25/08/2016

# Unit-5

## Power System Transient state stability Analysis :

### Transient state stability :

* It is the Ability of the power system to remain in synchronism followed by a large (or) sudden disturbance.

### Transient stability limit :

* It refers to the amount of power that can be transmitted with stability when the system is subjected to aperiodic disturbance
* The three principle types of Transient disturbances in the study of system steability in the order of increasing importance of
  1. Load variations <
  2. Switching operations >
  3. faults with subsequent circuit isolations.

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* Transient state stability problem is analysed by
  1. Solving system dynamics equations known as swing equation.
  2. Equal Area criterian using power angle characteristics which helps the system engineer to co-ordinate the circuit breaker operations

In another words it is a time delay that can be allowed to isolated the faulty section by the circuit breakers so that the system is transiently stable.

## Assumptions made in calculation of Transient Stability :

① The Resistance of synchronous machine and Transmission line is ignored.

② Damping term contributed by synchronous machine damping winding is ignored.

③ Rotor speed is Assumed to be constant.

④ Mechanical input to the machine is assumed to be remain constant.

⑤ Voltage behind Transient reactance is assumed to remain constant.

⑥ Loads are modelled as constant admittances.

## Dynamics of a synchronous Machine :

* The kinetic energy of synchronous machine is

$$KE = \frac{1}{2} J\omega_{sm}^{2} \times 10^{-6} \text{ MJ}$$

where J is moment of inertial expressed in $Kgm^2$

$\omega_{sm}$ is synchronous speed in mech rad/sec

But $\omega_s = \frac{P}{2}\omega_{sm}$

$\omega_s$ is rotor speed in electrical rad/sec

$$\omega_{sm} = \frac{2}{P}\omega_s$$

$$\therefore \; KE = \frac{1}{2} J \left(\frac{2}{P}\right)^2 \omega_s^2 \times 10^{-6}$$

$$= \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right] \omega_s$$

$$= \frac{1}{2} M \omega_s$$

* Where M is Moment of intertia in MJ/ele (rad/sec)

(or) $\frac{MJsec}{elec(rad)}$.

* We shall define Inertial constant 'H' as such that

$$GH = KE = \frac{1}{2} M\omega_s \; MJ$$

where G is Machine rating

H is Inertial constant in MJ/MVA

(or) $\frac{MW\text{-}S}{MVA}$

$$\Rightarrow M = \frac{2GH}{\omega_s}$$

$$= \frac{2GH}{2\pi f}$$

$$M = \frac{GH}{\pi f} \quad \frac{MJS}{elec.(rad)}$$

$$M = \frac{GH}{180f} \quad \frac{MJS}{elec\ degree}$$

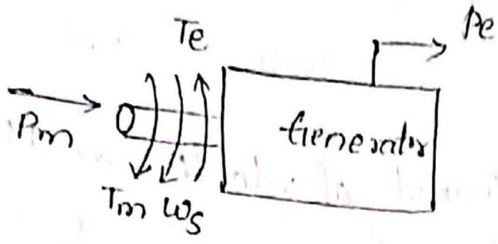
where M is inertia constant

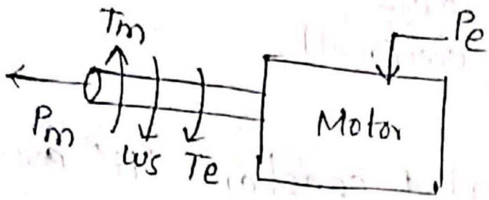
* Taking G as base, the inertia constant in per unit value is as

$$M_{pu} = \frac{H}{\pi f} \text{ in } \frac{S^2}{elec\ (rad)}$$

$$= \frac{H}{180f} \text{ in } \frac{Sec^2}{elec\ degree}$$

# Swing Equation




* Figure shows the torque, speed & flow of mechanical and electrical powers in a synchronous machine and it is assumed that friction windage and iron losses ~~track~~ is negligible

* The Differential Equation governing the rotor dynamics is

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \quad \text{—} \ 1$$

Where $\theta_m$ – angle in mech. radians.

$T_m$ – Turbine Torque in N-m and is assumed negative for motoring machine

$T_e$ – Electro magnetic Torque in N-m and is assumed negative for genera motoring machine.

* while the rotor undergoes dynamics as per the equation the rotor speed changes of insignificant magnitudes for time period of 1 sec.

* Therefore the equation can be converted into convinient form power form at synchronous speed $\omega_{sm}$.

* Multiplying with synchronous $\omega_{sm}$ on both side, the eq① is as

$$\Rightarrow 10^{-6} \times J\omega_{sm} \frac{d^2\theta_m}{dt^2} = (\omega_{sm} T_m - \omega_{sm} T_e) \times 10^{-6}$$

$$= P_m - P_e$$

where $P_m$ - mechanical power in MW

$P_e$ - electrical power in MW

$$\theta_e = \frac{P}{2}\theta_m$$

$$\Rightarrow J\left(\frac{2}{P}\right)\omega_s \frac{d^2}{dt^2}\left(\frac{2}{P}\theta_e\right) \times 10^{-6} = P_m - P_e$$

$$\Rightarrow \boxed{M \frac{d^2\theta_e}{dt^2} = P_m - P_e}$$

* It is more convinient to measure the angular position of the rotor with respect to the synchronously rotating reference speed.

* Let $\delta = \theta_e - \omega_s t$

which is called as power angle (or) Load angle (or) Torque angle.

$$\frac{d\delta}{dt} = \frac{d\theta_e}{dt} - \omega_s$$

$$\boxed{\frac{d^2\delta}{dt^2} = \frac{d^2\theta_e}{dt^2}}$$

Multiply both side by $M$

$$\boxed{M \frac{d^2\delta}{dt^2} = P_m - P_e} \Rightarrow \text{Swing Equation.}$$

* Actual value of $M = \frac{GH}{\pi f}$

$$\therefore \frac{GH}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

* In per unit value we have

$$M_{pu} \frac{d^2\delta}{dt^2} = (P_m - P_e) \quad pu$$

* This equation is called swing equation which describes rotor dynamics for synchronous machine both for motor and generator.

* It is the 2nd order differential equation where the damping torque is absent because of the assumption of loss less m/n and the damper wndg effect has been ignored.

* In a Multi winding system a common machine base system is choosen

* Let $G_{machine}$ = machine rating

$G_{system}$ = System base

$\therefore$ The above equation can be written as

$$\frac{G_{mach}}{G_{sys}} \left[\frac{H_{mach}}{\pi f}\right] \frac{d^2\delta}{dt^2} = (P_m - P_e) \frac{G_{mach}}{G_{syst}}$$

$$\left[\frac{H_{mach}}{\pi f}\right] \frac{d^2\delta}{dt^2} = (P_m - P_e) \; pu$$

$$H_{sys} = H_{mach} \times \frac{G_{mach}}{G_{sys}}$$

when $H_{mach}$ = Machine inertial constant in system base

$H_{sys}$

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* Let the swing equation of two machines of common base is

$$\frac{H_1}{\pi f} \frac{d^2\delta_1}{dt^2} = (P_{m_1} - P_{e_1}) \ pu$$

$$\frac{H_2}{\pi f} \frac{d^2\delta_2}{dt^2} = (P_{m_2} - P_{e_2}) \ pu$$

* With the machine rotors are rotating (or) swing coherently

$$\delta_1 = \delta_2 = \delta$$

$$\therefore \frac{H_{eq}}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

where $H_{eq} = H_1 + H_2$

$P_m = P_{m_1} + P_{m_2}$

$P_e = P_{e_1} + P_{e_2}$

* $H_{eq} = H_{1\,mach} \times \frac{G_{1\,mach}}{G_{1\,syste}} + H_{2\,mach} \times \frac{G_{2\,mach}}{G_{2\,syste}}$

* So this result can be extended for many number of machines which are swinging coherently

Problem

① A 50 Hz 4 pole turbo generator rated 100 MVA, 11kV has an Inertia constant of 8 MJ/MVA. Find the stored energy in the rotar at synchronous speed. If the mechanical input is suddenly rised to 80 MW for an electrical load of 50 MW find rotor Accelaration neglecting mechanical and electrical losses. If the Acceloration calculated is maintained for 10 cycle. Find the change in torque angle and rotor speed in rpm at the end of the period.

