FINITE IMPULSE RESPONSE FILTERS.

In many digital processing applications FIR filters are preferred over their IIR counterparts.

The following are the main advantages of the FIR filter over IIR filters.

1. FIR filters are always stable.
2. FIR filters with exactly linear phase can easily be designed.
3. FIR filters can be realized in both recursive & non-recursive structures.
4. FIR filters are free of limit cycle oscillations, when implemented on a finite word length digital systems.
5. Excellent design methods are available for various kinds of FIR filters.

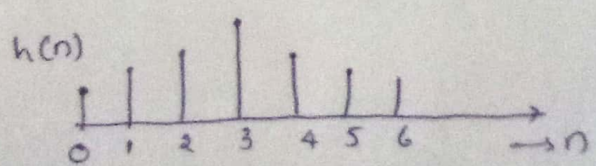
disadvantages of FIR filters.

1. The implementation of narrow transition band FIR filters are very costly, as it requires considerably more arithmetic operations and hardware components such as multipliers, adders and delay elements.
2. Memory requirement and execution time are very high.

Frequency response of linear phase FIR filters.

Case I : Symmetrical impulse response, N odd.

Consider fig



The frequency response of impulse response shown in fig can be written as

$$H(e^{j\omega}) = \sum_{n=0}^{6} h(n)\, e^{-j\omega n}$$

This can be written as

$$H(e^{j\omega}) = \sum_{n=0}^{2} h(n)\, e^{-j\omega n} + h(3)\, e^{-j3\omega} + \sum_{n=4}^{6} h(n)\, e^{-j\omega n}$$

In general, for N samples

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n)\, e^{-j\omega n}$$

let $n = N-1-m$, we have

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left[\frac{N-1}{2}\right] e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m)\, e^{-j\omega(N-1-m)}$$

for a symmetrical impulse response

$$h(n) = h(N-1-n).$$

$$\Rightarrow H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left[\frac{N-1}{2}\right] e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{j\omega\left(\frac{N-1}{2}-n\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega\left[\frac{N-1}{2}-n\right]} + h\left[\frac{N-1}{2}\right]\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=0}^{\frac{N-3}{2}} 2h(n) \cos\omega\left[\frac{N-1}{2}-n\right] + h\left[\frac{N-1}{2}\right]\right]$$

let $\frac{N-1}{2} - n = p$, then

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{p=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2}-p\right] \cos\omega p + h\left[\frac{N-1}{2}\right]\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2}-n\right] \cos\omega n + h\left(\frac{N-1}{2}\right)\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos\omega n$$

where

$$a(0) = h\left(\frac{N-1}{2}\right) \qquad a(n) = 2h\left(\frac{N-1}{2}-n\right)$$

let

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \bar{H}(e^{j\omega}) = \bar{H}(e^{j\omega})\, e^{j\theta(\omega)}$$

where $\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos\omega n$

$$\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega$$

Case II: Symmetric impulse response for N even.

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n) e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(N-1-n) e^{-j\omega(N-1-n)}$$

We know $h(n) = h(N-1-n)$.

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-2}{2}} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-2}{2}} h(n) e^{j\omega\left(\frac{N-1}{2}-n\right)} + \sum_{n=0}^{\frac{N-2}{2}} h(n) e^{-j\omega\left[\frac{N-1}{2}-n\right]} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-2}{2}} 2h(n) \cos\omega\left[\frac{N-1}{2}-n\right] \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=1}^{\frac{N}{2}} 2h\left[\frac{N}{2}-n\right] \cos\left[n-\frac{1}{2}\right]\omega \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N}{2}} b(n) \cos\left(\frac{n-1}{2}\right)\omega$$

where $b(n) = 2h\left[\frac{N}{2}-n\right]$

Let $H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \bar{H}(e^{j\omega}) = \bar{H}(e^{j\omega}) e^{j\theta(\omega)}$

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where $\bar{H}(e^{j\omega}) = \sum_{n=1}^{\frac{N}{2}} b(n) \cos\left(n-\frac{1}{2}\right)\omega$

$$\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega$$

Case iii : Anti symmetric : N odd.

for this type of sequence

$$h\left(\frac{N-1}{2}\right) = 0$$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) e^{-j\omega(N-1-n)}$$

We know $h(n) = -h(N-1-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} - \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n) e^{j\omega\left(\frac{N-1}{2}-n\right)} - \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \cdot j \left[\sum_{n=0}^{\frac{N-3}{2}} 2\,h(n) \sin\omega\left(\frac{N-1}{2} - n\right) \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2} \left[\sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin\omega n \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2} \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin\omega n$$

where $c(n) = 2h\left[\frac{N-1}{2} - n\right]$

$$H(e^{j\omega}) = \bar{H}(e^{j\omega})\, e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2} = \bar{H}(e^{j\omega})\, e^{j\theta(\omega)}$$

$$\theta(\omega) = \frac{\pi}{2} - \alpha\omega = \frac{\pi}{2} - \left(\frac{N-1}{2}\right)\omega$$

Case IV : N even

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{-j\omega n} + \sum_{n=N/2}^{N-1} h(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

We have $h(n) = -h(N-1-n)$

$$\Rightarrow H(e^{j\omega}) = \sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{-j\omega n} - \sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{j\omega\left(\frac{N-1}{2}-n\right)} + \sum_{n=0}^{\frac{N-2}{2}} h(n)\, e^{-j\omega\left(\frac{N-1}{2}-n\right)}\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2}\left[\sum_{n=1}^{N/2} 2h\left(\frac{N}{2}-n\right)\sin\omega\left(n-\tfrac{1}{2}\right)\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2}\left[\sum_{n=1}^{N/2} d(n)\sin\omega\left(n-\tfrac{1}{2}\right)\right]$$

where $d(n) = 2h\left(\frac{N}{2}-n\right)$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} e^{j\pi/2}\, \bar{H}(e^{j\omega})$$

$$= \bar{H}(e^{j\omega})\, e^{j\theta(\omega)}$$

$$\Rightarrow \bar{H}(e^{j\omega}) = \sum_{n=1}^{N/2} d(n)\sin\left(\omega\left(n-\tfrac{1}{2}\right)\right)$$

$$\theta(\omega) = \frac{\pi}{2} - \alpha\omega \quad ; \quad \alpha = \frac{N-1}{2}$$

Note : 1) The impulse response of symmetric with odd no. of samples can be used to design all types of filters.

