV terminals of transformer



sol  
1885 Am  
SLC MVA -

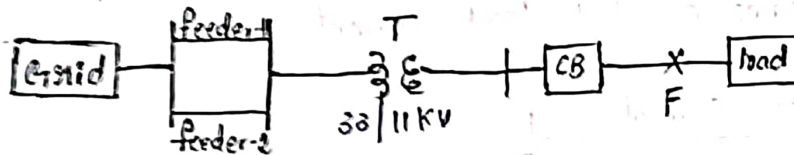
04-03-2017

Q Determine the required MVA rating of the circuit breaker for the system shown. Consider the grid as the infinite bus. choose 6MVA as base.

T/F - 3 $\phi$ , 33/11kV, 6MVA, 0.01 + j0.08 pu imp

Load - 3 $\phi$ , 11kV, 5.800 kVA, 0.8 lag, j0.2 pu imp

Impedance of each feeder - 9 + j18



Sol

Base MVA is 6MVA

pu impedance of T/F is 0.01 + j0.08

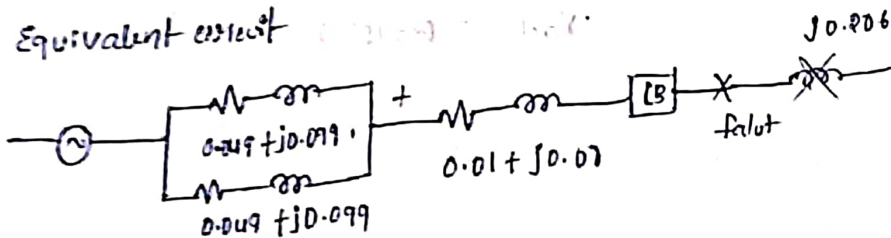
$$X_{pu} \text{ of feeder-1,2 is } = 9 + j18 \times \frac{6}{33^2}$$

$$= (0.049 + j0.099) \text{ pu}$$

$$X_{pu} \text{ of load } = j0.2 \times \frac{6}{5.8} \times \left(\frac{11}{11}\right)^2$$

$$= j0.206$$

Equivalent circuit



Equivalent impedance upto fault condition is

$$= \frac{0.049 + j0.099}{2} + 0.01 + j0.08$$

$$= (0.0345 + j0.179) \text{ pu}$$

$$Z_{eq} = 0.134 \angle 75.08^\circ \text{ pu}$$

$$\text{Short circuit MVA} = \frac{\text{Base MVA}}{(Z_{eq})_{pu}}$$

$$= \frac{6}{0.134}$$

$$= 44.77 \text{ MVA}$$



⑤ Find the values in ohms of the reactance per phase external to a 40 MVA, 10 kV, 50 Hz 3- $\phi$  generator such that the steady state current on short circuit shall not exceed the <sup>short</sup> full load current. The internal synchronous reactance of generator is 5%.

Sol reactance required to limit the short circuit current to 3 times of full load current

$$\frac{\text{full load current}}{\text{SC current}} = \frac{1}{3}$$

$$= 0.125 \text{ pu}$$

Internal synchronous reactance = 5% = 0.05 pu

External reactance =  $0.125 - 0.05$

$$X_{pu} = 0.075 \text{ pu}$$

$$X_{(\text{ohms})} = X_{pu} \times \frac{(KV_b)^2}{MVA_b} = 0.075 \times \frac{10^2}{40}$$

$$X_{act} = 0.375 \Omega$$

## Reactors

\* When every fault occurs in power systems, large currents flow. If the fault is a dead short circuit at the terminals of the busbar, enormous currents flow damaging the equipment and its components. To limit the flow of large currents and under these circumstances current limiting reactors are used which are large coils constructed for high self inductance and so located so that the effect of the fault doesn't affect other parts of the system and is thus localized.

\* From time to time new generating units are added to an existing system to augment the capacity, when this happens the fault level increases and it may become necessary to change the switch gear with proper use of Reactors. Addition of generating units doesn't necessitate the changes in existing switch gear.

## Construction of Reactors

\* These Reactors are built with non-magnetic core so that saturation of core with consequent reduction in inductance and increased short circuit current is avoided.

\* Alternatively it is possible to use the iron core with air gaps included in the magnetic core so that the saturation is avoided.

## Classification of Reactors

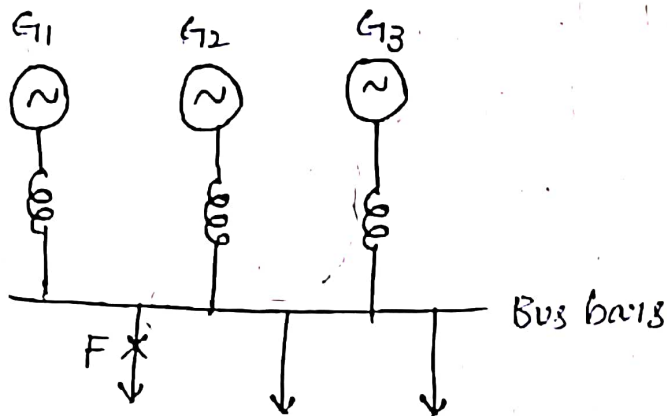
\* Reactors are classified into 3 types based on location

① Generator Reactors

② Feeder Reactors

③ Bus-bar Reactors

## ① Generator Reactor



\* The reactors are located in series with each of the generator as shown in the figure so that currents flowing into the fault 'F' from the generator is limited.

### Drawbacks

\* If the event of the fault occurring on the feeder, the voltage at remaining healthy feeders also may lose synchronism requiring resynchronization later.

\* There is a constant voltage drop in the reactors and also power loss even during the normal operation.

\* Since Modern generators are designed to withstand deadshort circuit at their terminals generator reactors are now a days not used except for 'old units' in operation.

## ② Feeder Reactor

Generators

\* In this method of protection each feeder is equipped with a series reactor.

\* In the event of a fault on any feeder the fault current drawn is restricted by the reactor.

### Drawbacks

\* Voltage drop, power loss still occurs in the reactor for a feeder fault. However the voltage drop occurs only in that particular feeder reactor.

\* Feeder reactors don't offer any protection for busbar faults, which are very rarely occur.

\* As series reactors inherently create voltage drop system voltage regulation will be impaired. Hence they are to be used only in special case such as for short feeders & large cross section.

### ③ Busbar reactors

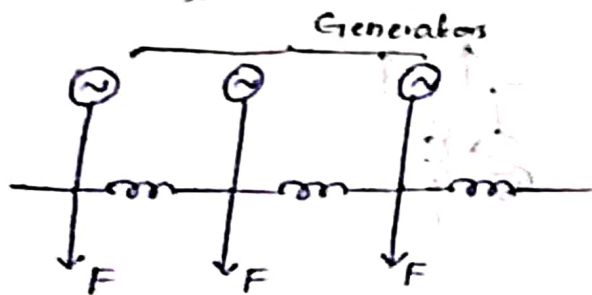


fig ①: Ring system

In the above two methods, the reactors carry full load current under normal operation, the consequent drawback of constant voltage drops & power loss can be avoided by dividing the busbars into sections and interconnect sections through protective reactors by two ways.

#### 1) Ring System

Here each feeder is fed by one generator. Very little power flows across the reactors during normal operation. Hence voltage drop and power loss

are negligible.

\* If a fault occurs on any feeder only the generator to which the feeder is connected is said to feed the fault and other generators are required to feed the fault through the reactor.

(ii) No-Bay System

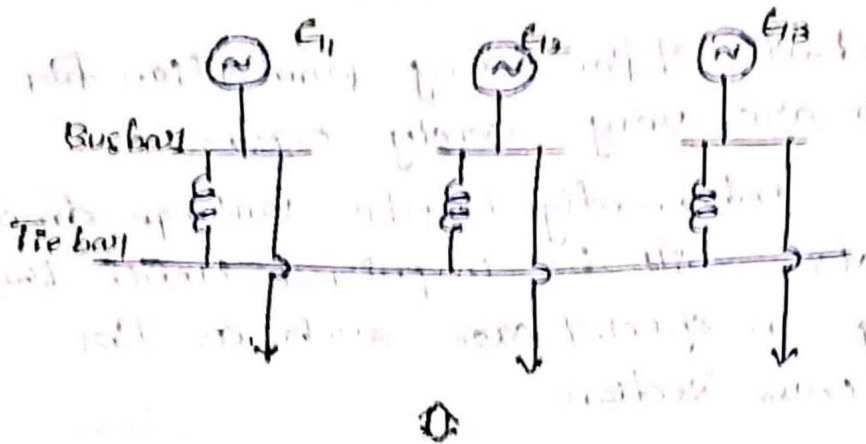
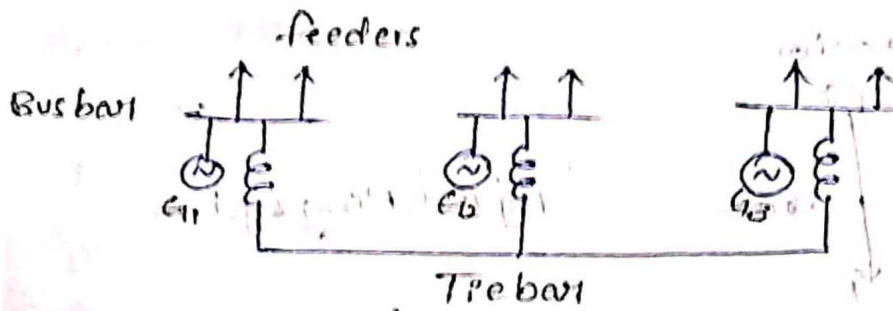


Fig (2)



\* This is an improvement over the ring system. Current fed into a fault has to pass through two reactors in series between the sections.

\* Additional generator may be connected to the system without requiring the changes in the system.

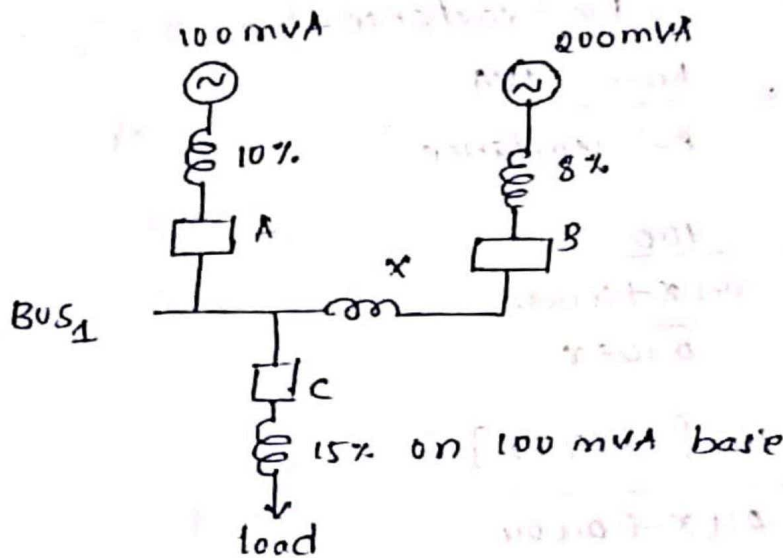
Disadvantages:

\* System requires an additional busbar & Tie busbar.

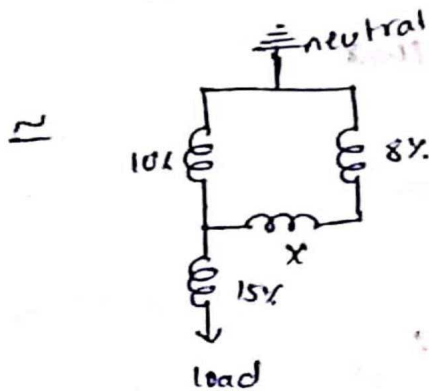
problems

① A 100MVA reactor with 10% reactance and 200MVA generation with 8% reactance on their own bases are connected as shown in figure. The fault level on Bus-1 is to be restricted to 1500 MVA. Calculate on 100 MVA base.

- i) the reactance of bus bar reactor  $x = ?$
- ii) fault level of bus-2 and
- iii) MVA level of circuit breaker 'C'



Sol



← single line diagram

Let Base MVA = 100

PU reactance of generation on 100MVA is  $\frac{10}{100} = 0.1$

PU reactance of generation on 200MVA is  $= \frac{8}{100} \times \frac{100}{200} = 0.04$

PU reactance of feeder =  $\frac{15}{100} \times \frac{100}{100} = 0.15$

i) When fault occurs on Bus-1 :

The reactance of 200MVA generator in series with Busbar reactor  $x$  acts in parallel with

the reactance of 100 MVA generator  
 pu reactance b/w fault point and neutral is

$$= \frac{0.1 \times (0.04 + X)}{0.1 + 0.04 + X}$$

$$= \frac{0.1X + 0.004}{0.14 + X} = \frac{0.1X + 0.004}{0.14 + X}$$

Short circuit MVA =  $\frac{\text{Base MVA}}{\text{pu reactance}}$

$$1500 = \frac{\text{Base MVA}}{\text{pu reactance}}$$

$$= \frac{100}{\frac{0.1X + 0.004}{0.14 + X}}$$

$$1500 = \frac{100 [0.14 + X]}{0.1X + 0.004}$$

$$150X + 6 = 100 [0.14 + 100X]$$

$$50X = 8$$

$$X = 0.16 \text{ pu}$$

ii) When fault is on Bus-2:

When fault occurs on Bus-2, the reactance of 100 MVA generator in series with bus bar reactor act in parallel with the reactance of 200 MVA generator.