Sol

Given data

$f = 50 Hz$

$P = 4$

$G = 100$ MVA

$H = 8$ MJ/MVA

$P_m = 80$ MW

$P_e = 50$ MW

$t = 10$ cycles $\times 0.02 = 0.2$ $\left[\frac{1}{f} = \frac{1}{50}\right]$

Rotor Accelaration

$P_a = P_m - P_e$

$= 80 - 50$

$= 30$ MW

Stored Energy

$GH = 100 \times 8$

$= 800$ MJ

Moment of inertial

$M = \frac{GH}{\pi f}$

$= \frac{800}{180 \times 50}$

$M = 0.088 \frac{MJ\,S}{Electrical\ degrees}$

$\alpha = \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$ $\left[\text{from } M\frac{d^2\delta}{dt^2} = P_m - P_e\right.$

$= \frac{30}{0.088}$

$= 337.8$ electrical degree/sec

change in value of $\delta$

$$\delta = \frac{1}{2}\alpha t^2$$

$$= \frac{1}{2}(337.8)(0.2)^2$$

$$\delta = 6.75 \text{ electrical degrees}$$

For complete revolution

$$\alpha = 60 \times \frac{337.8}{2 \times 360}$$

$$= 28.125 \text{ rpm/s}$$

After 10 cycles the rotor speed is

$$N = \frac{120 \times 50}{4} + (28.125 \times 0.2)$$

$$N = 1505.6 \text{ rpm}$$

**Note:-**

If the two M/c's of inertia constant $H_1$ & $H_2$ respectively not swinging coherently together. The resultant inertia constant is

$$H_{eq} = \frac{H_1 \times H_2}{H_1 + H_2}$$

**Two Finite Machines:-**

* A system having two machines can be replaced by an equivalent system having 1 m/c connected to an infinite bus such that the swing equation and swing curves of angular displacements b/n two m/c's are same for both the m/c's.

* Let $\delta_1$, $M_1$ & $P_{a1}$ be the Quantities referred to one machine.
* Let $\delta_2$, $M_2$ & $P_{a2}$ be the Quantities referred to another machine.
* The swing equations of two m/c's are

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{a1} \Rightarrow \frac{d^2\delta_1}{dt^2} = \frac{P_{m1}-P_{e1}}{M_1}$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{a2} \Rightarrow \frac{d^2\delta_2}{dt^2} = \frac{P_{m2}-P_{e2}}{M_2}$$

* Let the relative angle b/n the two rotors is

$$\delta = \delta_1 - \delta_2$$

$$\frac{d^2\delta}{dt^2} = \frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2}$$

$$\frac{d^2\delta}{dt^2} = \frac{P_{m1}-P_{e1}}{M_1} - \frac{P_{m2}-P_{e2}}{M_2}$$

* Multiply the above equation by $\frac{M_1 M_2}{M_1+M_2}$ on b.s

$$\frac{M_1 M_2}{M_1+M_2} \frac{d^2\delta}{dt^2} = \frac{M_1 M_2}{M_1+M_2}\left[\frac{P_{m1}-P_{e1}}{M_1} - \frac{P_{m2}-P_{e2}}{M_2}\right]$$

where $M_{eq} = \frac{M_1 M_2}{M_1+M_2}$

$$\therefore \quad M \frac{d^2\delta}{dt^2} = P_m - P_e$$

wher $P_m = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1+M_2}$

$P_e = \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1+M_2}$

* With the machines are swing together the net Accelaration power is given as.

*

$$\delta_1 - \delta_2 = \delta$$

$$P_a = P_{a1} + P_{a2}$$

$$M_1 \frac{d^2\delta}{dt^2} + M_2 \frac{d^2\delta}{dt^2} = P_{a1} + P_{a2}$$

$$(M_1 + M_2) \frac{d^2\delta}{dt} = P_a$$

Where $M_{eq} = M_1 + M_2$

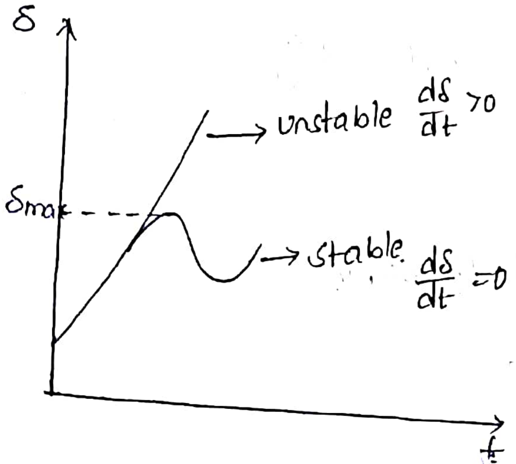
## Equal Area Criterion :-

* In a system where one machine is swing with respect to an infinite bus. It is possible to study Transient stability by means of single criterion without resorting to the neumerical solution of a swing Equation.

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{P_m - P_e}{M}$$

Where $P_a$ = Accelaration power

$$M = \frac{H}{\pi f}$$




* If the system is unstable, 'δ' continuous to increase indefinitly with time and the machine looses synchronism.

* On the other hand if the system is stable 'δ' performs oscillations whose amplitude is decreases in actual practise. because of damping terms (not included in the swing equation).

* It can be easily investigated now that for a stable system indication of steability will be given by observation of the first swing where 'δ' will go to a maximum and start to reduce. This fact can be stated as steability criterion:

* Hence the system is stable if $\frac{d\delta}{dt} = 0$ & is unstable if $\frac{d\delta}{dt} > 0$.

* For a sufficiently long time more than 1sec the steability criterion for P.S stated above can be converted into simple and easier applic-able for a single Machine infinite bus.

* Let the swing equation be

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Multiply both sides with $2\frac{d\delta}{dt}$

$$2\frac{d\delta}{dt}\left(\frac{d^2\delta}{dt^2}\right) = \frac{2P_a}{M}\frac{d\delta}{dt}$$

Now integrate on both sides

$$\frac{d^2\delta}{dt^2} = \frac{2}{M}\int_{\delta_0}^{\delta_m} P_a \, d\delta$$

$$\frac{d\delta}{dt} = \left[\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta\right]^{1/2}.$$

Where $\delta_0$ - Initial rotor angle before it begins to swing due to disturbance.

* The condition for steability is

$$\frac{d\delta}{dt} = 0$$

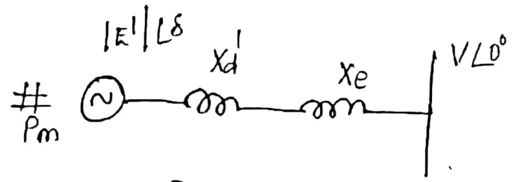
$$\int_{\delta_0}^{\delta} P_a \, d\delta = 0$$

* The condition for steability can therefore be stated as the system is stable If the area under $P_a\delta$ curve reduces to zero at some angle $\delta$.

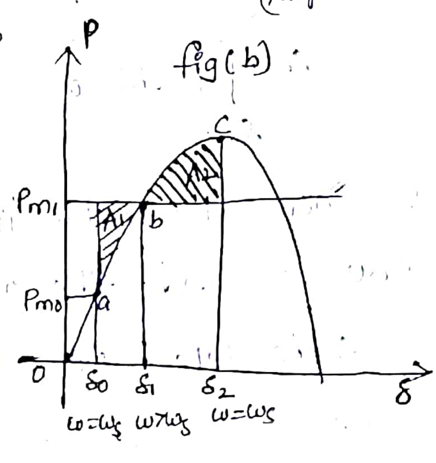
* In other words the positive accelarating area under $P_a\delta$ curve must equal to negative area i.e., decellarating area and Hence the name Equal Area Criterion.

