UNIT - I

CONTROL OF DC MOTORS BY LINE COMMUTATED CONVERTERS

Power Semi-conductor Drives: It is a course that deals with single phase and three phase Converter based Dc motor control, chopper based Control of dc motors.
It also deals with Dc drives four quadrant operation.

Electric Drive: The Combination of equipment to convert one form of energy to another form of energy and to provide a control for this process is called Electric drive.

The electric drive mainly consist of source, Power modulator, motor, Load and Control unit.

Block diagram of electric drive is as shown below.

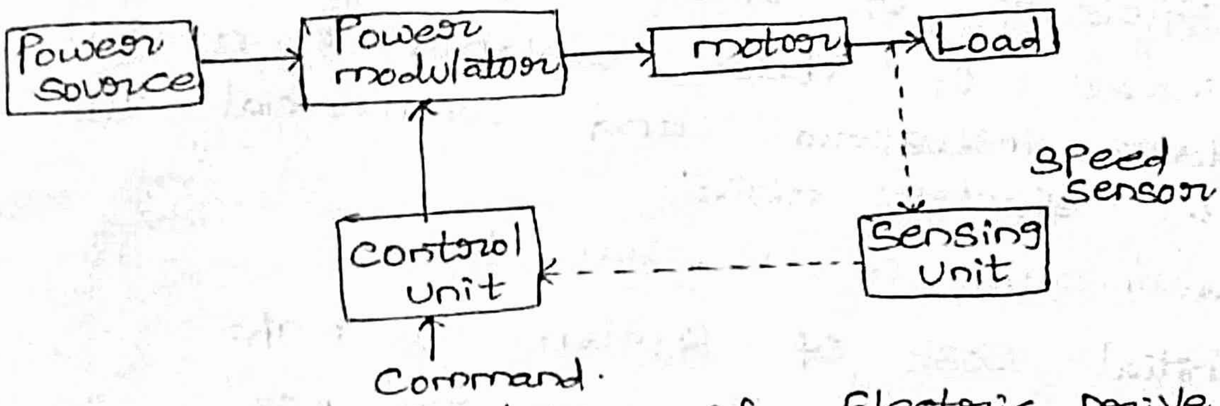


Fig. Block representation of Electric Drive.

1. Power source: The Power source may be either ac or dc as per the requirement

2. Power modulator:
Depending upon the choice of source and motor, the Power modulator was chosen.
It converts one form of energy into another form.
Ex: Rectifier, Inverter, chopper, cyclo converter etc...

3. Control Unit: It is mainly used to control the power modulator.
It is used for the safe operation of motor as well as power modulator.

4. Sensing Unit: It senses drive parameters like motor current and speed.
It is mainly required for protection and for closed loop operation.

Advantages of Electric Drive:

1. Electric drive has very large range of torque, speed and power.
2. Their working is independent of the environmental condition and are free from pollution.
3. They can be easily started.
4. Electric drive possesses four quadrant operation of speed-torque plane.
5. Efficiency of electric drive is high.

Because of these advantages all the modern industrial and commercial application employ electric drive.

DisAdvantages of Electric Drive:

1. Initial cost of system is high.
2. It has poor dynamic response.
3. It can cause noise pollution.
4. Output power obtained from drive is low.

Applications: Electric drives are widely used in industrial and domestic applications like

1. Transportation systems,
2. rolling mills, textile mills
3. Paper machines.
4. Fans, Pumps

Controlled converters are classified into two types. They are:

1. Single phase converters
2. Three phase converters.

1. Single phase converters: These are of two types:

(i) Single phase (semi) half controlled converter.
(ii) Single phase fully controlled converter.

2. Three phase converters:

(i) Three phase (semi) half controlled converter
(ii) Three phase fully controlled converter.

Single phase semi controlled converter connected to DC seperately excited motor:

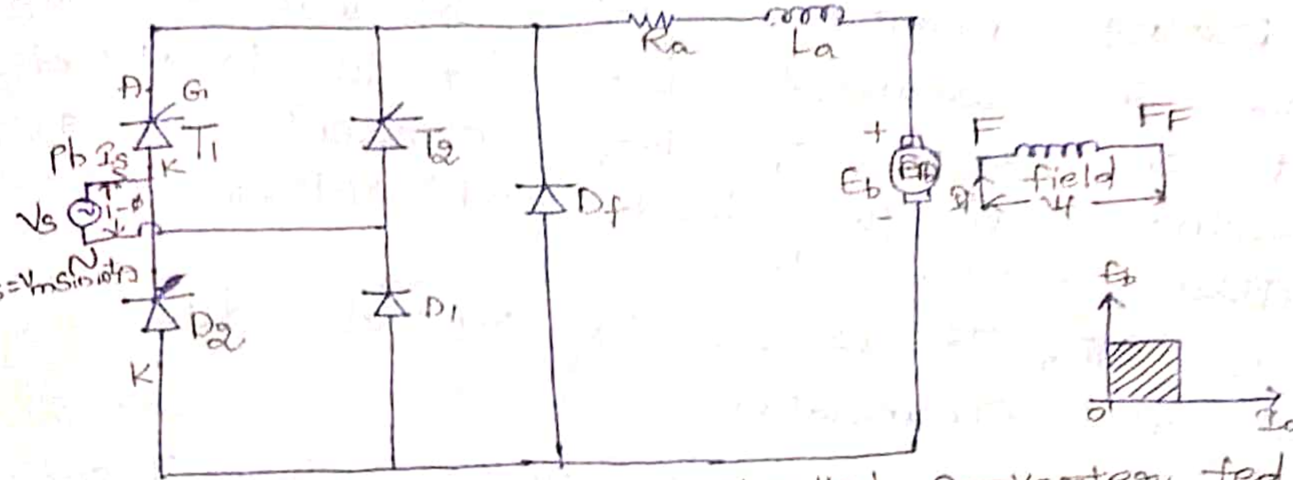


Fig. Single phase semi controlled converter fed DC seperately excited motor (It is single quadrant converter)

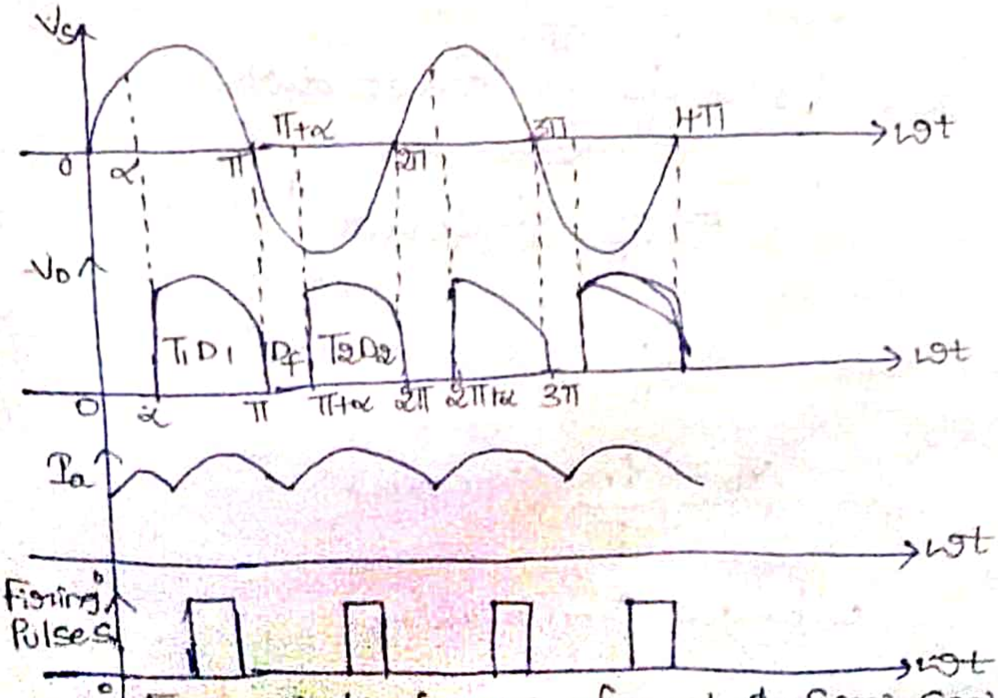


Fig. wave forms for 1-φ semi converter.

The circuit diagram, quadrant operation and wave forms for single phase semi controlled converter fed seperately excited DC motor is shown in figure.

Total number of thyristors in this converter are two and total number of diodes are two.

During positive half cycle T_1 and D_1 are in forward bias. T_1 is trigger at an angle of α, current starts flowing through the load and voltage will appear across load.

At π, T_1 comes to off state due to natural commutation.

During negative half cycle, T_2 and D_2 are in forward bias. T_2 is triggered at an angle of $\pi+\alpha$, current starts flowing through load and voltage will appear across load.

At 2π, T_2 comes to off state due to natural commutation.

For one cycle of output, two pulses are obtained. Hence it is a two pulse converter.

Output Voltage, $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \cdot d\omega t$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right]$$

$$V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$$

where $V_m = \sqrt{2}V_s$.

Speed - Torque characteristics:

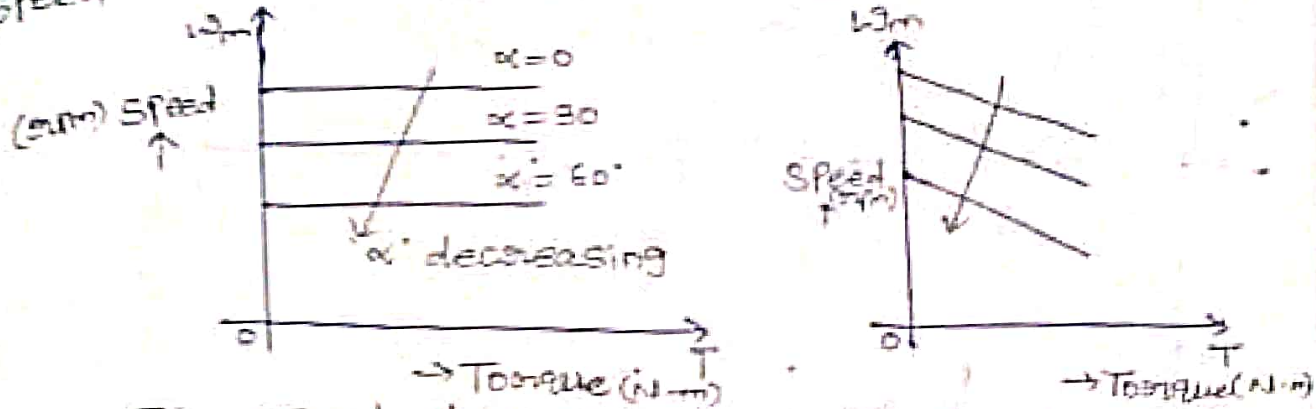


Fig. Speed - torque characteristics.