2) Symmetric impulse response having even no. of samples cannot be used to design Highpass filter.

3) The fq response of antisymmetric impulse response is imaginary and this type of filters are most suitable for Hilbert transform and differentiators

The Fourier series Method of Designing FIR filters.

The fq response $H(e^{j\omega})$ of a system is periodic in 2π.

→ from fourier series analysis we know that any periodic f^n can be expressed as linear combination of complex exponentials.

→ Therefore, the desired fq response of an FIR filter can be represented by fourier series.

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

where the Fourier coefficients $h_d(n)$ are the desired impulse response sequence of the filter.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, dn$$

The z-transform of the sequence is given by

$$H(z) = \sum_{n=-\infty}^{\infty} h_d(n) z^{-n}$$

The above eqn represents a non-causal digital filter of infinite duration. To get an FIR filter transfer function, the series can be truncated by assigning

$$h(n) = h_d(n) \quad \text{for} \quad |n| \leq \frac{N-1}{2}$$
$$= 0 \quad \text{otherwise.}$$

$$\Rightarrow H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n)\, z^{-n}$$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \cdots + h(1)z^{-1} + h(0) + h(-1)z$$

$$+ \cdots + h\left[-\left(\frac{N-1}{2}\right)\right] z^{\left(\frac{N-1}{2}\right)}$$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\, z^{-n} + h(-n)\, z^{n}\right]$$

For a symmetrical impulse response having symmetry at $n=0$

$$h(-n) = h(n)$$

$$\Rightarrow H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

→ The above transfer function is not physically realizable. Realizability can be brought by multiplying the above eqn by $z^{-\left(\frac{N-1}{2}\right)}$ where $\frac{N-1}{2}$ is delay in samples.

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= z^{-\left(\frac{N-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]\right]$$

Ex: Design an ideal lowpass filter with fq response

$$H_d(e^{j\omega}) = 1 \quad \text{for } -\pi/2 \le \omega \le \pi/2$$

$$= 0 \quad \text{for } \frac{\pi}{2} \le |\omega| \le \pi$$

find the values of h(n) for N = 11. ~~for~~ find H(z).

Soln:

Given $H_d(e^{j\omega}) = 1 \quad -\pi/2 \le \omega \le \pi/2$

$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi j n} e^{j\omega n}\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n(2j)}\left[e^{j\pi n/2} - e^{-j\pi n/2}\right]$$

$$= \frac{\sin \frac{\pi}{2} n}{\pi n} \qquad -\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples,

$$h(n) = \frac{\sin \frac{\pi}{2} n}{\pi n} \quad \text{for } |n| \le 5$$

$$= 0 \quad \text{otherwise.}$$

→ for n = 0, the above value become indeterminate, So

$$h(0) = \lim_{n\to 0} \frac{\sin \frac{\pi}{2} n}{\pi n} = \frac{1}{2}\lim_{n \to 0}\frac{\sin \frac{\pi}{2} n}{\frac{\pi n}{2}} \qquad \because \lim_{\theta \to 0}\frac{\sin\theta}{\theta} = 1$$

$$= \frac{1}{2}$$

Sol $n=1$ $h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183$

$||^{ly}$ $h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$

$h(3) = h(-3) = \frac{\sin \frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = -0.106$

$h(4) = h(-4) = \frac{\sin \frac{4\pi}{2}}{4\pi} = 0$

$h(5) = h(-5) = \frac{\sin \frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.06366$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\left[z^{n} + z^{-n}\right]\right]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.5 + 0.3183\,(z^{1} + z^{-1}) - 0.106\,(z^{3} + z^{-3}) + 0.06366\left[z^{5} + z^{-5}\right]$$

The transfer function of the realizable filter is

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= \otimes z^{-5}\left[0.5 + 0.3183\,(z + z^{-1}) - 0.106\,(z^{3} + z^{-3}) + 0.06366\,(z^{5} + z^{-5})\right]$$

$$= 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z^{-5} + 0.3183z^{-6} - 0.106z^{-8} + 0.06366z^{-10}$$

from the above eqn the filter coefficients of causal filter are given by

$h(0) = h(10) = 0.06366$; $h(1) = h(9) = 0$; $h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$; $h(4) = h(6) = 0.3183$; $h(5) = 0.5$

The frequency response is given by

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{5} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = 0.6366$$

$$a(2) = 2h(5-2) = 2h(3) = 0$$

$$a(3) = 2h(5-3) = 2h(2) = -0.212$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.127$$

$$\bar{H}(e^{j\omega}) = 0.5 + 0.6366 \cos \omega - 0.212 \cos 3\omega + 0.127 \cos 5\omega$$

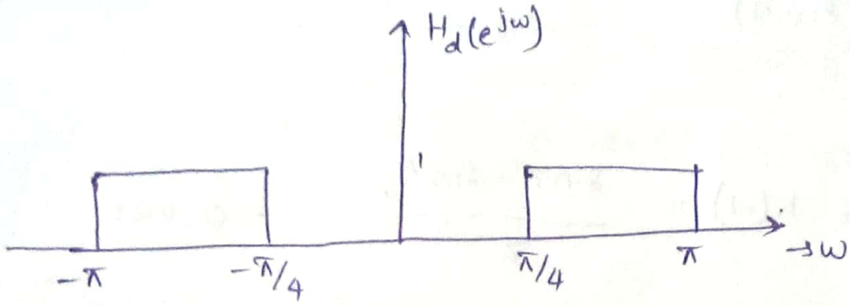
Ex: Design an ideal HPF with a frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad \frac{\pi}{4} \leq |\omega| \leq \pi$$

$$= 0 \quad \text{for} \quad |\omega| \leq \pi/4$$

find the values of $h(n)$ for $N = 11$. find $H(z)$.

