TURAN GÖNEN

ELECTRIC POWER DISTRIBUTION SYSTEM ENGINEERING

ELECTRIC POWER DISTRIBUTION SYSTEM ENGINEERING

McGraw-Hill Series in Electrical Engineering

Consulting Editor
Stephen W. Director, Carnegie–Mellon University

Circuits and Systems
Communications and Signal Processing
Control Theory
Electronics and Electronic Circuits
Power and Energy
Electromagnetics
Computer Engineering
Introductory
Radar and Antennas
VLSI

Previous Consulting Editors

Ronald N. Bracewell, Colin Cherry, James F. Gibbons, Willis W. Harman, Hubert Heffner, Edward W. Herold, John G. Linvill, Simon Ramo, Ronald A. Rohrer, Anthony E. Siegman, Charles Susskind, Frederick E. Terman, John G. Truxal, Ernst Weber, and John R. Whinnery

Power and Energy

Consulting Editor
Stephen W. Director, Carnegie–Mellon University

Chapman: *Electric Machinery Fundamentals*
Elgerd: *Electric Energy Systems Theory: An Introduction*
Fitzgerald, Kingsley, and Umans: *Electric Machinery*
Gönen: *Electric Power Distribution System Engineering*
Hu and White: *Solar Cells: From Basic to Advanced Systems*
Krause: *Analysis of Electric Machinery*
Krause and Wasynezuk: *Electromechanical Motion Devices*
Stevenson: *Elements of Power System Analysis*

ELECTRIC POWER DISTRIBUTION SYSTEM ENGINEERING

Turan Gönen
California State University, Sacramento

McGraw-Hill, Inc.
New York St. Louis San Francisco Auckland Bogotá
Caracas Lisbon London Madrid Mexico City Milan
Montreal New Delhi San Juan Singapore
Sydney Tokyo Toronto

This book was set in Times Roman.
The editors were Sanjeev Rao and J. W. Maisel;
the cover was designed by Rafael Hernandez;
the production supervisor was Marietta Breitwieser.
The drawings were done by Accurate Art, Inc.

ELECTRIC POWER DISTRIBUTION SYSTEM ENGINEERING

10 11 12 13 14 15 BRBBRB 95 9 8 7 6 5 4 3 2 1 0

ISBN 0-07-023707-7

Library of Congress Cataloging in Publication Data

Gönen, Turan.
Electric power distribution system engineering.

(McGraw-Hill series in electrical engineering.
Power and energy)
Includes bibliographies and index.
1. Electric power distribution. I. Title.
II Series.
TK3001 G58 1986 621 319 85-79
ISBN 0-07-023707-7

To an excellent engineer,
a great teacher, and a dear friend,
Dr. David D. Robb
and
in the memory of another
great teacher, my father

CONTENTS

PREFACE

Today, there are many excellent textbooks dealing with topics in power systems. Some of them are considered to be classics. However, they do not particularly address, nor concentrate on, topics dealing with electric power distribution engineering. Presently, to the author's knowledge, the only book available in the electric power systems literature that is totally devoted to power distribution engineering is the one by the Westinghouse Electric Corporation entitled *Electric Utility Engineering Reference Book—Distribution Systems*. However, as the title suggests, it is an excellent reference book but unfortunately not a textbook. Therefore the intention here is to fill the vacuum, at least partially, that has existed so long in power system engineering literature.

This book has evolved from the content of courses given by the author at the University of Missouri at Columbia, the University of Oklahoma, and Florida International University. It has been written for senior-level undergraduate and beginning-level graduate students, as well as practicing engineers in the electric power utility industry. It can serve as a text for a two-semester course, or by a judicious selection the material in the text can also be condensed to suit a single-semester course.

The book includes topics on distribution system planning, load characteristics, application of distribution transformers, design of subtransmission lines, distribution substations, primary systems, and secondary systems; voltage-drop and power-loss calculations; application of capacitors; harmonics on distribution systems; voltage regulation; and distribution system protection and reliability.

This book has been particularly written for students or practicing engineers who may want to teach themselves. Each new term is clearly defined when it is first introduced; also a glossary has been provided. Basic material has been explained carefully and in detail with numerous examples. Special features of the book include ample numerical examples and problems designed to use the information presented in each chapter. A special effort has been made to familiarize the reader with the vocabulary and symbols used by the industry. The addition of the appendixes and other back matter makes the text self-sufficient.

ACKNOWLEDGMENTS

This book could not have been written without the unique contribution of Dr. David D. Robb, of D. D. Robb and Associates, in terms of numerous problems and his kind encouragement and friendship over the years. The author also wishes to express his sincere appreciation to Dr. Paul M. Anderson of Power Math Associates and Arizona State University for his continuous encouragement and suggestions.

The author is most grateful to numerous colleagues, particularly Dr. John Thompson who provided moral support for this project, and Dr. James Hilliard of Iowa State University; Dr. James R. Tudor, Dr. Cyrus O. Harbourt, and Dr. Richard G. Hoft of the University of Missouri at Columbia; Dr. Earl M. Council of Louisiana Tech University; Dr. David K. Pantalone of the New England Power Service Company; Dr. Don O. Koval of the University of Alberta; Dr. John J. Grainger of North Carolina State University; Dr. Olle I. Elgerd of the University of Florida; Dr. Abdel-Aziz A. Fouad and Dr. Aly A. Mahmoud of Iowa State University; Enver Masud of the U.S. Department of Energy; Dr. James Story of Florida International University; and Dr. Kenan Bakin of Teledyne Electronics, Inc., for their interest, encouragement, and invaluable suggestions.

The author also wishes to thank Dr. William R. Kimel, Dean of the College of Engineering, and Dr. Charles Slivinsky, Chairman of the Electrical and Computer Engineering Department, at the University of Missouri at Columbia for creating the right environment for the author that enabled him to complete his project.

A special thank-you is extended to John Freed, Chief Distribution Engineer of the Oklahoma Gas & Electric Company; C. J. Baldwin, Advanced Systems Technology; Westinghouse Electric Corporation; W. O. Carlson, S & C Electric Company; L. D. Simpson, Siemens-Allis, Inc.; E. J. Moreau, Balteau Standard, Inc.; and T. Lopp, General Electric Company, for their kind help and encouragement.

The author is indebted to Prof. Bernard R. Sarchet, Dr. Derald J. Morgan, and Dr. Max Anderson of the University of Missouri at Rolla for their support and encouragement given to write this book. The author is also indebted to numerous students, particularly to F. Djavashi, M. Tahani, and D. C. Yu, who studied portions of the book in a graduate course and made countless contributions and valuable suggestions for improvements.

The author would also like to express his thanks for the many useful comments and suggestions provided by colleagues who reviewed this text during the course of its development, especially to J. J. Grainger, North Carolina State University; James P. Hilliard, Iowa State University; Syed Nasar, University of Kentucky; John Pavlat, Iowa State University; Lee Rosenthal, Fairleigh Dickinson University; Peter Sauer, University of Illinois; and R. L. Sullivan, University of Florida.

Finally, the author's deepest appreciation goes to his wife, Nilüfer Neel, and to his daughter, Sevil, for their limitless patience and understanding.

Turan Gönen

There is a Turkish proverb to the effect that
'the world belongs to the dissatisfied.'
I believe in this saying absolutely.
For me the one great underlying principle
of all human progress is that 'divine discontent'
makes men strive for better conditions
and improved methods.

CHARLES P. STEINMETZ

To make an end is to make a beginning.
The end is where we start from.

T. S. ELIOT

CHAPTER

ONE

DISTRIBUTION SYSTEM PLANNING AND AUTOMATION

1-1 INTRODUCTION

The electric utility industry was born in 1882 when the first electric power station, Pearl Street Electric Station in New York City, went into operation. The electric utility industry grew very rapidly, and generation stations and transmission and distribution networks have spread across the entire country. Considering the energy needs and available fuels that are forecasted for the next century, energy is expected to be increasingly converted to electricity after the year 2000. It is estimated that the installed generation capacity will be about 1200 GW in the United States by the year 2000.

In general, the definition of an electric power system includes a generating, a transmission, and a distribution system. In the past, the distribution system, on a national average, was estimated to be roughly equal in capital investment to the generation facilities, and together they represented over 80 percent of the total system investment [1]. In recent years, however, these figures have somewhat changed. For example, Fig. 1-1 shows electric utility plants in service for the years 1960 to 1978. The data represent the privately owned class A and class B utilities, which include 80 percent of all the electric utility in the United States. The percentage of electric plants represented by the production (i.e., generation), transmission, distribution, and general plant sector is shown in Fig. 1-2. The major investment has been in the production sector, with distribution a close

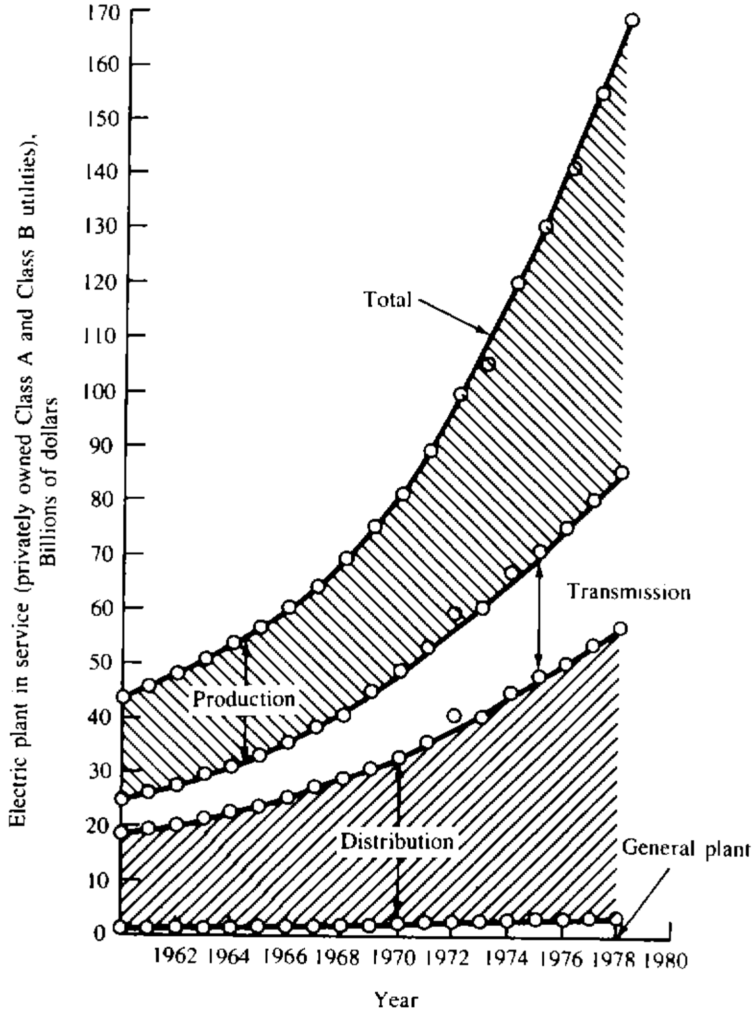


Figure 1-1 Electric utility plant in service (1960–1978). (*From* [2].)

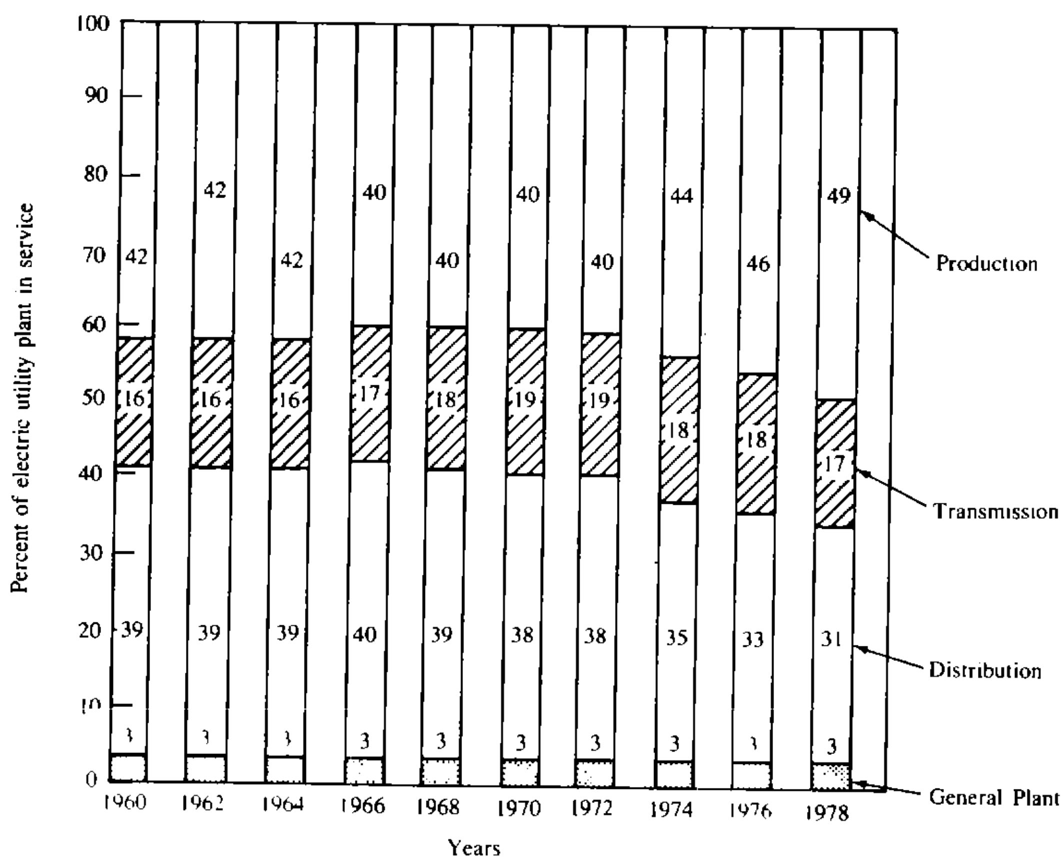


Figure 1-2 Electric utility plant in service by percent of sector (1960–1978). (*From* [2, 3].)

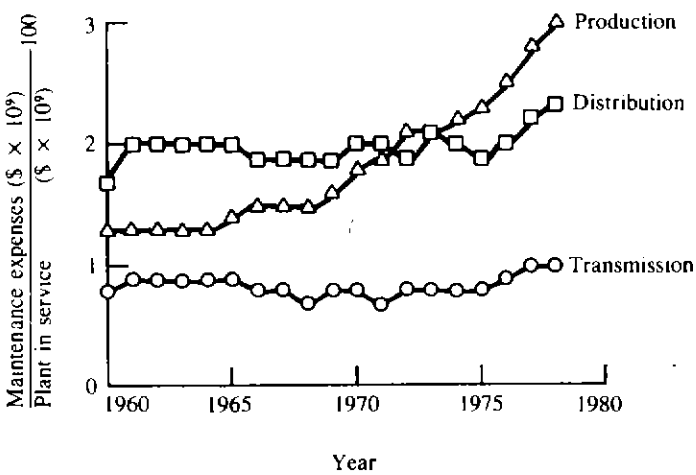


Figure 1-3 Ratio of maintenance expenses to plant in service for each utility sector (1968–1980). The data is for privately owned class A and class B electrical utilities. (*From* [2].)

second. Where expenditures for individual generation facilities are visible and receive attention due to their magnitude, the data indicate the significant investment in the distribution sector.

Furthermore, total operation and maintenance costs for the privately owned utilities have increased from $8.3 billion in 1969 to $40.2 billion in 1978 [4]. Production expense is the major factor in the total electrical operation and maintenance (O&M) expenses, representing 64 percent of total O&M expenses in 1978. The main reason for the increase has been rapidly escalating fuel costs. Figure 1-3 shows the ratio of maintenance expenses to the value of plant in service for each utility sector, namely, generation, transmission, and distribution. Again, the major O&M expense has been in the production sector, followed by the one for the distribution sector.

Succinctly put, the economic importance of the distribution system is very high, and the amount of investment involved dictates careful planning, design, construction, and operation.

1-2 DISTRIBUTION SYSTEM PLANNING

System planning is essential to assure that the growing demand for electricity can be satisfied by distribution system additions which are both technically adequate and reasonably economical. Even though considerable work has been done in the past on the application of some type of systematic approach to generation and transmission system planning, its application to distribution system planning has unfortunately been somewhat neglected. In the future, more than in the past, electric utilities will need a fast and economical planning tool to evaluate the consequences of different proposed alternatives and their impact on the rest of the system to provide the necessary economical, reliable, and safe electric energy to consumers.

The objective of distribution system planning is to assure that the growing demand for electricity, in terms of increasing growth rates and high load densities, can be satisfied in an optimum way by additional distribution systems, from the secondary conductors through the bulk power substations, which are both technically adequate and reasonably economical. All these factors and others, e.g., the scarcity of available land in urban areas and ecological considerations, can put the problem of optimal distribution system planning beyond the resolving power of the unaided human mind. Distribution system planners must determine the load magnitude and its geographic location. Then the distribution substations must be placed and sized in such a way as to serve the load at maximum cost effectiveness by minimizing feeder losses and construction costs, while considering the constraints of service reliability.

In the past, the planning for the other portions of the electric power supply system and distribution system frequently has been authorized at the company division level without review of or coordination with long-range plans. As a result of the increasing cost of energy, equipment, and labor, improved system planning through use of efficient planning methods and techniques is inevitable and necessary. The distribution system is particularly important to an electrical utility for two reasons: (1) its close proximity to the ultimate customer and (2) its high investment cost. Since the distribution system of a power supply system is the closest one to the customer, its failures affect customer service more directly than, for example, failures on the transmission and generating systems, which usually do not cause customer service interruptions.

Therefore, distribution system planning starts at the customer level. The demand, type, load factor, and other customer load characteristics dictate the type of distribution system required. Once the customer loads are determined, they are grouped for service from secondary lines connected to distribution transformers that step down from primary voltage. The distribution transformer loads are then combined to determine the demands on the primary distribution system. The primary distribution system loads are then assigned to substations that step down from transmission voltage. The distribution system loads, in turn, determine the size and location, or siting, of the substations as well as the routing and capacity of the associated transmission lines. In other words, each step in the process provides input for the step that follows.

The distribution system planner partitions the total distribution system planning problem into a set of subproblems which can be handled by using available, usually ad hoc, methods and techniques. The planner, in the absence of accepted planning techniques, may restate the problem as an attempt to minimize the cost of subtransmission, substations, feeders, laterals, etc., and the cost of losses. In this process, however, the planner is usually restricted by permissible voltage values, voltage dips, flicker, etc., as well as service continuity and reliability. In pursuing these objectives, the planner ultimately has a significant influence on additions to and/or modifications of the subtransmission network, locations and sizes of substations, service areas of substations, location of breakers and switches, sizes of feeders and laterals, voltage levels and voltage drops in the system, the location of capacitors and voltage regulators, and the loading of transformers and feeders.

There are, of course, some other factors that need to be considered such as transformer impedance, insulation levels, availability of spare transformers and mobile substations, dispatch of generation, and the rates that are charged to the customers. Furthermore, there are factors over which the distribution system planner has no influence but which, nevertheless, have to be considered in good long-range distribution systems planning, e.g., the timing and location of energy demands, the duration and frequency of outages, the cost of equipment, labor, and money, increasing fuel costs, increasing or decreasing prices of alternative energy sources, changing socioeconomic conditions and trends such as the growing demand for goods and services, unexpected local population growth or decline, changing public behavior as a result of technological changes, energy conservation, changing environmental concerns of the public, changing economic conditions such as a decrease or increase in gross national product (GNP) projections, inflation and/or recession, and regulations of federal, state, and local governments.

1-3 FACTORS AFFECTING SYSTEM PLANNING

The number and complexity of the considerations affecting system planning appears initially to be staggering. Demands for ever-increasing power capacity, higher distribution voltages, more automation, and greater control sophistication constitute only the beginning of a list of such factors. The constraints which circumscribe the designer have also become more onerous. These include a scarcity of available land in urban areas, ecological considerations, limitations on fuel choices, the undesirability of rate increases, and the necessity to minimize investments, carrying charges, and production charges.

Succinctly put, the planning problem is an attempt to minimize the cost of subtransmission, substations, feeders, laterals, etc., as well as the cost of losses. Indeed, this collection of requirements and constraints has put the problem of optimal distribution system planning beyond the resolving power of the unaided human mind.

1-3-1 Load Forecasting

The load growth of the geographical area served by a utility company is the most important factor influencing the expansion of the distribution system. Therefore, forecasting of load increases and system reaction to these increases is essential to the planning process. There are two common time scales of importance to load forecasting; long-range, with time horizons on the order of 15 or 20 years away, and short-range, with time horizons of up to 5 years distant. Ideally, these forecasts would predict future loads in detail, extending even to the individual customer level, but in practice, much less resolution is sought or required.

Figure 1-4 indicates some of the factors which influence the load forecast. As one would expect, load growth is very much dependent on the community

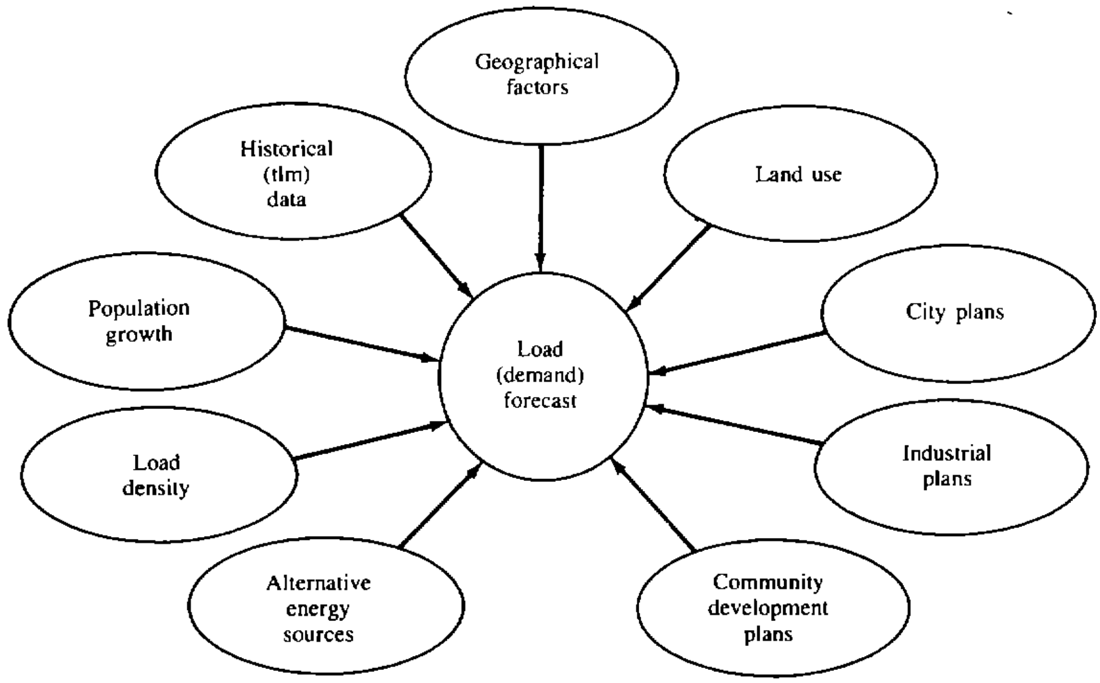


Figure 1-4 Factors affecting load forecast.

and its development. Economic indicators, demographic data, and official land use plans all serve as raw input to the forecast procedure. Output from the forecast is in the form of load densities (kilovoltamperes per unit area) for long-range forecasts. Short-range forecasts may require greater detail. Densities are associated with a coordinate grid for the area of interest. The grid data are then available to aid configuration design. The master grid presents the load forecasting data, and it provides a useful planning tool for checking all geographical locations and taking the necessary actions to accommodate the system expansion patterns.

1-3-2 Substation Expansion

Figure 1-5 presents some of the factors affecting the substation expansion. The planner makes a decision based on tangible or intangible information. For example, the forecasted load, load density, and load growth may require a substation expansion or a new substation construction. In the system expansion plan the present system configuration, capacity, and the forecasted loads can play major roles.

1-3-3 Substation Site Selection

Figure 1-6 shows the factors that affect substation site selection. The distance from the load centers and from the existing subtransmission lines as well as other limitations, such as availability of land, its cost, and land use regulations, are important.

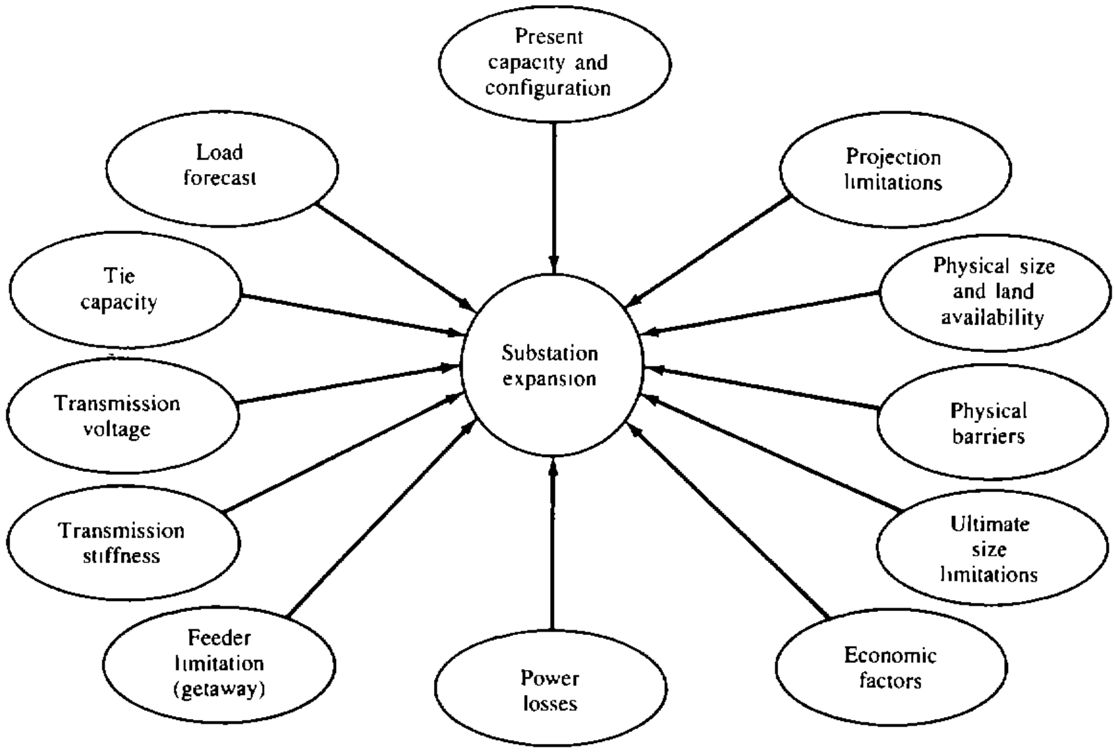


Figure 1-5 Factors affecting substation expansion.

The substation siting process can be described as a screening procedure through which all possible locations for a site are passed, as indicated in Fig. 1-7. The service region is the area under evaluation. It may be defined as the service territory of the utility. An initial screening is applied by using a set of considerations, e.g., safety, engineering, system planning, institutional, economics, aesthetics. This stage of the site selection mainly indicates the areas that are unsuitable for site development. Thus the service region is screened down to a set of candidate

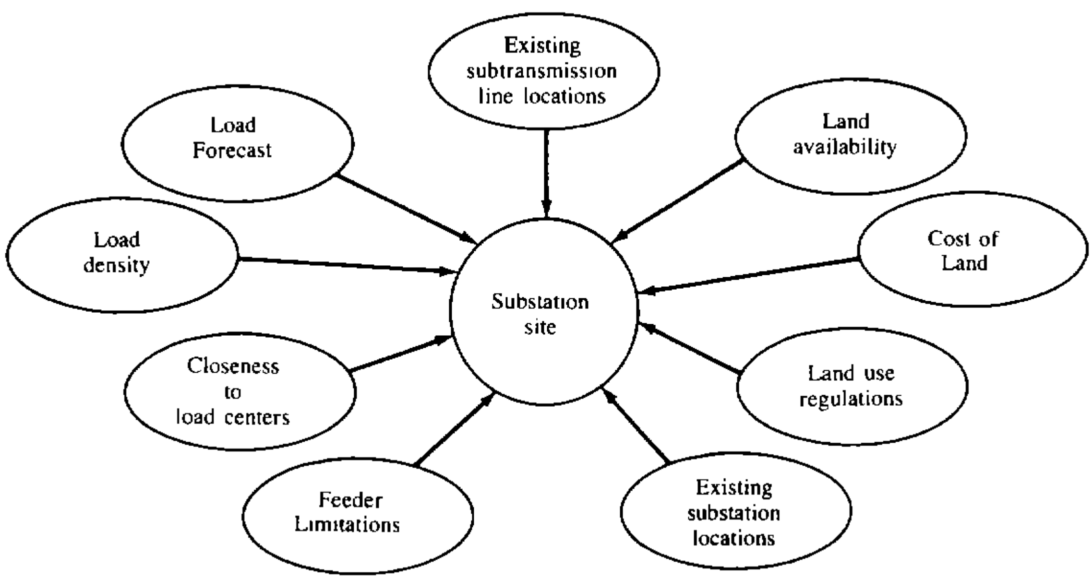


Figure 1-6 Factors affecting substation siting.

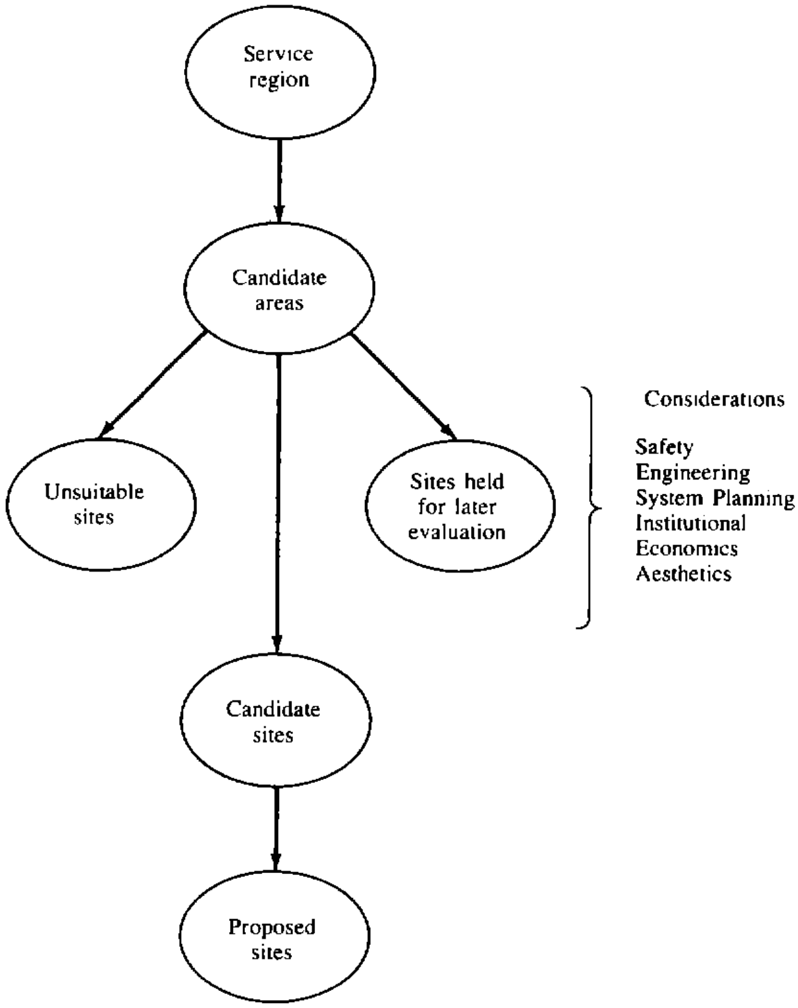


Figure 1-7 Substation site selection procedure.

sites for substation construction. Further, the candidate sites are categorized into three basic groups: (1) sites that are unsuitable for development in the foreseeable future, (2) sites that have some promise but are not selected for detailed evaluation during the planning cycle, and (3) candidate sites that are to be studied in more detail.

The emphasis put on each consideration changes from level to level and from utility to utility. Three basic alternative uses of the considerations are (1) quantitative vs. qualitative evaluation, (2) adverse vs. beneficial effects evaluation, and (3) absolute vs. relative scaling of effects. A complete site assessment should use a mix of all alternatives and attempt to treat the evaluation from a variety of perspectives.

1-3-4 Other Factors

Once the load assignments to the substations are determined, then the remaining factors affecting primary voltage selection, feeder route selection, number of feeders, conductor size selection, and total cost, as shown in Fig. 1-8, need to be considered.

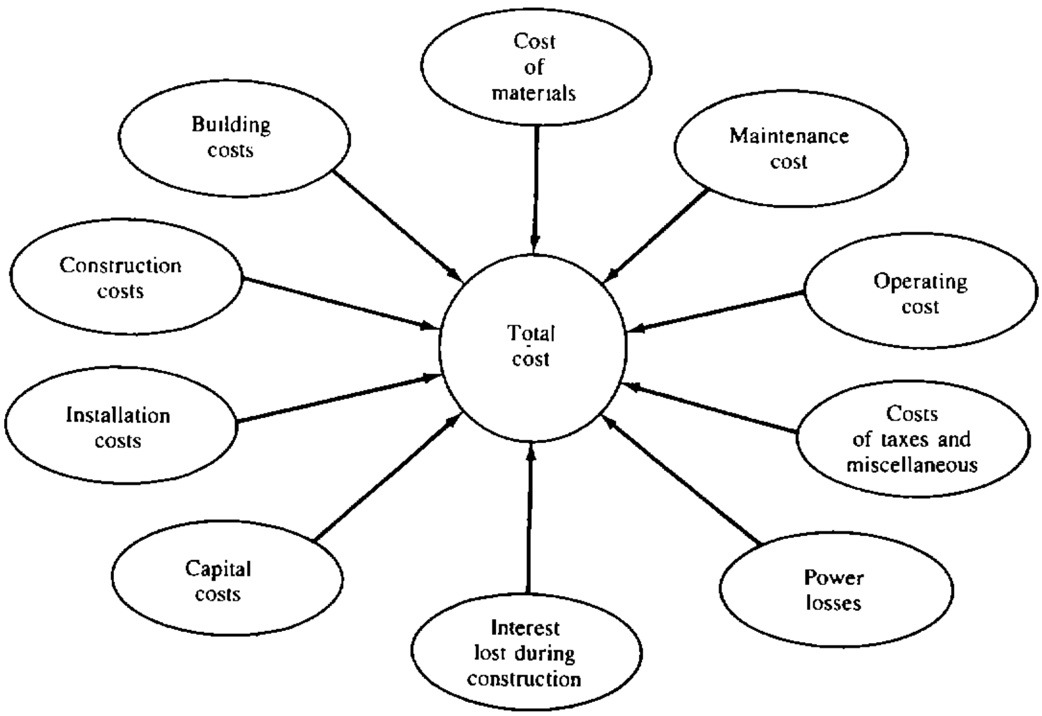


Figure 1-8 Factors affecting total cost of the distribution system expansion.

In general, the subtransmission and distribution system voltage levels are determined by company policies, and they are unlikely to be subject to change at the whim of the planning engineer unless the planner's argument can be supported by running test cases to show substantial benefits that can be achieved by selecting different voltage levels.

Further, because of the standardization and economy that are involved, the designer may not have much freedom in choosing the necessary sizes and types of capacity equipment. For example, the designer may have to choose a distribution transformer out of a fixed list of transformers that are presently stocked by the company for the voltage levels that are already established by the company. Any decision regarding addition of a feeder or adding on to an existing feeder will, within limits, depend on the adequacy of the existing system and the size, location, and timing of the additional loads that need to be served.

1-4 PRESENT DISTRIBUTION SYSTEM PLANNING TECHNIQUES

Today, many electric distribution system planners in the industry utilize computer programs, usually based on ad hoc techniques, such as load flow programs, radial or loop load flow programs, short-circuit and fault-current calculation programs, voltage drop calculation programs, and total system impedance calculation programs, as well as other tools such as load forecasting, voltage regulation, regulator setting, capacitor planning, reliability, and optimal siting and sizing algorithms. However, in general, the overall concept of using the output of each program as input for the next program is not in use. Of course, the computers do perform calculations more expeditiously than other methods and free the distribution engineer from detailed work. The engineer can then spend time reviewing results of the calculations, rather than actually making them. Nevertheless, there is no substitute for engineering judgment based on adequate planning at every stage of the development of power systems, regardless of how calculations are made. In general, the use of the aforementioned tools and their bearing on the system design is based purely on the discretion of the planner and overall company operating policy.

Figure 1-9 shows a functional block diagram of the distribution system planning process currently followed by most of the utilities. This process is repeated for each year of a long-range (15–20 years) planning period. In the development of this diagram, no attempt was made to represent the planning procedure of any specific company but rather to provide an outline of a typical planning process. As the diagram shows, the planning procedure consists of four major activities: load forecasting, distribution system configuration design, substation expansion, and substation site selection.

Configuration design starts at the customer level. The demand type, load factor, and other customer load characteristics dictate the type of distribution system required. Once customer loads are determined, secondary lines are defined which connect to distribution transformers. The latter provides the reduction from primary voltage to customer-level voltage. The distribution transformer loads are then combined to determine the demands on the primary distribution system. The primary distribution system loads are then assigned to substations that step down from subtransmission voltage. The distribution system loads, in turn, determine the size and location (siting) of the substations as well as the route and capacity of the associated subtransmission lines. It is clear that each step in this planning process provides input for the steps that follow.

Perhaps what is not clear is that in practice, such a straightforward procedure may be impossible to follow. A much more common procedure is the following. Upon receiving the relevant load projection data, a system performance analysis is done to determine whether the present system is capable of handling the new load increase with respect to the company's criteria. This analysis, constituting the second stage of the process, requires the use of tools such as a distribution load flow program, a voltage profile, and a regulation program. The acceptability criteria, representing the company's policies, obligations to the consumers, and additional constraints can include:

1. Service continuity
2. The maximum allowable peak-load voltage drop to the most remote customer on the secondary
3. The maximum allowable voltage dip occasioned by the starting of a motor of specified starting current characteristics at the most remote point on the secondary

4. The maximum allowable peak load
5. Service reliability
6. Power losses

As illustrated in Fig. 1-9, if the results of the performance analysis indicate that the present system is not adequate to meet future demand, then either the present system needs to be expanded by new, relatively minor, system additions,

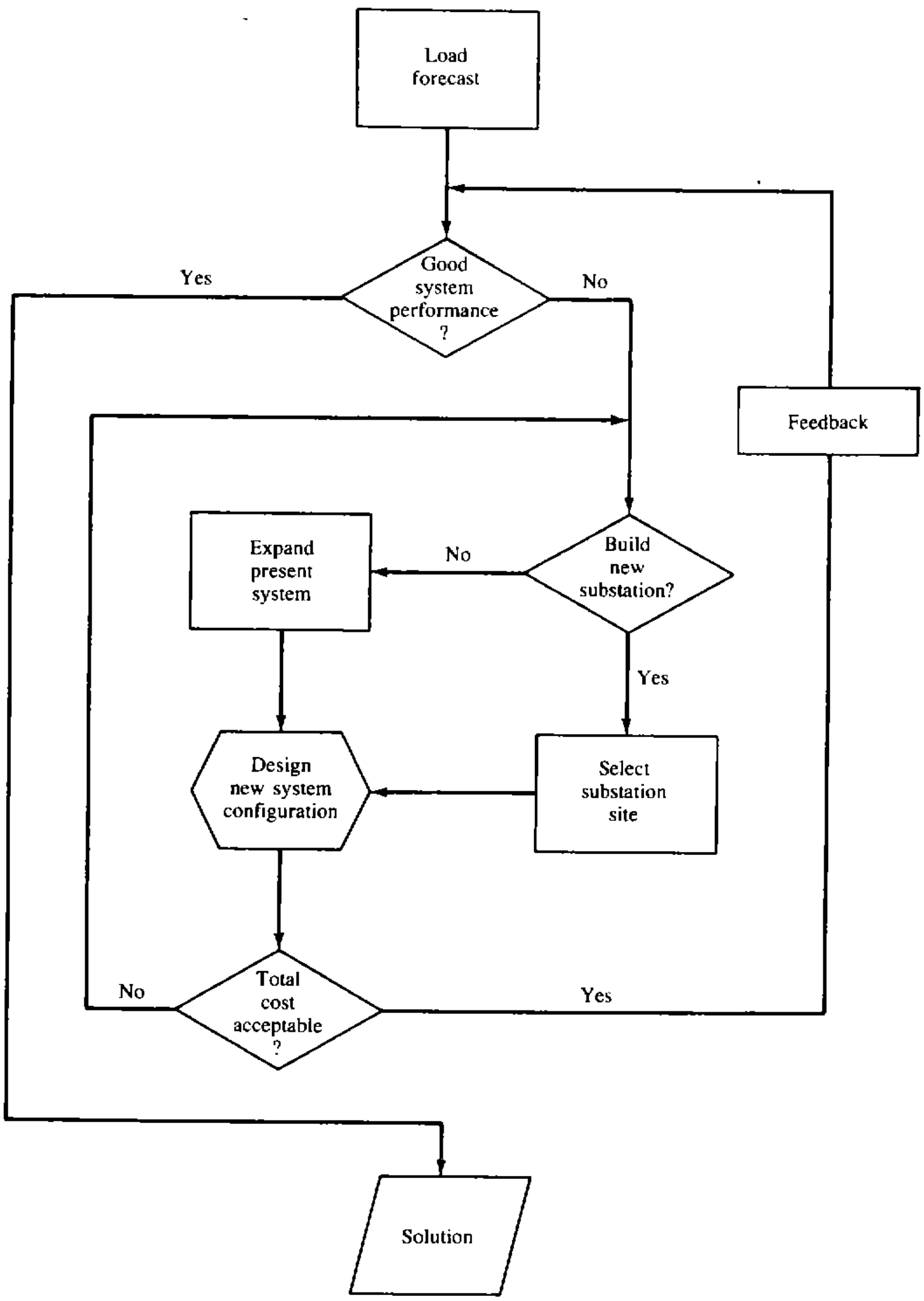


Figure 1-9 A block diagram of a typical distribution system planning process.

or a new substation may need to be built to meet the future demand. If the decision is to expand the present system with minor additions, then a new additional network configuration is designed and analyzed for adequacy. If the new configuration is found to be inadequate, another is tried, and so on, until a satisfactory one is found The cost of each configuration is calculated. If the cost is found to be too high, or adequate performance cannot be achieved, then the original expand-or-build decision is reevaluated. If the resulting decision is to build a new substation, a new placement site must be selected. Further, if the purchase price of the selected site is too high, the expand-or-build decision may need further reevaluation. This process terminates when a satisfactory configuration is attained which provides a solution to existing or future problems at a reasonable cost. Many of the steps in the above procedures can feasibly be done only with the aid of computer programs.

1-5 DISTRIBUTION SYSTEM PLANNING MODELS

In general, distribution system planning dictates a complex procedure due to a large number of variables involved and the difficult task of the mathematical presentation of numerous requirements and limitations specified by systems configuration.

Therefore, mathematical models are developed to represent the system and can be employed by distribution system planners to investigate and determine optimum expansion patterns or alternatives, for example, by selecting:

Optimum substation locations
Optimum substation expansions
Optimum substation transformer sizes
Optimum load transfers between substations and demand centers
Optimum feeder routes and sizes to supply the given loads

subject to numerous constraints to minimize the present worth of the total costs involved.

Some of the operations research techniques used in performing this task include:

1. The alternative-policy method, by which a few alternative policies are compared and the best one is selected
2. The decomposition method, in which a large problem is subdivided into several small problems and each one is solved separately
3. The linear-programming and integer-programming methods, which linearize constraint conditions
4. The dynamic-programming method

Each of these techniques has its own advantages and disadvantages. Especially in long-range planning, a great number of variables are involved, and thus there can be a number of feasible alternative plans which make the selection of the optimum alternative a very difficult one [10].

1-6 DISTRIBUTION SYSTEM PLANNING IN THE FUTURE

In the previous sections, some of the past and present techniques used by the planning engineers of the utility industry in performing the distribution systems planning have been discussed. Also, the factors affecting the distribution system planning decisions have been reviewed. Furthermore, the need for a systematic approach to distribution planning has been emphasized.

The following sections examine what today's trends are likely to portend for the future of the planning process.

1-6-1 Economic Factors

There are several economic factors which will have significant effects on distribution planning in the 1980s. The first of these is inflation. Fueled by energy shortages, energy source conversion cost, environmental concerns, and government deficits, inflation will continue to be a major factor.

The second important economic factor will be the increasing expense of acquiring capital. As long as inflation continues to decrease the real value of the dollar, attempts will be made by government to reduce the money supply. This in turn will increase the competition for attracting the capital necessary for expansions in distribution systems.

The third factor which must be considered is increasing difficulty in raising customer rates. This rate increase "inertia" also stems in part from inflation as well as from the results of customers being made more sensitive to rate increases by consumer activist groups.

1-6-2 Demographic Factors

Important demographic developments will affect distribution system planning in the near future. The first of these is a trend which has been dominant over the last 50 years: the movement of the population from the rural areas to the metropolitan areas. The forces which initially drove this migration—economic in nature—are still at work. The number of single-family farms has continuously declined during this century, and there are no visible trends which would reverse this population flow into the larger urban areas. As population leaves the countrysides, population must also leave the smaller towns which depend on the countrysides for economic life. This trend has been a consideration of distribution planners for years and represents no new effect for which account must be taken. However, the migration from the suburbs to the urban and near-urban areas is a new trend attributable to the energy crisis. This trend is just beginning to be visible, and it will result in an increase in multifamily dwellings in areas which already have high population densities.

1-6-3 Technological Factors

The final class of factors, which will be important to the distribution system planner, has arisen from technological advances that have been encouraged by the energy crisis. The first of these is the improvement in fuel-cell technology. The output power of such devices has risen to the point where in the areas with high population density, large banks of fuel cells could supply significant amounts of the total power requirements. Other nonconventional energy sources which might be a part of the total energy grid could appear at the customer level. Among the possible candidates would be solar and wind-driven generators. There is some pressure from consumer groups to force utilities to accept any surplus energy from these sources for use in the total distribution network. If this trend becomes important, it would change drastically the entire nature of the distribution system as it is known today.

1-7 FUTURE NATURE OF DISTRIBUTION PLANNING

Predictions about the future methods for distribution planning must necessarily be extrapolations of present methods. Basic algorithms for network analysis have been known for years and are not likely to be improved upon in the near future. However, the superstructure which supports these algorithms and the problem-solving environment used by the system designer is expected to change significantly to take advantage of new methods which technology has made possible. Before giving a detailed discussion of these expected changes, the changing role of distribution planning needs to be examined.

1-7-1 Increasing Importance of Good Planning

For the economic reasons listed above, distribution systems will become more expensive to build, expand, and modify. Thus it is particularly important that each distribution system design be as cost effective as possible. This means that the system must be optimal from many points of view over the time period from first day of operation to the planning-time horizon. In addition to the accurate load growth estimates, components must be phased in and out of the system so as to minimize capital expenditure, meet performance goals, and minimize losses. These requirments need to be met at a time when demographic trends are veering away from what have been their norms for many years in the past and when distribution systems are becoming more complex in design due to the appearance of more active components (e.g., fuel cells) instead of the conventional passive ones.

1-7-2 Impacts of Load Management

In the past, the power utility companies of this nation supplied electric energy to meet all customer demands when demands occurred. Recently, however, because of the financial constraints (i.e., high cost of labor, materials, and interest rates), environmental concerns, and the recent shortage (or high cost) of fuels, this basic philosophy has been reexamined and customer load management investigated as an alternative to capacity expansion.

Load management's benefits are systemwide. Alteration of the electric energy use patterns will not only affect the demands on system generating equipment but also alter the loading of distribution equipment. The load management may be used to reduce or balance loads on marginal substations and circuits, thus even extending their lives. Therefore, in the future, the implementation of load management policies may drastically affect the distribution of load, in time and in location, on the distribution system, subtransmission system, and the bulk power system. Since distribution systems have been designed to interface with controlled load patterns, the systems of the future will necessarily be designed somewhat differently to benefit from the altered conditions. However, the benefits of load management cannot be fully realized unless the system planners have the tools required to adequately plan incorporation into the evolving electric energy system. The evolution of the system in response to changing requirements and under changing constraints is a process involving considerable uncertainty.

The requirements of a successful load management program are specified by Delgado [19] as follows:

1. It must be able to reduce demand during critical system load periods.
2. It must result in a reduction in new generation requirements, purchased power, and/or fuel costs.
3. It must have an acceptable cost/benefit ratio.
4. Its operation must be compatible with system design and operation.
5. It must operate at an acceptable reliability level.
6. It must have an acceptable level of customer convenience.
7. It must provide a benefit to the customer in the form of reduced rates or other incentives.

1-7-3 Cost/Benefit Ratio for Innovation

In the utility industry, the most powerful force shaping the future is that of economics. Therefore, any new innovations are not likely to be adopted for their own sake but will be adopted only if they reduce the cost of some activity or provide something of economic value which previously had been unavailable for comparable costs. In predicting that certain practices or tools will replace current ones, it is necessary that one judge their acceptance on this basis.

The expected innovations which satisfy these criteria are planning tools implemented on a digital computer which deals with distribution systems in network terms. One might be tempted to conclude that these planning tools would be adequate for industry use throughout the 1980s. That this is not likely to be the case may be seen by considering the trends judged to be dominant during this period with those which held sway over the period in which the tools were developed.

1-7-4 New Planning Tools

Tools to be considered fall into two categories: network design tools and network analysis tools, The analysis tools may become more efficient but are not expected to undergo any major changes, although the environment in which they are used will change significantly. This environment will be discussed in the next section.

The design tools, however, are expected to show the greatest development since better planning could have a significant impact on the utility industry. The results of this development will show the following characteristics:

1. Network design will be optimized with respect to many criteria by using programming methods of operations research.
2. Network design will be only one facet of distribution system management directed by human engineers using a computer system designed for such management functions.
3. So-called network editors [7] will be available for designing trial networks; these designs in digital form will be passed to extensive simulation programs which will determine if the proposed network satisfies performance and load growth criteria.

1-8 THE CENTRAL ROLE OF THE COMPUTER IN DISTRIBUTION PLANNING

As is well known, distribution system planners have used computers for many years to perform the tedious calculations necessary for system analysis. However, it has only been in the past few years that technology has provided the means for planners to truly take a *system approach* to the total design and analysis. It is the central thesis of this book that the development of such an approach will occupy planners in the 1980s and will significantly contribute to their meeting the challenges previously discussed.

1-8-1 The System Approach

A collection of computer programs to solve the analysis problems of a designer does not necessarily constitute an efficient problem-solving system; nor does such a collection even when the output of one can be used as the input of another. The system approach to the design of a useful tool for the designer begins by examining the types of information required and its sources. The view taken is that this information generates decisions and additional information which pass from one stage of the design process to another. At certain points, it is noted that the human engineer must evaluate the information generated and add his or her input. Finally, the results must be displayed for use and stored for later reference. With this conception of the planning process, the system approach seeks to automate as much of the process as possible, ensuring in the process that the various transformations of information are made as efficiently as possible. One representation of this information flow is shown in Fig. 1-10, where the outer circle represents the interface between the engineer and the system. Analysis programs forming part of the system are supported by a data-base management system which stores, retrieves, and modifies various data on distribution systems. [11].

1-8-2 The Data-Base Concept

As suggested in Fig. 1-10, the data base plays a central role in the operation of such a system. It is in this area that technology has made some significant strides in the past 5 years so that not only is it possible to store vast quantities of data economically, but it is also possible to retrieve desired data with access times on

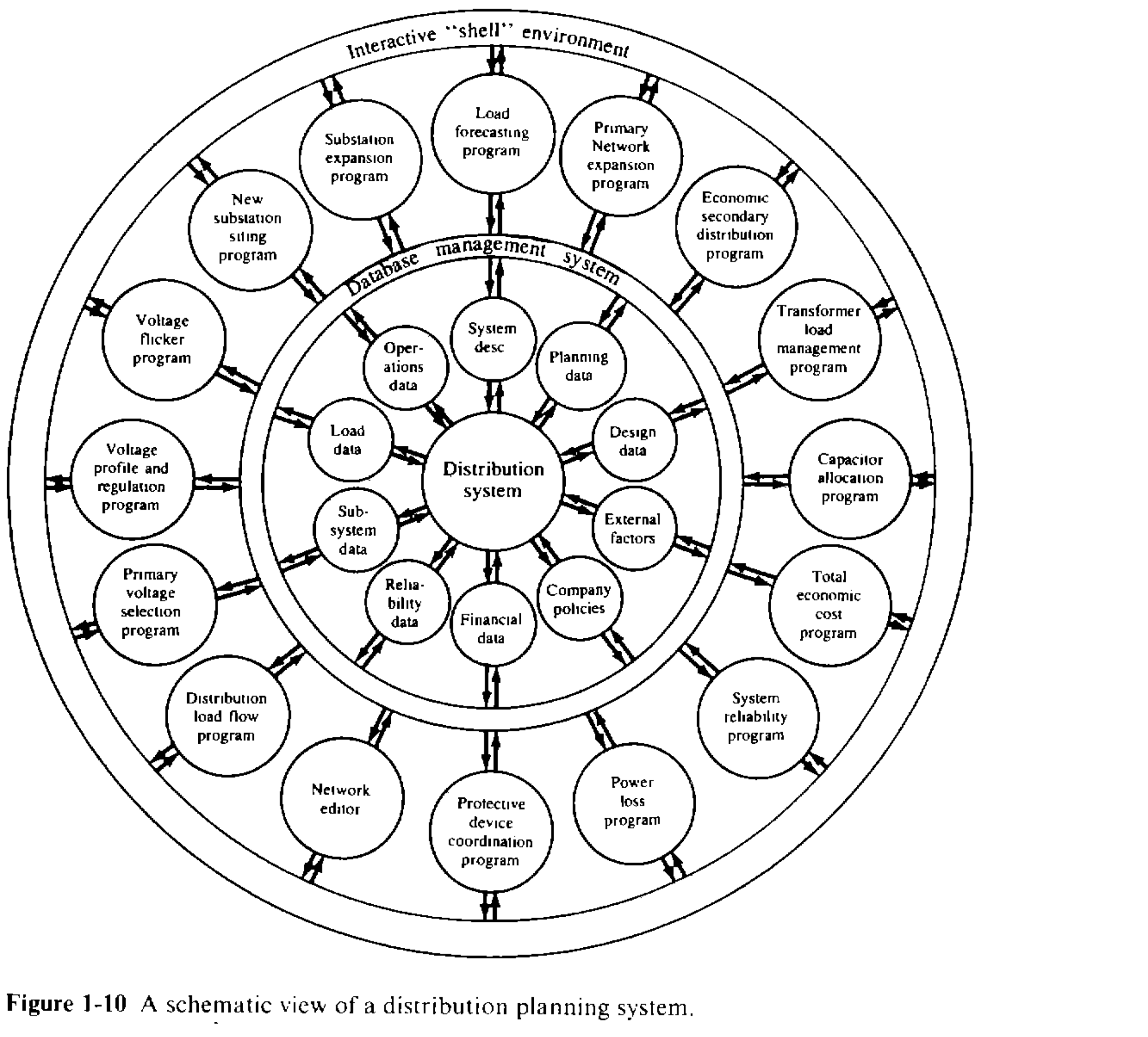


Figure 1-10 A schematic view of a distribution planning system.

the order of seconds. The data-base management system provides the interface between the process which requires access to the data and the data themselves. The particular organization which is likely to emerge as the dominant one in the near future is based on the idea of a relation. Operations on the data base are performed by the data-base management system (DBMS).

1-8-3 New Automated Tools

In addition to the data-base management program and the network analysis programs, it is expected that some new tools will emerge to assist the designer in arriving at the optimal design. One such new tool which has appeared in the literature is known as a *network editor* [7]. The network consists of a graph whose vertices are network components, such as transformers and loads, and edges which represent connections among the components.

The features of the network editor may include network objects, e.g., feeder line sections, secondary line sections, distribution transformers, or variable or fixed capacitors, control mechanisms, and command functions. A primitive network object comprises a name, an object class description, and a connection list. The control mechanisms may provide the planner with natural tools for correct network construction and modification. [11].

1-9 IMPACT OF DISPERSED STORAGE AND GENERATION

Following the oil embargo and the rising prices of oil, the efforts toward the development of alternative energy sources (preferably renewable resources) for generating electric energy have been increased. Furthermore, opportunities for small power producers and cogenerators have been enhanced by recent legislative initiatives, e.g., the Public Utility Regulatory Policies Act (PURPA) of 1978, and by the subsequent interpretations by the Federal Energy Regulatory Commission (FERC) in 1980 [20–21].

The following definitions of the criteria affecting facilities under PURPA are given in Section 201 of PURPA.

A small power production facility is one which produces electric energy solely by the use of primary fuels of biomass, waste, renewable resources, or any combination thereof. Furthermore, the capacity of such production sources together with other facilities located at the same site must not exceed 80 MW.

A cogeneration facility is one which produces electricity and steam or forms of useful energy for industrial, commercial, heating, or cooling applications.

A qualified facility is any small power production or cogeneration facility which conforms to the previous definitions and is owned by an entitity not primarily engaged in generation or sale of electric power

In general, these generators are small (typically ranging in size from 100 kW to 10 MW and connectable to either side of the meter) and can be economically connected only to the distribution system. They are defined as *dispersed-storage-and-generation* (DSG) devices. If properly planned and operated, DSG may

Table 1-1 Comparison of dispersed-storage-and-generation (DSG) Devices*

	Factors								
DSG devices	Size	Power source availability	Power source stability	Energy limitation	DSG voltage control	Response speed	Harmonic generation	Automatic start	Special DSG factors
Biomass	Variable	Good	Good	No	Yes	Fast	No	Yes	Yes
Geothermal	Medium	Good	Good	No	Yes	Medium	No	Yes	No
Pumped hydro	Large	Good	Good	Yes	Yes	Fast	No	Yes	No
Compressed air storage	Large	Good	Good	Yes	Yes	Fast	No	Yes	No
Solar thermal	Variable	Uncertain	Poor	No	Uncertain	Variable	Uncertain	Uncertain	Yes
Photovoltaics	Variable	Uncertain	Poor	No	Uncertain	Fast	Yes	Yes	Yes
Wind	Small	Uncertain	Poor	No	Uncertain	Fast	Uncertain	Yes	Yes
Fuel cells	Variable	Good	Good	No	Yes	Fast	Yes	Yes	No
Storage battery	Variable	Good	Good	Yes	Yes	Fast	Yes	Yes	No
Low-head hydro	Small	Variable	Good	No	Yes	Fast	No	Yes	No
Cogeneration:									
Gas turbine	Medium	Good	Good	No	Yes	Fast	No	Yes	No
Burning refuse	Medium	Good	Good	No	Yes	Fast	No	Yes	No
Landfill gas	Small	Good	Good	No	Yes	Fast	No	Yes	No

* From Kirkham and Klein [22]. Used by permission. © 1983 IEEE.

Table 1-2 Interaction between dispersed-storage-and-generation factors and energy management system functions*

	Factors								
Functions	Size	Power source availability	Power source stability	Energy limitation	DSG voltage control	Response speed	Harmonic generation	Automatic start	Special DSG factors
Automatic gen. control	1	1	1	1	0	1	0	0	0
Economic dispatch	1	1	1	1	?	0	0	1	0
Voltage control	1	0	1	0	1	1	?	0	0
Protection	1	0	1	0	1	1	1	1	1
State estimation	1	0	0	0	0	0	0	?	0
On-line load flow	1	0	0	0	0	0	0	0	0
Security monitoring	1	0	0	0	0	0	0	0	0

Key: 1 = interaction probable, 0 = interaction unlikely, ? = interaction possible.

* From Kirkham and Klein [22]. Used by permission. © 1983 IEEE.

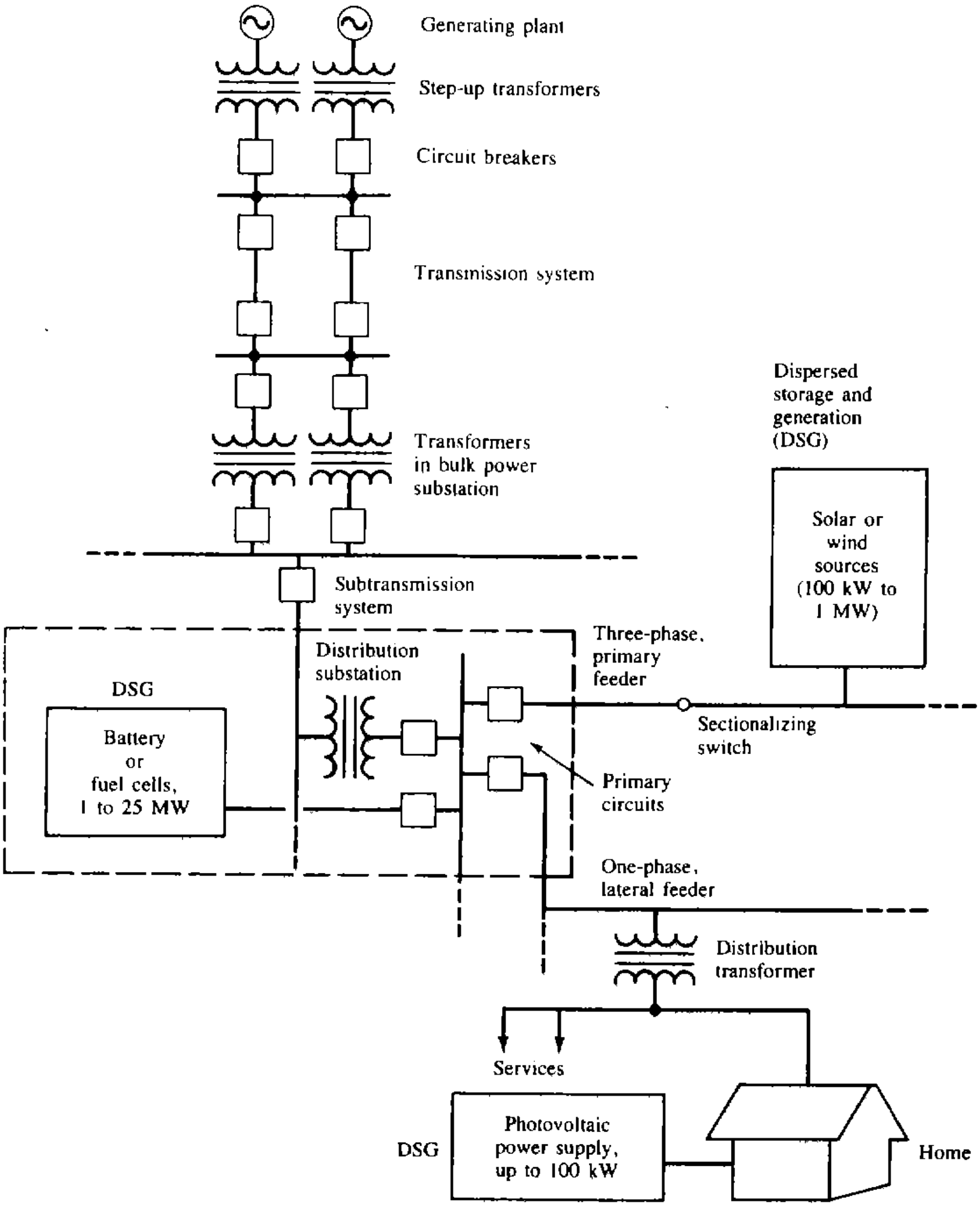


Figure 1-11 In the future, small, dispersed-energy-storage-and-generation units attached to a customer's home, a power distribution feeder, or a substation would require an increasing amount of automation and control. (*From* [*26*]. *Used by permission.* © *1982 IEEE.*)

provide benefits to distribution systems by reducing capacity requirements, improving reliability, and reducing losses. Examples of DSG technologies include hydroelectric, diesel generators, wind electric systems, solar electric systems, batteries, storage space and water heaters, storage air conditioners, hydroelectric pumped storage, photovoltaics, and fuel cells. Table 1-1 gives the results of a comparison of DSG devices with respect to the factors affecting the energy management System (EMS) of a utility system [22]. Table 1-2 gives the interactions between the DSG factors and the functions of the EMS or energy control center.

As mentioned before, it has been estimated that the installed generation capacity will be about 1200 GW in the United States by the year 2000. The contribution of the DSG systems to this figure has been estimated to be in a range of 4 to 10 percent. For example, if 5 percent of installed capacity is DSG in the year 2000, it represents a contribution of 60 GW.

According to Chen [26], as power-distribution systems become increasingly complex due to the fact that they have more DSG systems, as shown in Fig. 1-11, distribution automation will be indispensable for maintaining a reliable electric supply and for cutting down operating costs.

1-10 DISTRIBUTION SYSTEM AUTOMATION

The main purpose of an electric power system is to efficiently generate, transmit, and distribute electric energy. The operations involved dictate geographically dispersed and functionally complex monitoring and control systems, as shown in Fig. 1-12. As noted in the figure, the energy management system (EMS) exercises overall control over the total system. The *supervisory control and data acquisition* (SCADA) system involves generation and transmission systems. The *distribution automation and control* (DAC) system oversees the distribution system, including connected load. Automatic monitoring and control features have long been a part of the SCADA system. More recently automation has become a part of the overall energy management, including the distribution system. The motivating objectives of the DAC system are [25]:

1. Improved overall system efficiency in the use of both capital and energy
2. Increased market penetration of coal, nuclear, and renewable domestic energy sources
3. Reduced reserve requirements in both transmission and generation
4. Increased reliability of service to essential loads

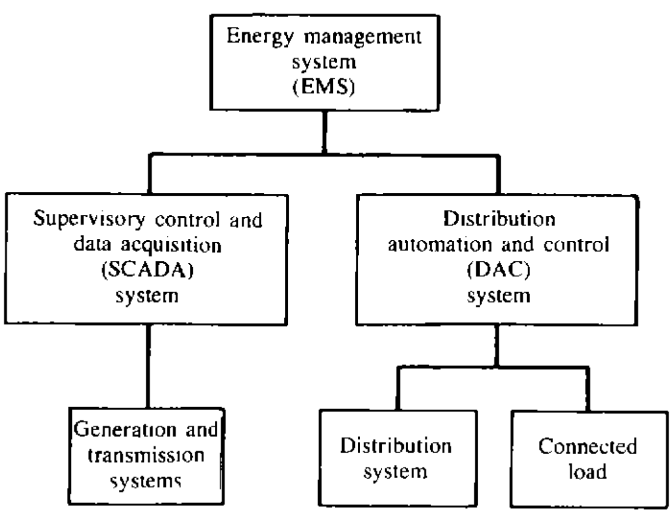


Figure 1-12 Monitoring and controlling an electric power system.

Advances in digital technology are making true distribution automation a reality. Recently, inexpensive minicomputers and powerful microprocessors (computer on a chip) have provided distribution system engineers with new tools that are making many distribution automation concepts achievable. Table 1-3 gives a profile of the electric utility industry in the United States in the year 2000. The given data clearly indicate that future distribution systems will be more complex than those of today. If the systems being developed are to be optimal with respect to construction cost, capitalization, performance reliability, and operating efficiency, better automation and control tools are required.

The term *distribution automation* has a very broad meaning, and additional applications are added every day. To some people, it may mean a communication system at the distribution level that can control customer load and can reduce peak-load generation through load management. To others, the distribution automation may mean an unattended distribution substation that could be considered attended through the use of an on-site microprocessor. The microprocessor, located at a distribution substation, can continuously monitor the system, make operating decisions, issue commands, and report any change in status to the distribution dispatch center (DDC), store it on-site for later use, or forget it, depending on the need of the utility.

1-10-1 Distribution Automation and Control Functions

There is no universal consensus among the utilities as to the types of functions which should be handled by a DAC system. Table 1-4 gives some of the automated distribution functions which can be categorized as the load management functions, real-time operational management functions, and remote meter reading functions. Some of these functions will be discussed in further detail.

Discretionary load switching This function is also called the *customer load management*. It involves direct control of loads at individual customer sites from a remote central location. Control may be exercised for the purpose of overall system peak-load reduction or to reduce the load on a particular substation or feeder that is

Table 1-3 A profile of the electric utility industry in the United States in the Year 2000*

Total U.S. population	250×10^6
Number of electric meters	110×10^6
Number of residences:	
With central air conditioners	33×10^6
With electric water heaters	25×10^6
With electric space heating	7×10^6
Number of electric utilities	3100

* From [24].

becoming overloaded. Customer loads that are appropriate for control are water heating, air conditioning, space heating, thermal storage heating, etc., and industrial loads supplied under interruptible survice contracts. While this function is similar to peak-load pricing, the dispatching center controls the individual customer loads rather than only the meters.

Peak-load pricing This function allows the implementation of peak-load pricing programs by remote switching of meter registers automatically for the purpose of time-of-day metering.

Load shedding This function permits the rapid dropping of large blocks of load, under certain conditions, according to an established priority basis.

Cold load pickup This function is a corollary to the load-shedding function. It entails the controlled pickup of dropped load.

Load reconfiguration This function involves remote control of switches and breakers to permit routine daily, weekly, or seasonal reconfiguration of feeders or feeder segments for the purpose of taking advantage of load diversity among feeders. It enables the system to effectively serve larger loads without requiring feeder reinforcement or new construction. It also enables routine maintenance on feeders without any customer load interruptions.

Voltage regulation This function allows the remote control of selected voltage regulators within the distribution network, together with network capacitor switching, to effect coordinated systemwide voltage control from a central facility.

Transformer load management (TLM) This function enables the monitoring and continuous reporting of transformer loading data and core temperature to prevent overloads, burnouts, or abnormal operation by timely reinforcement, replacement or reconfiguration.

Feeder load management (FLM) This function is similar to TLM, but the loads are monitored and measured on feeders and feeder segments (known as the *line sections*) instead. This function permits loads to be equalized over several feeders.

Capacitor control This function permits selective and remote-controlled switching of distribution capacitors.

Dispersed storage and generation Storage and generation equipment may be located at strategic places throughout the distribution system, and they may be used for peak shaving. This function enables the coordinated remote control of these sites.

Table 1-4 Automated distribution functions correlated with locations*

	Customer sites			Power system elements				
	Residential	Commercial and industrial	Agricultural	Distribution circuits	Industrial substation	Distribution substation	Bulk power substation	DSG facilities
Load management:								
Discretionary load switching	x	x	x					
Peak-load pricing	x	x	x					
Load shedding	x	x	x					
Cold load pickup	x	x	x					
Operational management:								
Load reconfiguration				x	x	x	x	
Voltage regulation				x	x	x	x	
Transformer load mgt.						x	x	
Feeder load mgt.					x	x		
Capacitor control				x	x			
Dispersed storage and generation								x
Fault detection, location, and isolation				x	x	x	x	
Load studies	x	x	x	x		x	x	
Condition and state monitoring		x	x	x	x	x	x	x
Remote meter reading:								
Automatic customer meter reading	x	x	x					

* From [24].

Fault detection, location, and isolation Sensors located throughout the distribution network can be used to detect and report abnormal conditions. This information, in turn, can be used to automatically locate faults, isolate the faulted segment, and initiate proper sectionalization and circuit reconfiguration. This function enables the dispatcher to send repair crews faster to the fault location and results in lesser customer interruption time.

Load studies This function involves the automatic on-line gathering and recording of load data for special off-line analysis. The data may be stored at the collection point, at the substation, or transmitted to a dispatch center. This function provides accurate and timely information for the planning and engineering of the power system.

Condition and state monitoring This function involves real-time data gathering and status reporting from which the minute-by-minute status of the power system can be determined.

Automatic customer meter reading This function allows the remote reading of customer meters for total consumption, peak demand, or time-of-day consumption and saves the otherwise necessary manhours involved in meter reading.

Remote service connect or disconnect This function permits remote control of switches to connect or disconnect an individual customer's electric service from a central control location.

1-10-2 The Level of Penetration of Distribution Automation

The level of penetration of distribution automation refers to how deeply into the distribution system the automation will go. Table 1-5 gives the present and near-future functional scope of power-distribution automation systems.

Recently, the need for gathering substation and power plant data has increased. According to Gaushell, Frisbie, and Kuchefski [27], this is due to

(*a*) Increased reporting requirements of reliability councils and government agencies
(*b*) Operation of the electric system closer to design limits
(*c*) Increased efficiency requirements because of much higher fuel prices
(*d*) The tendency of utilities to monitor lower voltages than previously

These needs have occurred simultaneously with the relative decline of the prices of computer and other electronic equipment. The result has been a quantum jump in the amount of data being gathered by a supervisory control and data acquisition (SCADA) system or energy management system (EMS).

A large portion of this data consists of analog measurements of electrical quantities, such as watts, vars, and volts, which are periodically sampled at a remote location, transmitted to a control center, and processed by computer for output on CRT displays, alarm logs, etc. However, as the amount of information to be reported grows, so do the number of communication channels and the amount of control center computer resources required.

Table 1-5 Functional scope of power-distribution automation systems.*

Present	Within up to 5 years	After 5 years
	Protection	
Excessive current over long time Instantaneous overcurrent Underfrequency Transformer protection Bus protection	Breaker-failure protection Synchronism check	Dispersed-storage-and-generation (DSG) protection Personnel safety
	Operational control and monitoring	
Automatic bus sectionalizing Alarm annunciation Transformer tap-change control Instrumentation Load control	Integrated voltage and var control: Capacitor bank control Transformer tap-change control Feeder deployment switching and automatic sectionalizing Load shedding Data acquisition, logging, and display Sequence-of-events recording Transformer monitoring Instrumentation and diagnostics	DSG command and control: power, voltage, synchronization DSG scheduling Automatic generation control Security assessment
	Data collection and system planning	
Remote supervisory control and data acquisition (SCADA) at a substation	Distribution SCADA Automatic meter reading	Distribution dispatching center Distribution system data base Automatic billing Service connecting and disconnecting
	Communications	
One-way load control	Two-way communication, using one medium	Two-way communication, using many media

* From Chen [26]. Used by permission. © 1982 IEEE

Therefore, as equipments are controlled or monitored further down the feeder, the utility obtains more information, can have greater control, and has greater flexibility. However, costs increase as well as benefits. As succinctly put by Markel and Layfield [28],

1. The number of devices to be monitored or controlled increases drastically.
2. The communication system must cover longer distances, connect more points, and transmit greater amounts of information.
3. The computational requirements increase to handle the larger amounts of data or to examine the increasing number of available switching options.
4. The time and equipment needed to identify and communicate with each individually controlled device increases as the addressing system becomes more finely grained.

Today, microprocessors use control algorithms which permit real-time control of distribution system configurations. For example, it has become a reality that normal loadings of substation transformers and of looped (via a normally open tie recloser) sectionalized feeders can be economically increased through software-controlled load-interrupting switches. SCADA remotes, often computer-directed, are being installed in increasing numbers in distribution substations. They provide advantages such as continuous scanning, higher speed of operation, and greater security. Furthermore, thanks to the falling prices of microprocessors, certain control practices (e.g., protecting power systems against circuit-breaker failures by energizing backup equipment, which is presently done only in transmission systems) are expected to become cost-effective in distribution systems.

The Electric Power Research Institute (EPRI) and the U.S. Department of Energy (DOE) singled out power-line, telephone, and radio carriers as the most promising systems for their research; other communication techniques are certainly possible. However, at the present time, these other techniques involve greater uncertainties [29].

In summary, the choice of a specific communication system or combination of systems depends upon the specific control or monitoring functions required, amount and speed of data transmission required, existing system configuration, density of control points, whether one-way or two-way communication is required, and, of course, equipment costs.

It is possible to use hybrid systems, i.e., two or more different communication systems, between utility and customer. For example, a radio carrier might be used between the control station and the distribution transformer, a power-line carrier between the transformer and the customer's meter. Furthermore, the command (forward) link might be one communication system, e.g., broadcast radio, and the return (data) link might be another system, such as VHF radio. An example of such a system is shown in Fig. 1-13. The forward (control) link of this system uses commercial broadcast radio. Utility phase-modulated (PM) digital signals are added to amplitude-modulated (AM) broadcast information. Standard AM receivers cannot detect the utility signals, and vice versa. The return data link uses VHF receivers that are synchronized by the broadcast station to significantly increase data rate and coverage range [30].

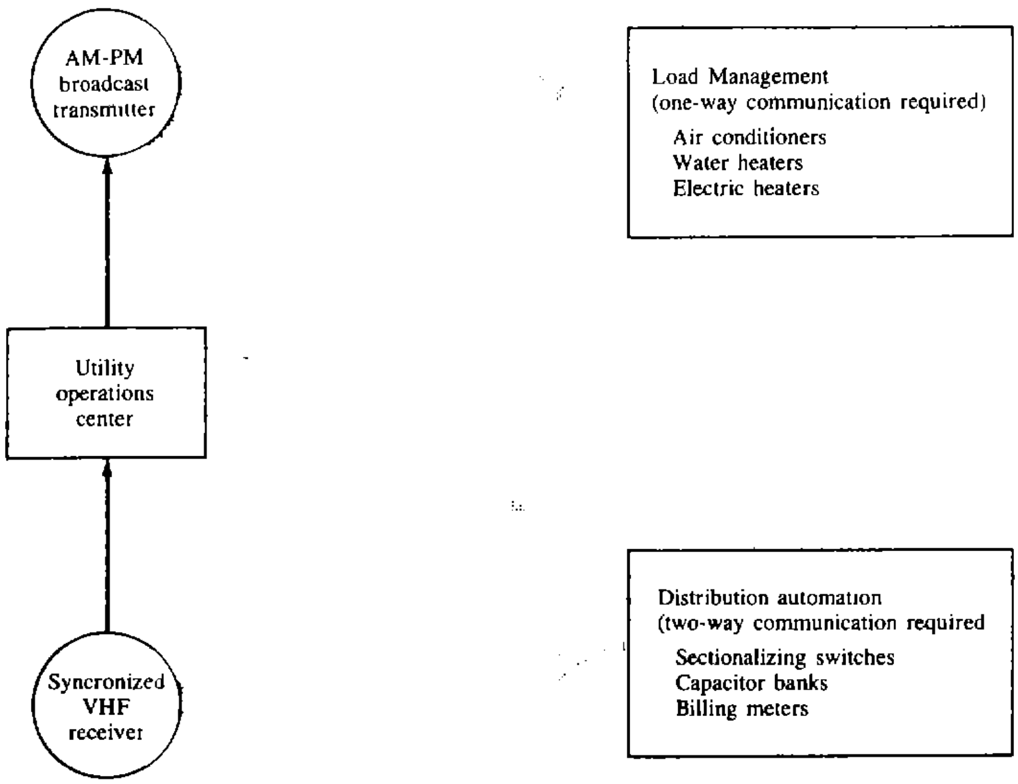


Figure 1-13 Applications of two-way radio communications. (*From* [*30*].)

Figure 1-14 shows an experimental system for automating power distribution at the LaGrange Park Substation of Commonwealth Edison Company of Chicago. The system includes two minicomputers, a commercial VHF radio transmitter and receiver, and other equipment installed at a special facility called *Probe*. Microprocessors atop utility poles can automatically connect or disconnect two sections of a distribution feeder upon instructions from the base station.

Figure 1-15 shows a substation control and protection system which has also been developed by EPRI. It features a common signal bus to control recording, comparison, and follow-up actions. It includes line protection and transformer protection. The project is directed toward developing microprocessor-based digital relays capable of interfacing with conventional current and potential transformers and of accepting digital data from the substation yard. These protective devices can also communicate with substation microcomputer controls capable of providing sequence of events, fault recording, and operator control display. They are also able to interface upward to the dispatcher's control and downward to the distribution system control [31].

Figure 1-16 shows an integrated distribution control and protection system developed by EPRI. The integrated system includes four subsystems: a substation integration module (SIM), a data acquisition system (DAS), a digital protection module (DPM), and a feeder remote unit (FRU). The substation integration module coordinates the functions of the data acquisition and control system, the digital protection module, and feeder remote units by collecting data from them and forming the real-time data base required for substation and feeder

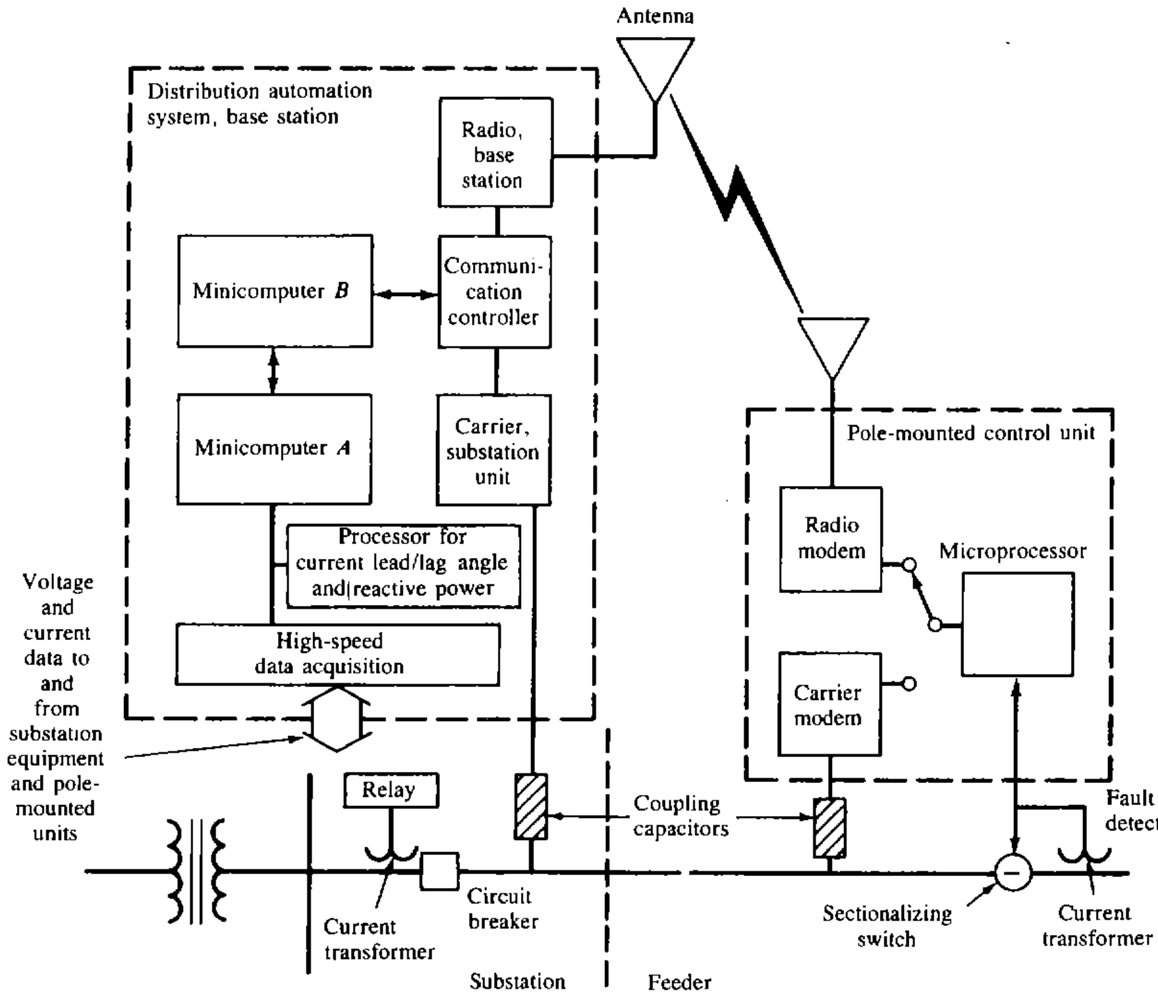


Figure 1-14 The research system consisted of two minicomputers with distributed high-speed data-acquisition processing units at the La Grange Park Substation. (*From* [*26*]. *Used by permission.* © *1982 IEEE.*)

control. The digital protection module operates in coordination with the data acquisition system and is also a stand-alone device.

1-10-3 Alternatives of Communication Systems

There are various types of communication systems available for distribution automation:

1. Power-line carrier
2. Radio carrier
3. Telephone (lines) carrier
4. Microwave
5. Private cables, including optical fibers

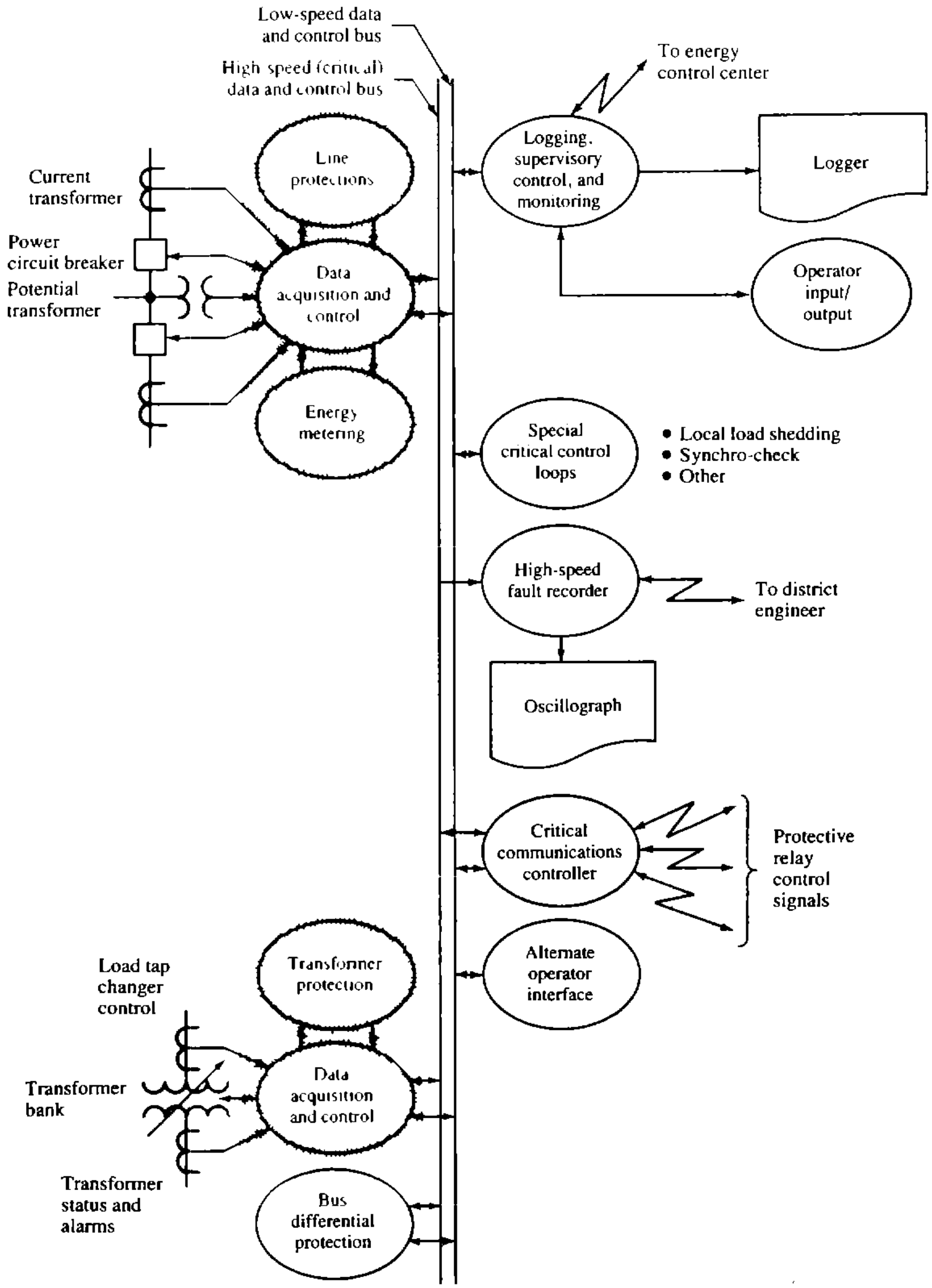


Figure 1-15 Substation control and protection system that features a common signal bus (center lines) to control recording, comparison, and follow-up actions (right). Critical processes are shaded. (*From* [*31*].)

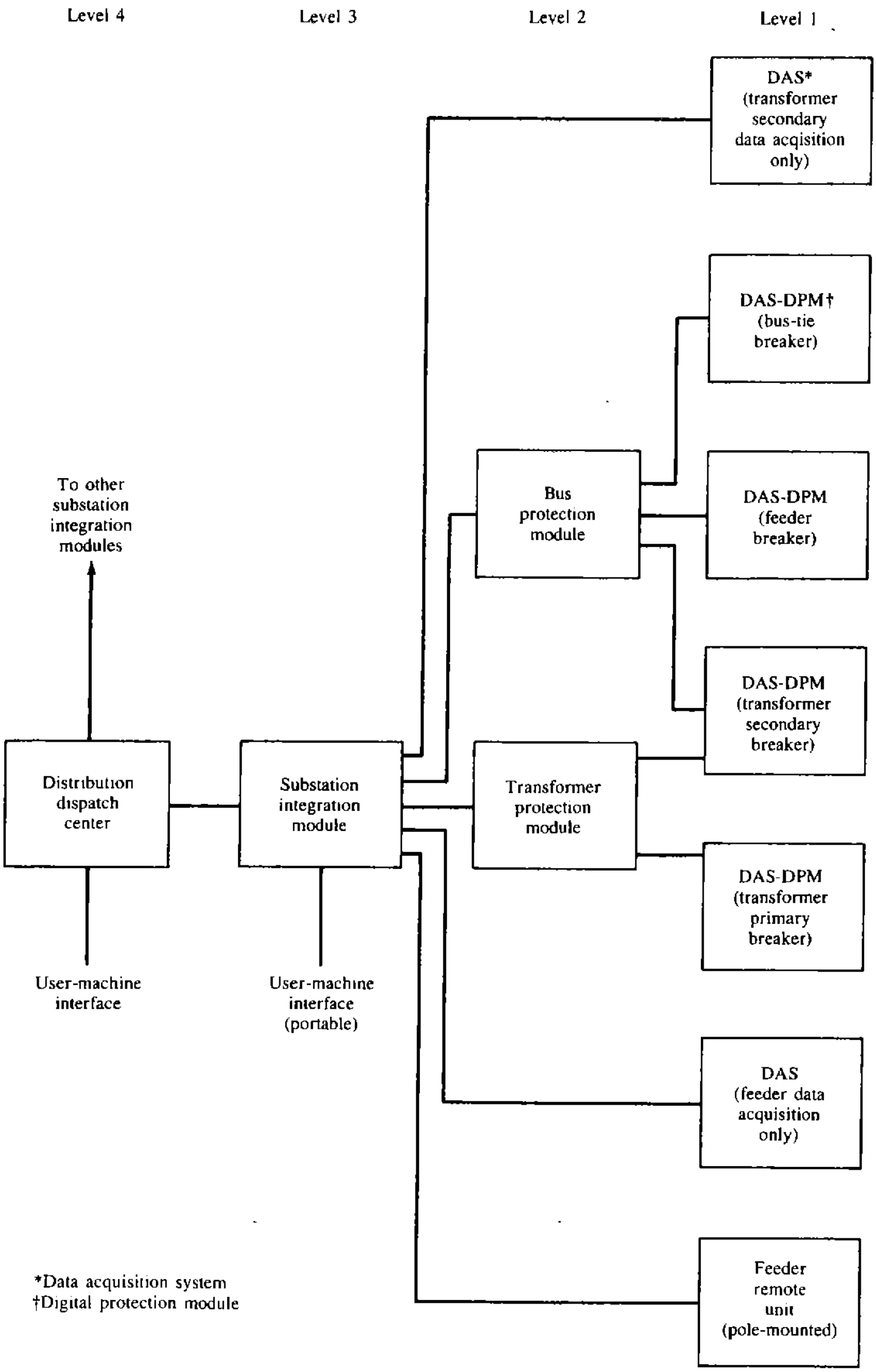


Figure 1-16 The integrated distribution control and protection system of EPRI. (*From* [32].)

Power-line carrier (PLC) systems use electric distribution lines for the transmission of communication signals. The advantages of the PLC system include complete coverage of the entire electric system and complete control by the utility. Its disadvantages include the fact that under mass failure or damage to the distribution system, the communication system could also fail, and that additional equipment must be added to the distribution system.

In radio carrier systems, communication signals are transmitted point to point via radio waves. Such systems would be owned and operated by electric utilities. It is a communication system which is separate and independent of the status of the distribution system. It can also be operated at a very high data rate. However, the basic disadvantage of the radio system is that the signal path can be blocked, either accidentally or intentionally.

Telephone carrier systems use existing telephone lines for signal communication, and therefore they are the least expensive. However, existing telephone tariffs probably make the telephone system one of the more expensive concepts at this time. Other disadvantages include the fact that the utility does not have complete control of the telephone system and that not all meters have telephone service at or near them. Table 1-6 summarizes the advantages and disadvantages of the aforementioned communication systems.

Furthermore, according to Chen [26], utilities would have to change their control hierarchies substantially in the future to accommodate the DSG systems in today's power-distribution systems, as shown in Fig. 1-17.

Table 1-6 Summary of advantages and disadvantages of the power-line, radio, and telephone carriers*

Advantages	Disadvantages
Power-line carrier	
Owned and controlled by utility	Utility system must be conditioned Considerable auxiliary equipment Communication system fails if poles go down
Radio carrier	
Owned and controlled by utility Point-to-point communication Terminal equipment only	Subject to interference by buildings and trees
Telephone carrier	
Terminal equipment only Carrier maintained by phone company	Utility lacks control Ongoing tariff costs New telephone drops must be added Installation requires house wiring Communication system fails if poles go down

* From [25].

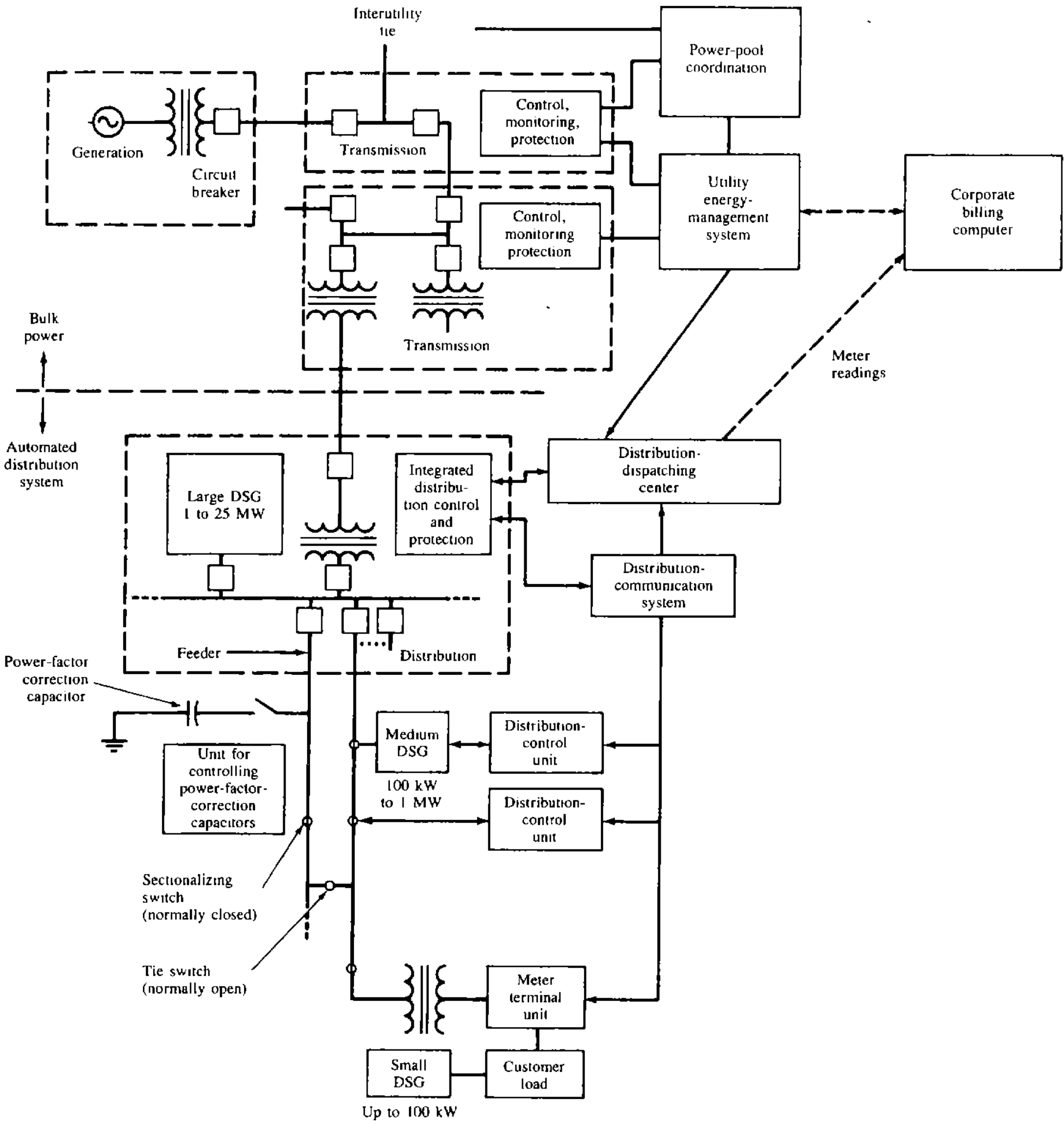


Figure 1-17 A control hierarchy envisaged for future utilities. (*From* [26]. *Used by permission. © 1982 IEEE.*)

1-11 SUMMARY AND CONCLUSIONS

In summary, future distribution systems will be more complex than those of today, which means that the distribution system planner's task will be more complex. If the systems being planned are to be optimal with respect to construction cost, capitalization, performance reliability, and operating efficiency, better planning and operation tools are required. While it is impossible to foresee all the effects that technology will have on the way in which distribution planning and engineering will be done, it is possible to identify the major forces which are beginning to institute a change in the methodology and extrapolate.

REFERENCES

1. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa, 1965.
2. Energy Information Administration: *Energy Data Reports—Statistics of Privately-Owned Electric Utilities in the United States*, U.S. Department of Energy, 1975–1978.
3. *The National Electric Reliability Study: Technical Study Reports*, U.S. Department of Energy, DOE/EP-0005, Office of Emergency Operations, April 1981.
4. *The National Power Grid Study*, vol. 2, U.S. Department of Energy, DOE/ERA-0056-2, Economic Regulatory Administration, Office of Utility Systems, September 1979.
5. Gönen, T., et al.: "Toward Automated Distribution Systems Planning," *Proceedings of the IEEE Control of Power Systems Conference*, Texas A&M University, College Station, Tex., Mar. 19–21, 1979, pp.23–30.
6. Munasinghe, M.: *The Economics of Power System Reliability and Planning*, Johns Hopkins, Baltimore, 1979.
7. Gönen, T., and J. C. Thompson "Distribution System Planning: The State-of-the-Art and the Future Trends," *Proceedings, Southwest Electrical Exposition and the IEEE Conference*, Houston, Tex., Jan. 22–24, 1980, pp. 13–18.
8. Gönen, T., and J. C. Thompson: "An Interactive Distribution System Planning Model," *Proceedings of the 1979 Modeling and Simulation Conference*, University of Pittsburgh, Pa., Apr. 25–27, 1979, vol. 10, pt. 3, pp. 1123–1131.
9. Sullivan, R. L.: *Power System Planning*, McGraw-Hill, New York, 1977.
10. Gönen, T., and B. L. Foote: "Application of Mixed-Integer Programming to Reduce Suboptimization in Distribution Systems Planning," *Proceedings of the 1979 Modeling and Simulation Conference*, University of Pittsburgh, Pa., Apr. 25–27, vol. 10, pt. 3, pp. 1133–1139.
11. Gönen, T., and D. C. Yu: "Bibliography of Distribution System Planning," *Proceedings of the IEEE Control of Power Systems Conference (COPS)*, Oklahoma City, Okla., Mar. 17–18, 1980, pp. 23–34.
12. Gönen, T., and B. L. Foote: "Distribution System Planning Using Mixed-Integer Programming," *IEEE Proc.*, vol. 128, pt. C, no. 2, March 1981, pp. 70–79.
13. Knight, U. G.: *Power Systems Engineering and Mathematics*, Pergamon, Oxford, England, 1972.
14. Gönen, T., B. L. Foote, and J. C. Thompson: *Development of Advanced Methods for Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979.
15. Gönen, T., and D. C. Yu: "A Distribution System Planning Model," *Proceedings of the IEEE Control of Power Systems Conference (COPS)*, Oklahoma City, Okla., Mar. 17–18, 1980, pp. 28–34.
16. Gönen, T., and B. L. Foote: "Mathematical Dynamic Optimization Model for Electrical Distribution System Planning," *Electr. Power Energy Syst.*, vol. 4, no. 2, April 1982, pp. 129–136.
17. Ludot, J. P., and M. C. Rubinstein: "Méthodes pour la Planification à Court Terme des Réseaux de Distribution," in *Proceedings of the Fourth PSCC*, Paper 1.1/12, Grenoble, France, 1972.

18. Launay, M.: "Use of Computer Graphics in Data Management Systems for Distribution Network Planning in Electricite De France (E.D.F.)," *IEEE Trans. Power Appar. Syst.*, vol. PAS-101, no. 2, 1982, pp. 276–283.
19. Delgado, R.: "Load Management—A Planner's View," *IEEE Trans. Power Appar. Syst.*, vol. PAS-102, no. 6, 1983, pp. 1812–13.
20. Public Utility Regulatory Policies Act (PURPA), House of Representatives, Report No. 95-1750, Conference Report, Oct. 10, 1980.
21. Federal Energy Regulatory Commission Regulations under Sections 201 and 210 of PURPA, Sections 292.101, 292.301–292.308, and 292.401–292.403.
22. Kirkham, H., and J. Klein: "Dispersed Storage and Generation Impacts on Energy Management Systems," *IEEE Trans. Power Appar. Syst.*, vol. PAS-102, no. 2, 1983, pp. 339–345.
23. Ma, F., L. Isaksen, and R. Patton: *Impacts of Dispersing Storage and Generation in Electric Distribution Systems*, final report, U.S. Department of Energy, July 1979.
24. Vaisnys, A.: *A Study of a Space Communication System for the Control and Monitoring of the Electric Distribution System*, JPL Publication 80-48, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif., May 1980.
25. *Distribution Automation and Control on the Electric Power System*, Proceedings of the Distribution Automation and Control Working Group, JPL Publication 79-35, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif., March 1979.
26. Chen, A. C. M.: "Automated Power Distribution," *IEEE Spectrum*, April 1982, pp. 55–60.
27. Gaushell, D. J., W. L. Frisbie, and M. H. Kuchefski: "Analysis of Analog Data Dynamics for Supervisory Control and Data Acquisition Systems," *IEEE Trans. Power Appar. Syst.*, vol. PAS-102, no. 2, February 1983, pp. 275–281.
28. Markel, L. C., and P. B. Layfield: "Economic Feasibility of Distribution Automation," *Proceedings of Control of Power Systems Conference*, Texas A&M University, College Station, Tex., Mar. 14–16, 1977, pp. 58–62.
29. "Two-Way Data Communication Between Utility and Customer," *EPRI J.*, May 1980, pp. 17–19.
30. "Distribution, Communication and Load Management," in "R&D Status Report—Electrical Systems Division," *EPRI J.*, September 1982, pp. 46–47.
31. "Control and Protection Systems," in "R&D Status Report—Electrical Systems Division," *EPRI J.*, June 1978, pp. 53–55.
32. "Distribution Automation," in "R&D Status Report—Electrical Systems Division," *EPRI J.*, May 1983, pp. 43–45.
33. Kaplan, G.: "Two-Way Communication for Load Management," *IEEE Spectrum*, August 1977, pp. 47–50.
34. Bunch, J. B., et al.: "Probe and its Implications for Automated Distribution Systems," *Proceedings of the American Power Conference*, Chicago, Ill., vol. 43, April 1981, pp. 683–688.
35. Castro, C. H., J. B. Bunch, and T. M. Topka: "Generalized Algorithms for Distribution Feeder Deployment and Sectionalizing," *IEEE Trans. Power Appar. Syst.*, vol. PAS-99, no. 2, March/April 1980, pp. 549–557.
36. Morgan, M. G., and S. N. Talukdar: "Electric Power Load Management: Some Technical, Economic, Regulatory and Social Issues," *Proc. IEEE*, vol. 67, no. 2, February 1979, pp. 241–313.
37. Bunch, J. B., R. D. Miller, and J. E. Wheeler: "Distribution System Integrated Voltage and Reactive Power Control," *Proceedings of the PICA Conference*, Philadelphia, Pa., May 5–8, 1981, pp. 183–188.
38. Redmon, J. R., and C. H. Gentz: "Effect of Distribution Automation and Control on Future System Configuration," *IEEE Trans. Power Appar. Syst.*, vol. PAS-100, no. 4, April 1981, pp. 1923–1931.
39. Chesnut, H., et al.: "Monitoring and Control Requirements for Dispersed Storage and Generation," *IEEE Trans. Power Appar. System.*, vol. PAS-101, no. 7, July 1982, pp. 2355–2363.
40. Inglis, D. J., D. L. Hawkins, and S. D. Whelan: "Linking Distribution Facilities and Customer Information System Data Bases," *IEEE Trans. Power Appar, Syst.*, vol. PAS-101, no. 2, February 1982, pp. 371–375.
41. Gönen, T., and J. C. Thompson: "Computerized Interactive Model Approach to Electrical Distribution System Planning," *Electr. Power Energy Syst.*, vol. 6, no. 1, January 1984, pp. 55–61.

CHAPTER

TWO

LOAD CHARACTERISTICS

2-1 BASIC DEFINITIONS

Demand "The demand of an installation or system is the load at the receiving terminals averaged over a specified interval of time" [1]. Here, the load may be given in kilowatts, kilovars, kilovoltamperes, kiloamperes, or amperes.

Demand interval It is the period over which the load is averaged. This selected Δt period may be 15 min, 30 min, 1 hr, or even longer. Of course, there may be situations where the 15- and 30-min demands are identical.

The demand statement should express the demand interval Δt used to measure it. Figure 2-1 shows a daily demand variation curve, or load curve, as a function of demand intervals. Note that the selection of both Δt and total time t is arbitrary. The load is expressed in per unit (pu) of peak load of the system. For example, the maximum of 15-min demands is 0.940 pu, and the maximum of 1-h demands is 0.884, whereas the average daily demand of the system is 0.254. The data given by the curve of Fig. 2-1 can also be expressed as shown in Fig. 2-2. Here, the time is given in per unit of the total time. The curve is constructed by selecting the maximum peak points and connecting them by a curve. This curve is called the *load-duration curve*. The load-duration curves can be daily, weekly, monthly, or annual. For example, if the curve is a plot of all the 8760 hourly loads during the year, it is called an *annual load-duration curve*. In that case, the curve shows the individual hourly loads during the year, but not in the order that they occurred, and the number of hours in the year that load exceeded the value shown.

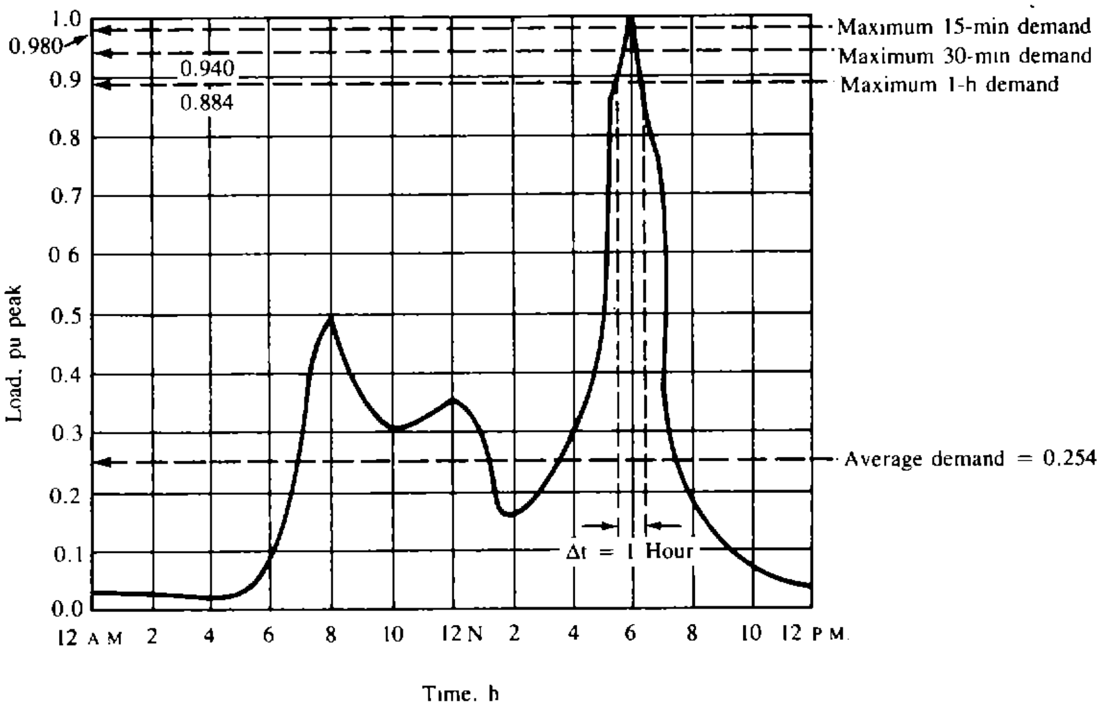


Figure 2-1 A daily demand variation curve.

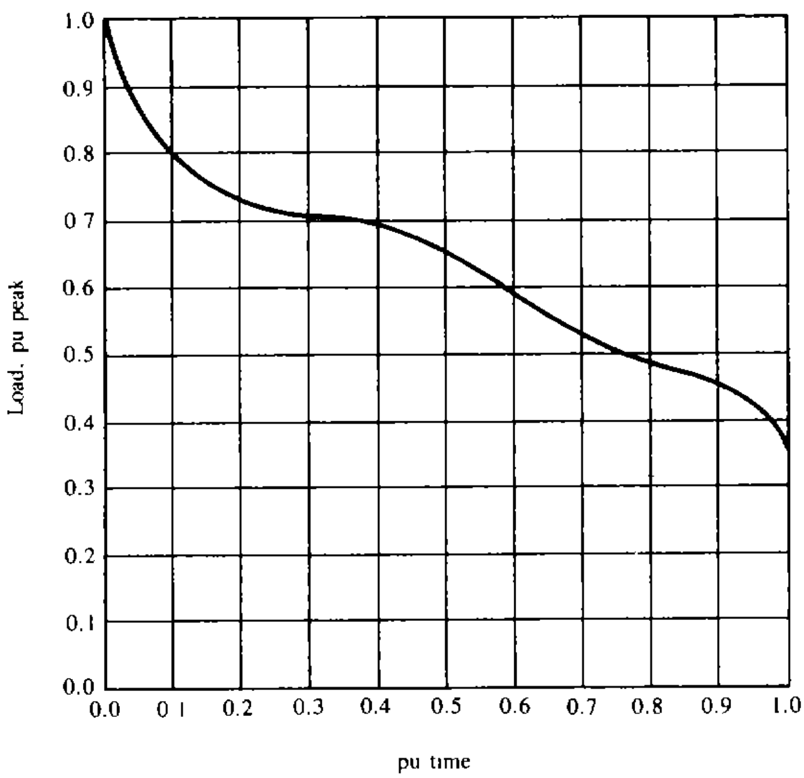


Figure 2-2 A load-duration curve.

The hour-to-hour load on a system changes over a wide range. For example, the daytime peak load is typically double the minimum load during the night. Usually, the annual peak load is, due to seasonal variations, about three times the annual minimum.

To calculate the average demand, the area under the curve has to be determined. This can easily be achieved by a computer program, for example, like the one given in Table 2-1.

Maximum demand "The maximum demand of an installation or system is the greatest of all demands which have occurred during the specified period of time" [1]. The maximum demand statement should also express the demand interval used to measure it. For example, the specific demand might be the maximum of all demands such as daily, weekly, monthly, or annual.

Example 2-1 Assume that the loading data given in Table 2-2 belongs to one of the primary feeders of the No Light & No Power (NL&NP) Company and that they are for a typical winter day. Develop the idealized daily load curve for the

Table 2-1 A computer program to calculate the area under a given consumption curve

```
C*****************************************************************
C                                                                *
C N=THE NUMBER OF GIVEN CONSUMPTION VALUE MINUS ONE.              *
C DELTAX=THE NUMBER OF THE YEARS BETWEEN INTERVALS.               *
C                                                                *
C*****************************************************************
      READ,N
      READ,DELTAX
      NN=N+1
      READ,(Y(I),I=1,NN)
      PRINT,NN,DELTAX,(Y(I),I=1,NN)
      SUM=Y(1)+Y(NN)
      NM=N-2
      DO 1 I=2,NM,2
      SUM=SUM+4.*Y(I)+2.*Y(I+1)
      IF((I+1).EQ.(N-1)) GO TO 20
    1 CONTINUE
   20 SUM=SUM+4.*Y(N)
    2 AREA=DELTAX*SUM/3.
      PRINT,AREA
      STOP
      END
$ENTRY
DATA
      N
      DELTAX.
Y.
/*
```

Table 2-2 Idealized load data for the NL&NP'S primary feeder

	Load, kW		
Time	Street lighting	Residential	Commercial
12 A.M.	100	200	200
1	100	200	200
2	100	200	200
3	100	200	200
4	100	200	200
5	100	200	200
6	100	200	200
7	100	300	200
8	—	400	300
9	—	500	500
10	—	500	1000
11	—	500	1000
12 noon	—	500	1000
1	—	500	1000
2	—	500	1200
3	—	500	1200
4	—	500	1200
5	—	600	1200
6	100	700	800
7	100	800	400
8	100	1000	400
9	100	1000	400
10	100	800	200
11	100	600	200
12 P.M.	100	300	200

given hypothetical primary feeder.

SOLUTION The solution is self-explanatory, as shown in Fig. 2-3.

Diversified demand (or coincident demand) It is the demand of the composite group, as a whole, of somewhat unrelated loads over a specified period of time. Here, the maximum diversified demand has an importance. It is the maximum sum of the contributions of the individual demands to the diversified demand over a specific time interval.

For example, "if the test locations can, in the aggregate, be considered statistically representative of the residential customers as a whole, a load curve for the entire residential class of customers can be prepared. If this same technique is used for other classes of customers, similar load curves can be prepared" [3]. As shown in Fig. 2-4, if these load curves are aggregated, the system load curve can be developed. The interclass coincidence relationships can be observed by comparing the curves.

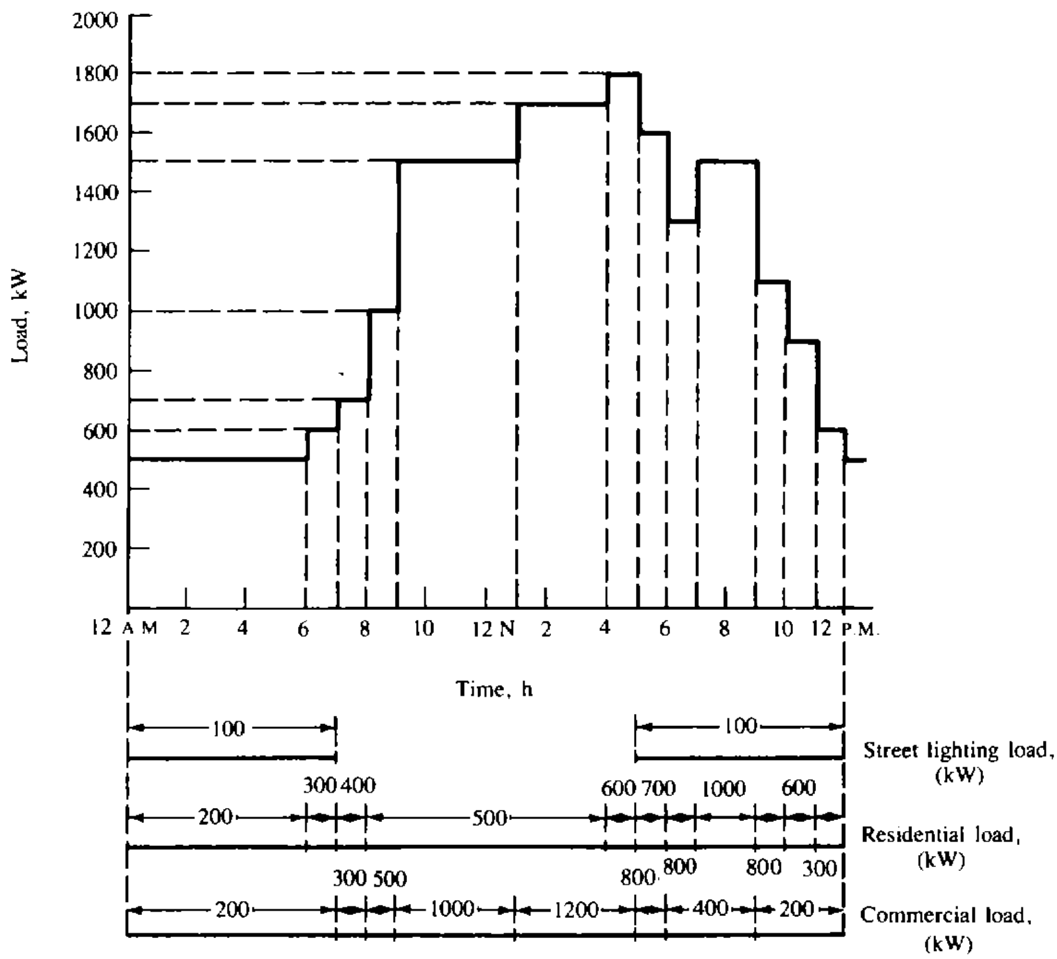


Figure 2-3

Noncoincident demand Manning [2] defines it as "the sum of the demands of a group of loads with no restrictions on the interval to which each demand is applicable." Here, again the maximum of the noncoincident demand is the value of some importance.

Demand factor It is the "ratio of the maximum demand of a system to the total connected load of the system" [1]. Therefore, the demand factor (DF) is

$$\mathrm{DF} \triangleq \frac{\text{maximum demand}}{\text{total connected demand}} \tag{2-1}$$

Of course, the demand factor can also be found for a part of the system, e.g., an industrial or commercial customer, instead of for the whole system. In either case, the demand factor is usually less than 1.0. It is an indicator of the simultaneous operation of the total connected load.

Connected load It is "the sum of the continuous ratings of the load-consuming apparatus connected to the system or any part thereof" [1]. When the maximum demand and total connected demand have the same units, the demand factor is dimensionless.

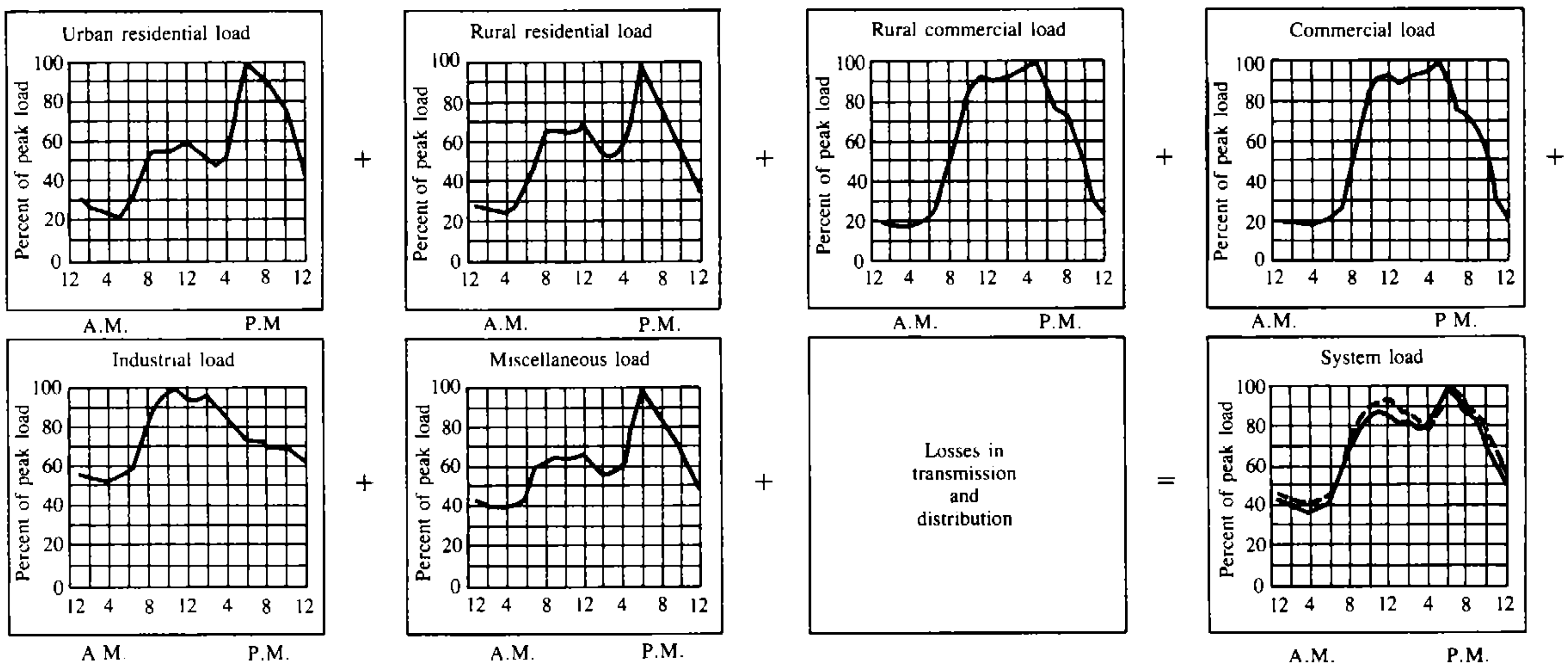


Figure 2-4 Development of aggregate load curves for a winter peak period. Miscellaneous load includes street lighting, sales to other agencies. Dashed curve shown on system load diagram is actual system generation sent out. Solid curve is based on group load study data. (*From* [*3*]. *Used by permission.*) (© *1957 IEEE.*)

Utilization factor It is "the ratio of the maximum demand of a system to the rated capacity of the system" [1]. Therefore, the utilization factor (F_u) is

$$F_u \triangleq \frac{\text{maximum demand}}{\text{rated system capacity}} \tag{2-2}$$

Of course, the utilization factor can also be found for a part of the system. The rated system capacity may be selected to be the smaller of thermal- or voltage-drop capacity [2].

Plant factor It is the ratio of the total actual energy produced or served over a designated period of time to the energy that would have been produced or served if the plant (or unit) had operated continuously at maximum rating. It is also known as the *capacity factor* or the *use factor*. Therefore,

$$\text{Plant factor} = \frac{\text{actual energy produced or served}}{\text{maximum plant rating} \times T} \tag{2-3}$$

It is mostly used in generation studies. For example,

$$\text{Annual plant factor} = \frac{\text{actual annual generation}}{\text{maximum plant rating}} \tag{2-4}$$

or

$$\text{Annual plant factor} = \frac{\text{actual annual energy generation}}{\text{maximum plant rating} \times 8760} \tag{2-5}$$

Load factor It is "the ratio of the average load over a designated period of time to the peak load occurring on that period" [1]. Therefore, the load factor F_{LD} is

$$F_{\text{LD}} \triangleq \frac{\text{average load}}{\text{peak load}} \tag{2-6}$$

or

$$\begin{aligned} F_{\text{LD}} &= \frac{\text{average load} \times T}{\text{peak load} \times T} \\ &= \frac{\text{units served}}{\text{peak load} \times T} \end{aligned} \tag{2-7}$$

where T = time, in days, weeks, months, or years. The longer the period T is the smaller the resultant factor. The reason for this is that for the same maximum demand, the energy consumption covers a larger time period and results in a smaller average load. Here, when time T is selected to be in days, weeks, months, or years, use it in 24, 168, 730, or 8760 h, respectively. It is less than or equal to 1.0.

Therefore, for example, the annual load factor is

$$\text{Annual load factor} = \frac{\text{total annual energy}}{\text{annual peak load} \times 8760} \tag{2-8}$$

Diversity factor It is "the ratio of the sum of the individual maximum demands of the various subdivisions of a system to the maximum demand of the whole system" [1]. Therefore, the diversity factor (F_D) is

$$F_D \triangleq \frac{\text{sum of individual maximum demands}}{\text{coincident maximum demand}} \tag{2-9}$$

or

$$F_D = \frac{D_1 + D_2 + D_3 + \cdots + D_n}{D_g} \tag{2-10}$$

or

$$F_D = \frac{\sum_{i=1}^{n} D_i}{D_g} \tag{2-11}$$

where

D_i = maximum demand of load i, disregarding time of occurrence

$D_g = D_{1+2+3+\cdots+n}$

= coincident maximum demand of group of n loads

The diversity factor can be equal to or greater than 1.0.

From Eq. (2-1), the demand factor is

$$\text{DF} = \frac{\text{maximum demand}}{\text{total connected demand}}$$

or

$$\text{Maximum demand} = \text{total connected demand} \times \text{DF} \tag{2-12}$$

Substituting Eq. (2-12) into Eq. (2-11), the diversity factor can also be given as

$$F_D = \frac{\sum_{i=1}^{n} \text{TCD}_i \times \text{DF}_i}{D_g} \tag{2-13}$$

where TCD_i = total connected demand of group, or class, i load

DF_i = demand factor of group, or class, i load

Coincidence factor It is "the ratio of the maximum coincident total demand of a group of consumers to the sum of the maximum power demands of individual consumers comprising the group both taken at the same point of supply for the same time" [1]. Therefore, the coincidence factor (F_c) is

$$F_c \triangleq \frac{\text{coincident maximum demand}}{\text{sum of individual maximum demands}} \tag{2-14}$$

or

$$F_c = \frac{D_g}{\sum_{i=1}^{n} D_i} \tag{2-15}$$

Thus, the coincidence factor is the reciprocal of diversity factor; that is,

$$F_c = \frac{1}{F_D} \tag{2-16}$$

These ideas on the diversity and coincidence are the basis for the theory and practice of north-to-south and east-to-west interconnections among the power pools in this country. For example, during wintertime, energy comes from south to north, and during summer, just the opposite occurs. Also, east-to-west interconnections help to improve the energy dispatch by means of sunset or sunrise adjustments, i.e., the setting of clocks 1 h late or early.

Load diversity It is "the difference between the sum of the peaks of two or more individual loads and the peak of the combined load" [1]. Therefore, the load diversity (LD) is

$$\mathrm{LD} \triangleq \left(\sum_{i=1}^{n} D_i\right) - D_g \tag{2-17}$$

Contribution factor Manning [2] defines c_i as "the contribution factor of the ith load to the group maximum demand." It is given in per unit of the individual maximum demand of the ith load. Therefore,

$$D_g \triangleq c_1 \times D_1 + c_2 \times D_2 + c_3 \times D_3 + \cdots + c_n \times D_n \tag{2.18}$$

Substituting Eq. (2-18) into Eq. (2-15),

$$F_c = \frac{c_1 \times D_1 + c_2 \times D_2 + c_3 \times D_3 + \cdots + c_n \times D_n}{\sum_{i=1}^{n} D_i} \tag{2-19}$$

or

$$F_c = \frac{\sum_{i=1}^{n} c_i \times D_i}{\sum_{i=1}^{n} D_i} \tag{2-20}$$

Special cases

Case 1: $D_1 = D_2 = D_3 = \cdots = D_n = D$. From Eq. (2-20),

$$F_c = \frac{D \times \sum_{i=1}^{n} c_i}{n \times D} \tag{2-21}$$

or

$$F_c = \frac{\sum_{i=1}^{n} c_i}{n} \tag{2-22}$$

That is, the coincident factor is equal to the average contribution factor.

Case 2: $c_1 = c_2 = c_3 = \cdots = c_n = c$. Hence, from Eq. (2-20),

$$F_c = \frac{c \times \sum_{i=1}^{n} D_i}{\sum_{i=1}^{n} D_i} \tag{2-23}$$

or

$$F_c = c \tag{2-24}$$

That is, the coincident factor is equal to the contribution factor.

Loss factor It is "the ratio of the average power loss to the peak-load power loss during a specified period of time" [1]. Therefore, the loss factor (F_{LS}) is

$$F_{LS} \triangleq \frac{\text{average power loss}}{\text{power loss at peak load}} \tag{2-25}$$

Equation (2-25) is applicable for the copper losses of the system but not for the iron losses.

Example 2-2 Assume that annual peak load of a primary feeder is 2000 kW, at which the power loss, i.e., total copper, or $\sum I^2R$, loss, is 80 kW per three-phase. Assuming an annual loss factor of 0.15, determine:
(*a*) The average annual power loss.
(*b*) The total annual energy loss due to the copper losses of the feeder circuits.

SOLUTION
(*a*) From Eq. (2-25),

$$\begin{aligned}\text{Average power loss} &= \text{power loss at peak load} \times F_{LS} \\ &= 80\text{ kW} \times 0.15 \\ &= 12\text{ kW}\end{aligned}$$

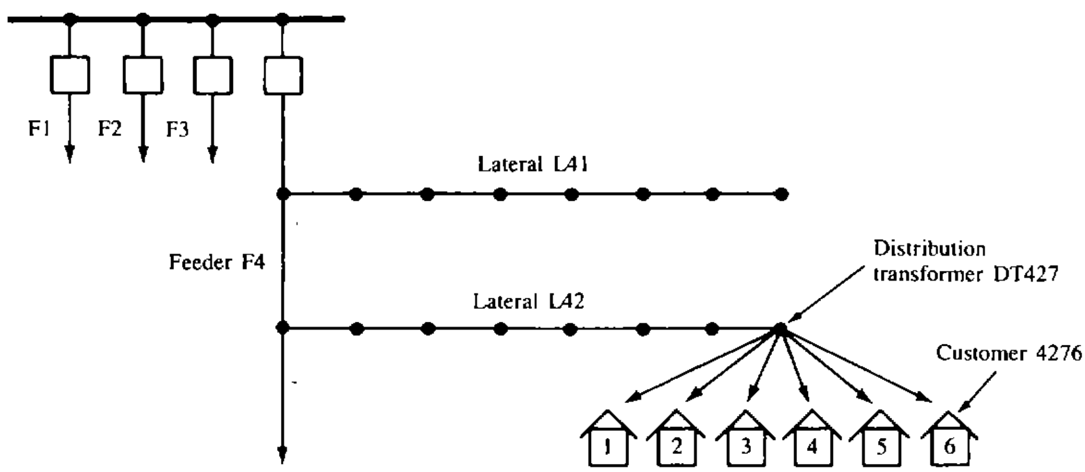


Figure 2-5

(*b*) The total annual energy loss is

$$\begin{aligned} \text{TAEL}_{Cu} &= \text{average power loss} \times 8760 \text{ h/yr} \\ &= 12 \times 8760 \\ &= 105{,}120 \text{ kWh} \end{aligned}$$

Example 2-3 Assume that there are six residential customers connected to a distribution transformer, as shown in Fig. 2-5. Notice the code in the customer account number, e.g., 4276. The first figure, 4, stands for feeder F4; the second figure, 2, indicates the lateral number connected to the F4 feeder; the third figure, 7, is for the distribution transformer on that lateral; and finally the last figure, 6, is for the house number connected to that distribution transformer.

Assume that the connected load is 9 kW per house and that the demand factor and diversity factor for the group of six houses, either from the NL&NP Company's records or from the relevant handbooks, have been decided as 0.65 and 1.10, respectively. Determine the diversified demand of the group of six houses on the distribution transformer DT427.

Solution From Eq. (2-13), the diversified demand of the group on the distribution transformer is

$$\begin{aligned} D_g &= \frac{\left(\sum_{i=1}^{6} \text{TCD}_i\right) \times \text{DF}_i}{F_D} \\ &= \frac{\sum_{i=1}^{6} 9 \text{ kW} \times 0.65}{1.1} \\ &= \frac{6 \times 9 \text{ kW} \times 0.65}{1.1} \\ &= 31.9 \text{ kW} \end{aligned}$$

Example 2-4 Assume that feeder 4 of Example 2-3 has a system peak of 3000 kVA per phase and a copper loss of 0.5 percent at the system peak. Determine the following:

(*a*) The copper loss of the feeder in kilowatts per phase.
(*b*) The total copper losses of the feeder in kilowatts per three-phase.

SOLUTION
(*a*) The copper loss of the feeder in kilowatts per phase is

$$\begin{aligned} I^2R &= 0.5\% \times \text{system peak} \\ &= 0.005 \times 3000 \text{ kVA} \\ &= 15 \text{ kW per phase} \end{aligned}$$

(*b*) The total copper losses of the feeder in kilowatts per three-phase is

$$\begin{aligned} 3I^2R &= 3 \times 15 \\ &= 45 \text{ kW per three-phase} \end{aligned}$$

Example 2-5 Assume that there are two primary feeders supplied by one of the three transformers located at the NL&NP's Riverside distribution substation, as shown in Fig. 2-6. One of the feeders supplies an industrial load which occurs primarily between 8 A.M. and 11 P.M., with a peak of 2000 kW at 5 P.M. The other one feeds residential loads which occur mainly between 6 A.M. and 12 P.M., with a peak of 2000 kW at 9 P.M., as shown in Fig. 2-7. Determine the following:

(*a*) The diversity factor of the load connected to transformer T3.
(*b*) The load diversity of the load connected to transformer T3.
(*c*) The coincidence factor of the load connected to transformer T3

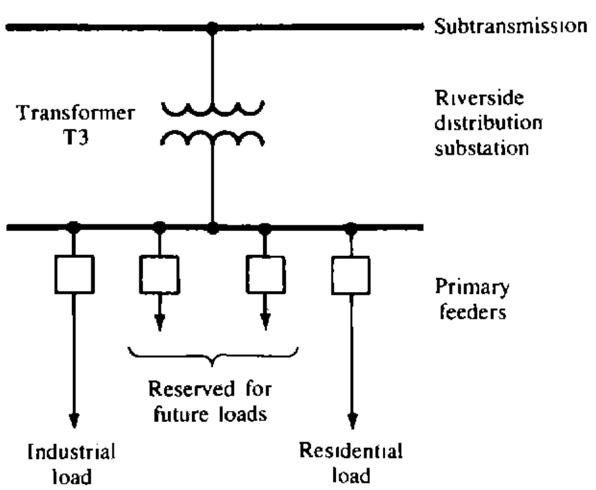


Figure 2-6

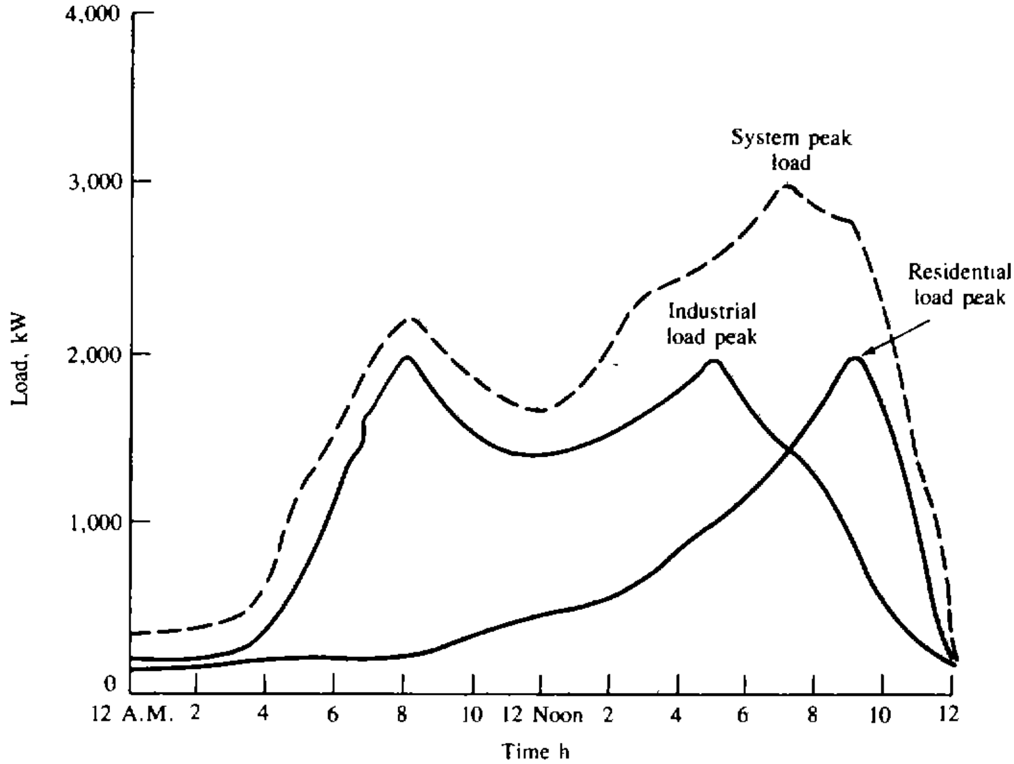


Figure 2-7

SOLUTION

(*a*) From Eq. (2-11) the diversity factor of the load is

$$F_D = \frac{\sum_{i=1}^{2} D_i}{D_g}$$

$$= \frac{2000 + 2000}{3000}$$

$$= 1.33$$

(*b*) From Eq. (2-17), the load diversity of the load is

$$\text{LD} = \sum_{i=1}^{2} D_i - D_g$$

$$= 4000 - 3000$$

$$= 1000 \text{ kW}$$

(*c*) From Eq. (3-16), the coincidence factor of the load is

$$F_c = \frac{1}{F_D}$$

$$= \frac{1}{1.33}$$

$$\cong 0.752$$

Example 2-6 Use the data given in Example 2-1 for the NL&NP's load curve. Note that the peak occurs at 5 P.M. Determine the following:
(a) The class contribution factors for each of the three load classes.
(b) The diversity factor for the primary feeder.
(c) The diversified maximum demand of the load group.
(d) The coincidence factor of the load group.

SOLUTION
(a) The class contribution factor is

$$c_i \triangleq \frac{\text{class demand at time of system (i.e., group) peak}}{\text{class noncoincident maximum demand}}$$

For street lighting, residential, and commercial loads,

$$c_{\text{street}} = \frac{0\text{ kW}}{100\text{ kW}} = 0$$

$$c_{\text{residential}} = \frac{600\text{ kW}}{1000\text{ kW}} = 0.6$$

$$c_{\text{commercial}} = \frac{1200\text{ kW}}{1200\text{ kW}} = 1.0$$

(b) From Eq. (2-11), the diversity factor is

$$F_D \triangleq \frac{\sum_{i=1}^{n} D_i}{D_g} \tag{2-11}$$

and from Eq. (2-18),

$$D_g \triangleq c_1 \times D_1 + c_2 \times D_2 + c_3 \times D_3 + \cdots + c_n \times D_n \tag{2-18}$$

Substituting Eq. (2-18) into Eq. (2-11),

$$F_D = \frac{\sum_{i=1}^{n} D_i}{\sum_{i=1}^{n} c_i \times D_i}$$

Therefore, the diversity factor for the primary feeder is

$$F_D = \frac{\sum_{i=1}^{3} D_i}{\sum_{i=1}^{3} c_i \times D_i}$$

$$= \frac{100 + 1000 + 1200}{0 \times 100 + 0.6 \times 1000 + 1.0 \times 1200}$$

$$= 1.278$$

(*c*) The diversified maximum demand is the coincident maximum demand, that is, D_g. Therefore, from Eq. (2-13), the diversity factor is

$$F_D = \frac{\sum_{i=1}^{n} \mathrm{TCD}_i \times \mathrm{DF}_i}{D_g} \tag{2-13}$$

where the maximum demand, from Eq. (2-12), is

$$\text{Maximum demand} = \text{total connected demand} \times \mathrm{DF} \tag{2-12}$$

Substituting Eq. (2-12) into Eq. (2-13),

$$F_D = \frac{\sum_{i=1}^{n} D_i}{D_g}$$

or

$$D_g = \frac{\sum_{i=1}^{n} D_i}{F_D}$$

Therefore the diversified maximum demand of the load group is

$$D_g = \frac{\sum_{i=1}^{3} D_i}{F_D}$$

$$= \frac{100 + 1000 + 1200}{1.278}$$

$$= 1800 \text{ kW}$$

(*d*) The coincidence factor of the load group, from Eq. (2-15), is

$$F_c = \frac{D_g}{\sum_{i=1}^{3} D_i}$$

or, from Eq. (2-16),

$$F_c = \frac{1}{F_D}$$

$$= \frac{1}{1.278}$$

$$= 0.7825$$

2-2 THE RELATIONSHIP BETWEEN THE LOAD AND LOSS FACTORS

In general, the loss factor cannot be determined from the load factor. However, the limiting values of the relationship can be found [2]. Assume that the primary feeder shown in Fig. 2-8 is connected to a variable load. Figure 2-9 shows an arbitrary and idealized load curve. However, it does not represent a daily load curve. Assume that the off-peak loss is $P_{LS,1}$ at some off-peak load P_1 and that the peak loss is $P_{LS,2}$ at the peak load P_2. The load factor is

$$F_{LD} = \frac{P_{av}}{P_{max}}$$

$$= \frac{P_{av}}{P_2} \tag{2-26}$$

From Fig. 2-9,

$$P_{av} = \frac{P_2 \times t + P_1 \times (T - t)}{T} \tag{2-27}$$

Substituting Eq. (2-27) into Eq. (2-26),

$$F_{LD} = \frac{P_2 \times t + P_1 \times (T - t)}{P_2 \times T}$$

or

$$F_{LD} = = \frac{t}{T} + \frac{P_1}{P_2} \times \frac{T - t}{T} \tag{2-28}$$

The loss factor is

$$F_{LS} = \frac{P_{LS,av}}{P_{LS,max}}$$

$$= \frac{P_{LS,av}}{P_{LS,2}} \tag{2-29}$$

where $P_{LS,av}$ = average power loss

$P_{LS,max}$ = maximum power loss

$P_{LS,2}$ = peak loss at peak load

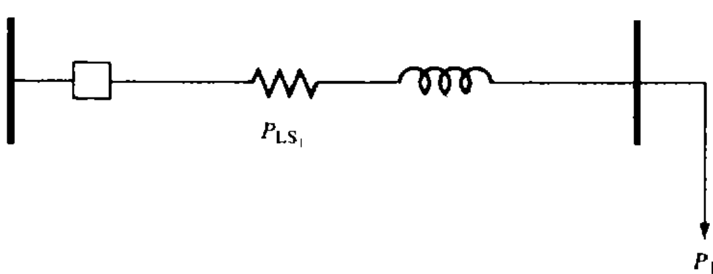


Figure 2-8

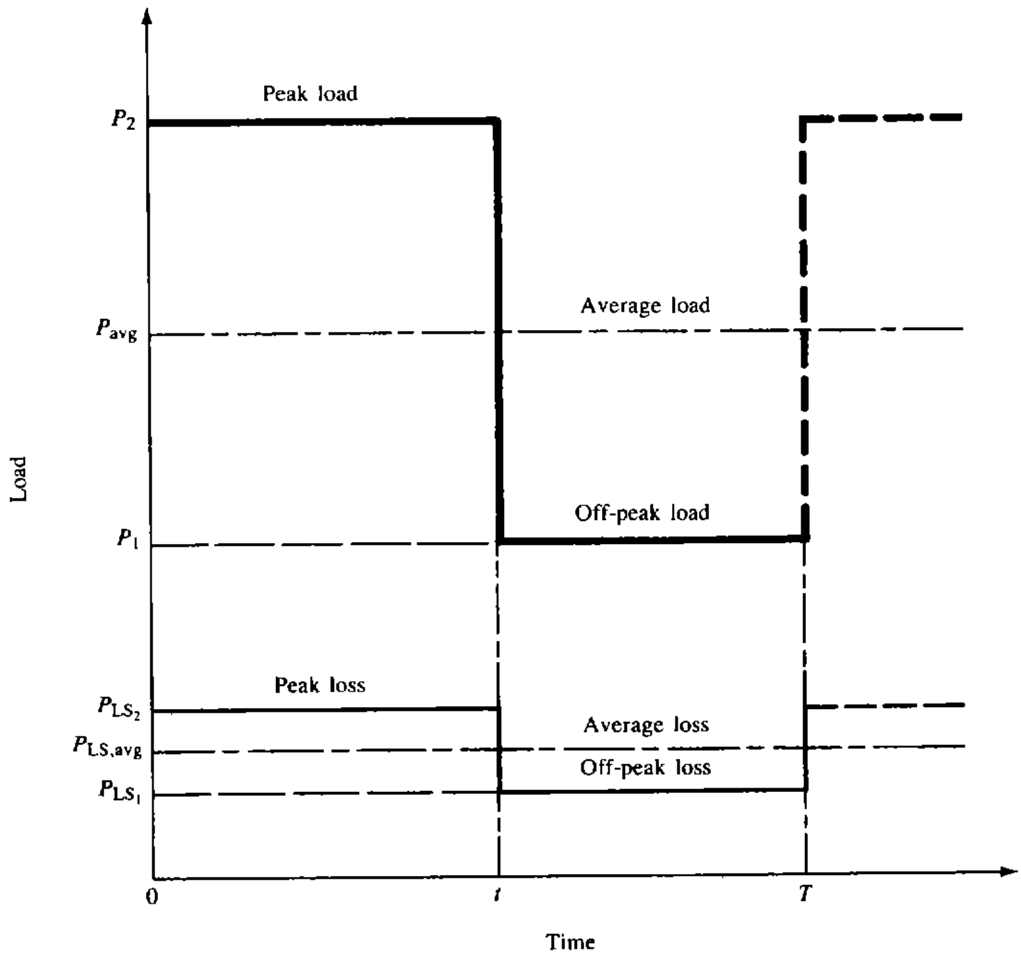


Figure 2-9

From Fig. 2-9,

$$P_{LS,av} = \frac{P_{LS,2} \times t + P_{LS,1} \times (T - t)}{T} \tag{2-30}$$

Substituting Eq. (2-30) into Eq. (2-29),

$$F_{LS} = \frac{P_{LS,2} \times t + P_{LS,1} \times (T - t)}{P_{LS,2} \times T} \tag{2-31}$$

where $P_{LS,1}$ = off-peak loss at off-peak load

t = peak load duration

$T - t$ = off-peak load duration

The copper losses are the function of the associated loads. Therefore, the off-peak and peak loads can be expressed, respectively, as

$$P_{LS,1} = k \times P_1^2 \tag{2-32}$$

and

$$P_{LS,2} = k \times P_2^2 \tag{2-33}$$

where the k is a constant. Thus, substituting Eqs. (2-32) and (2-33) into Eq. (2-31), the loss factor can be expressed as

$$F_{\mathrm{LS}} = \frac{(k \times P_2^2) \times t + (k \times P_1^2) \times (T - t)}{(k \times P_2^2) \times T} \tag{2-34}$$

or

$$F_{\mathrm{LS}} = \frac{t}{T} + \left(\frac{P_1}{P_2}\right)^2 \frac{T - t}{T} \tag{2-35}$$

By using Eqs. (2-28) and (2-35), the load factor can be related to loss factor for three different cases:

Case 1: *Off-peak load is zero.* Here,

$$P_{\mathrm{LS},1} = 0$$

since $P_1 = 0$. Therefore, from Eqs. (2-28) and (2-35),

$$F_{\mathrm{LD}} = F_{\mathrm{LS}} = \frac{t}{T} \tag{2-36}$$

That is, the load factor is equal to the loss factor and they are equal to the t/T constant.

Case 2: *Very short lasting peak.* Here,

$$t \to 0$$

hence, in Eqs. (2-28) and (2-35),

$$\frac{T - t}{T} \to 1.0$$

therefore

$$F_{\mathrm{LS}} \to (F_{\mathrm{LD}})^2 \tag{2-37}$$

That is, the value of the loss factor approaches the value of the load factor squared.

Case 3: *Load is steady.* Here,

$$t \to T$$

That is, the difference between the peak load and the off-peak load is negligible. For example, if the customer's load is a petrochemical plant, this would be the case. Thus, from Eqs. (2-28) and (2-35),

$$F_{\mathrm{LS}} \to F_{\mathrm{LD}} \tag{2-38}$$

That is, the value of the loss factor approaches the value of the load factor.

Therefore, in general, the value of the loss factor is

$$F_{LD}^2 < F_{LS} < F_{LD} \tag{2-39}$$

Therefore the loss factor cannot be determined directly from the load factor. The reason is that the loss factor is determined from losses as a function of time, which, in turn, are proportional to the time function of the square load [2–4].

However, Buller and Woodrow [5] developed an approximate formula to relate the loss factor to the load factor as

$$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2 \tag{2-40}$$

where F_{LS} = loss factor, pu
F_{LD} = load factor, pu

Equation (2-40) gives a reasonably close result. Figure 2-10 gives three different curves of loss factor as a function of load factor.

Example 2-7 Assume that the Riverside distribution substation of the NL&NP Company supplying Ghost Town, which is a small city, experiences an annual peak load of 3500 kW. The total annual energy supplied to the primary feeder circuits is 10,000,000 kWh. The peak demand occurs in July or August and is due to air-conditioning load.

(*a*) Find the annual average power demand.
(*b*) Find the annual load factor.

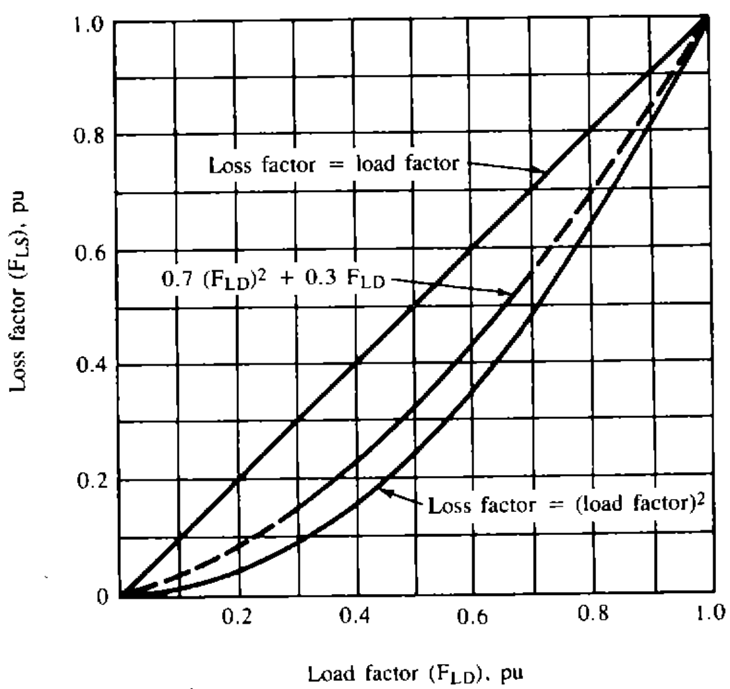


Figure 2-10 Loss factor curves as a function of load factor. (*From* [2].)

SOLUTION Assume a monthly load curve as shown in Fig. 2-11.

(a) The annual average power demand is

$$\text{Annual } P_{av} = \frac{\text{total annual energy}}{\text{year}}$$

$$= \frac{10^7 \text{ kWh/yr}}{8760 \text{ h/yr}}$$

$$= 1141 \text{ kW}$$

(b) From Eq. (2-6), the annual load factor is

$$F_{LD} = \frac{\text{annual average load}}{\text{peak monthly demand}}$$

$$= \frac{1141 \text{ kW}}{3500 \text{ kW}}$$

$$= 0.326$$

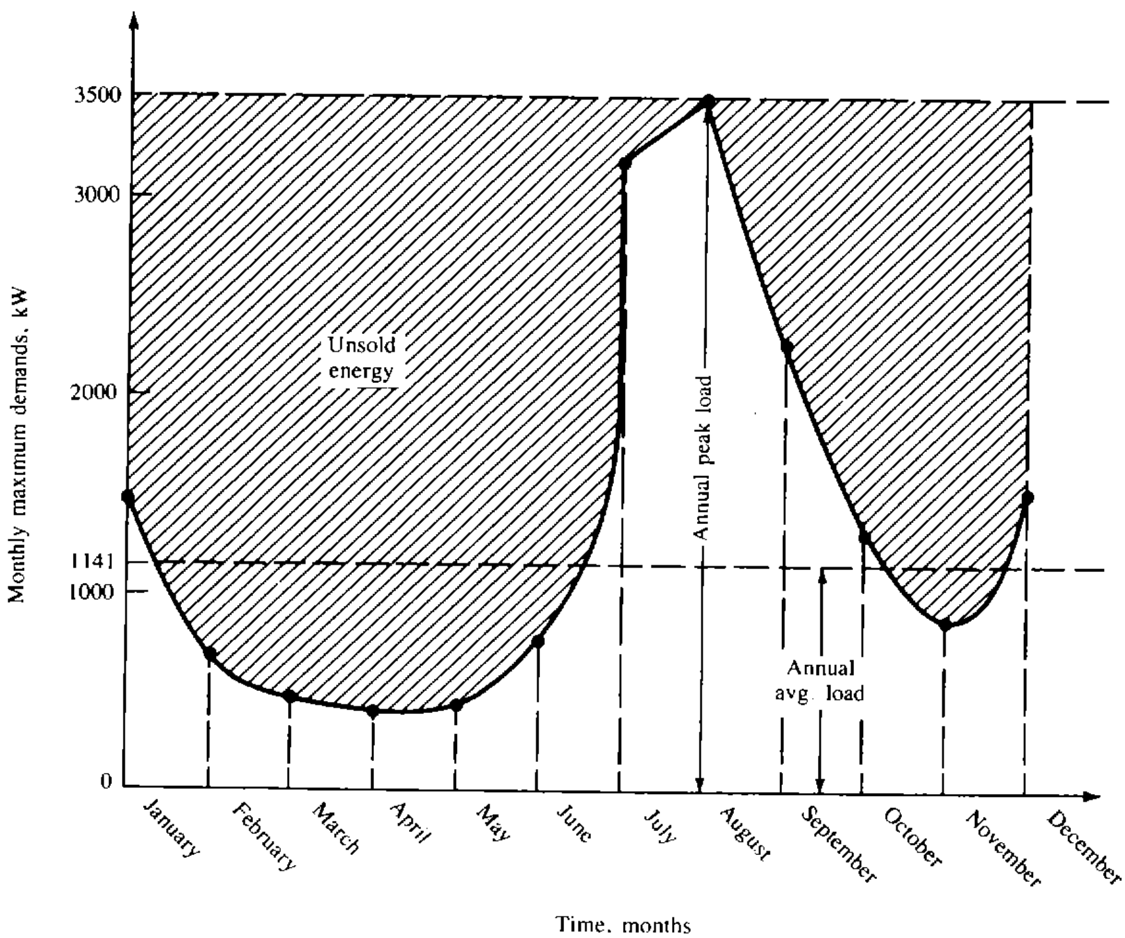


Figure 2-11

or, from Eq. (2-8),

$$\text{Annual load factor} = \frac{\text{total annual energy}}{\text{annual peak load} \times 8760}$$

$$= \frac{10^7 \text{ kWh/yr}}{3500 \text{ kW} \times 8760}$$

$$= 0.326$$

The unsold energy, as shown in Fig. 2-11, is a measure of capacity and investment cost. Ideally, it should be kept at a minimum.

Example 2-8 Use the data given in Example 2-7 and suppose that a new load of 100 kW with 100 percent annual load factor is to be supplied from the Riverside substation. The investment cost, or capacity cost, of the power system upstream, i.e., toward the generator, from this substation is \$3.00/kW per month. Assume that the energy delivered to these primary feeders costs the supplier, that is, NL&NP, \$0.03/kWh.
(*a*) Find the new annual load factor on the substation.
(*b*) Find the total annual cost to NL&NP to serve this load.

SOLUTION Figure 2-12 shows the new load curve after the addition of the new load of 100 kW with 100 percent load.
(*a*) The new annual load factor on the substation is

$$F_{\text{LD}} = \frac{\text{annual average load}}{\text{peak monthly load}}$$

$$= \frac{1141 + 100}{3500 + 100}$$

$$= \frac{1241}{3600}$$

$$= 0.344$$

(*b*) The total annual and additional cost to NL&NP to serve the additional 100-kW load has two cost components, namely, (1) annual capacity cost and (2) annual energy cost. Therefore

$$\text{Annual additional capacity cost} = \$3/\text{kW/mo} \times 12 \text{ mo/yr} \times 100 \text{ kW}$$

$$= \$3600$$

and

$$\text{Annual energy cost} = 100 \text{ kW} \times 8760 \text{ h/yr} \times \$0.03/\text{kWh}$$

$$= \$26280$$

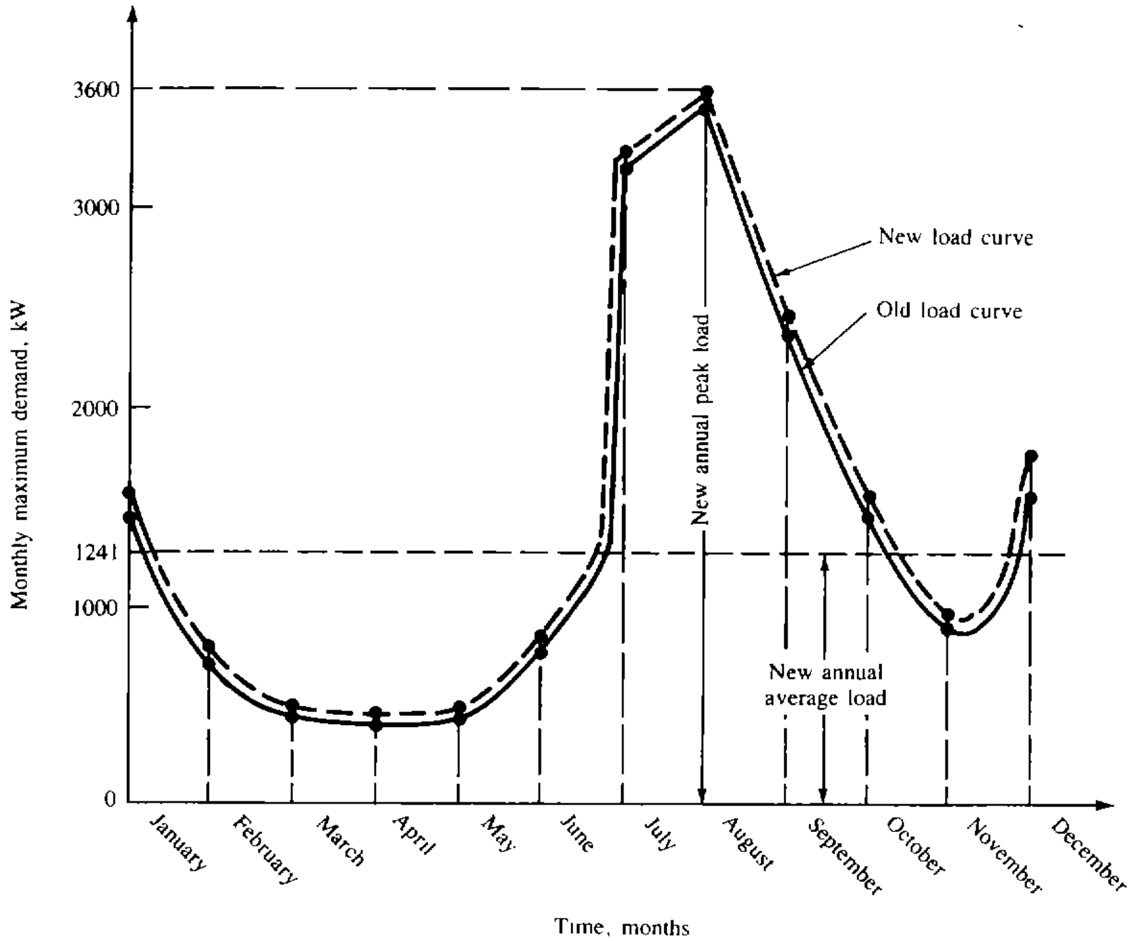


Figure 2-12

Therefore

$$\text{Total ann. additional costs} = \text{ann. capacity cost} + \text{ann. energy cost}$$
$$= \$3{,}600 + \$26{,}280$$
$$= \$29{,}880$$

Example 2-9 Assume that the annual peak-load input to a primary feeder is 2000 kW. A computer program which calculates voltage drops and I^2R losses shows that the total copper loss at the time of peak load is $\sum I^2R = 100$ kW. The total annual energy supplied to the sending end of the feeder is 5.61×10^6 kWh.