* To illustrate the Equal area criterion of steability we consider several types of disturbances that may occur in single machine infinite bus system.

(1) Sudden change in Mechanical input :- Modern PSA (Nagarath & Kothari)




fig(a)




fig(b)

* figure (a) represents a Transient model of single m/c

infinite bus system.

* The electrical power

$$P_e = \frac{|E||V|}{X_d' + X_e} \sin\delta$$

$$P_e = P_{max} \sin\delta$$

* Under steady state operation condition i.e., $\delta = \delta_0$

$$P_{m0} = P_{e0} = P_{max} \sin\delta_0$$

This is indicated by point 'a' in fig (b)

* Let the Mechanical input to the rotor is suddenl[y] increases from $P_{m0}$ to $P_{m1}$ by opening the steam valv[e]

$\therefore P_a = P_{m1} - P_e$ causes the rotor speed to increa[se]

So that $\omega > \omega_s$

$\therefore P_{m1} = P_{e1} \Rightarrow P_a = P_{m1} - P_{e1} = P_{max} \sin\delta_1 = 0$

This is indicated by point 'b' in fig (b)

* But the rotar angle continuous to increase as $\omega > \omega_s$ $P_a$ now becomes negative i.e., decellarating. The rotor speed begins to reduce but the angle continuos to increa[se] till $\delta_2$, where $\omega = \omega_s$ once again at state point 'c'.

* At 'c' the decellarating area $A_2$ becomes equal to Accellarating are $A_1$.

$$\therefore \int_{\delta_0}^{\delta_2} P_a \, d\delta = 0$$

* Since the rotor is decelerating the speed red[uces] below $\omega_s$ and the rotor angle begins to reduce.

$\therefore$ The state point now traverses the power angle curve in the opposite as indicated.

* It is easily seen that the system oscillates about the new steady state point 'b' i.e., $\delta = \delta_1$ with angle exception upto $\delta_0$ & $\delta_2$ on the two sides.

* These oscillations are similar to simple harmonic motions of an inertia spring system except that these are not sinusoidal.

* As oscillations decay out because of inherent system damping, the system settles to the new steady state

$$P_{m_1} = P_{e_1} = P_{max} \sin \delta_1$$

* From figure (b) Areas $A_1$ & $A_2$ are given as

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m_1} - P_e)\, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m_1})\, d\delta$$

* For the system to be stable it should be posible to find such that $A_1 = A_2$

* As $P_{m_1}$ is increases, the limiting condition is finally reached when $A_1$ equals the Area above $P_{m_1}$ line as shown in figure (b).

* Under this condition '$\delta_2$' aquires the Maximum value such that $\delta_2 = \delta_{max} = \pi - \delta_1$

$$= \pi - \sin^{-1}\left(\frac{P_{m_1}}{P_{max}}\right)$$

* Any further increases in $P_{m_1}$ means that $A_2 < A_1$, So that the excess kinetic energy causes '$\delta$' to increase beyond point 'c' and the decelerating power changes to accelerating power consequently the

system becomes unstable.

* It is shown by Equal Area criterion there is a upper limit to sudden increase in mechanical i/p for the system to remain stable.

* It may also be noted that the system will remain in stable even though rotor is oscillates, beyond $\delta=90°$, so long as the Equal Area Criterion is met.

* The condition for $\delta=90°$ is met for use-ing steady state steability only and doesn't apply to the Transient steability case.

## ③ Effect of clearing time on steability :

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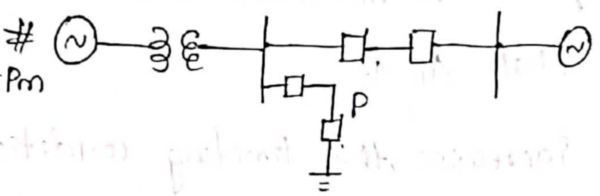



:- fig (a) :-

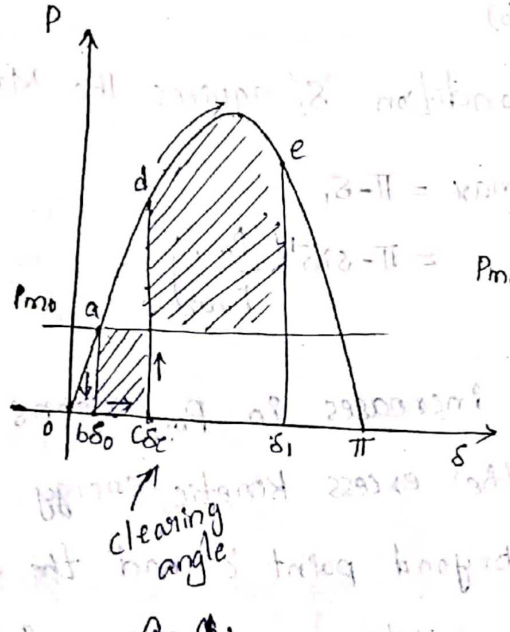



fig (b)

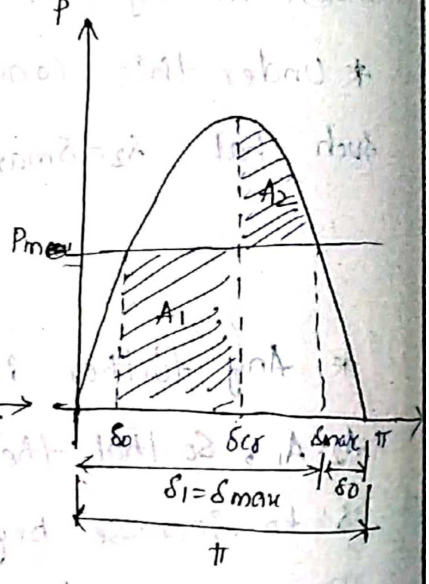



fig. (c)

* Let the System be Operating with mechanical i/p $P_{m0}$ and steady angle $\delta_0$. ~~as~~

$$P_{m0} = P_{e0}.$$

* As shown by point 'a' on the power angle curve.

* If a 3-φ fault occurs at point 'p' of outgoing radial line the electrical output is reduced to zero. i.e., $P_e = 0$ and the state point drops from 'a' to 'b'.

* The Acceleration area $A_1$ begins to increase and so ~~thus~~ does the rotor angle while the states point moves ~~does~~ ~~to~~ along bc.

* At time ~~t~~ $t_c$ corresponds to angle $\delta_c$, the faulted line is cleared, the values of $t_c$ & $\delta_c$ are known as clearing time and Clearing Angle.

* The System once again becomes healthy and transmits $P_e = P_{max} \sin\delta$ and state point shifts point 'c' on 'd'

* The rotor now decelerates and the decelerating area $A_2$ becomes. ~~mo~~ begins by state point moves along de.

* If an angle $\delta_1$ can be found such that $A_2 = A_1$ the system is found to be steable.