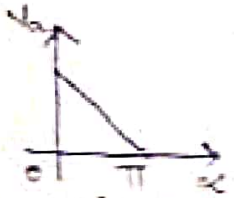


Fig. Variation of output Voltage as function of firing angle.

we Know that, For seperately excited DC motor,

Back emf, $E_b = V_o - I_a R_a$

But $E_b = K_m \cdot \omega_m$

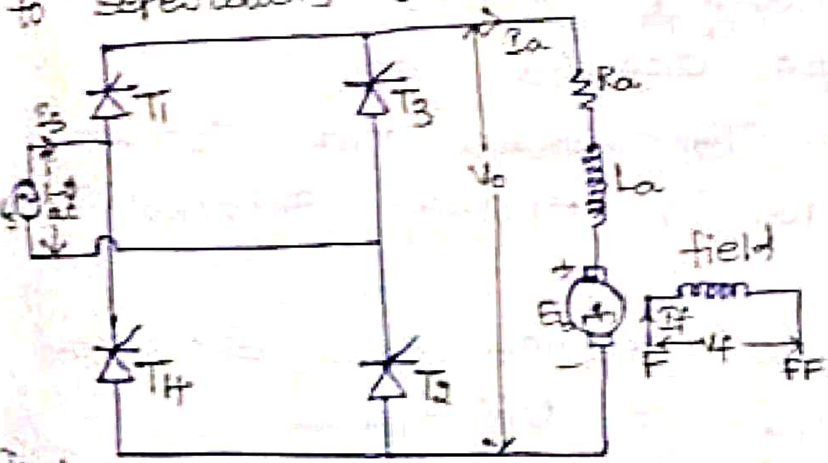
$$\Rightarrow K_m = \frac{E_b}{\omega_m} \text{ Volt sec/rad}$$

Developed Torque $\Upsilon = K_a \phi I_a$ N-m

$$\Rightarrow I_a = \frac{T}{K_a \phi}$$

Speed $N = \dfrac{V_o - I_a R_a}{K_a \phi}$ rad/sec

Single Phase Fully controlled converter connected to seperately excited DC motor:



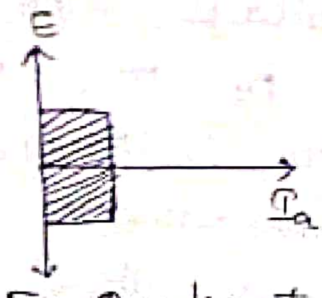


Fig. Quadrant operation.

Fig. 1-φ Fully converter fed seperately excited DC motor.

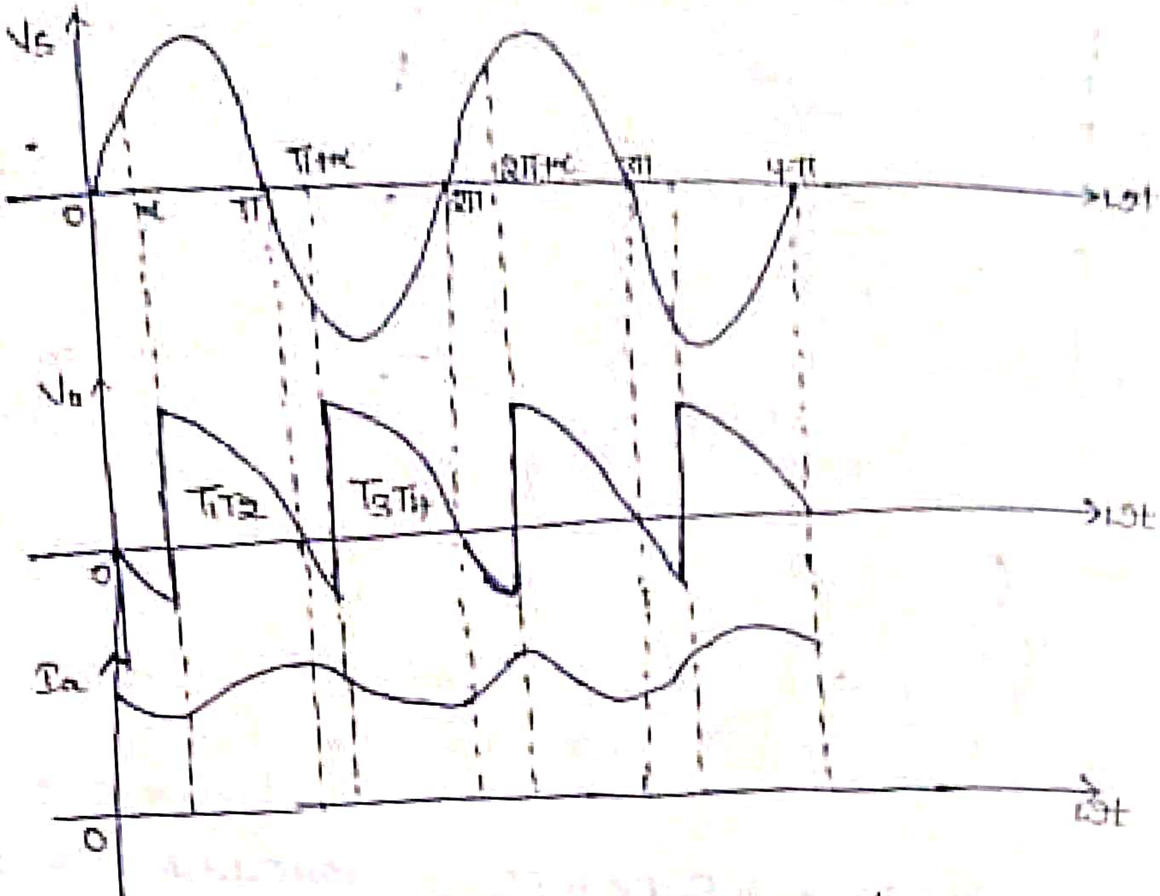


Fig. waveforms for single phase fully controlled converter fed seperately excited DC motor.

Circuit diagram, quadrant operation and waveforms for single phase half converter fed seperately excited DC motor is shown in figure.

It is a two-quadrant converter.

Total number of thyristors in this converter are four. Thyristors are triggered in sequence based upon their numbering.

During positive half cycle T_1 and T_2 are in forward bias and triggered at an angle α, current starts flowing through load and voltage will appears across load.

At π, T_1 and T_2 does not comes to off state due to energy stored in inductor.

At $\pi+\alpha$, complete energy gets dissipated and T_1, T_2 comes to off state.

During negative half cycle T_3 and T_4 are in forward bias and triggered at an angle $\pi+\alpha$, current starts flowing through load and voltage will appear across load. At 2π, T_3 and T_4 does not comes to off state due to energy stored in inductor. At $2\pi+\alpha$, complete energy gets dissipated and T_3, T_4 comes to off state.
It was observed that for one cycle of input, two pulses were obtained. Hence it is a two pulse converter.

output voltage, $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_s \cdot dt$

$$= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \, d\omega t$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[-\cos(\pi+\alpha) + \cos\alpha\right]$$

$$= \frac{V_m}{\pi}\left[-(-\cos\alpha) + \cos\alpha\right]$$

$$= \frac{V_m}{\pi}\left[2\cos\alpha\right]$$

$$V_0 = \frac{2V_m}{\pi}\cos\alpha \quad \text{where } V_m = \sqrt{2}V_s.$$

Speed - Torque characteristics:

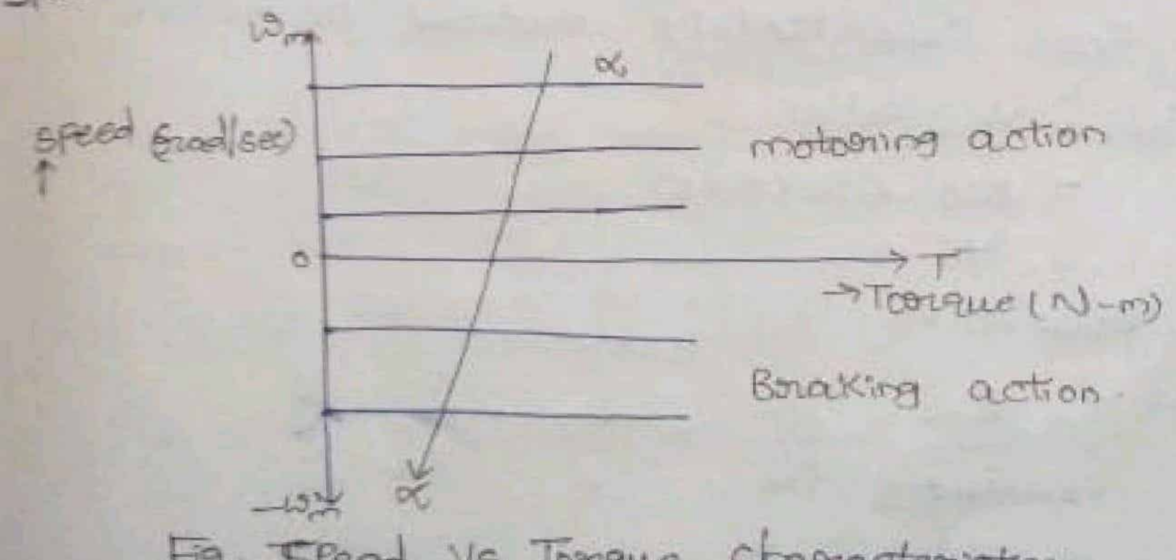


Fig. Speed Vs Torque characteristics.