(*a*) By using Eq. (2.40), determine the annual loss factor.

(*b*) Calculate the total annual copper loss energy and its value at \$0.03/kWh.

Solution

(*a*) From Eq. (2-40), the annual loss factor is

$$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$$

where

$$F_{LD} = \frac{5.61 \times 10^6 \text{ kWh}}{2000 \text{ kW} \times 8760 \text{ h/yr}}$$
$$= 0.32$$

Therefore

$$F_{LS} = 0.3 \times 0.32 + 0.7 \times 0.32^2$$
$$\cong 0.1681$$

(*b*) From Eq. (2-25),

$$F_{LS} = \frac{\text{average power loss}}{\text{power loss at peak load}}$$

or

$$\text{Average power loss} = 0.1681 \times 100 \text{ kW}$$
$$= 16.81 \text{ kW}$$

Therefore

$$\text{Total annual copper loss} = 16.81 \text{ kW} \times 8760 \text{ h/yr}$$
$$= 147{,}000 \text{ kWh}$$

and

$$\text{Cost of total annual copper loss} = 147{,}000 \text{ kWh} \times \$0.03/\text{kWh}$$
$$= \$4410$$

Example 2-10 Assume that one of the distribution transformers of the Riverside substation supplies three primary feeders. The 30-min annual maximum demands per feeder are listed in the following table, together with the power factor (PF) at the time of annual peak load.

Feeder	Demand, kW	PF
1	1800	0.95
2	2000	0.85
3	2200	0.90

Assume a diversity factor of 1.15 among the three feeders for both real power (P) and reactive power (Q).

(*a*) Calculate the 30-min annual maximum demand on the substation transformer in kilowatts and in kilovoltamperes.

(*b*) Find the load diversity in kilowatts.

(*c*) Select a suitable substation transformer size if zero load growth is expected and if company policy permits as much as 25 percent short-time overloads on the distribution substation transformers. Among the standard three-phase (3ϕ) transformer sizes available are
2500/3125 kVA self-cooled/forced-air-cooled
3750/4687 kVA self-cooled/forced-air-cooled
5000/6250 kVA self-cooled/forced-air-cooled
7500/9375 kVA self-cooled/forced-air-cooled

(*d*) Now assume that the substation load will increase at a constant percentage rate per year and will double in 10 years. If the 7599/9375-kVA-rated transformer is installed, in how many years will it be loaded to its *fans-on* rating?

SOLUTION

(*a*) From Eq. (2-10),

$$F_D = \frac{1800 + 2000 + 2200}{D_g} = 1.15$$

Therefore

$$D_g = \frac{6000}{1.15}$$
$$= 5217 \text{ kW} = P$$

To find power in kilovoltamperes, find the power factor angles. Therefore

$$\text{PF}_1 = \cos\theta_1 = 0.95 \rightarrow \theta_1 = 18.2^\circ$$
$$\text{PF}_2 = \cos\theta_2 = 0.85 \rightarrow \theta_2 = 31.79^\circ$$
$$\text{PF}_3 = \cos\theta_3 = 0.90 \rightarrow \theta_3 = 25.84^\circ$$

Thus the diversified reactive power (Q) is

$$Q = \left(\sum_{i=1}^{3} P_i \times \tan\theta_i\right) / F_D$$
$$= \frac{1800 \times \tan 18.2^\circ + 2000 \times \tan 31.79^\circ + 2200 \times \tan 25.84^\circ}{1.15}$$
$$= 2518.8 \text{ kvar}$$

Therefore

$$D_g = (P^2 + Q^2)^{1/2} = S$$
$$= (5217^2 + 2518.8^2)^{1/2}$$
$$= 5793.60 \text{ kVA}$$

(*b*) From Eq. (2-17), the load diversity is

$$\text{LD} = \sum_{i=1}^{3} D_i - D_g$$

$$= 6000 - 5217$$

$$= 783 \text{ kW}$$

(*c*) From the given transformer list, it is appropriate to choose the transformer with the 3750/4687-kVA rating since with the 25 percent short-time overload it has a capacity of

$$4687 \times 1.25 = 5858.8 \text{ kVA}$$

which is larger than the maximum demand of 5793.60 kVA as found in part *a*.

(*d*) Note that the term *fans-on* rating means the forced-air-cooled rating. To find the increase (g) per year,

$$(1 + g)^{10} = 2$$

hence

$$1 + g = 1.07175$$

or

$$g = 7.175\%/\text{y}$$

Therefore

$$(1.07175)^n \times 5793.60 = 9375 \text{ kVA}$$

or

$$(1.07175)^n = 1.6182$$

Therefore

$$n = \frac{\ln 1.6182}{\ln 1.07175}$$

$$= \frac{0.48130}{0.06929}$$

$$= 6.946, \text{ or } 7 \text{ years}$$

Therefore, if the 7500/9375-kVA-rated transformer is installed, it will be loaded to its *fans-on* rating in about 7 years.

2-3 MAXIMUM DIVERSIFIED DEMAND

Arvidson [7] developed a method of estimating distribution transformer loads in residential areas by the diversified-demand method which takes into account the diversity between similar loads and the noncoincidence of the peaks of different types of loads.

To take into account the noncoincidence of the peaks of different types of loads, Arvidson introduced the *hourly variation factor*. It is "the ratio of the demand of a particular type of load coincident with the group maximum demand to the maximum demand of that particular type of load [2]." Table 2-3 gives the hourly variation curves for various types of household appliances. Figure 2-13 shows a number of curves for various types of household appliances to determine the average maximum diversified demand per customer in kilowatts per load. In Figure 2-13, each curve represents a 100 percent saturation level for a specific demand.

To apply Arvidson's method to determine the maximum diversified demand for a given saturation level and appliance, the following steps are suggested [2]:

1. Determine the total number of appliances by multiplying the total number of customers by the per unit saturation.
2. Read the corresponding diversified demand per customer from the curve, in Fig. 2-13, for the given number of appliances.
3. Determine the maximum demand, multiplying the demand found in step 2 by the total number of appliances.
4. Finally, determine the contribution of that type load to the group maximum demand by multiplying the resultant value from step 3 by the corresponding hourly variation factor found from Table 2-3.

Example 2-11 Assume a typical distribution transformer (DT) that serves six residential loads, i.e., houses, through six service drops (SD) and two spans of secondary line (SL). Suppose that there are a total of 150 distribution transformers and 900 residences supplied by this primary feeder. Use Fig. 2-13 and Table 2-3. For the sake of illustration, assume that a typical residence contains a clothes dryer, a range, a refrigerator, and some lighting and miscellaneous appliances. Determine the following:

(*a*) The 30-min maximum diversified demand on the distribution transformer.
(*b*) The 30-min maximum diversified demand on the entire feeder.
(*c*) Use the typical hourly variation factors given in Table 2-3 and calculate the small portion of the daily demand curve on the distribution transformer, i.e., the total hourly diversified demands at 4, 5, and 6 P.M., on the distribution transformer, in kilowatts.

SOLUTION

(*a*) To determine the 30-min maximum diversified demand on the distribution transformer, the average maximum diversified demand per customer is

Table 2-3 Hourly variation factors

Hour	Lighting and misc.	Refrig- erator	Home freezer	Range	Air- condi- tioning*	Heat pump* Cooling season	Heat pump* Heating season	House* heating	Water heater† OPWH‡ Both elements re- stricted	Water heater† OPWH‡ Only bottom elements re- stricted	Water heater† Uncon- trolled	Clothes§ dryer
12 A.M.	0.32	0.93	0.92	0.02	0.40	0.42	0.34	0.11	0.41	0.61	0.51	0.03
1	0.12	0.89	0.90	0.01	0.39	0.35	0.49	0.07	0.33	0.46	0.37	0.02
2	0.10	0.80	0.87	0.01	0.36	0.35	0.51	0.09	0.25	0.34	0.30	0
3	0.09	0.76	0.85	0.01	0.35	0.28	0.54	0.08	0.17	0.24	0.22	0
4	0.08	0.79	0.82	0.01	0.35	0.28	0.57	0.13	0.13	0.19	0.15	0
5	0.10	0.72	0.84	0.02	0.33	0.26	0.63	0.15	0.13	0.19	0.14	0
6	0.19	0.75	0.85	0.05	0.30	0.26	0.74	0.17	0.17	0.24	0.16	0
7	0.41	0.75	0.85	0.30	0.41	0.35	1.00	0.76	0.27	0.37	0.46	0
8	0.35	0.79	0.86	0.47	0.53	0.49	0.91	1.00	0.47	0.65	0.70	0.08
9	0.31	0.79	0.86	0.28	0.62	0.58	0.83	0.97	0.63	0.87	1.00	0.20
10	0.31	0.79	0.87	0.22	0.72	0.70	0.74	0.68	0.67	0.93	1.00	0.65
11	0.30	0.85	0.90	0.22	0.74	0.73	0.60	0.57	0.67	0.93	0.99	1.00
12 noon	0.28	0.85	0.92	0.33	0.80	0.84	0.57	0.55	0.67	0.93	0.98	0.98
1	0.26	0.87	0.96	0.25	0.86	0.88	0.49	0.51	0.61	0.85	0.86	0.70
2	0.29	0.90	0.98	0.16	0.89	0.95	0.46	0.49	0.55	0.76	0.82	0.65
3	0.30	0.90	0.99	0.17	0.96	1.00	0.40	0.48	0.49	0.68	0.81	0.63
4	0.32	0.90	1.00	0.24	0.97	1.00	0.43	0.44	0.33	0.46	0.79	0.38
5	0.70	0.90	1.00	0.80	0.99	1.00	0.43	0.79	0	0.09	0.75	0.30
6	0.92	0.90	0.99	1.00	1.00	1.00	0.49	0.88	0	0.13	0.75	0.22
7	1.00	0.95	0.98	0.30	0.91	0.88	0.51	0.76	0	0.19	0.80	0.26
8	0.95	1.00	0.98	0.12	0.79	0.73	0.60	0.54	1.00	1.00	0.81	0.20
9	0.85	0.95	0.97	0.09	0.71	0.72	0.54	0.42	0.84	0.98	0.73	0.18
10	0.72	0.88	0.96	0.05	0.64	0.53	0.51	0.27	0.67	0.77	0.67	0.10
11	0.50	0.88	0.95	0.04	0.55	0.49	0.34	0.23	0.54	0.69	0.59	0.04
12 P.M.	0.32	0.93	0.92	0.02	0.40	0.42	0.34	0.11	0.44	0.61	0.51	0.03

* Load cycle and maximum diversified demand are dependent upon outside temperature, dwelling construction and insulation, among other factors.

† Load cycle and maximum diversified demands are dependent upon tank size, and heater element rating; values shown apply to 52-gal tank, 1500- and 1000-W elements.

‡ Load cycle dependent upon schedule of water heater restriction.

§ Hourly variation factor is dependent upon living habits of individuals; in a particular area, values may be different from those shown.

Source. From [2].

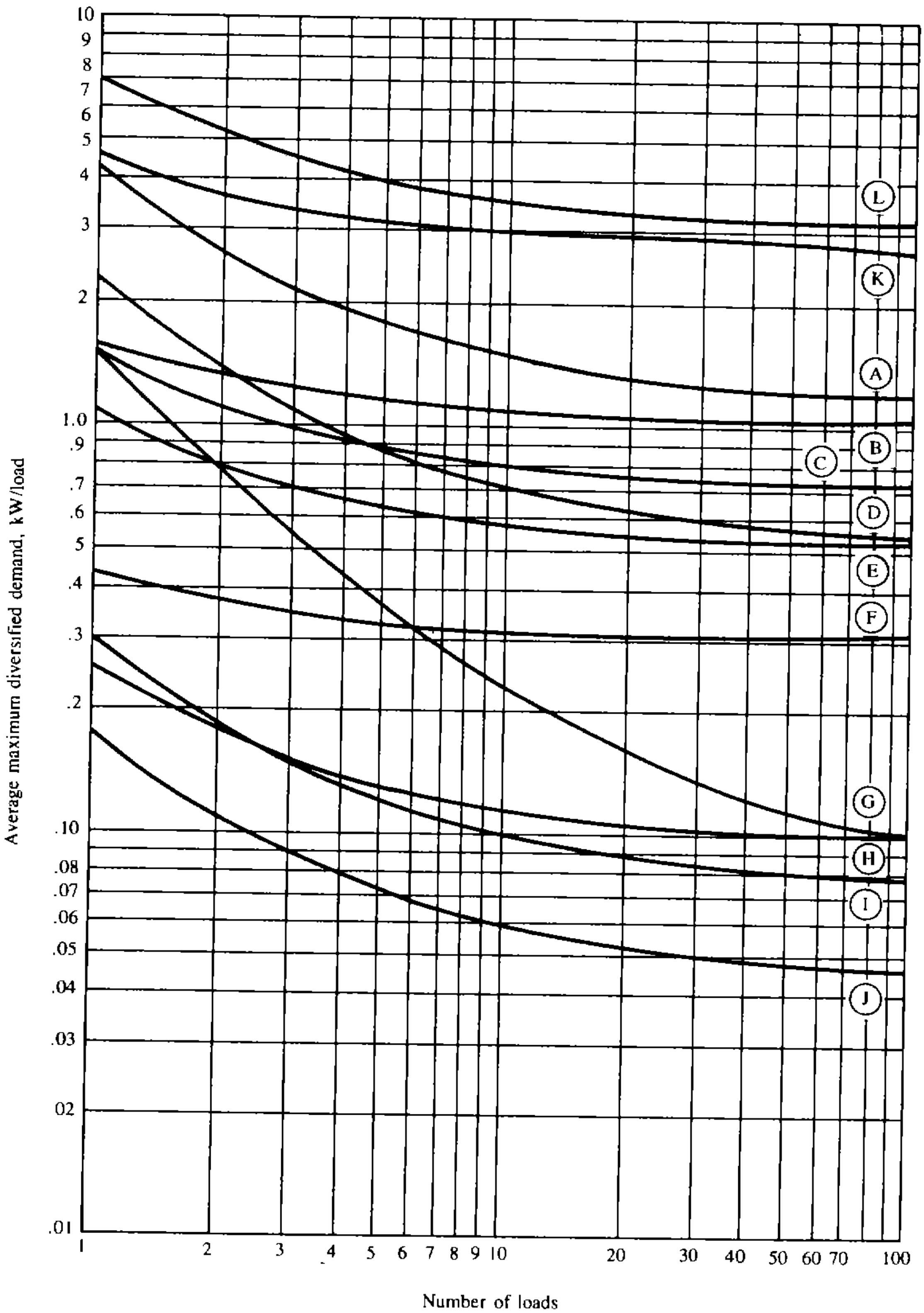


Figure 2-13 Maximum diversified 30-min demand characteristics of various residential loads: A = clothes dryer; B = off-peak water heater, "off-peak" load; C = water heater, uncontrolled, interlocked elements; D = range; E = lighting and miscellaneous appliances; F = 0.5-hp room coolers; G = off-peak water heater, "on-peak" load, upper element uncontrolled; H = oil burner; I = home freezer; J = refrigerator; K = central air-conditioning, including heat-pump cooling, 5-hp heat pump (4-ton air conditioner); L = house heating, including heat-pump-heating–connected load of 15-kW unit-type resistance heating or 5-hp heat pump. (*From* [*2*].)

found from Fig. 2-13. Therefore, when the number of loads is six, the average maximum diversified demands per customer are

$$P_{\text{av, max}} = \begin{cases} 1.6 \text{ kW/house} & \text{for dryer} \\ 0.8 \text{ kW/house} & \text{for range} \\ 0.066 \text{ kW/house} & \text{for refrigerator} \\ 0.61 \text{ kW/house} & \text{for lighting and misc. appliances} \end{cases}$$

Thus, the contributions of the appliances to the 30-min maximum diversified demand on the distribution transformer is approximately 18.5 kW.

(*b*) As in part *a*, the average maximum diversified demand per customer is found from Fig. 2-13. Therefore, when the number of loads is 900 (note that, due to the given curve characteristics, the answers would be the same as the ones for the number of loads of 100), then the average maximum diversified demands per customer are

$$P_{\text{av, max}} = \begin{cases} 1.2 \text{ kW/house} & \text{for dryer} \\ 0.53 \text{ kW/house} & \text{for range} \\ 0.044 \text{ kW/house} & \text{for refrigerator} \\ 0.52 \text{ kW/house} & \text{for lighting and misc. appliance} \end{cases}$$

Hence

$$\sum_{i=1}^{4} (P_{\text{av, max}})_i = 1.2 + 0.53 + 0.52 + 0.044$$

$$= 2.294 \text{ kW/house}$$

Therefore, the 30-min maximum diversified demand on the entire feeder is

$$\sum_{i=1}^{4} (P_{\text{av, max}})_i = 900 \times 2.294$$

$$= 2064.6 \text{ kW}$$

However, if the answer for the 30-min maximum diversified demand on one distribution transformer found in part *a* is multiplied by 150 to determine the 30-min maximum diversified demand on the entire feeder, the answer would be

$$150 \times 18.5 \cong 2775 \text{ kW}$$

which is greater than the 2064.6 kW found previously. This discrepancy is due to the application of the appliance diversities.

(c) From Table 2-3, the hourly variation factors can be found as 0.38, 0.24, 0.90, and 0.32 for dryer, range, refrigerator, and lighting and miscellaneous appliances. Therefore, the total hourly diversified demands on the distribution transformer can be calculated as given in the following table. Note that the results given in col. 6 are the sum of the values given in cols. 2 to 5.

	Contributions to demand by				
Time (1)	Dryers, kW (2)	Ranges, kW (3)	Refrigerators, kW (4)	Lighting & misc. appliances, kW (5)	Total hourly diversified demand, kW (6)
4 P.M.	9.6 × 0.38	4.8 × 0.24	0.4 × 0.90	3.7 × 0.32	6.344
5 P.M.	9.6 × 0.30	4.8 × 0.80	0.4 × 0.90	3.7 × 0.70	9.670
6 P.M.	9.6 × 0.22	4.8 × 1.00	0.4 × 0.90	3.7 × 0.92	10.674

2-4 LOAD GROWTH

The load growth of the geographical area served by a utility company is the most important factor influencing the expansion of the distribution system. Therefore, forecasting of load increases is essential to the planning process.

Fitting trends after transformation of data is a common practice in technical forecasting. An arithmetic straight line that will not fit the original data may fit, for example, the logarithms of the data as typified by the exponential trend

$$y_t = ab^x \tag{2-41}$$

This expression is sometimes called a *growth equation*, since it is often used to explain the phenomenon of growth through time. For example, if the load growth rate is known, the load at the end of the nth year is given by

$$P_n = P_0(1 + g)^n \tag{2-42}$$

where P_n = load at the end of the nth year

P_0 = initial load

g = annual growth rate

n = number of years

Now, if it is set so that $P_n = y_t$, $P_0 = a$, $1 + g = b$, and $n = x$, then Eq. (2-42) is identical to the exponential trend equation (2-41). Table 2-4 gives a computer program to forecast the future demand values if the past demand values are known.

Table 2-4 A demand-forecasting computer program

```
      DIMENSION RLXD(50),RLXC(50),Y(50)
C**************************************************************
C RLXD=READ PAST DEMAND VALUES IN MW.                          *
C RLXC=PREDICTED FUTURE DEMAND VALUES IN MW.                   *
C NP=NUMBER OF YEARS IN THE PAST UP TO THE PRESENT.            *
C NF=NUMBER OF YEARS FROM THE PRESENT TO THE FUTURE THAT       *
C     WILL BE PREDICTED.                                       *
C**************************************************************
      READ,NP,NF
      READ,(RLXD(I),I=1,NP)
      SXIYI=0.
      SXISQ=0.
      SXI=0.
      SYI=0.
      SYISQ=0.
      DO 1 I=1,NP
      XI=I-1
      Y(I)=ALOG(RLXD(I))
      SXIYI=SXIYI+XI*Y(I)
      SXI=SXI+XI
      SYI=SYI+Y(I)
      SXISQ=SXISQ+XI**2
      SYISQ=SYISQ+Y(I)**2
    1 CONTINUE
      A=(SXIYI-(SXI*SYI)/NP)/(SXISQ-(SXI**2)/NP)
      B=SYI/NP-A*SXI/NP
C**************************************************************
C A=ALOG(R);R=1+RATE OF GROWTH                                 *
C**************************************************************
      R=EXP(A)
C**************************************************************
C B=ALOG(RLXC(1))                                              *
C**************************************************************
      RLXC(1)=EXP(B)
C**************************************************************
C RG=RATE OF GROWTH                                            *
C**************************************************************
      RG=R-1
      PRINT,"RATE OF GROWTH=",RG
      NN=NP+NF
      DO 2 I=2,NN
      XI=I-1
      DY=A*XI+B
      RLXC(I)=EXP(DY)
    2 CONTINUE
      PRINT,"RLXD            RLXC"
      DO 4 I=1,NP
      PRINT,RLXD(I),RLXC(I)
    4 CONTINUE
      DO 3 I=1,NF
      IP=I+NP
      PRINT,"                ,RLXC(IP)"
    3 CONTINUE
      STOP
      END
$ENTRY
DATA
      NP
      NF
RLXD.
```

2-5 RATE STRUCTURE

Public utilities are monopolies, i.e., they have the exclusive right to sell their product in a given area. Their rates are subject to government regulation. The total revenue which a utility may be authorized to collect through the sales of its services should be equal to the company's total cost of service. Therefore

$$\begin{matrix}\text{Revenue} \\ \text{requirement}\end{matrix} = \begin{matrix}\text{operating} \\ \text{expenses}\end{matrix} + \begin{matrix}\text{depreciation} \\ \text{expenses}\end{matrix} + \text{taxes} + \begin{matrix}\text{rate base or} \\ \text{net valuation}\end{matrix} \times \begin{matrix}\text{rate of} \\ \text{return}\end{matrix} \tag{2-43}$$

The determination of the revenue requirement is a matter of regulatory commission decision. Therefore, designing schedules of rates which will produce the revenue requirement is a management responsibility subject to commission review. However, a regulatory commission cannot guarantee a specific rate of earnings; it can only declare that a public utility has been given the opportunity to try to earn it.

The rate of return is partly a function of local conditions and should correspond with the return being earned by comparable companies with similar risks. It should be sufficient to permit the utility to maintain its credit and attract the capital required to perform its tasks.

However, the rate schedules, by law, should avoid unjust and unreasonable discrimination, i.e., customers using the utility's service under similar conditions should be billed at similar prices. Of course, it is a matter of necessity to categorize the customers into classes and subclasses, but all customers in a given class should be treated the same. There are several types of rate structures used by the utilities, and some of them are:

1. Flat demand rate structure
2. Straight-line meter rate structure
3. Block meter rate structure
4. Demand rate structure
5. Season rate structure
6. Time-of-day (or peak-load pricing) structure

The flat rate structure provides a constant price per kilowatthour which does not change with the time of use, season, or volume. The rate is negotiated knowing connected load; thus metering is not required. It is sometimes used for parking lot or street lighting service. The straight-line meter rate structure is similar to the flat structure. It provides a single price per kilowatthour without considering customer demand costs.

The block meter rate structure provides lower prices for greater usage, i.e., it gives certain prices per kilowatthour for various kilowatthour blocks where the price per kilowatthour decreases for succeeding blocks. Theoretically, it does not encourage energy conservation and off-peak usage. Therefore it causes a greater than necessary peak and, consequently, excess idle generation capacity during most of the time, resulting in higher rates to compensate the cost of a greater peak-load capacity.

The demand rate structure recognizes load factor and consequently provides separate charges for demand and energy. It gives either a constant price per kilowatthour consumed or a decreasing price per kilowatthour for succeeding blocks of energy used.

The seasonal rate structure specifies higher prices per kilowatthour used during the season of the year in which the system peak occurs (on-peak season) and lower prices during the season of the year in which the usage is the lowest (off-peak season).

The time-of-day rate structure (or peak-load pricing) is similar to the seasonal load rate structure. It specifies higher prices per kilowatthour used during the peak-period of the day and lower prices during the off-peak period of the day.

The seasonal rate structure and the time-of-day rate structure are both designed to reduce the system's peak load and therefore reduce the system's idle stand-by capacity.

2-5-1 Customer Billing

Customer billing is done by taking the difference in readings of the meter at two successive times, usually at an interval of 1 month. The difference in readings indicates the amount of electricity, in kilowatthours, consumed by the customer in that period. This amount is multiplied by the appropriate rate or the series of rates and the adjustment factors, and the bill is sent to the customer.

Figure 2-14 shows a typical monthly bill rendered to a residential customer. The monthly bill includes the following items in the indicated spaces:

1. The customer's account number.
2. A code showing which of the rate schedules was applied to the customer's bill.
3. A code showing whether the customer's bill was estimated or adjusted.
4. Date on which the billing period ended.
5. Number of kilowatthours the customer's meter registered when the bill was tabulated.
6. Itemized list of charges. In this case, the only charge shown in box 6 of Fig. 2-14 is a figure determined by adding the price of the electricity the customer has used to the routine taxes and surcharges. However, had the customer received some special service during this billing period, a service charge would appear in this space as a separate entry.

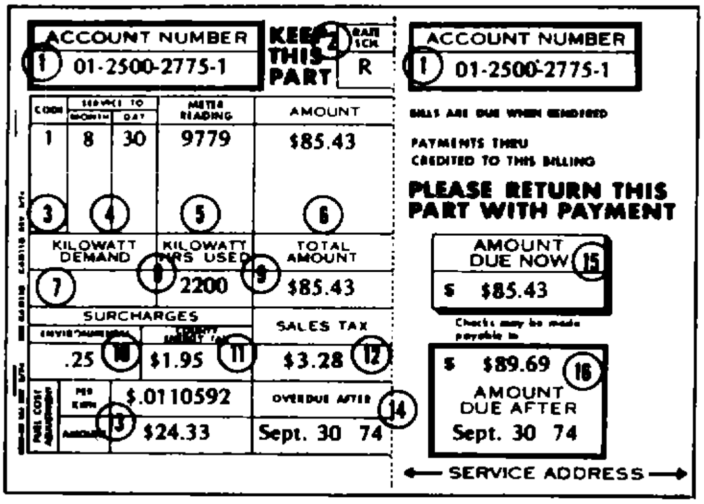


Figure 2-14 A customer's monthly electric bill.

7. Information appears in this box only when the bill is sent to a nonresidential customer using more than 6000 kWh electricity a month.
8. The number of kilowatthours the customer used during the billing period.
9. Total amount that the customer owes.
10. Environmental surcharge.
11. County energy tax.
12. State sales tax.
13. Fuel cost adjustment. Both the total adjustment and the adjustment per kilowatthour are shown.
14. Date on which bill, if unpaid, becomes overdue.
15. Amount due now.
16. Amount that the customer must pay if the bill becomes overdue.

The sample electrical bill, shown in Fig. 2-14 is based on the following rate schedule. Note that there is a minimum charge regardless of how little electricity the customer uses, and that the first 20 kWh that the customer uses is covered by this flat rate. Included in the minimum, or service, charge is the cost of providing service to the customer, including metering, meter reading, billing, and various overhead expenses.

Rate schedule

Minimum charge (including first 20 kWh or fraction thereof)	$2.25/month
Next 80 kWh	$0.0355/kWh
Next 100 kWh	$0.0321/kWh
Next 200 kWh	$0.0296/kWh
Next 400 kWh	$0.0265/kWh
Consumption in excess of 800 kwh	$0.0220/kWh

The sample bill shows a consumption of 2200 kWh which has been billed according to the following schedule:

First 20 kWh @ $2.25 (flat rate)	= $ 2.25
Next 80 kWh × 0.0355	= $ 2.84
Next 100 kWh × 0.0321	= $ 3.21
Next 200 kWh × 0.096	= $ 5.92
Next 400 kWh × 0.0265	= $10.60
Additional 1400 kWh × 0.0220	= $30.80
2200 kWh	= $55.62
Environmental surcharge	= $ 0.25
County energy tax	= $ 1.95
Fuel cost adjustment	= $24.33
State sales tax	= $ 3.28
Total amount	= $85.43

The customer is billed according to the utility company's rate schedule. In general, the rates vary according to the season. In most areas the demand for electricity increases in the warm months. Therefore, to meet the added burden, electric utilities are forced to use spare generators that are often less efficient and consequently more expensive to run. As an example, Table 2-5 gives a typical energy rate schedule for the on-peak and off-peak seasons for commercial users.

Table 2-5 A typical energy rate schedule for commercial users

On-peak season (June 1–October 31)	
First 50 kWh or less/month for $4.09	
Next 50 kWh/month	@ 5.509¢/kWh
Next 500 kWh/month	@ 4.843¢/kWh
Next 1400 kWh/month	@ 4.049¢/kWh
Next 3000 kWh/month	@ 3.878¢/kWh
All additional kWh/month	@ 3.339¢/kWh
Off-peak season (November 1–May 31)	
First 50 kWh or less/month for $4.09	
Next 50 kWh/month	@ 5.509¢/kWh
Next 500 kWh/month	@ 4.244¢/kWh
Next 1400 kWh/month	@ 3.122¢/kWh
Next 3000 kWh/month	@ 2.783¢/kWh
All additional kWh/month	@ 2.649¢/kWh

2-5-2 Fuel Cost Adjustment

The rates stated previously are based upon an average cost, in dollars per million Btu, for the cost of fuel burned at the NL&NP's thermal generating plants. The monthly bill as calculated under the previously stated rate is increased or decreased for each kilowatthour consumed by an amount calculated according to the following formula:

$$\text{FCAF} = A \times \frac{B}{10^6} \times C \times \frac{1}{1 - D} \tag{2-44}$$

where FCAF = fuel cost adjustment factor, \$/kWh, to be applied per kilowatthour consumed

A = weighted average Btu per kilowatthour for net generation from the NL&NP's thermal plants during the second calendar month preceding the end of the billing period for which the kilowatthour usage is billed

B = amount by which average cost of fuel per million Btu during the second calendar month preceding the end of the billing period for which the kilowatthour usage is billed exceeds or is less than \$1/million Btu

C = ratio, given in decimal, of the total net generation from all the NL&NP's thermal plants during the second calendar month preceding the end of the billing period for which the kilowatthour usage is billed to the total net generation from all the NL&NP's plants including hydro generation owned by the NL&NP, or kilowatthours produced by hydro generation and purchased by the NL&NP, during the same period

D = loss factor, which is the ratio, given in decimal, of kilowatthour losses (total kilowatthour losses less losses of 2.5 percent associated with off-system sales) to net system input, i.e., total system input less total kilowatthours in off-system sales, for the year ending December 31 preceding. This ratio is updated every year and applied for 12 months.

Example 2-12 Assume that the NL&NP Utility Company has the following, and typical, commercial rate schedule.

1. Monthly billing demand = 30-min monthly maximum kilowatt demand multiplied by the ratio of (0.85 ÷ monthly average PF). The PF penalty shall not be applied when the consumer's monthly average PF exceeds 0.85.
2. Monthy demand charge = \$2.00/kW of monthly billing demand.
3. Monthly energy charges shall be:
   2.50 cents/kWh for first 1000 kWh
   2.00 cents/kWh for next 3000 kWh
   1.50 cents/kWh for all kWh in excess of 4000

4. The total monthly charge shall be the sum of the monthly demand charge and the monthly energy charge.

Assume that two consumers, as shown in Fig. 2-15, each requiring a distribution transformer, are supplied from a primary line of the NL&NP.

(*a*) Assume that an average month is 730 h and find the monthly load factor of each consumer.

(*b*) Find a reasonable size, i.e., continuous kilovoltampere rating, for each distribution transformer.

(*c*) Calculate the monthly bill for each consumer.

(*d*) It is not uncommon to measure the average monthly PF on a monthly energy basis, where both kilowatthours and kilovarhours are measured. On this basis, what size capacitor, in kilovars, would raise the PF of customer B to 0.85?

(*e*) Secondary-voltage shunt capacitors, in small sizes, *may* cost about \$30/kvar installed with disconnects and short-circuit protection. Consumers sometimes install secondary capacitors to reduce their billings for utility service. Using the 30/kvar figure, find the number of months required for the PF correction capacitors found in part *d* to pay back for themselves with savings in demand charges.

SOLUTION

(*a*) From Eq. (2-7), the monthly load factors for each consumer are the following. For customer A,

$$F_{\text{LD}} = \frac{\text{units served}}{\text{peak load} \times T}$$

$$= \frac{7000\ \text{kWh}}{22\ \text{kW} \times 730\text{h}}$$

$$= 0.435$$

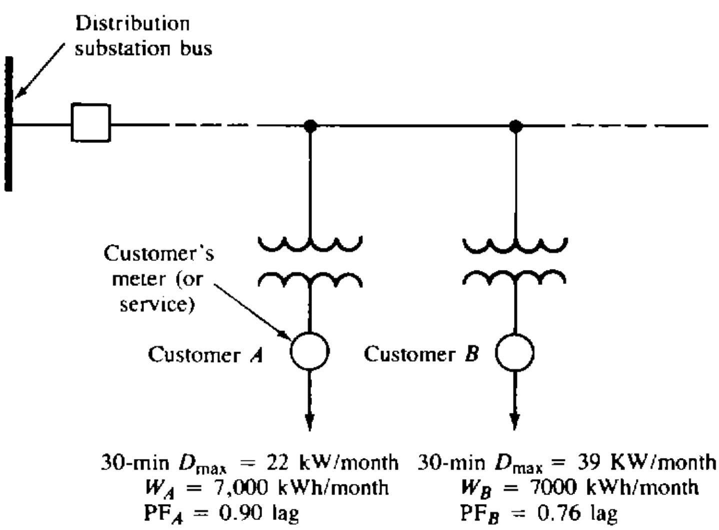


Figure 2-15

and for customer B,

$$F_{\text{LD}} = \frac{7000\text{ kWh}}{39\text{ kW} \times 730\text{ h}}$$

$$= 0.246$$

(*b*) The continuous kilovoltamperes for each distribution transformer are the following:

$$S_A = \frac{P_A}{\cos\theta}$$

$$= \frac{22\text{ kW}}{0.90}$$

$$= 24.4\text{ kVA}$$

and

$$S_B = \frac{P_B}{\cos\theta_B}$$

$$= \frac{39\text{ kW}}{0.76}$$

$$= 51.2\text{ kVA}$$

Therefore, the continuous sizes suitable for the distribution transformers A and B are 25 and 50 kVA ratings, respectively.

(*c*) The monthly bills for each customer are the following. For customer A:

$$\text{Monthly billing demand*} = 22\text{ kW} \times \frac{0.85}{0.90} \cong 22\text{ kW}$$

$$\text{Monthly demand charge} = 22\text{ kW} \times \$2.00/\text{kW} = \$44$$

Monthly energy charge:

$$\begin{aligned}
\text{First 1000 kWh} &= \$0.025/\text{kWh} \times 1000\text{ kWh} = \$25 \\
\text{Next 3000 kWh} &= \$0.02/\text{kWh} \times 3000\text{ kWh} = \$60 \\
\text{Excess kWh} &= \$0.015/\text{kWh} \times 3000\text{ kWh} = \$45
\end{aligned}$$

$$\text{Monthly energy charge} = \$130$$

Therefore

$$\begin{aligned}
\text{Total monthly bill} &= \text{monthly demand charge} + \text{monthly energy charge} \\
&= \$44 + \$130 \\
&= \$174
\end{aligned}$$

* It is calculated from $P\left(\frac{0.85}{\text{PF}}\right)$. However, if the PF is greater than 0.85, then still the actual amount of P is used, rather than, the resultant kW.

For customer B:

$$\text{Monthly billing demand} = 39 \text{ kW} \times \frac{0.85}{0.76} = 43.6 \text{ kW}$$

$$\text{Monthly demand charge} = 43.6 \text{ kW} \times \$2.00/\text{kW} = \$87.20$$

Monthly energy charge:

$$\begin{aligned} \text{First } 1000 \text{ kWh} &= \$0.025/\text{kWh} \times 1000 \text{ kWh} = \$25 \\ \text{Next } 3000 \text{ kWh} &= \$0.02/\text{kWh} \times 3000 \text{ kWh} = \$60 \\ \text{Excess kWh} &= \$0.015/\text{kWh} \times 3000 \text{ kWh} = \$45 \end{aligned}$$

$$\text{Monthly energy charge} = \$130$$

Therefore

$$\begin{aligned} \text{Total monthly bill} &= \$87.20 + \$130 \\ &= \$217.20 \end{aligned}$$

(*d*) Currently, customer B at the lagging power factor of 0.76 has

$$\frac{7000 \text{ kWh}}{0.76} \times \sin(\cos^{-1} 0.76) = 5986.13 \text{ kvarh}$$

If its power factor is raised to 0.85, customer B would have

$$\frac{7000 \text{ kWh}}{0.85} \times \sin(\cos^{-1} 0.85) = 4338 \text{ kvarh}$$

Therefore the capacitor size required is

$$\begin{aligned} \frac{5986.13 \text{ kvarh} - 4338 \text{ kvarh}}{730 \text{ h}} &= 2.258 \text{ kvar} \\ &\cong 2.3 \text{ kvar} \end{aligned}$$

(*e*) The new monthly bill for customer B would be

$$\text{Monthly billing demand} = 39 \text{ kW}$$

$$\text{Monthly demand charge} = 39 \text{ kW} \times \$2.00 = \$78$$

$$\text{Monthly energy charge} = \$130 \qquad \text{as before}$$

Therefore

$$\begin{aligned} \text{Total monthly bill} &= \$78 + \$130 \\ &= \$208 \end{aligned}$$

Hence the resultant savings due to the capacitor installation is the difference between the before-and-after total monthly bills. Thus

$$\text{Savings} = \$217.20 - 208$$
$$= \$9.20/\text{month}$$
$$\text{Savings} = \$87.20 - \$78$$
$$= \$9.20/\text{month}$$

The cost of the installed capacitor is

$$\$30/\text{kvar} \times 2.3\ \text{kvar} = \$69$$

Therefore the number of months required for the capacitors to "payback" for themselves with savings in demand charges can be calculated as

$$\text{Payback period} = \frac{\text{capacitor cost}}{\text{savings}}$$
$$= \frac{\$69}{\$9.20/\text{mo}}$$
$$= 7.5$$
$$\cong 8\ \text{months}$$

However, in practice, the available capacitor size is 3 kvar instead of 2.3 kvar. Therefore the realistic cost of the installed capacitor is

$$\$30/\text{kvar} \times 3\ \text{kvar} = \$90$$

Therefore

$$\text{Payback period} = \frac{\$90}{\$9.20/\text{mo}}$$
$$= 9.78$$
$$= \cong 10\ \text{months}$$

2-6 ELECTRIC METER TYPES

An electric meter is the device used to measure the electricity sold by the electric utility company. It is not only used to measure the electric energy delivered to residential, commercial, and industrial customers but also used to measure the electric energy passing through various parts of the generation, transmission, and distribution systems.

Figure 2-16 shows a single-phase watthour meter; Fig. 2-17 shows its basic parts; Fig. 2-18 gives a diagram of a typical motor and magnetic retarding system

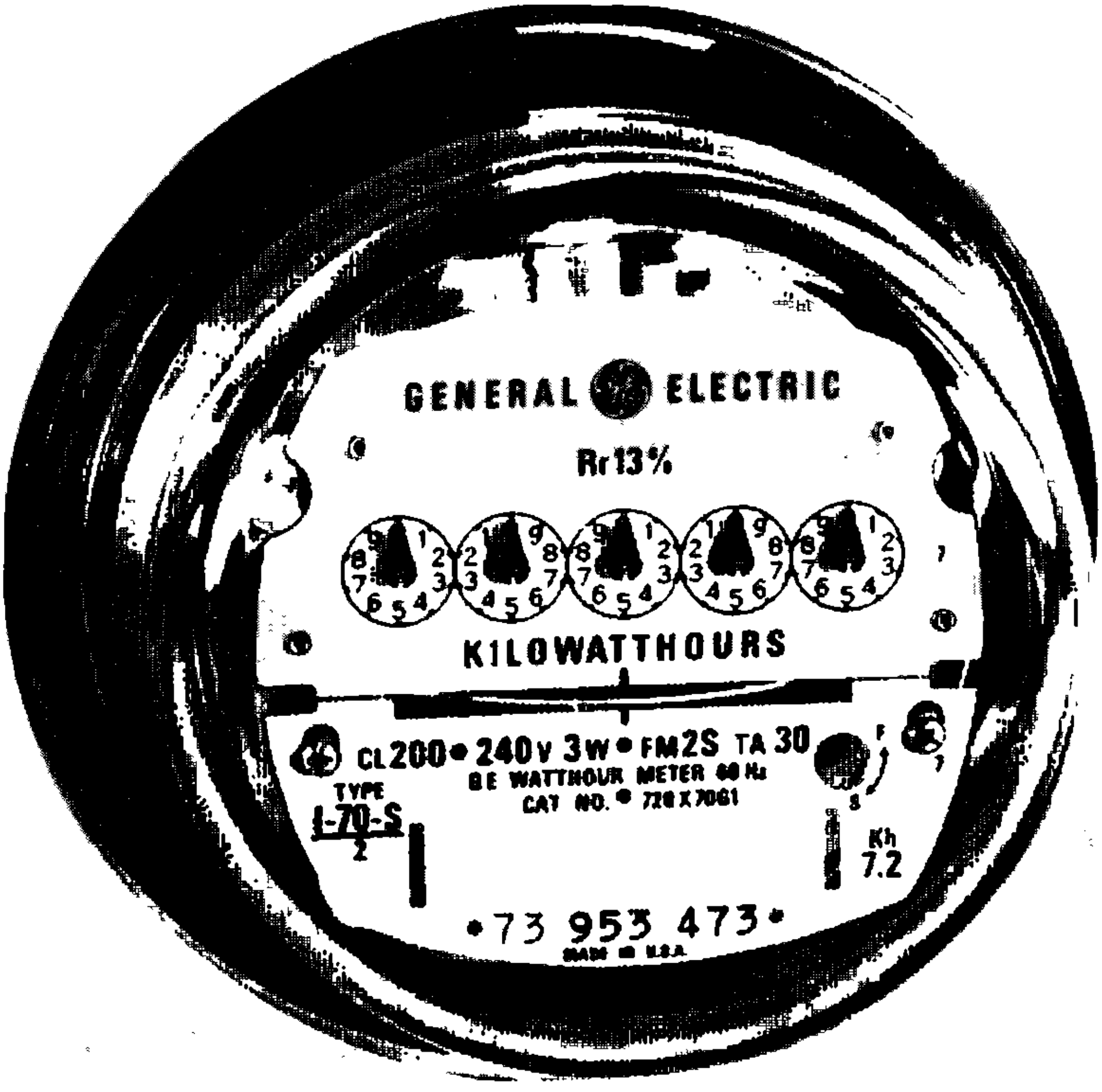


Figure 2-16 Single-phase watthour meter. (*General Electric Company*.)

for a single-phase watthour meter. The magnetic retarding system causes the rotor disk to establish, in combination with the stator, the speed at which the shaft will turn for a given load condition to determine the watthour constant. Figure 2-19*a* shows a typical socket-mounted two-stator polyphase watthour meter. It is a combination of single-phase watthour meter stators that drive a rotor at a speed proportional to the total power in the circuit.

The watthour meters employed to measure the electric energy passing through various parts of the generation, transmission, and distribution systems are required to measure large quantities of electric energy at relatively high voltages. For those applications, *transformer-rated* meters are developed. They are used in conjunction with standard instrument transformers, i.e., current transformers (CT) and potential transformers (PT). These transformers reduce the voltage and the current to values that are suitable for the low-voltage and low-current meters. Figure 2-19*b* shows a typical transformer-rated meter. Figure 2-20 shows a single-phase, two-wire watthour meter connected to a high-voltage circuit through current and potential transformers.

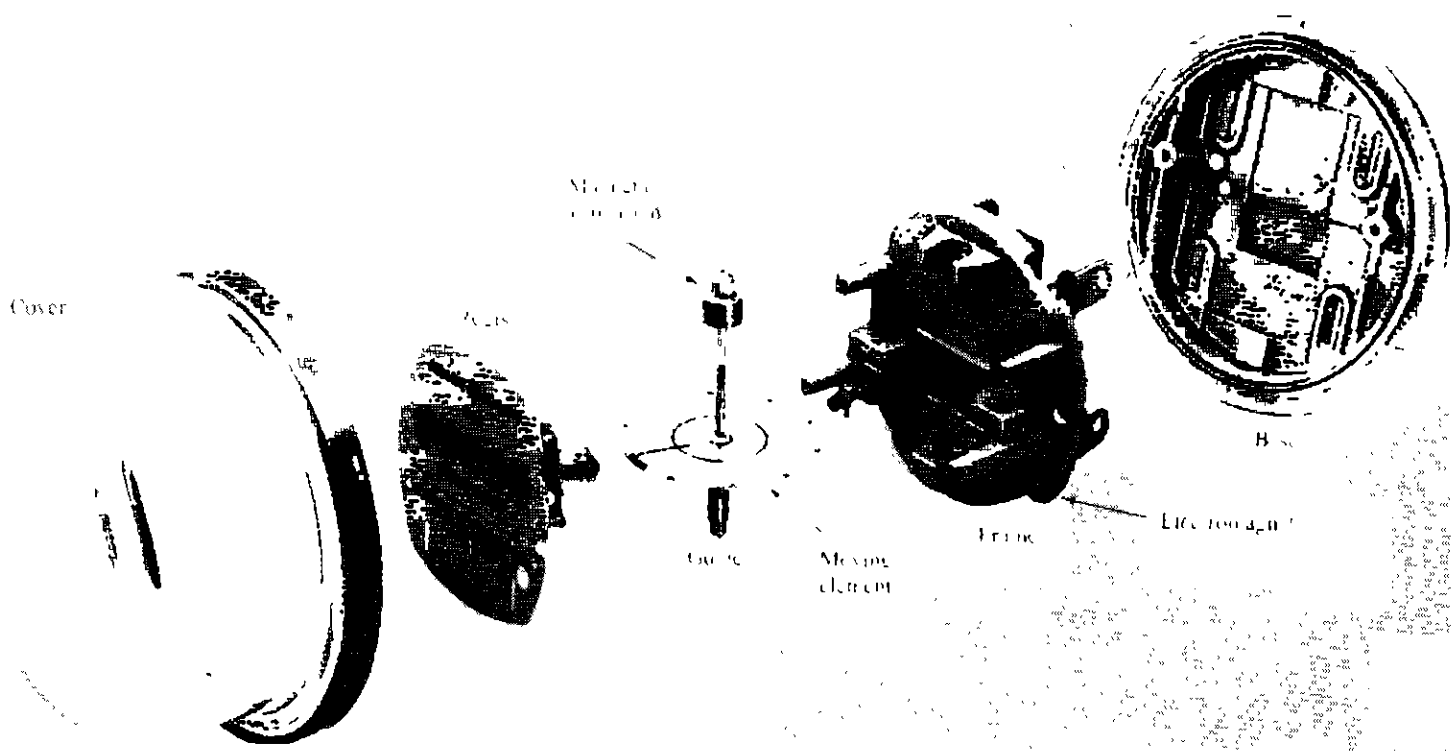


Figure 2-17 Basic parts of a single-phase watthour meter. (*General Electric Company.*)

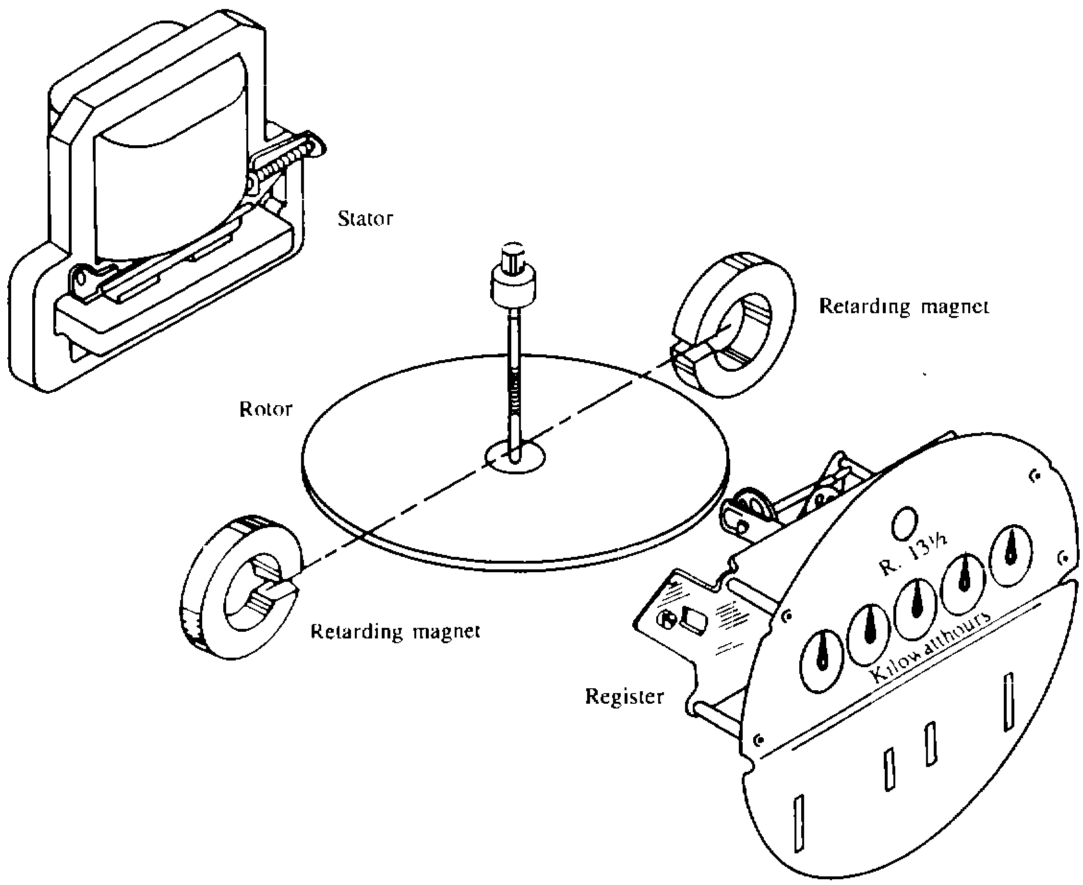


Figure 2-18 Diagram of typical motor and magnetic retarding system for a single-phase watthour meter. (*General Electric Company.*)

A demand meter is basically a watthour meter with a timing element added. The meter functions as an integrator and adds up the kilowatthours of energy used in a certain time interval, for example, 15, 30, or 60 min. Therefore, the demand meter indicates energy per time interval, or average power, which is expressed in kilowatts. Figure 2-21 shows a demand register.

2-6-1 Reading Electric Meters

By reading the register, i.e., the revolution counter, the customers' bills can be determined. There are primarily two different types of registers: (1) conventional dial and (2) cyclometer.

Figure 2-22 shows a conventional dial-type register. To interpret it, read the dials from left to right. (Note that numbers run clockwise on some dials and counterclockwise on others.) The figures above each dial show how many kilowatthours are recorded each time the pointer makes a complete revolution.

As shown in Fig. 2-22, if the pointer is between numbers, read the smaller one. The 0 stands for 10. If the pointer is pointed directly at a number, look at the dial to the right. If that pointer has not yet passed 0, record the smaller number; if it has passed 0, record the number the pointer is on. For example, in Fig. 2-22, the

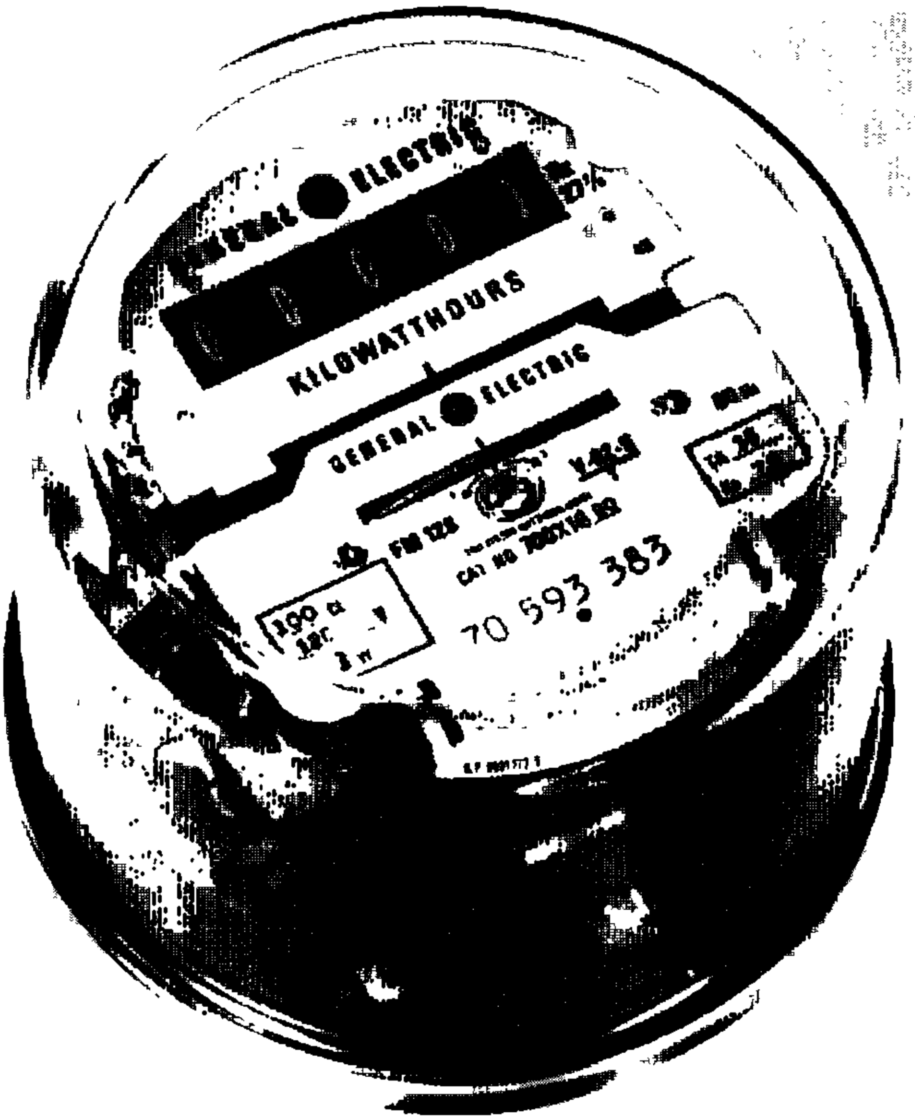


Figure 2-19 Typical polyphase watthour meters: (*a*) self-contained meter (socket-connected cyclometer type).

pointer on the first dial is between 8 and 9; therefore read 8. The pointer on the second dial is between 3 and 4; thus read 3. The pointer on the third dial is almost directly on 8, but the dial on the right has not reached 0 so the reading on the third dial is 7. The fourth dial is read 8. Therefore the total reading is 8378 kWh. The third dial would be read as 8 after the pointer on the 10-kWh dial reaches 0. This reading is based on a cumulative total; i.e., since the meter was last set at 0, 8378 kWh of electricity has been used.

To find the customer's monthly use, take two readings 1 month apart, and subtract the earlier one from the later one. Some electric meters have a constant, or multiplier, indicated on the meter. This type of meter is primarily for high-usage customers.

Figure 2-23 shows a cyclometer-type register. Here, even though the procedure is the same as in the conventional type, the wheels, which indicate numbers directly, replace the dials. Therefore, it makes possible the reading of the meter simply and directly, from left to right.

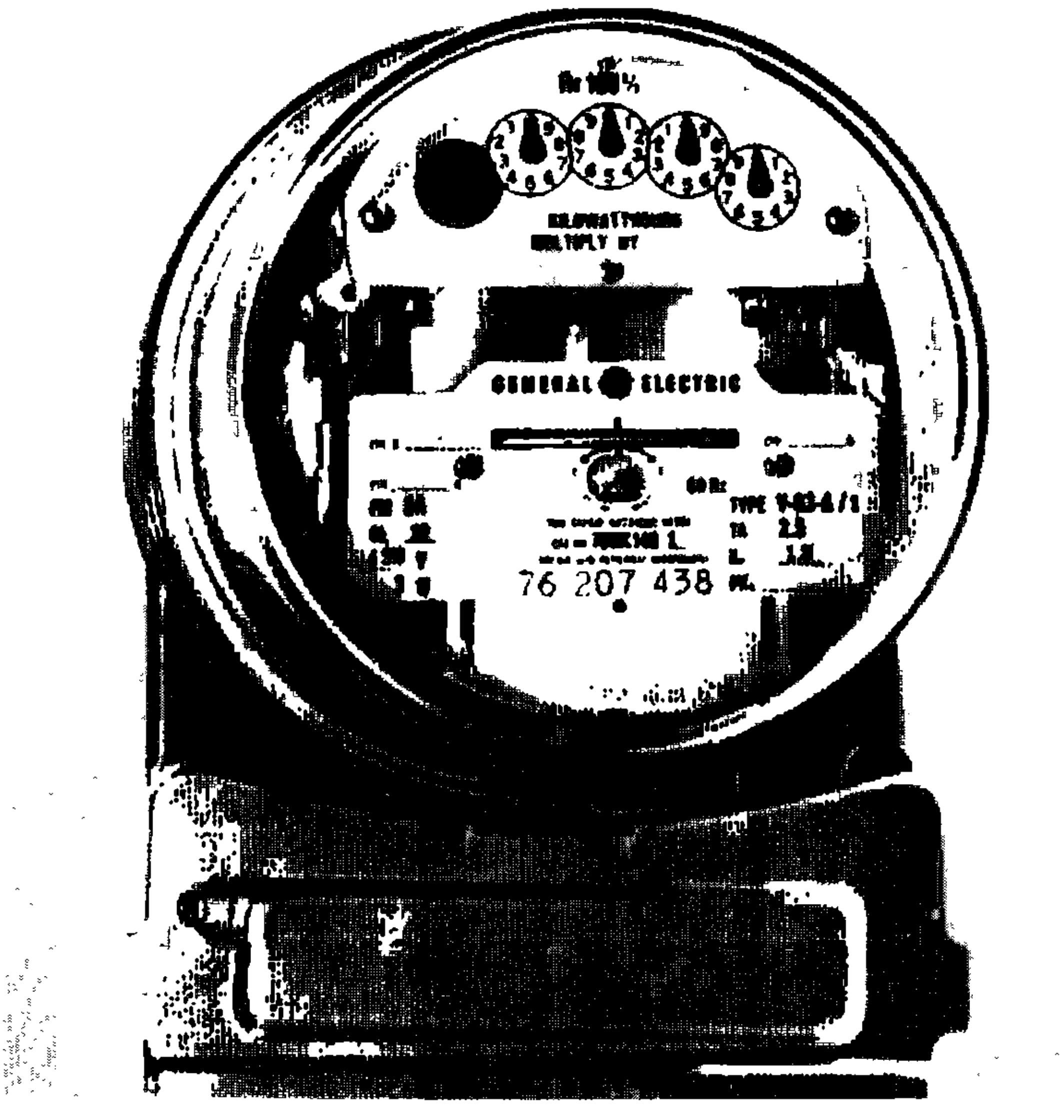


Figure 2-19 Typical polyphase watthour meters: (*b*) transformer-rated meter (bottom-connected pointer type). (*General Electric Company.*)

2-6-2 Instantaneous Load Measurements Using Watthour Meters

The instantaneous kilowatt demand of any customer may be determined by making field observations of the kilowatthour meter serving the customer. However, the instantaneous load measurement should not replace demand meters that record for longer time intervals. The instantaneous demand may be determined by using one of the following equations:

1. For a self-contained watthour meter,

$$D_i = \frac{3.6 \times K_r \times K_h}{T} \quad \text{kW} \tag{2-45}$$

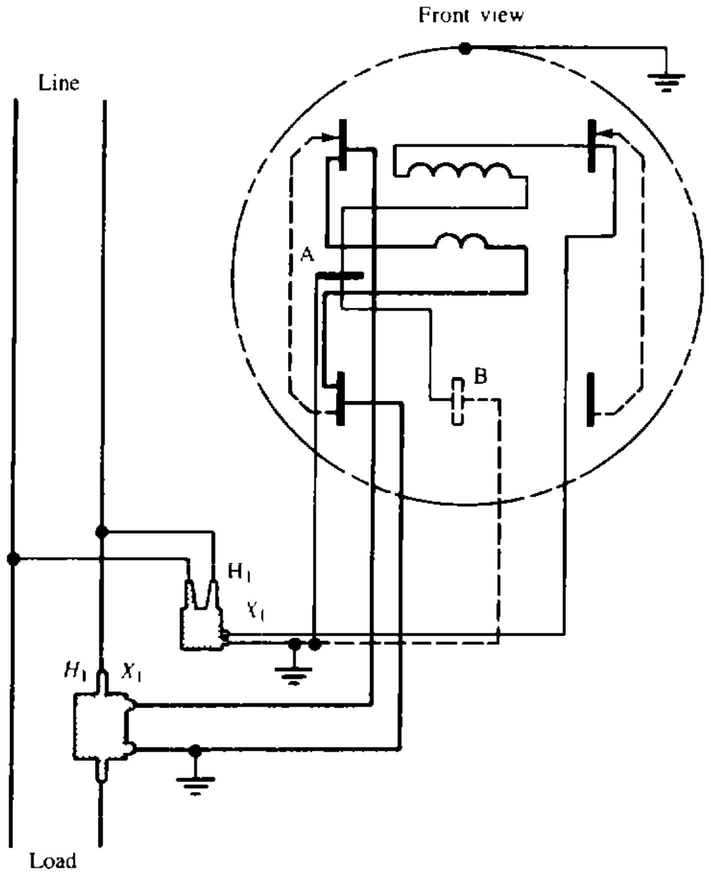


Figure 2-20 Single-phase, two-wire watthour meter connected to a high-voltage circuit through current and potential transformers. (*General Electric Company.*)

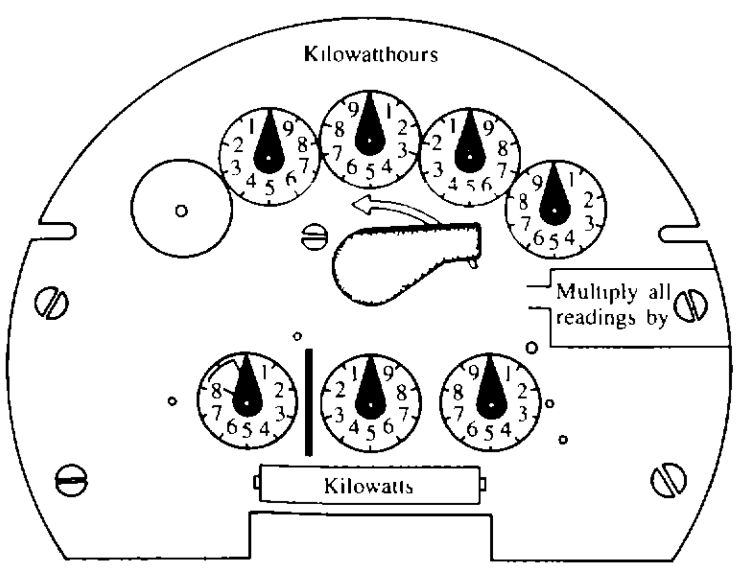


Figure 2-21 The register of a demand meter for large customers. (*General Electric Company.*)

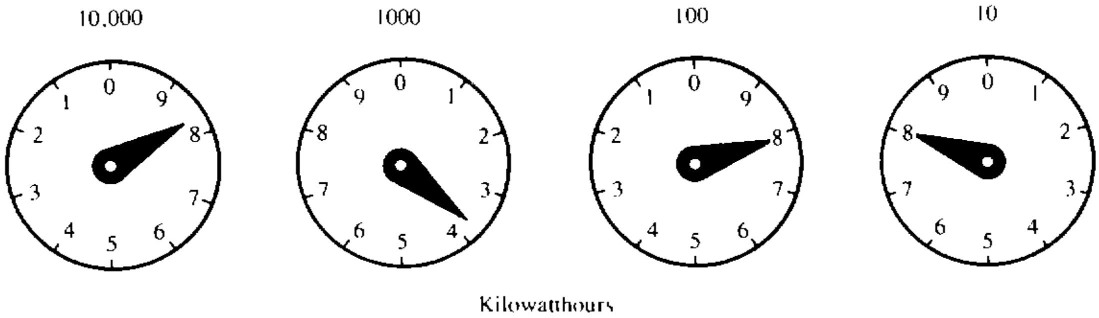


Figure 2-22 A conventional dial-type register.

2. For a transformer-rated meter (where instrument transformers are used with a watthour meter),

$$D_i = \frac{3.6 \times K_r \times K_h \times \text{CTR} \times \text{PTR}}{T} \quad \text{kW} \tag{2-46}$$

where D_i = instantaneous demand, kW
K_r = number of meter disk revolutions for a given time period
K_h = watthour meter constant (given on the register), Wh/rev
T = time, s
CTR = current transformer ratio
PTR = potential transformer ratio

Since the kilowatt demand is based on a short-time interval, two or more demand intervals should be measured. The average value of these demands is a good estimate of the given customer's kilowatt demand during the intervals measured.

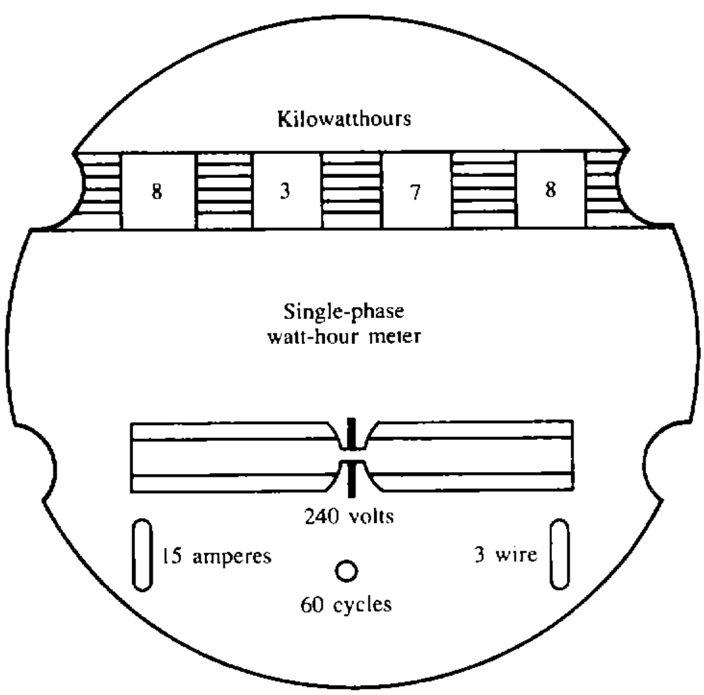


Figure 2-23 A cyclometer-type register.

Example 2-13 Assume that the load is measured twice with a watthour meter which has a meter constant of 7.2 and the following data are obtained:

	First reading	Second reading
Revolutions of disk	32	27
Time interval for revolutions of disks	59	40

Determine the instantaneous demands and the average demand.

SOLUTION From Eq. (2-45), for the first reading,

$$D_1 = \frac{3.6 \times K_r \times K_h}{T}$$

$$= \frac{3.6 \times 32 \times 7.2}{59}$$

$$= 14.058 \text{ kW}$$

and for the second reading,

$$D_2 = \frac{3.6 \times 27 \times 7.2}{40}$$

$$= 17.496 \text{ kW}$$

Therefore the average demand is

$$D_{av} = \frac{D_1 + D_2}{2}$$

$$= \frac{14.058 + 17.496}{2}$$

$$= 15.777 \text{ kW}$$

Example 2-14 Assume that the data given in Example 2-13 are the results of load measurement with watthour meters and instrument transformers. Suppose that the new meter constant is 1.8 and that the ratios of the current and potential transformers used are 200 and 1, respectively. Determine the instantaneous demands for both readings and the average demand.

SOLUTION Therefore, from Eq. (2-46),

$$D_1 = \frac{3.6 \times K_r \times K_h \times \text{CTR} \times \text{PTR}}{T}$$

$$= \frac{3.6 \times 32 \times 1.8 \times 200 \times 1}{59}$$

$$= 702.9 \text{ kW}$$

and

$$D_2 = \frac{3.6 \times 27 \times 1.8 \times 200 \times 1}{40}$$

$$= 874.8 \text{ kW}$$

Thus the average demand is

$$D_{\text{av}} = \frac{702.9 + 874.8}{2}$$

$$\cong 788.9 \text{ kW}$$

Example 2-15 Assume that the load is measured with watthour and varhour meters and instrument transformers and that the following readings are obtained:

	Watthour readings		Varhour readings	
	First set	Second set	First set	Second set
Revolutions of disk	20	30	10	20
Time interval for revolutions of disks	50	60	50	60

Assume that the new meter constants are 1.2 and that the ratios of the current and potential transformers used are 80 and 20, respectively. Determine the following:

(*a*) The instantaneous kilowatt demands.
(*b*) The average kilowatt demand.
(*c*) The instantaneous kilovar demands
(*d*) The average kilovar demand.
(*e*) The average kilovoltampere demand.

SOLUTION

(*a*) The instantaneous kilowatt demands are

$$D_1 = \frac{3.6 \times 20 \times 1.2 \times 80 \times 20}{50}$$

$$= 2764.8 \text{ kW}$$

and

$$D_2 = \frac{3.6 \times 30 \times 1.2 \times 80 \times 20}{60}$$

$$= 3456 \text{ kW}$$

(*b*) The average kilowatt demand is

$$D_{av} = \frac{2764.8 + 3456}{2}$$

$$= 3110.4 \text{ kW}$$

(*c*) The instantaneous kilovar demands are

$$D_1 = \frac{3.6 \times 10 \times 1.2 \times 80 \times 20}{50}$$

$$= 1382.4 \text{ kvar}$$

and

$$D_2 = \frac{3.6 \times 20 \times 1.2 \times 80 \times 20}{60}$$

$$= 2304 \text{ kvar}$$

(*d*) The average kilovar demand is

$$D_{av} = \frac{1382.4 + 2304}{2}$$

$$= 1843.2 \text{ kvar}$$

(*e*) The average kilovoltampere demand is

$$D_{av} = [(D_{av,\,kW})^2 + (D_{av,\,kvar})^2]^{1/2}$$

$$= (3110.4^2 + 1843.2)^{1/2}$$

$$\cong 3615.5 \text{ kVA}$$

PROBLEMS

2-1 Use the data given in Example 2-1 and assume that the feeder has the peak loss of 72 kW at peak load and an annual loss factor of 0.14. Determine the following:

(a) The daily average load of the feeder.
(b) The average power loss of the feeder.
(c) The total annual energy loss of the feeder.

2-2 Use the data given in Example 2-1 and the equations given in Sec. 2-2 and determine the load factor of the feeder.

2-3 Use the data given in Example 2-1 and assume that the connected demands for the street lighting load, the residential load, and the commercial load are 100, 2000 and 2000 kW, respectively. Determine the following:

(a) The demand factor of the street lighting load.
(b) The demand factor of the residential load.
(c) The demand factor of the commercial load.
(d) The demand factor of the feeder.

2-4 Using the data given in the following table for a typical summer day, repeat Example 2-1 and compare the results.

Time	Load, kW		
	Street lighting	Residential	Commercial
12 A.M.	100	250	300
1	100	250	300
2	100	250	300
3	100	250	300
4	100	250	300
5	100	250	300
6	100	250	300
7		350	300
8		450	400
9		550	600
10		550	1100
11		550	1100
12 noon		600	1100
1		600	1100
2		600	1300
3		600	1300
4		600	1300
5		650	1300
6		750	900
7		900	500
8	100	1100	500
9	100	1100	500
10	100	900	300
11	100	700	300
12 P.M.	100	350	300

2-5 Use the data given in Prob. 2-4 and repeat Prob. 2-2.

2-6 Use the data given in Prob. 2-4 and repeat Prob. 2-3.

2-7 Use the result of Prob. 2-2 and calculate the associated loss factor.

2-8 Assume that a load of 100 kW is connected at the Riverside substation of the NL&NP Company. The 15-min weekly maximum demand is given as 75 kW, and the weekly energy consumption is 4200 kWh. Assuming a week is 7 days, find the demand factor and the 15-min weekly load factor of the substation.

2-9 Assume that the total kilovoltampere rating of all distribution transformers connected to a feeder is 3000 kVA. Determine the following:

(*a*) If the average core loss of the transformers is 0.50 percent, what is the total annual core loss energy on this feeder?

(*b*) Find the value of the total core loss energy calculated in part *a* at \$0.025/kWh.

2-10 Use the data given in Example 2-6 and also consider the following added new load. Suppose that several buildings which have electric air-conditioning are converted from gas-fired heating to electric heating. Let the new electric heating load average 200 kW during 6 months of heating (and off-peak) season. Assume that off-peak energy delivered to these primary feeders costs the NL&NP Company 2 cents/kWh and that the capacity cost of the power system remains at \$3.00/kW per month.

(*a*) Find the new annual load factor on the substation.

(*b*) Find the total annual cost to NL&NP to serve this new load.

(*c*) Why is it that the hypothetical but illustrative energy cost is smaller in this problem than the one in Example 2-8?

2-11 The input to a subtransmission system is 87,600,000 kWh annually. On the peak-load day of the year, the peak is 25,000 kW and the energy input that day is 300,000 kWh. Find the load factors for the year and for the peak-load day.

2-12 The electric energy consumption of a residential customer has averaged 1150 kWh/month as follows, starting in January: 1400, 900, 1300, 1200, 800, 700, 1000, 1500, 700, 1500, 1400, and 1400 kWh. The customer is considering purchasing equipment for a hobby shop which he has in his basement. The equipment will consume about 200 kWh each month. Estimate the additional annual electric energy cost for operation of the equipment. Use the electrical rate schedule given in the following table.

Residential:	
Rate: (net) per month per meter	
Energy charge	
For first 25 kWh	6.0¢/kWh
For next 125 kWh	3.2¢/kWh
For next 850 kWh	2.0¢/kWh
All in excess of 1000 kWh	1.0¢/kWh
Minimum: \$1.50 per month	
Commercial	
A rate available for general, commercial, and miscellaneous power uses where consumption of energy does not exceed 10,000 kWh in any month during any calendar year.	
Rate: (net) per month per meter	
Energy charge	
For the first 25 kWh	6.0¢/kWh
For the next 375 kWh	4.0¢/kWh
For the next 3600 kWh	3.0¢/kWh
All in excess of 4000 kWh	1.5¢/kWh
Minimum: \$1.50 per month	
General power	
A rate available for service supplied to any commercial or industrial customer whose consumption in any month during the calendar year exceeds 10,000 kWh. A customer who exceeds 10,000 kWh per month in any 1 month may elect to receive power under this rate.	
A customer who exceeds 10,000 kWh in any 3 months or who exceeds 12,000 kWh in any 1 month during a calendar year shall be required to receive power under this rate at the option of the supplier.	

A customer who elects at his own option to receive power under this rate may not return to the commercial service rate except at the option of the supplier.

Rate: (net) per month per meter

kW is rate of flow. 1 kW for 1 h is 1kWh.

Demand charge	
For the first 30 kW of maximum demand per month	\$2.50/kW
For all maximum demand per month in excess of 30 kW	\$1.25/kW
Energy charge	
For the first 100 kWh per kW of maximum demand per month	2.0¢/kWh
For the next 200 kWh per kW of maximum demand per month	1.2¢/kWh
All in excess of 300 kWh per kW of maximum demand per month	0.5¢/kWh

Minimum charge: The minimum monthly bill shall be the demand charge for the month.

Determination of maximum demand: The maximum demand shall be either the highest integrated kW load during any 30-minute period occurring during the billing month for which the determination is made, or 75 percent of the highest maximum demand which has occurred in the preceding month, whichever is greater.

Water heating: 1.0¢/kWh with a minimum monthly charge of \$1.00.

2-13 The Zubits International Company, located in Ghost Town, consumed 16,000 kWh of electric energy for Zubit production this month. The company's monthly average energy consumption is also 16,000 kWh due to some unknown reasons. It has a 30-min monthly maximum demand of 200 kW and a connected demand of 580 kW. Use the electrical rate schedule given in Prob. 2-12.

(*a*) Find the Zubits International's total monthly electrical bill for this month.

(*b*) Find its 30-min monthly load factor.

(*c*) Find its demand factor.

(*d*) The company's newly hired plant engineer, who recently completed a load management course at Ghost University, suggested that, by shifting the hours of a certain production from the peak-load hours to off-peak hours, the maximum monthly demand can be reduced to 140 kW at a cost of \$50/month. Do you agree that this will save money? How much?

2-14 Repeat Example 2-11, assuming that there are eight houses connected on each distribution transformer and that there are a total of 120 distribution transformers and 960 residences supplied by the primary feeder.

2-15 Repeat Example 2-12, assuming that the 30-min monthly maximum demands of customers A and B are 27 and 42 kW, respectively. The new monthly energy consumptions by customers A and B are 8000 and 9000 kWh, respectively. The new lagging load power factors of A and B are 0.90 and 0.70, respectively.

REFERENCES

1. *American Standard Definitions of Electric Terms*, Group 35, Generation, Transmission and Distribution, ASA C42.35, 1957.
2. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
3. Sarikas, R. H., and H. B. Thacker: "Distribution System Load Characteristics and Their Use in Planning and Design," *AIEE Trans.*, no. 31, pt. III, August 1957, pp. 564–573.
4. Seelye, H. P.: *Electrical Distribution Engineering*, McGraw-Hill, New York, 1930.
5. Buller, F. H., and C. A. Woodrow: "Load Factor-Equivalent Hour Values Compared," *Electr. World*, vol. 92, no. 2, July 14, 1928, pp. 59–60.
6. General Electric Company: *Manual of Watthour Meters*, Bulletin GET-1840C.
7. Arvidson, C. E.: "Diversified Demand Method of Estimating Residential Distribution Transformer Loads," *Edison Electr. Inst. Bull.*, vol. 8, October 1940, pp. 469–79.

CHAPTER

THREE

APPLICATION OF DISTRIBUTION TRANSFORMERS

3-1 INTRODUCTION

In general, distribution transformers are used to reduce primary-system voltages (2.4 to 34.5 kV) to utilization voltages (120 to 600 V). Table 3-1 gives standard transformer capacity and voltage ratings according to ANSI Standard C57.12.20-1964 for single-phase distribution transformers. Other voltages are also available, for example, 2400 × 7200, which is used on a 2400-V system that is to be changed later to 7200 V.

The three primary symbols used are the dash (–), the slant (/), and the cross (×). The dash (–) is used to separate voltages of different windings, the slant (/) is used to separate voltages of the same winding, and the cross (×) is used to indicate a series-multiple connection.

Secondary symbols used are the letter Y, which indicates that the winding is connected or may be connected wye, and Gnd Y, which indicates that the winding has one end grounded to the tank or brought out through a reduced insulation bushing. Windings that are delta-connected or may be connected delta are designated by the voltage of the winding only.

In Table 3-1 further information is given by the order in which the voltages are written for low-voltage windings. To designate a winding with a mid-tap which will provide half the full-winding kilovoltampere rating at half the full-winding voltage, the full-winding voltage is written first, followed by a slant, and then the mid-tap voltage. For example, 240/120 is used for a three-wire connection to

Table 3-1 Standard transformer kilovoltamperes and voltages

kilovoltamperes		High voltages		Low voltages	
Single-phase	Three-phase	Single-phase	Three-phase	Single-phase	Three-phase
5	30	2400/4160Y	2400	120/240	208Y/120
10	45	4800/8320Y	4160Y/2400	240/480	240
15	75	4800Y/8320YX	4160Y	2400	480
25	112½	7200/12,470Y	4800	2520	480Y/277
37½	150	12,470GndY/7200	8320Y/4800	4800	240 × 480
50	225	7620/13,200Y	8320Y	5040	2400
75	300	13,200GndY/7620	7200	6900	4160Y/2400
100	500	12,000	12,000	7200	4800
167		13,200/22,860GndY	12,470Y/7200	7560	12,470Y/7200
250		13,200	12,470Y	7980	13,200Y/7620
333		13,800GndY/7970	13,200Y/7620		
500		13,800/23,900GndY	13,200Y		
		13,800	13,200		
		14,400/24,940GndY	13,800		
		16,340	22,900		
		19,920/34,500GndY	34,400		
		22,900	43,800		
		34,400	67,000		
		43,800			
		67,000			

designate a 120-V mid-tap voltage with a 240-V full-winding voltage. A winding which is appropriate for series, multiple, and three-wire connections will have the designation of multiple voltage rating followed by a slant and the series voltage rating, e.g., the notation 120/240 means that the winding is appropriate either for 120-V multiple connection, for 240-V series connection, and for 240/120 three-wire connection. When two voltages are separated by a cross (×), a winding is indicated which is appropriate for both multiple and series connection but not for three-wire connection. The notation 120 × 240 is used to differentiate a winding that can be used for 120-V multiple connection and for 240-V series connection, but not for a three-wire connection. Examples of all symbols used are given in Table 3-2.

To reduce installation costs to a minimum, small distribution transformers are made for pole mounting in overhead distribution. To reduce size and weight, preferred oriented steel is commonly used in their construction. Transformers 100 kVA and below are attached directly to the pole, and transformers larger than 100 up to 500 kVA are hung on crossbeams or support lugs. If three or more transformers larger than 100 kVA are used, they are installed on a platform supported by two poles.

In underground distribution, transformers are installed in street vaults, in manholes direct-buried, on pads at ground level, or within buildings. The type of

Table 3-2 Designation of voltage ratings for single- and three-phase distribution transformers

Single-phase		Three-phase	
Designation	Meaning	Designation	Meaning
120/240	Series, multiple, or three-wire connection.	2400/4160Y	Suitable for delta or wye connection
240/120	Series or three-wire connection only	4160Y	Wye connection only (no neutral)
240 × 480	Series or multiple connection only	4160Y2400	Wye connection only (with neutral available)
120/208Y	Suitable for delta or wye connection three-phase	12,470GndY/7200	Wye connection only (with reduced insulation neutral available)
12,470GndY/7200	One end of winding grounded to tank or brought out through reduced insulation bushing	4160	Delta connection only

transformer may depend upon soil content, lot location, public acceptance, or cost.

The distribution transformers and any secondary-service junction devices are installed within elements, usually placed on either the front or the rear lot lines of the customer's premises. The installation of the equipment to either front or rear locations may be limited by customer preference, local ordinances, and landscape conditions, etc. The rule of thumb requires that a transformer be centrally located with respect to the load it supplies in order to provide proper cable economy, voltage drop, and aesthetic effect.

Secondary-service junctions for an underground distribution system can be of the pedestal, hand-hole, or direct-buried splice types. No junction is required if the service cables are connected directly from the distribution transformer to the user's apparatus.

Secondary or service conductors can be either copper or aluminum. However, in general, aluminum conductors are mostly used due to cost savings. The cables are single-conductor or triplexed. Neutrals may be either bare or covered, installed separately, or assembled with the power conductors. All secondary or service conductors are rated 600 V, and their sizes differ from #6 AWG to 1000 kcmil.

3-2 TYPES OF DISTRIBUTION TRANSFORMERS

Heat is a limiting factor in transformer loading. Removing the coil heat is an important task. In liquid-filled types, the transformer coils are immersed in a smooth-surfaced, oil-filled tank. Oil absorbs the coil heat and transfers it to the tank surface which, in turn, delivers it to the surrounding air. For transformers 25 kVA and larger, the size of smooth tank surface required to dissipate the heat becomes larger than that required to enclose coils. Therefore the transformer tank may be corrugated to add surface, or external tubes may be welded to the tank. To further increase the heat-disposal capacity, air may be blown over the tube surface. Such designs are known as *forced-air-cooled*, with respect to *self-cooled* types. Presently, however, all distribution transformers are built to be self-cooled.

Therefore the distribution transformers can be classified as (1) dry-type and (2) liquid-filled-type. The dry-type distribution transformers are air-cooled and air-insulated. The liquid-filled-type distribution transformers can further be classified as (1) oil-filled and (2) inerteen-filled.

The distribution transformers employed in overhead distribution systems can be categorized as:

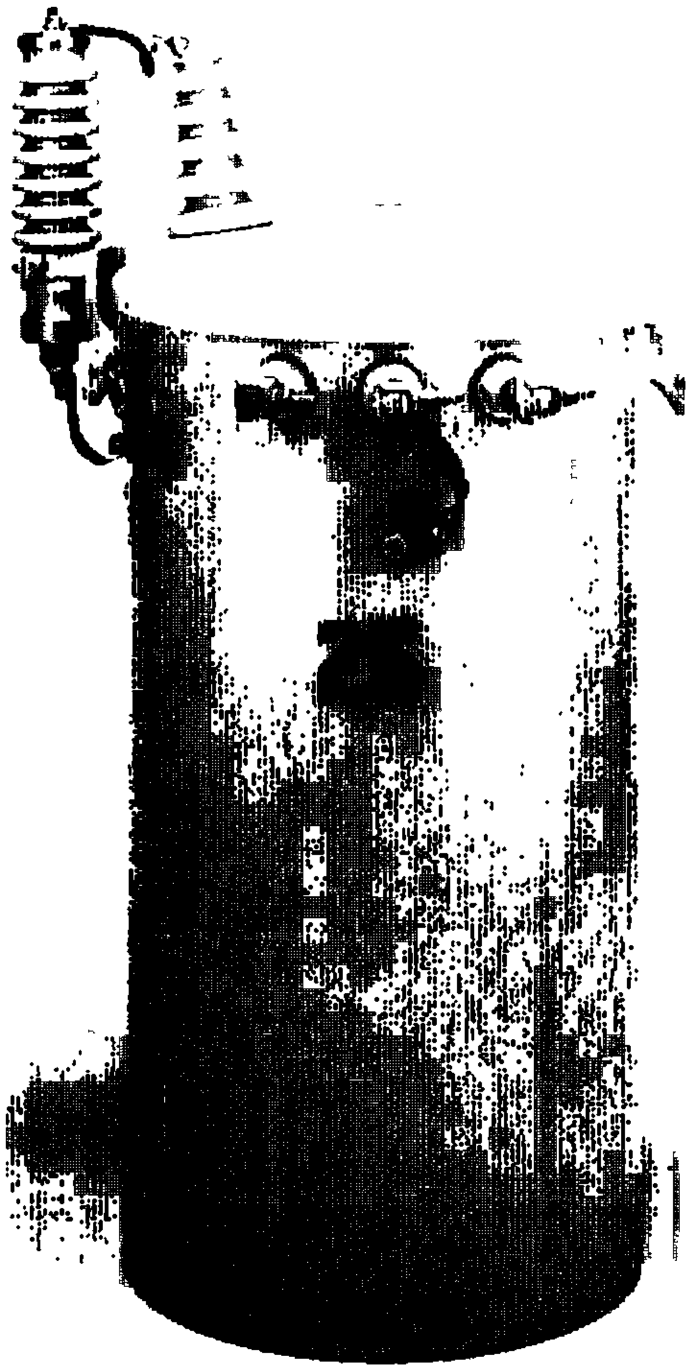
1. Conventional transformers
2. Completely self-protecting (CSP) transformers
3. Completely self-protecting for secondary banking (CSPB) transformers

The conventional transformers have no integral lightning, fault, or overload protective devices provided as a part of the transformer. The completely self-protecting (CSP) transformers are, as the name implies, self-protecting from lightning or line surges, overloads, and short circuits. Lightning arresters mounted directly on the transformer tank, as shown in Fig. 3-1, provide the primary winding against the lightning and line surges. The overload protection is provided by circuit breakers inside the transformer tank. The transformer is protected against an internal fault by internal protective links located between the primary winding and the primary bushings. Single-phase CSP transformers (oil-immersed, pole-mounted, 65°C, 60 Hz, 10–500 kVA) are available for a range of primary voltages from 2400 to 34,400 V. The secondary voltages are 120/240 or 240/480/277 V. The CSPB distribution transformers are designed for banked secondary service. They are built similar to the CSP transformers, but they are provided with two sets of circuit breakers. The second set is used to sectionalize the secondary when it is needed.

The distribution transformers employed in underground distribution systems can be categorized as:

1. Subway transformers
2. Low-cost residential transformers
3. Network transformers

Subway transformers are used in underground vaults. They can be conventional-type or current-protected-type. Low-cost residential transformers are similar to those conventional transformers employed in overhead distribution. Network transformers are employed in the secondary networks. They have the primary disconnecting and grounding switch and the network protector mounted



(a)

Figure 3-1 Overhead pole-mounted distribution transformers: (*a*) single-phase CSP (or conventional)

integrally on the transformer. They can be either liquid-filled, ventilated dry-type, or sealed dry-type.

Figure 3-2 shows various types of transformers. Figure 3-2*a* shows a typical secondary-unit substation with the high voltage and low voltage on opposite ends and full-length flanges for close coupling to high-voltage and low-voltage switchgear. These units are normally made in sizes from 75 to 2500 kVA, three-phase, to 35-kV class. A typical single-phase pole-type transformer for a normal utility application is shown in Fig. 3-2*b*. These are made from 10 to 500 kVA for delta and

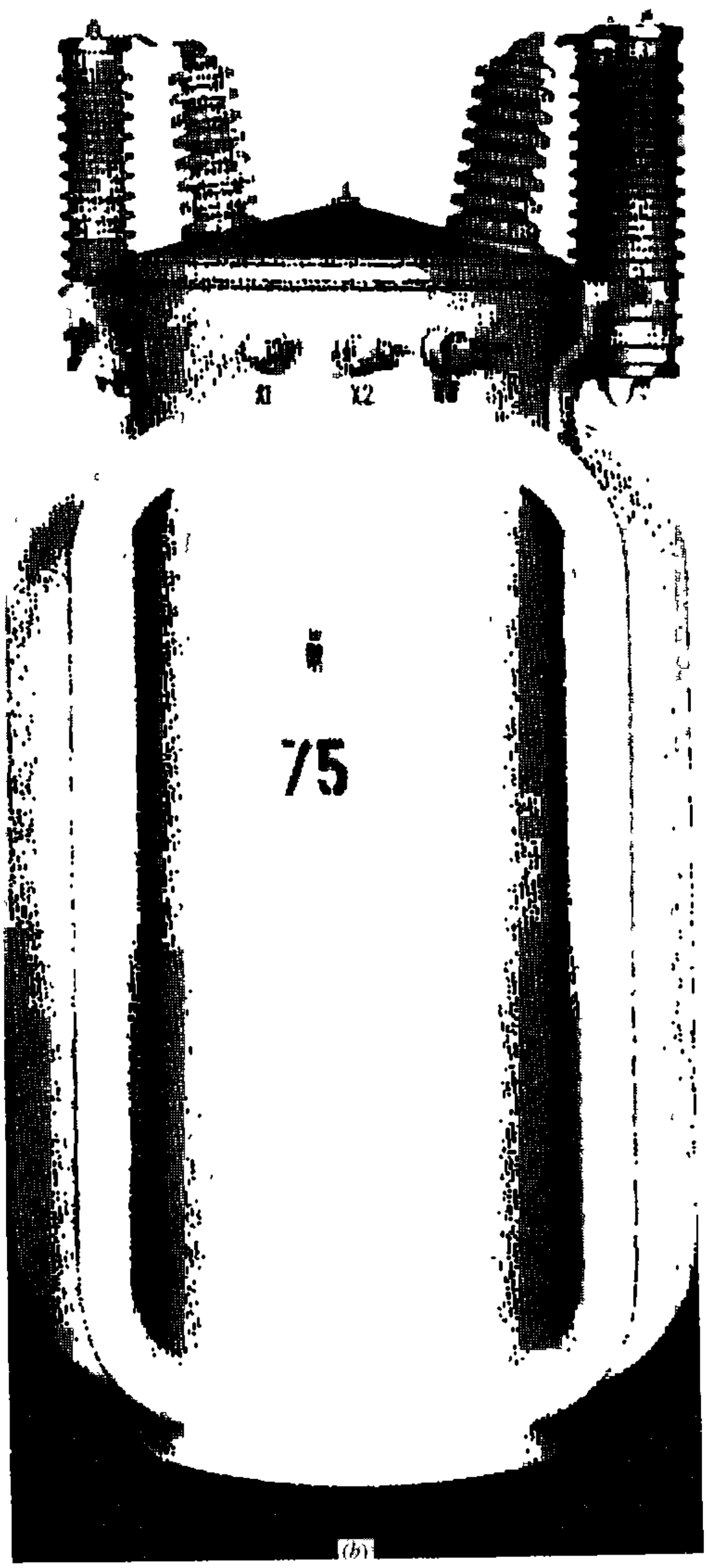


Figure 3-1 Overhead pole-mounted distribution transformers: (*b*) three-phase. (*Westinghouse Electric Corporation.*)

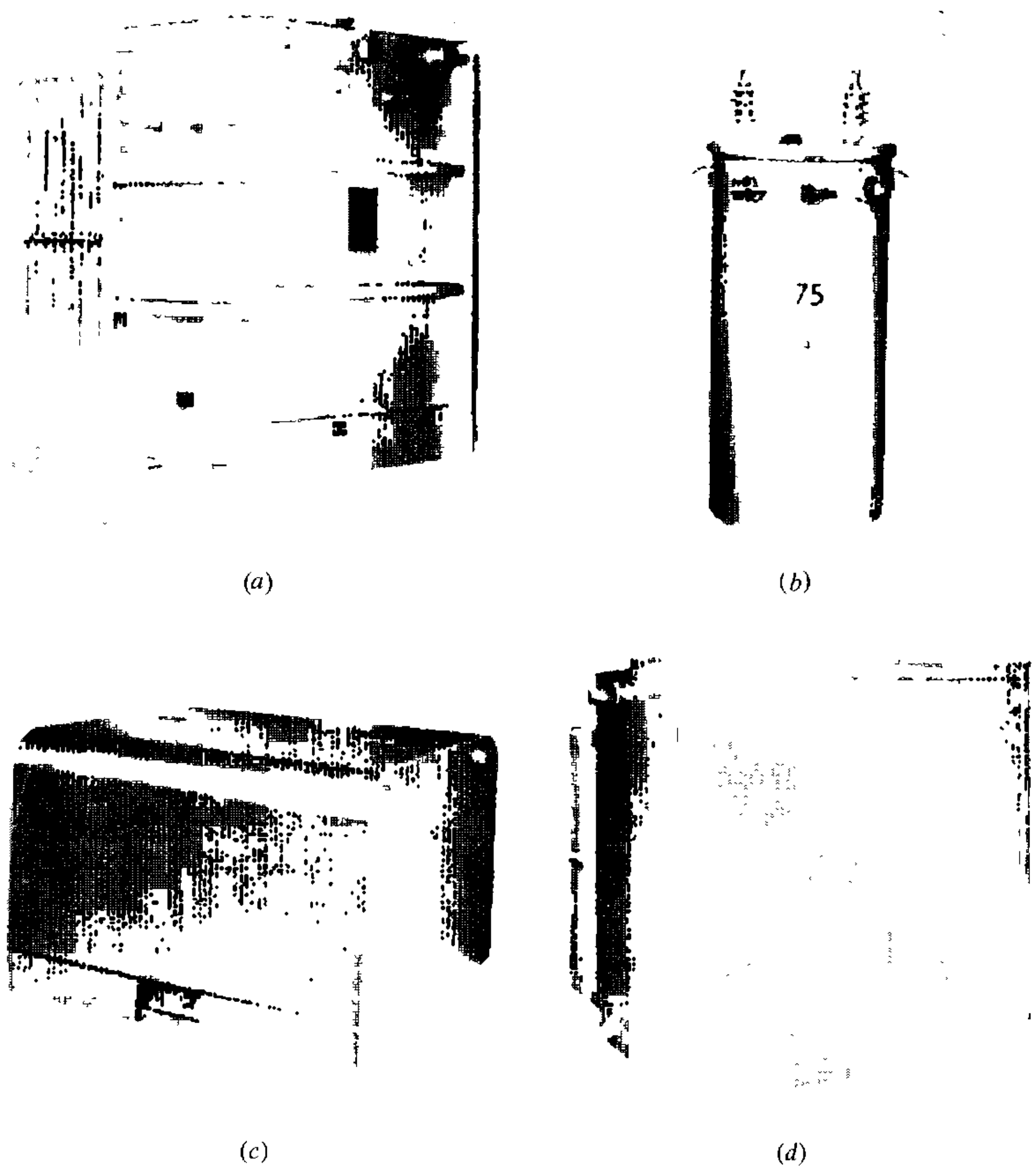
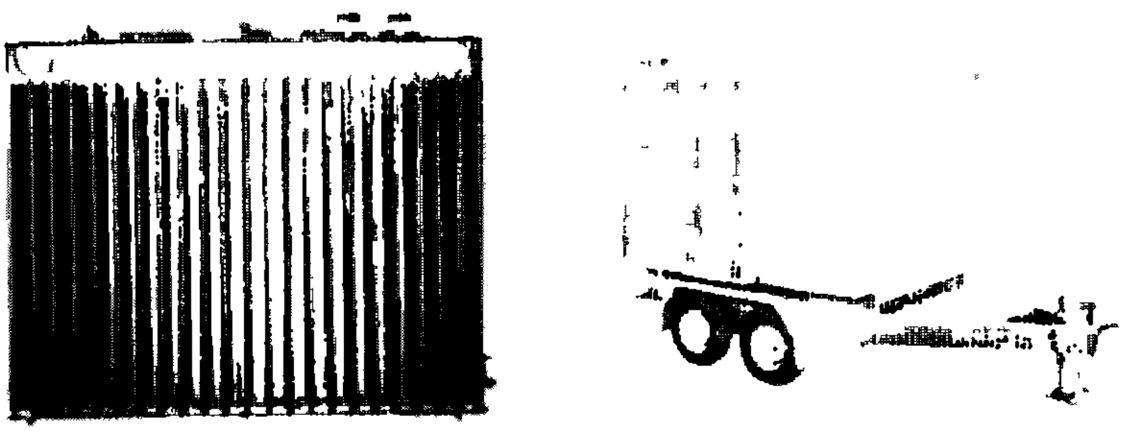


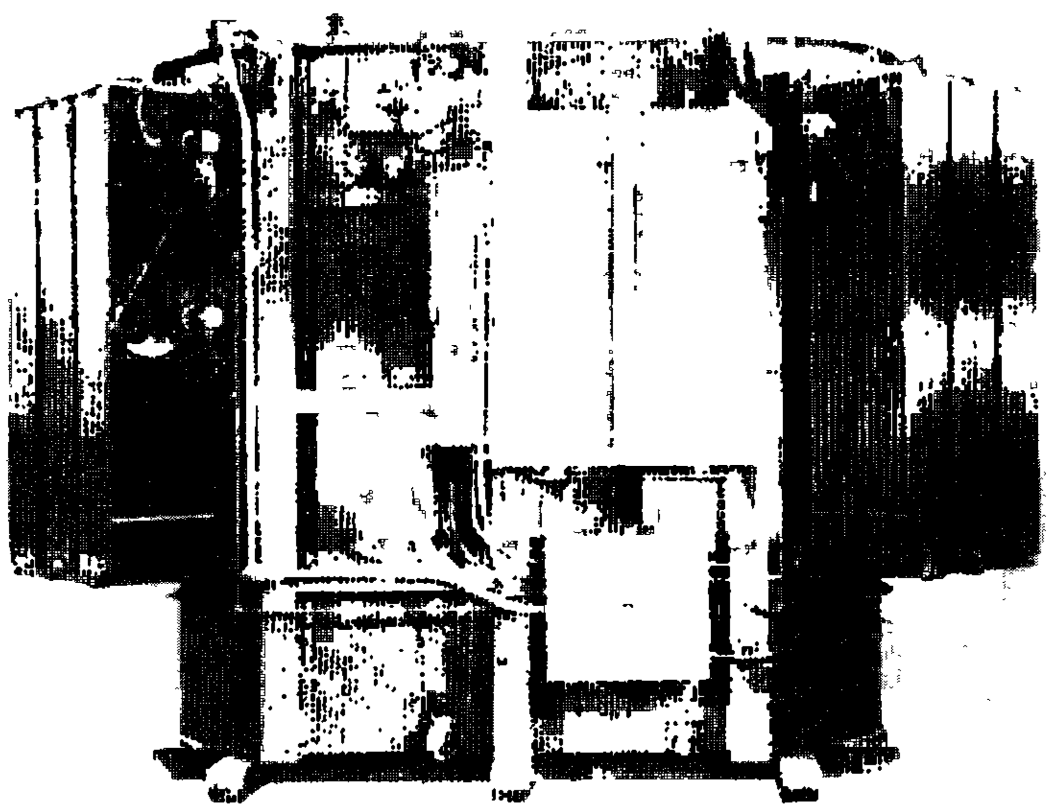
Figure 3-2 Various types of transformers. (*Balteau Standard Inc.*)

wye systems (one-bushing or two-bushing high voltage). Figure 3-2*c* shows a typical single-phase pad-mounted (minipad) utility-type transformer. These are made from 10 to 167 kVA. They are designed to do the same function as the pole type except they are for the underground distribution system where all cables are below grade. A typical three-phase pad-mounted (stan-pad) transformer used by utilities as well as industrial and commercial applications is shown in Fig. 3-2*d*. They are made from 45 to 2500 kVA normally, but have been made to 5000 kVA on special applications. They are also designed for underground service.

Figure 3-3*a* shows a typical three-phase subsurface-vault-type transformer used in utility applications in vaults below grade where there is no room to place the transformer elsewhere. These units are made for 75 to 2500 kVA and are made of a heavier gauge steel, special heavy corrugated radiators for cooling, and a



(a) (b)



(c)

Figure 3-3 Various types of transformers. (*Balteau Standard Inc.*)

special coal-tar type of paint. A typical mobile transformer is shown in Fig. 3-3*b*. These units are made for emergency applications and to allow utilities to reduce inventory. They are made typically for 500 to 2500 kVA. They can be used on underground service as well as overhead service. Normally they can have two or three primary voltages and two or three secondary voltages, so they may be used on any system the utility may have. For an emergency outage this unit is simply driven to the site, hooked up, and the power to the site is restored. This allows time to analyze and repair the failed unit. Figure 3-3*c* shows a typical power transformer.

Table 3-3 Electrical characteristics of typical single-phase distribution transformers*

kVA	% av. excit. curr.	120/240 V low voltage							240/480 and 277/480Y V low voltage						
		Watts loss		% regulation		% Z	% R	% X	Watts loss		% regulation		% Z	% R	% X
		No load	Total	1.0 PF	0.8 PF				No load	Total	1.0 PF	0.8 PF			
		2400/4160Y V high voltage													
5	2.4	34	137	2.06	2.12	2.2	2.1	0.8							
10	1.6	68	197	1.30	1.68	1.7	1.3	1.1	68	202	1.35	1.69	1.7	1.3	1.0
15	1.4	84	272	1.27	1.59	1.6	1.3	1.0	84	277	1.30	1.60	1.6	1.3	1.1
25	1.3	118	385	1.10	1.65	1.7	1.1	1.1	118	390	1.11	1.65	1.7	1.1	1.3
38	1.1	166	540	1.00	1.55	1.6	1.0	1.3	166	550	1.04	1.54	1.6	1.0	1.2
50	1.0	185	615	0.88	1.58	1.7	0.9	1.5	185	625	0.90	1.58	1.7	0.9	1.5
75	1.3	285	910	0.85	1.41	1.5	0.8	1.2	285	925	0.86	1.33	1.4	0.9	1.1
100	1.2	355	1175	0.84	1.55	1.7	0.8	1.5	355	1190	0.85	1.49	1.6	0.8	1.4
167	1.0	500	2100	0.99	1.75	1.9	1.0	1.6	500	2000	0.90	1.57	1.7	0.9	1.4
250	1.0	610	3390	1.16	2.16	2.4	1.1	2.1	610	3280	1.11	2.02	2.2	1.1	1.9
333	1.0	840	4200	1.08	2.51	3.0	1.0	2.8	840	3690	0.88	1.90	2.2	0.9	1.9
500	1.0	1140	5740	0.97	2.50	3.1	0.9	3.0	1140	4810	0.95	2.00	2.3	0.7	2.2

7200/12,470Y V high voltage

5	2.4	41	144	2.07	2.11	2.2	2.1	0.8	68	209	1.43	1.80	1.8	1.4	1.1
10	1.6	68	204	1.37	1.80	1.8	1.4	1.2	84	287	1.35	1.70	1.7	1.4	1.0
15	1.4	84	282	1.33	1.69	1.7	1.3	1.2	118	427	1.24	1.69	1.7	1.2	1.2
25	1.3	118	422	1.22	1.69	1.7	1.2	1.2	166	575	1.10	1.65	1.7	1.1	1.3
38	1.1	166	570	1.10	1.64	1.7	1.1	1.3	185	725	1.10	1.71	1.8	1.1	1.4
50	1.0	185	720	1.10	1.71	1.8	1.1	1.4	285	1000	0.97	1.52	1.6	1.0	1.3
75	1.3	285	985	0.95	1.60	1.7	0.9	1.4	355	1290	0.95	1.60	1.7	1.9	1.4
100	1.2	355	1275	0.95	1.72	1.9	1.9	1.7	500	2000	0.91	1.70	1.9	0.9	1.7
167	1.0	500	2100	0.98	1.90	2.1	1.0	1.9	610	3250	1.17	2.19	2.4	1.1	2.2
250	1.0	610	3490	1.22	2.45	2.8	1.2	2.6	840	3690	0.89	2.03	2.4	0.9	2.2
333	1.0	840	4255	1.07	2.50	3.0	1.0	2.8	1140	4810	0.78	1.99	2.4	0.7	2.3
500	1.0	1140	5640	0.95	2.55	3.2	0.9	3.1							

13,200/22,860GndY or 13,800/23,900GndY or 14,400/24,940GndY V high voltage

5	2.4	42	154	2.25	2.30	2.4	2.3	0.9	73	220	1.49	1.89	1.9	1.5	1.2
10	1.6	73	215	1.45	1.89	1.9	1.4	1.3	84	310	1.52	1.80	1.8	1.5	1.0
15	1.4	84	305	1.48	1.80	1.8	1.5	1.0	118	442	1.30	1.78	1.8	1.3	1.2
25	1.3	118	437	1.29	1.79	1.8	1.3	1.3	166	590	1.16	1.72	1.8	1.1	1.4
38	1.1	166	585	1.15	1.72	1.8	1.1	1.4	185	740	1.15	1.81	1.9	1.1	1.5
50	1.0	185	735	1.14	1.81	1.9	1.1	1.4	285	1065	1.06	1.78	1.8	1.0	1.5
75	1.4	285	1050	1.05	1.78	1.8	1.0	1.5	355	1310	0.98	1.74	1.9	1.0	1.6
100	1.3	355	1300	0.97	1.81	2.0	0.9	1.8	500	2060	0.95	1.80	2.0	0.9	1.8
167	1.0	500	2160	0.98	1.96	2.2	1.0	2.0	610	3285	1.11	2.16	2.5	1.1	2.3
250	1.0	610	3490	1.22	2.52	2.9	1.2	2.7	840	3750	0.91	2.05	2.4	0.9	2.2
333	1.0	840	4300	1.09	2.60	3.1	1.0	2.9	1140	4760	0.76	1.98	2.4	0.7	2.3
500	1.0	1140	5640	0.95	2.55	3.2	1.1	3.0							

* Data applies to 65°C rise transformers.

Table 3-4 Electrical characteristics of typical three-phase pad-mounted transformers

kVA	% av. excit. curr.	208Y/120 V low voltage: Watts loss, No load	208Y/120 V: Watts loss, Total	208Y/120 V: % regulation, 1.0 PF	208Y/120 V: % regulation, 0.8 PF	208Y/120 V: % Z	208Y/120 V: % R	208Y/120 V: % X	480Y/277 V low voltage: Watts loss, No load	480Y/277 V: Watts loss, Total	480Y/277 V: % regulation, 1.0 PF	480Y/277 V: % regulation, 0.8 PF	480Y/277 V: % Z	480Y/277 V: % R	480Y/277 V: % X
4160GndY/2400X12,470GndY/7200 V high voltage															
75	1.5	360	1,350	1.35	2.1	2.1	1.3	1.6	360	1,350	1.35	2.1	2.1	1.3	1.6
112	1.0	530	1,800	1.15	1.7	1.7	1.1	1.3	530	1,800	1.15	1.7	1.7	1.1	1.3
150	1.0	560	2,250	1.15	1.9	1.9	1.1	1.6	560	2,250	1.15	1.9	1.9	1.1	1.6
225	1.0	880	3,300	1.10	1.9	1.9	1.1	1.6	800	3,300	1.10	1.9	1.9	1.1	1.6
300	1.0	1050	4,300	1.10	1.9	2.0	1.1	1.7	1050	4,100	1.05	1.8	1.8	1.0	1.5
500	1.0	1600	6,800	1.15	2.2	2.3	1.0	2.1	1600	6,500	1.10	2.0	2.0	1.0	1.7
750	1.0	1800	10,200	1.28	4.4	5.7	1.1	5.6	1800	9,400	1.18	4.3	5.7	1.0	5.6
1000	1.0	2100	12,500	1.20	4.3	5.7	1.0	5.6	2100	10,900	1.04	4.2	5.7	0.9	5.7
1500	1.0	2900	19,400	1.26	4.3	5.7	1.1	5.6	3300	16,500	1.04	4.2	5.7	0.9	5.7
2500	1.0								4800	26,600	1.03	4.2	5.7	0.9	5.7
3750	1.0								6500	35,500	0.95	4.1	5.7	0.8	5.7
12,470GndY/7200 V high voltage															
75	1.5	360	1,350	1.4	1.7	1.7	1.3	1.1	360	1,350	1.4	1.5	1.5	1.3	0.8
112	1.0	530	1,800	1.2	1.5	1.5	1.1	1.0	530	1,800	1.2	1.3	1.3	1.1	0.7
150	1.0	560	2,250	1.2	1.8	1.9	1.1	1.6	560	2,250	1.2	1.7	1.7	1.1	1.3
225	1.0	880	3,300	1.1	1.8	1.8	1.1	1.4	880	3,300	1.1	1.6	1.6	1.1	1.2
300	1.0	1050	4,300	1.1	1.6	1.6	1.1	1.2	1050	4,100	1.1	1.4	1.4	1.0	1.0
500	1.0	1600	6,800	1.1	1.7	1.7	1.0	1.4	1600	6,500	1.1	1.4	1.4	1.0	1.0
750	1.0	1800	10,200	1.3	4.4	5.7	1.1	5.6	1800	9,400	1.2	4.3	5.7	1.0	5.6
1000	1.0	2100	12,500	1.2	4.3	5.7	1.0	5.6	2100	10,900	1.0	4.2	5.7	0.9	5.7
1500	1.0	2900	19,400	1.3	4.3	5.7	1.1	5.6	3300	16,500	1.0	4.2	5.7	0.9	5.7
2500	1.0								4800	26,600	1.0	4.2	5.7	0.9	5.7
3750	1.0								6500	35,500	0.9	4.1	5.7	0.8	5.7

		2400/4160Y/2400 V low voltage						
	% av excit. curr.	Watts loss		% regulation		% Z	% R	% X
kVA		No load	total	1.0 PF	0.8 PF			
12,470 delta V high voltage								
1000	1.38	2443	11,480	1.06	4.09	5.56	0.89	5.49
1500	1.33	3455	15,716	0.98	4.04	5.56	0.81	5.51
2500	1.29	4956	23,193	0.92	3.97	5.56	0.73	5.52
3750	1.37	6775	33,100	0.89	3.97	5.50	0.70	5.45
5000	1.33	8800	42,125	0.86	3.94	5.50	0.67	5.45
24,940 delta V high voltage								
1000	1.42	2533	11,588	1.07	4.09	5.56	0.91	5.49
1500	1.37	3625	15,213	0.96	4.03	5.56	0.80	5.50
2500	1.31	5338	23,213	0.88	3.98	5.56	0.72	5.52
3750	1.42	7075	33,700	0.90	3.97	5.50	0.71	5.44
5000	1.33	8725	43,550	0.88	3.96	5.50	0.69	5.44

This class of unit is manufactured from 3700 kVA to 30 MVA up to about 138-kV class. The picture shows removable radiators to allow for a smaller size during shipment, and fans for increased capacity when required, including an automatic on-load tap changer which changes as the voltage varies.

Table 3-3 presents electrical characteristics of typical single-phase distribution transformers. Table 3-4 gives electrical characteristics of typical three-phase pad-mounted transformers. (For more accurate values, consult the individual manufacturer's catalogs.)

To find the resistance (R') and reactance (X') of a transformer of equal size and voltage, which has a different impedance value (Z') than the one shown in tables, multiply the tabulated percent values of R and X by the ratio of the new impedance value to the tabulated impedance value, that is, Z'/Z. Therefore the resistance and the reactance of the new transformer can be found from

$$R' = R \times \frac{Z'}{Z} \qquad \% \Omega \tag{3-1}$$

and

$$X' = X \times \frac{Z'}{Z} \qquad \% \Omega \tag{3-2}$$

3-3 REGULATION

To calculate the transformer regulation for a kilovoltampere load of power factor $\cos \theta$, at rated voltage, any one of the following formulas can be used:

$$\% \text{ regulation} = \frac{S_L}{S_T}\left[\% IR \cos\theta + \% IX \sin\theta + \frac{(\% IX \cos\theta - \% IR \sin\theta)^2}{200}\right] \tag{3-3}$$

or

$$\% \text{ regulation} = \frac{I_{\text{op}}}{I_{\text{ra}}}\left[\% R \cos\theta + \% X \sin\theta + \frac{(\% X \cos\theta - \% R \sin\theta)^2}{200}\right] \tag{3-4}$$

or

$$\% \text{ regulation} = V_R \cos\theta + V_X \sin\theta + \frac{(V_X \cos\theta - V_R \sin\theta)^2}{200} \tag{3-5}$$

where θ = power factor angle of load

V_R = percent resistance voltage

$$= \frac{\text{copper loss}}{\text{output}} \times 100$$

S_L = apparent load power
S_T = rated apparent power of transformer
I_{op} = operating current
I_{ra} = rated current
V_X = percent leakage reactance voltage
$= (V_Z^2 - V_R^2)^{1/2}$
V_Z = percent impedance voltage

Note that the percent regulation at unity power factor is

$$\%\,\text{regulation} = \frac{\text{copper loss}}{\text{output}} \times 100 + \frac{(\%\,\text{reactance})^2}{200} \tag{3-6}$$

3-4 TRANSFORMER EFFICIENCY

The efficiency of a transformer can be calculated from

$$\%\,\text{efficiency} = \frac{\text{Output in watts}}{\text{output in watts} + \text{total losses in watts}} \times 100 \tag{3-7}$$

The total losses include the losses in the electric circuit, magnetic circuit, and dielectric circuit. Stigant and Franklin [3, p. 97] state that a transformer has its highest efficiency at a load at which the iron loss and copper loss are equal. Therefore the load at which the efficiency is highest can be found from

$$\%\,\text{load} = \left(\frac{\text{iron loss}}{\text{copper loss}}\right)^{1/2} \times 100 \tag{3-8}$$

Figures 3-4 and 3-5 show nomograms for quick determination of the efficiency of a transformer. (For more accurate values, consult the individual manufacturer's catalogs.) With the cost of electric energy presently 5 to 6 cents/kWh and projected to double within the next 10 to 15 years, as shown in Fig. 3-6, the cost efficiency of transformers now shifts to align itself with energy efficiency.

Note that the iron losses (or core losses) include (1) hysteresis loss and (2) eddy-current loss. The hysteresis loss is due to the power requirement of maintaining the continuous reversals of the elementary magnets (or individual molecules) of which the iron is composed as a result of the flux alternations in a transformer core. The eddy-current loss is the loss due to circulating currents in the core iron, caused by the magnetic flux in the iron cutting the iron, which is a conductor. The eddy-current loss is proportional to the square of the frequency and the square of the flux density. The core is built up of thin laminations insulated from each other by an insulating coating on the iron to reduce the eddy-current loss. Also, in order to reduce the hysteresis loss and the eddy-current loss, special grades of steel alloyed with silicon are used. The iron or core losses are practically independent of the load. On the other hand, the copper losses are due to the resistance of the primary and secondary windings.

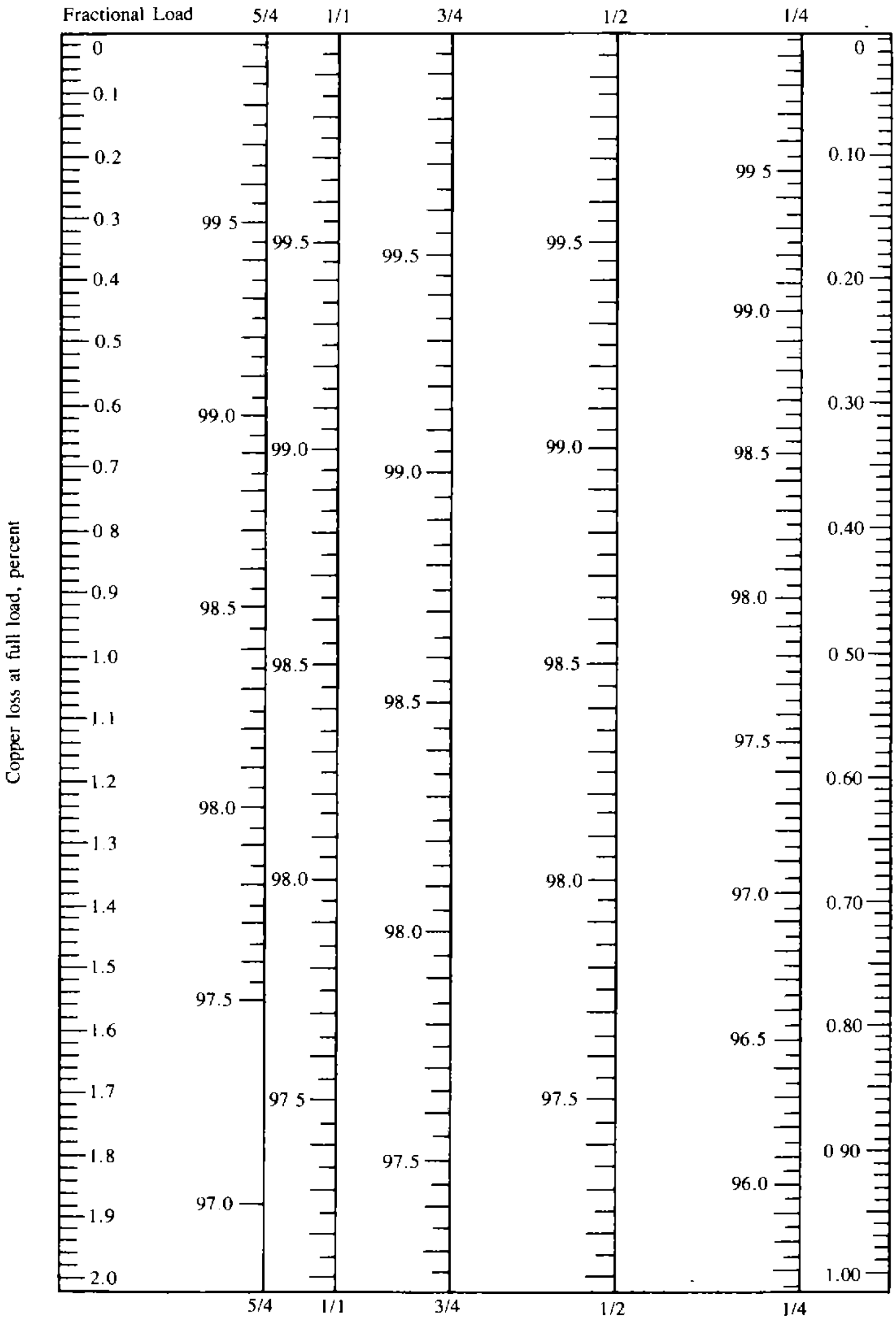


Figure 3-4 Transformer efficiency chart applicable only to the unity PF condition. To obtain the efficiency at a given load, lay a straightedge across the iron and copper loss values and read the efficiency at the point where the straightedge cuts the required load ordinate. (*From* [3].)

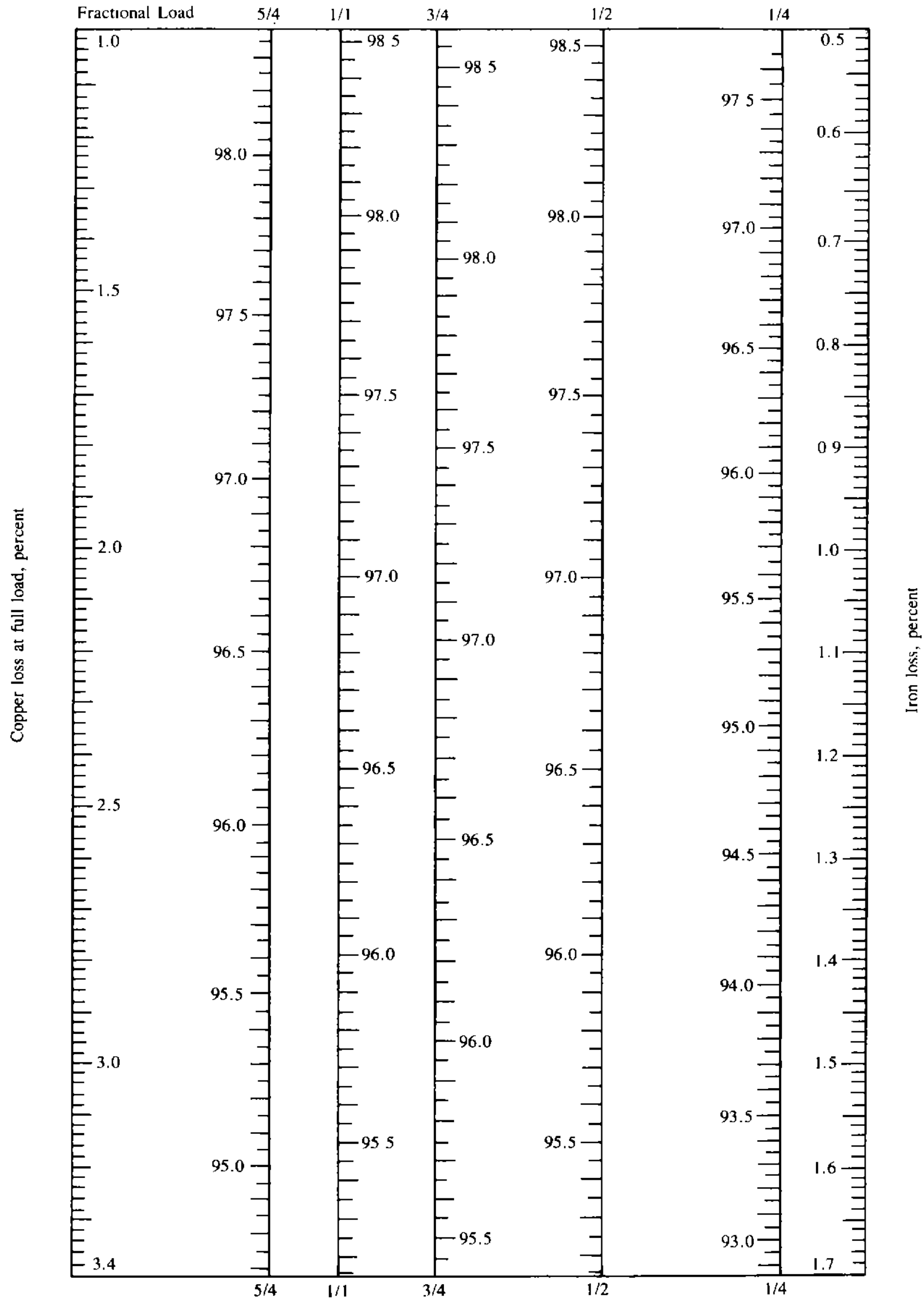


Figure 3-5 Transformer efficiency chart applicable only to the unity PF condition. To obtain the efficiency at a given load, lay a straightedge across the iron and copper loss values and read the efficiency at the point where the straightedge cuts the required load ordinate. (*From* [*3*].)

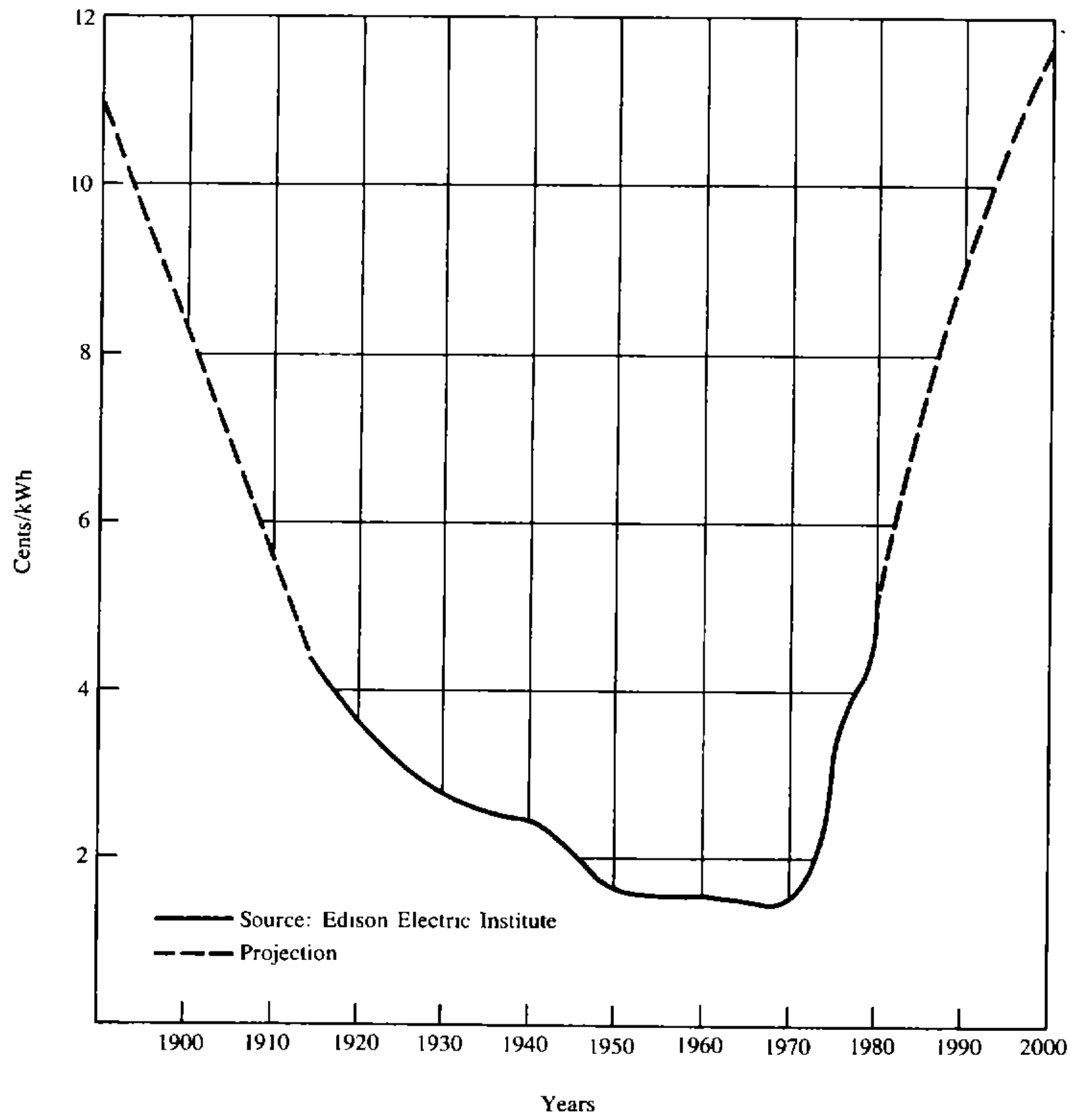


Figure 3-6 Cost of electric energy.

In general, the distribution transformer costs can be classified as (1) the cost of investment, (2) the cost of lost energy due to the losses in the transformer, and (3) the cost of demand lost (i.e., the cost of lost capacity) due to the losses in the transformer. Of course, the cost of investment is the largest cost component, and it includes the cost of the transformer itself and the costs of material and labor involved in the transformer installation.

Figure 3-7 shows the annual cost per unit load v. load level. At low-load levels, the relatively high costs result basically from the investment cost, whereas at high-load levels, they are due to the cost of additional loss of life of the transformer, the cost of lost energy, and the cost of demand loss in addition to the investment cost. Figure 3-7 indicates an operating range close to the bottom of the curve. Usually, it is economical to install a transformer at approximately 80 percent of its nameplate rating and to replace it later, at approximately 180 percent, by one with a larger capacity. However, presently, increasing costs of capital, plant and equipment, and energy tend to reduce these percentages.

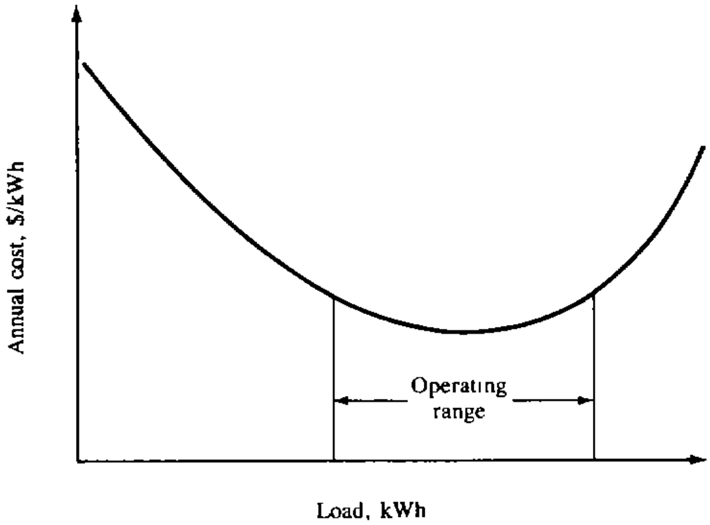


Figure 3-7 Annual cost per unit load vs. load level.

3-5 TERMINAL OR LEAD MARKINGS

The terminals or leads of a transformer are the points to which external electric circuits are connected. According to NEMA and ASA standards, the higher-voltage winding is identified by HV or H, and the lower-voltage winding is identified by LV or x. Transformers with more than two windings have the windings identified as H, x, y, and z, in order of decreasing voltage. The terminal H_1 is located on the right-hand side when facing the high-voltage side of the transformer. On single-phase transformers the leads are numbered so that when H_1 is connected to x_1, the voltage between the highest-numbered H lead and the highest-numbered x lead is less than the voltage of the high-voltage winding.

On three-phase transformers, the terminal H_1 is on the right-hand side when facing the high-voltage winding, with the H_2 and H_3 terminals in numerical sequence from right to left. The terminal x_1 is on the left-hand side when facing the low-voltage winding, with the x_2 and x_3 terminals in numerical sequence from left to right.

3-6 TRANSFORMER POLARITY

Transformer-winding terminals are marked to show polarity, to indicate the high- from the low-voltage side. Primary and secondary are not identified as such because which is which depends on input and output connections.

Transformer polarity is an indication of the direction of current flowing through the high-voltage leads with respect to the direction of current flow through the low-voltage leads at any given instant. In other words, the transformer polarity simply refers to the relative direction of induced voltages between the high-voltage leads and the low-voltage terminals. The polarity of a single-phase distribution transformer may be additive or subtractive. With standard markings, the voltage from H_1 to H_2 is always in the same direction or in phase with the voltage from

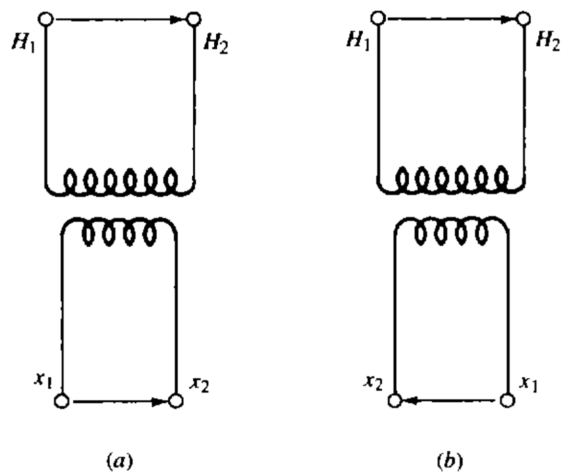


Figure 3-8 Additive and subtractive polarity connections: (*a*) subtractive polarity and (*b*) additive polarity.

x_1 to x_2. In a transformer where H_1 and x_1 terminals are adjacent, as shown in Fig. 3-8*a*, the transformer is said to have *subtractive* polarity. On the other hand, when terminals H_1 and x_1 are diagonally opposite, as shown in Fig. 3-8*b*, the transformer is said to have *additive* polarity.

Transformer polarity can be determined by performing a simple test in which two adjacent terminals of the high- and low-voltage windings are connected together and a moderate voltage is applied to the high-voltage winding, as shown in Fig. 3-9, and then the voltage between the high- and low-voltage winding terminals that are not connected together are measured. The polarity is subtractive if the voltage read is less than the voltage applied to the high-voltage winding, as shown in Fig. 3-9*a*. The polarity is additive if the voltage read is greater than the applied voltage, as shown in Fig. 3-9*b*.

By industry standards, all single-phase distribution transformers 200 kVA and smaller, having high voltages 8660 V and below (winding voltages), have additive polarity. All other single-phase transformers have a subtractive polarity. Polarity markings are very useful when connecting transformers into three-phase banks.

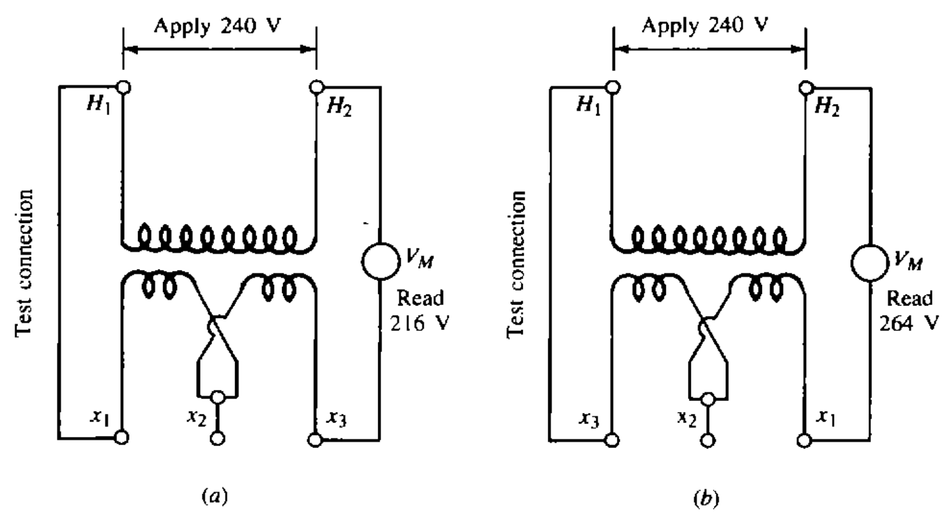


Figure 3-9 Polarity test: (*a*) subtractive polarity and (*b*) additive polarity.

3-7 DISTRIBUTION TRANSFORMER LOADING GUIDES

The rated kilovoltamperes of a given transformer is the output which can be obtained continuously at rated voltage and frequency without exceeding the specified temperature rise. Temperature rise is used for rating purposes rather than actual temperature, since the ambient temperature may vary considerably under operating conditions. The life of insulation commonly used in transformers depends upon the temperature the insulation reaches and the length of time that this temperature is sustained. Therefore, before the overload capabilities of the transformer can be determined, the ambient temperature, preload conditions, and the duration of peak loads must be known.

Based on Appendix C57.91 entitled *The Guide for Loading Mineral Oil–Immersed Overhead-Type Distribution Transformers with 55°C and 65°C Average Winding Rise* [4], which is an appendix to the ANSI Overhead Distribution Standard C57.12, twenty transformer insulation-life curves were developed. These curves indicate a minimum life expectancy of 20 years at 95°C and 110°C hot-spot temperatures for 55°C and 65°C rise transformers. Previous transformer loading guides were based on the so-called 8°C insulation-life rule. For example, for transformers with class A insulation (usually oil-filled), the rate of deterioration doubles approximately with each 8°C increase in temperature. In other words, if a class A insulation transformer were operated 8°C above its rated temperature, its life would be cut in half.

3-8 EQUIVALENT CIRCUITS OF A TRANSFORMER

It is possible to use several equivalent circuits to represent a given transformer. However, the general practice is to choose the simplest one which would provide the desired accuracy in calculations.

Figure 3-10 shows an equivalent circuit of a single-phase two-winding transformer. It represents a practical transformer with an iron core and connected to a load (Z_L). When the primary winding is excited, a flux is produced through the iron core. The flux that links both primary and secondary is called the *mutual flux*, and its maximum value is denoted as ϕ_m. However, there are also leakage fluxes

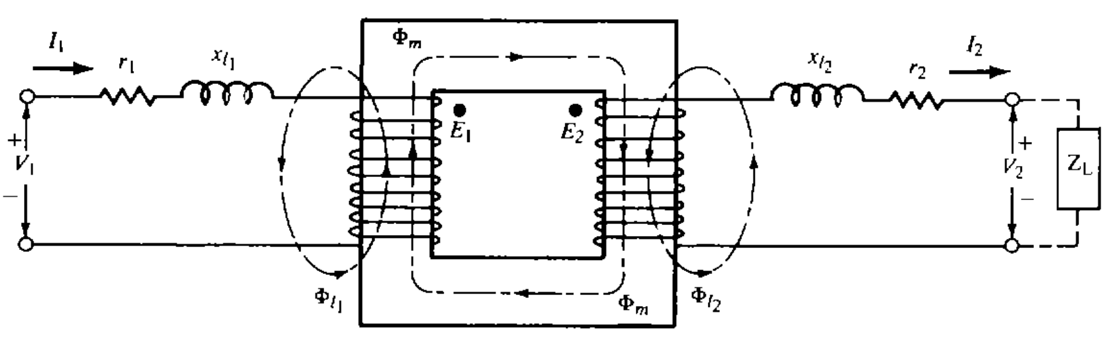


Figure 3-10 Basic circuit of a practical transformer.

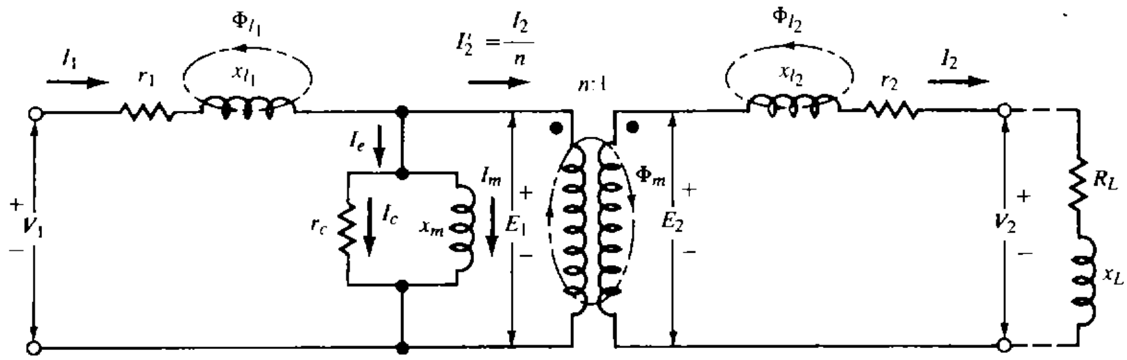


Figure 3-11 Equivalent circuit of a loaded transformer.

ϕ_{l1} and ϕ_{l2} that are produced at the primary and secondary windings, respectively. In turn, the ϕ_{l1} and ϕ_{l2} leakage fluxes produce x_{l1} and x_{l2}, that is, primary and secondary inductive reactances, respectively. Of course, the primary and secondary windings also have their internal resistances of r_1 and r_2.

Figure 3-11 shows an equivalent circuit of a loaded transformer. Note that I_2' current is a primary-current (or load) component which exactly corresponds to the secondary current I_2, as it does for an ideal transformer. Therefore

$$I_2' = \frac{n_2}{n_1} I_2 \tag{3-9}$$

or

$$I_2' = \frac{I_2}{n} \tag{3-10}$$

where I_2 = secondary current
n_1 = number of turns in primary winding
n_2 = number of turns in secondary winding

$$n = \text{turns ratio} = \frac{n_1}{n_2} \tag{3-11}$$

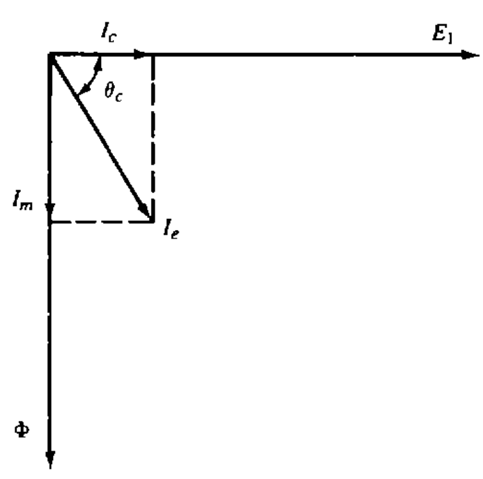


Figure 3-12 Phasor diagram corresponding to the excitation-current components.

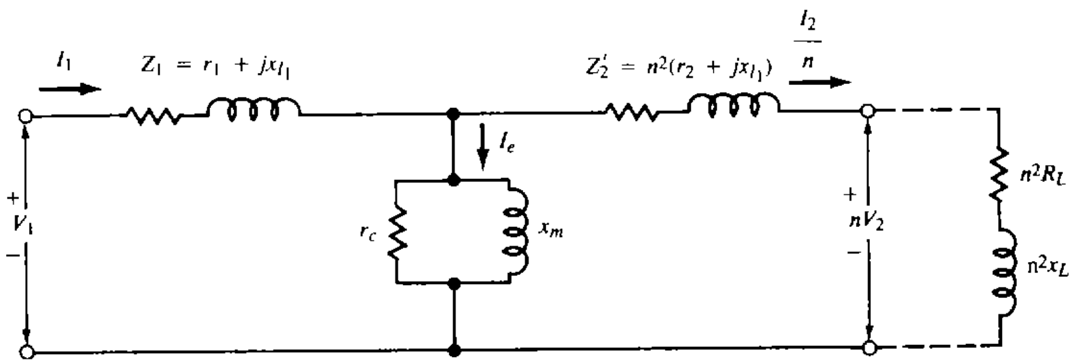


Figure 3-13 Equivalent circuit with the referred secondary values.

The I_e current is the excitation-current component of the primary current I_1 that is needed to produce the resultant mutual flux. As shown in Fig. 3-12, the excitation current I_e also has two components, namely, (1) the magnetizing-current component I_m and (2) the core-loss component I_c. The r_c represents the equivalent transformer power loss due to (hysteresis and eddy current) iron losses in the transformer core as a result of the magnetizing current I_m. The x_m represents the inductive reactance components of the transformer with an open secondary.

Figure 3-13 shows an approximate equivalent circuit with combined primary and reflected secondary and load impedances. Note that the secondary current I_2 is seen by the primary side as I_2/n and that the secondary and load impedances are transferred (or referred) to the primary side as $n^2(r_2 + jx_{l2})$ and $n^2(R_L + jX_L)$, respectively. Also note that the secondary-side terminal voltage V_2 is transferred as nV_2.

Since the excitation current I_e is very small with respect to I_2/n for a loaded transformer, the former may be ignored, as shown in Fig. 3-14. Therefore the equivalent impedance of the transformer referred to the primary is

$$\begin{aligned} Z_{eq} &= Z_1 + Z_2' \\ &= Z_1 + n^2 Z_2 \\ &= r_{eq} + jx_{eq} \end{aligned} \tag{3-12}$$

where

$$Z_1 = r_1 + jx_{l1} \tag{3-13}$$

$$Z_2 = r_2 + jx_{l2} \tag{3-14}$$

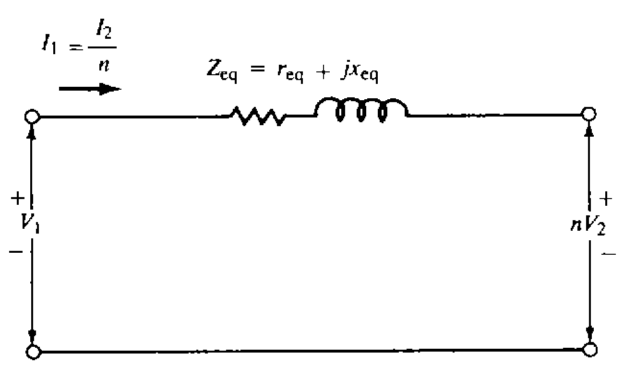


Figure 3-14 Simplified equivalent circuit assuming negligible excitation current.

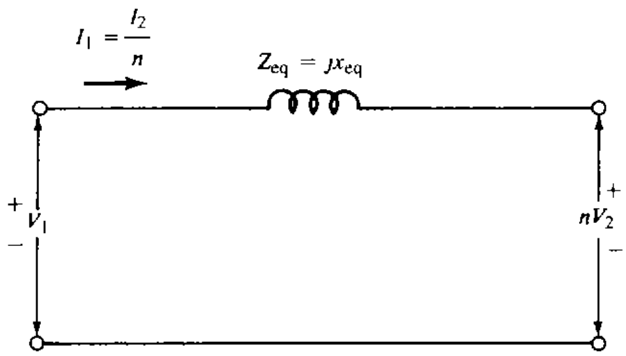


Figure 3-15 Simplified equivalent circuit for a large-size power transformer

and therefore the equivalent resistance and reactance of the transformer referred to the primary are

$$r_{eq} = r_1 + n^2 r_2 \tag{3-15}$$

and

$$x_{eq} = x_1 + n^2 x_2 \tag{3-16}$$

As before in Fig. 3-15, for large-size power transformers,

$$r_{eq} \to 0$$

therefore the equivalent impedance of the transformer becomes as

$$Z_{eq} = jx_{eq} \tag{3-17}$$

3-9 SINGLE-PHASE TRANSFORMER CONNECTIONS

3-9-1 General

At the present time, the single-phase distribution transformers greatly outnumber the polyphase ones. This is partially due to the fact that lighting and the smaller power loads are supplied at single-phase from single-phase secondary circuits. Also, most of the time, even polyphase secondary systems are supplied by single-phase transformers which are connected as polyphase banks.

Single-phase distribution transformers have one high-voltage primary winding and two low-voltage secondary windings which are rated at a nominal 120 V. Earlier transformers were built with four insulated secondary leads brought out of the transformer tank, the series or parallel connection being made outside the tank. Presently, in modern transformers, the connections are made inside the tank, with only three secondary terminals being brought out of the transformer.

Single-phase distribution transformers have one high-voltage primary winding and two low-voltage secondary windings. Figure 3-16 shows various connection diagrams for single-phase transformers supplying single-phase loads. Secondary coils each rated at a nominal 120 V may be connected in parallel to supply a two-wire 120-V circuit, as shown in Fig. 3-16*a* and *b*, or they may be connected in series

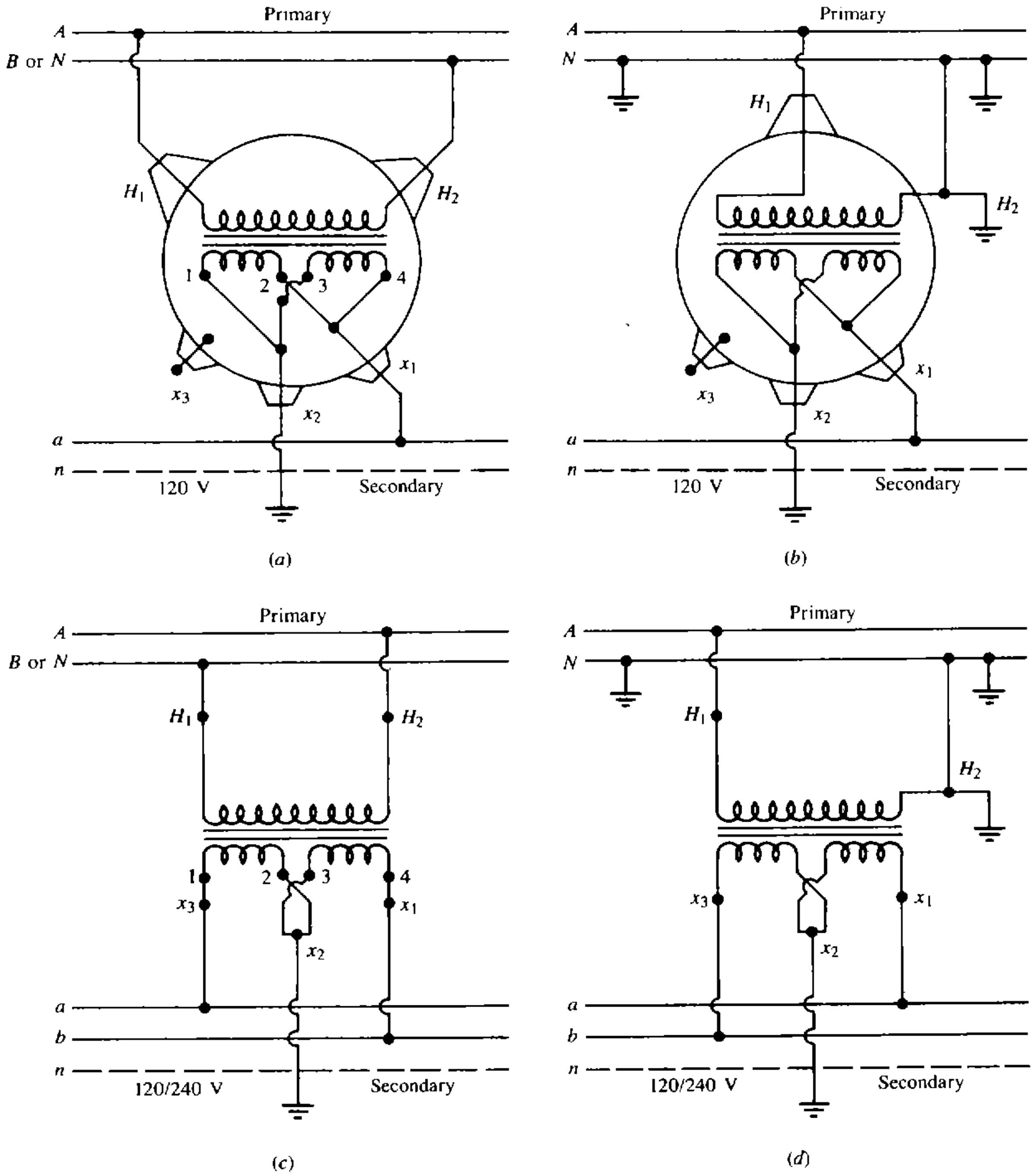


Figure 3-16 Single-phase transformer connections.

to supply a three-wire 120/240-V single circuit, as shown in Fig. 3-16*c* and *d*. The connections shown in Fig. 3-16*a* and *b* are used where the loads are comparatively small and the length of the secondary circuits is short. It is often used for a single customer who requires only 120-V single-phase power. However, for modern homes, this connection usually is not considered adequate. If a mistake is made in polarity when connecting the two secondary coils in parallel (see Fig. 3-16*a*) so that the low-voltage terminal 1 is connected to terminal 4 and terminal 2 to terminal 3, the result will be a short-circuited secondary which will blow the fuses that are installed on the high-voltage side of the transformer (they are not

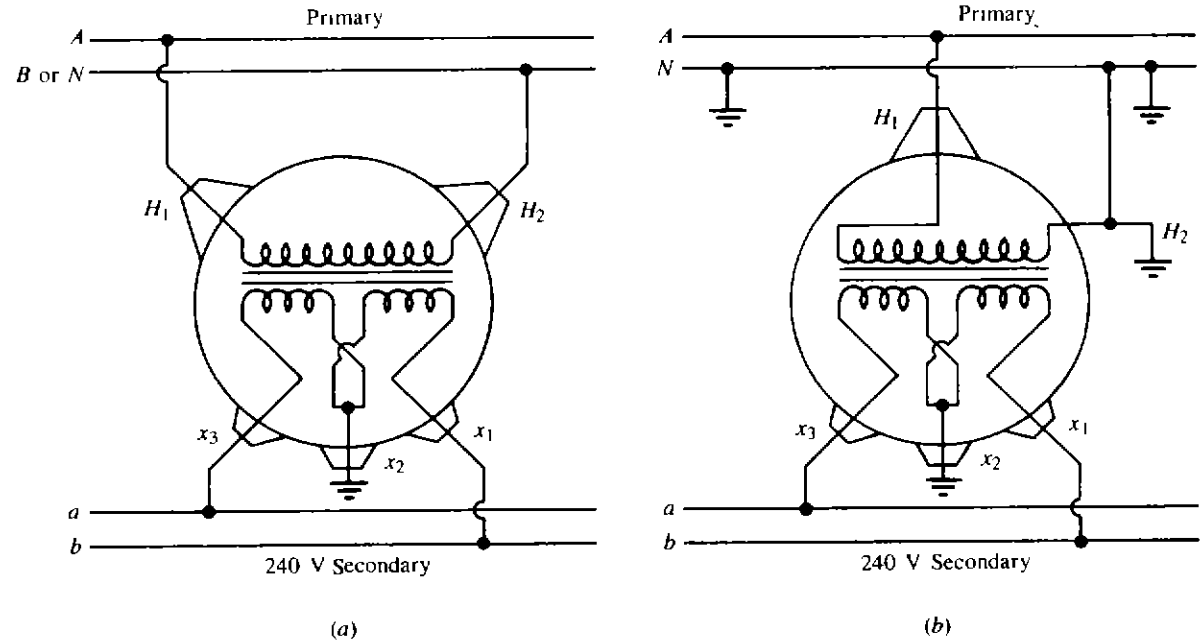


Figure 3-17 Single-phase transformer connections.

shown in the figure). On the other hand, a mistake in polarity when connecting the coils in series (see Fig. 3-16*c*) will result in the voltage across the outer conductors being zero instead of 240 V. Taps for voltage adjustment, if provided, are located on the high-voltage winding of the transformer. Figure 3-16*b* and *d* shows single-bushing transformers connected to a multigrounded primary. They are used on 12,470GndY/7,200, 13,200GndY/7,620, and 24,940GndY/14,400 V multigrounded neutral systems. It is crucial that good and solid grounds are maintained on the transformer and on the system. Figure 3-17 shows single-phase transformer connections for single- and two-bushing transformers to provide customers who require only 240-V single-phase power. These connections are used for small industrial applications.

In general, however, the 120/240-V three-wire connection system is preferred since it has twice the load capacity of the 120-V system with only $1\frac{1}{2}$ times the amount of conductor. Here, each 120-V winding has one-half the total kilovoltampere rating of the transformer. Therefore, if the connected 120-V loads are equal, the load is balanced and no current flows in the neutral conductor. Thus the loads connected to the transformer must be held as nearly balanced as possible to provide the most economical usage of transformer capacity and to keep regulation to a minimum. Normally, one leg of the 120-V two-wire system and the middle leg of the 240-V two-wire or 120/240-V three-wire system is grounded to limit the voltage to ground on the secondary circuit to a minimum.

3-9-2 Single-phase Transformer Paralleling

When greater capacity is required in emergency situations, two single-phase transformers of the same or different kilovoltampere ratings can be connected in parallel. The single-phase transformers can be of either additive or subtractive polarity as long as the following conditions are observed and connected, as shown in Fig. 3-18.

1. All transformers have the same turns ratio.
2. All transformers are connected to the same primary phase.
3. All transformers have identical frequency ratings.
4. All transformers have identical voltage ratings.
5. All transformers have identical tap settings.
6. Per unit impedance of one transformer is between 0.925 and 1.075 of the other in order to maximize capability.

However, paralleling two single-phase transformers is not economical since the total cost and losses of two small transformers are much larger than one large transformer with the same capacity. Therefore, it should be used only as a temporary remedy to provide for increased demands for single-phase power in emergency situations. Figure 3-19 shows two single-phase transformers, each with

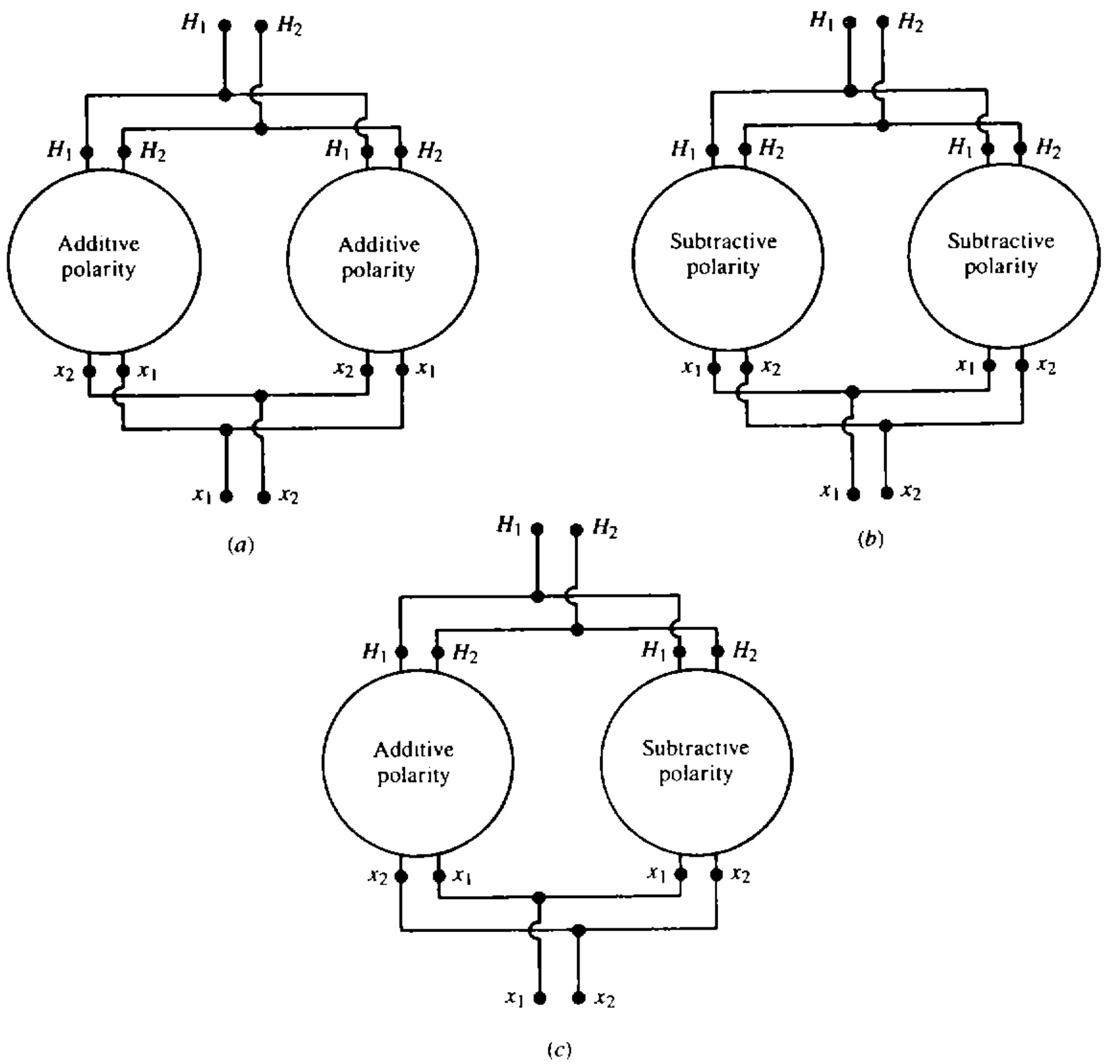


Figure 3-18 Single-phase transformer paralleling.