* The system finally settle down to the steady operating point 'a' in an oscillatory manner because of inherent damping

* As the clearing of faulty Line is delayed $A_1$ increases. and so does $\delta_1$

* To find $A_2 = A_1$ till $\delta_1 = \delta_{max}$ as shown in figure.
* For the clearing time (or) angle larger than this value, the system would be unstable as $A_2 < A_1$
* The Maximum Allowable value of clearing time & Angle for which the system remain stable are known as Critical clearing time ($t_{cr}$) & angle ($\delta_{cr}$)
* From figure 'c'

$$\delta_{max} = \pi - \delta_0$$

$$P_m = P_e = P_{max} \sin \delta_0$$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta$$

$$A_1 = P_m (\delta_{cr} - \delta_0)$$

$$\text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_e - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m)\, d\delta$$

$$= P_{max} (\cos\delta_{cr} - \cos\delta_{max}) - P_m (\delta_{max} - \delta_{cr})$$

* For the system to be stable $A_2 = A_1$

$$P_m (\delta_{cr} - \delta_0) = P_{max} (\cos\delta_{cr} - \cos\delta_{max}) - P_m (\delta_{max} - \delta_{cr})$$

$$P_m (\delta_{cr} - \delta_0 + \delta_{max} - \delta_{cr}) = P_{max} (\cos\delta_{cr} - \cos\delta_{max})$$

$$P_m (\delta_{max} - \delta_0) = P_{max} (\cos\delta_{cr} - \cos\delta_{max})$$

Substitute $\delta_{max} = \pi - \delta_0$

$P_m = P_{max} \sin \delta_0$

$P_{max} \sin\delta_0 (\pi - \delta_0 - \delta_0) = P_{max} (\cos\delta_{cr} - \cos(\pi - \delta_0))$

$\sin\delta_0 (\pi - 2\delta_0) = \cos\delta_{cr} + \cos\delta_0$

$\sin(\pi\delta_0 - 2\delta_0^2)$

$\left[\delta_{cr} = \cos^{-1}\left((\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right)\right]$

$\cos\delta_{cr} = \sin\delta_0 (\pi - 2\delta_0) - \cos\delta_0$

$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]$

$$\boxed{\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]} \quad \text{—(a)}$$

* $\Rightarrow M \frac{d^2\delta}{dt^2} = P_a$

$= P_m - P_e \qquad (P_e = 0)$

$= P_m$

$$\frac{d^2\delta}{dt^2} = P_m\left(\frac{\pi f}{H}\right) \quad \text{—(b)}$$

During the period the fault is persisting the swing equation is as above.

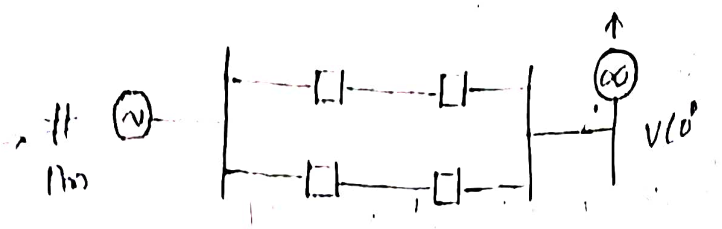
* Integrate eq (b) two times

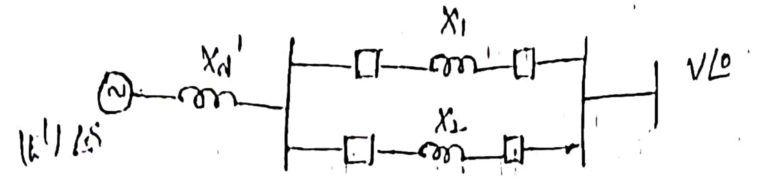
$$\iint \frac{d^2\delta}{dt^2} = \iint P_m\left(\frac{\pi f}{H}\right) dt\, dt$$

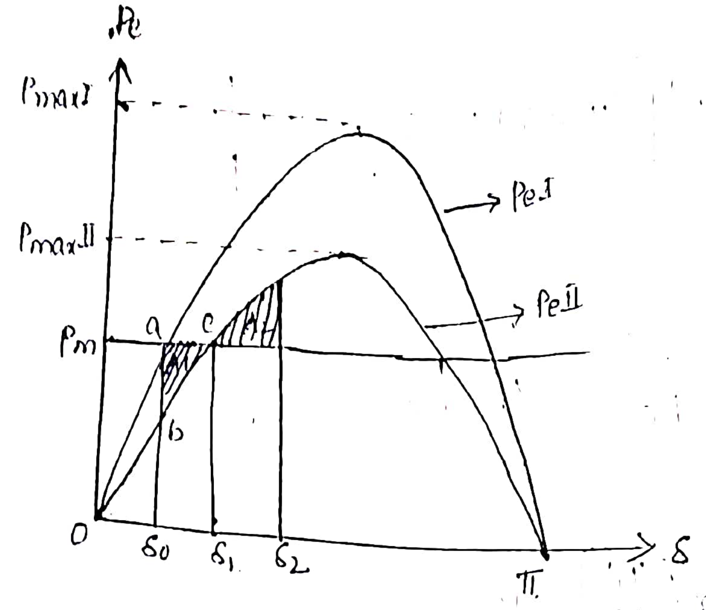
$$\delta_{cr} = \frac{\pi f}{H}\left(P_m t_{cr}^2 + \delta_0\right)$$

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}$$

(3) Sudden Loss of One of parallel lines :-




* Consider a Single Machine tied to infinite bus through the parallel line. The Transient steability of the system when one of the line say line 2 is suddenly switched off with system operating at steady load.

$$Pe_I = P_{max\,I} \sin\delta$$

$$= \frac{|E'||V|}{X_d' + (X_1 \| X_2)} \sin\delta$$

* Immediatly on switching off the line 2

$$P_{eII} = \frac{|E||V|}{X_d' + X_1} \sin\delta$$

$$= P_{max\,II} \sin\delta$$

* Now $P_{max\,II} < P_{max\,I}$ because $(X_d'+X_1) > (X_d' + (X_1 \| X_2))$

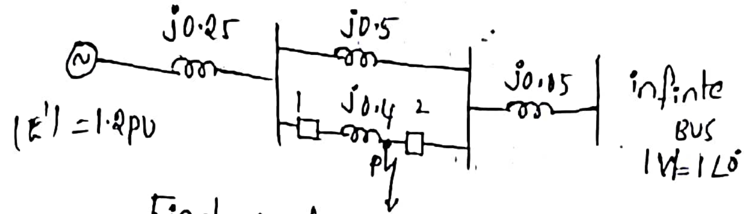
* The system is operating initially with the steady power transfer at torque angle $\delta_0$ on curve-1.
* Immediatly after switch off line-2 the operating point shifts to curve-2 i.e., point b.
* Now Accelarating Energy corresponding to $A_1$ is put into rotor followed by decellarating energy for $\delta > \delta_1$

* Assuming that an area $A_2$ corresponding to decellarating energy can be found such that $A_1 = A_2$ the System will be stable and finally operates at point c. corresponding to new rotor angle $\delta_1 > \delta_0$.

* This is so because, A single line offers larger reactance and larger rotor angle is needed to transfer same steady state power.
* It is Also easy to see that with the steady load is increased $P_m$ is shifted to upwards
* A limit is finally reached beyond which decellarating area $A_2$ can't be formed and $\therefore$ the system behaves as an unstable one
* For the limiting case of shifting $\delta$ has a maximum value of $\delta_1 = \delta_{max}$ i.e., $\delta_{max} = \delta_1 = \pi - \delta_c$

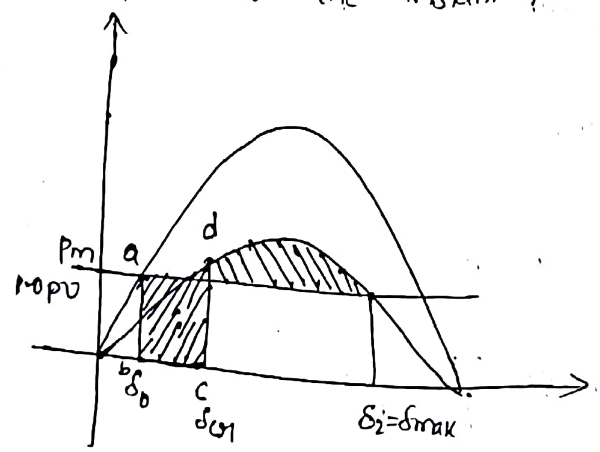
Problem:

(1) Given a system where a 3-$\phi$ -fault is applied at point A, at point B, at point C as shown in figure




Find Critical clearing Angle for clearing the fault with the simultaneous opening of the breakers 1 & 2. The values of reactances of various components are indicated on the diagram, the generator is generating 1.0pu power at the instant proceeding the fault.