∴ The pu reactance b/w fault point and the neutral

$$R_s = \frac{(0.1 + X) \times 0.04}{(0.1 + X) + 0.04}$$

$$= \frac{(0.1 + X) \times 0.04}{0.14 + X}$$

where  $X = 0.16$

$$X_1 = 0.0346 \text{ pu}$$

- fault level of bus-2

$$= \frac{\text{Base MVA}}{\text{pu reactance}}$$

$$= \frac{100}{0.0346}$$

$$= 2885 \text{ MVA}$$

(iii) MVA rating of circuit breaker = C

pu reactance from the neutral bus to the

- fault point:

$$X_{eq} = 0.15 + \frac{0.1 \times (0.04 + 0.16)}{0.1 + 0.04 + 0.16}$$

$$= 0.216 \text{ pu}$$

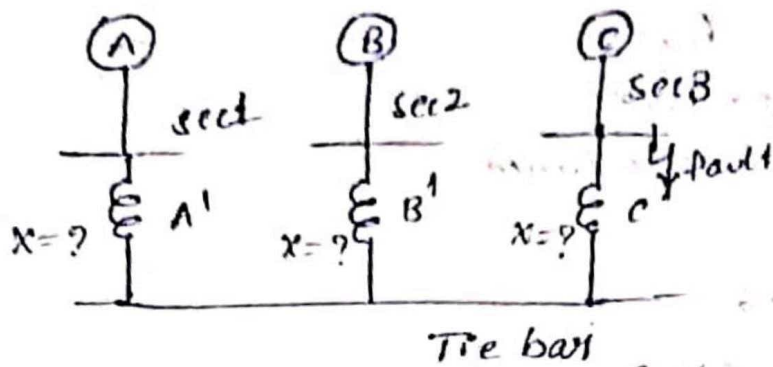
$$\text{MVA rating} = \frac{\text{Base MVA}}{\text{pu reactance}}$$

$$= \frac{100}{0.216}$$

$$= 461.6 \text{ MVA}$$

③ The Main Busbars in a generating station are divided into three sections each section being connected to Tie bar by a similar reactor of 20 MVA, 3- $\phi$ , 50Hz, 11KV generator with a short circuit reactance of 15% is connected to each section bus-bar. When a short circuit takes place between the phases of one of the section busbar. The voltage of the remaining sections falls to 55% of normal value. Calculate the reactance of each reactor.

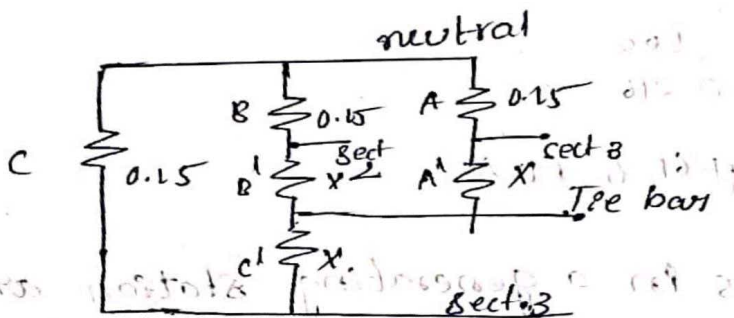




- \* Assume fault is occurred on sect-3
- \* A & A' are in series and are in parallel with B & B' in series
- \* ∴ The reactances of A, B, A', B' b/n the tie bar and neutral

$$= \frac{0.15 + X}{2}$$

$$= [0.5X + 0.075] \text{ pu}$$



- \* The reactance from neutral to fault via tie bar is  $[0.5X + 0.075 + X]$
- \* The voltage of section 1, 2 drops 55% i.e., 0.55 times of the normal value, therefore times of normal value is drop in reactance of A & B is

$= \frac{0.15}{2}$  is 0.45 times the total reactance from neutral to fault

$$\frac{0.15}{2} = 0.45 (1.5X + 0.075)$$

$$= 0.675X + 0.03375 = 0.075$$

$$X = \frac{0.075 - 0.03375}{0.675}$$

$$X = 0.0611 \text{ pu}$$

\* The reactance of each reactor is

$$\frac{\text{pu reactance} \times \text{kv}^2}{\text{base mVA}}$$

$$= \frac{0.0611 \times 11^2}{20}$$

$$X = \underline{\underline{0.372}}$$

03/03/2017

## Symmetrical Component Transformation

\* The system becomes unbalanced due to short circuit faults i.e., unsymmetrical faults and unbalanced loads.

\* Analysis under unbalanced condition has to be carried on a 3- $\phi$  basis.

\* A more convenient method of analysing the unbalanced operation is through the symmetrical components. where the 3- $\phi$  voltages and currents which may be unbalanced are transformed into 3 sets of balanced voltages and currents respectively called Symmetrical components (or) sequence components.

\* An unbalanced system of 'n' phases can be resolved into 'n' system of balanced phases. i.e. A set of 3 unbalanced vectors can be resolved into 3 sets of balanced vectors.

① Symmetrical components.

②

③

## Symmetrical Components

\* According to Fortescue Theorem 3 unbalanced phases of 3- $\phi$  system can be resolved into 3 component set of balanced phases

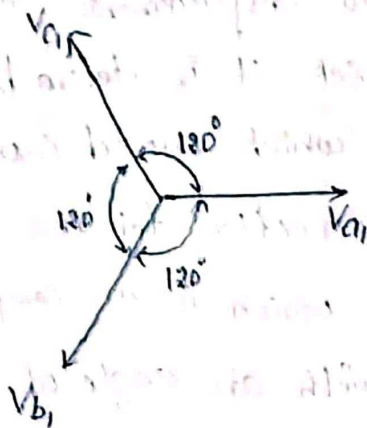
① positive sequence components

② Negative sequence components

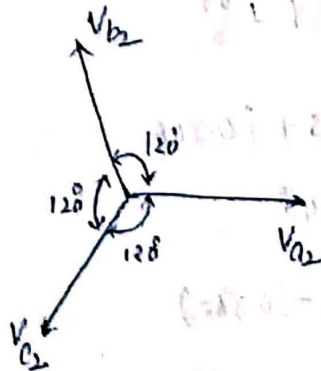
### ⑤ Zero Sequence Components

#### ① Positive Sequence Components

\* A set of 3 phasors equal in magnitude and displaced each other by  $120^\circ$ . And having the same phase sequence as that of the original phasors which are denoted as  $V_{a1}, V_{b1}$  &  $V_{c1}$ .

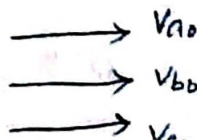


#### ② Negative Sequence Components



\* A set of 3 phasors equal in magnitude and displaced each other by  $120^\circ$  and having the phase sequence opposite to the original phasors.

#### ③ Zero Sequence Components



• The total of 3 phases is equal to 360 degrees. If we divide this by 3, we get 120 degrees. This is the angle between any two phases.

Complex representation of 3 phases

• The three symmetrical components through which we can represent 3 phase displacement in position of magnitude component are it is desirable to choose a reference phase which should cause 120 degree rotation in anticlockwise direction. For this purpose we use a complex number which is an complex number of unit magnitude to add an angle of 120 degrees.

$$\begin{aligned}
 1 &= 1 \angle 0^\circ \\
 &= \cos 0^\circ + j \sin 0^\circ \\
 &= 1 + j0 \\
 a &= 1 \angle 120^\circ \\
 &= \cos 120^\circ + j \sin 120^\circ \\
 &= -0.5 + j0.866 \\
 a^2 &= 1 \angle 240^\circ \\
 &= \cos 240^\circ + j \sin 240^\circ \\
 &= -0.5 - j0.866
 \end{aligned}$$

- 120 degrees
- 240 degrees

Symmetrical Component Transformation

• Let the 3 phases represented by a, b, c with phase sequence a, b, c. Then phase sequence is a, b, c.

\* Let the subscripts 0, 1, 2 denotes zero, positive and negative sequence respectively.

\* If  $V_a, V_b$  &  $V_c$  represents an unbalance set of phasor voltage the three balanced set of phases voltages are

$$V_{a0}, V_{b0}, V_{c0} \rightarrow \text{zero}$$

$$V_{a1}, V_{b1}, V_{c1} \rightarrow \text{+ve}$$

$$V_{a2}, V_{b2}, V_{c2} \rightarrow \text{-ve}$$

\* Each of original unbalance phasor is equal to sum of the sequence components

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{b0} + V_{b1} + V_{b2}$$

$$V_c = V_{c0} + V_{c1} + V_{c2}$$

\* Consider the <sup>+ve</sup> sequence components

$$V_{a1} = V_{a1} \angle 0^\circ = V_{a1} \angle 0^\circ$$

$$V_{b1} = \alpha^2 V_{a1} = V_{a1} \angle 240^\circ$$

$$V_{c1} = \alpha V_{a1} = V_{a1} \angle 120^\circ$$

\* Consider the <sup>-ve</sup> sequence components

$$V_{a2} = V_{a2} \angle 0^\circ$$

$$V_{b2} = \alpha V_{a2} = V_{a2} \angle 120^\circ$$

$$V_{c2} = \alpha^2 V_{a2} = V_{a2} \angle 240^\circ$$

\* Consider the zero sequence components

$$V_{a0} = V_{b0} = V_{c0}$$

\* Now unbalance set of phasor voltages become balance set of voltages as below

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$

$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

\* Now it can be write in matrix form as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [A] \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\therefore [A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = A^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a0} = \frac{1}{3} (V_a + V_b + V_c)$$

$$V_{a1} = \frac{1}{3} (V_a + \alpha V_b + \alpha^2 V_c)$$

$$V_{a2} = \frac{1}{3} (V_a + \alpha^2 V_b + \alpha V_c)$$

\* Ify 
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

we get as

$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c]$$

$$I_{a1} = \frac{1}{3} [I_a + \alpha I_b + \alpha^2 I_c]$$

$$I_{a2} = \frac{1}{3} [I_a + \alpha^2 I_b + \alpha I_c]$$

### Power in Symmetrical Components :-

\* The total complex power flowing into 3- $\phi$  circuit in all the 3 lines a, b, c is

$$S = P + jQ$$

$$= V_a I_a^* + V_b I_b^* + V_c I_c^*$$

\* Where

$V_a, V_b, V_c$  are phase voltage

$I_a, I_b, I_c$  are phase currents.

\* Power in Matrix form as

$$S = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

(DS1)

$$S = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

\* Let  $V = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$ ,  $I = \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$

\*  $\therefore S = [AV]^T [AI]^*$

where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$S = A^T V^T A^* I^*$$



\* where  $A^T = A$

$$A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

\*  $\therefore S = AV^T A^* I^*$

$$S = AA^* V^T I^*$$

\* Now  $AA^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$

$$= \begin{bmatrix} 3 & 1+\alpha+\alpha^2 & 1+\alpha^2+\alpha \\ 1+\alpha^2+\alpha & 1+\alpha^3+\alpha^3 & 1+\alpha^4+\alpha^2 \\ 1+\alpha+\alpha^2 & 1+\alpha^2+\alpha^4 & 1+\alpha^3+\alpha^3 \end{bmatrix}$$

$$AA^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I$$

\*  $\therefore S = [V^T] 3I I^*$

$$= 3 \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} I_{a0}^* \\ I_{a1}^* \\ I_{a2}^* \end{bmatrix}$$

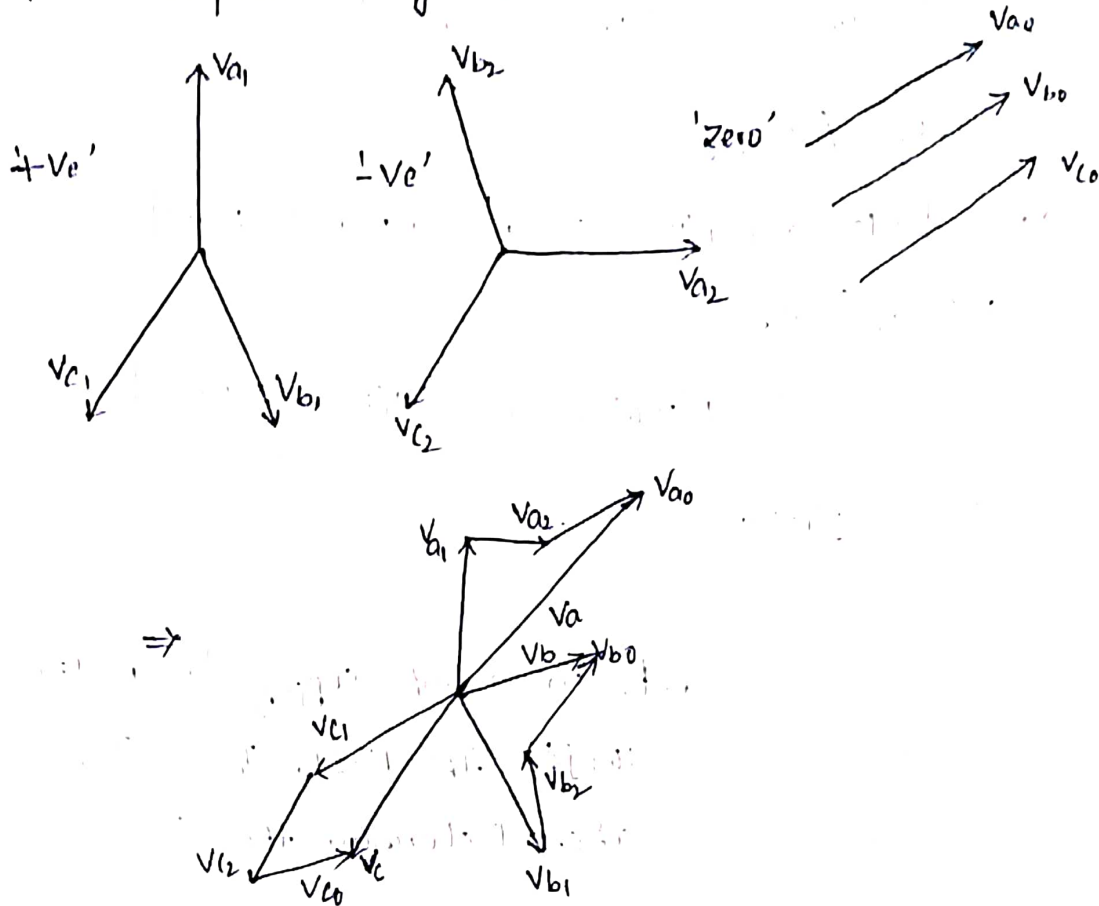
$$= 3 [V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*]$$

$$= 3V_a I_a \cos \phi$$

$$S = 3V_a I_a \cos \phi_0 + 3V_a I_{a1} \cos \phi_1 + 3V_a I_{a2} \cos \phi_2$$

$\therefore$  The sum of powers of 3 symmetrical components equal to a-p power. So symmetrical power component is power invariant