Soln :



The desired fq response is shown above.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4}n\right]$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \quad \text{for } |n| \le 5$$

$$= 0 \quad \text{otherwise.}$$

for $n = 0$

$$h(0) = \lim_{n\to 0} \frac{\sin \pi n}{\pi n} - \lim_{n \to 0} \frac{\sin \frac{\pi}{4} n}{\pi n}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore \quad h(n) = h(-n)$

for $n=1$

$$h(1) = h(-1) = \frac{\sin\pi - \sin\pi/4}{\pi} = -0.225$$

$||^{ly}$ $$h(2) = h(-2) = \frac{\sin 2\pi - \sin\frac{\pi}{2}}{2\pi} = -0.159$$

$$h(3) = h(-3) = \frac{\sin 3\pi - \sin\frac{3\pi}{4}}{3\pi} = -0.075$$

$$h(4) = h(-4) = \frac{\sin 4\pi - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)(z^n + z^{-n})$$

$$= 0.75 + \sum_{n=1}^{5} h(n)(z^n + z^{-n})$$

$$= 0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2})$$
$$- 0.075(z^3 + z^{-3}) + 0.045(z^5 + z^{-5})$$

The transfer f^n of a realizable filter is

$$H'(z) = z^{-5} H(z)$$
$$= z^{-5}\left[0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2})\right.$$
$$\left. - 0.075(z^3 + z^{-3}) + 0.045(z^5 + z^{-5})\right]$$

$$H'(z) = 0.045 - 0.075 z^{-2} - 0.159 z^{-3} - 0.225 z^{-4}$$
$$+ 0.75 z^{-5} - 0.225 z^{-6} - 0.159 z^{-7} - 0.075 z^{-8}$$
$$+ 0.045 z^{-10}$$

from the above eqn, the filter coefficients of causal filter are

$h(0) = h(10) = 0.045$; $h(1) = h(9) = 0$; $h(2) = h(8) = -0.075$

$h(3) = h(7) = -0.159$ $h(4) = h(6) = -0.225$; $h(5) = 0.75$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = -0.45$$

$$a(2) = 2h(5-2) = 2h(3) = -0.318$$

$$a(3) = 2h(5-3) = 2h(2) = -0.15$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

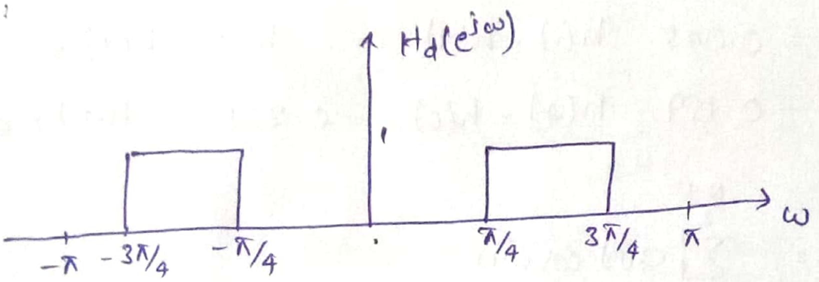
$$a(5) = 2h(5-5) = 2h(0) = 0.09$$

$$\bar{H}(e^{j\omega}) = a(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega$$
$$+ a(4)\cos 4\omega + a(5)\cos 5\omega$$
$$= 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega$$
$$+ 0.09\cos 5\omega .$$

Ex: Design an ideal BPF with fq response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4}$$
$$= 0 \quad \text{otherwise.}$$

Soln:



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}$$

$$= \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{3\pi/4} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi j n}\left[e^{-j\pi n/4} - e^{-j3\pi n/4} + 8\, e^{j3\pi n/4} + e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right] \quad -\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \quad \text{for} \quad |n| \le 5$$
$$= 0 \quad \text{otherwise.}$$

The filter coefficients are symmetrical about $n=0$ satisfying the condition $h(n) = h(-n)$

for $n=0 \Rightarrow h(0) = \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} d\omega + \int_{\pi/4}^{3\pi/4} d\omega\right]$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{4}+\frac{3\pi}{4}+\frac{3\pi}{4}-\frac{\pi}{4}\right] = \frac{1}{2} = 0.5$$

$$h(1) = h(-1) = \frac{\sin\frac{3\pi}{4} - \sin\pi/4}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin 3\pi/2 - \sin\pi/2}{2\pi} = \frac{-2}{2\pi} = -0.3183$$

$$h(3) = h(-3) = \frac{\sin 9\pi/4 - \sin\frac{3\pi}{4}}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 3\pi - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\frac{15\pi}{4} - \sin\frac{5\pi}{4}}{5\pi} = 0$$

The transfer f^n of the filter is

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}}\left[h(n)\left(z^{n}+z^{-n}\right)\right]$$

$$= 0.5 - 0.3183\left(z^{2}+z^{-2}\right)$$

The transfer f^n of the realizable filter is

$$H'(z) = z^{-5}\left[0.5 - 0.3183\left(z^{2}+z^{-2}\right)\right]$$

$$= -0.3183z^{-3} + 0.5z^{-5} - 0.3183z^{-7}$$

The filter coefficients of the causal filters are.

$$h(0) = h(10) = h(1) = h(9) = h(2) = h(8) = h(4) = h(6) = 0$$

$$h(3) = h(7) = -0.3183$$

$$h(5) = 0.5$$

$$\bar{H}(e^{j\omega}) = \sum_{n=1}^{\frac{N-1}{2}} a(n) \cos\omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = 0$$

$$a(2) = 2h(5-2) = 2h(3) = -0.6366$$

$$a(3) = 2h(5-3) = 2h(2) = 0$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0$$

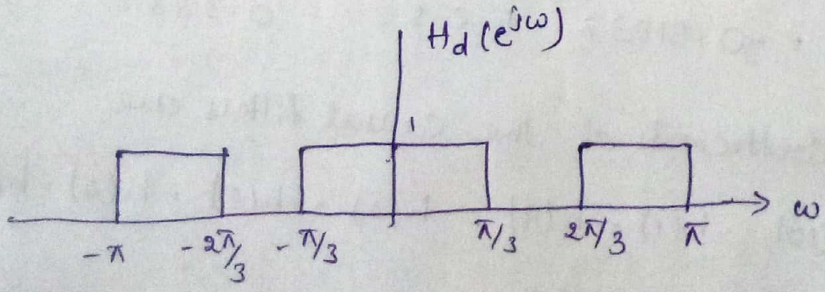
$$\bar{H}(e^{j\omega}) = 0.5 - 0.6366 \cos 2\omega$$

Ex: Design an ideal band reject filter with a desired fq response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad |\omega| \leq \pi/3 \ \& \ |\omega| \geq \frac{2\pi}{3}$$
$$= 0 \quad \text{otherwise.}$$

find $h(n)$ for $N = 11$. find $H(z)$.

Soln:



fq. response of Bandreject filter.