Sol




$\Rightarrow$ Under normal operating conditions (or) Prefault condition

$$X_1 = j0.25 + (j0.5 \,||\, j0.4) + j0.05$$

$$= j0.522 \text{ pu}$$

$$P_m = P_{eI} = \frac{|E'||V|}{X_1} \sin\delta$$

$$= \frac{1.2 \times 1}{j0.522} \sin\delta$$

$$= 2.29 \sin\delta = 1 \text{ pu}$$

$$\sin\delta = 0.436$$

$$\delta_0 = 25.8^\circ \times \frac{\pi}{180}$$

$$= 0.45 \text{ rad}$$

which is same condition as in previous case.

Problem:

① Given a System
point A, at point

j0.25

$|E'| = 1.2pu$

Sol Find
with the
The Values
indicate
1.0pu pow

Sol

$P_m$
1.0pu.

⇒ Und

$P_m$

Now, During fault conditions no power is transferred

i.e., $P_{eII} = 0$

⇒ Now, Under Post fault condition.

$$X_{III} = j0.25 + j0.5 + j0.05$$

$$= j0.8 \text{ pu}$$

$$P_{eIII} = \frac{|E||V|}{X_{III}} \sin\delta$$

$$= \frac{1.2 \times 1}{j0.8} \sin\delta$$

$$P_{eIII} = 1.5 \sin\delta$$

⇒ The Maximum permissable angle $\delta_{max}$ for area $A_1 = A_2$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right)$$

$$= \pi - \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$\delta_{max} = 138.18^\circ$$

$$\delta_{max} = 2.412 \text{ rad}$$

⇒ Applying Equal Area Criterian for Critical clearing Angle

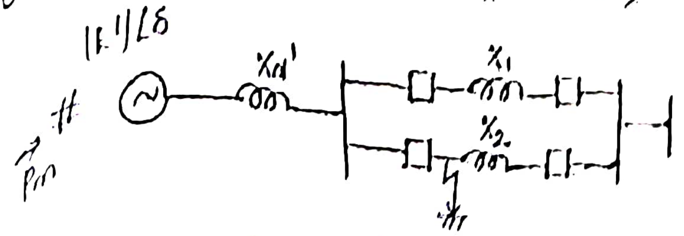
$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta$$

$$= P_m(\delta_{cr} - \delta_0)$$

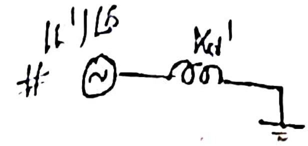
$$= 1(\delta_{cr} - 0.45)$$

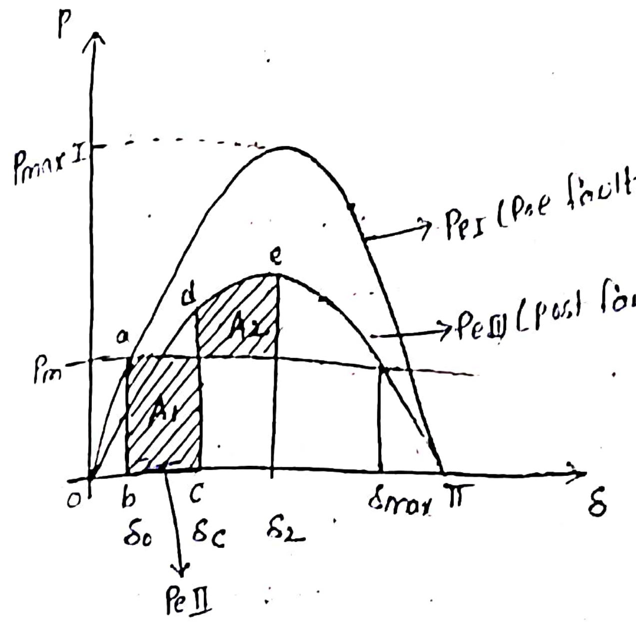
$$A_1 = \delta_{cr} - 0.45$$

(1) Sudden short circuit on one of parallel lines




During fault




* The system will be stable if the decelerating area $A_2$ can be found equal to Accelerating Area $A_1$ before $\delta$ reaches $\delta_{max}$.

Case (a)

* short circuit at one end of the line

$$* \; P_{eI} = \frac{|E'||V|}{X_d' + (X_1 \| X_2)} \sin\delta$$

$$= P_{max\,I} \sin\delta \quad \text{(Pre fault)}$$

$$* \; P_{eII} = 0 \quad \text{(P During fault)}$$

$$* \; P_{eIII} = \frac{|E'|\,|V|}{X_d' + 0} \sin\delta$$

$$= P_{max\,III} \sin\delta \quad \text{(post fault)}$$

* When circuit breaker operation is included the value of $P_{e\,III}$ is

$$* \; P_{e\,III} = \frac{|E'||V|}{X_d' + X_1} \sin\delta$$

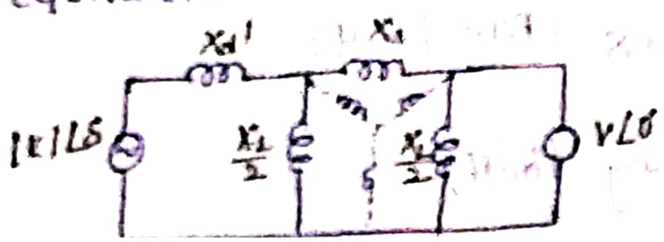
$$= P_{max\,III} \sin\delta \quad \text{(post fault)}$$

* As Area $A_1$ depends on clearing timing to corresponding to delay angle $\delta_c$. clearing time must be less than critical value i.e., critical clearing time for the system to be stable.
* It is to be observed that the equal area criterion helps to determine critical clearing angle & not critical clearing time. critical clearing time can be obtained by neumerical solution of swing equation
* It is Also easily follows that larger initial loading $P_m$ increases $A_1$ for a given clearing angle & time
* ∴ quicker fault clearing will be needed to maintain stable operation.

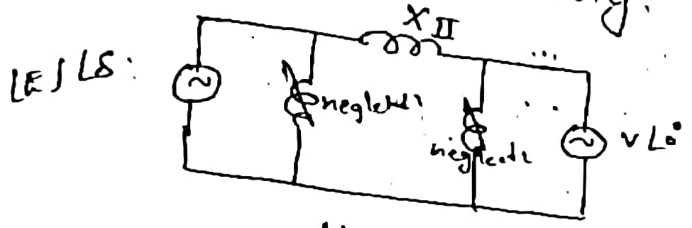
Case(b)

* short circuit at middle of the line (or) Away from line ends.