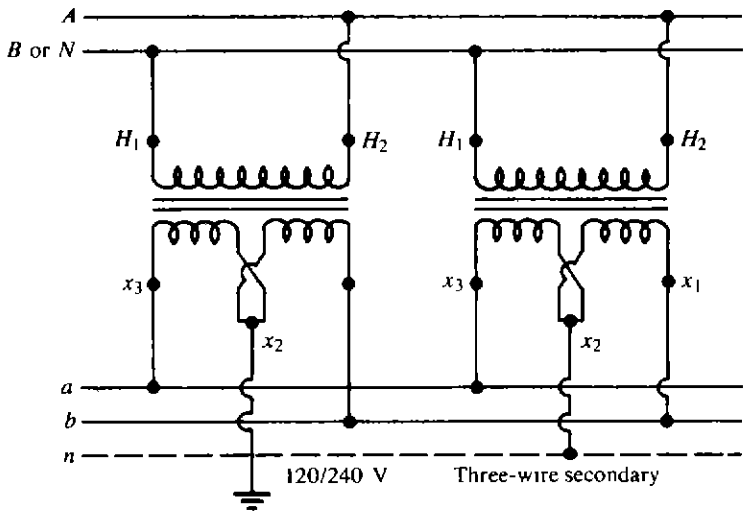


Figure 3-19 Parallel operation of two single-phase transformers.

two bushings, connected to a two-conductor primary to supply 120/240-V single-phase power on a three-wire secondary.

To illustrate load division among the parallel-connected transformers, consider the two transformers connected in parallel and feeding a load, as shown in Fig. 3-20. Assume that the aforementioned conditions for paralleling have already been met.

Figure 3-21 shows the corresponding equivalent circuit referred to as the *low-voltage side*. Since the transformers are connected in parallel, the voltage drop through each transformer must be equal. Therefore

$$I_1(Z_{eq,T1}) = I_2(Z_{eq,T2}) \tag{3-18}$$

from which

$$\frac{I_1}{I_2} = \frac{Z_{eq,T2}}{Z_{eq,T1}} \tag{3-19}$$

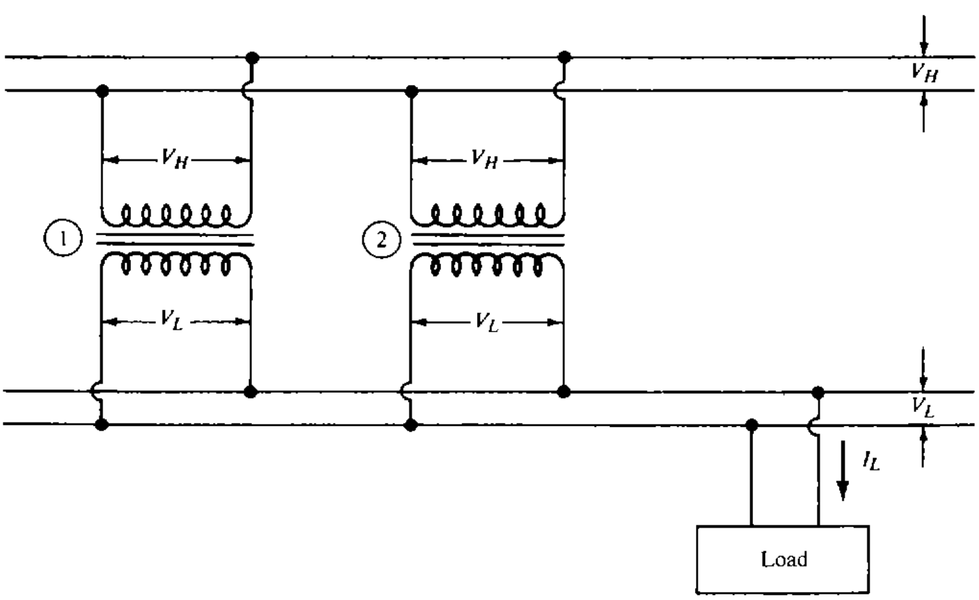


Figure 3-20

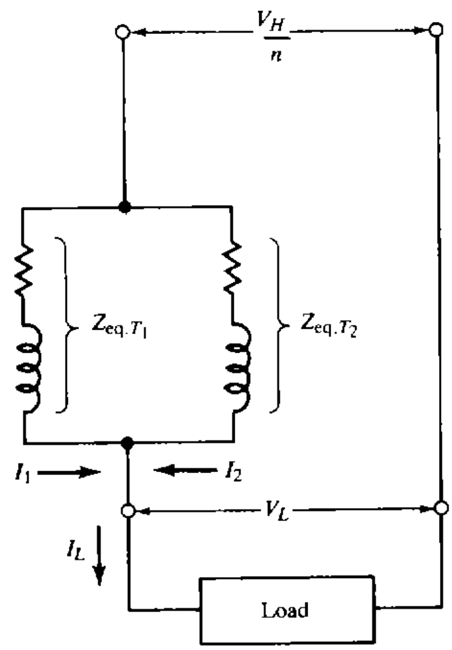


Figure 3-21

where I_1 = secondary current of transformer 1
I_2 = secondary current of transformer 2
I_L = load current
$Z_{eq,T1}$ = equivalent impedance of transformer 1
$Z_{eq,T2}$ = equivalent impedance of transformer 2

From Eq. (3-19) it can be seen that the load division is determined only by the relative ohmic impedance of the transformers. If the ohmic impedances in Eq. (3-19) are replaced by their equivalent in terms of percent impedance, the following equation can be found,

$$\frac{I_1}{I_2} = \frac{(\%Z)_{T2}}{(\%Z)_{T1}} \frac{S_{T1}}{S_{T2}} \tag{3-20}$$

where $(\% Z)_{T1}$ = percent impedance of transformer 1
$(\% Z)_{T2}$ = percent impedance of transformer 2
S_{T1} = kilovoltampere rating of transformer 1
S_{T2} = kilovoltampere rating of transformer 2

Equation (3-20) can be expressed in terms of kilovoltamperes supplied by each transformer since the primary and the secondary voltages for each transformer are the same, respectively. Therefore

$$\frac{S_{L1}}{S_{L2}} = \frac{(\%Z)_{T2}}{(\%Z)_{T1}} \frac{S_{T1}}{S_{T2}} \tag{3-21}$$

where S_{L1} = kilovoltamperes supplied by transformer 1 to the load
S_{L2} = kilovoltamperes supplied by transformer 2 to the load

Example 3-1 Figure 3-22 shows an equivalent circuit of a single-phase transformer with three-wire secondary for three-wire single-phase distribution. The typical distribution transformer is rated as 25 kVA, 7200–120/240 V, 60 Hz, and has the following per unit impedance based on the transformer ratings and based on use of the entire low-voltage (LV) winding with zero neutral current:

$$R_T = 0.014 \text{ pu}$$

and

$$X_T = 0.012 \text{ pu}$$

Here, the two halves of the low voltage may be independently loaded, and, in general, the three-wire secondary load will not be balanced. Therefore, in general, the equivalent circuit needed is that of a three-winding single-phase transformer as shown in Fig. 3-22, when voltage drops and/or fault currents are to be computed. Thus use the meager amount of data (it is all that is usually available) and evaluate numerically all the impedances shown in Fig. 3-22.

SOLUTION Figure 3-22 is based on the reference by Lloyd [1]. To determine $\bar{Z}_{HX_{1-2}}$ approximately, Lloyd gives the following formula:

$$\bar{Z}_{HX_{1-2}} = 1.5R_T + j1.2x_T \tag{3-22}$$

where $\bar{Z}_{HX_{1-2}}$ = the transformer impedance referred to high-voltage winding when the section of the low-voltage winding between the terminals x_2 and x_3 is short-circuited

From Fig. 3-22, the turns ratio of the transformer is

$$n = \frac{V_H}{V_x} = \frac{7200 \text{ V}}{120 \text{ V}} = 60$$

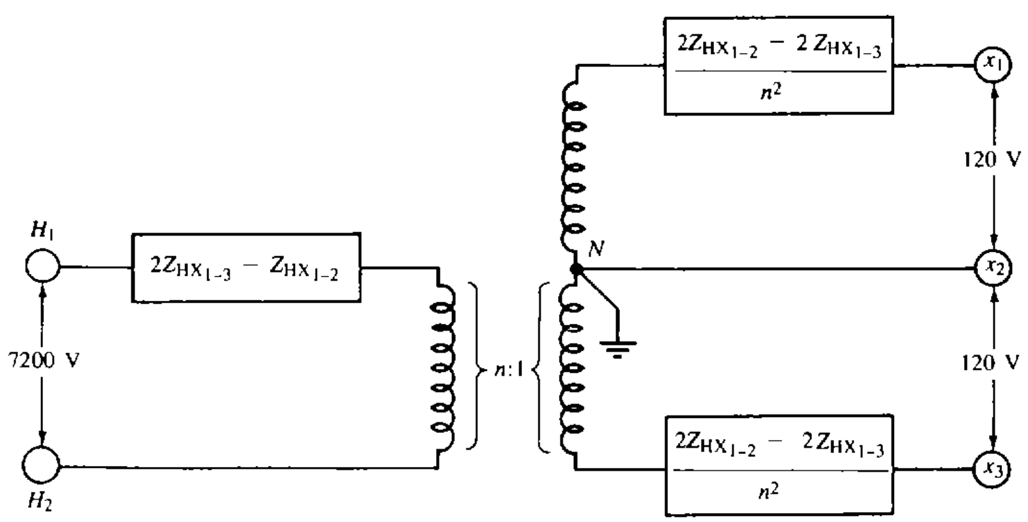


Figure 3-22 (*Based on* [1].)

Since the given per unit impedances of the transformer are based on the use of the entire low-voltage winding,

$$\begin{aligned}\bar{Z}_{HX_{1-3}} &= R_T + jX_T \\ &= 0.014 + j0.012 \text{ pu}\end{aligned}$$

Also, from Eq. (3-22),

$$\begin{aligned}\bar{Z}_{HX_{1-2}} &= 1.5R_T + j1.2X_T \\ &= 1.5 \times 0.014 + j1.2 \times 0.012 \\ &= 0.021 + j0.0144 \text{ pu}\end{aligned}$$

Therefore

$$\begin{aligned}2\bar{Z}_{HX_{1-3}} - \bar{Z}_{HX_{1-2}} &= 2(0.014 + j0.012) - (0.021 + j0.0144) \\ &= 0.007 + j0.0096 \text{ pu} \\ &= 14.515 + j19.906 = 24.637\underline{/53.9^\circ}\ \Omega\end{aligned}$$

and

$$\begin{aligned}\frac{2\bar{Z}_{HX_{1-2}} - 2\bar{Z}_{HX_{1-3}}}{n^2} &= \frac{2(0.021 + j0.0144) - 2(0.014 + j0.012)}{60^2} \\ &= 3.89 \times 10^{-6} + j1.334 \times 10^{-6} \text{ pu} \\ &= 0.008064 + j0.0028 = 8.525 \times 10^{-3}\underline{/18.9^\circ}\ \Omega\end{aligned}$$

Example 3-2 Using the transformer equivalent circuit found in Example 3-1, determine the line-to-neutral (120 V) and line-to-line (240 V) fault currents in three-wire single-phase 120/240-V secondaries shown in Fig. 3-23 and 3-24, respectively. In the figures, R represents the resistance of service-drop cable per conductor. Usually R is much larger than X for such cable and therefore X may be neglected.

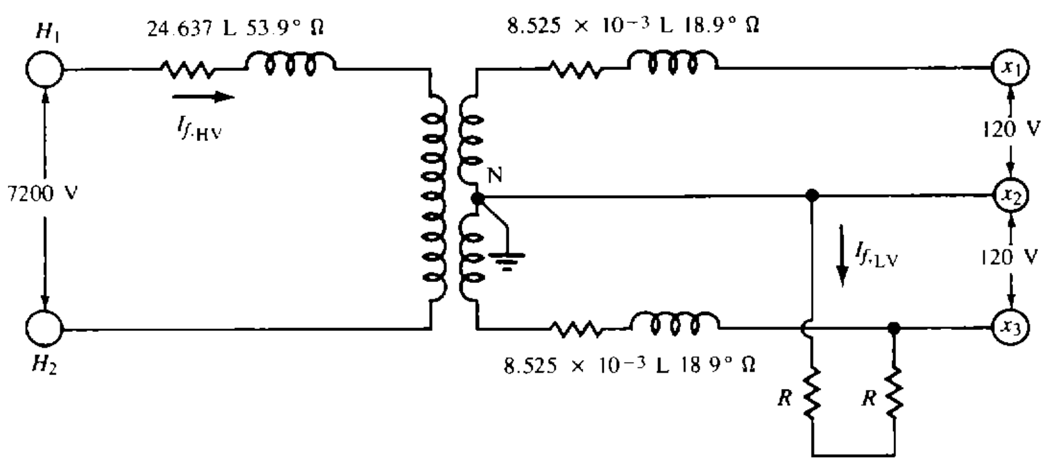


Figure 3-23 Secondary line-to-neutral fault.

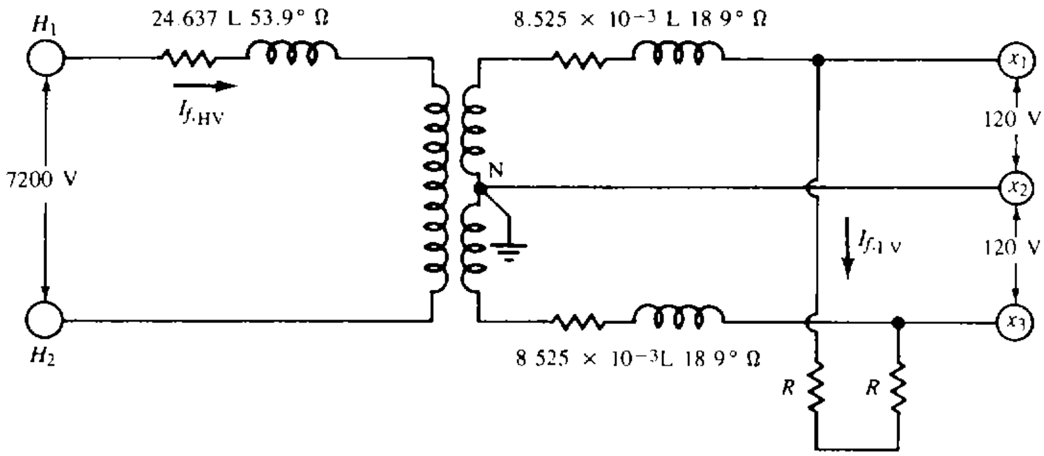


Figure 3-24 Secondary line-to-line fault.

Using the given data, determine the following:

(*a*) Find the symmetrical rms fault currents in the high-voltage (HV) and low-voltage (LV) circuits for a 120-V fault if the R of the service-drop cable is zero.

(*b*) Find the symmetrical rms fault currents in the high-voltage and low-voltage circuits for a 240-V fault if the R of the service-drop cable is zero.

(*c*) If the transformer is a CSPB type, find the minimum allowable interrupting capacity (in symmetrical rms amperes) for a circuit breaker connected to the transformer's low-voltage terminals.

SOLUTION

(*a*) When $R = 0$, from Fig. 3-23, the line-to-neutral fault current in the secondary side of the transformer is

$$\bar{I}_{f,\mathrm{LV}} = \frac{120}{8.525 \times 10^{-3}\angle 18.9^\circ + (\frac{1}{60})^2(24.637\angle 53.9^\circ)}$$

$$= 8181.7\angle -34.4^\circ \text{ A}$$

Thus the fault current in the high-voltage side is

$$\bar{I}_{f,\mathrm{HV}} = \frac{\bar{I}_{f,\mathrm{LV}}}{n}$$

$$= \frac{8181.7}{60} = 136.4 \text{ A}$$

Note that the turns ratio is found as

$$n = \frac{7200 \text{ V}}{120 \text{ V}} = 60$$

(*b*) When $R = 0$, from Fig. 3-24, the line-to-line fault current in the secondary side of the transformer is

$$\bar{I}_{f,\text{LV}} = \frac{240}{2(8.525 \times 10^{-3}\underline{/18.9^\circ}) + (\frac{1}{30})^2(24.637\underline{/53.9^\circ})}$$

$$= 5649\underline{/-40.6^\circ} \text{ A}$$

Thus the fault current in the high-voltage side is

$$\bar{I}_{f,\text{HV}} = \frac{\bar{I}_{f,\text{LV}}}{n}$$

$$= \frac{5649}{30}$$

$$= 188.3 \text{ A}$$

Note that the turns ratio is found as

$$n = \frac{7200 \text{ V}}{240 \text{ V}} = 30$$

(*c*) Therefore the minimum allowable interrupting capacity for a circuit breaker connected to the transformer low-voltage terminals is 8181.7 A.

Example 3-3 Using the data given in Example 3-2, determine the following:

(*a*) Estimate approximately the value of the R, that is, the service-drop cable's resistance, that will produce equal line-to-line and line-to-neutral fault currents.

(*b*) If the conductors of the service-drop cable are aluminum, find the length of the service-drop cable that would correspond to the resistance R found in part *a* in case of (1) #4 AWG conductors with a resistance of 2.58 Ω/mi and (2) #1/0 AWG conductors with a resistance of 1.03 Ω/mi.

SOLUTION

(*a*) Since the line-to-line and the line-to-neutral fault currents are supposed to be equal to each other,

$$\frac{240}{2R + 0.032256 + j0.02765} = \frac{120}{2R + 0.012096 + j0.0083}$$

or

$$R \cong 0.0075 \ \Omega$$

(*b*) The length of the service-drop cable is:

(i) If #4 AWG aluminum conductors with a resistance of 2.58 Ω/mi or 4.886×10^{-4} Ω/ft are used,

$$\text{Service-drop length} = \frac{R}{4.886 \times 10^{-4}}$$
$$= \frac{0.0075\ \Omega}{4.886 \times 10^{-4}\ \Omega/\text{ft}}$$
$$\cong 15.35 \text{ ft}$$

(ii) If #1/0 AWG aluminum conductors with a resistance of 1.03 Ω/mi or 1.9508×10^{-4} Ω/ft are used,

$$\text{Service-drop length} = \frac{0.0075\ \Omega}{1.9508 \times 10^{-4}\ \Omega/\text{ft}}$$
$$\cong 38.45 \text{ ft}$$

Example 3-4 Assume that a 250-kVA transformer with 2.4 percent impedance is paralleled with a 500-kVA transformer with 3.1 percent impedance. Determine the maximum load that can be carried without overloading either transformer. Assume that the maximum allowable transformer loading is 100 percent of the rating.

SOLUTION Designating the 250- and 500-kVA transformers as transformers 1 and 2, respectively, and using Eq. (3-21),

$$\frac{S_{L1}}{S_{L2}} = \frac{(\%Z)_{T2}}{(\%Z)_{T1}} \frac{S_{T1}}{S_{T2}}$$
$$= \frac{3.1}{2.4} \times \frac{250}{500} = 0.6458$$

Assume a load of 500 kVA on the 500-kVA transformer. The preceding result shows that the load on the 250-kVA transformer will be 193.5 kVA when the load on the 500-kVA transformer is 500 kVA. Therefore the 250-kVA transformer becomes overloaded before the 500-kVA transformer. The load on the 500-kVA tranformer when the 250-kVA transformer is carrying rated load is

$$S_{L2} = \frac{S_{L1}}{0.6458}$$
$$= \frac{250}{0.6458}$$
$$= 387.1 \text{ kVA}$$

Thus the total load is

$$\sum_{i=1}^{2} S_{Li} = S_{L1} + S_{L2}$$
$$= 250 + 387.1$$
$$= 637.1 \text{ kVA}$$

3-10 THREE-PHASE CONNECTIONS

To raise or lower the voltages of three-phase distribution systems, either single-phase transformers can be connected to form three-phase transformer banks or three-phase transformers (having all windings in the same tank) are used.

Common methods of connecting three single-phase transformers for three-phase transformations are the delta-delta (Δ-Δ), wye-wye (Y-Y), wye-delta (Y-Δ), and delta-wye (Δ-Y) connections. Here, it is assumed that all transformers in the bank have the same kilovoltampere rating.

3-10-1 The Δ-Δ Transformer Connection

Figures 3-25 and 3-26 show the delta-delta connection formed by tying together single-phase transformers to provide 240-V service at 0 and 180° angular displacements, respectively.

This connection is often used to supply a small single-phase lighting load and three-phase power load simultaneously. To provide this type of service the mid-tap of the secondary winding of one of the transformers is grounded and connected to the secondary neutral conductor, as shown in Fig. 3-27. Therefore the single-phase loads are connected between the phase and neutral conductors. Thus the transformer with the mid-tap carries two-thirds of the 120/240-V single-phase load and one-third of the 240-V three-phase load. The other two units each carry one-third of both the 120/240- and 240-V loads.

There is no problem from third-harmonic overvoltage or telephone interference. However, high circulating currents will result unless all three single-phase transformers are connected on the same regulating taps and have the same voltage ratios. The transformer bank rating is decreased unless all transformers have identical impedance values. The secondary neutral bushing can be grounded on only one of the three single-phase transformers, as shown in Fig. 3-27.

Therefore, to get balanced transformer loading, the conditions include the following:

1. All transformers have identical voltage ratios.
2. All transformers have identical impedance values.
3. All transformers are connected on identical taps.

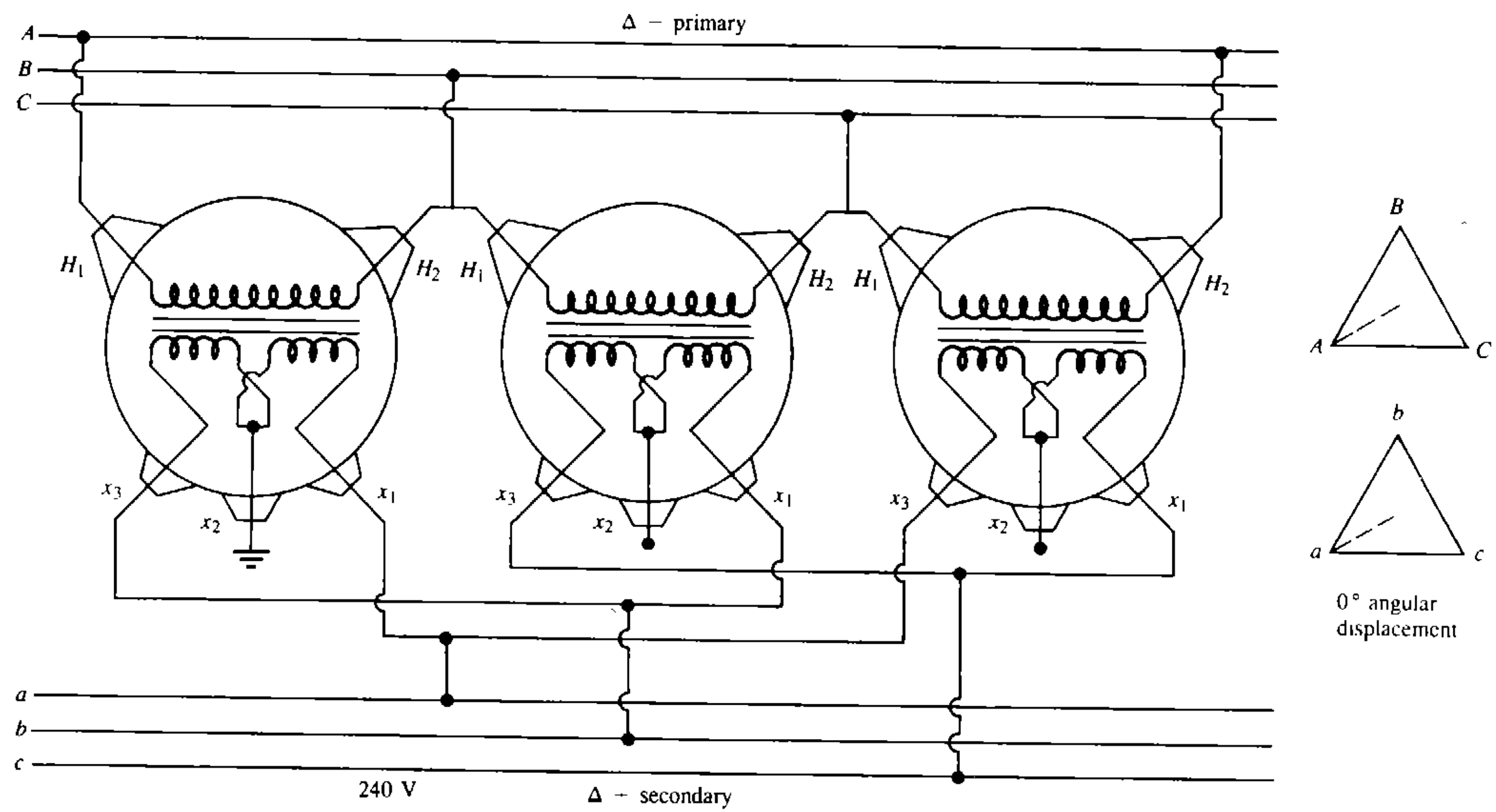


Figure 3-25 Delta-delta transformer bank connection with 0° angular displacement.

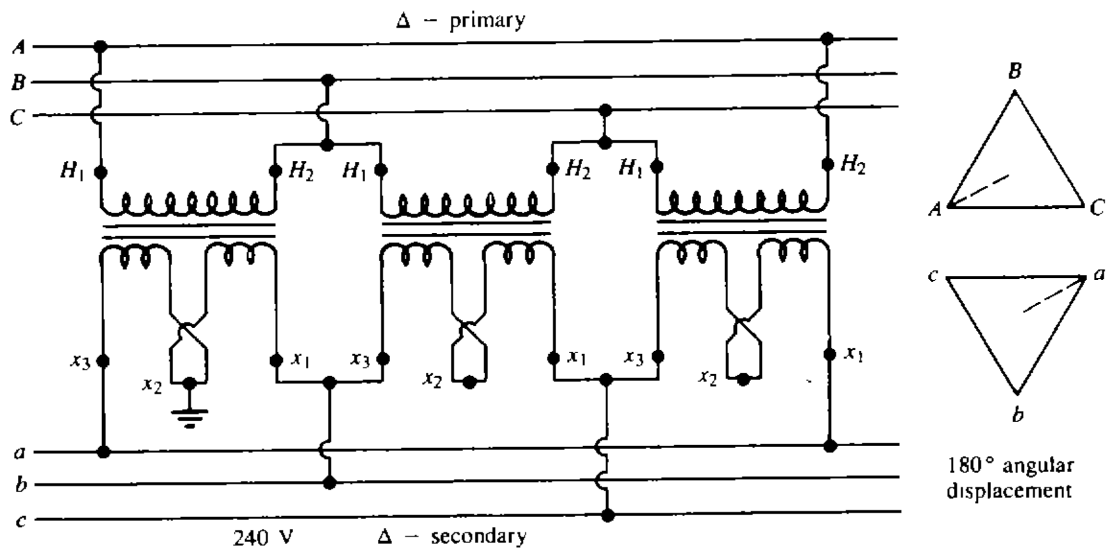


Figure 3-26 Delta-delta transformer bank connection with 180° angular displacement.

However, if two of the units have the identical impedance values and the third unit has an impedance value which is within, plus or minus, 25 percent of the impedance value of the like transformers, it is possible to operate the delta-delta bank, with a small unbalanced transformer loading, at reduced bank output capacity. Table 3-5 gives the permissible amounts of load unbalanced on the odd and like transformers. Note that Z_1 is the impedance of the odd transformer unit and Z_2 is the impedance of the like transformer units. Therefore, with unbalanced transformer loading, the load values have to be checked against the values of the table so that no one transformer is overloaded.

Assume that Fig. 3-28 shows the equivalent circuit of a delta-delta-connected transformer bank referred to the low-voltage side. A voltage-drop equation can be written for the low-voltage windings as

$$\bar{V}_{ba} + \bar{V}_{ac} + \bar{V}_{cb} = \bar{I}_{ba}\bar{Z}_{ab} + \bar{I}_{ac}\bar{Z}_{ca} + \bar{I}_{cb}\bar{Z}_{bc} \tag{3-23}$$

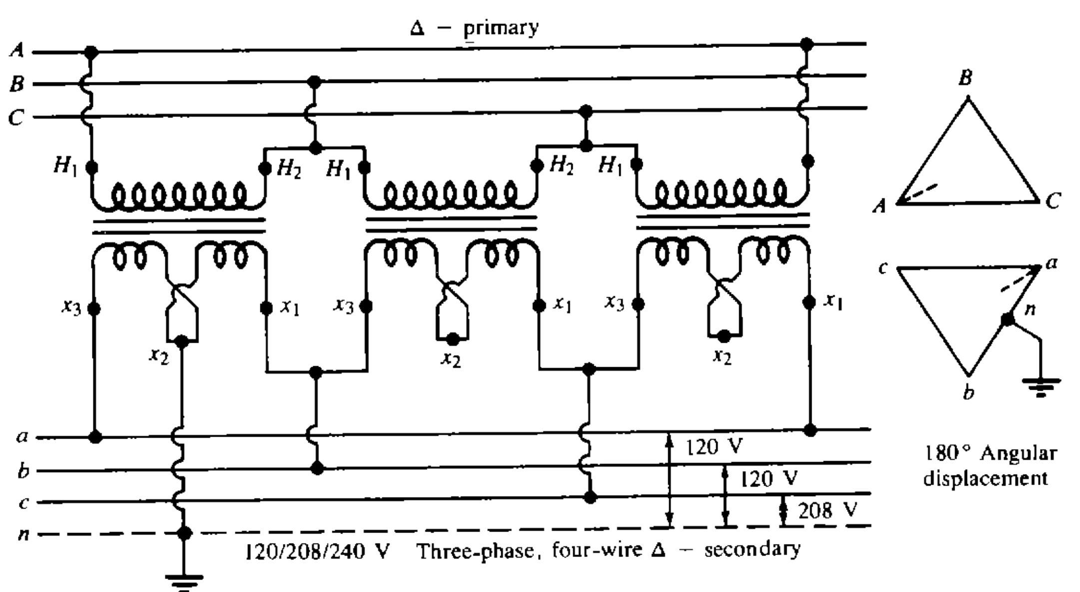


Figure 3-27 Delta-delta connection to provide 120/208/240 V three-phase four-wire service.

Table 3-5 The permissable percent loading on odd and like transformers as a function of the Z_1/Z_2 ratio

$\frac{Z_1}{Z_2}$ ratio	% load on Odd unit	% load on Like unit
0.75	109.0	96.0
0.80	107.0	96.5
0.85	105.2	97.3
0.90	103.3	98.3
1.10	96.7	102.0
1.15	95.2	102.2
1.20	93.8	103.1
1.25	92.3	103.9

where

$$\bar{V}_{ba} + \bar{V}_{ac} + \bar{V}_{cb} = 0 \tag{3-24}$$

Therefore Eq. (3-23) becomes

$$\bar{I}_{ba}\bar{Z}_{ab} + \bar{I}_{ac}\bar{Z}_{ca} + \bar{I}_{cb}\bar{Z}_{bc} = 0 \tag{3-25}$$

For the delta-connected secondary,

$$\bar{I}_a = \bar{I}_{ba} - \bar{I}_{ac} \tag{3-26}$$

$$\bar{I}_b = \bar{I}_{cb} - \bar{I}_{ba} \tag{3-27}$$

$$\bar{I}_c = \bar{I}_{ac} - \bar{I}_{cb} \tag{3-28}$$

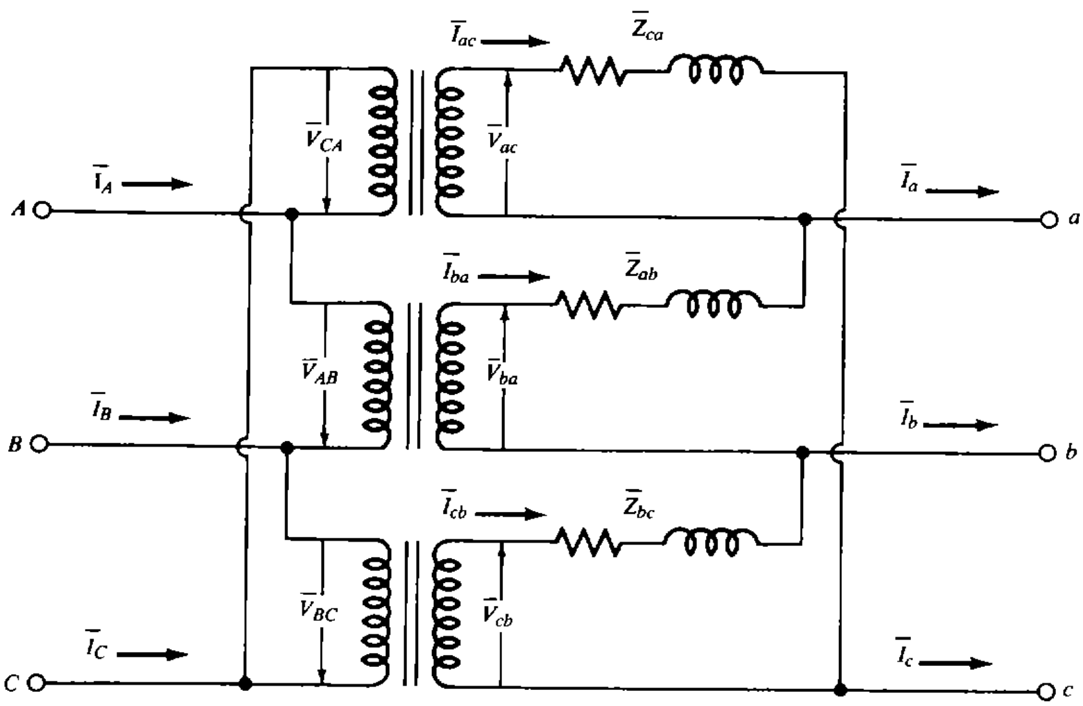


Figure 3-28 Equivalent circuit of a delta-delta-connected transformer bank.

From Eq. (3-25),

$$\bar{I}_{ba}\bar{Z}_{ab} = -\bar{I}_{ac}\bar{Z}_{ca} - \bar{I}_{cb}\bar{Z}_{bc} \tag{3-29}$$

Adding the terms of $\bar{I}_{ba}\bar{Z}_{bc}$ and $\bar{I}_{ba}\bar{Z}_{ca}$ to either side of Eq. (3-29) and substituting Eq. (3-26) into the resultant equation,

$$\bar{I}_{ba} = \frac{\bar{I}_a\bar{Z}_{ca} - \bar{I}_b\bar{Z}_{bc}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{3-30}$$

and similarly,

$$\bar{I}_{ac} = \frac{\bar{I}_c\bar{Z}_{bc} - \bar{I}_a\bar{Z}_{ab}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{3-31}$$

and

$$\bar{I}_{cb} = \frac{\bar{I}_b\bar{Z}_{ab} - \bar{I}_c\bar{Z}_{ca}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{3-32}$$

If the three transformers shown in Fig. 3-28 have equal percent impedance and equal ratios of percent reactance to percent resistance, then Eq. (3-30) to (3-32) can be expressed as

$$\bar{I}_{ba} = \frac{\dfrac{\bar{I}_a}{S_{T,ca}} - \dfrac{\bar{I}_b}{S_{T,bc}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3-33}$$

$$\bar{I}_{ac} = \frac{\dfrac{\bar{I}_c}{S_{T,bc}} - \dfrac{\bar{I}_a}{S_{T,ab}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3-34}$$

$$\bar{I}_{cb} = \frac{\dfrac{\bar{I}_b}{S_{T,ab}} - \dfrac{\bar{I}_c}{S_{T,ca}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3-35}$$

where $S_{T,ab}$ = kilovoltampere rating of the single-phase transformer connected between phases a and b

$S_{T,bc}$ = kilovoltampere rating of the single-phase transformer connected between phases b and c

$S_{T,ca}$ = kilovoltampere rating of the single-phase transformer connected between phases c and a

Example 3-5 Three single-phase transformers are connected delta-delta to provide power for a three-phase wye-connected 200-kVA load with a 0.80 lagging power factor and a 80-kVA single-phase light load with a 0.90 lagging power factor, as shown in Fig. 3-29.

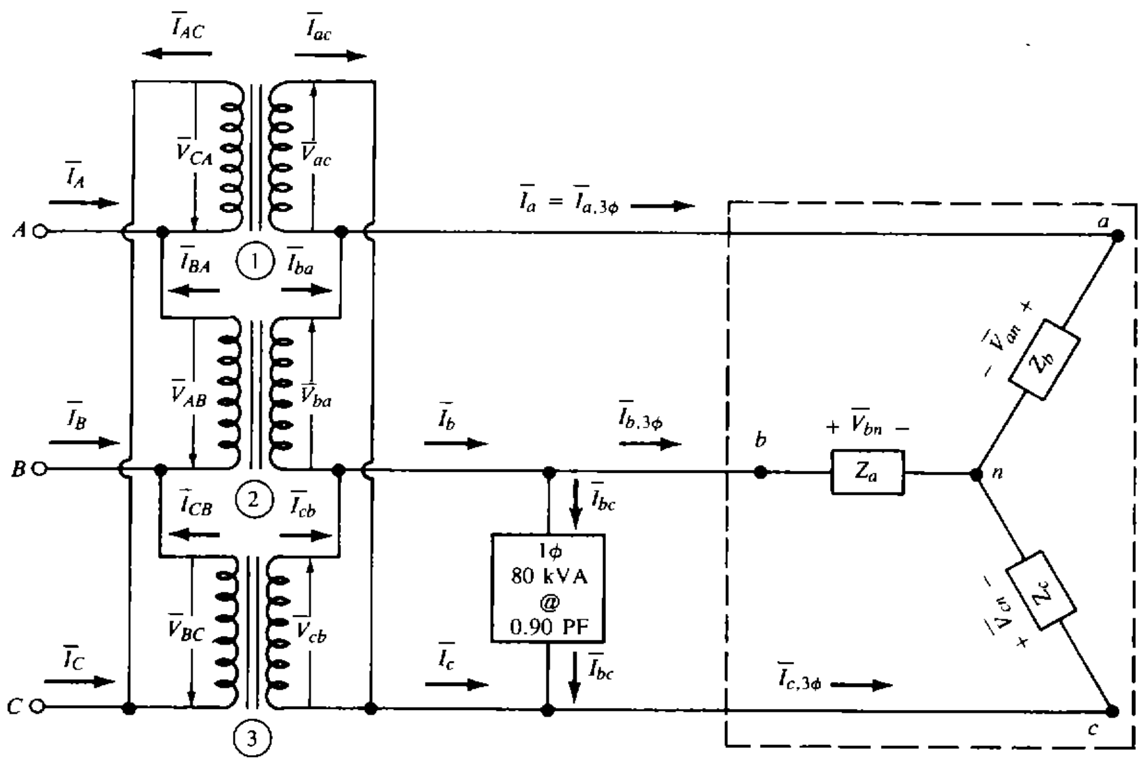


Figure 3-29

Assume that the three single-phase transformers have equal percent impedance and equal ratios of percent reactance to percent resistance. The primary-side voltage of the bank is 7620/13,200 V and the secondary-side voltage is 240 V. Assume that the single-phase transformer connected between phases b and c is rated at 100 kVA and the other two are rated at 75 kVA. Determine the following:

(*a*) The line current flowing in each secondary-phase wire.
(*b*) The current flowing in the secondary winding of each transformer.
(*c*) The load on each transformer in kilovoltamperes
(*d*) The current flowing in each primary winding of each transformer.
(*e*) The line current flowing in each primary-phase wire.

SOLUTION

(*a*) Using the voltage drop $\bar{V}_{an}$ as the reference, the three-phase components of the line currents can be found as

$$
\begin{aligned}
|\bar{I}_{a,3\phi}| &= |\bar{I}_{b,3\phi}| \\
&= |\bar{I}_{c,3\phi}| \\
&= \frac{S_{L,3\phi}}{\sqrt{3} \times V_{(L-L)}} \\
&= \frac{200}{\sqrt{3} \times 0.240} \\
&= 481.7 \text{ A}
\end{aligned}
$$

Since the three-phase load has a lagging power factor of 0.80,

$$
\begin{aligned}
\bar{I}_{a,3\phi} &= |\bar{I}_{a,3\phi}|(\cos\theta - j\sin\theta) \\
&= 481.7(0.80 - j0.60) \\
&= 385.36 - j289.02 \\
&= 481.7\underline{/-36.9^\circ}\ \text{A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_{b,3\phi} &= a^2\bar{I}_{a,3\phi} \\
&= (1\underline{/240^\circ})(481.7\underline{/-36.9^\circ}) \\
&= 481.7\underline{/203.1^\circ} \\
&= -443.08 - j188.99\ \text{A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_{c,3\phi} &= a\bar{I}_{a,3\phi} \\
&= (1\underline{/120^\circ})(481.7\underline{/-36.9^\circ}) \\
&= 481.7\underline{/83.1^\circ} \\
&= 57.87 + j478.21\ \text{A}
\end{aligned}
$$

The single-phase component of the line currents can be found as

$$
|I_{bc}| = \frac{S_{L,1\phi}}{V_{(L-L)}} = \frac{80}{0.240} = 333.33\ \text{A}
$$

Since the single-phase load has a lagging power factor of 0.90, the current phasor I_{bc} lags its voltage phasor V_{bc} by -25.8°. Also, since the voltage phasor V_{bc} lags the voltage reference V_{an} by 90° (see Fig. 3-26), then the current phasor I_{bc} will lag the voltage reference V_{an} by -115.8° $(= -25.8^\circ - 90^\circ)$. Therefore

$$
\begin{aligned}
\bar{I}_{bc} &= 333.33\underline{/-115.8^\circ} \\
&= -145.3 - j300\ \text{A}
\end{aligned}
$$

Hence the line currents flowing in each secondary-phase wire can be found as

$$
\begin{aligned}
\bar{I}_a &= \bar{I}_{a,3\phi} \\
&= 481.7\underline{/-36.9^\circ}\ \text{A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_b &= \bar{I}_{b,3\phi} + \bar{I}_{bc} \\
&= 481.7\underline{/203.1^\circ} + 333.33\underline{/-115.8^\circ} \\
&= -588.38 - j488.99 \\
&= 765.05\underline{/219.7^\circ}\ \text{A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_c &= \bar{I}_{c,3\phi} - \bar{I}_{bc} \\
&= 481.7\underline{/83.1^\circ} - 333.33\underline{/-115.8^\circ} \\
&= -87.43 + j178.21 \\
&= 198.5\underline{/-63.8^\circ}\ \text{A}
\end{aligned}
$$

(b) By using Eq. (3-34), the current flowing in the secondary winding of transformer 1 can be found as

$$\bar{I}_{ac} = \frac{\dfrac{\bar{I}_c}{S_{T,bc}} - \dfrac{\bar{I}_a}{S_{T,ab}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}}$$

$$= \frac{\dfrac{198.5\angle -63.8^\circ}{100} - \dfrac{481.7\angle -36.9^\circ}{75}}{\frac{1}{75} + \frac{1}{100} + \frac{1}{75}}$$

$$= \frac{1.985\angle -63.8^\circ - 6.4227\angle -36.9^\circ}{0.0367}$$

$$= -116.07 + j56.55$$

$$= 129.11\angle -33.1^\circ \text{ A}$$

Similarly, by using Eq. (3-33),

$$\bar{I}_{ba} = \frac{\dfrac{\bar{I}_a}{S_{T,ca}} - \dfrac{\bar{I}_b}{S_{T,bc}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}}$$

$$= \frac{\dfrac{481.7\angle -36.9^\circ}{75} - \dfrac{765.05\angle 219.7^\circ}{100}}{\frac{1}{75} + \frac{1}{100} + \frac{1}{75}}$$

$$= \frac{6.4227\angle -36.9^\circ - 7.6505\angle 219.7^\circ}{0.0367}$$

$$= 300.34 + j28.08$$

$$= 301.65\angle 5.3^\circ \text{ A}$$

and using Eq. (3-35),

$$\bar{I}_{cb} = \frac{\dfrac{\bar{I}_b}{S_{T,ab}} - \dfrac{\bar{I}_c}{S_{T,ca}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}}$$

$$= \frac{\dfrac{765.05\angle 219.7^\circ}{75} - \dfrac{198.5\angle -63.8^\circ}{75}}{0.0367}$$

$$= -245.6 - j112.95$$

$$= 270.3\angle 204.7^\circ \text{ A}$$

(*c*) The kilovoltampere load on each transformer can be found as

$$\begin{aligned}
S_{L,ab} &= V_{ba} \times |\bar{I}_{ba}| \\
&= 0.240 \times 301.65 \\
&= 72.4 \text{ kVA}
\end{aligned}$$

$$\begin{aligned}
S_{L,bc} &= V_{cb} \times |\bar{I}_{cb}| \\
&= 0.240 \times 270.33 \\
&= 64.88 \text{ kVA}
\end{aligned}$$

$$\begin{aligned}
S_{L,ca} &= V_{ac} \times |\bar{I}_{ac}| \\
&= 0.240 \times 129.11 \\
&= 30.99 \text{ kVA}
\end{aligned}$$

(*d*) The current flowing in the primary winding of each transformer can be found by dividing the current flow in each secondary winding by the turns ratio. Therefore

$$n = \frac{7620 \text{ V}}{240 \text{ V}} = 31.75$$

and hence

$$\begin{aligned}
\bar{I}_{AC} &= \frac{\bar{I}_{ac}}{n} \\
&= \frac{129.11\underline{/-33.1^\circ}}{31.75} \\
&= 4.07\underline{/-33.1^\circ} \text{ A}
\end{aligned}$$

$$\begin{aligned}
\bar{I}_{BA} &= \frac{\bar{I}_{ba}}{n} \\
&= \frac{301.65\underline{/5.3^\circ}}{31.75} \\
&= 9.50\underline{/5.3^\circ}
\end{aligned}$$

$$\begin{aligned}
\bar{I}_{CB} &= \frac{\bar{I}_{cb}}{n} \\
&= \frac{270.3\underline{/204.7^\circ}}{31.75} \\
&= 8.51\underline{/204.7^\circ} \text{ A}
\end{aligned}$$

(*e*) The line current flowing in each primary-phase wire can be found as

$$\begin{aligned}
\bar{I}_A &= \bar{I}_{AC} - \bar{I}_{BA} \\
&= 4.07\underline{/-33.1^\circ} - 9.50\underline{/5.3^\circ} \\
&= -6.05 - j3.1 \\
&= 6.80\underline{/270.1^\circ}
\end{aligned}$$

$$\begin{aligned}
\bar{I}_B &= \bar{I}_{BA} - \bar{I}_{CB} \\
&= 9.50\underline{/5.3^\circ} - 8.51\underline{/204.7^\circ} \\
&= 17.19 + j4.44 \\
&= 17.76\underline{/14.5^\circ}
\end{aligned}$$

$$\begin{aligned}
\bar{I}_C &= \bar{I}_{CB} - \bar{I}_{AC} \\
&= 8.51\underline{/204.7^\circ} - 4.07\underline{/33.1^\circ} \\
&= -11.14 - j1.34 \\
&= 11.22\underline{/186.8^\circ}\,\text{A}
\end{aligned}$$

3-10-2 The Open-Δ Open-Δ Transformer Connection

The delta-delta connection is the most flexible of the various connection forms. One of the advantages of this connection is that if one transformer becomes damaged or is removed from service, the remaining two can be operated in what is known as the *open-delta* or *V connection*, as shown in Fig. 3-30.

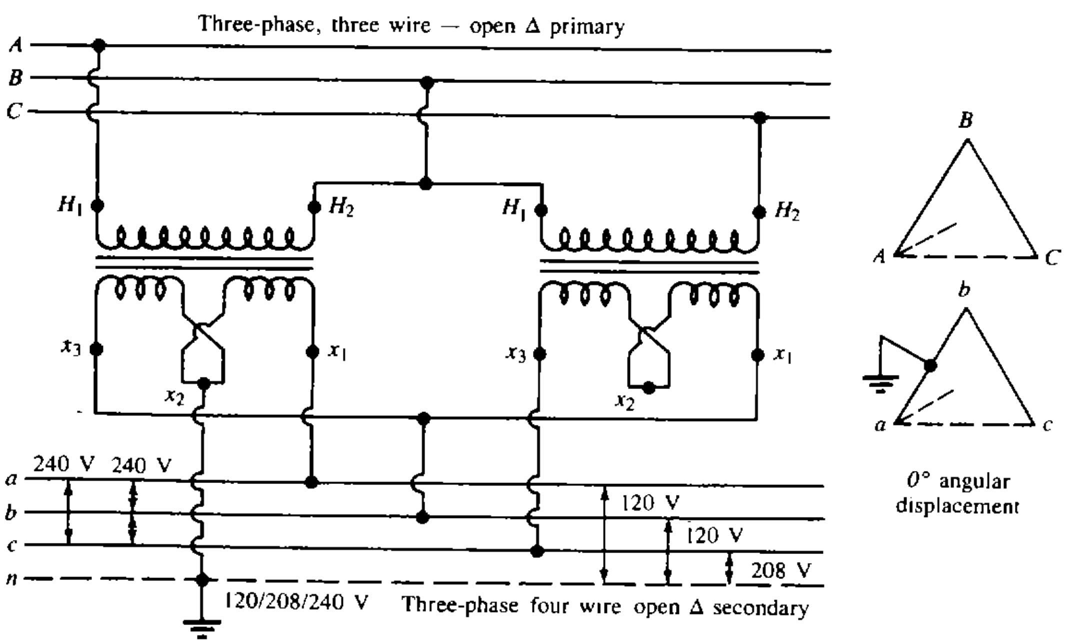


Figure 3-30 Three-phase four-wire open-delta connection. (Note that 3ϕ–4W means a three-phase system made up of four wires.)

Assume that a balanced three-phase load with unity power factor is served by all three transformers of a delta-delta bank. The taking out of one of the transformers from the service will result in having the currents in the other two transformers increase by a ratio of 1.73, even though the output of the transformer bank bank is the same with a unity power factor as before. However, the individual transformers now function at a power factor of 0.866. One of the transformers delivers a leading load and the other a lagging load. To operate the remaining portion of the delta-delta transformer bank (i.e., the open-delta open-delta bank) safely, the connected load has to be decreased by the 57.7 percent which can be found as follows:

$$S_{\Delta\text{-}\Delta} = \frac{\sqrt{3}\,V_{L\text{-}L} I_L}{1000} \qquad \text{kVA} \tag{3-36}$$

and

$$S_{\angle\text{-}\angle} = \frac{\sqrt{3}\,V_{L\text{-}L} I_L}{\sqrt{3} \times 1000} \qquad \text{kVA} \tag{3-37}$$

Therefore, by dividing Eq. (3-36) by Eq. (3-37), side by side,

$$\begin{aligned} \frac{S_{\angle\text{-}\angle}}{S_{\Delta\text{-}\Delta}} &= \frac{1}{\sqrt{3}} \\ &= 0.577, \text{ or } 57.7\% \end{aligned} \tag{3-38}$$

where $S_{\Delta\text{-}\Delta}$ = kilovoltampere rating of the delta-delta bank
$S_{\angle\text{-}\angle}$ = kilovoltampere rating of the open-delta bank
$V_{L\text{-}L}$ = line-to-line voltage, V
I_L = line (or full load) current, A

Note that the two transformers of the open-delta bank make up 66.6 percent of the installed capacity of the three transformers of the delta-delta bank, but they can supply only 57.7 percent of the three. Here, the ratio of 57.7/66.6 = 0.866 is the power factor at which the two transformers operate when the load is at unity power factor. By being operated in this way, the bank still delivers three-phase currents and voltages in their correct phase relationships, but the capacity of the bank is reduced to 57.7 percent of what it was with all three transformers in service since it has only 86.6 percent of the rating of the two units making up the three-phase bank. Open-delta banks are quite often used where the load is expected to grow, and when the load does grow, the third transformer may be added to complete a delta-delta bank.

Figure 3-31 shows an open-delta connection for 240-V three-phase three-wire secondary service at 0° angular displacement. The neutral point n shown in the low-voltage phasor diagram exists only on the paper.

For the sake of illustration, assume that a balanced three-phase load, e.g., an induction motor as shown in the figure, with a lagging power factor is connected

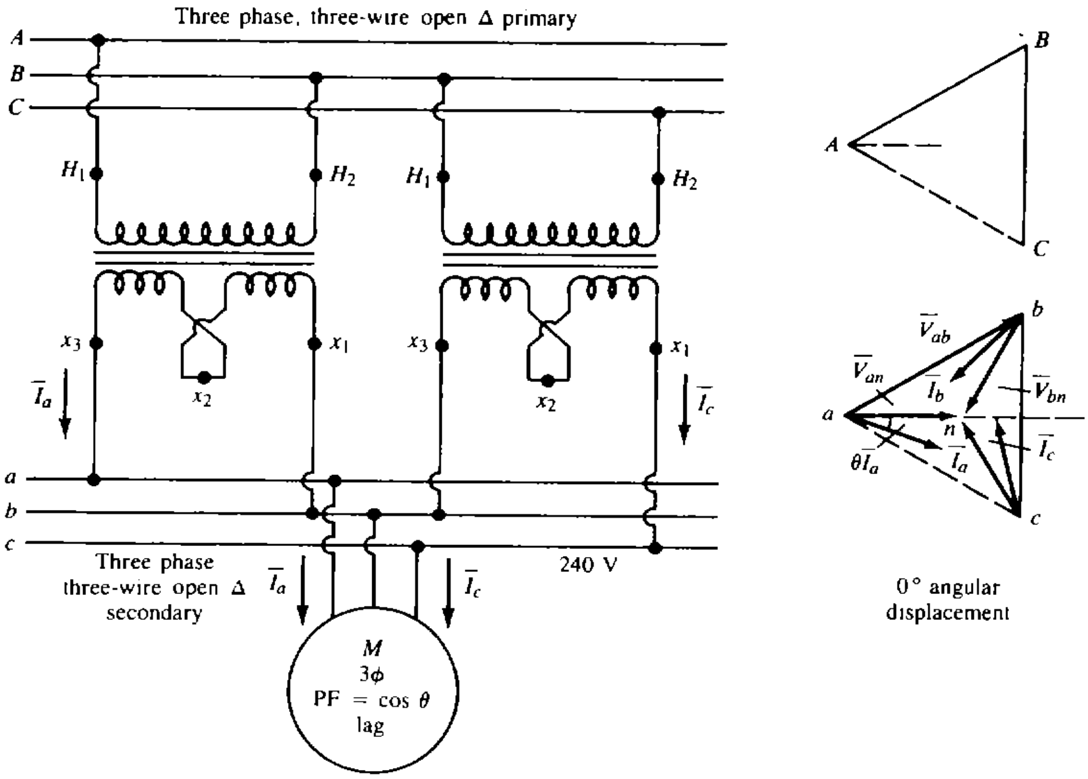


Figure 3-31 Three-phase three-wire open-delta connection.

to the secondary. Therefore the a, b, c phase currents in the secondary can be found as

$$\bar{I}_a = \frac{S_{3\phi}}{\sqrt{3}V_{L\text{-}L}} \angle \theta_{\bar{I}_a} \quad \text{A} \tag{3-39}$$

$$\bar{I}_b = \frac{S_{3\phi}}{\sqrt{3}V_{L\text{-}L}} \angle \theta_{\bar{I}_b} \quad \text{A} \tag{3-40}$$

$$\bar{I}_c = \frac{S_{3\phi}}{\sqrt{3}V_{L\text{-}L}} \angle \theta_{\bar{I}_c} \quad \text{A} \tag{3-41}$$

The transformer kilovoltampere loads can be calculated as follows. The kilovoltampere load on the first transformer is

$$S_{T_1} = V_{L\text{-}L} \times |\bar{I}_a|$$

$$= V_{L\text{-}L} \times \frac{S_{3\phi}}{\sqrt{3}V_{L\text{-}L}}$$

$$= \frac{S_{3\phi}}{\sqrt{3}} \quad \text{kVA} \tag{3-42}$$

and the kilovoltampere load on the second transformer is

$$S_{T2} = V_{L\text{-}L} \times |\bar{I}_b|$$

$$= V_{L\text{-}L} \times \frac{S_{3\phi}}{\sqrt{3}V_{L\text{-}L}}$$

$$= \frac{S_{3\phi}}{\sqrt{3}} \quad \text{kVA} \tag{3-43}$$

Therefore, the total load that the transformer bank can be loaded to (or the total "effective" transformer bank capacity) is

$$\sum_{i=1}^{2} S_{T_i} = \frac{2 \times S_{3\phi}}{\sqrt{3}} \tag{3-44}$$

and hence,

$$S_{3\phi} = \frac{\sqrt{3}}{2} \sum_{i=1}^{2} S_{T_i} \quad \text{kVA} \tag{3-45}$$

For example, if there are two 50-kVA transformers in the open-delta bank, even though the total transformer bank capacity appears to be

$$\sum_{i=1}^{2} S_{T_i} = 100 \text{ kVA}$$

in reality the bank's "effective" maximum capacity is

$$S_{3\phi} = \frac{\sqrt{3}}{2} \times 100 = 86.6 \text{ kVA}$$

On the other hand, if there are three 50-kVA transformers in the delta-delta bank, the bank's maximum capacity is

$$S_{3\phi} = \sum_{i=1}^{3} S_{T_i} = 150 \text{ kVA}$$

which shows an increase of 73 percent over the 86.6-kVA load capacity.

Assume that the load power factor is cos θ and its angle can be calculated as

$$\theta = \theta_{\bar{V}_{an}} - \theta_{\bar{I}_a} \tag{3-46}$$

or using $\bar{V}_{an}$ as the reference,

$$\theta = 0^\circ - \theta_{\bar{I}_a} \tag{3-47}$$

If θ_{I_a} is negative, then θ is positive which means it is the angle of a lagging load power factor. Also, it can be shown that

$$\theta = \theta_{\bar{V}_{bn}} - \theta_{\bar{I}_b} \tag{3-48}$$

or

$$\theta = -120^\circ - \theta_{\bar{I}_b} \tag{3-49}$$

and

$$\theta = \theta_{\bar{V}_{cn}} - \theta_{\bar{I}_c} \tag{3-50}$$

or

$$\theta = +120^\circ - \theta_{\bar{I}_c} \tag{3-51}$$

On the other hand, the transformer power factors for transformers 1 and 2 can be calculated as

$$\cos\theta_{T1} = \cos(\theta_{\bar{V}_{ab}} - \theta_{\bar{I}_a}) \tag{3-52}$$

or if $\theta_{\bar{I}_a} = -30^\circ$,

$$\cos\theta_{T1} = \cos(\theta_{\bar{V}_{bc}} + 30^\circ) \tag{3-53}$$

and

$$\cos\theta_{T2} = \cos(\theta_{\bar{V}_{bc}} - \theta_{\bar{I}_c}) \tag{3-54}$$

or if $\theta_{\bar{I}_c} = +30^\circ$

$$\cos\theta_{T2} = \cos(\theta_{\bar{V}_{bc}} - 30^\circ) \tag{3-55}$$

Therefore, the total real power output of the bank is

$$\begin{aligned} P_T &= P_{T1} + P_{T2} \\ &= V_{L\text{-}L}|\bar{I}_a|\cos(\theta + 30^\circ) + V_{L\text{-}L}|\bar{I}_c|\cos(\theta - 30^\circ) \\ &= \sqrt{3}V_{L\text{-}L}I_L\cos\theta \qquad \text{kW} \end{aligned} \tag{3-56}$$

and, similarly, the total reactive power output of the bank is

$$\begin{aligned} Q_T &= Q_{T1} + Q_{T2} \\ &= \sqrt{3}V_{L\text{-}L}I_L\sin\theta \qquad \text{kvar} \end{aligned} \tag{3-57}$$

As shown in Table 3-6 when the connected bank load has a lagging power factor of 0.866, it has a 30° power factor angle and, therefore, transformer 1, from Eq. (3.53), has a 0.5 lagging power factor and transformer 2, from Eq. (3-55), has a unity power factor. However, when the bank load has a unity power factor, of course its angle is zero, and therefore transformer 1 has a 0.866 lagging power factor and transformer 2 has a 0.866 leading power factor.

Table 3-6 The effects of the load power factor on the transformer power factors

Load power factor		Transformer power factors	
$\cos\theta$	θ	$\cos\theta_{T1} = \cos(\theta + 30^\circ)$	$\cos\theta_{T2} = \cos(\theta - 30^\circ)$
0.866 lag	$+30^\circ$	0.5 lag	1.0
1.0	0°	0.866 lag	0.866 lead

3-10-3 The Y-Y Transformer Connection

Figure 3-32 shows three transformers connected wye-wye on a typical three-phase four-wire multigrounded system to provide for 120/208Y-V service at 0° angular displacement. This particular system provides a 208-V three-phase power supply for three-phase motors and a 120-V single-phase power supply for lamps and other small single-phase loads. Of course, an attempt should be made to distribute the single-phase loads reasonably equally among the three phases. One of the advantages of the wye-wye connection is that when a system has changed from delta to a four-wire wye to increase system capacity, existing transformers can be used. For example, assume that the old distribution system was 2.4-kV delta and the new distribution system is 2.4/4.16Y kV. Here the existing 2.4/4.16Y-kV transformers can be connected in wye and employed.

In the wye-wye transformer bank connection, only 57.7 percent (or 1/1.73) of the line voltage affects each winding, but full line current flows in each transformer winding. Power-distribution circuits supplied from a wye-wye bank often create series disturbances in communication circuits (e.g., telephone interference) in their immediate vicinity. Also, the primary neutral point should be solidly grounded and tied firmly to the system neutral; otherwise, excessive voltages may be developed on the secondary side. For example, if the neutral of the transformer is isolated from the system neutral, an unstable condition results at the transformer neutral, caused primarily by third-harmonic voltages. If the transformer neutral is connected to ground, the possibility of telephone interference is greatly enhanced and there is also a possibility of resonance between the line capacitance to ground and the magnetizing impedance of the transformer.

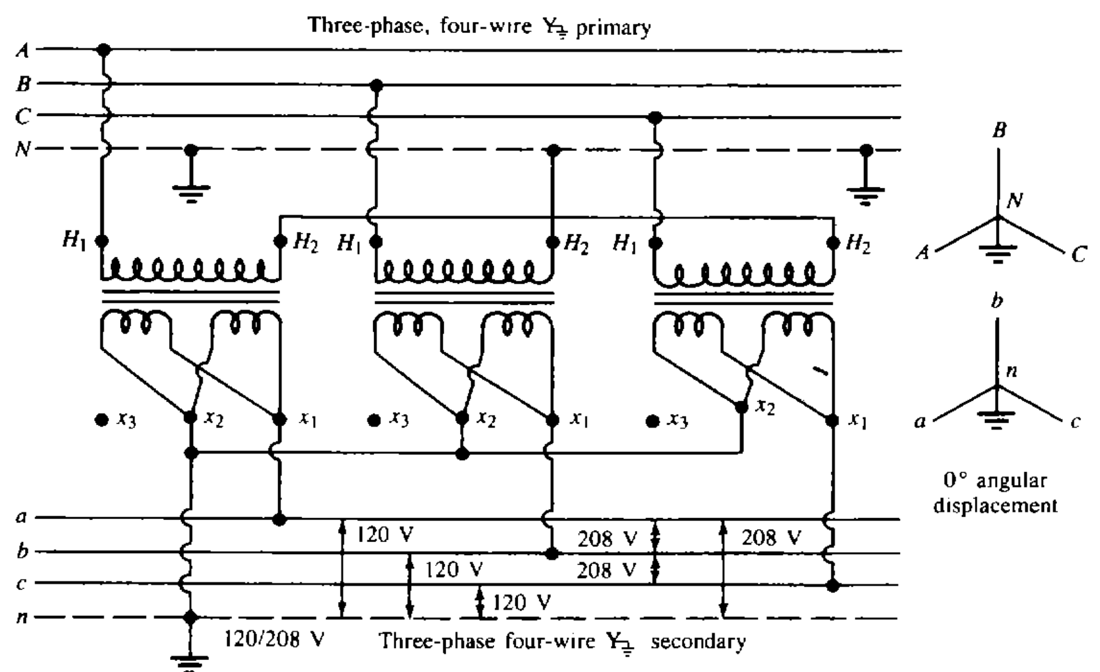


Figure 3-32 Wye-wye connection to provide a 120/208-V grounded-wye three-phase four-wire multi-grounded service.

3-10-4 The Y-Δ Transformer Connection

Figure 3-33 shows three single-phase transformers connected in wye-delta on a three-phase three-wire ungrounded-wye, primary system to provide for 120/208/240-V three-phase four-wire delta secondary service at 30° angular displacement.

Figure 3-34 shows three transformers connected in wye-delta on a typical three-phase four-wire grounded-wye primary system to provide for 240-V three-phase three-wire delta secondary service at 210° angular displacement.

The wye-delta connection is advantageous in many cases because the voltage between the outside legs of the wye is 1.73 times the voltage to neutral, so that higher distribution voltage can be gained by using transformers with primary winding of only the voltage between any leg and the neutral. For example, 2.4-kV primary single-phase transformers can be connected in wye on the primary to a 4.16-kV three-phase wye circuit. In the wye-delta connection the voltage/transformation ratio of the bank is 1.73 times the voltage/transformation ratio of the individual transformers. When transformers of different capacities are used, the maximum safe bank rating is three times the capacity of the smallest transformers.

The primary supply, usually a grounded wye circuit, may be either three-wire or four-wire including a neutral wire. The neutral wire, running from the neutral of the wye-connected substation transformer bank supplying the primary circuit, may be completely independent of the secondary or may be united with the neutral of the secondary system. In the case of having the primary neutral independent of the secondary system, it is used as an isolated neutral and is grounded at the

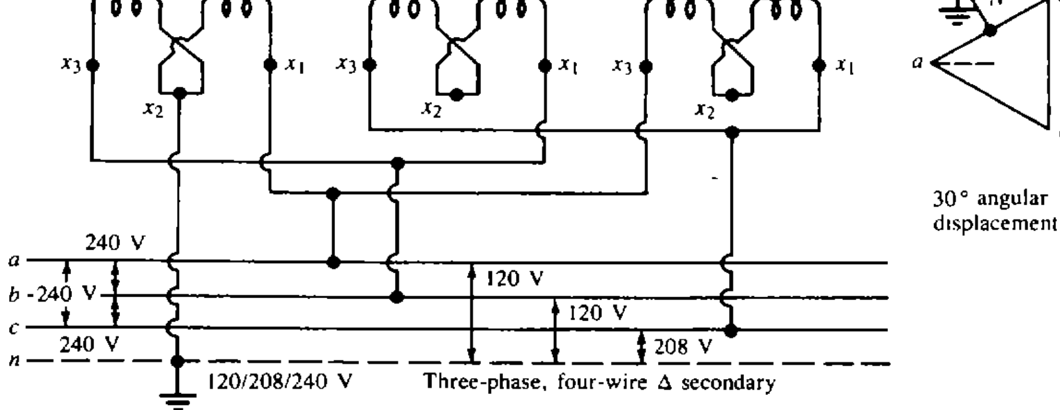


Figure 3-33 Wye-delta connection to provide a 120/208/240-V three-phase four-wire secondary service.

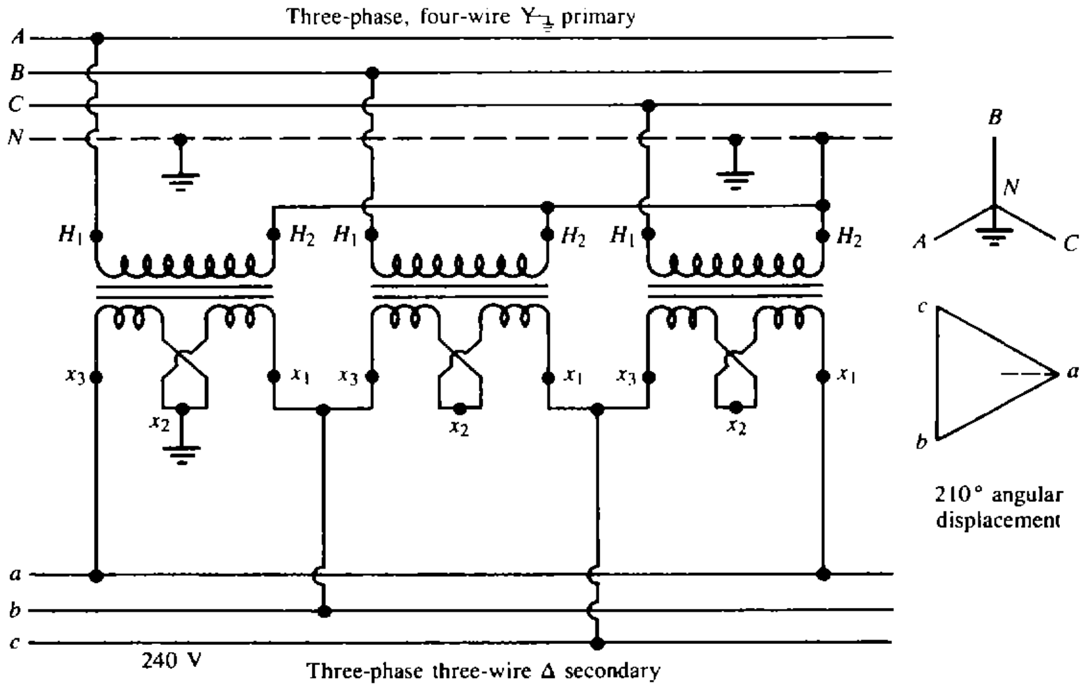


Figure 3-34 Wye-delta connection to provide a 240-V three-phase three-wire secondary service.

substation only. In the case of having the same wire serving as both a primary neutral and the secondary neutral, it is grounded at many points, including each customer's service and is a multigrounded common neutral. However, in either case, the primary-bank neutral is usually not connected to the primary-circuit neutral since it is not necessary and prevents a burned-out transformer winding during phase-to-ground faults and extensive blowing of fuses throughout the system.

In the case of the wye-wye connection, neglecting the neutral on the primary side causes the voltages to be deformed from the sine-wave form. In the case of the wye-delta connection, if the neutral is spared on the primary side the voltage waveform tends to deform, but this deformation causes circulating currents in the delta, and these currents act as magnetizing currents to correct the deformation. Thus there is no objection to neglecting the neutral. However, if the transformer supplies a motor load, a damaging overcurrent is produced in each three-phase motor circuit, causing an equal amount of current to flow in two wires of the motor branch circuit and the total of the two currents to flow in the third. If the highest of the three currents occurs in the unprotected circuit, motor burnout will probably happen. This applies to ungrounded wye-delta and delta-wye banks.

If the transformer bank is employed to supply three-phase and single-phase load, and if the bank neutral is solidly connected, disconnection of the large transformer by fuse operation causes an even greater overload on the remaining two transformers. Here, the blowing of a single fuse is hard to detect since no decrease in service quality is noticeable right away, and one of the two remaining transformers may be burned out by the overload. On the other hand, if the bank neutral is not connected to the primary-circuit neutral, but left isolated, disconnection of one transformer results in a partial service interruption without danger of a transformer burnout. The approximate rated capacity required in a wye-delta-connected bank with an isolated bank neutral to serve a combined three-phase and single-phase load, assuming unity power factor, can be found as

$$\frac{2S_{1\phi} + S_{3\phi}}{3}$$

which is equal to rated transformer capacity across lighting phase, where

$$S_{1\phi} = \text{single-phase load, kVA}$$

$$S_{3\phi} = \text{three-phase load, kVA}$$

In summary, when the primary-side neutral of the transformer bank is not isolated but connected to the primary-circuit neutral, the wye-delta transformer bank may burn out due to the following reasons:

1. The transformer bank may act as a grounding transformer bank for unbalanced primary conditions and may supply fault current to any fault on the circuit to which it is connected, reducing its own capacity for connected load.
2. The transformer bank may be overloaded if one of the protective fuses opens on a line-to-ground fault, leaving the bank with only the capacity of an open-wye open-delta bank.
3. The transformer bank causes circulating current in the delta in an attempt to balance any unbalanced load connected to the primary line.
4. The transformer bank provides a delta in which triple-harmonic currents circulate.

All the aforementioned effects can cause the transformer bank to carry current in addition to its normal load current, resulting in the burnout of the transformer bank.

3-10-5 The Open-Y Open-Δ Transformer Connection

As shown in Fig. 3-35, in the case of having one phase of the primary supply opened, the transformer bank becomes open-wye open-delta and continues to serve the three-phase load at a reduced capacity.

Example 3-6 Two single-phase transformers are connected open-wye open-delta to provide power for a three-phase wye-connected 100-kVA load with a 0.80 lagging power factor and a 50-kVA single-phase load with a 0.90 lagging power factor, as shown in Fig. 3-36. Assume that the primary-side voltage of the bank is 7620/13,200 V and the secondary-side voltage is 240 V. Using the given information, calculate the following:

(*a*) The line current flowing in each secondary-phase wire.
(*b*) The current flowing in the secondary winding of each transformer.
(*c*) The kilovoltampere load on each transformer.
(*d*) The current flowing in each primary-phase wire and in the primary neutral.

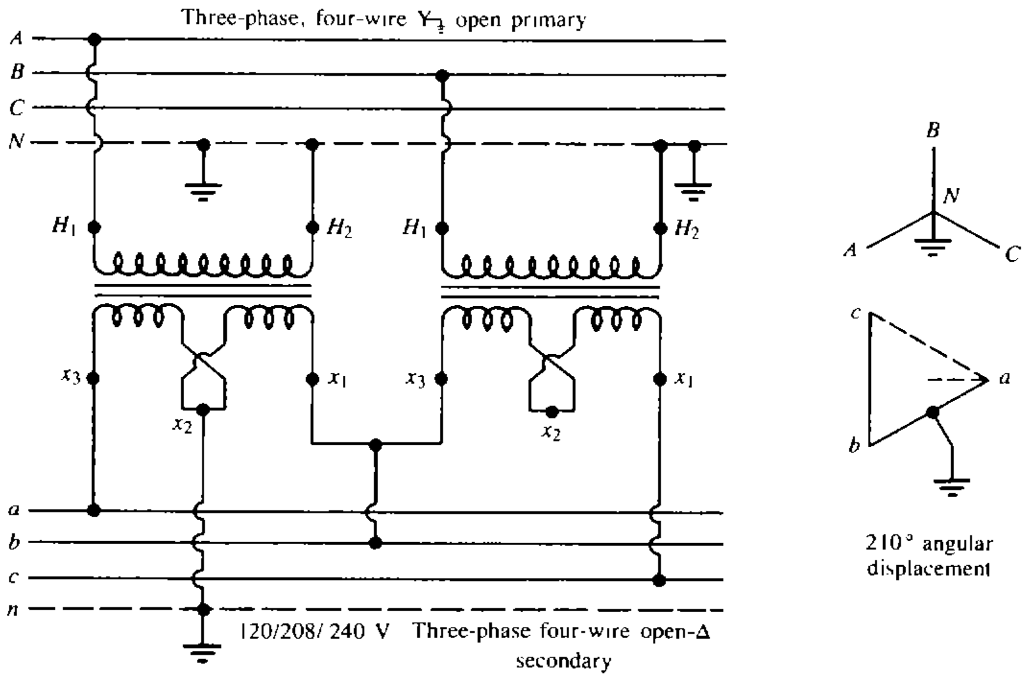


Figure 3-35 Open-wye open-delta connection.

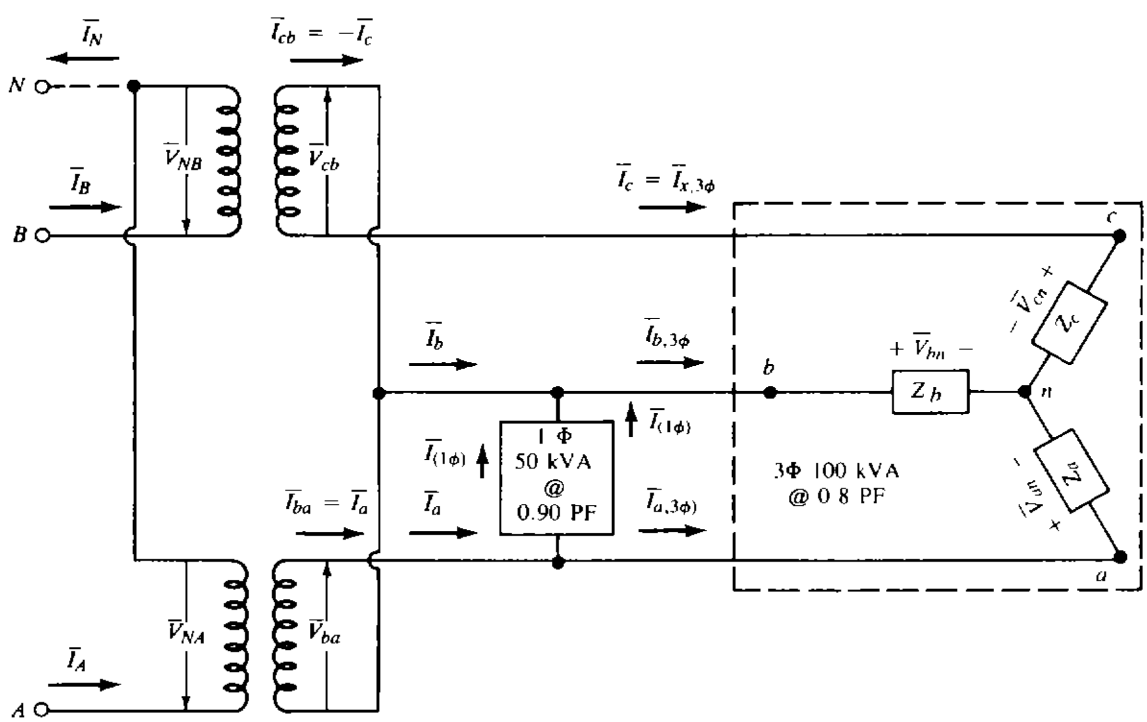


Figure 3-36

SOLUTION

(*a*) Using the voltage drop $\bar{V}_{an}$ as the reference, the three-phase components of the line currents can be found as

$$
\begin{aligned}
|\bar{I}_{a,3\phi}| &= |\bar{I}_{b,3\phi}| \\
&= |\bar{I}_{c,3\phi}| \\
&= \frac{S_{L,3\phi}}{\sqrt{3} \times V_{L\text{-}L}} \qquad (3\text{-}58) \\
&= \frac{100}{\sqrt{3} \times 0.240} \\
&= 240.8 \text{ A}
\end{aligned}
$$

Since the three-phase load has a lagging power factor of 0.80,

$$
\begin{aligned}
\bar{I}_{a,3\phi} &= |\bar{I}_{a,3\phi}|(\cos\theta_3 - j\sin\theta_3) \qquad (3\text{-}59) \\
&= 240.8(0.80 - j0.60) \\
&= 192.68 - j144.5 \\
&= 240.8\underline{/-36.9^\circ} \text{ A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_{b,3\phi} &= a^2 \bar{I}_{a,3\phi} \qquad (3\text{-}60) \\
&= (1\underline{/240^\circ})(240.8\underline{/-36.9^\circ}) \\
&= 240.8\underline{/203.1^\circ} \\
&= -221.5 - j94.5 \text{ A}
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_{c,3\phi} &= a\bar{I}_{a,3\phi} \qquad (3\text{-}61) \\
&= (1\underline{/120^\circ})(240.8\underline{/-36.9^\circ}) \\
&= 240.8\underline{/83.1^\circ} \\
&= 28.9 + j239.1 \text{ A}
\end{aligned}
$$

The single-phase component of the line currents can be found as

$$
\begin{aligned}
|\bar{I}_{1\phi}| &= \frac{S_{L,1\phi}}{V_{L\text{-}L}} \qquad (3\text{-}62) \\
&= \frac{50}{0.240} \\
&= 208.33 \text{ A}
\end{aligned}
$$

therefore

$$
\begin{aligned}
\bar{I}_{1\phi} &= |\bar{I}_{1\phi}|[\cos(30^\circ - \theta_1) + j\sin(30^\circ - \theta_1)] \qquad (3\text{-}63) \\
&= 208.33[\cos(30^\circ - 25.8^\circ) + j\sin(30^\circ - 25.8^\circ)] \\
&= 208.33(\cos 4.2 + j\sin 4.2^\circ) \\
&= 207.78 + j15.26 \text{ A}
\end{aligned}
$$

Hence the line currents flowing in each secondary-phase wire can be found as

$$\begin{aligned}\bar{I}_a &= \bar{I}_{a,3\phi} + \bar{I}_{1\phi}\\ &= 192.68 - j144.5 + 207.78 + j15.26\\ &= 400.46 - j129.24\\ &= 420.8\angle -17.9^\circ \text{ A}\end{aligned}$$

$$\begin{aligned}\bar{I}_b &= \bar{I}_{b,3\phi} - \bar{I}_{1\phi}\\ &= -221.5 - j94.5 - 207.78 - j15.26\\ &= -429.28 - j109.76\\ &= 442.8\angle -165.7^\circ \text{ A}\end{aligned}$$

$$\begin{aligned}\bar{I}_c &= I_{c,3\phi}\\ &= 240.8\angle 83.1^\circ \text{ A}\end{aligned}$$

(*b*) The current flowing in the secondary winding of each transformer is

$$\begin{aligned}\bar{I}_{ba} &= \bar{I}_a\\ &= 420.8\angle -17.9^\circ \text{ A}\end{aligned}$$

$$\begin{aligned}\bar{I}_{cb} &= -\bar{I}_c\\ &= -240.8\angle 83.1^\circ\\ &= 240.8\angle 83.1^\circ + 180^\circ\\ &= 240.8\angle 263.1^\circ \text{ A}\end{aligned}$$

(*c*) The kilovoltampere load on each transformer can be found as

$$\begin{aligned}S_{L,ba} &= V_{ba} \times |\bar{I}_{ba}| \qquad (3\text{-}64\text{a})\\ &= 0.240 \times 420.8\\ &= 101 \text{ kVA}\end{aligned}$$

$$\begin{aligned}S_{L,cb} &= V_{cb} \times |\bar{I}_{cb}| \qquad (3\text{-}64\text{b})\\ &= 0.240 \times 240.8\\ &= 57.8 \text{ kVA}\end{aligned}$$

(*d*) The current flowing in each primary-phase wire can be found by dividing the current flow in each secondary winding by the turns ratio. Therefore

$$n = \frac{7620 \text{ V}}{240 \text{ V}} = 31.75$$

and hence

$$\bar{I}_A = \frac{\bar{I}_{ba}}{n} \tag{3-65a}$$

$$= \frac{420.8\angle -17.9^\circ}{31.75}$$

$$= 13.25\angle -17.9^\circ$$

$$= 12.6 - j4.07 \text{ A}$$

$$\bar{I}_B = \frac{\bar{I}_{cb}}{n} \tag{3-65b}$$

$$= \frac{240.8\angle 263.1^\circ}{31.75}$$

$$= 7.58\angle 263.1^\circ$$

$$= -0.91 - j7.53 \text{ A}$$

Therefore the current in the primary neutral is

$$\bar{I}_N = \bar{I}_A + \bar{I}_B \tag{3-66}$$

$$= 13.25\angle -17.9^\circ + 7.58\angle 263.1^\circ$$

$$= 11.69 - j11.6$$

$$= 16.47\angle -44.8^\circ \text{ A}$$

3-10-6 The Δ-Y Transformer Connection

Figures 3-37 and 3-38 show three single-phase transformers connected in delta-wye to provide for 120/208-V three-phase four-wire grounded-wye service at 30° and 210° angular displacements, respectively.

In the previously mentioned transformer banks the single-phase lighting load is all on one phase, resulting in unbalanced primary currents in any one bank. To eliminate this difficulty, the delta-wye system finds many uses. Here the neutral of the secondary three-phase system is grounded and single-phase loads are connected between the different phase wires and the neutral while the three-phase loads are connected to the phase wires. Therefore the single-phase loads can be balanced on three phases in each bank, and banks may be paralleled if desired.

When transformers of different capacities are used, maximum safe transformer bank rating is three times the capacity of the smallest transformer. If one transformer becomes damaged or is removed from service, the transformer bank becomes inoperative.

With both the wye-wye and the delta-delta connections, the line voltages on the secondaries are in phase with the line voltages on the primaries, but with the wye-delta or the delta-wye connections, the line voltages on the secondaries are at 30° to the line voltages on the primaries. Consequently a wye-delta or delta-wye transformer bank cannot be operated in parallel with a delta-delta or wye-wye

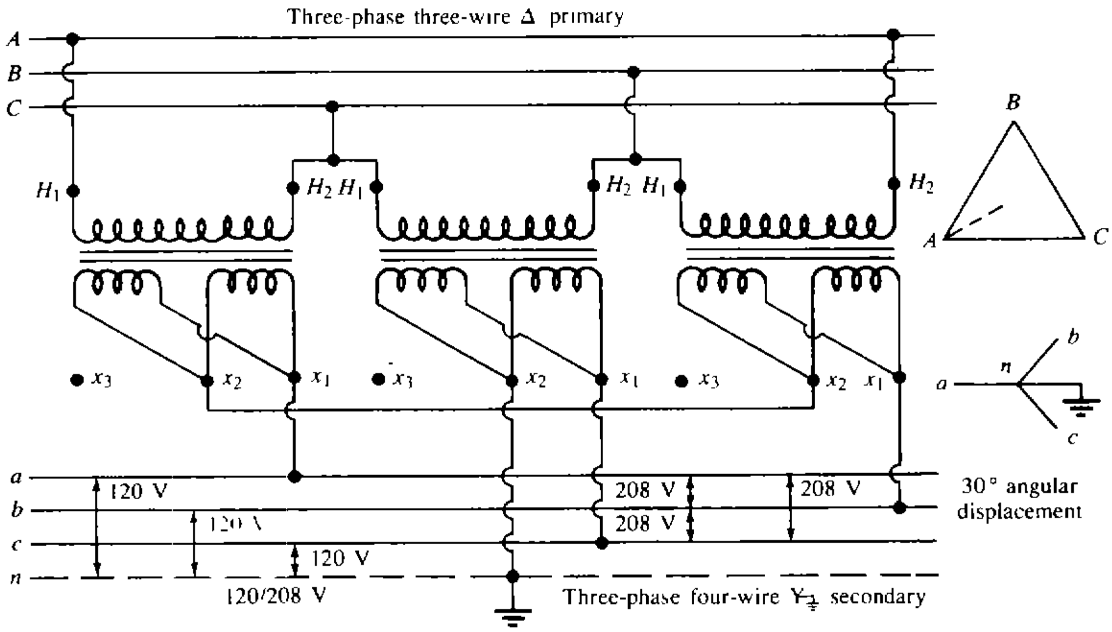


Figure 3-37 Delta-wye connection with 30° angular displacement.

transformer bank. Having the identical angular displacements becomes especially important when three-phase transformers are interconnected into the same secondary system or paralleled with three-phase banks of single-phase transformers. The additional conditions to successfully parallel three-phase distribution transformers are the following:

1. All transformers have identical frequency ratings.
2. All transformers have identical voltage ratings.
3. All transformers have identical tap settings.
4. Per unit impedance of one transformer is between 0.925 and 1.075 of the other.

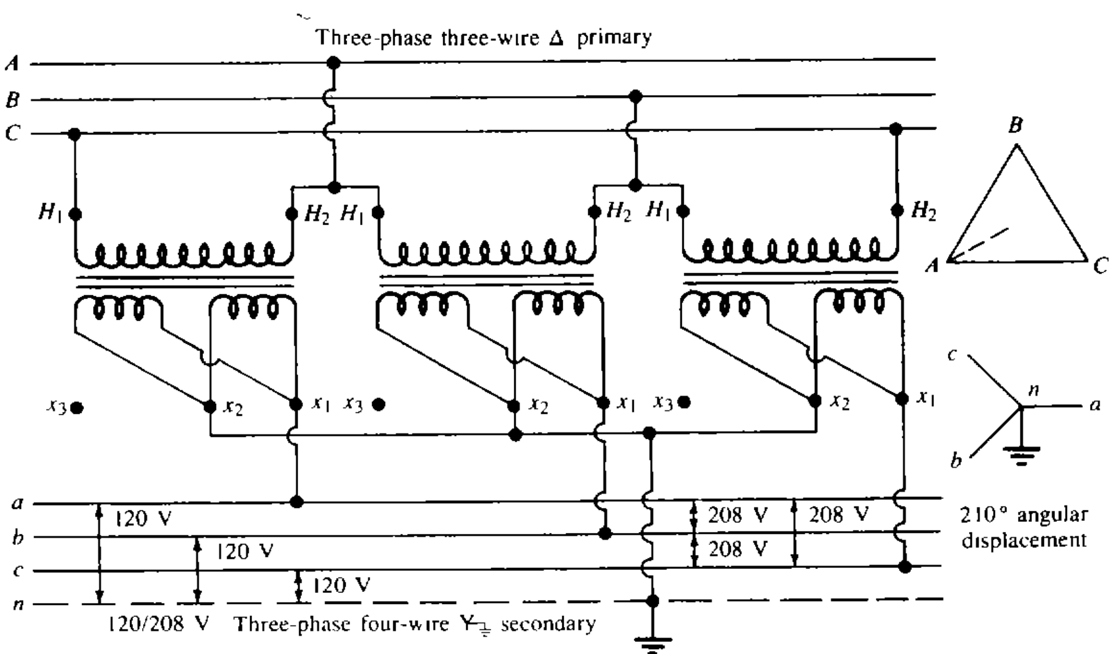


Figure 3-38 Delta-wye connection with 210° angular displacement.

The delta-wye step-up and wye-delta step-down connections are especially suitable for high-voltage transmission systems. They are economical in cost, and they supply a stable neutral point to be solidly grounded or grounded through resistance of such value as to damp the system critically and prevent the possibility of oscillation.

3-11 THREE-PHASE TRANSFORMERS

Three-phase voltages may be transformed by means of three-phase transformers. The core of a three-phase transformer is made with three legs, a primary and secondary winding of one phase being placed on each leg. It is possible to construct the core with only three legs since the fluxes established by the three windings are 120° apart in time phase. Two core legs act as the return for the flux in the third leg. For example, if flux is at a maximum value in one leg at some instant, the flux is half that value and in the opposite direction through the other two legs at the same instant.

The three-phase transformer takes less space than do three single-phase transformers having the same total capacity rating since the three windings can be placed together on one core. Furthermore, three-phase transformers are usually more efficient and less expensive than the equivalent single-phase transformer banks. This is especially noticeable at the larger ratings. On the other hand, if one phase winding becomes damaged the entire three-phase transformer has to be removed from the service. Three-phase transformers can be connected in any of the aforementioned connection types. The difference is that all connections are made inside the tank.

Figures 3-39 to 3-43 show various connection diagrams for three-phase transformers. Figure 3-39 shows a delta-delta connection for 120/208/240-V

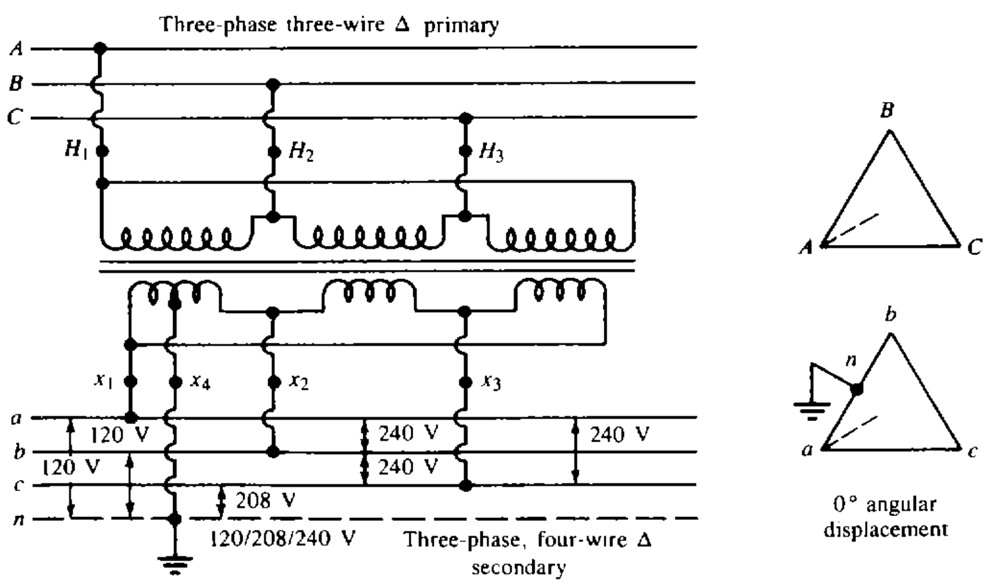


Figure 3-39 Three-phase transformer connected in delta-delta.

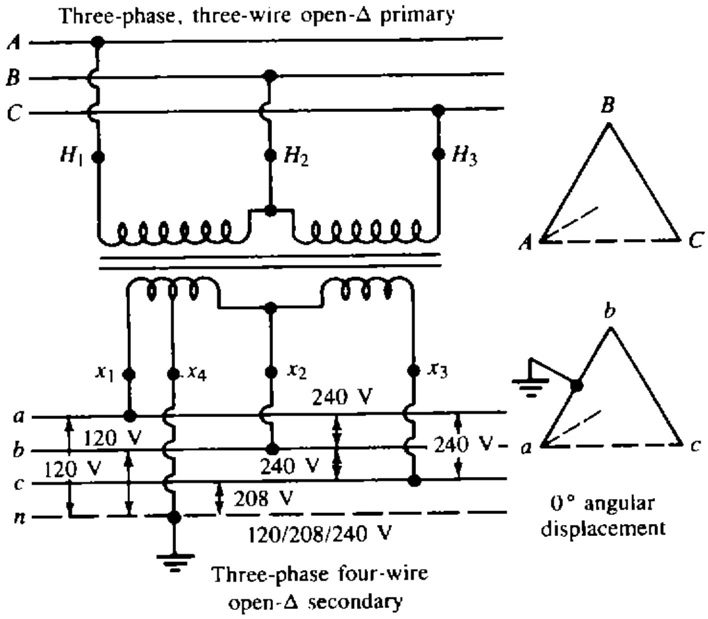


Figure 3-40 Three-phase transformer connected in open-delta.

three-phase four-wire secondary service at 0° angular displacement. It is used to supply 240-V three-phase loads with small amounts of 120-V single-phase load. Usually, transformers with a capacity of 150 kVA or less are built in such a design that when 5 percent of the rated kilovoltamperes of the transformer is taken from the 120-V tap on the 240-V connection, the three-phase capacity is decreased by 25 percent.

Figure 3-40 shows a three-phase open-delta connection for 120/240-V service. It is used to supply large 120- and 240-V single-phase loads simultaneously with small amounts of three-phase load. The two sets of windings in the transformer are of different capacity sizes in terms of kilovoltamperes. The transformer efficiency is low especially for three-phase loads. The transformer is rated only 86.6

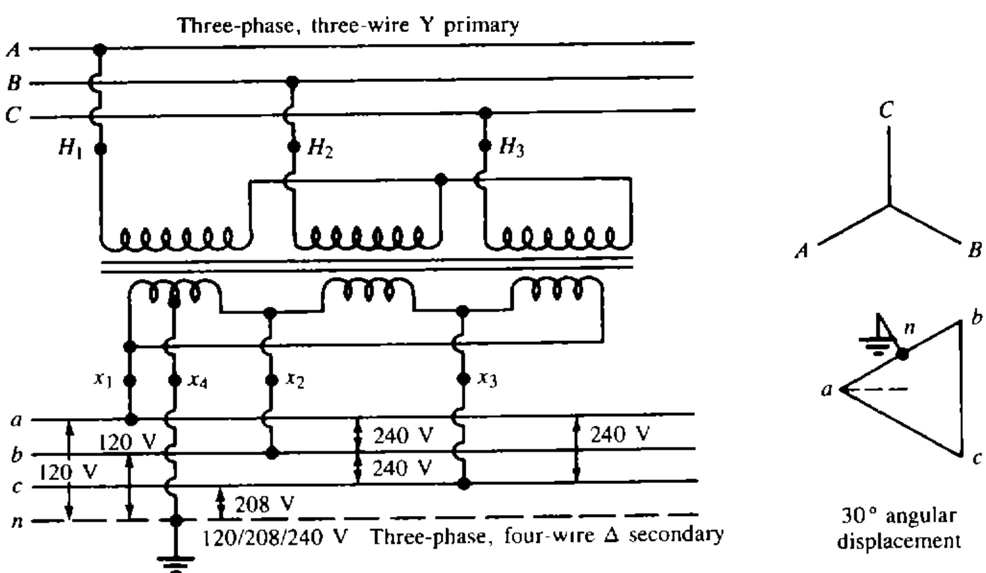


Figure 3-41 Three-phase transformer connected in wye-delta.

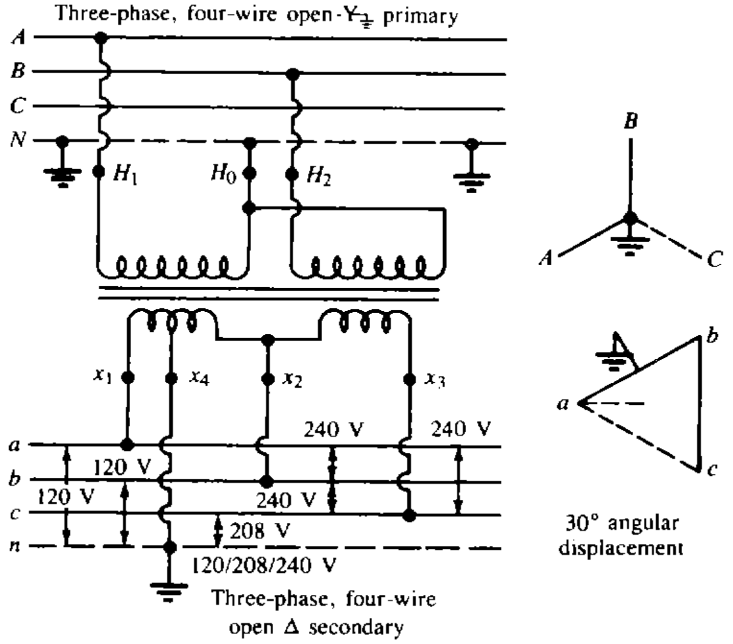


Figure 3-42 Three-phase transformer connected in open-wye open-delta.

percent of the rating of two sets of windings when they are equal in size, and less than this when they are unequal.

Figure 3-41 shows a three-phase wye-delta connection for 120/240-V service at 30° angular displacement. It is used to supply three-phase 240-V loads and small amounts of 120-V single-phase loads. Figure 3-42 shows a three-phase open-wye open-delta connection for 120/240-V service at 30° angular displacement. The statements on efficiency and capacity for three-phase open-delta connection are also applicable for this connection. Figure 3-43 shows a three-phase transformer connected in wye-wye for 120/208Y-V service. The connection allows single-phase loads to balance among the three phases.

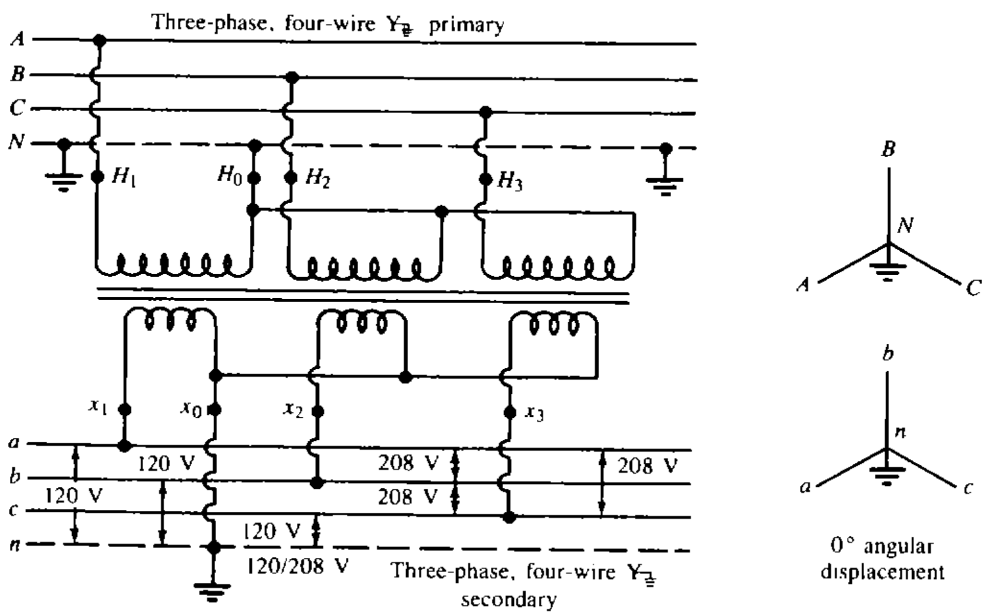


Figure 3-43 Three-phase transformer connected in grounded wye-wye.

3-12 THE T OR SCOTT CONNECTION

In some localities, two-phase is required from a three-phase system. The T or Scott connection, which employs two transformers, is the most frequently used connection for three-phase to two-phase (or even three-phase) transformations. In general, the T connection is primarily used for getting a three-phase transformation, whereas the Scott connection is mainly used for getting a two-phase transformation. In either connection type, the basic design is the same. Figures 3-44 to 3-46 show various types of the Scott connection. This connection type requires two single-phase transformers with Scott taps. The first transformer is called the *main* transformer and connected from line to line, and the second one is called the *teaser* transformer and connected from the midpoint of the first transformer to the third line. It dictates that the midpoints of both primary and secondary windings be available for connections. The secondary may be either three-, four-, or five-wire, as shown in the figures.

In either case, the connection needs specially wound, single-phase transformers. The main transformer has a 50 percent tap on the primary-side winding, whereas the teaser transformer has an 86.6 percent tap. (In usual design practice, both transformers are built to be identical so that both have a 50 percent and an 86.6 percent tap in order to be used interchangeably as main and teaser transformers.) Although only two single-phase transformers are required, their total rated kilovoltampere capacity must be 15.5 percent greater if the transformers are interchangeable, or 7.75 percent greater if noninterchangeable, than the actual load supplied (or than the standard single-phase transformer of the same kilovoltampere and voltage). It is very important to keep the relative phase sequence of the windings the same so that the impedance between the two half windings is a minimum to prevent excessive voltage drop and the resultant voltage unbalance between phases.

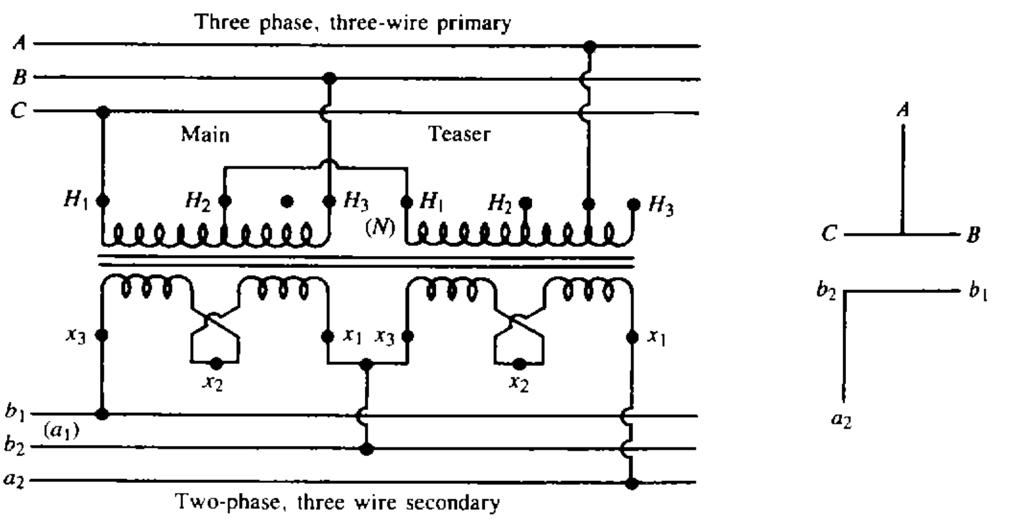


Figure 3-44 The T or Scott connection for three-phase to two-phase three-wire, transformer.

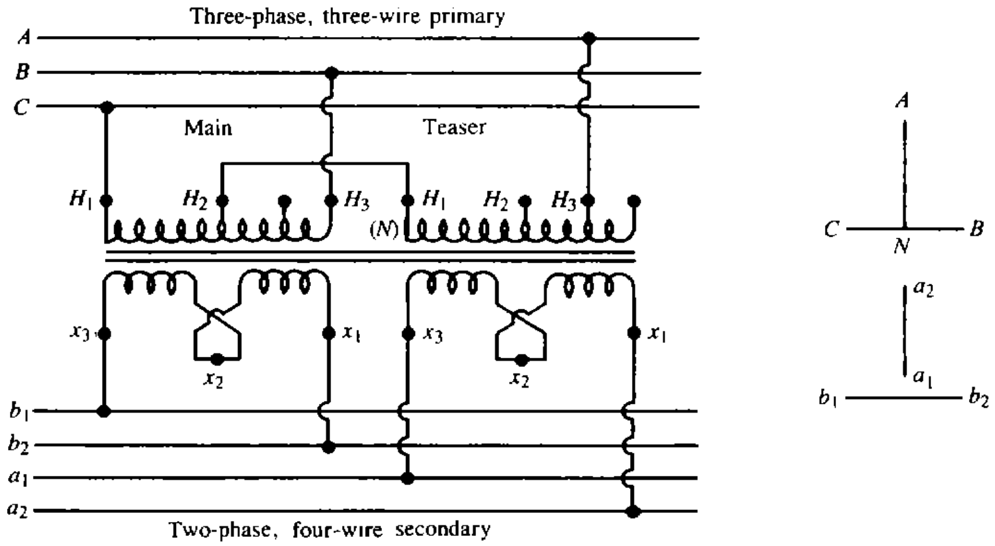


Figure 3-45 The T or Scott connection for three-phase to two-phase, four-wire, transformation.

The T or Scott connections change the number of phases but not the power factor, which means that a balanced load on the secondary will result in a balanced load on the primary. When the two-phase load at the secondary has a unity power factor, the main transformer operates at 86.6 percent power factor and the teaser transformer operates at unity power factor. These connections can transform power in either direction, i.e., from three-phase to two-phase or from two-phase to three-phase.

Example 3-7 Two transformer banks are sometimes used in distribution systems, as shown in Fig. 3-47, especially to supply customers having large single-phase lighting loads and small three-phase (motor) loads.

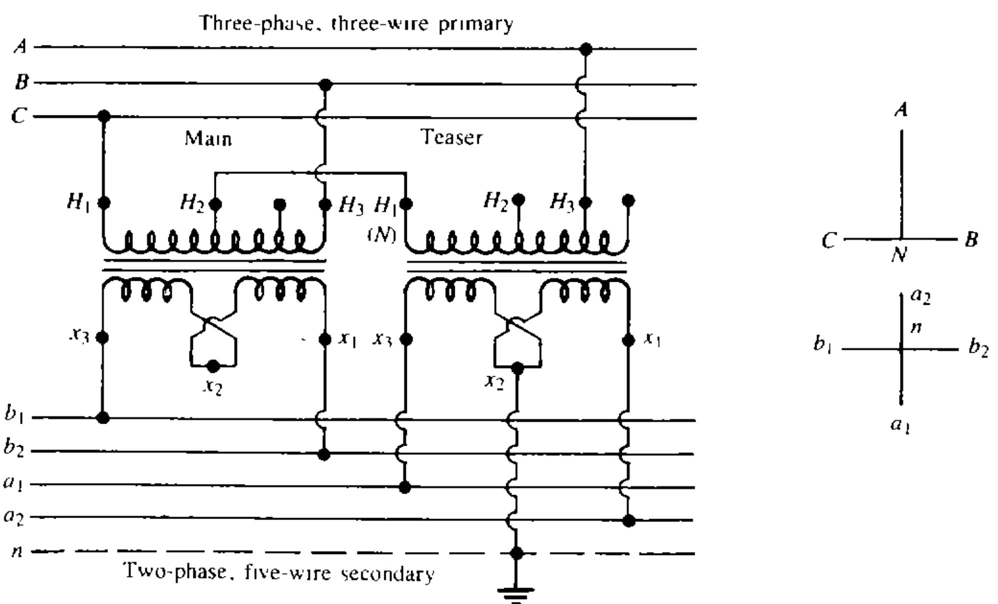


Figure 3-46 The T or Scott connection for three-phase to two-phase, five-wire, transformation.

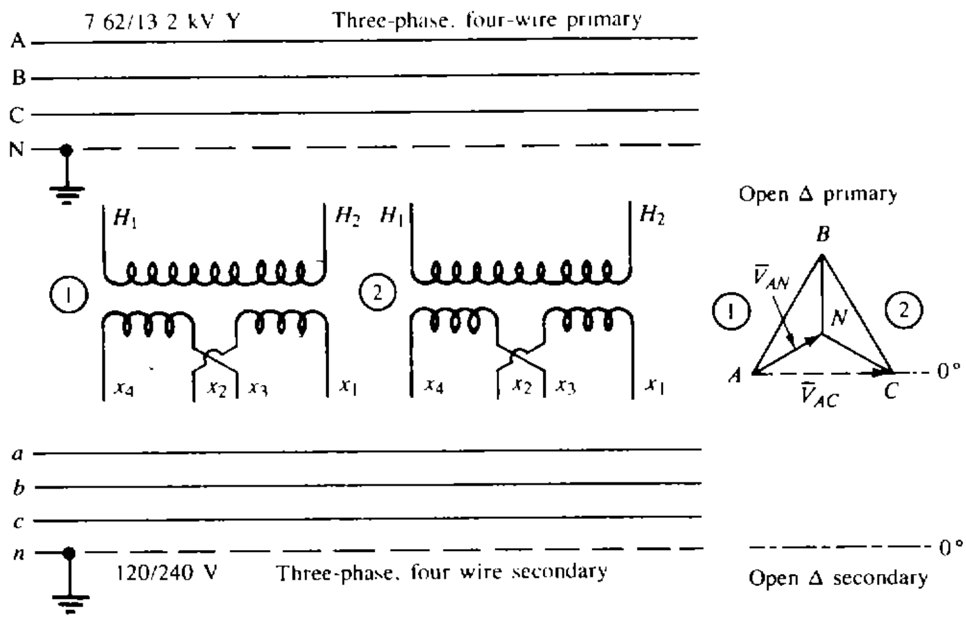


Figure 3-47

The low-voltage connections are three-phase four-wire 120/240 V open-delta. The high-voltage connections are either open-delta or open-wye. If it is open-delta, the transformer-rated high voltage is the primary line-to-line voltage. If it is open-wye, the transformer-rated high voltage is the primary line-to-neutral voltage.

In preparing wiring diagrams and phasor diagrams, it is important to understand that all odd-numbered terminals of a given transformer, that is, H_1, x_1, x_3, etc., have the same instantaneous voltage polarity. For example, if all the odd-numbered terminals are positive (+) at a particular instant of time, then all the even-numbered terminals are negative (−) at the same instant. In other words, the no-load phasor voltages of a given transformer, for example, $\bar{V}_{H_1H_2}$, $\bar{V}_{x_1x_2}$, and $\bar{V}_{x_3x_4}$, are all in phase.

Assume that ABC phase sequence is used in the connections for both high voltage and low voltage and the phasor diagrams and

$$\bar{V}_{AC} = 13{,}200\underline{/0^\circ} \text{ V}$$

and

$$\bar{V}_{AN} = 7620\underline{/30^\circ} \text{ V}.$$

Also assume that the left-hand transformer is used for lighting. To establish the two-transformer bank with open-delta primary and open-delta secondary:

(*a*) Draw and/or label the voltage phasor diagram required for the open-delta primary and open-delta secondary on the 0° references given.

(*b*) Show the connections required for the open-delta primary and open-delta secondary.

SOLUTION Figure 3-48 illustrates the solution. Note that, because of Kirchhoff's voltage law, there are $\overline{V}_{AC}$ and $\overline{V}_{ac}$ voltages between A and C and between a and c, respectively. Also note that the midpoint of the left-hand transformer is grounded to provide the 120 V for lighting loads.

Example 3-8 Figure 3-49 shows another two-transformer bank which is known as the T-T connection. Today, some of the so-called three-phase distribution transformers now marketed contain two single-phase cores and coils mounted in one tank and connected T-T. The performance is substantially like banks of three identical single-phase transformers or classical core- or shell-type three-phase transformers. However, perfectly balanced secondary voltages do not occur even though the load and the primary voltages are perfectly balanced. In spite of that, the unbalance in secondary voltages is small.

Figure 3-49 shows a particular T-T connection diagram and an arbitrary set of balanced three-phase primary voltages. Assume that the no-load line-to line and line-to-neutral voltages are 480 and 277 V, respectively, exactly like wye circuitry, and *abc* sequence.

Based on the given information and Fig. 3-49, determine the following:

(*a*) Draw the low-voltage phasor diagram, correctly oriented on the 0° reference shown.

(*b*) Find the value of the $\overline{V}_{ab}$ phasor.

(*c*) Find the magnitudes of the following rated winding voltages,

(i) The voltage $V_{H_1H_2}$ on transformer 1.

(ii) The voltage $V_{x_1x_2}$ on transformer 1.

(iii) The voltage $V_{x_2x_3}$ on transformer 1.

(iv) The voltage $V_{H_1H_2}$ on transformer 2.

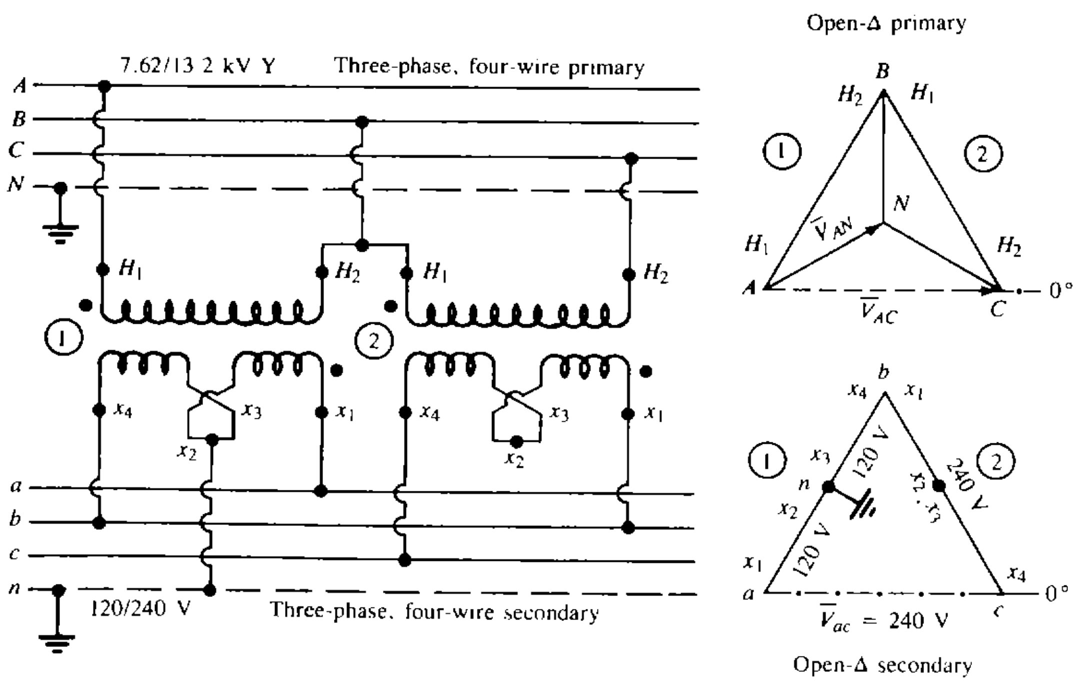


Figure 3-48

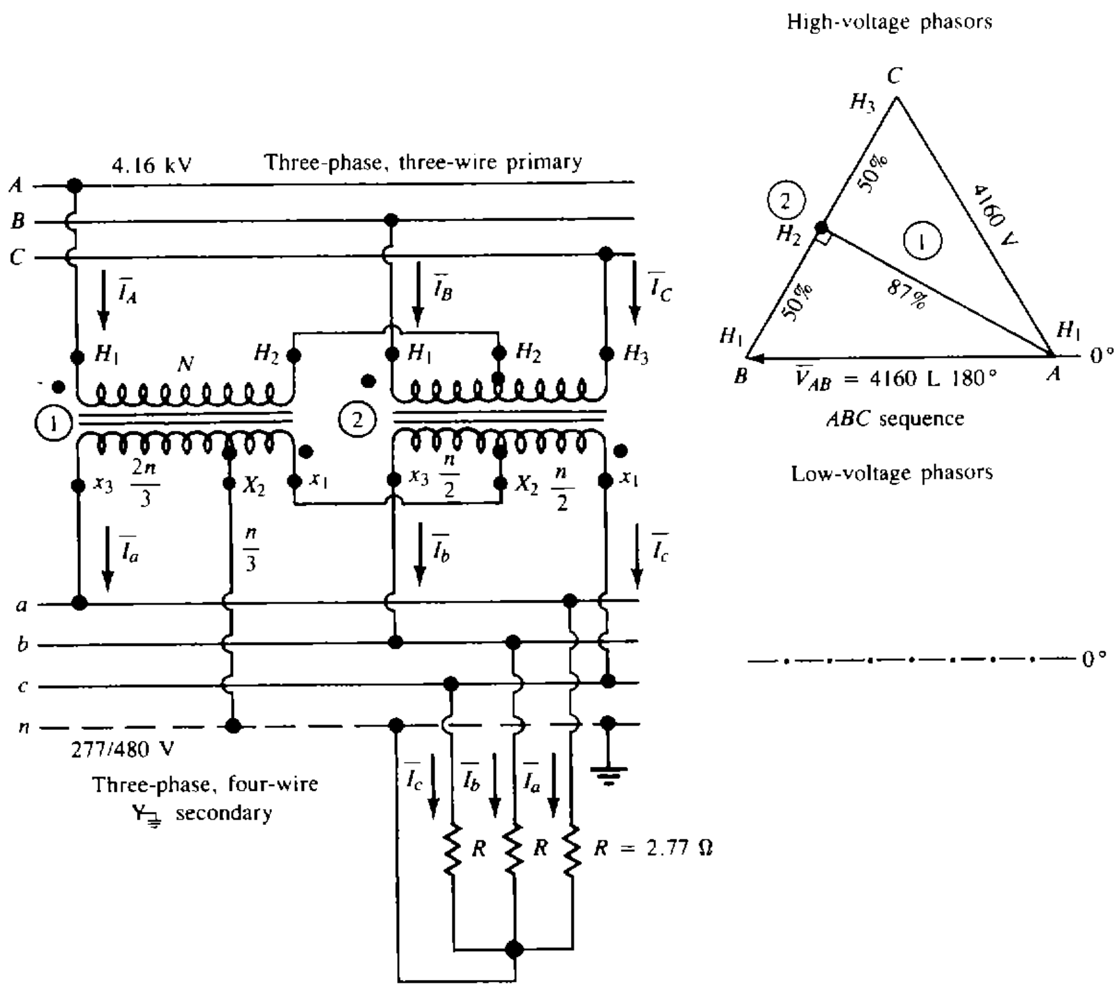


Figure 3-49

(v) The voltage $V_{H_2H_3}$ on transformer 2.
(vi) The voltage $V_{x_1x_2}$ on transformer 2.
(vii) The voltage $V_{x_2x_3}$ on transformer 2.

(*d*) Would it be possible to parallel a T-T transformer bank with:
(i) A delta-delta bank?
(ii) A wye-wye bank?
(iii) A delta-wye bank?

SOLUTION

(*a*) Figure 3-50 shows the required low-voltage phasor diagram. Note the 180° phase shift among the corresponding phasors.

(*b*) The value of the voltage phasor is

$$\bar{V}_{ab} = 480\angle 0^\circ \text{ V}$$

(*c*) The magnitudes of the rated winding voltages:
(i) From the high-voltage phasor diagram shown in Fig. 3-49,

$$|\bar{V}_{H_1H_2}| = (4160^2 - 2080^2)^{1/2}$$
$$= 3600 \text{ V}$$

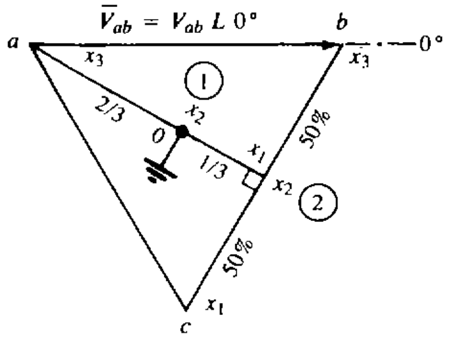


Figure 3-50

(ii) From Fig. 3-49 and 3-50,

$$|\bar{V}_{x_1x_2}| = \tfrac{1}{3}(480^2 - 240^2)^{1/2}$$
$$= 139 \text{ V}$$

(iii) From Fig. 3-49 and 3-50,

$$|\bar{V}_{x_2x_3}| = \tfrac{2}{3}(480^2 - 240^2)^{1/2}$$
$$= 277 \text{ V}$$

(iv) From Fig. 3-49,

$$|\bar{V}_{H_1H_2}| = 50\%(4160 \text{ V})$$
$$= 2080 \text{ V}$$

(v) From Fig. 3-49,

$$|\bar{V}_{H_2H_3}| = 2080 \text{ V}$$

(vi) From Fig. 3-50,

$$|\bar{V}_{x_1x_2}| = 50\%(480 \text{ V})$$
$$= 240 \text{ V}$$

(vii) From Fig. 3-50,

$$|\bar{V}_{x_2x_3}| = 240 \text{ V}$$

(*d*) (i) No; (ii) no; (iii) yes.

Example 3-9 Assume that the T-T transformer bank of Example 3-8 is to be loaded with the balanced resistors ($R = 2.77\ \Omega$) shown in Fig. 3-49. Also assume that the secondary voltages are to be perfectly balanced and that the necessary high-voltage applied voltages then are not perfectly balanced. Determine the following:

(*a*) The low-voltage current phasors.
(*b*) The low-voltage current-phasor diagram.
(*c*) At what power factor does the transformer operate?
(*d*) What power factor is seen by winding x_3x_2 of transformer 2?
(*e*) What power factor is seen by winding x_1x_2 of transformer 2?

SOLUTION

(*a*) The low-voltage phasor diagram of Fig. 3-50 can be redrawn as shown in Fig. 3-51*a*. Therefore, from Fig. 3-51*a*, the low-voltage current phasors are

$$\bar{I}_a = \frac{\bar{V}_{a0}}{R} = \frac{277\angle -30^\circ}{2.77} = 100\angle -30^\circ \text{ A}$$

$$\bar{I}_b = \frac{\bar{V}_{b0}}{R} = \frac{-277\angle +30^\circ}{2.77} = -100\angle +30^\circ = 100\angle -150^\circ \text{ A}$$

$$\bar{I}_c = \frac{\bar{V}_{c0}}{R} = \frac{277\angle 90^\circ}{2.77} = 100\angle 90^\circ \text{ A}$$

(*b*) Figure 3-51*b* shows the low-voltage current-phasor diagram.

(*c*) From part *a*, the power factor of transformer 1 can be found as

$$\cos\theta_{T1} = \cos(\theta_{\bar{V}_{a0}} - \theta_{I_a}) = \cos[(-30^\circ) - (-30^\circ)] = 1.0$$

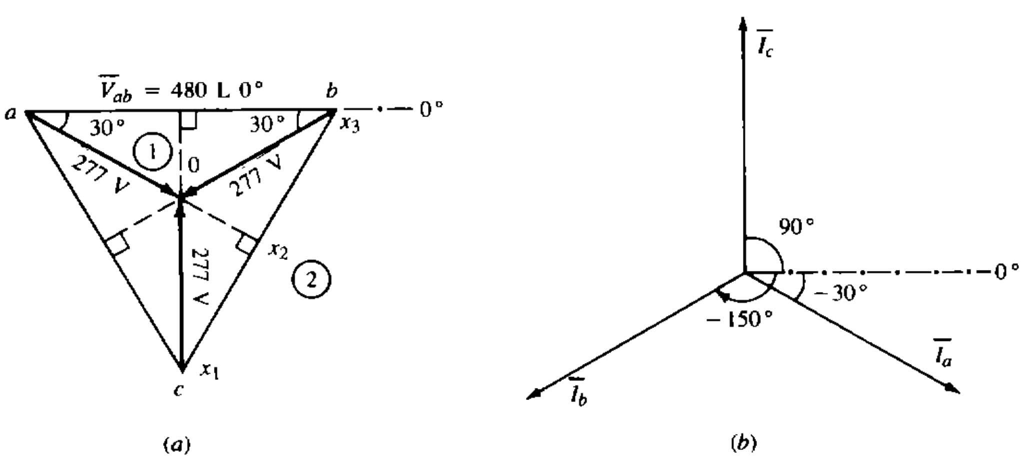


Figure 3-51

(*d*) The power factor seen by the winding x_3x_2 of transformer 2 is 0.866, lagging.
(*e*) The power factor seen by the winding x_1x_2 of transformer 2 is 0.866, leading.

Example 3-10 Consider Example 3-9 and Fig. 3-50, and determine the following:
(*a*) The necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 1.
(*b*) The necessary voltampere rating of the x_2x_1 low-voltage winding of transformer 1.
(*c*) Total voltampere output from transformer 1.
(*d*) The necessary voltampere rating of the x_1x_2 low-voltage winding of transformer 2.
(*e*) The necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 2.
(*f*) Total voltampere output from transformer 2.
(*g*) The ratio of total voltampere rating of all low-voltage windings in the transformer bank to maximum continuous voltampere output from the bank.

SOLUTION
(*a*) From Fig. 3-50, the necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 1 is

$$S_{x_2x_3} = \frac{2}{3}\left(\frac{\sqrt{3}}{2}V\right)I$$

$$= \frac{VI}{\sqrt{3}} \qquad VA$$

(*b*) Similarly,

$$S_{x_2x_1} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}V\right)I$$

$$= \frac{\mathrm{VI}}{2\sqrt{3}} \qquad VA$$

(*c*) Therefore total voltampere output rating from transformer 1 is

$$\sum S_{T1} = S_{x_2x_1} + S_{x_2x_3}$$

$$= \frac{\sqrt{3}}{2}VI \qquad \mathrm{VA}$$

(*d*) From Fig. 3-50, the necessary voltampere rating of the X_1X_2 low-voltage winding of transformer 2 is

$$S_{x_1x_2} = \frac{V}{2} \times I \qquad \text{VA}$$

(*e*) Similarly,

$$S_{x_2x_3} = \frac{V}{2} \times I \qquad \text{VA}$$

(*f*) Therefore, total voltampere output rating from transformer 2 is

$$\begin{aligned}\sum S_{T2} &= S_{x_1x_2} + S_{x_2x_3} \\ &= VI \qquad \text{VA}\end{aligned}$$

(*g*) The ratio is

$$\begin{aligned}\frac{\sum \text{Installed core and coil capacity}}{\text{Max continuous output}} &= \frac{(\sqrt{3}/2)VI + VI}{\sqrt{3}\,VI} \\ &= \frac{(\sqrt{3}/2) + 1}{\sqrt{3}} = 1.078\end{aligned}$$

The same ratio for two-transformer banks connected in open-delta HV open-delta LV, or open-wye HV open-delta LV is 1.15.

Example 3-11 In general, except for unique unbalanced loads, two-transformer banks do not deliver balanced three-phase low-voltage terminal voltages even when the applied high-voltage terminal voltages are perfectly balanced. Also the two transformers do not, in general, operate at the same power factor or at the same percentages of their rated kilovoltamperes. Hence, the two transformers are likely to have unequal percentages of voltage regulation.

Figure 3-52 shows two single-phase transformers connected in open-wye high-voltage and open-delta low-voltage. The two-transformer bank supplies a large amount of single-phase lighting and some small amount of three-phase power loads. Both transformers have 7200/120–240-V ratings and have equal transformer impedance of

$$\bar{Z}_T = 0.01 + j0.03 \text{ pu}$$

based on their ratings. Here, neglect transformer magnetizing currents.

Figure 3-53 shows the low-voltage phasor diagram. In this problem the secondary voltages are to be assumed to be perfectly balanced and the primary voltages then unbalanced as required. Note that, in Fig. 3-53, the 0 indicates

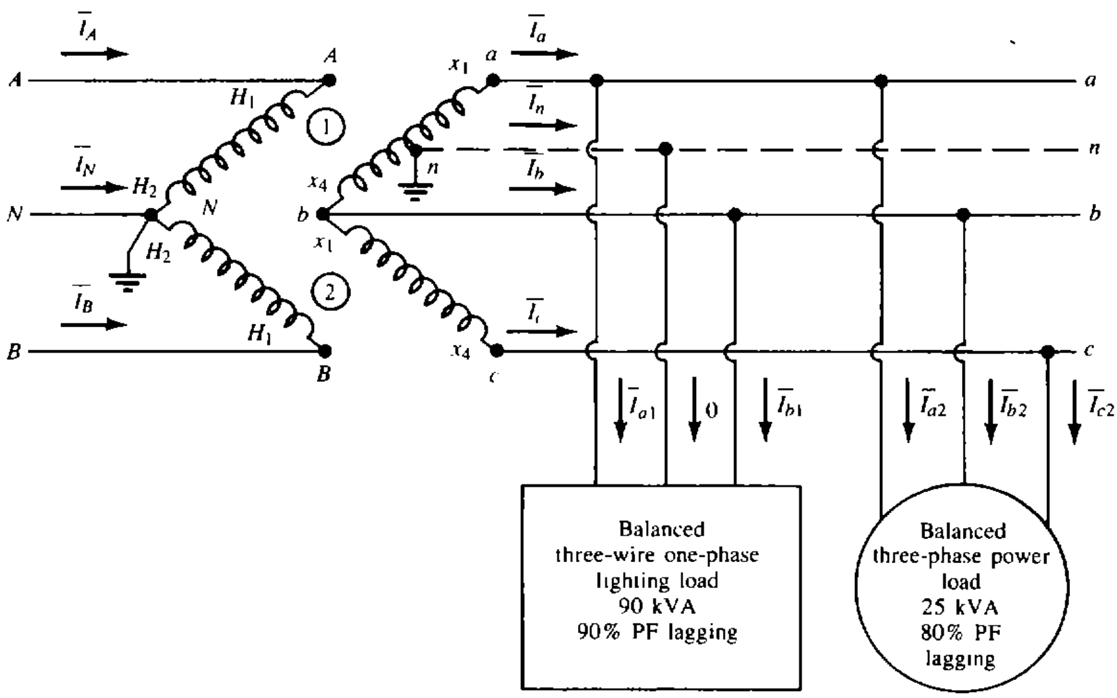


Figure 3-52

the three-phase neutral point. Based on the given information, determine the following:

(*a*) Find the phasor currents $\bar{I}_a$, $\bar{I}_b$, and $\bar{I}_c$.

(*b*) Select suitable standard kilovoltampere ratings for both transformers. Overloads, as much as 10 percent, will be allowable as an arbitrary criterion.

(*c*) Find the per unit kilovoltampere load on each transformer.

(*d*) Find the power factor of the output of each transformer.

(*e*) Find the phasor currents $\bar{I}_A$, $\bar{I}_B$, and $\bar{I}_N$ in the high-voltage leads.

(*f*) Find the high-voltage terminal voltages $\bar{V}_{AN}$ and $\bar{V}_{BN}$. Therefore this part of the question can indicate the amount of voltage unbalance that may be encountered with typical equipment and typical loading conditions.

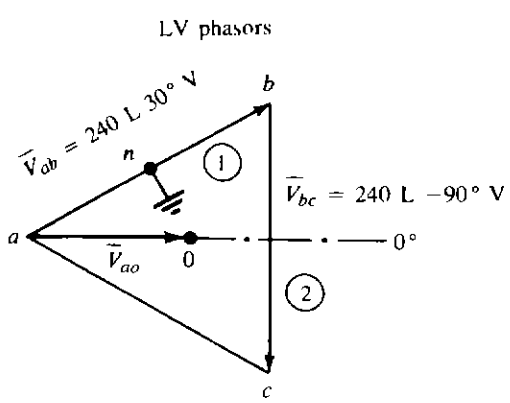


Figure 3-53

SOLUTION

(*a*) For the three-wire single-phase balanced lighting load,

$$\cos\theta = 0.90 \text{ lagging} \qquad \text{or} \qquad \theta = 25.8^\circ$$

therefore, using the symmetrical-components theory,

$$\begin{aligned}\bar{I}_{a1} &= \frac{90 \text{ kVA}}{0.240 \text{ kV}} \underline{/\theta_{\bar{V}ab} - \theta} \\ &= 375\underline{/30^\circ - 25.8^\circ} \\ &= 375(\cos 4.2^\circ + j\sin 4.2^\circ) \\ &= 374 + j27.5 \text{ A} \\ &= 375\underline{/4.2^\circ} \text{ A}\end{aligned}$$

Also

$$\begin{aligned}\bar{I}_{b1} &= -\bar{I}_{a1} \\ &= -374 - j27.5 \text{ A} \\ &= -375\underline{/4.2^\circ} \text{ A}\end{aligned}$$

For the three-phase balanced power load,

$$\cos\theta = 0.80 \text{ lagging} \qquad \text{or} \qquad \theta = 36.8^\circ$$

therefore

$$\begin{aligned}\bar{I}_{a2} &= \frac{25 \text{ kVA}}{\sqrt{3} \times 0.240 \text{ kV}} \underline{/\theta_{\bar{V}_{a0}} - \theta} \\ &= 60.2\underline{/0^\circ - 36.8^\circ} \\ &= 60.2\underline{/-36.8^\circ} \text{ A.}\end{aligned}$$

Also

$$\begin{aligned}\bar{I}_{b2} &= a^2 \bar{I}_{a2} \\ &= 1\underline{/240^\circ} \times 60.2\underline{/-36.8^\circ} \\ &= 60.2\underline{/203.2^\circ} \text{ A}\end{aligned}$$

and

$$\begin{aligned}\bar{I}_{c2} &= a\bar{I}_{a2} \\ &= 1\underline{/120^\circ} \times 60.2\underline{/-36.8^\circ} \\ &= 60.2\underline{/83.2^\circ} \text{ A}\end{aligned}$$

Therefore the phasor currents in the transformer secondary are

$$\begin{aligned}\bar{I}_a &= \bar{I}_{a1} + \bar{I}_{a2}\\ &= 375\underline{/4.2^\circ} + 60.2\underline{/-36.8^\circ}\\ &= 422.04 - j8.44\\ &= 422.12\underline{/-1.15^\circ}\ \text{A}\end{aligned}$$

$$\begin{aligned}\bar{I}_b &= \bar{I}_{b1} + \bar{I}_{b2}\\ &= -375\underline{/4.2^\circ} + 60.2\underline{/203.2^\circ}\\ &= -429.33 - j51.22\\ &= 432.37\underline{/-173.2^\circ}\ \text{A}\end{aligned}$$

$$\begin{aligned}\bar{I}_c &= \bar{I}_{c1} + \bar{I}_{c2}\\ &= 0 + 60.2\underline{/83.2^\circ}\\ &= 60.2\underline{/83.2^\circ}\ \text{A}\end{aligned}$$

(*b*) For transformer 1.

$$\begin{aligned}S_{T1} &= 0.240\ \text{kV} \times I_a\\ &= 0.240 \times 422.12\\ &= 101.3\ \text{kVA}\end{aligned}$$

If a transformer with 100 kVA is selected,

$$S_{T1} = 1.013\ \text{pu kVA}$$

with an overload of 1.3 percent. For transformer 2,

$$\begin{aligned}S_{T2} &= 0.240\ \text{kV} \times I_c\\ &= 0.240 \times 60.2\\ &= 14.4\ \text{kVA}\end{aligned}$$

If a transformer with 15 kVA is selected,

$$S_{T2} = 0.96\ \text{pu kVA}$$

with a 4 percent excess capacity.

(*c*) From part *b*,

$$S_{T1} = 1.013\ \text{pu kVA}$$

$$S_{T2} = 0.96\ \text{pu kVA}$$

(*d*) Since the power factor that a transformer sees is not the power factor that the load sees, for transformer 1,

$$\begin{aligned}\cos\theta_{T1} &= \cos(\theta_{\bar{V}_{ab}} - \theta_{\bar{I}_a})\\ &= \cos[30^\circ - (-1.15^\circ)]\\ &= \cos 31.15^\circ\\ &= 0.856 \text{ lagging}\end{aligned}$$

and for transformer 2,

$$\begin{aligned}\cos\theta_{T2} &= \cos(\theta_{\bar{V}_{cb}} - \theta_{\bar{I}_c})\\ &= \cos(90^\circ - 83.2^\circ)\\ &= \cos 6.8^\circ\\ &= 0.993 \text{ lagging}\end{aligned}$$

(*e*) The turns ratio is

$$n = \frac{7200\text{ V}}{240\text{ V}} = 30$$

therefore

$$\begin{aligned}\bar{I}_A &= \frac{\bar{I}_a}{n}\\ &= \frac{422.12\underline{/-1.15^\circ}}{30}\\ &= 14.07\underline{/-1.15^\circ}\text{ A}\end{aligned}$$

$$\begin{aligned}\bar{I}_C &= -\frac{\bar{I}_C}{n}\\ &= -\frac{60.2\underline{/83.2^\circ}}{30}\\ &\cong -2\underline{/83.2^\circ}\text{ A}\end{aligned}$$

Thus

$$\begin{aligned}\bar{I}_N &= -(\bar{I}_A + \bar{I}_B)\\ &= -(14.07\underline{/-1.15^\circ} - 2\underline{/83.2^\circ}\\ &= -14.02\underline{/-9.3^\circ}\text{ A.}\end{aligned}$$

(*f*) In per units,

$$\bar{V}_{AN,\text{pu}} = \bar{V}_{ab,\text{pu}} + \bar{I}_{a,\text{pu}} \times \bar{Z}_{T,\text{pu}}$$

where

$$\bar{I}_{\text{base, LV}} = \frac{100 \text{ kVA}}{0.240 \text{ kV}}$$
$$= 416.67 \text{ A}$$

$$\bar{I}_{a,\text{pu}} = \frac{\bar{I}_a}{I_{\text{base, LV}}}$$
$$= \frac{422.12\angle -1.15^\circ}{416.67}$$
$$= 1.013\angle -1.15^\circ \text{ pu A}$$

$$\bar{V}_{ab,\text{pu}} = \frac{\bar{V}_{ab}}{\bar{V}_{\text{base, LV}}}$$
$$= \frac{0.240\angle 30^\circ}{0.240}$$
$$= 1.0\angle 30^\circ \text{ pu V}$$

$$\bar{Z}_{T,\text{pu}} = 0.01 + j0.03 \text{ pu } \Omega$$

Therefore

$$\bar{V}_{AN,\text{pu}} = 1.0\angle 30^\circ + (1.013\angle -1.15^\circ)(0.01 + j0.03)$$
$$= 1.024\angle 31.15^\circ \text{ pu V}$$

or

$$\bar{V}_{AN} = \bar{V}_{AN,\text{pu}} \times \bar{V}_{\text{base, HV}}$$
$$= (1.024\angle 31.15^\circ)(7200 \text{ V})$$
$$= 7372.8\angle 31.15^\circ \text{ V}$$

Also

$$\bar{V}_{BN,\text{pu}} = \bar{V}_{bc,\text{pu}} - \bar{I}_{c,\text{pu}} \times \bar{Z}_{T,\text{pu}}$$

where

$$\bar{I}_{c,\text{pu}} = \frac{\bar{I}_c}{I_{\text{base, LV}}}$$
$$= \frac{60.2\angle 83.2^\circ}{416.67}$$
$$= 0.144\angle 83.2^\circ \text{ pu A}$$

$$\overline{V}_{bc,\,\text{pu}} = \frac{\overline{V}_{bc}}{\overline{V}_{\text{base, LV}}}$$
$$= \frac{0.240\angle{-90^\circ}}{0.240}$$
$$= 1.0\angle{-90^\circ}\ \text{V}$$

Therefore

$$\overline{V}_{BN,\,\text{pu}} = 1.0\angle{-90^\circ} - (0.144\angle{83.2^\circ})(0.01 + j0.03)$$
$$= 1.0129\angle{-88.44^\circ}\ \text{pu V}$$

or

$$\overline{V}_{BN} = \overline{V}_{BN,\,\text{pu}} \times \overline{V}_{\text{base, HV}}$$
$$= (1.0129\angle{-88.44^\circ})(7200\ \text{V})$$
$$= 7292.9\angle{-88.44^\circ}\ \text{V}$$

Note that the difference between the phase angles of the $\overline{V}_{AN}$ and $\overline{V}_{BN}$ voltages is almost 120° and the difference between their magnitudes is almost 80 V.

3-13 THE AUTOTRANSFORMER

The usual transformer has two windings (not including a tertiary, if there is any) which are not connected to each other, whereas an autotransformer is a transformer in which one winding is connected in series with the other as a single winding. In this sense, an autotransformer is a normal transformer connected in a special way. It is rated on the basis of output kilovoltamperes rather than the transformer's kilovoltamperes. It has lower leakage reactance, lower losses, smaller excitation current requirements, and, most of all, it is cheaper than the equivalent two-winding transformer (especially when the voltage ratio is 2:1 or less).

Figure 3-54 shows the wiring diagram of a single-phase autotransformer. Note that S and C denote the series and common portions of the winding. There

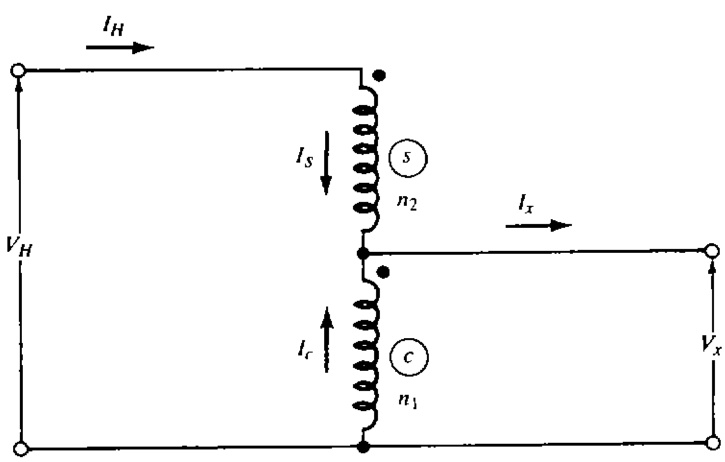


Figure 3-54 Wiring diagram of a single-phase autotransformer.

are two voltage ratios, namely, circuit and winding ratios. The circuit ratio is

$$\begin{aligned}\frac{V_H}{V_x} &= n \\ &= \frac{n_1 + n_2}{n_1} \\ &= 1 + \frac{n_2}{n_1}\end{aligned} \tag{3-67}$$

where V_H = high-voltage-side voltage
V_x = low-voltage-side voltage
n = turns ratio of the autotransformer
n_1 = number of turns in common winding
n_2 = number of turns in series winding

As can be observed from Eq. (3-67), the circuit ratio is always larger than 1.

On the other hand, the winding-voltage ratio is

$$\begin{aligned}\frac{V_S}{V_C} &= \frac{n_2}{n_1} \\ &= n - 1\end{aligned} \tag{3-68}$$

where V_S = voltage across the series winding
V_C = voltage across the common winding

Of course, the current ratio is

$$\begin{aligned}\frac{I_C}{I_S} &= \frac{I_C}{I_H} \\ &= \frac{I_x - I_H}{I_H} \\ &= n - 1\end{aligned} \tag{3-69}$$

where I_C = current in common winding
I_S = current in series winding
I_x = output current at the low-voltage side
I_H = input current at the high-voltage side

Therefore the circuits voltampere rating for an ideal autotransformer is

$$\begin{aligned}\text{Circuits VA rating} &= V_H I_H \\ &= V_x I_x\end{aligned} \tag{3-70}$$

and the windings voltampere rating is

$$\begin{aligned}\text{Windings VA rating} &= V_S I_S \\ &= V_C I_C\end{aligned} \tag{3-71}$$

which describes the capacity of the autotransformer in terms of core and coils.

Therefore the capacity of an autotransformer can be compared to the capacity of an equivalent two-winding transformer (assuming the same core and coils are used) as

$$\frac{\text{Capacity as autotransformer}}{\text{Capacity as two-winding transformer}} = \frac{V_H I_H}{V_S I_S}$$

$$= \frac{V_H I_H}{(V_H - V_x) I_H}$$

$$= \frac{V_H / V_x}{(V_H - V_x)/V_x}$$

$$= \frac{n}{n-1} \tag{3-72}$$

For example, if n is given as 2, the ratio, given by Eq. (3-72), is 2, which means that

Capacity as autotransformer = 2 × capacity as two-winding transformer.

Therefore one can use a 500-kVA autotransformer instead of using a 1000-kVA two-winding transformer. Note that as n approaches 1, which means that the voltage ratios approach 1, such as 7.2 kV/6.9 kV, then the savings, in terms of the core and coil sizes of autotransformer, increases. An interesting case happens when the voltage ratio (or the turns ratio) is unity: the maximum savings is achieved but then there is no need for any transformer since the high and low voltages are the same.

Figure 3-55 shows a single-phase autotransformer connection used in distribution systems to supply 120/240-V single-phase power from an existing 208Y/120-V three-phase system, the most economically.

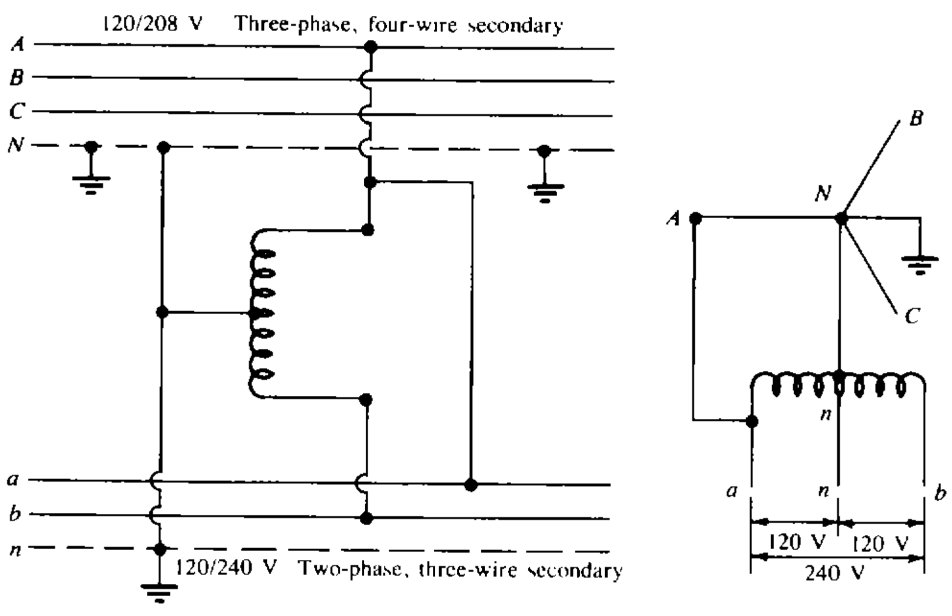


Figure 3-55 Single-phase autotransformer.

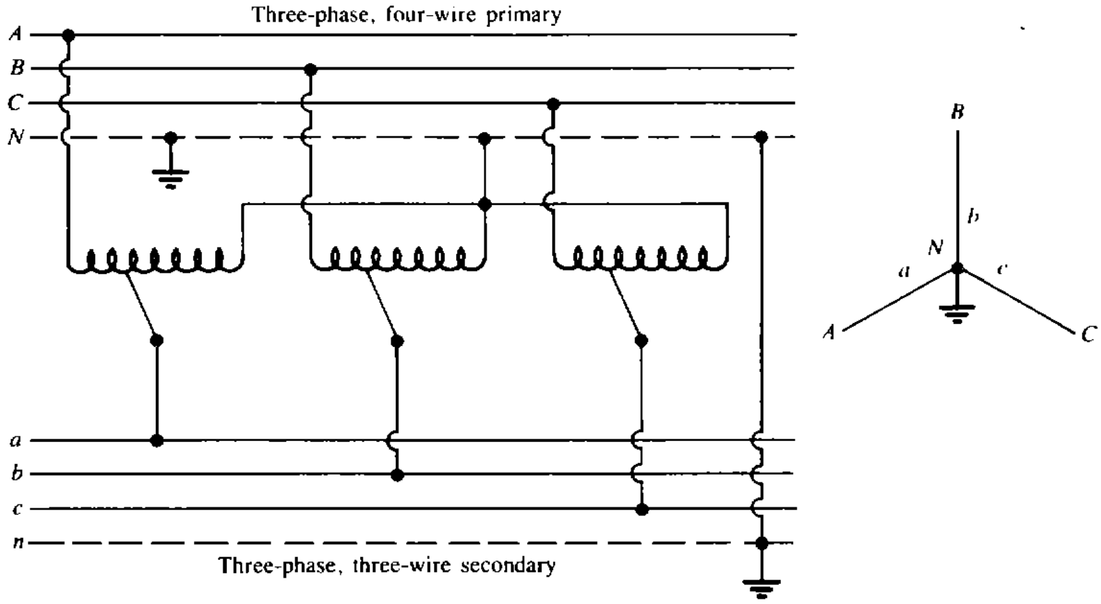


Figure 3-56 Three-phase autotransformer.

Figure 3-56 shows a three-phase autotransformer wye-wye connection used in distribution systems to increase voltage at the ends of feeders or where extensions are being made to existing feeders. It is also the most economical way of stepping down the voltage. It is necessary that the neutral of the autotransformer bank be connected to the system neutral to prevent excessive voltage development on the secondary side. Also, the system impedance should be large enough to restrict the short-circuit current to about 20 times the rated current of the transformer to prevent any transformer burnouts.

3-14 THE BOOSTER TRANSFORMERS

Booster transformers are also called the *buck-and-boost transformers* and provide a fixed buck or boost voltage to the primary of a distribution system when the line voltage drop is excessive. The transformer connection is made in such a way that the secondary is in series and in phase with the main line.

Figure 3-57 shows a single-phase booster transformer connection. The connections shown in Fig. 3-57*a* and *b* boost the voltage 5 and 10 percent, respectively. In Figure 3-57*a*, if the lines to the low-voltage bushings x_3 and x_1 are interchanged, a 5 percent buck in the voltage results. Figure 3-58 shows a three-phase three-wire booster transformer connection using two single-phase booster transformers. Figure 3-59 shows a three-phase four-wire booster transformer connection using three single-phase booster transformers. Both low- and high-voltage windings and bushings have the same level of insulation. To prevent harmful voltage induction by the series winding, the transformer primary must never be open under any circumstances before opening or unloading the secondary. Also, the primary side of the transformer should not have any fuses or disconnecting devices. Boosters are often used in distribution feeders where the cost of tap-changing transformers is not justified.

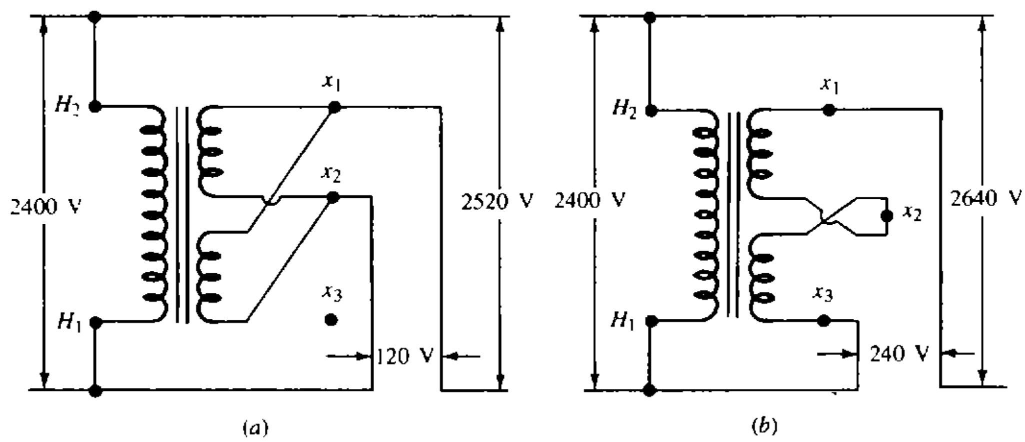


Figure 3-57 Single-phase booster transformer connection: (*a*) for 5 percent boost and (*b*) for 10 percent boost.

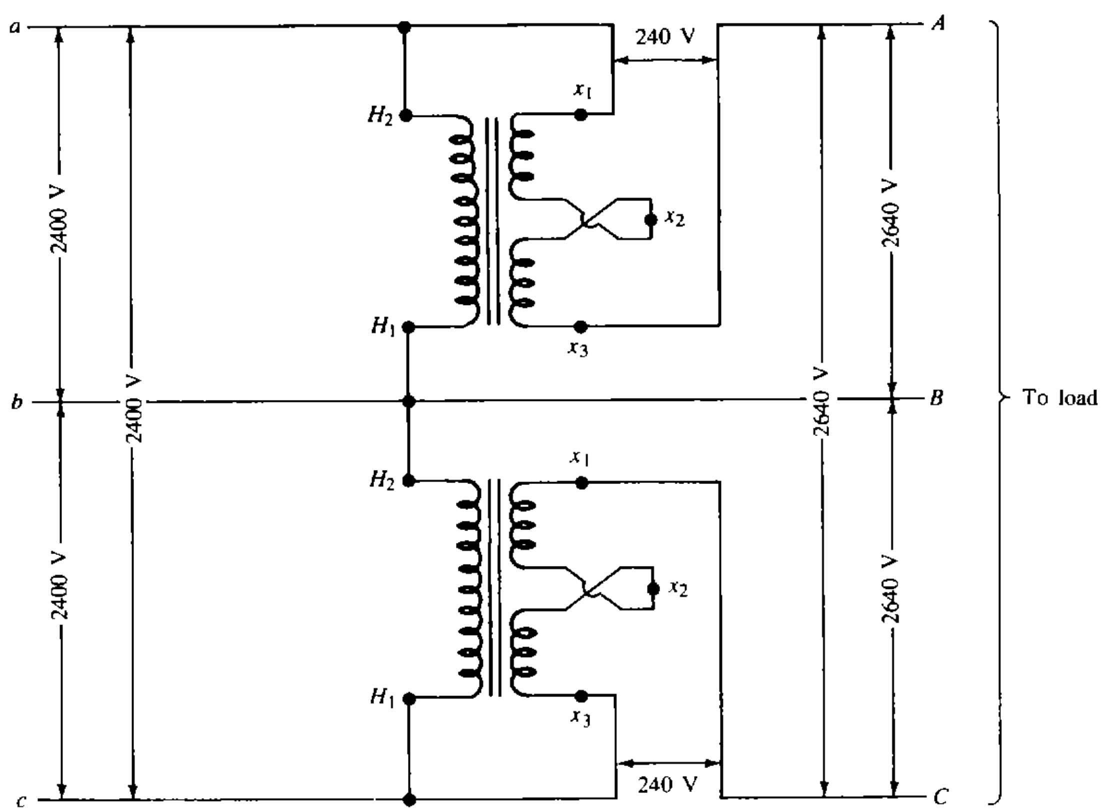


Figure 3-58 Three-phase three-wire booster transformer connection using two single-phase booster transformers.

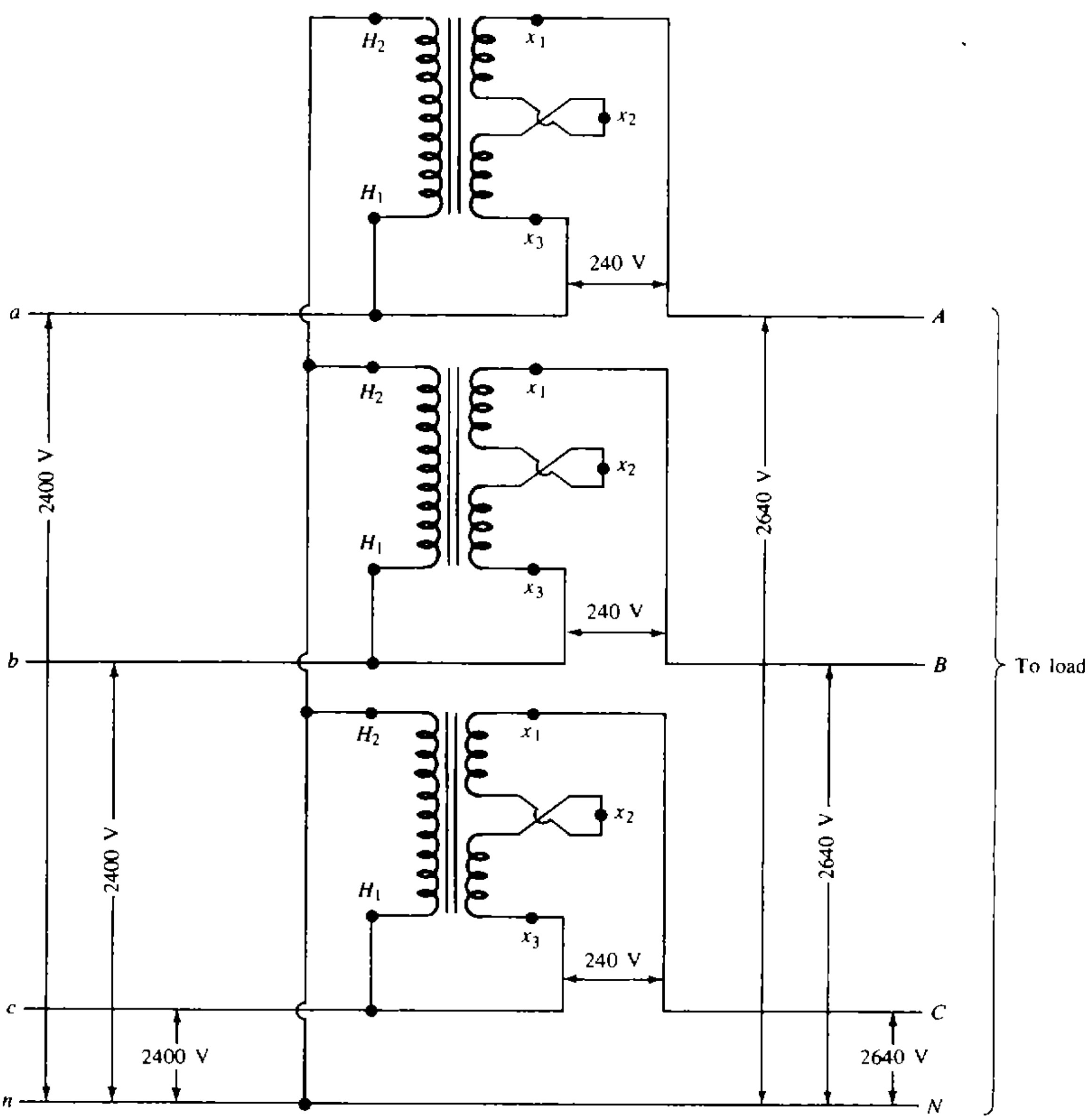


Figure 3-59 Three-phase four-wire booster transformer connection using three single-phase booster transformers.