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$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n} d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n} d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi j n}\left[e^{-j2\pi n/3} - e^{-j\pi n} + e^{j\pi n/3} - e^{-j\pi n/3} + e^{j\pi n} - e^{j2\pi n/3} \right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n + \sin \frac{\pi}{3} n - \sin \frac{2\pi}{3} n \right] \quad -\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \quad \text{for} \quad |n| \le 5$$
$$= 0 \quad \text{otherwise}$$

The filter coefficients are symmetrical about $n=0$ satisfying the condition $h(n) = h(-n)$.

for $n=0$ $\quad h(0) = \lim_{n \to 0} \left[\frac{\sin \pi n}{\pi n} + \frac{\sin \frac{\pi}{3} n}{\pi n} - \frac{\sin \frac{2\pi}{3} n}{\pi n} \right]$

$$= 1 + \frac{1}{3} - \frac{2}{3} = 0.667$$

$$h(1) = h(-1) = \frac{\sin \pi + \sin \frac{\pi}{3} - \sin \frac{2\pi}{3}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{1}{2\pi}\left[\sin 2\pi + \sin \frac{2\pi}{3} - \sin \frac{4\pi}{3} \right] = 0.2757$$

$$h(3) = h(-3) = \frac{1}{3\pi}\left[\sin 3\pi + \sin \pi - \sin 2\pi \right] = 0$$

$$h(4) = h(-4) = \frac{1}{4\pi}\left[\sin 4\pi + \sin \frac{4\pi}{3} - \sin \frac{8\pi}{3} \right] = -0.1378$$

$$h(5) = h(-5) = \frac{\sin 5\pi + \sin \frac{5\pi}{3} - \sin \frac{10\pi}{3}}{5\pi} = 0$$

The transfer function of the filter is

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.667 + 0.2757(z^2 + z^{-2}) - 0.1378(z^4 + z^{-4})$$

The transfer function of a realizable filter

$$H'(z) = z^{-5} H(z)$$

$$= -0.1378 z^{-1} + 0.2757 z^{-3} + 0.667 z^{-5} + 0.2757 z^{-7} - 0.1378 z^{-9}$$

The filter coefficients of the causal filter are

$h(0) = h(10) = h(2) = h(8) = h(4) = h(6) = 0$

$h(1) = h(9) = -0.1378$

$h(3) = h(7) = 0.2757$

$h(5) = 0.667$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$a(0) = 2h(5-1) = 2h(4) = 0$

$a(2) = 2h(5-2) = 2h(3) = 0.5514$

$a(3) = 2h(5-3) = 2h(2) = 0$

$a(4) = 2h(5-4) = 2h(1) = -0.2756$

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$a(5) = 2h(5-5) = 2h(0) = 0$

$\bar{H}(e^{j\omega}) = 0.667 + 0.5514 \cos 2\omega - 0.2756 \cos 4\omega$.

Design of FIR filters using windows

→ Abrupt truncation of the fourier series results in oscillation in the passband and stopband.

→ These oscillations are due to slow convergence of the fourier series and this effect is known as Gibbs phenomenon.

→ To reduce these oscillations, the fourier coefficients of the filter are modified by multiplying the infinite impulse response with a finite weighing sequence $w(n)$ called a window

→ After multiplying window sequence $w(n)$ with $h_d(n)$, we get a finite duration sequence $h(n)$, that satisfies the desired magnitude response

$$h(n) = h_d(n)\, w(n) \quad \text{for all } |n| \leq \left[\frac{N-1}{2}\right]$$

$$= 0 \quad \text{for } |n| > \left(\frac{N-1}{2}\right)$$

1. Rectangular window

The rectangular window sequence is given by

$$w_R(n) = 1 \quad \text{for } -(N-1)/2 \leq n \leq (N-1)/2$$

$$= 0 \quad \text{otherwise}.$$

→ For the rectangular window, the amplitude of the side lobes is unaffected by the length of the window.

→ So the amplitude of ~~sidelobes~~ side lobes can be reduced by using a less abrupt truncation of filter coefficients.

→ This can be achieved using a window f^n that tapers smoothly towards zero at both ends. ~~Ones~~ such type of window is <u>triangular window</u>.

2. <u>Triangular or Bartlett window</u>

The N-point triangular window is given by

$$w_T(n) = 1 - \frac{2|n|}{N-1} \quad \text{for} \quad -\left(\frac{N-1}{2}\right) \leq n \leq \left(\frac{N-1}{2}\right)$$

→ The triangular window produces a smooth magnitude response in both passband and stopband, but it has following <u>dis adv</u> compared to rectangular window.

1. The transition region is more
2. The attenuation in stop band is ~~less~~ less.

→ Because of the above <u>char</u>, the triangular window is not usually a good choice.

3. <u>Raised cosine window</u>.

Raised cosine window multiplies central Fourier coefficients by approximately unity and smoothly truncate the Fourier coefficients toward the ends of the filter.

The window sequence is of the form

$$w_\alpha(n) = \alpha + (1-\alpha)\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2}$$

$$= 0 \quad \text{otherwise}$$

4. Hanning window

The Hanning window sequence can be obtained by substituting ~~α is~~ $\alpha = 0.5$ in Raised cosine window equation:

$$W_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise}.$$

→ Hanning Window results in smaller ripples in both passband and stop band of the LPF design.

5. Hamming Window

The eqn for Hamming window can be obtained by substituting $\alpha = 0.54$ in the Raised cosine window eqn.

$$W_H(n) = 0.54 + 0.46\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise}.$$

→ Since Hamming window generates lesser oscillation in the side lobes than the Hanning window for the same main lobe width, Hamming window is generally preferred.

6. Blackman window

The Blackman window sequence is given by

$$w_B(n) = 0.42 + 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1}$$

$$\text{for } -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \qquad \text{otherwise}$$

→ The additional cosine term compared with the Hamming and Hanning windows, reduces the side lobes, but increases the main lobe width.

7. Kaiser Window

The Kaiser window is given by

$$w_K(n) = \frac{I_0\left[\alpha \sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right]}{I_0(\alpha)} \qquad \text{for } |n| \le \frac{N-1}{2}$$

$$= 0 \qquad \text{otherwise}$$

where α is the adjustable parameter and $I_0(x)$ is the modified zeroth-order Bessel f^n of the first kind of order zero given by

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

Desirable characteristics of a window:

1. The central lobe of the fq response of the window should contain most of the energy & should be narrow.
2. The highest side lobe level of the fq response should be small.
3. The side lobes of the fq response should decrease in energy rapidly as ω tends to π.