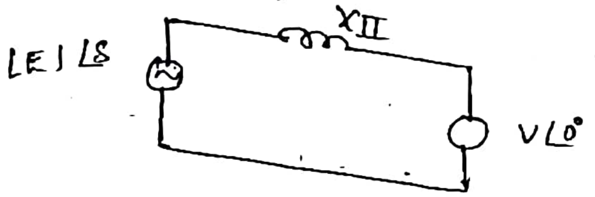
* Equivalent circuit becomes



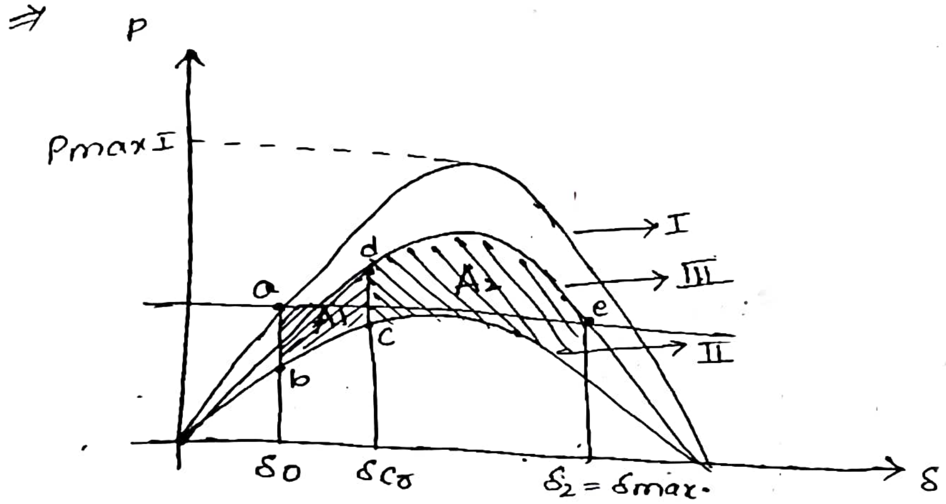
↓ Converting Y to Δ




↓




⇒




* $$P_{eI} = \frac{|E'||V|}{X_d' + (X_1 \| X_2)} \sin\delta$$

$$= P_{max\,I} \sin\delta \quad \text{(Prefault)}$$

* $$P_{eII} = \frac{|E'||V|}{X_{II}} \sin\delta$$

$$= P_{max\,II} \sin\delta \quad \text{(During fault)}$$

* $$P_{eIII} = \frac{|E'||V|}{X_d' + X_1} \sin\delta$$

$$= P_{max\,III} \sin\delta \quad \text{(Post fault)}$$

* Applying Equal Area Criterion $A_1 = A_2$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{max\,II} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max\,III} \sin\delta - P_m)\, d\delta$$

we get

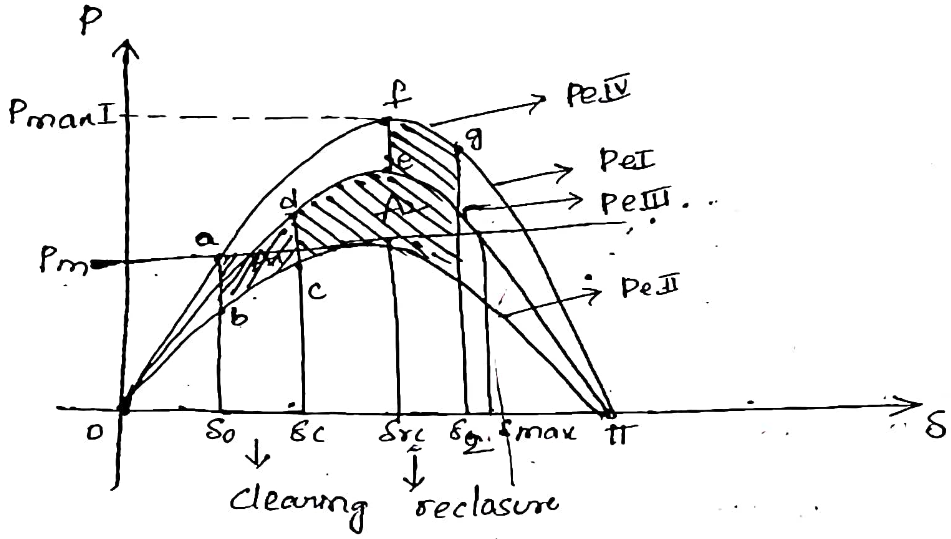
$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right)$$

$$\cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) + P_{max\,III} \cdot \cos\delta_{max} - P_{max\,II} \cos\delta_0}{P_{max\,III} - P_{max\,II}}$$

* When we have reclosure

$$Pe_{IV} = Pe_{I} = P_{max\,I} \sin\delta$$

$$\text{where } P_{max\,I} = \frac{|E'||V|}{X_d' + (X_1 \| X_2)}$$




* Now $\delta_1 = \delta_{max} - \sin^{-1}\left(\frac{P_m}{P_{max\,I}}\right)$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{max\,II}) \sin\delta \, d\delta = \int_{\delta_c}^{\delta_{rc}} (P_{max\,II} \sin\delta - P_m) \, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{max\,I} \sin\delta - P_m) \, d\delta$$

* Reclosure time $t_{rc} = t_{cr} + T$

where $T$ = time b/n clearing & reclosure

$t_{cr}$ = clearing time

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e_{III}} - P_m) \, d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} (1.5 \sin\delta - 1) \, d\delta$$

$$= \left[-1.5\cos\delta - \delta\right]_{\delta_{cr}}^{\delta_{max} = 2.41}$$

$$= -1.5\cos 2.41 - 2.41 + 1.5\cos\delta_{cr} + \delta_{cr}$$

$$= -1.498 - 2.41 + 1.5\cos\delta_{cr} + \delta_{cr}$$

$$A_2 = -3.908 + 1.5\cos\delta_{cr} + \delta_{cr}$$

$\Rightarrow$ Now $\quad A_1 = A_2$

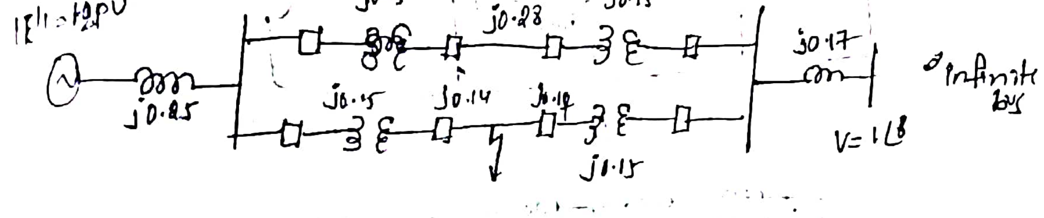
$$\cancel{\delta_{cr}} - 0.45 = -3.908 + 1.5\cos\delta_{cr} + \cancel{\delta_{cr}}$$

$$\cos\delta_{cr} = \frac{3.908 - 0.45}{1.5}$$

$$= 2.305$$

$$\delta_{cr} = 0.973 \text{ rad}$$

(2) Find the critical clearing angle the 3-$\phi$ system shown in figure at point where generater is delivering 1PU under prefault conditions