PROBLEMS

3-1 Repeat Example 3-7, assuming an open-wye primary and an open-delta secondary and using the 0° references given in Fig. P3-1.
Also, determine:

(*a*) The value of the open-delta high-voltage phasor between A and B, that is, $\overline{V}_{AB}$.

(*b*) The value of the open-wye high-voltage phasor between A and N, that is, $\overline{V}_{AN}$.

3-2 Repeat Example 3-10, if the low-voltage line current I is 100 A and the line-to-line low voltage is 480 V.

3-3 Consider the T-T connection given in Fig. P3-3 and determine the following:

(*a*) Draw the low-voltage diagram, correctly oriented on the 0° reference shown.

(*b*) Find the value of the $\overline{V}_{ab}$ and $\overline{V}_{an}$ phasors.

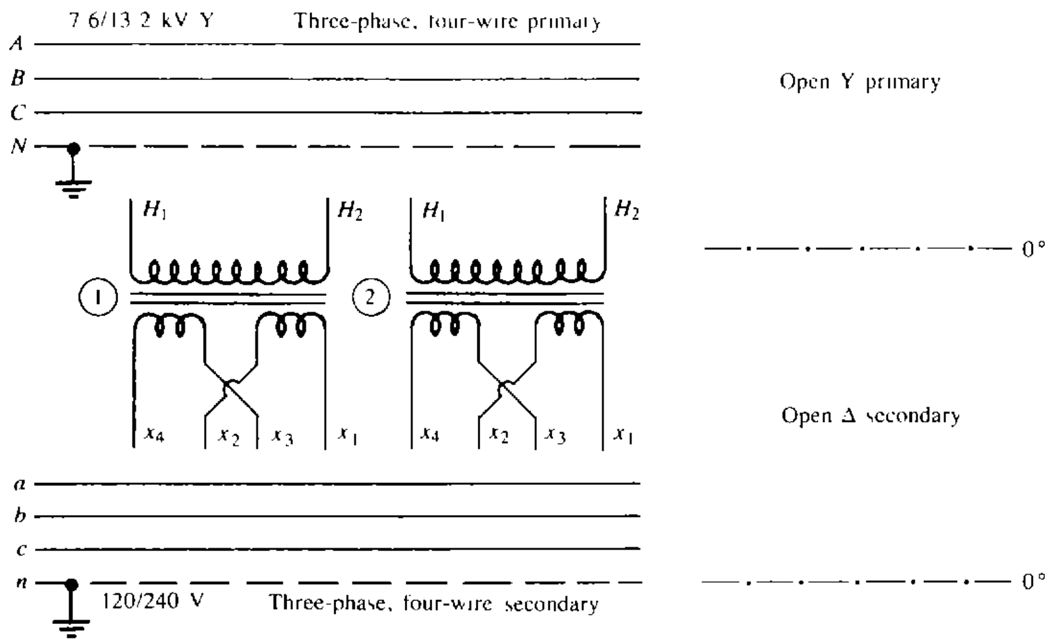


Figure P3-1

(*c*) Find the magnitudes of the following rated winding voltages:

(i) The voltage $V_{H_1H_2}$ on transformer 1.

(ii) The voltage $V_{x_1\phi}$ on transformer 1.

(iii) The voltage $V_{\phi x_2}$ on transformer 1.

(iv) The voltage $V_{H_1 0}$ on transformer 2.

(v) The voltage V_{0H} on transformer 2.

(vi) The voltage $V_{x_1 n}$ on transformer 2.

(vii) The voltage V_{nx} on transformer 2.

3-4 Assume that the T-T transformer bank given in Prob. 3-3 is loaded with the balanced resistors given. Assume that the secondary voltages are perfectly balanced; the necessary HV applied voltages then are not perfectly balanced. Use secondary voltages of 480 V and neglect magnetizing currents. Determine the following:

(*a*) The low-voltage current phasors.

(*b*) The high-voltage current phasors.

3-5 Use the results of Probs. 3-3 and 3-4 and apply the complex power formula $\bar{S} = P + jQ = \bar{V}\bar{I}^*$ four times, once for each low-voltage winding, e.g., a part of the output of transformer 1 is $\bar{V}_{X_1X_2}\bar{I}_a^*$. Based on these results, find:

(*a*) Total complex power output from the T-T bank. (Does your result agree with that which is easily computed as input to the resistors?)

(*b*) The necessary kilovoltampere ratings of both low-voltage windings of both transformers.

(*c*) The ratio of total kilovoltampere ratings of all low-voltage windings in the transformer bank to total kilovoltampere output from the bank.

3-6 Consider Fig. P3-6 and assume that the motor is rated 25 hp and is mechanically loaded so that it draws 25.0-kVA three-phase input at $\cos\theta = 0.866$ lagging power factor.

(*a*) Draw the necessary high-voltage connections so that the low voltages shall be as shown, i.e., of *abc* phase sequence.

(*b*) Find the power factors $\cos\theta_{T_1}$ and $\cos\theta_{T_2}$ at which each transformer operates.

(*c*) Find the ratio of voltampere load on one transformer to total voltamperes delivered to the load.

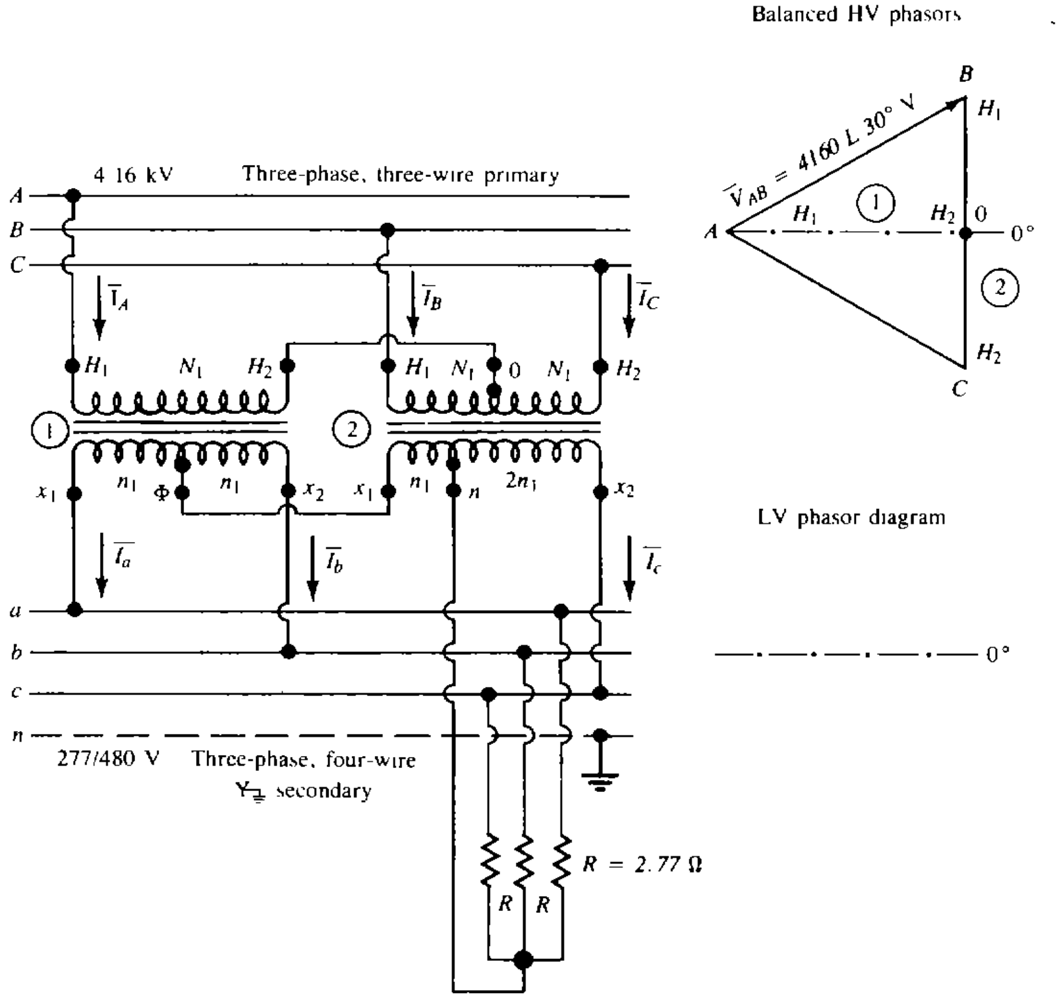


Figure P3-3

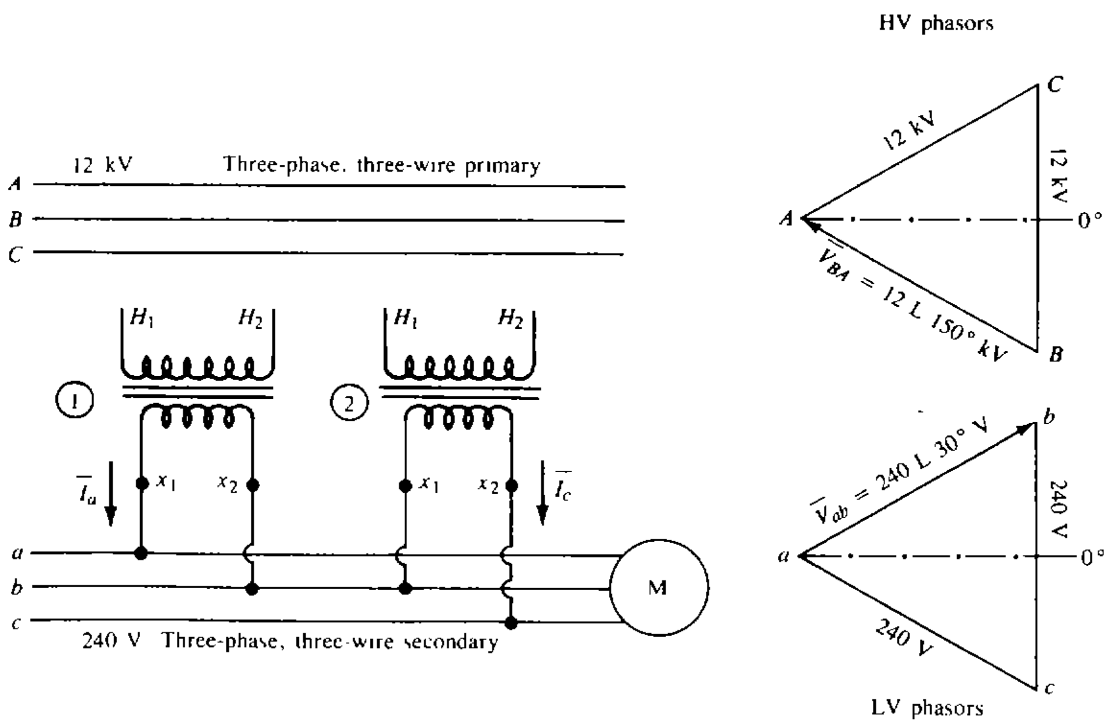


Figure P3-6

3-7 Consider Fig. P3-7 and assume that the two-transformer T-T bank delivers 120/208 V three-phase four-wire service from a three-phase three-wire 4160 V primary line. The problem is to determine if this bank can carry unbalanced loads even though the primary neutral terminal N is not connected to the source neutral. (If it can, the T-T performance is quite different from the three-transformer wye-grounded wye bank.) Employ the ideal-transformer theory and pursue the question as follows:

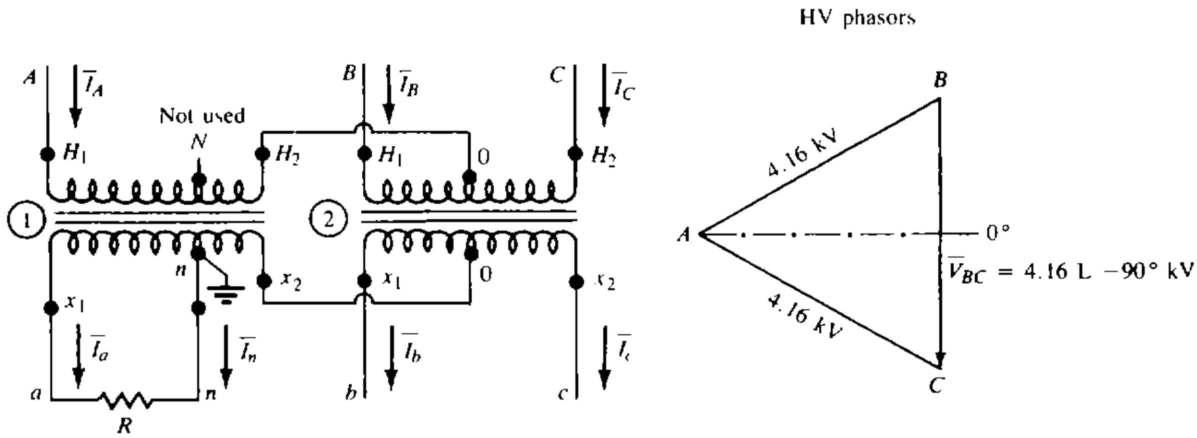


Figure P3-7

(*a*) Load phase *an* with $R = 1.20\ \Omega$ resistance and then find the following six complex currents numerically: $\bar{I}_a$, $\bar{I}_b$, $\bar{I}_c$, $\bar{I}_A$, $\bar{I}_B$, and $\bar{I}_C$.

(*b*) Find the following complex powers of windings by using the $\bar{S} = P + jQ = \bar{V}\bar{I}^*$ equation numerically:

$$\bar{S}_{T_1(x_1-n)} = \text{complex power of } x_1 - n \text{ portion of transformer 1,}$$

$$\bar{S}_{T_1(H_1-H_2)} = \text{complex power of } H_1 - H_2 \text{ portion of transformer 1,}$$

$$\bar{S}_{T_2(H_1-O)} = \text{complex power of } H_1 - O \text{ portion of transformer 2,}$$

$$\bar{S}_{T_2(H_2-O)} = \text{complex power of } H_2 - O \text{ portion of transformer 2.}$$

(*c*) Do your results indicate that this bank will carry unbalanced loads successfully? Why?

3-8 Figure P3-8 shows two single-phase transformers, each with a 7620-V high-voltage winding and two 120-V low-voltage windings. The diagram shows the proposed connections for an open-wye to open-delta bank and the high-voltage applied phasor-voltage drops. Here, *abc* phase sequence at low-voltage and high-voltage sides and 120/240 V are required.

(*a*) Sketch the low-voltage phasor diagram, correctly oriented on the 0° reference line. Label it adequately with x's ①, and ②, etc. to identify.

(*b*) State whether or not the proposed connections will output the required three-phase four-wire 120/240-V delta low voltage.

3-9 A large number of 25-kVA distribution transformers are to be purchased. Two competitive bids have been received. The bid data are tabulated below.

Transformer	Cost of transformer delivered to NL&NP's warehouse	Copper loss at rated load	Core loss at rated voltage and frequency	Per unit exciting current
A	$355	360 W	130 W	0.015
B	$345	380 W	150 W	0.020

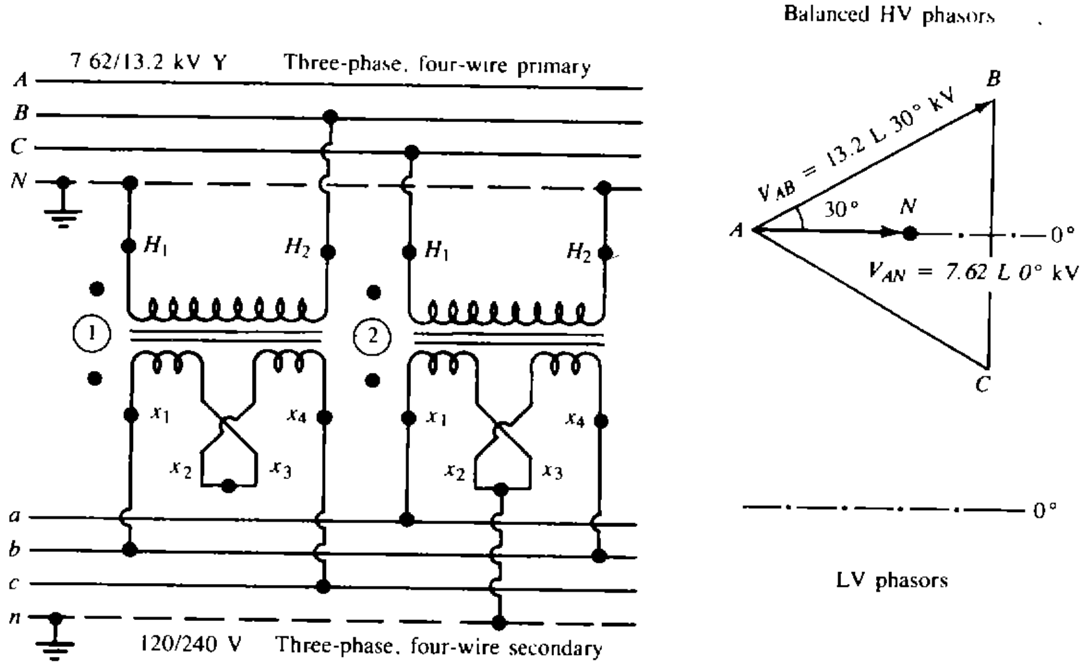


Figure P3-8

Evaluate the bids on the basis of total annual cost (TAC) and recommend the purchase of the one having the least TAC. The cost of installing a transformer is not to be included in this study. The following system data are given:

Annual peak load on transformer = 35 kVA
Annual loss factor = 0.15
Per unit annual fixed charge rate = 0.15
Installed cost of shunt capacitors = \$10/kvar
Incremental cost of off-peak energy = \$0.01/kWh
Incremental cost of on-peak energy = \$0.012/kWh
Investment cost of power system upstream from distribution transformers = \$300/kVA.

Calculate the TAC of owning and operating one such transformer, and state which transformer should be purchased. (*Hint*: Study the relevant equations in Chap. 6 before starting to calculate.)

3-10 Assume that a 250-kVA distribution transformer is used for single-phase pole mounting. The transformer is connected phase-to-neutral 7200 V on the primary, and 2520 V phase-to-neutral on the secondary side. The leakage impedance of the transformer is 3.5 percent. Based on the given information, determine the following:

(*a*) Assume that the transformer has 0.7 puA in the high-voltage winding. Find the actual current values in the high- and low-voltage windings. What is the value of the current in the low-voltage winding in per units?

(*b*) Find the impedance of the transformer as referred to the high- and low-voltage windings in ohms.

(*c*) Assume that the low-voltage terminals of the transformer are short-circuited and 0.22 per unit voltage is applied to the high-voltage winding. Find the high- and low-voltage winding currents that exist as a result of the short circuit in per units and amps.

(*d*) Determine the internal voltage drop of the transformer, due to its leakage impedance, if a 1.2 per unit current flows in the high-voltage winding. Give the result in per units and volts.

REFERENCES

1. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3 East Pittsburgh, Pa., 1965.
2. Westinghouse Electric Corporation: *Electrical Transmission and Distribution Reference Book*, East Pittsburgh, Pa., 1964.
3. Stigant, S. A., and A. C. Franklin: *The J&P Transformer Book*, Butterworth, London, 1973.
4. *The Guide for Loading Mineral Oil-Immersed Overhead-Type Distribution Transformers with 55°C and 65°C Average Winding Rise*, American National Standards Institute,. Appendix C57.91-1969.
5. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*, 11th ed., McGraw-Hill, New York, 1978.
6. Clarke, E.: *Circuit Analysis of AC Power Systems*, vol. 1, General Electric Series, Schenectady, N.Y., 1943.
7. General Electric Company: *Distribution Transformer Manual*, Hickory, N.C., 1975.

CHAPTER

FOUR

DESIGN OF SUBTRANSMISSION LINES AND DISTRIBUTION SUBSTATIONS

4-1 INTRODUCTION

In a broad definition, the distribution system is that part of the electric utility system between the bulk power source and the customers' service switches. This definition of the distribution system includes the following components:

1. Subtransmission system
2. Distribution substations
3. Distribution or primary feeders
4. Distribution transformers
5. Secondary circuits
6. Service drops

However, some distribution system engineers prefer to define the distribution system as that part of the electric utility system between the distribution substations and the consumers' service entrance.

Figure 4-1 shows a one-line diagram of a typical distribution system. The subtransmission circuits deliver energy from bulk power sources to the distribution substations. The subtransmission voltage is somewhere between 12.47 and 245 kV. The distribution substation, which is made of power transformers together with the necessary voltage-regulating apparatus, buses, and switchgear,

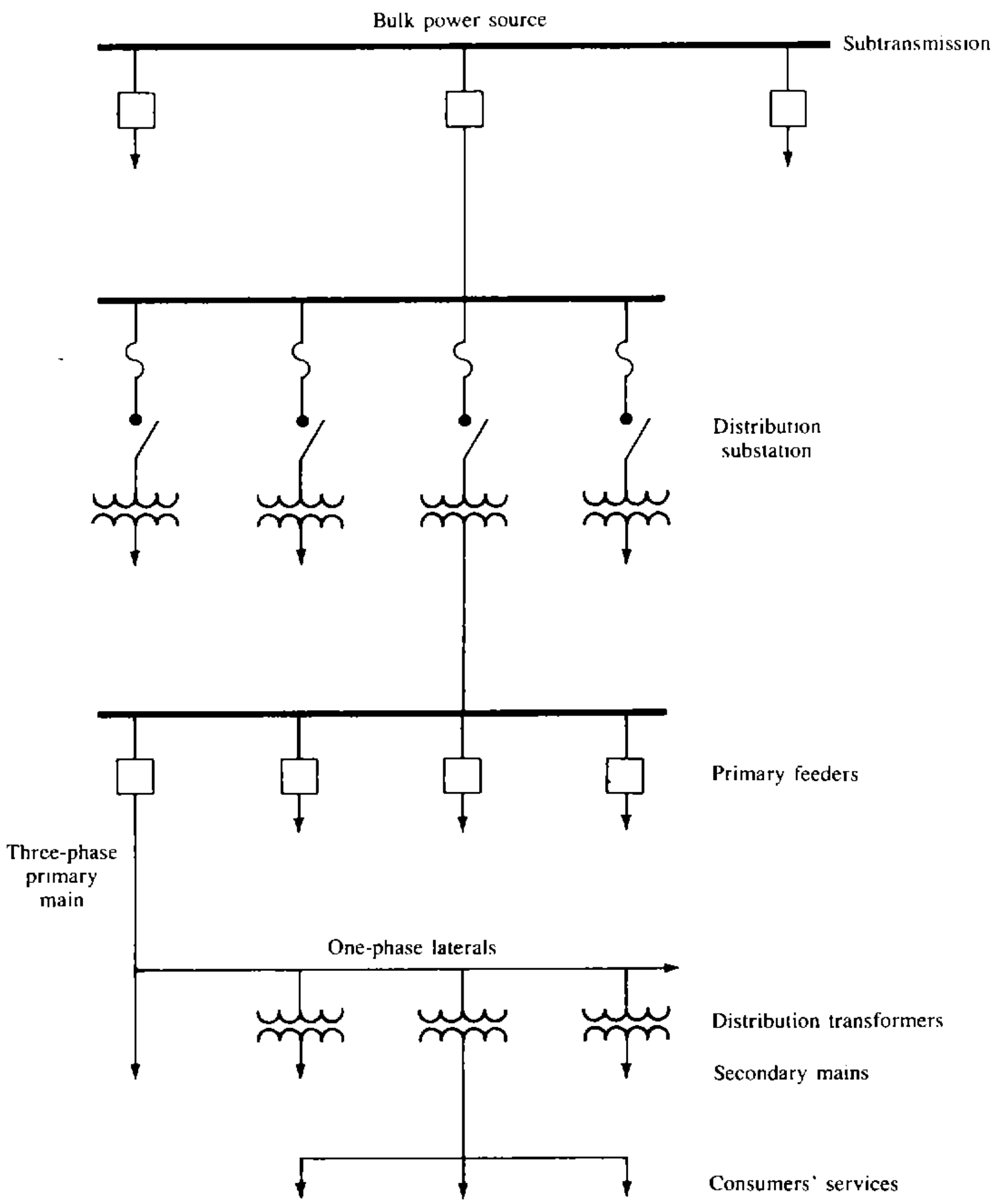


Figure 4-1 One-line diagram of a typical distribution system.

reduces the subtransmission voltage to a lower primary system voltage for local distribution. The three-phase primary feeder, which is usually operating in the range of 4.16 to 34.5 kV, distributes energy from the low-voltage bus of the substation to its load center where it branches into three-phase subfeeders and single-laterals.

Distribution transformers, in ratings from 10 to 500 kVA, are usually connected to each primary feeder, subfeeders, and laterals. They reduce the distribution voltage to the utilization voltage. The secondaries facilitate the path to distribute energy from the distribution transformer to consumers through service drops.

This chapter covers briefly the design of subtransmission and distribution substations.

4-2 SUBTRANSMISSION

The subtransmission system is that part of the electric utility system which delivers power from bulk power sources, such as large transmission substations. The subtransmission circuits may be made of overhead open-wire construction on wood poles or of underground cables. The voltage of these circuits varies from 12.47 to 245 kV, with the majority at 69-, 115-, and 138-kV voltage levels. There is a continuous trend in the usage of the higher voltage as a result of the increasing use of higher primary voltages.

The subtransmission system designs vary from simple radial systems to a subtransmission network. The major considerations affecting the design are cost and reliability.

Figure 4-2 shows a radial subtransmission system. In the radial system, as the name implies, the circuits radiate from the bulk power stations to the distribution substations. The radial system is simple and has a low first cost but it also has a low service continuity. Because of this reason, the radial system is not generally used. Instead, an improved form of radial-type subtransmission design is preferred, as shown in Fig. 4-3. It allows relatively faster service restoration when a fault occurs on one of the subtransmission circuits.

In general, due to higher service reliability, the subtransmission system is designed as loop circuits or multiple circuits forming a subtransmission grid or network. Figure 4-4 shows a loop-type subtransmission system. In this design, a single circuit originating from a bulk power bus runs through a number of substations and returns to the same bus.

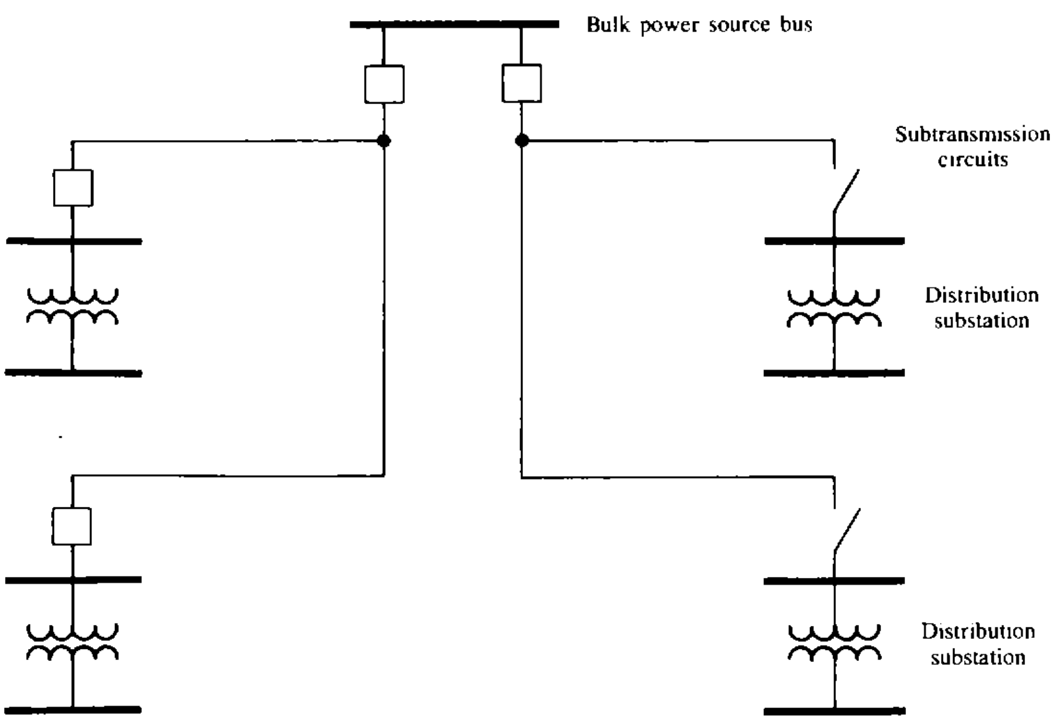


Figure 4-2 Radial-type subtransmission.

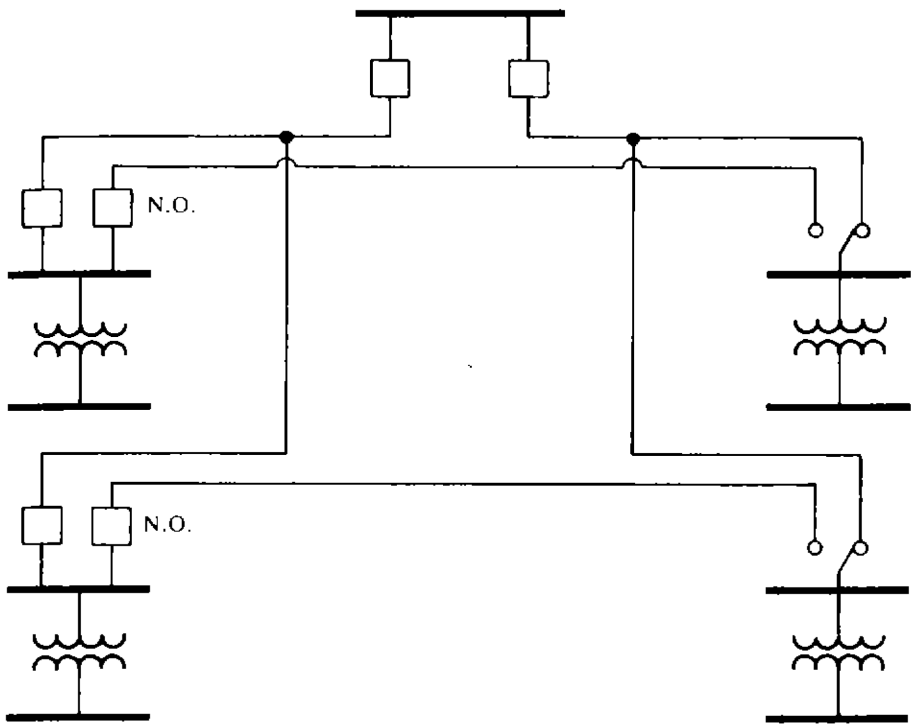


Figure 4-3 Improved form of radial-type subtransmission.

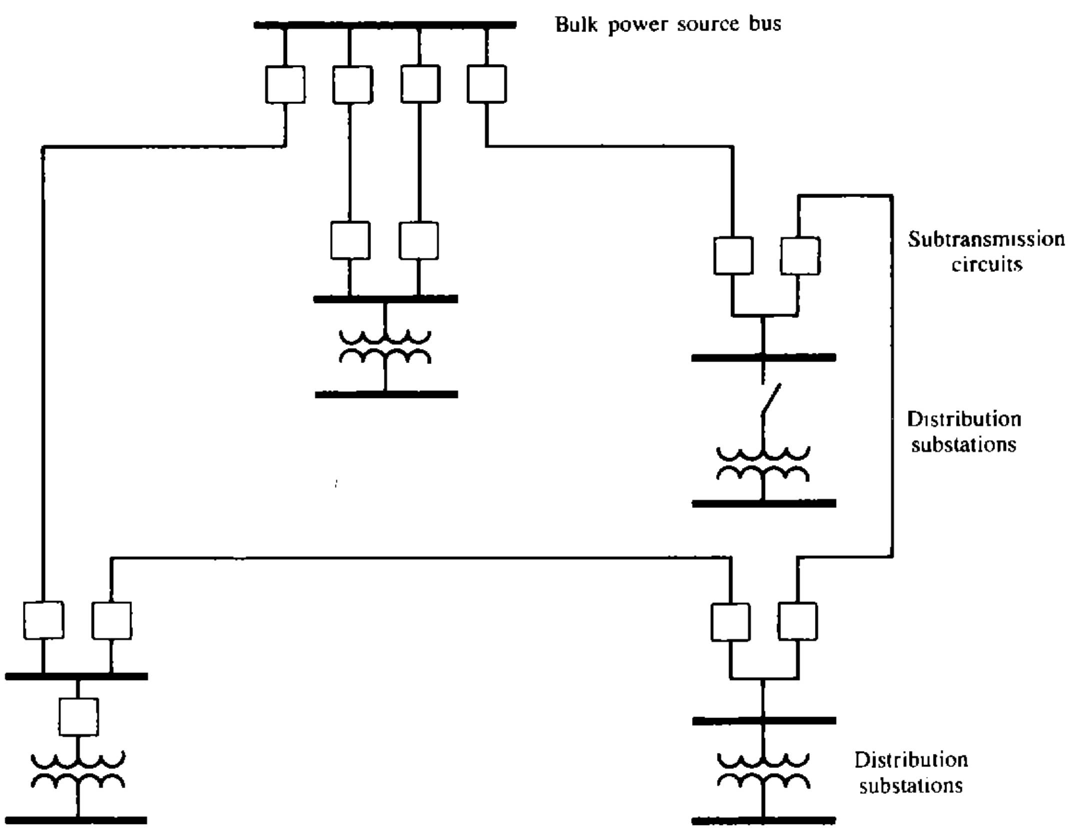


Figure 4-4 Loop-type subtransmission.

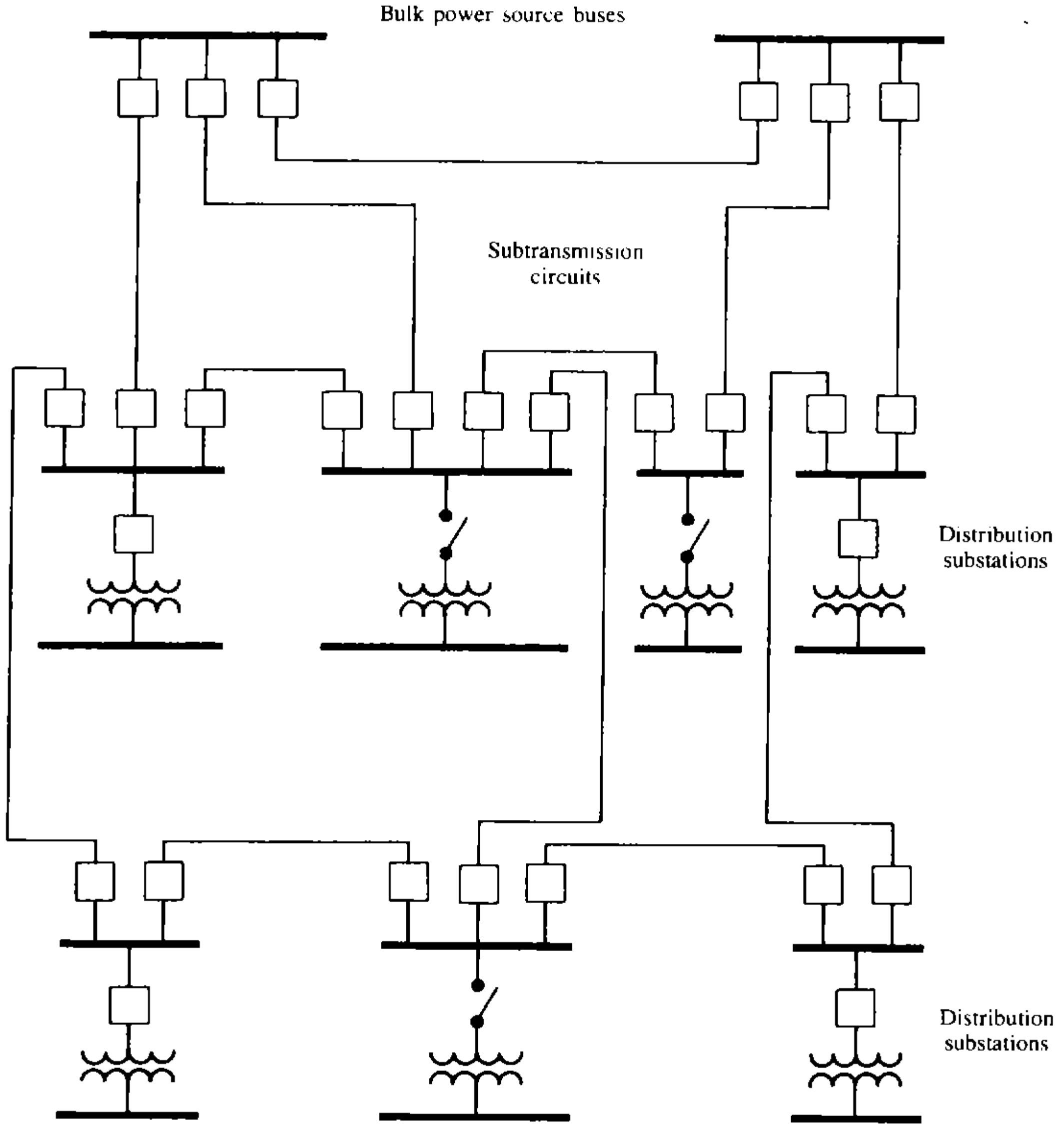


Figure 4-5 Grid- or network-type subtransmission.

Figure 4-5 shows a grid-type subtransmission which has multiple circuits. The distribution substations are interconnected, and the design may have more than one bulk power source. Therefore, it has the greatest service reliability, and it requires costly control of power flow and relaying. It is the most commonly used form of subtransmission.

4-3 DISTRIBUTION SUBSTATIONS

Distribution substation design has been somewhat standardized by the electric utility industry based upon past experiences. However, the process of standardiztion is a continuous one.

Figures 4-6 and 4-7 show typical distribution substations. The attractive appearance of these substations is enhanced by the use of underground cable in

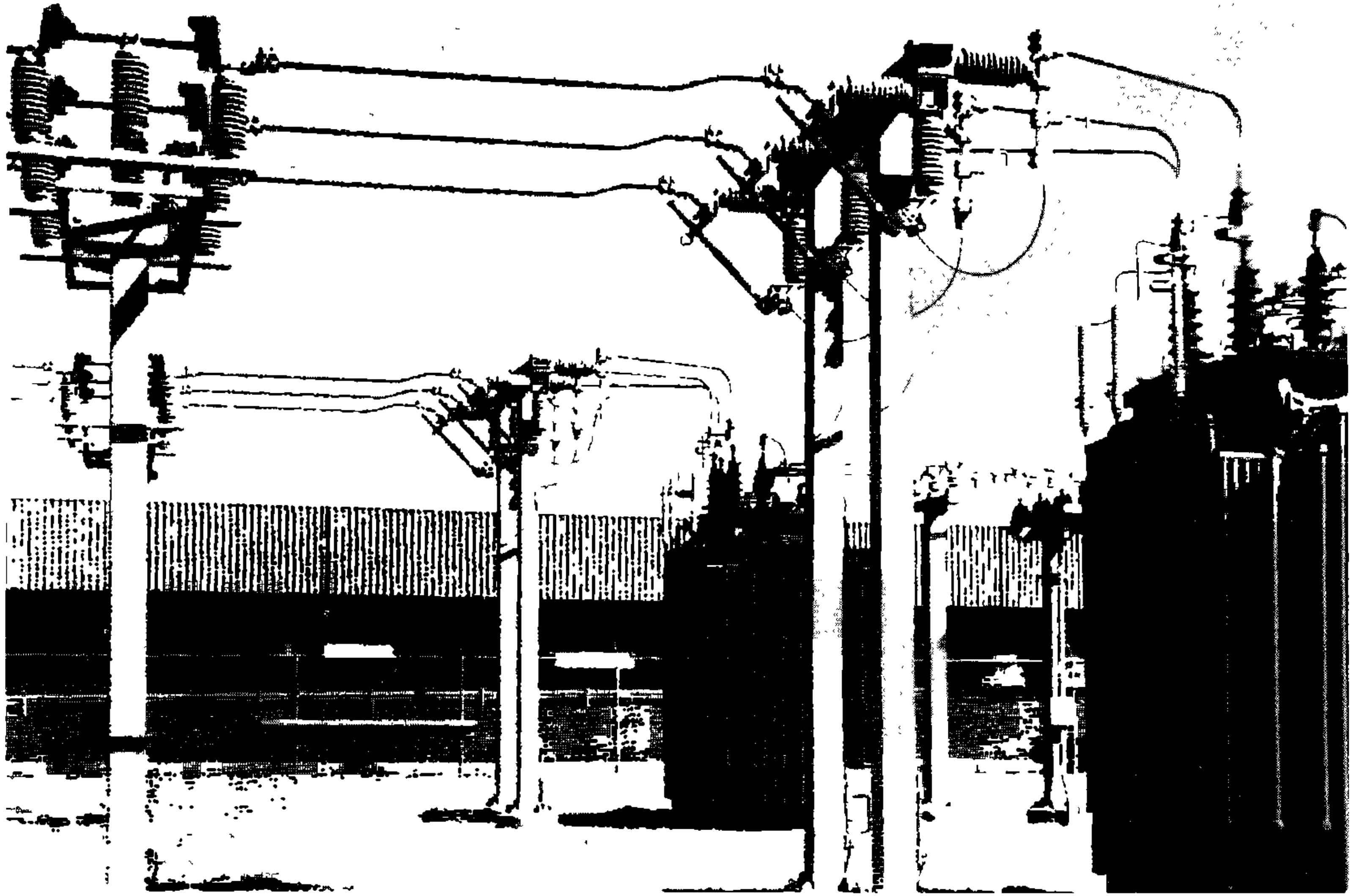


Figure 4-6 A typical distribution subtransmission. (*S&C Electric Company.*)

Figure 4-7 A typical small distribution substation. (*S&C Electric Company*.)

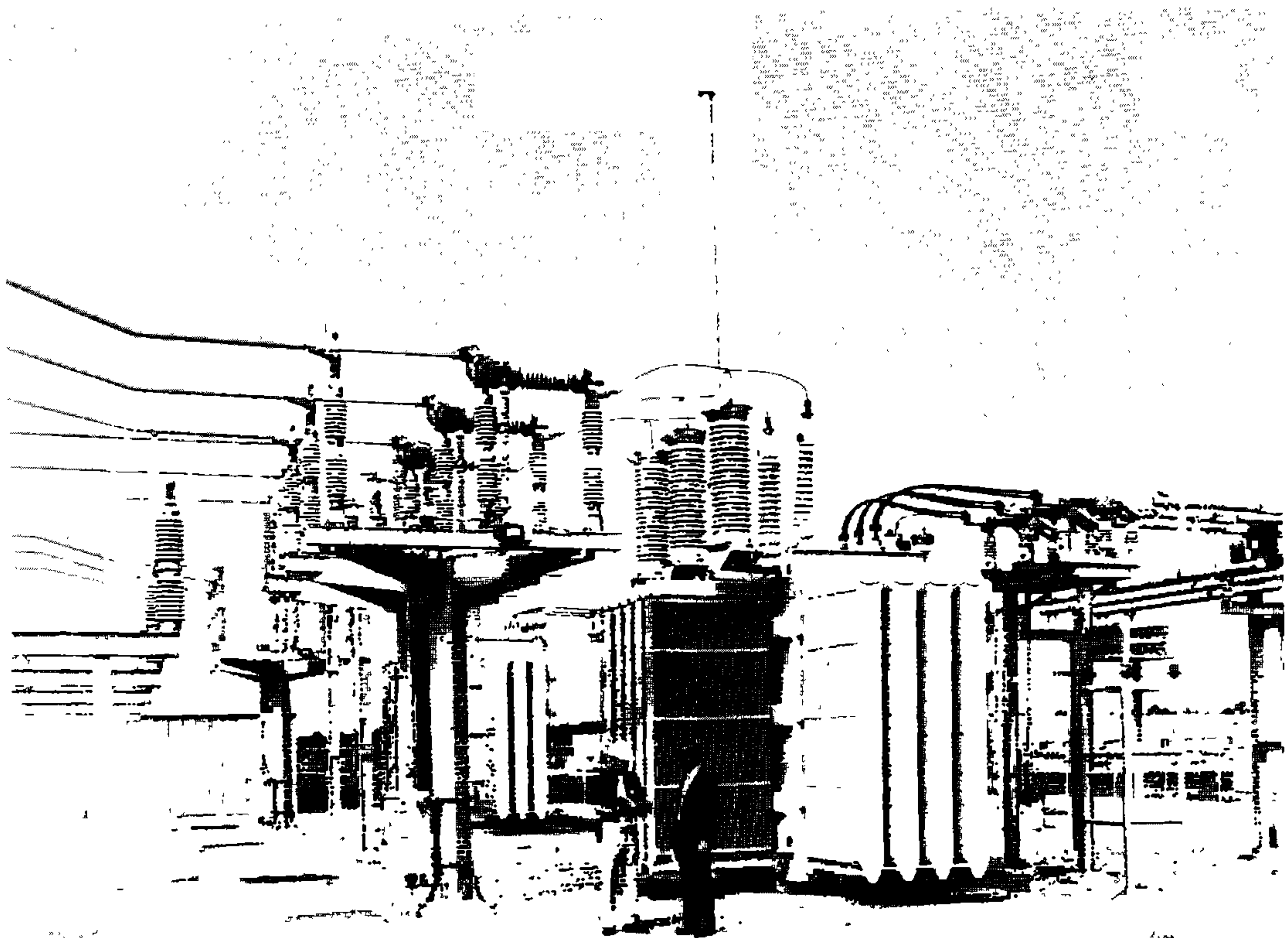


Figure 4-8 Overview of a modern substation. (*S&C Electric Company.*)

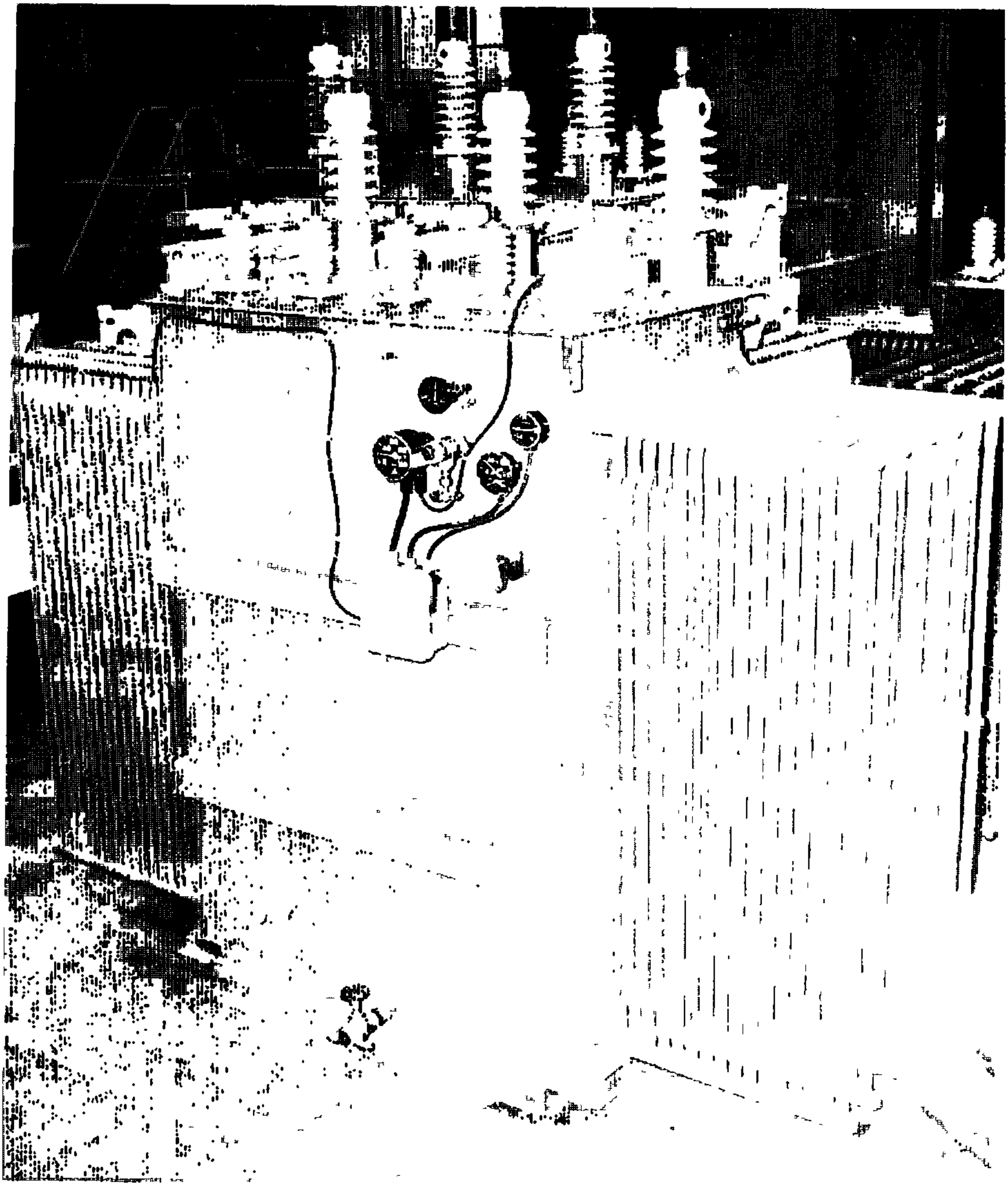


Figure 4-9 Close view of a typical modern distribution substation transformer. (*Westinghouse Electric Company.*)

and out of the station as well as between the transformer secondary and the low-voltage bus structure. Automatic switching is used for sectionalizing in some of these stations and for preferred-emergency automatic transfer in others.

Figure 4-8 shows an overall view of a modern substation. The figure shows two 115-kV 1200-A vertical-break-style circuit switchers to switch and protect two transformers supplying power to a large tire manufacturing plant. The transformer located in the foreground is rated 15/20/28 MVA, 115/4.16 kV, 8.8 percent impedance, and the second transformer is rated 15/20/28 MVA, 115/13.8 kV, 9.1 percent impedance. Figure 4-9 shows a close view of a typical modern distribution substation transformer.

A typical substation may include the following equipment: (1) power transformers, (2) circuit breakers, (3) disconnecting switches, (4) station buses and insulators, (5) current-limiting reactors, (6) shunt reactors, (7) current transformers, (8) potential transformers, (9) capacitor voltage transformers, (10) coupling capacitors, (11) series capacitors, (12) shunt capacitors, (13) grounding system, (14) lightning arresters and/or gaps, (15) line traps, (16) protective relays, (17) station batteries, and (18) other apparatus.

4-4 SUBSTATION BUS SCHEMES

The electrical and physical arrangements of the switching and busing at the subtransmission voltage level are determined by the selected substation scheme (or diagram). On the other hand, the selection of a particular substation scheme is based upon safety, reliability, economy, simplicity, and other considerations.

The most commonly used substation bus schemes include (1) single bus scheme; (2) double bus–double breaker (or double main) scheme; (3) main-and-transfer bus scheme; (4) double bus–single breaker scheme; (5) ring bus scheme; and (6) breaker-and-a-half scheme.

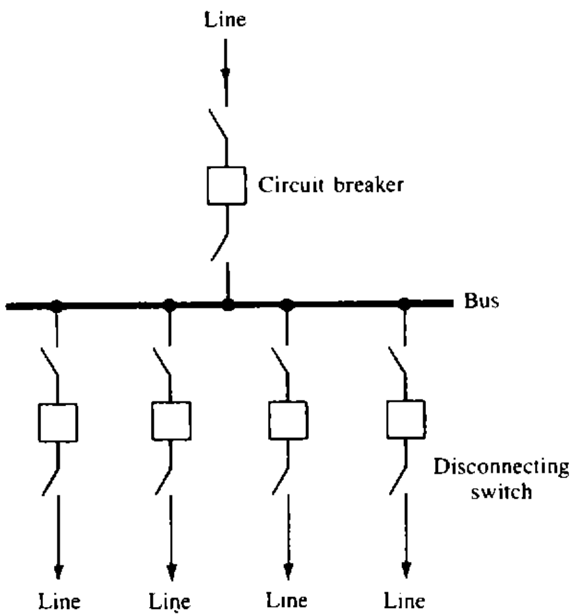


Figure 4-10 A typical single-bus scheme.

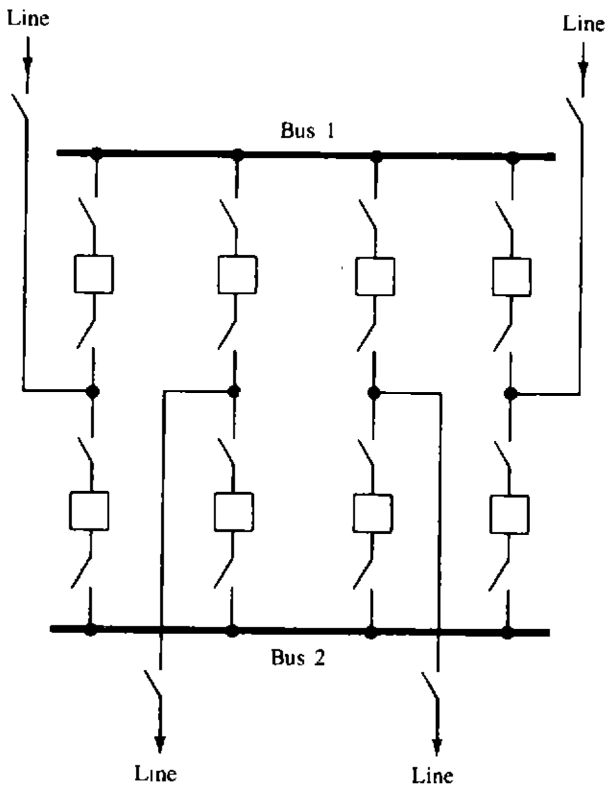


Figure 4-11 A typical double-bus–double-breaker scheme.

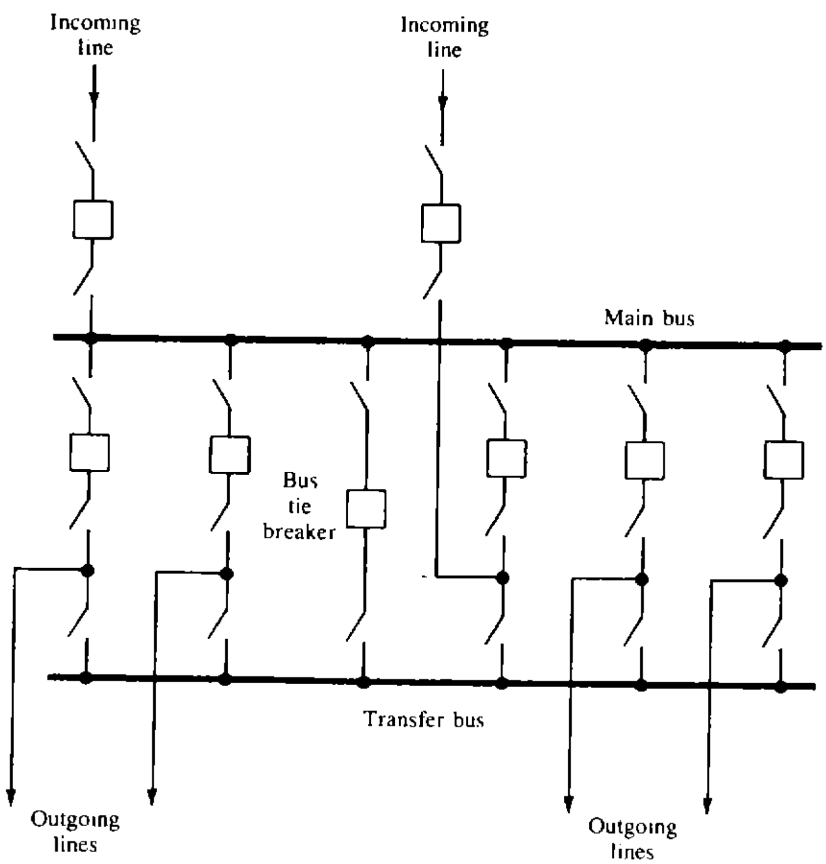


Figure 4-12 A typical main-and-transfer bus scheme.

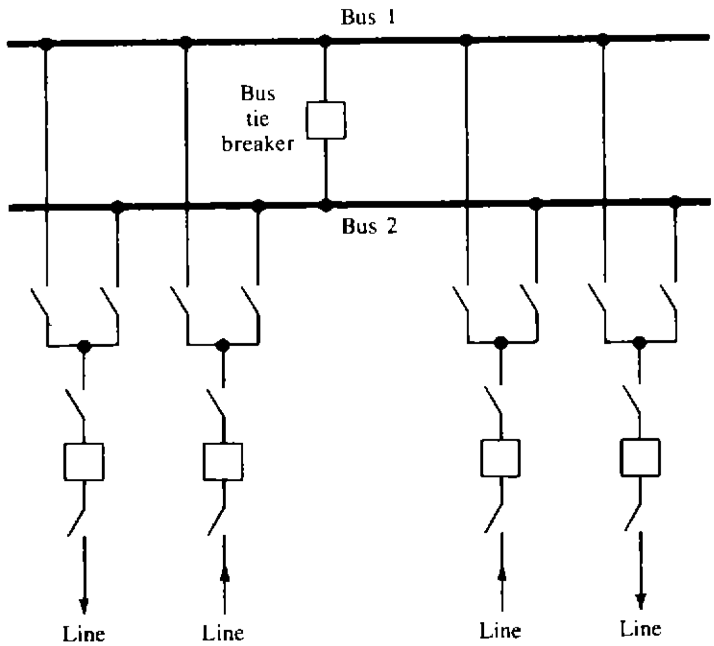


Figure 4-13 A typical double bus–single breaker scheme.

Figure 4-10 shows a typical single bus scheme; Fig. 4-11 gives a typical double bus–double breaker scheme; Fig. 4-12 illustrates a typical main-and-transfer bus scheme; Fig. 4-13 shows a typical double bus–single breaker scheme; Fig. 4-14 gives a typical ring bus scheme; Fig. 4-15 illustrates a typical breaker-and-a-half scheme.

Each scheme has some advantages and disadvantages depending upon economical justification of a specific degree of reliability. Table 4-1 gives a summary of switching schemes' advantages and disadvantages.

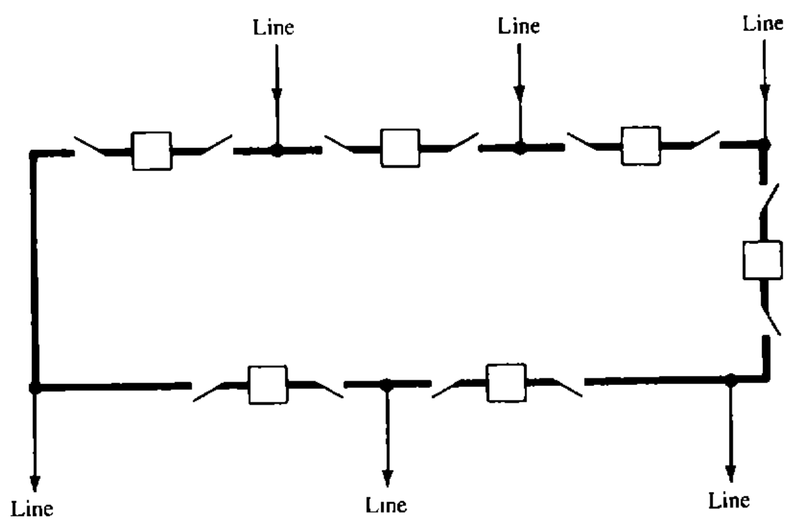


Figure 4-14 A typical ring bus scheme.

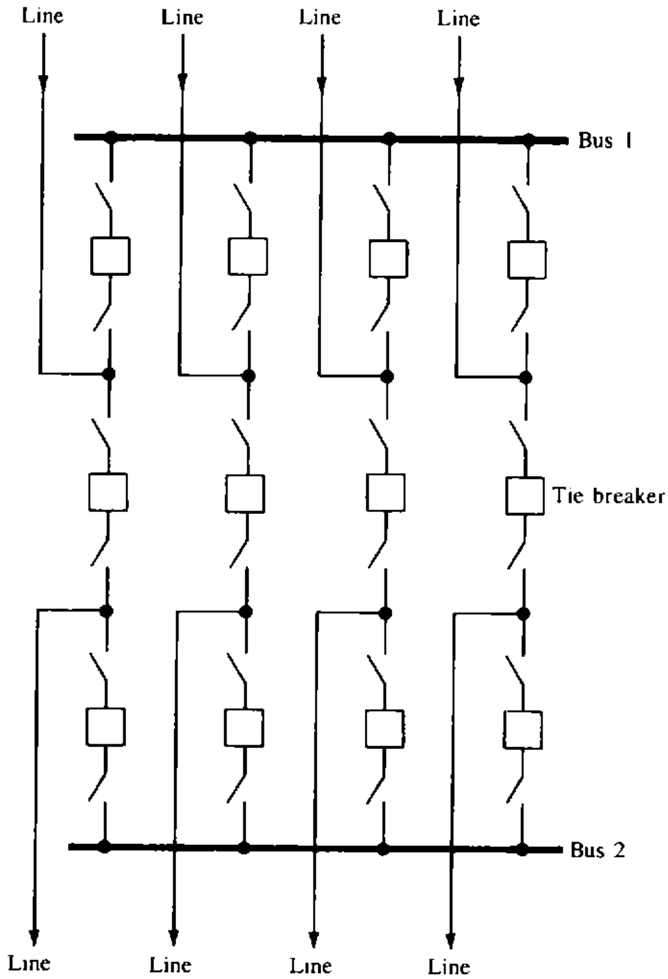


Figure 4-15 A typical breaker-and-a-half scheme.

Table 4-1 Summary of comparison of switching schemes*

Switching scheme	Advantages	Disadvantages
1. Single bus	1. Lowest cost.	1. Failure of bus or any circuit breaker results in shutdown of entire substation. 2. Difficult to do any maintenance. 3. Bus cannot be extended without completely deenergizing substation. 4. Can be used only where loads can be interrupted or have other supply arrangements.
2. Double bus–double breaker	1. Each circuit has two dedicated breakers. 2. Has flexibility in permitting feeder circuits to be connected to either bus. 3. Any breaker can be taken out of service for maintenance. 4. High reliability.	1. Most expensive. 2. Would lose half the circuits for breaker failure if circuits are not connected to both buses.
3. Main-and-transfer	1. Low initial and ultimate cost. 2. Any breaker can be taken out of service for maintenance.	1. Requires one extra breaker for the bus tie. 2. Switching is somewhat complicated when maintaining a breaker

	3. Potential devices may be used on the main bus for relaying.	3. Failure of bus or any circuit breaker results in shutdown of entire substation.
4. Double bus–single breaker	1. Permits some flexibility with two operating buses. 2. Either main bus may be isolated for maintenance. 3. Circuit can be transferred readily from one bus to the other by use of bus-tie breaker and bus selector disconnect switches.	1. One extra breaker is required for the bus tie. 2. Four switches are required per circuit. 3. Bus protection scheme may cause loss of substation when it operates if all circuits are connected to that bus. 4. High exposure to bus faults. 5. Line breaker failure takes all circuits connected to that bus out of service. 6. Bus-tie breaker failure takes entire substation out of service.
5. Ring bus	1. Low initial and ultimate cost. 2. Flexible operation for breaker maintenance. 3. Any breaker can be removed for maintenance without interrupting load. 4. Requires only one breaker per circuit. 5. Does not use main bus. 6. Each circuit is fed by two breakers. 7. All switching is done with breakers.	1. If a fault occurs during a breaker maintenance period, the ring can be separated into two sections. 2. Automatic reclosing and protective relaying circuitry rather complex. 3. If a single set of relays is used, the circuit must be taken out of service to maintain the relays. (Common on all schemes.) 4. Requires potential devices on all circuits since there is no definite potential reference point. These devices may be required in all cases for synchronizing, live line, or voltage indication. 5. Breaker failure during a fault on one of the circuits causes loss of one additional circuit owing to operation of breaker-failure relaying.
6. Breaker-and-a-half	1. Most flexible operation. 2. High reliability. 3. Breaker failure of bus side breakers removes only one circuit from service. 4. All switching is done with breakers 5. Simple operation; no disconnect switching required for normal operation. 6. Either main bus can be taken out of service at any time for maintenance. 7. Bus failure does not remove any feeder circuits from service.	1. $1\frac{1}{2}$ breakers per circuit. 2. Relaying and automatic reclosing are somewhat involved since the middle breaker must be responsive to either of its associated circuits.

* From [1].

4-5 SUBSTATION LOCATION

The location of a substation is dictated by the voltage levels, voltage regulation considerations, subtransmission costs, substation costs, and the costs of primary feeders, mains, and distribution transformers. It is also restricted by other factors, as explained in Chap. 1, which may not be technical in nature.

However, to select an ideal location for a substation, the following rules should be observed [2]:

1. Locate the substation as much as feasible close to the load center of its service area, so that the addition of load times distance from the substation is a minimum.
2. Locate the substation such that proper voltage regulation can be obtainable without taking extensive measures.
3. Select the substation location such that it provides proper access for incoming subtransmission lines and outgoing primary feeders and also allows for future growth.
4. The selected substation location should provide enough space for the future substation expansion.
5. The selected substation location should not be opposed by land use regulations, local ordinances, and neighbors.
6. The selected substation location should help to minimize the number of customers affected by any service discontinuity.
7. Other considerations, such as adaptability, emergency, etc.

4-6 THE RATING OF A DISTRIBUTION SUBSTATION

The additional capacity requirements of a system with increasing load density can be met by:

1. Either holding the service area of a given substation constant and increasing its capacity
2. Or developing new substations and thereby holding the rating of the given substation constant

It is helpful to assume that the system changes (1) at constant load density for short-term distribution planning and (2) at increasing load density for long-term planning. Further, it is also customary and helpful to employ geometric figures to represent substation service areas, as suggested by Van Wormer [3], Denton and Reps [4], and Reps [5]. It simplifies greatly the comparison of alternative plans which may require different sizes of distribution substation, different numbers of primary feeders, and different primary-feeder voltages.

Reps [5] analyzed a square-shaped service area representing a part of, or the entire service area of, a distribution substation. It is assumed that the square area is served by four primary feeders from a central feed point, as shown in Fig. 4-16. Each feeder and its laterals are of three-phase. Dots represent balanced three-phase loads lumped at that location and fed by distribution transformers.

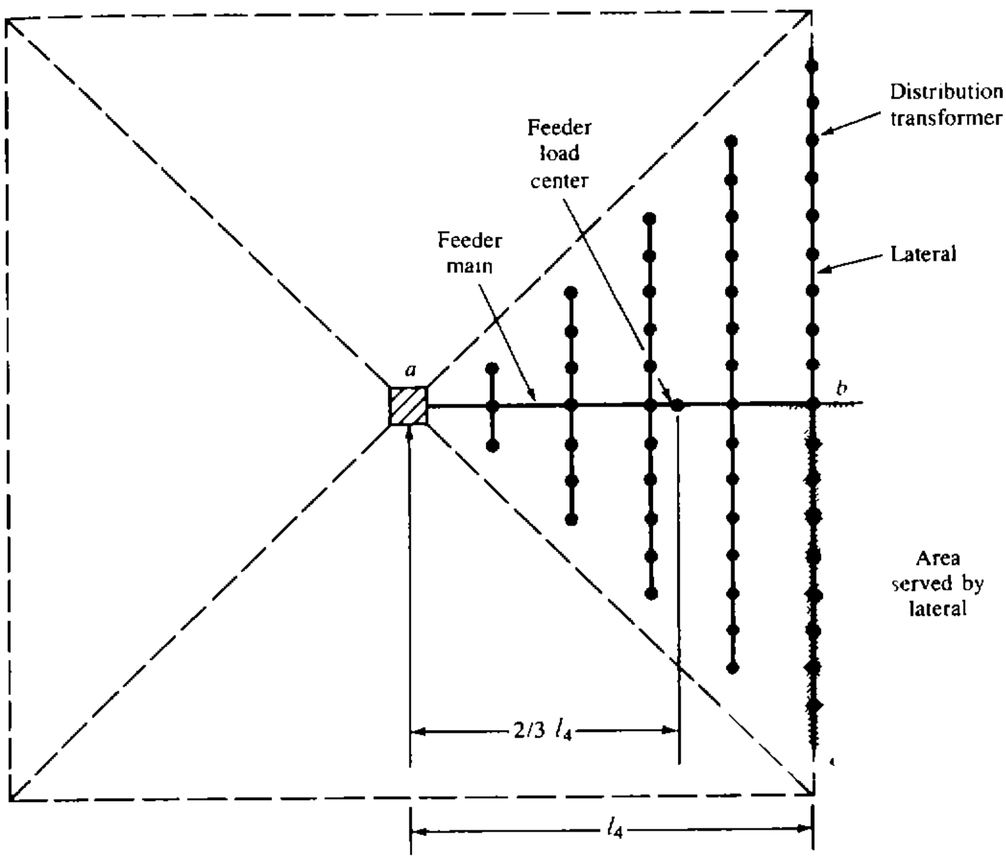


Figure 4-16 Square-shaped distribution substation service area. (*Based on* [5].)

Here, the percent voltage drop from the feed point a to the end of the last lateral at c is

$$\%\,\mathrm{VD}_{ac} = \%\,\mathrm{VD}_{ab} + \%\,\mathrm{VD}_{bc}$$

Reps [5] simplified the above voltage-drop calculation by introducing a constant K which can be defined as *percent voltage drop per kilovoltampere-mile.* Figure 4-17 gives the K constant for various voltages and copper conductor sizes. Figure 4-17 is developed for three-phase overhead lines with an equivalent spacing of 37 in between phase conductors. The following analysis is based on the work done by Denton and Reps [4] and Reps [5].

In Fig. 4-16, each feeder serves a total load of

$$S_4 = A_4 \times D \qquad \mathrm{kVA} \tag{4-1}$$

where S_4 = kilovoltampere load served by one of four feeders emanating from a feed point

A_4 = area served by one of four feeders emanating from a feed point, mi^2

D = load density, kVA/mi^2

Equation (4-1) can be rewritten as

$$S_4 = l_4^2 \times D \qquad \mathrm{kVA} \tag{4-2}$$

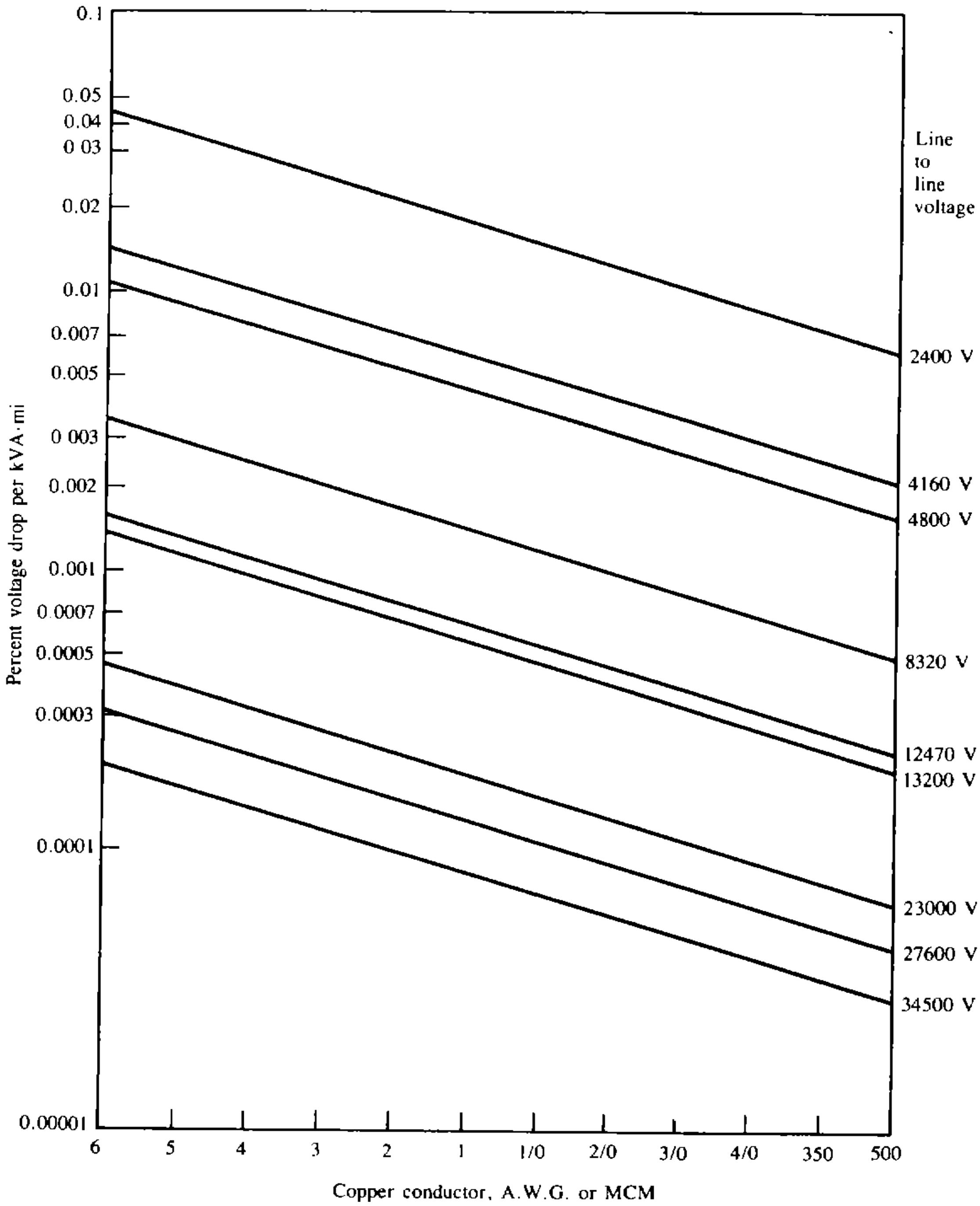


Figure 4-17 The K constant for copper conductors, assuming a lagging-load power factor of 0.9.

since

$$A_4 = l_4^2 \tag{4-3}$$

where l_4 is the linear dimension of the primary-feeder service area in miles.

Assuming uniformly distributed load, i.e., equally loaded and spaced distribution transformers, the voltage drop in the primary-feeder main is

$$\% \text{VD}_{4,\text{main}} = \tfrac{2}{3} \times l_4 \times K \times S_4 \tag{4-4}$$

or substituting Eq. (4-2) into Eq. (4-4),

$$\% \text{VD}_{4,\text{main}} = 0.667 \times K \times D \times l_4^3 \tag{4-5}$$

In Eq. (4-4) and (4-5), it is assumed that the total or lumped-sum load is located at a point on the main feeder at a distance of $\frac{2}{3} \times l_4$ from the feed point a.

Reps [5] extends the discussion to a hexagonally shaped service area supplied by six feeders from the feed point which is located at the center, as shown in Fig. 4-18. Assume that each feeder service area is equal to one-sixth of the hexagonally shaped total area, or

$$A_6 = \frac{l_6}{\sqrt{3}} \times l_6$$

$$= 0.578 \times l_6^2 \tag{4-6}$$

where A_6 = area served by one of six feeders emanating from a feed point, mi^2
l_6 = linear dimension of a primary-feeder service area, mi

Here, each feeder serves a total load of

$$S_6 = A_6 \times D \qquad \text{kVA} \tag{4-7}$$

or substituting Eq. (4-6) into Eq. (4-7),

$$S_6 = 0.578 \times D \times l_6^2 \tag{4-8}$$

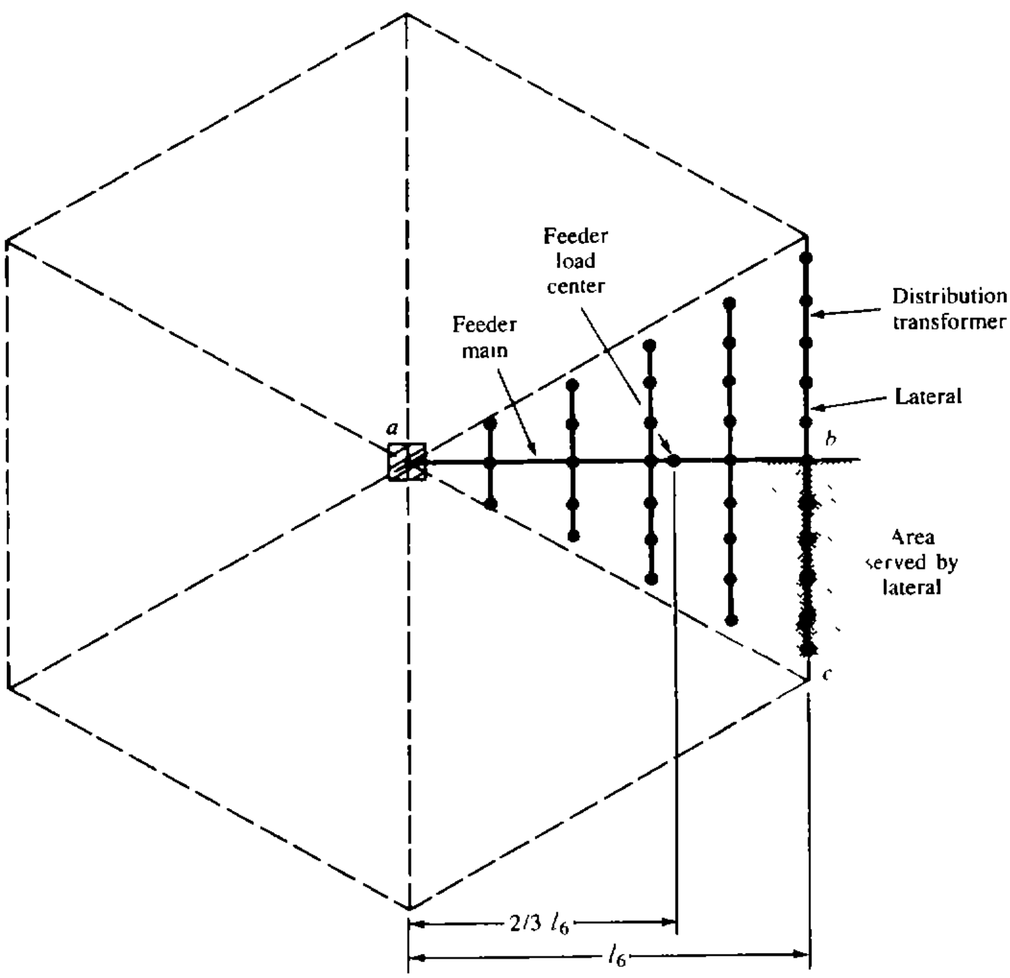


Figure 4-18 Hexagonally shaped distribution substation area. (*Based on* [5].)

As before, it is assumed that the total or lump-sum is located at a point on the main feeder at a distance of $\frac{2}{3} \times l_6$ from the feed point. Therefore, the percent voltage drop in the main feeder is

$$\% \, \mathrm{VD}_{6,\text{main}} = \tfrac{2}{3} \times l_6 \times K \times S_6 \tag{4-9}$$

or substituting Eq. (4-8) into Eq. (4-9),

$$\% \, \mathrm{VD}_{6,\text{main}} = 0.385 \times K \times D \times l_6^3 \tag{4-10}$$

4-7 GENERAL CASE: SUBSTATION SERVICE AREA WITH *n* PRIMARY FEEDERS

Denton and Reps [4] and Reps [5] extend the discussion to the general case in which the distribution substation service area is served by n primary feeders emanating from the point, as shown in Fig. 4-19. Assume that the load in the service area is uniformly distributed and each feeder serves an area of triangular shape. The differential load served by the feeder in a differential area of dA is

$$dS = D\,dA \qquad \text{kVA} \tag{4-11}$$

where dS = differential load served by the feeder in the differential area of dA, kVA
D = load density, kVA/mi^2
dA = differential service area of the feeder, mi^2

In Fig. 4-19, the following relationship exists:

$$\tan\theta = \frac{y}{x + dx} \tag{4-12}$$

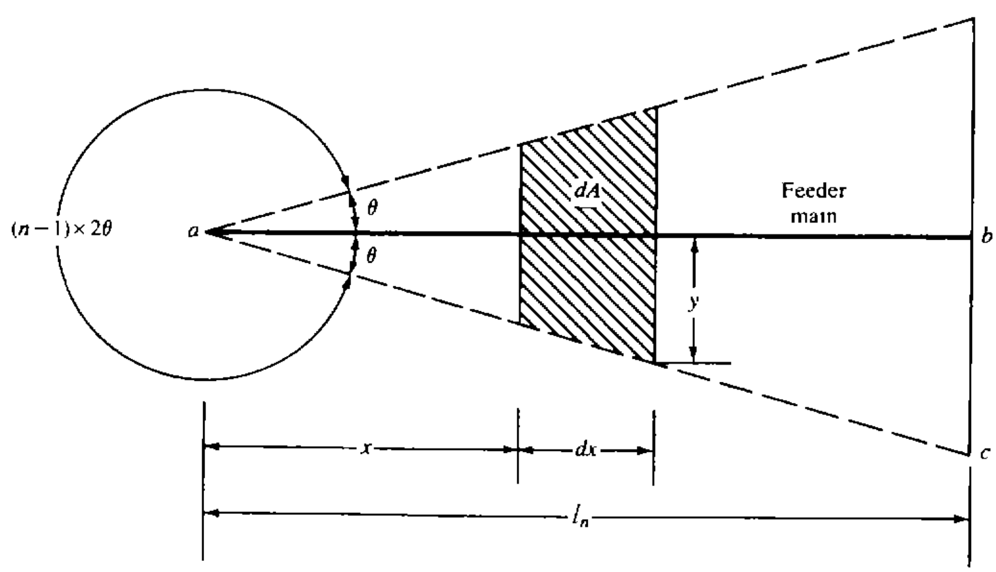


Figure 4-19 Distribution substation service area served by n primary feeders.

or

$$\begin{aligned} y &= (x + dx)\tan\theta \\ &\cong x \cdot \tan\theta \end{aligned} \tag{4-13}$$

Therefore the total service area of the feeder can be calculated as

$$\begin{aligned} A_n &= \int_{x=0}^{l_n} dA \\ &= l_n^2 \times \tan\theta \end{aligned} \tag{4-14}$$

Hence, the total kilovoltampere load served by one of n feeders can be calculated as

$$\begin{aligned} S_n &= \int_{x=0}^{l_n} dS \\ &= D \times l_n^2 \times \tan\theta \end{aligned} \tag{4-15}$$

This total load is located, as a lump-sum load, at a point on the main feeder at a distance of $\frac{2}{3} \times l_4$ from the feed point a. Therefore, the summation of the percent voltage contributions of all such areas is

$$\%\,\mathrm{VD}_n = \tfrac{2}{3} \times l_n \times K \times S_n \tag{4-16}$$

or, substituting Eq. (4-15) into Eq. (4-16),

$$\%\,\mathrm{VD}_n = \tfrac{2}{3} \times K \times D \times l_n^3 \times \tan\theta \tag{4-17}$$

or, since

$$n(2\theta) = 360° \tag{4-18}$$

Eq. (4-17) can also be expressed as

$$\%\,\mathrm{VD}_n = \tfrac{2}{3} \times K \times D \times l_n^3 \times \tan\frac{360°}{2n} \tag{4-19}$$

Equations (4-18) and (4-19) are only applicable when $n \geq 3$. Table 4-2 gives the results of the application of Eq. (4-17) to square and hexagonal areas.

For $n = 1$ the percent voltage drop in the feeder main is

$$\%\,\mathrm{VD}_1 = \tfrac{1}{2} \times K \times D \times l_1^3 \tag{4-20}$$

Table 4-2 Application results of Eq. (4-17)

n	θ	$\tan\theta$	$\%\,\mathrm{VD}_n$
4	45°	1.0	$\frac{2}{3}K \times D \times l_4^3$
6	30°	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}\left(\frac{2}{3}K \times D \times l_6^3\right)$

and for $n = 2$ it is

$$\%\text{VD}_2 = \tfrac{1}{2} \times K \times D \times l_2^3 \tag{4-21}$$

To compute the percent voltage drop in uniformly loaded lateral, lump and locate its total load at a point halfway along its length, and multiply the kilovoltampere-mile product for that line length and loading by the appropriate K constant [5].

4-8 COMPARISON OF THE FOUR- AND SIX-FEEDER PATTERNS

For a square-shaped distribution substation area served by four primary feeders, that is, $n = 4$, the area served by one of the four feeders is

$$A_4 = l_4^2 \qquad \text{mi}^2 \tag{4-22}$$

The total area served by all four feeders is

$$\begin{aligned} TA_4 &= 4A_4 \\ &= 4l_4^2 \qquad \text{mi}^2 \end{aligned} \tag{4-23}$$

The kilovoltampere load served by one of the feeders is

$$S_4 = D \times l_4^2 \qquad \text{kVA} \tag{4-24}$$

Thus the total kilovoltampere load served by all four feeders is

$$TS_4 = 4D \times l_4^2 \qquad \text{kVA} \tag{4-25}$$

The percent voltage drop in the main feeder is

$$\%\text{VD}_{4,\text{main}} = \tfrac{2}{3} K \times D \times l_4^3 \tag{4-26}$$

The load current in the main feeder at the feed point a is

$$I_4 = \frac{S_4}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{4-27}$$

or

$$I_4 = \frac{D \times l_4^2}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{4-28}$$

Of course, the ampacity, i.e., the current-carrying capacity, of a conductor selected for the main feeder should be larger than the current values that can be obtained from Eq. (4-27) and (4-28).

On the other hand, for a hexagonally shaped distribution substation area served by six primary feeders, that is, $n = 6$, the area served by one of the six feeders is

$$A_6 = \frac{1}{\sqrt{3}} l_6^2 \qquad \text{mi}^2 \tag{4-29}$$

The total area served by all six feeders is

$$TA_6 = \frac{6}{\sqrt{3}} l_6^2 \qquad \text{mi}^2 \tag{4-30}$$

The kilovoltampere load served by one of the feeders is

$$S_6 = \frac{1}{\sqrt{3}} D \times l_6^2 \qquad \text{kVA} \tag{4-31}$$

Therefore the total kilovoltampere load served by all six feeders is

$$TS_6 = \frac{6}{\sqrt{3}} D \times l_6^2 \qquad \text{kVA} \tag{4-32}$$

The percent voltage drop in the main feeder is

$$\%\,\text{VD}_{6,\,\text{main}} = \frac{2}{3\sqrt{3}} K \times D \times l_6^3 \tag{4-33}$$

The load current in the main feeder at the feed point a is

$$I_6 = \frac{S_6}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{4-34}$$

or

$$I_6 = \frac{D \times l_6^2}{3 \times V_{L\text{-}L}} \qquad \text{A} \tag{4-35}$$

The relationship between the service areas of the four- and six-feeder patterns can be found under two assumptions: (1) feeder circuits are thermally limited and (2) feeder circuits are voltage-drop-limited.

1. Thermally limited feeder circuits. For a given conductor size and neglecting voltage drop,

$$I_4 = I_6 \tag{4-36}$$

Substituting Eq. (4-28) and (4-35) into Eq. (4-36),

$$\frac{D \times l_4^2}{\sqrt{3} \times V_{L\text{-}L}} = \frac{D \times l_6^2}{3 \times V_{L\text{-}L}} \tag{4-37}$$

from Eq. (4-37),

$$\left(\frac{l_6}{l_4}\right)^2 = \sqrt{3} \tag{4-38}$$

Also, by dividing Eq. (4-30) by Eq. (4-23),

$$\begin{aligned}\frac{TA_6}{TA_4} &= \frac{6/\sqrt{3}\, l_6^2}{4l_4^2} \\ &= \frac{\sqrt{3}}{2}\left(\frac{l_6}{l_4}\right)^2\end{aligned} \tag{4-39}$$

Substituting Eq. (4-38) into Eq. (4-39),

$$\frac{TA_6}{TA_4} = \frac{3}{2} \tag{4-40}$$

or

$$TA_6 = 1.50 \times TA_4 \tag{4-41}$$

Therefore the six feeders can carry 1.50 times as much load as the four feeders if they are thermally loaded.

2. Voltage-drop-limited feeder circuits. For a given conductor size and assuming equal percent voltage drop,

$$\%\,\mathrm{VD}_4 = \%\,\mathrm{VD}_6 \tag{4-42}$$

Substituting Eq. (4-26) and (4-33) into Eq. (4-42) and simplifying the result,

$$l_4 = 0.833 \times l_6 \tag{4-43}$$

From Eq. (4-30), the total area served by all six feeders is

$$TA_6 = \frac{6}{\sqrt{3}}\, l_6^2 \tag{4-44}$$

Substituting Eq. (4-43) into Eq. (4-23), the total area served by all four feeders is

$$TA_4 = 2.78 \times l_6^2 \tag{4-45}$$

Dividing Eq. (4-44) by Eq. (4-45),

$$\frac{TA_6}{TA_4} = \frac{5}{4} \tag{4-46}$$

or

$$TA_6 = 1.25 \times TA_4 \tag{4-47}$$

Therefore the six feeders can carry only 1.25 times as much load as the four feeders if they are voltage-drop-limited.

4-9 DERIVATION OF THE K CONSTANT

Consider the primary-feeder main shown in Fig. 4-20. Here, the effective impedance $\bar{Z}$ of the three-phase main depends upon the nature of the load. For example, when a lumped-sum load is connected at the end of the main, as shown in the figure, the effective impedance is

$$\bar{Z} = z \times l \qquad \Omega/\text{phase} \tag{4-48}$$

where z = impedance of three-phase main line, $\Omega/(\text{mi} \cdot \text{phase})$
l = length of the feeder main, mi

When the load is uniformly distributed, the effective impedance is

$$\bar{Z} = \tfrac{1}{2} z \times l \qquad \Omega/\text{phase} \tag{4-49}$$

When the load has an increasing load density, the effective impedance is

$$\bar{Z} = \tfrac{2}{3} z \times l \qquad \Omega/\text{phase} \tag{4-50}$$

Taking the receiving-end voltage as the reference phasor,

$$\bar{V}_r = V_r \underline{/0^\circ} \qquad \text{V} \tag{4-51}$$

from the phasor diagram given in Fig. 4-21, the sending-end voltage is

$$\bar{V}_s = V_s \underline{/\delta^\circ} \qquad \text{V} \tag{4-52}$$

The current is

$$\bar{I} = I \underline{/-\theta_I^\circ} \qquad \text{A}$$

and the power-factor angle is

$$\begin{aligned} \theta^\circ &= \theta_{V_r}^\circ - \theta_I^\circ \\ &= -\theta_I^\circ \end{aligned} \tag{4-53}$$

and the power factor is a lagging one. When the real power P and the reactive power Q flow in opposite directions, the power factor is a leading one.

Here, the per unit voltage regulation is defined as

$$\text{VR}_{\text{pu}} \triangleq \frac{V_s - V_r}{V_r} \tag{4-54}$$

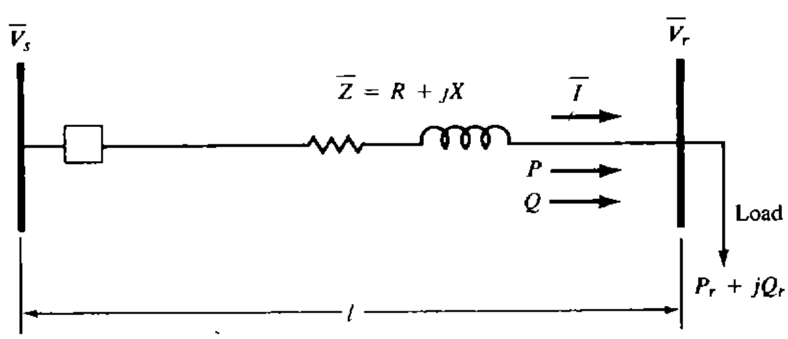


Figure 4-20

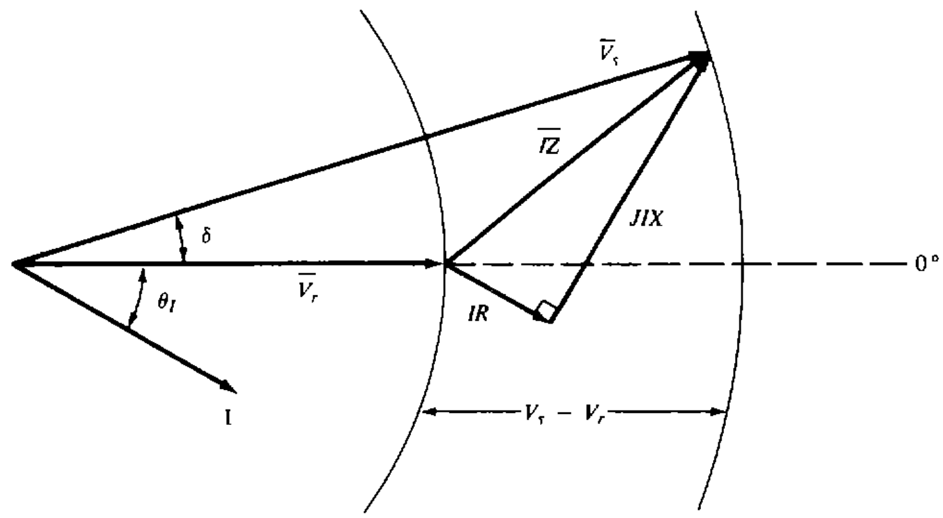


Figure 4-21

and the percent voltage regulation is

$$\% \, \mathrm{VR} = \frac{V_s - V_r}{V_r} \times 100 \tag{4-55}$$

or

$$\% \, \mathrm{VR} = 100 \times \mathrm{VR}_{\mathrm{pu}} \tag{4-56}$$

whereas the per unit voltage drop is defined as

$$\mathrm{VD}_{\mathrm{pu}} \triangleq \frac{V_s - V_r}{V_b} \tag{4-57}$$

and the percent voltage drop is

$$\% \, \mathrm{VD} = \frac{V_s - V_r}{V_b} \times 100 \tag{4-58}$$

or

$$\% \, \mathrm{VD} = 100 \times \mathrm{VD}_{\mathrm{pu}} \tag{4-59}$$

where V_b is the arbitrary base voltage. The base secondary voltage is usually selected as 120 V. The base primary voltage is usually selected with respect to the potential transformation (PT) ratio used.

Common PT ratios	V_b
20	2,400 V
60	7,200 V
100	12,000 V

From Fig. 4-20 and 4-21, the sending-end voltage is

$$\bar{V}_s = \bar{V}_r + \bar{I}\bar{Z} \tag{4-60}$$

or

$$V_s(\cos\delta + j\sin\delta) = V_r\underline{/0^\circ} + I(\cos\theta - j\sin\theta)(R + jX) \tag{4-61}$$

The quantities in Eq. (4-61) can be either all in per units or in the mks (or SI) system. Use line-to-neutral voltages for single-phase three-wire or three-phase three- or four-wire systems.

In typical distribution circuits,

$$R \cong X$$

and the voltage angle δ is closer to zero or typically

$$0^\circ \leq \delta \leq 4^\circ$$

whereas in typical transmission circuits,

$$\delta \cong 0^\circ$$

since X is much larger than R.

Therefore, for a typical distribution circuit, the $\sin\delta$ can be neglected in Eq. (4-61). Hence

$$V_s \cong V_s \cos\delta$$

and Eq. (4-61) becomes

$$V_s \cong V_r + IR\cos\theta + IX\sin\theta \tag{4-62}$$

Therefore the per unit voltage drop, for a lagging power factor, is

$$\mathrm{VD}_{\mathrm{pu}} = \frac{IR\cos\theta + IX\sin\theta}{V_b} \tag{4-63}$$

and it is a positive quantity. The $\mathrm{VD}_{\mathrm{pu}}$ is negative when there is a leading power factor due to shunt capacitors or when there is a negative reactance X due to series capacitors installed in the circuits.

The complex power at the receiving end is

$$P_r + jQ_r = \bar{V}_r\bar{I}^* \tag{4-64}$$

Therefore

$$\bar{I} = \frac{P_r - jQ_r}{\bar{V}_r} \tag{4-65}$$

since

$$\bar{V}_r = V_r\underline{/0^\circ}$$

Substituting Eq. (4-65) into Eq. (4-61), which is the exact equation since the voltage angle δ is not neglected, the sending-end voltage can be written as

$$\bar{V}_s = V_r\angle 0^\circ + \frac{RP_r + XQ_r}{V_r\angle 0^\circ} - j\frac{RQ_r - XP_r}{V_r\angle 0^\circ} \tag{4-66}$$

or approximately,

$$V_s \cong V_r + \frac{RP_r + XQ_r}{V_r} \tag{4-67}$$

Substituting Eq. (4-67) into Eq. (4-57),

$$\text{VD}_{\text{pu}} \cong \frac{RP_r + XQ_r}{V_r V_b} \tag{4-68}$$

or

$$\text{VD}_{\text{pu}} \cong \frac{(S_r/V_r)R\cos\theta + (S_r/V_r)X\sin\theta}{V_b} \qquad \text{pu V} \tag{4-69}$$

or

$$\text{VD}_{\text{pu}} \cong \frac{S_r \times R\cos\theta + S_r \times X\sin\theta}{V_r V_b} \qquad \text{pu V} \tag{4-70}$$

since

$$P_r = S_r\cos\theta \qquad \text{W} \tag{4-71}$$

and

$$Q_r = S_r\sin\theta \qquad \text{var} \tag{4-72}$$

Equations (4-69) and (4-70) can also be derived from Eq. (7-63), since

$$S_r = V_r I \qquad \text{VA} \tag{4-73}$$

The quantities in Eqs. (4-68) and (4-70) can be either all in per units or in the SI system. Use the line-to-neutral voltage values and per phase values for the P_r, Q_r, and S_r.

To determine the K constant, use Eq. (4-68),

$$\text{VD}_{\text{pu}} \cong \frac{RP_r + XQ_r}{V_r V_b} \qquad \text{pu V}$$

or

$$\text{VD}_{\text{pu}} \cong \frac{(S_{3\phi})(s)(r\cos\theta + x\sin\theta)(\frac{1}{3}\times 1000)}{V_r V_b} \qquad \text{pu V} \tag{4-74}$$

or

$$\text{VD}_{\text{pu}} = s \times K \times S_{3\phi} \qquad \text{pu V} \tag{4-75}$$

or

$$\mathrm{VD}_{\mathrm{pu}} = s \times K \times S_n \qquad \text{pu V} \tag{4-76}$$

where

$$K \cong \frac{(r \cos\theta + x \sin\theta)(\frac{1}{3} \times 1000)}{V_r V_b} \tag{4-77}$$

Therefore

$$K = f(\text{conductor size, spacing, } \cos\theta,\ V_b)$$

and it has the unit of

$$\frac{\mathrm{VD}_{\mathrm{pu}}}{\text{Arbitrary no. of kVA} \cdot \text{mi}}$$

To get the percent voltage drop, multiply the right side of Eq. (4-77) by 100, so that

$$K \cong \frac{(r \cos\theta + x \sin\theta)(\frac{1}{3} \times 1000)}{V_r V_b} \times 100 \tag{4-78}$$

which has the unit of

$$\frac{\%\ \mathrm{VD}}{\text{Arbitrary no. of kVA} \cdot \text{mi}}$$

In Eqs. (4-74) to (4-76), s is the effective length of the feeder main which depends upon the nature of the load. For example, when the load is connected at the end of the main as lumped-sum, the effective feeder length is

$$s = l \qquad \text{unit length}$$

when the load is uniformly distributed along the main,

$$s = \tfrac{1}{2} \times l \qquad \text{unit length}$$

when the load has an increasing load density,

$$s = \tfrac{2}{3} \times l \qquad \text{unit length}$$

Example 4-1 Assume that a three-phase 4.16-kV wye-grounded feeder main has #4 copper conductors with an equivalent spacing of 37 in. between phase conductors and a lagging-load power factor of 0.9.

(*a*) Determine the K constant of the main by employing Eq. (4-77).

(*b*) Determine the K constant of the main by using the precalculated percent voltage drop per kilovoltampere-mile curves and compare it with the one found in part *a*.

SOLUTION

(*a*) From Eq. (4-77),

$$K \cong \frac{(r\cos\theta + x\sin\theta)(\frac{1}{3}\times 1000)}{V_r V_b} \qquad \text{pu V}$$

where $r = 1.503\ \Omega/\text{mi}$ from Table A-1 for 50°C and 60 Hz

$x_L = x_a + x_d = 0.7456\ \Omega/\text{mi}$

$x_a = 0.609\ \Omega/\text{mi}$ from Table A-1 for 60 Hz

$x_d = 0.1366\ \Omega/\text{mi}$ from Table A-10 for 60 Hz and 37-inch spacing

$\cos\theta = 0.9$, lagging

$V_r = V_b = 2400$ V, line-to-neutral voltage

Therefore the per unit voltage drop per kilovoltampere-mile is

$$K \cong \frac{(1.503\times 0.9 + 0.7456\times 0.4359)(\frac{1}{3}\times 1000)}{2400^2}$$

$$\cong 0.0001\ \text{VD}_{\text{pu}}/(\text{kVA}\cdot\text{mi})$$

or

$$K \cong 0.01\ \%\ \text{VD}/(\text{kVA}\cdot\text{mi})$$

(*b*) From Fig. 4-17, the K constant for #4 copper conductors is

$$K = 0.01\%\ \text{VD}/(\text{kVA}\cdot\text{mi})$$

which is the same as the one found in part *a*.

Example 4-2 Assume that the feeder shown in Fig. 4-22 has the same characteristics as the one in Example 4-1 and a lumped-sum load of 500 kVA with a lagging-load power factor of 0.9 is connected at the end of a 1-mi-long feeder main. Calculate the percent voltage drop in the main.

SOLUTION The percent voltage drop in the main is

$$\begin{aligned}\%\ \text{VD} &= s \times K \times S_n\\ &= 1.0\ \text{mi} \times 0.01\%\ \text{VD}/(\text{kVA}\cdot\text{mi}) \times 500\ \text{kVA}\\ &= 5.0\%\end{aligned}$$

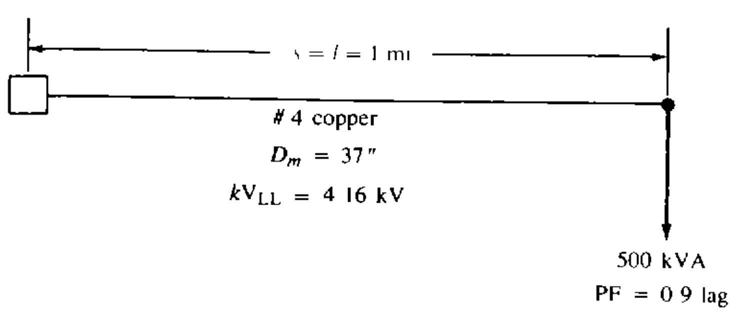


Figure 4-22

Example 4-3 Assume that the feeder shown in Fig. 4-23 has the same characteristics as the one in Example 4-2, but the 500-kVA load is uniformly distributed along the feeder main. Calculate the percent voltage drop in the main.

SOLUTION The percent voltage drop in the main is

$$\% \text{ VD} = s \times K \times S_n$$

where the effective feeder length s is

$$s = \tfrac{1}{2} = 0.5 \text{ mi}$$

Therefore

$$\begin{aligned}\% \text{ VD} &= \tfrac{1}{2} \times K \times S_n \\ &= 0.5 \text{ mi} \times 0.01/\% \text{ VD/(kVA} \cdot \text{mi)} \times 500 \text{ kVA} \\ &= 2.5\%\end{aligned}$$

Therefore, it can be seen that the negative effect of the lumped-sum load on the % VD is worse than the one for the uniformly distributed load. Figure 4-23 also shows the conversion of the uniformly distributed load to a lumped-sum load located at point a for the voltage-drop calculation.

Example 4-4 Assume that the feeder shown in Fig. 4-24 has the same characteristics as the one in Example 4-2, but the 500-kVA load has an increasing load density. Calculate the percent voltage drop in the main.

SOLUTION The percent voltage drop in the main is

$$\% \text{ VD} = s \times K \times S_n$$

where the effective feeder length s is

$$s = \tfrac{2}{3}l = 0.6667 \text{ mi}$$

Therefore

$$\begin{aligned}\% \text{ VD} &= \tfrac{2}{3}l \times K \times S_n \\ &= 0.6667 \text{ mi} \times 0.01\% \text{ VD/(kVA} \cdot \text{mi)} \times 500 \text{ kVA} \\ &= 3.33\%\end{aligned}$$

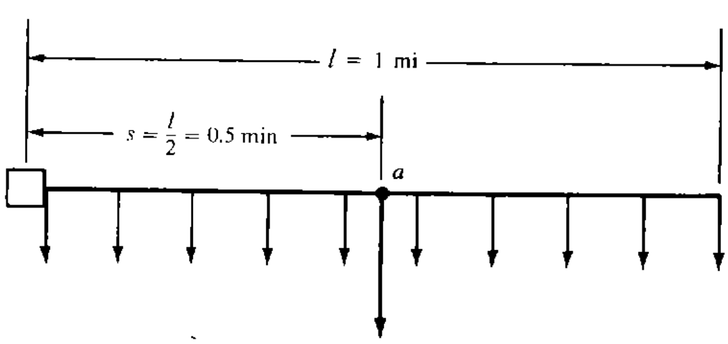


Figure 4-23

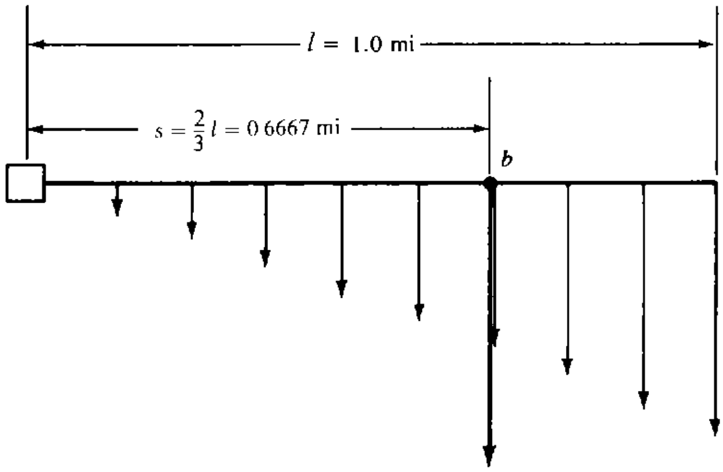


Figure 4-24

Thus it can be seen that the negative effect of the load with an increasing load density is worse than the one for the uniformly distributed load but is better than the one for the lumped-sum load. Figure 4-24 also shows the conversion of the load with an increasing load density to a lumped-sum load located at point b for the voltage-drop calculation.

Example 4-5 Use the results of the calculations of Examples 4-2 to 4-4 to calculate and compare the percent-voltage-drop ratios, and reach conclusions.

SOLUTION

(*a*) The ratio of the percent voltage drop for the lumped-sum load to the one for the uniformly distributed load is

$$\frac{\%\,\mathrm{VD}_{\mathrm{lumped}}}{\%\,\mathrm{VD}_{\mathrm{uniform}}} = \frac{5.0}{2.5} = 2.0 \tag{4-79}$$

Therefore

$$\%\,\mathrm{VD}_{\mathrm{lumped}} = 2.0\,\%\,\mathrm{VD}_{\mathrm{uniform}} \tag{4-80}$$

(*b*) The ratio of the percent voltage drop for the lumped-sum load to the percent voltage drop for the load with increasing load density is

$$\frac{\%\,\mathrm{VD}_{\mathrm{lumped}}}{\%\,\mathrm{VD}_{\mathrm{increasing}}} = \frac{5.0}{3.33} = 1.5 \tag{4-81}$$

Therefore

$$\%\,\mathrm{VD}_{\mathrm{lumped}} = 1.5\,\%\,\mathrm{VD}_{\mathrm{increasing}} \tag{4-82}$$

(*c*) The ratio of the percent voltage drop for the load with increasing load density to the one for the uniformly distributed load is

$$\frac{\%\,\mathrm{VD}_{\mathrm{increasing}}}{\%\,\mathrm{VD}_{\mathrm{uniform}}} = \frac{3.33}{2.50} = 1.33 \tag{4-83}$$

Therefore

$$\%\,\mathrm{VD}_{\text{increasing}} = 1.33\%\,\mathrm{VD}_{\text{uniform}} \tag{4-84}$$

4-10 SUBSTATION APPLICATION CURVES

Reps [5] derived the following formula to relate the application of distribution substations to load areas:

$$\%\,\mathrm{VD}_n = \frac{(\frac{2}{3} \times l_n)K(n \times D \times A_n)}{n} \tag{4-85}$$

where $\%\,\mathrm{VD}_n$ = percent voltage drop in primary-feeder circuit
$\frac{2}{3} \times l_n$ = effective length of primary feeder
K = $\%\,\mathrm{VD}/(\mathrm{kVA} \cdot \mathrm{mi})$ of the feeder
A_n = area served by one feeder
n = number of primary feeders
D = load density

Reps [5] and Denton and Reps [4] developed an alternative form of Eq. (4-85) as

$$\%\mathrm{VD}_n = \frac{TS_n^{3/2}}{n^{3/2} \times D^{1/2}} \frac{\frac{2}{3} \times K}{(\tan\theta)^{1/2}} \tag{4-86}$$

where TS_n = total kVA supplied from a substation ($= n \times D \times A_n$).

Based on Eq. (4-86), they have developed the distribution substation application curves, as shown in Fig. 4-25 and 4-26. These application curves relate the load density, substation load kilovoltamperes, primary-feeder voltage, and permissible feeder loading.

The distribution substation application curves are based on the following assumptions [5]:

1. #4/0 AWG copper conductors are used for the three-phase primary-feeder mains.
2. #4 AWG copper conductors are used for the three-phase primary-feeder laterals.
3. The equivalent spacing between phase conductors is 37 in.
4. A lagging-load power factor of 0.9.

The curves are the plots of number of primary feeders n vs. load density D for numerous values of TS_n, that is, total kilovoltampere loading of all n primary feeders including a pattern serving the load area of a substation or feed point. In Fig. 4-25 and 4-26, the curves for n vs. D are given for both constant TS_n or constant TA_n, that is, total area served by all n feeders emanating from the feed point or substation. The curves are drawn for five primary-feeder voltage levels and for two different percent voltage drops, that is, 3 and 6 percent. The percent voltage drop is [5] .

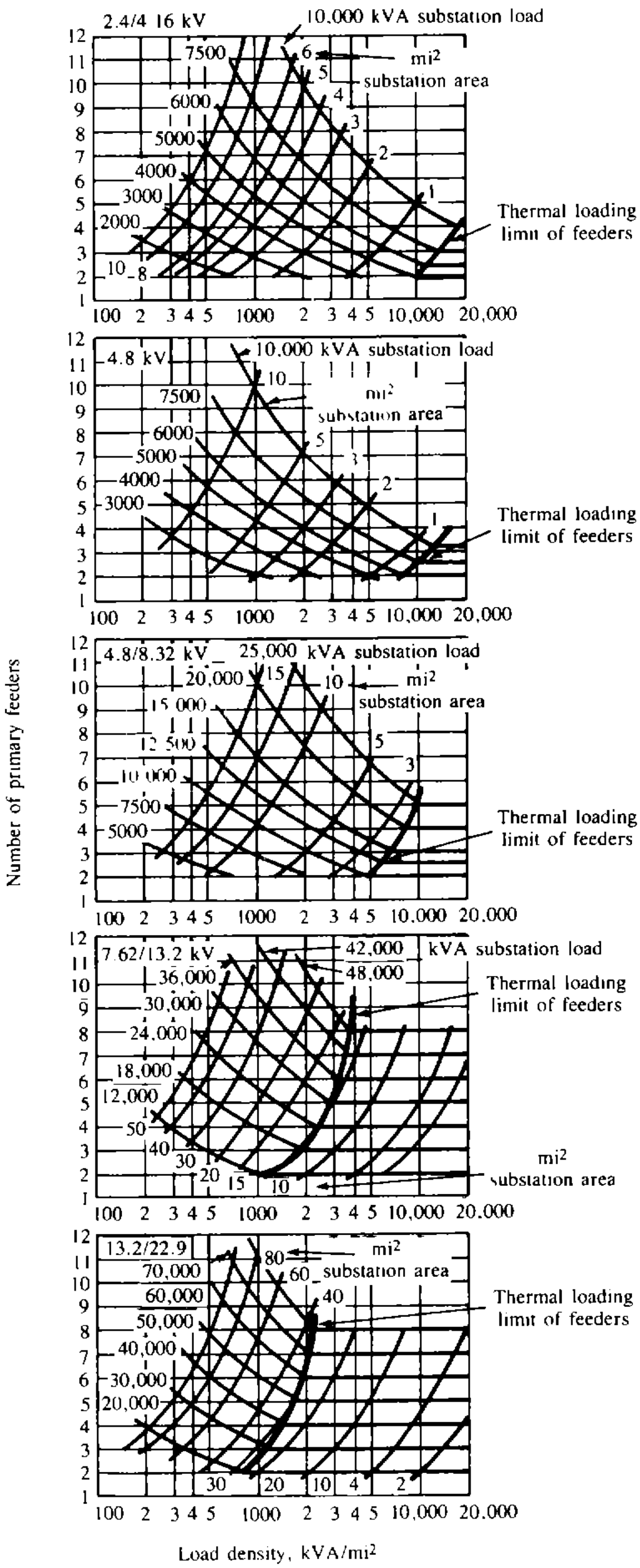


Figure 4-25 Distribution substation application curves for 3 percent voltage drop. (*From* [5].)

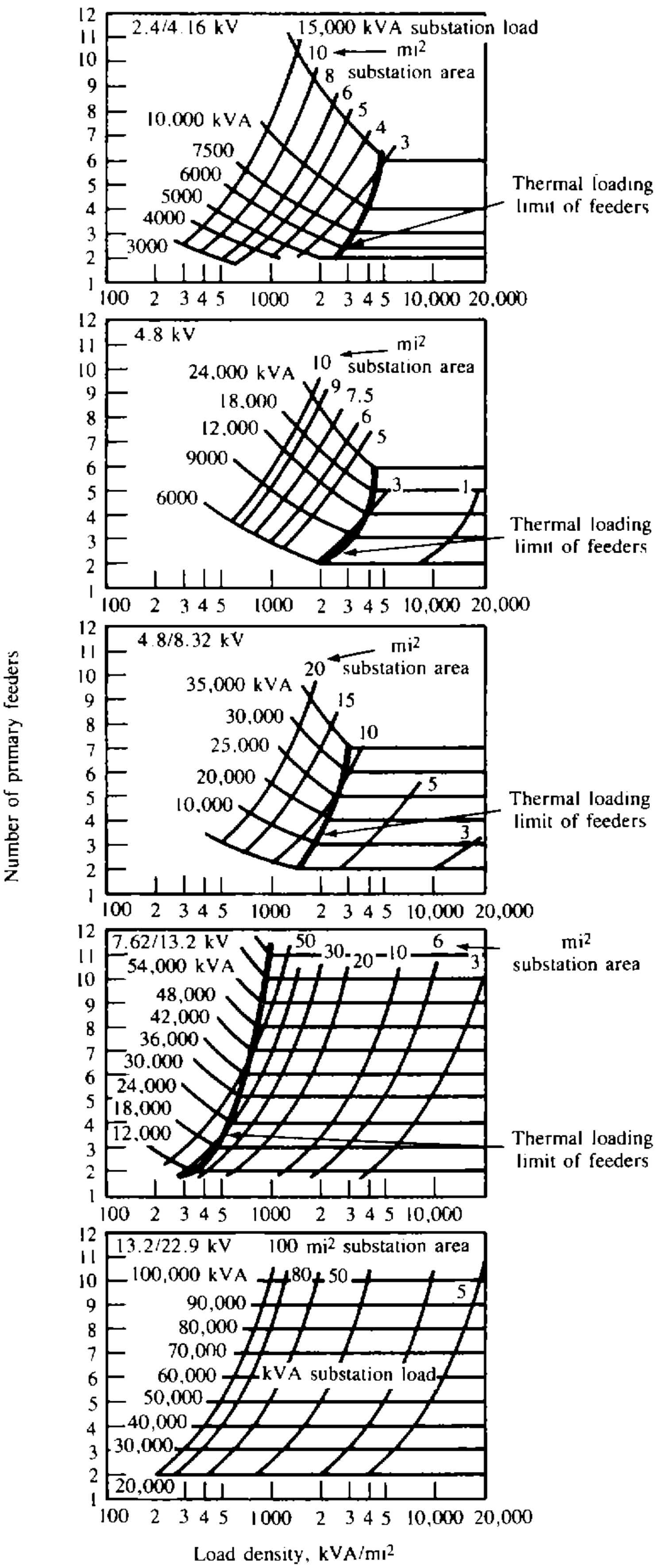


Figure 4-26 Distribution substation application curves for 6 percent voltage drop. (*From* [5].)

. . . measured from the feed-point or distribution-substation bus to the last distribution transformer on the farthest lateral on a feeder.

The combination of distribution substations and primary feeders applied in a given system are generally designed to give specified percent voltage drop or a specified kVA loading in primary feeders. In areas where load density is light and primary feeders must cover long distances, the allowable maximum percent voltage in a primary feeder usually determines the kVA-loading limit on that feeder. In areas where load density is relatively heavy and primary feeders are relatively short, the maximum-allowable loading on a primary feeder is usually governed by its current-carrying capacity, which may be attained as a feeder becomes more heavily loaded, and before voltage drop becomes a problem.

The application curves readily show whether the loading of primary feeders in a given substation area is limited by voltage drop or feeder current-carrying capacity. For each substation or feed-point kVA loading, a curve of constant loading may be followed (from upper-left toward lower-right) as load density increases. As such a curve is followed, load density increases, and the number of primary feeders required to serve that load decreases. But eventually the number of primary feeders diminishes to the minimum number required to carry the given kVA load from the standpoint of feeder current-carrying, or kVA-thermal, capacity. Further decrease in the number of primary feeders is not permissible, and the line of constant feed-point loading abruptly changes slope and becomes horizontal. For the horizontal portion of the curve, feeder loading is constant but percent voltage drop decreases as load density increases. Hence each set of planning curves may be divided into two general regions, one region in which voltage drop is constant, and the other region within which primary-feeder loading is constant. In the region of constant primary-feeder loading, percent voltage drop decreases as load density increases.

Example 4-6 Refer to previous text and note that the distribution substation application curves, given in Fig. 4-25 and 4-26, are valid only for the conductor sizes, spacing, and load power factor stated.

(*a*) Use the substation application curves and the data given in Table 4-3 for eight different cases and determine (1) the substation sizes, (2) the required number of feeders, and (3) whether the feeders are thermally limited (TL) or voltage-drop-limited (VDL). Tabulate the results.

(*b*) In case thermally loaded or thermally limited (TL) feeders are encountered, attempt to deduce if it is the #4/0 AWG copper main or the #4 AWG copper lateral that is thermally limited. Show and explain your reasoning and calculations.

Table 4-3

Case no.	Load density D, kVA/mi^2	Substation area coverage TA_n, mi^2	Maximum total primary feeder, % VD	Base feeder voltage kV$_{L\text{-}L}$
1	500	6.0	3.0	4.16
2	500	6.0	6.0	4.16
3	2,000	3.0	3.0	4.16
4	2,000	3.0	6.0	4.16
5	10,000	1.0	3.0	4.16
6	10,000	1.0	6.0	4.16
7	2,000	15.0	3.0	13.2
8	2,000	15.0	6.0	13.2

SOLUTION

(*a*) For case 1, The total substation kilovoltampere load is

$$TS_n = D \times TA_n$$
$$= 500 \times 6.0$$
$$= 3000 \text{ kVA}$$

From the appropriate figure (the one with 3.0 percent voltage drop and 4.16-kV line-to-line voltage base) among the figures given in Fig. 4-25, for 3000-kVA substation load, 500 kVA/mi^2 load density, and 6.0-mi^2 substation area coverage, the number of required feeders can be found as 3.8, or 4. Since the corresponding point in the figure is located on the left-hand side of the curve for the thermal-loading limit of feeders (the one with darker line), the feeders are voltage-drop-limited. The remaining cases can be answered in a similar manner as given in Table 4-4. Note that cases 6 and 8 are of TL feeders since their corresponding points are located on the right-hand side of the thermal-loading limit curves.

(*b*) Cases 6 and 8 have feeders which are thermally loaded. From Table A-1 the conductor ampacities for a #4/0 copper main and a #4 copper lateral can be found as 480 A and 180 A, respectively.

For case 6, the kilovoltampere load of one feeder is

$$S_n = \frac{TS_n}{n}$$
$$= \frac{10{,}000 \text{ kVA}}{4} = 2500 \text{ kVA}$$

Therefore, the load current is

$$I = \frac{S_n}{\sqrt{3} \times V_{L\text{-}L}}$$
$$= \frac{2500 \text{ kVA}}{\sqrt{3} \times 4.16 \text{ kV}} = 347.4 \text{ A}$$

Table 4-4

Case no.	Substation size TS_n	Required no. of feeders n	Voltage-drop-limited (VDL) or thermally limited (TL) feeders
1	3,000	3.8 (or 4)	VDL
2	3,000	2	VDL
3	6,000	5	VDL
4	6,000	3	VDL
5	10,000	5	VDL
6	10,000	4	TL
7	30,000	5.85 (or 6)	VDL
8	30,000	5	TL

Since the conductor ampacity of the lateral is less than the load current, it is thermally limited but not the main feeder.

For case 8, the kilovoltampere load of one feeder is

$$S_n = \frac{30{,}000 \text{ kVA}}{5} = 6000 \text{ kVA}$$

The load current is

$$I = \frac{6000 \text{ kVA}}{\sqrt{3} \times 13.2 \text{ kV}} = 262.4 \text{ A}$$

Therefore, only the lateral is thermally limited.

4-11 INTERPRETATION OF THE PERCENT-VOLTAGE-DROP FORMULA

Equation (4-85) can be rewritten in alternative forms to illustrate the interrelationship of several parameters guiding the application of distribution substations to load areas:

$$\begin{aligned} \% \text{ VD}_n &= \frac{(\frac{2}{3} \times l_n \times K)n \times D \times A_n}{n} \\ &= \frac{(\frac{2}{3} \times l_n \times K)TS_n}{n} \\ &= (\tfrac{2}{3} \times l_n \times K)S_n \end{aligned}$$

where $\% \text{ VD}_n$ = percent voltage drop in primary-feeder circuit

K = $\%$ VD/(kVA · mi) characteristic of the feeder

$\frac{2}{3} \times l_n$ = effective length of primary feeder

$TS_n = n \times D \times A_n$ = total kilovoltamperes supplied from feed point

S_n = kilovoltampere load served by one of n feeders

n = number of primary feeders

D = load density, kVA/mi^2

A_n = area served by one feeder

To illustrate the use and interpretation of the equation, assume five different cases, as shown in Table 4-5.

Case 1 represents an increasing service area as a result of geographic extensions of a city. If the length of the primary feeder is doubled (shown in the table by $\times 2$), holding everything else constant, the service area A_n of the feeder increases four times, which in turn increases TS_n and S_n four times, causing the $\%$ VD$_n$ in the feeder to increase eight times. Therefore increasing the feeder length should be avoided as a remedy due to the severe penalty.

Case 2 represents load growth due to load density growth. For example, if the load density is doubled, it causes TS_n and S_n to be doubled, which in turn

Table 4-5 Illustration of the use and interpretation of Eq. (4-85)

Case	l_n	K	Base kV_{L-L}	n	D	A_n	TS_n	S_n	$\%\ VD_n$
1. Geographic extensions	$\times 2\uparrow$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 4\uparrow$	$\times 4\uparrow$	$\times 4\uparrow$	$\times 8\uparrow$
2. Load growth	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 2\uparrow$	$\times 1$	$\times 2\uparrow$	$\times 2\uparrow$	$\times 2\uparrow$
3. Add new feeders	$\times 1$	$\times 1$	$\times 1$	$\times 2\uparrow$	$\times 1$	$\times 1$	$\times 1$	$\times \frac{1}{2}\downarrow$	$\times \frac{1}{2}\downarrow$
4. Feeder reconductoring	$\times 1$	$\times \frac{1}{2}\downarrow$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times \frac{1}{2}\downarrow$
5. Δ-to-$\mathrm{Y}_{\overline{\overline{\perp}}}$ conversion	$\times 1$	$\times \frac{1}{3}\downarrow$	$\times \sqrt{3}\uparrow$	$\times 1$	$\times 1$	$\times \frac{1}{2}$	$\times 1$	$\times 1$	$\times \frac{1}{3}\downarrow$

increases the $\%\ VD_n$ in the feeder to be doubled. Therefore increasing load density also has a negative effect on the voltage drop.

Case 3 represents the addition of new feeders. For example, if the number of the feeders is doubled, it causes S_n to be reduced by half, which in turn causes the $\%\ VD_n$ to be reduced by half. Therefore, new feeder additions help to reduce the voltage drop.

Case 4 represents feeder reconductoring. For example, if the conductor size is doubled, it reduces the K constant by half, which in turn reduces the $\%\ VD_n$ by half.

Case 5 represents the delta-to-grounded-wye conversion. It increases the line-to-line base kilovoltage by $\sqrt{3}$, which in turn decreases the K constant, causing the $\%$ VD to decrease to one-third its previous value.

Example 4-7 To illustrate distribution substation sizing and spacing, assume a square-shaped distribution substation service area as shown in Fig. 4-16. Assume that the substation is served by four three-phase four-wire 2.4/4.16-kV grounded-wye primary feeders. The feeder mains are made of either #2 AWG copper or #1/0 ACSR conductors. The three-phase open-wire overhead lines have a geometric mean spacing of 37 in between phase conductors. Assume a lagging-load power factor of 0.9 and a 1000 kVA/mi^2 uniformly distributed load density. Calculate the following:

(*a*) Consider thermally loaded feeder mains and find:
- (i) Maximum load per feeder
- (ii) Substation size
- (iii) Substation spacing, both ways
- (iv) Total percent voltage drop from the feed point to the end of the main

(*b*) Consider voltage-drop-limited feeders which have 3 percent voltage drop and find:
- (i) Substation spacing, both ways
- (ii) Maximum load per feeder
- (iii) Substation size
- (iv) Ampere loading of the main in per unit of conductor ampacity

SOLUTION From Tables A-1 and A-5 of Appendix A, the conductor ampacities for #2 AWG copper and #1/0 ACSR conductors can be found as 230 A.

(*a*) Thermally loaded mains:

(i) Maximum load per feeder is

$$S_n = \sqrt{3} \times V_{L\text{-}L} \times I_{\max}$$
$$= \sqrt{3} \times 4.16 \times 230$$
$$= 1657.2 \text{ kVA}$$

(ii) Substation size is

$$TS_n = 4 \times S_n$$
$$= 4 \times 1657.2$$
$$= 6628.8 \text{ kVA}$$

(iii) Substation spacing, both ways, can be found from

$$S_n = A_n \times D$$
$$= l_4^2 \times D$$

or

$$l_4 = \left(\frac{S_n}{D}\right)^{1/2}$$
$$= \left(\frac{1657.2 \text{ kVA}}{1000 \text{ kVA/mi}^2}\right)^{1/2}$$
$$= 1.287 \text{ mi}$$

Therefore

$$2l_4 = 2 \times 1.287$$
$$= 2.575 \text{ mi}$$

(iv) Total percent voltage drop in the main is

$$\% \text{ VD}_n = \tfrac{2}{3} \times K \times D \times l_4^3$$
$$= \tfrac{2}{3} \times 0.007 \times 1000 \times (1.287)^3$$
$$= 9.95\%$$

where K is 0.007 and found from Fig. 4-17.

(*b*) Voltage-drop-limited feeders:

(i) Substation spacing, both ways, can be found from

$$\% \text{ VD}_n = \tfrac{2}{3} \times K \times D \times l_4^3$$

or

$$l_4 = \left(\frac{3 \times \% \text{ VD}_n}{2 \times K \times D}\right)^{1/3}$$
$$= \left(\frac{3 \times 3}{2 \times 0.007 \times 1000}\right)^{1/3}$$
$$= 0.86 \text{ mi}$$

Therefore

$$2l_4 = 2 \times 0.86$$
$$= 1.72 \text{ mi}$$

(ii) Maximum load per feeder is

$$S_n = D \times l_4^2$$
$$= 1000 \times (0.86)^2$$
$$= 750 \text{ kVA}$$

(iii) Substation size is

$$TS_n = 4 \times S_n$$
$$= 4 \times 750$$
$$= 3000 \text{ kVA}$$

(iv) Ampere loading of the main is

$$I = \frac{S_n}{\sqrt{3} \times V_{L\text{-}L}}$$
$$= \frac{750}{\sqrt{3} \times 4.16}$$
$$= 104.09\text{A}$$

Therefore the ampere loading of the main in per unit of conductor ampacity is

$$I_{\text{pu}} = \frac{104.09 \text{ A}}{230 \text{ A}}$$
$$= 0.4526 \text{ pu}$$

Example 4-8 Assume a square-shaped distribution substation service area as shown in Fig. 4-27. The square area is 4 mi^2 and has numerous three-phase laterals. The designing distribution engineer has the following design data which are assumed to be satisfactory estimates.

The load is uniformly distributed, and the connected load density is 2000 kVA/mi^2. The demand factor, which is an average value for all loads, is 0.60. The diversity factor among all loads in the area is 1.20. The load power factor is 0.90 lagging, which is an average value applicable for all loads.

For some unknown reasons (perhaps, due to the excessive distance from load centers or transmission lines, or other limitations, such as availability of land, its cost, and land use ordinances and regulations), the only available substation sites are at locations A and B. If the designer selects site A as the substation location, there will be a 2-mi-long feeder main and 16 three-phase 2-mi-long laterals. On the other hand, if the designer selects site B as the

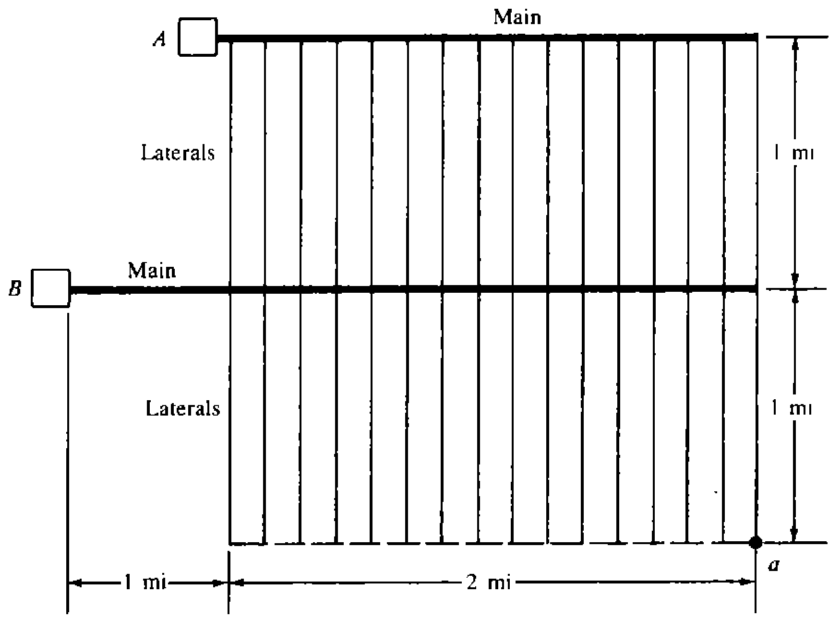


Figure 4-27

substation location, there will be a 3-mi-long feeder main (including a 1-mi-long express feeder main) and 32 three-phase 1-mi-long laterals.

The designer wishes to select the better one of the given two sites by investigating the total peak-load voltage drop at the end of the most remote lateral, i.e., at point a.

Assume 7.62/13.2-kV three-phase four-wire grounded-wye primary-feeder mains which are made of #2/0 copper overhead conductors. The laterals are of #4 copper conductors, and they are all three-phase, four-wire, and grounded-wye.

Using the precalculated percent voltage drop per kilovoltampere-mile curves given in Fig. 4-17, determine the better substation site by calculating the percent voltage drops at point a that correspond to each substation site and select the better one.

SOLUTION Maximum diversified demand is

$$\text{Diversified demand} = \frac{\sum_{i=1}^{n} \text{demand factor}_i \times \text{connected load}_i}{\text{diversity factor}}$$

$$= \frac{0.60 \times 2000 \text{ kVA/mi}^2}{1.20}$$

$$= 1000 \text{ kVA/mi}^2$$

The peak loads of the substations A and B are the same:

$$TS_n = 1000 \text{ kVA/mi}^2 \times 4 \text{ mi}^2$$

$$= 4000 \text{ kVA}$$

From Fig. 4-17, the K constants for #2/0 and #4 conductors are found as 0.0004 and 0.00095, respectively.

The maximum percent voltage drop for substation A occurs at point a, and it is the summation of the percent voltage drops in the main and the last lateral. Therefore†

$$\begin{aligned}\%\,\mathrm{VD}_a &= \frac{l}{2}K_m S_m + \frac{l}{2}K_l S_l \\ &= \tfrac{2}{2} \times 0.0004 \times 4000 + \tfrac{2}{2} \times 0.00095 \times \frac{4000}{16} \\ &\cong 1.84\,\%\end{aligned}$$

The maximum percent voltage drop for substation B also occurs at point a. Therefore

$$\%\,\mathrm{VD}_a = 2 \times 0.0004 \times 4000 + \tfrac{1}{2} \times 0.00095 \times \frac{4000}{32} \cong 3.26\,\%$$

Therefore substation site A is better than substation site B from the voltage-drop point of view.

Example 4-9 Assume a square-shaped distribution substation service area as shown in Fig. 4-28. The four-feeder substation serves a square area of $2a \times 2a$ mi^2.

The load density distribution is D kVA/mi^2 and is uniformly distributed. Each feeder main is three-phase four-wire grounded-wye with multigrounded common neutral open-wire line.

Since dimension d is much smaller than dimension a, assume that the length of each feeder main is approximately a mi, and the area served by the last lateral, which is indicated in the figure as the cross-hatched area, is approximately $a \times d$ mi^2. The power factor of all loads is $\cos\theta$ lagging. The impedance of the feeder main line per phase is

$$z_m = r_m + jx_m \qquad \Omega/\text{mi}$$

The impedance of the lateral line per phase is

$$z_l = r_l + jx_l \qquad \Omega/\text{mi}$$

The $V_{L\text{-}L}$ is the base line-to-line voltage in kilovolts, which is also the nominal operating voltage.

(*a*) Assume that laterals are also three-phase four-wire grounded-wye with multigrounded common neutral open-wire line. Show that the percent voltage drop at the end of the last lateral is

$$\%\,\mathrm{VD} = \frac{2D \times a^3(r_m\cos\theta + x_m\sin\theta)}{30 \times V_{L\text{-}L}^2} + \frac{D \times a^2 \times d(r_l\cos\theta + x_l\sin\theta)}{20 \times V_{L\text{-}L}^2} \tag{4-87}$$

† Note that there has been a shift in notation and the symbol S_m now stands for S_n.

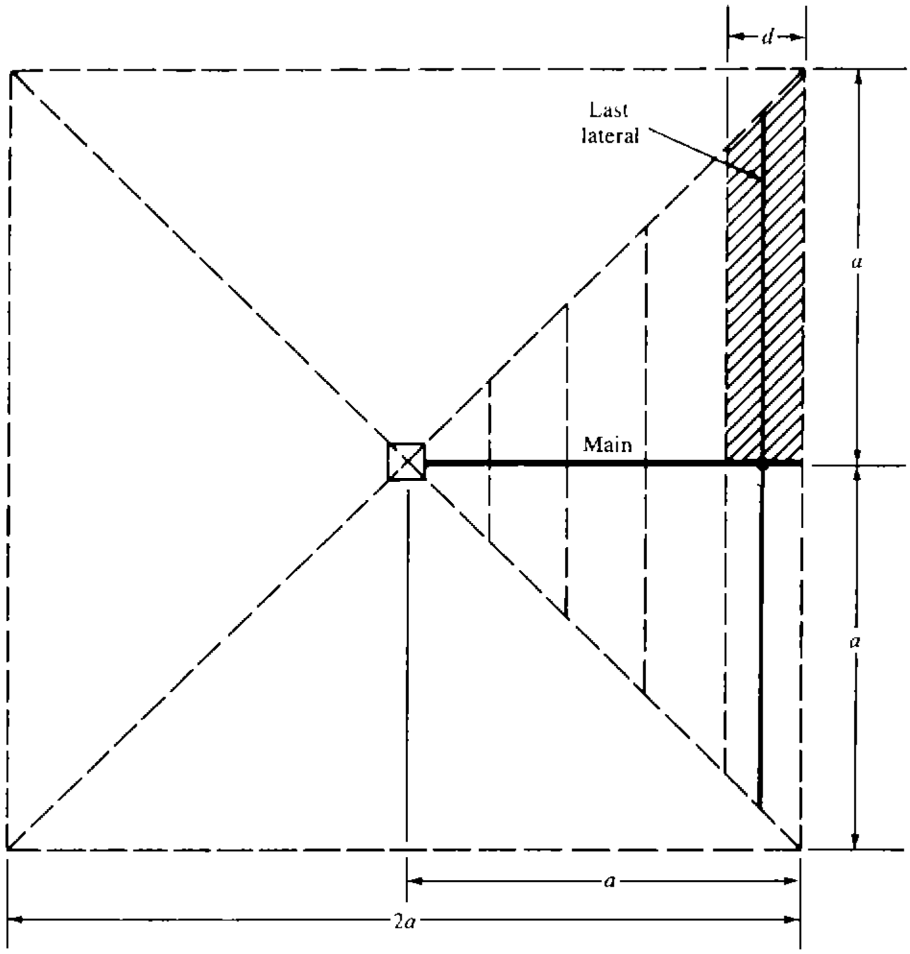


Figure 4-28

(b) Assume that the laterals are single-phase two-wire with multigrounded common neutral open-wire line. Apply Morrison's approximation [6] and modify the equation given in part a.

SOLUTION

(a) The total kilovoltampere load served by one main is

$$S_m = D \times \frac{(2a)^2}{4}$$

$$= D \times a^2 \qquad \text{kVA} \tag{4-88}$$

The current in the main of the substation is

$$I_m = \frac{D \times a^2}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{4-89}$$

Therefore, the percent voltage drop at the end of the main is

$$\%\,\mathrm{VD}_m = \frac{D \times a^2}{\sqrt{3} \times V_{L\text{-}L}} (r_m \cos\theta + x_m \sin\theta) \frac{\sqrt{3}}{1000 \times V_{L\text{-}L}} (\tfrac{2}{3}a)\,100$$

$$= \frac{2D \times a^3}{30 \times V_{L\text{-}L}^2} (r_m \cos\theta + x_m \sin\theta) \tag{4-90}$$

The kilovoltampere load served by the last lateral is

$$S_l = D \times a \times d \qquad \text{kVA} \tag{4-91}$$

The current in the lateral is

$$I_l = \frac{D \times a \times d}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{4-92}$$

Thus the percent voltage drop at the end of the lateral is

$$\% \, \text{VD}_l = \frac{D \times a \times d}{\sqrt{3} \times V_{L\text{-}L}} (r_l \cos \theta + x_l \sin \theta) \frac{\sqrt{3}}{1000 \times V_{L\text{-}L}} (\tfrac{1}{2}a)100$$

$$= \frac{D \times a^2 \times d}{20 \times V_{L\text{-}L}^2} (r_l \cos \theta + x_l \sin \theta) \tag{4-93}$$

Therefore, the addition of Eq. (4-90) and (4-93) gives Eq. (4-87).

(*b*) According to Morrison [6], the percent voltage drop of a single-phase circuit is approximately four times that for a three-phase circuit, assuming the usage of the same-size conductors. Therefore

$$\% \, \text{VD}_{1\phi} = 4 \times \% \, \text{VD}_{3\phi} \tag{4-94}$$

Hence, the percent voltage drop in the main is the same as given in part *a*, but the percent voltage drop for the lateral is not the same and is

$$\% \, \text{VD}_{l,1\phi} = 4 \times \frac{D \times a^2 \times d}{20 \times V_{L\text{-}L}^2} (r_l \cos \theta + x_l \sin \theta)$$

$$= \frac{D \times a^2 \times d}{5 \times V_{L\text{-}L}^2} (r_l \cos \theta + x_l \sin \theta) \tag{4-95}$$

Thus the total percent voltage drop will be the sum of the percent voltage drop in the three-phase main, given by Eq. (4-90), and the percent voltage drop in the single-phase lateral, given by Eq. (4-95). Therefore the total voltage drop is

$$\% \, \text{VD} = \frac{2D \times a^3 (r_m \cos \theta + x_m \sin \theta)}{30 \times V_{L\text{-}L}^2} + \frac{D \times a^2 \times d}{5 \times V_{L\text{-}L}^2} (r_l \cos \theta + x_l \sin \theta) \tag{4-96}$$

Example 4-10 Figure 4-29 shows a pattern of service area coverage (not necessarily a good pattern) with primary-feeder mains and laterals. There are five substations shown in the figure, each with two feeder mains. For example, substation *A* has two mains like *A*, and each main has many closely spaced laterals such as *a-a*.

If the laterals are not three-phase, the load in the main is assumed to be well-balanced among the three phases. The load tapped off the main decreases linearly with the distance *s*, as shown in Fig. 4-30.

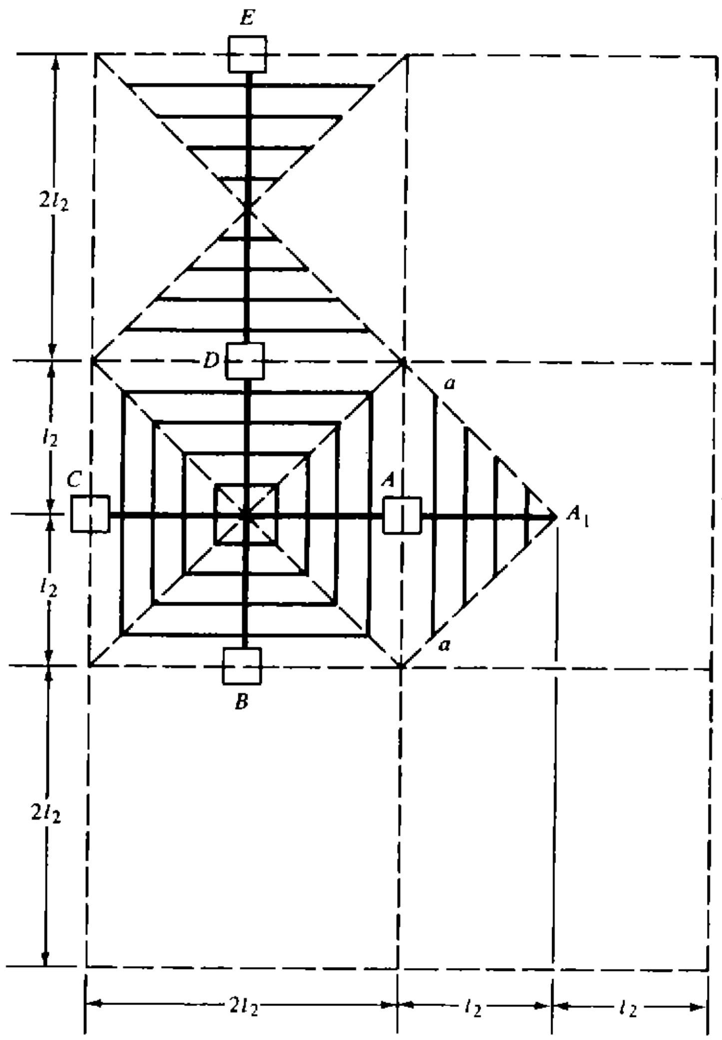


Figure 4-29

Using the following notation and the notation given in the figures, analyze a feeder main.

D = uniformly distributed load density, kVA/mi^2

$V_{L\text{-}L}$ = base voltage and nominal operating voltage, line-to-line kV

A_2 = area supplied by one feeder main

TA_2 = area supplied by one substation

S_2 = kVA input at the substation to one feeder main

TS_2 = total kVA load supplied by one substation

K_2 = % VD/(kVA · mi) for conductors and load power factor being considered

z_2 = impedance of three-phase main line, Ω/(mi · phase)

VD_2 = voltage drop at end of main, for example, A_1

(*a*) Find the differential area dA and the differential kilovoltampere-load-supplied $d(S)$ shown in Fig. 4-30.

(*b*) Find the kVA load flow in the main at any point s, that is, S_s. Express the S_s in terms of S_2, s, and l_2.

(*c*) Find the differential voltage drop at point s and then show that the total load may be concentrated at $s = l_2/3$ for the purpose of computing the VD_2.

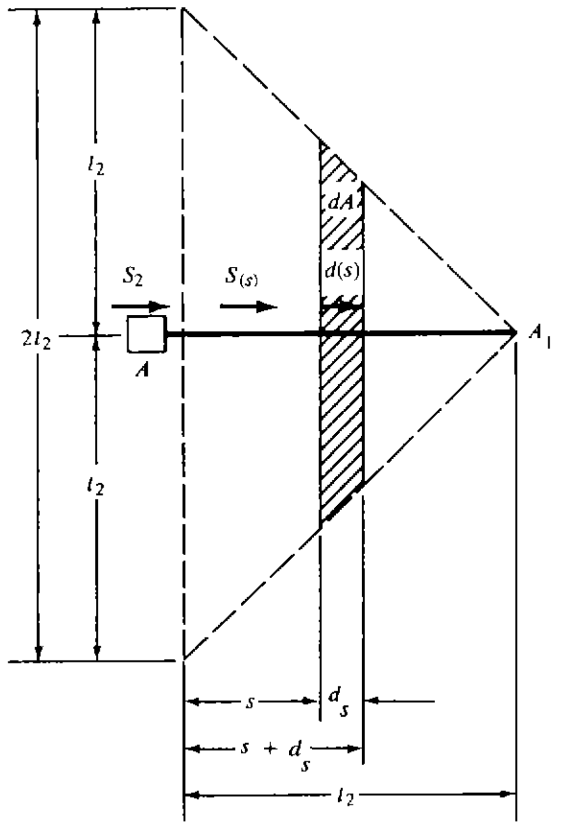


Figure 4-30

(*d*) Suppose that this two-feeders-per-substation pattern is to be implemented with thermally limited, i.e., ampacity-loaded, feeders.

Assume that the load density is 500 kVA/mi^2, the line-to-line voltage is 12.47 kV, and the feeder mains are #4/0 AWG ACSR open-wire lines. Find the substation spacing, both ways, that is, $2l_2$, and the load on the substation transformers, that is, TS_2.

SOLUTION

(*a*) From Fig. 4-30, the differential area is

$$dA = 2(l_2 - s)\,ds \qquad \text{mi}^2 \tag{4-97}$$

Therefore, the differential kilovoltampere load supplied is

$$d(S) = 2D(l_2 - s)\,ds \qquad \text{kVA} \tag{4-98}$$

(*b*) The kilovoltampere load flow in the main at any point s is

$$S_s = 2(l_2 - s)^2 D$$

$$= 2(l_2 - s)^2 \times \frac{S_2}{2l_2^2}$$

$$= \left(\frac{l_2 - s}{l_2}\right)^2 \cdot S_2 \qquad \text{kVA} \tag{4-99}$$

(c) The differential current at any point s is

$$I_s = \frac{S_s}{\sqrt{3} \times V_{L\text{-}L}} \quad \text{A} \tag{4-100}$$

Hence, the differential voltage drop at point s is

$$\begin{aligned} d(\text{VD})_s &= I_s \times z_2 \, ds \\ &= \frac{S_s}{\sqrt{3} \times V_{L\text{-}L}} \times z_2 \, ds \\ &= \left(\frac{l_2 - s}{l_2}\right)^2 \left(\frac{S_2}{\sqrt{3} \times V_{L\text{-}L}}\right) z_2 \, ds \\ &= \frac{S_2 \times z_2}{\sqrt{3} \times V_{L\text{-}L} \times l_2^2} \times (l_2 - s)^2 \, ds \end{aligned} \tag{4-101}$$

The integration of either side of Eq. (4-101) gives the voltage drop at point s:

$$\begin{aligned} \text{VD}_s &= \int_0^s d(\text{VD})_s \\ &= \int_0^s \frac{S_2 \times z_2}{\sqrt{3} \times V_{L\text{-}L} \times l_2^2} (l_2 - s)^2 \, ds \\ &= \frac{S_2 \times z_2}{\sqrt{3} \times V_{L\text{-}L} \times l_2^2} \left.\frac{(l_2 - s)^3}{3}\right|_0^s \\ &= \frac{S_2 \times z_2}{\sqrt{3} \times V_{L\text{-}L} \times l_2^2} \frac{l_2^3}{3} - \frac{S_2 \times z_2}{\sqrt{3} \times V_{L\text{-}L} \times l_2^2} \frac{(l_2 - s)^3}{3} \\ &= \frac{S_2 \times z_2}{3\sqrt{3} \times V_{L\text{-}L} \times l_2^2} [l_2^3 - (l_2 - s)^3] \end{aligned} \tag{4-102}$$

When $s = l_2$, Eq. (4-102) becomes

$$\begin{aligned} \text{VD}_2 &= \frac{S_2 \times z_2 \times l_2^3}{3\sqrt{3} \times V_{L\text{-}L} \times l_2^2} \\ &= \frac{S_2 \times z_2 \times l_2}{3\sqrt{3} \times V_{L\text{-}L}} \\ &= \frac{S_2}{\sqrt{3} V_{L\text{-}L}} \times z_2 \times \frac{l_2}{3} \end{aligned} \tag{4-103}$$

Therefore, the load has to be lumped at $l_2/3$.

(*d*) From Table A-5 of Appendix A, the conductor ampacity for #4/0 AWG ACSR conductor can be found as 340 A. Therefore

$$S_2 = \sqrt{3} \times 12.47\,\text{kV} \times 340\,\text{A}$$
$$\cong 7343.5\,\text{kVA}$$

Since

$$S_2 = D \times l_2^2$$

then

$$l_2 = \left(\frac{S_2}{D}\right)^{1/2}$$
$$= \left(\frac{7335\,\text{kVA}}{500\,\text{kVA/mi}^2}\right)^{1/2} \quad (4\text{-}104)$$
$$= 3.83\,\text{mi}$$

Therefore the substation spacing, both ways, is

$$2l_2 = 2 \times 3.83$$
$$= 7.66\,\text{mi}$$

Total load supplied by one substation is

$$TS_2 = 2 \times S_2$$
$$= 2 \times 7343.5$$
$$= 14{,}687\,\text{kVA}$$

Example 4-11 Compare the method of service area coverage given in Example 4-10 with the four-feeders-per-substation pattern of Sec. 4-6 (see Fig. 4-16). Use the same feeder main conductors so that $K_2 = K_4$, and the same line-to-line nominal operating voltage $V_{L\text{-}L}$.

Here, let S_4 be the kilovoltampere input to one feeder main of the four-feeder substation, and let TS_4, A_4, VD_4, K_4, etc., all pertain similarly to the four-feeder substation. Investigate the voltage-drop-limited feeders and determine the following:

(*a*) Ratio of substation spacings $= 2l_2/2l_4$
(*b*) Ratio of areas covered per feeder main $= A_2/A_4$
(*c*) Ratio of substation loads $= TS_2/TS_4$

SOLUTION

(*a*) Assuming the percent voltage drops and the K constants are the same in both cases,

$$\%\,\text{VD}_2 = \%\,\text{VD}_4$$

and

$$K_2 = K_4$$

where

$$\begin{aligned}\%\,\mathrm{VD}_2 &= (D \times l_2^2)(K_2)(\tfrac{1}{3} l_2)\\ &= \tfrac{1}{3} K_2 \times D \times l_2^3\end{aligned}$$

and

$$\begin{aligned}\%\,\mathrm{VD}_4 &= (D \times l_4^2)(K_4)(\tfrac{2}{3} l_4)\\ &= \tfrac{2}{3} K_4 \times D \times l_4^3\end{aligned}$$

Therefore

$$l_2^3 = 2 l_4^3$$

or the ratio of substation spacings is

$$\left(\frac{l_2}{l_4}\right)^3 = 2 \tag{4-105}$$

or for both ways,

$$\frac{2 l_2}{2 l_4} \cong 1.26 \tag{4-106}$$

(*b*) The ratio of areas covered per feeder main is

$$\begin{aligned}\frac{A_2}{A_4} &= \frac{l_2^2}{l_4^2}\\ &= \left(\frac{l_2}{l_4}\right)^2\\ &= 2^{2/3}\\ &\cong 1.59\end{aligned} \tag{4-107}$$

(*c*) The ratio of substation loads is

$$\begin{aligned}\frac{TS_2}{TS_4} &= \frac{2 \times D \times l_2^2}{4 \times D \times l_4^2}\\ &= \frac{1}{2}\left(\frac{l_2}{l_4}\right)^2\\ &\cong 0.8\end{aligned} \tag{4-108}$$

PROBLEMS

4-1 Verify Eq. (4-17).

4-2 Derive Eq. (4-44).

4-3 Prove that doubling feeder voltage level causes the percent voltage drop in the primary-feeder circuit to be reduced to one-fourth of its previous value.

4-4 Repeat Example 4-1, parts *a* and *b*, assuming a three-phase 34.5-kV wye-grounded feeder main which has 350-kcmil copper conductors with an equivalent spacing of 37 in between phase conductors and a lagging-load power factor of 0.9.

4-5 Repeat part *a* of Prob. 4-4, assuming 300-kcmil ACSR conductors.

4-6 Repeat Prob. 4-5, assuming a lagging-load power factor of 0.7.

4-7 Repeat Prob. 4-6, assuming AWG #4/0 conductors.

4-8 Repeat Example 4-2, assuming ACSR conductors.

4-9 Repeat Example 4-3, assuming ACSR conductors.

4-10 Repeat Example 4-4, assuming ACSR conductors.

4-11 Repeat Example 4-5, assuming ACSR conductors.

4-12 Repeat Example 4-7, assuming a 13.2/22.9-kV voltage level.

4-13 Repeat Example 4-8 for a load density of 1000 kVA/mi^2.

4-14 Repeat part *d* of Example 4-10 for a load density of 1000 kVA/mi^2.

REFERENCES

1. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*, 11th ed., McGraw-Hill, New York, 1978.
2. Seely, H. P., *Electrical Distribution Engineering*. 1st ed., McGraw-Hill, New York, 1930.
3. Van Wormer, F. C.: "Some Aspects of Distribution Load Area Geometry," *AIEE Trans.*, December 1954, pp. 1343–1349.
4. Denton, W. J., and D. N. Reps: "Distribution Substation and Primary Feeder Planning," *AIEE Trans.*, June 1955, pp. 484–499.
5. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
6. Morrison, C.: "A Linear Approach to the Problem of Planning New Feed Points into a Distribution System," *AIEE Trans.*, pt. III (PAS), December 1963, pp. 819–832.

CHAPTER

FIVE

DESIGN CONSIDERATIONS OF PRIMARY SYSTEMS

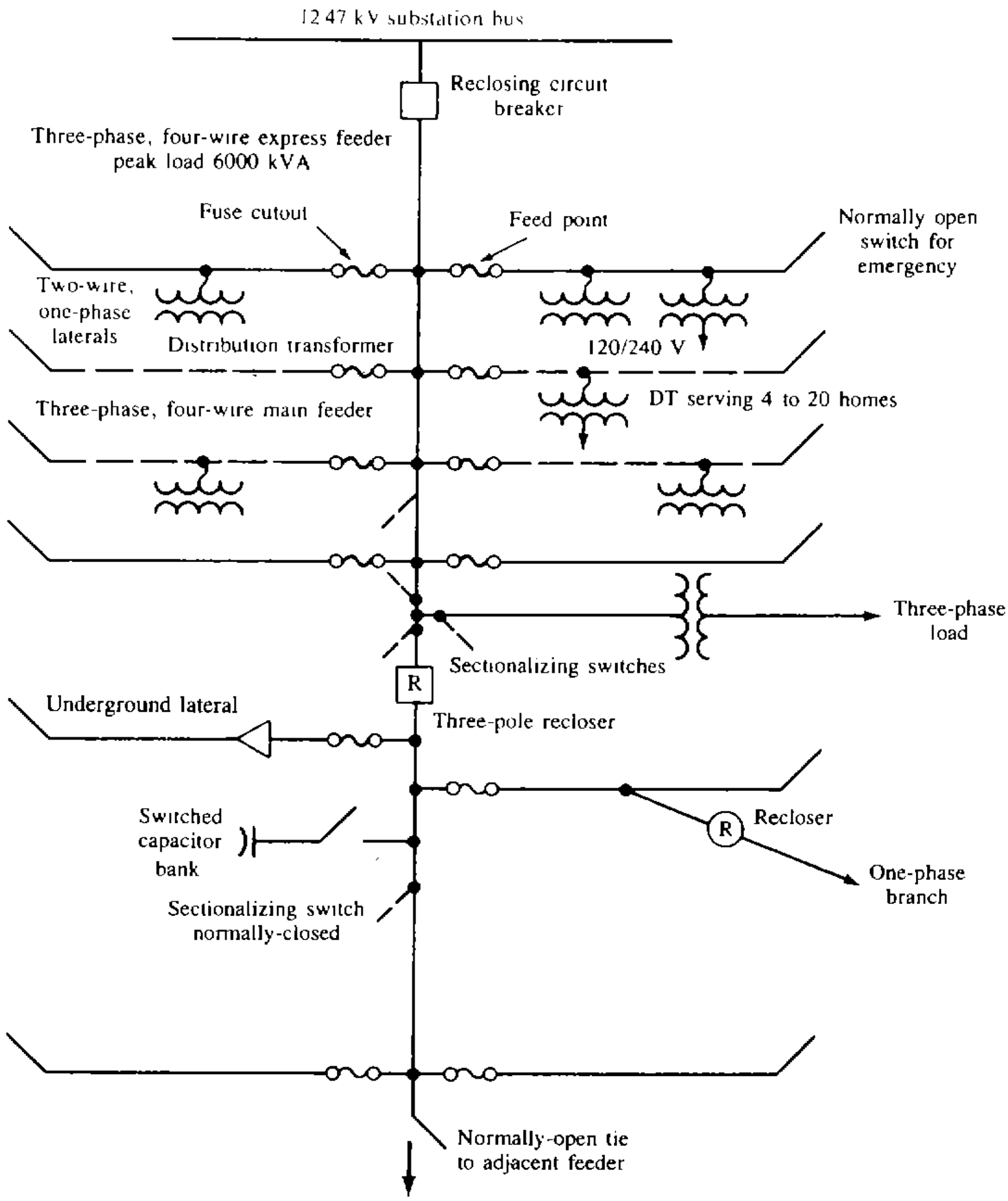
5-1 INTRODUCTION

The part of the electric utility system which is between the distribution substation and the distribution transformers is called the *primary system*. It is made of circuits known as *primary feeders* or *primary distribution feeders*.