\* The sequence diagrams are



### Problem\*

① The line to ground voltages on HV side of a step-up Transformer are 100 kV, 33 kV, 38 kV on phases A, B & C respectively, the voltage of phase-A lead the phase of 'B' by 100 degrees and lags that of 'C' by  $175.5^\circ$ . Determine the symmetrical components of the voltage phasors

Sol

To find

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

- Given

$$V_a = 100 \angle 0^\circ$$

$$V_b = 33 \angle -100^\circ$$

$$V_c = 38 \angle 176.5^\circ$$

$$V_{a0} = \frac{1}{3} (100 \angle 0^\circ + 33 \angle -100^\circ + 38 \angle 176.5^\circ)$$

$$= 21.304 \angle -28.175^\circ$$

$$V_{a1} = \frac{1}{3} (100 \angle 0^\circ + 1 \angle 120^\circ \times 33 \angle -100^\circ + 1 \angle 240^\circ \times 38 \angle 176.5^\circ)$$

$$= 52.64 \angle 15.788^\circ$$

$$V_{a2} = \frac{1}{3} (100 \angle 0^\circ + 1 \angle 240^\circ \times 33 \angle -100^\circ + 1 \angle 120^\circ \times 38 \angle 176.5^\circ)$$

$$= 30.85 \angle -7.945^\circ$$

② The phase currents in a 3- $\phi$  supply to an unbalanced load are  $I_a = 10 + j20$ ,  $I_b = 12 - j1$ ,  $I_c = -3 - j5$ , the phase sequence is abc. Determine the sequence components of currents.

$$I_{a0} = \frac{1}{3} (I_a + I_b + I_c) =$$

$$I_{a1} = \frac{1}{3} (I_a + \alpha I_b + \alpha^2 I_c)$$

$$I_{a2} = \frac{1}{3} (I_a + \alpha^2 I_b + \alpha I_c)$$

$$I_a = 10 + j20$$

$$I_b = 12 - j1$$

$$I_c = -3 - j5$$

$$I_{a0} = 5.83 \angle -120.96^\circ$$

$$I_{a1} = 7.867 \angle 36.38^\circ = 6.33 + j4.66$$

$$I_{a2} = 0.679 + j11.99$$

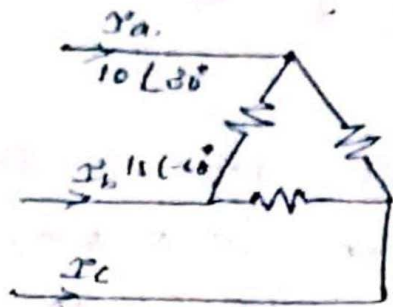
$$= 12.014 \angle 86.75^\circ$$

$$I_{a0} = 2.992 \angle 33.33^\circ$$

$$= 4.45 \angle 118.05^\circ$$

⑤ A Delta connected balanced resistive load is connected across an unbalanced 3- $\phi$  supply as shown with current in line a & b specified find the symmetrical components of the currents.

Sol



$$I_{00} = \frac{1}{3} (I_a + I_b + I_c)$$

$$I_{a1} = \frac{1}{3} (I_a + \alpha I_b + \alpha^2 I_c)$$

$$I_{a2} = \frac{1}{3} (I_a + \alpha^2 I_b + \alpha I_c)$$

As since the load is balanced

$$I_a + I_b + I_c = 0$$

$$I_c = -(I_a + I_b)$$

$$= 18.02 \angle 153.69^\circ$$

$$I_{a0} = \frac{1}{3} (10 \angle 30^\circ + 15 \angle -60^\circ + 18.02 \angle 153.69^\circ)$$

$$= 2.320 \times 10^{-3} - j1.139$$

$$= 2.585 \times 10^{-3} \angle -26.15^\circ$$

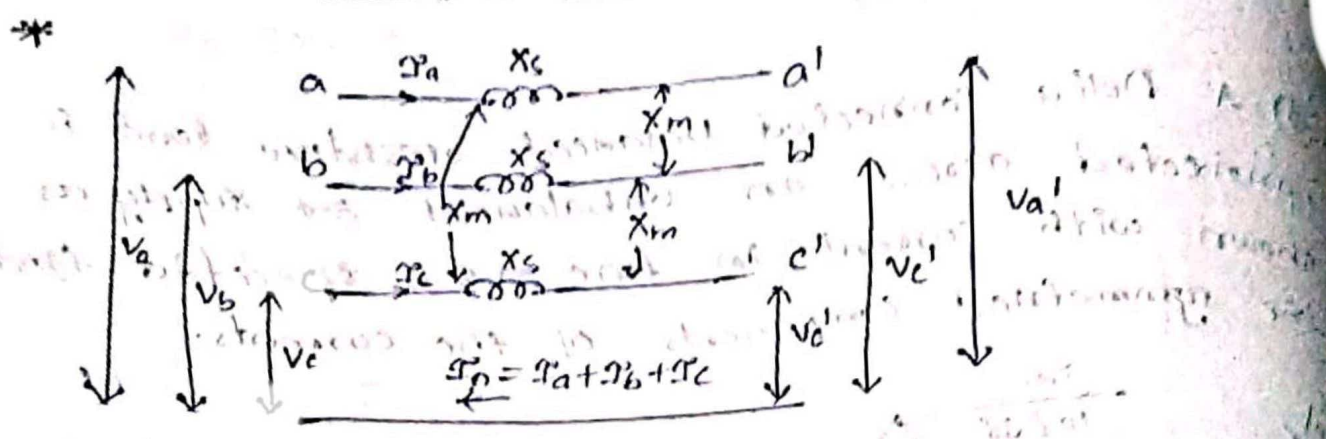
$$I_{a1} = \frac{1}{3} (10 \angle 30^\circ + 1(120 \times 15 \angle -60^\circ) + 1(240 \times 18.02 \angle 153.69^\circ))$$

$$= 10.384 + j9.328$$

$$= 13.959 \angle 41.93^\circ$$

$$I_{a2} = -1.72 - j4.32 = 4.65 \angle -111.75^\circ$$

# Sequence Impedances of Transmission Lines



\* A 3- $\phi$  circuit of a Transmission line as shown in figure carrying unbalanced current  $I_a, I_b$  &  $I_c$ , the return path for the  $I_n$  is sufficiently away from the mutual effect can be neglected.

\* Applying KVL for 'a' phase

$$V_a - V_{a'} = j I_a X_s + j I_b X_m + j I_c X_m$$

for 'b' phase

$$V_b - V_{b'} = j I_b X_s + j I_a X_m + j I_c X_m$$

for 'c' phase

$$V_c - V_{c'} = j I_c X_s + j I_a X_m + j I_b X_m$$

\* In Matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_{a'} \\ V_{b'} \\ V_{c'} \end{bmatrix} = \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\Rightarrow [V_p - V_p'] = [Z] [I_p]$$

\* Where  $V_p$  &  $I_p$  are phase voltages and currents  
 Let  $V_s$  &  $I_s$  are sequence voltages & sequence currents.

\* The above equation can be write in sequence components as.

$$[A] [V_s - V_s'] = [Z][A] [I_s]$$

$$[V_s - V_s'] = [A]^{-1} [Z][A] [I_s]$$

$$[V_s - V_s'] = j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix} [I_s]$$

$$[V_s - V_s'] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} [I_s]$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} - \begin{bmatrix} V_{a0}' \\ V_{a1}' \\ V_{a2}' \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

where  $Z_0 = X_s + 2X_m$  [zero sequence]

$Z_1 = X_s - X_m$  [ +ve sequence]

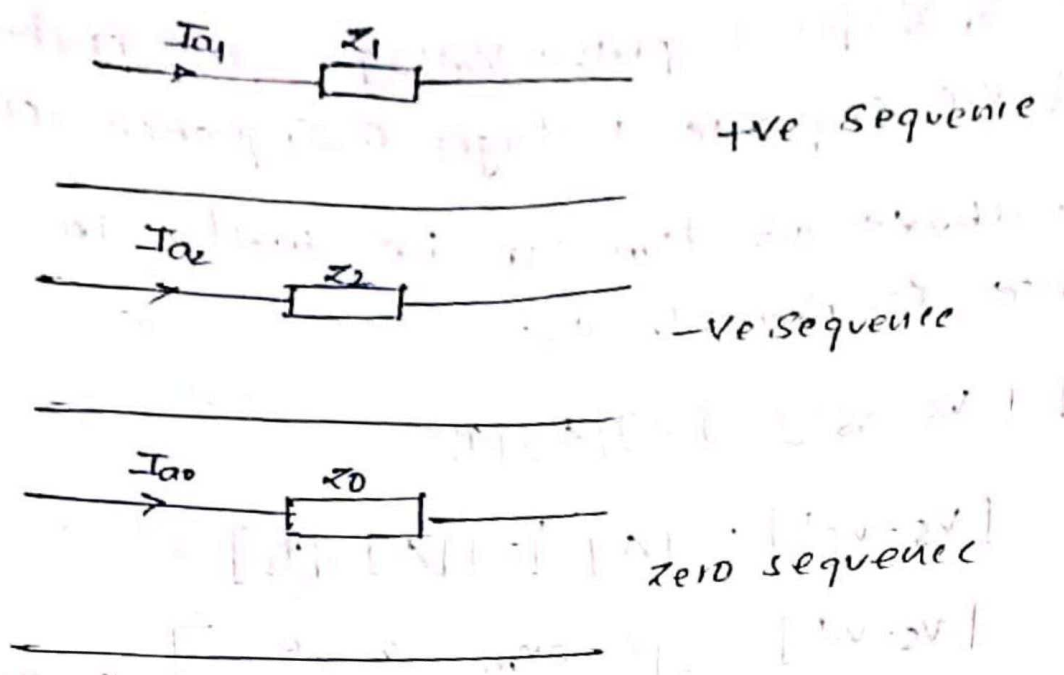
$Z_2 = X_s - X_m$  [ -ve sequence]

\* From the above equations positive & negative sequence impedances are equal.

\*  $Z_0 > Z_1$  &  $Z_2$

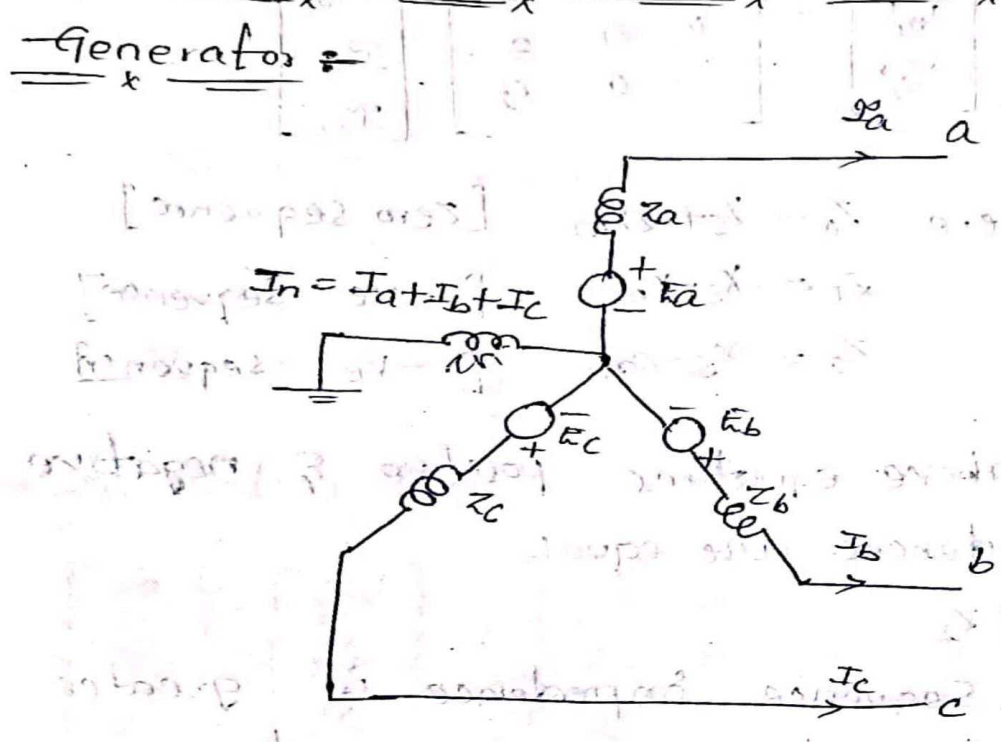
i.e., zero sequence impedance is greater than positive & negative sequence impedance.

\* Equivalent network as shown in figure



- \* There is no mutual impedance i.e.,  $Z_m = 0$  in the Transmission line.
- \* If there is no mutual impedance i.e.,  $X_m = 0$
- $\therefore Z_0 = Z_1 = Z_2 = X_s$

Sequence Impedance Networks of a Synchronous Generator



- \* Figure shows an Unloaded Synchronous Generator whose neutral is grounded through an impedance  $Z_n$ .  $E_a, E_b$  &  $E_c$  are induced EMFs of 3 phases.
- \* If Any fault occurs on the terminals of Alternator or the currents  $I_a, I_b$  &  $I_c$  flow in the lines.
- \* Whenever fault involves ground current  $I_n$  flows through ground impedance  $Z_n$ .
- \* Unbalanced currents can be resolved into balance sequence components i.e.,  $I_{a0}, I_{a1}, I_{a2}$ .
- \* Because of winding symmetry, currents of particular sequence provided, the voltage drop at the sequence only.
- \* There is no coupling between equivalent circuits of various sequences.