For seperately excited motor,

output Voltage, $V_0 = \frac{2V_m}{\pi} \cos\alpha$

Back emf, $E_b = V_0 - I_a R_a$

But $E_b = K_m \omega_m$

$$K_m = \frac{E_b}{\omega_m}$$

$K_m = K_a\phi$ Volt sec/rad

Developed Torque, $T = K_a\phi I_a$

speed, $N = \omega_m = \frac{V_0 - I_a R_a}{K_a\phi}$ rad/sec

Problems:

1. A 220V, 875 rpm, 150A seperately excited DC motor has $R_a = 0.06\Omega$. It is fed from single phase fully controlled rectifier. Calculate
(i) Firing angle for rated motor torque at -500rpm
(ii) Find speed at 160° at rated torque
(iii) Calculate firing angle at 750 rpm for rated motor torque

Sol Given data,

$V_a = 220V$

$I_a = 150A$

(i) Given,

$N_1 = 875$ ~~rad/sec~~ rpm, $N_2 = -500$ rpm

w.K.T, For seperatetely excited DC motor,

$E_{b1} = V_0 - I_a R_a$

$= 220 - (150 \times 0.06)$

$= 220 - 9$

$E_{b1} = 211V$

w.K.T, $\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2}$

$\Rightarrow \frac{211}{E_{b2}} = \frac{875}{-500}$

$\Rightarrow 211x - 500 = 875\,E_{b2}$

$\Rightarrow E_{b2} = \frac{211x - 500}{875} = -120.571\,V$

$E_{b2} = V_0 - I_a R_a$

$V_0 = E_{b2} + I_a R_a$

$\frac{2V_m}{\pi}\cos\alpha = -120.571 + 150 \times 0.06$

$\frac{2\sqrt{2} \times 220}{\pi}\cos\alpha = -111.571$

$198.069 \cos\alpha = -111.571$

$\cos\alpha = \frac{-111.571}{198.069} \Rightarrow \alpha = \cos^{-1}\left(\frac{-111.571}{198.069}\right)$

$\alpha = 124.283^\circ$

(ii) speed at 160° at rated torque:

W.K.T, $V_0 = \frac{2V_m}{\pi}\cos\alpha$

$V_m = \sqrt{2}V_s = \sqrt{2} \times 220 = 311.126\,V$

$V_0 = \frac{2 \times 311.126}{\pi}\cos(160^\circ) = -186.124\,V$

W.K.T, $E_{b2} = V_0 - I_a R_a$

$= -186.124 - (150 \times 0.06) = -195.124\,V$

$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \Rightarrow N_2 = \frac{E_{b2} \times N_1}{E_{b1}} = \frac{875 \times -195.124}{211}$

$N_2 = -809.163$ rpm

$N_2 = -809.163 \times \frac{2\pi}{60}$

$= -84.73$ rad/sec

(iii) $\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2}$

$\Rightarrow E_{b2} = \frac{E_{b1} \times N_2}{N_1} = \frac{211 \times 750}{875} = 180.85\,V$

$E_{b2} = V_0 - I_a R_a \Rightarrow V_0 = E_{b2} + I_a R_a$

$= 180.85 + 150 \times 0.06$

$V_0 = 189.85\,V$

$$\Rightarrow \frac{2V_m}{\pi}\cos\alpha = 189.85V$$

$$\cos\alpha = \frac{189.85}{\frac{2V_m}{\pi}}$$

$$\alpha = \cos^{-1}\left(\frac{189.85}{\frac{2\sqrt{2}\times 220}{\pi}}\right) = \cos^{-1}\left(\frac{189.85}{198.069}\right)$$

$$\alpha = 16.563^\circ$$

2. A 220V, 1500 rpm, 10A seperately excited DC motor is controlled by single phase bridge rectifier with $R_a = 1.2\Omega$. 230V, 50Hz AC supply. Calculate
(i) Speed for firing angle of 30° and torque of 5 N-m.

Sol Given data,

$V_o = 220V$

$I_a = 10A$

$R_a = 1.2\Omega$

w.K.T, $E_{b1} = V_o - I_aR_a = 220 - (10\times1.2)$
$= 220 - 12 = 208V.$

Torque, $\tau = K_a\phi I_a = K_m I_a$

$$\Rightarrow I_a = \frac{\tau}{K_m}$$

$$E_b = K_m \cdot \omega_m \Rightarrow K_m = \frac{E_{b1}}{\omega_m} = \frac{208}{157.07}$$

$= 1.324$ Voltsec/rad

$$N = 1500\ rpm = 1500\times\frac{2\pi}{60} = 157.07\ rad/sec$$

$$\therefore I_a = \frac{5}{1.324} = 3.776A.$$

$$E_{b2} = V_0 - I_a R_a = \frac{2V_m}{\pi}\cos\alpha - I_a R_a$$

$$= \frac{2\sqrt{2}\times 230}{\pi}\cos(30) - (8.216\times 1.5)$$

$$= 167.002\,V$$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \Rightarrow N_2 = \frac{E_{b2}\cdot N_1}{E_{b1}} = \frac{167.002\times 1500}{208}$$

$$N_2 = 1204.341\ rpm = 1204.341\times\frac{2\pi}{60}$$

$$N_2 = 126.112\ rad/sec$$

(ii) Calculate developed torque for a firing angle of 45° and speed of 1000 rpm.

Rated speed, current and voltage means K_m is same i.e $K_m = 1.324$ Voltsec/rad.

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 230}{\pi}\cos(45°) = 146.42\,V$$

$$E_b = V_0 - I_a R_a = K_m \omega_m$$

$$\omega_m = 1000\,rpm = 1000\times\frac{2\pi}{60} = 104.719\ rad/sec$$

$$E_b = 104.719\times 1.324 = 138.648\,V.$$

$$E_b = V_0 - I_a R_a$$

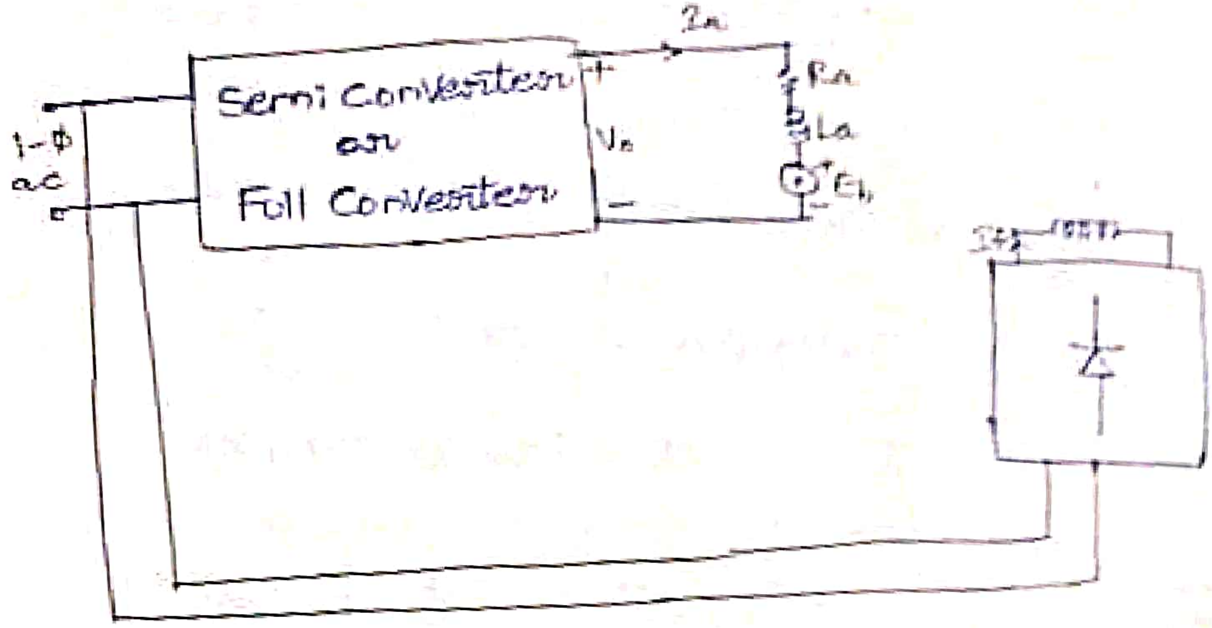
$$V_0 = E_b + I_a R_a$$

$$V_0 - E_b - I_a R_a = 0$$

$$V_0 - E_b = I_a R_a$$

$$I_a = \frac{V_0 - E_b}{R_a} = 6.48\,A.$$

Generalised Analysis:



From Figure,

Back Voltage or emf = $K_a \phi_n$

Average Back emf, $E_g = K_a \phi N$

'α' on Armature side, we get speed control below rated speed.

'α' ~~Gator~~ Vary on field side, speed control above rated speed obtained.

Developed torque $t = K_a \phi i_a$

Average developed torque $T = K_a \phi I_a$

Armature circuit voltage equation is

$$e_a = R_a i_a + L_a \frac{di_a}{dt} + e_g$$

In terms of average values,

$$E_a = E_g \mp I_a R_a \ V$$

$$N = \frac{E_a - I_a R_a}{K_a \phi} \ rad/sec$$

Single Phase Semi Converter Connected to Series DC motor:

It is Single quadrant Converter circuit diagram, quadrant operation and waveforms for single phase semi converter fed Series DC motor is shown below.