Figure 5-1 shows a one-line diagram of a typical primary distribution feeder. A feeder includes a "main" or main feeder, which usually is a three-phase four-wire circuit, and branches or laterals, which usually are single-phase or three-phase circuits tapped off the main. Also sublaterals may be tapped off the laterals as necessary. In general, laterals and sublaterals located in residential and rural areas are single-phase and consist of one phase conductor and the neutral. The majority of the distribution transformers are single-phase and connected between the phase and the neutral through fuse cutouts.

A given feeder is sectionalized by reclosing devices at various locations in such a manner as to remove as little as possible of the faulted circuit so as to hinder service to as few consumers as possible. This can be achieved through the coordination of the operation of all the fuses and reclosers.

It appears that, due to growing emphasis on the service reliability, the protection schemes in the future will be more sophisticated and complex, ranging from manually operated devices to remotely controlled automatic devices based on supervisory controlled or computer-controlled systems.



Residential area. Approximately 1,000 homes per square mile
Feeder area. 1 to 4 mi^2 depending on load density
15 to 30 single-phase laterals per feeder
150 to 500 MVA short-circuit available at substation bus

Figure 5-1 One-line diagram of typical primary distribution feeders. (*From* [*1*].)

The congested and heavy-load locations in metropolitan areas are served by using underground primary feeders. They are usually radial three-conductor cables. The improved appearance and less-frequent trouble expectancy are among the advantages of this method. However, it is more expensive, and the repair time is longer than the overhead systems. In some cases, the cable can be employed as suspended on poles. The cost involved is greater than that of open-wire but much less than that of underground installation.

There are various and yet interrelated factors affecting the selection of a primary-feeder rating. Examples are:

1. The nature of the load connected
2. The load density of the area served
3. The growth rate of the load
4. The need for providing spare capacity for emergency operations
5. The type and cost of circuit construction employed
6. The design and capacity of the substation involved
7. The type of regulating equipment used
8. The quality of service required
9. The continuity of service required

The voltage conditions on distribution systems can be improved by using shunt capacitors which are connected as near the loads as possible to derive the greatest benefit. The use of shunt capacitors also improves the power factor involved which in turn lessens the voltage drops and currents, and therefore losses, in the portions of a distribution system between the capacitors and the bulk power buses. The capacitor ratings should be selected carefully to prevent the occurrence of excessive overvoltages at times of light loads due to the voltage rise produced by the capacitor currents.

The voltage conditions on distribution systems can also be improved by using series capacitors. But the application of series capacitors does not reduce the currents and therefore losses, in the system.

5-2 RADIAL-TYPE PRIMARY FEEDER

The simplest and the lowest-cost and therefore the most common form of primary feeder is the radial-type primary feeder as shown in Fig. 5-2. The main primary feeder branches into various primary laterals which in turn separates into several sublaterals to serve all the distribution transformers. In general, the main feeder and subfeeders are three-phase three- or four-wire circuits and the laterals are three-phase or single-phase. The current magnitude is the greatest in the circuit conductors that leave the substation. The current magnitude continually lessens out toward the end of the feeder as laterals and sublaterals are tapped off the feeder. Usually, as the current lessens, the size of the feeder conductors is also reduced. However, the permissible voltage regulation may restrict any feeder size reduction which is based only on the thermal capability, i.e., current-carrying capacity, of the feeder.

The reliability of service continuity of the radial primary feeders is low. A fault occurrence at any location on the radial primary feeder causes a power outage for every consumer on the feeder unless the fault can be isolated from the source by a disconnecting device such as a fuse, sectionalizer, disconnect switch, or recloser.

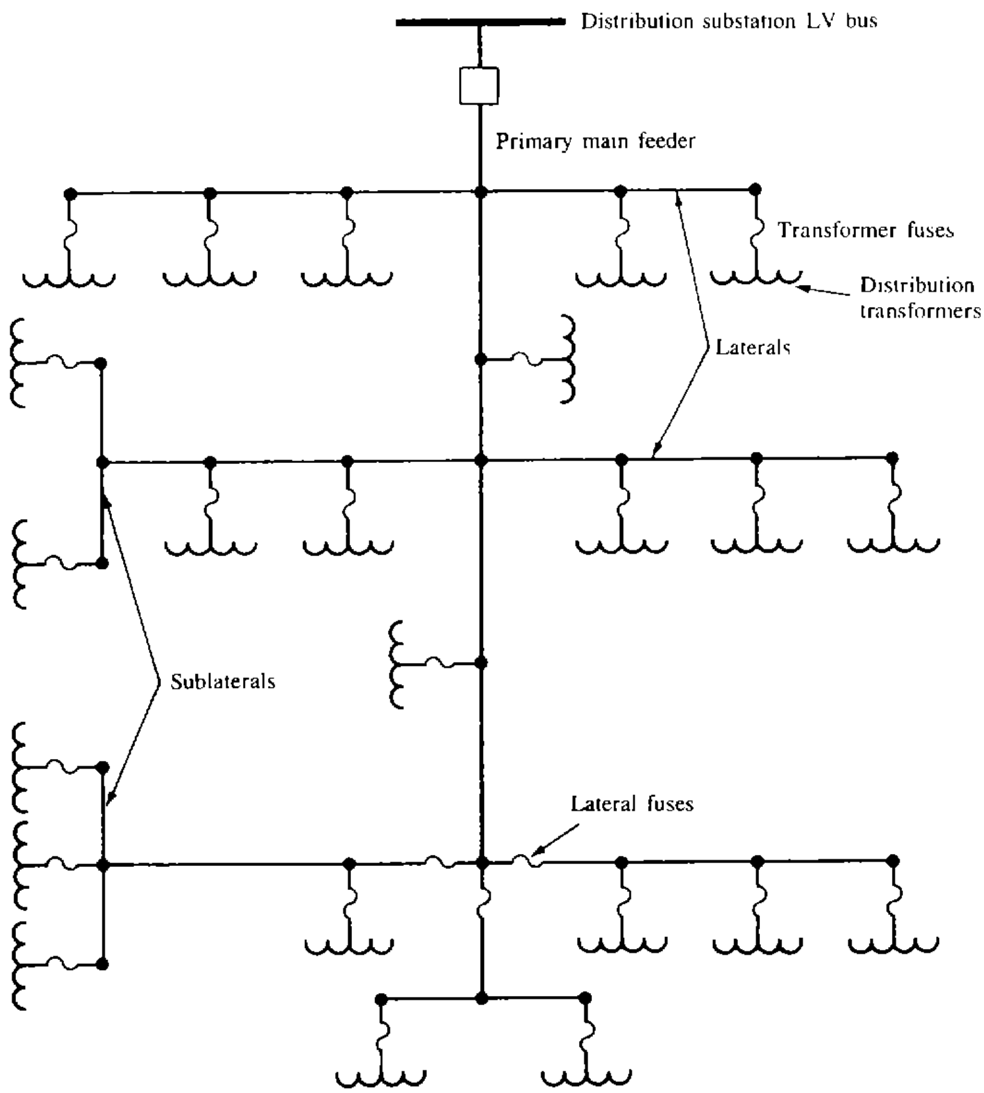


Figure 5-2 Radial-type primary feeder.

Figure 5-3 shows a modified radial-type primary feeder with tie and sectionalizing switches to provide fast restoration of service to customers by switching unfaulted sections of the feeder to an adjacent primary feeder or feeders. The fault can be isolated by opening the associated disconnecting devices on each side of the faulted section.

Figure 5-4 shows another type of radial primary feeder with express feeder and backfeed. The section of the feeder between the substation low-voltage bus and the load center of the service area is called an *express feeder*. No subfeeders or laterals are allowed to be tapped off the express feeder. However, a subfeeder is allowed to provide a backfeed toward the substation from the load center.

Figure 5-5 shows a radial-type phase-area feeder arrangement in which each phase of the three-phase feeder serves its own service area. In Fig. 5-4 and 5-5, each dot represents a balanced three-phase load lumped at that location.

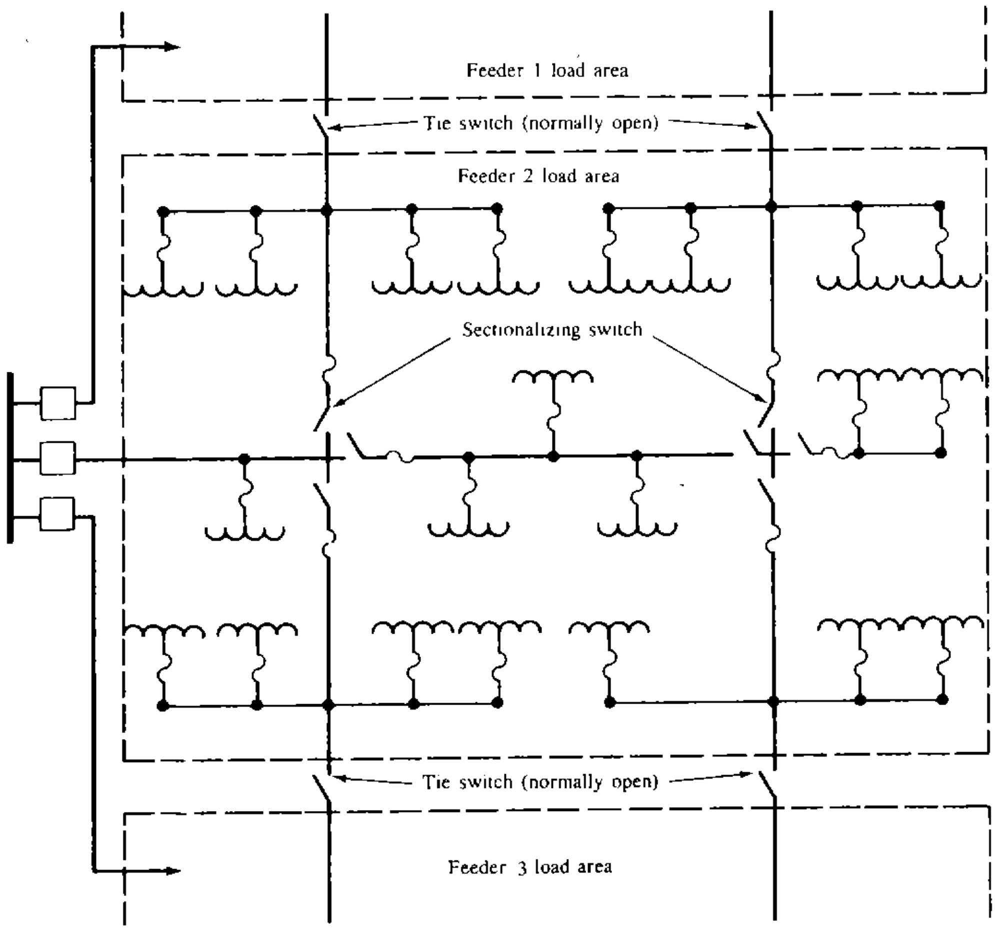


Figure 5-3 Radial-type primary feeder with tie and sectionalizing switches.

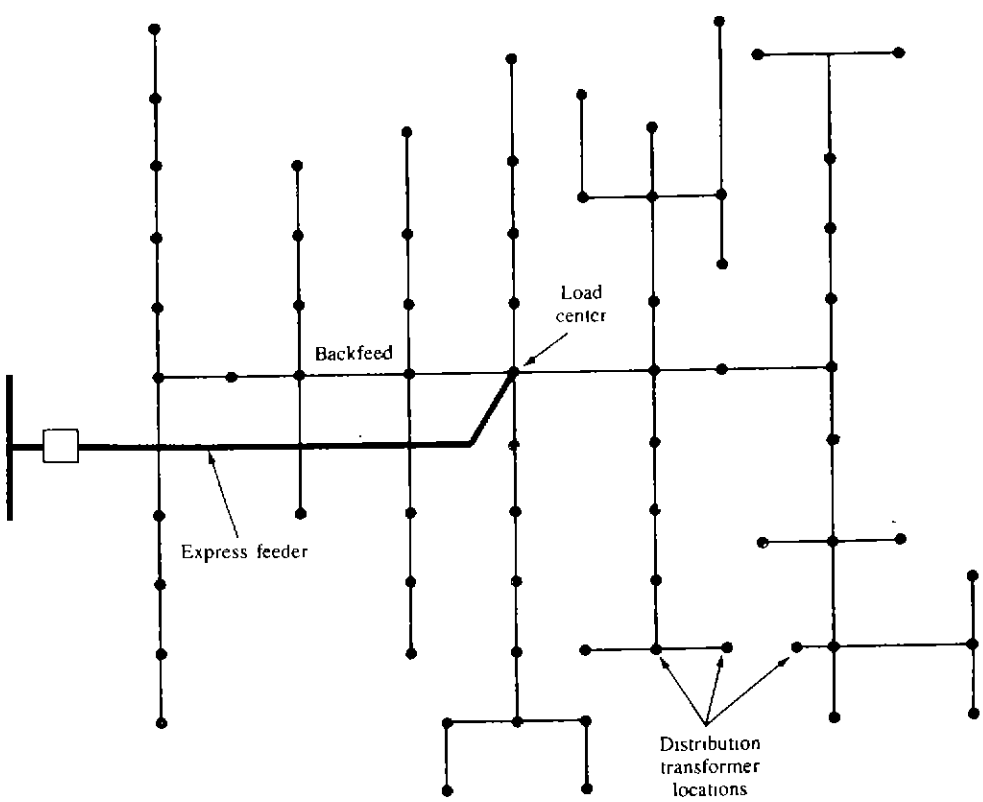


Figure 5-4 Radial-type primary feeder with express feeder and backfeed.

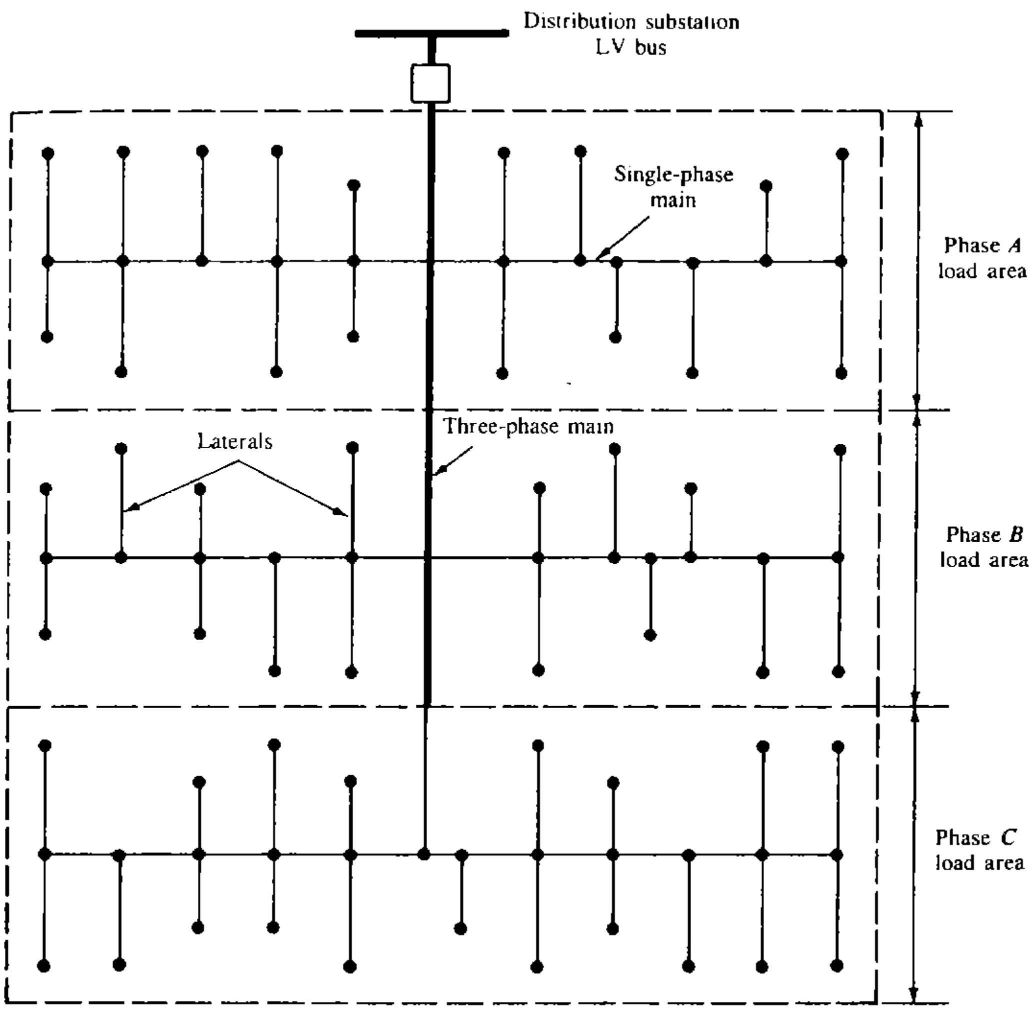


Figure 5-5 Radial-type phase-area feeder.

5-3 LOOP-TYPE PRIMARY FEEDER

Figure 5-6 shows a loop-type primary feeder which loops through the feeder load area and returns back to the bus. Sometimes the loop tie disconnect switch is replaced by a loop tie breaker due to the load conditions. In either case, the loop can function with the tie disconnect switches or breakers normally open or normally closed.

Usually, the size of the feeder conductor is kept the same throughout the loop. It is selected to carry its normal load plus the load of the other half of the loop. This arrangement provides two parallel paths from the substation to the load when the loop is operated with normally open tie breakers or disconnect switches.

A primary fault causes the feeder breaker to be open. The breaker will remain open until the fault is isolated from both directions. The loop-type primary-feeder arrangement is especially beneficial to provide service for loads where high service reliability is important. In general, a separate feeder breaker on each end of the loop is preferred, despite the cost involved. The parallel feeder paths can also be

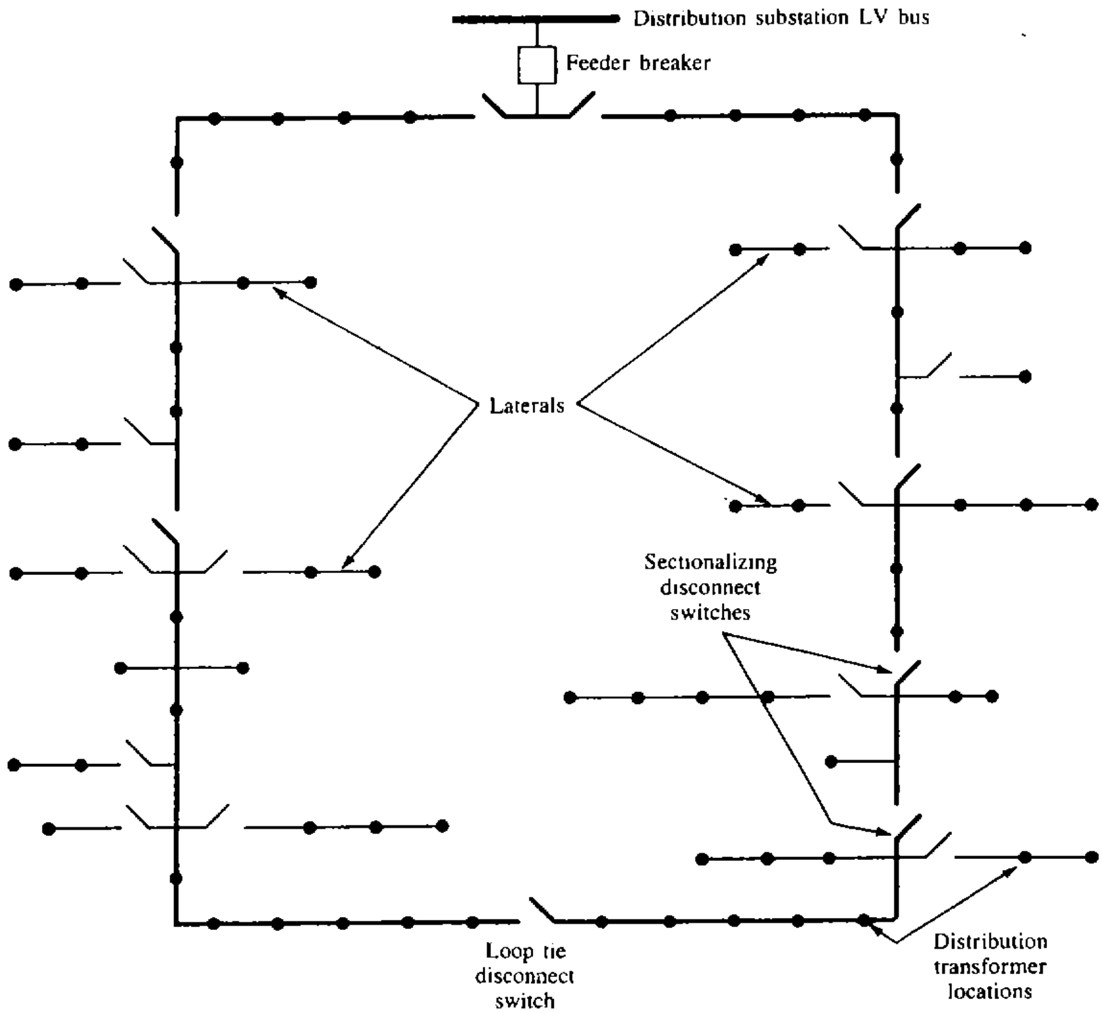


Figure 5-6 Loop-type primary feeder.

connected to separate bus sections in the substation and supplied from separate transformers. In addition to main feeder loops, normally open lateral loops are also used, particularly in underground systems.

5-4 PRIMARY NETWORK

As shown in Fig. 5-7, a primary network is a system of interconnected feeders supplied by a number of substations. The radial primary feeders can be tapped off the interconnecting tie feeders. They can also be served directly from the substations. Each tie feeder has two associated circuit breakers at each end in order to have less load interrupted due to a tie-feeder fault.

The primary-network system supplies a load from several directions. Proper location of transformers to heavy-load centers and regulation of the feeders at the substation buses provide for adequate voltage at utilization points. In general, the losses in a primary network are lower than those in a comparable radial system due to load division.

The reliability and the quality of service of the primary-network arrangement is much higher than the radial and loop arrangements. However, it is more difficult to design and operate than the radial or loop systems.

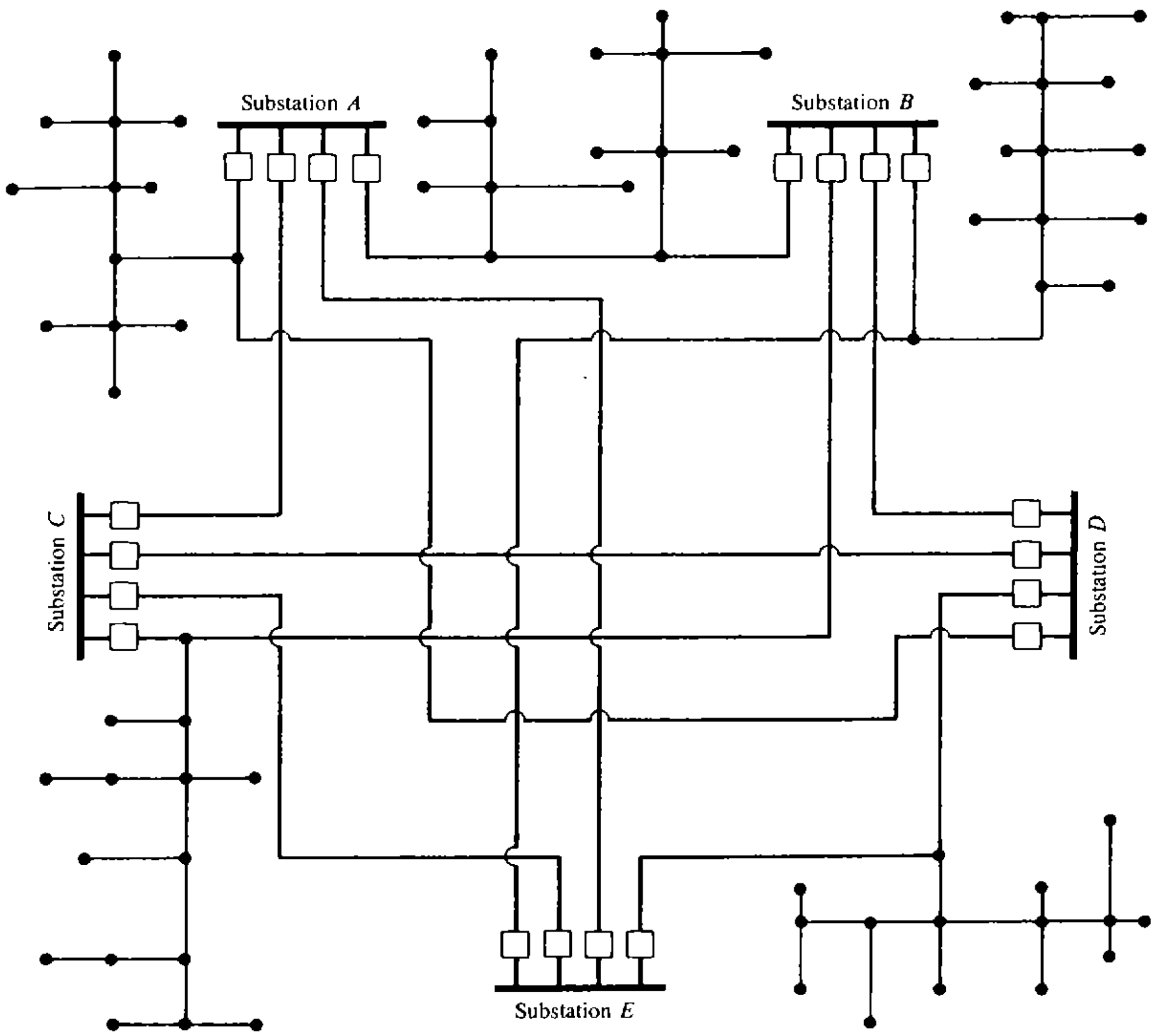


Figure 5-7 Primary network.

5-5 PRIMARY-FEEDER VOLTAGE LEVELS

The primary-feeder voltage level is the most important factor affecting the system design, cost, and operation. Some of the design and operation aspects affected by the primary-feeder voltage level are [2]:

1. Primary-feeder length
2. Primary-feeder loading
3. Number of distribution substations
4. Rating of distribution substations
5. Number of subtransmission lines
6. Number of customers affected by a specific outage
7. System maintenance practices
8. The extent of tree trimming
9. Joint use of utility poles
10. Type of pole-line design and construction
11. Appearance of the pole line

There are additional factors affecting the decisions for primary-feeder voltage-level selection, as shown in Fig. 5-8.

Table 5-1 gives typical primary voltage levels used in the United States. Three-phase four-wire multigrounded common-neutral primary systems, for example, 12.47Y/7.2 kV, 24.9Y/14.4 kV, and 34.5Y/19.92 kV, are employed almost exclusively. The fourth wire is used as the multigrounded neutral for both the primary and the secondary systems. The 15-kV-class primary voltage levels are most commonly used. However, the current trend is toward higher voltages, e.g., the 34.5-kV class is gaining rapid acceptance. The 5-kV class continues to decline in usage. California is one of the few states which has three-phase three-wire primary systems. The four-wire system is economical, especially for underground residential distribution (URD) systems, since each primary lateral has only one insulated phase wire and the bare neutral instead of having two insulated wires.

Usually, primary feeders located in low-load density areas are restricted in length and loading by permissible voltage drop rather than by thermal restrictions, whereas primary feeders located in high-load density areas, e.g., industrial and commercial areas, may be restricted by the thermal limitations.

In general, for a given percent voltage drop, the feeder length and loading are direct functions of the feeder voltage level. This relationship is known as the *voltage-square rule*. For example, if the feeder voltage is doubled, for the same percent voltage drop, it can supply the same power four times the distance However, as Lokay [2] explains it clearly, the feeder with the increased length feeds more load. Therefore the advantage obtained by the new and higher-voltage level through the voltage-square factor, i.e.,

$$\text{Voltage-square factor} = \left(\frac{V_{L\text{-}N,\,\text{new}}}{V_{L\text{-}N,\,\text{old}}}\right)^2 \tag{5-1}$$

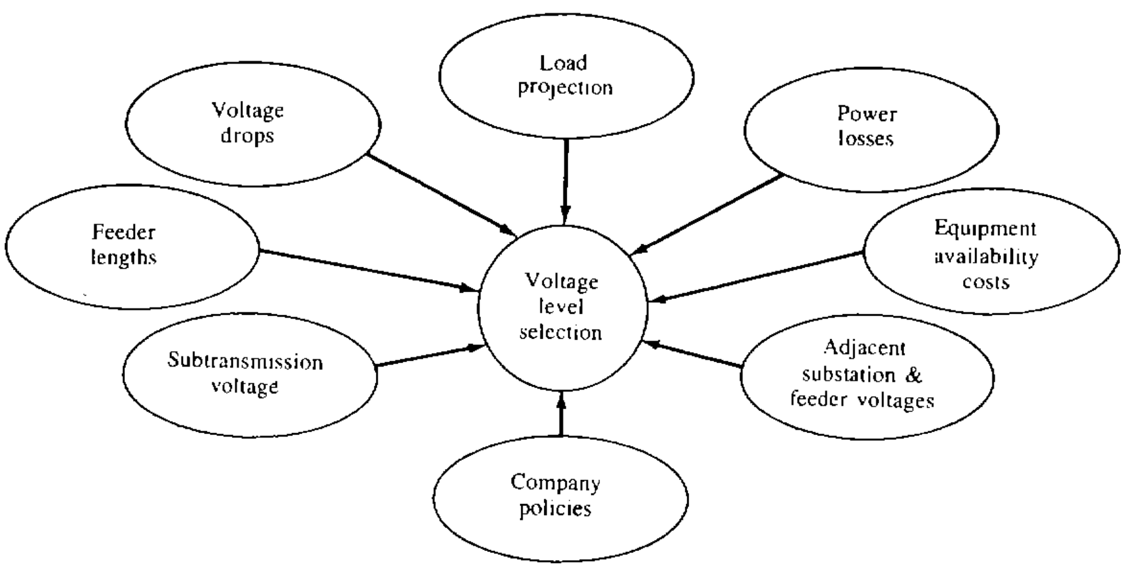


Figure 5-8 Factors affecting primary-feeder voltage-level selection decision.

Table 5-1 Typical primary voltage levels

Class	3ϕ voltage	
2.5 kV	2300	3W-Δ
	2400*	3W-Δ
5.0 kV	4000	3W-Δ or 3W-Y
	4160*	4W-Y
	4330	3W-Δ
	4400	3W-Δ
	4600	3W-Δ
	4800	3W-Δ
8.66 kV	6600	3W-Δ
	6900	3W-Δ or 4W-Y
	7200*	3W-Δ or 4W-Y
	7500	4W-Y
	8320	4W-Y
15 kV	11000	3W-Δ
	11500	3W-Δ
	12000	3W-Δ or 4W-Y
	12470*	4W-Y
	13200*	3W-Δ or 4W-Y
	13800*	3W-Δ
	14400	3W-Δ
25 kV	22900*	4W-Y
	24940*	4W-Y
34.5 kV	34500*	4W-Y

* Most common voltage in the individual classes.

has to be allocated between the growth in load and in distance. Further, the same percent voltage drop will always result provided that the following relationship exists:

$$\text{Distance ratio} \times \text{load ratio} = \text{voltage-square factor} \tag{5-2}$$

where

$$\text{Distance ratio} = \frac{\text{new distance}}{\text{old distance}} \tag{5-3}$$

and

$$\text{Load ratio} = \frac{\text{new feeder loading}}{\text{old feeder loading}} \tag{5-4}$$

The relationship between the voltage-square factor rule and the *feeder distance-coverage principle* is further explained in Fig. 5-9.

There is a relationship between the area served by a substation and the voltage-rule. Lokay [2] defines it as the *area-coverage principle.* As illustrated in

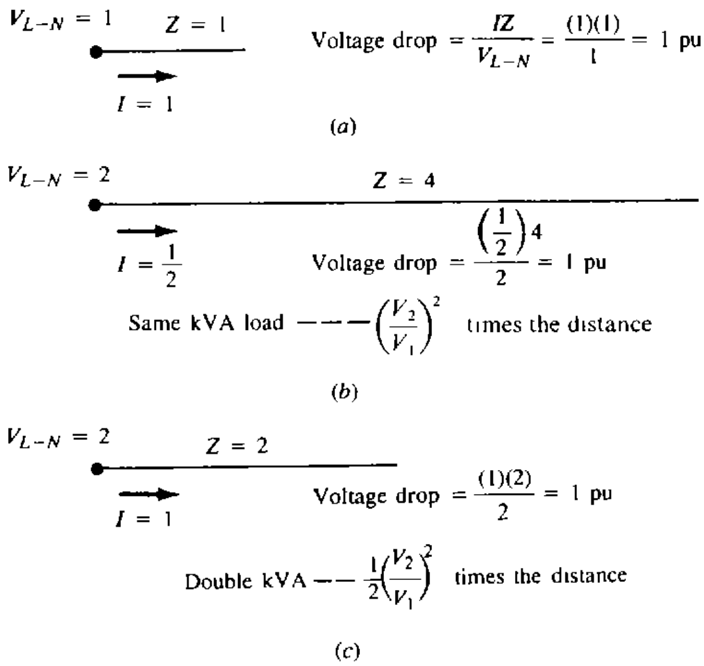


Figure 5-9 Illustration of the voltage-square rule and the feeder distance-coverage principle as a function of feeder voltage level and a single load. (*From* [2].)

Fig. 5-10, for a constant percent voltage drop and a uniformly distributed load, the feeder service area is proportional to

$$\left[\left(\frac{V_{L\text{-}N,\,\text{new}}}{V_{L\text{-}N,\,\text{old}}}\right)^2\right]^{2/3} \tag{5-5}$$

provided that both dimensions of the feeder service area change by the same proportion. For example, if the new feeder voltage level is increased to twice the previous voltage level, the new load and area that can be served with the same percent voltage drop is

$$\left[\left(\frac{V_{L\text{-}N,\,\text{new}}}{V_{L\text{-}N,\,\text{old}}}\right)^2\right]^{2/3} = (2^2)^{2/3} = 2.52 \tag{5-6}$$

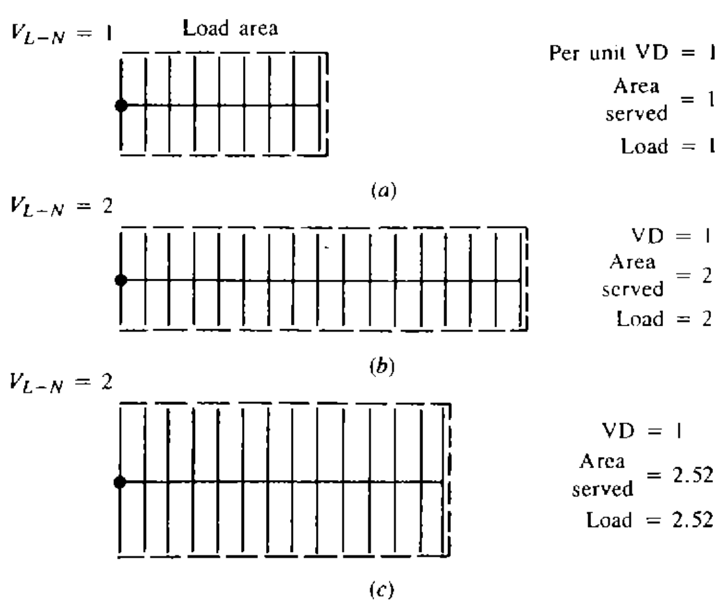


Figure 5-10 Feeder area-coverage principle as related to feeder voltage and a uniformly distributed load. (*From* [2].)

times the original load and area. If the new feeder voltage level is increased to three times the previous voltage level, the new load and area that can be served with the same percent voltage drop is

$$\left[\left(\frac{V_{L\text{-}N,\,\text{new}}}{V_{L\text{-}N,\,\text{old}}}\right)^2\right]^{2/3} = (3^2)^{2/3} = 4.32 \qquad (5\text{-}7)$$

times the original load and area.

5-6 PRIMARY-FEEDER LOADING

Primary-feeder loading is defined as the loading of a feeder during peak-load conditions as measured at the substation [2]. Some of the factors affecting the design loading of a feeder are:

1. The density of the feeder load
2. The nature of the feeder load
3. The growth rate of the feeder load
4. The reserve-capacity requirements for emergency
5. The service-continuity requirements
6. The service-reliability requirements
7. The quality of service
8. The primary-feeder voltage level
9. The type and cost of construction.
10. The location and capacity of the distribution substation
11. The voltage regulation requirements

There are additional factors affecting the decisions for feeder routing, the number of feeders, and feeder conductor-size selection, as shown in Figs. 5-11 to 5-13.

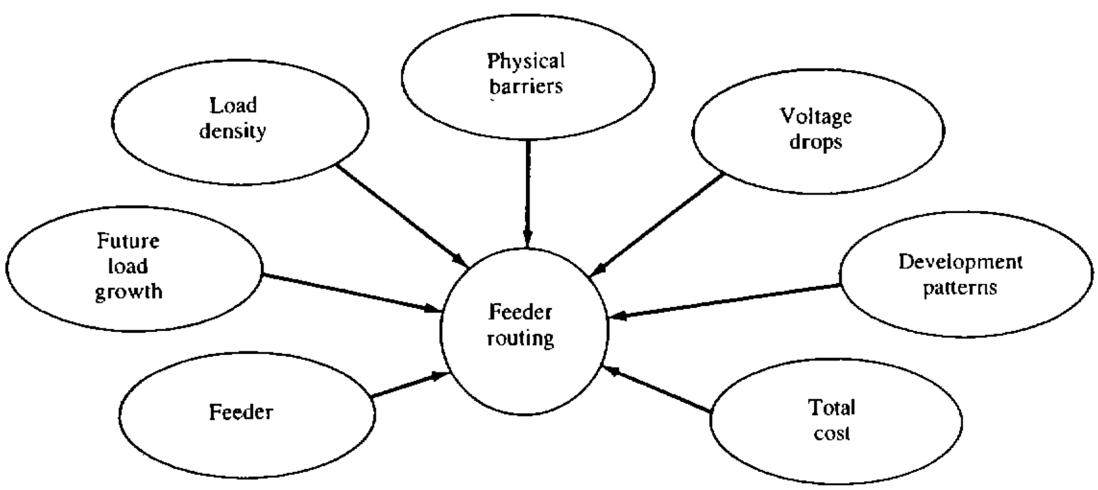


Figure 5-11 Factors affecting feeder routing decisions.

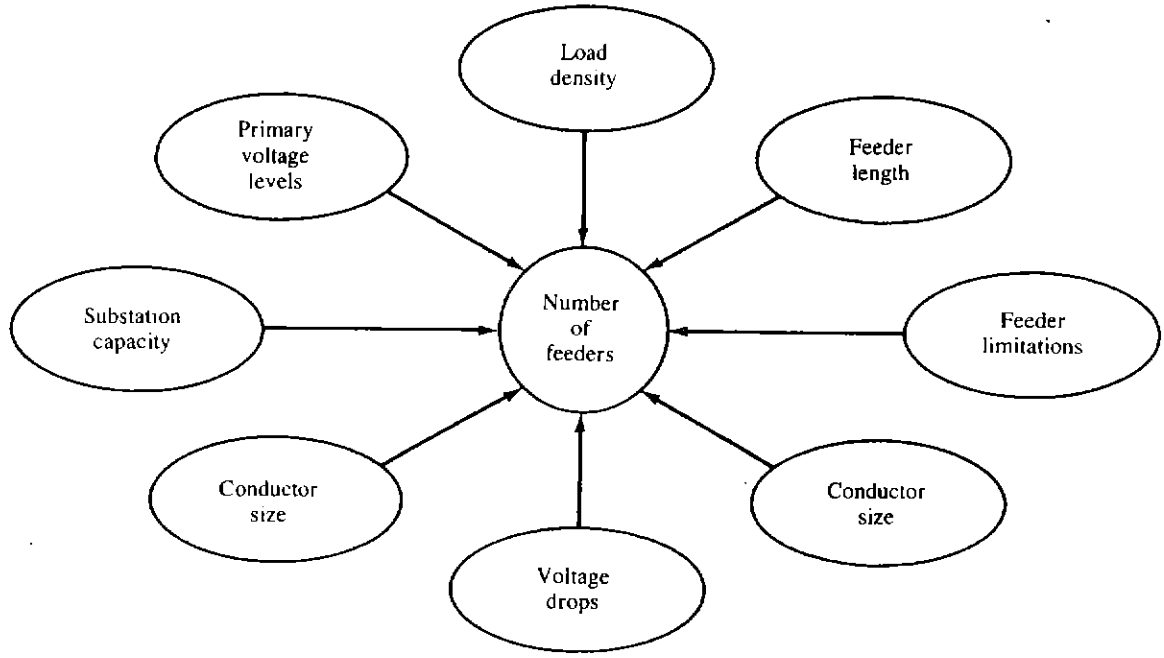


Figure 5-12 Factors affecting number of feeders.

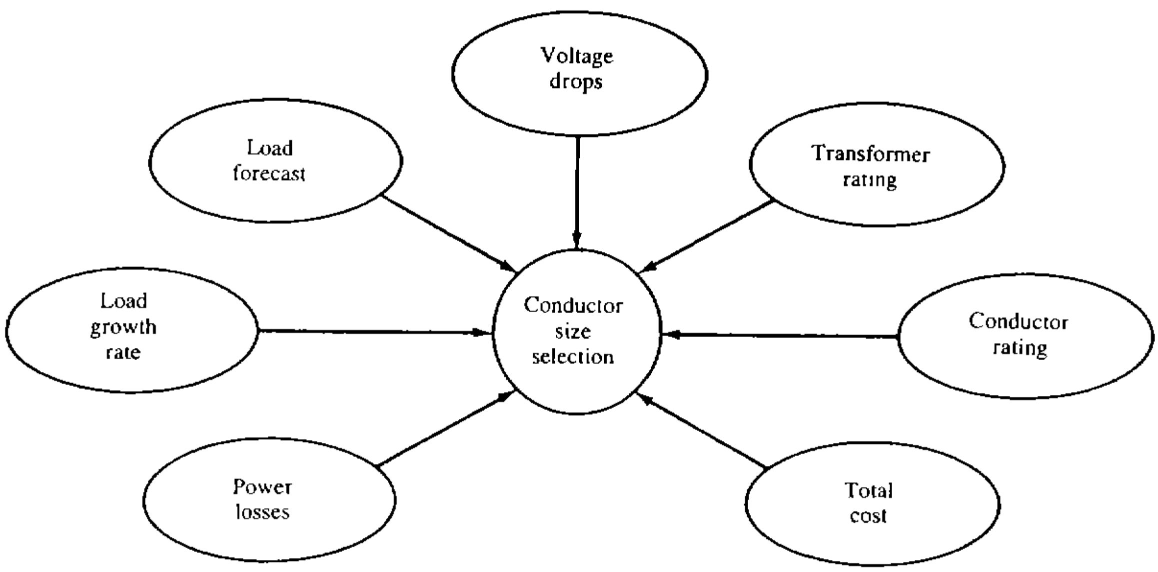


Figure 5-13 Factors affecting conductor-size selection.

5-7 TIE LINES

A tie line is a line that connects two supply systems to provide emergency service to one system from another, as shown in Fig. 5-14. Usually, a tie line provides service for area loads along its route as well as providing for emergency service to adjacent areas or substations. Therefore tie lines are needed to perform either of the following two functions:

1. To provide emergency service for an adjacent feeder for the reduction of outage time to the customers during emergency conditions

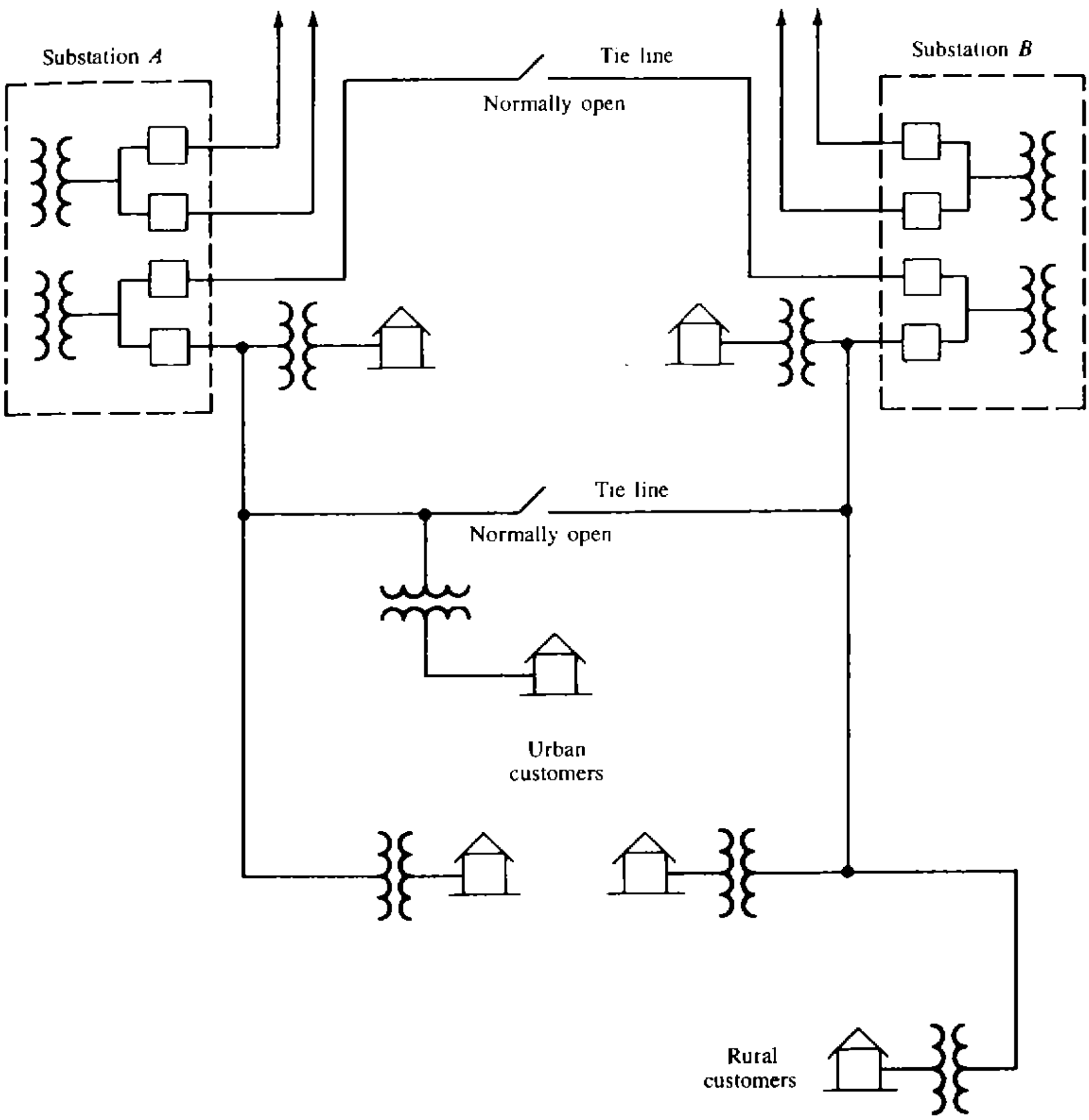


Figure 5-14 One-line diagram of typical two-substation area supply with tie lines.

2. To provide emergency service for adjacent substation systems, thereby eliminating the necessity of having an emergency backup supply at every substation. Tie lines should be installed when more than one substation is required to serve the area load at one primary distribution voltage.

Usually the substation primary feeders are designed and installed in such an arrangement as to have the feeders supplied from the same transformer extend in opposite directions so that all required ties can be made with circuits supplied from different transformers. For example, a substation with two transformers and four feeders might have the two feeders from one transformer extending north and south. The two feeders from the other transformer may extend east and west. All tie lines should be made to circuits supplied by other transformers. This would make it much easier to restore service to an area that is affected by a transformer failure.

Disconnect switches are installed at certain intervals in main feeder tie lines to facilitate load transfer and service restoration. The location of disconnect switches needs to be selected carefully to obtain maximum operating flexibility. Not only the physical arrangement of the circuit but also the size and nature of loads between switches are important. Loads between the disconnect switches should be balanced as much as possible so that load transfers between circuits do not adversely affect circuit operation. The optimum voltage conditions are obtained only if the circuit is balanced as closely as possible throughout its length.

5-8 DISTRIBUTION FEEDER EXIT: RECTANGULAR-TYPE DEVELOPMENT

The objective of this section is to provide an example for a uniform area development plan to minimize the circuitry changes associated with the systematic expansion of the distribution system.

Assume that underground feeder exits are extended out of a distribution substation into an existing overhead system. Also assume that at the ultimate development of this substation, a 6-mi^2 service area will be served with a total of 12 feeder circuits, four per transformer. Assuming uniform load distribution, each of the 12 circuits would serve approximately $\frac{1}{2}$ mi^2 in a fully developed service area. This is called the *rectangular-type development* and illustrated in Figs. 5-15 to 5-18.

In general, adjacent service areas are served from different transformer banks in order to provide for transfer to adjacent circuits in the event of transformer outages. The addition of new feeder circuits and transformer banks requires circuit number changes as the service area develops. The center transformer bank

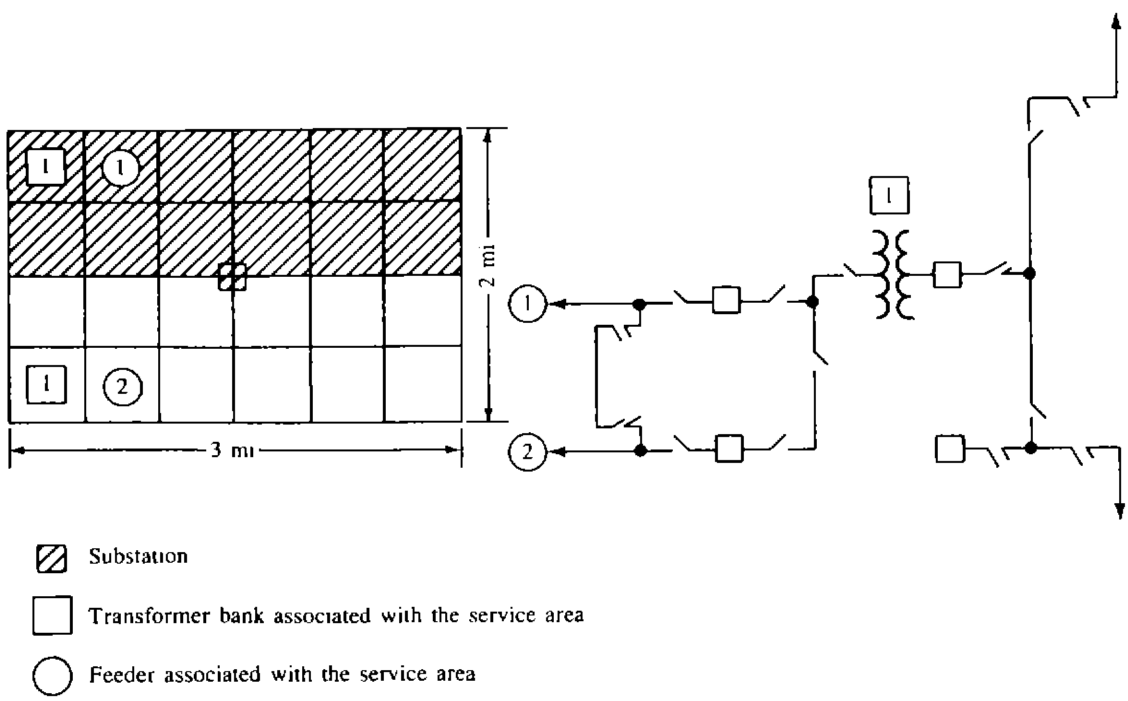


Figure 5-15

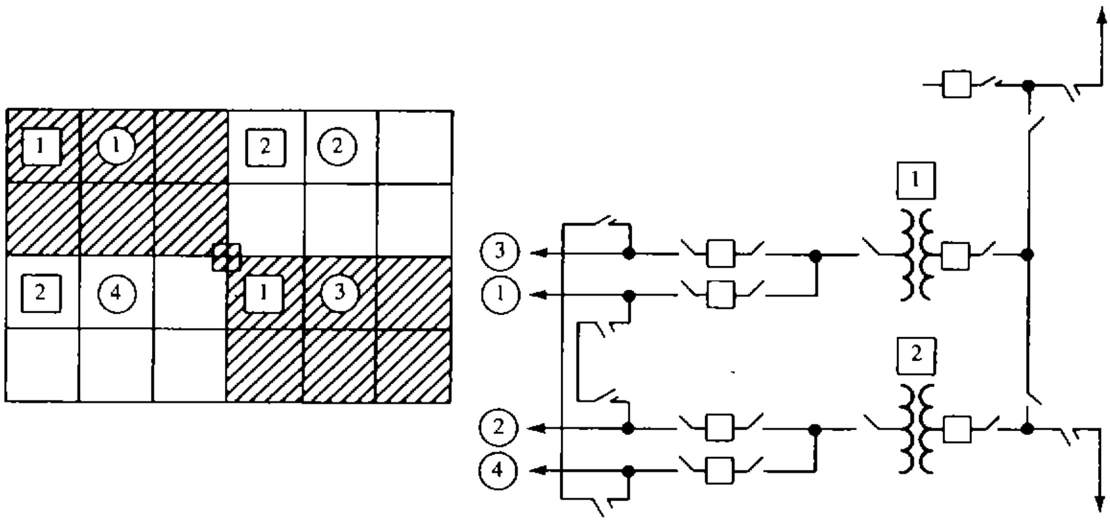


Figure 5-16

is always fully developed when the substation has eight feeder circuits. As the service area develops, the remaining transformer banks develop to full capacity. There are two basic methods of development, depending upon the load density of a service area, namely, the 1-2-4-8-12 feeder circuit method and the 1-2-4-6-8-12 feeder circuit method. The numbers shown for feeders and transformer banks in the following figures represent only the sequence of installation as the substation develops.

Method of development for high-load density areas In service areas with high-load density, the adjacent substations are developed similarly to provide for adequate load-transfer capability and service continuity. Here, for example, a two-transformer-bank substation can carry a firm rating of the emergency rating of one bank plus circuit ties, plus reserve considerations. Since sufficient circuit ties must be available to support the loss of a large transformer unit, the 1-2-4-8-12 feeder

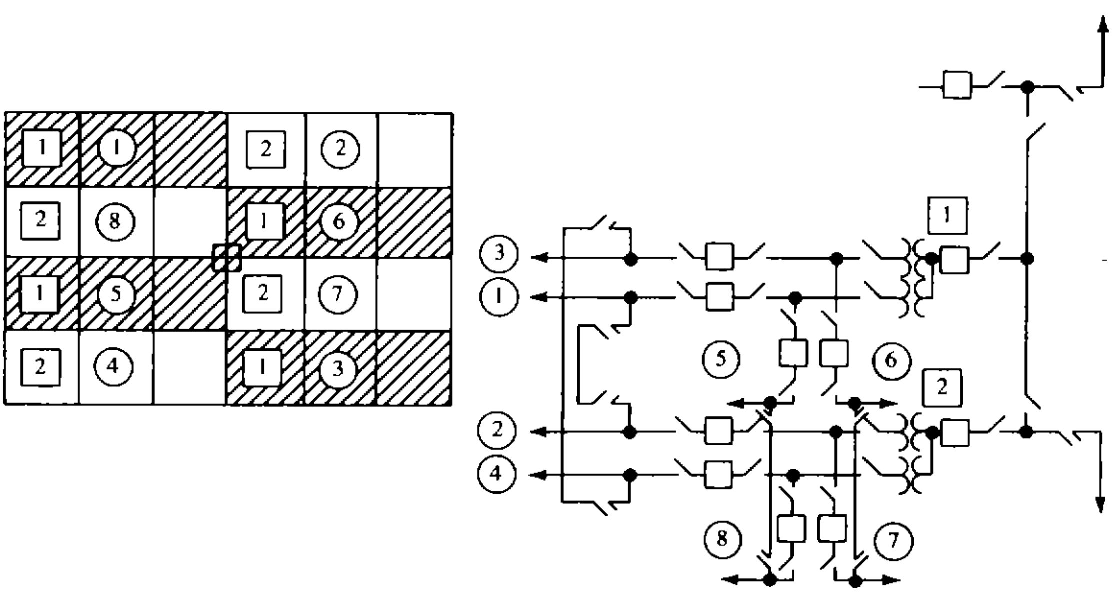


Figure 5-17

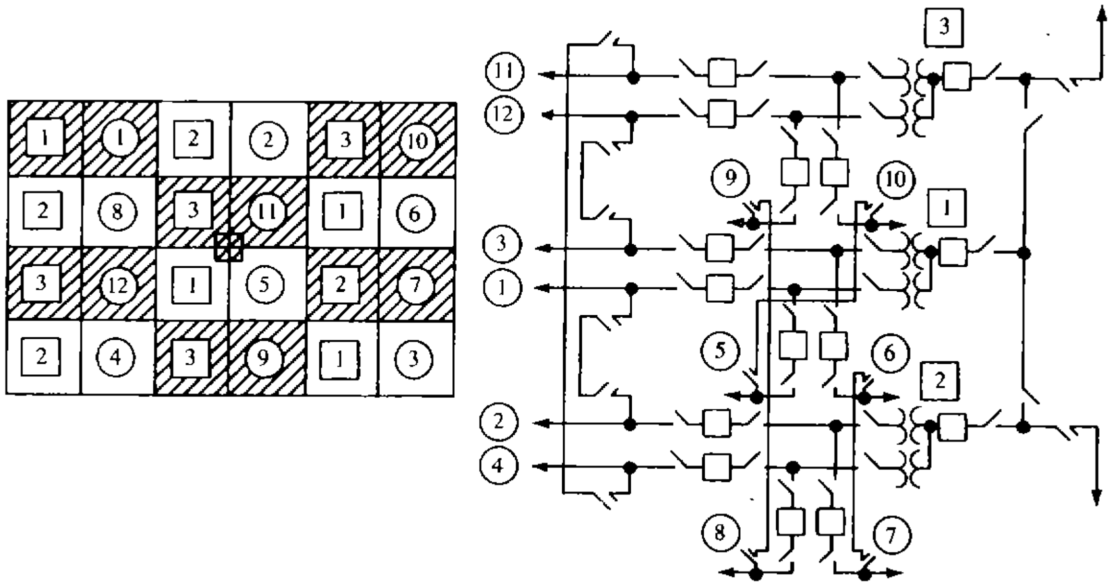


Figure 5-18

method is especially desirable for a high-load density area. Figures 5-15 to 5-18 show the sequence of installing additional transformers and feeders.

Method of development for low-load density areas In low-load density areas, where adjacent substations are not adequately developed and circuit ties are not available due to excessive distances between substations, the 1-2-4-6-8-12 circuit-developing substation scheme is more suitable. These large distances between substations generally limit the amount of load that can be transferred between substations without objectionable outage time due to circuit switching and guarantee that minimum voltage levels are maintained. This method requires

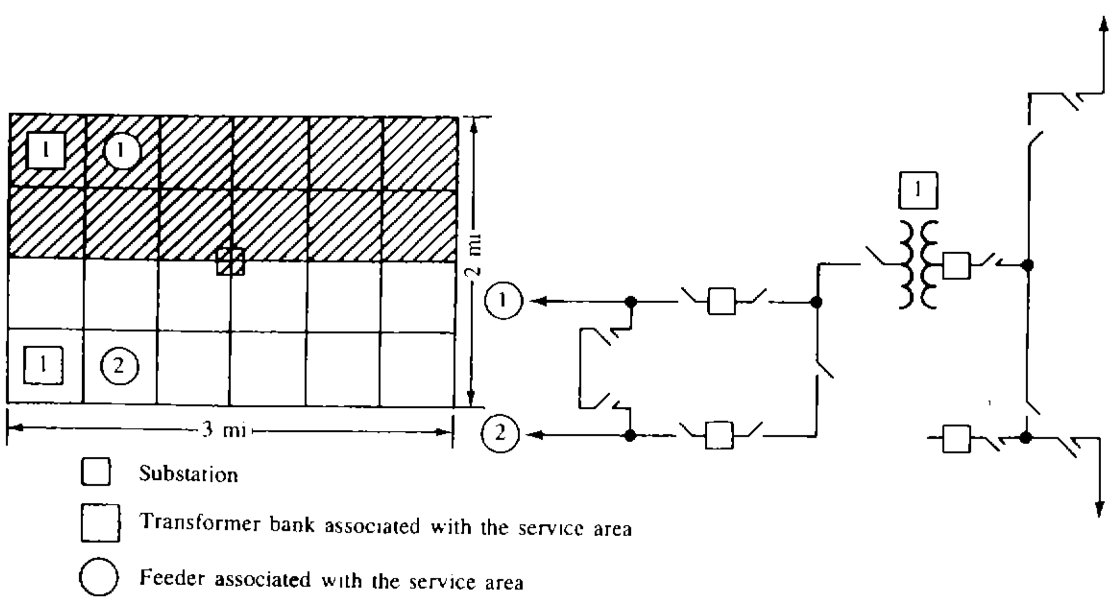


Figure 5-19

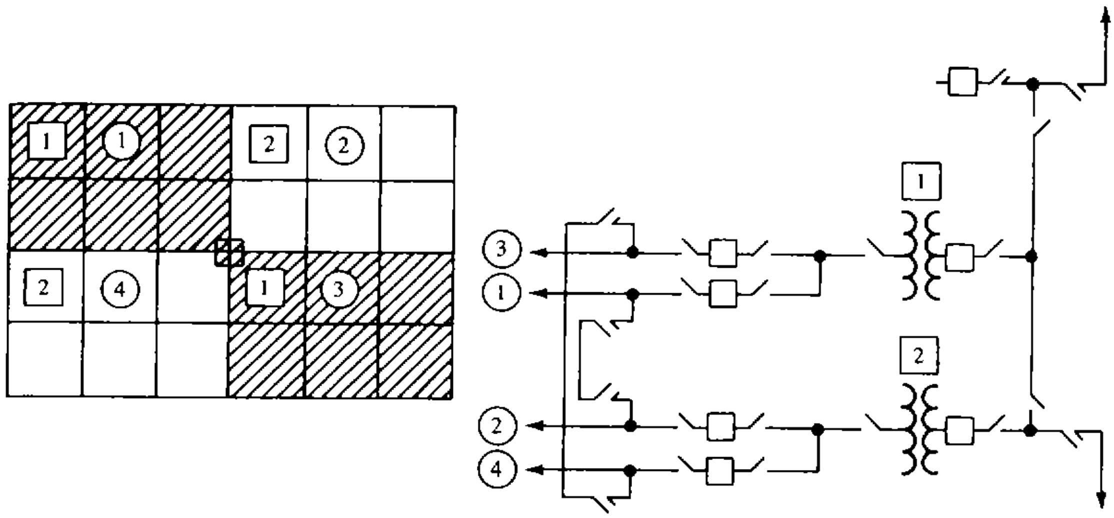


Figure 5-20

the substation to have all three transformer banks before using the larger transformers in order to provide a greater firming capability within the individual substation.

As illustrated in Figs. 5-19 to 5-23, once three, for example, 12/16/20-MVA, transformer units and six feeders are reached in the development of this type of substation, there are two alternatives for further expansion: (1) either remove one of the banks and increase the remaining two bank sizes to the larger, for example, 24/32/40 MVA, transformer units employing the low-side bays of the third transformer as part of the circuitry in the development of the remaining two banks, or (2) completely ignore the third transformer-bank area and complete the development of two remaining sections similar to the previous method.

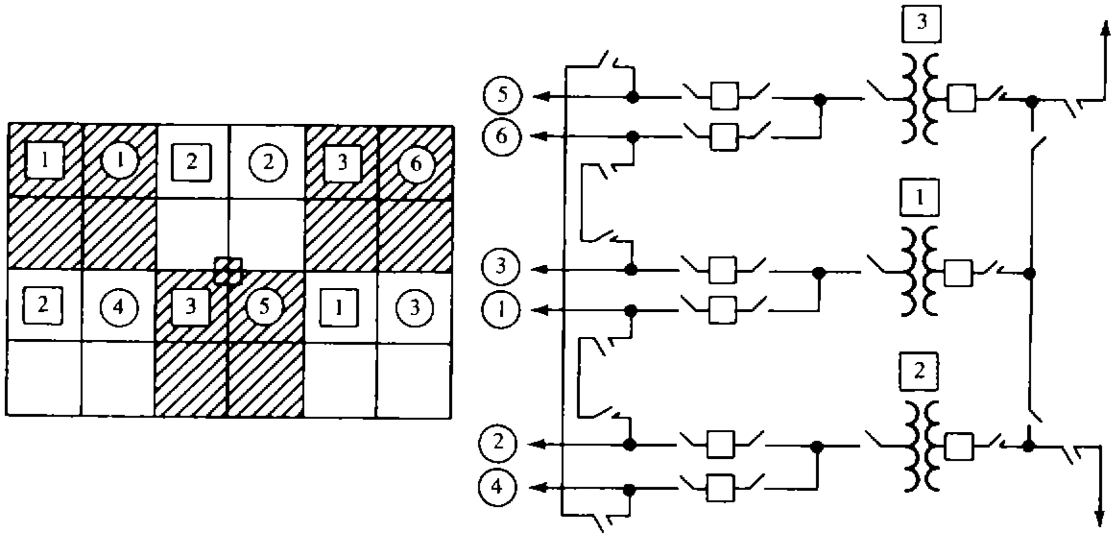


Figure 5-21

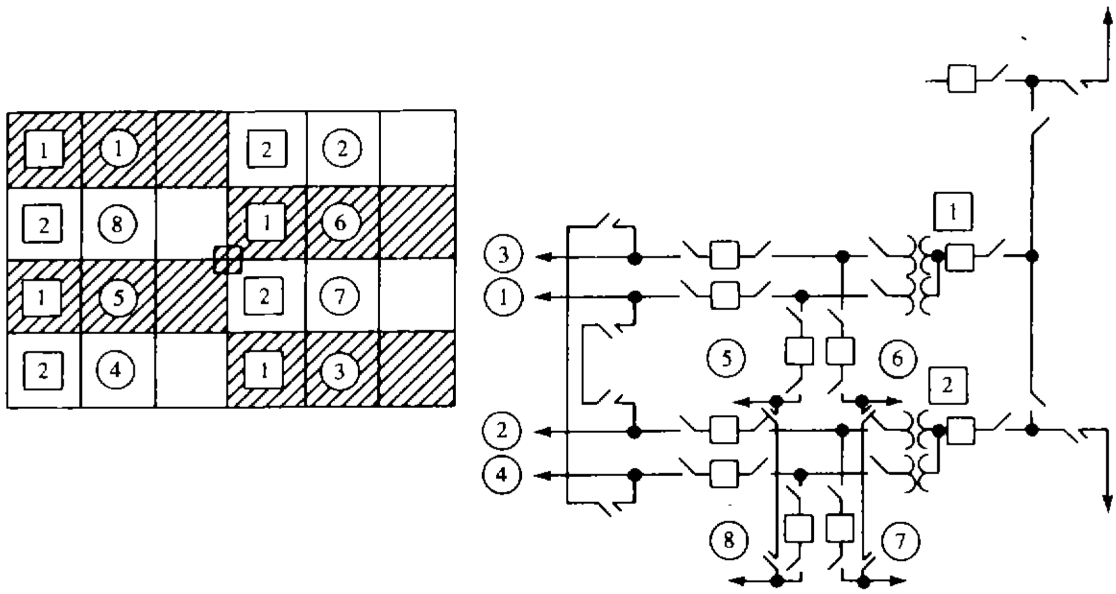


Figure 5-22

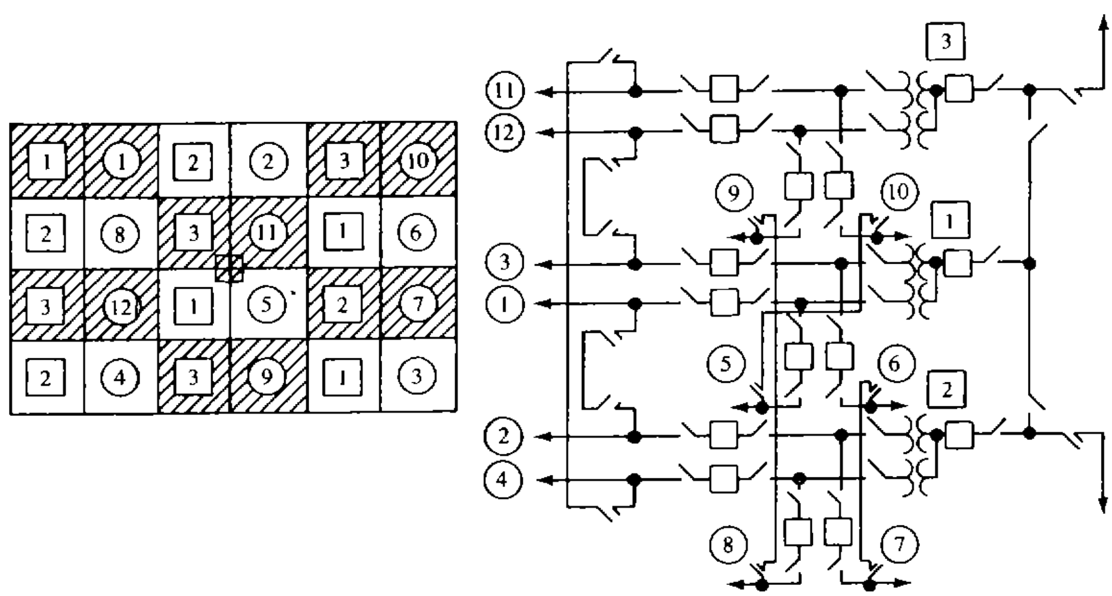
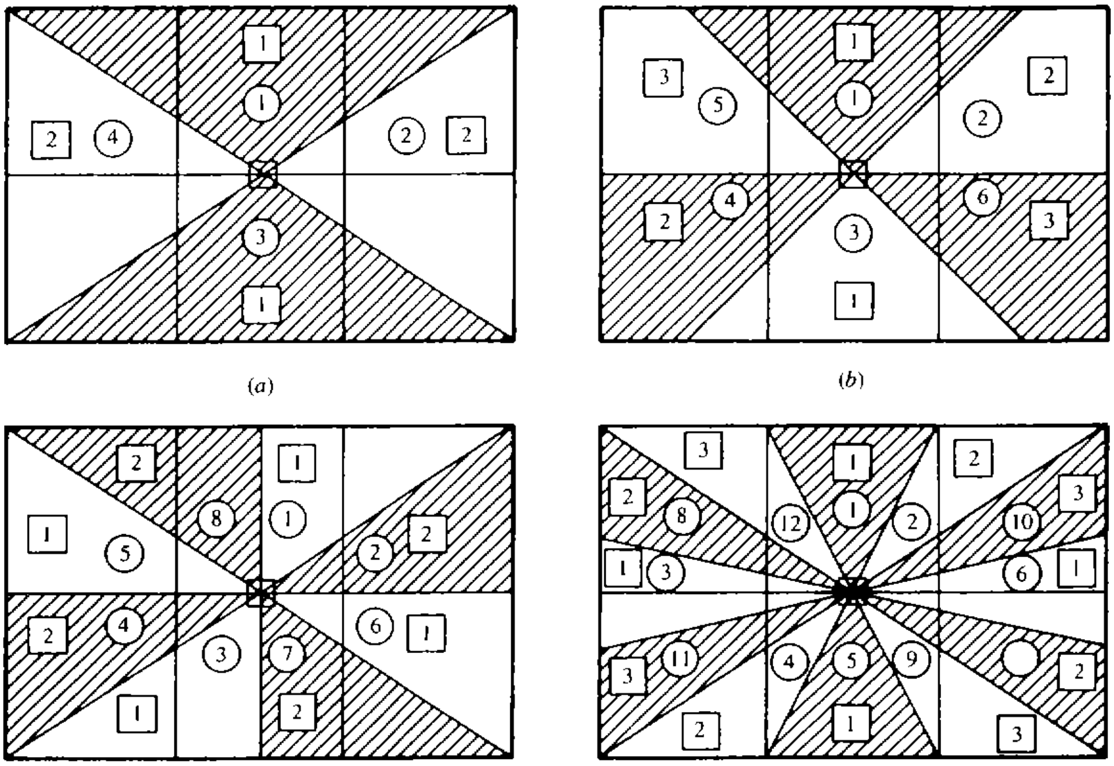


Figure 5-23

5.9 RADIAL-TYPE DEVELOPMENT

In addition to the rectangular-type development associated with overhead expansion, there is a second type of development that is due to the growth of underground residential distribution (URD) subdivisions with underground feeders serving local load as they exit into the adjacent service areas. At these locations the overhead feeders along the quarter section lines are replaced with underground cables, and as these underground lines extend outward from the substation, the area load is served. These underground lines extend through the



(a) (b) (c) (d)

Figure 5-24

platted service area developments and terminate usually on a remote overhead feeder along a section line. This type of development is called radial-type development, and it resembles a wagon wheel with the substation as the hub and the radial spokes as the feeders, as shown in Fig. 5-24.

5-10 RADIAL FEEDERS WITH UNIFORMLY DISTRIBUTED LOAD

The single-line diagram, shown in Fig. 5-25, illustrates a three-phase feeder main having the same construction, i.e., in terms of cable size or open-wire size and spacing, along its entire length l. Here, the line impedance is $z = r + jx$ per unit length.

The load flow in the main is assumed to be perfectly balanced and uniformly distributed at all locations along the main. In practice, a reasonably good phase balance sometimes is realized when single-phase and open-wye laterals are wisely distributed among the three phases of the main.

Assume that there are many closely spaced loads and/or lateral lines connected to the main but not shown in Fig. 5-25. Since the load is uniformly distributed along the main, as shown in Fig. 5-26, the load current in the main is a function of the distance. Therefore, in view of the many closely spaced small loads, a differential tapped-off load current $d\bar{I}$, which corresponds to a dx differential distance, is to be

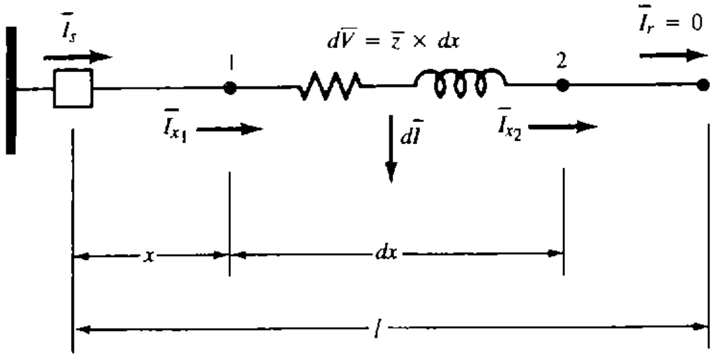


Figure 5-25

used as an idealization. Here, l is the total length of the feeder and x is the distance of the point 1 on the feeder from the beginning end of the feeder. Therefore the distance of point 2 on the feeder from the beginning end of the feeder is $x + dx$. I_s is the sending-end current at the feeder breaker, and I_r is the receiving-end current. I_{x1}, and I_{x2} are the currents in the main at points 1 and 2, respectively. Assume that all loads connected to the feeder have the same power factor.

The following equations are valid both in per unit or per phase (line-to-neutral) dimensional variables. The circuit voltage is of either primary or secondary, and therefore shunt capacitance currents may be neglected.

Since the total load is uniformly distributed from $x = 0$ to $x = l$,

$$\frac{d\bar{I}}{dx} = \bar{k} \tag{5-8}$$

which is a constant.

Therefore $\bar{I}_x$, that is, the current in the main of some x distance away from the circuit breaker, can be found as a function of the sending-end current $\bar{I}_s$ and the

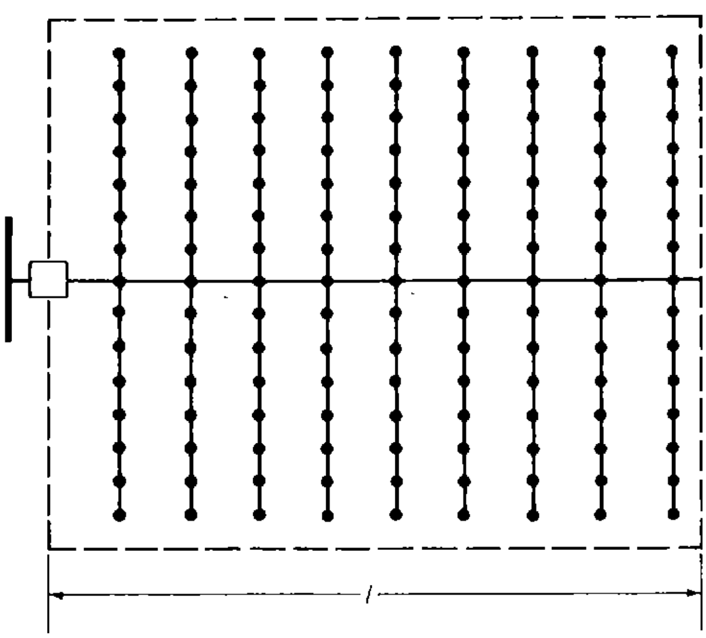


Figure 5-26

distance x. This can be accomplished either by inspection or by writing a current equation containing the integration of the $d\bar{I}$. Therefore, for the dx distance,

$$\bar{I}_{x1} = \bar{I}_{x2} + d\bar{I} \tag{5-9}$$

or

$$\bar{I}_{x2} = \bar{I}_{x1} - d\bar{I} \tag{5-10}$$

From Eq. (5-10),

$$\begin{aligned} \bar{I}_{x2} &= \bar{I}_{x1} - d\bar{I}\frac{dx}{dx} \\ &= \bar{I}_{x1} - \frac{d\bar{I}}{dx}\,dx \end{aligned} \tag{5-11}$$

or

$$\bar{I}_{x2} = \bar{I}_{x1} - \bar{k}\,dx \tag{5-12}$$

where

$$\bar{k} = \frac{d\bar{I}}{dx}$$

or, approximately,

$$I_{x2} = I_{x1} - k\,dx \tag{5-13}$$

and

$$I_{x1} = I_{x2} + k\,dx \tag{5-14}$$

Therefore, for the total feeder,

$$I_r = I_s - k \times l \tag{5-15}$$

and

$$I_s = I_r + k \times l \tag{5-16}$$

When $x = l$, from Eq. (5–15),

$$I_r = I_s - k \times l = 0$$

hence

$$k = \frac{I_s}{l} \tag{5-17}$$

and since $x = l$,

$$I_r = I_s - k \times x \tag{5-18}$$

Therefore, substituting Eq. (5-17) into Eq. (5-18),

$$I_r = I_s\left(1 - \frac{x}{l}\right) \tag{5-19}$$

For a given x distance,

$$I_x = I_r$$

thus Eq. (5-19) can be written as

$$I_x = I_s\left(1 - \frac{x}{l}\right) \tag{5-20}$$

which gives the current in the main at some x distance away from the circuit breaker. Note that from Eq. (5-20),

$$I_x = \begin{cases} I_r = 0 & \text{at } x = l \\ I_r = I_s & \text{at } x = 0 \end{cases}$$

The differential series voltage drop $d\bar{V}$ and the differential power loss dP_{LS} due to I^2R losses can also be found as a function of the sending-end current I_s and the distance x in a similar manner.

Therefore the differential series voltage drop can be found as

$$d\bar{V} = I_x \times z\,dx \tag{5-21}$$

or substituting Eq. (5-20) into Eq. (5-21),

$$d\bar{V} = I_s \times z\left(1 - \frac{x}{l}\right)dx \tag{5-22}$$

Also, the differential power loss can be found as

$$dP_{LS} = I_x^2 \times r\,dx \tag{5-23}$$

or substituting Eq. (5-20) into Eq. (5-23),

$$dP_{LS} = \left[I_s\left(1 - \frac{x}{l}\right)\right]^2 r\,dx \tag{5-24}$$

The series voltage drop VD_x due to I_x current at any point x on the feeder is

$$VD_x = \int_0^x dV \tag{5-25}$$

Substituting Eq. (5-22) into Eq. (5-25),

$$VD_x = \int_0 I_s \times z\left(1 - \frac{x}{l}\right)dx \tag{5-26}$$

or

$$VD_x = I_s \times z \times x\left(1 - \frac{x}{2l}\right) \tag{5-27}$$

Therefore, the total series voltage drop $\sum VD_x$ on the main feeder when $x = l$ is

$$\sum VD_x = I_s \times z \times x\left(1 - \frac{l}{2l}\right)$$

$$= \tfrac{1}{2}z \times l \times I_s \tag{5-28}$$

The total copper loss per phase in the main due to I^2R losses is

$$\sum P_{LS} = \int_0^l dP_{LS} \tag{5-29}$$

or

$$\sum P_{LS} = \tfrac{1}{3}I_s^2 \times r \times l \tag{5-30}$$

Therefore, from Eq. (5-28), the distance x from the beginning of the main feeder at which location the total load current I_s may be concentrated, i.e., lumped for the purpose of calculating the total voltage drop, is

$$x = \frac{l}{2}$$

whereas, from Eq. (5-30), the distance x from the beginning of the main feeder at which location the total load current I_s may be lumped for the purpose of calculating the total power loss is

$$x = \frac{l}{3}$$

5-11 RADIAL FEEDERS WITH NONUNIFORMLY DISTRIBUTED LOAD

The single-line diagram, shown in Fig. 5-27, illustrates a three-phase feeder main which has the tapped-off load increasing linearly with the distance x. Note that the load is zero when $x = 0$. The plot of the sending-end current vs. the x distance along the feeder main gives the curve shown in Fig. 5-28.

From Fig. 5-28, the negative slope can be written as

$$\frac{dI_x}{dx} = -k \times I_s \times x \tag{5-31}$$

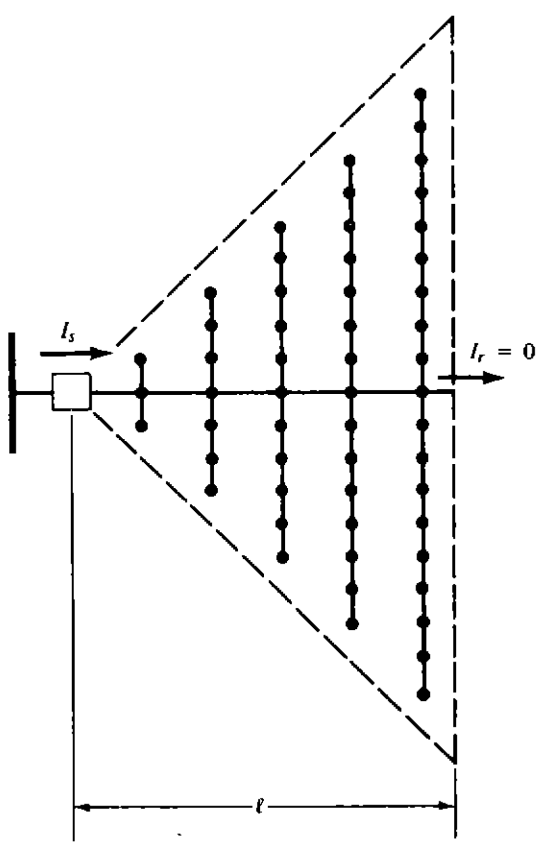


Figure 5-27

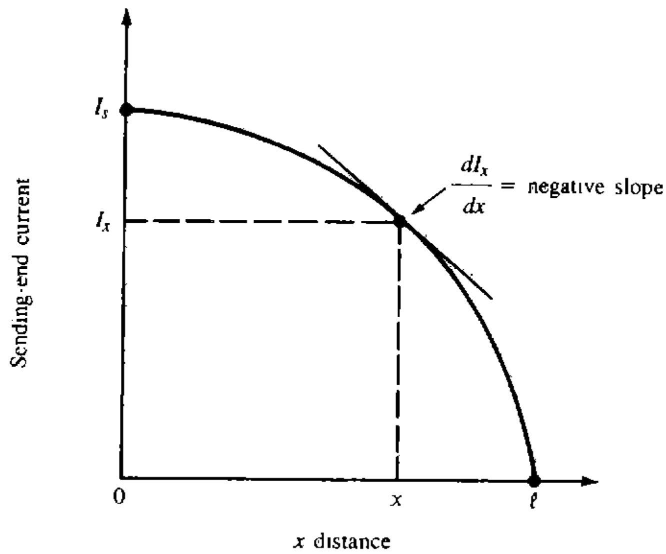


Figure 5-28

Here, the k constant can be found from

$$I_s = \int_{x=0}^{l} -dI_x$$

$$= \int_{x=0}^{l} k \times I_s \times x\,dx \tag{5-32}$$

or

$$I_s = k \times I_s \times \frac{l^2}{2} \tag{5-33}$$

From Eq. (5-33), the k constant is

$$k = \frac{2}{l^2} \tag{5-34}$$

Substituting Eq. (5-34) into Eq. (5-31),

$$\frac{dI_x}{dx} = -2I_s \times \frac{x}{l^2} \tag{5-35}$$

Therefore the current in the main at some x distance away from the circuit breaker can be found as

$$I_x = I_s\left(1 - \frac{x^2}{l^2}\right) \tag{5-36}$$

Hence the differential series voltage drop is

$$d\overline{V} = I_x \times z\,dx \tag{5-37}$$

or

$$d\overline{V} = I_s \times z\left(1 - \frac{x^2}{l^2}\right) dx \tag{5-38}$$

Also, the differential power loss can be found as

$$dP_{LS} = I_x^2 \times r\,dx \tag{5-39}$$

or

$$dP_{LS} = I_s^2 \times r\left(1 - \frac{x^2}{l^2}\right)^2 dx \tag{5-40}$$

The series voltage drop due to I_x current at any point x on the feeder is

$$\text{VD}_x = \int_0^x dV \tag{5-41}$$

Substituting Eq. (5-38) into Eq. (5-41) and integrating the result,

$$\text{VD}_x = I_s \times z \times x\left(1 - \frac{x^2}{3l^2}\right) \tag{5-42}$$

Therefore the total series voltage drop on the main feeder when $x = l$ is

$$\sum \text{VD}_x = \tfrac{2}{3} z \times l \times I_s \tag{5-43}$$

The total copper loss per phase in the main due to I^2R losses is

$$\sum P_{LS} = \int_0^l dP_{LS} \tag{5-44}$$

or

$$\sum P_{LS} = \tfrac{8}{15} I_s^2 \times r \times l \tag{5-45}$$

5-12 APPLICATION OF THE *A*, *B*, *C*, *D* GENERAL CIRCUIT CONSTANTS TO RADIAL FEEDERS

Assume a single-phase or balanced three-phase transmission or distribution circuit characterized by the $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ general circuit constants, as shown in Fig. 5-29. The mixed data assumed to be known, as commonly encountered in system design, are $|V_S|$, P_r, and $\cos\theta_r$. Assume that all data represent either per phase dimensional values or per unit values.

As shown in Fig. 5-30, taking phasor $\bar{V}_r$ as the reference,

$$\bar{V}_r = V_r \underline{/0^\circ} \tag{5-46}$$

$$\bar{V}_s = V_s \underline{/\delta^\circ} \tag{5-47}$$

$$\bar{I}_r = I_r \underline{/-\theta_r^\circ} \tag{5-48}$$

where $\bar{V}_r$ = receiving-end voltage phasor
$\bar{V}_s$ = sending-end voltage phasor
$\bar{I}_r$ = receiving-end current phasor

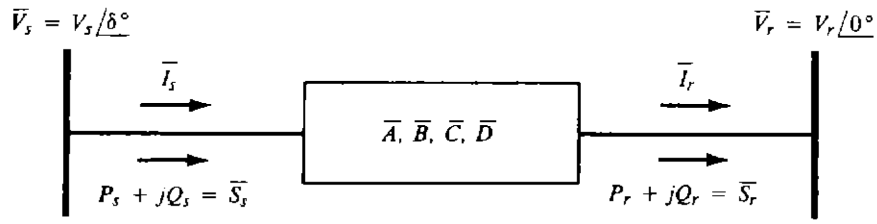


Figure 5-29

The sending-end voltage in terms of the general circuit constants can be expressed as

$$\bar{V}_s = \bar{A} \times \bar{V}_r + \bar{B} \times \bar{I}_r \tag{5-49}$$

where

$$\bar{A} = A_1 + jA_2 \tag{5-50}$$

$$\bar{B} = B_1 + jB_2 \tag{5-51}$$

$$\bar{I}_r = I_r(\cos\theta_r - j\sin\theta_r) \tag{5-52}$$

$$\bar{V}_r = V_r\underline{/0°} = V_r \tag{5-53}$$

$$\bar{V}_s = V_s(\cos\delta + j\sin\delta) \tag{5-54}$$

Therefore Eq. (5-49) can be written as

$$V_s \times \cos\delta + jV_s \times \sin\delta = (A_1 + jA_2)V_r + (B_1 + jB_2)(I_r \times \cos\theta_r - jI_r \times \sin\theta_r)$$

from which

$$V_s \times \cos\delta = A_1 \times V_r + B_1 \times I_r \times \cos\theta_r + B_2 \times I_r \times \sin\theta_r \tag{5-55}$$

and

$$V_s \times \sin\delta = A_2 \times V_r + B_2 \times I_r \times \cos\theta_r - B_1 \times I_r \times \sin\theta_r \tag{5-56}$$

By taking squares of Eq. (5-55) and (5-56), and adding them side by side,

$$\begin{aligned} V_s^2 = {} & (A_1 \times V_r + B_1 \times I_r \times \cos\theta_r + B_2 \times I_r \times \sin\theta_r)^2 \\ & + (A_2 \times V_r + B_2 \times I_r \times \cos\theta_r - B_1 \times I_r \times \sin\theta_r)^2 \end{aligned} \tag{5-57}$$

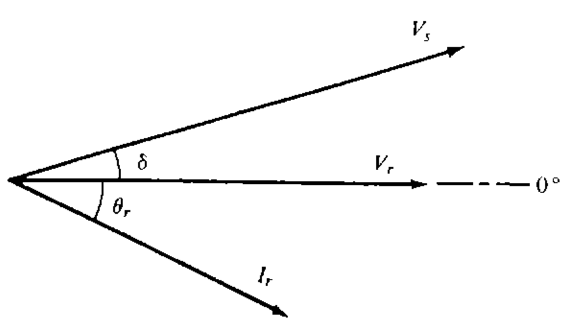


Figure 5-30

or

$$\begin{aligned} V_s^2 = {} & V_r^2(A_1^2 + A_2^2) + 2V_r \times I_r \times \cos\theta_r(A_1 \times B_1 + A_2 \times B_2) \\ & + B_1^2(V_r^2 \times \cos^2\theta_r + I_r^2 \times \sin^2\theta_r) \\ & + B_2^2(I_r^2 \times \sin^2\theta_r + I_r^2 \times \cos^2\theta_r) \\ & + 2V_r \times I_r \times \sin\theta_r(A_1 \times B_2 - B_1 \times A_2) \end{aligned} \tag{5-58}$$

Since

$$P_r = V_r \times I_r \times \cos\theta_r \tag{5-59}$$

$$Q_r = V_r \times I_r \times \sin\theta_r \tag{5-60}$$

and

$$Q_r = P_r \times \tan\theta_r \tag{5-61}$$

Eq. (5-58) can be rewritten as

$$\begin{aligned} & V_r^2(A_1^2 + A_2^2) + (B_1^2 + B_2^2)(1 + \tan^2\theta_r)\frac{P_r^2}{V_r^2} \\ & \quad = V_s^2 - 2P_r[(A_1 \times B_1 + A_2 \times B_2) + (A_1 \times B_2 - B_1 \times A_2)\tan\theta_r] \end{aligned} \tag{5-62}$$

Let

$$\hat{K} = V_s^2 - 2P_r[(A_1 \times B_1 + A_2 \times B_2) + (A_1 \times B_2 - B_1 \times A_2)\tan\theta_r] \tag{5-63}$$

then Eq. (8.65) becomes

$$(A_1^2 + A_2^2)V_r^2 + (B_1^2 + B_2^2)(1 + \tan^2\theta_r)\frac{P_r^2}{V_r^2} - \hat{K} = 0 \tag{5-64}$$

or

$$(A_1^2 + A_2^2)V_r^2 + (B_1^2 + B_2^2)(\sec^2\theta_r)\frac{P_r^2}{V_r^2} - \hat{K} = 0 \tag{5-65}$$

Therefore, from Eq. (8.67), the receiving-end voltage can be found as

$$V_r = \left\{\frac{\hat{K} \pm [\hat{K}^2 - 4(A_1^2 + A_2^2)(B_1^2 + B_2^2) \times P_r^2 \times \sec^2\theta_r]^{1/2}}{2(A_1^2 + A_2^2)}\right\}^{1/2} \tag{5-66}$$

Also, from Eq. (5-55) and (5-56),

$$V_s \times \sin\delta = A_2 \times V_r + B_2 \times I_r \times \cos\theta_r - B_1 \times I_r \times \sin\theta_r$$

and

$$V_s \times \cos\delta = A_1 \times V_r + B_1 \times I_r \times \cos\theta_r + B_2 \times I_r \times \sin\theta_r$$

where

$$I = \frac{P_r}{V_r \times \cos\theta_r} \tag{5-67}$$

Therefore

$$V_s \times \sin\delta = A_2 \times V_r + \frac{B_2 \times P_r}{V_r} - \frac{B_1 \times P_r}{V_r}\tan\theta_r \tag{5-68}$$

and

$$V_s \times \cos \delta = A_1 \times V_r + \frac{B_1 \times P_r}{V_r} + \frac{B_2 \times P_r}{V_r} \tan \theta_r \tag{5-69}$$

By dividing Eq. (5-68) by Eq. (5-69),

$$\tan \delta = \frac{A_2 \times V_r^2 + B_2 \times P_r - B_1 \times P_r \times \tan \theta_r}{A_1 \times V_r^2 + P_r \times B_1 + B_2 \times P_r \times \tan \theta_r} \tag{5-70}$$

or

$$\tan \delta = \frac{A_2 \times V_r^2 + P_r(B_2 - B_1 \times \tan \theta_r)}{A_1 \times V_r^2 + P_r(B_1 + B_2 \times \tan \theta_r)} \tag{5-71}$$

Equations (5-66) and (5-71) are found for a general transmission system. They could be adapted to the simpler transmission consisting of a short primary-voltage feeder where the feeder capacitance is usually negligible, as shown in Fig. 5-31.

To achieve the adaptation, Eq. (5-63), (5-66), and (5-71) can be written in terms of R and X. Therefore, for the feeder shown in Fig. 5-31,

$$[\bar{I}] = [\bar{Y}][\bar{V}] \tag{5-72}$$

or

$$\begin{bmatrix} \bar{I}_s \\ \bar{I}_r \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix} \begin{bmatrix} \bar{V}_s \\ \bar{V}_r \end{bmatrix} \tag{5-73}$$

where

$$\bar{Y}_{11} = \frac{1}{\bar{Z}} \tag{5-74}$$

$$\bar{Y}_{21} = \bar{Y}_{12} - \frac{1}{\bar{Z}} \tag{5-75}$$

$$\bar{Y}_{22} = \frac{1}{\bar{Z}} \tag{5-76}$$

Therefore

$$\bar{A}_1 = -\frac{\bar{Y}_{22}}{\bar{Y}_{21}} = 1 \tag{5-77}$$

or

$$A_1 + jA_2 = 1 \tag{5-78}$$

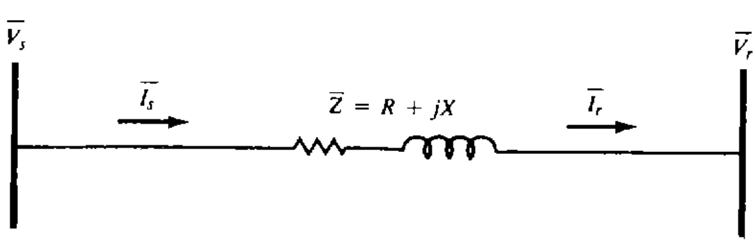


Figure 5-31

where

$$A_1 = 1 \tag{5-79}$$

and

$$A_2 = 0 \tag{5-80}$$

Similarly,

$$\bar{B}_1 = -\frac{1}{\bar{Y}_{21}} = \bar{Z} \tag{5-81}$$

or

$$B_1 + jB_2 = R + jX \tag{5-82}$$

where

$$B_1 = R \tag{5-83}$$

and

$$B_2 = X \tag{5-84}$$

Substituting Eq. (5-79), (5-80), (5-83), and (5-84) into Eq. (5-66),

$$V_r = \left\{\frac{\hat{K} \pm [\hat{K}^2 - 4(R^2 + X^2)P_r^2 \times \sec^2\theta]^{1/2}}{2}\right\}^{1/2} \tag{5-85}$$

or

$$V_r = \left(\frac{\hat{K}}{2}\left\{1 \pm \left[1 - \frac{4(R^2 + X^2)P_r^2}{\hat{K}^2 \times \cos^2\theta_r}\right]^{1/2}\right\}\right)^{1/2} \tag{5-86}$$

or

$$V_r = \left(\frac{\hat{K}}{2}\left\{1 \pm \left[1 - \left(\frac{2 \times Z \times P_r}{\hat{K} \times \cos\theta_r}\right)^2\right]^{1/2}\right\}\right)^{1/2} \tag{5-87}$$

where

$$\hat{K} = V_s^2 - 2 \times P_r(R + X \times \tan\theta_r) \tag{5-88}$$

Also, from Eq. (5-71),

$$\tan\delta = \frac{P_r(X - R \times \tan\theta_r)}{V_r^2 + P_r(R + X \times \tan\theta_r)} \tag{5-89}$$

Example 5-1 Assume that the radial express feeder, shown in Fig. 5-31, is used on rural distribution and is connected to a lumped-sum (or concentrated) load at the receiving end. Assume that the feeder impedance is $0.10 + j0.10$ pu, the sending-end voltage is 1.0 pu, P_r is 1.0 pu constant power load, and the power factor at the receiving end is 0.80 lagging. Use the given data and the exact equations for K, V_r, and $\tan\delta$ given previously and determine the following:

(*a*) Compute V_r and δ by using the exact equations and find also the corresponding values of the I_r and I_s currents.

(*b*) Verify the numerical results found in part *a* by using those results in

$$\bar{V}_s = \bar{V}_r + (R + jX)\bar{I}_r \tag{5-90}$$

SOLUTION

(*a*) From Eq. (5-88),

$$\begin{aligned}\hat{K} &= V_s^2 - 2 \times P_r(R + X \times \tan\theta_r)\\ &= 1.0^2 - 2 \times 1[0.10 + 0.1 \times \tan(\cos^{-1} 0.80)]\\ &= 0.65 \text{ pu}\end{aligned}$$

From Eq. (5-87),

$$\begin{aligned}V_r &= \left(\frac{\hat{K}}{2}\left\{1 \pm \left[1 - \left(\frac{2 \times Z \times P_r}{\hat{K} \times \cos\theta_r}\right)^2\right]^{1/2}\right\}\right)^{1/2}\\ &= \left(\frac{0.65}{2}\left\{1 \pm \left[1 - \left(\frac{2 \times 0.141 \times 1.0}{0.65 \times 0.8}\right)^2\right]^{1/2}\right\}\right)^{1/2}\\ &= 0.7731 \text{ pu}\end{aligned}$$

From Eq. (5-89),

$$\begin{aligned}\tan\delta &= \frac{P_r(X - R \times \tan\theta_r)}{V_r^2 + P_r(R + X \times \tan\theta_r)}\\ &= \frac{1.0[0.10 - 0.10 \times \tan(\cos^{-1} 0.80)]}{0.7731^2 + 1.0[0.10 + 0.10 \times \tan(\cos^{-1} 0.80)]}\\ &= 0.0323\end{aligned}$$

therefore

$$\delta \cong 1.85^\circ$$

$$\begin{aligned}\bar{I}_r = \bar{I}_s &= \frac{P_r}{V \times \cos\theta_r}\underline{/-\theta_r}\\ &= \frac{1.0}{0.7731 \times 0.80}\underline{/-36.8^\circ}\\ &= 1.617\underline{/-36.8^\circ} \text{ pu}\end{aligned}$$

(*b*) From the given equation,

$$\begin{aligned}\bar{V}_r &= \bar{V}_s - (R + jX)\bar{I}_r\\ &= 1.0\underline{/1.85^\circ} - (0.10 + j0.10)(1.617\underline{/-36.8^\circ})\\ &\cong 0.7731\underline{/0^\circ} \text{ pu}\end{aligned}$$

5-13 THE DESIGN OF RADIAL PRIMARY DISTRIBUTION SYSTEMS

The radial primary distribution systems are designed in several different ways: (1) overhead primaries with overhead laterals or (2) underground residential distribution (URD), e.g., with mixed distribution of overhead primaries and underground laterals.

5-13-1 Overhead Primaries

For the sake of illustration, Fig. 5-32 shows an arrangement for overhead distribution which includes a main feeder and 10 laterals connected to the main with sectionalizing fuses. Assume that the distribution substation, shown in the figure, is arbitrarily located; it may also serve a second area, which is not shown in the figure, that is equal to the area being considered and, for example, located "below" the shown substation site.

Here, the feeder mains are three-phase and of 10 short blocks length or less. The laterals, on the other hand, are all of six long blocks length and are protected with sectionalizing fuses. In general, the laterals may be either single-phase, open wye-grounded, or three-phase.

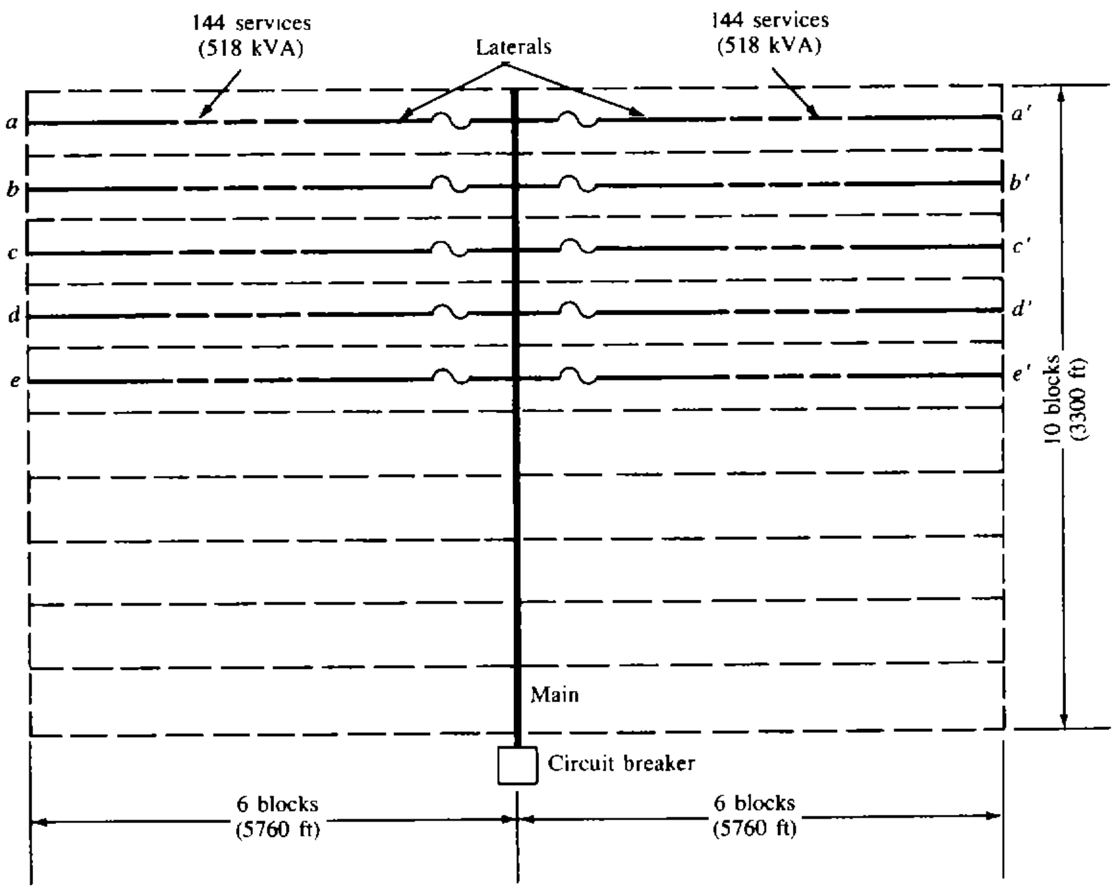


Figure 5-32 An overhead radial distribution system.

Here, in the event of a permanent fault on a lateral line, only a relatively small fraction of the total area is outaged. Ordinarily permanent faults on the overhead line can be found and repaired quickly.

5-13-2 Underground Residential Distribution

Even though an underground residential distribution costs somewhere between 1.25 to 10 times more than a comparable overhead system, due to its certain advantages it is used commonly [4, 5]. Among the advantages of the underground system are:

1. The lack of outages caused by the abnormal weather conditions such as ice, sleet, snow, severe rain and storms, and lightning
2. The lack of outages caused by accidents, fires, and foreign objects
3. The lack of tree trimming and other preventative maintenance tasks
4. The aesthetic improvement

For the sake of illustration, Fig. 5-33 shows an underground residential distribution for a typical overhead and underground primary distribution system of the two-way feed type. The two arbitrarily located substations are assumed to be supplied from the same subtransmission line, which is not shown in the figure, so that the low-voltage buses of the two substations are nominally in phase. In the figure, the two overhead primary-feeder mains carry the total load of the area being considered, i.e., the area of the 12 block by 10 block. Of course, the other two overhead feeder mains carry the other equally large area. Therefore, in this example, each area has 120 blocks.

The laterals, in residential areas, typically are single-phase and consist of directly buried (rather than located in ducts) concentric neutral-type PE-insulated cable. Such cable usually insulated for 15-kV line-to-line solidly grounded neutral service and the commonly used single-phase line-to-neutral operating voltages are nominally 7200 or 7620 V.

The distribution transformers now often used are of the *pad-mounted* or *submersible* type. The pad-mounted distribution transformers are completely enclosed in strong, locked sheet metal enclosures and mounted on grade on a concrete slab. The submersible-type distribution transformers are placed in a cylindrical excavation that is lined with a concrete, bituminized fiber or corrugated sheet metal tube. The tubular liner is secured after near-grade level with a locked cover.

Ordinarily each lateral line is operated normally open (N.O.) at or near the center as Fig. 5-33 suggests. An excessive amount of time may be required to locate and repair a fault in a directly buried primary cable. Therefore it is desirable to provide switching so that any one run of primary cable can be deenergized for cable repair or replacement while still maintaining service to all (or nearly all) distribution transformers.

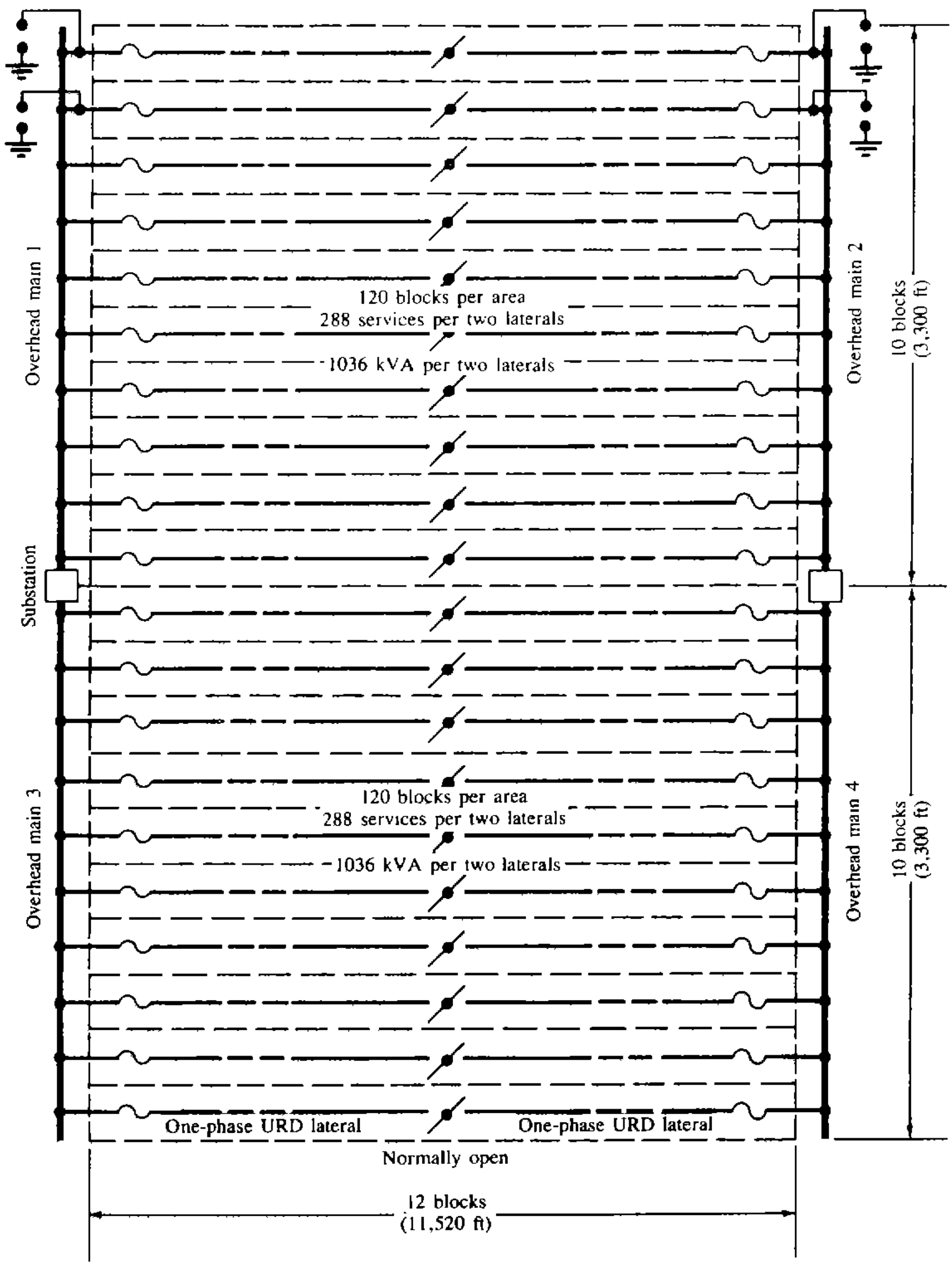


Figure 5-33 A two-way feed-type underground residential distribution system.

Figure 5-34 shows apparatus, suggested by Lokay [2], that is or has been used to accomplish the desired switching or sectionalizing. The figure shows a single-line diagram of loop-type primary-feeder circuit for a low-cost underground distribution system in residential areas. Figure 5-34*a* shows it with a disconnect switch at each transformer, whereas Fig. 5-34*b* shows the similar setup without a disconnect switch at each transformer. In Fig. 5-34*a*, if the cable "above" *C* is faulted, the switch at *C* and the switch or cutout "above" *C* are opened, and, at the same time, the sectionalizing switch at B is closed. Therefore, the faulted cable above C and the distribution transformer at C are then out of service.

Figure 5-35 shows a distribution transformer with internal high-voltage fuse and with stick-operated plug-in type of high-voltage load-break connectors. Some of the commonly used plug-in types of load-break connector ratings include 8.66-kV line-to-neutral, 200-A continuous 200-A load break, and 10,000-A symmetrical fault close-in rating.

Figure 5-36 shows a distribution transformer with internal high-voltage fuse and with stick-operated high-voltage load-break switches that can be used in Fig. 5-34*a* to allow four modes of operation, namely:

1. The transformer is energized and the loop is closed.
2. The transformer is energized and the loop is open to the right.
3. The transformer is energized and the loop is open to the left.
4. The transformer is deenergized and the loop is open.

In Fig 5-33, note that, in case of trouble, the open may be located near one of the underground feed points. Therefore, at least in this illustrative design, the single-phase underground cables should be at least ampacity-sized for the load of 12 blocks, not merely six blocks.

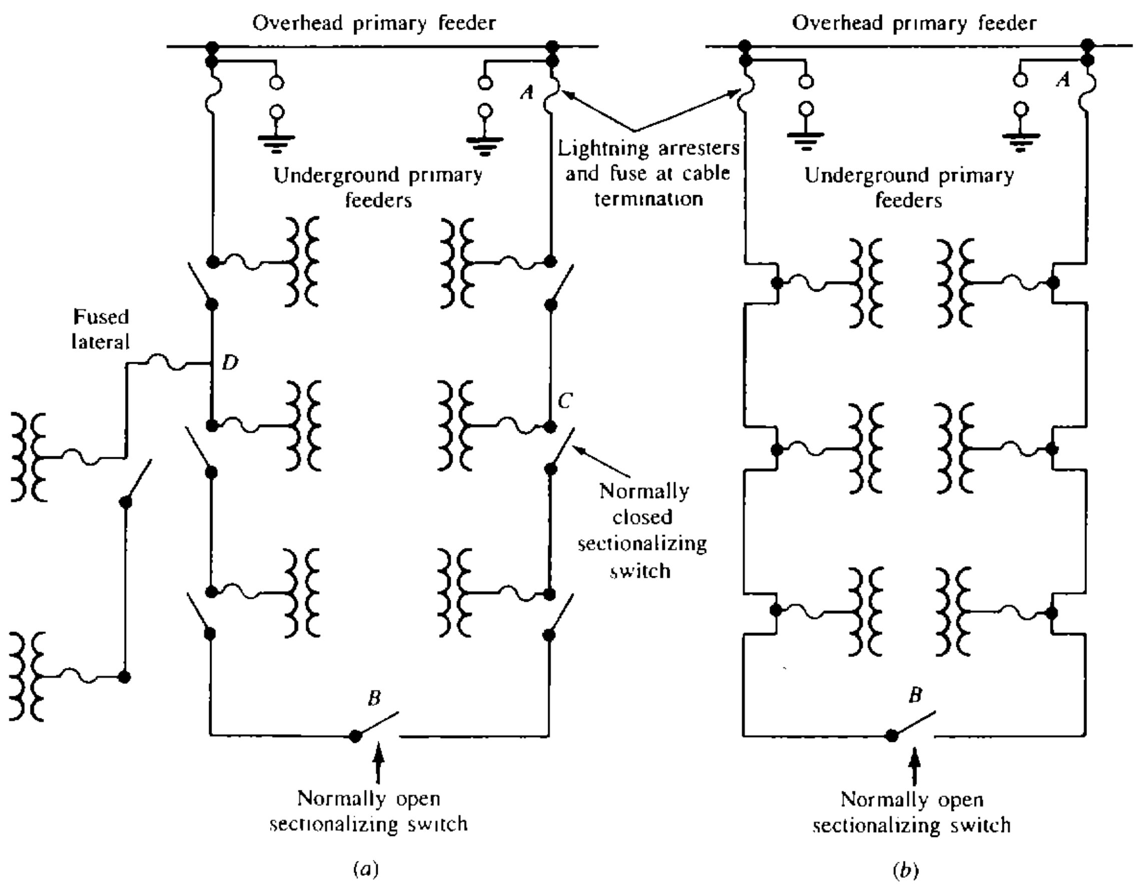


Figure 5-34 Single-line diagram of loop-type primary-feeder circuits: (*a*) with a disconnect switch at each transformer and (*b*) without a disconnect switch at each transformer. (*From* [2].)

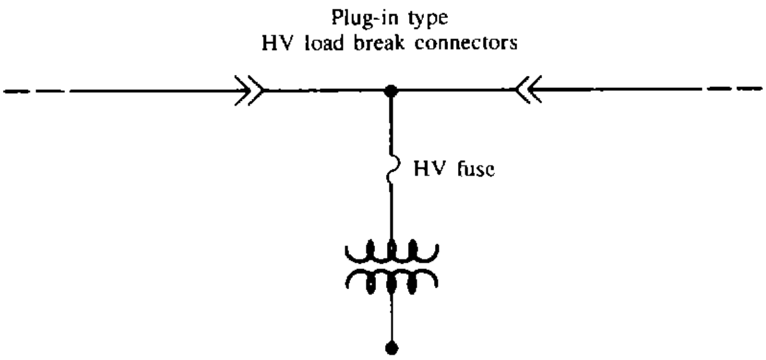


Figure 5-35 A distribution transformer with internal high-voltage fuse and load-break connectors.

In Fig. 5-33, note further the difficulty in providing abundant overvoltage protection to cable and distribution transformers by placing lightning arresters at the open cable ends. The location of the open *moves* because of switching, whether for repair purposes or for load balancing.

Example 5-2 Consider the layout of the area and the annual peak demands shown in Fig. 5-32. Note that the peak demand per lateral is found as

$$144 \text{ customers} \times 3.6 \text{ kVA/customer} \cong 518 \text{ kVA}$$

Assume a lagging-load power factor of 0.90 at all locations in all primary circuits at the time of the annual peak load. For purposes of computing voltage drop in mains and in three-phase laterals, assume that the single-phase load is perfectly balanced among the three phases. Idealize the voltage-drop calculations further by assuming uniformly distributed load along all laterals. Assume nominal operating voltage when computing current from the kilovoltampere load.

For the open-wire overhead copper lines, compute the percent voltage drops, using the precalculated percent voltage drop per kilovoltampere-mile curves given in Chap. 4. Note that $D_m = 37$ in is assumed.

The joint EEI-NEMA report [6] defines *favorable* voltages at the point of utilization, inside the buildings, to be from 110 to 125 V. Here, for illustrative purposes, the lower limit is arbitrarily raised to 116 V at the meter, i.e., at the end of the service-drop cable. This allowance may compensate for additional voltage drops, not calculated, due to:

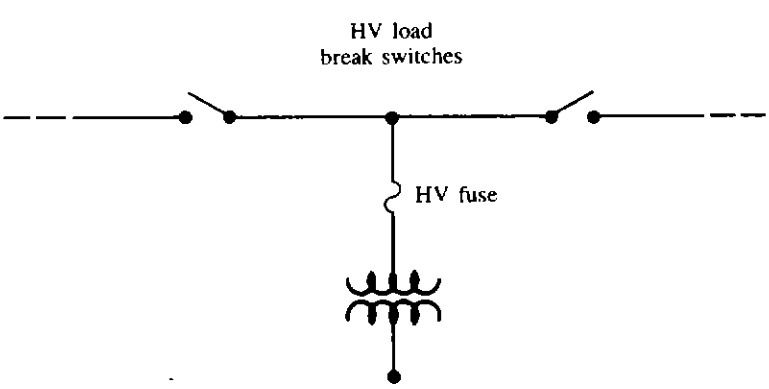


Figure 5-36 A distribution transformer with internal high-voltage fuses and load-break switches.

1. Unbalanced loading in three-wire single-phase secondaries
2. Unbalanced loading in four-wire three-phase primaries
3. Load growth
4. Voltage drops in building wiring

Therefore the voltage criteria that are to be used in this problem are

$$V_{\max} = 125 \text{ V} = 1.0417 \text{ pu}$$

and

$$V_{\min} = 116 \text{ V} = 0.9667 \text{ pu}$$

at the meter. The maximum voltage drop, from the low-voltage bus of the distribution substation to the most remote meter, is 7.50 percent. It is assumed that a 3.5 percent maximum steady-state voltage drop in the secondary distribution system is reasonably achievable. Therefore the maximum allowable primary voltage drop for this problem is limited to 4.0 percent.

Assume open-wire overhead primaries with three-phase four-wire laterals, and that the nominal voltage is used as the base voltage and is equal to 2400/4160 V for the three-phase four-wire grounded-wye primary system with copper conductors and $D_m = 37$ in. Consider the "longest" primary circuit, consisting of a 3300-ft main and the two most remote laterals, like the laterals a and a' of Fig. 5-32. Use ampacity-sized conductors but in no case smaller than AWG #6 for reasons of mechanical strength. Determine the following:

(a) The percent voltage drops at the ends of the laterals and the main.

(b) If the 4 percent maximum voltage-drop criterion is exceeded, find a reasonable combination of larger conductors for main and for lateral that will meet the voltage-drop criterion.

SOLUTION

(a) Figure 5-37 shows the "longest" primary circuit, consisting of the 3300-ft main and the most remote laterals a and a'. In Fig. 5-37 the signs / / / /indicate that there are three-phase and one neutral conductors in that portion of the one-line diagram. The current in the lateral is

$$I_{\text{lateral}} = \frac{S_l}{\sqrt{3} \times V_{L\text{-}L}} \tag{5-91}$$

$$= \frac{518}{\sqrt{3} \times 4.16} \cong 72 \text{ A}$$

Thus, from Table A-1, AWG #6 copper conductor with 130-A ampacity is selected for the laterals. The current in the main is

$$I_{\text{main}} = \frac{S_m}{\sqrt{3} \times V_{L\text{-}L}} \tag{5-92}$$

$$= \frac{1036}{\sqrt{3} \times 4.16} \cong 144 \text{ A}$$

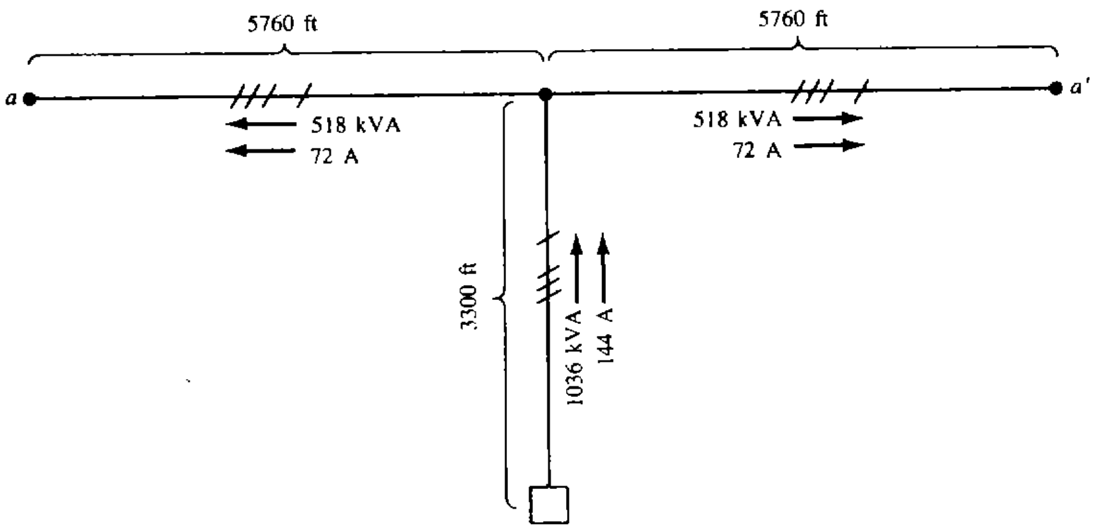


Figure 5-37

Hence, from Table A-1, AWG #4 copper conductor with 180-A ampacity is selected for the mains. Here, note that the AWG #5 copper conductors with 150-A ampacity is not selected due to the resultant too-high total voltage drop.

From Fig. 4-17, the K constants for the AWG #6 laterals and the AWG #4 mains can be found as 0.015 and 0.01, respectively. Therefore, since the load is assumed to be uniformly distributed along the lateral,

$$\% \, \mathrm{VD}_{\text{lateral}} = \frac{l}{2} \times K \times S \tag{5-93}$$

$$= \frac{1}{2} \times \frac{5760 \text{ ft}}{5280 \text{ ft/mi}} \times 0.015 \times 518 \text{ kVA}$$

$$= 4.24$$

and since the main is considered to have a lumped-sum load of 1036 kVA at the end of its length,

$$\% \, \mathrm{VD}_{\text{main}} = l \times K \times S \tag{5-94}$$

$$= \frac{3300 \text{ ft}}{5280 \text{ ft/mi}} \times 0.01 \times 1036 \text{ kVA}$$

$$= 6.48$$

Therefore the total percent primary voltage drop is

$$\sum \% \, \mathrm{VD} = \% \, \mathrm{VD}_{\text{main}} + \% \, \mathrm{VD}_{\text{lateral}} \tag{5-95}$$

$$= 6.48 + 4.24$$

$$= 10.72$$

which exceeds the maximum primary voltage-drop criterion of 4.00 percent.

Here, note that if single-phase laterals were used instead of the three-phase laterals, according to Morrison [7] the percent voltage drop of a single-phase circuit is approximately four times that for a three-phase circuit, assuming the use of the same-size conductors. Hence, for the laterals,

$$\begin{aligned} \sum \% \mathrm{VD}_{1\phi} &= 4(\% \mathrm{VD}_{3\phi}) \\ &= 4 \times 4.2 \\ &\cong 16.96 \end{aligned} \tag{5-96}$$

Therefore, from Eq. (5-95), the new total percent voltage drop would be

$$\begin{aligned} \sum \% \mathrm{VD} &= \% \mathrm{VD}_{\text{main}} + \% \mathrm{VD}_{\text{lateral}} \\ &= 6.48 + 16.96 \\ &= 23.44 \end{aligned}$$

which would be far exceeding the maximum primary voltage-drop criterion of 4.00 percent.

(*b*) Therefore, to meet the maximum primary voltage-drop criterion of 4.00 percent, from Table A-1 select 4/0 and AWG #1 copper conductors with ampacities of 480 A and 270 A for the main and laterals, respectively. Hence, from Eq. (5-93),

$$\begin{aligned} \% \mathrm{VD}_{\text{lateral}} &= \frac{l}{2} \times K \times S \\ &= \frac{1}{2} \times \frac{5760 \text{ ft}}{5280 \text{ ft/mi}} \times 0.006 \times 518 \text{ kVA} \\ &\cong 1.695 \end{aligned}$$

and from Eq. (5-94),

$$\begin{aligned} \% \mathrm{VD}_{\text{main}} &= l \times K \times S \\ &= \frac{3300 \text{ ft}}{5280 \text{ ft/mi}} \times 0.003 \times 1036 \text{ kVA} \\ &\cong 1.943 \end{aligned}$$

Therefore, from Eq. (5-95),

$$\begin{aligned} \sum \% \mathrm{VD} &= \% \mathrm{VD}_{\text{main}} + \% \mathrm{VD}_{\text{lateral}} \\ &= 1.943 + 1.695 \\ &= 3.638 \end{aligned}$$

which meets the maximum primary voltage-drop criterion of 4.00 percent.

Example 5-3 Repeat Example 5-2 but assume that, instead of the open-wire overhead primary system, a self-supporting aerial messenger cable with aluminum conductors is being used. This is to be considered one step toward the improvement of the aesthetics of the overhead primary system, since, in general, very few crossarms are required.

For the voltage-drop calculations in the self-supporting aerial messenger cable, use Table A-23 for its resistance and reactance values. For ampacities, use Table 5-2 which gives data for cross-linked polyethylene (XLPE)-insulated aluminum conductor, grounded neutral + 3/0 aerial cables. These ampacities are based on 40°C ambient and 90°C conductor temperatures and are taken from the General Electric Company's Publication No. PD-16.

SOLUTION

(*a*) The voltage drop, due to the uniformly distributed load, at the lateral is

$$\mathrm{VD}_{\text{lateral}} = I(r \times \cos\theta + x_L \times \sin\theta)\frac{l}{2} \quad \mathrm{V} \tag{5-97}$$

where $I = 72$ A, from Example 5-2
$r = 4.13\ \Omega/\text{mi}$, for AWG #6 aluminum conductors from Table A-23
$x_L = 0.258\ \Omega/\text{mi}$, for AWG #6 aluminum conductors from Table A-23
$\cos\theta = 0.90$
$\sin\theta = 0.436$

Therefore

$$\mathrm{VD}_{\text{lateral}} = 72(4.13 \times 0.9 + 0.258 \times 0.436)\frac{5760\ \text{ft}}{5280\ \text{ft/mi}} \times \frac{1}{2}$$

$$= 150.4\ \mathrm{V}$$

Table 5-2 Current-carrying capacity of XLPE aerial cables

Conductor size	Ampacity, A	
	5-kV cable	15-kV cable
6 AWG	75	
4 AWG	99	
2 AWG	130	135
1 AWG	151	155
1/0 AWG	174	178
2/0 AWG	201	205
3/0 AWG	231	237
4/0 AWG	268	273
250 kcmil	297	302
350 kcmil	368	372
500 kcmil	459	462

or, in percent,

$$\% \text{ VD}_{\text{lateral}} = \frac{150.4 \text{ V}}{2400 \text{ V}}$$

$$= 6.27$$

The voltage drop due to the lumped-sum load at the end of main is

$$\text{VD}_{\text{main}} = I(r \times \cos\theta + x_L \times \sin\theta)l \qquad \text{V} \tag{5-98}$$

where I = 144 A, from Example 5-2
r = 1.29 Ω/mi, for AWG #1 aluminum conductors from Table A-23
x_L = 0.211 Ω/mi, for AWG #1 aluminum conductors from Table A-23

Therefore

$$\text{VD}_{\text{main}} = 144(1.29 \times 0.9 + 0.211 \times 0.436)\frac{3300 \text{ ft}}{5280 \text{ ft/mi}}$$

$$\cong 112.8 \text{ V}$$

or, in percent,

$$\% \text{ VD}_{\text{main}} = \frac{112.8 \text{ V}}{2400 \text{ V}}$$

$$= 4.7$$

Thus, from Eq. (5-95), the total percent primary voltage drop is

$$\sum \% \text{ VD} = \% \text{ VD}_{\text{main}} + \% \text{ VD}_{\text{lateral}}$$
$$= 4.7 + 6.27$$
$$= 10.97$$

which far exceeds the maximum primary voltage-drop criterion of 4.00 percent.

(*b*) Therefore, to meet the maximum primary voltage-drop criterion of 4.00 percent, from Tables 5-2 and A-23, select 4/0 and 1/0 aluminum conductors with ampacities of 268 A and 174 A for the main and laterals, respectively. Hence, from Eq. (5-97),

$$\text{VD}_{\text{lateral}} = 72(1.03 \times 0.9 + 0.207 \times 0.436)\frac{5760 \text{ ft}}{5280 \text{ ft/mi}} \times \frac{1}{2}$$

$$= 39.95 \text{ V}$$

or, in percent,

$$\% \text{ VD}_{\text{lateral}} = \frac{39.95 \text{ V}}{2400 \text{ V}}$$

$$= 1.66$$

From Eq. (5-98),

$$\mathrm{VD}_{\text{main}} = 144(0.518 \times 0.9 + 0.191 \times 0.436)\frac{3300\ \text{ft}}{5280\ \text{ft/mi}}$$

$$= 49.45\ \text{V}$$

or, in percent,

$$\%\ \mathrm{VD}_{\text{main}} = \frac{49.45\ \text{V}}{2400\ \text{V}}$$

$$= 2.06$$

Thus, from Eq. (5-95), the total percent primary voltage drop is

$$\sum \%\ \mathrm{VD} = 2.06 + 1.66$$

$$= 3.72$$

which meets the maximum primary voltage-drop criterion of 4.00 percent.

Example 5-4 Repeat Example 5-2 but assume that the nominal operating voltage is used as the base voltage and is equal to 7200/12,470 V for the three-phase four-wire grounded-wye primary system with copper conductors. Use $D_m = 37$ in although $D_m = 53$ in is more realistic for this voltage class. This simplification allows the use of the precalculated percent voltage drop per kilovoltampere-mile curves given in Chap. 4.

Consider serving the $12 \times 10 = 120$-block area, shown in Fig. 5-32, with two feeder mains so that the longest of the two feeders would consist of a 3300-ft main and 10 laterals, i.e., the laterals a through e and the laterals a' through e'. Use ampacity-sized conductors, but not smaller than AWG #6, and determine the following:

(*a*) Repeat part *a* of Example 5-2.
(*b*) Repeat part *b* of Example 5-2.
(*c*) The deliberate use of too-small D_m leads to small errors in what and why?

SOLUTION

(*a*) The assumed load on the longer feeder is

$$518\ \text{kVA/lateral} \times 10\ \text{laterals/feeder} = 5180\ \text{kVA}$$

Therefore the current in the main is

$$I_{\text{main}} = \frac{5180\ \text{kVA}}{\sqrt{3} \times 12.47\ \text{kV}}$$

$$= 240.1\ \text{A}$$

Thus, from Table A-1, AWG #2, three-strand copper conductor, is selected for the mains. The current in the lateral is

$$I_{\text{lateral}} = \frac{518 \text{ kVA}}{\sqrt{3} \times 12.47 \text{ kV}}$$

$$= 24.01 \text{ A}$$

Hence, from Table A-1, AWG #6 copper conductor is selected for the laterals.

From Fig. 4-17, the K constants for the AWG #6 laterals and the AWG #2 mains can be found as 0.00175 and 0.0008, respectively. Therefore, since the load is assumed to be uniformly distributed along the lateral, from Eq. (5-93),

$$\%\text{VD}_{\text{lateral}} = \frac{l}{2} \times K \times S$$

$$= \frac{1}{2} \times \frac{5760 \text{ ft}}{5280 \text{ ft/mi}} \times 0.00175 \times 518 \text{ kVA}$$

$$= 0.50$$

and since, due to the peculiarity of this new problem, one-half of the main has to be considered as an express feeder and the other half is connected to a uniformly distributed load of 5180 kVA,

$$\%\text{ VD}_{\text{main}} = \tfrac{3}{4} \times l \times K \times S \tag{5-99}$$

$$= \frac{3}{4} \times \frac{3300 \text{ ft}}{5280 \text{ ft/mi}} \times 0.0008 \times 5180 \text{ kVA}$$

$$= 1.94$$

Therefore, from Eq. (5-95), the total percent primary voltage drop is

$$\sum \% \text{ VD} = 1.94 + 0.50$$

$$= 2.44$$

(*b*) It meets the maximum primary voltage-drop criterion of 4.00 percent.

(*c*) Since the inductive reactance of the line is

$$x_L = 0.1213 \times \ln \frac{1}{D_s} + 0.1213 \times \ln D_m \qquad \Omega/\text{mi}$$

or

$$x_L = x_a + x_d \qquad \Omega/\text{mi}$$

when $D_m = 37$ in,

$$x_d = 0.1213 \times \ln \frac{37 \text{ in}}{12 \text{ in/ft}}$$

$$= 0.1366 \ \Omega/\text{mi}$$

and when $D_m = 53$ in,

$$x_d = 0.1213 \times \ln \frac{53 \text{ in}}{12 \text{ in/ft}}$$

$$= 0.1802 \ \Omega/\text{mi}$$

Hence, there is a difference of

$$\Delta x_d = 0.0436 \ \Omega/\text{mi}$$

which calculates a smaller voltage-drop value than it really is.

Example 5-5 Consider the layout of the area and the annual peak demands shown in Fig. 5-33. The primary distribution system in the figure is a mixed system with overhead mains and underground residential distribution (URD) system. Assume that open-wire overhead mains are used with 7200/12,470-V three-phase four-wire grounded-wye ACSR conductors and that $D_m = 53$ in. Also assume that concentric neutral XLPE-insulated underground cable with aluminum conductors is used for single-phase and 7200-V underground cable laterals.

For voltage-drop calculations and ampacity of concentric neutral XLPE-insulated URD cable with aluminum conductors, use Table 5-3.

The foregoing data are for a currently used 15-kV solidly grounded-neutral class of cable construction consisting of (1) Al phase conductor, (2) extruded semiconducting conductor shield, (3) 175 mils thickness of cross-linked PE insulation, (4) extruded semiconducting sheath and insulation shield, and (5) bare copper wires spirally applied around the outside to serve as the current-carrying grounded neutral. The data given are for a cable intended for single-phase service, hence the number and size of concentric neutral are selected to have "100 percent neutral" ampacity. When three such cables are to be installed to make a three-phase circuit, the number and/or size of copper concentric neutral strands on each cable are reduced to 33 percent (or less) neutral ampacity per cable.

Another type of insulation in current use is high-molecular-weight PE (HMWPE). It is rated for only 75°C conductor temperature and, therefore, provides a little less ampacity than XLPE insulation on the same conductor size. The HMWPE requires 220 mils insulation thickness in lieu of 175 mil. Cable reactances are, therefore, slightly higher when HMWPE is used. However, the Δx_L is negligible for ordinary purposes.

The determination of correct $r + jx_L$ values of these relatively new concentric-neutral cables is a subject of current concern and research. A portion of the neutral current remains in the bare concentric-neutral conductors; the remainder returns in the earth (Carson's equivalent conductor). More detailed information about this matter is available in Refs. 8 and 9. Use the given data and determine the following:

(*a*) Size each of the overhead mains 1 and 2, of Fig. 5-33, with enough ampacity to serve the entire 12 × 10 block area. Size each single-phase lateral URD cable with ampacity for the load of 12 blocks.

Table 5-3 15-kV concentric neutral XLPE-insulated Al URD cable

Al conductor size	Cu neutral	Ω/1000 ft*		Ampacity, A	
		$r^{\dagger}$	x_L	Direct burial	In duct
4 AWG	6–#14	0.526	0.0345	128	91
2 AWG	10–#14	0.331	0.0300	168	119
1 AWG	13–#14	0.262	0.0290	193	137
1/0 AWG	16–#14	0.208	0.0275	218	155
2/0 AWG	13–#12	0.166	0.0260	248	177
3/0 AWG	16–#12	0.132	0.0240	284	201
4/0 AWG	20–#12	0.105	0.0230	324	230
250 kcmil	25–#12	0.089	0.0220	360	257
300 kcmil	18–#10	0.074	0.0215	403	291
350 kcmil	20–#10	0.063	0.0210	440	315

* For single-phase circuitry.
† At 90°C conductor temperature.
Source: Data abstracted from Rome Cable Company, *URD Technical Manual*, 4th ed.

(*b*) Find the percent voltage drop at the ends of the most remote laterals under *normal operation*, i.e., all laterals open at the center and both mains are energized.
(*c*) Find the percent voltage drop at the most remote lateral under *the worst possible emergency operation*; i.e., one main is outaged and all laterals are fed full length from the one energized main.
(*d*) Is the voltage-drop criterion met for normal operation and for the worst emergency operation?

SOLUTION
(*a*) Since under the emergency operation the remaining energized main supplies the doubled number of laterals, the assumed load is

$$2 \times 518 \text{ kVA/lateral} \times 10 \text{ laterals/feeder} = 10{,}360 \text{ kVA}$$

Therefore the current in the main is

$$I_{\text{main}} = \frac{10{,}360 \text{ kVA}}{\sqrt{3} \times 12.47 \text{ kV}}$$

$$= 480.2 \text{ A}$$

Thus, from Table A-5, 300-kcmil ACSR conductors, with 500-A ampacity, are selected for the mains. Since under the emergency operation, due to doubled load, the current in the lateral is doubled,

$$I_{\text{lateral}} = \frac{2 \times 518 \text{ kVA}}{7.2 \text{ kV}}$$

$$\cong 144 \text{ A}$$

Therefore, from Table 5-3, AWG #2 XLPE Al URD cable, with 168-A ampacity, is selected for the laterals.

(*b*) Under *normal operation*, all laterals are open at the center and both mains are energized. Thus the voltage drop, due to uniformly distributed load, at the main is

$$\mathrm{VD}_{\text{main}} = I(r \times \cos\theta + x_L \times \sin\theta)\frac{l}{2} \qquad \mathrm{V} \tag{5-100}$$

or

$$\mathrm{VD}_{\text{main}} = I[r \times \cos\theta + (x_a + x_d) \times \sin\theta]\frac{l}{2} \qquad \mathrm{V} \tag{5-101}$$

where $I = 480.2/2 = 240.1$ A
$r = 0.342\ \Omega/\text{mi}$ for 300-kcmil ACSR conductors from Table A-5
$x_a = 0.458\ \Omega/\text{mi}$ for 300-kcmil ACSR conductors from Table A-5
$x_d = 0.1802\ \Omega/\text{mi}$ for $D_m = 53$ in from Table A-10
$\cos\theta = 0.90$
$\sin\theta = 0.436$

Therefore

$$\mathrm{VD}_{\text{main}} = 240.1[0.342 \times 0.9 + (0.458 + 0.1802)0.436]\frac{3300\ \text{ft}}{5280\ \text{ft/mi}} \times \frac{1}{2}$$

$$\cong 44\ \mathrm{V}$$

or, in percent,

$$\%\ \mathrm{VD}_{\text{main}} = \frac{44\ \mathrm{V}}{7200\ \mathrm{V}}$$

$$= 0.61.$$

The voltage drop at the lateral, due to the uniformly distributed load, from Eq. (5-97) is

$$\mathrm{VD}_{\text{lateral}} = I(r \times \cos\theta + x \times \sin\theta)\frac{l}{2} \qquad \mathrm{V}$$

where $I = 144/2 = 72$ A
$r = 0.331\ \Omega/1000\ \text{ft}$ for AWG #2 XLPE Al URD cable from Table 5-3
$x_L = 0.0300\ \Omega/1000\ \text{ft}$ for AWG #2 XLPE Al URD cable from Table 5-3

Therefore

$$\mathrm{VD}_{\text{lateral}} = 72(0.331 \times 0.9 + 0.0300 \times 0.436)\frac{5760\ \text{ft}}{1000\ \text{ft}} \times \frac{1}{2}$$

$$\cong 64.5\ \mathrm{V}$$

or, in percent,

$$\% \mathrm{VD}_{\text{lateral}} = \frac{64.5 \text{ V}}{7200 \text{ V}}$$

$$= 0.9$$

Thus, from Eq. (5-95), the total percent primary voltage drop is

$$\sum \% \mathrm{VD} = 0.61 + 0.9$$
$$= 1.51$$

(*c*) Under *the worst possible emergency operation*, one main is outaged and all laterals are supplied full length from the remaining energized main. Thus the voltage drop in the main, due to uniformly distributed load, from Eq. (5-101) is

$$\mathrm{VD}_{\text{main}} = 480.2(0.3078 + 0.2783)\,\frac{3300 \text{ ft}}{5280 \text{ ft/mi}} \times \frac{1}{2}$$

$$\cong 88 \text{ V}$$

or, in percent,

$$\% \mathrm{VD}_{\text{main}} = 1.22$$

The voltage drop at the lateral, due to uniformly distributed load, from Eq. (5-97) is

$$\mathrm{VD}_{\text{lateral}} = 144(0.331 \times 0.9 + 0.03 \times 0.435)\,\frac{5760 \text{ ft}}{1000 \text{ ft}}$$
$$\cong 258 \text{ V}$$

or, in percent,

$$\% \mathrm{VD}_{\text{lateral}} = \frac{258 \text{ V}}{7200 \text{ V}}$$

$$= 3.5$$

Therefore, from Eq. (5-95), the total percent primary voltage drop is

$$\sum \% \mathrm{VD} = 1.22 + 3.5$$
$$= 4.72$$

(*d*) The primary voltage-drop criterion is met for normal operation but is not met for the worst emergency operation.

PROBLEMS

5-1 Repeat Example 5-2, assuming a 30-min annual maximum demand of 4.4 kVA per customer.

5-2 Repeat Example 5-3, assuming the nominal operating voltage to be 7200/12,470 V.

5-3 Repeat Example 5-3, assuming a 30-min annual maximum demand of 4.4 kVA per customer.

5-4 Repeat Example 5-4, and find the exact solution by using $D_m = 53$ in.

5-5 Repeat Example 5-5, assuming a lagging-load power factor of 0.80 at all locations.

5-6 Assume that a radial express feeder used in rural distribution is connected to a concentrated and static load at the receiving end. Assume that the feeder impedance is $0.15 + j0.30$ pu, the sending end voltage is 1.0 pu, the constant power load at the receiving end is 1.0 pu with a lagging power factor of 0.85. Use the given data and the exact equations for $\hat{K}$, V_r, and $\tan\delta$ given in Sec. 5-12 and determine the following:

(*a*) The values V_r and δ by using the exact equations.

(*b*) The corresponding values of the I_r and I_s currents.

5-7 Use the results found in Prob. 5-6 and Eq. (5-90) and determine the receiving-end voltage $\bar{V}_r$.

5-8 Assume that a three-phase 34.5-kV radial express feeder is used in rural distribution and that the receiving-end voltages at full load and no load are 34.5 and 36.9 kV, respectively. Determine the percent voltage regulation of the feeder.

5-9 A three-phase radial express feeder has a line-to-line voltage of 22.9 kV at the receiving end, a total impedance of $5.25 + 10.95\ \Omega$/phase, and a load of 5 MW with a lagging power factor of 0.90. Determine the following:

(*a*) The line-to-neutral and line-to-line voltages at the sending end.

(*b*) The load angle.

5-10 Use the results of Prob. 5-9 and determine the percent voltage regulation of the feeder.

5-11 Assume that a wye-connected three-phase load is made up of three impedances of $50\angle 25°\ \Omega$ each and that the load is supplied by a three-phase four-wire primary express feeder. The balanced line-to-neutral voltages at the receiving end are:

$$\bar{V}_{an} = 7630\angle 0°\ \text{V}$$

$$\bar{V}_{bn} = 7630\angle 240°\ \text{V}$$

$$\bar{V}_{cn} = 7630\angle 120°\ \text{V}$$

Determine the following:

(*a*) The phasor currents in each line.

(*b*) The line-to-line phasor voltages.

(*c*) The total active and reactive power supplied to the load.

5-12 Repeat Prob. 5-11, if the same three load impedances are connected in a delta connection.

5-13 Assume that the service area of a given feeder is increasing as a result of new residential developments. Determine the new load and area that can be served with the same percent voltage drop if the new feeder voltage level is increased to 34.5 kV from the previous voltage level of 12.47 kV.

5-14 Assume that the feeder in Prob. 5-13 has a length of 2 mi and that the new feeder uniform loading has increased to three times the old feeder loading. Determine the new maximum length of the feeder with the same percent voltage drop.

REFERENCES

1. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*. 11th ed., McGraw-Hill, New York, 1978.
2. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.

3. Gönen, T., et al.: *Development of Advanced Methods For Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va.
4. Edison Electric Institute: *Underground Systems Reference Book*, 2d ed., New York, 1957.
5. Andrews, F. E.: "Residential Underground Distribution Adaptable," *Electr. World*, Dec. 12, 1955, pp. 107–113.
6. EEI-NEMA: *Preferred Voltage Ratings for AC Systems and Equipment*, EEI Publication No. R-6, NEMA Publication No. 117, May 1949.
7. Morrison, C.: "A Linear Approach to the Problem of Planning New Feed Points into a Distribution System," *AIEE Trans.*, pt. III (PAS), December 1963, pp. 819–832.
8. Smith, D. R., and J. V. Barger: "Impedance and Circulating Current Calculations for URD Multi-Wire Concentric Neutral Circuits," *IEEE Trans. Power Appar. Syst.*, vol. PAS-91, no. 3. May/June 1972, pp. 992–1006.
9. Stone, D. L.: "Mathematical Analysis of Direct Buried Rural Distribution Cable Impedance," *IEEE Trans. Power Appar. Syst.*, vol. PAS-91, no. 3, May/June 1972, pp. 1015–1022.

CHAPTER

SIX

DESIGN CONSIDERATIONS OF SECONDARY SYSTEMS

6-1 INTRODUCTION

A realistic view of the power distribution systems should be based on "gathering" functions rather than on "distributing" since the size and locations of the customer demands are not determined by the distribution engineer but by the customers. Customers install all types of energy-consuming devices which can be connected in every conceivable combination and at times of customers' choice. This concept of distribution starts with the individual customers and loads, and proceeds through several gathering stages where each stage includes various groups of increasing numbers of customers and their loads. Ultimately the generating stations themselves are reached through services, secondaries, distribution transformers, primary feeders, distribution substation, subtransmission and bulk power stations, and transmission lines.

In designing a system, distribution engineers should consider not only the immediate, i e , short-range, factors but also the long-range problems. The designed system should not only solve the problems of economically building and operating the systems to serve the loads of today but also should require a long-range projection into the future to determine the most economical distribution system components and practices to serve the higher levels of the customers' demands which will then exist. Therefore, the present design practice should be influenced by the requirements of the future system.

Of course, distribution engineers, who have to consider the many factors, variables, and alternative solutions of the complex distribution design problems, need a technique that will enable them to select the most economical size combination of distribution transformers, secondary conductors, and service drops.

The recent developments in high-speed digital computers, through the use of computer programs, have provided (1) the fast and economic consideration of many feasible alternatives and (2) the economic and engineering evaluation of these alternatives as they evolve with different strategies throughout the study period. The strategies may include, for example, cutting the secondary, changing the transformers, and possibly adding capacitors.

Naturally, each designed system should meet a specified performance criterion throughout the study period. The most optimum, i.e., most economical, system design which corresponds to a load-growth projection schedule can be selected. Also, through periodic use of the programs, distribution engineers can determine whether strategies adopted continue to be desirable or whether they require some modification as a result of some changes in economic considerations and load-growth projections.

To minimize the secondary-circuit lengths, distribution engineers locate the distribution transformers close to the load centers and try to have the secondary service drops to the individual customers as short as possible.

Since only a small percentage of the total service interruptions are due to failures in the secondary system, distribution engineers, in their system design decisions of the secondary distribution, are primarily motivated by the considerations of economy, copper losses (I^2R) in the transformer and secondary circuit, permissible voltage drops, and voltage flicker of the system. Of course, there are some other engineering and economic factors affecting the selection of the distribution transformer and the secondary configurations, such as permissible transformer loading, balanced phase loads for the primary system, investment costs of the various secondary system components, cost of labor, cost of capital, and inflation rates.

Distribution transformers represent a significant part of the secondary system cost. Therefore, one of the major concerns of distribution engineers is to minimize the investment in distribution transformers. In general, the present practice in the power industry is to plan the distribution transformer loading on the basis that there should not be excessive spare capacity installed, and transformers should be exchanged, or banked, as the secondary load grows.

Usually, a transformer load management (TLM) system is desirable for consistent loading practices and economical expansion plans. Distribution engineers, recognizing the impracticality of obtaining complete demand information on all customers, have attempted to combine a limited amount of demand data with the more complete, and readily available, energy consumption data available in the customer account files. A typical demand curve is scaled according to the energy consumed, and the resultant information is used to estimate the peak loading on specific pieces of equipment, such as distribution transformers, in which case it is known as *transformer load management* (TLM), feeders, and substations [4–7].

However, in general, residential, commercial, and industrial customers are categorized in customer files by rate classification only; i.e., potentially useful and important subclassifications are not distinguished. Therefore, demand data is generally collected for the purpose of generating typical curves only for each rate classification.

6-2 SECONDARY VOLTAGE LEVELS

Today, the standard (or preferred) voltage levels for the electric power systems are given by the American National Standards Institute's (ANSI) Standard C84.1-1977, entitled *Voltage Ratings for Electric Power Systems and Equipment (60 Hz)*.

Accordingly, the standard voltage level for single-phase residential loads is 120/240 V. It is supplied through three-wire single-phase services, from which both 120-V lighting and 240-V single-phase power connections are made to large household appliances such as ranges, clothes dryers, and water heaters. For grid- or mesh-type secondary-network systems, used usually in the areas of commercial and residential customers with high-load densities, the voltage level is 208Y/120 V. It is also supplied through three-wire single-phase services, from which both 120-V lighting and 208-V single-phase power connections are made. For "spot" networks used in downtown areas for high-rise buildings with super-high-load densities, and also for areas of industrial and/or commercial customers, the voltage level is 480Y/277 V. It is supplied through four-wire three-phase services, from which both 277 V for fluorescent lighting and other single-phase loads and 480-V three-phase power connections are made

Today, one can also find other voltage levels in use contrary to the ANSI standards, for example, 120/240-V four-wire three-phase; 240-V three-wire three-phase; 480-V three-wire three-phase; 240/416-V four-wire three-phase; or 240/480-V four-wire three-phase.

To increase the service reliability for critical loads, such as hospitals, computer centers, crucial industrial loads, some backup systems, e.g., emergency generators and/or batteries, with automatic switching devices are provided.

6-3 THE PRESENT DESIGN PRACTICE

The part of the electric utility system which is between the primary system and the consumer's property is called the *secondary system*. Secondary distribution systems include step-down distribution transformers, secondary circuits (secondary mains), consumer services (or service drops), and meters to measure consumer energy consumption.

Generally, the secondary distribution systems are designed in single-phase for areas of residential customers and in three-phase for areas of industrial or commercial customers with high-load densities. The types of the secondary distribution systems include:

1. The separate-service system for each consumer with separate distribution transformer and secondary connection
2. The radial system with a common secondary main which is supplied by one distribution transformer and feeding a group of consumers
3. The secondary-bank system with a common secondary main that is supplied by several distribution transformers which are all fed by the same primary feeder
4. The secondary-network system with a common grid-type main that is supplied by a large number of distribution transformers which may be connected to various feeders for their supplies

The separate-service system is seldom used and serves industrial- or rural-type service areas. Generally speaking, most of the secondary systems for serving residential, rural, and light-commercial areas are radial-designed. Figure 6-1 shows the one-line diagram of a radial secondary system. It has a low cost and is simple to operate.

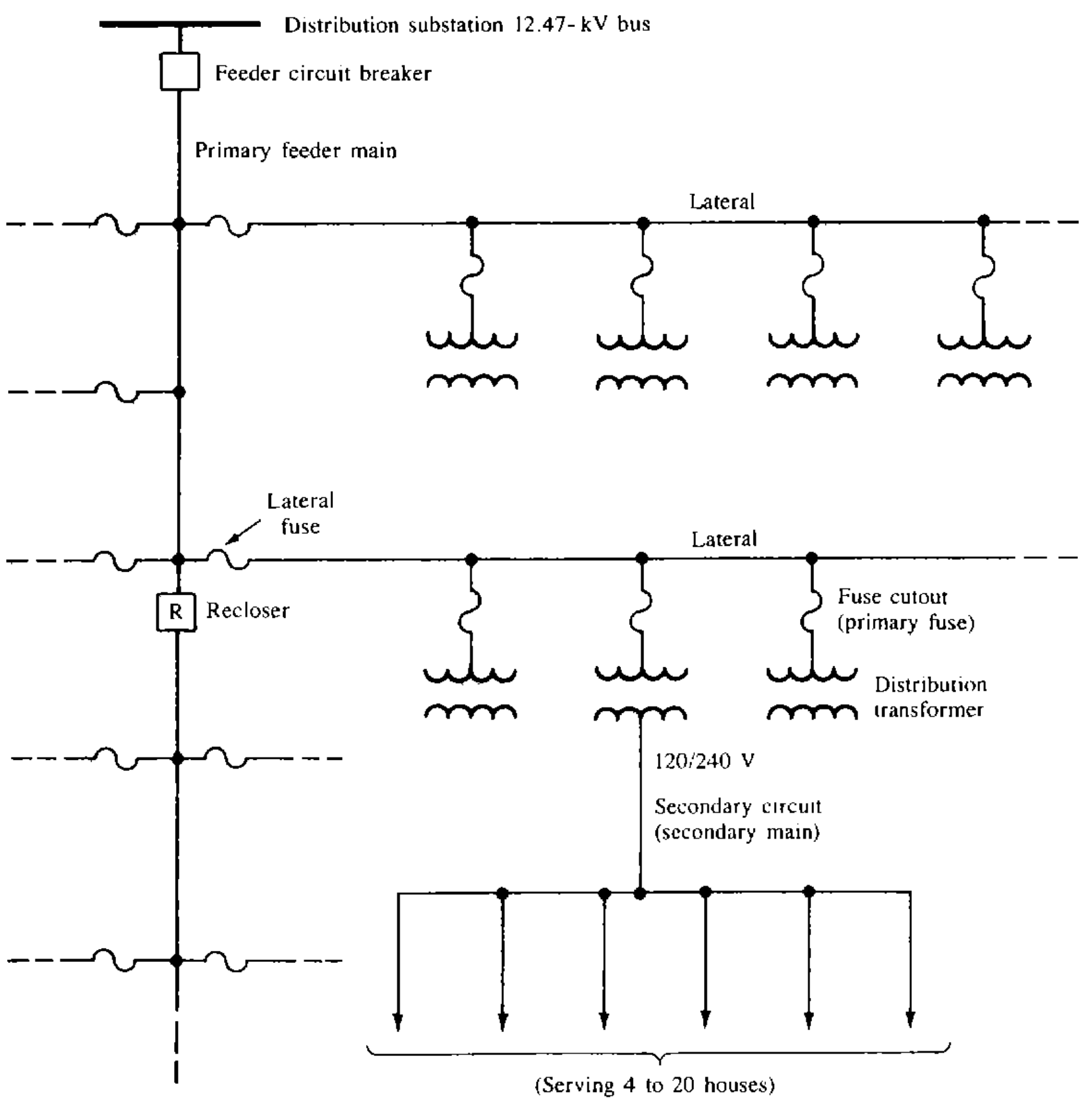


Figure 6-1 One-line diagram of a simple radial secondary system.

6-4 SECONDARY BANKING

The "banking" of distribution transformers, i.e., parallel connection, or, in other words, *interconnection*, of the secondary sides of two or more distribution transformers which are supplied from the same primary feeder is sometimes practiced in residential and light-commercial areas where the services are relatively close to each other, and therefore the required spacing between transformers is little. However, many utilities prefer to keep the secondary of each distribution transformer separate from all others. In a sense, secondary banking is a special form of network configuration on a radial distribution system. The advantages of the banking of distribution transformers include:

1. Improved voltage regulation
2. Reduced voltage dip or light flicker due to motor starting, by providing parallel supply paths for motor-starting currents
3. Improved service continuity or reliability
4. Improved flexibility in accommodating load-growth, at low cost, i.e., possible increase in the average loading of transformers without corresponding increase in the peak load

Banking the secondaries of distribution transformers allows us to take advantage of the load diversity existing among the greater number of consumers, which, in turn, induces a savings in the required transformer kilovoltamperes. This savings can be as large as 35 percent according to Lokay [3], depending upon the load types and the number of consumers.