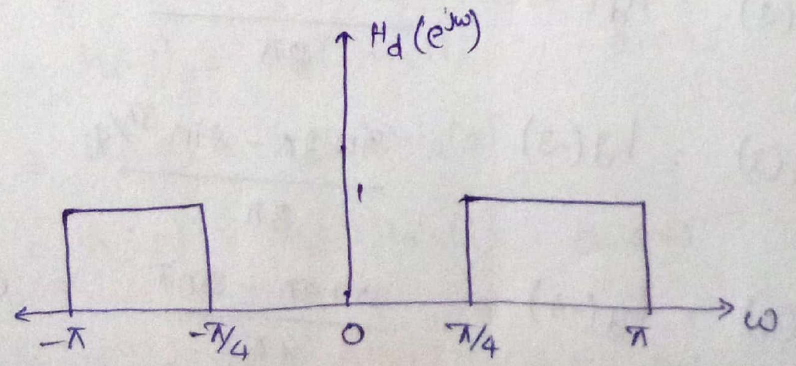
Ex: Design an ideal high pass filter with a fq response

$$H_d(e^{j\omega}) = 1 \quad \text{for } \frac{\pi}{4} \le |\omega| \le \pi$$
$$= 0 \quad \text{for } |\omega| \le \pi/4$$

find the values of $h(n)$ for $N = 11$. find $H(z)$

(i) using Rectangular window
(ii) Hanning window
(iii) Hamming window.

Soln:



We know

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n} dn + \int_{\pi/4}^{\pi} e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi j n}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-jn\pi} + e^{jn\pi} - e^{jn\pi/4}\right]$$

$$h_d(n) = \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4} n\right] \quad -\infty \le n \le \infty$$

The filter coefficients are obtained from above eqn

$$h_d(0) = \lim_{n \to 0}\left[\frac{1}{\pi n}\left(\sin \pi n - \sin \frac{\pi}{4} n\right)\right] = 0.75$$

$$h_d(1) = h_d(-1) = \frac{\sin \pi - \sin \pi/4}{\pi} = -0.225$$

$$h_d(2) = h_d(-2) = \frac{\sin 2\pi - \sin \pi/2}{2\pi} = -0.159$$

$$h_d(3) = h_d(-3) = \frac{\sin 3\pi - \sin 3\pi/4}{3\pi} = -0.075$$

$$h_d(4) = h_d(-4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$$

$$h_d(5) = h_d(-5) = \frac{1}{5\pi}\left[\sin 5\pi - \sin \frac{5\pi}{4}\right] = 0.045$$

(i) Rectangular window

The Rectangular window sequence is given by

$$w_R(n) = 1 \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

ie., $-5 \le n \le 5$

$w_R(0) = 1$

$w_R(-1) = w_R(1) = 1$

$w_R(-2) = w_R(+2) = 1$

$w_R(-3) = w_R(+3) = 1$

$w_R(-4) = w_R(4) = 1$

$w_R(-5) = w_R(5) = 1$

The filter coefficients of Rectangular window sequence

$$h(n) = h_d(n)\, w_R(n) \quad \text{for} \quad -5 \le n \le 5$$

$$= 0 \quad \text{otherwise}$$

$h(0) = h_d(0)\, w_R(0) = 0.75$

$h(1) = h(-1) = h_d(1)\, w_R(1) = -0.225$

$h(2) = h(-2) = h_d(2)\, w_R(2) = -0.159$

$h(3) = h(-3) = h_d(3)\, w_R(3) = -0.075$

$h(4) = h(-4) = h_d(4)\, w_R(4) = 0$

$h(5) = h(-5) = h_d(5)\, w_R(5) = 0.045$

The transfer f^n of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\,(z^n + z^{-n})$$

$$H(z) = 0.75 + \sum_{n=1}^{5} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.75 - 0.225(z+z^{-1}) - 0.159(z^{2}+z^{-2}) - 0.075(z^{3}+z^{-3}) + 0.045(z^{5}+z^{-5})$$

The transfer function of the realizable filter is

$$H'(z) = z^{-5} H(z)$$

$$= z^{-5}\left[0.75 - 0.225(z+z^{-1}) - 0.159(z^{2}+z^{-2}) - 0.075(z^{3}+z^{-3}) + 0.045(z^{5}+z^{-5})\right]$$

$$= 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.225z^{-4} + 0.75z^{-5} - 0.225z^{-6} - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10}$$

from the above eqn, the filter coefficient of causal filter are

$h(0) = h(10) = 0.045$; $h(1) = h(9) = 0$;

$h(2) = h(8) = -0.075$; $h(3) = h(7) = -0.159$

$h(4) = h(6) = -0.225$; $h(5) = 0.75$

$$\bar{H}(e^{jw}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos wn \quad \text{where}$$

$$a(0) = h\left[\frac{N-1}{2}\right] = h(5) = 0.75$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$a(1) = 2h(5-1) = 2h(4) = -0.45$

$a(2) = 2h(5-2) = 2h(3) = -0.318$

$a(3) = 2h(5-3) = 2h(2) = -0.15$

$a(4) = 2h(5-4) = 2h(1) = 0$

$a(5) = 2h[5-5] = 2h(0) = 0.09$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$= a(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega$$

$$= 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 5\omega$$

(ii) Hanning window

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise}$$

for $N = 11$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{5} \quad -5 \le n \le 5$$

$$= 0 \quad \text{otherwise}$$

$w_{Hn}(0) = 0.5 + 0.5 = 1$

$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5\cos\frac{\pi}{5} = 0.9045$

$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5\cos\frac{2\pi}{5} = 0.655$

$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5\cos\frac{3\pi}{5} = 0.345$

$\omega_{Hn}(4) = \omega_{Hn}(-4) = 0.5 + 0.5\cos\frac{4\pi}{5} = 0.0945$

$\omega_{Hn}(5) = \omega_{Hn}(-5) = 0.5 + 0.5\cos\pi = 0$

filter coefficients using Hanning Window are

$$h(n) = h_d(n)\,W_{Hn}(n) \quad \text{for } -5 \le n \le 5$$
$$= 0 \quad \text{otherwise}$$

$h(0) = h_d(0)\,\omega_{Hn}(0) = (0.75)(1) = 0.75$

$h(-1) = h(1) = h_d(1)\,\omega_{Hn}(1) = (-0.225)(0.905) = -0.204$

$h(-2) = h(2) = h_d(2)\,\omega_{Hn}(2) = (-0.159)(0.655) = -0.104$

$h(-3) = h(3) = h_d(3)\,\omega_{Hn}(3) = (-0.075)(0.345) = -0.026$

$h(-4) = h(4) = h_d(4)\,W_{Hn}(4) = 0(0.8145) = 0$

$h(-5) = h(5) = h_d(5)\,W_{Hn}(5) = 0.045(0) = 0$

The transfer f^n of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$= 0.75 - 0.204(z + z^{-1}) - 0.104(z^2 + z^{-2}) - 0.026(z^3 + z^{-3})$$