Sol $\Rightarrow$ Before the fault is occured (or) prefault conditions

$$X_1 = j0.25 + \frac{j0.15 + j0.28 + j0.15}{2} + j0.17$$

$$= j0.71 \text{ pu}$$

$$P_{eI} = \frac{|E'||V|}{X_1} \sin\delta$$

$$= \frac{1.2 \times 1}{0.71} \sin\delta$$

$$= 1.69 \sin\delta$$

$$P_m = P_{eI} = 1.69 \sin\delta = 1$$

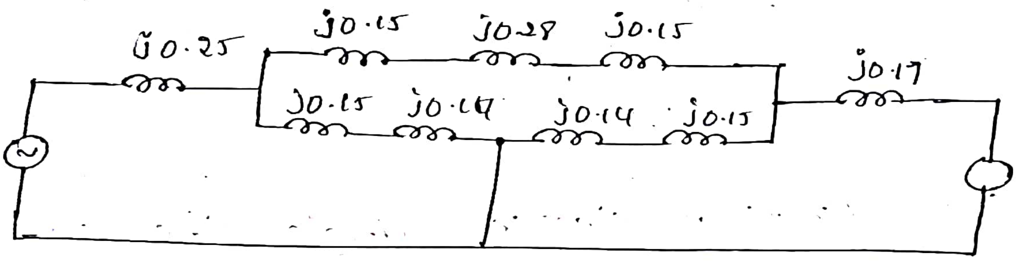
$$\sin\delta = \frac{1}{1.69}$$

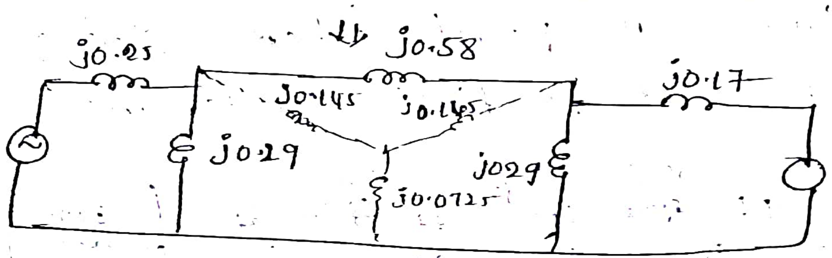
$$\delta_0 = 0.633 \text{ rad}$$

$$= 36.26^\circ$$

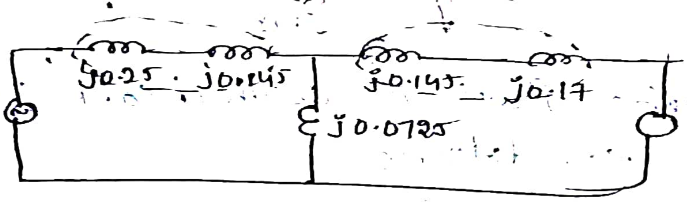
⇒ During fault condition

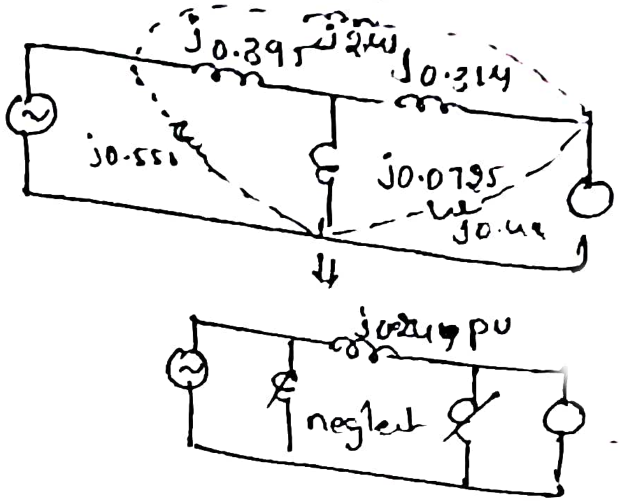
The positive sequence network is




⇓






$X_{II} = j0.24$ pu

$$Pe_{II} = \frac{1.2 \times 1}{j0.224} \sin\delta$$

$$= 0.495 \sin\delta$$

⇒ Under post fault condition

$$X_{III} = 0.25 + 0.15 + 0.23 + 0.15 + 0.17 = 1 \text{ pu}$$

$$Pe_{III} = \frac{1.2 \times 1}{1} \sin\delta$$

$$= 1.2 \sin\delta$$

$$\delta_2 = \delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right)$$

$$= \pi - \sin^{-1}\left(\frac{1}{1.2}\right)$$

$$= 123.5^0$$

$$\delta_2 = 2.156 \text{ rad}$$

⇒ Now $A_1 = A_2$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - Pe_{II})\, d\delta = \int_{0.633}^{\delta_{cr}} (1 - 0.495 \sin\delta)\, d\delta$$

$$= 1.(\delta_{cr} - 0.633) - 0.4$$

$$= (\delta_{cr} - 0.633) - 0.499\,(0.997 - \quad)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e III} - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} (1.2 \sin\delta - 1)\, d\delta$$

$$= \int_{\delta_{cr}}^{2.16} (1.2 \sin\delta - 1)\, d\delta$$

$$= 1.2(\cos\delta_{cr} - \cos 2.16) - (2.16 - \delta_{cr})$$

$\Rightarrow$ Now $A_1 = A_2$

$$\cancel{\delta_{cr}} - 0.633 - 0.495(0.999 - \cos\delta_{cr}) = 1.2(\cos\delta_{cr} - 0.99) - 2.16 + \delta_{cr}$$

$$-1.123 + \cos\delta_{cr}(0.495) = 1.2\cos\delta_{cr} - 1.198 - 2.16$$

$$0.705 \cos\delta_{cr} = -2.235 \qquad 0.055$$

$$\cos\delta_{cr} = -3.17 \qquad 86.84$$

$$=$$

$$\delta_{cr} = 0.858 \text{ rad}$$

③ A Gntr operating at 50 Hz delivering 1 PU power to an infinite bus through a Transmission circuit with negligible resistance. A Fault takes place reducing the maximum power transferred to 0.5 PU; where as before the fault his ~~covered~~ Power of 2 pu and after clearing the fault it is 1.5 PU. By the use of Equal Area Criteria Determine the Critical Clearing angle.

Given data | $P_m = 1.0\,pu$

$P_{max\,I} = 2\,pu$

$P_{max\,II} = 0.5\,pu$

$P_{max\,III} = 1.5\,pu$

$$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max\,I}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 30^\circ$$

$$= 0.523\ rad$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right)$$

$$= 2.41\ rad$$

$$\cos\delta_{cr} = \frac{P_m(\delta_{max}-\delta_0) + P_{max\,III}\cos\delta_{max} - P_{max\,III}\cos\delta_0}{P_{max\,III} - P_{max\,II}}$$

$= \cancel{-0.5528}$

$= \cancel{1.385}$

$\cancel{\delta_{cr}} = \cancel{1.2291}$

$\cancel{\cos}\ \cancel{\delta_{cr}} = \cancel{70.35}$

$\cancel{\delta_{cr}} =$

$\cos\delta_{cr} = 0.999$

$\delta_{cr} = 70.35^\circ$

$= 1.2\ rad$

## Methods to Improve Transient Stability :-

1. Use of High inertia Machines
2. Use of High speed Governers which can quickly adjust generator input to the load.
3. Use of Quick acting Voltage regulators
4. Exitation system is designed to give close voltage reggulation under Transient Consideration.
5. Reducing the severity of the faults made possible by protecting against lightening by the use of quick acting and auto reclosing circuit brakers and use of relays having small time of operation.
6. By the use of High neutral grounding impedence.
7. Reduction of Transfer reactance.

## Recent Methods to improve the stability :-

### 1. HVDC Links :-

* The increased use of HVDC Links employing thyristers would deviate stability problems.
* A DC link is a Asynchronous link in which there is no risk of fault in one system causing loss of stability in the other system.

### 2. Braking Resistors :-

* For stability improvement where clearing is delayed (or) Large load is suddenly lossed a Resistive load called Braking Resistor is connected at (or) near the

generator bus.

③ **Bypass Valving :-**

* In this method the stability of the unit is improved by decreasing Mechanical input to the turbine which can be Achieved by Fast valving. In which closing of a turbine valve to reduce the power input over 0.1 to 0.2 sec and immediatly reopened.

④ **Full Load Rejection Technique :-**

* Fast valving combined with high speed clearing time is sufficient to maintain stability in most cases; Howevere There are still situations where stability is difficult to maintain the Remedy of the situation Is a full load rejection speed could be utilised after the unit is separated from the system.

* To do this the unit has to be equiped with a large by passed steam system after system has recovered from the shock caused by the fault the Unit should be resynchronised and reloaded.

* The Main disadvantage is that it requires extra cost of Large Bypassed system.

⑤ **Neumerical Solution of Swing Equation by Point by Point Method (or) Step by step Method :-**

* It is a convensional approximate method but a well tried and proven one. It can be used to analyse Multisystem stability.

* Point by point Method is obtained here for on one machine Tied to an infinite bus.