$\alpha \leq \omega t \leq \pi \rightarrow V_0 = +ve$

$\pi \leq \omega t \leq \pi+\alpha \rightarrow V_0 = 0$

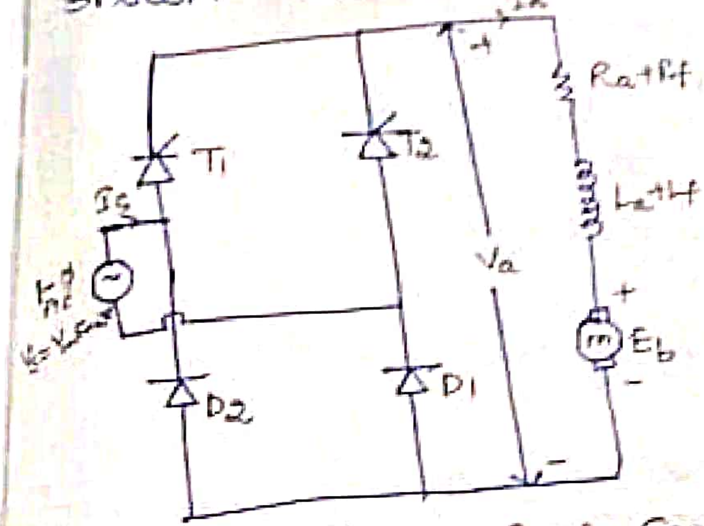


Fig. Single Phase Semi Converter fed DC Series motor

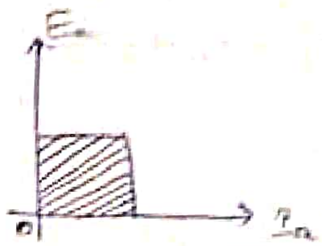


Fig. Quadrant operation

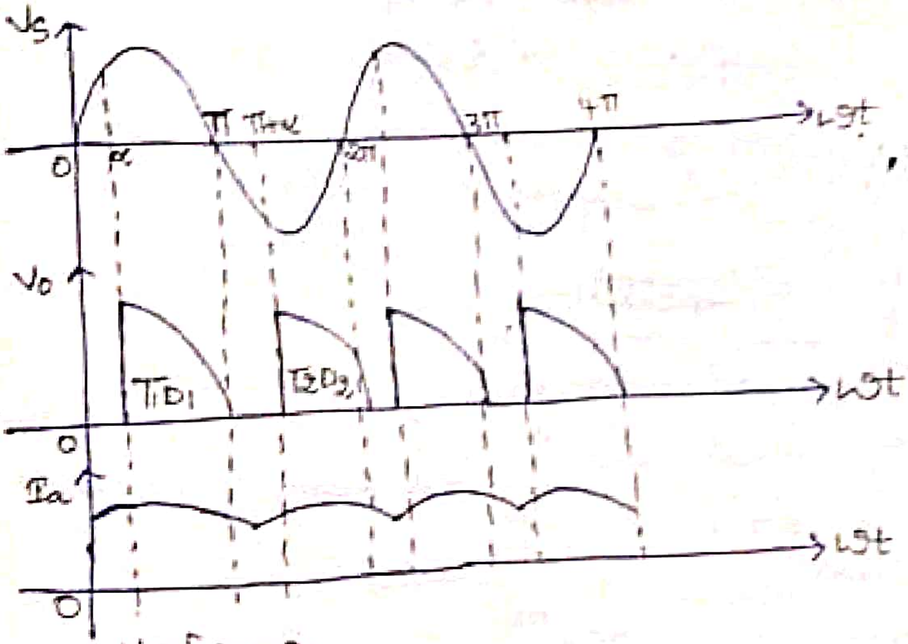


Fig. waveforms.

Total number of thyristors in this converter are two and total number of diodes are two.

During positive half cycle T_1 and D_1 are in forward bias and T_1 is triggered at an angle α, current start flowing through load and voltage will appear across load.

At π, T_1 and D_1 come to off state due to natural commutation.

During negative half cycle T_2 and D_2 are in forward bias and T_2 is triggered at an angle $\pi+\alpha$, current start flowing through the load and voltage will appear across the load at 2π, T_2 and D_2 comes to off state due to natural commutation.

It was observed that for one cycle of input, two pulses were obtained. Hence it is a two pulse converter.

output Voltage, $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \cdot d\omega t$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right]$$

$$V_0 = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

where $V_m = \sqrt{2}\,V_s$

Speed - Torque characteristics:

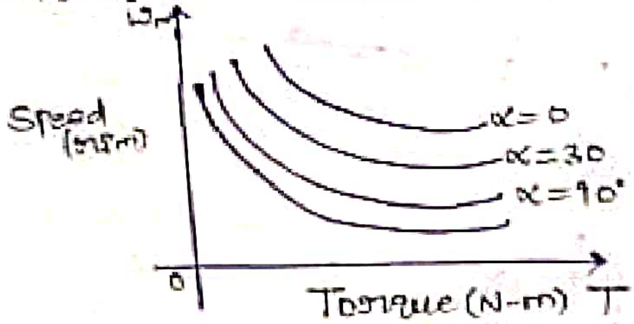


Fig. Torque speed characteristics for Single phase semi converter fed DC series motor.

we Know that,

Back Emf, $E_b = K_a \phi N$

For Series motor,

Total flux = Armature flux + Residual flux

i.e $\phi = \phi_a + \phi_r$

But $\phi_a = K_f \cdot I_a$

$\phi_{res} = K_{res}$

$\therefore E_b = K_a (K_f + K_{res}) I_a \cdot N$

$E_b = K_a(\phi_a + \phi_r)N$

$= K_a K_f I_a N + K_a K_{res} N$

$E_b = K_{af} \cdot I_a N + K_{res} \cdot N \rightarrow ①$

where $K_{af} = K_a K_f$ and $K_{res} = K_a K_{res}$

$E_b = V_o - I_a(R_a + R_f) \rightarrow ②$

From ① and ②,

$V_o = I_a(R_a + R_f) + K_{af} \cdot I_a \cdot N + K_{res} \cdot N$

Torque, $T = K_{a\phi} I_a$

$\phi = \phi_a + \phi_r$

$T_a = K_{af} \cdot I_a^2 \Rightarrow I_a = \sqrt{\frac{T_a}{K_{af}}}$

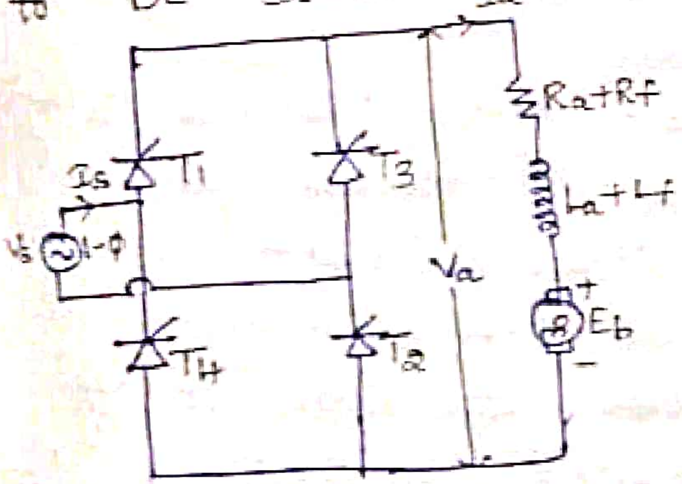
$T \propto I_a^2$

$\therefore$ speed $\Rightarrow N = \dfrac{V_o - I_a(R_a + R_f)}{K_{af} I_a + K_{res}}$

$$T = K_{af}\left[\frac{\frac{V_m}{\pi}(1+\cos\alpha) - K_{res} \cdot N}{R_a + R_f + K_{af} N}\right]^2$$

$$= K_{af}\left[\frac{V_o - K_{res} \cdot N}{R_a + R_f + K_{af} \cdot N}\right]^2$$

Single Phase Fully controlled converter connected to DC series motor:



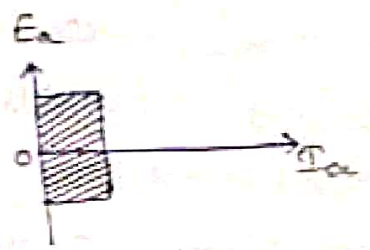


Fig. Quadrant operation.

Fig. Single phase fully controlled converted fed DC series motor

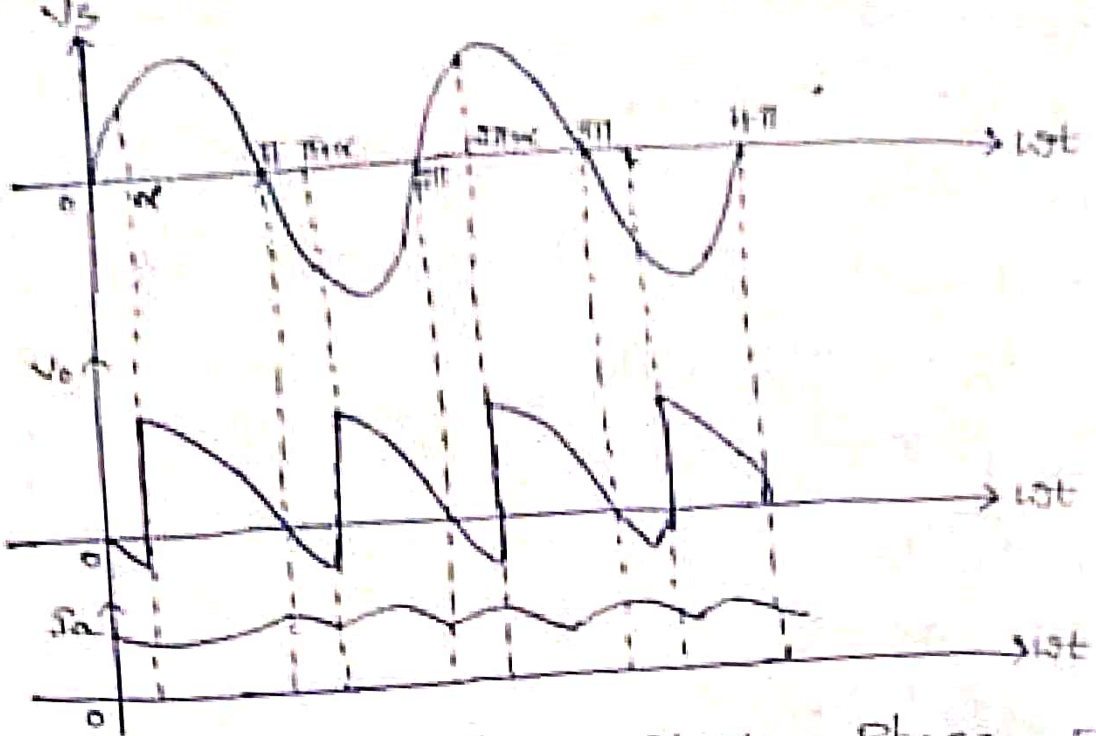


Fig. waveforms for single Phase Fully Converter fed DC series motor

It is a two quadrant converter.

Total number of thyristors in this converter are four.

During positive half cycle thyristors T_1 and T_2 are in forward bias and triggered at an angle α, current start flowing through load and Voltage will appear across load.

At π, T_1 and T_2 does not come to off state due to energy stored in inductor.

At $\pi+\alpha$, T_1 and T_2 comes to off state due to natural commutation.

During negative half cycle, T_3 and T_4 are in forward bias and triggered at an angle $\pi+\alpha$, current start flowing through load and Voltage will appear across load.

At 2π, T_3 and T_4 does not come to off state due to energy stored in inductor.

At $2\pi+\alpha$, T_3 and T_4 comes to off state due to natural commutation.

It was observed that for one cycle of input two pulses were obtained. Hence it is a two pulse converter.