Figure 6-2 shows two different methods of banking secondaries. The method illustrated in Fig. 6-2*a* is commonly used and is generally preferred because it permits the use of a lower-rated fuse on the high-voltage side of the transformer,

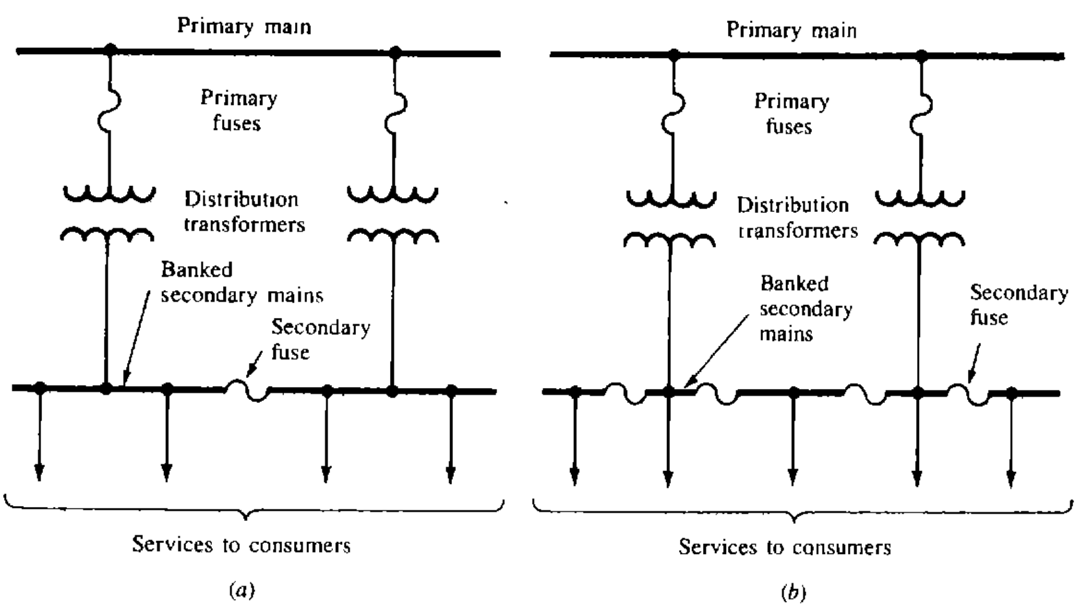


Figure 6-2 Two different methods of banking secondaries.

and it prevents the occurrence of cascading the fuses. This method also simplifies the coordination with primary-feeder sectionalizing fuses by having a lower-rated fuse on the high side of the transformer. Furthermore, it provides the most economical system.

Figure 6-3 gives two other methods of banking secondaries. The method shown in Fig. 6-3*a* is the oldest one and offers the least protection, whereas the method shown in Fig. 6-3*b* offers the greatest protection. Therefore the methods illustrated in Figs. 6-2*a*, *b*, and 6-3*a* have some definite disadvantages which include:

1. The requirement for careful policing of the secondary system of the banked transformers to detect blown fuses
2. The difficulty in coordination of secondary fuses
3. Furthermore, the method illustrated in Fig. 6-2*b* has the additional disadvantage of being difficult to restore service after a number of fuses on adjacent transformers have been blown

Today, due to the aforementioned difficulties, many utilities prefer the method given in Fig. 6-3*b*. The special distribution transformer known as the *completely self-protecting-bank* (CSPB) *transformer* has, in its unit, a built-in high-voltage protective link, secondary breakers, signal lights for overload warnings, and lightning protection.

CSPB transformers are built in both single-phase and three-phase. They have two identical secondary breakers which trip independently of each other upon excessive current flows. In case of a transformer failure, the primary protective links and the secondary breakers will both open. Therefore, the service interruption will be minimum and restricted only to those consumers who are supplied from the secondary section which is in fault. However, all the methods of secondary

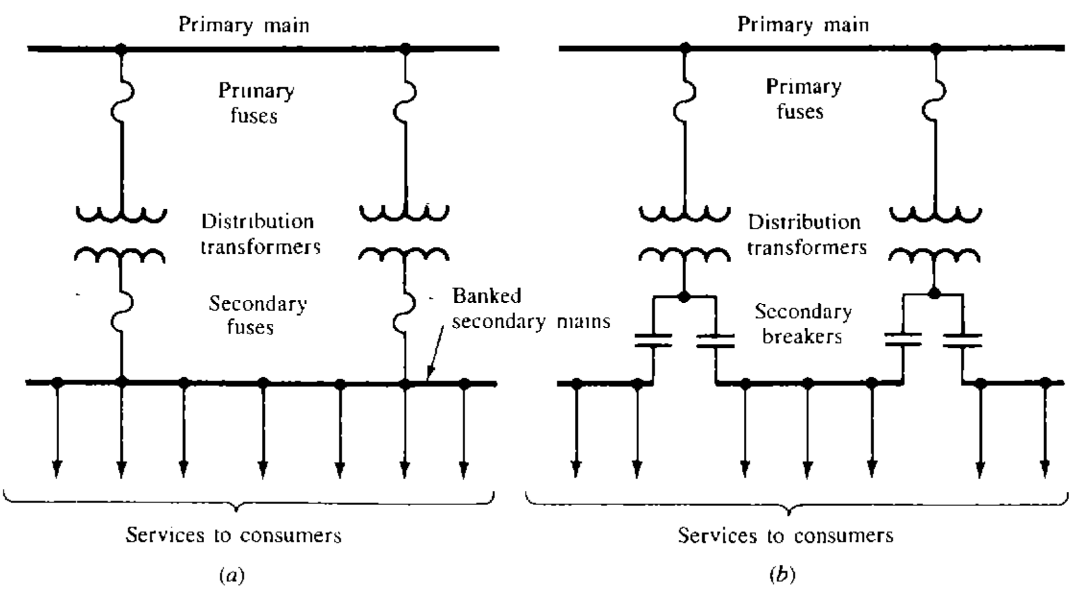


Figure 6-3 Two additional methods of banking secondaries.

banking have an inherent disadvantage: the difficulty in performing transformer load management (TLM) to keep up with changing load conditions. The main concern when designing a banked secondary system is the equitable load division among the transformers. It is desirable that transformers whose secondaries are banked in a straight line be within one size of each other. For other types of banking, transformers may be within two sizes of each other to prevent excessive overload in case the primary fuse of an adjacent larger transformer should blow. Today, in general, the banking is applied to the secondaries of single-phase transformers, and all transformers in a bank must be supplied from the same phase of the primary feeder.

6-5 THE SECONDARY NETWORKS

Generally speaking, most of the secondary systems are radial-designed except for some specific service areas (e.g., downtown areas or business districts, some military installations, hospitals) where the reliability and service-continuity considerations are far more important than the cost and economic considerations. Therefore the secondary systems may be designed in grid- or mesh-type network configurations in those areas. The low-voltage secondary networks are particularly well-justified in the areas of high-load density. They can also be built in underground to avoid overhead congestion. The overhead low-voltage secondary networks are economically preferable over underground low-voltage secondary networks in the areas of medium-load density. However, the underground secondary networks give a very high degree of service reliability. In general, where the load density justifies an underground system, it also justifies a secondary network system.

Figure 6-4 shows a one-line diagram of a small segment of a secondary network supplied by three primary feeders. In general, the usually low-voltage (208Y/120 V) grid- or mesh-type secondary-network system is supplied through network-type transformers by two or more primary feeders to increase the service reliability. In general, these are radial-type primary feeders. However, the loop-type primary feeders are also in use to a very limited extent. The primary feeders are interlaced in a way to prevent the supply to any two adjacent transformer banks from the same feeder. As a result of this arrangement, if one primary feeder is out of service for any reason (single contingency), the remaining feeders can feed the load without overloading and without any objectionable voltage drop. The primary-feeder voltage levels are in the range of 4.16 to 34.5 kV. However, there is a tendency toward the use of higher primary voltages. Currently, the 15-kV class is predominating. The secondary network must be designed in such a manner as to provide at least one of the primary feeders as a spare capacity together with its transformers. To achieve even load distribution between transformers and minimum voltage drop in the network, the network transformers must be located accordingly throughout the secondary network.

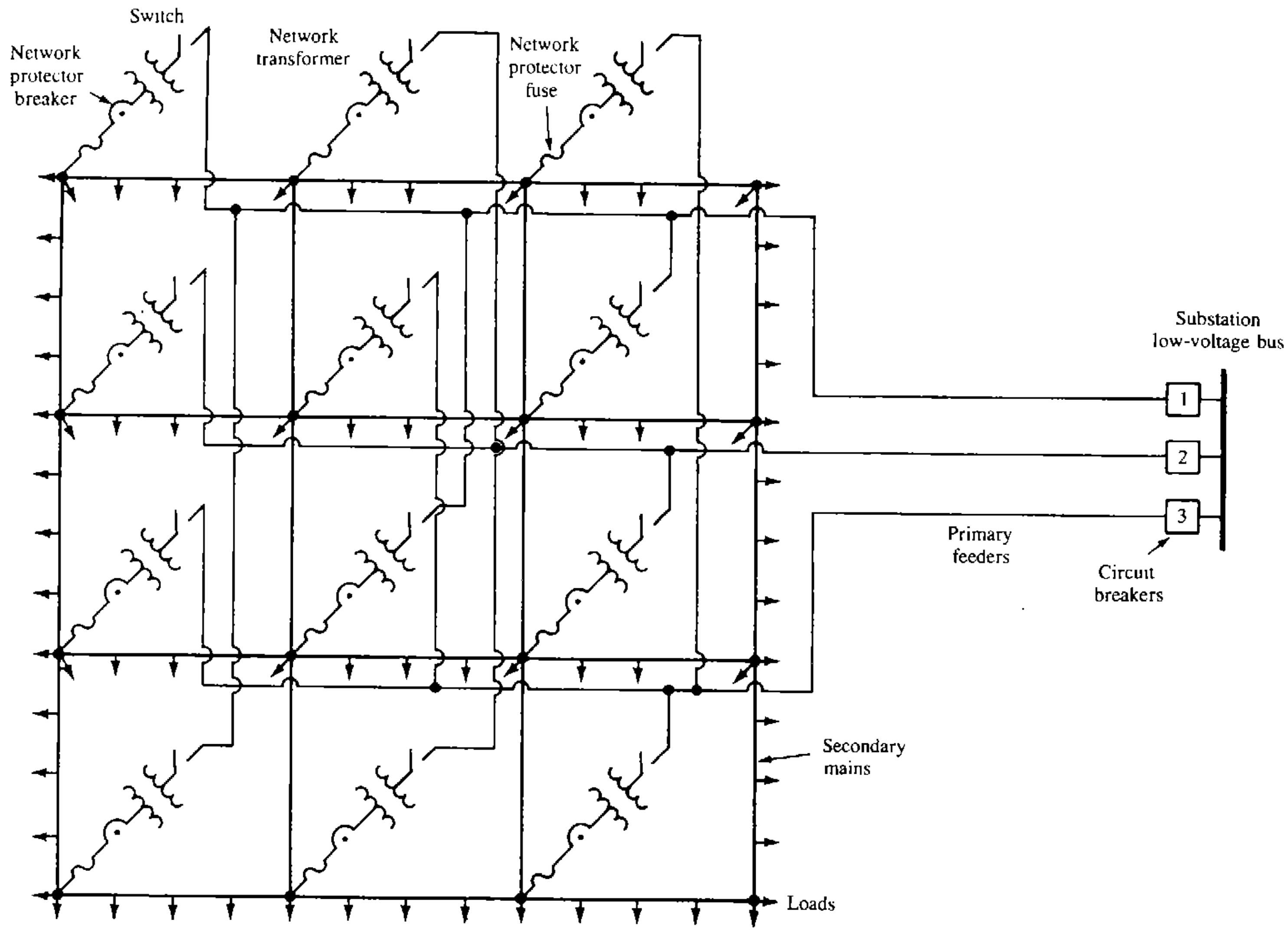


Figure 6-4 One-line diagram of small segment of a secondary-network system.

As explained previously, the smaller secondary networks are designed based 1 single contingency, i.e., the outage of one primary feeder. However, larger ndary-network systems must be designed based upon double contingency or nd contingency, i.e., having two feeder outages simultaneously. According to s [3], the factors affecting the probability of occurrence of double outages are:

The total number of primary feeders
The total mileage of the primary-feeder circuit
The number of accidental feeder outages per year
The scheduled feeder-outage time per year
The time duration of a feeder outage

Even though theoretically the primary feeders may be supplied from different irces such as distribution substations, bulk power substations, or generating ints, it is generally preferred to have the feeders supplied from the same sub-tion to prevent voltage magnitude and phase-angle differences among the ders, which can cause a decrease in the capacities of the associated transformers ie to improper load division among them. Also, during light-load periods, the ower flow in a reverse direction in some feeders connected to separate sources is an lditional concern.

-5-1 Secondary Mains

eelye [13] suggested that the proper size and arrangement of the secondary iains should provide for:

. The proper division of the normal load among the network transformers
:. The proper division of the fault current among the network transformers
i. Good voltage regulation to all consumers
4. Burning off short circuits or grounds at any point without interrupting service

All secondary mains (underground or overhead) are routed along the streets and are three-phase four-wire wye-connected with solidly grounded neutral conductor. In the underground networks, the secondary mains usually consist of single-conductor cables which may be either metallic- or nonmetallic-sheathed. Secondary cables commonly have been rubber-insulated, but PE cables are now used to a considerable extent. They are installed in duct lines or duct banks. Man-holes at street intersections are constructed with enough space to provide for various cable connections and limiters, and to permit any necessary repair activities by workers.

On the other hand, the secondary mains in the overhead secondary networks usually are open-wire circuits with weatherproof conductors. The conductor sizes depend upon the network-transformer ratings. For a grid-type secondary main, the minimum conductor size must be able to carry about 60 percent of the full-load current to the largest network transformer. This percentage is much less for the underground secondary mains. The most frequently used cable sizes for secondary mains are 4/0 or 250 kcmil, and, to a certain extent, 350 and 500 kcmil. The selection of the sizes of the mains is also affected by the consideration of burning faults clear. In case of a phase-to-phase or phase-to-ground short circuit, the secondary network is designed to burn itself clear without using sectionalizing fuses or other overload protective devices. Here, "burning clear" of a faulted secondary-network cable refers to a burning away of the metal forming the contact between phases or from phase to ground until the low voltage of the secondary network can no longer support the arc. To achieve fast clearing, the secondary network must be able to provide for high current values to the fault. Of course, the larger the cable, the higher the short-circuit current value needed to achieve the burning clear of the faulted cable. Therefore, conductors of 500 kcmil are about the largest conductors used for secondary-network mains. The conductor size is also selected keeping in mind the voltage-drop criterion, so that the voltage drop along the mains under normal load conditions does not exceed a maximum of 3 percent.

6-5-2 Limiters

Most of the time the method permitting secondary-network conductors to burn clear, especially in 120/208 V, gives good results without loss of service. However, under some circumstances, particularly at higher voltages, for example, 480 V, this method may not clear the fault due to insufficient fault current, and, as a result, extensive cable damage, manhole fires, and service interruptions may occur.

To have fast clearing of such faults, so-called limiters are used. The limiter is a high-capacity fuse with a restricted copper section, and it is installed in each phase conductor of the secondary main at each junction point. The limiter's fusing or time-current characteristics are designed to allow the normal network load current to pass without melting but to operate and clear a faulted section of main before the cable insulation is damaged by the heat generated in the cable by the fault current. The fault should be cleared away by the limiters rapidly, before the network protector fuses blow. Therefore, the time-current characteristics of the selected limiters should be coordinated with the time-current characteristics of the network protectors and the insulation-damage characteristics of the cable. The distribution engineer's decision of using limiters should be based upon two considerations: (1) minimum service interruption, and (2) whether the saving in damage to cables pays more than the cost of the limiters. Figure 6-5 shows the time-current characteristics of limiters used in 120/208-V systems and the insulation-damage characteristics of the underground-network cables (paper- or rubber-insulated).

6-5-3 Network Protectors

As shown in Fig. 6-4, the network transformer is connected to the secondary network through a network protector (NP). The network protector consists of an air circuit breaker with a closing and tripping mechanism controlled by a network master and phasing relay, and backup fuses. All these are enclosed in a

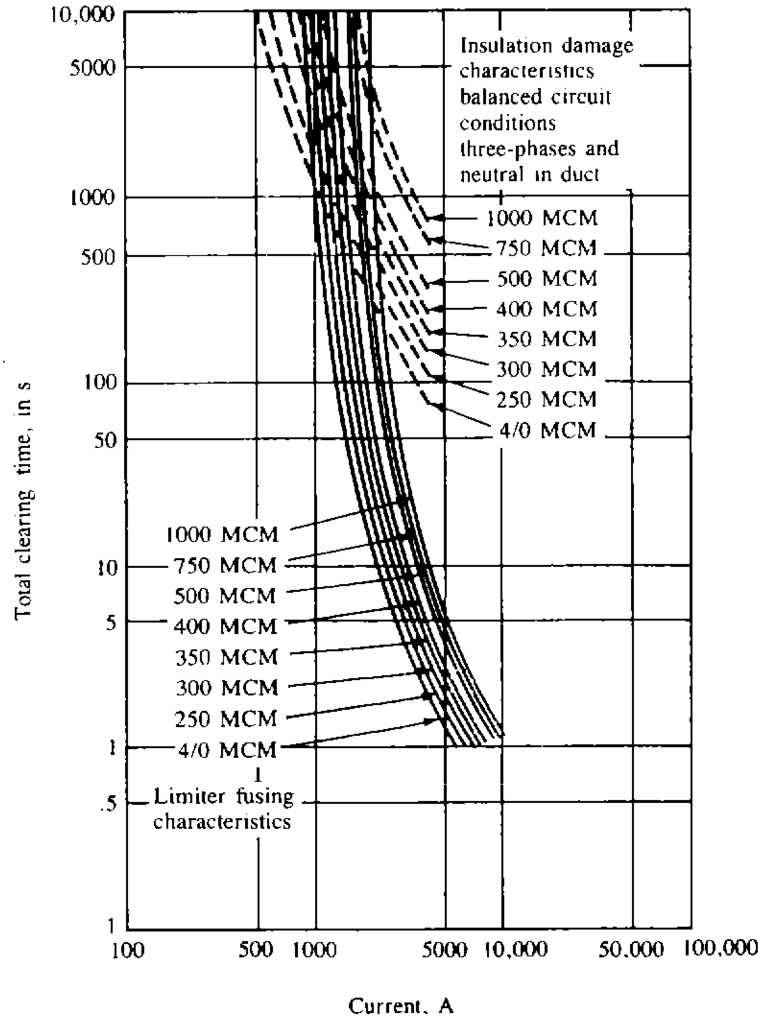


Figure 6-5 Limiter characteristics in terms of time to fuse vs. current and insulation-damage characteristics of the underground-network cables. (*From* [3].)

metal case which may be mounted on the transformer or separately mounted. The fuses provide backup protection to disconnect the network transformer from the network if the network protector fails to do so during a fault. The functions of a network protector include:

1. To provide automatic isolation of faults occurring in the network transformer or in the primary feeder. For example, when a fault occurs in one of the high-voltage feeders, it causes the feeder circuit breaker, at the substation, to be open. At the same time, a current flows to the feeder fault point from the secondary network through the network transformers normally supplied by the faulted feeder. This reverse power flow triggers the circuit breakers of the network protectors connected to the faulty feeder to open. Therefore the fault becomes isolated without any service interruption to any of the consumers connected to the network.
2. To provide automatic closure under the predetermined conditions, i.e., when the primary-feeder voltage magnitude and the phase relation with respect to the network voltage are correct. For example, the transformer voltage should

be slightly higher (about 2 V) than the secondary-network voltage in order to achieve power flow from the network transformer to the secondary-network system, and not the reverse. Also, the low-side transfer voltage should be in phase with, or leading, the network voltage.

3. To provide its reverse power relay to be adequately sensitive to trip the circuit breaker with currents as small as the exciting current of the transformer. For example, this is important for the protection against line-to-line faults occurring in unigrounded three-wire primary feeders feeding network transformers with delta connections.
4. To provide protection against the reverse power flow in some feeders connected to separate sources. For example, when a network is fed from two different substations, under certain conditions the power may flow from one substation to the other through the secondary network and network transformers. Therefore the network protectors should be able to detect this reverse power flow and to open. Of course, here, the best protection is not to employ more than one substation as the source.

As previously explained, each network contains backup fuses, one per phase. These fuses provide backup protection for the network transformer if the network protector breakers fail to operate. Figure 6-6 illustrates an ideal coordination of secondary-network protective apparatus. The coordination is achieved by proper selection of time delays for the successive protective devices placed in

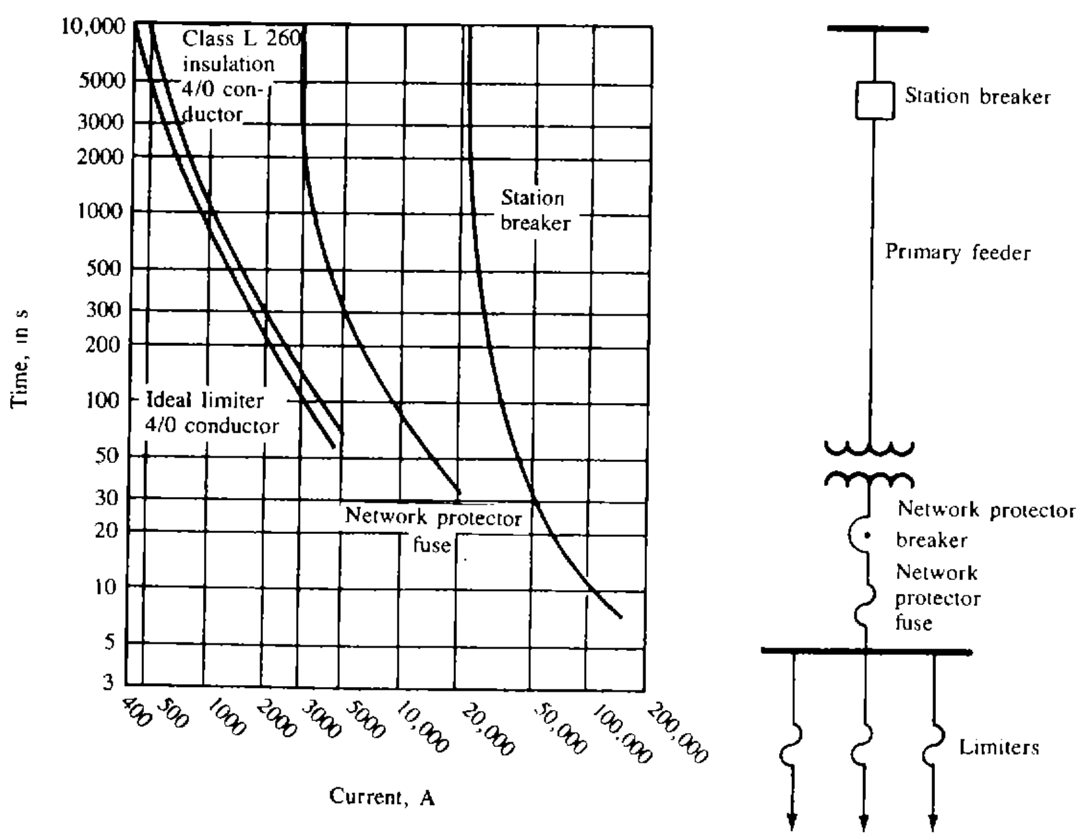


Figure 6-6 An ideal coordination of secondary-network overcurrent protection devices. (*From* [3].)

series. Table 6-1 indicates the required action or operation of each protective equipment under different fault conditions associated with the secondary-network system. For example, in case of a fault in a given secondary main, only the associated limiters should isolate the fault, whereas in case of a transformer internal fault, both the network protector breaker and the substation breaker should trip.

6-5-4 High-Voltage Switch

Figures 6-4 and 6-7 show three-position switches electrically located at the high-voltage side of the network transformers. They are physically mounted on one end of the network transformer.

As shown in Fig. 6-7, position 2 is for normal operation, position 3 is for disconnecting the network transformer, and position 1 is for grounding the primary circuit. In any case the switch is manually operated and is not designed to interrupt current. The first step is to open the primary-feeder circuit breaker at the substation before opening the switch and taking the network unit out of service. After taking the unit out, the feeder circuit breaker may be closed to reestablish service to the rest of the network. However, the switch cannot be operated, due to an electric interlock system, unless the network transformer is first deenergized. The grounding position provides safety for the workers during any work on the deenergized primary feeders. To facilitate the disconnection of the transformer from an energized feeder, sometimes a special disconnecting switch which has an interlock with the associated network protector is used, as shown in Fig. 6-7. Therefore, the switch cannot be opened unless the load is first removed by the network protector from the network transformer.

6-5-5 Network Transformers

In the overhead secondary networks, the transformers can be mounted on poles or platforms, depending on their sizes. For example, small ones (75 or 150 kVA) can be mounted on poles, whereas larger transformers (300 kVA) are mounted on platforms. The transformers are either single-phase or three-phase distribution transformers.

In the underground secondary networks, the transformers are installed in vaults. The network protector is mounted on one side of the transformer and the three-position high-voltage switch on the other side. This type of arrangement is

Table 6-1 The required operation of the protective apparatus

Fault type	Limiter	NP fuse	NP breaker	Substation circuit breaker
Mains	Yes	No	No	No
Low-voltage bus	Yes	Yes	No	No
Transformer internal fault	No	No	Trips	Trips
Primary feeder	No	No	Trips	Trips

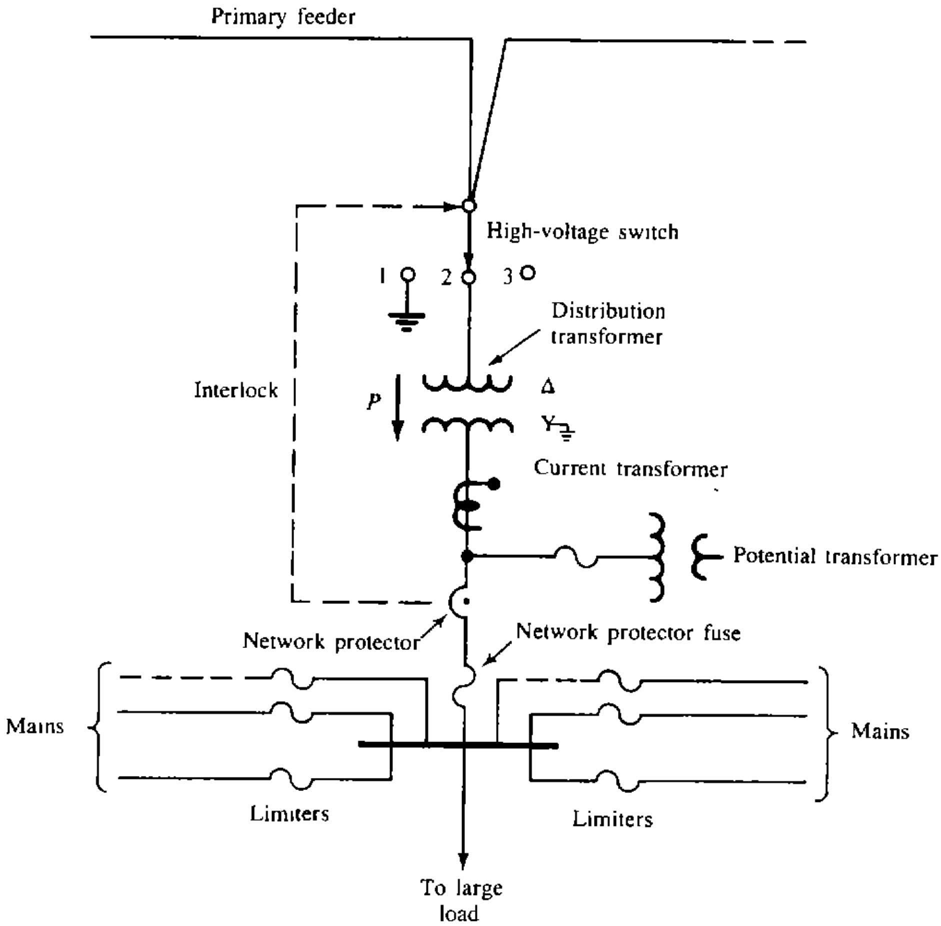


Figure 6-7 High-voltage switch.

called a *network unit*. A typical network transformer is three-phase, with a low-voltage rating of 216Y/125 V, and can be as large as 1000 kVA. Table 6-2 gives standard ratings for three-phase transformers which are used as secondary-network transformers. Because of the savings in vault space and in installation costs, network transformers are now built as three-phase units.

In general, the network transformers are submersible and oil- or askarel-cooled. However, because of environmental concerns, askarel is not used as an insulating medium in new installations any more. Depending upon the locale of the installation, the network transformers can also be ventilated dry-type or sealed dry-type, submersible.

6-5-6 Transformer Application Factor

Reps [3] defines the application factor as "the ratio of installed netwc transformer to load." Therefore, by the same token,

$$\text{Application factor} = \frac{\sum S_T}{\sum S_L} \tag{6-1}$$

where $\sum S_T$ = total capacity of network transformers

$\sum S_L$ = total load of secondary network

Preferred nominal system voltage	Transformer high voltage				Standard kVA ratings for low-voltage rating of 216Y/125 V
			Taps		
	Rating	BIL (kV)	Above	Below	
2400/4160Y	4160* 4160Y/2400*† 4330 4330Y/2500†	60	None None None None	None None None None	300, 500, 750
4800	5000	60	None	4875/4750/4625/4500	300, 500, 750
7200	7200* 7500	75	None None	7020/6840/6660/6480 7313/7126/6939/6752	300, 500, 750
7200	11,500	95	None	11,213/10,926/10,639/10,352	300, 500, 750. 1000
12,000	12,000* 12,500	95	None None	11,700/11,400/11,100/10,800 12,190/11,875/11,565/11,250	300, 500, 750, 1000
7200/12,470Y	13,000Y/7500†	95	None	12,675/12,350/12,025/11,700	300, 500, 750, 1000
13,200 7620/13,200Y	13,200* 13,200Y/7620*† 13,750 13,750Y/7940†	95	None None None None	12,870/12,540/12,210/11,880 12,870/12,540/12,210/11,880 13,406/13,063/12,719/12,375 13,406/13,063/12,719/12,375	300, 500, 750, 1000
14,440	14,400*	95	None	14,040/13,680/13,320/12,960	300, 500, 750, 1000
23,000	22,900* 24,000	150	24,100/23,500 25,200/24,600	22,300/21,700 23,400/22,800	500, 750, 1000

Note: All windings are delta-connected unless otherwise indicated.
* Preferred ratings which should be used when establishing new networks.
† High-voltage and low-voltage neutrals are internally connected by a removable link.
Source: From [3].

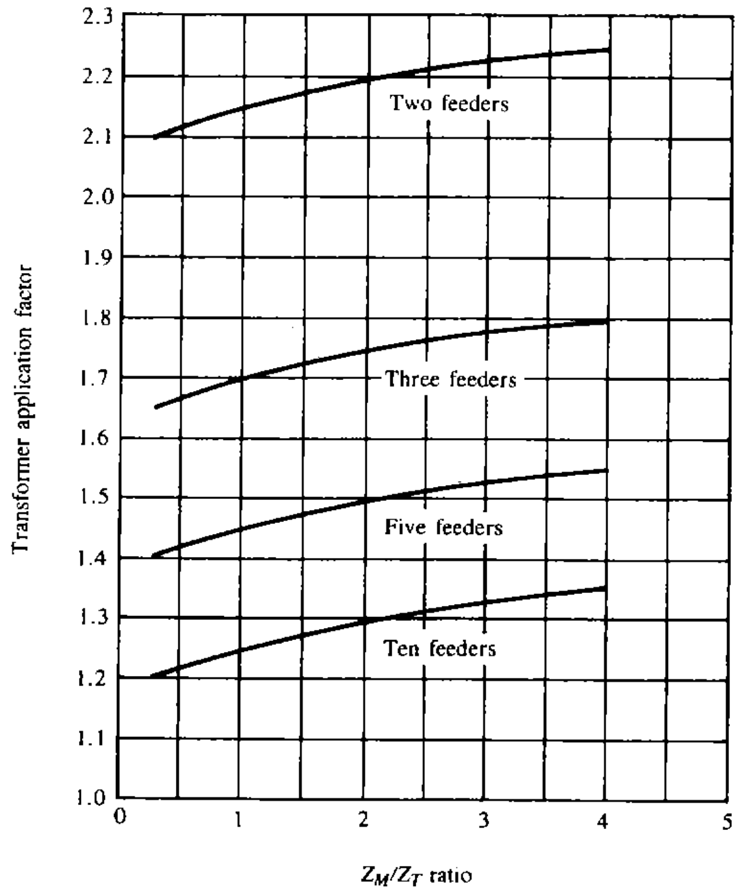


Figure 6-8 Network-transformer application factors as a function of Z_M/Z_T ratio and number of feeders used. (*From* [*3*].)

The application factor is based upon single contingency, i.e., the loss of one of the primary feeders. According to Reps [3], the application factor is a function of the following:

1. The number of primary feeders used
2. The ratio of Z_M/Z_T, where Z_M is the impedance of each section of secondary main and Z_T is the impedance of the secondary-network transformer
3. The extent of nonuniformity in load distribution among the network transformers under the single contingency

Figure 6-8 gives the plots of the transformer application factor vs. the ratio of Z_M/Z_T for different numbers of feeders. For a given number of feeders and a given Z_M/Z_T ratio, the required capacity of network transformers to supply a given amount of load can be found by using Fig. 6-8.

6-6 SPOT NETWORKS

A spot network is a special type of network which may have two or more network units feeding a common bus from which services are tapped. The transformer capacity utilization is better in the spot networks than in the distributed networks

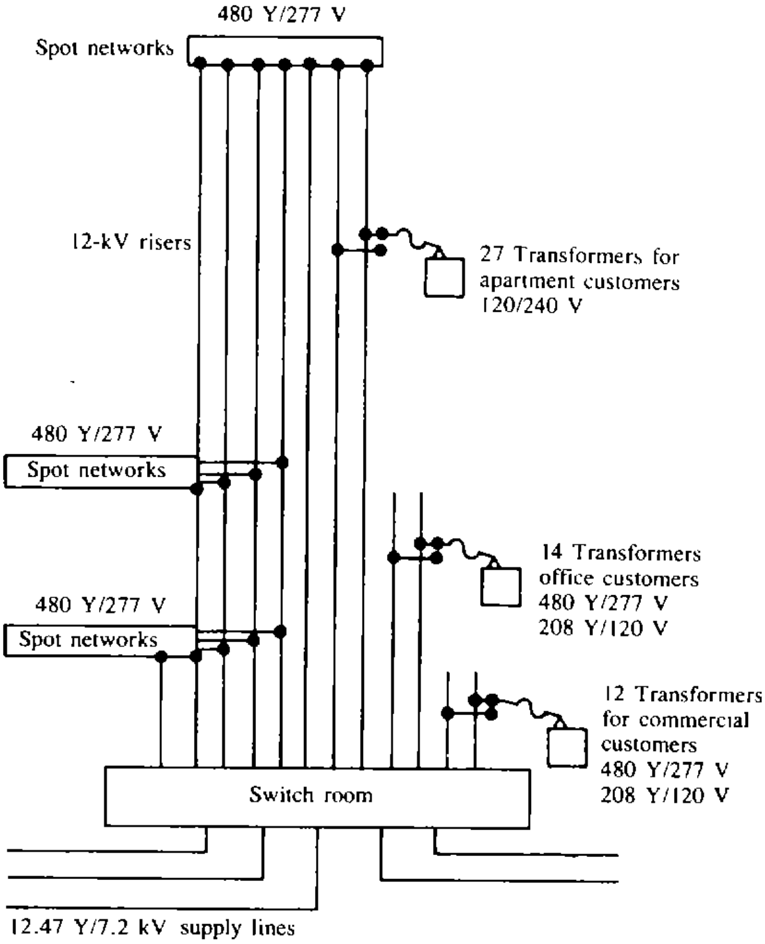


Figure 6-9 One-line diagram of the multiple primary system for the John Hancock Center. (*From* [8].)

due to equal load division among the transformers regardless of a single-contingency condition. The impedance of the secondary main, between transformers, is zero in the spot networks. The spot networks are likely to be found in new high-rise commercial buildings. Even though spot networks with light loads can utilize 208Y/120 V as the nominal low voltage, the commonly used nominal low voltage of the spot networks is 480Y/277 V. Figure 6-9 shows a one-line diagram of the primary system for the John Hancock Center.

6-7 ECONOMIC DESIGN OF SECONDARIES

In this section a method for (at least approximately) minimizing the total annual cost (TAC) of owning and operating the secondary portion of a three-wire single-phase distribution system in a residential area is presented. The method can be applied either to overhead (OH) or underground residential distribution (URD) construction. Naturally, it is hoped that a design for satisfactory voltage-drop and voltage-dip performance will agree at least reasonably well with the design which yields minimum TAC.

6-7-1 The Patterns and Some of the Variables

Figure 6-10 illustrates the layout and one particular pattern having one span of secondary line each way from the distribution transformer. The system is assumed to be built in a straight line along an alley or along rear lot lines. The lots are assumed to be of uniform width d so that each span of secondary line (SL) is of length $2d$. If secondary lines are not used, then there is a distribution transformer on every pole, OH construction, and every transformer supplies four service drops (SD). The primary line, which obviously must be installed along the alley, is not shown in Fig. 6-10. The number of spans of secondary lines each way from a transformer is an important variable. Sometimes no secondary line is used in high-load density areas. In light-load density areas, three or more spans of SL each way from the transformer may be encountered in practice.

If Fig. 6-10 represents an OH system, the transformer, with its arrester(s) and fuse cutout(s), is pole-mounted. The secondary line and the service drop may be of either open-wire or triplex cable construction. If Fig. 6-10 represents a typical URD design, the transformer is grade-mounted on a concrete slab and completely enclosed in a grounded metal housing, or else it is submersibly installed in a hole lined with concrete, Transite, or equivalent material. Both secondary line and service drops are triplexed or twin concentric neutral direct-burial cable laid in narrow trenches which are backfilled after the installation of the cable. The distribution transformers have the parameters defined below:

S_T = transformer capacity, continuously rated kVA

I_{exc} = per unit exciting current (based on S_T)

$P_{T,\text{Fe}}$ = transformer core loss at rated voltage and rated frequency, kW

$P_{T,\text{Cu}}$ = transformer copper loss at rated kVA load, kW

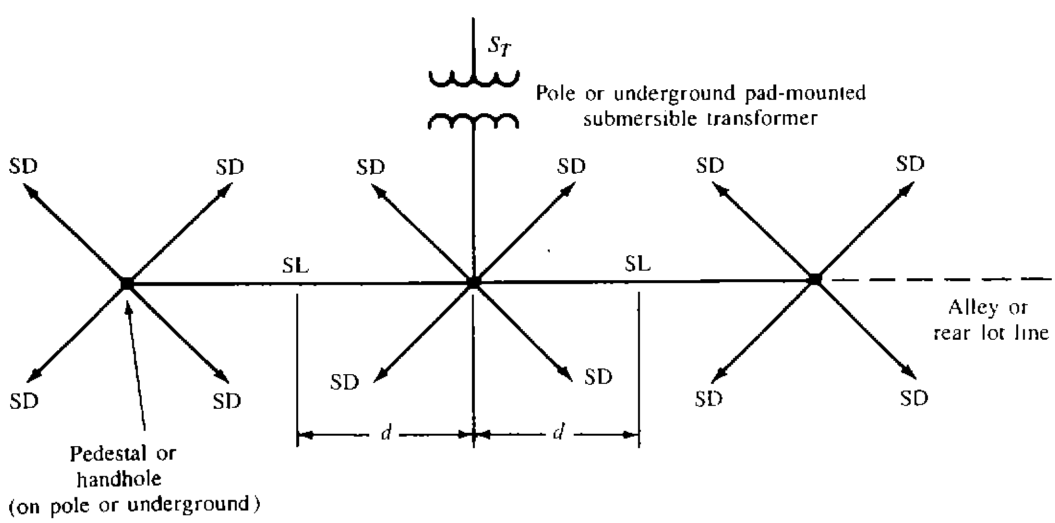


Figure 6-10 Illustration of a typical pattern.

The SL has the parameters defined below:

$$A_{\mathrm{SL}} = \text{conductor area, kcmil}$$

$$\rho = \text{conductor resistivity, } (\Omega\cdot\text{cmil})/\text{ft}$$

$$= 20.5 \text{ at } 65°\text{C for aluminum cable}$$

The SD have the parameters A_{SD} and ρ with meanings that correspond to those given for secondary lines.

6-7-2 Further Assumptions

1. All secondaries and services are single-phase three-wire and nominally 120/240 V.
2. Perfectly balanced loading obtains in all three-wire circuits.
3. The system is energized 100 percent of the time, that is, 8760 h/yr.
4. The annual loss factor is estimated by using Eq. (2-40), that is,

$$F_{\mathrm{LS}} = 0.3F_{\mathrm{LD}} + 0.7F_{\mathrm{LD}}^2 \tag{2-40}$$

5. The annual peak-load kilovoltampere loading in any element of the pattern, i.e., SD, section of SL, or transformer, is estimated by using the maximum diversified demand of the particular number of customers located downstream from the circuit element in question. This point is illustrated later.
6. Current flows are estimated in kilovoltamperes and nominal operating voltage, usually 240 V.
7. All loads have the same (and constant) power factor.

6-7-3 The General TAC Equation

The total annual cost (TAC) of owning and operating one pattern of the secondary system is a summation of investment (fixed) costs (IC) and operating (variable) costs (OC). The costs to be considered are contained in the equation below.

$$\begin{aligned}\mathrm{TAC} = {} & \sum \mathrm{IC}_T + \sum \mathrm{IC}_{\mathrm{SL}} + \sum \mathrm{IC}_{\mathrm{SD}} + \sum \mathrm{IC}_{\mathrm{PH}} + \sum \mathrm{OC}_{\mathrm{exc}} \\ & + \sum \mathrm{OC}_{T,\mathrm{Fe}} + \sum \mathrm{OC}_{T,\mathrm{Cu}} + \sum \mathrm{OC}_{\mathrm{SL,Cu}} + \sum \mathrm{OC}_{\mathrm{SD,Cu}}\end{aligned} \tag{6-2}$$

The summations are to be taken for the one standard pattern being considered, like Fig. 6-10, but modified appropriately for the number of spans of SL being considered. It is apparent that the TAC so found may be divided by the number of customers per pattern so that the TAC can be allocated on a *per-customer* basis.

6-7-4 Illustrating the Assembly of Cost Data

The following cost data are sufficient for illustrative purposes but not necessarily of the accuracy required for engineering design in commercial practice. Some of the cost data given may be quite inaccurate because of recent, severe inflation. The data are intended to represent an OH system using three-conductor triplex aluminum cable for both secondary lines and service drops. The important aspect of the following procedures is the finding of equations for all costs so that analytical methods can be employed to minimize the TAC.

1. IC_T = annual installed cost of distribution transformer + associated protective equipment
   $= (250 + 7.26 \times S_T)i$ $/transformer (6-3)
   where $15\ \mathrm{kVA} \leq S_T \leq 100\ \mathrm{kVA}$
   S_T = transformer-rated kVA
2. $\mathrm{IC}_{\mathrm{SL}}$ = annual installed cost of triplex aluminum SL cable
   $= (60 + 4.50 \times A_{\mathrm{SL}})i$ $/1000 ft (6-4)
   where A_{SL} = conductor area, kcmil
   i = pu fixed charge rate on investment
   Note that this cost is 1000 ft of *cable*, that is, 3000 feet of conductor.
3. $\mathrm{IC}_{\mathrm{SD}}$ = annual installed cost of triplex aluminum SD cable
   $= (60 + 4.50 \times A_{\mathrm{SD}})i$ $/1000 ft (6-5)
   In this example Eqs. (6-4) and (6-5) are alike because the same material, i.e., triplex aluminum cable, is assumed to be used for both SL and SD construction.
4. $\mathrm{IC}_{\mathrm{PH}}$ = annual installed cost of pole and hardware on it, but excluding transformer and transformer protective equipment
   $= \$160i$ $/pole (6-6)
   in case of URD design, the cost item $\mathrm{IC}_{\mathrm{PH}}$ would designate the annual installed cost of a secondary pedestal or handhole.
5. $\mathrm{OC}_{\mathrm{exc}}$ = annual operating cost of transformer exciting current
   $= I_{\mathrm{exc}} \times S_T \times \mathrm{IC}_{\mathrm{cap}} \times i$ $/transformer (6-7)
   where $\mathrm{IC}_{\mathrm{cap}}$ = total installed cost of primary-voltage shunt capacitors
   = $5.00/kvar
   I_{exc} = an average value of transformer exciting current based on S_T kVA rating
   = 0.015 pu
6. $\mathrm{OC}_{T,\mathrm{Fe}}$ = annual operating cost of transformer due to core (iron) losses
   $= (\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{off}})P_{T,\mathrm{Fe}}$ $/transformer (6-8)
   where $\mathrm{IC}_{\mathrm{sys}}$ = average investment cost of power system upstream, i.e., toward generator, from distribution transformers
   = $350/kVA
   $\mathrm{EC}_{\mathrm{off}}$ = incremental cost of electric energy (off-peak)
   = $0.008/kWh
   $P_{T,\mathrm{Fe}}$ = annual transformer core loss, kW
   $= 0.004 \times S_T$ $\quad 15\ \mathrm{kVA} \leq S_T \leq 100\ \mathrm{kVA}$
7. $\mathrm{OC}_{T,\mathrm{Cu}}$ = annual operating cost of transformer due to copper losses

$$= (\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{on}} \times F_{\mathrm{LS}})\left(\frac{S_{\max}}{S_T}\right)^2 P_{T,\mathrm{Cu}} \qquad \$/\text{transformer} \tag{6-9}$$

where EC_{on} = incremental cost of electric energy (on-peak)
= \$0.010/kWh
S_{max} = annual maximum kVA demand on transformer
$P_{T,Cu}$ = transformer copper loss, kW at rated kVA load

$$= 0.073 + 0.00905 \times S_T \quad \text{where } 15\text{ kVA} \leq S_T \leq 100\text{ kVA} \tag{6-10}$$

F_{LS} = annual loss factor

8. $OC_{SL,Cu}$ = annual operating cost of copper loss in a unit length of SL

$$= (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS})P_{SL,Cu} \tag{6-11}$$

where $P_{SL,Cu}$ = power loss in a unit of SL at time of annual peak load due to copper losses, kW

$P_{SL,Cu}$ is an I^2R loss, and it must be related to conductor area A_{SL} with $R = \rho L/A_{SL}$. One has to decide carefully whether L should represent length of conductor or length of cable. When establishing $\sum OC_{SL,Cu}$ for the particular pattern being used, one has to remember that different sections of secondary lines may have different values of current and, therefore, different $P_{SL,Cu}$.

9. $OC_{SD,Cu}$ = annual operating cost of copper loss in a unit length of SD. $OC_{SD,Cu}$ is handled like $OC_{SL,Cu}$ as described in Eq. (6-11). When developing $\sum OC_{SD,Cu}$, it is important to relate $P_{SD,Cu}$ properly to the total length of service drops in the entire pattern.

6-7-5 Illustrating the Estimation of Circuit Loading

The simplifying assumptions 5 and 6 above describe one method for estimating the loading of each element of the pattern. It is important to find reasonable estimates for the current loads in each SD, in each section of SL, and in the transformer so that reasonable approximations will be used for the copper-loss costs $OC_{T,Cu}$, $\sum OC_{SL,Cu}$, and $\sum OC_{SD,Cu}$.

To proceed, it is necessary to have data for the annual maximum diversified kilovoltampere demand per customer vs. the number of customers being diversified. The illustrative data tabulated in Table 6-3 have been taken from Lawrence, Reps, and Patton's paper entitled "Distribution System Planning Through Optimized Design, I—Distribution Transformers and Secondaries" [9, Fig. 3]. As explained in that paper, the maximum diversified demand data were developed with the appliance diversity curves and the hourly variation factors.

It is apparent that the data could be plotted and the demand per customer for intermediate numbers of customers could then be read from the curve. Alternately, if a digital computer is programmed to perform the work described here, a linear interpolation might reasonably be used to estimate the per-customer demand for intermediate numbers of customers.

Figure 6-11 shows a pattern having two secondary lines each way from the transformer. The reader can apply the foregoing data and with linear interpolation find the flows shown in Fig. 6-11. The nominal voltage used is 240 V.

Table 6-3 Illustrative load data*

No. of customers being diversified	Ann. max. demand, kVA/customer
1	5.0
2	3.8
4	3.0
8	2.47
10	2.2
20	2.1
30	2.0
100	1.8

* From [9, fig. 3].

6-7-6 The Developed Total Annual Cost Equation

Upon expanding all the cost items 1 to 9 in Sec. 6-7-4, taking the correct summations for the pattern being used, and introducing the results into Eq. (6-2), one finds that

$$\mathrm{TAC} = A + \frac{B}{S_T^2} + \frac{C}{S_T} + D \times S_T + E \times A_{\mathrm{SD}} + \frac{F}{A_{\mathrm{SD}}} + G \times A_{\mathrm{SL}} + \frac{H}{A_{\mathrm{SL}}} \tag{6-12}$$

In Eq. (6-12), the coefficients A to H are numerical constants. It is important to note that TAC has been reduced to a function of three design variables, that is,

$$\mathrm{TAC} = f(S_T, A_{\mathrm{SD}}, \text{and } A_{\mathrm{SL}}) \tag{6-13}$$

However, one has to remember that many parameters, such as the fixed charge rate i, transformer core and copper losses, installed costs of poles and lines, are contained in coefficients A to H. It should be further noted that the variables

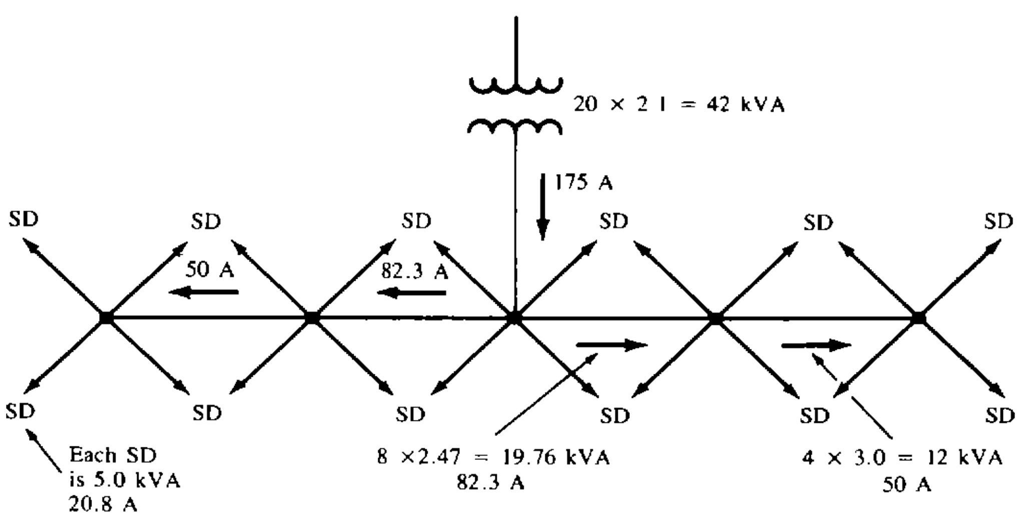


Figure 6-11 Estimated circuit loading for copper-loss determinations.

S_T, A_{SD}, and A_{SL} are in fact discrete variables. They are not continuous variables. For example, if theory indicates that $S_T = 31$ kVA is the optimum transformer size, the designer must choose rather arbitrarily between the standard commercial sizes of 25 and 37.5 kVA. The same ideas apply to conductor sizes for A_{SL} and A_{SD}.

6-7-7 Minimization of the Total Annual Costs

One may commence by using Eq. (6-12), taking three partial derivatives, and setting each derivative to zero:

$$\frac{\partial(\mathrm{TAC})}{\partial S_T} = 0 \tag{6-14}$$

$$\frac{\partial(\mathrm{TAC})}{\partial A_{SL}} = 0 \tag{6-15}$$

$$\frac{\partial(\mathrm{TAC})}{\partial A_{SD}} = 0 \tag{6-16}$$

The work required by Eq. (6-14) is formidable. The roots of a cubic must be found. At this point one has the minimum TAC if only S_T is varied, and similarly for only A_{SL} and A_{SD} variables. There is no assurance that the true, grant minimum of TAC will be achieved if the results of Eqs. (6-14) to (6-16) are applied simultaneously.

Having in fact discrete variables in this problem, one now discards continuous-variable methods. The results of Eqs. (6-14) to (6-16) are used henceforth merely as indicators of the region that contains the minimum TAC achievable with standard commercial equipment sizes. The problem is continued by computing TAC for the standard commercial sizes of equipment nearest to the results of Eqs. (6-14) to (6-16) and then for one (or more?) standard sizes both larger and smaller than those indicated by Eqs. (6-14) to (6-16).

The results at this point are a reasonable number of computed TAC values, all close to the idealized, continuous variable TAC. Designers can easily scan these final few TAC results and select the (S_T, A_{SL}, and A_{SD}) combinations that they think best.

6-7-8 Other Constraints

There are additional criteria which must be met in the total design of the distribution system, whether or not minimum TAC is realized. The further criteria involve quality of utility service. Minimum TAC designs may be encountered which will violate one or more of the commonly used criteria:

1. A minimum allowable steady-state voltage at the most remote service entrance may have been set by law, public utility commission order, or company policy.

2. A maximum allowable motor-starting voltage dip at the most remote service entrance similarly may have been established.
3. Ordinarily the ampacity of no section of secondary lines or service drops should be exceeded by the designer.
4. The maximum allowable distribution transformer loading, in per unit of the transformer continuous rating, should not be exceeded by the designer.

Example 6-1 This example deals with the costs of a single-phase overhead secondary distribution system in a residential area. Figures 6-12 and 6-13 show the layouts and the service arrangement to be considered. Note that equal lot widths, hence uniform load spacings, are assumed. All service drops are assumed to be 70 ft long. The calculations should be done for *one block* of the residential area.

In case of overhead secondary distribution system, assume that there are 12 services per transformers, i.e., there are two transformers per block which are at poles 2 and 5, as shown in Fig. 6-12.

Subsequent problems of succeeding chapters will deal with the voltage-drop constraints which are used to set a minimum standard of quality of service. Naturally it is hoped that a design for satisfactory voltage-drop performance will agree at least reasonably well with the design for minimum total annual cost.

Table 6-4 gives load data to be used in this example problem. Use 30-min annual maximum demands for customer class 2 for this problem.

Use the following data and assumptions:

1. All secondaries and services are single-phase three-wire, nominally 120/240 V.

Table 6-4 Load data for Example 6-1*

No. of customers being diversified	30-min ann. max. demands, kVA/customer		
	Class 1	Class 2	Class 3
1	18.0	10.0	2.5
2	14.4	7.6	1.8
4	12.0	6.0	1.5
12	10.0	4.4	1.2
100	8.4	3.6	1.1

* Data based on figure 3 of Ref. 9. The kilovolt-ampere demands cited have been doubled arbitrarily in an effort to modernize the data. It is explained in the reference cited that the original maximum demand data were developed from appliance diversity curves and hourly variation factors.

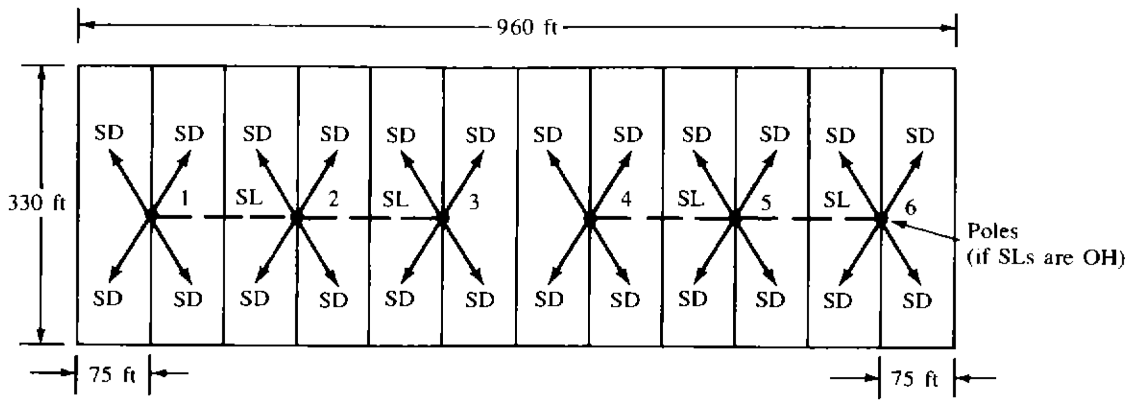


Figure 6-12 Residential area lot layout and service arrangement.

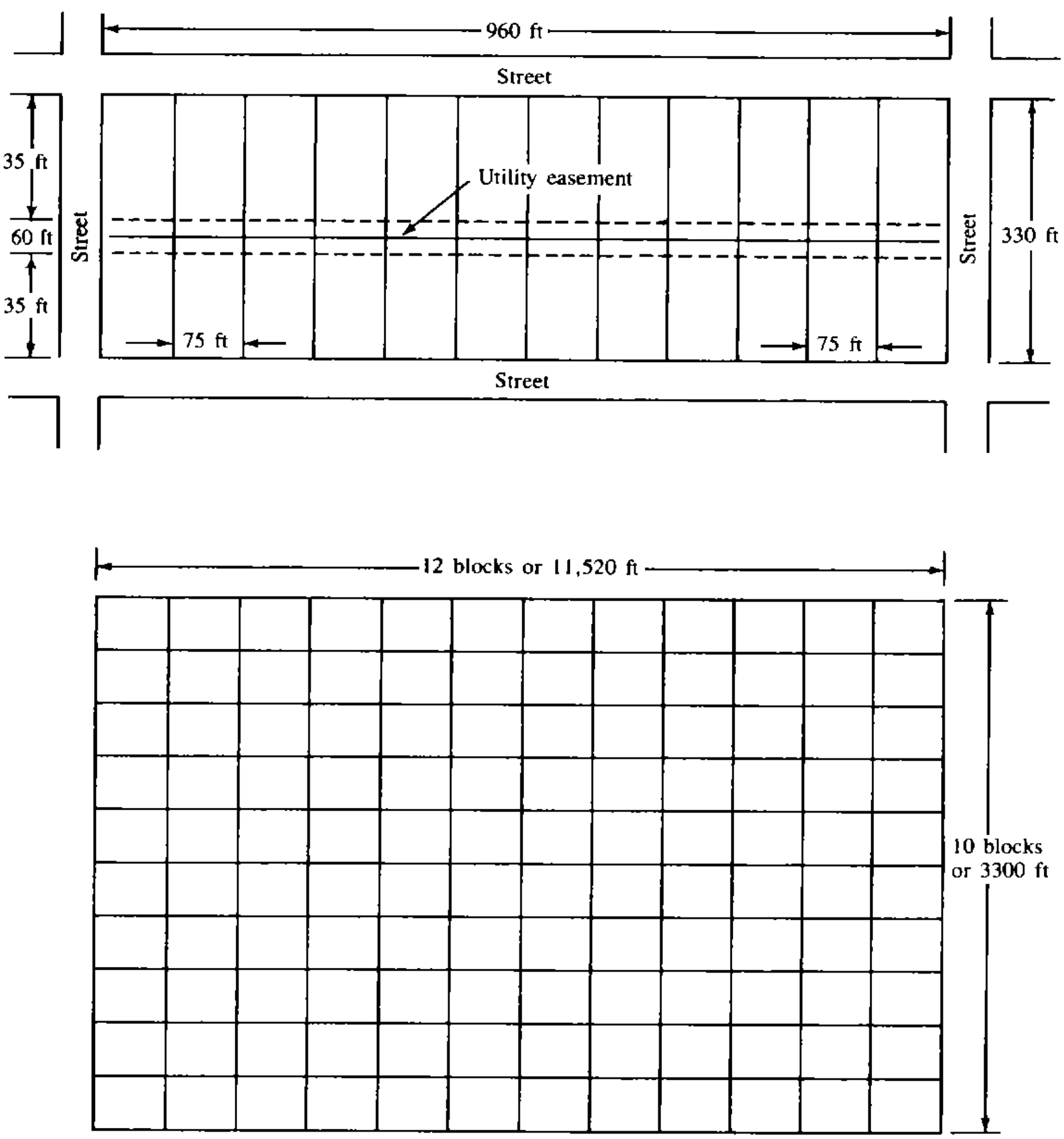


Figure 6-13 Residential area lot layout and utility easement arrangement.

2. Assume perfectly balanced loading in all single-phase three-wire circuits.
3. Assume that the system is energized 100 percent of the time, that is, 8760 h/yr.
4. Assume the annual load factor to be $F_{\text{LD}} = 0.35$.
5. Assume the annual loss factor to be

$$F_{\text{LS}} = 0.3F_{\text{LD}} + 0.7F_{\text{LD}}^2$$

6. Assume that the annual peak-load copper losses are properly evaluated ($\sum I^2R$) by applying the given class 2 loads as:
   a. One consumer per service drop
   b. Four consumers per section of secondary line
   c. Twelve consumers per transformer
   Here, $P_{\text{SL,Cu}}$ is an I^2R loss, and it must be related to conductor area A_{SL} with

$$R = \frac{\rho \times L}{1000 \times A_{\text{SL}}}$$

   where A_{SL} = conductor area, kcmil
   ρ = 20.5 ($\Omega\cdot$cmil)/ft at 65°C for aluminum cable
   L = length of conductor wire involved (not cable length)
   (The designer must be careful to establish a correct relation between $\sum \text{OC}_{\text{SL,Cu}}$, that is, the annual operating cost per block, and the amount of secondary line for which $P_{\text{SL,Cu}}$ is evaluated.)
7. Assume nominal operating voltage of 240 V when computing currents.
8. Assume a 90 percent power factor for all loads.
9. Assume a fixed charge (capitalization) rate of 0.15.

Using the given data and assumptions, develop a numerical total annual cost equation applicable to one block of these residential areas for the case of 12 services per transformer, i.e., two transformers per block. The equation should contain the variables of S_T, A_{SD}, and A_{SL}. Also determine the following:

(*a*) The most economical SD size (A_{SD}) and the nearest larger standard AWG wire size.

(*b*) The most economical SL size (A_{SL}) and the nearest larger standard AWG wire size.

(*c*) The most economical distribution transformer size (S_T) and the nearest larger standard transformer size.

(*d*) The total annual cost per block for the theoretically most economical sizes of equipment.

(*e*) The total annual cost per block for the nearest larger standard commercial sizes of equipment.

(*f*) The total annual cost per block for the nearest larger transformer size and for the second larger sizes of A_{SD} and A_{SL}.

(*g*) Fixed charges per customer per month for the design using the nearest larger standard commercial sizes of equipment.

(*h*) The variable (operating) costs per customer per month for the design using the nearest larger standard commercial sizes of equipment.

SOLUTION From Eq. (6-2), the total annual cost is

$$\mathrm{TAC} = \sum \mathrm{IC}_T + \sum \mathrm{IC}_{\mathrm{SL}} + \sum \mathrm{IC}_{\mathrm{SD}} + \sum \mathrm{IC}_{\mathrm{PH}} + \sum \mathrm{OC}_{\mathrm{exc}} + \sum \mathrm{OC}_{T,\mathrm{Fe}} + \sum \mathrm{OC}_{T,\mathrm{Cu}} + \sum \mathrm{OC}_{\mathrm{SL,Cu}} + \sum \mathrm{OC}_{\mathrm{SD,Cu}} \quad (6\text{-}2)$$

Since there are two transformers per block and 12 services per transformer, from Eq. (6-3) the annual installed cost of the two distribution transformers and associated protective equipment is

$$\begin{aligned} \mathrm{IC}_T &= 2(250 + 7.26 \times S_T)i \\ &= 2(250 + 7.26 \times S_T)0.15 \\ &= 75 + 2.178 S_T \qquad \$/\text{block} \end{aligned} \quad (6\text{-}17)$$

From Eq. (6-4), the annual installed cost of the triplex aluminum cable used for 300 ft per transformer (since there is 150 ft SL at each side of each transformer) in the secondary lines is

$$\begin{aligned} \mathrm{IC}_{\mathrm{SL}} &= 2(60 + 4.50 \times A_{\mathrm{SL}})i \\ &= 2(60 + 4.50 \times A_{\mathrm{SL}}) \times 0.15 \times \frac{300 \text{ ft/trf}}{1000 \text{ ft}} \\ &= 5.4 + 0.405 A_{\mathrm{SL}} \qquad \$/\text{block} \end{aligned} \quad (6\text{-}18)$$

From Eq. (6-5) the annual installed cost of triplex aluminum 24-service drops per block (each SD is 70 ft long) is

$$\begin{aligned} \mathrm{IC}_{\mathrm{SD}} &= 2(60 + 4.50 \times A_{\mathrm{SD}})i \\ &= 2(60 + 4.50 \times A_{\mathrm{SD}}) \times 0.15 \times \frac{12 \times 70 \text{ ft/SD}}{1000 \text{ ft}} \\ &= 15.12 + 1.134 A_{\mathrm{SD}} \qquad \$/\text{block} \end{aligned} \quad (6\text{-}19)$$

From Eq. (6-6), the annual cost of pole and hardware for the six poles per block is

$$\begin{aligned} \mathrm{IC}_{\mathrm{PH}} &= \$160 \times i \times 6 \text{ poles/block} \\ &= \$160 \times 0.15 \times 6 \\ &= \$144/\text{block} \end{aligned} \quad (6\text{-}20)$$

From Eq. (6-7), the annual operating cost of transformer exciting current per block is

$$\begin{aligned} \mathrm{OC}_{\mathrm{exc}} &= 2I_{\mathrm{exc}} \times S_T \times \mathrm{IC}_{\mathrm{cap}} \times i \\ &= 2(0.015) \times S_T \times \$5/\text{kvar} \times 0.15 \\ &= 0.0225\, S_T \qquad \$/\text{block} \end{aligned} \quad (6\text{-}21)$$

From Eq. (6-8), the annual operating cost of core (iron) losses of the two transformers per block is

$$
\begin{aligned}
\mathrm{OC}_{T,\mathrm{Fe}} &= 2(\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{off}})0.004 \times S_T \\
&= 2(\$350/\mathrm{kVA} \times 0.15 + 8760 \times \$0.008/\mathrm{kWh})0.004 \times S_T \\
&= 0.98 S_T \qquad \$/\mathrm{block} \qquad (6\text{-}22)
\end{aligned}
$$

From Eq. (6-9), the annual operating cost of transformer copper losses of the two transformers per block is

$$\mathrm{OC}_{T,\mathrm{Cu}} = 2(\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{on}} \times F_{\mathrm{LS}})\left(\frac{S_{\max}}{S_T}\right)^2 P_{T,\mathrm{Cu}}$$

where

$$
\begin{aligned}
F_{\mathrm{LS}} &= 0.3F_{\mathrm{LD}} + 0.7F_{\mathrm{LD}}^2 \\
&= 0.70(0.35)^2 + 0.30(0.35) \\
&= 0.1904 \\
S_{\max} &= 12 \text{ customers/trf} \times 4.4 \text{ kVA/customer} \\
&= 52.8 \text{ kVA/transformer}
\end{aligned}
$$

Here, the figure of 4.4 kVA/customer is found from Table 6-4 for 12 class 2 customers.

From Eq. (6-10), the transformer copper loss in kilowatts at rated kilovoltampere load is found as

$$P_{T,\mathrm{Cu}} = 0.073 + 0.00905 S_T$$

Therefore

$$
\begin{aligned}
\mathrm{OC}_{T,\mathrm{Cu}} &= 2[(\$350/\mathrm{kVA}) \times 0.15 + 8760 \times (\$0.01/\mathrm{kWh}) \times 0.1904] \\
&\quad \times \left(\frac{52.8\mathrm{kVA/trf}}{S_T}\right)^2 (0.073 + 0.00905 \times S_T) \\
&= \frac{28{,}170}{S_T^2} + \frac{3492}{S_T} \qquad \$/\mathrm{block} \qquad (6\text{-}23)
\end{aligned}
$$

From Eq. (6-11), the annual operating cost of copper losses in the four secondary lines is

$$\mathrm{OC}_{\mathrm{SL,Cu}} = 2(\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{on}} \times F_{\mathrm{LS}})P_{\mathrm{SL,Cu}}$$

where $P_{\mathrm{SL,Cu}}$ = copper losses in two secondary lines at time of annual peak load, kW/transformer (see Fig. 6-14)

$$= I^2 \times R$$

where

$$
\begin{aligned}
R &= \frac{\rho \times L}{1000 \times A_{\mathrm{SL}}} \\
&= \frac{20.5(\Omega \cdot \mathrm{cmil})/\mathrm{ft} \times 300 \text{ ft wire} \times 2}{1000 \times A_{\mathrm{SL}}} \\
&= \frac{12.3}{A_{\mathrm{SL}}} \qquad (\Omega \cdot \mathrm{kcmil})/\mathrm{transformer}
\end{aligned}
$$

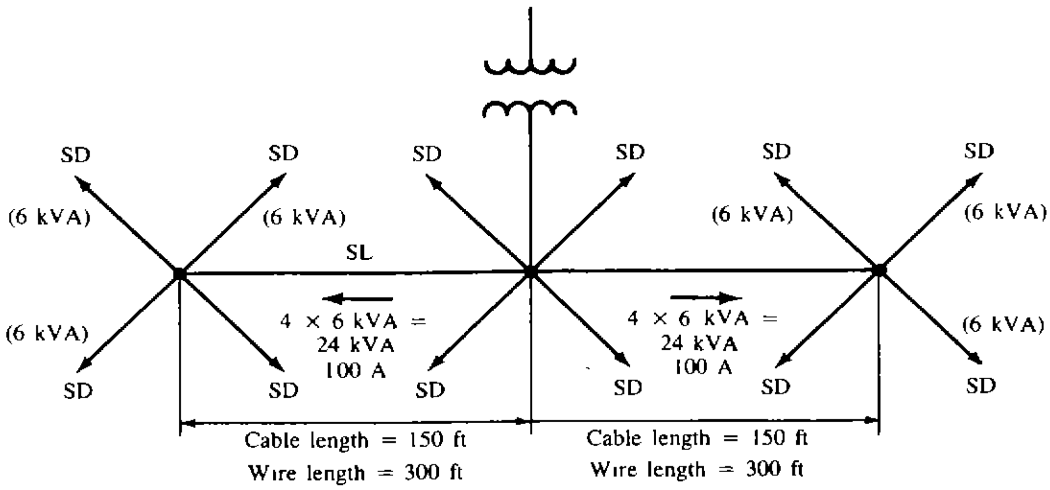


Figure 6-14

Therefore

$$P_{\mathrm{SL,Cu}} = \left(\frac{24\ \mathrm{kVA}}{240\ \mathrm{V}}\right)^2 \times \frac{12.3}{A_{\mathrm{SL}}} \times \frac{1}{1000}$$

$$= \frac{123}{A_{\mathrm{SL}}} \qquad \mathrm{kW/transformer}$$

Thus

$$\mathrm{OC}_{\mathrm{SL,Cu}} = 2[(\$350/\mathrm{kVA}) \times 0.15 + 8760 \times (\$0.01/\mathrm{kWh}) \times 0.1904]\frac{123}{A_{\mathrm{SL}}}$$

$$= \frac{17{,}018}{A_{\mathrm{SL}}} \qquad \$/\mathrm{block} \tag{6-24}$$

Also from Eq. (6-11), the annual operating cost of copper losses in the 24 service drops is

$$\mathrm{OC}_{\mathrm{SD,Cu}} = (\mathrm{IC}_{\mathrm{sys}} \times i + 8760 \times \mathrm{EC}_{\mathrm{on}} \times F_{\mathrm{LS}})P_{\mathrm{SD,Cu}}$$
$$= (69.179)P_{\mathrm{SD,Cu}}$$

where $P_{\mathrm{SD,Cu}}$ = copper losses in the 24 secondary drops at the time of annual peak load, kW
$= I^2 \times R$

where

$$R = \frac{\rho \times L}{1000 \times A_{\mathrm{SD}}}$$

$$= \frac{20.5(\Omega \cdot \mathrm{cmil/ft})(70\ \mathrm{ft}) \times (24\ \mathrm{SD/block}) \times (2\ \mathrm{wires/SD})}{1000 \times A_{\mathrm{SD}}}$$

$$= \frac{68.88}{A_{\mathrm{SD}}} \qquad (\Omega \cdot \mathrm{kcmil})/\mathrm{block}$$

From Table 6-4, the 30-min annual maximum demand for one service drop per one class 2 customer can be found as 10 kVA. Therefore

$$P_{\text{SD,Cu}} = \left(\frac{10\ kVA}{240}\right)^2 \times \frac{68.88}{A_{\text{SD}}} \times \frac{1}{1000}$$

$$= \frac{119.58}{A_{\text{SD}}} \quad \text{kW/block}$$

Thus

$$\text{OC}_{\text{SD,Cu}} = 69.179 \times \frac{119.58}{A_{\text{SD}}}$$

$$\cong \frac{8273}{A_{\text{SD}}} \quad \text{\$/block} \tag{6-25}$$

Substituting Eqs. (6-17) to (6-25) into Eq. (6-2), the total annual cost equation can be found as

$$\text{TAC} = (75 + 2.178 \times S_T) + (5.4 + 0.405 \times A_{\text{SL}}) + (15.12 + 1.134 \times A_{\text{SD}})$$
$$+ (144 + 0.0225 \times S_T) + (0.98 \times S_T) + \left(\frac{28{,}170}{S_T^2} + \frac{3492}{S_T}\right)$$
$$+ \frac{17{,}108}{A_{\text{SL}}} + \frac{8273}{A_{\text{SD}}}$$

After simplifying,

$$\text{TAC} = 239.52 + 3.1805 \times S_T + \frac{3{,}492}{S_T} + \frac{28{,}170}{S_T^2} + 0.405$$
$$\times A_{\text{SL}} + \frac{17{,}018}{A_{\text{SL}}} + 1.134 \times A_{\text{SD}} + \frac{8273}{A_{\text{SD}}} \tag{6-26}$$

(*a*) By partially differentiating Eq. (6-26) with respect to A_{SD} and equating the resultant to zero,

$$\frac{\partial(\text{TAC})}{\partial A_{\text{SD}}} = 1.134 - \frac{8273}{A_{\text{SD}}^2} = 0$$

from which the most economical service-drop size can be found as

$$A_{\text{SD}} = \left(\frac{8273}{1.134}\right)^{1/2}$$

$$= 85.41 \text{ kcmil}$$

Therefore, the nearest larger standard AWG wire size can be found from tables as 1/0, that is, 105,500 cmil.

(*b*) Similarly, the most economical secondary-line size can be found from

$$\frac{\partial(\mathrm{TAC})}{\partial A_{\mathrm{SL}}} = 0.405 - \frac{17{,}018}{A_{\mathrm{SL}}^2} = 0$$

as

$$A_{\mathrm{SL}} = \left(\frac{17{,}018}{0.405}\right)^{1/2}$$

$$= 204.99 \text{ kcmil}$$

Therefore the nearest larger AWG wire size is 4/0, that is, 211.6 kcmil.

(*c*) The most economical distribution transformer size can be found from

$$\frac{\partial(\mathrm{TAC})}{\partial S_T} = 3.1805 - \frac{3492}{S_T^2} - \frac{56{,}340}{S_T^3} = 0$$

as

$$S_T \cong 39 \text{ KVA}$$

Therefore the nearest larger standard transformer size is 50 kVA.

(*d*) By substituting the found values of A_{SD}, A_{SL}, and S_T into Eq. (6-26), the total annual cost per block for the theoretically most economical sizes of equipment can be found as

$$\mathrm{TAC} = 239.52 + 3.1805(39) + \frac{3492}{39} + \frac{28{,}170}{(39)^2} + 0.405(204.99)$$

$$+ \frac{17{,}018}{204.99} + 1.134(85.41) + \frac{8273}{85.41}$$

$$\cong \$838/\text{block}$$

(*e*) By substituting the found standard values of A_{SD}, A_{SL}, and S_T into Eq. (6-26), the total annual cost per block for the nearest larger standard commercial sizes of equipment can be found as

$$\mathrm{TAC} = 239.52 + 3.1805(50) + \frac{3492}{50} + \frac{28{,}170}{(50)^2} + 0.405(211.6)$$

$$+ \frac{17{,}018}{211.6} + 1.134(105.5) + \frac{8273}{105.5}$$

$$\cong \$844/\text{block}$$

(*f*) The second larger sizes of A_{SD} and A_{SL} are 133.1 kcmil and 250 kcmil, respectively. Therefore

$$\mathrm{TAC} = 239.52 + 3.1805(50) + \frac{3492}{50} + \frac{28{,}170}{(50)^2} + 0.405(250)$$

$$+ \frac{17{,}018}{250} + 1.134(133.1) + \frac{8273}{133.1}$$

$$\cong \$862/\text{block/year}$$

(g) The fixed charges per customer per month for the design using the nearest larger standard commercial sizes of equipment is

$$\text{TAC} = \left(\sum \text{IC}_T + \sum \text{IC}_{\text{SL}} + \sum \text{IC}_{\text{SD}} + \sum \text{IC}_{\text{PH}}\right)$$
$$\times \frac{1}{24 \text{ customers/block} \times 12 \text{ mon/yr}}$$
$$\cong \$1.9225/\text{customer/month}$$

(h) The variable (operating) costs per customer per month for the design using the nearest larger standard commercial sizes of equipment is

$$\text{TAC} = \left(\sum \text{OC}_{\text{exc}} + \sum \text{OC}_{T,\text{Fe}} + \sum \text{OC}_{T,\text{Cu}} + \sum \text{OC}_{\text{SL,Cu}} + \sum \text{OC}_{\text{SD,Cu}}\right)$$
$$\times \frac{1}{24 \times 12}$$
$$= \left[0.0225(50) + 0.98(50) + \frac{28{,}170}{(50)^2} + \frac{3492}{50} + \frac{17{,}018}{211.6} + \frac{8273}{105.5}\right]$$
$$\times \frac{1}{24 \times 12}$$
$$= \$1.0084/\text{customer/month}$$

Note that the fixed charges are larger than the operating costs.

6-8 UNBALANCED LOAD AND VOLTAGES

A single-phase three-wire circuit is regarded as unbalanced if the neutral current is not zero. This happens when the loads connected, e.g., between line and neutral, are not equal. The result is unsymmetrical current and voltages and a nonzero current in the neutral line. In that case, the necessary calculations can be done by using the method of symmetrical components.

Example 6-2 This example and Examples 6-3 and 6-4 deal with the computation of voltages in unbalanced single-phase three-wire secondary circuits, as shown in Fig. 6-15. Here, both the mutual-impedance methods and the flux-linkage methods are applicable as alternative methods for computing the voltage drops in the secondary line. This example deals with the computation of the complex linkages due to the line currents in the conductors a, b, and n. Assume that the distribution transformer used for this single-phase three-wire distribution is rated as 7200/120–240 V, 25 kVA, 60 Hz, and the n_1 and n_2 turns ratios are 60 and 30. As Fig. 6-15 suggests, the two halves of the low-voltage winding of the distribution transformer are independently loaded with unequal secondary loads. Therefore the single-phase three-wire

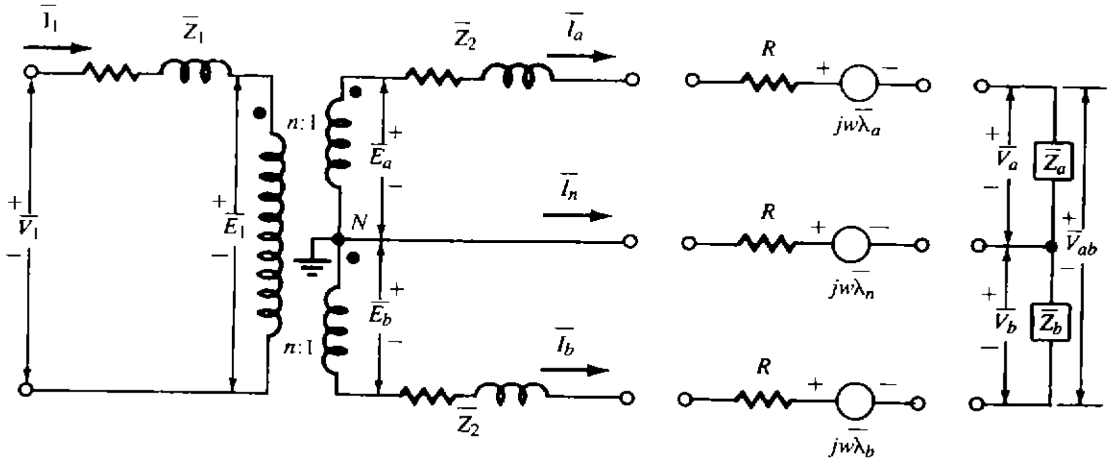


Figure 6-15

secondaries are unbalanced. The vertical spacing between the secondary wires is as illustrated in Fig. 6-16. Assume that the secondary wires are made of #4/0 seven-strand hard-drawn aluminum conductors and 400 ft of line length. Use 50°C resistance in finding the line impedances.

Furthermore, assume that (1) the load impedances $\bar{Z}_a$ and $\bar{Z}_b$ are independent of voltage, (2) the primary-side voltage is $\bar{V}_1 = 7272\underline{/0^\circ}$ V and is maintained constant, and (3) the line capacitances and transformer exciting current are negligible. Use the given information and develop numerical equations for the phasor expressions of the flux linkages $\bar{\lambda}_a$, $\bar{\lambda}_b$, and $\bar{\lambda}_n$ in terms of $\bar{I}_a$ and $\bar{I}_b$. In other words, find the coefficient matrix, numerically, in the equation

$$\begin{bmatrix} \bar{\lambda}_a \\ \bar{\lambda}_b \\ \bar{\lambda}_n \end{bmatrix} = \begin{bmatrix} \text{coefficient} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \end{bmatrix} \tag{6-27}$$

SOLUTION The phasor expressions of the complex flux linkages $\bar{\lambda}_a$, $\bar{\lambda}_b$, and $\bar{\lambda}_n$ due to the line currents in the conductors a, b, and n can be written as[†]

$$\bar{\lambda}_a = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{1}{D_{aa}} + \bar{I}_b \times \ln \frac{1}{D_{ab}} + \bar{I}_n \times \ln \frac{1}{D_{an}}\right) \quad \frac{\text{Wb}\cdot\text{T}}{\text{m}} \tag{6-28}$$

$$\bar{\lambda}_b = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{1}{D_{ab}} + \bar{I}_b \times \ln \frac{1}{D_{aa}} + \bar{I}_n \times \ln \frac{1}{D_{bn}}\right) \quad \frac{\text{Wb}\cdot\text{T}}{\text{m}} \tag{6-29}$$

$$\bar{\lambda}_n = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{1}{D_{na}} + \bar{I}_b \times \ln \frac{1}{D_{nb}} + \bar{I}_n \times \ln \frac{1}{D_{nn}}\right) \quad \frac{\text{Wb}\cdot\text{T}}{\text{m}} \tag{6-30}$$

Since

$$\bar{I}_a + \bar{I}_b + \bar{I}_n = 0 \tag{6-31}$$

[†] The notation "ln" is used for "log to the base e."

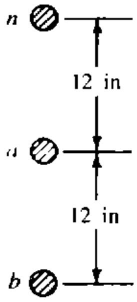


Figure 6-16

the current in the neutral conductor can be written as

$$\bar{I}_n = -\bar{I}_a - \bar{I}_b \tag{6-32}$$

Thus, substituting Eq. (6-32) into Eqs. (6-28) to (6-30),

$$\bar{\lambda}_a = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{D_{an}}{D_{aa}} + \bar{I}_b \times \ln \frac{D_{an}}{D_{ab}}\right) \qquad \frac{\text{Wb} \cdot \text{T}}{\text{m}} \tag{6-33}$$

$$\bar{\lambda}_b = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{D_{bn}}{D_{ab}} + \bar{I}_b \times \ln \frac{D_{bn}}{D_{bb}}\right) \qquad \frac{\text{Wb} \cdot \text{T}}{\text{m}} \tag{6-34}$$

$$\bar{\lambda}_n = 2 \times 10^{-7}\left(\bar{I}_a \times \ln \frac{D_{nn}}{D_{na}} + \bar{I}_b \times \ln \frac{D_{nn}}{D_{nb}}\right) \qquad \frac{\text{Wb} \cdot \text{T}}{\text{m}} \tag{6-35}$$

Therefore, from Eqs. (6-33) to (6-35),

$$\begin{bmatrix} \bar{\lambda}_a \\ \\ \bar{\lambda}_b \\ \\ \bar{\lambda}_c \end{bmatrix} = \begin{bmatrix} 2 \times 10^{-7} \times \ln \dfrac{D_{an}}{D_{aa}} & 2 \times 10^{-7} \times \ln \dfrac{D_{an}}{D_{ab}} \\ 2 \times 10^{-7} \times \ln \dfrac{D_{bn}}{D_{ab}} & 2 \times 10^{-7} \times \ln \dfrac{D_{bn}}{D_{bb}} \\ 2 \times 10^{-7} \times \ln \dfrac{D_{nn}}{D_{na}} & 2 \times 10^{-7} \times \ln \dfrac{D_{nn}}{D_{nb}} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \end{bmatrix} \qquad \frac{\text{Wb} \cdot \text{T}}{\text{m}} \tag{6-36}$$

Thus, from Eq. (6-36), the coefficient matrix can be found numerically as

$$\begin{bmatrix} \text{coefficient} \\ \text{matrix} \end{bmatrix} = \begin{bmatrix} 2 \times 10^{-7} \times \ln \dfrac{1}{0.01577} & 2 \times 10^{-7} \times \ln \dfrac{1}{1} \\ 2 \times 10^{-7} \times \ln \dfrac{2}{1} & 2 \times 10^{-7} \times \ln \dfrac{2}{0.01577} \\ 2 \times 10^{-7} \times \ln \dfrac{0.01577}{1} & 2 \times 10^{-7} \times \ln \dfrac{0.01577}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8.2992 \times 10^{-7} & 0 \\ 1.3862 \times 10^{-7} & 9.6855 \times 10^{-7} \\ -8.2992 \times 10^{-7} & -9.6855 \times 10^{-7} \end{bmatrix} \qquad \frac{\text{Wb} \cdot \text{T}}{\text{m}}$$

Note that the elements in the coefficient matrix can be converted to weber-teslas per foot if they are multiplied by 0.3048 m/ft.

Example 6-3 Assume that, in Example 6-2, $\bar{I}_a$, $\bar{I}_b$, and $\bar{V}_1$ are specified but not the load impedances $\bar{Z}_a$ and $\bar{Z}_b$. Develop symbolic equations that will give solutions for the load voltages $\bar{V}_a$, $\bar{V}_b$, and $\bar{V}_{ab}$ in terms of the voltage $\bar{V}_1$, the impedances, and the flux linkages.

SOLUTION Since the transformation ratio of the distribution transformer is

$$n = \frac{E_1}{E_a} = \frac{E_1}{E_b} = \frac{7200 \text{ V}}{120 \text{ V}} = 60$$

the primary-side current can be written as

$$\bar{I}_1 = \frac{\bar{I}_a - \bar{I}_b}{n} \tag{6-37}$$

Here,

$$\bar{E}_1 = \bar{V}_1 - \bar{I}_1 \bar{Z}_1 \tag{6-38}$$

Substituting Eq. (6-37) into Eq. (6-38),

$$\bar{E}_1 = \bar{V}_1 - \bar{Z}_1 \times \frac{\bar{I}_a - \bar{I}_b}{n} \tag{6-39}$$

Also,

$$\bar{E}_a = \bar{E}_b = \frac{\bar{E}_1}{N} \tag{6-40}$$

Substituting Eq. (6-39) into Eq. (6-40),

$$\bar{E}_a = \bar{E}_b = \frac{\bar{V}_1}{n} - \frac{\bar{Z}_1}{n^2}(\bar{I}_a - \bar{I}_b) \tag{6-41}$$

By writing a loop equation for the secondary side of the equivalent network of Fig. 6-15,

$$-\bar{E}_a + \bar{Z}_2 \bar{I}_a + R\bar{I}_a + jw\bar{\lambda}_a + \bar{V}_a - jw\bar{\lambda}_n + R(\bar{I}_a + \bar{I}_b) = 0 \tag{6-42}$$

Substituting Eq. (6-41) into Eq. (6-42),

$$-\frac{\bar{V}_1}{n} + \frac{\bar{Z}_1}{n^2}(\bar{I}_a - \bar{I}_b) + \bar{Z}_2 \bar{I}_a + R\bar{I}_a + jw\bar{\lambda}_a + \bar{V}_a - jw\bar{\lambda}_n + R(\bar{I}_a + \bar{I}_b) = 0$$

or

$$\bar{V}_a = \frac{\bar{V}_1}{n} + \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_b - \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + 2R\right)\bar{I}_a - jw(\bar{\lambda}_a - \bar{\lambda}_n) \quad (6\text{-}43)$$

Also, by writing a second loop equation,

$$\bar{E}_b + \bar{Z}_2 \bar{I}_b + R\bar{I}_b + jw\bar{\lambda}_b - \bar{V}_b + R(\bar{I}_a + \bar{I}_b) - jw\bar{\lambda}_n = 0 \quad (6\text{-}44)$$

Substituting Eq. (6-41) into Eq. (6-55),

$$\bar{V}_b = \frac{\bar{V}_1}{n} - \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_a + \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + 2R\right)\bar{I}_b + jw(\bar{\lambda}_b - \bar{\lambda}_n) \quad (6\text{-}45)$$

However, from Fig. 6-15,

$$\bar{V}_{ab} = \bar{V}_a + \bar{V}_b \quad (6\text{-}46)$$

therefore, substituting Eqs. (6-43) and (6-45) into Eq. (6-46),

$$\bar{V}_{ab} = 2\frac{\bar{V}_1}{n} - \left(\frac{2\bar{Z}_1}{n^2} + R + \bar{Z}_2\right)\bar{I}_a + \left(\frac{2\bar{Z}_1}{n^2} + R + \bar{Z}_2\right)\bar{I}_b + jw(\bar{\lambda}_b - \bar{\lambda}_a) \quad (6\text{-}47)$$

Example 6-4 Assume that in Example 6-3 the given voltages are

$$\bar{V}_1 = 7272\underline{/0^\circ} \text{ V}$$
$$\bar{E}_a = 120\underline{/0^\circ} \text{ V}$$
$$\bar{E}_b = 120\underline{/0^\circ} \text{ V}$$

and the load impedances are

$$\bar{Z}_a = 0.80 + j0.60\ \Omega$$
$$\bar{Z}_b = 0.80 + j0.60\ \Omega$$

determine the following:
(*a*) The secondary currents $\bar{I}_a$ and $\bar{I}_b$.
(*b*) The secondary neutral current $\bar{I}_n$.
(*c*) The secondary voltages $\bar{V}_a$ and $\bar{V}_b$.
(*d*) The secondary voltage $\bar{V}_{ab}$.

SOLUTION From Eq. (6-43),

$$\bar{V}_a = \bar{I}_a\bar{Z}_a = \frac{\bar{V}_1}{n^2} + \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_b - \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + 2R\right)\bar{I}_a - jw(\bar{\lambda}_a - \bar{\lambda}_n) \quad (6\text{-}43)$$

or

$$\frac{\bar{V}_1}{n} = \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + 2R + \bar{Z}_a\right)\bar{I}_a - \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_b + jw(\bar{\lambda}_a - \bar{\lambda}_n) \quad (6\text{-}48)$$

Similarly, from Eq. (6-45),

$$\bar{V}_b = -\bar{I}_b\bar{Z}_b = \frac{\bar{V}_1}{n} - \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_a + \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + 2R\right)\bar{I}_b + jw(\bar{\lambda}_b - \bar{\lambda}_n)$$

or

$$\frac{\bar{V}_1}{n} = \left(\frac{\bar{Z}_1}{n^2} - R\right)\bar{I}_a - \left(\frac{\bar{Z}_1}{n^2} + \bar{Z}_2 + \bar{Z}_b + 2R\right)\bar{I}_b - jw(\bar{\lambda}_b - \bar{\lambda}_n) \quad (6\text{-}49)$$

Substituting the given values into Eq. (6-48),

$$\frac{7272}{60} = \bar{I}_a\left[\frac{14.5152}{60^2} + j\frac{19.90656}{60^2} + 0.008064 + j0.027648 + 0.8 + j0.6\right.$$

$$\left.+\frac{2(400)(0.486)}{5280}\right] + \bar{I}_b\left[\frac{(400)(0.486)}{5280} - \frac{14.5152}{60^2} - j\frac{19.90656}{60^2}\right]$$

$$+ j377(0.3048)(400) \times 10^{-7}(8.299\bar{I}_a + 8.299\bar{I}_a + 9.686\bar{I}_b)$$

or

$$121.2 = \bar{I}_a(0.8857 + j0.6846) + \bar{I}_b(0.03279 + j0.03899) \quad (6\text{-}50)$$

Also, substituting the given values into Eq. (6-49),

$$\frac{7272}{60} = \bar{I}_a\left[\frac{14.5152}{60^2} + j\frac{19.90656}{60^2} - \frac{(400)(0.486)}{60^2}\right]$$

$$+ \bar{I}_b\left[-0.8 + j0.6 - \frac{14.5152}{60^2} - j\frac{19.90656}{60^2} - 0.008064\right.$$

$$\left.-j0.0027648 - \frac{2(400)(0.486)}{5280}\right] - j(377)(0.3048)(400)$$

$$\times 10^{-7}(1.386\bar{I}_a + 9.686\bar{I}_b + 8.299\bar{I}_a + 9.686\bar{I}_b)$$

or

$$121.2 = \bar{I}_a(-0.03279 - j0.03899) + \bar{I}_b(-0.88574 + j0.50267) \quad (6\text{-}51)$$

Therefore, from Eqs. (6-50) and (6-51),

$$\begin{bmatrix}121.2\\121.2\end{bmatrix} = \begin{bmatrix}0.8857 + j0.6846 & 0.03279 + j0.03899\\ -0.03279 - j0.03899 & -0.88574 + j0.50267\end{bmatrix}\begin{bmatrix}\bar{I}_a\\ \bar{I}_b\end{bmatrix} \quad (6\text{-}52)$$

By solving Eq. (6-52),

$$\begin{bmatrix}\bar{I}_a\\ \bar{I}_b\end{bmatrix} = \begin{bmatrix}89.8347 - j62.393\\ -107.387 - j62.5885\end{bmatrix}\text{A} \quad (6\text{-}53)$$

(*a*) From Eq. (6-53), the secondary currents are

$$\bar{I}_a = 89.8347 - j62.393$$
$$= 109.376\angle{-34.78^\circ}\ \text{A}$$

and

$$\bar{I}_b = -107.387 - j62.5885$$
$$= 124.295\angle{210.24^\circ}\ \text{A}$$

(*b*) Therefore, the secondary neutral current is

$$\bar{I}_n = -\bar{I}_a - \bar{I}_b$$
$$= 17.5523 + j124.9815\ \text{A}$$

(*c*) The secondary voltages are

$$\bar{V}_a = \bar{I}_a \times \bar{Z}_a$$
$$= (109.376\angle{-34.78^\circ})(1\angle{36.87^\circ})$$
$$= 109.376\angle{2.09^\circ}\ \text{V}$$

and

$$\bar{V}_b = -\bar{I}_b \times \bar{Z}_b$$
$$= -(124.295\angle{210.24^\circ})(1\angle{-36.87^\circ})$$
$$= 124.295\angle{-6.63^\circ}\ \text{V}$$

(*d*) Therefore, the secondary voltage $\bar{V}_{ab}$ is

$$\bar{V}_{ab} = \bar{V}_a + \bar{V}_b$$
$$= 109.376\angle{2.09^\circ} + 124.295\angle{-6.63^\circ}$$
$$= 232.997\angle{-2.55^\circ}\ \text{V}$$

Example 6-5 Figure 6-17 shows an ac secondary network which has been adapted from Ref. 3. The loads shown in Fig. 6-17 are in three-phase kilowatts and kilovars, with a lagging power factor of 0.85. The nominal voltage is 208 V. All distribution transformers are rated 500 kVA three-phase, with 4160-V delta high voltage and 125/216-V wye-grounded low voltage. They have leakage impedance Z_T of 0.0086 + j0.0492 pu based on transformer ratings.

All secondary underground mains have copper 3-#4/0 per phase and 3-#3/0 neutral cables in nonmagnetic conduits. The positive-sequence impedance Z_M of 500 ft of main is 0.181 + j0.115 pu on a 1000-kVA base.

All primary feeder circuits are 1.25 mi long. Three single-conductor 500-kcmil 5-kV shielded-copper PE-insulated underground cables are used at 90° conductor temperature. Their impedances within the small area of the network are neglected. The positive-sequence impedance Z_F of the feeder cable is 0.01 + j0.017 pu on a 1000-kVA base for 1.25-mi-long feeders. The approximate ampacities are 473 A for one circuit per duct bank and 402 A for four equally loaded circuits per duct bank.

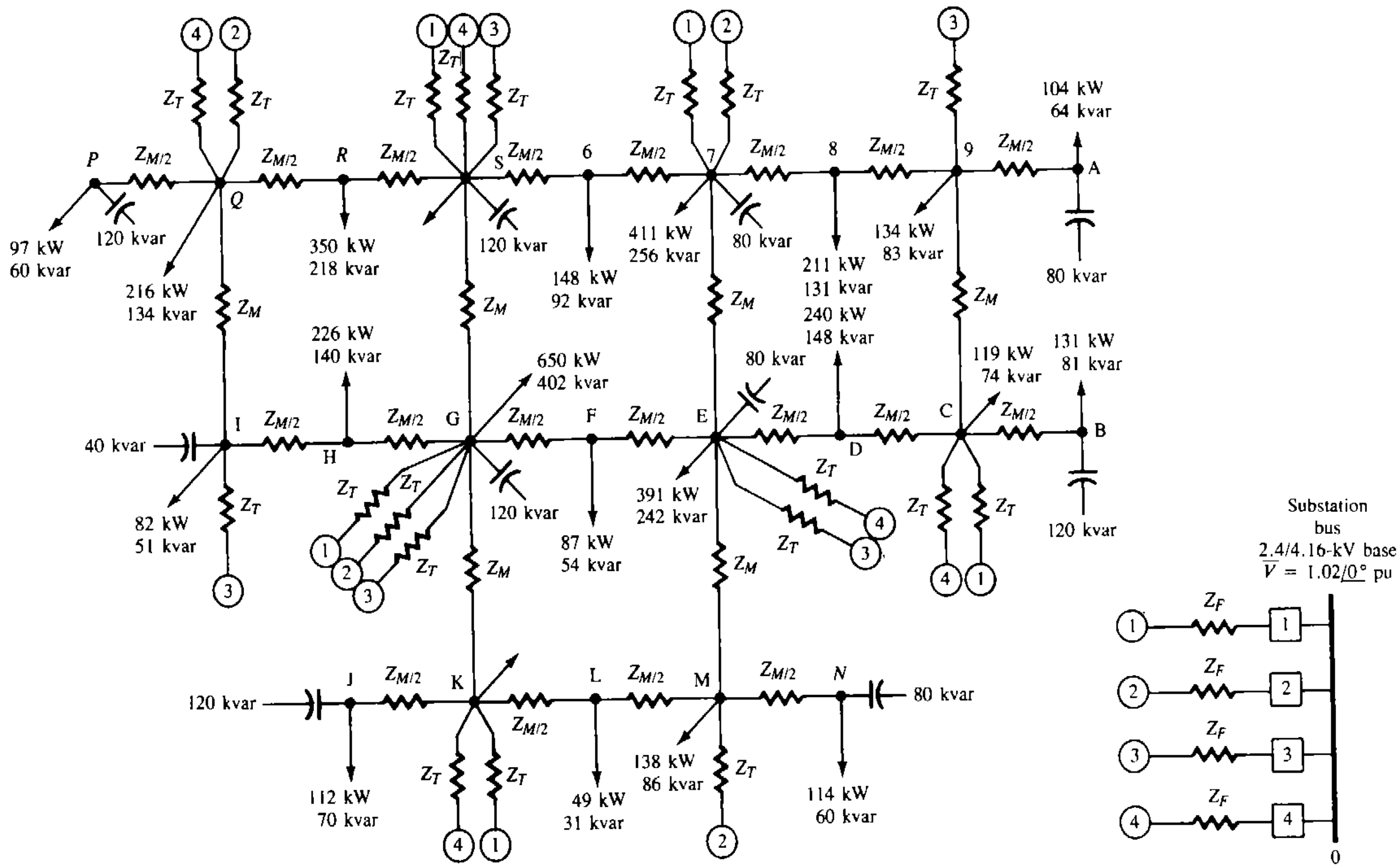


Figure 6-17 (*Adapted from* [3].)

The bases used are (1) three-phase power base of 1000 kVA; (2) for secondaries, 125/216 V, 2666.7 A, 0.04687 Ω; and (3) for primaries, 2400/4160 V, 138.9 A, 17.28 Ω.

The standard 125/216-V network-capacitor sizes used are 40, 80, and 120 kvar. In this study, these capacitors are not switched. Ordinarily it is desired that distribution circuits not get into leading power factor operation during off-peak load periods. Therefore, the total magnetizing vars generated by unswitched shunt capacitors should not exceed the total magnetizing vars taken by the off-peak load. In this example, the total reactive load is 3150 kvar at peak load, and it is assumed that off-peak load is one-third of peak load, or 1050 kvar. Therefore a total capacitor size of 960 kvar has been used. It has been distributed arbitrarily throughout the network in standard sizes, but with the larger capacitor banks generally being located at the larger-load buses and at the ends of radial stubs from the network.

Using the given data, four separate load flow solutions have been obtained for the following operating conditions in the example secondary network:

Case 1: Normal switching. Normal loads, and all shunt capacitors are off.

Case 2: Normal switching. Normal loads, and all shunt capacitors are on.

Case 3: First-contingency outage. Primary feeder 1 is out. Normal loads and all shunt capacitors are on.

Case 4: Second-contingency outage. Primary feeders 1 and 4 are out. Normal loads and all shunt capacitors are on. Note that this second-contingency outage is very severe, causing the largest load (at bus 5) to lose two-thirds of its transformer capacity.

To make a voltage study, Table 6-5 has been developed based on the load flow studies for the four cases. The values given in the table are per unit bus voltage values. Here, the buses selected for the study are the ones located at the ends of radials or else the ones which are badly disturbed by the second-contingency outage of case 4.

Table 6-5 Bus voltage values, pu

Buses	Case 1	Case 2	Case 3	Case 4
A	0.951	0.967	0.954	0.915
B	0.958	0.975	0.955	0.860
C	0.976	0.986	0.966	0.873
J	0.959	0.976	0.954	0.864
K	0.974	0.984	0.962	0.875
N	0.958	0.973	0.963	0.924
P	0.960	0.977	0.966	0.926
R	0.945	0.954	0.938	0.890
S	0.964	0.972	0.951	0.898

Use the given data and determine the following:

(a) If the lowest "favorable" and the lowest "tolerable" voltages are defined as 114 V and 111 V, respectively, what are the pu voltages, based on 125 V, that correspond to the lowest favorable voltage and the lowest tolerable voltage for nominally 120/208Y systems?

(b) List the buses given in Table 6-5, for the first-contingency outage, that have (1) less than favorable voltage and (2) less than tolerable voltage.

(c) List the buses given in Table 6-5, for the second-contingency outage, that have (1) less than favorable voltage and (2) less than tolerable voltage.

(d) Find Z_M/Z_T, $\frac{1}{2}(Z_M/Z_T)$, and using Fig. 6-8, find the value of the "application factor" for this example network and make an approximate judgment about the sufficiency of the design of this network.

SOLUTION

(a) The lowest favorable voltage in per unit is

$$\frac{114 \text{ V}}{125 \text{ V}} = 0.912 \text{ pu}$$

and the lowest tolerable voltage in per unit is

$$\frac{111 \text{ V}}{125 \text{ V}} = 0.888 \text{ pu}$$

(b) There are no buses in Table 6-5, for the first-contingency outage, that have (1) less than favorable voltage or (2) less than tolerable voltage.

(c) For the second-contingency outage, the buses in Table 6-5 that have (1) less than favorable voltage are B, C, J, K, R, and S and (2) less than tolerable voltage are B, C, J, K.

(d) The given transformer impedance of $0.0086 + j0.0492$ pu is based on 500 kVA. Therefore it corresponds to

$$Z_T = 0.0172 + j0.0984 \text{ pu } \Omega$$

which is based on 1000 kVA. Therefore the ratios are

$$\frac{Z_M}{Z_T} = \frac{0.181 + j0.115}{0.0172 + j0.0984}$$
$$= 2.147$$

or

$$\frac{1}{2}\left(\frac{Z_M}{Z_T}\right) = 1.0735$$

Thus, from Fig. 6-8, the corresponding average transformer application factor for four feeders can be found as 1.6. To verify this value for the given design, the actual application factor can be recalculated as

$$\text{Actual application factor} = \frac{\text{total installed network-transformer capacity}}{\text{total load}}$$

$$= \frac{19 \text{ transformers} \times 500 \text{ kVA/transformer}}{5096 + j3158}$$

$$= 1.5846$$

Therefore, the design of this network is sufficient.

PROBLEMS

6-1 Repeat Example 6-1. Assume that there are four services per transformer per block, i.e., one transformer on each pole.

6-2 Repeat Example 6-1. Assume that the annual load factor is 0.65.

6-3 Repeat Prob. 6-1. Assume that the annual load factor is 0.65.

6-4 Consider Prob. 6-1 and find the following:

(*a*) The most economical service-drop size (A_{SD}) and the nearest larger commercial wire size.

(*b*) The most economical secondary-line size (A_{SL}) and the nearest larger standard transformer size.

(*c*) The total annual cost per block for the nearest larger standard sizes of equipment.

6-5 Repeat Example 6-4, assuming the load impedances are

$$Z_a = 1.0 + j0.0\ \Omega$$

and

$$Z_b = 1.5 + j0.0\ \Omega$$

6-6 Repeat Example 6-4, assuming the load impedances are

$$Z_a = 1.0 + j0.0\ \Omega$$

and

$$Z_b = 3.0 + j0.0\ \Omega$$

6-7 Repeat Example 6.4, assuming the load impedances are

$$Z_a = 0.80 + j0.60\ \Omega$$

and

$$Z_b = 1.0 + j0.0\ \Omega$$

6-8 The following table gives the total real and reactive power losses for the secondary network given in Example 6-5. Explain the circumstances which cause minimum and maximum losses. Bear in mind that the total $P + jQ$ power delivered to the loads is identical in all cases.

Case no.	$\sum P_L$, MW	$\sum Q_L$, Mvar
1	0.16379	0.38807
2	0.14160	0.33142
3	0.19263	0.46648
4	0.36271	0.82477

6-9 The following table gives the primary-feeder circuit loading for the primary feeders given in Example 6-5.

Case no.	$P + jQ$, pu MVA			
	Feeder 1	Feeder 2	Feeder 3	Feeder 4
1	$1.3575 - j0.9012$	$1.186 - j0.8131$	$1.3822 - j0.9381$	$1.3341 - j0.8857$
2	$1.3496 - j0.6540$	$1.1854 - j0.5894$	$1.375 - j0.6936$	$1.3278 - j0.6308$
3	Out	$1.5965 - j0.8468$	$1.8427 - j0.952$	$1.8495 - j0.9354$
4	Out	$2.5347 - j1.4587$	$2.924 - j1.7285$	Out

Determine the ampere loads of each feeder and complete the following table.

Case no.	Percent of ampacity rating			
	Feeder 1	Feeder 2	Feeder 3	Feeder 4
1				
2				
3	Out			
4	Out			

6-10 Assume that the following table gives the transformer loading for transformers 1, 3, and 4, using bus S data, for Example 6-5.

Case no.	Transformer loading, kVA		
	Trf. 1	Trf. 3	Trf. 4
1	380.365	374.00	385.450
2	358.475	352.31	363.375
3		509.42	508.921
4		812.61	

Complete the following table. Note that bus S not only has the largest load but also loses two-thirds of its transformer capacity in the event of the second-contingency outage being considered here.

Case no.	Load-in percent of transformer rating		
	Trf. 1	Trf. 3	Trf. 4
1			
2			
3			
4			

6-11 Assume that the following table gives the loading of the secondary mains close to bus S in Example 6-5.

	Loading of secondary mains, pu MVA				
Case no.	S–R	R–Q	S–6	6–7	S–G
1	0.1715	0.2516	0.0699	0.1065	0.0361
2	0.1662	0.2560	0.0692	0.1072	0.0364
3	0.1252	0.3110	0.0816	0.0945	0.0545
4	0.0872	0.3778	0.0187	0.1901	0.1430

Determine the ampere loading of the mains close to bus S and also complete the following table.

	Loading of secondary mains, % of rated ampacity				
Case no.	S–R	R–Q	S–6	6–7	S–G
1					
2					
3					
4					

REFERENCES

1. Gönen, T., et al.: *Development of Advanced Methods For Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va.
2. Westinghouse Electric Corporation: *Electrical Transmission and Distribution Reference Book*, East Pittsburgh, Pa., 1964.
3. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3. East Pittsburgh, Pa., 1965.
4. Davey, J., et al.: "Practical Application of Weather Sensitive Load Forecasting to System Planning," *Proceedings, the IEEE PES Summer Meeting*, San Francisco, Calif., July 9–14, 1972.
5. Chang, N. E.: "Loading Distribution Transformers," *Transmission and Distribution*, no. 26, August 1974, pp. 58–59.
6. Chang, N. E.: "Determination and Evaluation of Distribution Transformer Losses of the Electric System Through Transformer Load Monitoring," *IEEE Trans. Power Appar. Syst.*, vol. PAS-89, July/August 1970, pp. 1282–84.
7. Electric Power Research Institute: *Analysis of Distribution R&D Planning*, EPRI Report 329, Palo Alto, Calif., October 1975.
8. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*, 11th ed., McGraw-Hill, New York, 1978.
9. Lawrence, R. F., D. N. Reps, and A. D. Patton: "Distribution System Planning Through Optimal Design, I—Distribution Transformers and Secondaries," *AIEE Trans.*, pt. III, vol. PAS-79, June 1960, pp. 199–204.

10. Chang, S. H.: "Economic Design of Secondary Distribution System by Computer," M.S. thesis, Iowa State University, Ames, 1974.
11. Robb, D. D.: *ECDES Program User Manual*. Power System Computer Service, Iowa State University, Ames, 1975.
12. Edison Electric Institute–National Electric Manufacturers Association: *EEI-NEMA Standards for Secondary Network Transformers*, EEI Publication no. 57-7, NEMA Publication No. TR4-1957.
13. Seelye, H. P.: *Electrical Distribution Engineering*. 1st ed., McGraw-Hill, New York, 1930.

CHAPTER
SEVEN

VOLTAGE-DROP AND POWER-LOSS CALCULATIONS

7-1 THREE-PHASE BALANCED PRIMARY LINES

As discussed in Chap. 5, a utility company strives to achieve a well-balanced distribution system in order to improve system voltage regulation by means of equally loading each phase. Figure 7-1 shows a primary system with either a three-phase three-wire or a three-phase four-wire main. The laterals can be either (1) three-phase three-wire, (2) three-phase four-wire, (3) single-phase with line-to-line voltage, ungrounded, (4) single-phase with line-to-neutral voltage, grounded, or (5) two-phase plus neutral, open-wye.

7-2 NON-THREE-PHASE PRIMARY LINES

Usually there are many laterals on a primary feeder which are not necessarily in three-phase, e.g., single-phase which causes the voltage drop and power loss due to load current not only in the phase conductor but also in the return path.

7-2-1 Single-Phase Two-Wire Laterals with Ungrounded Neutral

Assume that an overloaded single-phase lateral is to be changed to an equivalent three-phase three-wire and balanced lateral, holding the load constant. Since the power input to the lateral is the same as before,

$$S_{1\phi} = S_{3\phi} \tag{7-1}$$

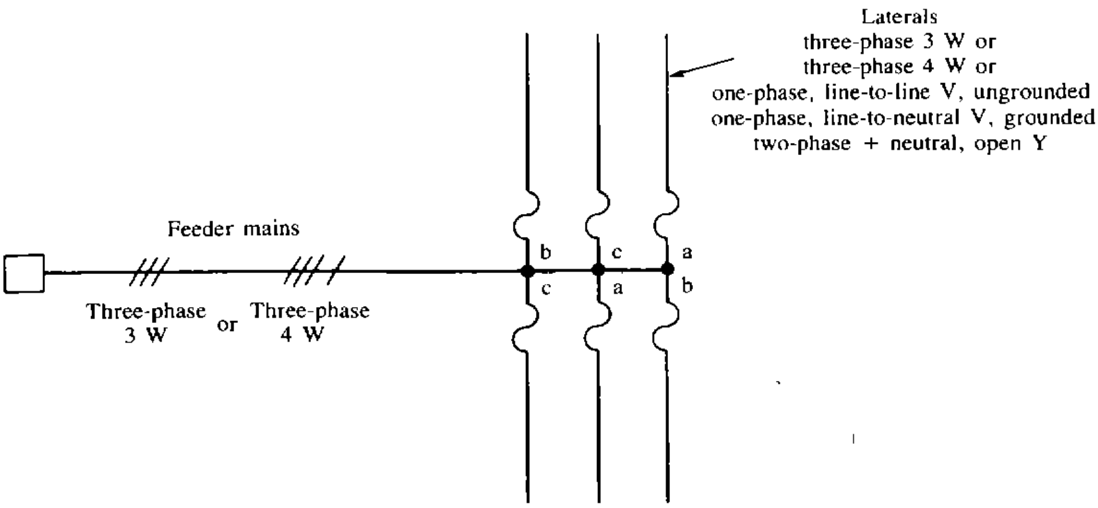


Figure 7-1

where the subscripts 1ϕ and 3ϕ refer to the single-phase and three-phase circuits, respectively. Equation (7-1) can be rewritten as

$$(\sqrt{3} \times V_s)I_{1\phi} = 3V_sI_{3\phi} \tag{7-2}$$

where V_s is the line-to-neutral voltage. Therefore, from Eq. (7-2),

$$I_{1\phi} = \sqrt{3} \times I_{3\phi} \tag{7-3}$$

which means that the current in the single-phase lateral is 1.73 times larger than the one in the equivalent three-phase lateral.

The voltage drop in the three-phase lateral can be expressed as

$$\mathrm{VD}_{3\phi} = I_{3\phi}(R \cos \theta + X \sin \theta) \qquad \mathrm{V} \tag{7-4}$$

and in the single-phase lateral as

$$\mathrm{VD}_{1\phi} = I_{1\phi}(K_R R \cos \theta + K_X X \sin \theta) \tag{7-5}$$

where K_R and K_X are conversion constants of R and X and are used to convert them from their three-phase values to the equivalent single-phase values.

$K_R = 2.0$

$K_X = 2.0$ when underground cable is used

$K_X \cong 2.0$ when overhead line is used, with approximately a $\pm 10\%$ accuracy

Therefore Eq. (7-5) can be rewritten as

$$\mathrm{VD}_{1\phi} = I_{1\phi}(2R \cos \theta + 2X \sin \theta) \qquad \mathrm{V} \tag{7-6}$$

or substituting Eq. (7-3) into Eq. (7-6),

$$\mathrm{VD}_{1\phi} = 2\sqrt{3} \times I_{3\phi}(R \cos \theta + X \sin \theta) \qquad \mathrm{V} \tag{7-7}$$

By dividing Eq. (7-7) by Eq. (7-4) side by side,

$$\frac{\mathrm{VD}_{1\phi}}{\mathrm{VD}_{3\phi}} = 2\sqrt{3} \tag{7-8}$$

which means that the voltage drop in the single-phase ungrounded lateral is approximately 3.46 times larger than the one in the equivalent three-phase lateral. Since base voltages for the single-phase and three-phase laterals are

$$V_{B(1\phi)} = \sqrt{3} \times V_{s,\,L\text{-}N} \qquad \text{V} \tag{7-9}$$

and

$$V_{B(3\phi)} = V_{s,\,L\text{-}N} \qquad \text{V} \tag{7-10}$$

Eq. (7-8) can be expressed in per units as

$$\frac{\mathrm{VD}_{\mathrm{pu},\,1\phi}}{\mathrm{VD}_{\mathrm{pu},\,3\phi}} = 2.0 \tag{7-11}$$

which means that the per unit voltage drop in the single-phase ungrounded lateral is two times larger than the one in the equivalent three-phase lateral. For example, if the per unit voltage drop in the single-phase lateral is 0.10, it would be 0.05 in the equivalent three-phase lateral.

The power losses due to the load currents in the conductors of the single-phase lateral and the equivalent three-phase lateral are

$$P_{\mathrm{L,S}1\phi} = 2 \times I_{1\phi}^2 R \qquad \text{W} \tag{7-12}$$

and

$$P_{\mathrm{LS},\,3\phi} = 3 \times I_{3\phi}^2 R \qquad \text{W} \tag{7-13}$$

respectively. Substituting Eq. (7-3) into Eq. (7-12),

$$P_{\mathrm{LS},\,1\phi} = 2(\sqrt{3} \times I_{3\phi})^2 R \tag{7-14}$$

and dividing the resultant Eq. (7-14) by Eq. (7-13) side by side,

$$\frac{P_{\mathrm{LS},\,1\phi}}{P_{\mathrm{LS},\,3\phi}} = 2.0 \tag{7-15}$$

which means that the power loss due the load currents in the conductors of the single-phase lateral is two times larger than the one in the equivalent three-phase lateral.

Therefore, one can conclude that by changing a single-phase lateral to an equivalent three-phase lateral both the per unit voltage drop and the power loss due to copper losses in the primary line are approximately halved.

7-2-2 Single-Phase Two-Wire Unigrounded Laterals

In general, this system is presently not used due to the following disadvantages. There is no earth current in this system. It can be compared to a three-phase four-wire balanced lateral in the following manner. Since the power input to the lateral is the same as before,

$$S_{1\phi} = S_{3\phi} \tag{7-16}$$

or
$$V_s \times I_{1\phi} = 3 \times V_s \times I_{3\phi} \tag{7-17}$$

from which
$$I_{1\phi} = 3 \times I_{3\phi} \tag{7-18}$$

The voltage drop in the three-phase lateral can be expressed as
$$\text{VD}_{3\phi} = I_{3\phi}(R \cos \theta + X \sin \theta) \qquad \text{V} \tag{7-19}$$

and in the single-phase lateral as
$$\text{VD}_{1\phi} = I_{1\phi}(K_R R \cos \theta + K_X X \sin \theta) \qquad \text{V} \tag{7-20}$$

where $K_R = 2.0$ when a full-capacity neutral is used i.e., if the wire size used for neutral conductor is the same as the size of the phase wire.
$K_R > 2.0$ when a reduced-capacity neutral is used
$K_X \cong 2.0$ when overhead line is used

Therefore, if $K_R = 2.0$ and $K_X = 2.0$, Eq. (7-20) can be rewritten as
$$\text{VD}_{1\phi} = I_{1\phi}(2R \cos \theta + 2X \sin \theta) \qquad \text{V} \tag{7-21}$$

or substituting Eq. (7-18) into Eq. (7-21),
$$\text{VD}_{1\phi} = 6 \times I_{3\phi}(R \cos \theta + X \sin \theta) \qquad \text{V} \tag{7-22}$$

Dividing Eq. (7-22) by Eq. (7-19) side by side,
$$\frac{\text{VD}_{1\phi}}{\text{VD}_{3\phi}} = 6.0 \tag{7-23}$$

which means that the voltage drop in the single-phase two-wire unigrounded lateral with full-capacity neutral is six times larger than the one in the equivalent three-phase four-wire balanced lateral.

The power losses due to the load currents in the conductors of the single-phase two-wire unigrounded lateral with full-capacity neutral and the equivalent three-phase four-wire balanced lateral are
$$P_{\text{LS},1\phi} = I_{1\phi}^2(2R) \qquad \text{W} \tag{7-24}$$

and
$$P_{\text{LS},3\phi} = 3 \times I_{3\phi}^2 R \qquad \text{W} \tag{7-25}$$

respectively. Substituting Eq. (7-18) into Eq. (7-24),
$$P_{\text{LS},1\phi} = (3 \times I_{3\phi})^2(2R) \qquad \text{W} \tag{7-26}$$

and dividing Eq. (7-26) by Eq. (7-25) side by side,
$$\frac{P_{\text{LS},1\phi}}{P_{\text{LS},3\phi}} = 6.0 \tag{7-27}$$

Therefore, the power loss due to load currents in the conductors of the single-phase two-wire unigrounded lateral with full-capacity neutral is six times larger than the one in the equivalent three-phase four-wire lateral.

7-2-3 Single-Phase Two-Wire Laterals with Multigrounded Common Neutrals

Figure 7-2 shows a single-phase two-wire lateral with multigrounded common neutral. As shown in the figure, the neutral wire is connected in parallel (i.e., multigrounded) with the ground wire at various places through ground electrodes in order to reduce the current in the neutral wire. I_a is the current in the phase conductor, I_w is the return current in the neutral wire, and I_d is the return current in the Carson's equivalent ground conductor. According to Morrison [1], the return current in the neutral wire is

$$I_n = \zeta_1 I_a \qquad \text{where } \zeta_1 = 0.25 \text{ to } 0.33 \tag{7-28}$$

and it is almost independent of size of the neutral conductor.

In Fig. 7-2, the constant K_R is less than 2.0 and the constant K_X is more or less equal to 2.0 because of conflictingly large D_m (i.e., mutual geometric mean distance or geometric mean radius, GMR) of the Carson's equivalent ground (neutral) conductor.

Therefore, Morrison's data [1] (probably empirical) indicate that

$$\text{VD}_{\text{pu},\,1\phi} = \zeta_2 \times \text{VD}_{\text{pu},\,3\phi} \qquad \text{where } \zeta_2 = 3.8 \text{ to } 4.2 \tag{7-29}$$

and

$$P_{\text{LS},\,1\phi} = \zeta_3 \times P_{\text{LS},\,3\phi} \quad \text{W} \qquad \text{where } \zeta_3 = 3.5 \text{ to } 3.75 \tag{7-30}$$

Therefore, assuming that the data from Morrison [1] are accurate,

$$K_R < 2.0 \qquad \text{and} \qquad K_X < 2.0$$

the per unit voltage drops and the power losses due to load currents can be approximated as

$$\text{VD}_{\text{pu},\,1\phi} \cong 4.0 \times \text{VD}_{\text{pu},\,3\phi} \tag{7-31}$$

and

$$P_{\text{LS},\,1\phi} \cong 3.6 \times P_{\text{LS},\,3\phi} \quad \text{W} \tag{7-32}$$

for the illustrative problems.

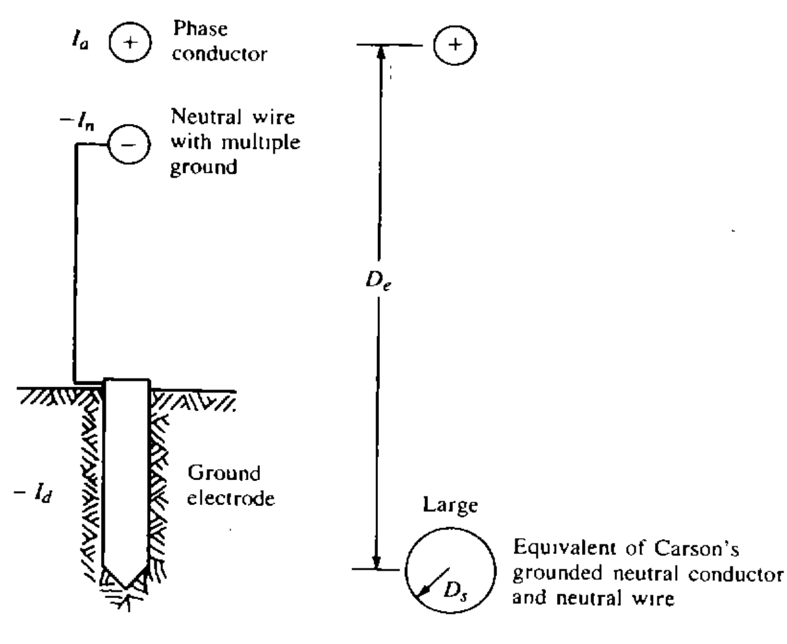


Figure 7-2

7-2-4 Two-Phase Plus Neutral (Open-Wye) Laterals

Figure 7-3 shows an open-wye-connected lateral with two-phase and neutral. Of course, the neutral conductor can be unigrounded or multigrounded, but because of disadvantages the unigrounded neutral is generally not used. If the neutral is unigrounded, all neutral current is in the neutral conductor itself. Theoretically, it can be expressed that

$$\bar{\mathbf{V}} = \bar{\mathbf{Z}}\bar{\mathbf{I}} \tag{7-33}$$

where

$$\bar{V}_a = \bar{Z}_a \bar{I}_a \tag{7-34}$$

$$\bar{V}_b = \bar{Z}_b \bar{I}_b \tag{7-35}$$

It is correct for equal load division between the two phases.

Assuming equal load division among phases, the two-phase plus neutral lateral can be compared to an equivalent three-phase lateral, holding the total kilovoltampere load constant. Therefore

$$S_{2\phi} = S_{3\phi} \tag{7-36}$$

or

$$2V_s I_{2\phi} = 3V_s I_{3\phi} \tag{7-37}$$

from which

$$I_{2\phi} = \tfrac{3}{2} I_{3\phi} \tag{7-38}$$

The voltage-drop analysis can be performed depending upon whether the neutral is unigrounded or multigrounded. If the neutral is unigrounded and the neutral-conductor impedance (Z_n) is zero, the voltage drop in each phase is

$$\mathrm{VD}_{2\phi} = I_{2\phi}(K_R R \cos\theta + K_X X \sin\theta) \qquad \mathrm{V} \tag{7-39}$$

$$\text{where } K_R = 1.0$$

$$K_X = 1.0$$

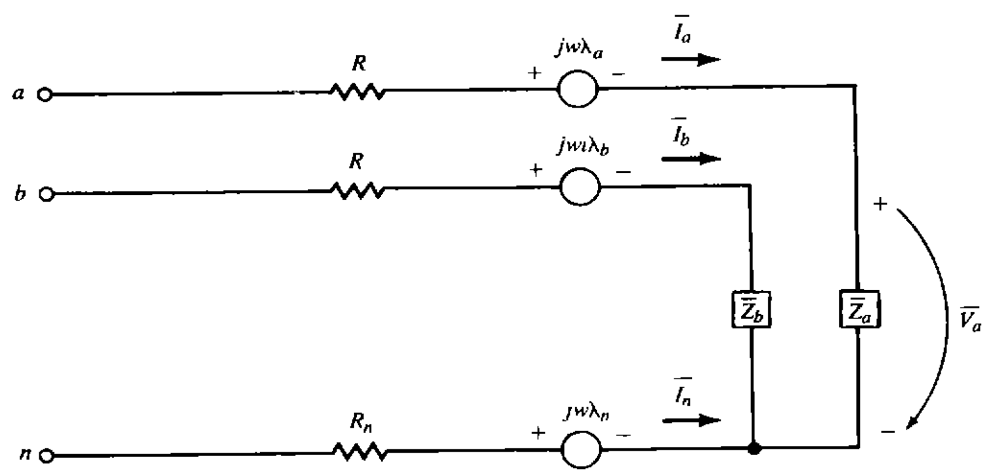


Figure 7-3

Therefore

$$\mathrm{VD}_{2\phi} = I_{2\phi}(R\cos\theta + X\sin\theta) \qquad \mathrm{V} \tag{7-40}$$

or substituting Eq. (7-38) into Eq. (7-40),

$$\mathrm{VD}_{2\phi} = \tfrac{3}{2} I_{3\phi}(R\cos\theta + X\sin\theta) \qquad \mathrm{V} \tag{7-41}$$

Dividing Eq. (7-41) by Eq. (7-19), side by side,

$$\frac{\mathrm{VD}_{2\phi}}{\mathrm{VD}_{3\phi}} = \frac{3}{2} \tag{7-42}$$

However, if the neutral is unigrounded and the neutral-conductor impedance (Z_n) is larger than zero,

$$\frac{\mathrm{VD}_{2\phi}}{\mathrm{VD}_{3\phi}} > \frac{3}{2} \tag{7-43}$$

therefore in this case some unbalanced voltages are inherent.

However, if the neutral is multigrounded and $Z_n > 0$, the data from Morrison [1] indicate that the per unit voltage drop in each phase is

$$\mathrm{VD}_{\mathrm{pu},\,2\phi} = 2.0 \times \mathrm{VD}_{\mathrm{pu},\,3\phi} \tag{7-44}$$

when a full-capacity neutral is used and

$$\mathrm{VD}_{\mathrm{pu},\,2\phi} = 2.1 \times \mathrm{VD}_{\mathrm{pu},\,3\phi} \tag{7-45}$$

when a reduced-capacity neutral (i.e., when the neutral conductor employed is one or two sizes smaller than the phase conductors) is used.

The power loss analysis also depends upon whether the neutral is unigrounded or multigrounded. If the neutral is unigrounded, the power loss is

$$P_{\mathrm{LS},\,2\phi} = I^2_{2\phi}(K_R R) \tag{7-46}$$

where $K_R = 3.0$ when a full-capacity neutral is used
$K_R > 3.0$ when a reduced-capacity neutral is used

Therefore, if $K_R = 3.0$,

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} = \frac{3I^2_{2\phi}R}{3I^2_{3\phi}R} \tag{7-47}$$

or

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} = 2.25 \tag{7-48}$$

On the other hand, if the neutral is multigrounded,

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} < 2.25 \tag{7-49}$$

Based on the data from Morrison [1], the approximate value of this ratio is

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} \cong 1.64 \tag{7-50}$$

which means that the power loss due to load currents in the conductors of the two-phase three-wire lateral with multigrounded neutral is approximately 1.64 times larger than the one in the equivalent three-phase lateral.

7-3 FOUR-WIRE MULTIGROUNDED COMMON-NEUTRAL DISTRIBUTION SYSTEM

Figure 7-4 shows a typical four-wire multigrounded common-neutral distribution system. Because of the economic and operating advantages, this system is used extensively. The assorted secondaries can be, for example, either (1) 120/240-V

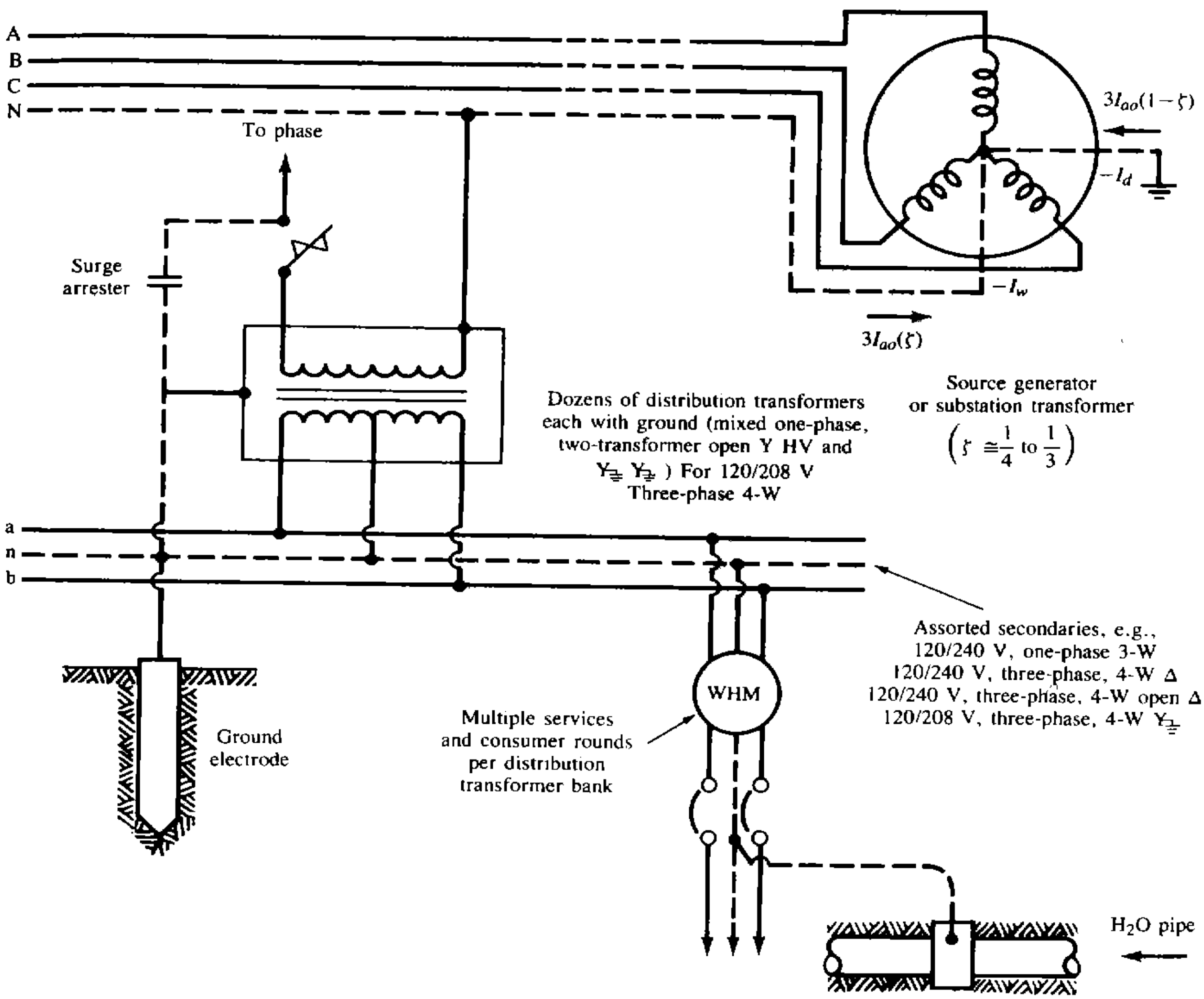


Figure 7-4 A four-wire multigrounded common-neutral distribution system.

single-phase three-wire, (2) 120/240-V three-phase four-wire connected in delta, (3) 120/240-V three-phase four-wire connected in open-delta, or (4) 120/208-V three-phase four-wire connected in grounded-wye. Where primary and secondary systems are both existent, the same conductor is used as the "common" neutral for both systems. The neutral is grounded at each distribution transformer, at various places where no transformers are connected, and to water pipes or driven ground electrodes at each user's service entrance. The secondary neutral is also grounded at the distribution transformer and the service drops. Typical values of the resistances of the ground electrodes are 5, 10, or 15 Ω. Under no circumstances should they be larger than 25 Ω. Usually, a typical metal water pipe system has a resistance value of less than 3 Ω. A part of the unbalanced, or zero sequence, load current flows in the neutral wire, and the remaining part flows in the ground and/or the water system. Usually the same conductor size is used for both phase and neutral conductors.

Example 7-1 Assume that the circuit shown in Fig. 7-5 represents a single-phase circuit if dimensional variables are used; it represents a balanced three-phase circuit if per unit variables are used. The $R + jX$ represents the total impedance of lines and/or transformers. The power factor of the load is $\cos \theta = \cos(\theta_{\bar{V}_R} - \theta_{\bar{I}})$. Find the load power factor for which the voltage drop is maximum.

SOLUTION The line voltage drop is

$$\mathrm{VD} = I(R \cos \theta + X \sin \theta)$$

By taking its partial derivative with respect to the θ angle and equating the result to zero,

$$\frac{\partial(\mathrm{VD})}{\partial \theta} = -IR \sin \theta + IX \cos \theta = 0$$

or

$$\frac{X}{R} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

therefore

$$\theta_{\max} = \tan^{-1} \frac{X}{R}$$

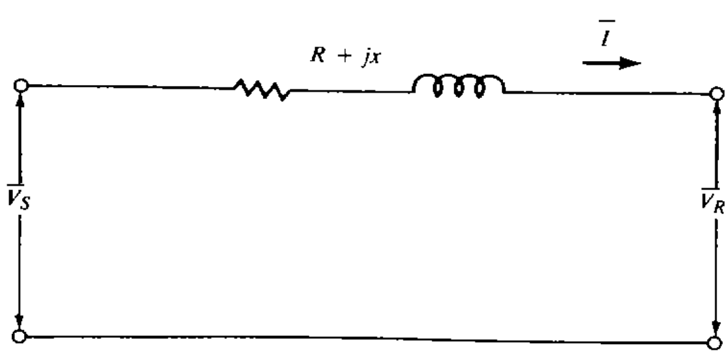


Figure 7-5

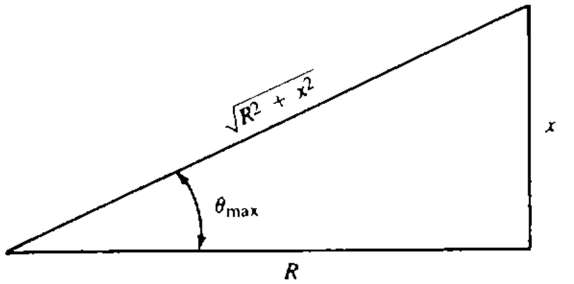


Figure 7-6

and from the impedance triangle shown in Fig. 7-6, the load power factor for which the voltage drop is maximum is

$$\text{PF} = \cos \theta_{\max} = \frac{R}{(R^2 + X^2)^{1/2}} \tag{7-51}$$

also

$$\cos \theta_{\max} = \cos \left(\tan^{-1} \frac{X}{R}\right) \tag{7-52}$$

Example 7-2 Consider the three-phase three-wire 416-V secondary system with balanced loads at A, B, and C as shown in Fig. 7-7. Determine the following:

(*a*) Calculate the total voltage drop, or as it is sometimes called, *voltage regulation*, in one phase of the lateral by using the approximate method.
(*b*) Calculate the real power per phase for each load.
(*c*) Calculate the reactive power per phase for each load.
(*d*) Calculate the kilovoltampere output and load power factor of the distribution transformer.

SOLUTION
(*a*) Using the approximate voltage-drop equation, that is,

$$\text{VD} = I(R \cos \theta + X \sin \theta)$$

the voltage drop for each load can be calculated as

$$\text{VD}_A = 30(0.05 \times 1.0 + 0.01 \times 0) = 1.5 \text{ V}$$

$$\text{VD}_B = 20(0.15 \times 0.5 + 0.03 \times 0.866) = 2.02 \text{ V}$$

$$\text{VD}_C = 50(0.20 \times 0.9 + 0.08 \times 0.436) = 10.744 \text{ V}$$

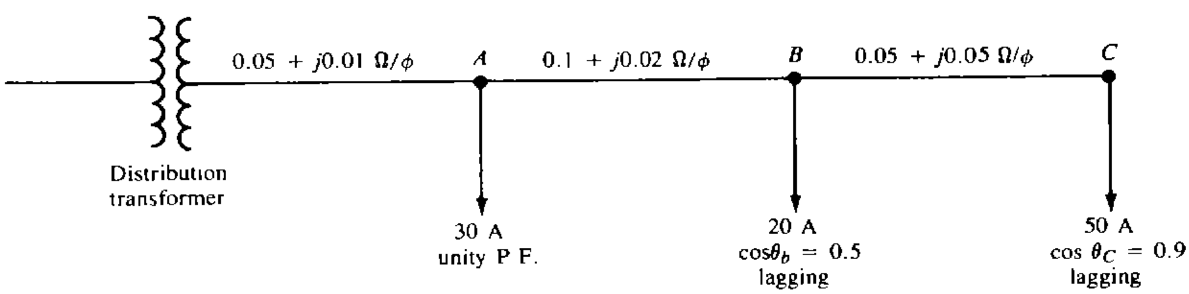


Figure 7-7

Therefore the total voltage drop is

$$\begin{aligned}\sum \mathrm{VD} &= \mathrm{VD}_A + \mathrm{VD}_B + \mathrm{VD}_C \\ &= 1.5 + 2.02 + 10.744 \\ &= 14.264 \text{ V}\end{aligned}$$

or

$$\frac{14.264 \text{ V}}{240 \text{ V}} = 0.0594 \text{ pu V}$$

(*b*) The real power per phase for each load can be calculated from

$$P = VI \cos \theta$$

or

$$P_A = 240 \times 30 \times 1.0 = 7.2 \text{ kW}$$

$$P_B = 240 \times 20 \times 0.5 = 2.4 \text{ kW}$$

$$P_C = 240 \times 50 \times 0.9 = 10.8 \text{ kW}$$

Therefore the total real power per phase is

$$\begin{aligned}\sum P &= P_A + P_B + P_C \\ &= 7.2 + 2.4 + 10.8 \\ &= 20.4 \text{ kW}\end{aligned}$$

(*c*) The reactive power per phase for each load can be calculated from

$$Q = VI \sin \theta$$

or

$$Q_A = 240 \times 30 \times 0 = 0 \text{ kvar}$$

$$Q_B = 240 \times 20 \times 0.866 = 4.156 \text{ kvar}$$

$$Q_C = 240 \times 50 \times 0.436 = 5.232 \text{ kvar}$$

Therefore the total reactive power per phase is

$$\begin{aligned}\sum Q &= Q_A + Q_B + Q_C \\ &= 0 + 4.156 + 5.232 \\ &= 9.389 \text{ kvar}\end{aligned}$$

(*d*) Therefore the kilovoltampere output of the distribution transformer is

$$\begin{aligned}S &= (P^2 + Q^2)^{1/2} \\ &= (20.4^2 + 9.389^2)^{1/2} \\ &\cong 22.457 \text{ kVA/three phase}\end{aligned}$$

Thus the total kilovoltampere output of the distribution transformer is

$$22.457 \text{ kVA}$$

Hence, the load power factor of the distribution transformer is

$$\cos\theta = \frac{\sum P}{S}$$

$$= \frac{20.4 \text{ kW}}{22.457 \text{ kVA}}$$

$$= 0.908 \text{ lagging}$$

Example 7-3 This example is a continuation of Example 6-1. It deals with voltage drops in the secondary distribution system. In this and the following examples, a single-phase three-wire 120/240-V directly buried underground residential distribution (URD) secondary system will be analyzed, and calculations will be made for motor-starting voltage dip and for steady-state voltage drops at the time of annual peak load. Assume that the cable impedances given in Table 7-2 are correct for a typical URD secondary cable.

Transformer data The data given in Table 7-1 are for modern single-phase 65°C OISC distribution transformers of the 7200–120/240-V class The data were taken from a recent catalog of a manufacturer. All given per unit values are based on the transformer-rated kilovoltamperes and voltages.

The 2400-V-class transformers of the sizes being considered have about 15 percent less R and about 7 percent less X than the 7200-V transformers. Ignore the small variation of impedance with rated voltage and assume that voltage drop calculated with the given data will suffice for whichever primary voltage is used.

Table 7-1 Single-phase 7200–120/240-V distribution transformer data at 65°C

Rated kVA	Core loss,* kW	Copper loss,† kW	R, pu	X, pu	Excitation current, A
15	0.083	0.194	0.0130	0.0094	0.014
25	0.115	0.309	0.0123	0.0138	0.015
37.5	0.170	0.400	0.0107	0.0126	0.014
50	0.178	0.537	0.0107	0.0139	0.014
75	0.280	0.755	0.0101	0.0143	0.014
100	0.335	0.975	0.0098	0.0145	0.014

* At rated voltage and frequency.

† At rated voltage and kilovoltampere load.

Figure 7-8 Triplexed cable assembly.

URD secondary cable data Cable insulations and manufacture are constantly being improved, especially for high-voltage cables. Therefore, any cable data soon become obsolete. The following information and data have been abstracted from recent cable catalogs.

Much of the 600-V-class cable now commonly used for secondary lines and services has Al conductor and cross-linked PE insulation which can stand 90°C conductor temperature. The triplexed cable assembly shown in Fig. 7-8 (quadruplexed for three-phase four-wire service) has three or four insulated conductors when aluminum is used. When copper is used, the one grounded neutral conductor is bare. The neutral conductor typically is two AWG sizes smaller than the phase conductors.

The twin concentric cable assembly shown in Fig. 7-9 has two insulated copper or aluminum phase conductors plus several spirally served small bare copper binding conductors which act as the current-carrying grounded neutral. The number and size of the spiral neutral wires vary so that the ampacity of the neutral circuit is equivalent to two AWG wire sizes smaller than the phase conductors. Table 7-2 gives data for twin concentric aluminum/copper XLPE 600-V-class cable.

The triplex and twin concentric assemblies obviously have the same resistance for a given size of phase conductors. The triplex assembly has very slightly higher reactance than the concentric assembly. The difference in reactances is too small to be noted unless precise computations are undertaken for some special purpose. The reactances of those cables should be increased by about 25 percent if they are installed in iron conduit. The reactances given below are valid only for balanced loading (where the neutral current is zero).

The triplex assembly has about 15 percent smaller ampacity than the concentric assembly, but the exact amount of reduction varies with wire size. The ampacities given are for 90°C conductor temperature, 20°C ambient earth temperature, direct burial in earth, and 10 percent daily load factor. When installed in buried duct, the ampacities are about 70 percent of those listed below. For load factors less than 100 percent, consult current literature or cable standards. The increased ampacities are significantly large.

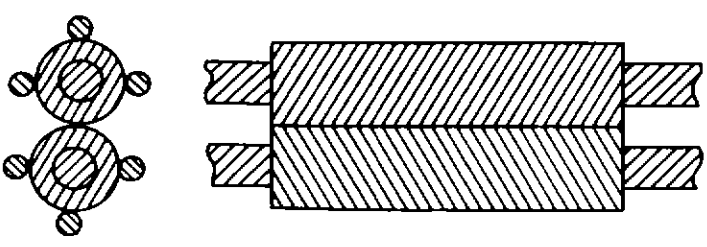


Figure 7-9 Twin concentric cable assembly.

Table 7-2 Twin-concentric Al/Cu XLPE 600-V cable data

Size	R (Ω/1000 ft) per cond. Phase cond. 90°C	R (Ω/1000 ft) per cond. Neut. cond. 80°C	X(Ω/1000 ft) per phase conductor	Direct-burial ampacity, A	$\bar{K}^*$ 90% PF	$\bar{K}^*$ 50% PF
2 AWG	0.334	0.561	0.0299	180	0.02613	0.01608
1 AWG	0.265	0.419	0.0305	205	0.02098	0.01324
1/0 AWG	0.210	0.337	0.0297	230	0.01683	0.01089
2/0 AWG	0.167	0.259	0.0290	265	0.01360	0.00905
3/0 AWG	0.132	0.211	0.0280	300	0.01092	0.00752
4/0 AWG	0.105	0.168	0.0275	340	0.00888	0.00636
250 kcmil	0.089	0.133	0.0280	370	0.00769	0.00573
350 kcmil	0.063	0.085	0.0270	445	0.00571	0.00458
500 kcmil	0.044	0.066	0.0260	540	0.00424	0.00371

* Per unit voltage drop per 10^4 A · ft (amperes per conductor times feet of cable) based on 120 V line-to-neutral or 240 V line-to-line. Valid for the two power factors shown and for perfectly balanced 3-wire loading.

Arbitrary criteria

1. Use the approximate voltage-drop equation, that is,

$$\text{VD} = I(R\cos\theta + X\sin\theta)$$

   and adapt it to per unit data when computing transformer voltage drops and adapt it to ampere and ohm data when computing service-drop (SD) and secondary-line (SL) voltage drops. Obtain all voltage-drop answers in per unit based on 240 V.
2. Maximum allowable motor-starting voltage dip (VDIP) = 3 percent = 0.03 pu = 3.6 V based on 120 V. This figure is arbitrary; utility practices vary.
3. Maximum allowable steady-state voltage drop in the secondary system (transformer + SL + SD) = 3.50 percent = 0.035 pu = 4.2 V based on 120 V. This figure also is quite arbitrary; regulatory commission rules and utility practices vary. More information about favorable and tolerable amounts of voltage drop will be discussed in connection with subsequent examples, which will involve voltage drops in the primary lines.
4. The loading data for computation of steady-state voltage drop is given in Table 7-3.
5. As loading data for transient motor-starting VDIP, assume an air-conditioning compressor motor located most unfavorably. It has a 3-hp single-phase 240-V 80-A locked rotor current, with a 50 percent PF locked rotor.

Assumptions

1. Assume perfectly balanced loading in all three-wire single-phase circuits.
2. Assume nominal operating voltage of 240 V when computing currents from kilovoltampere loads.

Table 7-3 Load data

Circuit element	Load, kVA
SD	1 class 2 load (10 kVA)
SL	1 class 2 load (10 kVA) + 3 diversified class 2 loads (6.0 kVA each)
Transformer	1 class 2 load (10 kVA) + either 3 diversified class 2 loads (6.0 kVA each) or 11 diversified class 2 loads (4.4 kVA each)

3. Assume 90 percent lagging power factor for all loads.

Using the given data and assumptions, calculate the K constant for any one of the secondary cable sizes, hoping to verify one of the given values in Table 7-2.

SOLUTION Let the secondary cable size be #2 AWG, arbitrarily. Also let the I current be 100 A and the length of the secondary line be 100 ft. Using the values from Table 7-2, the resistance and reactance values for 100 ft of cable can be found as

$$
\begin{aligned}
R &= 0.334\ \Omega/1000\ \text{ft} \times \frac{100\ \text{ft}}{1000\ \text{ft}} \\
&= 0.0334\ \Omega
\end{aligned}
$$

and

$$
\begin{aligned}
X &= 0.0299\ \Omega/1000\ \text{ft} \times \frac{100\ \text{ft}}{1000\ \text{ft}} \\
&= 0.00299\ \Omega
\end{aligned}
$$

Therefore, using the approximate voltage-drop equation,

$$
\begin{aligned}
\text{VD} &= I(R\cos\theta + X\sin\theta) \\
&= 100(0.0334 \times 0.9 + 0.00299 \times 0.435) \\
&= 3.136\ \text{V}
\end{aligned}
$$

or, in per unit volts,

$$
\frac{3.136\ \text{V}}{120\ \text{V}} = 0.0261\ \text{pu V}
$$

which is very close to the value given in Table 7-2 for the $\tilde{K}$ constant, that is, 0.02613 pu V/(10^4 A · ft) of cable

Example 7-4 Use the information and data given in Examples 6-1 and 7-3. Assume an underground residential distribution (URD) system. Therefore, the secondary lines shown in Fig. 6-12 are made of underground (UG) secondary cables. Assume 12 services per distribution transformer and two transformers per block which are at the locations of poles 2 and 5, as shown in Fig. 6-12. Service pedestals are at the locations of poles 1, 3, 4, and 6. Assume that the selected equipment sizes (for S_T, A_{SL}, A_{SD}) are of the nearest standard size which are larger than the theoretically most economical sizes and determine the following:

(*a*) Find the steady-state voltage drop in per units at the most remote consumer's meter for the annual maximum system loads given in Table 7-3.

(*b*) Find the voltage dip in per units for motor starting at the most unfavorable location.

(*c*) If the voltage-drop and/or voltage-dip criteria are not met, select larger equipment and find a design that will meet these arbitrary criteria. Do not, however, immediately select the largest sizes of S_T, A_{SL}, and A_{SD} equipment and call that a worthwhile design. In addition, contemplate the data and results and attempt to be wise in selecting A_{SL} or A_{SD} (or both) for enlarging to meet the voltage criteria.

SOLUTION

(*a*) Due to the diversity factors involved, the load values given in Table 7-3 are different for service drops, secondary lines, and transformers. For example, the load on the transformer is selected as

$$\text{Transformer load} = 10 + 11 \times 4.4$$
$$= 58.4 \text{ kVA}$$

Therefore, selecting a 50-kVA transformer,

$$I = \frac{58.4 \text{ kVA}/240 \text{ V}}{S_T/240 \text{ V}}$$
$$= \frac{58.4 \text{ kVA}}{50 \text{ kVA}}$$
$$= 1.168 \text{ pu A}$$

Thus the per unit voltage drop in the transformer is

$$\text{VD}_T = I(R \cos \theta + X \sin \theta)$$
$$= (1.168 \text{ pu A})(0.0107 \times 0.9 + 0.0139 \times 0.435)$$
$$= 0.0183 \text{ pu V}$$

As shown in Fig. 7-10, the load on each secondary line (that portion of the wiring between the transformer and the service pedestal) is similarly calculated as

$$\text{SL load} = 10 + 3 \times 6$$
$$= 28 \text{ kVA}$$

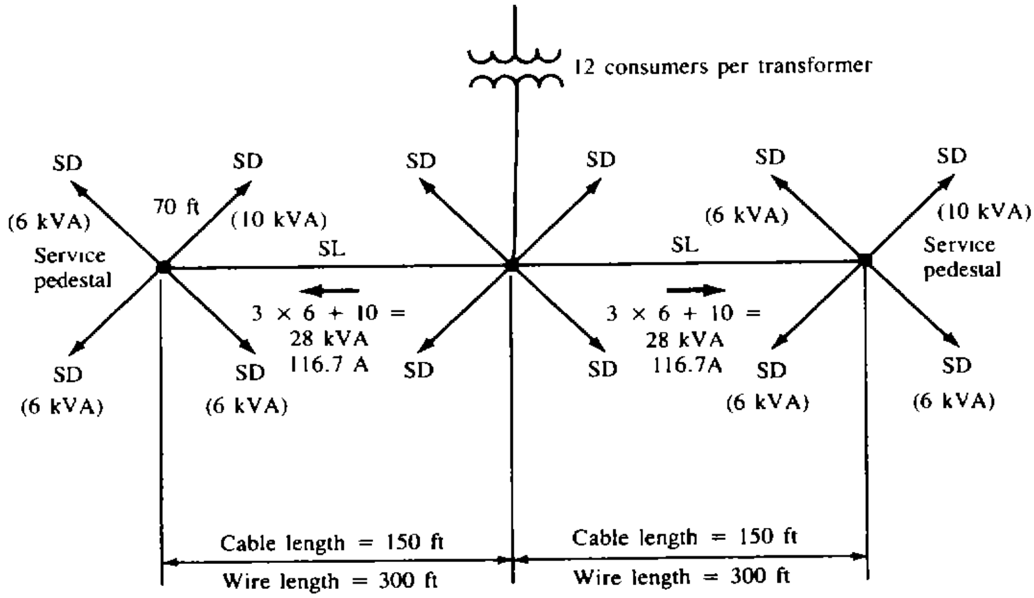


Figure 7-10

or 116.7 A. If the secondary line is selected to be #4/0 AWG with the $\tilde{K}$ constant of 0.0088 from Table 7-2, the per unit voltage drop in each secondary line is

$$\mathrm{VD_{SL}} = \tilde{K}\left(\frac{I \times l}{10^4}\right)$$

$$= 0.0088\left(\frac{116.7 \times 150\,\text{ft}}{10^4}\right)$$

$$= 0.01554 \text{ pu V}$$

The load on each service drop is given to be 10 kVA or 41.6 A from Table 7-3. If each service drop of 70-ft length is selected to be #1/0 AWG with the $\tilde{K}$ constant of 0.01683 from Table 7-2, the per unit voltage drop in each service drop is

$$\mathrm{VD_{SD}} = 0.01683\left(\frac{41.6\text{A} \times 70\,\text{ft}}{10^4}\right)$$

$$= 0.0049 \text{ pu V}$$

Therefore the total steady-state voltage drop in per units at the most remote consumer's meter is

$$\sum \mathrm{VD} = \mathrm{VD}_T + \mathrm{VD_{SL}} + \mathrm{VD_{SD}}$$

$$= 0.0183 + 0.01554 + 0.0049$$

$$= 0.0388 \text{ pu V}$$

which exceeds the given criterion of 0.035 pu V (*b*) To find the voltage dip in per units for motor starting at the most unfavorable location, the given starting current of 80 A can be converted to a kilovoltampere load of 19.2 kVA (80 A × 240 V). Therefore the per unit voltage dip in the 50-kVA transformer is

$$\mathrm{VDIP}_T = (R\cos\theta + X\sin\theta)\,\frac{19.2\ \mathrm{kVA}}{50\ \mathrm{kVA}}$$

$$= (0.0107 \times 0.5 + 0.0139 \times 0.866)\,\frac{19.2\ \mathrm{kVA}}{50\ \mathrm{kVA}}$$

$$= 0.00668\ \mathrm{pu\ V}$$

The per unit voltage dip in the secondary line of #4/0 AWG cable is

$$\mathrm{VDIP}_{\mathrm{SL}} = \tilde{K}\left(\frac{80\ \mathrm{A} \times 150\ \mathrm{ft}}{10^4}\right)$$

$$= 0.00636\left(\frac{80 \times 150}{10^4}\right)$$

$$= .0.00763\ \mathrm{pu\ V}$$

The per unit voltage dip in the service drop of #1/0 AWG cable is

$$\mathrm{VDIP}_{\mathrm{SD}} = \tilde{K}\left(\frac{80\ \mathrm{A} \times 70\ \mathrm{ft}}{10^4}\right)$$

$$= 0.01089\left(\frac{80 \times 70}{10^4}\right)$$

$$= 0.0061\ \mathrm{pu\ V}$$

Therefore the total voltage dip in per units due to motor starting at the most unfavorable location is

$$\sum \mathrm{VDIP} = \mathrm{VDIP}_T + \mathrm{VDIP}_{\mathrm{SL}} + \mathrm{VDIP}_{\mathrm{SD}}$$

$$= 0.00668 + 0.00763 + 0.0061$$

$$= 0.024\ \mathrm{pu\ V}$$

which meets the given criterion of 0.03 pu V.

(*c*) Since in part *a* the voltage-drop criterion has not been met, select the secondary-line cable size to be one size larger than the previous #4/0 AWG size, that is, 250 kcmil, keeping the size of the transformer the same. Therefore, the new per unit voltage drop in the secondary line becomes

$$\mathrm{VD}_{\mathrm{SL}} = 0.00769\left(\frac{116.7\ \mathrm{A} \times 150\ \mathrm{ft}}{10^4}\right)$$

$$= 0.01347\ \mathrm{pu\ V}$$

Also, selecting one-size-larger cable, that is, #2/0 AWG, for the service drop, the new per unit voltage drop in the service drop becomes

$$\begin{aligned} VD_{SD} &= 0.0136\left(\frac{41.6\text{ A} \times 70\text{ ft}}{10^4}\right) \\ &= 0.00396 \text{ pu V} \end{aligned}$$

Therefore the new total steady-state voltage drop in per units at the most remote consumer's meter is

$$\begin{aligned} \sum VD &= VD_T + VD_{SL} + VD_{SD} \\ &= 0.0183 + 0.01347 + 0.00396 \\ &= 0.03573 \text{ pu V} \end{aligned}$$

which is still larger than the criterion. Thus, select 350-kcmil cable size for the secondary lines and #2/0 AWG cable size for the service drops to meet the criteria.