### Positive Sequence

- \* The Synchronous Generator has symmetrical winding. Therefore the induced EMFs are positive sequence but 've' and 'zero' sequence voltages are zero.
- \* When the machine carries the 've' sequence currents only, the Armature reaction flux caused by positive sequence currents rotates at synchronous speed in the same direction as that of rotor.
- \* The machine equivalently offers a direct axis reactance whose value reduces from  $X_d''$  to  $X_d'$  and finally to steady state reactance  $X_d$ .
- \* The Armature resistance is assumed to be zero, the positive sequence impedance is assumed to be zero  $Z_0 = jX_d''$ ,  $Z_1 = jX_d'$ ,  $Z_2 = jX_d$



\* The positive sequence impedance of generator is

$$Z_1 = jX_d''$$

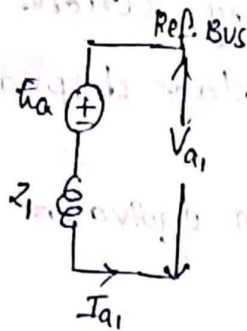
$$Z_2 = jX_d'$$

$$Z_3 = jX_d$$

$$X_d'' < X_d' < X_d$$

\*  $Z_n$  doesn't appear in the model. +ve sequence network as  $I_n = 0$  for +ve sequence currents.

\*  $\therefore$  3- $\phi$  +ve sequence n/w model of synchronous machine is as shown.



$$V_{a1} \Rightarrow V_{a1} = -I_{a1} Z_1 + E_a$$

$$V_{a1} = E_a - I_{a1} Z_1$$

### Negative Sequence

\* A Syn machine doesn't generate any -ve sequence voltage (or) syn generator has zero, +ve seq induced with the flow of negative seq currents in the stator the rotating magnetic flux is created which rotates in opposite direction to that of the +ve seq speed at double synchronous speed w.r.t the rotor.

\* The currents at double the stator frequency are therefore induced in stator field and damper windg.

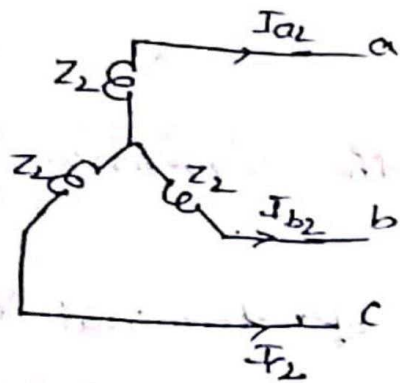
\* The '-ve' seq. MMF moves alternatively in direct and Quadrature Axis sets up varying the Armature reaction effect.

\* ∴ '-ve' seq. reactance is taken as, the avg of direct and quadrature Axis sub transient reactance i.e.,

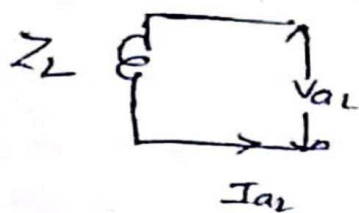
$$X_2 = \left[ \frac{X_d'' + X_q''}{2} \right] = Z_2$$

$$|Z_2| < |Z_1|$$

\* -ve sequence 3-φ model.



\* 1-φ model is



$$\therefore V_{a2} = -I_{a2} Z_2$$

### Zero Sequence Impedance Network

\* The zero sequence Induced Voltages are also zero in synchronous generator (or) the generator doesn't generate negative sequence voltage, the flow of zero sequence currents is created by 3 MMFs

\* which are in phase but are distributed in space phase by  $120^\circ$ .

\* The motor windg's present leakage reactance on the flow of zero sequence currents.

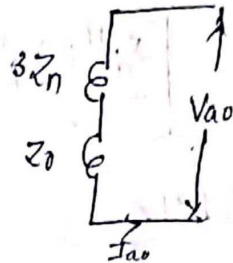
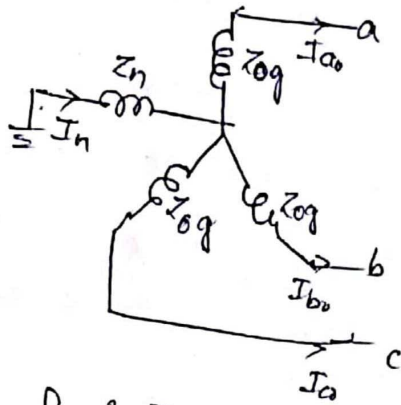


Fig:  $I_n$  3- $\phi$

Fig:  $I_n$  3- $\phi$

\* The currents flowing in the impedance  $Z_n$

$$I_n = 3I_{a0}$$

\* The zero seq. voltage of terminal 'a' w.r.t to ground is

$$V_{a0} = 3Z_n I_{a0} + Z_{og} I_{a0}$$

$$= -3Z_n I_{a0} - Z_{og} I_{a0}$$

$$V_{a0} = -(3Z_n I_{a0} + Z_{og} I_{a0})$$

$$\therefore (Z_0 = 3Z_n + Z_{og})$$

$$V_{a0} = -Z_0 I_{a0}$$

### Sequence Impedance at Networks of Transformers

\* The +ve seq. series impedance of a transformer equals to its leakage impedance.

Since transformer is static device the leakage impedance doesn't change with alteration of phase sequence of balanced applied voltage.

1) - dence doesn't change with alteration of phase sequence of balanced applied voltage.

\* The negative sequence impedance of a transformer is also equal to leakage impedance of T.F.

$$Z_1 = Z_2 = Z_{\text{leakage}}$$

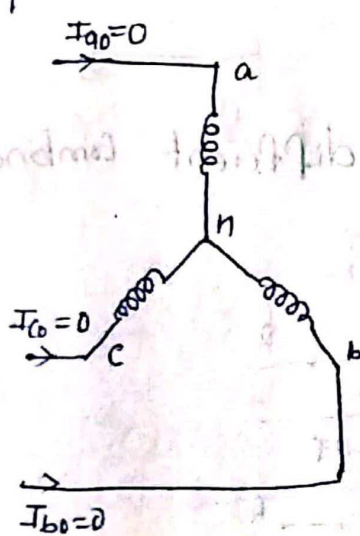
\* A transformer offers a zero sequence impedance which may differ slightly from corresponding positive and negative sequence values.

Zero Sequence Network Impedance of Transformer:

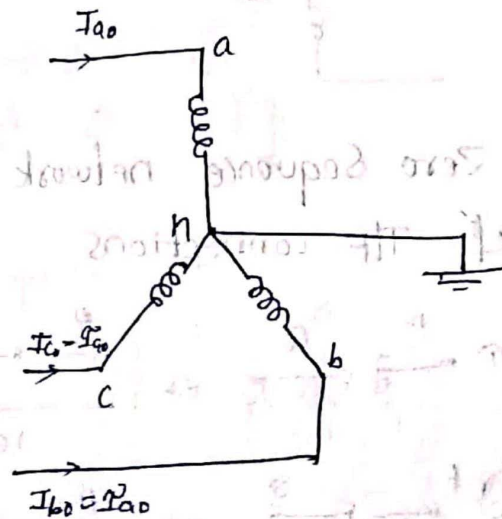
Before considering the zero sequence N/w's of various types of T.F's connections three important observations are made.

\* When magnetising current is neglected transformer primary would carry current, only if there is current flow on the secondary side. i.e., transformer is loaded.

\* zero sequence currents can flow in the legs of star connection only if the star point is grounded which provides the necessary return path for zero sequence currents.



Ungrounded

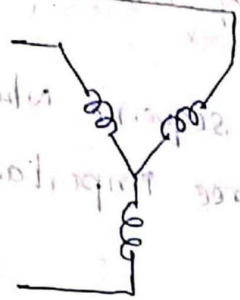


grounded.

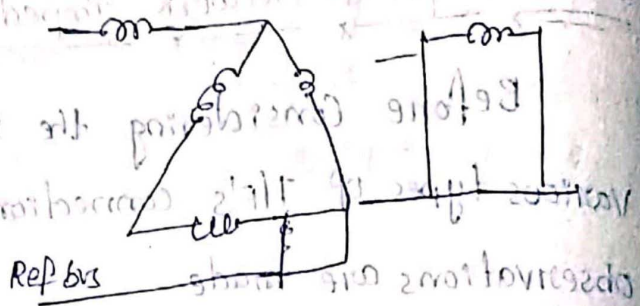
No zero sequence currents can flow in the lines, connected to a delta connection and no return path is available in this currents.

Zero sequence currents flow in the legs of delta, such currents are caused by presence of zero sequence in  $\Delta$ -connection.

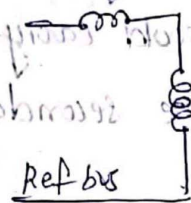
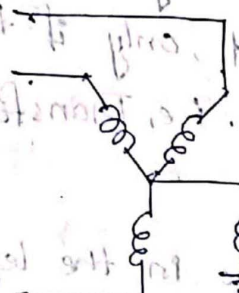
Y connection



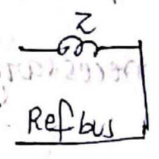
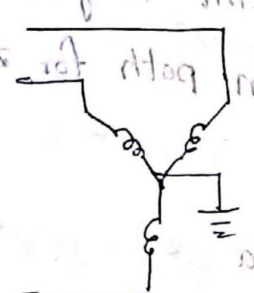
Zero sequence connection



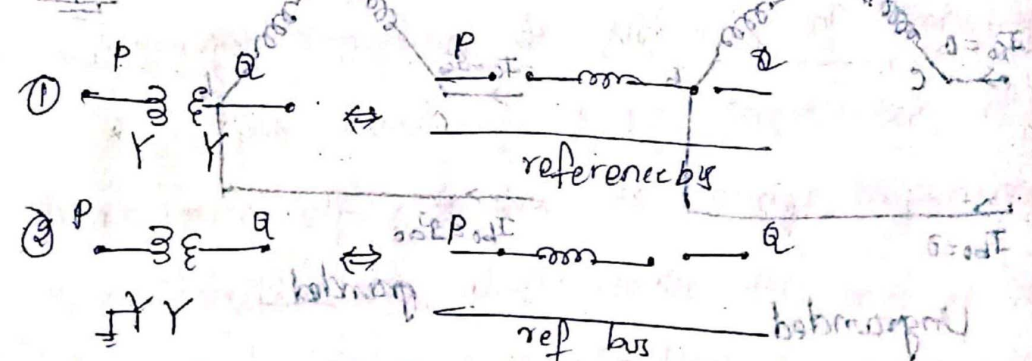
\* When inductive current is supplied from transformer primary current flow on secondary side.  $3\omega$  is added.

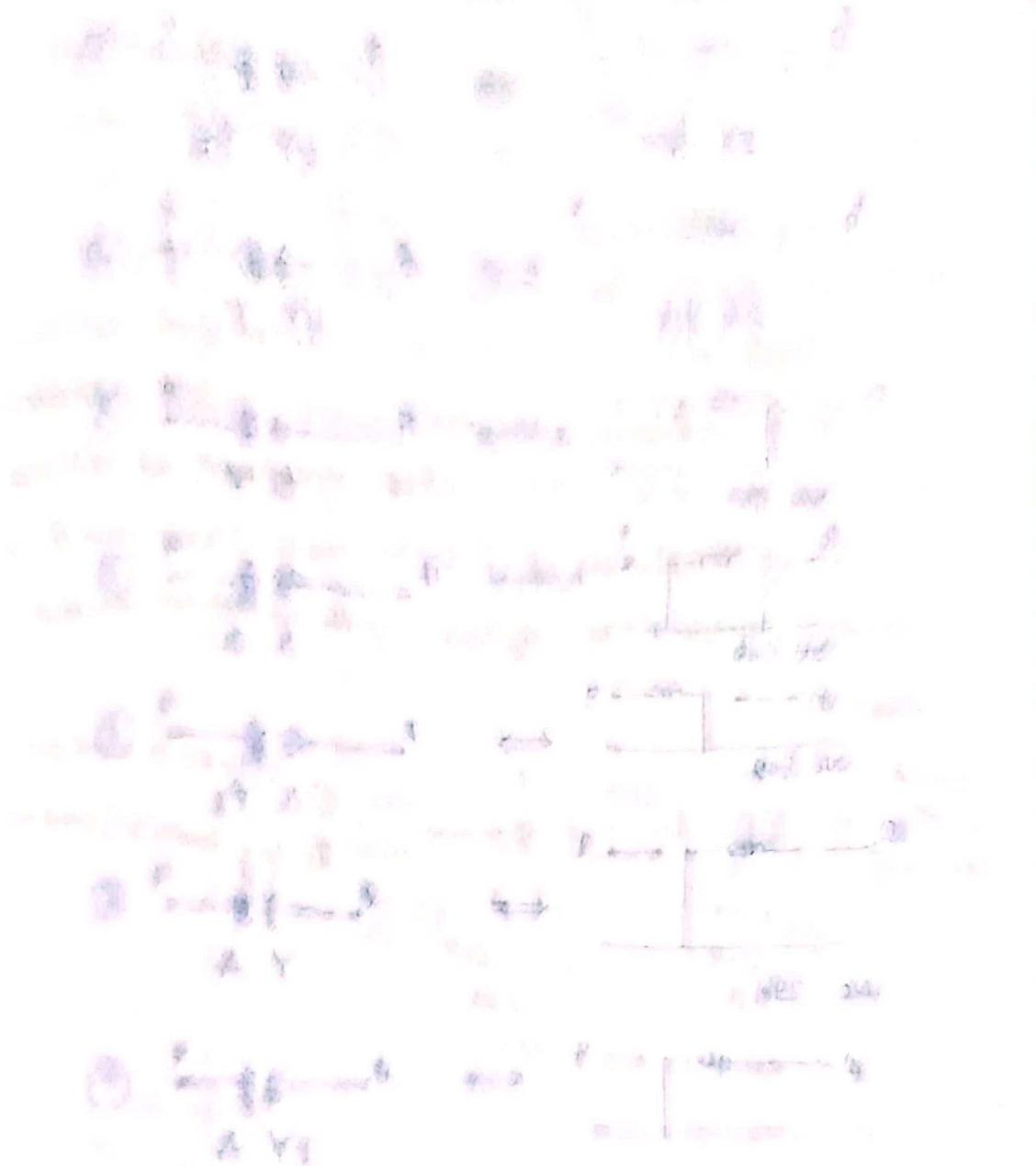


\* Zero sequence currents can flow in the legs of star connection only if the star point is grounded which provides the return path for zero sequence currents.