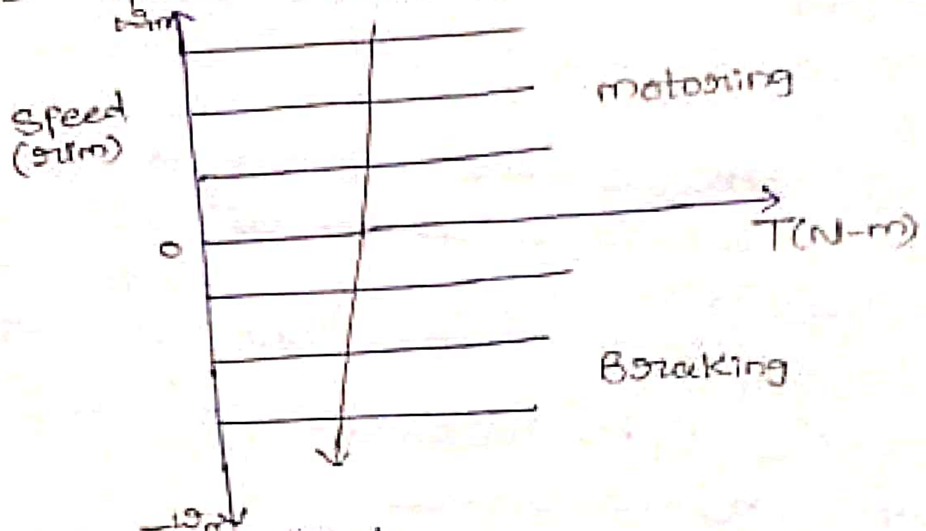
$\therefore$ output voltage, $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \, d\omega t$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[-\cos(\pi+\alpha) + \cos\alpha\right]$$

$$V_0 = \frac{2V_m}{\pi}\cos\alpha$$

Torque - speed characteristics:



we know that,

$$E_b = K_a \phi N$$

But $\phi = \phi_a + \phi_r$

$$\phi_a = K_f \cdot I_a$$

$$\phi_r = K_{res}$$

$$\therefore E_b = K_a(\phi_a + \phi_{res}) \cdot I_a \cdot N$$

$$= K_{af} I_a \cdot N + K_{res} \cdot N \rightarrow (1)$$

$$E_b = V_0 - I_a(R_a + R_f) \rightarrow (2)$$

From (1) and (2), we get

$$V_0 = I_a(R_a + R_f) + K_{af} I_a N + K_{res} \cdot N$$

Developed torque, $T = K_a \phi \cdot I_a$

$\therefore T_a = K_{af} \cdot I_a^2$

$T \propto I_a^2$

Speed $N = \dfrac{V_0 - I_a(R_a+R_f)}{K_{af} I_a + K_{res}}$

$$T = K_{af}\left[\frac{V_0 - K_{res} N}{R_a+R_f+K_{af} N}\right]^2$$

Problems:

1. The speed of 220V, 900 rpm dc series motor is controlled by a single phase semi converter. The total field and armature circuit resistance is 0.3Ω. The motor constants being $K_{af} = 0.03$ Nm/A² and $K_{res} = 0.075$ Vs/rad, find the motor torque and motor current if the firing angle of thyristors in semi-converter is 30°. The motor operates at 1000 rpm and input supply voltage is 250V ac. Also obtain motor terminal voltage

Sol Given that 1-φ semi converter,

$V_s = 250V$, $R_a+R_f = 0.3\Omega$

output voltage, $V_0 = \dfrac{V_m}{\pi}(1+\cos\alpha)$

$$= \frac{\sqrt{2}\times 250}{\pi}(1+\cos 30^\circ)$$

$$V_0 = 200V$$

Speed $N = 900$ rpm $= 900 \times \dfrac{2\pi}{60} = 94.24$ rad/sec

W.K.T, $I_a = \dfrac{V_0 - K_{res} \cdot N}{K_{af} N + R_a+R_f}$

$$\Rightarrow I_a = \frac{V_0 - K_{res} N}{K_{af} N + R_a+R_f} = \frac{200 - 0.075\times 94.24}{0.03\times 94.24 + 0.3}$$

$$I_a = 61.694 \text{ A}$$

$\tau = K_{af} \cdot I_a^2 = 0.03 \times (61.694)^2 = 114.184$ N-m

Terminal Voltage, $E_b = V_o - I_a(R_a+R_f)$

$$E_b = 200 - 61.694 \times 0.3$$

$$E_b = 181.49\,V$$

2. A DC Series motor has $R_a = 3\Omega$, $R_f = 3\Omega$, $K_{af} = 0.15$ V/rad. motor speed is controlled by single phase semi-converter, firing angle is 45°, speed of motor is 1450 rpm, applied voltage is 300 sinωt. Find steady state motor current and Torque.

Sol Given that 1-φ Semi converter fed DC series motor.

$$V_o = \frac{V_m}{\pi}(1+\cos\alpha), \quad \alpha = 45^\circ, \; R_a = 3\Omega, \; R_f = 3\Omega$$

$$V_s = V_m \sin\omega t = 300 \sin\omega t \Rightarrow V_m = 300V.$$

$$\therefore V_o = \frac{300}{\pi}(1+\cos 45) = 179.318V.$$

$$\text{motor current}, \; I_a = \frac{V_o - K_{res} N}{R_a + R_f + K_{af} \cdot N}$$

$$N = 1450 \text{ rpm} = 1450 \times \frac{2\pi}{60} = 151.843 \text{ rad/sec}$$

Note:*** K_{res} not given. Hence it neglected.

$$\therefore I_a = \frac{179.318}{3+3+0.15 \times 151.843} = 6.231\,A$$

Torque, $\tau = K_{af} \cdot I_a^2$

$$= 0.15 \times (6.231)^2$$

$$\tau = 5.823 \text{ N-m}$$

3. A 220V, 1000 rpm, 10A dc series motor has $R_a + R_f = 0.6\Omega$. It is fed from single phase fully controlled rectifier with ac voltage of 250V, 50Hz. Determine
(i) motor speed at half of rated motor torque when $\alpha = 150°$.

Sol Given data,

1-ϕ full converter fed dc series motor

$$V_o = \frac{2V_m}{\pi} \cos\alpha$$

$$= \frac{2\sqrt{2} \times 250}{\pi} \cos(150)$$

$$V_o = -194.924 \, V.$$

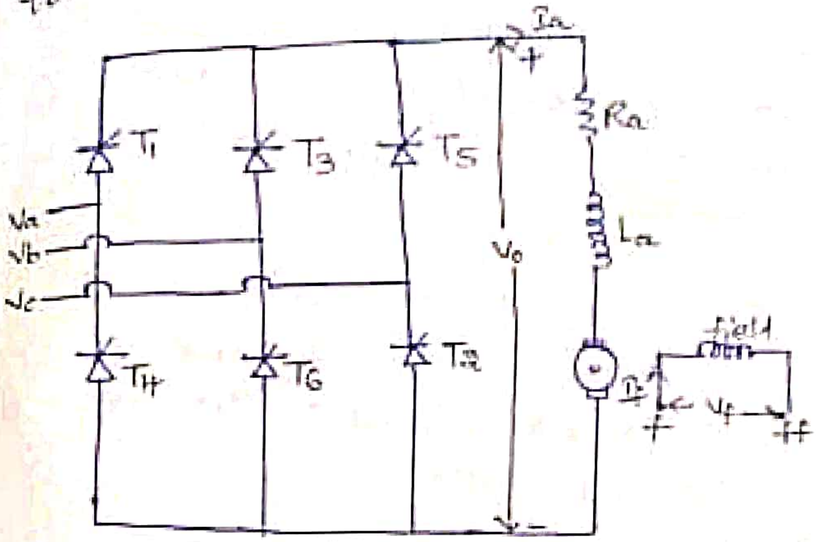
Half rated motor torque, $\Rightarrow I_a = \frac{I_a}{2} = \frac{10}{2} = 5A.$

$$T = K_{af} \cdot I_a^2$$

$$I_a = \frac{V_o - K_{res} \cdot N}{K_{af} \cdot N + R_a + R_f}$$

$$\text{Speed}, \; N = \frac{V_o - I_a(R_a + R_f)}{K_{af} \cdot I_a + K_{res}}$$

Three phase fully controlled converter connected to seperately excited DC motor:



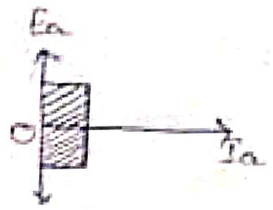
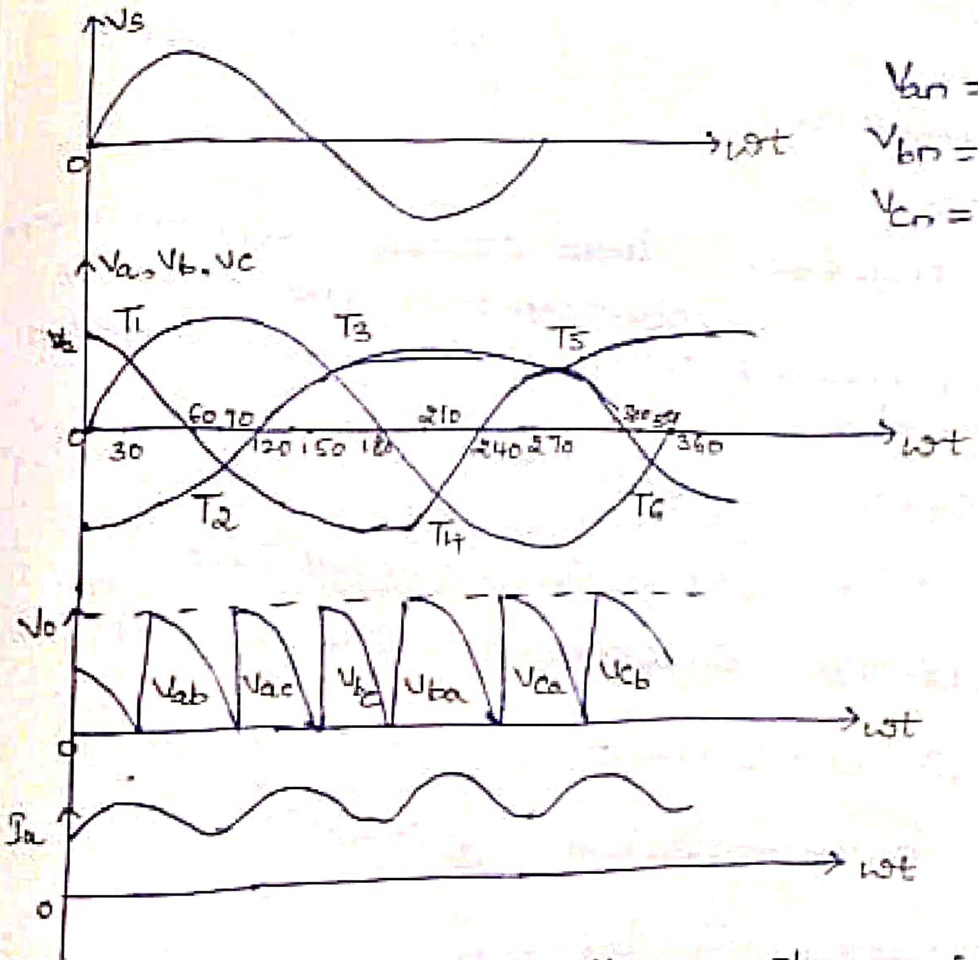


Fig. Quadrant operation

Fig circuit diagram of 3-ϕ full converter fed seperately excited DC motor.