Example 7-5 Figure 7-11 shows a residential secondary distribution system. Assume that the distribution transformer capacity is 75 kVA (use Table 7-1), all secondaries and services are single-phase three-wire, nominally 120/240 V, and all secondary lines are of #2/0 Al/Cu XLPE cable, and service drops are of #1/0 Al/Cu XLPE cable (use Table 7-2). All service drops are 100 ft long, and all secondary lines are 200 ft long. Assume an average lagging-load power

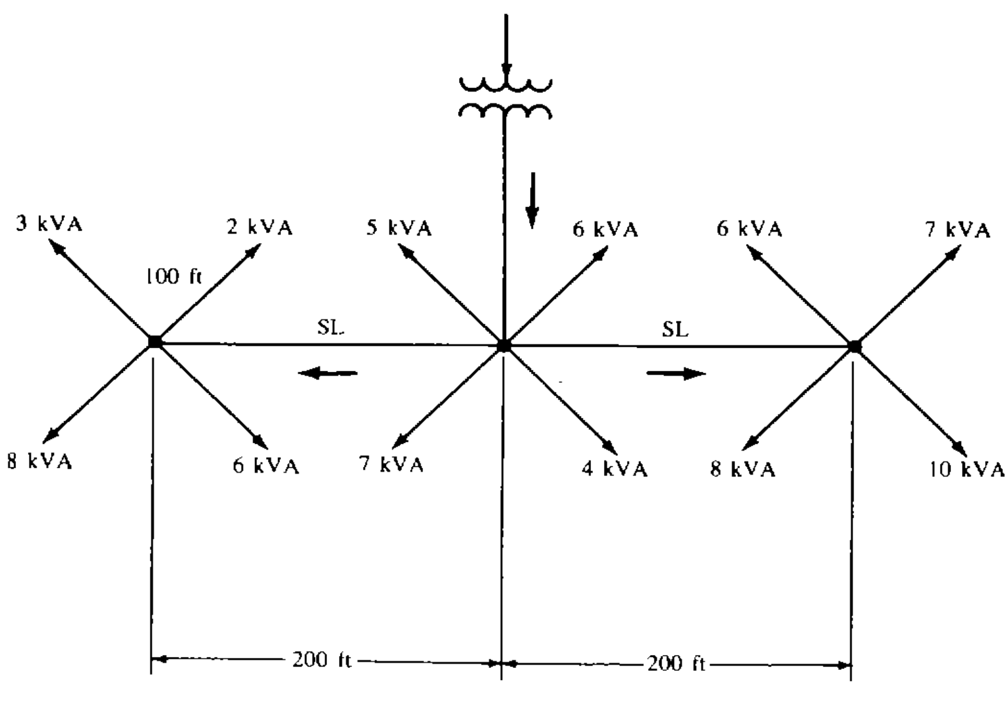


Figure 7-11

factor of 0.9 and 100 percent load diversity factors and determine the following:

(a) Find the total load on the transformer in kilovoltamperes and in per units.

(b) Find the total steady-state voltage drop in per units at the most remote and severe customer's meter for the given annual maximum system loads.

SOLUTION

(a) Assuming a diversity factor of 100 percent, the total load on the transformer is

$$S_T = \underbrace{3 + 2 + 8 + 6}_{19\text{ kVA}} + \underbrace{5 + 6 + 7 + 4}_{22\text{ kVA}} + \underbrace{6 + 7 + 8 + 10}_{31\text{ kVA}}$$

$$= 72 \text{ kVA}$$

or, in per units,

$$\begin{aligned} I &= \frac{S_T}{S_B} \\ &= \frac{72\text{ kVA}}{75\text{ kVA}} \\ &= 0.96 \text{ pu A} \end{aligned}$$

(b) To find the total voltage drop in per units at the most remote and severe customer's meter, calculate the per unit voltage drops in the transformer, the service line, and the service drop of the most remote and severe customer. Therefore

$$\begin{aligned} \mathrm{VD}_T &= I(R\cos\theta + X\sin\theta) \\ &= 0.96(0.0101 \times 0.90 + 0.0143 \times 0.4359) \\ &= 0.0147 \text{ pu V} \end{aligned}$$

$$\begin{aligned} \mathrm{VD}_{\mathrm{SL}} &= \tilde{K}\left(\frac{I \times l}{10^4}\right) \\ &= 0.0136\left(\frac{129.17\text{ A} \times 200\text{ ft}}{10^4}\right) \\ &= 0.03513 \text{ pu V} \end{aligned}$$

$$\begin{aligned} \mathrm{VD}_{\mathrm{SD}} &= 0.01683\left(\frac{41.67\text{ A} \times 100\text{ ft}}{10^4}\right) \\ &= 0.0070 \text{ pu V} \end{aligned}$$

Therefore the total voltage drop is

$$\begin{aligned}\sum \mathrm{VD} &= \mathrm{VD}_T + \mathrm{VD}_{\mathrm{SL}} + \mathrm{VD}_{\mathrm{SD}} \\ &= 0.0147 + 0.03513 + 0.0070 \\ &= 0.0568 \text{ pu V}\end{aligned}$$

Example 7-6 Figure 7-12 shows a three-phase four-wire grounded-wye distribution system with multigrounded neutral, supplied by an express feeder and mains. In the figure, d and s are the width and length of a primary lateral, where s is much larger than d. Main lengths are equal to $cb = ce = s/2$. The number of the primary laterals can be found as s/d. The square-shaped service area (s^2) has a uniformly distributed load density, and all loads are presumed

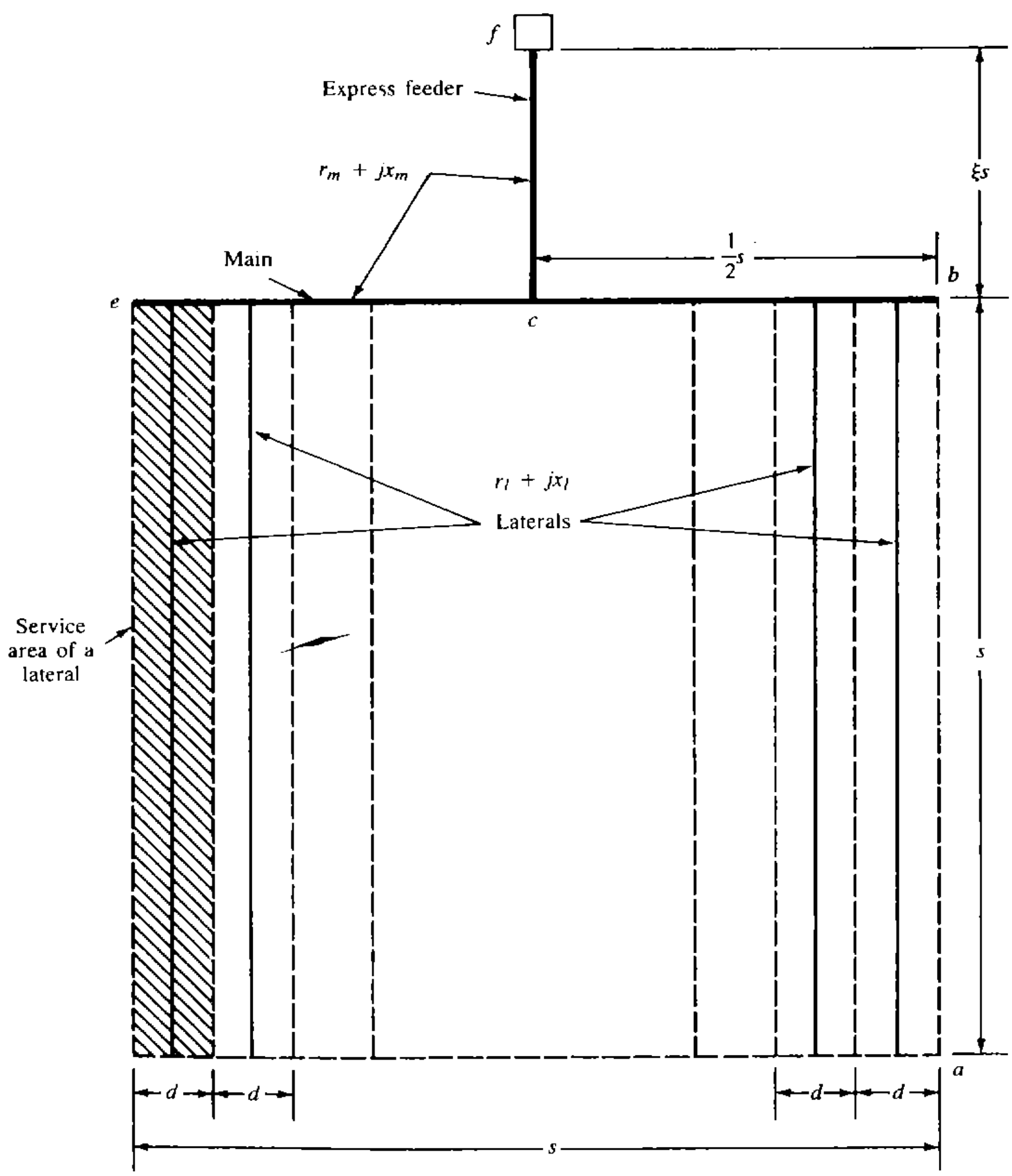


Figure 7-12

to have the same lagging power factor. Each primary lateral, such as *ba*, serves an area of length s and width d. Assume that

D = uniformly distributed load density, kVA/(unit length)^2

$V_{L\text{-}L}$ = nominal operating voltage which is also base voltage, line-to-line kV

$r_m + jx_m$ = impedance of three-phase express and mains, Ω/(phase · unit length)

$r_l + jx_l$ = impedance of a three-phase lateral line, Ω/(phase · unit length)

Use the given information and data and determine the following:

(*a*) Assume that the laterals are in three-phase and find the per unit voltage drop expressions for:

(i) The express feeder *fc*, that is, $\text{VD}_{\text{pu},fc}$

(ii) The main *cb*, that is, $\text{VD}_{\text{pu},cb}$

(iii) The primary lateral *ba*, that is, $\text{VD}_{\text{pu},ba}$

Note that the equations to be developed should contain the constants D, s, d, impedances, θ load power-factor angle, $V_{L\text{-}L}$, etc., but not current variable I.

(*b*) Change all the laterals from the three-phase four-wire system to an open-wye system so that investment costs will be reduced but three-phase secondary service can still be rendered where needed. Assume that the phasing connections of the many laterals are well-balanced on the mains. Employ Morrison's approximations and modify the equations derived in part *a*.

SOLUTION

(*a*) Total kVA load served $= D \times s^2$ kVA (7-53)

$$\text{Current at point } f = \frac{D \times s^2}{\sqrt{3} \times V_{L\text{-}L}} \tag{7-54}$$

$$\text{Voltage drop} = I \times z \times l_{\text{eff}} \tag{7-55}$$

Therefore

$$\text{(i)}\quad \text{VD}_{\text{pu},fc} = \frac{D \times s^2}{\sqrt{3} \times V_{L\text{-}L}} (r_m \cos\theta + x_m \sin\theta) \frac{\sqrt{3}}{1000 \times V_{L\text{-}L}} (\xi \times s)$$

$$= \frac{\xi \times D \times s^3}{1000 \times V_{L\text{-}L}^2} (r_m \cos\theta + x_m \sin\theta) \quad \text{pu V} \tag{7-56}$$

$$\text{(ii)}\quad \text{VD}_{\text{pu},cb} = \frac{\frac{1}{2}D \times s^2}{\sqrt{3} \times V_{L\text{-}L}} (r_m \cos\theta + x_m \sin\theta) \frac{\sqrt{3}}{1000 \times V_{L\text{-}L}} (\tfrac{1}{4}s)$$

$$= \frac{D \times s^3}{8000 \times V_{L\text{-}L}^2} (r_m \cos\theta + x_m \sin\theta) \quad \text{pu V} \tag{7-57}$$

$$\text{(iii) } \mathrm{VD}_{\mathrm{pu},ba} = \frac{D(d \times s)}{\sqrt{3} \times V_{L\text{-}L}} (r_l \cos\theta + x_l \sin\theta) \frac{\sqrt{3}}{1000 \times V_{L\text{-}L}} (\tfrac{1}{2}s)$$

$$= \frac{D \times d \times s^2}{2000 \times V_{L\text{-}L}^2} (r_L \cos\theta + x_l \sin\theta) \qquad \text{pu V} \qquad (7\text{-}58)$$

(*b*) There would not be any change in the equations given in part *a*.

Example 7-7 Figure 7-13 shows a square-shaped service area ($A = 4$ mi^2) with a uniformly distributed load density of D kVA/mi^2 and 2 mi of #4/0 AWG copper overhead main from *a* to *b*. There are many closely spaced primary laterals which are not shown in the square-shaped service area of the figure. In this voltage-drop study, use the precalculated voltage-drop curves of Fig. 4-17 when applicable. Use the nominal primary voltage of 7620/13,200 V for a three-phase four-wire wye-grounded system. Assume that at peak loading the load density is 1000 kVA/mi^2 and the lumped load is 2000 kVA, and that at off-peak loading the load density is 333 kVA/mi^2 and the lumped load is still 2000 kVA. The lumped load is of a small industrial plant working three shifts a day. The substation bus voltages are 1.025 pu V of 7620 base volts at peak load and 1.000 pu V during off-peak load.

The transformer located between buses *c* and *d* has a three-phase rating of 2000 kVA and a delta-rated high voltage of 13,200 V and grounded-wye-rated low voltage of 277/480 V. It has $0 + j0.05$ per unit impedance based on the transformer ratings. It is tapped up to raise the low voltage 5.0 percent relative to the high voltage, i.e., the equivalent turns ratio in use is (7620/277) $\times$ 0.95. Use the given information and data for peak loading and determine the following:

(*a*) The percent voltage drop from the substation to point *a*, from *a* to *b*, from *b* to *c*, and from *c* to *d* on the main.

(*b*) The per unit voltages at the points *a*, *b*, *c*, and *d* on the main.

(*c*) The line-to-neutral voltages at the points *a*, *b*, *c*, and *d*.

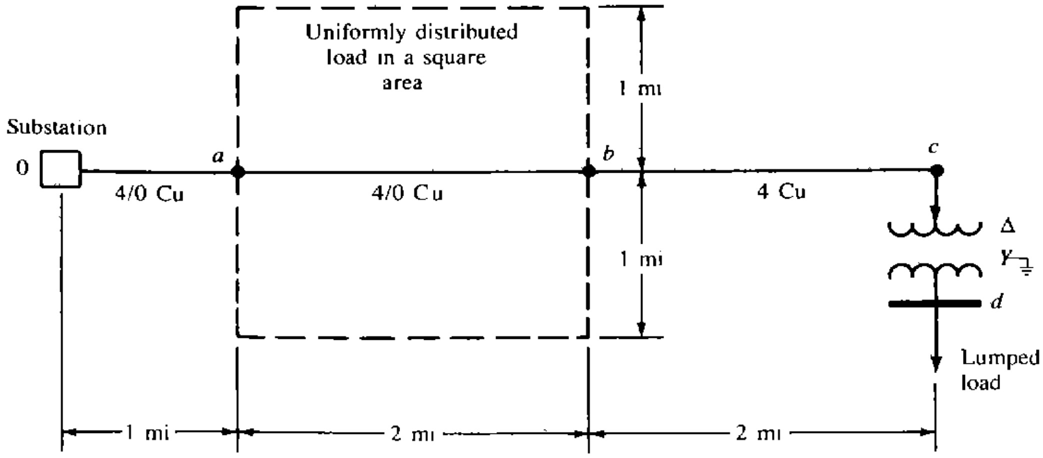


Figure 7-13

SOLUTION

(*a*) The load connected in the square-shaped service area is

$$\begin{aligned} S_n &= D \times A_n \\ &= 1000 \times 4 \\ &= 4000 \text{ kVA} \end{aligned}$$

Thus the total kilovoltampere load on the main is

$$\begin{aligned} S_m &= 4000 + 2000 \\ &= 6000 \text{ kVA} \end{aligned}$$

From Fig. 4-17, for #4/0 copper, the K constant is found to be 0.0003. Therefore the percent voltage drop from the substation to point a is

$$\begin{aligned} \%\,\text{VD}_{0a} &= K \times S_m \times l \\ &= 0.0003 \times 6000 \times 1 \\ &= 1.8\,\%\text{V} \qquad \text{or} \qquad 0.018 \text{ pu V} \end{aligned}$$

The percent voltage drop from point a to point b is

$$\begin{aligned} \%\,\text{VD}_{ab} &= K \times S_n \times \frac{l}{2} + K \times S_{\text{lump}} \times l \\ &= 0.0003 \times 4000 \times 1 + 0.0003 \times 2000 \times 2 \\ &= 2.4\,\%\text{ V} \qquad \text{or} \qquad 0.024 \text{ pu V} \end{aligned}$$

The percent voltage drop from point b to point c is

$$\begin{aligned} \%\,\text{VD}_{bc} &= K \times S_{\text{lump}} \times l \\ &= 0.0009 \times 2000 \times 2 \\ &= 3.6\,\%\text{ V} \qquad \text{or} \qquad 0.036 \text{ pu V} \end{aligned}$$

To find the percent voltage drop from point c to bus d,

$$\begin{aligned} I &= \frac{2000 \text{ kVA}}{\sqrt{3} \times V_{L\text{-}L} \text{ at point } c} \\ &= \frac{2000 \text{ kVA}}{\sqrt{3} \times (0.947 \times 13.2 \text{ kV})} \\ &= 92.373 \text{ A} \end{aligned}$$

$$\begin{aligned} I_B &= \frac{2000 \text{ kVA}}{\sqrt{3} \times 13.2 \text{ kV}} \\ &= 87.477 \text{ A} \end{aligned}$$

$$\begin{aligned} I_{\text{pu}} &= \frac{I}{I_B} \\ &= 1.056 \text{ pu A} \end{aligned}$$

Note that usually in a simple problem like this the reduced voltage at point c is ignored. In that case, for example, the per unit current would be 1.0 pu A rather than 1.056 pu A. Since

$$Z_{T,\mathrm{pu}} = 0 + j0.05 \text{ pu } \Omega$$

and

$$\cos\theta = 0.9 \qquad \text{or} \qquad \theta = 25.84° \text{ lagging}$$

therefore

$$I_{\mathrm{pu}} = 1.056\underline{/25.84°} \text{ pu A}$$

Thus, to find the percent voltage drop at bus d, first it can be found in per unit as

$$\mathrm{VD}_{cd} = \frac{I(R\cos\theta + X\sin\theta)}{V_B} \qquad \text{pu V}$$

but since the low voltage has been tapped up 5 percent,

$$\mathrm{VD}_{cd} = \frac{I(R\cos\theta + X\sin\theta)}{V_B} - 0.05 \qquad \text{pu V}$$

Therefore

$$\mathrm{VD}_{cd} = \frac{1.056(0 \times 0.9 + 0.05 \times 0.4359)}{1.0} - 0.05$$

$$= -0.0267 \text{ pu V}$$

or

$$\%\ \mathrm{VD}_{cd} = -2.67\ \%\ \mathrm{V}$$

Here, the negative sign of the voltage drop indicates that it is in fact a voltage rise rather than a voltage drop.

(*b*) The per unit voltages at the points a, b, c, and d on the main are

$$V_a = V_0 - V_{0a}$$
$$= 1.025 - 0.018$$
$$= 1.007 \text{ pu V} \qquad \text{or} \qquad 100.7\ \%\ \mathrm{V}$$

$$V_b = V_a - V_{ab}$$
$$= 1.007 - 0.024$$
$$= 0.983 \text{ pu V} \qquad \text{or} \qquad 98.3 \text{ to V}$$

$$V_c = V_b - V_{bc}$$
$$= 0.983 - 0.036$$
$$= 0.947 \text{ pu V} \qquad \text{or} \qquad 94.7\ \%\ \mathrm{V}$$

$$V_d = V_c - V_{cd}$$
$$= 0.947 - (-0.0267)$$
$$= 0.9737 \text{ pu V} \qquad \text{or} \qquad 97.37\ \%\ \mathrm{V}$$

(c) The line-to-neutral voltages are

$$V_a = 7620 \times 1.007$$
$$= 7673.3 \text{ V}$$

$$V_b = 7620 \times 0.983$$
$$= 7490.5 \text{ V}$$

$$V_c = 7620 \times 0.947$$
$$= 7216.1 \text{ V}$$

$$V_d = 277 \times 0.9737$$
$$= 269.7 \text{ V}$$

Example 7-8 Use the relevant information and data given in Example 7-7 for off-peak loading and repeat Example 7-7, and find the V_d voltage at bus d in line-to-neutral volts.

SOLUTION

(a) At off-peak loading, the load connected in the square-shaped service area is

$$S_n = D \times A_n$$
$$= 333 \times 4$$
$$= 1332 \text{ kVA}$$

Thus the total kilovoltampere load on the main is

$$S_m = 1332 + 2000$$
$$= 3332 \text{ kVA}$$

Therefore the percent voltage drop from the substation to point a is

$$\% \text{VD}_{0a} = K \times S_m \times l$$
$$= 0.0003 \times 3332 \times 1$$
$$= 1.0\% \text{ V} \qquad \text{or} \qquad 0.01 \text{ pu V}$$

The percent voltage drop from point a to point b is

$$\% \text{VD}_{ab} = K \times S_n \times \frac{l}{2} + K \times S_{\text{lump}} \times l$$
$$= 0.003 \times 1332 \times 1 + 0.0003 \times 2000 \times 2$$
$$= 1.6\% \text{ V} \qquad \text{or} \qquad 0.016 \text{ pu V}$$

The percent voltage drop from point b to point c is

$$\% \text{VD}_{bc} = K \times S_{\text{lump}} \times l$$
$$= 0.0009 \times 2000 \times 2$$
$$= 3.6\% \text{ V} \qquad \text{or} \qquad 0.036 \text{ pu V}$$

To find the percent voltage drop from point c to bus d, the percent voltage drop at bus d can be found as before:

$$\%\,\text{VD}_{cd} = -0.0267 \text{ pu V} \qquad \text{or} \qquad -2.67\%\text{ V}$$

(b) The per unit voltages at points a, b, c, and d on the main are

$$
\begin{aligned}
V_a &= V_0 - V_{0a}\\
&= 1.0 - 0.01\\
&= 0.99 \text{ pu V} \qquad \text{or} \qquad 99\%\text{ V}\\
V_b &= V_a - V_{ab}\\
&= 0.99 - 0.016\\
&= 0.974 \text{ pu V} \qquad \text{or} \qquad 97.4\%\text{ V}\\
V_c &= V_b - V_{bc}\\
&= 0.974 - 0.036\\
&= 0.938 \text{ pu V} \qquad \text{or} \qquad 93.8\%\text{ V}\\
V_d &= V_c - V_{cd}\\
&= 0.938 - (-0.0267)\\
&= 0.9647 \text{ pu V} \qquad \text{or} \qquad 96.47\%\text{ V}
\end{aligned}
$$

(c) The line-to-neutral voltages are

$$
\begin{aligned}
V_a &= 7620 \times 0.99\\
&= 7543.8 \text{ V}\\
V_b &= 7620 \times 0.974\\
&= 7421.9 \text{ V}\\
V_c &= 7620 \times 0.938\\
&= 7147.6 \text{ V}\\
V_d &= 277 \times 0.9647\\
&= 267.2 \text{ V}
\end{aligned}
$$

Note that the voltages at bus d during peak and off-peak loading are nearly the same.

Example 7-9 Figure 7-14 shows that a large number of small loads are closely spaced along the length l. If the loads are single-phase, they are assumed to be well-balanced among the three phases.

A three-phase four-wire wye-grounded 7.62/13.2-kV primary line is to be built along the length l and fed through a distribution substation transformer

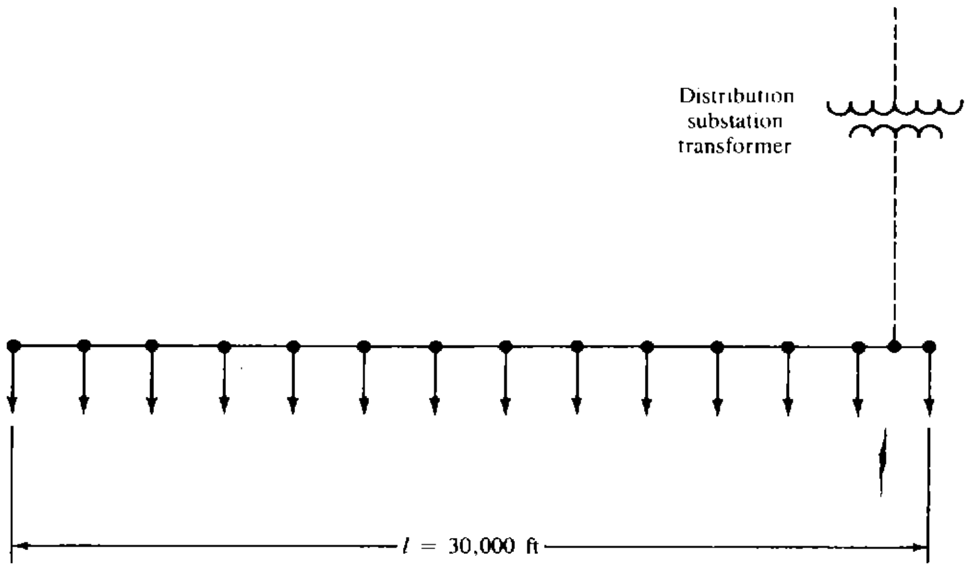


Figure 7-14

from a high-voltage transmission line. Assume that the uniform (or linear) distribution of the connected load along the length l is

$$\frac{S_{\text{connected load}}}{l} = 0.45 \text{ kVA/ft}$$

The 30-min annual demand factor (DF) of all loads is 0.60, the diversity factor (F_D) among all loads is 1.08, and the annual loss factor (F_{LS}) is 0.20. Assuming a lagging power factor of 0.9 for all loads and a 37-in geometric mean spacing of phase conductors, use Fig. 4-17 for voltage-drop calculations for copper conductors. Use the relevant tables in Appendix A for additional data about copper and ACSR conductors and determine the following:

Locate the distribution substation where you think it would be the most economical, considering only the 13.2-kV system, and then find:

(*a*) The minimum ampacity-sized copper and ACSR phase conductors

(*b*) The percent voltage drop at the location having the lowest voltage at the time of the annual peak load, using the ampacity-sized copper conductor found in part *a*.

SOLUTION To achieve minimum voltage drop, the substation should be located at the middle of the line l, and therefore:

(*a*) From Eq. (2-13), the diversified maximum demand of the group of the load is

$$D_g = \frac{\sum_{i=1}^{n} \text{TCD}_i \times \text{DF}_i}{F_D}$$

$$= \frac{0.45 \text{ kVA/ft} \times 0.60}{1.08}$$

$$= 0.250 \text{ kVA/ft}$$

Thus the peak load of each main on the substation transformer is

$$S_{PK} = 0.250 \text{ kVA/ft} \times 15{,}000 \text{ ft}$$
$$= 3750 \text{ kVA}$$

or

$$3750 \text{ kVA} = \sqrt{3} \times 13.2 \text{ kV} \times I$$

hence

$$I = \frac{3750 \text{ kVA}}{\sqrt{3} \times 13.2 \text{ kV}}$$
$$= 164.2 \text{ A}$$

in each main out of the substation. Therefore, from the tables of Appendix A, it can be recommended that either #4 AWG copper conductor or #2 AWG ACSR conductor be used.

(*b*) Assuming that #4 AWG copper conductor is used, the percent voltage drop at the time of the annual peak load is

$$\% \text{ VD} = K\% \text{ VD/(kVA} \cdot \text{mi)} \times S_{PK} \text{ kVA} \times \frac{l \text{ ft}}{2} \frac{1}{5280 \text{ ft/mi}}$$
$$= 0.0009 \times 3750 \times \frac{15{,}000}{2 \times 5280}$$
$$= 5.3\% \text{ V}$$

Example 7-10 Now suppose that the line in Example 7-9 is arbitrarily constructed with #4/0 AWG ACSR phase conductor and that the substation remains where you put it in part *a*. Assume 50°C conductor temperature and find the total annual I^2R energy loss (TAEL_{Cu}), in kilowatthours, in the entire line length.

SOLUTION The total I^2R loss in the entire line length is

$$\sum I^2R = 3I^2\left(r \times \frac{l}{3}\right)$$
$$= 3(164.2)^2(0.592 \ \Omega/\text{mi}) \frac{30{,}000 \text{ ft}}{3 \times 5280 \text{ ft/mi}}$$
$$= 90{,}689.2 \text{ W}$$

Therefore the total I^2R energy loss is

$$\text{TAEL}_{\text{Cu}} = [(\textstyle\sum I^2R)F_{LS}](8760 \text{ h/yr})$$
$$= \frac{90{,}689.2}{10^3} \times 0.20 \times 8760$$
$$= 158{,}887.4 \text{ kWh}$$

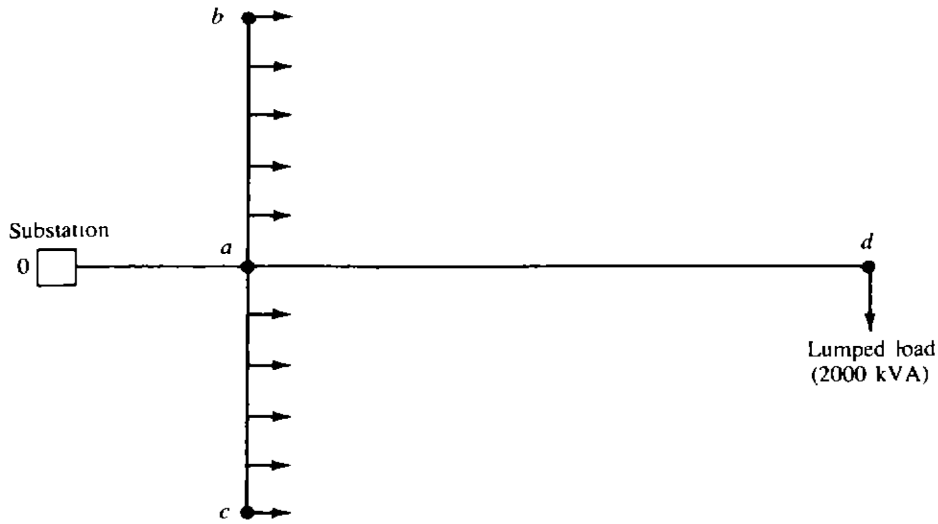


Figure 7-15

Example 7-11 Figure 7-15 shows a single-line diagram of a simple three-phase four-wire wye-grounded primary feeder. The nominal operating voltage and the base voltage is given as 7200/12,470 V. Assume that all loads are balanced three-phase and all have 90 percent power factor, lagging. The given values of the constant K in Table 7-4 are based on 7200/12,470 V. There is a total of a 3000-kVA uniformly distributed load over a 4-mi line between b and c. Use the given data and determine the following:

(*a*) Find the total percent voltage drop at points a, b, c, and d.

(*b*) If the substation bus voltages are regulated to 7300/12,650 V, what are the line-to-neutral and line-to-line voltages at point a?

SOLUTION

(*a*) The total load flowing through the line between points 0 and a is

$$\sum S = 2000\text{ kVA} + 3000\text{ kVA} = 5000\text{ kVA}$$

therefore the percent voltage drop at point a is

$$\begin{aligned}\%\,\text{VD}_a &= K\left(\sum S\right)l \\ &= 0.0005 \times 5000 \times 1.0 \\ &= 2.5\%\text{ V}\end{aligned}$$

Table 7-4 The K constants

Run	Conductor type	Distance, mi	K, % VD/(kVA · mi)
Sub. to a	#4/0 ACSR	1.0	0.0005
a to b	#1 ACSR	2.0	0.0010
a to c	#1 ACSR	2.0	0.0010
a to d	#1 ACSR	2.0	0.0010

Similarly, the load flowing through the line between points a and b is

$$S = 1500 \text{ kVA}$$

therefore

$$\begin{aligned}
\% \text{VD}_b &= K \times S \times \frac{l}{2} + \% \text{VD}_a \\
&= 0.0010 \times 1500 \times 1 + 2.5\% \\
&= 4\% \text{ V} \\
\% \text{VD}_c &= \% \text{VD}_b = 4\% \text{ V} \\
\% \text{VD}_d &= K \times S_{\text{lump}} \times l + \% \text{VD}_a \\
&= 0.0010 \times 2000 \times 2 + 2.5\% \\
&= 6.5\% \text{ V}
\end{aligned}$$

(*b*) If the substation bus voltages are regulated to 7300/12,650 V at point a the line-to-neutral voltage is

$$\begin{aligned}
V_{a,\,L\text{-}N} &= 7300 - \text{VD}_{a,\,L\text{-}N} \\
&= 7300 - 7300 \times 0.025 \\
&= 7117.5 \text{ V}
\end{aligned}$$

and the line-to-line voltage is

$$\begin{aligned}
V_{a,\,L\text{-}L} &= 12{,}650 - \text{VD}_{a,\,L\text{-}L} \\
&= 12{,}650 - 12{,}650 \times 0.025 \\
&= 12{,}333.8 \text{ V}
\end{aligned}$$

7-4 PERCENT POWER (OR COPPER) LOSS

The percent power (or conductor) loss of a circuit can be expressed as

$$\begin{aligned}
\% I^2R &= \frac{P_{\text{LS}}}{P_r} \times 100 \\
&= \frac{I^2R}{R_r} \times 100
\end{aligned} \tag{7-59}$$

where P_{LS} = power loss of a circuit, kW
$= I^2R$

P_r = power delivered by the circuit, kW

The conductor I^2R losses can readily be found from Tables 7-5 to 7-9 for various voltage levels and load factors.

At times, in ac circuits, the ratio of percent power, or conductor, loss to percent voltage regulation can be used, and it is given by the following approximate expression:

$$\frac{\% I^2R}{\% \mathrm{VD}} = \frac{\cos \phi}{\cos \theta \times \cos (\phi - \theta)} \tag{7-60}$$

where $\% I^2R$ = percent power loss of a circuit
$\%$ VD = percent voltage drop of the circuit
ϕ = impedance angle = $\tan^{-1}(X/R)$
θ = power-factor angle

7-5 A METHOD TO ANALYZE DISTRIBUTION FEEDER COSTS

To make any meaningful feeder-size selection, the distribution engineer should make a cost study associated with feeders in addition to the voltage-drop and power-loss considerations. The cost analysis for each feeder size should include (1) investment cost of the installed feeder, (2) cost of energy lost due to I^2R losses in the feeder conductors, and (3) cost of demand lost, i.e., the cost of useful system capacity lost (including generation, transmission, and distribution systems), in order to maintain adequate system capacity to supply the I^2R losses in the distribution feeder conductors. Therefore the total annual feeder cost of a given size feeder can be expressed as

$$\mathrm{TAC} = \mathrm{AIC} + \mathrm{AEC} + \mathrm{ADC} \qquad \$/\mathrm{mi} \tag{7-61}$$

where TAC = total annual equivalent cost of feeder, \$/mi
AIC = annual equivalent of investment cost of installed feeder, \$/mi
AEC = annual equivalent of energy cost due to I^2R losses in feeder conductors, \$/mi
ADC = annual equivalent of demand cost incurred to maintain adequate system capacity to supply I^2R losses in feeder conductors, \$/mi

7-5-1 Annual Equivalent of Investment Cost

The annual equivalent of investment cost of a given size feeder can be expressed as

$$\mathrm{AIC} = \mathrm{IC}_F \times i_F \qquad \$/\mathrm{mi} \tag{7-62}$$

where AIC = annual equivalent of investment cost of a given size feeder, \$/mi
IC_F = cost of installed feeder, \$/mi
i_F = annual fixed charge rate applicable to feeder

The general utility practice is to include cost of capital, depreciation, taxes, insurance, and operation and maintenance (O&M) expenses in the annual fixed charge rate or so-called carrying charge rate. It is given as a decimal.

Table 7-5 Conductor I^2R losses, kWh/(mi · yr), at 7.2/12.5 kV and a load factor of 0.25*

For 7.62/13.2 kV, multiply these values by 0.893; for 14.4/24.9 kV, multiply by 0.25.

Ann. peak load, kW	Single-phase				"V"-phase				Three-phase					
	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	1 copper 2/0 ACSR	1/0 copper 3/0 ACSR	2/0 copper 4/0 ACSR
20	29	19	13	9	14	10								
40	115	76	51	35	57	38	25	17	23					
60	259	172	114	78	129	86	57	39	52	33				
80	460	305	203	138	230	153	101	69	92	58	36			
100	719	477	317	216	359	239	158	108	144	91	57	45		
120	1030	687	456	312	517	344	228	156	208	131	82	65	52	
140	1410	935	621	424	704	468	311	212	283	178	112	89	70	56
160	1840	1220	811	554	920	611	406	277	369	232	146	116	92	73
180	2330	1550	1030	701	1160	773	513	350	467	294	185	146	116	92
200	2870	1910	1270	866	1440	954	634	433	577	363	228	181	143	114
225	3640	2420	1600	1090	1820	1210	802	548	730	460	289	229	181	144
250	4490	2980	1980	1350	2250	1490	990	676	902	568	356	283	224	178
275	5440	3610	2400	1640	2720	1800	1200	818	1090	687	431	342	271	215
300	6470	4290	2850	1950	3230	2150	1430	974	1300	817	513	407	322	256
325	7590	5040	3350	2290	3800	2520	1670	1140	1520	959	602	478	378	300
350	8800	5850	3880	2650	4400	2920	1940	1330	1770	1110	698	554	439	348
375	10100	6710	4460	3040	5050	3360	2230	1520	2030	1280	801	636	503	399
400	11500	7640	5070	3460	5750	3820	2530	1730	2310	1450	912	723	573	454
450	14500	9660	6420	4380	7280	4830	3210	2190	2920	1840	1150	915	725	575
500	18000	11900	7920	5410	8980	5970	3960	2700	3610	2270	1420	1130	895	710

550	21700	14400	9580	6550	10900	7220	4790	3270	4360	2750	1720	1370	1080	859
600		17200	11400	7790	12900	8590	5700	3890	5190	3270	2050	1630	1290	1020
650		20200	13400	9140	15200	10100	6690	4570	6090	3840	2410	1910	1510	1200
700			15500	10600	17600	11700	7760	5300	7070	4450	2790	2220	1750	1390
750			17800	12200	20200	13400	8910	6090	8110	5110	3210	2540	2010	1600
800			20300	13800		15300	10100	6920	9230	5810	3650	2890	2290	1820
850				15600		17200	11400	7820	10400	6560	4120	3270	2590	2050
900				17500		19300	12800	8760	11700	7360	4620	3660	2900	2300
950				19500		21500	14300	9760	13000	8200	5140	4080	3230	2560
1000				21600			15800	10800	14400	9080	5700	4520	3580	2840
1100							19200	13100	17500	11000	6900	5470	4330	3440
1200							22800	15600	20800	13100	8210	6510	5160	4090
1300								18300	24400	15300	9630	7640	6050	4800
1400								21200	28300	17800	11200	8860	7020	5570
1500								24300	32500	20400	12800	10200	8060	6390
1600								27700	36900	23200	14600	11600	9160	7270
1700									41700	26200	16500	13100	10300	8210
1800									46700	29400	18500	14600	11600	9200
1900									52100	32800	20600	16300	12900	10300
2000									57700	36300	22800	18100	14300	11400
2200									69800	43900	27600	21900	17300	13700
2400										52300	32800	26000	20600	16400
2600										61400	38500	30600	24200	19200
2800										71200	44700	35400	28100	22300
3000										81700	51300	40700	32200	25600

Note: This table is calculated for a PF of 90 percent. To adjust for a different PF, multiply these values by the factor $k = (90)^2/(PF)^2$.

* From [6].

Table 7-6 Conductor I^2R losses, kWh (mi · yr), at 7.2/12.5 kV and a load factor of 0.35*

For 7.62/13.2 kV, multiply these values by 0.893; for 14.4/24.9 kV, multiply by 0.25.

Ann. peak load, kW	Single-phase				"V"-phase				Three-phase					
	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	1 copper 2/0 ACSR	1/0 copper 3/0 ACSR	2/0 copper 4/0 ACSR
20	49	33	22	15	25	16	11							
40	197	131	87	59	99	66	43	30	40					
60	444	295	196	134	222	147	98	67	89	56				
80	789	524	348	238	395	262	174	119	158	100	63			
100	1230	819	544	371	617	409	272	186	248	156	98	78		
120	1780	1180	783	535	888	590	391	267	356	224	141	112	88	
140	2420	1600	1070	728	1210	802	533	364	485	305	192	152	120	96
160	3160	2100	1390	950	1580	1050	696	475	634	399	250	199	157	125
180	4000	2650	1760	1200	2000	1330	881	601	802	505	317	251	199	158
200	4930	3280	2170	1490	2470	1640	1090	743	990	623	391	310	246	195
225	6240	4150	2750	1880	3120	2070	1380	940	1250	789	495	393	311	247
250	7710	5120	3400	2320	3850	2560	1700	1160	1550	974	611	485	384	305
275	9330	6190	4110	2810	4660	3100	2060	1400	1870	1180	740	587	465	368
300	11100	7370	4890	3340	5550	3680	2450	1670	2230	1400	880	698	553	438
325	13000	8650	5740	3920	6510	4320	2870	1960	2610	1650	1030	819	649	515
350	15100	10000	6660	4550	7550	5020	3330	2270	3030	1910	1200	950	752	597
375	17300	11500	7650	5220	8670	5760	3820	2610	3480	2190	1380	1090	864	685
400	19700	13100	8700	5940	9870	6550	4350	2970	3960	2490	1560	1240	983	780
450		16600	11000	7520	12500	8290	5500	3760	5010	3160	1980	1570	1240	987
500		20500	13600	9280	15400	10200	6800	4640	6200	3890	2440	1940	1540	1220

550			16400	11200	18700	12400	8220	5620	7500	4710	2960	2350	1860	1470
600			19600	13400	22200	14700	9790	6680	8910	5600	3520	2790	2210	1750
650			22900	15700	26100	17300	11500	7840	10500	6580	4130	3280	2600	2060
700				18200		20100	13300	9090	12100	7630	4790	3800	3010	2390
750				20900		23000	15300	10400	13900	8760	5500	4360	3460	2740
800				23800		26200	17400	11900	15800	9970	6260	4960	3930	3120
850							19700	13400	17900	11300	7070	5600	4440	3520
900							22000	15000	20000	12600	7920	6280	4980	3950
950							24500	16800	22300	14100	8830	7000	5540	4400
1000							27200	18600	24800	15600	9800	7760	6140	4870
1100							32900	22500	29900	18900	11800	9390	7430	5900
1200							39100	26700	35600	22400	14100	11200	8850	7020
1300								31400	41800	26300	16500	13100	10400	8240
1400								36400	48500	30500	19200	15200	12000	9550
1500								41800	55700	35100	22000	17500	13800	10900
1600								47500	63400	39900	25000	19900	15700	12500
1700									71500	45000	28300	22400	17800	14100
1800									80200	50500	31700	25100	19900	15800
1900									89400	56200	35300	28000	22200	17600
2000									99000	62300	39100	31000	24600	19500
2200									119800	75400	47300	37500	29700	23600
2400										89700	56300	44700	35400	28100
2600										105300	66100	52400	41500	32900
2800											76700	60800	48200	38200
3000											88000	69800	55300	43800

Note: This table is calculated for a PF of 90 percent. To adjust for a different PF, multiply these values by the factor $k = (90)^2/(PF)^2$.

* From [6].

Table 7-7 Conductor I^2R losses, kWh/(mi · yr), at 7.2/12.5 kV and a load factor of 0.5*

For 7.62/13.2 kV, multiply these values by 0.893; for 14.4/24.9 kV, multiply by 0.25.

Ann. peak load, kW	Single-phase				"V"-phase				Three-phase					
	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	1 copper 2/0 ACSR	1/0 copper 3/0 ACSR	2/0 copper 4/0 ACSR
20	90	60	40	27	45	30	20	14	18					
40	360	239	159	108	180	120	80	54	73	45				
60	810	538	357	244	405	270	179	122	160	102	64			
80	1440	956	635	434	720	478	317	217	290	182	114	91		
100	2250	1490	992	678	1130	747	496	339	454	284	178	142	113	
120	3240	2150	1430	976	1620	1080	714	488	653	409	257	204	162	128
140	4410	2930	1940	1330	2210	1460	972	664	889	557	350	277	220	174
160	5760	3820	2540	1730	2880	1910	1270	867	1160	728	457	362	288	228
180	7290	4840	3210	2190	3650	2420	1610	1100	1470	921	578	459	365	288
200	9000	5980	3970	2710	4500	2990	1980	1360	1820	1140	714	566	450	356
225	11400	7560	5020	3430	5700	3780	2510	1720	2300	1440	904	717	570	450
250	14100	9340	6200	4230	7030	4670	3100	2120	2840	1780	1115	885	703	556
275		11300	7500	5120	8510	5650	3750	2560	3430	2150	1350	1070	851	673
300		13450	8930	6100	10130	6720	4460	3050	4080	2560	1610	1270	1010	800
325		15800	10480	7160	11900	7890	5240	3580	4790	3000	1890	1490	1190	939
350			12150	8300	13800	9150	6080	4150	5560	3480	2190	1730	1380	1090
375			13950	9530		10510	6980	4760	6380	4000	2510	1990	1580	1250
400				10840		11960	7940	5420	7260	4550	2860	2270	1800	1420
450				13700		15100	10050	6860	9190	5760	3610	2870	2280	1800
500				16900		18700	12400	8470	11300	7110	4460	3540	2810	2220

550						22600	15000	10250	13700	8600	5400	4280	3400	2690
600							17900	12200	16300	10200	6430	5100	4050	3200
650							21000	14300	19200	12000	7540	5980	4750	3760
700							24300	16600	22200	13900	8750	6940	5510	4360
750								19100	25500	16000	10000	7960	6330	5000
800								21700	29000	18200	11400	9060	7200	5690
850								24500	32800	20500	12900	10200	8130	6430
900								27400	36800	23000	14500	11500	9110	7200
950								30600	41000	25700	16100	12800	10200	8030
1000									45400	28400	17800	14200	11300	8890
1100									54900	34400	21600	17100	13600	10800
1200									65300	40900	25700	20400	16200	12800
1300									76700	48100	30200	23900	19000	15000
1400									88900	55700	35000	27700	22000	17400
1500									102100	64000	40200	31900	25300	20000
1600										72800	45700	36200	28800	22800
1700										82200	51600	40900	32500	25700
1800										92100	57800	45900	36500	28800
1900										102600	64400	51100	40600	32100
2000										113800	71400	56600	45000	35600
2200											86400	68500	54500	43000
2400											102800	81500	64800	51200
2600											120600	95700	76100	60100
2800												111000	88200	69700
3000												127000	101300	80100

Note: This table is calculated for a PF of 90 percent. To adjust for a different PF, multiply these values by the factor $k = (90)^2/(PF)^2$.

* From [6].

Table 7-8 Conductor I^2R losses, kWh/(mi · yr), at 7.2/12.5 kV and a load factor of 0.55*

For 7.62/13.2 kV, multiply these values by 0.893; for 14.4/24.9 kV, multiply by 0.25.

Ann. peak load, kW	Single-phase				"V"-phase				Three-phase					
	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	1 copper 2/0 ACSR	1/0 copper 3/0 ACSR	2/0 copper 4/0 ACSR
20	106	71	47	32	53	35	23	16	21	13				
40	426	283	188	128	213	141	94	64	85	54	34			
60	958	636	422	288	479	318	211	144	192	121	76	60		
80	1700	1130	750	513	851	565	375	256	342	215	135	108	85	
100	2660	1770	1170	801	1330	883	586	400	534	336	211	167	132	105
120	3830	2540	1690	1150	1920	1270	844	577	769	484	304	241	191	151
140	5210	3460	2300	1570	2610	1730	1150	785	1050	659	413	328	260	206
160	6810	4520	3000	2050	3400	2260	1500	1030	1370	860	540	428	339	269
180	8620	5720	3800	2590	4310	2860	1900	1300	1730	1090	683	542	429	341
200	10600	7060	4690	3200	5320	3530	2350	1600	2140	1340	844	669	530	420
225	13500	8940	5940	4050	6730	4470	2970	2030	2700	1700	1070	847	671	532
250	16600	11000	7330	5010	8310	5520	3660	2500	3330	2100	1320	1050	828	657
275	20100	13400	8870	6060	10100	6680	4430	3030	4040	2540	1600	1270	1000	795
300		15900	10600	7210	12000	7950	5280	3600	4810	3020	1900	1510	1190	946
325		18700	12400	8460	14000	9330	6190	4230	5640	3550	2230	1770	1400	1110
350		21600	14400	9810	16300	10800	7180	4900	6540	4120	2580	2050	1620	1290
375			16500	11300	18700	12400	8240	5630	7510	4730	2970	2350	1860	1480
400			18800	12800	21300	14100	9380	6410	8540	5380	3370	2680	2120	1680
450			23700	16200	26900	17900	11900	8110	10800	6810	4270	3390	2680	2130
500				20000		22100	14700	10000	13300	8400	5270	4180	3310	2630

550				24200			17700	12100	16200	10200	6380	5060	4010	3180
600							21100	14400	19200	12100	7590	6020	4770	3780
650							24800	16900	22600	14200	8910	7070	5600	4440
700								19600	26200	16500	10300	8200	6490	5150
750								22500	30000	18900	11900	9410	7450	5910
800								25600	34200	21500	13500	10700	8480	6730
850								28900	38600	24300	15200	12100	9570	7590
900								32400	43200	27200	17100	13600	10700	8510
950								36100	48200	30300	19000	15100	11900	9490
1000									53400	33600	21100	16700	13200	10500
1100									64600	40700	25500	20200	16000	12700
1200									76900	48400	30400	24100	19100	15100
1300									90200	56800	35600	28300	22400	17800
1400									105000	65900	41300	32800	26000	20600
1500										75600	47500	37600	29800	23600
1600										86000	54000	42800	33900	26900
1700										97200	61000	48400	38300	30400
1800										109000	68300	54200	42900	34100
1900											76100	60400	47800	37900
2000											84400	66900	53000	42000
2200											102000	81000	64100	50900
2400												96400	76300	60500
2600												113000	89600	71100
2800													103900	82400
3000													119000	94600

Note: This table is calculated for a PF of 90 percent. To adjust for a different PF, multiply these values by the factor $k = (90)^2/(PF)^2$.

* From [6].

Table 7-9 Conductor I^2R losses, kWh/(mi · yr), at 7.2/12.5 kV and a load factor of 0.6*

For 7.62/13.2 kV, multiply these values by 0.893; for 14.4/24.9 kV, multiply by 0.25.

Ann. peak load, kW	Single-phase				"V"-phase				Three-phase					
	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	8 copper	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	6 copper 4 ACSR	4 copper 2 ACSR	2 copper 1/0 ACSR	1 copper 2/0 ACSR	1/0 copper 3/0 ACSR	2/0 copper 4/0 ACSR
20	124	82	55	37	62	41	27	19	25	16	10			
40	495	329	218	149	248	164	109	75	99	63	39	31		
60	1110	740	491	335	557	370	246	168	224	141	88	70	56	
80	1980	1320	873	596	990	658	437	298	398	250	157	125	99	78
100	3100	2060	1360	932	1550	1030	682	466	621	391	245	195	154	122
120	4460	2960	1960	1340	2230	1480	982	671	895	563	353	280	222	176
140	6070	4030	2670	1830	3030	2010	1340	913	1220	766	481	382	302	240
160	7920	5260	3490	2390	3960	2630	1750	1190	1590	1000	628	498	395	313
180	10000	6660	4420	3020	5010	3330	2210	1510	2010	1270	795	631	500	396
200	12400	8220	5460	3730	6190	4110	2730	1860	2490	1560	982	779	617	489
225	15700	10400	6910	4720	7830	5200	3450	2360	3150	1980	1240	986	780	619
250	19300	12800	8530	5820	9670	6420	4260	2910	3880	2440	1530	1220	964	764
275	23400	15500	10300	7050	11700	7770	5160	3520	4700	2960	1860	1470	1170	925
300		18500	12300	8390	13900	9250	6140	4190	5590	3520	2210	1750	1390	1100
325		21700	14400	9840	16300	10900	7210	4920	6560	4130	2590	2060	1630	1280
350			16700	11400	18900	12600	8360	5710	7610	4790	3010	2380	1890	1500
375			19200	13100	21800	14400	9590	6550	8740	5500	3450	2740	2170	1720
400			21800	14900	24800	16400	10900	7450	9940	6260	3930	3120	2470	1960
450				18900		20800	13800	9430	12600	7920	4970	3940	3120	2480
500				23300		25700	17100	11600	15500	9780	6140	4870	3850	3060

550							20600	14100	18800	11800	7420	5890	4660	3700
600							24600	16800	22400	14100	8840	7010	5550	4400
650							28800	19700	26200	16500	10400	8220	6510	5170
700							33400	22800	30400	19200	12000	9540	7550	6000
750								26200	34900	22000	13800	10900	8670	6880
800								29800	39800	25000	15700	12500	9870	7830
850								33700	44900	28300	17700	14100	11100	8840
900								37700	50300	31700	19900	15800	12500	9900
950								42000	56100	35300	22200	17600	13900	11000
1000									62100	39100	24500	19500	15400	12200
1100									75200	47300	29700	23600	18700	14800
1200									89500	56300	35300	28000	22200	17600
1300									105000	66100	41500	32900	26100	20700
1400										76600	48100	38200	30200	24000
1500										88000	55200	43800	34700	27500
1600										100100	62800	49800	39500	31300
1700											70900	56300	44600	35300
1800											79500	63100	50000	39600
1900											88600	70300	55700	44200
2000											98200	77900	61700	48900
2200											118800	94200	74600	59200
2400												112100	88800	70400
2600													104200	82700
2800													120100	95900
3000													138800	110000

Note: This table is calculated for a PF of 90 percent. To adjust for a different PF, multiply these values by the factor $k = (90)^2/(PF)^2$.

* From [6].

7-5-2 Annual Equivalent of Energy Cost

The annual equivalent of energy cost due to I^2R losses in feeder conductors can be expressed as

$$\text{AEC} = 3I^2R \times \text{EC} \times F_{\text{LL}} \times F_{\text{LSA}} \times 8760 \qquad \$/\text{mi} \tag{7-63}$$

where AEC = annual equivalent of energy cost due to I^2R losses in feeder conductors, $/mi
EC = cost of energy, $/kWh
F_{LL} = load-location factor
F_{LS} = loss factor
F_{LSA} = loss-allowance factor

The load-location factor of a feeder with uniformly distributed load can be defined as

$$F_{LL} = \frac{s}{l} \tag{7-64}$$

where F_{LL} = load-location factor in decimal
s = distance of point on feeder where total feeder load can be assumed to be concentrated for purpose of calculating I^2R losses,
l = total feeder length, mi

The loss factor can be defined as the ratio of the average annual power loss to the peak annual power loss and can be found approximately for urban areas from

$$F_{\text{LS}} = 0.3F_{\text{LD}} + 0.7F_{\text{LD}}^2 \tag{7-65}$$

and for rural areas [6],

$$F_{\text{LS}} = 0.16F_{\text{LD}} + 0.84F_{\text{LD}}^2$$

The loss-allowance factor is an allocation factor that allows for the additional losses incurred in the total power system due to the transmission of power from the generating plant to the distribution substation.

7-5-3 Annual Equivalent of Demand Cost

The annual equivalent of demand cost incurred to maintain adequate system capacity to supply the I^2R losses in the feeder conductors can be expressed as

$$\begin{aligned}\text{ADC} = 3I^2R \times F_{\text{LL}} \times F_{\text{PR}} \times F_R \\ \times F_{\text{LSA}}[(C_G \times i_G) + (C_T \times i_T) + (C_S \times i_S)] \qquad \$/\text{mi}\end{aligned} \tag{7-66}$$

where ADC = annual equivalent of demand cost incurred to maintain adequate system capacity to supply I^2R losses in feeder conductors, \$/mi

F_{LL} = load-location factor

F_{PR} = peak-responsibility factor

F_R = reserve factor

F_{LSA} = loss-allowance factor

C_G = cost of (peaking) generation system \$/kVA

C_T = cost of transmission system, \$/kVA

C_S = cost of distribution substation, \$/kVA

i_G = annual fixed charge rate applicable to generation system

i_T = annual fixed charge rate applicable to transmission system

i_S = annual fixed charge rate applicable to distribution substation

The reserve factor is the ratio of total generation capability to the total load and losses to be supplied. The peak-responsibility factor is a per unit value of the peak feeder losses that are coincident with the system peak demand.

7-5-4 Levelized Annual Cost

In general, the costs of energy and demand and even O&M expenses vary from year to year during a given time, as shown in Fig. 7-16*a*; therefore it becomes necessary to *levelize* these costs over the expected economic life of the feeder, as shown in Fig. 7-16*b*.

Assume that the costs occur discretely at the end of each year, as shown in Fig. 7-16*a*. The levelized annual cost† of equal amounts can be calculated as

$$A = [F_1(P/F)_1^i + F_2(P/F)_2^i + F_3(P/F)_3^i + \cdots + F_n(P/F)_n^i](A/P)_n^i \quad (7\text{-}67)$$

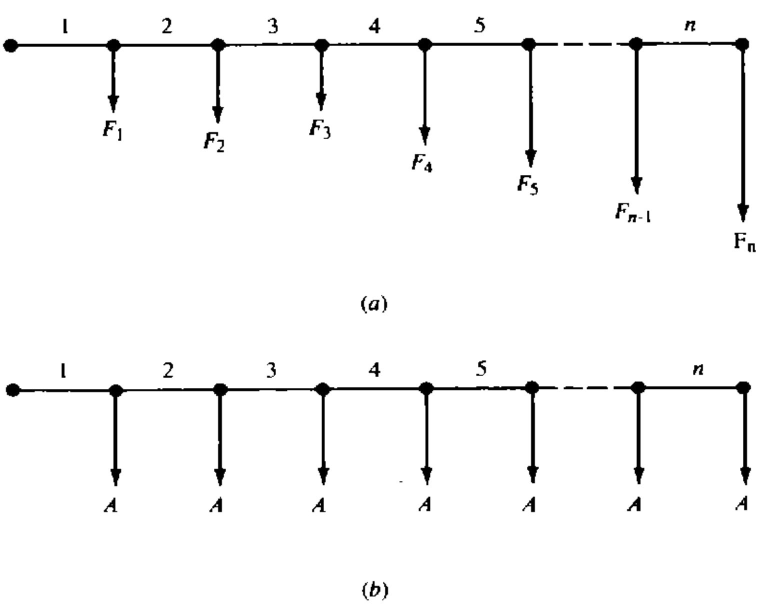


Figure 7-16 Illustration of the levelized annual cost concept: (*a*) unlevelized annual cost flow diagram, and (*b*) levelized cost flow diagram.

† Also called the *annual equivalent* or *annual worth*.

or

$$A = \left[\sum_{j=1}^{n} F_i (P/F)_j^i \right] (A/P)_n^i \tag{7-68}$$

where A = levelized annual cost, \$/yr
F_i = unequal (or actual or unlevelized) annual cost, \$/yr
n = economic life, yr
i = interest rate
$(P/F)_n^i$ = present worth (or present equivalent) of a future sum factor (with i interest rate and n years of economic life); also known as *single-payment discount factor*
$(A/P)_n^i$ = uniform series worth of a present sum factor; also known as *capital-recovery factor*

The single-payment discount factor and the capital-recovery factor can be found from the compounded-interest tables or from the following equations, respectively,

$$(P/F)_n^i = \frac{1}{(1 + i)^n} \tag{7-69}$$

and

$$(A/P)_n^i = \frac{i(1 + i)^n}{(1 + i)^n - 1} \tag{7-70}$$

Example 7-12 Assume that the following data have been gathered for the system of the NL&NP Company.

Feeder length = 1 mi
Cost of energy = 20 mills/kWh (or \$0.02/kWh)
Cost of generation system = \$200/kW
Cost of transmission system = \$65/kW
Cost of distribution substation = \$20/kW
Annual fixed charge rate for generation = 0.21
Annual fixed charge rate for transmission = 0.18
Annual fixed charge rate for substation = 0.18
Annual fixed charge rate for feeders = 0.25
Interest rate = 12 percent
Load factor = 0.4
Loss-allowance factor = 1.03
Reserve factor = 1.15
Peak-responsibility factor = 0.82

Table 7-10 gives cost data for typical ACSR conductors used in rural areas at 12.5 and 24.9 kV. Table 7-11 gives cost data for typical ACSR conductors used in urban areas at 12.5 and 34.5 kV. Using the given data, develop nomographs that can be readily used to calculate the annual equivalent energy costs and demand costs due to I^2R losses, and the total annual equivalent cost of the feeder in dollars per mile.

Table 7-10 Typical ACSR conductors used in rural areas

Conductor size	Ground wire size	Conductor wt, lb	Ground wire weight, lb	Cost, \$/lb	Installation & hardware cost, \$	Total installed feeder cost, \$
			At 12.5 kV			
#4	#4	356	356	0.6	6,945.6	7,800
1/0	#2	769	566	0.6	7,176.2	8,900
3/0	1/0	1223	769	0.6	7,737.2	10,400
4/0	1/0	1542	769	0.6	8,563	11,800
266.8 kcmil	1/0	1802	769	0.6	9,985	13,690
477 kcmil	1/0	3642	769	0.6	10,967	17,660
			At 24.9 kV			
#4	#4	356	356	0.6	7,605.6	8,460
1/0	#2	769	566	0.6	7,856.2	9,580
3/0	1/0	1223	769	0.6	8,217.2	10,880
4/0	1/0	1542	769	0.6	8,293	11,530
266.8 kcmil	1/0	1802	769	0.6	9,615	13,320
477 kcmil	1/0	3462	769	0.6	11,547	18,240

Solution Using the given and additional data and appropriate equations from Sec. 7-5, the following nomographs have been developed. Figures 7-17 to 7-20 give nomographs to calculate the annual energy costs incurred due to I^2R losses in ACSR feeder conductors of various sizes in hundreds of dollars per mile. Figures 7-21 to 7-24 give nomographs to calculate the annual costs incurred due to I^2R losses in ACSR feeder conductors of various sizes in hundreds of dollars per mile. Figures 7-25 to 7-27 give nomographs to calculate the total annual equivalent cost of ACSR feeders of various sizes for rural and urban areas, respectively, in thousands of dollars per mile.

Table 7-11 Typical ACSR conductors used in urban areas

Conductor size	Ground wire size	Conductor wt, lb	Ground wire weight, lb	Cost, \$/lb	Installation & hardware cost, \$	Total installed feeder cost, \$
			At 12.5 kV			
#4	#4	356	356	0.6	21,145.6	22,000
1/0	#4	769	356	0.6	22,402.2	24,000
3/0	#4	1223	356	0.6	24,585	27,000
477 kcmil	1/0	3462	769	0.6	28,307	35,000
			At 34.5 kV			
#4	#4	356	356	0.6	21,375.6	22,230
1/0	#4	769	356	0.6	22,632.2	24,230
3/0	#4	1223	356	0.6	24,815	27,230
477 kcmil	1/0	3462	769	0.6	28,537	35,230

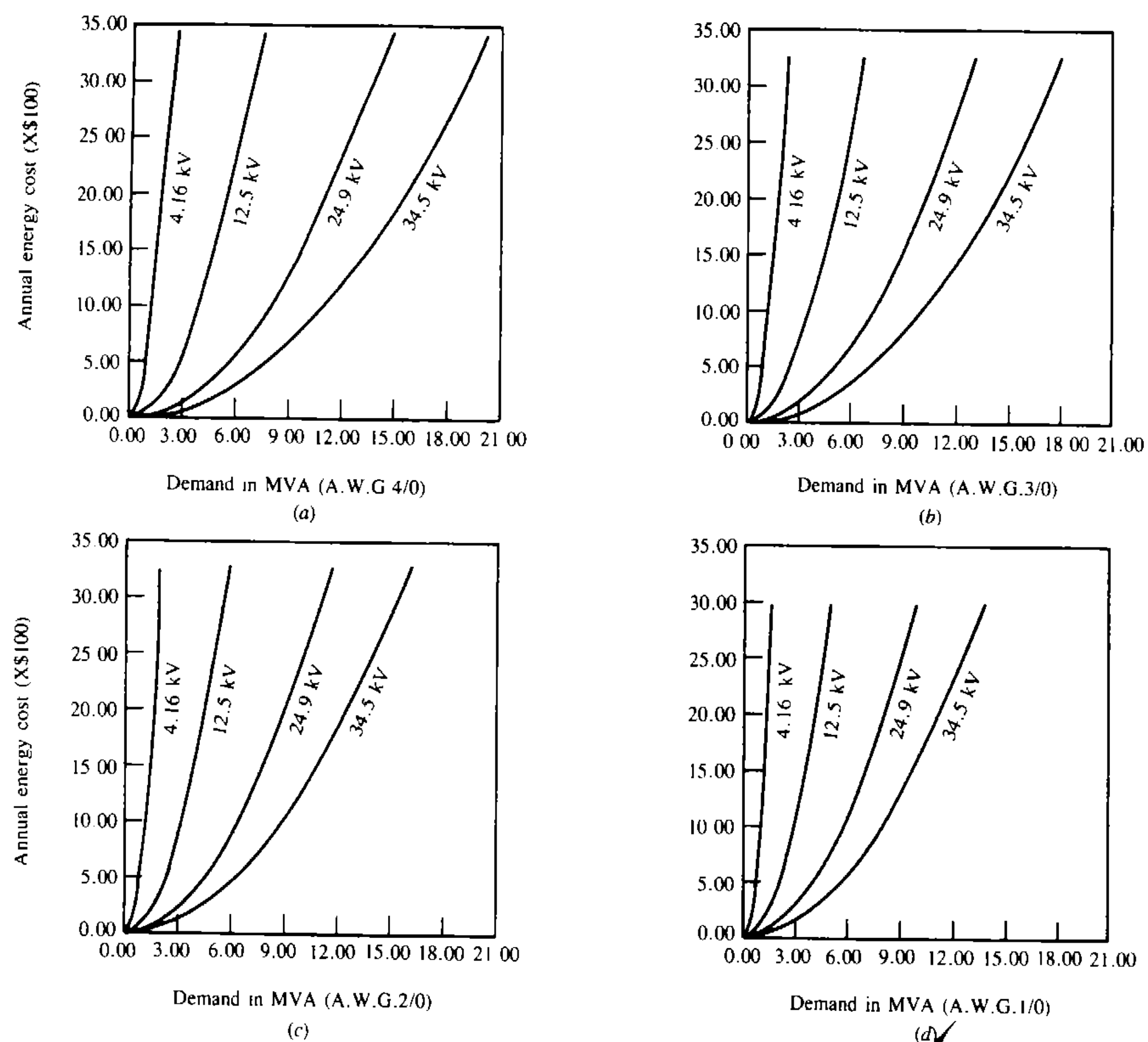


Figure 7-17 Annual energy cost due to I^2R losses in ACSR feeders in hundreds of dollars per mile: (*a*) AWG 4/0, (*b*) AWG 3/0, (*c*) AWG 2/0 and AWG 1/0.

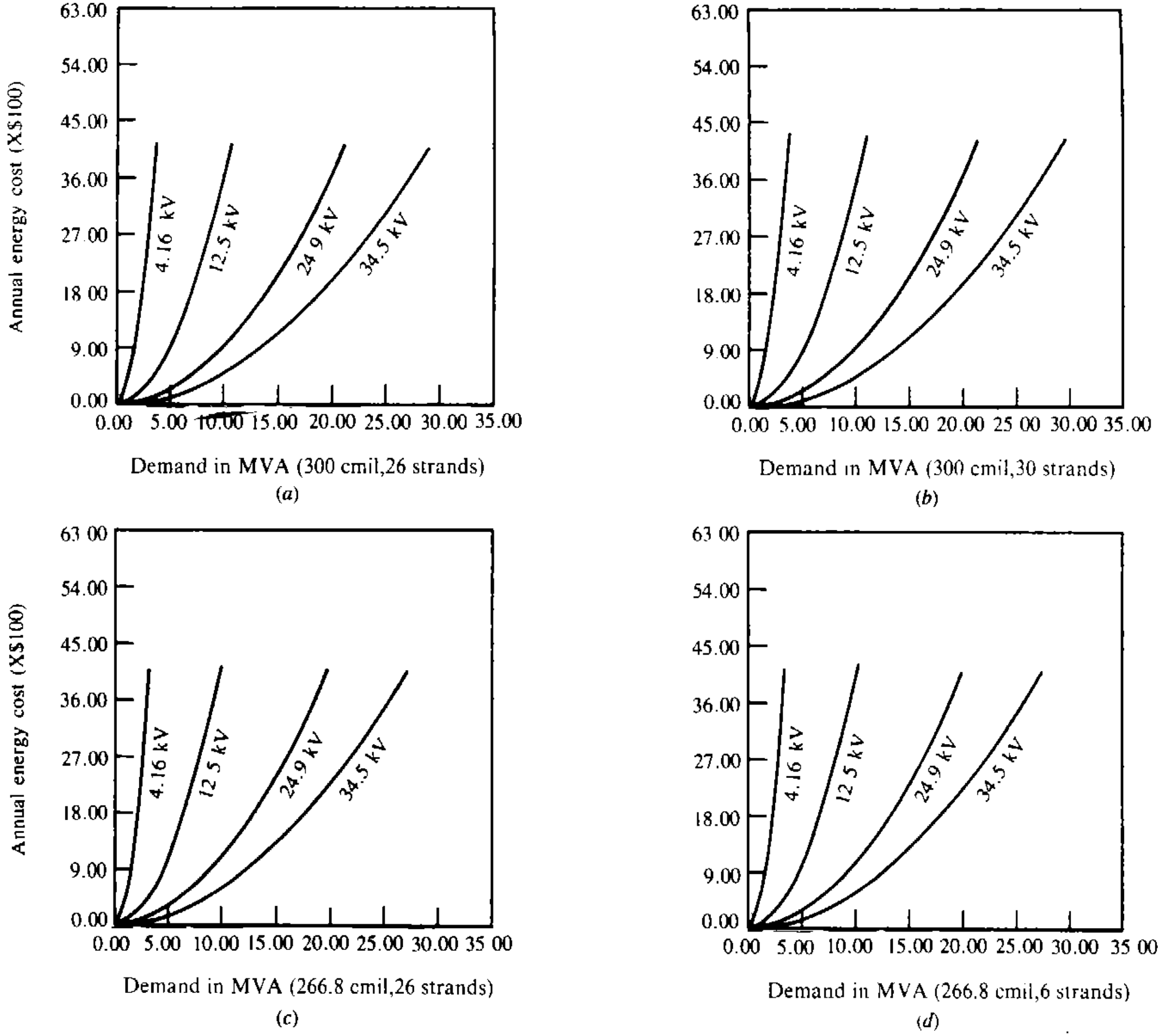


Figure 7-18 Annual energy cost due to I^2R losses in ACSR feeders in hundreds of dollars per mile: (*a*) 300 cmil, 26 strands, (*b*) 300 cmil, 30 strands, (*c*) 266.8 cmil, 26 strands, and (*d*) 266.8 cmil, 6 strands.

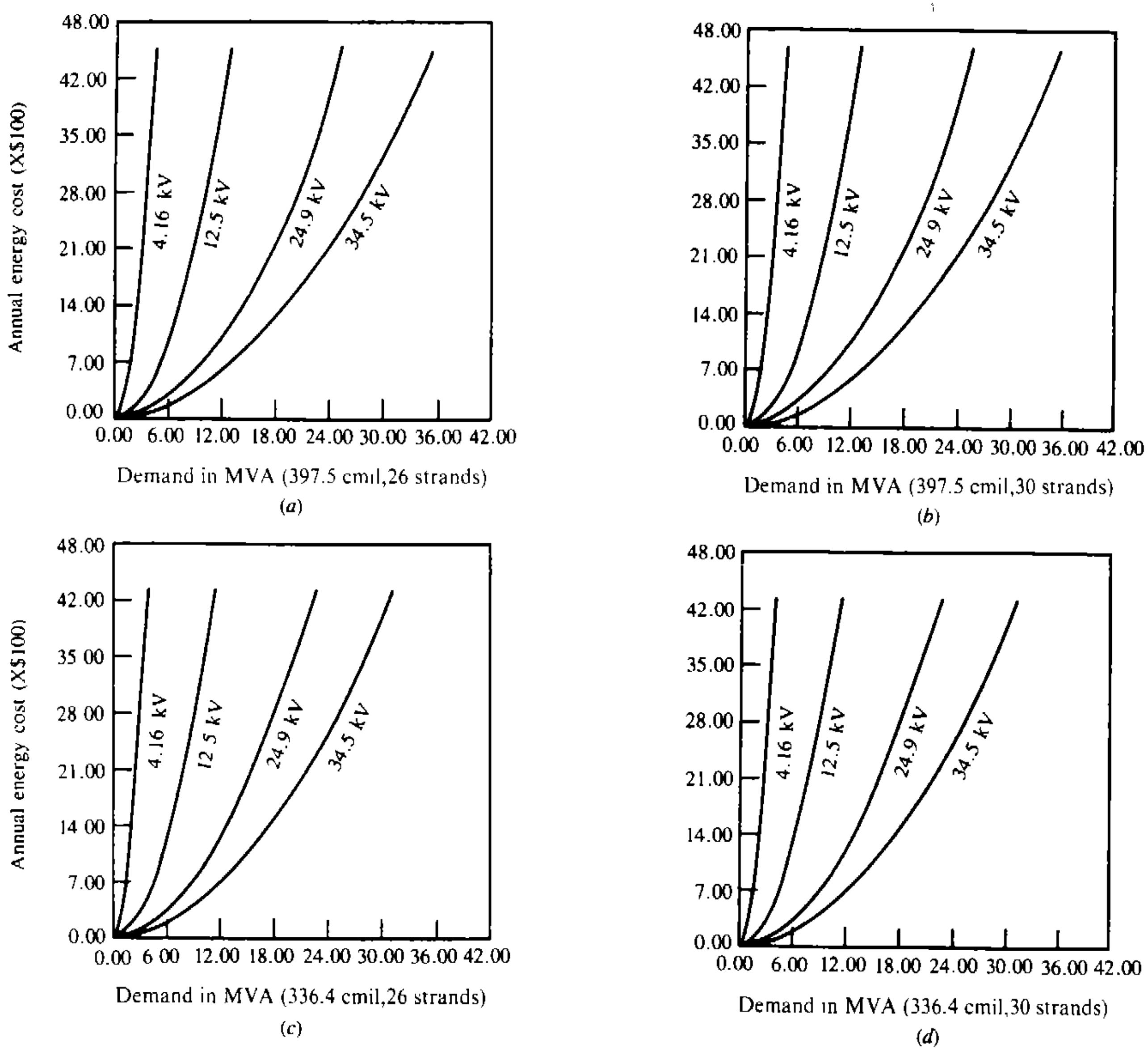


Figure 7-19 Annual energy cost due to I^2R losses in ACSR feeders in hundreds of dollars per mile: (*a*) 397.5 cmil, 26 strands, (*b*) 397.5 cmil, 30 strands, (*c*) 336.4 cmil, 26 strands, and (*d*) 336.4 cmil, 30 strands.

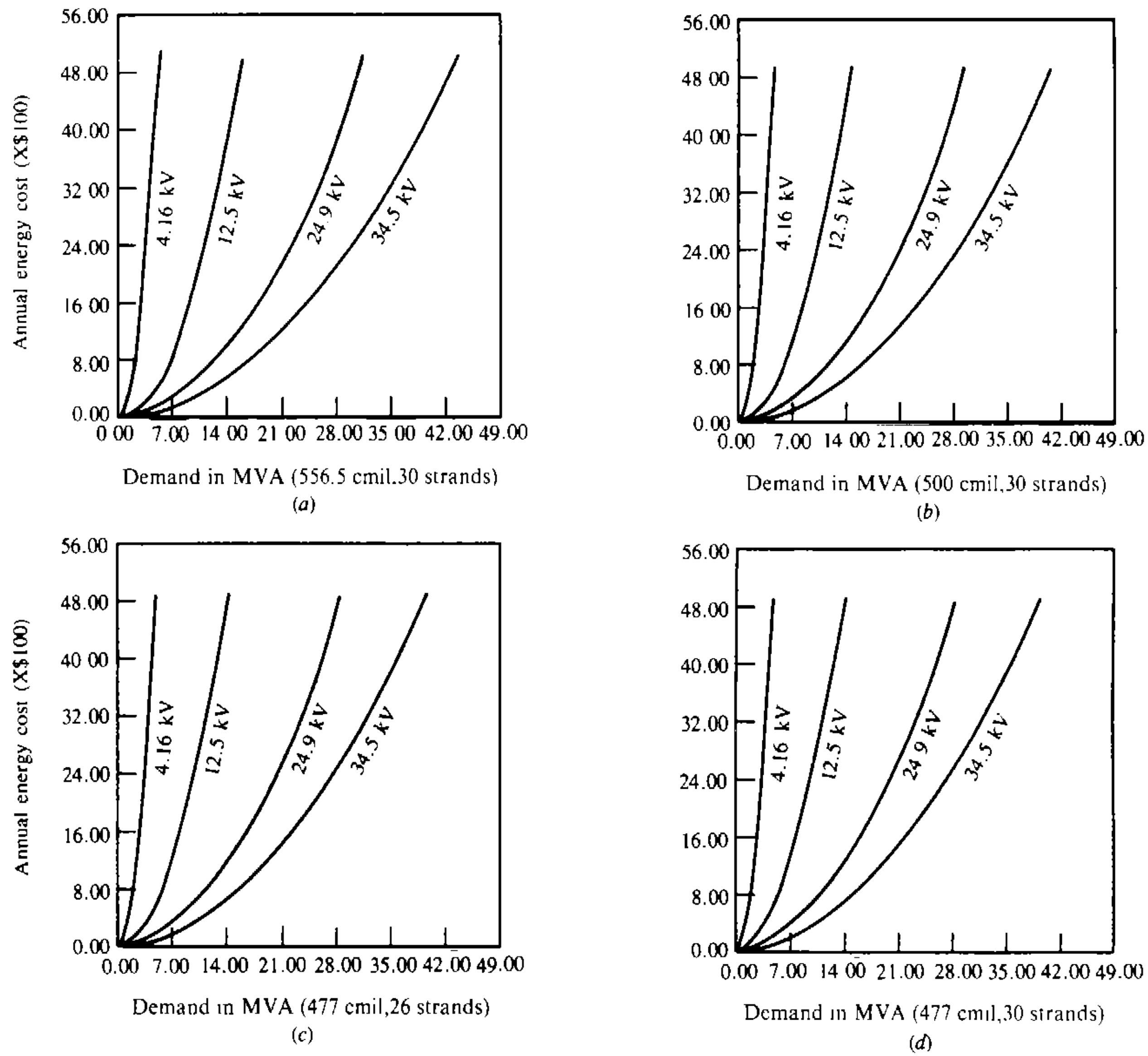


Figure 7-20 Annual energy cost due to I^2R losses in ACSR feeders in hundreds of dollars per mile: (*a*) 556.5 cmil, 30 strands, (*b*) 500 cmil, 30 strands, (*c*) 477 cmil, 26 strands, and (*d*) 477 cmil, 30 strands.

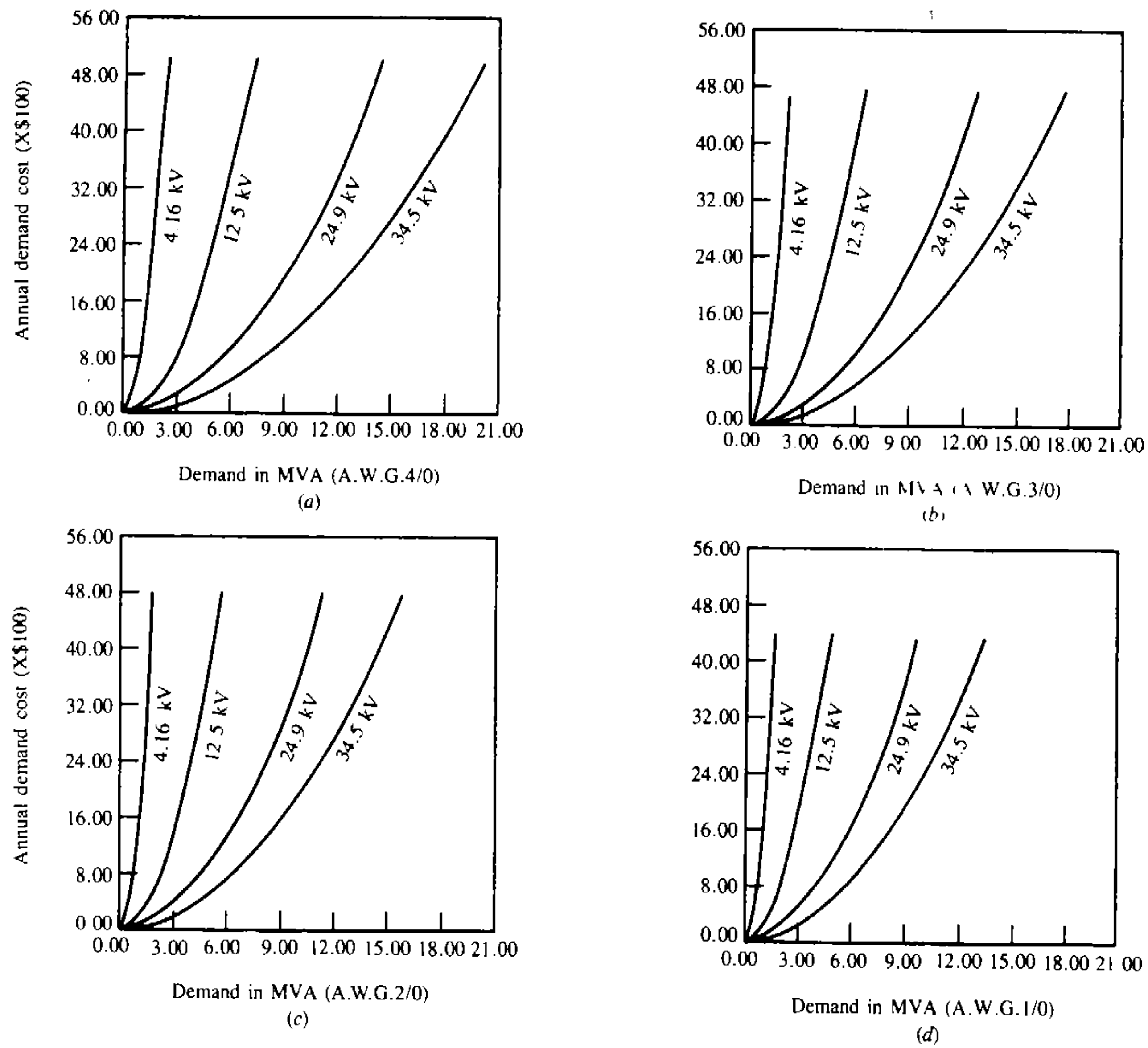


Figure 7-21 Annual demand cost due to energy losses in ACSR feeders in hundreds of dollars per mile: (*a*) AWG 4/0, (*b*) AWG 3/0, (*c*) AWG 2/0 and AWG 1/0.

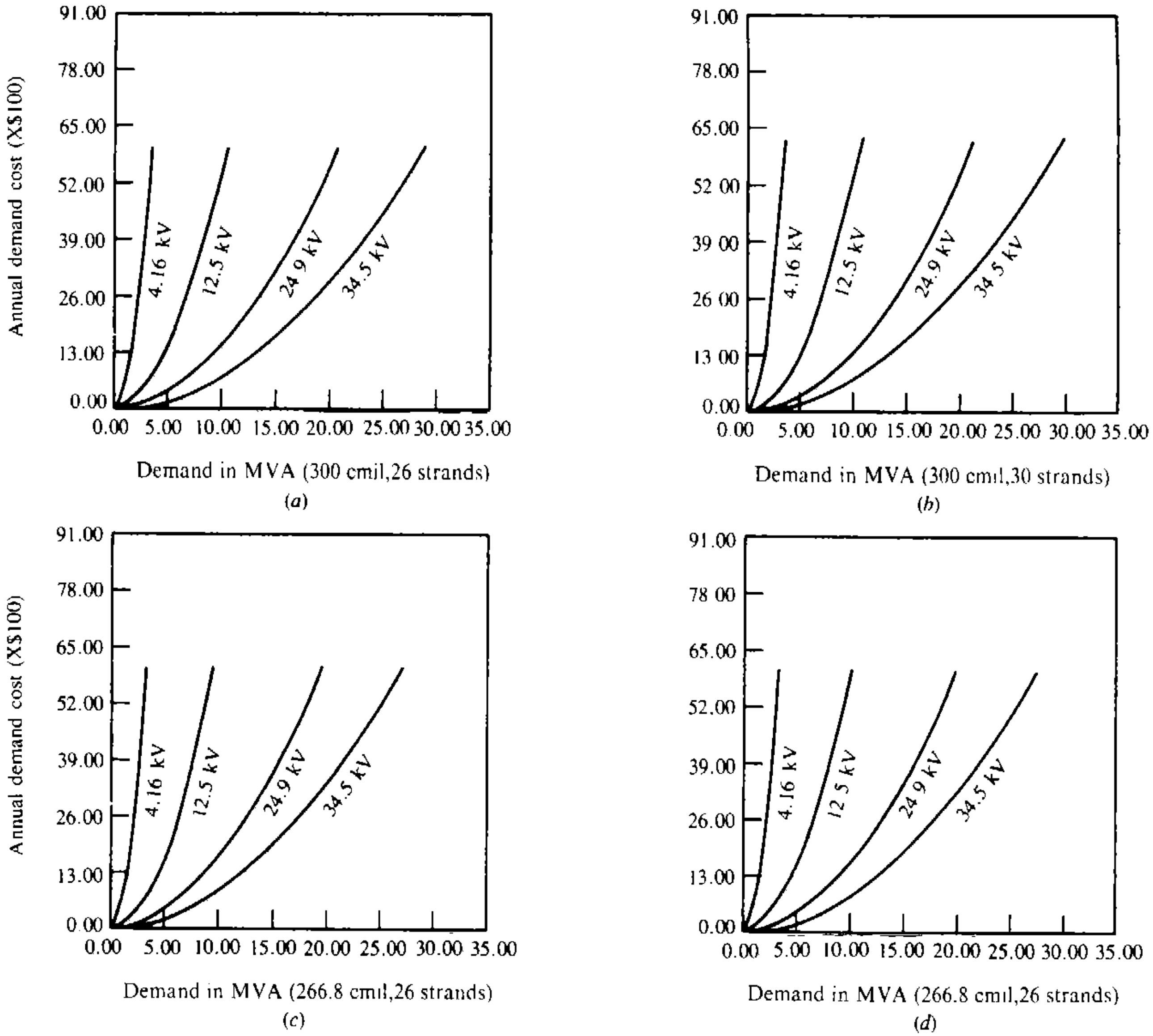


Figure 7-22 Annual demand cost due to energy losses in ACSR feeders in hundreds of dollars per mile: (*a*) 300 cmil, 26 strands, (*b*) 300 cmil, 30 strands, (*c*) 266.8 cmil, 26 strands, and (*d*) 266.8 cmil, 6 strands.

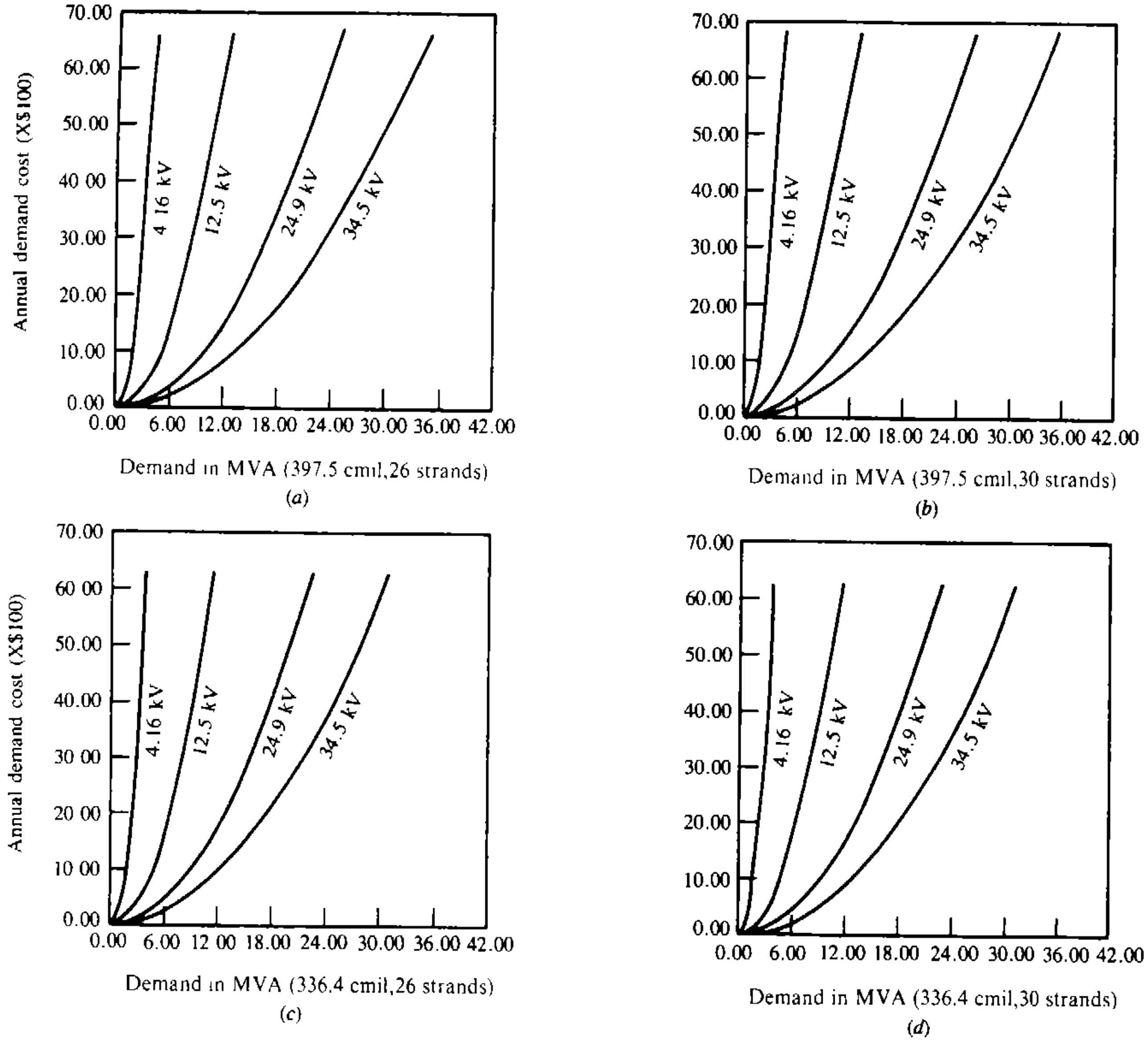


Figure 7-23 Annual demand cost due to energy losses in ACSR feeders in hundreds of dollars per mile: (*a*) 397.5 cmil, 26 strands, (*b*) 397.5 cmil, 30 strands, (*c*) 336.4 cmil, 26 strands, and (*d*) 336.4 cmil, 30 strands.

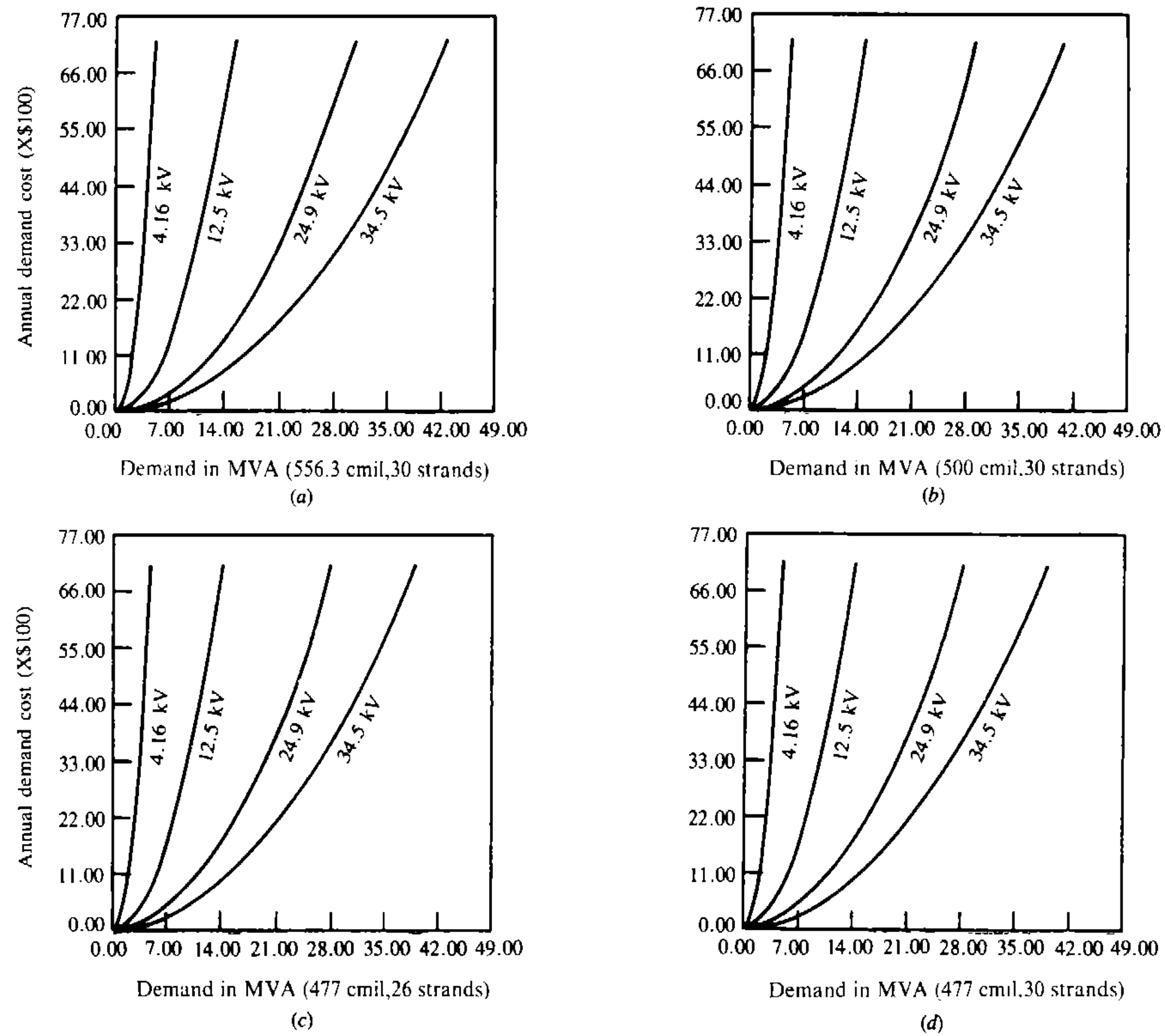


Figure 7-24 Annual demand cost due to energy losses in ACSR feeders in hundreds of dollars per mile: (*a*) 556.5 cmil, 30 strands, (*b*) 500 cmil, 30 strands, (*c*) 477 cmil, 26 strands, and (*d*) 477 cmil, 30 strands.

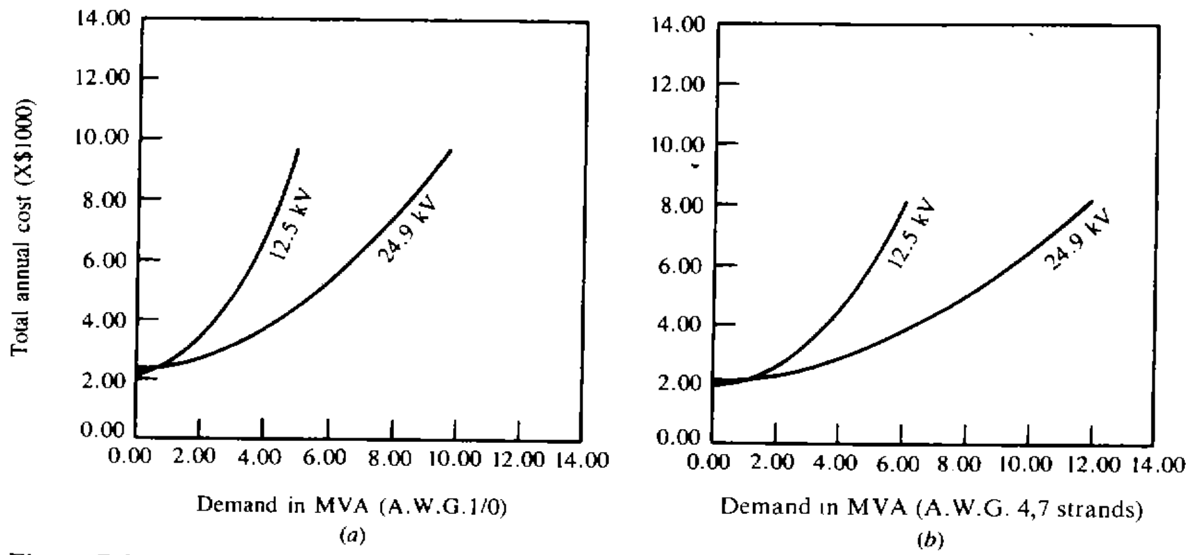


Figure 7-25 Total annual equivalent cost of ACSR feeders for rural areas in thousands of dollars per mile: (*a*) AWG 1/0, and (*b*) AWG 4, 7 strands.

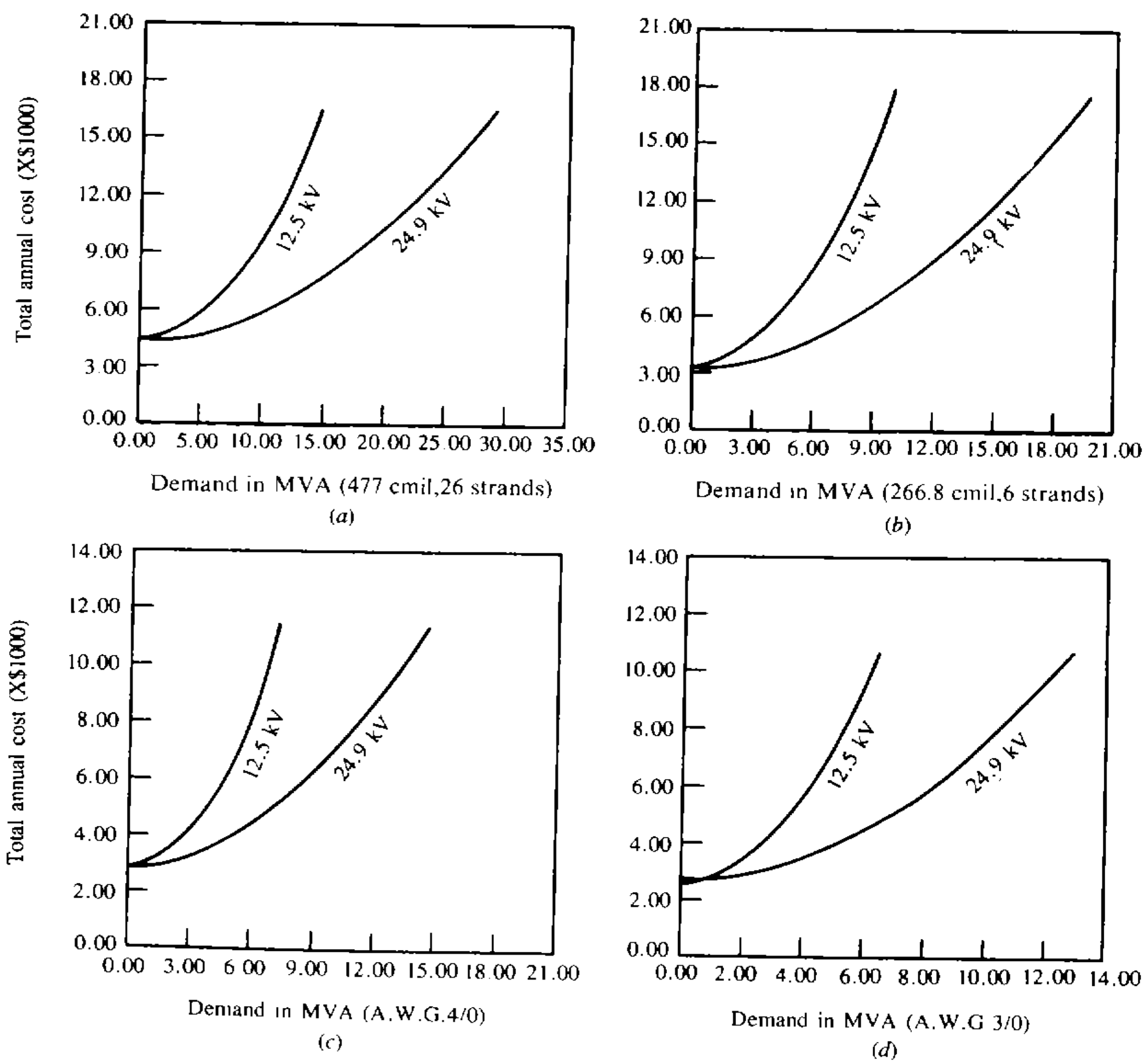


Figure 7-26 Total annual equivalent cost of ACSR feeders for rural areas in thousands of dollars per mile: (*a*) 477 cmil, 26 strands, (*b*) 266.8 cmil, 6 strands, (*c*) AWG 4/0, and (*d*) AWG 3/0.

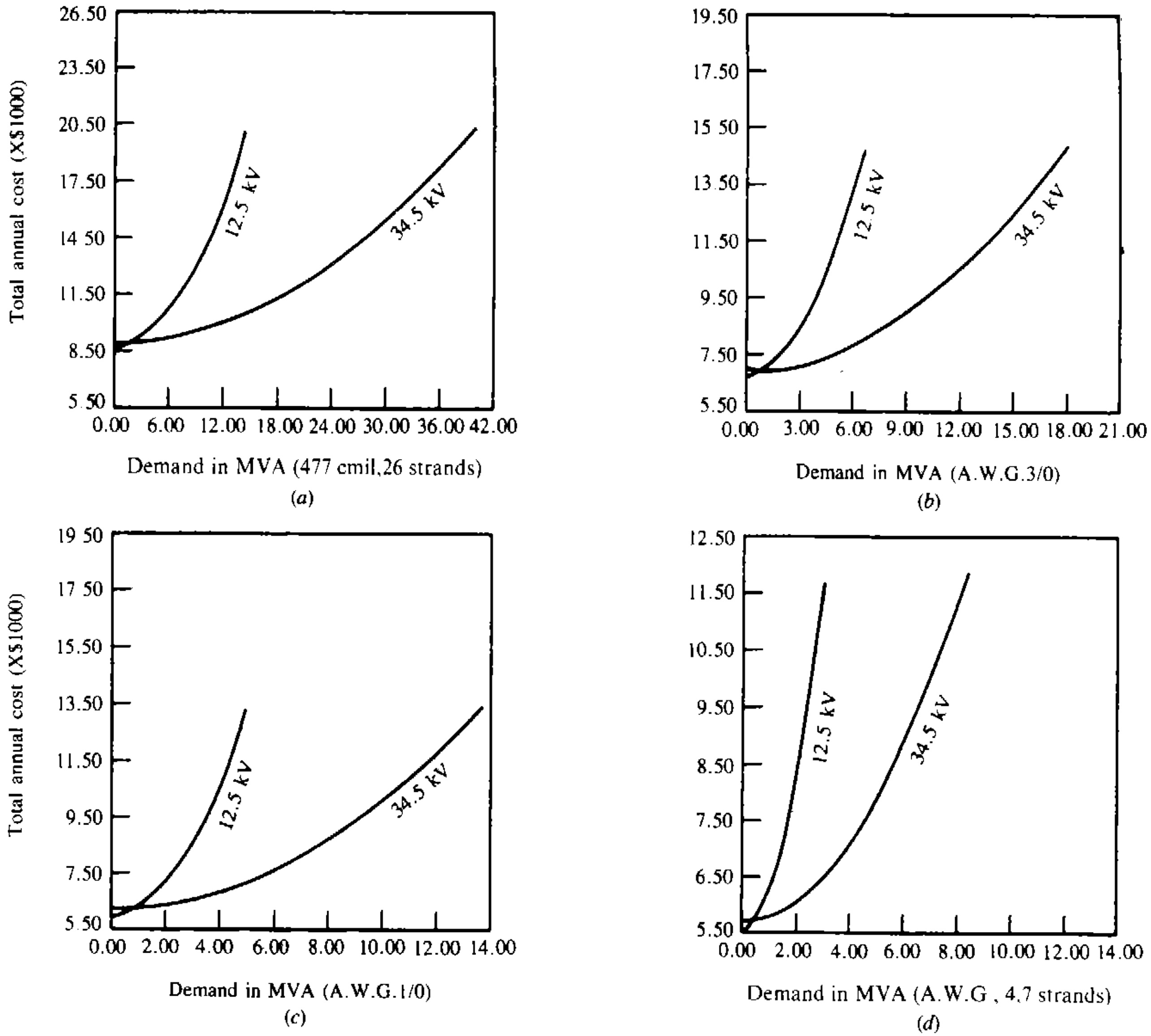


Figure 7-27 Total annual equivalent cost of ACSR feeders for urban areas in thousands of dollars per mile: (*a*) 477 cmil, 26 strands, (*b*) AWG 3/0, (*c*) AWG 1/0, and (*d*) AWG 4, 7 strands.

7-6 ECONOMIC ANALYSIS OF EQUIPMENT LOSSES

Today, the substantially escalating plant, equipment, energy, and capital costs make it increasingly more important to evaluate losses of electric equipment (e.g., power or distribution transformers) before making any final decision for purchasing new equipment and/or replacing (or retiring) existing ones. For example, nowadays it is not uncommon to find out that a transformer with lower losses but higher initial price tag is less expensive than the one with higher losses but lower initial price when total cost over the life of the transformer is considered.

However, in the replacement or retirement decisions, the associated cost savings in operating and maintenance costs in a given life-cycle analysis† or life-cycle cost study† must be greater than the total purchase price of the more-efficient replacement transformer. Based on the "sunk cost" concept of engineering economy, the carrying charges of the existing equipment do not affect the retirement decision, regardless of the age of the existing unit. In other words, the fixed, or carrying, charges of an existing equipment must be amortized (written off) whether the unit is retired or not.

The transformer cost study should include the following factors:

1. Annual cost of copper losses
2. Annual cost of core losses
3. Annual cost of exciting current
4. Annual cost of regulation
5. Annual cost of fixed charges on the first cost of the installed equipment

These annual costs may be different from year to year during the economical lifetime of the equipment. Therefore, it may be required to levelize them, as explained in Sec. 7-5-4. Read Sec. 6-7 for further information on the cost study of the distribution transformers. For the economic replacement study of the power transformers, the following simplified technique may be sufficient. Dodds [10] summarizes the economic evaluation of the cost of losses in an old and a new transformer step by step as:

1. Determine the power ratings for the transformers as well as the peak and average system loads.
2. Obtain the load and no-load losses for the transformers under rated conditions.
3. Determine the original cost of the old transformer and the purchase price of the new one.
4. Obtain the carrying charge rate, system capital cost rate, and energy cost rate for your particular utility.

† These phrases are used by some governmental agencies and other organizations to specifically require that bid evaluations or purchase decisions be based not just on first cost but on all factors (such as future operating costs) that influence which alternative is the more economical.

5. Calculate the transformer carrying charge and the cost of losses for each transformer. The cost of losses is equal to the system carrying charge plus the energy charge.
6. Compare the total cost per year for each transformer. The total cost is equal to the sum of the transformer carrying charge and the cost of losses.
7. Compare the total cost per year of the old and the new transformer. If the total cost per year of the new transformer is less, replacement of the old transformer can be economically justified.

PROBLEMS

7-1 Consider Fig. P7-1 and repeat Example 7-2.

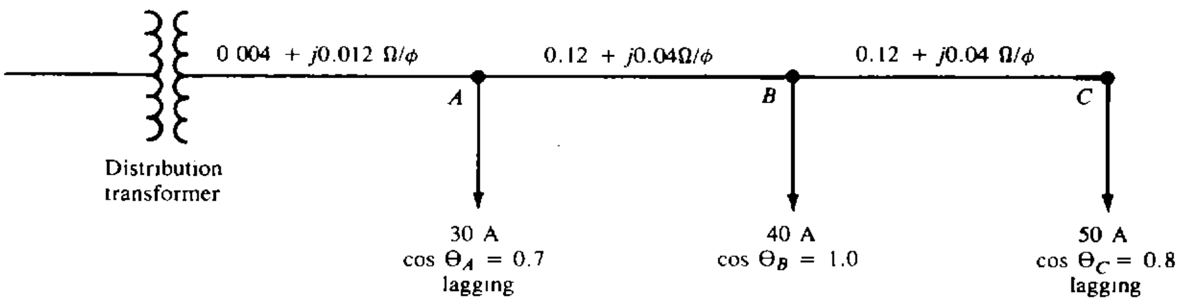


Figure P7-1

7-2 Repeat Example 7-4, using a transformer with 75-kVA capacity.

7-3 Repeat Example 7-4, assuming four services per transformer. Here, omit the UG secondary line. Assume that there are six transformers per block, i.e., one transformer at each pole location.

7-4 Repeat Prob. 7-3, using a 75-kVA transformer.

7-5 Repeat Example 7-5, using a 100-kVA transformer and #3/0 AWG and #2 AWG cables for the secondary lines and service drops, respectively.

7-6 Repeat Example 7-7. Use the nominal primary voltage of 19,920/34,500 V and assume that the remaining data are the same.

7-7 Assume that a three-conductor dc overhead line with equal conductor sizes (see Fig. P7-7) is considered to be employed to transmit three-phase three-conductor ac energy at 0.92 power factor. If voltages to ground and transmission line efficiencies are the same for both direct and alternating current, and the load is balanced, determine the change in the power transmitted in percent.

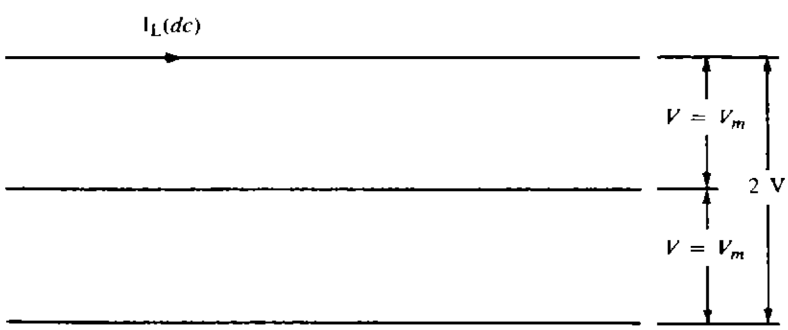


Figure P7-7

7-8 Assume that a single-phase feeder circuit has a total impedance of $1 + j3\ \Omega$ for lines and/or transformers. The receiving-end voltage and load current are $2400\angle 0°$ V and $50\angle -30°$ A, respectively. Determine the following:

(*a*) The power factor of the load.

(*b*) The load power factor for which the voltage drop is maximum, using Eq. (7-51).

(*c*) Repeat part *b*, using Eq. (7-52).

7-9 An unbalanced three-phase wye-connected and grounded load is connected to a balanced three-phase four-wire source. The load impedance Z_a, Z_b, and Z_c are given as $70\angle 30°$, $85\angle -40°$, and $50\angle 35°\ \Omega$/phase, respectively, and the phase *a* line voltage has an effective value of 13.8 kV. Use the line-to-neutral voltage of phase *a* as the reference and determine the following:

(*a*) The line and neutral currents.

(*b*) The total power delivered to the loads.

7-10 Consider Fig. P7-1 and assume that the impedances of the three line segments from left to right are $0.1 + j0.3$, $0.1 + j0.1$, and $0.08 + j0.12\ \Omega$/phase, respectively. Also assume that this three-phase three-wire 480-V secondary system supplies balanced loads at *A*, *B*, and *C*. The loads at *A*, *B*, and *C* are represented by 50 A with a lagging power factor of 0.85, 30 A with a lagging power factor of 0.90, and 50 A with a lagging power factor of 0.95, respectively. Determine the following:

(*a*) The total voltage drop in one phase of the lateral using the approximate method.

(*b*) The real power per phase for each load.

(*c*) The reactive power per phase for each load.

(*d*) The kilovoltampere output and load power factor of the distribution transformer.

7-11 Assume that bulk power substation 1 supplies substations 2 and 3, as shown in Fig. P7-11, through three-phase lines. Substations 2 and 3 are connected to each other over a tie line, as shown. Assume that the line-to-line voltage is 69 kV and determine the following:

(*a*) The voltage difference between substations 2 and 3 when tie line 23 is open-circuited.

(*b*) The line currents when all three lines are connected as shown in the figure.

(*c*) The total power loss in part *a*.

(*d*) The total power loss in part *b*.

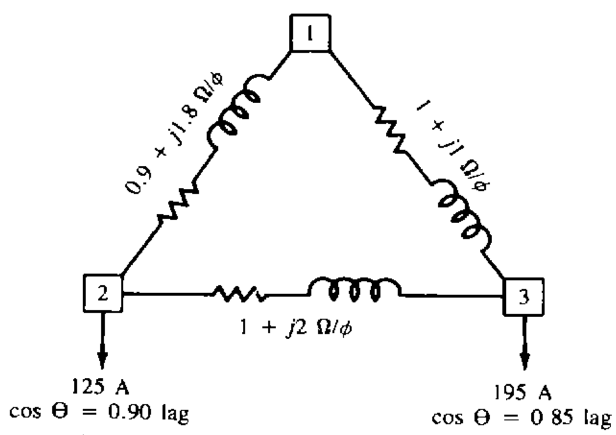


Figure P7-11

7-12 Repeat Example 7-3, assuming 50 percent lagging power factor for all loads.

REFERENCES

1. Morrison, C.: "A Linear Approach to the Problem of Planning New Feed Points Into a Distribution System," *AIEE Trans.*, p. III (PAS) December 1963, pp. 819–832.
2. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.

3. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*. 11th ed., McGraw-Hill, New York, 1978.
4. Gönen, T., et al.: *Development of Advanced Methods for Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va.
5. Gönen, T., and D. C. Yu: "A Comparative Analysis of Distribution Feeder Costs," a paper presented at the Southwest Electrical Exposition and IEEE Conference, Houston, Tex., Jan. 22–24, 1980.
6. Rural Electrification Administration, U.S. Department of Agriculture: *Economic Design of Primary Lines for Rural Distribution Systems*, REA Bulletin 60-9, May 1960.
7. Gönen, T., and D. C. Yu: "A Distribution System Planning Model," *Control of Power Systems Conference Proceedings*, Oklahoma City, Okla., Mar. 17–18, 1980, pp. 28–34.
8. Schlegel, M. C.: "New Selection Method Reduces Conductor Losses," *Electr. World*, Feb. 1, 1977, pp. 43–44.
9. Light, J.: "An Economic Approach to Distribution Conductor Size Selection," paper presented at the Missouri Valley Electric Association 49th Annual Engineering Conference, Kansas City, Mo., Apr. 12–14, 1978.
10. Dodds, T. H.: "Costs of Losses Can Economically Justify Replacement of an Old Transformer with a New One," *The Line*, vol. 80, no. 2, July 1980, pp. 25–28.
11. Smith, R. W., and D. J. Ward: "Does Early Distribution Transformer Retirement Make Sense?" *Electric. Forum*, vol. 6, no. 3, 1980, pp. 6–9.
12. Delaney, M. B.: "Economic Analysis of Electrical Equipment Losses," *The Line*, vol. 74, no. 4, 1974, pp. 7–8.
13. Klein, K. W.: "Evaluation of Distribution Transformer Losses and Loss Ratios," *Elec. Light Power*, July 15, 1960, pp. 56–61.
14. Jeynes, P. H.: "Evaluation of Capacity Differences in the Economic Comparison of Alternative Facilities," *AIEE Trans.*, pt. III (PAS), January 1952, pp. 62–80.

CHAPTER

EIGHT

APPLICATION OF CAPACITORS TO DISTRIBUTION SYSTEMS

8-1 BASIC DEFINITIONS

Capacitor element: an indivisible part of a capacitor consisting of electrodes separated by a dielectric material

Capacitor unit: an assembly of one or more capacitor elements in a single container with terminals brought out

Capacitor segment: a single-phase group of capacitor units with protection and control system

Capacitor module: a three-phase group of capacitor segments

Capacitor bank: a total assembly of capacitor modules electrically connected to each other

8-2 POWER CAPACITORS

At a casual look a capacitor seems to be a very simple and unsophisticated apparatus, i.e., two metal plates separated by a dielectric insulating material. It has no moving parts, but instead functions by being acted upon by electric stress. In reality, however, a power capacitor is a highly technical and complex device in that very thin dielectric materials and high electric stresses are involved, coupled with highly sophisticated processing techniques. Figure 8-1 shows a cutaway view of a power-factor-correction capacitor. Figure 8-2 shows a typical capacitor utilization in a switched pole-top rack.

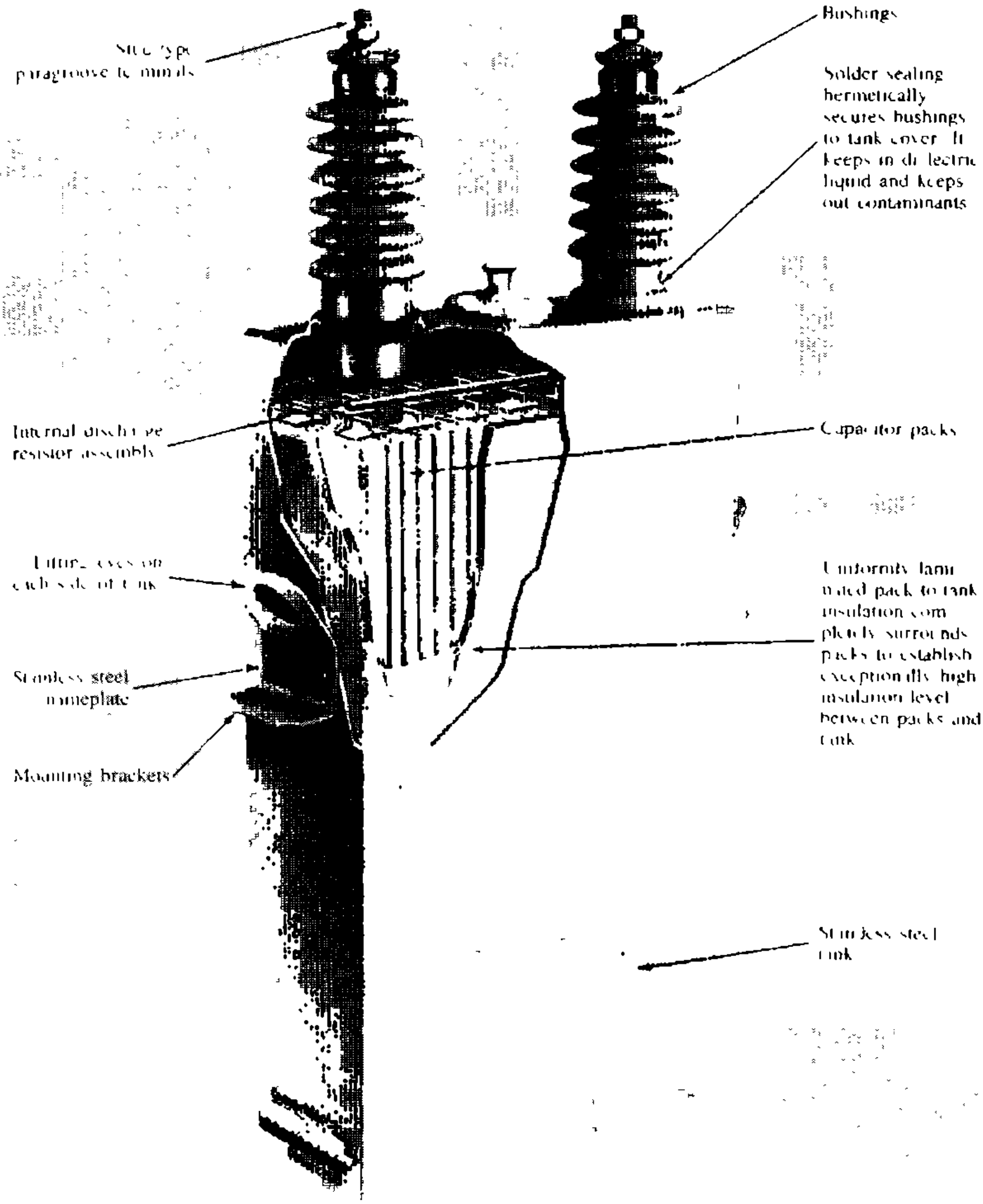


Figure 8-1 A cutaway view of a power-factor-correction capacitor. (*McGraw-Edison Company.*)

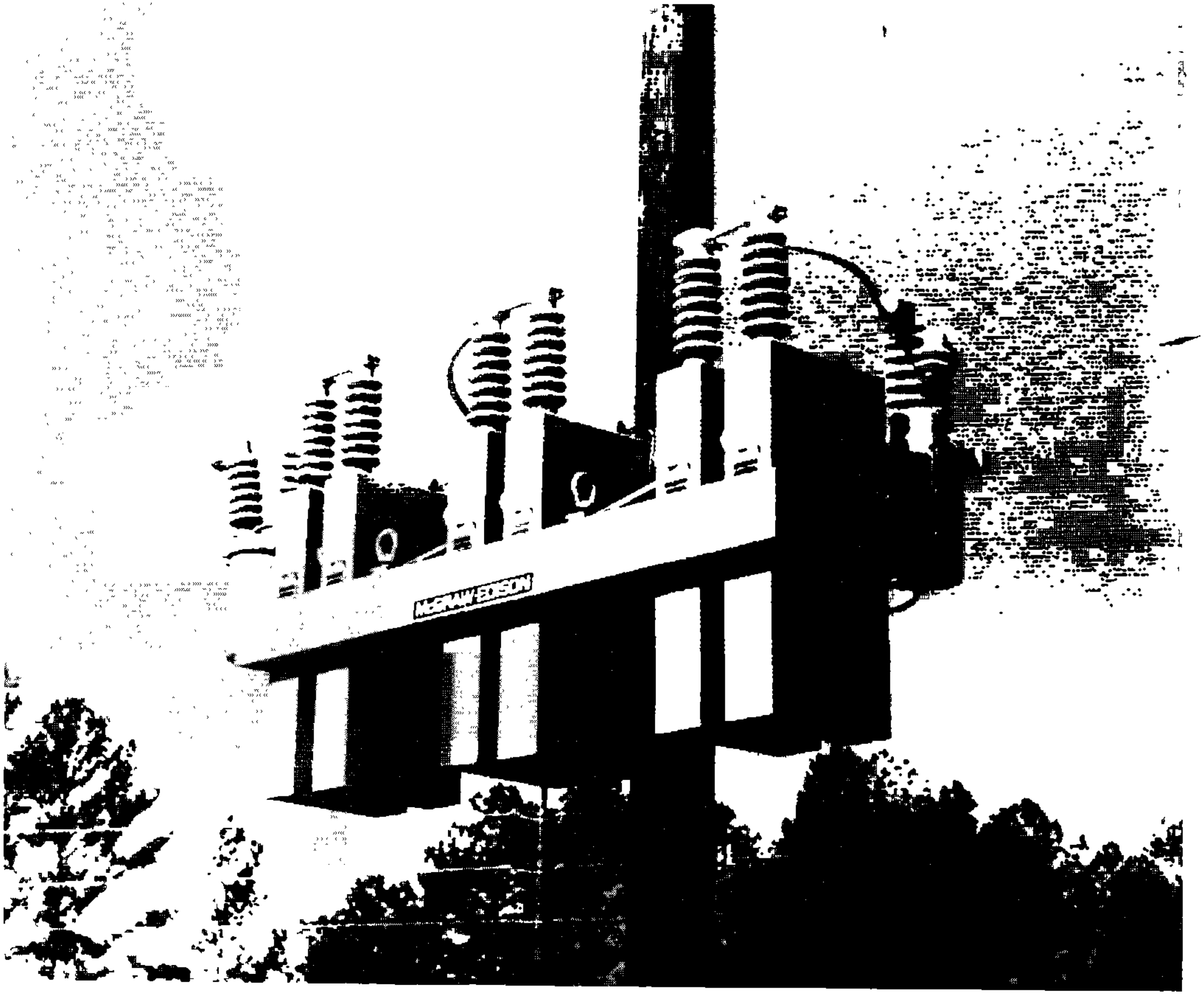


Figure 8-2 Typical capacitor utilization in a switched pole-top rack. (*McGraw-Edison Company.*)

In the past, most power capacitors were constructed with two sheets of pure aluminum foil separated by three or more layers of chemically impregnated kraft paper. Power capacitors have been improved tremendously over the last 30 years or so, partly due to improvements in the dielectric materials and their more efficient utilization and partly due to improvements in the processing techniques involved. Capacitor sizes have increased from the 15–25 kvar range to the 200–300 kvar range (capacitor banks are usually supplied in sizes ranging from 300 to 1800 kvar). Nowadays, power capacitors are much more efficient than those of 30 years ago and are available to the electric utilities at a much lower cost per kilovar. In general, capacitors are getting more attention today than ever before, partly due to a new dimension added in the analysis: changeout economics. Under certain circumstances, even replacement of older capacitors can be justified on the basis of lower-loss evaluations of the modern capacitor design. Capacitor technology has evolved to extremely low loss designs employing the all-film concept; as a result, the utilities can make economic loss evaluations in choosing between the presently existing capacitor technologies.

8-3 EFFECTS OF SERIES AND SHUNT CAPACITORS

As mentioned earlier, the fundamental function of capacitors, whether they are series or shunt, installed as a single unit or as a bank, is to regulate the voltage and reactive power flows at the point where they are installed. The shunt capacitor does it by changing the power factor of the load, whereas the series capacitor does it by directly offsetting the inductive reactance of the circuit to which it is applied.

8-3-1 Series Capacitors

Series capacitors, i.e., capacitors connected in series with lines, have been used to a very limited extent on distribution circuits due to being a more specialized type of apparatus with a limited range of application. Also, because of the special problems associated with each application, there is a requirement for a large amount of complex engineering investigation. Therefore, in general, utilities are reluctant to install series capacitors, especially of small sizes.

As shown in Fig. 8-3, a series capacitor compensates for inductive reactance. In other words, a series capacitor is a negative (capacitive) reactance in series with the circuit's positive (inductive) reactance with the effect of compensating for part or all of it. Therefore, the primary effect of the series capacitor is to minimize, or even suppress, the voltage drop caused by the inductive reactance in the circuit. At times, a series capacitor can even be considered as a voltage regulator that provides for a voltage boost which is proportional to the magnitude and power factor of the through current. Therefore, a series capacitor provides for a voltage rise which increases automatically and instantaneously as the load grows. Also, a series capacitor produces more net voltage rise than a shunt capacitor at lower power factors, which creates more voltage drop. However, a series capacitor betters the

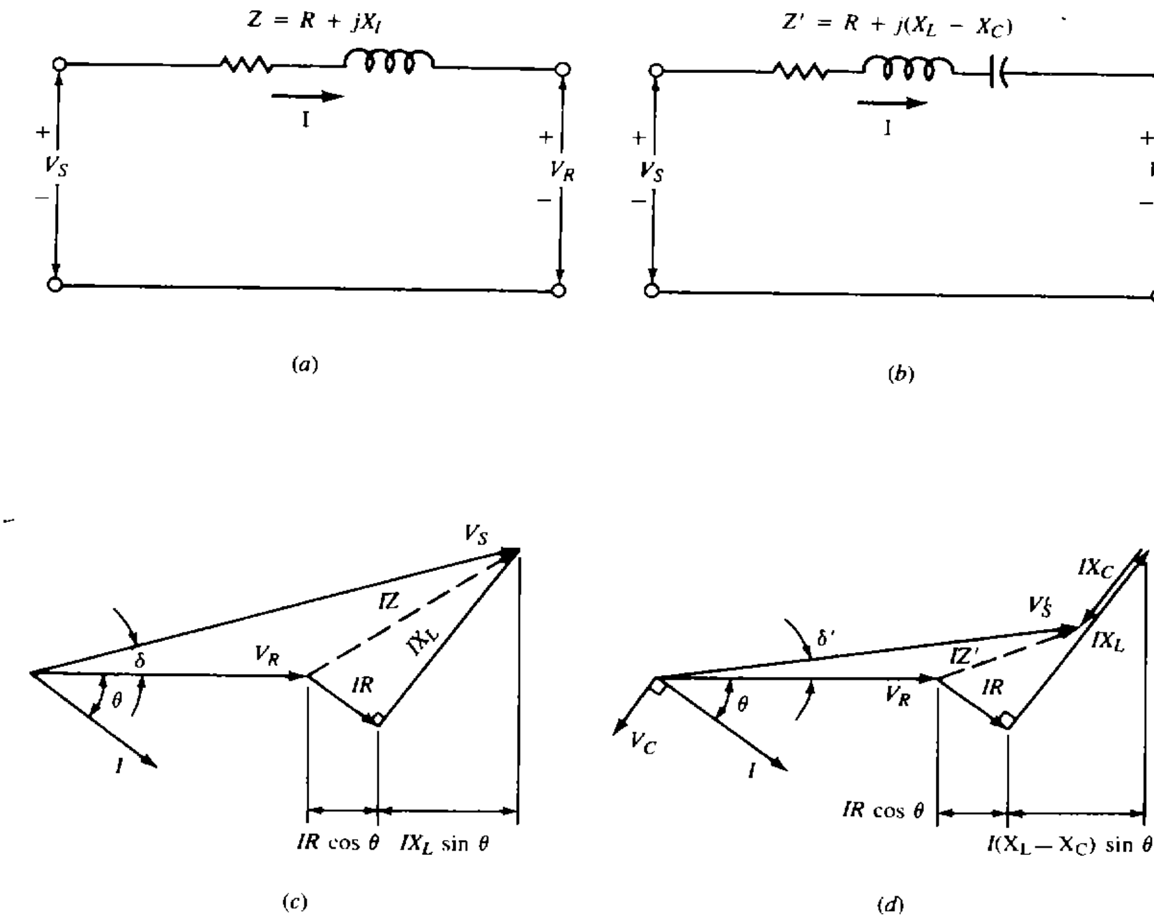


Figure 8-3 Voltage-phasor diagrams for a feeder circuit of lagging power factor: (*a*) and (*c*) without and (*b*) and (*d*) with series capacitors.

system power factor much less than a shunt capacitor and has little effect on the source current.

Consider the feeder circuit and its voltage-phasor diagram as shown in Fig. 8-3*a* and *c*. The voltage drop through the feeder can be expressed approximately as

$$\mathrm{VD} = IR\cos\theta + IX_L\sin\theta \tag{8-1}$$

where R = resistance of feeder circuit
X_L = inductive reactance of feeder circuit
$\cos\theta$ = receiving-end power factor
$\sin\theta$ = sine of the receiving-end power-factor angle

As can be observed from the phasor diagram, the magnitude of the second term in Eq. (8-1) is much larger than the first. The difference gets to be much larger when the power factor is smaller and the ratio of R/X_L is small.

However, when a series capacitor is applied, as shown in Fig. 8-3*b* and *d*, the resultant lower voltage drop can be calculated as

$$\mathrm{VD} = IR\cos\theta + I(X_L - X_c)\sin\theta \tag{8-2}$$

where X_c = capacitive reactance of the series capacitor.

Overcompensation Usually, the series-capacitor size is selected for a distribution feeder application in such a way that the resultant capacitive reactance is smaller than the inductive reactance of the feeder circuit. However, in certain applications (where the resistance of the feeder circuit is larger than its inductive reactance), the reverse might be preferred so that the resultant voltage drop is

$$\mathrm{VD} = IR \cos\theta - I(X_c - X_L)\sin\theta \tag{8-3}$$

The resultant condition is known as *overcompensation*. Figure 8-4*a* shows a voltage-phasor diagram for overcompensation at normal load. At times, when the selected level of overcompensation is strictly based on normal load, the resultant overcompensation of the receiving-end voltage may not be pleasing at all because the lagging current of a large motor at start can produce an extraordinarily large voltage rise, as shown in Fig. 8-4*b*, which is especially harmful to lights (shortening their lives) and causes light flicker, resulting in consumers' complaints.

Leading power factor To decrease the voltage drop considerably between the sending and receiving ends by the application of a series capacitor, the load current must have a lagging power factor. As an example, Fig. 8-5*a* shows a voltage-phasor diagram with a leading-load power factor without having series capacitors in the line. Figure 8-5*b* shows the resultant voltage-phasor diagram with the same leading-load power factor but this time with series capacitors in the line. As can be seen from the figure, the receiving-end voltage is reduced as a result of having series capacitors.

When $\cos\theta = 1.0$, $\sin\theta \cong 0$, and therefore

$$I(X_L - X_c)\sin\theta \cong 0$$

hence Eq. (8-2) becomes

$$\mathrm{VD} \cong IR \tag{8-4}$$

Thus, in such applications, series capacitors practically have no value.

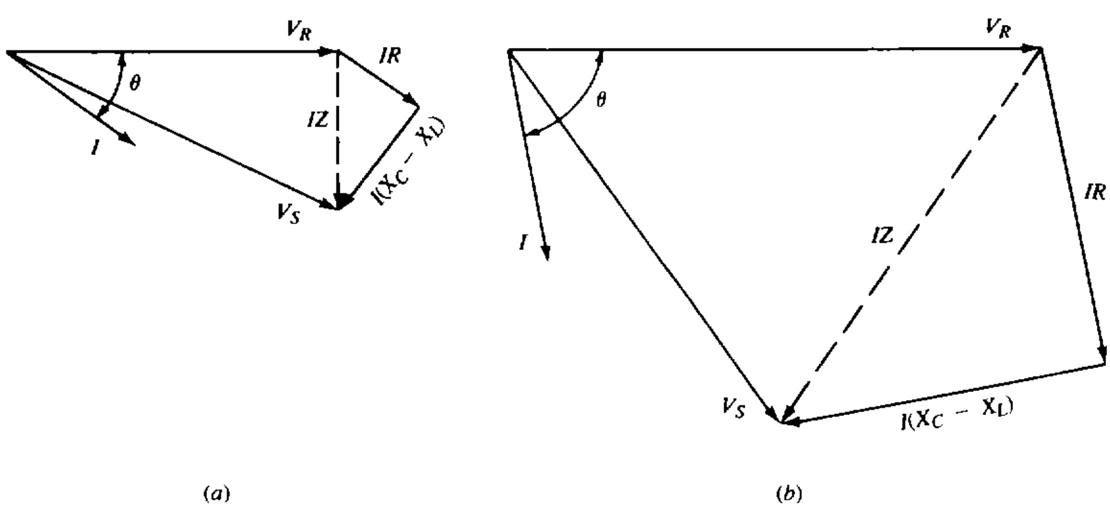


Figure 8-4 Overcompensation of the receiving-end voltage: (*a*) at normal load and (*b*) at the start of a large motor.

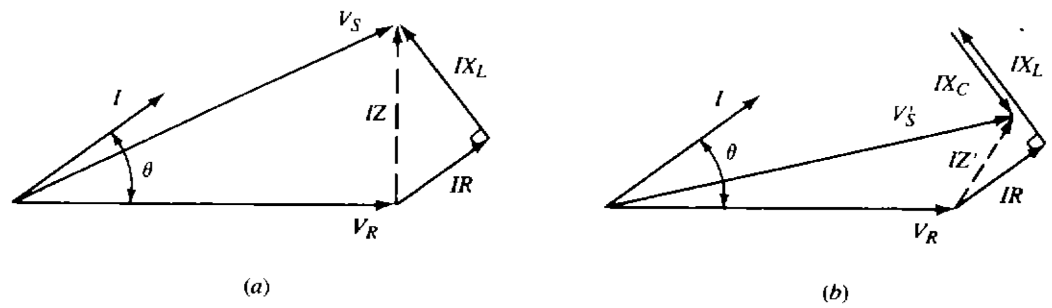


Figure 8-5 Voltage-phasor diagram with leading power factor: (*a*) without series capacitors and (*b*) with series capacitors.

Because of the aforementioned reasons and others (e.g., ferroresonance in transformers, subsynchronous resonance during motor starting, shunting of motors during normal operation, and difficulty in protection of capacitors from system fault current), series capacitors do not have large applications in distribution systems. However, they are employed in subtransmission systems to modify the load division between parallel lines. For example, often a new subtransmission line with larger thermal capability is parallel with an already existing line. It may be very difficult, if not impossible, to load the subtransmission line without overloading the old line. Here, series capacitors can be employed to offset some of the line reactance with greater thermal capability. They are also employed in subtransmission systems to decrease the voltage regulation.

8-3-2 Shunt Capacitors

Shunt capacitors, i.e., capacitors connected in parallel with lines, are used extensively in distribution systems. Shunt capacitors supply the type of reactive power or current to counteract the out-of-phase component of current required by an inductive load. In a sense, shunt capacitors modify the characteristic of an inductive load by drawing a leading current which counteracts some or all of the lagging component of the inductive load current at the point of installation. Therefore a shunt capacitor has the same effect as an overexcited synchronous condenser, generator, or motor.

As shown in Fig. 8-6, by the application of shunt capacitor to a feeder, the magnitude of the source current can be reduced, the power factor can be improved, and consequently the voltage drop between the sending end and the load is also reduced. However, shunt capacitors do not affect current or power factor beyond their point of application. Figures 8-6*a* and *c* show the single-line diagram of a line and its voltage-phasor diagram before the addition of the shunt capacitor, and Fig. 8-6*b* and *d* show them after the addition.

Voltage drop in feeders, or in short transmission lines, with lagging power factor can be approximated as

$$\mathrm{VD} = I_R R + I_X X_L \qquad \mathrm{V} \tag{8-5}$$

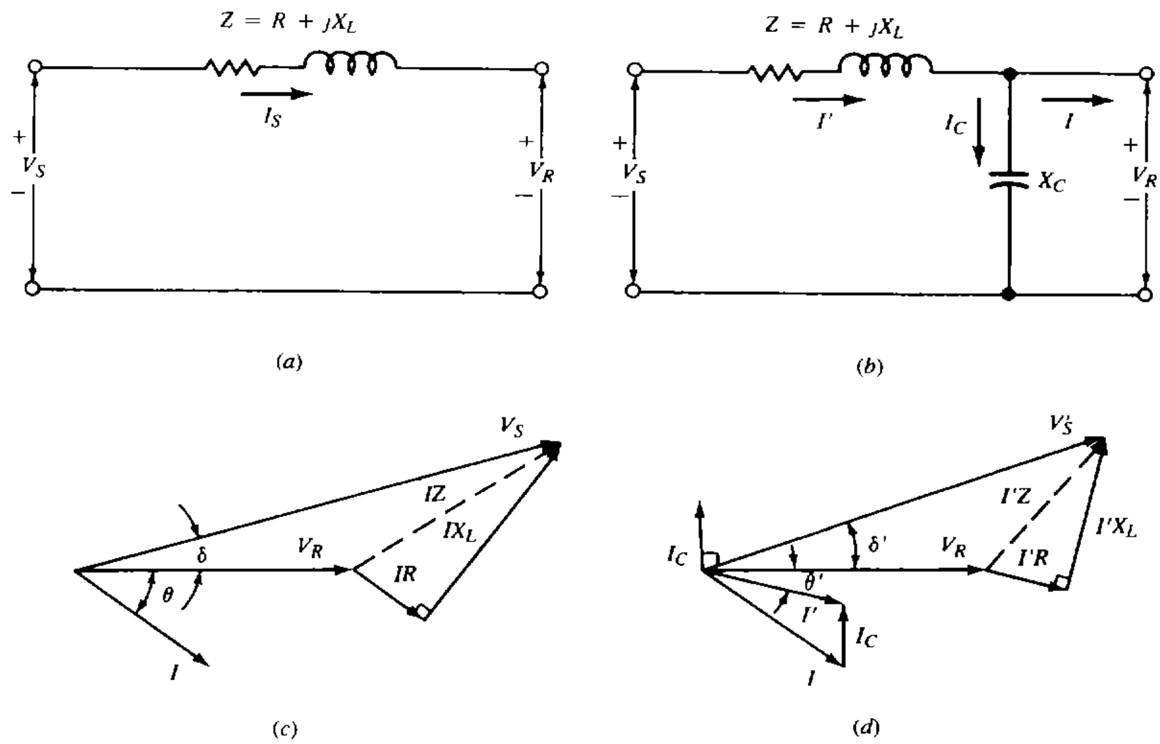


Figure 8-6 Voltage-phasor diagrams for a feeder circuit of lagging power factor: (*a*) and (*c*) without and (*b*) and (*d*) with shunt capacitors.

where R = total resistance of feeder circuit, Ω
X_L = total inductive reactance of feeder circuit, Ω
I_R = real power (or in-phase) component of current, A
I_X = reactive (or out-of-phase) component of current lagging the voltage by 90°, A

When a capacitor is installed at the receiving end of the line, as shown in Fig. 8-6*b*, the resultant voltage drop can be calculated approximately as

$$\mathrm{VD} = I_R R + I_X X_L - I_c X_L \qquad \mathrm{V} \tag{8-6}$$

where I_c = reactive (or out-of-phase) component of current leading the voltage by 90°, A.

The difference between the voltage drops calculated by using Eqs. (8-5) and (8-6) is the voltage rise due to the installation of the capacitor and can be expressed as

$$\mathrm{VR} = I_c X_L \qquad \mathrm{V} \tag{8-7}$$

8-4 POWER-FACTOR CORRECTION

8-4-1 General

A typical utility system would have a reactive load at 80 percent power factor during summer months. Therefore, in typical distribution loads, the current lags the voltage, as shown in Fig. 8-7*a*. The cosine of the angle between current and

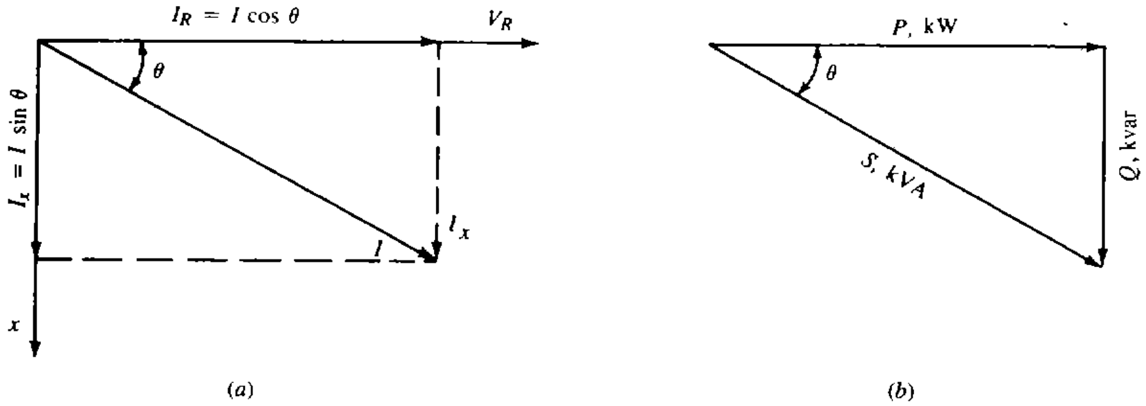


Figure 8-7 (*a*) Phasor diagram and (*b*) power triangle for a typical distribution load.

sending voltage is known as the *power factor* of the circuit. If the in-phase and out-of-phase components of the current I is multiplied by the receiving-end voltage V_R, the resultant relationship can be shown on a triangle known as the *power triangle*, as shown in Fig. 8-7*b*. Figure 8-7*b* shows the triangular relationship that exists between kilowatts, kilovoltamperes, and kilovars. Note that, by adding the capacitors, the reactive power component Q of the apparent power S of the load can be reduced or totally suppressed. Figures 8-8 and 8-9 illustrate how the reactive power component Q increases with each 10 percent change of power factor. Note that, as illustrated in Fig. 8-8, even an 80 percent power factor of the reactive power (kilovar) size is quite large, causing a 25 percent increase in the total apparent power (kilovoltamperes) of the line. At this power factor, 75 kvar of capacitors is needed to cancel out the 75 kvar of lagging component.

As previously mentioned, the generation of reactive power at a power plant and its supply to a load located at a far distance is not economically feasible, but it can easily be provided by capacitors located at the load centers. Figure 8-10 illustrates the power-factor correction for a given system. As illustrated in the

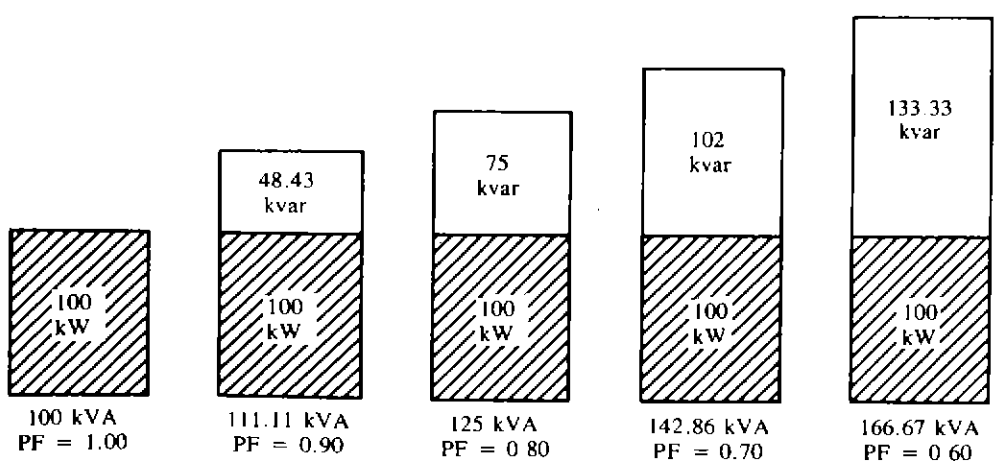


Figure 8-8 Illustration of the required increase in the apparent and reactive powers as a function of the load power factor, holding the real power of the load constant.

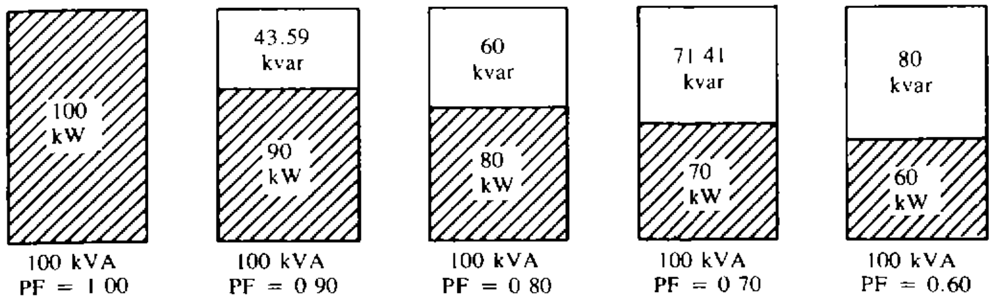


Figure 8-9 Illustration of the change in the real and reactive powers as a function of the load power factor, holding the apparent power of the load constant.

figure, capacitors draw leading reactive power from the source; i.e., they supply lagging reactive power to the load. Assume that a load is supplied with a real power P, lagging reactive power Q_1, and apparent power S_1 at a lagging power factor of

$$\cos \theta_1 = \frac{P}{S_1}$$

or

$$\cos \theta_1 = \frac{P}{(P^2 + Q_1^2)^{1/2}} \tag{8-8}$$

When a shunt capacitor of Q_c kVA is installed at the load, the power factor can be improved from $\cos \theta_1$ to $\cos \theta_2$, where

$$\begin{aligned}\cos \theta_2 &= \frac{P}{S_2} \\ &= \frac{P}{(P^2 + Q_2^2)^{1/2}} \\ &= \frac{P}{[P^2 + (Q_1 - Q_c)^2]^{1/2}}\end{aligned} \tag{8-9}$$

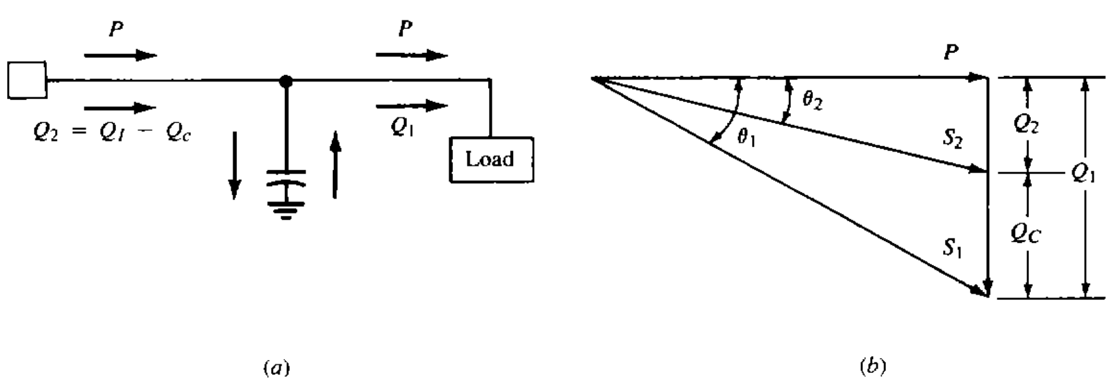


Figure 8-10 Illustration of power-factor correction.

Table 8-1 Power-factor correction

Rcactive factor	Orig. power factor, %	Correcting factor: Desired power factor, %																				
		80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
0.800	60	0.584	0.610	0.636	0.662	0.688	0.714	0.741	0.767	0.794	0.822	0.850	0.878	0.905	0.939	0.971	1.005	1.043	1.083	1.311	1.192	1.334
0.791	61	0.549	0.575	0.601	0.627	0.653	0.679	0.706	0.732	0.759	0.787	0.815	0.843	0.870	0.904	0.936	0.970	1.008	1.048	1.096	1.157	1.299
0.785	62	0.515	0.541	0.567	0.593	0.619	0.645	0.672	0.698	0.725	0.753	0.781	0.809	0.836	0.870	0.902	0.936	0.974	1.014	1.062	1.123	1.265
0.776	63	0.483	0.509	0.535	0.561	0.587	0.613	0.640	0.666	0.693	0.721	0.749	0.777	0.804	0.838	0.870	0.904	0.942	0.982	1.030	1.091	1.233
0.768	64	0.450	0.476	0.502	0.528	0.554	0.580	0.607	0.633	0.660	0.688	0.716	0.744	0.771	0.805	0.837	0.871	0.909	0.949	0.997	1.058	1.200
0.759	65	0.419	0.445	0.471	0.479	0.523	0.549	0.576	0.602	0.629	0.657	0.685	0.713	0.740	0.774	0.806	0.840	0.878	0.918	0.966	1.027	1.169
0.751	66	0.388	0.414	0.440	0.466	0.492	0.518	0.545	0.571	0.598	0.626	0.654	0.682	0.709	0.743	0.775	0.809	0.847	0.887	0.935	0.996	1.138
0.744	67	0.358	0.384	0.410	0.436	0.462	0.488	0.515	0.541	0.568	0.596	0.624	0.652	0.679	0.713	0.745	0.779	0.817	0.857	0.905	0.966	1.108
0.733	68	0.329	0.355	0.381	0.407	0.433	0.459	0.486	0.512	0.539	0.567	0.595	0.623	0.650	0.684	0.716	0.750	0.788	0.828	0.876	0.937	1.079
0.725	69	0.299	0.325	0.351	0.377	0.403	0.429	0.456	0.482	0.509	0.537	0.565	0.593	0.620	0.654	0.686	0.720	0.758	0.798	0.840	0.907	1.049
0.714	70	0.270	0.296	0.322	0.348	0.374	0.400	0.427	0.453	0.480	0.508	0.536	0.564	0.591	0.625	0.657	0.691	0.729	0.769	0.811	0.878	1.020
0.704	71	0.242	0.268	0.294	0.320	0.346	0.372	0.399	0.425	0.452	0.480	0.508	0.536	0.563	0.597	0.629	0.663	0.700	0.741	0.783	0.850	0.992
0.694	72	0.213	0.239	0.265	0.291	0.317	0.343	0.370	0.396	0.423	0.451	0.479	0.507	0.534	0.568	0.600	0.634	0.672	0.712	0.754	0.821	0.963
0.682	73	0.186	0.212	0.238	0.264	0.290	0.316	0.343	0.369	0.396	0.424	0.452	0.480	0.507	0.541	0.573	0.607	0.645	0.685	0.727	0.794	0.936
0.673	74	0.159	0.185	0.211	0.237	0.263	0.289	0.316	0.342	0.369	0.397	0.425	0.453	0.480	0.514	0.546	0.580	0.618	0.658	0.700	0.767	0.909
0.661	75	0.132	0.158	0.184	0.210	0.236	0.262	0.289	0.315	0.342	0.370	0.398	0.426	0.453	0.487	0.519	0.553	0.591	0.631	0.673	0.740	0.882
0.650	76	0.105	0.131	0.157	0.183	0.209	0.235	0.262	0.288	0.315	0.343	0.371	0.399	0.426	0.460	0.492	0.526	0.564	0.604	0.652	0.713	0.855
0.637	77	0.079	0.105	0.131	0.157	0.183	0.209	0.236	0.262	0.289	0.317	0.345	0.373	0.400	0.434	0.466	0.500	0.538	0.578	0.620	0.687	0.829
0.626	78	0.053	0.079	0.105	0.131	0.157	0.183	0.210	0.236	0.263	0.291	0.319	0.347	0.374	0.408	0.440	0.474	0.512	0.552	0.594	0.661	0.803
0.613	79	0.026	0.052	0.078	0.104	0.130	0.156	0.183	0.209	0.236	0.264	0.292	0.320	0.347	0.381	0.413	0.447	0.485	0.525	0.567	0.634	0.776

0.600	80	0.000	0.026	0.052	0.078	0.104	0.130	0.157	0.183	0.210	0.238	0.266	0.294	0.321	0.355	0.387	0.421	0.459	0.499	0.541	0.608	0.750
0.588	81		0.000	0.026	0.052	0.078	0.104	0.131	0.157	0.184	0.212	0.240	0.268	0.295	0.329	0.361	0.395	0.433	0.473	0.515	0.582	0.724
0.572	82			0.000	0.026	0.052	0.078	0.105	0.131	0.158	0.186	0.214	0.242	0.269	0.303	0.335	0.369	0.407	0.447	0.489	0.556	0.698
0.559	83				0.000	0.026	0.052	0.079	0.105	0.132	0.160	0.188	0.216	0.243	0.277	0.309	0.343	0.381	0.421	0.463	0.530	0.672
0.543	84					0.000	0.026	0.053	0.079	0.106	0.134	0.162	0.190	0.217	0.251	0.283	0.317	0.355	0.395	0.437	0.504	0.646
0.529	85						0.000	0.027	0.053	0.080	0.108	0.136	0.164	0.191	0.225	0.257	0.291	0.329	0.369	0.417	0.478	0.620
0.510	86							0.000	0.026	0.053	0.081	0.109	0.137	0.167	0.198	0.230	0.265	0.301	0.342	0.390	0.451	0.593
0.497	87								0.000	0.027	0.055	0.083	0.111	0.141	0.172	0.204	0.239	0.275	0.316	0.364	0.425	0.567
0.475	88									0.000	0.028	0.056	0.083	0.113	0.144	0.176	0.211	0.247	0.288	0.336	0.397	0.540
0.455	89										0.000	0.028	0.055	0.086	0.117	0.149	0.183	0.221	0.262	0.309	0.370	0.512
0.443	90											0.000	0.028	0.058	0.089	0.121	0.155	0.193	0.234	0.281	0.342	0.484
0.427	91												0.000	0.030	0.061	0.093	0.127	0.165	0.206	0.253	0.314	0.456
0.392	92													0.000	0.031	0.063	0.097	0.135	0.176	0.223	0.284	0.426
0.386	93														0.000	0.032	0.066	0.104	0.145	0.192	0.253	0.395
0.341	94															0.000	0.035	0.072	0.113	0.160	0.221	0.363
0.327	95																0.000	0.036	0.078	0.125	0.186	0.328
0.280	96																	0.000	0.041	0.089	0.150	0.292
0.242	97																		0.000	0.048	0.109	0.251
0.199	98																			0.000	0.061	0.203
0.137	99																				0.000	0.142

Therefore, as can be observed from Fig. 8-10*b*, the apparent power and the reactive power are decreased from S_1 kVA to S_2 kVA and from Q_1 kvar to Q_2 kvar (by providing a reactive power of Q_c), respectively. Of course, the reduction of reactive current results in a reduced total current, which in turn causes less power losses. Thus the power-factor correction produces economic savings in capital expenditures and fuel expenses through a release of kilovoltamperage capacity and reduction of power losses in all the apparatus between the point of installation of the capacitors and the source power plants, including distribution lines, substation transformers, and transmission lines. The economic power factor is the point at which the economic benefits of adding shunt capacitors just equals the cost of capacitors. In the past, this economic power factor was around 95 percent. Today's high plant and fuel costs have pushed the economic power factor toward unity. However, as the corrected power factor moves nearer to unity, the effectiveness of capacitors in improving the power factor, decreasing the line kilovoltamperes transmitted, increasing the load capacity, or reducing line copper losses by decreasing the line current sharply decreases. Therefore the correction of power factor to unity becomes more expensive with regard to the marginal cost of capacitors installed.

Table 8-1 is a power-factor-correction table to simplify the calculations involved in determining the capacitor size necessary to improve the power factor of a given load from original to desired value.

Example 8-1 Assume that a 700-kVA load has a 65 percent power factor. It is desired to improve the power factor to 92 percent. Using Table 8-1, determine the following:

(*a*) The correction factor required.
(*b*) The capacitor size required.
(*c*) What would be the resulting power factor if the next higher standard capacitor size is used?

SOLUTION

(*a*) From Table 8-1, the correction factor required can be found as 0.74.
(*b*) The 700-kVA load at 65 percent power factor is

$$\begin{aligned} P_L &= S_L \times \cos\theta \qquad (8\text{-}10) \\ &= 700 \times 0.65 \\ &= 455 \text{ kW} \end{aligned}$$

The capacitor size necessary to improve the power factor from 65 to 92 percent can be found as

$$\begin{aligned} \text{Capacitor size} &= P_L \text{ (correlation factor)} \qquad (8\text{-}11) \\ &= 455(0.74) \\ &= 336.7 \text{ kvar} \end{aligned}$$

(*c*) Assume that the next higher standard capacitor size (or rating) is selected to be 360 kvar. Therefore the resulting new correction factor can be found from

$$\text{New correction factor} = \frac{\text{standard capacitor rating}}{P_L} \tag{8-12}$$

$$= \frac{360 \text{ kvar}}{455 \text{ kW}}$$

$$= 0.7912$$

From the table by linear interpolation, the resulting corrected percent power factor, with an original power factor of 65 percent and a correction factor of 0.7912, can be found as

$$\text{New corrected \% power factor} = 93 + \frac{172}{320}$$

$$\cong 93.5$$

8-4-2 A Computerized Method to Determine the Economic Power Factor

As suggested by Hopkinson [3], a load flow digital computer program can be employed to determine the kilovoltamperes, kilovolts, and kilovars at annual peak level for the whole system (from generation through the distribution substation buses) as the power factor is varied. As a start, shunt capacitors are applied to each substation bus for correcting to an initial power factor, for example, 90 percent. Then, a load flow run is performed to determine the total system kilovoltamperes, and kilowatt losses (from generator to load) at this level and capacitor kilovars are noted. Later, additional capacitors are applied to each substation bus to increase the power factor by 1 percent, and another load flow run is made. This process of iteration is repeated until the power factor becomes unity. As a final step, the benefits and costs are calculated at each power factor. The economic power factor is determined as the value at which benefits and costs are equal. After determining the economic power factor, the additional capacitor size required can be calculated as

$$\Delta Q_c = P_{\text{PK}}(\tan \phi - \tan \theta) \tag{8-13}$$

where ΔQ_c = required capacitor size, kvar
P_{PK} = system demand at annual peak, kW
$\tan \phi$ = tangent of original power-factor angle
$\tan \theta$ = tangent of economic power-factor angle

An illustration of this method is given in Example 8-5.