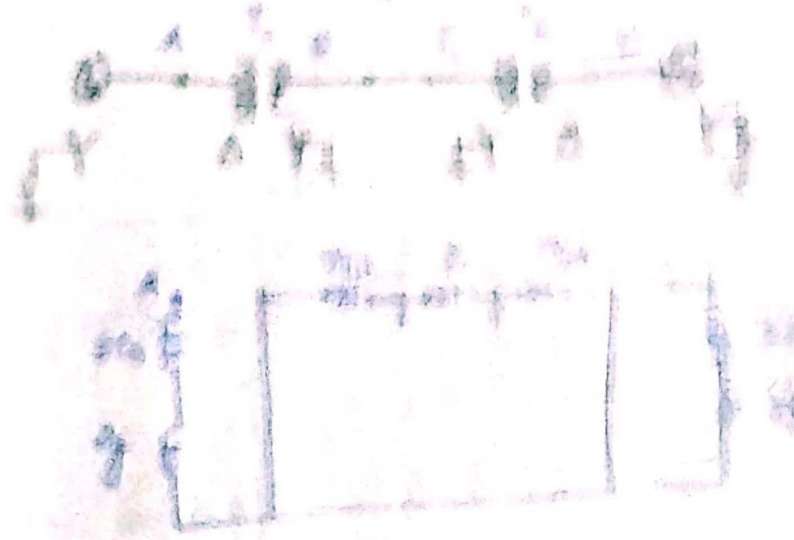


Zero Sequence network of different combination of Y connections

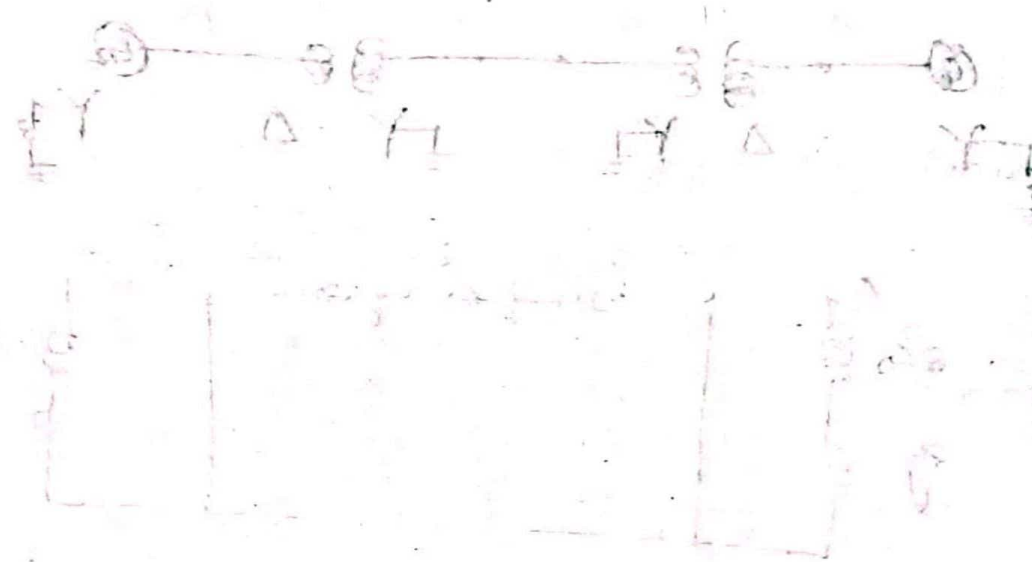
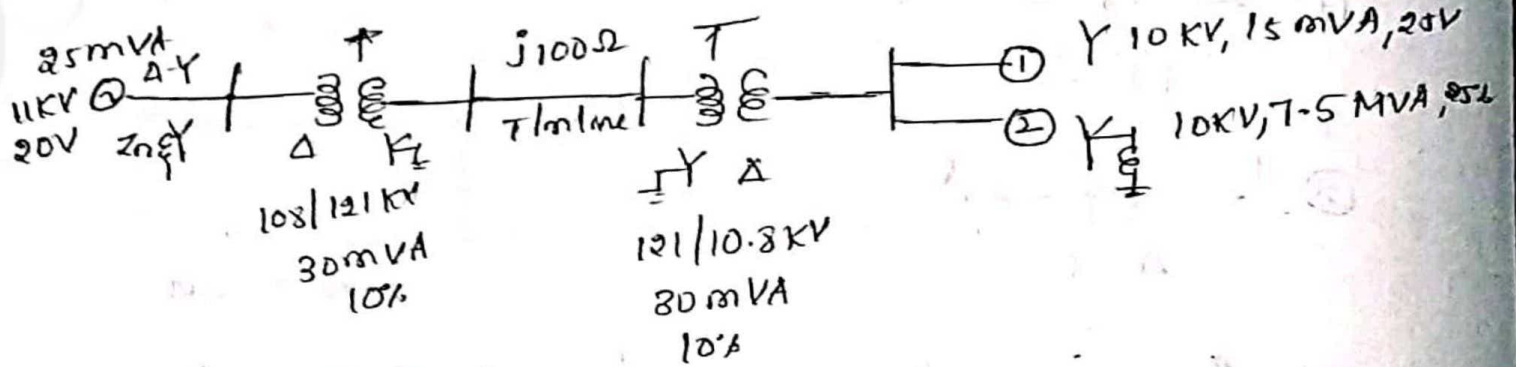




power system single line diagram shown  
 in above case equivalent circuit



2) A 25 MVA, 11 kV 3 $\phi$  generator has a sub-transient reactance of 20% the generator supplies 2 motors over a transmission line with transformers is as shown in figure. The motors have rated r.p.s of 15 & 7.5 mVA both 10 kV with 25% sub transient reactance. The 3 $\phi$  transformers are both rated 30 MVA, 10.8/10 kV connected in  $\Delta$ -Y with leakage reactance of 10% each. The series reactance of line is  $100\ \Omega$ . Draw the  $\xi$ -ve sequence network of system. with reactance marked in figure.



Ans

$$MVA_B = 25 \text{ MVA}$$

$$KV_B = 11 \text{ KV}$$

$$X_G = j0.2$$

$$X_{T1} = 0.1 \times \frac{25}{30} \times \left(\frac{10.8}{11}\right)^2 = j0.0803 \text{ pu}$$

$$X_L =$$

$$\Rightarrow \text{New KV} = 11 \times \frac{121}{10.8} = 123.24 \text{ KV}$$

$$X_L = j100 \times \frac{25}{123.24^2}$$

$$= j0.164 \text{ pu}$$

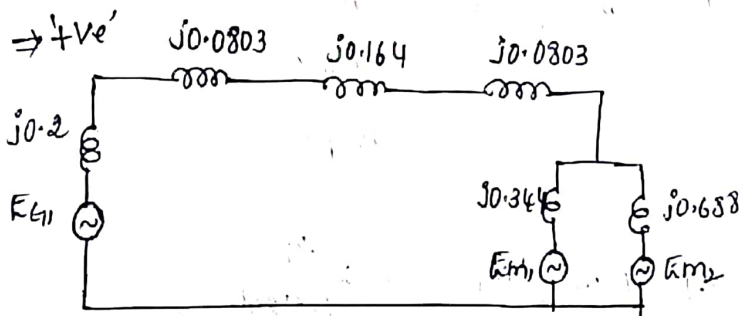
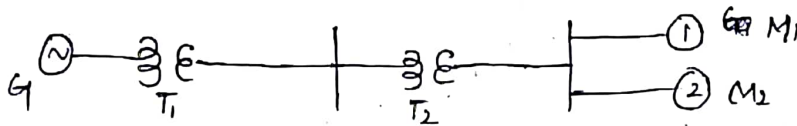
$$X_{T2} = 0.1 \times \frac{25}{30} \times \left(\frac{121}{123.24}\right)^2$$

$$= j0.0803 \text{ pu}$$

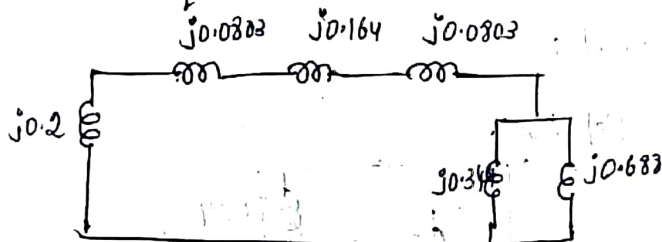
$$\Rightarrow \text{New KV for } T_2 = 123.24 \times \frac{10.8}{121} = 11 \text{ KV}$$

$$X_{M1} = 0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = j0.344 \text{ pu}$$

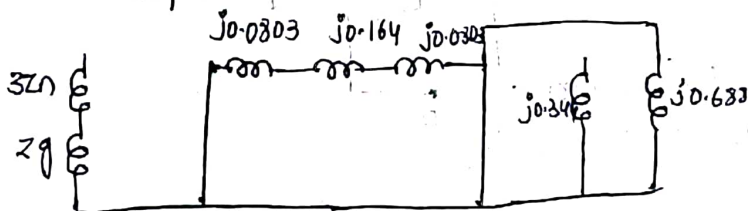
$$X_{M2} = 0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = j0.688 \text{ pu}$$



$\Rightarrow$  '-Ve' Sequence



$\Rightarrow$  'zero' Sequence





Q) Draw the zero sequence N/W for the above system described as shown in figure. Assume zero sequence reactance of generator & motor 0.06 pu current limiting reactor are 2.5 each. are connected in the neutral of generator and motor. The 0 seq. reactance of 71m line is 300.

Sol

zero sequence reactance of 71F's =  $j0.0803$

zero sequence reactance of Generator = 0.06 pu

zero sequence reactance of motor-1 =  $j0.06 \times \frac{25}{75} \times \left(\frac{10}{11}\right)^2$

$M_1 = 0.0826$  pu

zero sequence reactance of motor-2 =  $j0.06 \times \frac{25}{75} \times \left(\frac{10}{11}\right)^2$

$M_2 = 0.164$  pu

Reactance of CLR =  $\frac{2.5 \times 25}{11^2}$

= 0.516 pu

zero sequence reactor of CLR =  $3Z_n$

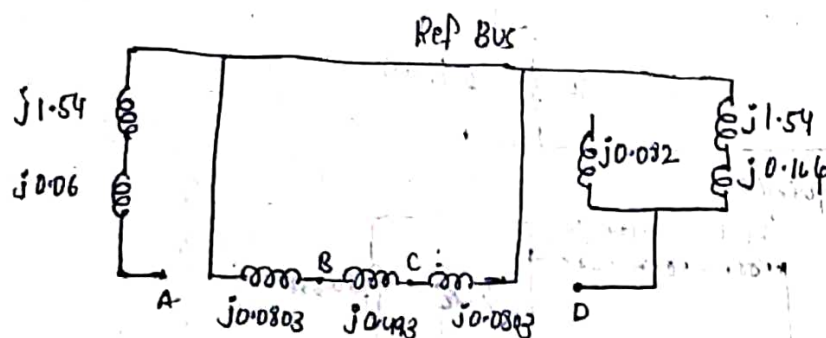
=  $3 \times 0.516$

= 1.54 pu

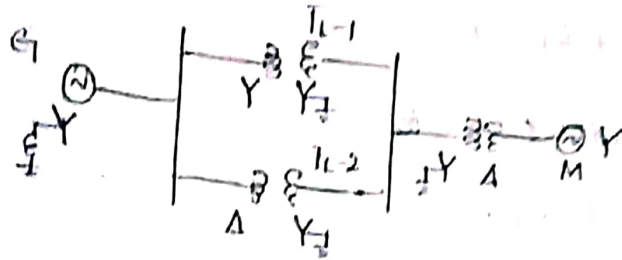
zero sequence of 71m line =  $300 \times \frac{25}{12804}$

= 0.493 pu

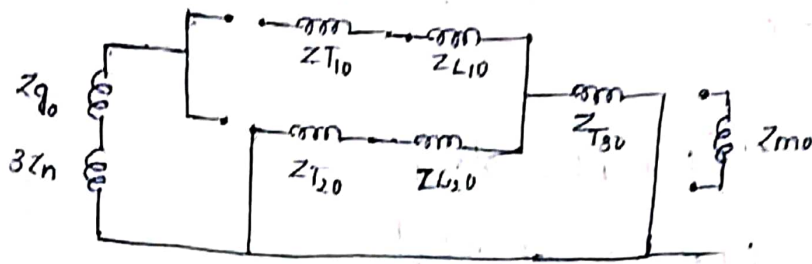
zero sequence network:



Q) Draw ZSN of the ps as shown in figure



Sol



### Unsymmetrical fault Analysis :-

\* Most of the faults that occur on the power system are unsymmetrical faults which causes unbalanced currents to flow in the system.

\* The various unsymmetrical faults are LG, LL & LLG faults these comes under the shunt (or) short circuit faults. one conductor, two conductor open comes under open circuit faults.