$V_{an} = V_m \sin\omega t$

$V_{bn} = V_m \sin(\omega t - 120)$

$V_{cn} = V_m \sin(\omega t - 240)$

Fig. wave forms of three Phase full Converter fed seperately excited DC motor:

It is a two Quadrant Converter.

Total number of thyristors in this Converter are six.

Thyristors are divided into two groups.

Positive group of thyristors are T_1, T_3 and T_5.

Negative group of thyristors are T_5, T_4 and T_2.
Thyristors are triggered in sequence according to their numbering.
Each thyristor is triggered at an angle of 60°. Period of conduction of each thyristor is 120°.

Already T_6 is in conduction state. T_1 is triggered at an angle $\alpha+30^\circ$, T_1 and T_6 are in conduction. Current starts flowing and voltage will appear across load.

$$V_o = V_{ab}$$

T_2 is triggered at angle $\alpha+90^\circ$, then T_6 comes to off state due to natural commutation. T_3 is triggered at $\alpha+150^\circ$. T_2, T_3 are in conduction.

$$V_o = V_{ac}$$

In this manner thyristors are triggered based on their numbering. For one cycle of input six pulses obtained. It is a six pulse converter.

$$\therefore V_{ab} = V_{an} - V_{bn}$$

$$= V_m \sin\omega t - V_m \sin(\omega t - 120)$$

$$= V_m [\sin\omega t - (\sin\omega t \cdot \cos 120^\circ + \cos\omega t \cdot \sin 120^\circ)]$$

$$= V_m [\sin\omega t - \sin\omega t \cdot \cos(90+30) + \cos\omega t \sin(90+30)]$$

$$= V_m [\sin\omega t - [\sin\omega t(-\sin 30) + \cos\omega t \cdot \sin 30]]$$

$$= V_m [\sin\omega t + \tfrac{1}{2}\sin\omega t + \tfrac{1}{2}\cos\omega t]$$

$$= V_m \left[\tfrac{3}{2}\sin\omega t + \tfrac{1}{2}\cos\omega t\right]$$

$$= \sqrt{3}V_m \left[\tfrac{\sqrt{3}}{2}\sin\omega t + \tfrac{1}{2}\cos\omega t\right]$$

$$V_{ab} = \sqrt{3}V_m [\sin(\omega t+30)]$$

Output voltage, $V_o = \dfrac{1}{\frac{\pi}{3}} \displaystyle\int_{\alpha+30}^{\alpha+90} \sqrt{3}V_m \sin(\omega t+30) \cdot d\omega t$

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\left[-\cos(\omega t+30)\right]_{\alpha+90}^{\alpha+90}$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[-\cos(\alpha+60)+\cos(\alpha+120)\right]$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[-\cos(\alpha+120)+\cos(\alpha+60)\right]$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[-\cos[180+(\alpha-60)]+\cos(\alpha+60)\right]$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[\cos(\alpha-60)+\cos(\alpha+60)\right]$$

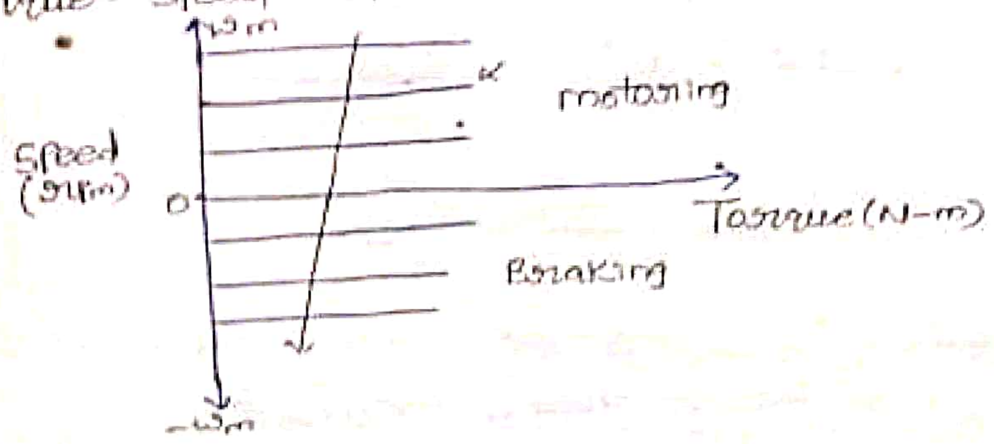
$$= \frac{3\sqrt{3}V_m}{\pi}(2\cos\alpha\cdot\cos 60)$$

$$= \frac{3\sqrt{3}V_m}{\pi}\cdot\frac{1}{\cancel{2}}\cos\alpha\cdot\cancel{2}$$

$$V_o = \frac{3\sqrt{3}V_m}{\cancel{2}\pi}\cos\alpha$$

where $V_m = \sqrt{2}V_s$ and $V_s = \frac{V_L}{\sqrt{3}}$

Torque - speed characteristics:



Back emf, $E_b = V_o - I_a R_a$

$$E_b = K_m \cdot \omega_m$$

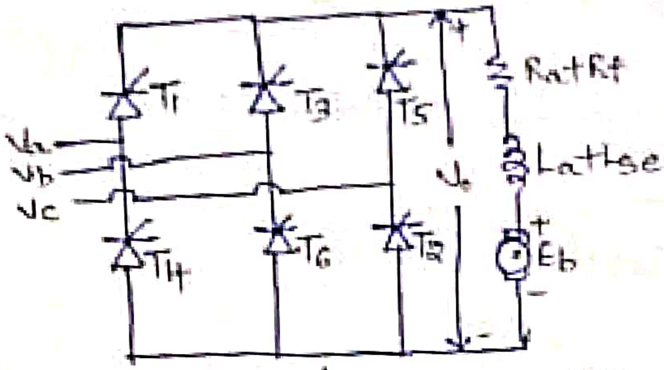
$$\tau = K_a\phi I_a$$

$$N = \frac{V_o - I_a R_a}{K_a\phi}$$

Three phase Fully Controlled Converter Connected to DC series motor:

It is a two Quadrant Converter.

Circuit diagram, quadrant operation and waveforms are shown below.



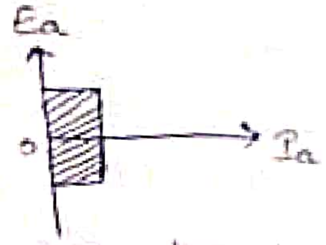


Fig. Quadrant operation

Fig. Circuit diagram for three phase fully controlled Converter fed DC Series motor.

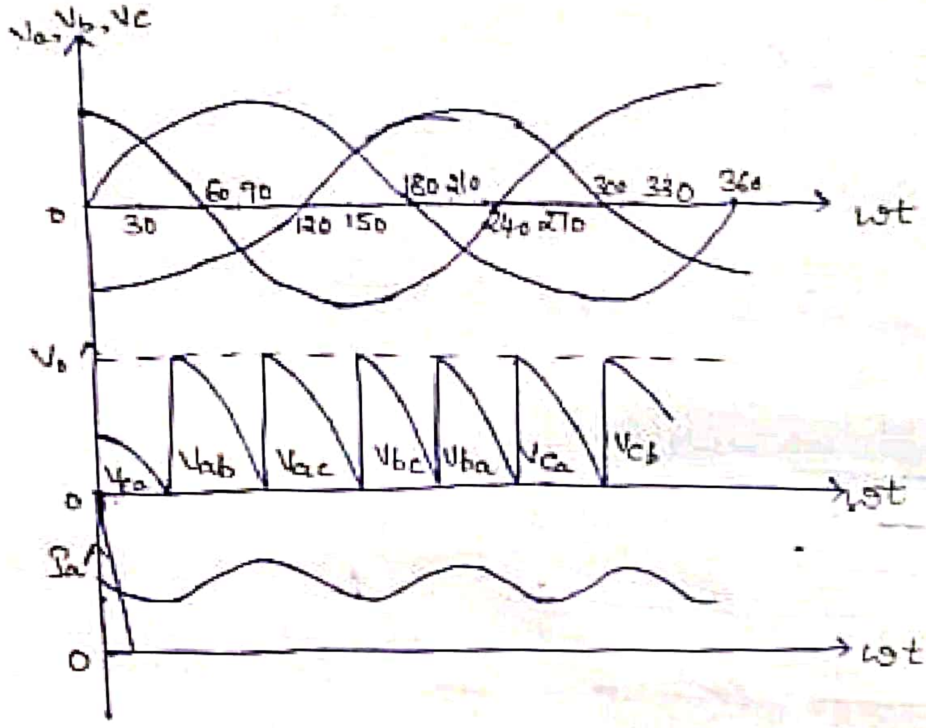


Fig. waveforms.

It is a two Quadrant Converter.

Total number of thyristors are six. Thyristors are divided into two groups i.e., positive and negative group of thyristors. Positive group of thyristors are T_1, T_3 and T_5. Negative group of thyristors are T_2, T_4 and T_6.

Thyristors are triggered in sequence based upon their numbering.

Each thyristor is triggered at an angle of 60°. period of conduction of each thyristor is 120°.

Already T_6 is in conduction state. T_1 is triggered at an angle of $\alpha+30$. T_1 and T_6 are in conduction, voltage will appear across load.

$$V_0 = V_{ab}.$$

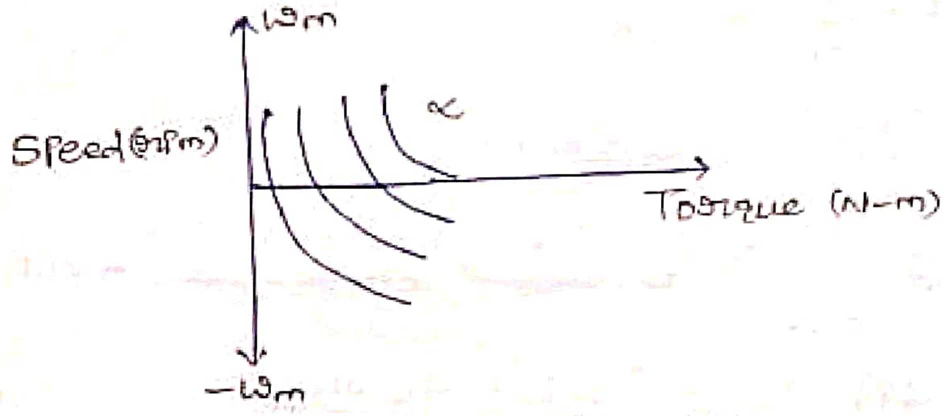
In this manner thyristors are triggered in sequence according to their numbering.