The transfer function of realizable filter is

$$H'(z) = z^{-5} H(z)$$

$$= -0.026 z^{-2} - 0.104 z^{-3} - 0.204 z^{-4} + 0.75 z^{-5} - 0.204 z^{-6} - 0.104 z^{-7} - 0.026 z^{-8}.$$

The causal filter coefficients are

$h(0) = h(1) = h(9) = h(10) = 0$

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$h(2) = h(8) = -0.026$; $h(3) = h(7) = -0.104$

$h(4) = h(6) = -0.204$; $h(5) = 0.75$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$a(0) = h\left[\frac{N-1}{2}\right] = h(5) = 0.75$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = -0.408$$

$$a(2) = 2h(5-2) = 2h(3) = -0.208$$

$$a(3) = 2h(5-3) = 2h(2) = -0.052$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0$$

$$\bar{H}(e^{j\omega}) = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$$

(iii) Hamming window

The Hamming window sequence is given by

$$w_H(n) = 0.54 + 0.46\cos\frac{2\pi n}{N-1} \quad \text{for } -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise}$$

The window sequence for $N = 11$

$$w_H(n) = 0.54 + 0.46\cos\frac{\pi n}{5} \quad \text{for } -5 \le n \le 5$$

$$= 0 \quad \text{otherwise}$$

$$w_H(0) = 0.54 + 0.46 = 1$$

$$w_H(-1) = w_H(1) = 0.54 + 0.46 \cos\frac{\pi}{5} = 0.912$$

$$w_H(-2) = w_H(2) = 0.54 + 0.46 \cos\frac{2\pi}{5} = 0.682$$

$$w_H(-3) = w_H(3) = 0.54 + 0.46 \cos\frac{3\pi}{5} = 0.398$$

$$w_H(-4) = w_H(4) = 0.54 + 0.46 \cos\frac{4\pi}{5} = 0.1678$$

$$w_H(-5) = w_H(5) = 0.54 + 0.46 \cos\pi = 0.08.$$

filter coefficients using Hamming window sequence are

$$h(n) = h_d(n)\, w_H(n) \quad \text{for} \quad -5 \le n \le 5$$
$$= 0 \quad \text{otherwise}$$

$$h(0) = h_d(0)\, w_H(0) = 1(0.75) = 0.75$$

$$h(-1) = h(1) = h_d(1)\, w_{Hm}(1) = (-0.225)(0.912) = -0.2052$$

$$h(-2) = h(2) = h_d(2)\, w_H(2) = (-0.159)(0.682) = -0.1084$$

$$h(-3) = h(3) = h_d(3)\, w_H(3) = (-0.075)(0.398) = -0.03$$

$$h(-4) = h(4) = h_d(4)\, w_H(4) = (0)(0.1678) = 0$$

$$h(-5) = h(5) = h_d(5)\, w_H(5) = (-0.045)(0.08) = 0.0036$$

The transfer f^n of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$= 0.75 - 0.2052\,(z^{-1} + z) - 0.1084\,(z^{-2} + z^{2})$$
$$- 0.03\,(z^{-3} + z^{3}) + 0.0036\,(z^{-5} + z^{5})$$

The transfer f^n of the realizable filter is

$$H'(z) = z^{-5} H(z)$$

$$= 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.2052z^{-4} + 0.75z^{-5}$$

$$- 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10}$$

The filter coefficients of causal filter are

$h(0) = h(10) = 0.0036$; $h(1) = h(9) = 0$; $h(2) = h(8) = -0.03$

$h(3) = h(7) = -0.1084$; $h(4) = h(6) = -0.2052$; $h(5) = 0.75$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = -0.4104$$

$$a(2) = 2h(5-2) = 2h(3) = -0.2168$$

$$a(3) = 2h(5-3) = 2h(2) = -0.06$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.0072$$

$$\bar{H}(e^{j\omega}) = 0.75 - 0.4104 \cos\omega - 0.2168 \cos 2\omega - 0.06 \cos 3\omega$$

$$+ 0.0072 \cos 5\omega$$

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Q) Design a filter with

$$H_d(e^{j\omega}) = e^{-j5\omega} \qquad -\frac{\pi}{2} \le \omega \le \frac{\pi}{2}$$

$$= 0 \qquad \frac{\pi}{2} < |\omega| \le \pi$$

using (i) Triangular window (ii) Blackman window with $N = 11$.

Summary of windows

→ The triangular window has a transition width twice that of rectangular window.

→ The attenuation in stopband for triangular window is less, so it not very popular for FIR filter design.

→ The Hanning and Hamming windows have same transition width. But Hamming window is most widely used, because, it generates less ringing in the side lobes.

→ The Blackman window reduces the side lobe level at the cost of increase in transition width.

→ The Kaiser window is superior to other windows, because, for given specifications its transition width is always small. By varying 'α' the side lobe level and main lobe peak can be achieved. The main lobe width can be varied by varying the length N. So, Kaiser window is favorite window for many digital filter designers.

Advantages and disadvantages of FIR filter

Adv : 1. The filter coefficients can be obtained with minimum computation effort.

2. The window functions are readily available in closed-form expression.

3. The ripples in both passband and in stopband are almost completely removed.

disadv : 1. It is not always not possible to obtain a closed form expression for the fourier series coefficients $h(n)$.

2. Windows provides limited flexibility in the design.

3. It is some what difficult to determine, in advance, the type of window and the duration N required to meet a given prescribed fq specification.