### Sequence Voltages of a generator :-

\* Consider a symmetrically designed 3 $\phi$  generator. Let  $E_A, E_B$  &  $E_C$  are the generated voltage

$E_{A0}, E_{A1}, E_{A2}$  are 0, +ve, -ve sequence voltage of

phase 'a'.

We have

$$E_a = E_a$$

$$E_b = a^2 E_a$$

$$E_c = a E_a$$

$$\rightarrow k_{a0} = \frac{1}{3} [k_a + k_b + k_c]$$

$$= \frac{1}{3} [k_a + a^2 k_a + a k_c]$$

$$= \frac{1}{3} k_a [1 + a^2 + a]$$

$$k_{a0} = 0 \quad (\because 1 + a^2 + a = 0)$$

$$\Rightarrow k_{a1} = \frac{1}{3} [k_a + a k_b + a^2 k_c]$$

$$= \frac{1}{3} [k_a + a^3 k_a + a^3 k_a]$$

$$= \frac{k_a}{3} [1 + a^3 + a^3]$$

$$= \frac{k_a}{3} [1 + 2a^3]$$

$$= \frac{k_a}{3} [1 + 2(1)]$$

$$= \frac{k_a}{3} (3)$$

$$k_{a1} = k_a$$

$$\Rightarrow k_{a2} = \frac{1}{3} [k_a + a^2 k_b + a k_c]$$

$$= \frac{1}{3} [k_a + a^4 k_a + a^2 k_a]$$

$$= \frac{k_a}{3} [1 + a^4 + a^2]$$

$$k_{a2} = 0$$

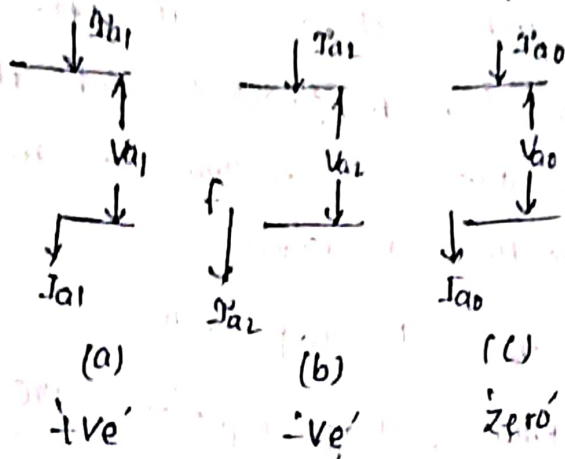
\* It is observed from above that a symmetrical

designed generator generates only positive sequence

voltage &  $-k_e'$ , zero sequence voltages are zero.

$$\begin{bmatrix} E_{a0} \\ E_{a1} \\ E_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ k_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Equivalent networks are as follows



\* Depending upon the type of fault the sequence currents and voltages are constrained leading to a particular connection of sequence networks.

\* Voltage of the neutral

$$V_n = -I_n z_n$$

$$= -3 I_{a0} z_n$$

$$I_n = I_a + I_b + I_c$$

$$= (I_{a0} + I_{a1} + I_{a2}) +$$

$$(I_{b0} + I_{b1} + I_{b2}) +$$

$$(I_{c0} + I_{c1} + I_{c2})$$

$$= (I_{a0} + I_{a1} + I_{a2}) +$$

$$(a I_{a0} + a I_{a1} + a^2 I_{a2})$$

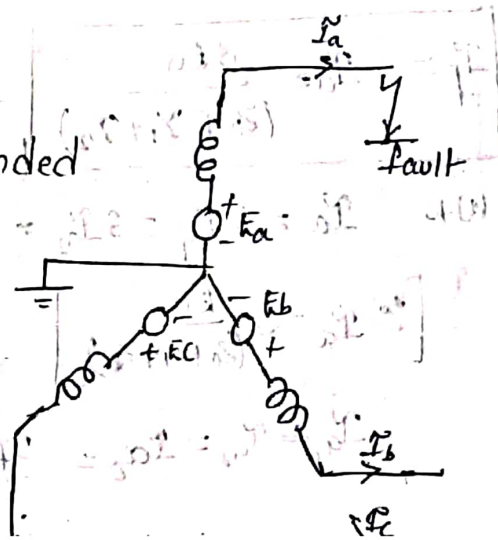
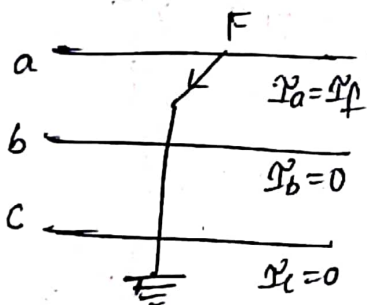
$$+ (I_{a1} + a I_{a1} + a^2 I_{a2})$$

$$= I_{a0}(1+1+1) + I_{a1}(0) + I_{a2}(0)$$

$$= 3 I_{a0}$$

Single line fault (or)

(a) When neutral is grounded



\* A single-line to ground fault occurs at phase-A the fault current flows through phase-A and the remaining phase currents are zero because the machine is (unloaded) under no load condition.

$$V_a = 0; I_b = 0; I_c = 0; I_a = I_f \rightarrow \textcircled{1}$$

\* The sequence N/w voltage equations are

$$\left. \begin{aligned} V_{a0} &= -I_{a0} Z_0 \\ V_{a1} &= E_a - I_{a1} Z_1 \\ V_{a2} &= -I_{a2} Z_2 \end{aligned} \right\} \rightarrow \textcircled{2}$$

\* The symmetrical components of currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3} I_a = I_{a1} = I_{a2} \rightarrow \textcircled{3}$$

$$(\because I_b = I_c = 0)$$

\* The symmetrical components of voltages are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

$$E_a = I_{a0} Z_0 + I_{a1} Z_1 + I_{a2} Z_2$$

$$E_a = \frac{1}{3} I_a (Z_0 + Z_1 + Z_2) \quad (\text{from eq } \textcircled{3})$$

$$\boxed{I_f = I_a = \frac{3 E_a}{(Z_0 + Z_1 + Z_2)}} \rightarrow \textcircled{4}$$

fault current  
for LG fault

$$\text{W.K } I_a = 3 I_{a1} = 3 I_{a2} = 3 I_{a0}$$

$$\therefore I_a = 3 \frac{E_a}{(Z_0 + Z_1 + Z_2)}$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_f}{3}$$

b) When neutral is not grounded  
 If neutral of generator is not grounded.  
 then  $z_0$  tends to infinity. so  $I_f = 0$   
 $\therefore I_a = 0$

c) When neutral is grounded through impedance

\* If the neutral of generator is connected through an impedance  $z_n$  then  $z_0$  is replaced by  $z_0 + 3z_n$ .

\* For the voltages on the healthy lines are

$$V_b = V_{b0} + V_{b1} + V_{b2}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} \quad (\text{from eq ③})$$

$$= -\frac{I_a z_0}{3} + a^2 (E_a - I_{a1} z_1) + a (E_a - I_{a2} z_2)$$

$$= -\frac{I_a z_0}{3} + a^2 E_a - a^2 \frac{I_a z_1}{3} + a \frac{I_a z_2}{3}$$

$$= -\frac{I_a}{3} (z_0 + a^2 z_1 + a z_2) + a^2 E_a$$

Sub eq ④ in above eq,

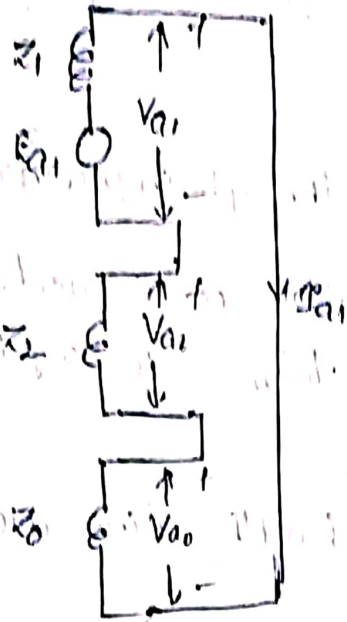
$$-\frac{E_a}{z_0 + z_1 + z_2} z_0 + a^2 \left( E_a - \frac{E_a z_1}{z_0 + z_1 + z_2} \right) +$$

$$a \left( -\frac{E_a z_2}{z_0 + z_1 + z_2} \right)$$

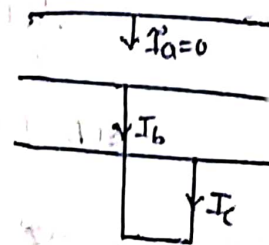
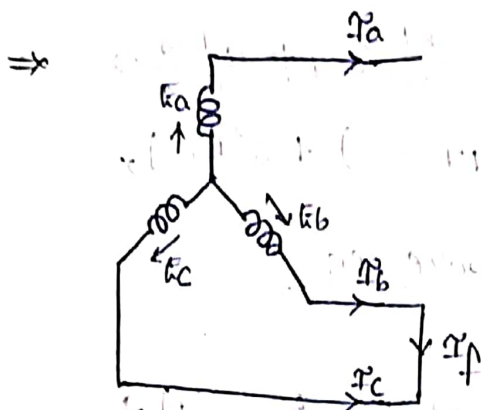
$$V_b = E_a \left[ \frac{(a^2 - a) z_2 + (a^2 - 1) z_0}{z_0 + z_1 + z_2} \right]$$

$$\text{By } V_c = E_a \left[ \frac{(a - 1) z_0 + (a - a^2) z_2}{z_0 + z_1 + z_2} \right]$$

The equivalent network is



Line to Line fault  $\div$  (L-L Fault)



\* Let the L-L fault occurs between lines b & c

$$\therefore I_a = 0, I_b + I_c = 0 \Rightarrow I_b = -I_c$$

\* Now the sequence currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a0} = 0$$

$$I_{a1} = \frac{1}{3} [aI_b - a^2I_b] = \frac{I_b}{3} (a - a^2)$$

$$I_{a2} = \frac{1}{3} [a^2I_b - aI_b] = \frac{I_b}{3} (a^2 - a)$$

$$\therefore I_{a1} = -I_{a2}$$

\* Symmetrical components of Voltages.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_{a1} = \frac{1}{3}(V_a + V_b(a + a^2))$$

$$V_{a2} = \frac{1}{3}(V_a + V_b(a^2 + a))$$

$$\therefore V_{a1} = V_{a2}$$

\* But  $V_{a0} = -I_{a0}Z_0 = \frac{1}{3}(V_a + V_b + V_c)$

$$= 0 = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_{a0} = 0$$

$$\therefore V_{a1} = V_{a2} \quad \& \quad V_{a0} = 0$$

\* But  $V_{a1} = E_a - I_{a1}Z_1$  [  $V_{a1}$  for synchronous generator ]

$$V_{a2} = -I_{a2}Z_2$$

$$\therefore V_{a1} = V_{a2}$$

$$\therefore E_a - I_{a1}Z_1 = -I_{a2}Z_2$$

$$E_a = I_{a1}Z_1 - I_{a2}Z_2$$

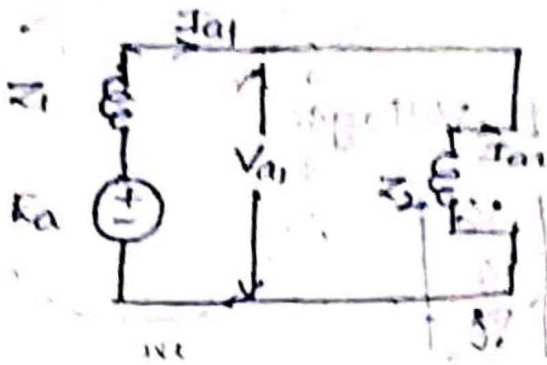
$$= (Z_1 + Z_2)I_{a1}$$

$$[\because I_{a1} = -I_{a2}]$$

$$\boxed{\tilde{I}_{a1} = \frac{E_a}{Z_1 + Z_2} = -I_{a2}}$$

\* The sequence network from L-L faults is connected in opposite and parallel.