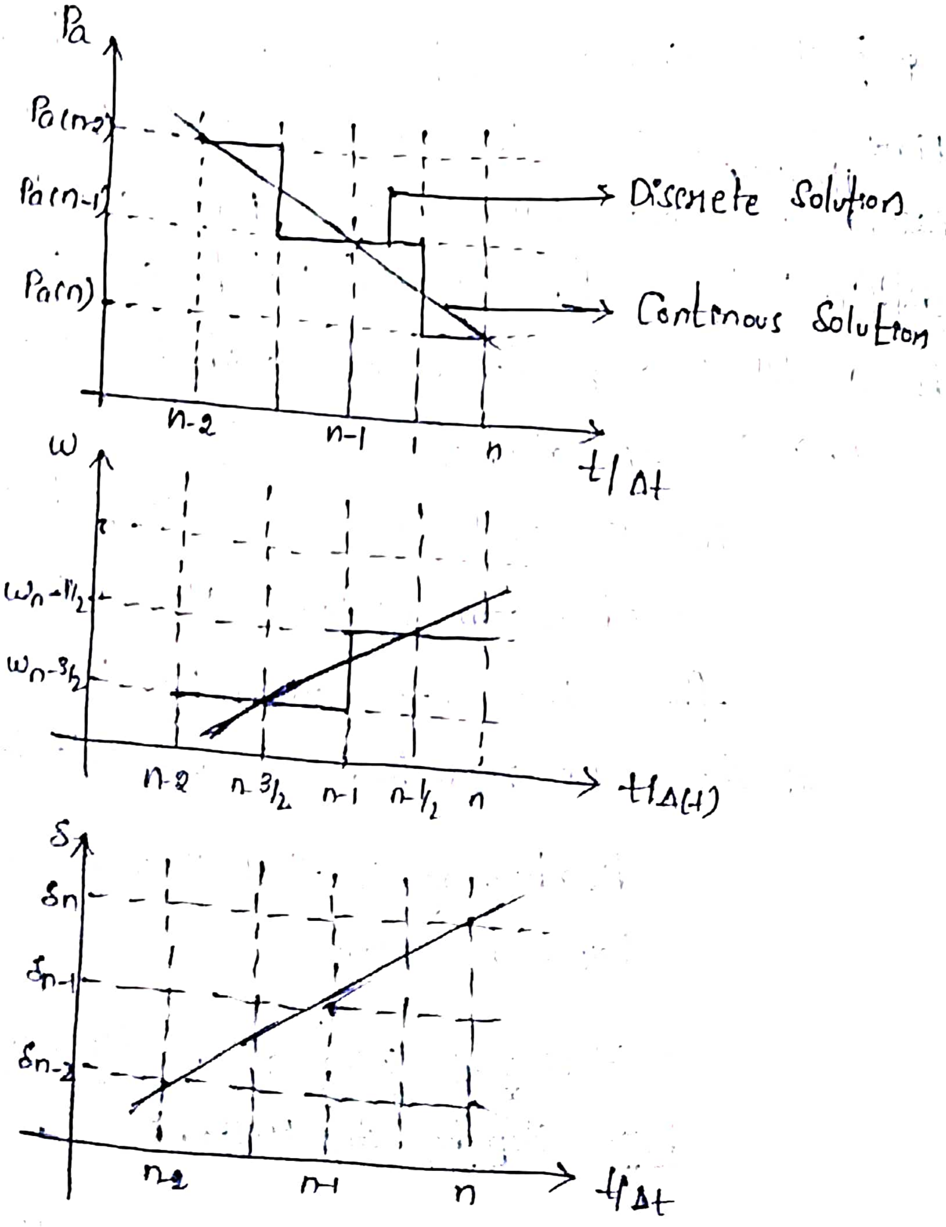
* Consider the swing equation.

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

where $M = \frac{GH}{\pi f}$

$M_{pu} = \frac{H}{\pi f}$

* The solution $\delta(t)$ is obtained at discret intervals of time with intervals spread at $\Delta t$ uniform through-out.



* The Accelarating Power & the change in speed are the discretized which are as the above

* The Pa computed at the begining of the interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered.

* The Angular Rotor velocity $\omega = \frac{d\delta}{dt}$ is assumed constant through out any interval at the value computed for the middle of interval as shown in figure for over and above synchronous velocity.

* At the end of $(n-1)^{th}$ interval the accelarating power $P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1}$

where $\delta_{n-1}$ has been previously calculated.

* The change in velocity $\omega = \frac{d\delta}{dt}$ caused by $P_{a(n-1)}$ assumed constant over $\Delta t$ from $n-3/2$ to $n-1/2$

$$\omega_{(n-1/2)} - \omega_{(n-3/2)} = \frac{\Delta t}{m} P_{a(n-1)}$$

* The change in $\Delta$ during $(n-1)^{th}$ interval is

$$\Delta\delta_{(n-1)} = \delta_{(n-1)} - \delta_{(n-2)} = \Delta t\, \omega_{(n-3/2)} \quad \text{—} (1)$$

& During $n^{th}$ Interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\, \omega_{(n-1/2)} \quad \text{—} (2)$$

subtract eq(1) from (2)

$$\Delta\delta_n - \Delta\delta_{n-1} = \Delta t \left[ \omega_{(n-1/2)} - \omega_{(n-3/2)} \right]$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \Delta t \left[ \frac{\Delta t}{m} P_{a(n-1)} \right]$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)}$$

$$\delta_n = \delta_{n-1} + \Delta\delta_n$$

$$\text{Ily } \delta_{n+1} = \delta_n + \Delta\delta_{n+1}$$

* The occurence (or) removal of fault causes a discontinuty in accelarating power $P_a$, if such a discontinuty occurs at a begining of an interval then the average of the values of $P_a$ before and after the discontinuty must be used. thus in computing the incremend of angle occuring during the first interval after a fault is applied at $t=0$ is

$$\Rightarrow \Delta\delta_1 = \frac{\Delta t^2}{M} + \frac{P_{a0+}}{2}$$

⇒ where $P_{a0+}$ is Accelarting power immediately after the occurance of fault.

$P_{a0-}$ is Accelarting power immediately before the occurance of fault.

* If the fault is cleared at begining of $n^{th}$ interval then

$$\Rightarrow P_{a(n-1)} = \frac{1}{2}\left[(P_{a(n-1)-}) + (P_{a(n-1)+})\right]$$

⇒ where $P_{a(n-1)-}$ is Accelarating power immediately before clearing the fault

$P_{a(n-1)+}$ is Accelarating power immediately after clearing the fault.

* If the Discontinuity occurs at the middle of an interval no special procedure is needed, the increment of an angle during such an interval is calculated as usual from the value of Pa at the beging of an interval.

## Factors effecting Transient Stability :

Transient Stability is greatly effected by the type ... cation of the fault.

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} Pa_{(n-1)}$$

$$\delta_n = \delta_{n-1} + \Delta\delta_n$$

* From the above equation an increase in inertial constant 'M' of the machine, reduces the angle through which it swings in a given time interval. offering there by a method of improving the stability. But this can't be employed in practice because of economic region and for the region of slowing down. the response of the speed governer loop apart from an excessive rotor ~~(weunit)~~ weight).

* As the Maximum power limit of various power angle curves is raised for the given critical angle the Accelarating area decreases but the decelarting area increases. There by adding to the Transient stability of the system.

* The Maximum power transfer of the system can be increased by rising the voltage profile of the system and reducing the transformer reactors.

Transient stability can be increased by the system voltage increases by the use of AVR [Automatic Voltage Reggulators]

→ By the use of High speed exitation system.

→ Reduction in system Transfer reactance.

→ By the use of High speed closures

* When a fault takes place in a system, the voltages decreases at generator terminals these are sensed by AVR's which helps to restore Generator terminal Voltag by Acting with in the exitation system.

* Modern exitor systems using solid state controls quickly respond to Bus voltage reduction and can achieve from 1/2 to 1 1/2 cycles gain in critical clearing times for 3-$\phi$ faults on HT side of the Transformer.

* Reducing the Transfer reactance is an another imp-ortant practical method of increasing the stability limits Consequently this also increases the system voltage profile. The reactance is reduced by

→ By reducing the Conductor spacing

→ By increasing the Conductor diameter.

However the Conductor spacing is determined by the lightenning protection & Minimum clearence to protect the arc from one phase moving to another phase.

→The conductor diameter can be increased by using material of low conductivity (or) by hallow conductors.

→The use of bundle conductors is an effective means of reducing the series reactance.

→Increasing the number of parallel lines between T/m lines is quite often used to reduce Transfer reactance which also improves the reliability of the power system.