Frequency sampling method of designing FIR filters :

Let $h(n)$ is the filter coefficients of an FIR filter and $H(k)$ is the DFT of $h(n)$. Then, we have

$$h(n) = \frac{1}{N} \sum_{K=0}^{N-1} H(k) e^{j 2\pi kn/N} \qquad n = 0, 1 \cdots N-1$$

$$H(k) = \sum_{K=0}^{N-1} h(n) e^{-j 2\pi kn/N} \qquad K = 0 \cdots N-1$$

The DFT samples $H(k)$ for an FIR sequence can be regarded as samples of the filter z-transform evaluated at N-points spaced equally around unit circle, i.e,

$$H(k) = H(z)\Big|_{z = e^{j2\pi k/N}}$$

The transfer f^n $H(z)$ of an FIR filter with impulse response is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$\Rightarrow \quad H(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{j2\pi kn/N} \right] z^{-n}$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \sum_{n=0}^{N-1} H(k) \left(e^{j2\pi k/N} z^{-1} \right)^n$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \left(\frac{1 - \left(e^{j2\pi k/N} z^{-1}\right)^N}{1 - e^{j2\pi k/N} z^{-1}} \right)$$

$$1 + r + \cdots + r^{N-1} = \frac{1 - r^N}{1 - r}$$

$$= \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi k/N} z^{-1}}$$

We know:

$$H(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} = H\left(e^{j2\pi kn/N}\right) = H(k)$$

ie., $H(k)$ is kth DFT component obtained by sampling fq resp $H(e^{j\omega})$. Hence this method is fq sampling method.

Frequency response

The fq response of the FIR filter can be obtained by setting $z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi k/N} \cdot e^{-j\omega}}$$

$$= \frac{e^{-j\omega N/2}\left(e^{j\omega N/2} - e^{-j\omega N/2}\right)}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{-j(\omega - 2\pi k/N)}}$$

$$= \frac{e^{-j\omega N/2}}{N} \sum_{k=0}^{N-1} \frac{H(k)\left[e^{j\omega N/2} - e^{-j\omega N/2}\right]}{e^{-j(\omega/2 - \pi k/N)}\left[e^{j(\omega/2 - \pi k/N)} - e^{-j(\omega/2 - \pi k/N)}\right]}$$

$$= \frac{e^{-j\omega(N-1)/2}}{N} \sum_{k=0}^{N-1} \frac{H(k)\, e^{-j\pi k/N} \sin \omega N/2}{\sin(\omega/2 - \pi k/N)}$$

$$= \frac{e^{-j\omega\left(\frac{N-1}{2}\right)}}{N} \sum_{k=0}^{N-1} \frac{H(k)\,(-1)^k e^{-j\pi k/N} \sin N(\omega/2 - k\pi/N)}{\sin(\omega/2 - \pi k/N)}$$

$$\because \sin\left(\frac{\omega N}{2} - k\pi\right) = (-1)^k \sin\frac{\omega N}{2}$$

Design

We exploit the basic symmetry property of the sampled fq response to simplify the computations in designing FIR filter. There are two types of design based on set of samples that we choose from fq response.

Type 1. design

In this type of design the fq samples of desired response $H_d(e^{jw})$ are determined, using the relation

$$H(k) = H_d(e^{jw})\Big|_{\omega = \frac{2\pi}{N}k} \qquad k = 0, 1 \ldots N-1$$

The fq samples can be expressed in the form

$$H(k) = |H(k)| e^{j\theta(k)} \qquad \text{— (A)}$$

for linear phase

$$\theta(k) = -\alpha\omega\Big|_{\omega = \frac{2\pi}{N}k} \qquad k = 0, 1 \ldots N-1$$

$$= -\left(\frac{N-1}{2}\right)\frac{2\pi}{N}k$$

$$= -\left(\frac{N-1}{N}\right)\pi k \qquad k = 0, 1 \ldots N-1$$

The filter coefficients $h(n)$ can be obtained by finding IDFT of $H(k)$ ie.,

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{j2\pi nk/N} \qquad n = 0, 1, \ldots N-1 \quad \text{— (B)}$$

If $h(n)$, the impulse response of the filter is to be a real valued signal, the fq samples $H(k)$ must satisfy the symmetry requirement ie.,

for N odd or even

$$H(N-k) = H^*(k) \qquad k = 0, 1 \ldots N-1 \quad \text{— (C)}$$

in addition, for N even $H(\frac{N}{2}) = 0$

With the fq response $H(k)$, the magnitude response is even f^n.

$$|H(k)| = |H(N-k)| \qquad k = 0 \cdots N-1 \quad — (D)$$

and the phase is an odd f^n.

$$\theta(k) = -\theta(N-k) \qquad k = 0, 1 \cdots N-1 \quad — (1)$$

In the above eqn replacing k by N-k, we get

$$\theta(N-k) = -\left(\frac{N-1}{N}\right)\pi(N-k)$$

$$= -(N-1)\pi + \left(\frac{N-1}{N}\right)\pi k$$

To satisfy the requirements of eqn (1), $\theta(k)$ for N odd

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \qquad k = 0 \cdots \frac{N-1}{2}$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \qquad k = \frac{N+1}{2} \cdots N-1 \quad — (2)$$

//ly for N even

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \qquad k = 0 \cdots \frac{N}{2}-1$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \qquad k = \frac{N}{2}+1 \cdots N-1$$

$$= 0 \qquad k = \frac{N}{2} \quad — (3)$$

Substituting eqn (2) in eqn (A)

$$H(k) = |H(k)| \cdot e^{-j(N-1)\pi k/N} \qquad k = 0, 1 \cdots \frac{N-1}{2}$$

$$= |H(k)| \, e^{j[(N-1)\pi - (N-1)\pi k/N]} \qquad k = \frac{N+1}{2} \cdots N-1$$

Sub ③ in Ⓐ, we get for N-even

$$H(k) = |H(k)|\, e^{-j(N-1)\pi k/N} \qquad k = 0, 1, \ldots \frac{N}{2}-1$$

$$= |H(k)|\, e^{j[(N-1)\pi - (N-1)\pi k/N]} \qquad k = \frac{N}{2}+1 \ldots N-1$$

$$= 0 \qquad k = \frac{N}{2}.$$

If the filter is to be linear phase, the h(n) must also satisfy the symmetry condition

$$h(n) = h(N-1-n)$$

This symmetry condition, along with the symmetry condition for H(k), can be used to reduce the fq specifications from N points to $\frac{N+1}{2}$ pts for N-odd, and $\frac{N}{2}$ pts for N-even.