Note: output voltage derivation is as same as three phase fully controlled converter fed seperately excited DC motor.

$$V_0 = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$$

where $V_m = \sqrt{2}V_s$ and $V_s = \frac{V_L}{\sqrt{3}}$

Torque - speed characteristics:



Back emf, $E_b = V_0 - I_a(R_a+R_f)$

$$E_b = K_{af}\cdot I_a\cdot N + K_{res}\cdot N$$

$$T = K_{af}\cdot I_a^2$$

$$N = \frac{V_0 - I_a(R_a+R_f)}{K_{af}\cdot I_a + K_{res}}$$

$$T = K_{af}\left[\frac{V_0 - K_{res}\cdot N}{K_{af}N + R_a+R_f}\right]^2$$

where $I_a = \frac{V_0 - K_{res}N}{K_{af}\cdot N + R_a + R_f}$

Problems:

1. The speed of seperately excited Dc motor is controlled by a three phase fully controlled rectifier. Calculate speed of motor at a torque of 50 N-m for firing angle of 45°. 3-ϕ, 415V, 50Hz, $K_m = 1.5$ Vs/rad, $R_a = 0.9\Omega$

Sol Given data,

3-ϕ fully controlled fed seperately excited dc motor.

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$$

$$V_m = \sqrt{2}V_s \Rightarrow V_s = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6V$$

$$V_m = \sqrt{2} \times 239.6 = 338.845V.$$

$$T = K_m \cdot I_a$$

$$I_a = \frac{T}{K_m} = \frac{50}{1.5} = 33.33 A$$

$$V_o = \frac{3\sqrt{3} \times 338.845}{\pi}\cos 45 = 396.294 V$$

$$N = \frac{V_o - I_a R_a}{K_a\phi} = \frac{396.294 - 33.33 \times 0.9}{1.5} = 244.19 \text{ rad/sec}$$

$$N = 244.19 \times \frac{60}{2\pi} = 2331.84 \text{ rpm.}$$

2. A 220V, 1500rpm, 50A seperately excited dc motor is fed from three phase fully controlled rectifier with AC source voltage of 440V, 50HZ. Determine firing angle when motor running at -800rpm twice the rated torque. $R_a = 0.5\Omega$.

Sol Given that,

3-ϕ fully controlled rectifier fed seperately excited dc motor.

$V_L = 440V$

$N_1 = 1500$ rpm

$N_2 = -800$ rpm

$I_a = 50\,A$

$R_a = 0.5\,\Omega$

$$V_0 = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha \rightarrow ①$$

$V_m = \sqrt{2}\,V_{ph}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.034\,V$$

$V_m = \sqrt{2} \times 254.034 = 359.25V$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2}$$

$E_{b1} = V_0 - I_aR_a = 220 - 50\times0.5 = 195V$

$E_{b2} = V_0 - I_aR_a = 440 - 50\times0.5 = 415V$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \Rightarrow E_{b2} = \frac{E_{b1}\cdot N_2}{N_1} = \frac{195\times -800}{1500} = -104V$$

$V_0 = E_{b2} + I_aR_a = -104 + 50\times0.5 = -54V$

$$① \Rightarrow -54 = \frac{3\sqrt{3}\times 359.25}{\pi}\cos\alpha$$

$-54 = 594.19\cos\alpha$

$$\cos\alpha = \frac{-54}{594.19} = 95.21^\circ$$

Note:

1. $\alpha > 90 \rightarrow$ Speed is Negative
2. $\alpha < 90 \rightarrow$ Speed is Positive

3. A 80 Kw, 440V, 800 rpm seperately excited DC motor is operating at 600 rpm and developing 75% of rated torque controlled by 3-ϕ six pulse converter. If back emf at rated speed is 410V, Determine firing angle of armature converter. Input is 3-ϕ, 450V, 50Hz supply.

Sol Given that,

3-ϕ six Pulse Converter fed seperately excited DC motor.

Power $P = 80\,Kw$.

$V_L = 450V$

$V_0 = 440\ V$

$N_1 = 800\ rpm$

$N_2 = 600\ rpm$

$P = 3V_{ph} I_a \Rightarrow I_a = \frac{P}{3V_{ph}}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{450}{\sqrt{3}} = 259.807\ V$$

$$I_a = \frac{80 \times 10^3}{3 \times 259.807} = 102.64\ A$$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \Rightarrow E_{b2} = \frac{E_{b1} N_2}{N_1}$$

$$E_{b1} = V_0 - I_a R_a \Rightarrow 440 - 102.64 \times R_a = 410$$

$$R_a = \frac{410 - 440}{-102.64} = 0.29\Omega$$

$$E_{b2} = \frac{410 \times 600}{800} = \cancel{308.27\ V}\ 307.5\ V$$

$$E_{b2} = V_0 - I_a R_a$$

$$307.5 - V_0 = -0.75 \times 102.64 \times 0.29$$

$$-V_0 = -0.75 \times 102.64 \times 0.29 - 308.27$$

$$V_0 = 330.594\ V$$

$$\frac{3\sqrt{2}V_m}{\pi} \cos\alpha = 330.594\ V$$

$$\frac{3\sqrt{2} \times \sqrt{3} \times 259.807}{\pi} \cos\alpha = 330.594$$

$$330.594 = 607.71 \cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{330.594}{607.71}\right)$$

$$\alpha = 57.04^\circ.$$

Three Phase (half) Semi Controlled Converter fed Seperately Excited DC motor:

It is a Single quadrant converter.

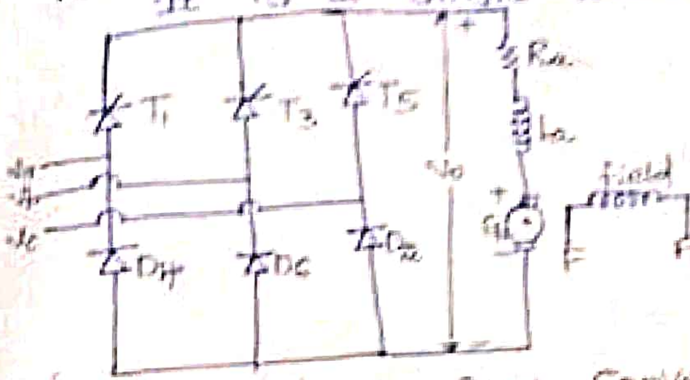


Fig. Three Phase Semi Converter fed Seperately Excited DC motor.

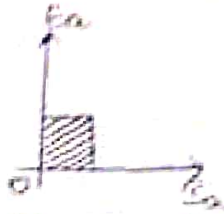


Fig. quadrant operation.

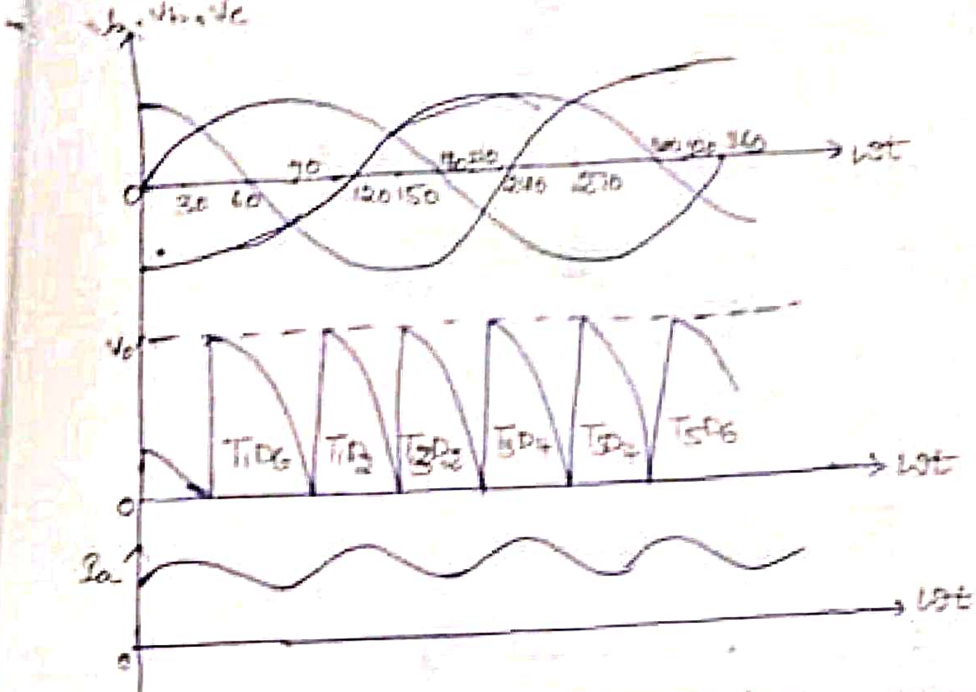


Fig. wave forms for three Phase Semi Controlled Converter Connected to Seperately excited DC motor.

Total number of thyristors in this Converter are three and total number of diodes are three.

Period of Conduction of each thyristor is 120°.

Already T_6 is in Conduction. T_1 is triggered at angle of $\alpha+30$, d_6 starts conducting. Current starts flowing through load and Voltage will appear across load.

$$V_0 = V_{ab}$$

D_6 Comes to off state at $\frac{\pi}{2}$ due to reverse bias. Next D_2 start Conducting with T_1.

$$V_0 = V_{ac}$$

In this manner, thyristors are triggered
sequence based upon their numbering.
For one cycle of input, six pulses were
obtained. Hence it is a six pulse converter.