\* We know that

$$\begin{aligned}
 I_b &= I_{a0} + a^2 I_{a1} + a I_{a2} \\
 I_f = I_b &= 0 + a^2 I_{a1} + a (-I_{a1}) \\
 &= (a^2 - a) I_{a1} \\
 &= (a^2 - a) \frac{E_a}{Z_1 + Z_2} \\
 I_f &= \frac{-j\sqrt{3} E_a}{Z_1 + Z_2} = I_b = I_c
 \end{aligned}$$

\* Now

$$\begin{aligned}
 V_a &= V_{a0} + V_{a1} + V_{a2} \\
 &= 0 + E_a - I_{a1} Z_1 - I_{a2} Z_2 \\
 &= E_a - I_{a1} Z_1 + I_{a1} Z_2 \\
 &= E_a - I_{a1} (Z_1 - Z_2)
 \end{aligned}$$

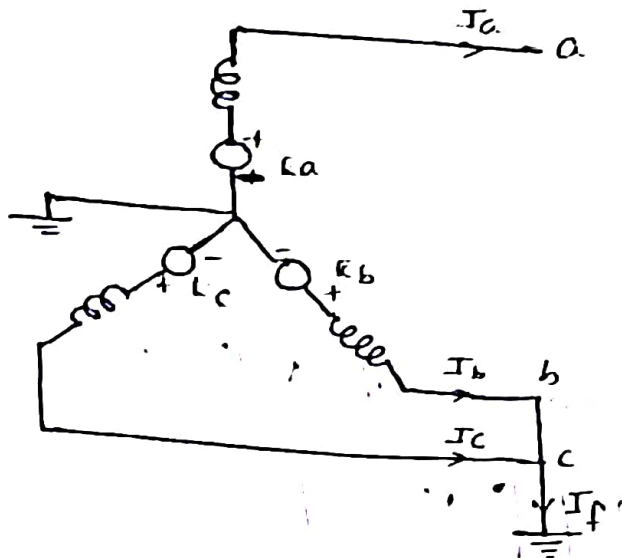
$$\begin{aligned}
 V_a &= E_a - E_a \frac{Z_1 - Z_2}{Z_1 + Z_2} \\
 &= E_a \frac{Z_1 + Z_2 - Z_1 + Z_2}{Z_1 + Z_2} \\
 &= E_a \frac{2Z_2}{Z_1 + Z_2}
 \end{aligned}$$

$$\begin{aligned}
 V_a &= 2 E_a \left( \frac{Z_2}{Z_1 + Z_2} \right) \\
 V_b &= -E_a \left( \frac{Z_2}{Z_1 + Z_2} \right) = V_c
 \end{aligned}$$

\* The presence (or absence) of the grounded generator neutral doesn't effect the fault current. The sequence networks connected to opposite

Double line to ground fault (L-L-G Fault)

\* The equivalent network is



\* The figure shows double-line to ground fault of phases 'b' & 'c'. The fault current is passed through ground.

$$\begin{aligned} \therefore I_a &= 0 \\ V_b &= V_c = 0 \\ I_b + I_c &= I_f \end{aligned} \quad \left. \vphantom{\begin{aligned} \therefore I_a &= 0 \\ V_b &= V_c = 0 \\ I_b + I_c &= I_f \end{aligned}} \right\} \text{---} \textcircled{1}$$

\* Symmetrical Components of current

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a = 0 \\ I_b = I_b \\ I_c = I_c \end{bmatrix}$$

$$I_{a0} = 0$$

$$I_{a1} = \frac{1}{3} [a + aI_b + a^2I_c]$$

$$I_{a2} = \frac{1}{3} [a + a^2I_b + aI_c]$$

\* Now  $I_a = I_{a0} + I_{a1} + I_{a2}$

$$= 0 + I_{a1} + I_{a2} = 0$$

$$I_{a1} + I_{a2} = 0 = -I_{a0}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$I_f = I_b + I_c$$

$$= 2I_{a0} + (a^2 + a)I_{a1} + (a + a^2)I_{a2}$$

$$= 2I_{a0} - (I_{a1} + I_{a2}) = 3I_{a0}$$

$$I_f = 3I_{a0}$$

\* Now Symmetrical components of voltage

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b = 0 \\ V_c = 0 \end{bmatrix}$$

$$V_{a0} = \frac{V_a}{3} = V_{a1} = V_{a2}$$

$$I_f = 3I_{a0}$$

⇒ Let  $V_{a1} = V_{a2}$

$$E_a - I_{a1}Z_1 = -I_{a2}Z_2$$

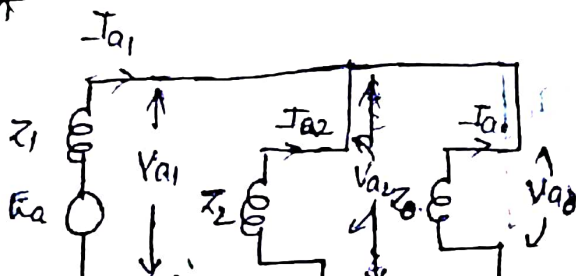
$$I_{a2} = \frac{-(E_a - I_{a1}Z_1)}{Z_2}$$

$$\Rightarrow V_{a2} = V_{a1}$$

$$-I_{a2}Z_2 = -I_{a0}Z_0$$

$$I_{a0} = I_{a2} \left( \frac{Z_2}{Z_0} \right)$$

\*



$$\begin{aligned}
 \mathcal{I}_{a1} &= \frac{E_a}{z_1 + (z_2 \parallel z_0)} \\
 &= \frac{E_a}{z_1 + \frac{z_2 z_0}{z_2 + z_0}} \\
 &= \frac{E_a (z_2 + z_0)}{z_1 z_0 + z_1 z_2 + z_2 z_0}
 \end{aligned}$$

$$\mathcal{I}_{a1} = \frac{E_a (z_0 + z_2)}{z_1 z_0 + z_1 z_2 + z_2 z_0}$$

Now sub  $\mathcal{I}_{a1}$  in  $\mathcal{I}_{a2}$

$$\mathcal{I}_{a2} = \frac{-1}{z_2} \left[ E_a - \left( \frac{E_a (z_0 + z_2)}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right) z_1 \right]$$

$$= \frac{-1}{z_2} \left[ \frac{E_a (z_1 z_0 + z_1 z_2 + z_2 z_0) - E_a (z_0 z_1 + z_2 z_1)}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right]$$

$$= \frac{-1}{z_2} \left[ \frac{E_a (z_2 z_0)}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right]$$

$$\mathcal{I}_{a2} = - \left[ \frac{E_a z_0}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right]$$

Now sub  $\mathcal{I}_{a2}$  in  $\mathcal{I}_{a0}$

$$\mathcal{I}_{a0} = \frac{E_a z_0}{z_1 z_0 + z_1 z_2 + z_2 z_0} \left[ \frac{z_1}{z_0} \right]$$

$$\mathcal{I}_{a0} = - \left[ \frac{E_a z_2}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right]$$

Now  $V_a = V_{a0} + V_{a1} + V_{a2}$

$$= \mathcal{I}_{a0} z_0 + E_a - \mathcal{I}_{a1} z_1 - \mathcal{I}_{a2} z_2$$

$$= \left[ \frac{E_a z_2}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right]$$

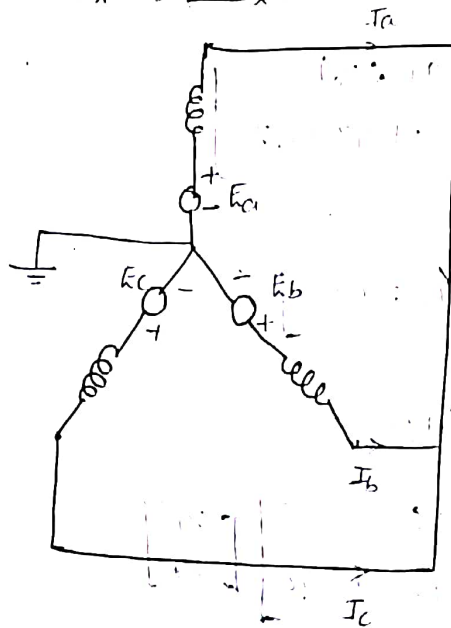
$$V_a = \left[ \frac{E_a z_2}{z_0 z_0 + z_1 z_2 + z_2 z_0} \right] z_0 + E_a \left[ \frac{E_a (z_0)}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right] z_1 +$$

$$\left[ \frac{E_a z_0}{z_1 z_0 + z_1 z_2 + z_2 z_0} \right] z_2$$

$$= \frac{1}{z_1 z_0 + z_1 z_2 + z_2 z_0} \left[ E_a z_2 z_0 + E_a - E_a z_0 z_1 - E_a z_2 z_1 + E_a z_0 z_2 \right]$$

$$V_0 = \left[ \frac{3 z_2 z_0}{z_1 z_2 + z_1 z_0 + z_2 z_0} \right] E_a$$

Three phase short circuit fault



\* If all the 3- $\phi$ s are short circuit the terminal conditions are as shown below.

$$V_a = V_b = V_c$$

$$I_a = I_b = I_c = 0$$

\* Zero sequence current is

$$I_{a0} = \frac{1}{3} (I_a + I_b + I_c) = 0$$

\* +ve seq current

$$I_{a1} = \frac{1}{3} (I_a + a I_b + a^2 I_c)$$

$$\hat{I}_b = a^2 \hat{I}_a, \hat{I}_c = a \hat{I}_a$$

$$\hat{I}_{a1} = \frac{1}{3} [\hat{I}_a + a^3 \hat{I}_a + a^3 \hat{I}_a] \quad a^3 = 1$$

$$= \frac{1}{3} \hat{I}_a$$

$$\hat{I}_{a1} = \hat{I}_a$$

\* Negative sequence current

$$\hat{I}_{a2} = \frac{1}{3} [\hat{I}_a + a^2 \hat{I}_b + a \hat{I}_c]$$

$$= \frac{1}{3} [\hat{I}_a + a^4 \hat{I}_b + a^2 \hat{I}_c]$$

$$= \frac{1}{3} \hat{I}_a (0)$$

$$\hat{I}_{a2} = 0$$

\* Which means that for a 3 $\phi$  slc fault zero & -ve sequence currents are absent, +ve sequence current is equal to the phase current of phase A

\* Now, +ve seq. voltage is

$$V_{a1} = \frac{1}{3} [V_a + aV_b + a^2V_c]$$

$$= \frac{1}{3} [V_a + aV_a + a^2V_a] \quad (V_a = V_b = V_c)$$

$$V_{a1} = 0$$

\* -ve seq. voltage is

$$V_{a2} = \frac{1}{3} [V_a + a^2V_b + aV_c]$$

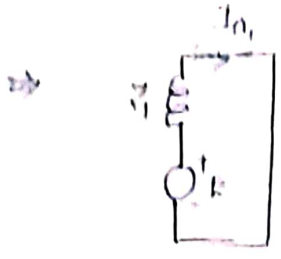
$$= \frac{1}{3} [V_a + a^2V_a + aV_a]$$

$$= 0$$

$$* \quad V_{a0} = 0 = -\hat{I}_{a0} z_0 = 0(z_0) = 0$$

$$* \quad V_{a1} = E_a - \hat{I}_{a1} z_1$$

$$\hat{I}_{a1} = \frac{E_a}{z_1} \quad \because V_{a1} = 0$$



\* From the Analysis of faults, following observations are made.

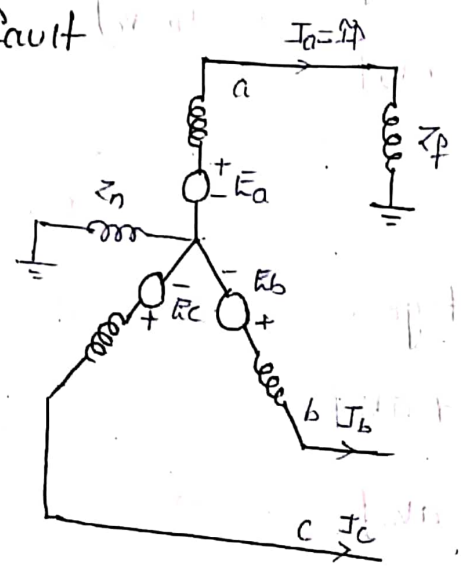
① Positive Sequence currents are present in all types of faults.

② Negative sequence currents are present in all the unsymmetrical faults.

③ Zero sequence currents are present when neutral is present. Fault involves the ground. and magnitude is  $I_n = 3I_{a0}$ .

Faults through Impedance:

(i) LG fault



\*  $I_b = I_c = 0$

$V_a = I_a Z_p$

\* +ve Seq voltage

$V_{a1} = E_{a1} - I_{a1} Z_1$

seq voltage

$$V_{a2} = -I_{a2} z_2$$

zero seq voltage

$$V_{a0} = -I_{a0} z_0$$

\* Here  $z_0 = z_g + 3z_n$

\* Now currents are

zero seq current

$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c] = \frac{I_a}{3} = \frac{I_f}{3}$$

$$\Rightarrow I_f = 3 I_{a0}$$

$$\frac{I_f}{3} =$$

+ve seq current & -ve sequence current

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_f}{3}$$

\* Now  $V_a$  is

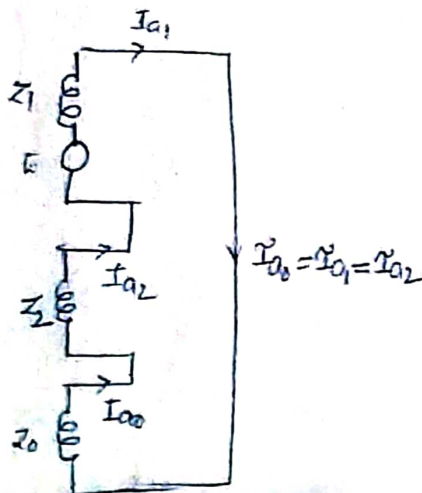
$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= I_{a0} z_0 = 3 I_{a1} z_f = 3 I_{a2} z_f = 3 I_{a0} z_f$$

$$\left[ V_a = z_0 I_f = z_0 I_a = z_0 3 I_{a0} \right]$$

$$(I_{a0} = I_{a1} = I_{a2})$$

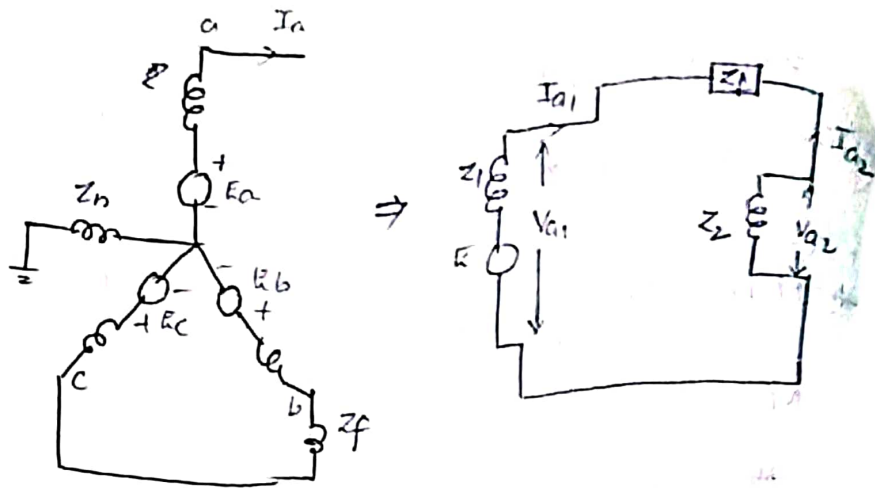
$$V_a = 3 I_{a1} z_f = 3 I_{a2} z_f = 3 I_{a0} z_f$$