Sub eqn Ⓒ in Ⓑ, the filter coefficients can be written as

$$h(n) = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{(N-1)/2} \mathrm{Re}\left[H(k)\, e^{j2\pi kn/N}\right]\right\} \text{ - N odd}$$

and for N-even

$$h(n) = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} \mathrm{Re}\left[H(k)\, e^{j2\pi kn/N}\right]\right.$$

The system fn of the filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

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Ex: Determine the filter coeff $h(n)$ obtained by sampling

$$H_d(e^{j\omega}) = e^{-j(N-1)\omega/2} \qquad 0 \le |\omega| \le \frac{\pi}{2}$$

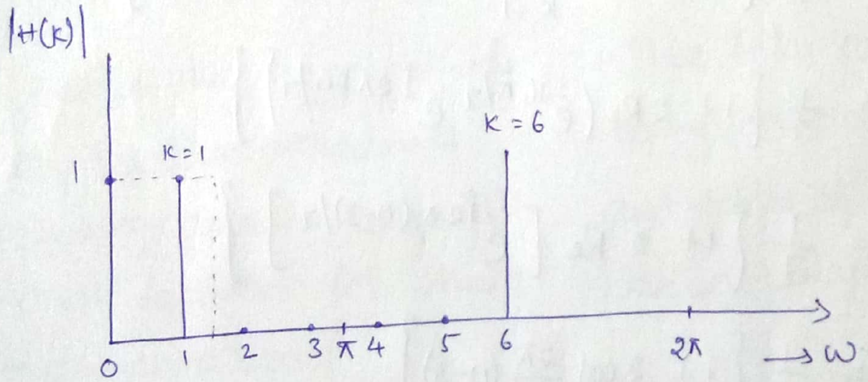
$$= 0 \qquad \frac{\pi}{2} \le |\omega| \le \pi$$

for $N = 7$

Soln :

magnitude response with samples for the given specification is shown

$$H(k) = \left|H_d(e^{j\omega})\right|\Big|_{\omega = \frac{2\pi k}{N}}$$



Given $N = 7$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{7}} \qquad k = 0, 1 \ldots 6$$

from fig, we have.

$$|H(k)| = 1 \quad \text{for} \quad k = 0, 1, 6$$

$$= 0 \quad \text{for} \quad k = 2, 3, 4, 5$$

we have $\quad \theta(k) = -\left(\frac{N-1}{N}\right)\pi k = -\frac{6}{7}\pi k \qquad k = 0, 1, 2, 3$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k = \frac{6\pi}{7}(7-k) \qquad k = 4, 5, 6$$

Now the fq response of the linear phase filter can be written by substituting $\theta(k)$ & $|H(k)|$ in $H(k)$

$$H(k) = |H(k)|\, e^{j\theta(k)}$$

$$= e^{-j6\pi k/7} \qquad k = 0, 1$$

$$= 0 \qquad k = 2, 3, 4, 5$$

$$= e^{-j6\pi(k-7)/7} \qquad k = 6$$

The filter coefficients for N odd are given by

$$h(n) = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{j2\pi kn/7}\right]\right\} \qquad n = 0 \cdots N-1$$

$$= \frac{1}{7}\left\{1 + 2\,Re\left(e^{-j6\pi/7}\, e^{j2\pi kn/7}\right)\right\}$$

$$= \frac{1}{7}\left\{1 + 2\,Re\left[e^{j2\pi(n-3)/7}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\cos\frac{2\pi}{7}(n-3)\right\}$$

$$h(0) = h(6) = \frac{1}{7}\left(1 + 2\cos\frac{6\pi}{7}\right) = -0.11456$$

$$h(1) = h(5) = \frac{1}{7}\left(1 + 2\cos\frac{4\pi}{7}\right) = 0.07928$$

$$h(2) = h(4) = \frac{1}{7}\left(1 + 2\cos\frac{2\pi}{7}\right) = 0.321$$

$$h(3) = \frac{1}{7}(1+2) = 0.42857.$$

4 . 22.

Comparison between FIR & IIR filters

FIR filter	IIR filter
1. Impulse response of this filter is restricted to finite no. of samples.	1. Impulse response of this filter entends over infinite duration.
2. They can have precisely linear phase.	2. They don't have linear phase.
3. Closed form design eqns do not exist	3. Variety of fq selective filters can be designed using closed-form design formulas.
4. Most of the design Methods are iterative procedures, requiring powerful computati- -onal facilities for their implementation.	4. These filter can be designed using only a hand Calculator and tables of analog filter design parameters
5. Greater flexibility to control the shape of their magni- -tude response.	5. less flexibility specially for obtaining non-standard fq responses.
6. In these filters, the poles are fixed at the origin, high selectivity can be achieved by using a relatively high order for transfer f^n	6. The poles are placed any- -where inside the unit O^{le}, high selectivity can be achieved with low order transfer f^n.
7. Always stable	7. Not always stable

8. Errors due to round off noise are less severe

8. IIR filters are more susceptible to errors due to round off noise.

Q). Det the coefficients of linear phase FIR filter of length $M = 15$ has a symmetric unit sample response and fq response that satisfies the conditions

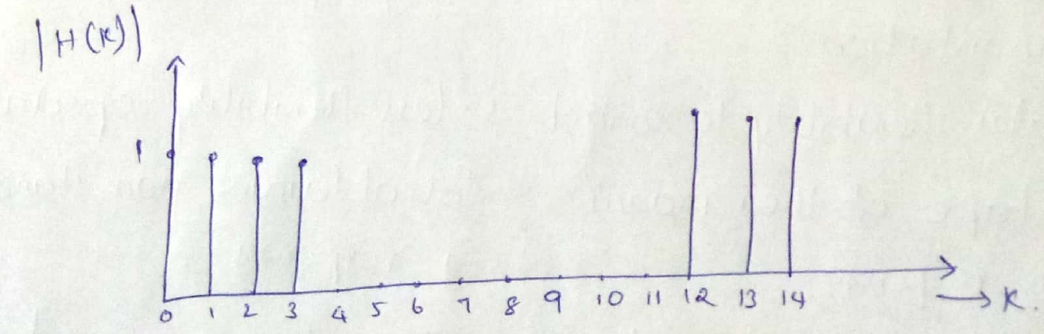
$$H\left[\frac{2\pi k}{15}\right] = 1 \qquad k = 0, 1, 2, 3$$

$$= 0 \qquad k = 4, 5, 6, 7$$

Soln:

$$|H(k)| = 1 \quad \text{for} \quad 0 \le k \le 3 \quad \& \quad 12 \le k \le 14$$

$$= 0 \quad \text{for} \quad 4 \le k \le 11$$



Hint: to determine $H(k)$ samples from $8 \le k \le 14$ use symmetry property of $H(k)$

ie→ $\overset{*}{H}(k) = H(15-k)$

$$|H(15-k)| = |\overset{*}{H}(k)| = |H(k)|$$