Average output voltage,

$$V_0 = \frac{1}{\frac{2\pi}{3}} \int_{\alpha+30}^{120°\,\frac{\pi}{2}} V_{ab}\cdot d\omega t + \int_{60°\,\frac{\pi}{2}}^{\alpha+180°} V_{ac}\cdot d\omega t$$

$$= \frac{3}{2\pi} \int_{\alpha+30}^{120\,\frac{\pi}{2}} \sqrt{3}V_m \sin\omega t\cdot d\omega t + \int_{60\,\frac{\pi}{2}}^{\alpha+150°} \sqrt{3}V_m \sin\omega t\cdot d\omega t$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \int_{\alpha+30}^{\frac{\pi}{2}} \sin\omega t\cdot d\omega t + \int_{\frac{\pi}{2}}^{\alpha+150} \sin\omega t\cdot d\omega t$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \left[\left[-\cos\omega\right]_{\alpha+30}^{\frac{\pi}{2}} + \left[-\cos\omega\right]_{\frac{\pi}{2}}^{\alpha+150} \right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \left[0 + \cos(\alpha+30) + -\cos(\alpha+150) - 0\right]$$

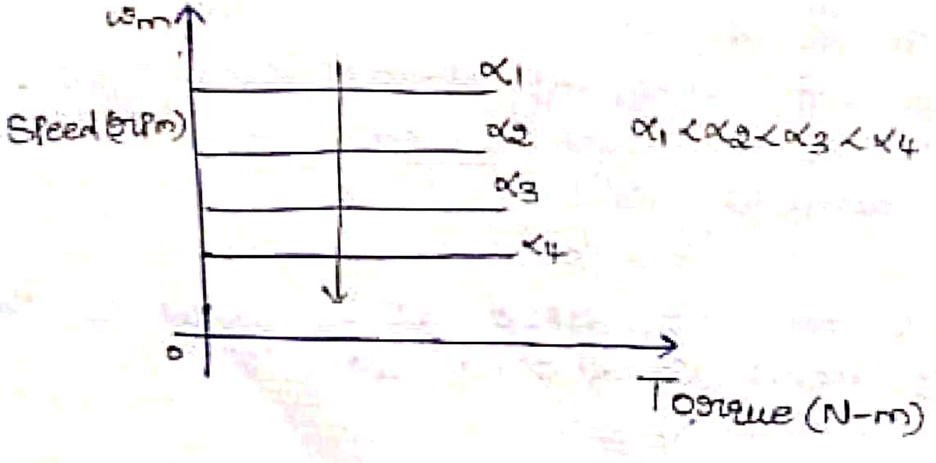
$$= \frac{3\sqrt{3}V_m}{2\pi} \left[\cos(\alpha+30) + -\cos(\alpha+150)\right]$$

$$V_0 = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

where $V_m = \sqrt{2}V_s$

$$V_s = \frac{V_L}{\sqrt{3}}$$

Torque - Speed characteristics:



$\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$

For Seperately Excited DC motor,

$$E_b = V_0 - I_a R_a$$

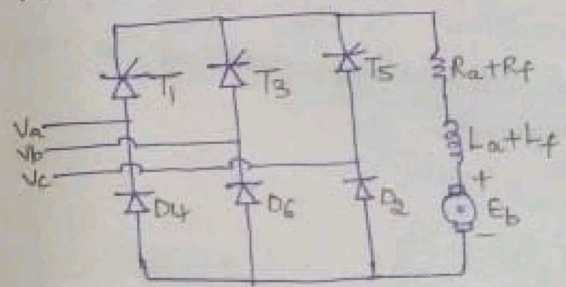
$$E_b = K_m \cdot \omega_m$$

$$T = K_a \phi I_a$$

$$I_a = \frac{T}{K_a \phi}$$

Speed $N = \dfrac{V_0 - I_a R_a}{K_a \phi}$

Three phase semi controlled converter connected to Series DC motor:



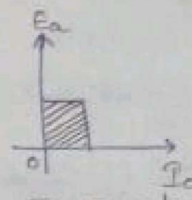


Fig. Quadrant operation

Fig. 3-φ Semi Converter Connected to DC Series motor

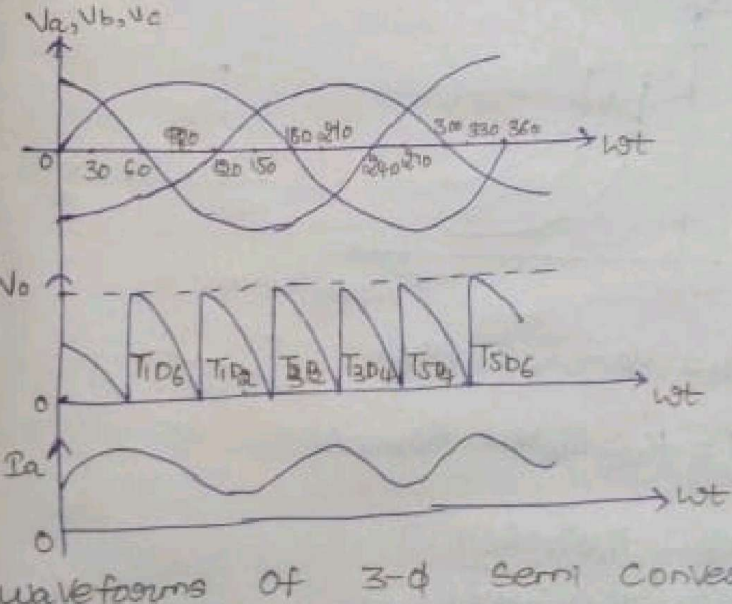


Fig. waveforms of 3-φ Semi Converter fed DC Series motor

It is a single Quadrant Converter.

Total number of thyristors in this Converter are three and total number of diodes are three. Period of conduction is 120°

Thyristors are triggered in sequence based upon their numbering.

Already T_6 is in conduction T_1 is triggered at $\alpha+30$, T_1 and D6 are in conduction

$$V_o = V_{ab}$$

Due to reverse bias D6 comes to off state at $\frac{\pi}{6}$.
Next D2 start conducting with T_1.

$$V_o = V_{ac}.$$

In this manner thyristors are triggered in sequence based upon their numbering.

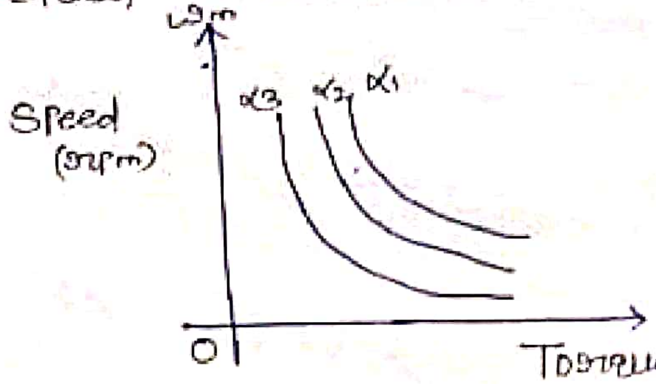
For one cycle of input, six pulses obtained. Hence it is a six pulse converter.

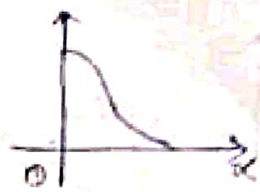
Note: output voltage derivation is as same as 3-ϕ semi converter fed separately excited DC motor.

$$\therefore V_o = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$V_m = \sqrt{2}V_s \quad \text{and} \quad V_s = \frac{V_L}{\sqrt{3}}$$

Torque - speed characteristics:





Back emf, $E_b = V_o - I_a(R_a+R_f)$

$$\tau = K_{af} I_a N + K_{res} \cdot N$$

$$N = \frac{V_o - I_a(R_a+R_f)}{K_{af} \frac{T}{I_a} + \cancel{R_a+R_f} K_{res}}$$

S.No	Type of motor	Output voltage & back emf	Torque and speed
1	1-ϕ Semi Converter fed Seperately excited DC motor	$V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$ $E_b = V_0 - I_a R_a$ $E_b = K_a \phi N$	$T = K_a \phi I_a = K_a \phi I_a$ $N = \frac{V_0 - I_a R_a}{K_a \phi}$
2	1-ϕ Semi Converter fed DC Series motor	$V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$ $E_b = V_0 - I_a(R_a+R_f)$ $E_b = K_{af} I_a N + K_{res} N$	$T = K_a \phi I_a$ $T = K_{af} I_a^2$ $N = \frac{V_0 - I_a(R_a+R_f)}{K_{af} I_a + K_{res}}$
3	1-ϕ fully Converter fed Seperately excited DC motor	$V_0 = \frac{2V_m}{\pi}\cos\alpha$ $E_b = V_0 - I_a R_a$ $E_b = K_m \omega_m$	$T = K_a \phi I_a$ $N = \frac{V_0 - I_a R_a}{K_a \phi}$
4	1-ϕ fully Converter fed DC series motor	$V_0 = \frac{2V_m}{\pi}\cos\alpha$ $E_b = V_0 - I_a(R_a+R_f)$ $E_b = K_{af} \cdot I_a \cdot N + K_{res} N$	$T = K_a \phi I_a$ $T = K_{af} \cdot I_a^2$ $N = \frac{V_0 - I_a(R_a+R_f)}{K_{af} I_a + K_{res}}$
5	3-ϕ Semi Converter fed Seperately excited motor	$V_0 = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$ $T = K_a \phi I_a$ $E_b = V_0 - I_a R_a$; $V_S = \frac{V_L}{\sqrt{3}}$	$T = K_a \phi I_a$ $N = \frac{V_0 - I_a R_a}{K_a \phi}$
6.	3-ϕ fully Converter fed Seperately excited DC motor	$V_0 = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ $T = K_a \phi I_a$ $E_b = V_0 - I_a R_a$	"
7.	3-ϕ Semi Converter fed DC series motor	$V_0 = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$ $E_b = K_a \phi I_a N + K_{res} \cdot N$ $E_b = V_0 - I_a(R_a+R_f)$	$T = K_{af} \cdot I_a^2$ $N = \frac{V_0 - I_a(R_a+R_f)}{K_{af} I_a + K_{res}}$
8.	3-ϕ full Converter fed DC Series motor	$V_0 = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ $E_b = V_0 - I_a(R_a+R_f)$ $E_b = K_{af} I_a N + K_{res} \cdot N$	"

Notes:

Parameter	Unit
Back emf (E_b)	Volts
Torque (T)	N-m
K_m	Volt sec/rad
Speed (N)	rpm $= \frac{2\pi}{60}$ rad/sec

* If Kres value is not given in problem it has to be neglected.
* Two speed were given and to calculate back emf

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2}$$

1. By using Armature control method, speed below rated speed was obtained.
2. By field control method, speeds above rated speed was obtained.