$$I_f = \frac{E}{z_1 + z_2 + (z_g + 3z_n) + 3z_f}$$



(ii) L-L Fault with fault impedance  $Z_f$



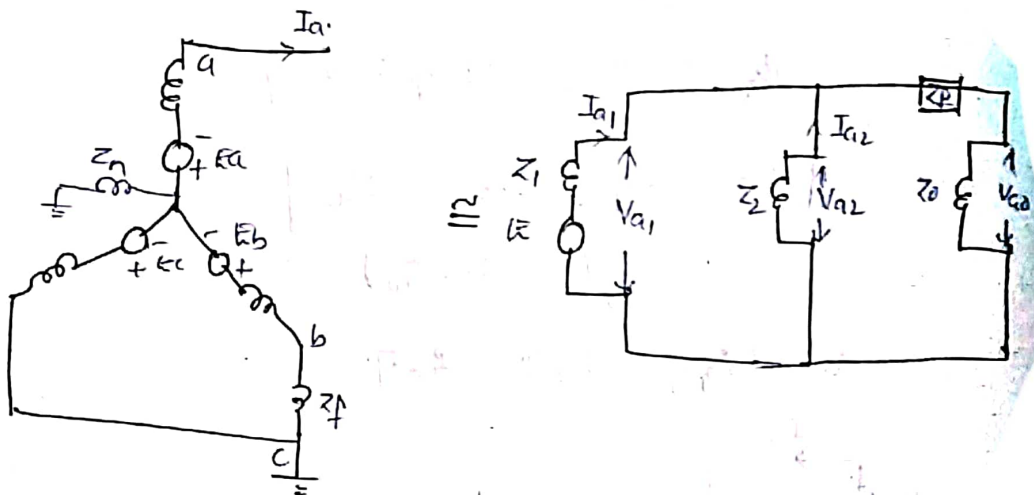
$$I_f = I_b = -I_c = \frac{-j\sqrt{3} E}{Z_1 + Z_2 + Z_f}$$

$$V_{a0} = 0$$

$$V_{a1} = V_{a2}$$

$$I_{a1} = \frac{E}{Z_1 + Z_2 + Z_f}$$

(iii) L-L-G Fault with fault impedance  $Z_f$



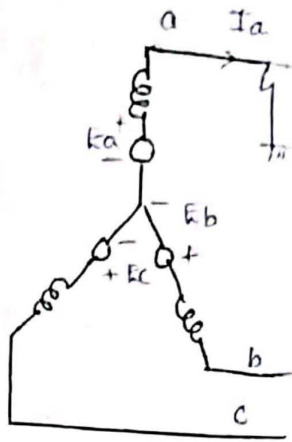
$$I_{a1} = \frac{E}{Z_1 + \frac{Z_2 (Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}$$

$$I_f = 3I_{a0}$$

$$I_{a0} = -I_{a1} \times \frac{Z_2}{Z_0 + Z_2 + 3Z_f}$$

Problem

① A generator rated, 100 MVA, 20 KV has  $X_1 = X_2 = 20\%$  and  $X_0 = 5\%$ , its neutral is grounded through a reactor of  $0.32 \Omega$ . The generator is operating at rated voltage with load and it is disconnected from the system when the ~~it~~<sup>single line</sup> to ground fault occurs at its terminals. Find the subtransient currents in the faulted phase and line-to-line p voltages.



Given  $X_1 = X_2 = 20\% = 0.2 \text{ pu}$

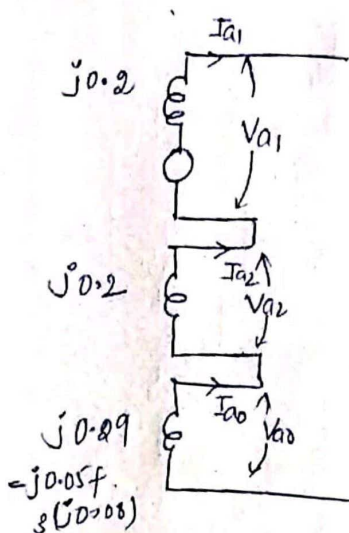
$X_0 = 5\% = 0.065 \text{ pu}$

$X_n = 0.32 \Omega$

$(X_n)_{\text{pu}} = 0.32 \times \frac{100}{20^2}$

$= 0.08 \text{ pu}$

$E = 1 \text{ pu}$



$$I_f = \frac{3L}{Z_0 + Z_1 + Z_2}$$

$$= \frac{3 \times 1}{j0.29 + j0.21 + j0.2}$$

$$I_f = -j4.347 \text{ pu}$$

$$I_f(\text{Actual}) = I_f(\text{base}) \times I_f(\text{pu})$$

$$I_f(\text{base}) = \frac{100 \times 10^3}{\sqrt{3} \times 20}$$

$$= 2886.7 \text{ Amps} \times 10^3$$

$$\sim 2886.7 \text{ A}$$

$$I_f(\text{actual}) = 2886.7 \times -j4.347$$

$$= \underline{\underline{j12551.6 \text{ Amps}}}$$



$$I_f = 3 I_{a_1}$$

$$V_{a_0} = -I_{a_0} (X_g + 3X_n)$$

$$V_{b_0} = V_{a_0} = V_{c_0}$$

$$E = I_{a_1} X_1$$

$$= -j3.14 \times j0.2$$

$$= 0.628 \angle -90^\circ \text{ pu}$$

$$V_{a_1} = V_{a_2} = V_{c_1} = 0.372 \text{ pu}$$

$$\Rightarrow V_{a_0} = -I_{a_0} (X_g + 3X_n)$$

$$I_{a_0} = \frac{-V_{a_0}}{(X_g + 3X_n)}$$

$$= \frac{-0.372}{0.32 + 3(0.08)}$$

$$= -j1.23 \text{ pu}$$

$$I_f = 3 I_{a_1}$$

$$= 3(-j1.23)$$

$$= -j3.69 \text{ pu}$$

$$I_f(\text{actual}) = I_f \text{ pu} \times I_{f_b}$$

$$= -j3.69 \times 120 \angle 0^\circ$$

$$= -j442.8 \text{ A}$$

$$\approx 442.8 \text{ A}$$

$$V_a = V_{a_0} + V_{a_1} + V_{a_2} = 3V_{a_0} = 3(0.372) = 1.116 \text{ pu}$$

$$V_b = V_{a_0} + a^2 V_{a_1} + a V_{a_2} = 0$$

$$V_c = V_{a_0} + a V_{a_1} + a^2 V_{a_2} = 0$$

$$V_{ab} = 1.116 \text{ pu}$$

$$V_{bc} = 0$$

$$V_{ca} = -1.116 \text{ pu}$$

② The following sequence currents are registered in power system under an unbalanced fault condition.  $I_{\text{positive}} = -j1.653 \text{ pu}$ ,  $I_{\text{negative}} = +j0.5 \text{ pu}$ ,  $I_{\text{zero}} = +j0.153 \text{ pu}$ . Identify

the type of fault assuming the pre-fault voltage is 1 pu and post-fault positive sequence voltage is 0.175 pu find the seq. impedances for the system under the above condition.

Given

$$I_{a1} = -j1.653 \text{ pu}$$

$$I_{a2} = j0.5 \text{ pu}$$

$$I_{a0} = j1.153 \text{ pu}$$

\* Here  $I_{a0} \neq I_{a1} \neq I_{a2}$  ( $\therefore$  this is not LL fault)

$\therefore$  The fault is not be align to a gr a line to ground fault.

$$* I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} = 2.543 \angle 137^\circ$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} = 2.54 \angle 42.5^\circ$$

$$\therefore I_b \neq -I_c \quad I_a \neq 0$$

$\therefore$  the fault is not LL fault.

\*  $V_b = V_c = 0$  in case of LL fault

Check:

$$V_{a1} = V_{a2} = V_{a0}$$

$$V_{a1} = E - I_{a1} Z_1$$

$$= 1 - (j1.653) Z_1$$

$$* Z_1 = \frac{(V_{a1} - E)}{I_{a1}}$$

$$= \frac{-(0.175 - 1)}{-j1.653}$$

$$= 0.499 \text{ pu}$$

$$V_{a2} = -I_{a2} Z_2$$

$$Z_2 = \frac{-V_{a2}}{I_{a2}}$$

$$= \frac{-0.175}{j0.5}$$

$$= j0.35 \text{ pu}$$

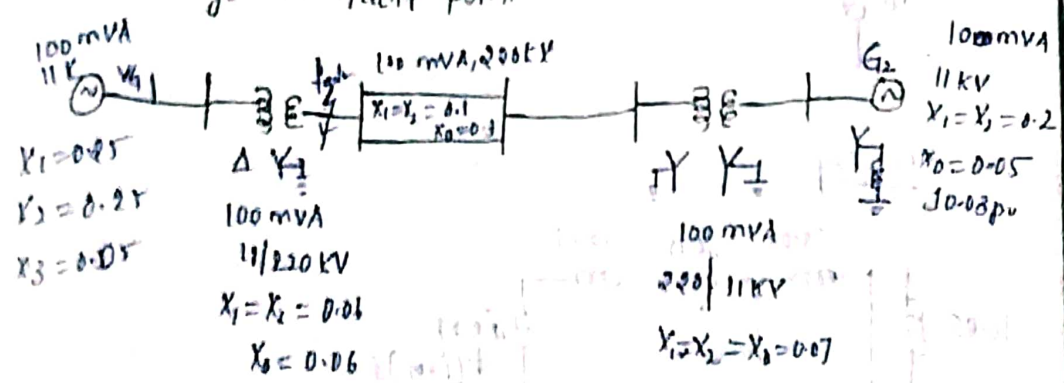
$$V_{a0} = -I_{a0} Z_0$$

$$Z_0 = \frac{-0.175}{j1.153}$$

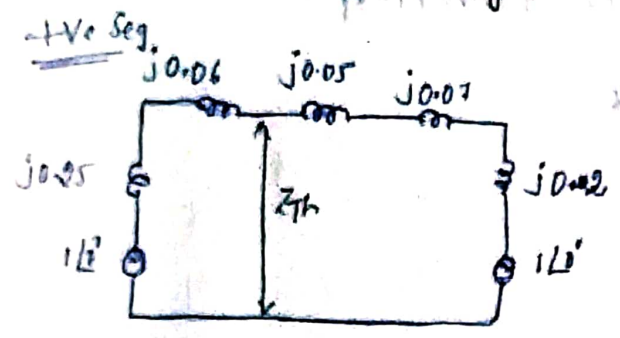
$$= j0.151 \text{ pu}$$

So  $\therefore V_{a0} = V_{a1} = V_{a2}$   
 $\downarrow$  It is L-L-G fault.

③ For the given power system, the single line to ground fault occurs at bus-2 of the network find i) Line to neutral voltage at fault point



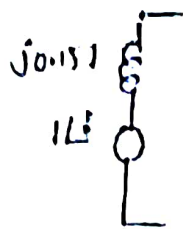
Reduce the seq. n/w to give the Thevenin equivalent circuit looking into Bus-2.



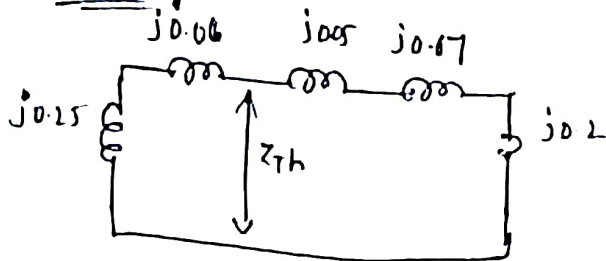
$$Z_{th} = (j0.06 + j0.27) \parallel (j0.05 + j0.07 + j0.9)$$

$$= j0.31 \parallel j0.32$$

$$= j0.157$$



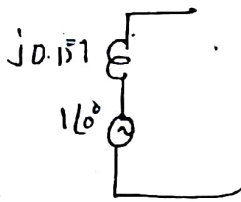
⇒ -ve seq.



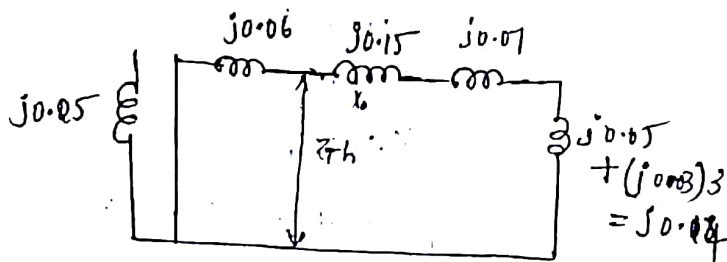
$$(j0.25 + j0.06) \parallel (j0.05 + j0.07 + j0.2)$$

$$Z_{th} = j0.31 \parallel j0.32$$

$$= j0.157 \text{ pu.}$$



⇒ zero seq.

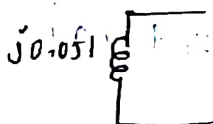


$$Z_{th} = j0.06 \parallel j0.15 + j0.07 + j0.08$$

$$= j0.06 \parallel j0.30$$

$$= 0.051 j$$

$$= j0.051$$



$$I_{a_0} = \frac{E}{j\omega L (z_0 + z_1 + z_2)}$$

$$= \frac{1 \angle 0^\circ}{j0.157 + j0.157 + j0.051}$$

$$= -j2.737$$

$$I_A = 3I_{a_0}$$

$$= j8.217$$

$$(I_{a_1} = I_{a_2} = I_{a_0} = -j2.737)$$

$$I_{base} = \frac{\text{Base MVA}}{KV}$$

$$= \frac{100 \times 10^3}{\sqrt{3} \times 220}$$

$$= 262.43$$

$$I_A \text{ actual} = I_A (\text{pu}) \times I_{base}$$

$$= -j8.217 \times 262.43$$

$$= -2156.92 \text{ A}$$