8-5 APPLICATION OF CAPACITORS

In general, capacitors can be applied at almost any voltage level. As illustrated in Fig. 8-11, individual capacitor units can be added in parallel to achieve the desired kilovar capacity and can be added in series to achieve the required kilovolt voltage. They are employed at or near rated voltage for economic reasons.

The cumulative data gathered for the whole utility industry indicate that approximately 60 percent of the capacitors is applied to the feeders, 30 percent to the substation buses, and the remaining 10 percent to the transmission system [3].

The application of capacitors to the secondary systems is very rare due to small economic advantages. Zimmerman [4] has developed a nomograph, shown in Fig. 8-12, to determine the economic justification, if any, of the secondary capacitors considering only the savings in distribution transformer cost.

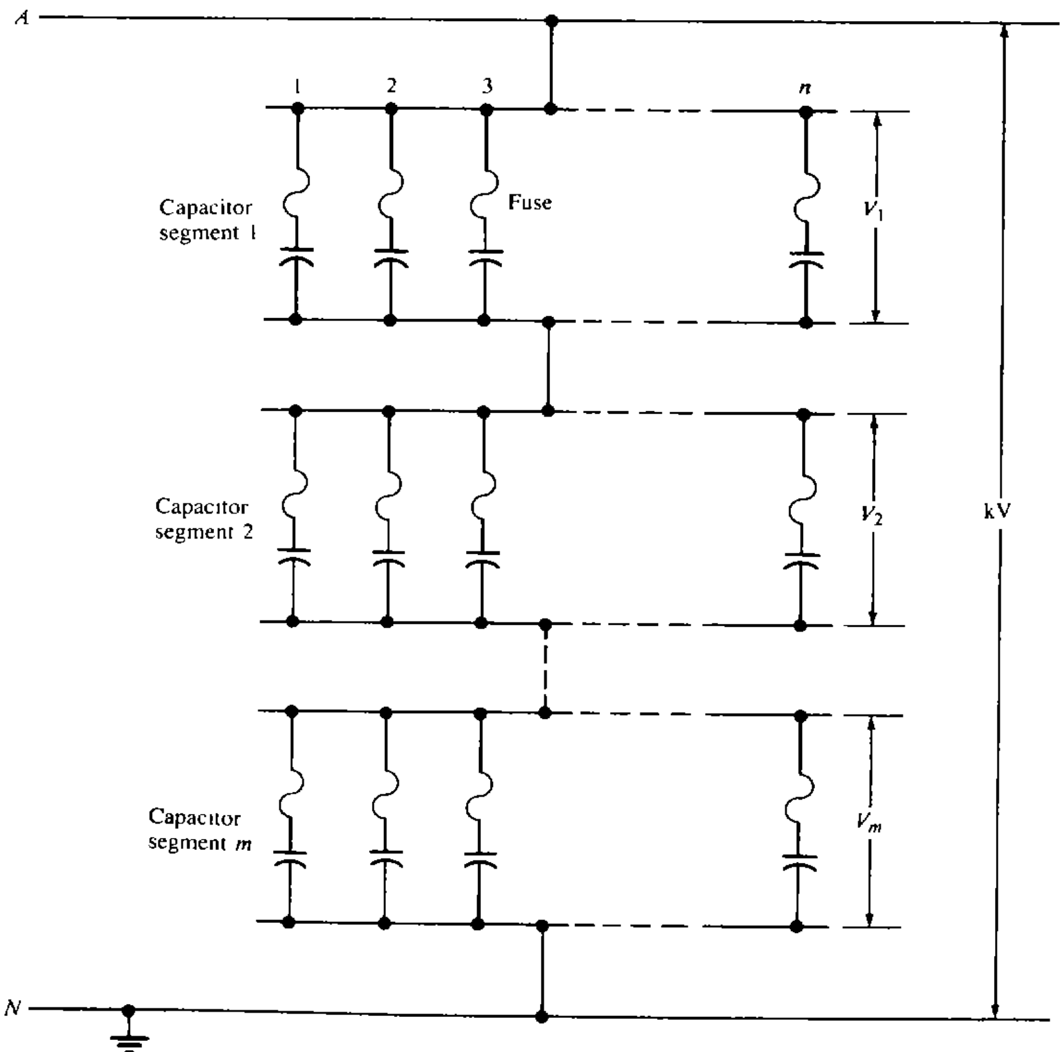


Figure 8-11 Connection of capacitor units for one phase of a three-phase wye-connected bank.

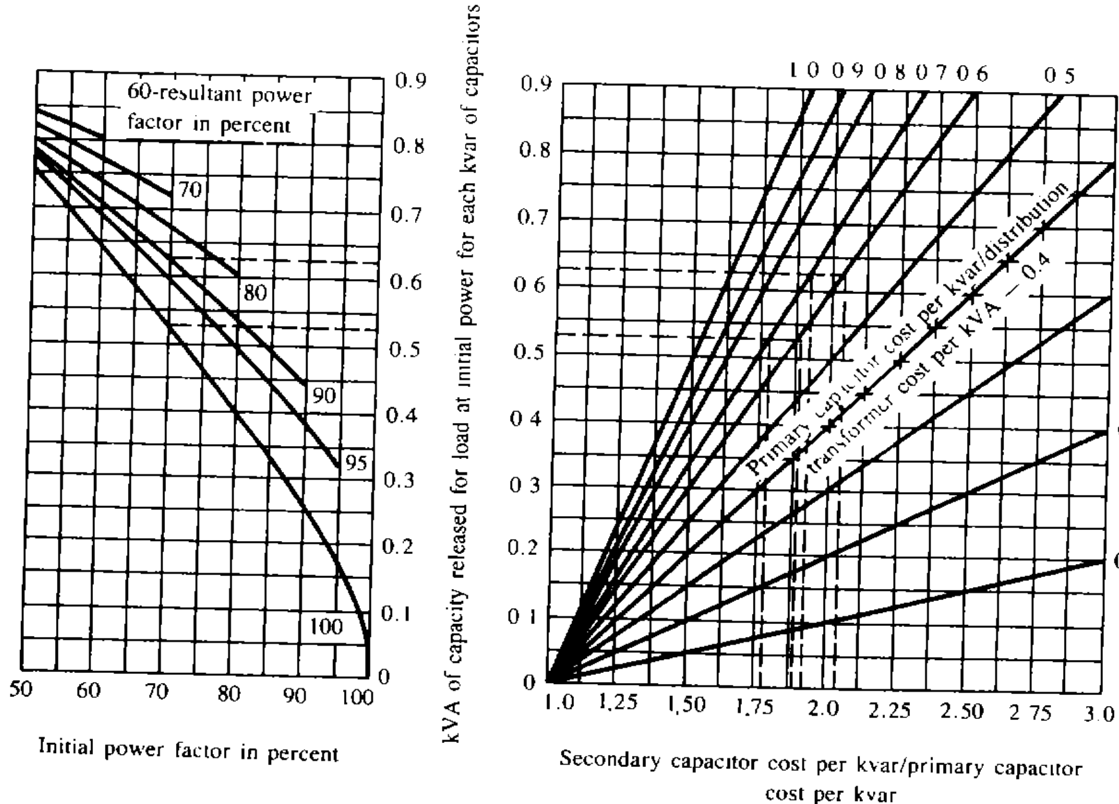


Figure 8-12 Secondary capacitor economics considering only savings in distribution transformer cost. (*From* [4]. *Used by permission.* © *1953 IEEE.*)

Example 8-2 Assume that a three-phase 500-hp 60-Hz 4160-V wye-connected induction motor has a full-load efficiency of 88 percent, a lagging power factor of 0.75, and is connected to a feeder. If it is desired to correct the power factor of the load to a lagging power factor of 0.9 by connecting three capacitors at the load, determine the following:

(*a*) The rating of the capacitor bank, in kilovars.

(*b*) The capacitance of each unit if the capacitors are connected in delta, in microfarads.

(*c*) The capacitance of each unit if the capacitors are connected in wye, in microfarads.

SOLUTION

(*a*) The input power of the induction motor can be found as

$$P = \frac{(500\text{ hp})(0.7457\text{ kW/hp})}{0.88}$$

$$= 423.69\text{ kW}$$

The reactive power of the motor at the uncorrected power factor is

$$\begin{aligned} Q_1 &= P \tan \theta_1 \\ &= 423.69 \tan(\cos^{-1} 0.75) \\ &= 423.69 \times 0.8819 \\ &= 373.7\text{ kvar} \end{aligned}$$

The reactive power of the motor at the corrected power factor is

$$\begin{aligned} Q_2 &= P \tan \theta_2 \\ &= 423.69 \tan(\cos^{-1} 0.90) \\ &= 423.69 \times 0.4843 \\ &= 205.2 \text{ kvar} \end{aligned}$$

Therefore the reactive power provided by the capacitor bank is

$$\begin{aligned} Q_c &= Q_1 - Q_2 \\ &= 373.7 - 205.2 \\ &= 168.5 \text{ kvar} \end{aligned}$$

Hence, assuming the losses in the capacitors are negligible, the rating of the capacitor bank is 168.5 kvar.

(*b*) If the capacitors are connected in delta as shown in Fig. 8-13*a*, the line current is

$$\begin{aligned} I_L &= \frac{Q_c}{\sqrt{3} \times V_{L\text{-}L}} \\ &= \frac{168.5}{\sqrt{3} \times 4.16} \\ &= 23.39 \text{ A} \end{aligned}$$

and therefore

$$\begin{aligned} I_c &= \frac{I_L}{\sqrt{3}} \\ &= \frac{23.39}{\sqrt{3}} \\ &= 13.5 \text{ A} \end{aligned}$$

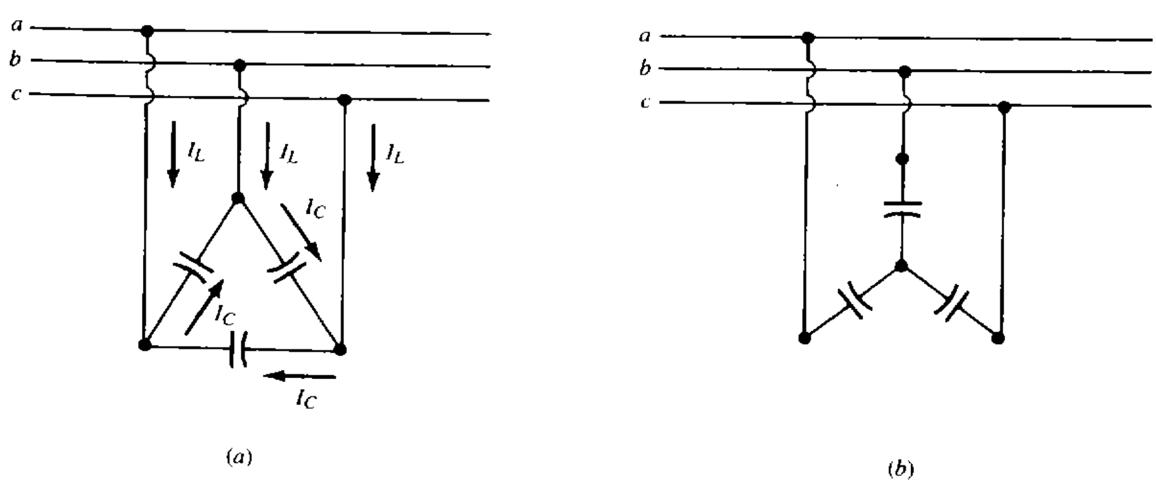


Figure 8-13

Thus, the reactance of each capacitor is

$$X_c = \frac{V_{L\text{-}L}}{I_c}$$

$$= \frac{4160}{13.53}$$

$$= 308.11\ \Omega$$

and hence the capacitance of each unit, if the capacitors are connected in delta, is

$$C = \frac{10^6}{\omega X_c}$$

or

$$C = \frac{10^6}{2\pi f X_c}$$

$$= \frac{10^6}{2\pi \times 60 \times 308.11}$$

$$= 8.61\ \mu\text{F}$$

(*c*) If the capacitors are connected in wye as shown in Fig. 8-13*b*,

$$I_c = I_L = 23.39\ \text{A}$$

and therefore

$$X_c = \frac{V_{L\text{-}N}}{I_c}$$

$$= \frac{4160}{\sqrt{3} \times 23.39}$$

$$= 102.70\ \Omega$$

Thus the capacitance of each unit, if the capacitors are connected in wye, is

$$C = \frac{10^6}{2\pi f X_c}$$

$$= \frac{10^6}{2\pi \times 60 \times 102.70}$$

$$= 25.83\ \mu\text{F}$$

Example 8-3 Assume that a 2.4-kV single-phase circuit feeds a load of 360 kW (measured by a wattmeter) at a lagging load factor and the load current is 200 A. If it is desired to improve the power factor, determine the following:
(*a*) The uncorrected power factor and reactive load.
(*b*) The new corrected power factor after installing a shunt capacitor unit with a rating of 300 kvar.

SOLUTION
(*a*) Before the power-factor correction,

$$S_1 = V \times I$$
$$= 2.4 \times 200$$
$$= 480 \text{ kVA}$$

therefore the uncorrected power factor can be found as

$$\cos \theta_1 = \frac{P}{S_1}$$
$$= \frac{360 \text{ kW}}{480 \text{ kVA}}$$
$$= 0.75$$

and the reactive load is

$$Q_1 = S_1 \times \sin(\cos^{-1} PF_1)$$
$$= 480 \times 0.661$$
$$= 317.5 \text{ kvar}$$

(*b*) After the installation of the 300-kvar capacitors,

$$Q_2 = Q_1 - Q_c$$
$$= 317.5 - 300$$
$$= 17.5 \text{ kvar}$$

and therefore, the new power factor can be found from Eq (8-9) as

$$\cos \theta_2 = \frac{P}{[P^2 + (Q_1 - Q_c)^2]^{1/2}}$$
$$= \frac{360}{(360^2 + 17.5^2)^{1/2}}$$
$$= 0.9989 \qquad \text{or} \qquad 99.89 \text{ percent}$$

Example 8-4 Assume that the Riverside Substation of the NL&NP Company has a bank of three 2000-kVA transformers that supplies a peak load of 7800 kVA at a lagging power factor of 0.89. All three transformers have a thermal capability of 120 percent of the nameplate rating. It has already been planned to install 1000 kvar of shunt capacitors on the feeder to improve the voltage regulation. Determine the following:

(*a*) Whether or not to install additional capacitors on the feeder to decrease the load to the thermal capability of the transformer.

(*b*) The rating of the additional capacitors.

SOLUTION

(*a*) Before the installation of the 1000-kvar capacitors,

$$\begin{aligned} P &= S_1 \times \cos\theta \\ &= 7800 \times 0.89 \\ &= 6942 \text{ kW} \end{aligned}$$

and

$$\begin{aligned} Q_1 &= S_1 \times \sin\theta \\ &= 7800 \times 0.456 \\ &= 3556.8 \text{ kvar} \end{aligned}$$

Therefore, after the installation of the 1000-kvar capacitors,

$$\begin{aligned} Q_2 &= Q_1 - Q_c \\ &= 3556.8 - 1000 \\ &= 2556.8 \text{ kvar} \end{aligned}$$

and using Eq. (8-9),

$$\begin{aligned} \cos\theta_2 &= \frac{P}{[P^2 + (Q_1 - Q_c)^2]^{1/2}} \\ &= \frac{6942}{(6942^2 + 2556.8^2)^{1/2}} \\ &= 0.938 \qquad \text{or} \qquad 93.8 \text{ percent} \end{aligned}$$

and the corrected apparent power is

$$\begin{aligned} S_2 &= \frac{P}{\cos\theta_2} \\ &= \frac{6942}{0.938} \\ &= 7397.9 \text{ kVA} \end{aligned}$$

On the other hand, the transformer capability is

$$\begin{aligned} S_T &= 6000 \times 1.20 \\ &= 7200 \text{ kVA} \end{aligned}$$

Therefore the capacitors installed to improve the voltage règulation are not adequate; additional capacitor installation is required.

(*b*) The new or corrected power factor required can be found as

$$\cos\theta_{2,\text{new}} = \frac{P}{S_T} = \frac{6942}{7200} = 0.9642 \qquad \text{or} \qquad 96.42 \text{ percent}$$

and thus the new required reactive power can be found as

$$\begin{aligned} Q_{2,\text{new}} &= P \times \tan\theta_{2,\text{new}} \\ &= P \times \tan(\cos^{-1} PF_{2,\text{new}}) \\ &= 6942 \times 0.2752 \\ &= 1910 \text{ kvar} \end{aligned}$$

Therefore the rating of the additional capacitors required is

$$\begin{aligned} Q_{c,\text{add}} &= Q_2 - Q_{2,\text{new}} \\ &= 2556.8 - 1910 \\ &= 646.7 \text{ kvar} \end{aligned}$$

8-5-1 Capacitor Installation Types

In general, capacitors installed on feeders are pole-top banks with necessary group fusing. The fusing applications restrict the size of the bank that can be used Therefore the maximum sizes used are about 1800 kvar at 15 kV and 3600 kvar at higher voltage levels. Usually, utilities do not install more than four capacitor banks (of equal sizes) on each feeder.

Figure 8-14 illustrates the effects of a fixed capacitor on the voltage profiles of a feeder with uniformly distributed load at heavy load and light load. If only fixed-type capacitors are installed, as can be observed in Fig. 8-14*c*, the utility will experience an excessive leading power factor and voltage rise at that feeder Therefore, as shown in Fig. 8-15, some of the capacitors are installed as switched-capacitor banks so they can be switched off during light-load conditions. Thus the fixed capacitors are sized for light load and connected permanently. As shown in the figure, the switched capacitors can be switched as a block or in several consecutive steps as the reactive load becomes greater from light-load level to peak load and sized accordingly. However, in practice, the number of steps or blocks is selected to be much less than the ones shown in the figure due to the additional expenses involved in the installation of the required switchgear and control equipment.

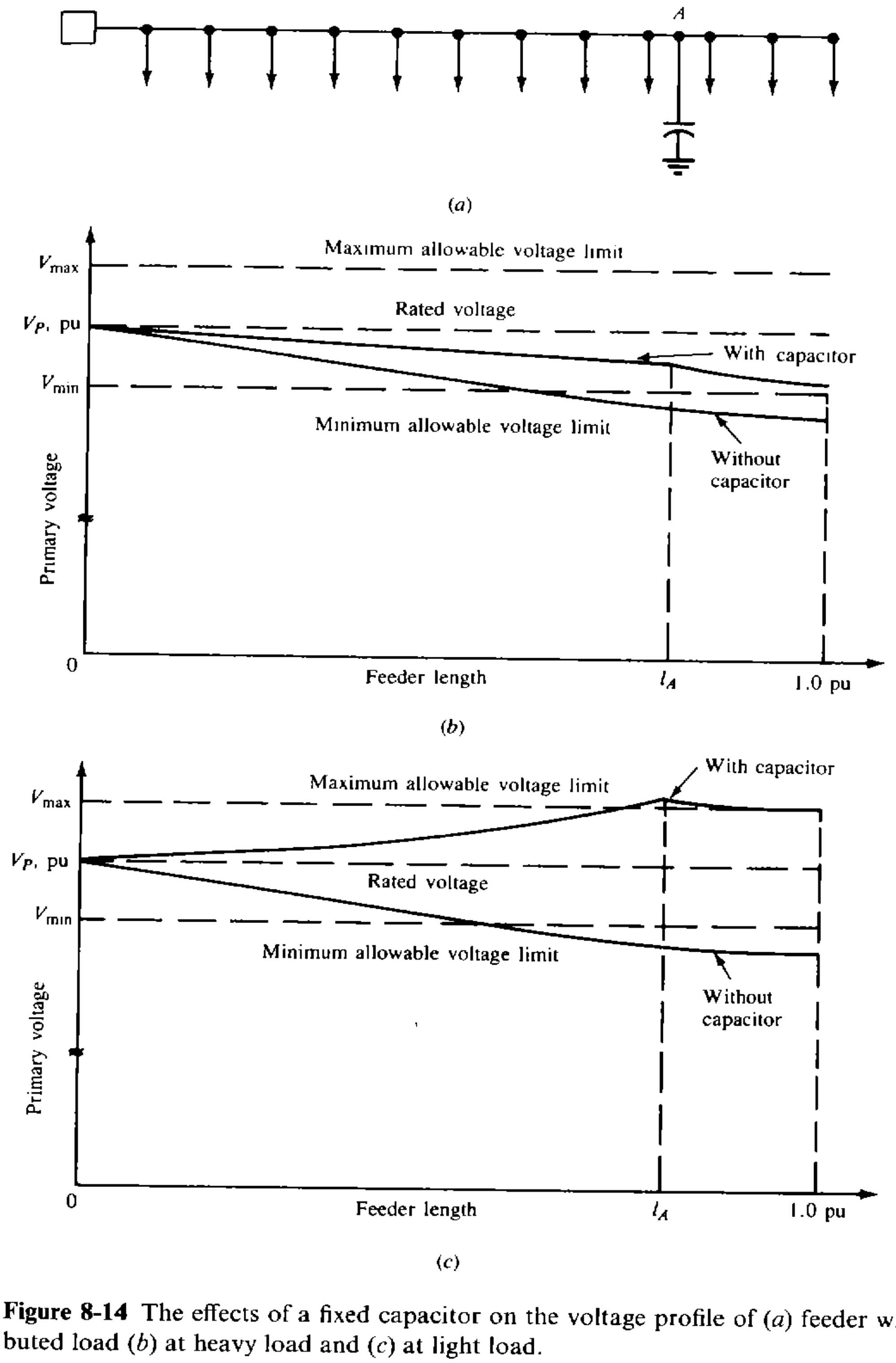


Figure 8-14 The effects of a fixed capacitor on the voltage profile of (*a*) feeder with uniformly distributed load (*b*) at heavy load and (*c*) at light load.

A system survey is required in choosing the type of capacitor installation. As a result of load flow program runs or manual load studies on feeders or distribution substations, the system's lagging reactive loads (i.e., power demands) can be determined and the results can be plotted on a curve as shown in Fig. 8-15. This curve is called the *reactive load duration curve* and is the cumulative sum of the reactive loads (e.g., fluorescent lights, household appliances, and motors) of

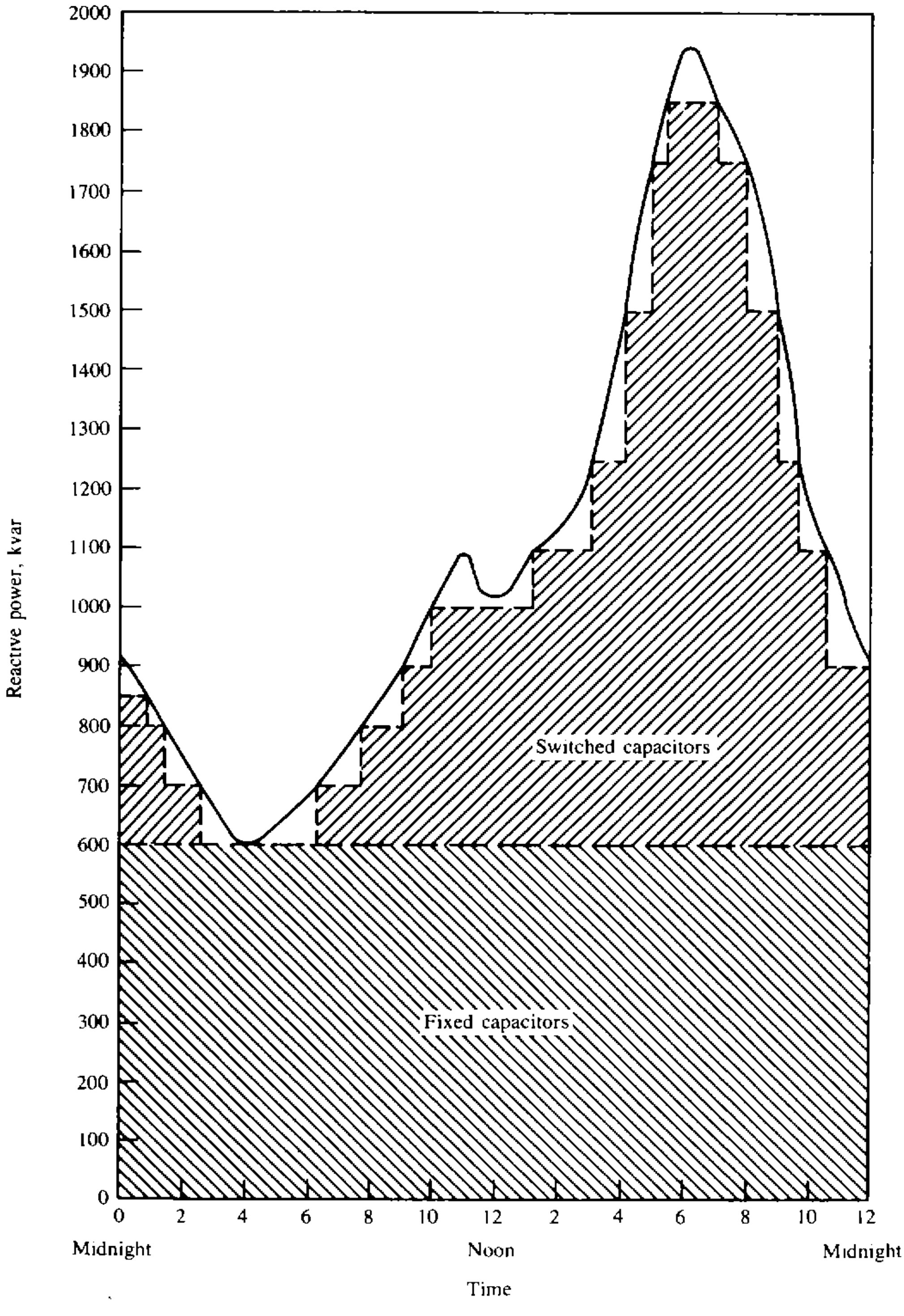


Figure 8-15 Sizing of the fixed and switched capacitors to meet the daily reactive power demands.

consumers and the reactive power requirements of the system (e.g., transformers and regulators). Once the daily reactive load duration curve is obtained, then by visual inspection of the curve the size of the fixed capacitors can be determined to meet the minimum reactive load. For example, from Fig. 8-15 one can determine that the size of the fixed capacitors required is 600 kvar. The remaining kilovar demands of the loads are met by the generator or preferably by the switched capacitors. However, since meeting the kilovar demands of the system from the generator is too expensive and may create problems in the system stability, capacitors are used. Capacitor sizes are selected to match the remaining load characteristics from hour to hour.

Many utilities apply the following rule of thumb to determine the size of the switched capacitors: Add switched capacitors until

$$\frac{\text{kvar from switched + fixed capacitors}}{\text{kvar of peak reactive feeder load}} \geq 0.70 \tag{8-14}$$

From the voltage regulation point of view, the kilovars needed to raise the voltage at the end of the feeder to the maximum allowable voltage level at minimum load (25 percent of peak load) is the size of the fixed capacitors that should be used. On the other hand, if more than one capacitor bank is installed, the size of each capacitor bank at each location should have the same proportion, that is,

$$\frac{\text{kvar of load center}}{\text{kvar of total feeder}} = \frac{\text{kVA of load center}}{\text{kVA of total feeder}} \tag{8-15}$$

However, the resultant voltage rise must not exceed the light-load voltage drop. The approximate value of the percent voltage rise can be calculated from

$$\%\,\text{VR} = \frac{Q_{c,\,3\phi}\,xl}{10 \times V_{L\text{-}L}^2} \qquad \%\,\text{V} \tag{8-16}$$

where $\%\,\text{VR}$ = percent voltage rise
$Q_{c,\,3\phi}$ = three-phase reactive power due to fixed capacitors applied, kvar
x = line reactance, Ω/mi
l = length of feeder from sending end of feeder to fixed-capacitor location, mi
$V_{L\text{-}L}$ = line-to-line voltage, kV

Of course, the percent voltage rise can also be found from

$$\%\,\text{VR} = \frac{I_c\,xl}{V_{L\text{-}L}} \qquad \%\,\text{V} \tag{8-17}$$

where

$$I_c = \frac{Q_{c,\,3\phi}}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \tag{8-18}$$

= current drawn by fixed-capacitor bank

If the fixed capacitors are applied to the end of the feeder and if the percent voltage rise is already determined, the maximum value of the fixed capacitors can be determined from

$$\text{Max } Q_{c,\,3\phi} = \frac{10(\%\,\text{VR})\,V_{L\text{-}L}^2}{xl} \qquad \text{kvar} \tag{8-19}$$

Equations (8-16) and (8-17) can also be used to calculate the percent voltage rise due to the switched capacitors. Therefore, once the percent voltage rises due to both fixed and switched capacitors are found, the total percent voltage rise can be calculated as

$$\sum \% \,\mathrm{VR} = \% \,\mathrm{VR}_{\mathrm{NSW}} + \% \,\mathrm{VR}_{\mathrm{SW}} \tag{8-20}$$

where $\sum \% \,\mathrm{VR}$ = total percent voltage rise
$\% \,\mathrm{VR}_{\mathrm{NSW}}$ = percent voltage rise due to fixed (or nonswitched) capacitors
$\% \,\mathrm{VR}_{\mathrm{SW}}$ = percent voltage rise due to switched capacitors

Some utilities use the following rule of thumb: The total amount of fixed and switched capacitors for a feeder is the amount necessary to raise the receiving-end feeder voltage to maximum at 50 percent of peak feeder load.

Once the kilovars of capacitors necessary for the system is determined, there remains only the question of proper location. The rule of thumb for locating the fixed capacitors on feeders with uniformly distributed loads is to locate them approximately at two-thirds of the distance from the substation to the end of the feeder. For the uniformly decreasing loads, fixed capacitors are located approximately halfway out on the feeder. On the other hand, the location of the switched capacitors is basically determined by the voltage regulation requirements, and it usually turns out to be the last one-third of the feeder away from the source.

8-5-2 Types of Controls for Switched Shunt Capacitors

The switching process of capacitors can be done by manual control or by automatic control using some type of control intelligence. Manual control (at the location or as remote control) can be employed at distribution substations. The intelligence types that can be used in automatic control include time-switch, voltage, current, voltage-time, voltage-current, and temperature. The most popular types are the time-switch control, voltage control, and voltage-current control. The time-switch control is the least-expensive one. Some combinations of these controls are also used to follow the reactive load duration curve more closely, as illustrated in Fig. 8-16.

8-5-3 Types of Three-Phase Capacitor-Bank Connections

A three-phase capacitor bank on a distribution feeder can be connected in (1) delta, (2) grounded-wye, or (3) ungrounded-wye. The type of connection used depends upon:

1. System type, i.e., whether it is a grounded or an ungrounded system
2. Fusing requirements
3. Capacitor-bank location
4. Telephone interference considerations

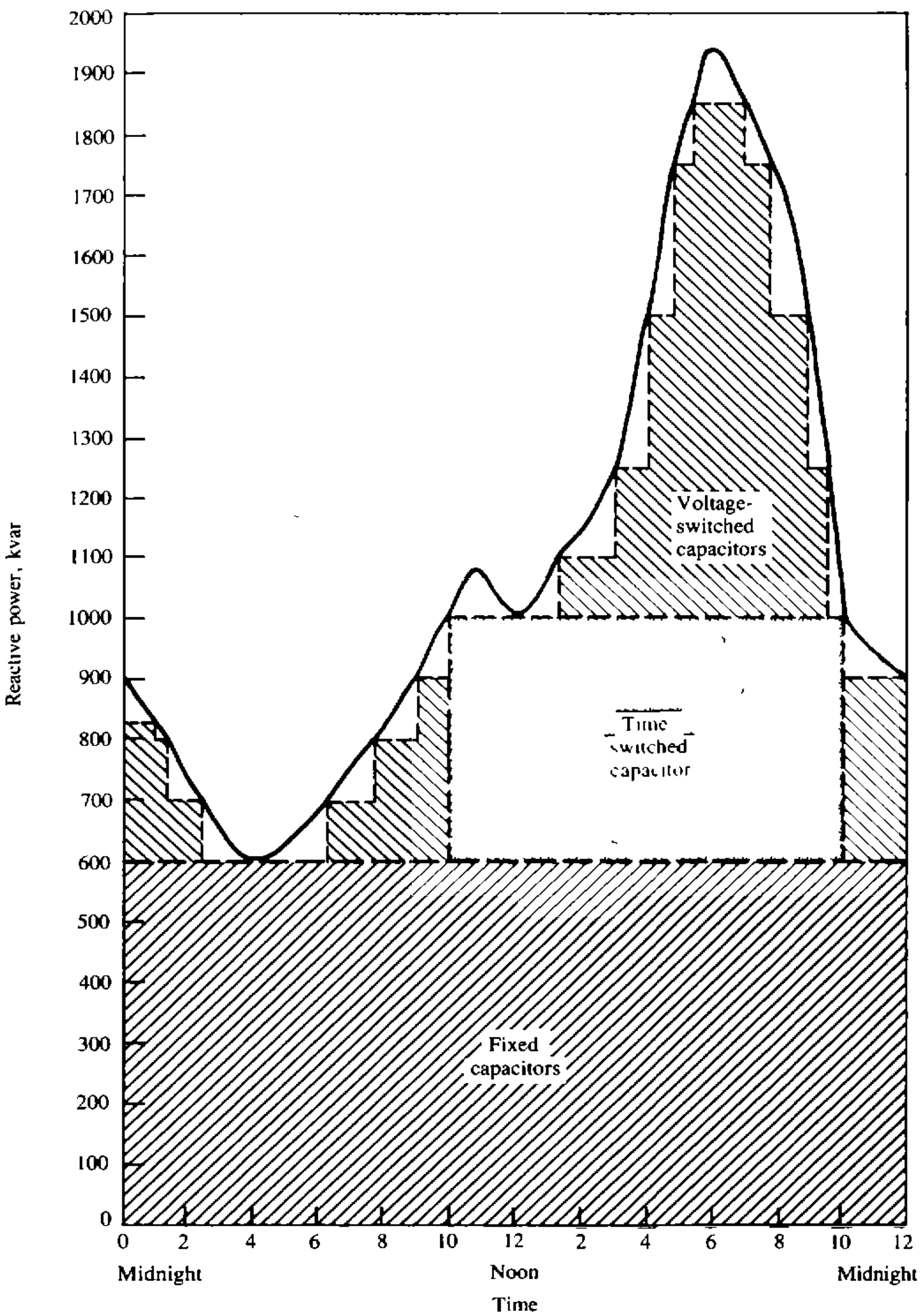


Figure 8-16 Meeting the reactive power requirements with fixed, voltage-control, and time-control capacitors.

A resonance condition may occur in delta and ungrounded-wye (floating neutral) banks when there is a one- or two-line open-type fault that occurs on the source side of the capacitor bank due to the maintained voltage on the open phase which backfeeds any transformers located on the load side of the open conductor through the series capacitor. As a result of this condition, the single-phase distribution transformers on four-wire systems may be damaged. Therefore ungrounded-wye capacitor banks are *not* recommended under the following conditions:

1. On feeders with light load where the minimum load per phase beyond the capacitor bank does not exceed 150 percent of the per phase rating of the capacitor bank
2. On feeders with single-phase breaker operation at the sending end.
3. On fixed-capacitor banks
4. On feeder sections beyond a sectionalizing-fuse or single-phase recloser
5. On feeders with emergency load transfers

However, the ungrounded-wye capacitor banks are recommended if one or more of the following conditions exist.

1. Excessive harmonic currents in the substation neutral can be precluded.
2. Telephone interferences can be minimized.
3. Capacitor-bank installation can be made with two single-phase switches rather than with three single-pole switches.

Usually, grounded-wye capacitor banks are employed only on four-wire three-phase primary systems. Otherwise, if a grounded-wye capacitor bank is used on a three-phase three-wire ungrounded-wye or delta system, it furnishes a ground current source which may disturb sensitive ground relays.

8-6 ECONOMIC JUSTIFICATION FOR CAPACITORS

Loads on electric utility systems include two components: active power (measured in kilowatts) and reactive power (measured in kilovars). Active power has to be generated at power plants, whereas reactive power can be provided by either power plants or capacitors. It is a well-known fact that shunt power capacitors are the most economical source to meet the reactive power requirements of inductive loads and transmission lines operating at a lagging power factor.

When reactive power is provided only by power plants, each system component (i.e., generators, transformers, transmission and distribution lines, switchgear, and protective equipment) has to be increased in size accordingly. Capacitors can mitigate these conditions by decreasing the reactive power demand all the way back to the generators. Line currents are reduced from capacitor locations all the way back to generation equipment. As a result, losses and loadings are reduced in distribution lines, substation transformers, and transmission lines. Depending upon the uncorrected power factor of the system, the installation of capacitors can increase generator and substation capability for additional load at least 30 percent and can increase individual circuit capability, from the voltage regulation point of view, approximately 30 to 100 percent. Furthermore, the current reduction in transformer and distribution equipment and lines reduces the load on these kilovoltampere-limited apparatus and consequently delays the new facility installations. In general, the ecomonic benefits force capacitor banks to be installed on the primary distribution system rather than on the secondary.

It is a well-known rule of thumb that the optimum amount of capacitor kilovars to employ is always the amount at which the economic benefits obtained from the addition of the last kilovar exactly equals the installed cost of the kilovars of capacitors. The methods used by the utilities to determine the economic benefits derived from the installation of capacitors vary from company to company, but the determination of the total installed cost of a kilovar of capacitors is easy and straightforward.

In general, the economic benefits that can be derived from capacitor installation can be summarized as:

1. Released generation capacity
2. Released transmission capacity
3. Released distribution substation capacity
4. Additional advantages in distribution system:
   a. Reduced energy (copper) losses
   b. Reduced voltage drop and consequently improved voltage regulation
   c. Released capacity of feeder and associated apparatus
   d. Postponement or elimination of capital expenditure due to system improvements and/or expansions
   e. Revenue increase due to voltage improvements

8-6-1 Benefits Due to Released Generation Capacity

The released generation capacity due to the installation of capacitors can be calculated approximately from

$$\Delta S_G = \begin{cases} \left[\left(1 - \dfrac{Q_c^2 \times \cos^2\theta}{S_G^2}\right)^{1/2} + \dfrac{Q_c \times \sin\theta}{S_G} - 1\right] S_G & \text{when } Q_c > 0.10 S_G \quad (8\text{-}21) \\ Q_c \times \sin\theta & \text{when } Q_c \le 0.10 S_G \quad (8\text{-}22) \end{cases}$$

where ΔS_G = released generation capacity beyond maximum generation capacity at original power factor, kVA
S_G = generation capacity, kVA
Q_c = reactive power due to corrective capacitors applied, kvar
$\cos\theta$ = original (or uncorrected or old) power factor before application of capacitors

Therefore the annual benefits due to the released generation capacity can be expressed as

$$\Delta\$_G = \Delta S_G \times C_G \times i_G \tag{8-23}$$

where $\Delta\$_G$ = annual benefits due to released generation capacity, \$/yr

ΔS_G = released generation capacity beyond maximum generation capacity at original power factor, kVA

C_G = cost of (peaking) generation, \$/kW

i_G = annual fixed charge rate† applicable to generation

8-6-2 Benefits Due to Released Transmission Capacity

The released transmission capacity due to the installation of capacitors can be calculated approximately as

$$\Delta S_T = \begin{cases} \left[\left(1 - \dfrac{Q_c^2 \times \cos^2\theta}{S_T^2}\right)^{1/2} + \dfrac{Q_c \times \sin\theta}{S_T} - 1\right] S_T & \text{when } Q_c > 0.10 S_T \quad (8\text{-}24) \\ Q_c \times \sin\theta & \text{when } Q_c \le 0.10 S_T \quad (8\text{-}25) \end{cases}$$

where ΔS_T = released transmission capacity‡ beyond maximum transmission capacity at original power factor, kVA

S_T = transmission capacity, kVA

Thus the annual benefits due to the released transmission capacity can be found as

$$\Delta\$_T = \Delta S_T \times C_T \times i_T \tag{8-26}$$

where $\Delta\$_T$ = annual benefits due to released transmission capacity, \$/yr

ΔS_T = released transmission capacity beyond maximum transmission capacity at original power factor, kVA

C_T = cost of transmission line and associated apparatus, \$/kVA

i_T = annual fixed charge rate applicable to transmission

8-6-3 Benefits Due to Released Distribution Substation Capacity

The released distribution substation capacity due to the installation of capacitors can be found approximately from

$$\Delta S_S = \begin{cases} \left[\left(1 - \dfrac{Q_c^2 \times \cos^2\theta}{S_S^2}\right)^{1/2} + \dfrac{Q_c \times \sin\theta}{S_S} - 1\right] S_S & \text{when } Q_c > 0.10 S_S \quad (8\text{-}27) \\ Q_c \times \sin\theta & \text{when } Q_c \le 0.10 S_S \quad (8\text{-}28) \end{cases}$$

† Also called *carrying charge rate*. It is defined as that portion of the annual revenue requirements which results from a plant investment. Total carrying charges include (1) return (on equity and debt), (2) book depreciation, (3) taxes (including amount paid currently and amounts deferred to future years) (4) insurance, and (5) operations and maintenance. It is expressed as a decimal.

‡ Note that the symbol S_T now stands for *transmission* capacity rather than *transformer* capacity.

where ΔS_S = released distribution substation capacity beyond maximum substation capacity at original power factor, kVA
S_S = distribution substation capacity, kVA

Hence the annual benefits due to the released substation capacity can be calculated as

$$\Delta\$_S = \Delta S_S \times C_S \times i_S \tag{8-29}$$

where $\Delta\$_S$ = annual benefits due to the released substation capacity, \$/yr
ΔS_S = released substation capacity, kVA
C_S = cost of substation and associated apparatus, \$/kVA
i_S = annual fixed charge rate applicable to substation

8-6-4 Benefits Due to Reduced Energy Losses

The annual energy losses are reduced as a result of decreasing copper losses due to the installation of capacitors. The conserved energy can be expressed as

$$\Delta\text{ACE} = \frac{Q_{c,3\phi}R(2S_{L,3\phi}\sin\theta - Q_{c,3\phi})8760}{1000 \times V_{L\text{-}L}^2} \qquad \text{kWh/yr} \tag{8-30}$$

where ΔACE = annual conserved energy, kWh/yr
$Q_{c,3\phi}$ = three-phase reactive power due to corrective capacitors applied, kvar
R = total line resistance to load center, Ω
$S_{L,3\phi}$ = original, i.e., uncorrected, three-phase load, kVA
$\sin\theta$ = sine of original (uncorrected) power-factor angle
$V_{L\text{-}L}$ = line-to-line voltage, kV

Therefore the annual benefits due to the conserved energy can be calculated as

$$\Delta\$_{\text{ACE}} = \Delta\text{ACE} \times \text{EC} \tag{8-31}$$

where $\Delta\$_{\text{ACE}}$ = annual benefits due to conserved energy, \$/yr
EC = cost of energy, \$/kWh

8-6-5 Benefits Due to Reduced Voltage Drops

The following advantages can be obtained by the installation of capacitors into a circuit:

1. The effective line current is reduced, and consequently both IR and IX_L voltage drops are decreased, which results in improved voltage regulation.
2. The power-factor improvement further decreases the effect of reactive line voltage drop.

The percent voltage drop that occurs in a given circuit can be expressed as

$$\% \text{ VD} = \frac{S_{L,3\phi}(r \cos \theta + x \sin \theta)l}{10 \times V_{L\text{-}L}^2} \tag{8-32}$$

where $\% \text{ VD}$ = percent voltage drop
$S_{L,3\phi}$ = three-phase load, kVA
r = line resistance, Ω/mi
x = line reactance, Ω/mi
l = length of conductors, mi
$V_{L\text{-}L}$ = line-to-line voltage, kV

The voltage drop that can be calculated from Eq. (8-32) is the basis for the application of the capacitors. After the application of the capacitors, the system yields a voltage rise due to the improved power factor and the reduced effective line current. Therefore the voltage drops due to IR and IX_L are minimized. The approximate value of the percent voltage rise along the line can be calculated as

$$\% \text{ VR} = \frac{Q_{c,3\phi} \times l}{10 \times V_{L\text{-}L}^2} \tag{8-33}$$

Furthermore, an additional voltage-rise phenomenon through every transformer from the generating source to the capacitors occurs due to the application of capacitors. It is independent of load and power factor of the line and can be expressed as

$$\% \text{ VR}_T = \left(\frac{Q_{c,3\phi}}{S_{T,3\phi}}\right) x_T \tag{8-34}$$

where $\% \text{ VR}_T$ = percent voltage rise through transformer
$S_{T,3\phi}$ = total three-phase transformer rating, kVA
x_T = percent transformer reactance (approximately equal to transformer's nameplate impedance).

8-6-6 Benefits Due to Released Feeder Capacity

In general, feeder capacity is restricted by allowable voltage drop rather than by thermal limitations (as seen in Chap. 4). Therefore the installation of capacitors decreases the voltage drop and consequently increases the feeder capacity. Without including the released regulator or substation capacity, this additional feeder capacity can be calculated as

$$\Delta S_F = \frac{(Q_{c,3\phi})x}{x \sin \theta + r \cos \theta} \qquad \text{kVA} \tag{8-35}$$

Therefore the annual benefits due to the released feeder capacity can be calculated as

$$\Delta \$_F = \Delta S_F \times C_F \times i_F \tag{8-36}$$

where $\Delta\$_F$ = annual benefits due to released feeder capacity, \$/yr
ΔS_F = released feeder capacity, kVA
C_F = cost of installed feeder, \$/kVA
i_F = annual fixed charge rate applicable to feeder

8-6-7 Financial Benefits Due to Voltage Improvement

The revenues to the utility are increased as a result of increased kilowatthour energy consumption due to the voltage rise produced on a system by the addition of the corrective capacitor banks. This is especially true for residential feeders. The increased energy consumption depends on the nature of the apparatus used. For example, energy consumption for lighting increases as the square of the voltage. As an Example, Table 8-2 gives the additional kilowatthour energy increase (in percent) as a function of the ratio of the average voltage after the addition of capacitors to the average voltage before the addition of capacitors (based on a typical load diversity).

Thus the increase in revenues due to the increased kilowatthour energy consumption can be calculated as

$$\Delta\$_{\text{BEC}} = \Delta\text{BEC} \times \text{BEC} \times \text{EC} \tag{8-37}$$

where $\Delta\$_{\text{BEC}}$ = additional annual revenue due to increased kWh energy consumption, \$/yr
ΔBEC = additional kWh energy consumption increase, %
BEC = original (or base) annual kWh energy consumption, kWh/yr

8-6-8 Total Financial Benefits Due to Capacitor Installations

Therefore the total benefits due to the installation of capacitor banks can be summarized as

$$\sum \Delta\$ = \underbrace{\Delta\$_G + \Delta\$_T + \Delta\$_S + \Delta\$_F}_{\substack{\text{Demand}\\\text{reduction}}} + \underbrace{\Delta\$_{\text{ACE}}}_{\substack{\text{Energy}\\\text{reduction}}} + \underbrace{\Delta\$_{\text{BCE}}}_{\substack{\text{Revenue}\\\text{increase}}} \tag{8-38}$$

Table 8-2

$\dfrac{V_{\text{av, after}}}{V_{\text{av, before}}}$	ΔkWh increase, %
1.00	0
1.05	8
1.10	16
1.15	25
1.20	34
1.25	43
1.30	52

The total benefits obtained from Eq. (8-38) should be compared against the annual equivalent of the total cost of the installed capacitor banks. The total cost of the installed capacitor banks can be found from

$$\mathrm{AEIC}_c = \Delta Q_c \times \mathrm{IC}_c \times i_c \tag{8-39}$$

where AEIC_c = annual equivalent of total cost of installed capacitor banks, \$/yr
ΔQ_c = required amount of capacitor-bank additions, kvar
IC_c = cost of installed capacitor banks, \$/kvar
i_c = annual fixed charge rate applicable to capacitors

In summary, capacitors can provide the utility industry with a very effective cost-reduction instrument. With plant costs and fuel costs continually increasing, electric utilities benefit whenever new plant investment can be deferred or eliminated and energy requirements reduced. Thus capacitors aid in minimizing operating expenses and allow the utilities to serve new loads and customers with a minimum system investment. Today, utilities in the United States have approximately 1 kvar of power capacitors installed for every 2 kW of installed generation capacity in order to take advantage of the economic benefits involved [5].

Example 8-5† Assume that a large power pool is presently operating at 90 percent power factor. It is desired to improve the power factor to 98 percent. To improve the power factor to 98 percent, a number of load flow runs are made, and the results are summarized in Table 8-3.

Assume that the average fixed charge rare is 0.20, average demand cost is \$250/kW, energy cost is \$0.045/kWh, the system loss factor is 0.17, and an average capacitor cost is \$4.75/kvar. Use responsibility factors of 1.0 and 0.9 for capacitors installed on the substation buses and on feeders, respectively. Determine the following:

(*a*) The resulting additional savings in kilowatt losses at the 98 percent power factor when all capacitors are applied to substation buses.

Table 8-3

Comment	At 90% PF	At 98% PF
Total loss reduction due to capacitors applied to substation buses, kW	495,165	491,738
Additional loss reduction due to capacitors applied to feeders, kW	85,771	75,342
Total demand reduction due to capacitors applied to substation buses and feeders, kVA	22,506,007	21,172,616
Total required capacitor additions at buses and feeders, kvar	9,810,141	4,213,297

† Based on ref. 3.

(*b*) The resulting additional savings in kilowatt losses at the 98 percent power factor when some capacitors are applied to feeders.
(*c*) The total additional savings in kilowatt losses.
(*d*) The additional savings in the system kilovoltampere capacity.
(*e*) The additional capacitors required, in kilovars.
(*f*) The total annual savings in demand reduction due to additional capacitors applied to substation buses and feeders, in dollars per year.
(*g*) The annual savings due to the additional released transmission capacity, in dollars per year.
(*h*) The total annual savings due to the energy loss reduction, in dollars per year.
(*i*) The total annual cost of the additional capacitors, in dollars per year.
(*j*) The total net annual savings, in dollars per year.
(*k*) Is the 98 percent power factor the economic power factor?

SOLUTION

(*a*) From Table 8-3, the resulting additional savings in kilowatt losses due to the power-factor improvement at the substation buses is

$$\Delta P_{LS} = 495{,}165 - 491{,}738$$
$$= 3427 \text{ kW}$$

(*b*) From Table 8-3 for feeders,

$$\Delta P_{LS} = 85{,}771 - 75{,}342$$
$$= 10{,}429 \text{ kW}$$

(*c*) Therefore the total additional kilowatt savings is

$$\sum \Delta P_{LS} = 3427 + 10{,}429$$
$$= 13{,}856 \text{ kW}$$

As can be observed, the additional kilowatt savings due to capacitors applied to the feeders is more than three times that of capacitors applied to the substation buses. This is due to the fact that power losses are larger at the lower voltages.

(*d*) From Table 8-3, the additional savings in the system kilovoltampere capacity is

$$\Delta S_{sys} = 22{,}506{,}007 - 21{,}172{,}616$$
$$= 1{,}333{,}391 \text{ kVA}$$

(*e*) From Table 8-3, the additional capacitors required are

$$\Delta Q_c = 9{,}810{,}141 - 4{,}213{,}297$$
$$= 5{,}596{,}844 \text{ kvar}$$

(*f*) The annual savings in demand reduction due to capacitors applied to distribution substation buses is approximately

$$(3427 \text{ kW})(1.0)(\$250/\text{kW})(0.20/\text{yr}) = \$171{,}350/\text{yr}$$

and due to capacitors applied to feeders is

$$(10{,}429 \text{ kW})(0.9)(\$250/\text{kW})(0.20/\text{yr}) = \$469{,}305/\text{yr}$$

Therefore the total annual savings in demand reduction is

$$\$171{,}350 + \$469{,}305 = \$640{,}655/\text{yr}$$

(*g*) The annual savings due to the additional released transmission capacity is

$$(1{,}333{,}391 \text{ kVA})(\$27/\text{kVA})(0.20/\text{yr}) = \$7{,}200{,}311/\text{yr}$$

(*h*) The total annual savings due to the energy loss reduction is

$$(\$13{,}856 \text{ kW})(8760 \text{ hr/yr})(0.17)(\$0.045/\text{kWh}) = \$928{,}546/\text{yr}$$

(*i*) The total annual cost of the additional capacitors is

$$(5{,}596{,}844 \text{ kvar})(\$4.75/\text{kvar})(0.20/\text{yr}) = \$5{,}317{,}002/\text{yr}$$

(*j*) The total annual savings is

$$\underbrace{\$640{,}655}_{\substack{\text{Savings in}\\ \text{demand}}} + \underbrace{\$7{,}200{,}311}_{\substack{\text{Savings in}\\ \text{capacity}}} + \underbrace{\$928{,}546}_{\substack{\text{Savings in}\\ \text{energy}}} = \$8{,}769{,}512/\text{yr}$$

Therefore the total net annual savings is

$$\$8{,}769{,}512 - \$5{,}317{,}002 = \$3{,}452{,}510/\text{yr}$$

(*k*) No, since the total net annual savings is not zero.

8-7 A PRACTICAL PROCEDURE TO DETERMINE THE BEST CAPACITOR LOCATION

In general, the best location for capacitors can be found by optimizing power loss and voltage regulation. A feeder voltage profile study is performed to warrant the most effective location for capacitors and the determination of a voltage which is within recommended limits. Usually, a 2-V rise on circuits used in urban areas and a 3-V rise on circuits used in rural areas are approximately the maximum voltage changes that are allowed when a switched-capacitor bank is placed into operation The general iteration process involved is summarized in the following steps:

1. Collect the following circuit and load information:
   a. Any two of the following for each load: kilovoltamperes, kilovars, kilowatts, and load power factor
   b. Desired corrected power of circuit

   c. Feeder circuit voltage
   d. A feeder circuit map which shows locations of loads and presently existing capacitor banks
2. Determine the kilowatt load of the feeder and the power factor.
3. From Table 8-1, determine the kilovars per kilowatts of load (i.e., the correction factor) necessary to correct the feeder-circuit power factor from the original to the desired power factor. To determine the kilovars of capacitors required, multiply this correction factor by the total kilowatts of the feeder circuit.
4. Determine the individual kilovoltamperes and power factor for each load or group of loads.
5. To determine the kilovars on the line, multiply individual load or groups of loads by their respective reactive factors that can be found from Table 8-1.
6. Develop a nomograph to determine the line loss in watts per thousand feet due to the inductive loads tabulated in steps 4 and 5. Multiply these line losses by their respective line lengths in thousands of feet. Repeat this process for all loads and line sections and add them to find the total inductive line loss.
7. In the case of having presently existing capacitors on the feeder, perform the same calculations as in step 6, but this time subtract the capacitive line loss from the total inductive line loss. Use the capacitor kilovars determined in step 3 and the nomograph developed for step 6 and find the line loss in each line section due to capacitors.
8. To find the distance to capacitor location, divide total inductive line loss by capacitive line loss per thousand feet. If this quotient is greater than the line section length:
   a. Divide the remaining inductive line loss by capacitive line loss in the next line section to find the location.
   b. If this quotient is still greater than the line section length, repeat step 8*a*.
9. Prepare a voltage profile by hand calculations or by using a computer program for voltage profile and load analysis to determine the circuit voltages. If the profile shows that the voltages are inside the recommended limits, then the capacitors are installed at the location of minimum loss. If not, then use engineering judgment to locate them for the most effective voltage control application.

8-8 A MATHEMATICAL PROCEDURE TO DETERMINE THE OPTIMUM CAPACITOR ALLOCATION

The optimum application of shunt capacitors on distribution feeders to reduce losses has been studied in numerous papers such as those by Neagle and Samson [7], Schmidt [8], Maxwell [1, 9], Cook [10], Schmill [11], Chang [12–14], Bae [15], Gönen and Djavashi [17], and Grainger et al. [21–24]. Figure 8-17 shows a realistic representation of a feeder which contains a number of line segments with a combination of concentrated (or lumped-sum) and uniformly distributed loads, as suggested by Chang [13]. Each line segment represents a part of the feeder

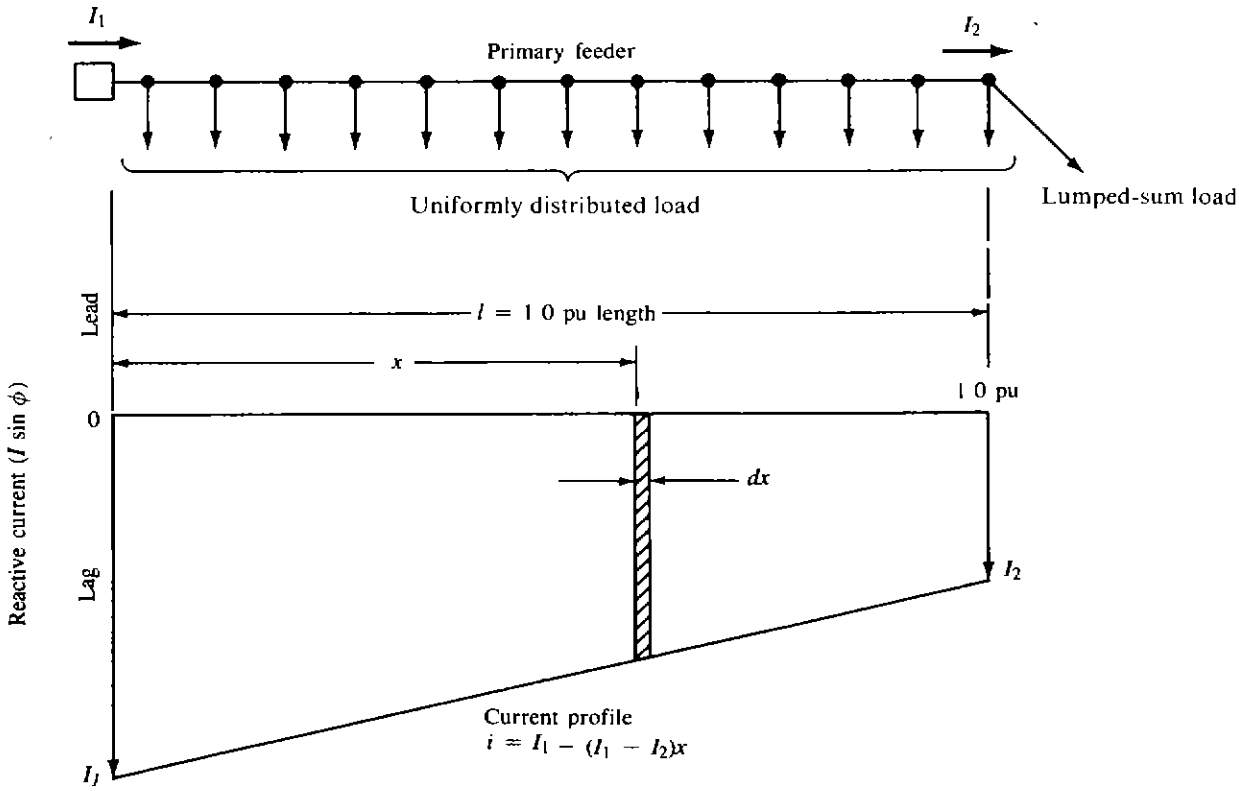


Figure 8-17 Primary feeder with lumped-sum (or concentrated) and uniformly distributed loads, and reactive current profile before adding capacitor.

between sectionalizing devices, voltage regulators, or other points of significance. For the sake of convenience, the load or line current and the resulting I^2R loss can be assumed to have two components, namely, (1) those due to the in-phase or active component of current and (2) those due to the out-of-phase or reactive component of current. Since losses due to the in-phase or active component of line current are not signficantly affected by the application of shunt capacitors, they are not considered. This can be verified as follows

Assume that the I^2R losses are caused by a lagging line current I flowing through the circuit resistance R. Therefore it can be shown that

$$I^2R = (I \cos \phi)^2R + (I \sin \phi)^2R \tag{8-40}$$

After adding a shunt capacitor with current I_c, the resultants are a new line current I_1 and a new power loss I_1^2R. Hence

$$I_1^2R = (I \cos \phi)^2R + (I \sin \phi - I_c)^2R \tag{8-41}$$

Therefore the loss reduction as a result of the capacitor addition can be found as

$$\Delta P_{\text{LS}} = I^2R - I_1^2R \tag{8-42}$$

or by substituting Eq. (8-40) and (8-41) into Eq. (8-42),

$$\Delta P_{\text{LS}} = 2(I \sin \phi)I_cR - I_c^2R \tag{8-43}$$

Thus only the out-of-phase or reactive component of line current, that is, $I \sin \theta$, should be taken into account for I^2R loss reduction as a result of a capacitor addition.

Assume that the length of a feeder segment is 1.0 per unit length, as shown in Fig. 8-17. The current profile of the line current at any given point on the feeder is a function of the distance of that point from the beginning end of the feeder. Therefore the differential I^2R loss of a dx differential segment located at a distance x can be expressed as

$$dP_{LS} = 3[I_1 - (I_1 - I_2)x]^2 R\, dx \tag{8-44}$$

Therefore the total I^2R loss of the feeder can be found as

$$\begin{aligned} P_{LS} &= \int_{x=0}^{1.0} dP_{LS} \\ &= 3 \int_{x=0}^{1.0} [I_1 - (I_1 - I_2)x]^2 R\, dx \\ &= (I_1^2 + I_1 I_2 + I_2^2)R \end{aligned} \tag{8-45}$$

where P_{LS} = total I^2R loss of feeder before adding capacitor
I_1 = reactive current at beginning of feeder segment
I_2 = reactive current at end of feeder segment
R = total resistance of feeder segment
x = per unit distance from beginning of feeder segment

8-8-1 Loss Reduction Due to Capacitor Allocation

Case 1: One capacitor bank The insertion of one capacitor bank on the primary feeder causes a break in the continuity of the reactive load profile, modifies the reactive current profile, and consequently reduces the loss, as shown in Fig. 8-18.

Therefore the loss equation after adding one capacitor bank can be found as before,

$$P'_{LS} = 3 \int_{x=0}^{x_1} [I_1 - (I_1 - I_2)x - I_c]^2 R\, dx + 3 \int_{x=x_1}^{1.0} [I_1 - (I_1 - I_2)x]^2 R\, dx \tag{8-46}$$

or

$$P'_{LS} = (I_1^2 + I_1 I_2 + I_2^2)R + 3x_1[(x_1 - 2)I_1 I_c - x_1 I_2 I_c + I_c^2]R \tag{8-47}$$

Thus the per unit power loss reduction as a result of adding one capacitor bank can be found from

$$\Delta P_{LS} = \frac{P_{LS} - P'_{LS}}{P_{LS}} \tag{8-48}$$

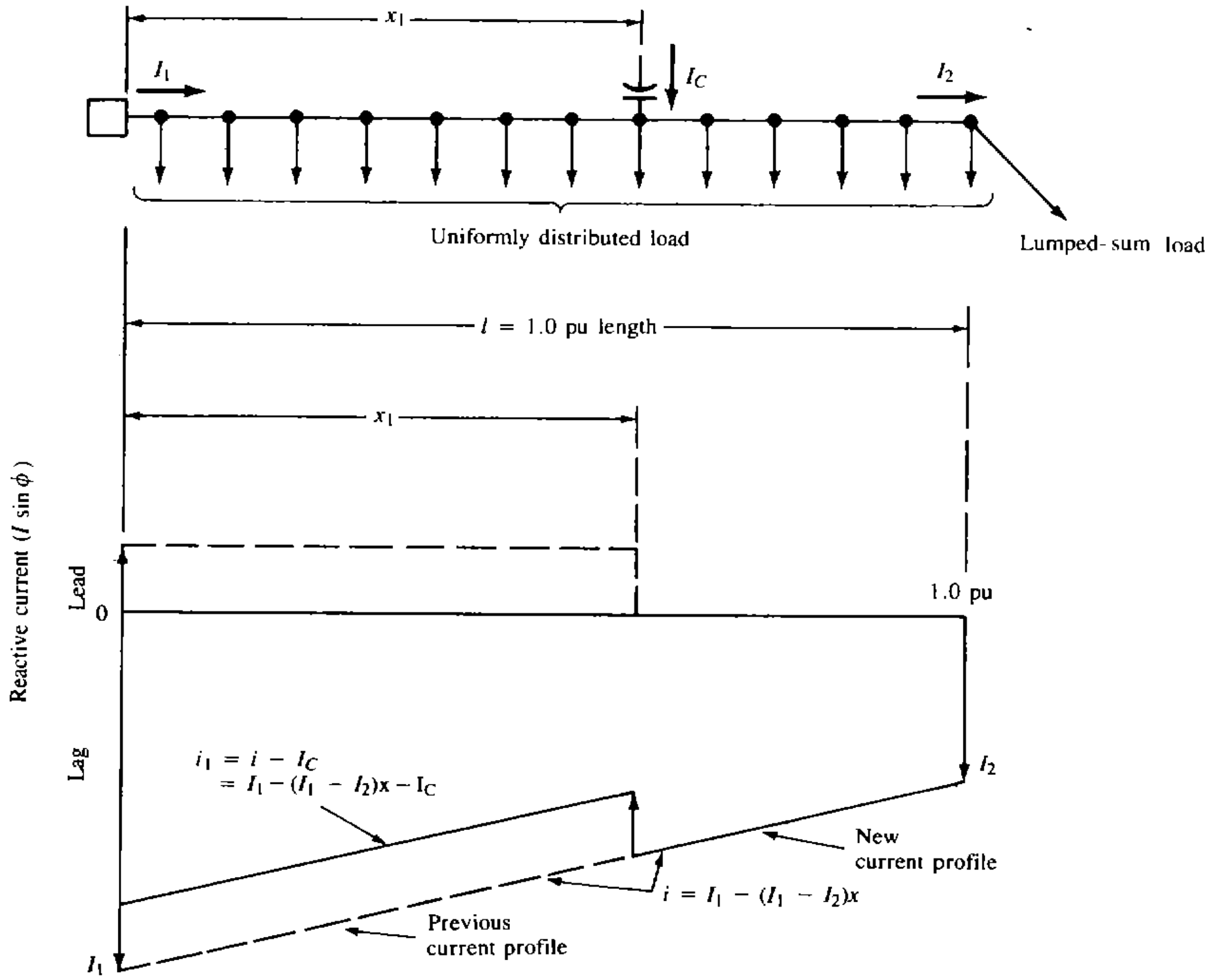


Figure 8-18 Loss reduction with one capacitor bank.

or substituting Eq. (8-45) and (8-46) into Eq. (8-48),

$$\Delta P_{\text{LS}} = \frac{-3x_1[(x_1 - 2)I_1 I_c - x_1 I_2 I_c + I_c^2]R}{(I_1^2 + I_1 I_2 + I_2^2)R} \tag{8-49}$$

or rearranging Eq. (8-49) by dividing its numerator and denominator by I_1^2 so that

$$\Delta P_{\text{LS}} = \frac{3x_1}{1 + (I_2/I_1) + (I_2/I_1)^2}\left[(2 - x_1)\left(\frac{I_c}{I_1}\right) + x_1\left(\frac{I_2}{I_1}\right)\left(\frac{I_c}{I_1}\right) - \left(\frac{I_c}{I_1}\right)^2\right] \tag{8-50}$$

If c is defined as the ratio of the capacitive kilovoltamperes (ckVA) of the capacitor bank to the total reactive load, that is,

$$c = \frac{\text{ckVA of capacitor installed}}{\text{total reactive load}} \tag{8-51}$$

then

$$c = \frac{I_c}{I_1} \tag{8-52}$$

and if λ is defined as the ratio of the reactive current at the end of the line segment to the reactive current at the beginning of the line segment, that is,

$$\lambda = \frac{\text{reactive current at the end of line segment}}{\text{reactive current at the beginning of line segment}} \tag{8-53}$$

then

$$\lambda = \frac{I_2}{I_1} \tag{8-54}$$

Therefore, substituting Eq. (8-52) and (8-54) into Eq. (8-50), the per unit power loss reduction can be found as

$$\Delta P_{\text{LS}} = \frac{3x_1}{1 + \lambda + \lambda^2}\left[(2 - x_1)c + x_1\lambda c - c^2\right] \tag{8-55}$$

or

$$\Delta P_{\text{LS}} = \frac{3cx_1}{1 + \lambda + \lambda^2}\left[(2 - x_1) + x_1\lambda - c\right] \tag{8-56}$$

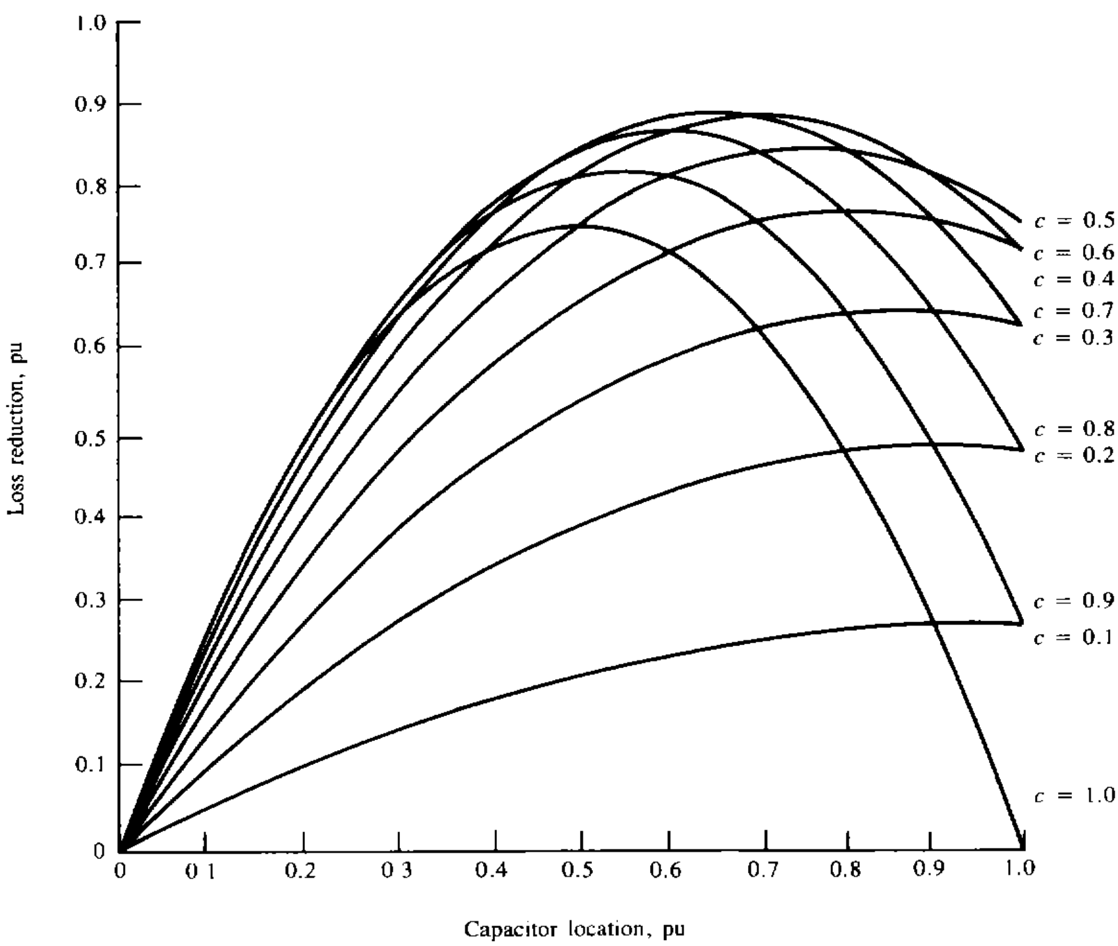


Figure 8-19 Loss reduction as a function of the capacitor-bank location and capacitor compensation ratio for a line segment with uniformly distributed loads ($\lambda = 0$).

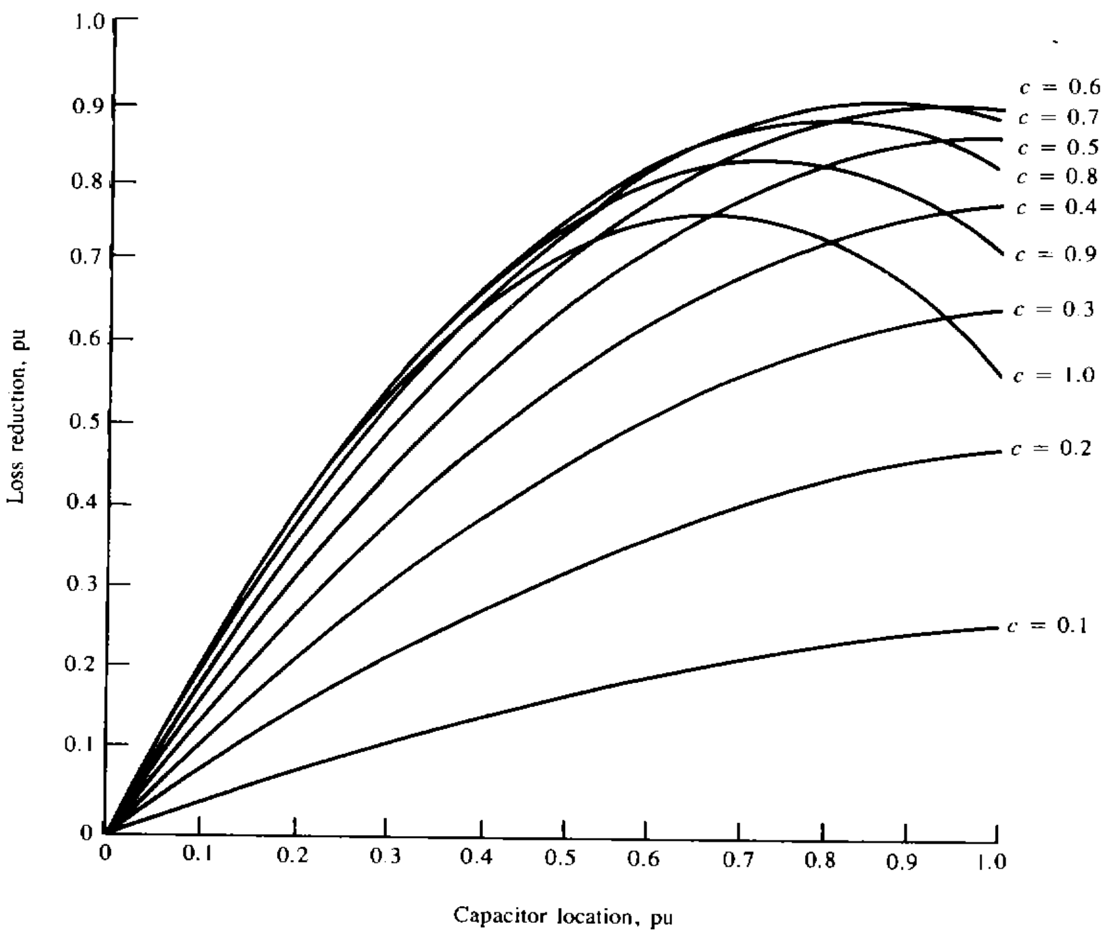


Figure 8-20 Loss reduction as a function of the capacitor-bank location and capacitor compensation ratio for a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{1}{4}$).

where x_1 = per unit distance of capacitor-bank location from beginning of feeder segment (between 0 and 1.0 pu).

If α is defined as the reciprocal of $1 + \lambda + \lambda^2$, that is,

$$\alpha = \frac{1}{1 + \lambda + \lambda^2} \tag{8-57}$$

then Eq. (8-56) can also be expressed as

$$\Delta P_{LS} = 3\alpha c x_1[(2 - x_1) + \lambda x_1 - c] \tag{8-58}$$

Figures 8-19 to 8-23 give the loss reduction that can be accomplished by changing the location of a single capacitor bank with any given size for different capacitor compensation ratios along the feeder for different representative load patterns, e.g., uniformly distributed loads ($\lambda = 0$), concentrated or lumped-sum loads ($\lambda = 1$), or a combination of concentrated and uniformly distributed loads ($0 < \lambda < 1$). To use these nomographs for a given case, the following factors must be known:

1. Original losses due to reactive current
2. Capacitor compensation ratio
3. The location of the capacitor bank

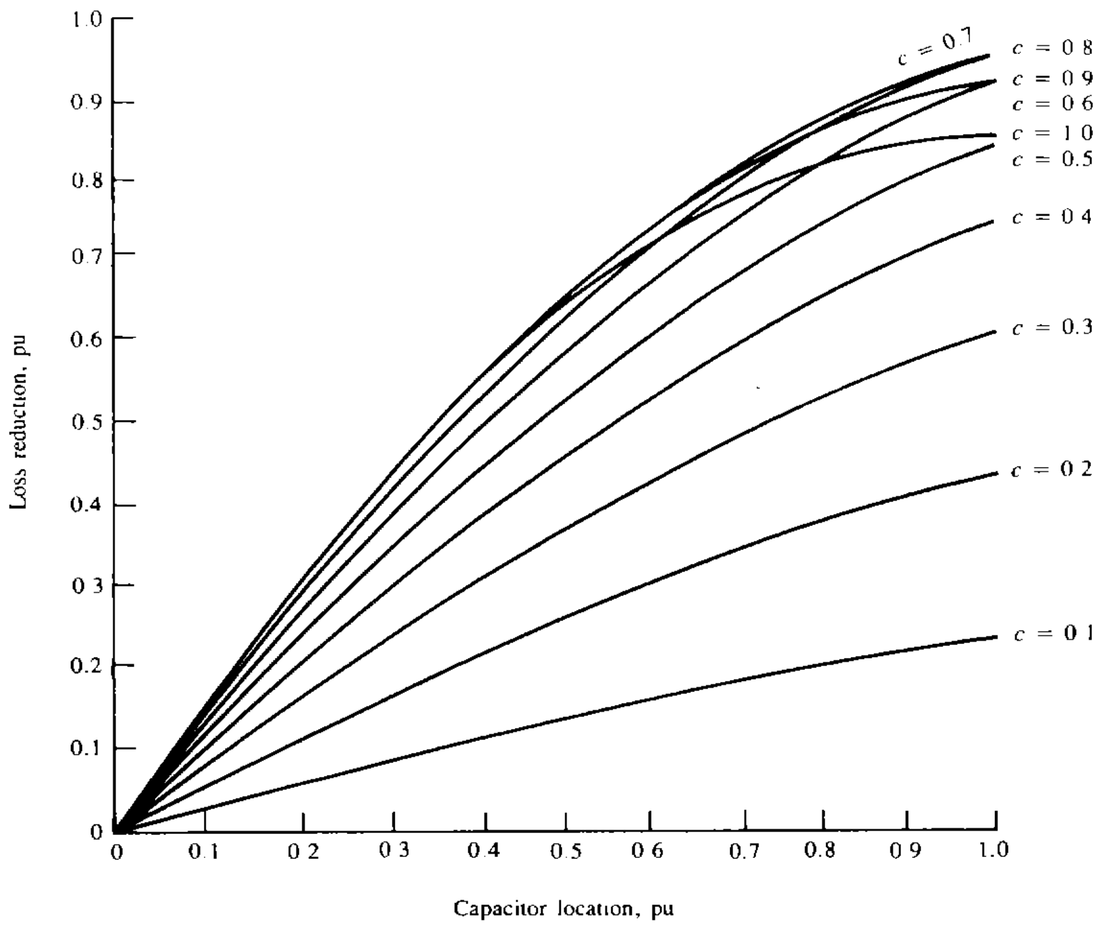


Figure 8-21 Loss reduction as a function of the capacitor-bank location and capacitor compensation ratio for a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{1}{2}$).

As an example, assume that the load on the line segment is uniformly distributed and the desired compensation ratio is 0.5. From Fig. 8-19, it can be found that the maximum loss reduction can be obtained if the capacitor bank is located at 0.75 per unit length from the source. The associated loss reduction is 0.85 per unit or 85 percent. If the bank is located anywhere else on the feeder, however, the loss reduction would be less than the 85 percent. In other words, there is only one location for any any given-size capacitor bank to achieve the maximum loss reduction. Table 8-4 gives the optimum location and percent loss reduction for a given-size capacitor bank located on a feeder with uniformly distributed load ($\lambda = 0$). From the table it can be observed that the maximum loss reduction can be achieved by locating the single capacitor bank at the two-thirds length of the feeder away from the source.

Figure 8-24 gives the loss reduction for a given capacitor bank of any size and located at the optimum location on a feeder with various combinations of load types based on Eq. (8-58).

Figure 8-25 gives the loss reduction due to an optimum-sized capacitor bank located on a feeder with various combinations of load types.

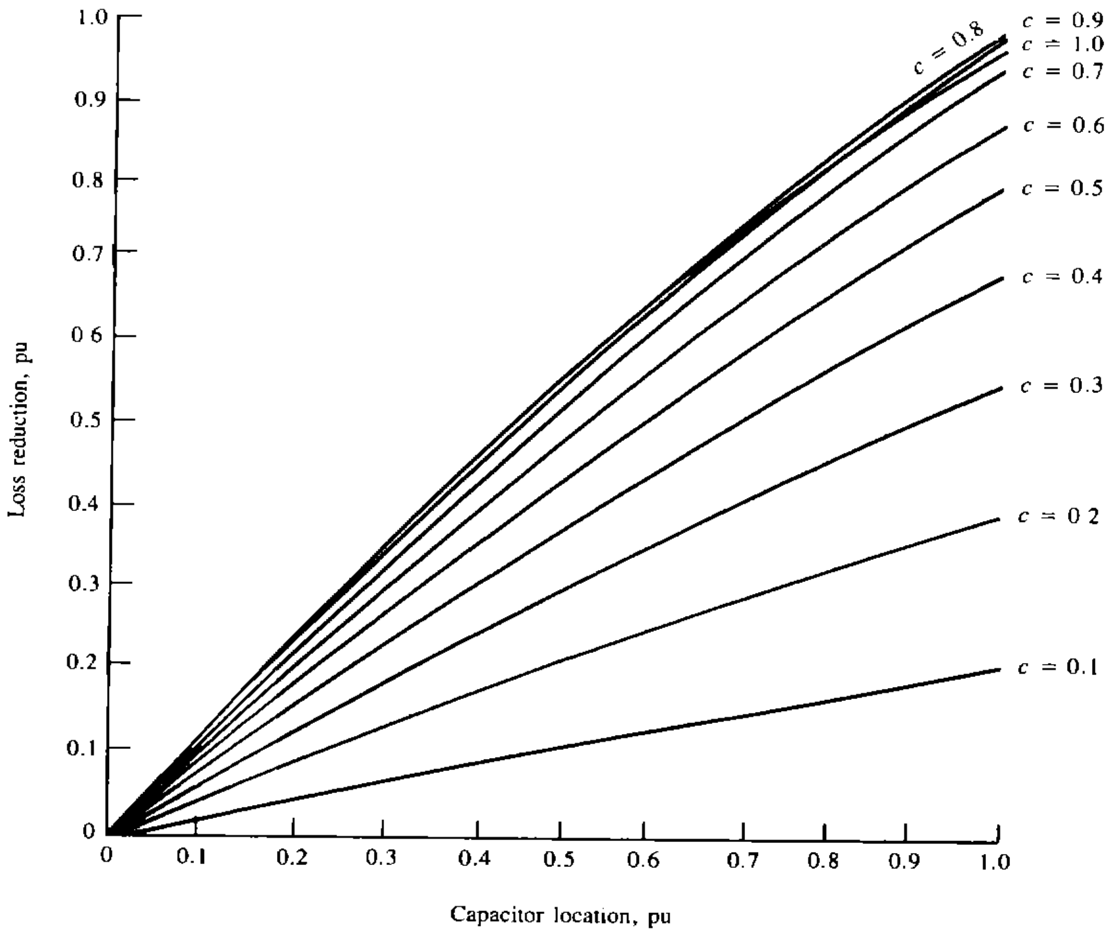


Figure 8-22 Loss reduction as a function of the capacitor-bank location and capacitor compensation ratio for a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{3}{4}$)

Case 2: Two capacitor banks Assume that two capacitor banks of equal size are inserted on the feeder, as shown in Fig. 8-26. The same procedure can be followed as before, and the new loss equation becomes

$$P'_{LS} = 3\int_{x=0}^{x_1} [I_1 - (I_1 - I_2)x - 2I_c]^2 R\,dx + 3\int_{x=x_1}^{x_2} [I_1 - (I_1 I_2)x - I_c]^2 R\,dx$$

$$+ 3\int_{x=x_2}^{1.0} [I_1 - (I_1 - I_2)x]^2 R\,dx \quad (8\text{-}59)$$

Therefore, substituting Eqs. (8-45) and (8-59) into Eq. (8-48), the new per unit loss reduction equation can be found as

$$\Delta P_{LS} = 3\alpha c x_1[(2 - x_1) + \lambda x_1 - 3c] + 3\alpha c x_2[(2 - x_2) + \lambda x_2 - c] \quad (8\text{-}60)$$

or

$$\Delta P_{LS} = 3\alpha c\{x_1[(2 - x_1) + \lambda x_1 - 3c] + x_2[(2 - x_2) + \lambda x_2 - c]\} \quad (8\text{-}61)$$

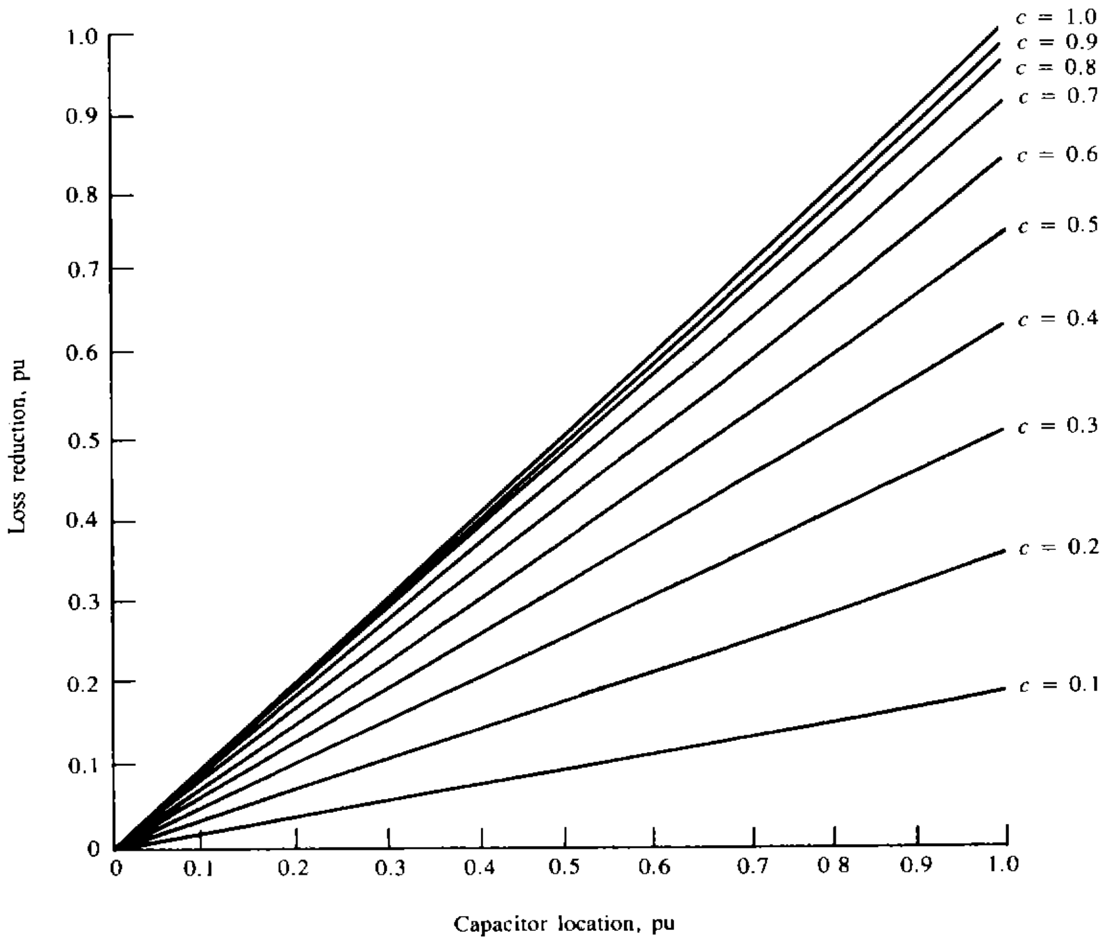


Figure 8-23 Loss reduction as a function of the capacitor-bank location and capacitor compensation ratio for a line segment with concentrated loads ($\lambda = 1$).

Table 8-4

Capacitor-bank rating, pu	Optimum location, pu	Optimum loss reduction, %
0.0	1.0	0
0.1	0.95	27
0.2	0.90	49
0.3	0.85	65
0.4	0.80	77
0.5	0.75	84
0.6	0.70	88
0.7	0.65	89
0.8	0.60	86
0.9	0.55	82
1.0	0.50	75

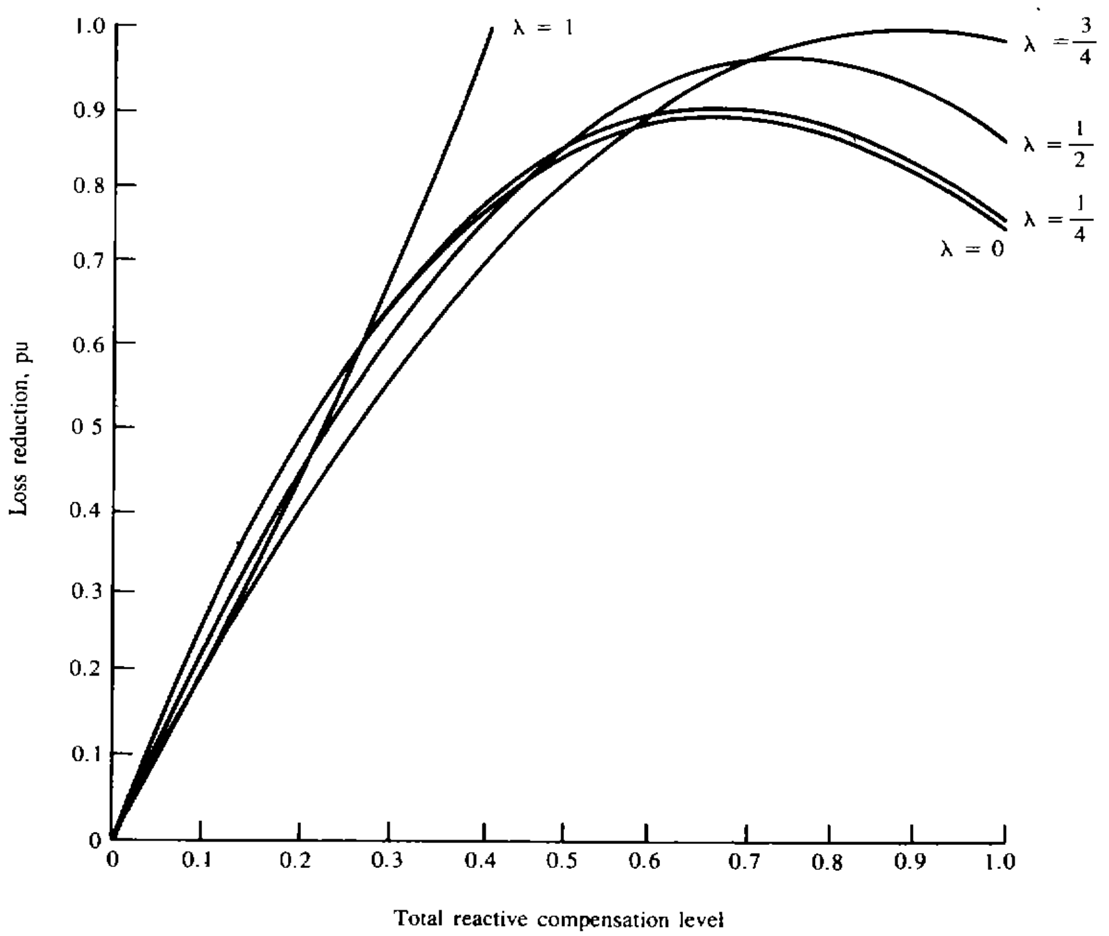


Figure 8-24 Loss reduction due to a capacitor bank located at the optimum location on a line section with various combinations of concentrated and uniformly distributed loads.

Case 3: Three capacitor banks Assume that three capacitor banks of equal size are inserted on the feeder, as shown in Fig. 8-27. The relevant per unit loss reduction equation can be found as

$$\Delta P_{LS} = 3\alpha c\{x_1[(2 - x_1) + \lambda x_1 - 5c] + x_2[(2 - x_2) + \lambda x_2 - 3c] + x_3[(2 - x_3) + \lambda x_3 - c]\} \quad (8\text{-}62)$$

Case 4: Four capacitor banks Assume that four capacitor banks of equal size are inserted on the feeder, as shown in Fig. 8-28. The relevant per unit loss reduction equation can be found as

$$\Delta P_{LS} = 3\alpha c\{x_1[(2 - x_1) + \lambda x_1 - 7c] + x_2(2 - x_2) + \lambda x_2 - 5c] + x_3[(2 - x_3) + \lambda x_3 - 3c] + x_4[(2 - x_4) + \lambda x_4 - c]\} \quad (8\text{-}63)$$

Case 5: *n* capacitor banks As the aforementioned results indicate, the per unit loss reduction equations follow a definite pattern as the number of capacitor banks increases. Therefore, the general equation for per unit loss reduction, for an *n*

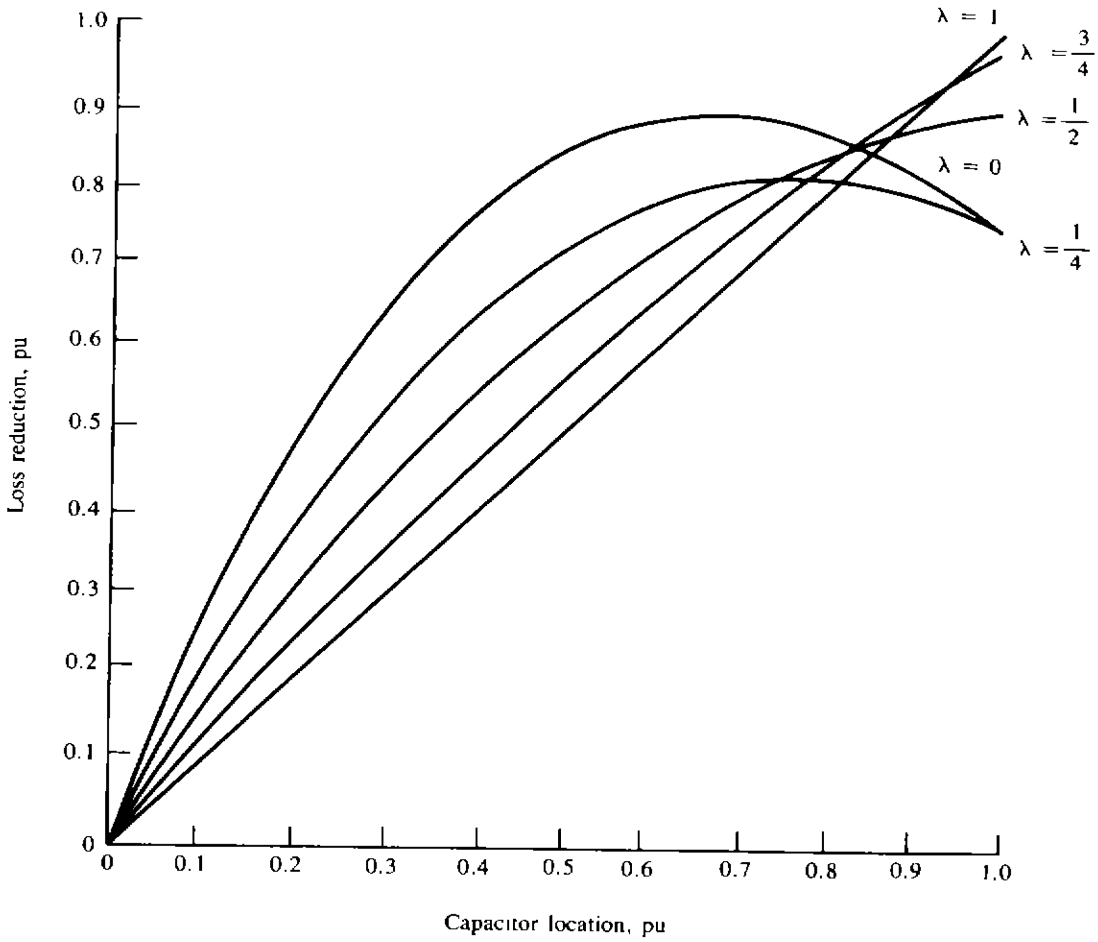


Figure 8-25 Loss reduction due to an optimum-sized capacitor bank located on a line segment with various combinations of concentrated and uniformly distributed loads.

capacitor-bank feeder, can be expressed as

$$\Delta P_{LS} = 3\alpha c \sum_{i=1}^{n} x_i[(2 - x_i) + \lambda x_i - (2i - 1)c] \tag{8-64}$$

where c = capacitor compensation ratio at each location (determined from Eq. 8-51)

x_i = per unit distance of ith capacitor-bank location from source

n = total number of capacitor banks

8-8-2 Optimum Location of a Capacitor Bank

The optimum location for the ith capacitor bank can be found by taking the first-order partial derivative of Eq. (8-64) with respect to x_i and setting the resulting expression equal to zero. Therefore

$$x_{i,\text{opt}} = \frac{1}{1-\lambda} - \frac{(2i-1)c}{2(1-\lambda)} \tag{8-65}$$

where $x_{i,\text{opt}}$ = optimum location for ith capacitor bank in per unit length.

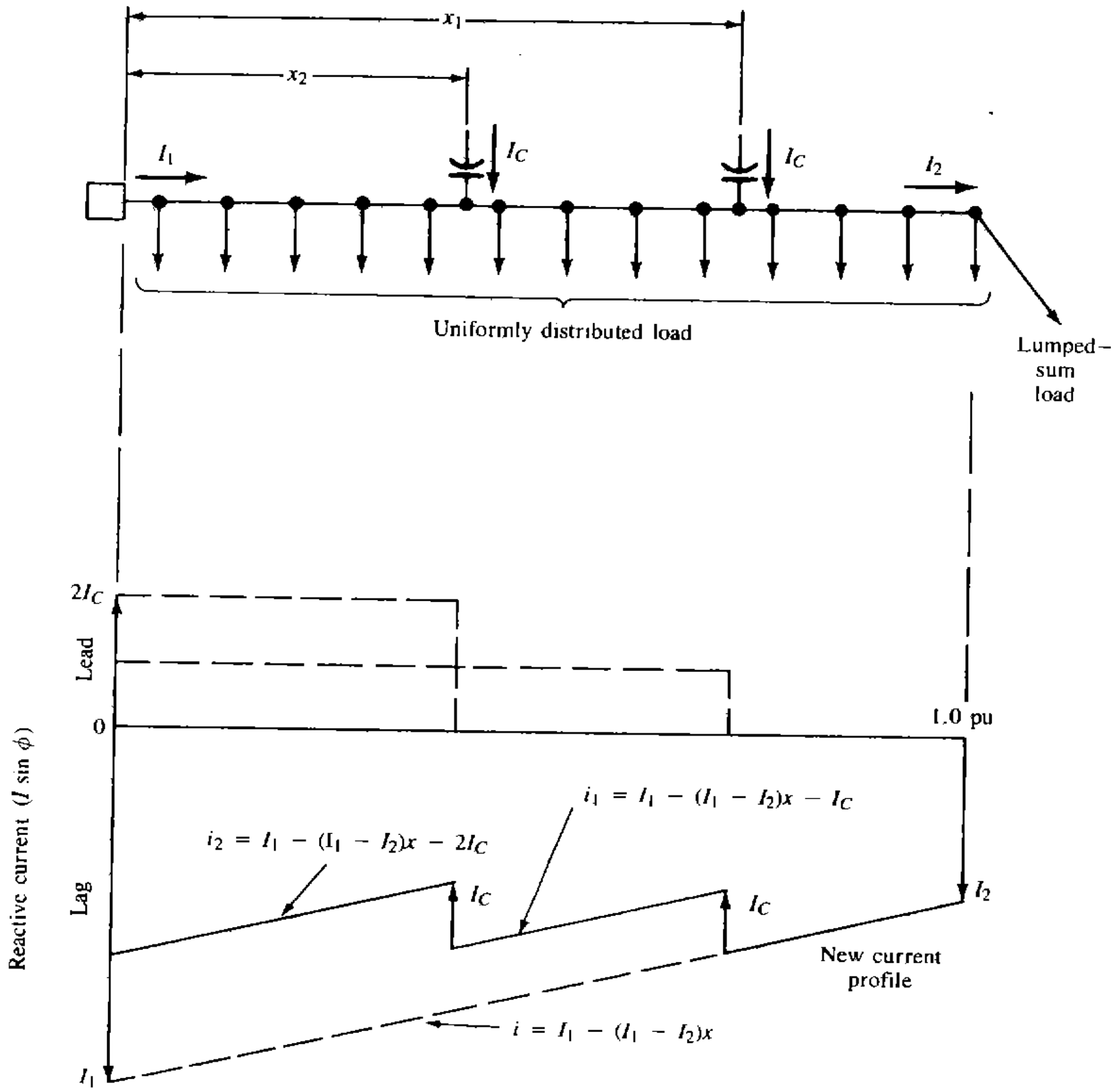


Figure 8-26 Loss reduction with two capacitor banks.

By substituting Eq. (8-65) into Eq. (8-64), the optimum loss reduction can be found as

$$\Delta P_{\text{LS, opt}} = 3\alpha c \sum_{i=1}^{n} \left[\frac{1}{1-\lambda} - \frac{(2i-1)c}{1-\lambda} + \frac{i^2c^2}{1-\lambda} - \frac{c^2}{4(1-\lambda)} - \frac{ic^2}{1-\lambda} \right] \quad (8\text{-}66)$$

Equation (8-66) is an infinite series of algebraic form which can be simplified by using the following relations:

$$\sum_{i=1}^{n} (2i-1) = n^2 \quad (8\text{-}67)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad (8\text{-}68)$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad (8\text{-}69)$$

$$\sum_{i=1}^{n} \frac{1}{1-\lambda} = \frac{n}{1-\lambda} \quad (8\text{-}70)$$

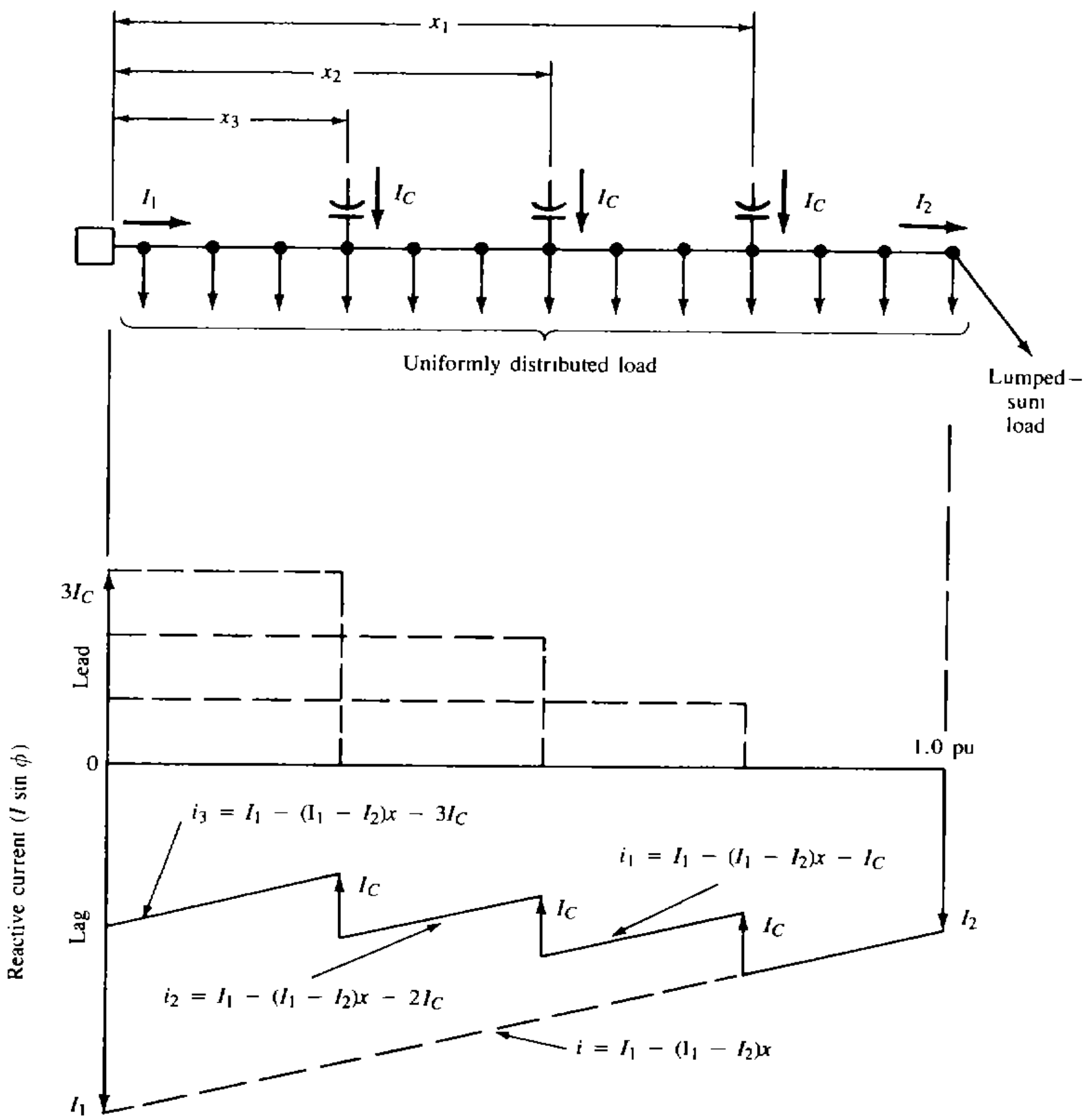


Figure 8-27 Loss reduction with three capacitor banks.

Therefore

$$\Delta P_{\mathrm{LS,\,opt}} = 3\alpha c\left[\frac{n}{1-\lambda} - \frac{n^2 c}{1-\lambda} + \frac{nc^2(n+1)(2n+1)}{6} - \frac{nc^2}{4(i-\lambda)} - \frac{nc^2(n+1)}{2(1-\lambda)}\right] \tag{8-71}$$

or

$$\Delta P_{\mathrm{LS,\,opt}} = \frac{3\alpha c}{1-\lambda}\left[n - cn^2 + \frac{c^2 n(4n^2-1)}{12}\right] \tag{8-72}$$

The capacitor compensation ratio at each location can be found by differentiating Eq. (8-72) with respect to c and setting it equal to zero as

$$c = \frac{2}{2n+1} \tag{8-73}$$

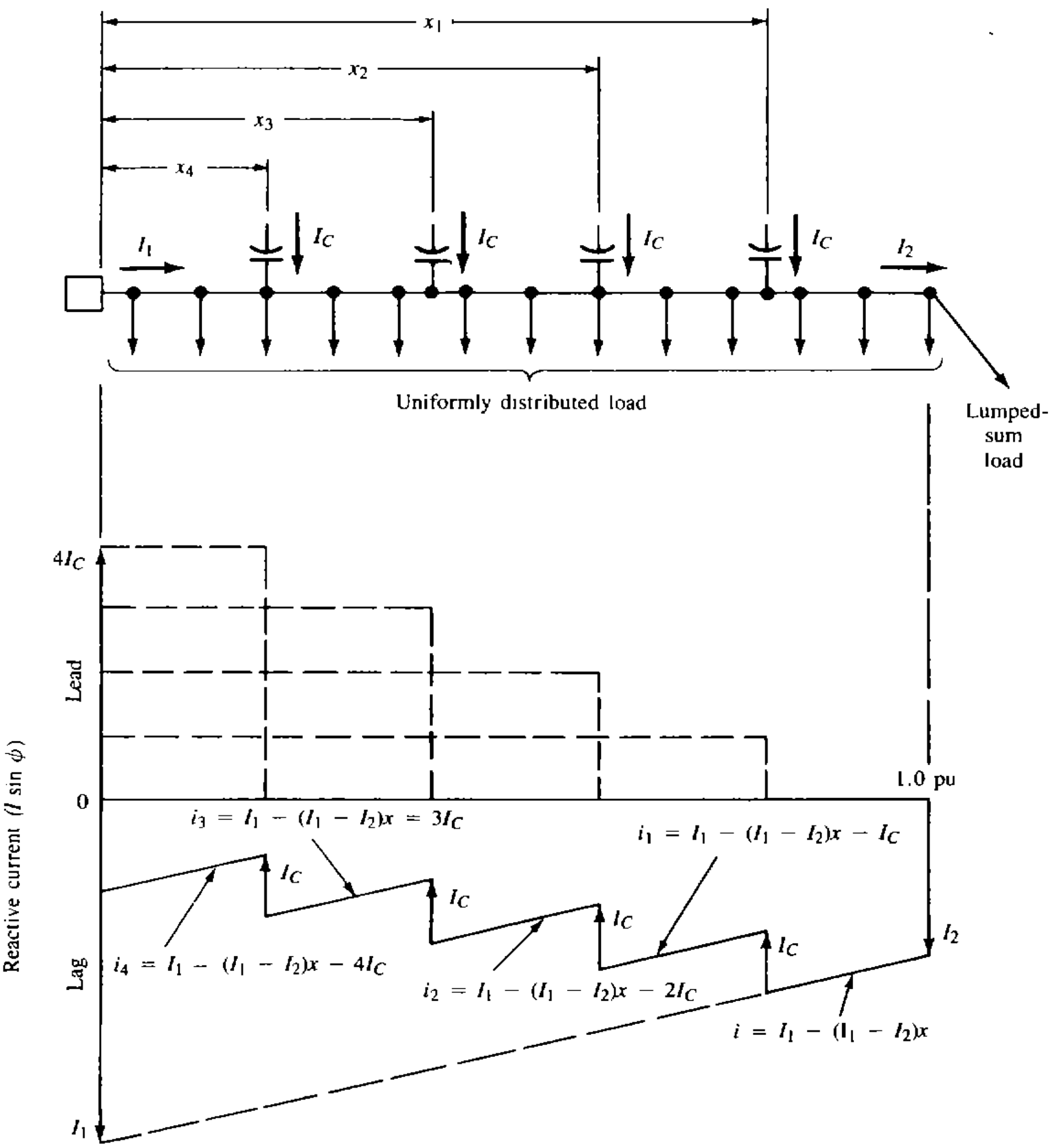


Figure 8-28 Loss reduction with four capacitor banks.

Equation (8-73) can be called the $2/(2n + 1)$ *rule*. For example, for $n = 1$, the capacitor rating is two-thirds of the total reactive load which is located at

$$x_1 = \frac{2}{3(1 - \lambda)} \tag{8-74}$$

of the distance from the source to the end of the feeder, and the peak loss reduction is

$$\Delta P_{\mathrm{LS,opt}} = \frac{8\alpha}{9(1 - \lambda)} \tag{8-75}$$

For a feeder with a uniformly distributed load, the reactive current at the end of the line is zero (that is, $I_2 = 0$); therefore

$$\lambda = 0 \qquad \text{and} \qquad \alpha = 1$$

Thus for the optimum loss reduction of

$$\Delta P_{\mathrm{LS,opt}} = \tfrac{8}{9}\ \mathrm{pu} \tag{8-76}$$

the optimum value of x_1 is

$$x_1 = \tfrac{2}{3}\ \mathrm{pu} \tag{8-77}$$

and the optimum value of c is

$$c = \tfrac{2}{3}\ \mathrm{pu} \tag{8-78}$$

Figure 8-29 gives a maximum loss reduction comparison for capacitor banks, with various total reactive compensation levels, and located optimally on a line segment which has uniformly distributed load ($\lambda = 0$), based on Eq. (8-72). The given curves are for one, two, three, and infinite number of capacitor banks. For example, from the curve given for one capacitor bank, it can be observed that a capacitor bank rated two-thirds of the total reactive load and located at two-thirds of the distance out on the feeder from the source provides for a loss reduction of 89 percent. In the case of two capacitor banks, with a four-fifths of total reactive compensation, located at four-fifths of the distance out on the feeder, the maximum loss reduction is 96 percent. Figure 8-30 gives similar curves for a combination of concentrated and uniformly distributed loads ($\lambda = \frac{1}{4}$).

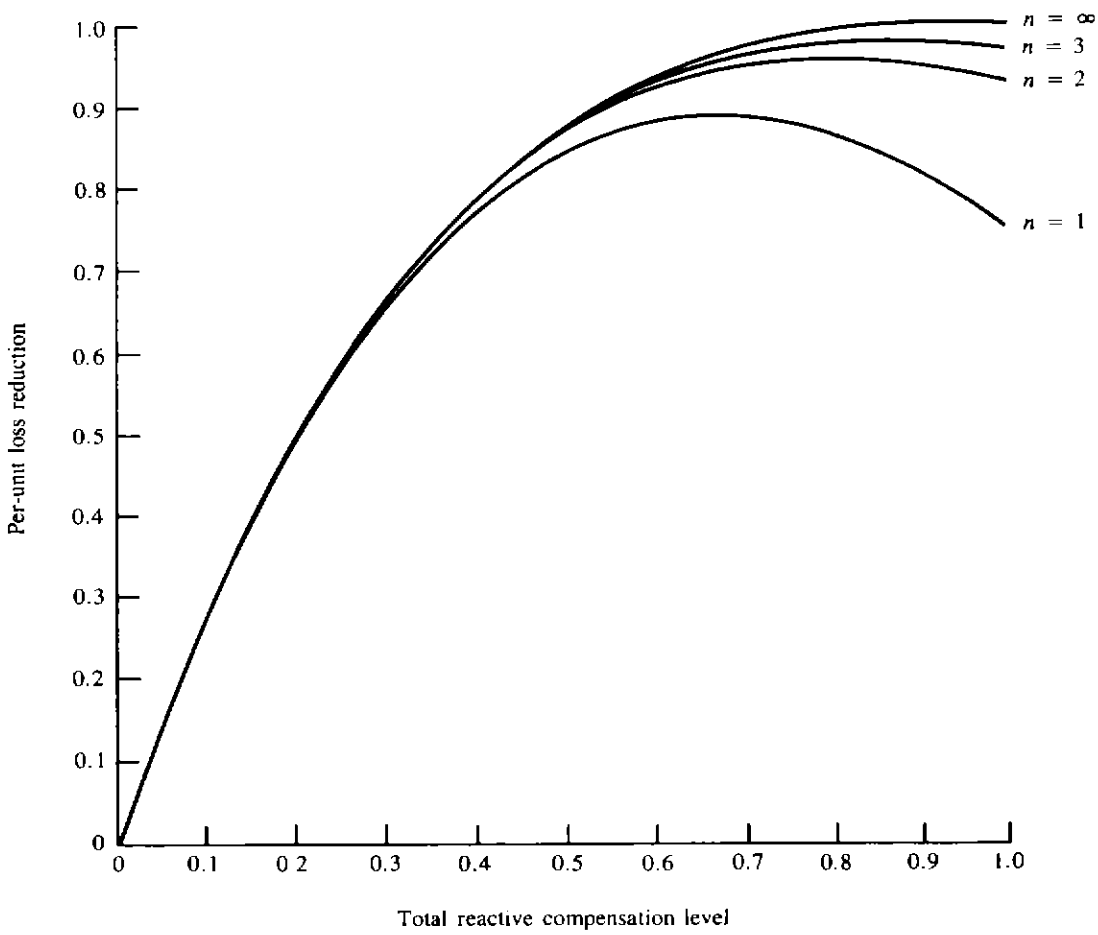


Figure 8-29 Comparison of loss reduction obtainable from $n = 1, 2, 3$, and ∞ number of capacitor banks, with $\lambda = 0$.

8-8-3 Energy Loss Reduction Due to Capacitors

The per unit energy loss reduction in a three-phase line segment with a combination of concentrated and uniformly distributed loads due to the allocation of fixed shunt capacitors is

$$\Delta \mathrm{EL} = 3\alpha c \sum_{i=1}^{n} x_i[(2 - x_i)F'_{\mathrm{LD}} + x_i \lambda F'_{\mathrm{LD}} - (2i - 1)c]T \qquad (8\text{-}79)$$

where F'_{LD} = reactive load factor $= Q/S$

T = total time period during which fixed-shunt-capacitor banks are connected

$\Delta\mathrm{EL}$ = energy loss reduction, pu

The optimum locations for the fixed shunt capacitors for the maximum energy loss reduction can be found by differentiating Eq. (8-79) with respect to x_i and setting the result equal to zero. Therefore

$$\frac{\partial(\Delta \mathrm{EL})}{\partial x_i} = 3\alpha c[2F'_{\mathrm{LD}}(\lambda - 1)x_i + 2F'_{\mathrm{LD}} - (2i - 1)c] \qquad (8\text{-}80)$$

$$\frac{\partial^2(\Delta \mathrm{EL})}{\partial x_i^2} = -2F'_{\mathrm{LD}}(1 - \lambda) < 0 \qquad (8\text{-}81)$$

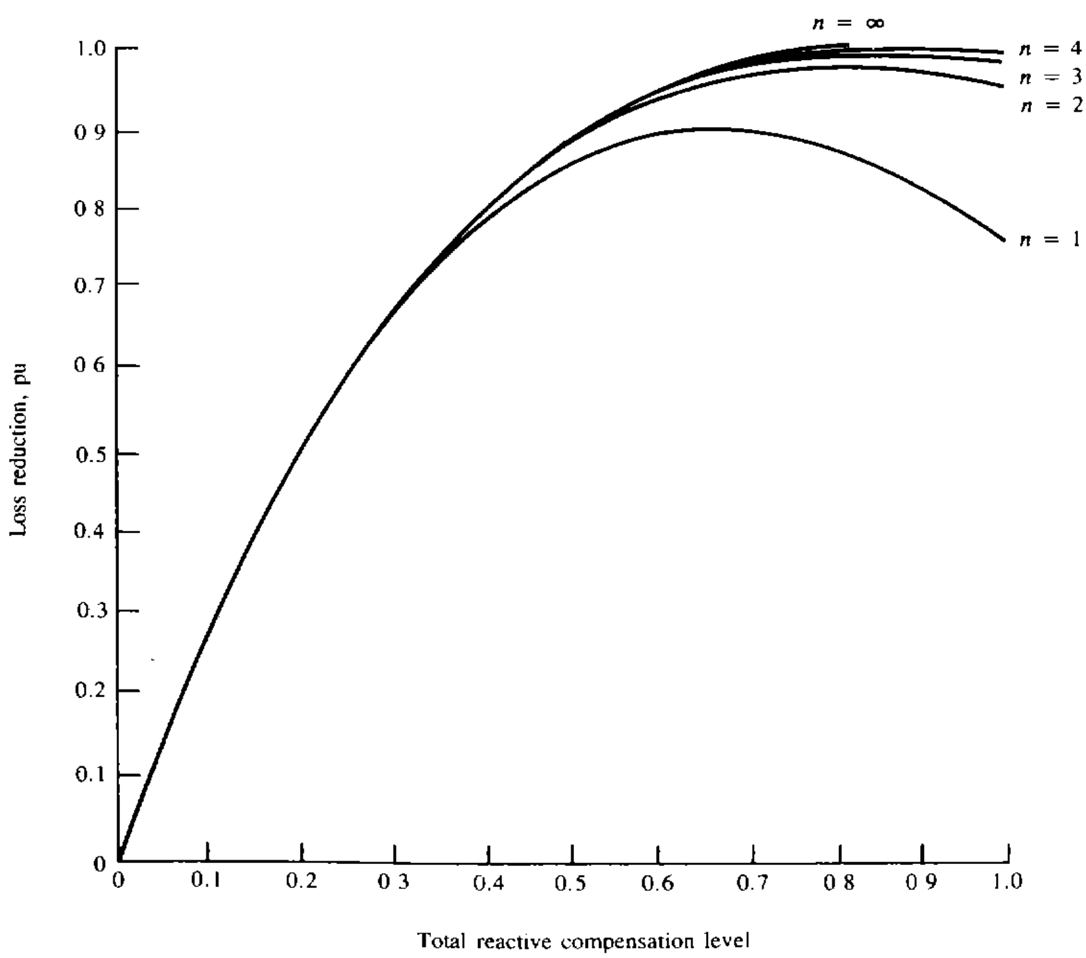


Figure 8-30 Comparison of loss reduction obtainable from $n = 1, 2, 3, 4$, and ∞ number of capacitor banks, with $\lambda = \frac{1}{4}$.

The optimum capacitor location for the maximum energy loss reduction can be found by setting Eq. (8-80) to zero, so that

$$x_{i,\,\mathrm{opt}} = \frac{1}{1-\lambda} - \frac{(2i-1)c}{2(1-\lambda)F'_{\mathrm{LD}}} \tag{8-82}$$

Similarly, the optimum total capacitor rating can be found as

$$C_T = \frac{2n}{2n+1}F'_{\mathrm{LD}} \tag{8-83}$$

From Eq. (8-83), it can be observed that if the total number of capacitor banks approaches infinity, then the optimum total capacitor rating becomes equal to the reactive load factor.

If only one capacitor bank is used, the optimum capacitor rating to provide for the maximum energy loss reduction is

$$C_T = \tfrac{2}{3}F'_{\mathrm{LD}} \tag{8-84}$$

This equation gives the well-known *two-thirds rule* for fixed shunt capacitors. Figure 8-31 shows the relationship between the total capacitor compensation ratio and the reactive load factor, in order to achieve maximum energy loss reduction, for a line segment with uniformly distributed load where $\lambda = 0$ and $\alpha = 1$.

By substituting Eq. (8-82) into Eq. (8-79), the optimum energy loss reduction can be found as

$$\begin{aligned}
\Delta\mathrm{EL}_{\mathrm{opt}} &= \frac{3\alpha c}{1-\lambda}\left[nF'_{\mathrm{LD}} - cn^2 + \frac{c^2 n(4n^2-1)}{12F'_{\mathrm{LD}}}\right]T \\
&= \frac{3\alpha cn}{1-\lambda}\left[F'_{\mathrm{LD}} - cn + \frac{c^2 n^2(4n^2-1)}{12n^2F'_{\mathrm{LD}}}\right]T \\
&= \frac{3\alpha C_T}{1-\lambda}\left[F'_{\mathrm{LD}} - C_T + \frac{C_T^2(4n^2-1)}{12n^2F'_{\mathrm{LD}}}\right]T
\end{aligned} \tag{8-85}$$

where C_T = total reactive compensation level = cn.

Based on Eq. (8-85), the optimum energy loss reductions with any size capacitor bank located at the optimum location for various reactive load factors have been calculated, and the results have been plotted on Fig. 8-32 to 8-36. It is important to note the fact that, for all values of λ, when reactive load factors are 0.2 or 0.4, the use of a fixed-capacitor bank with corrective ratios of 0.4 and 0.8, respectively, gives a zero energy loss reduction.

Figures 8-37 to 8-41 show the effects of various reactive load factors on the maximum energy loss reductions for a feeder with different load patterns.

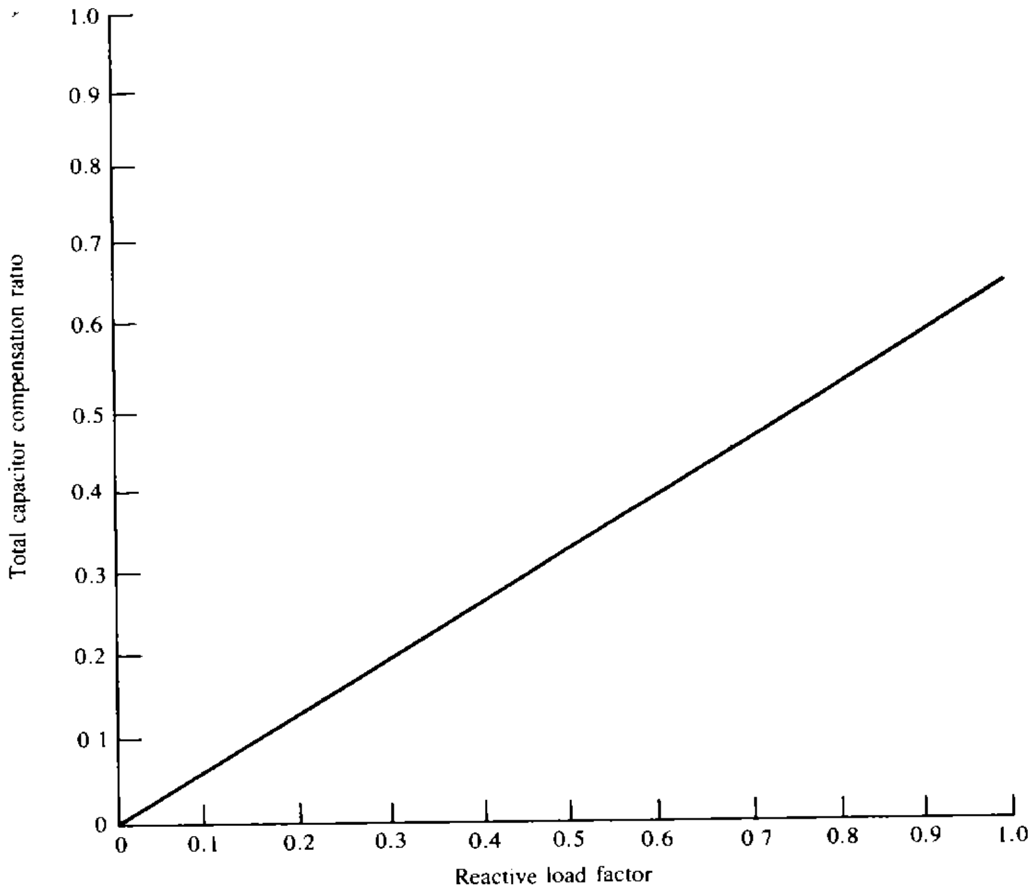


Figure 8-31 Relationship between the total capacitor compensation ratio and the reactive load factor for uniformly distributed load ($\lambda = 0$ and $\alpha = 1$).

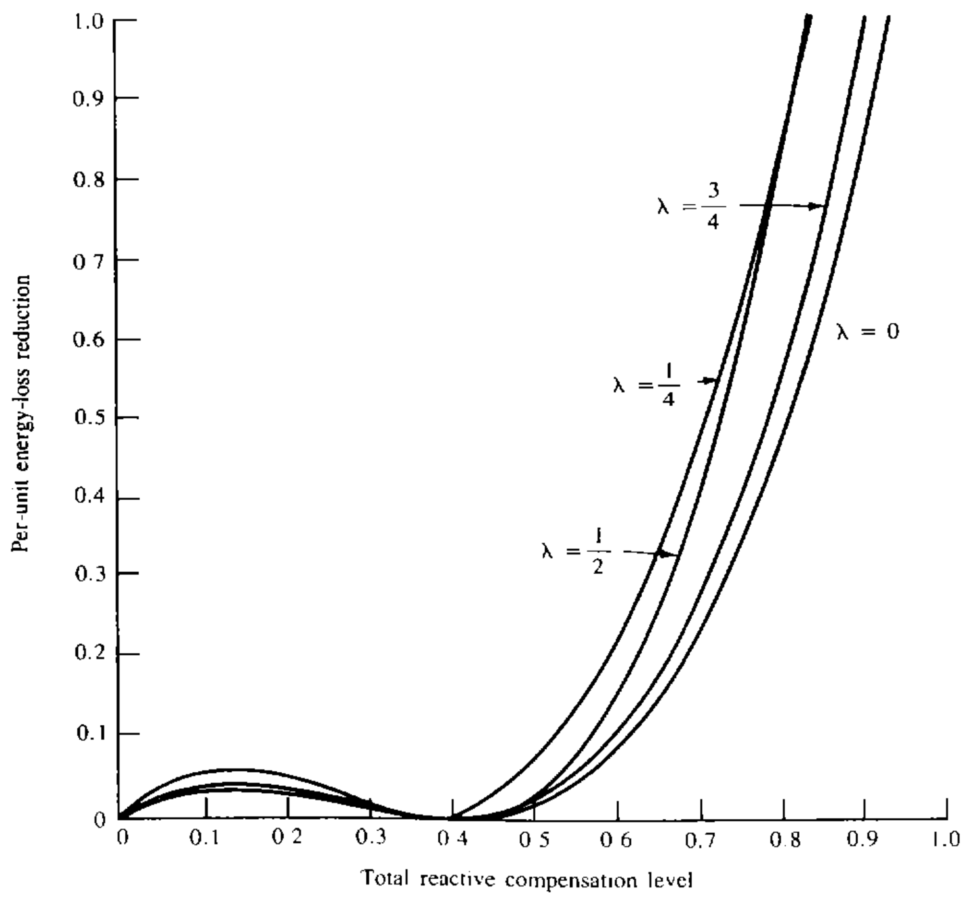


Figure 8-32 Energy loss reduction with any capacitor-bank size, located at optimum location $F'_{LD} = 0.2$).

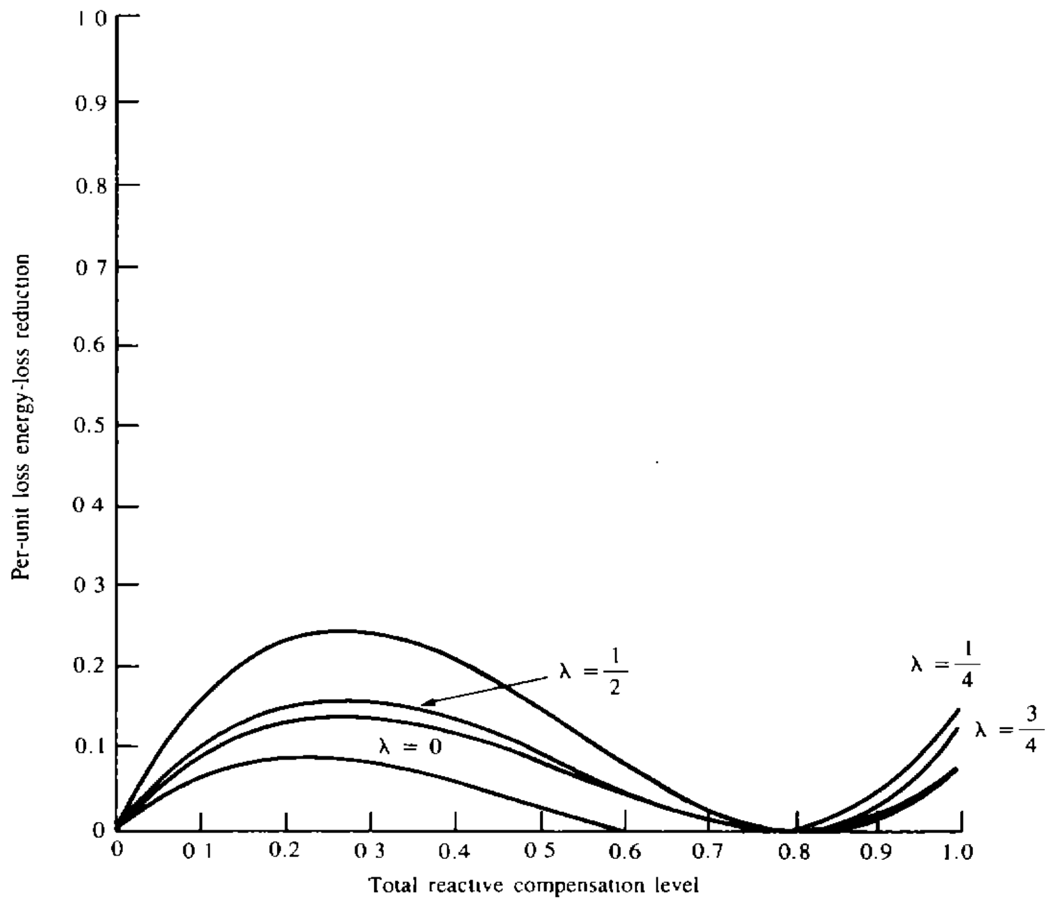


Figure 8-33 Energy loss reduction with any capacitor-bank size, located at the optimum location ($F'_{LD} = 0.4$).

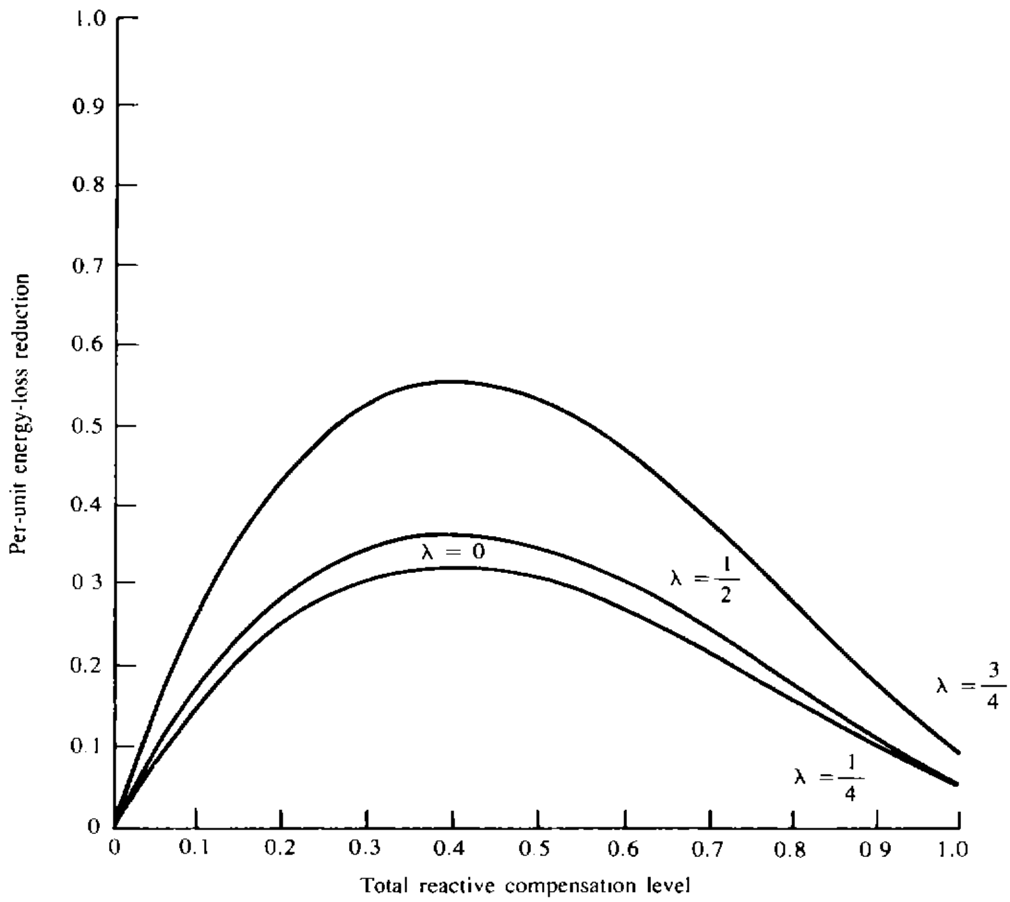


Figure 8-34 Energy loss reduction with any capacitor-bank size, located at the optimum location ($F'_{LD} = 0.6$).

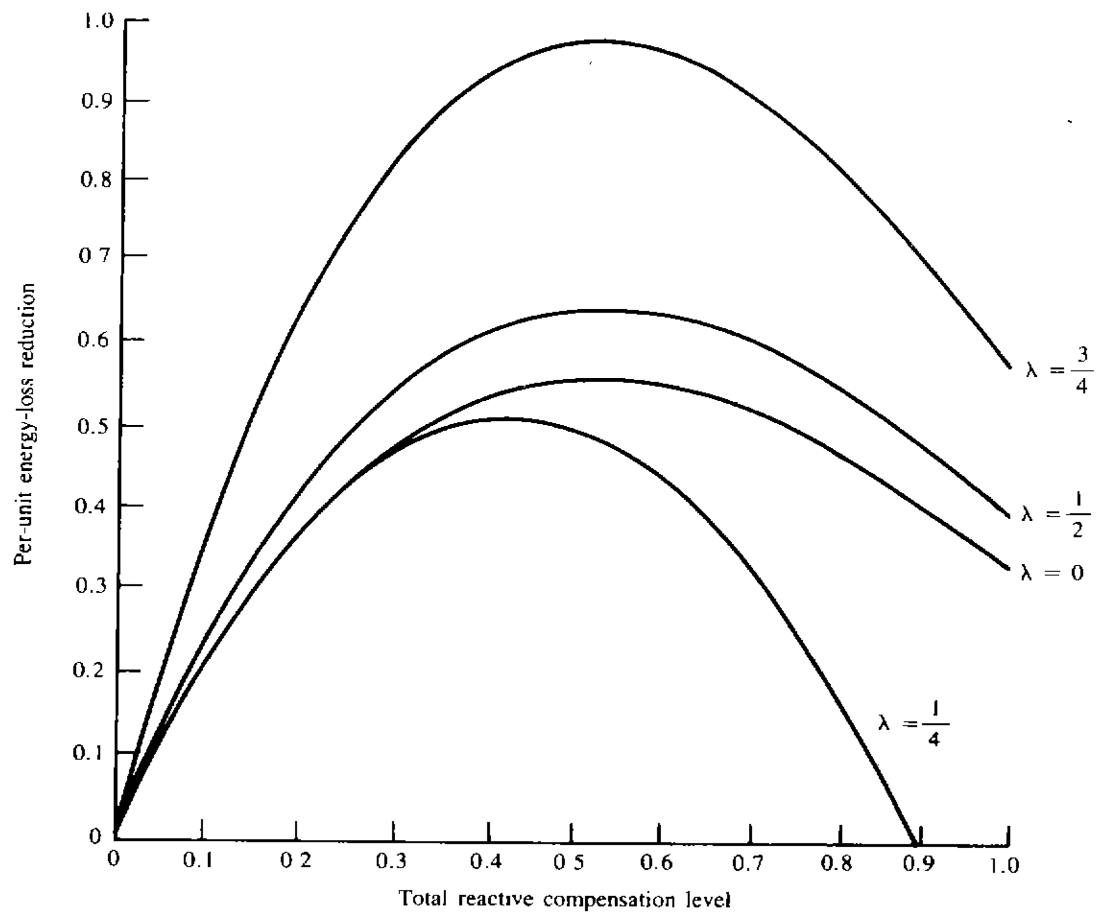


Figure 8-35 Energy loss reduction with any capacitor-bank size, located at the optimum location ($F'_{LD} = 0.8$).

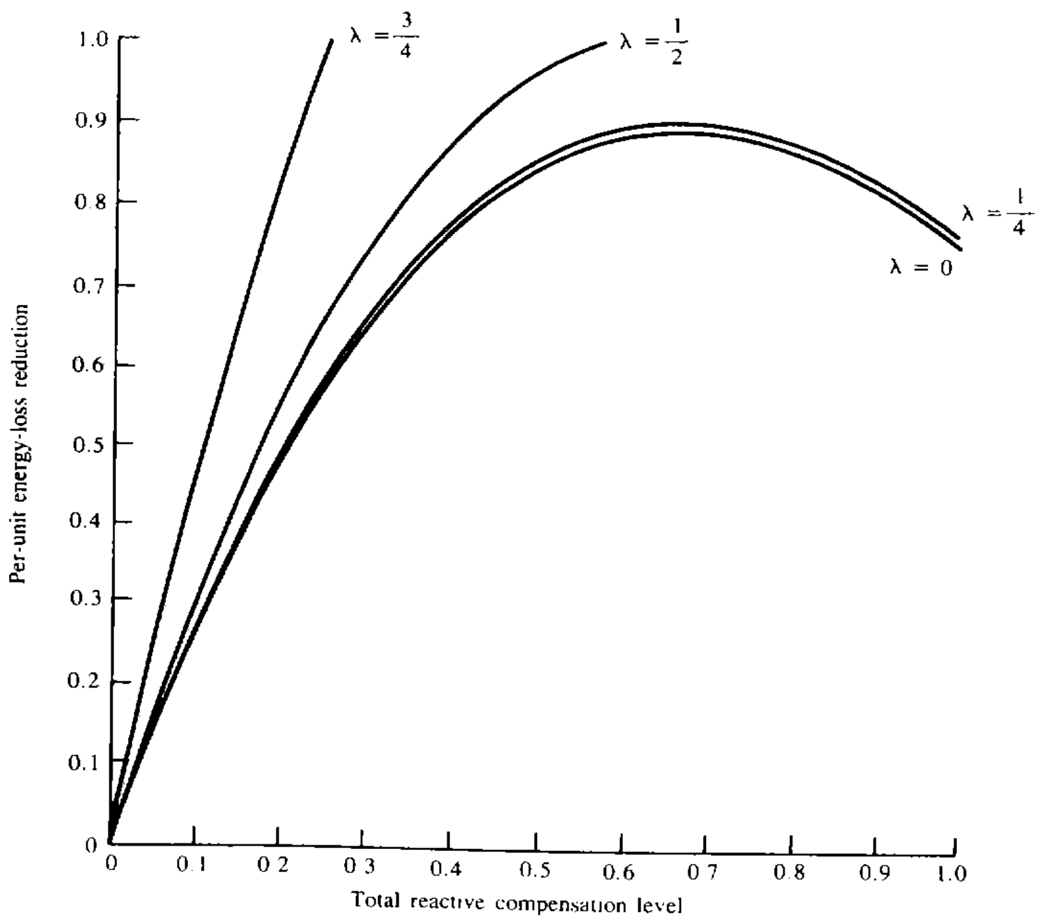


Figure 8-36 Energy loss reduction with any capacitor-bank size, located at the optimum location ($F'_{LD} = 1.0$).

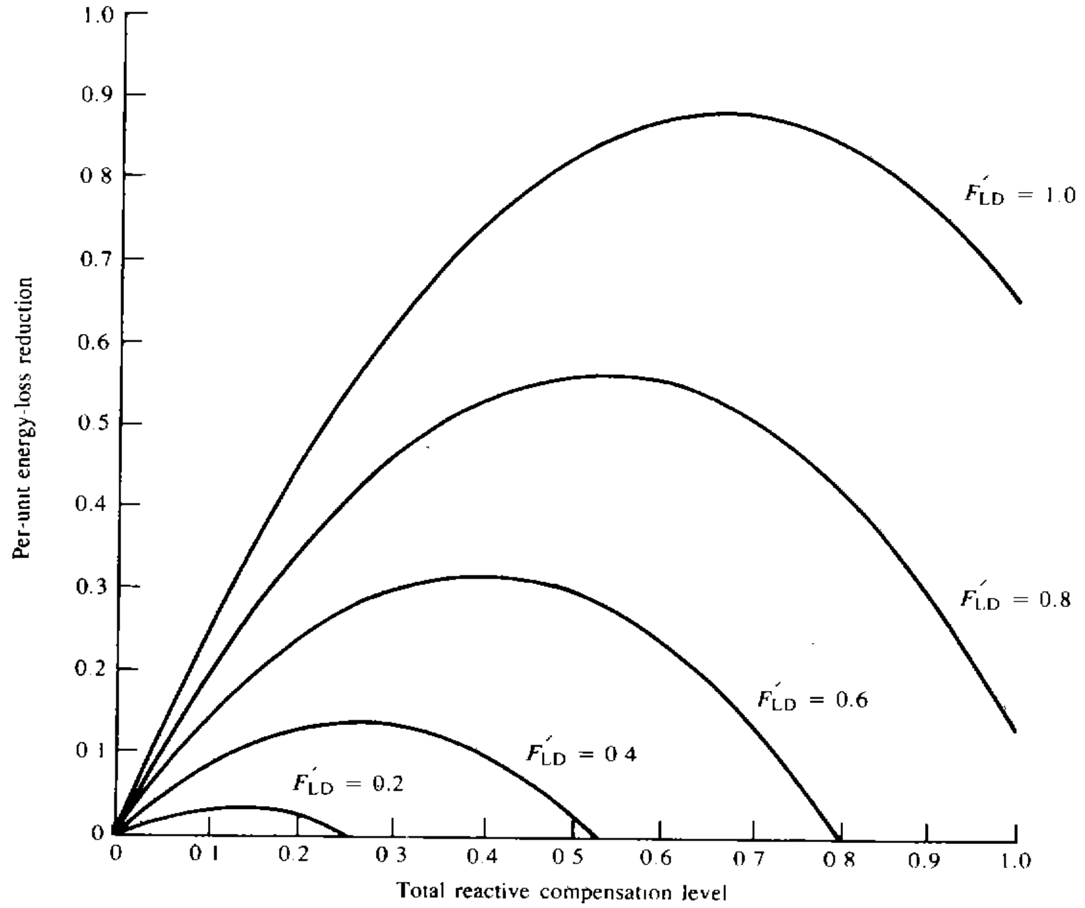


Figure 8-37 Effects of reactive load factors on energy loss reduction due to capacitor-bank installation on a line segment with uniformly distributed load ($\lambda = 0$).

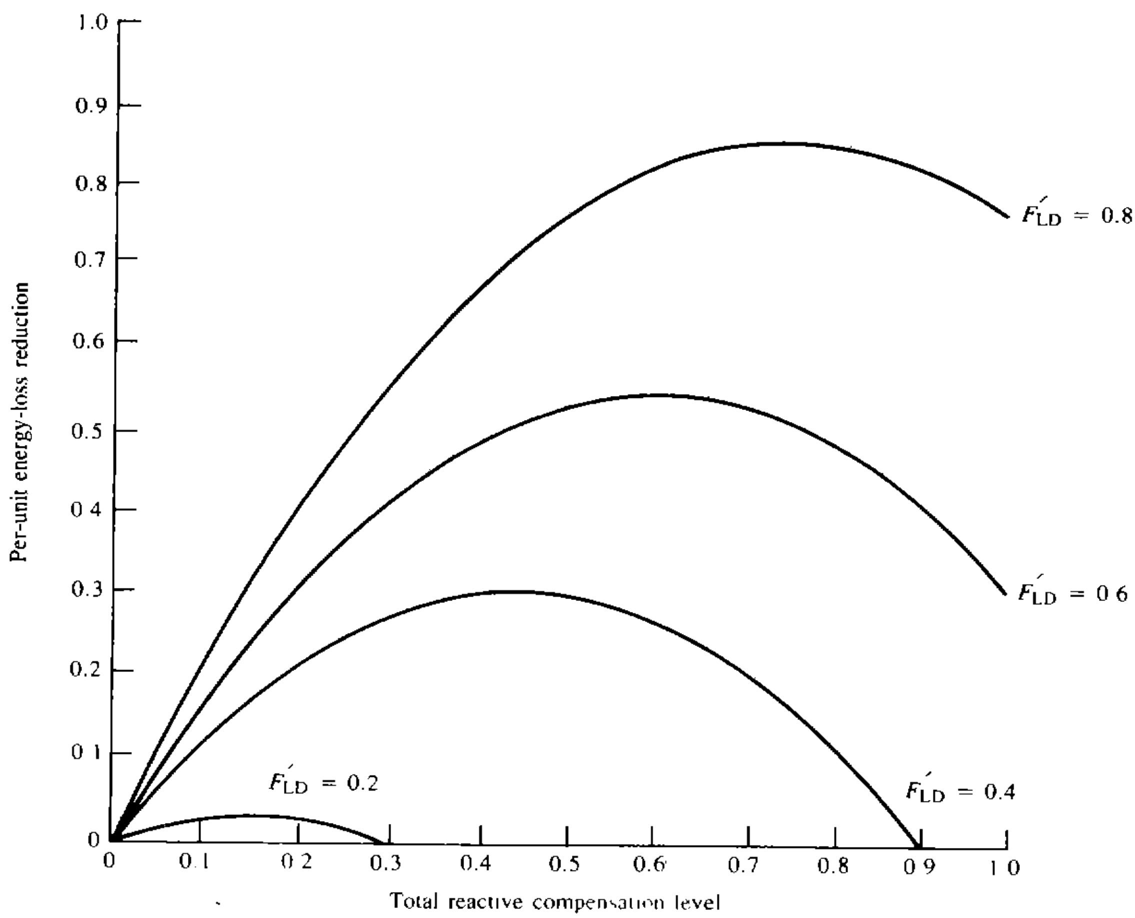


Figure 8-38 Effects of reactive load factors on energy loss reduction due to capacitor-bank installation on a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{1}{4}$).

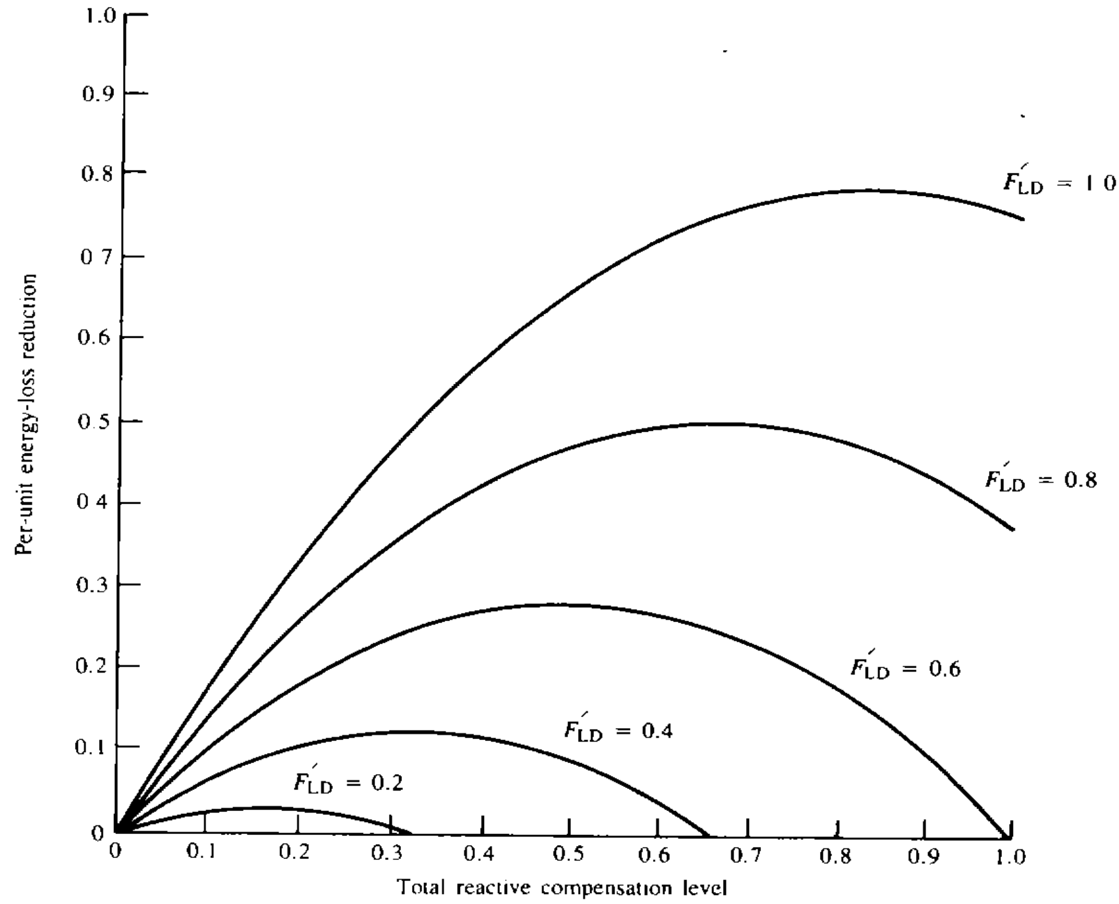


Figure 8-39 Effects of reactive load factors on energy loss reduction due to capacitor-bank installation on a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{1}{2}$).

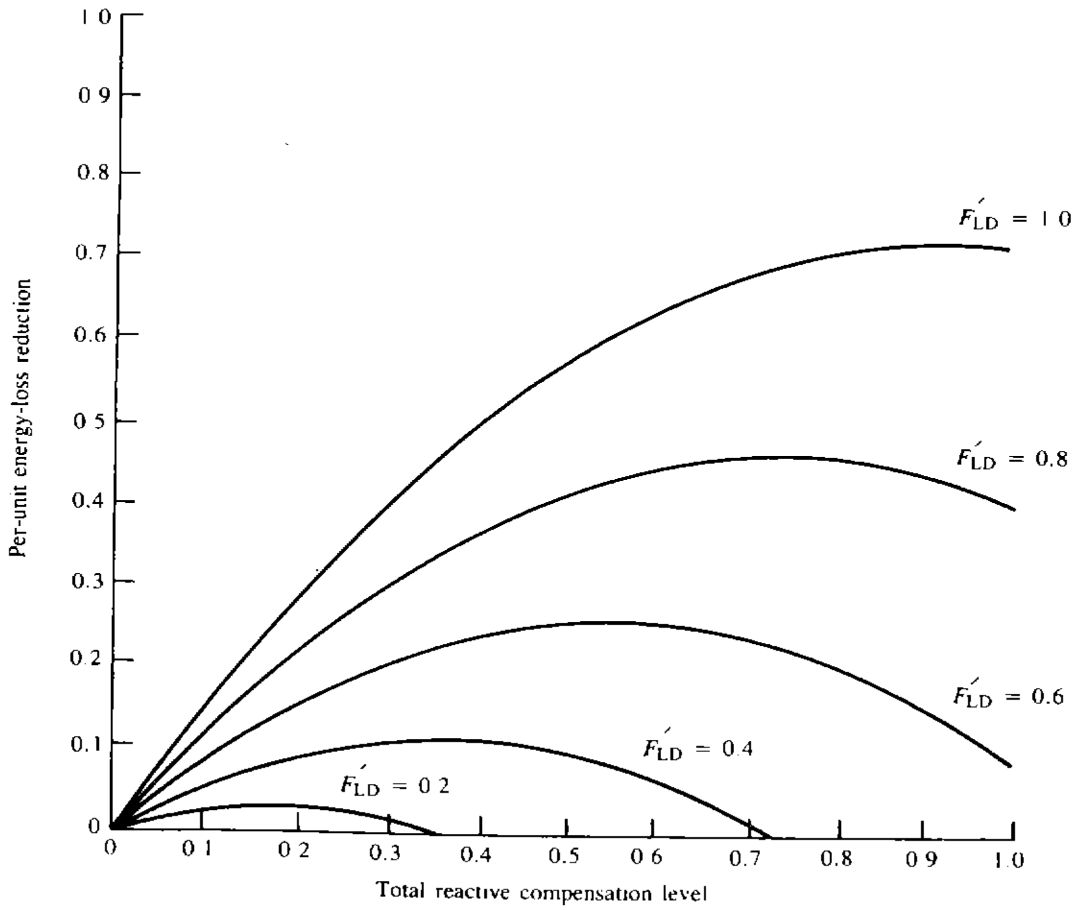


Figure 8-40 Effects of reactive load factors on loss reduction due to capacitor-bank installation on a line segment with a combination of concentrated and uniformly distributed loads ($\lambda = \frac{3}{4}$).

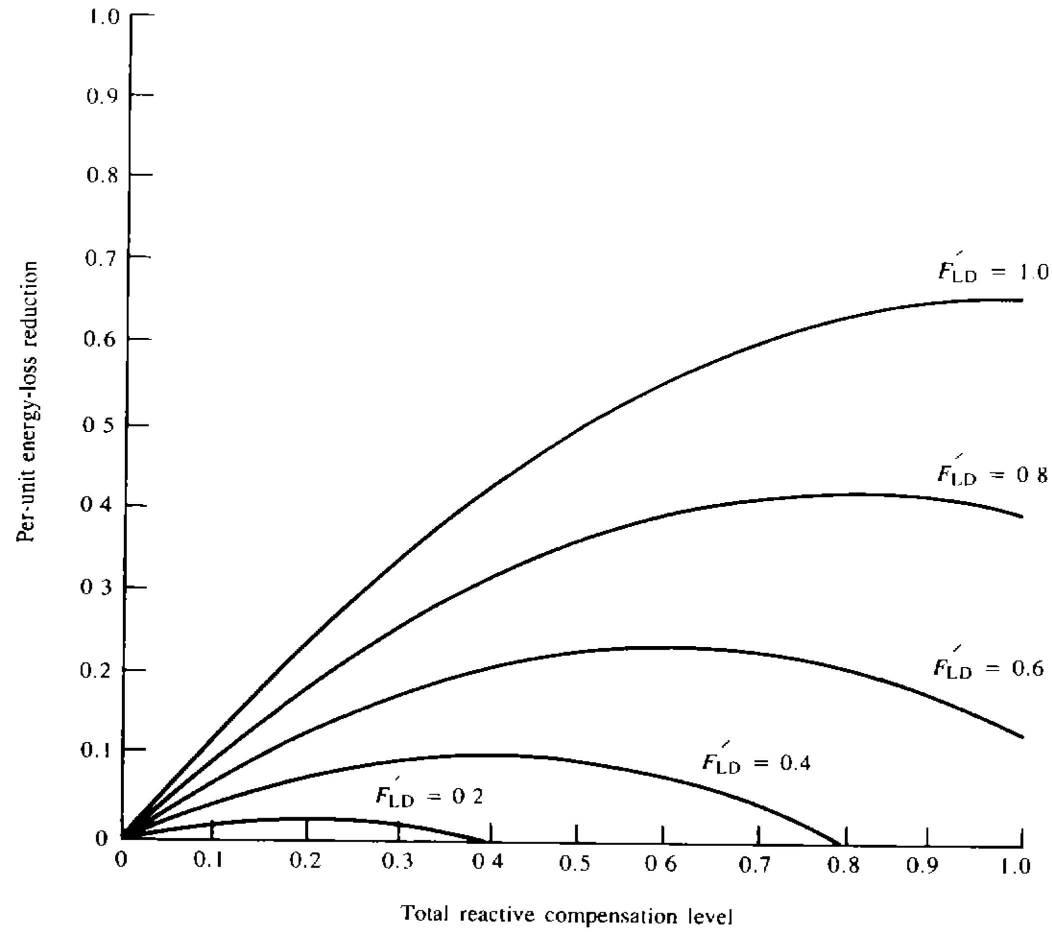


Figure 8-41 Effects of reactive load factors on energy loss reduction due to capacitor-bank installation on a line segment with a concentrated load ($\lambda = 1$).

8-8-4 Relative Ratings of Multiple Fixed Capacitors

The total savings due to having two fixed-shunt-capacitor banks located on a feeder with uniformly distributed load can be found as

$$\sum \$ = 3c_1\left(1 - c_1 + \frac{c_1^2}{4}\right)K_2 + 3c_2\left(1 - c_2 + \frac{c_2^2}{4}\right)K_2 + 3c_1\left(F'_{\text{LD}} - c_1 + \frac{c_1^2}{4F'_{\text{LD}}}\right)K_1 T$$
$$+ 3c_2\left(F'_{\text{LD}} - c_2 + \frac{c_2^2}{4F'_{\text{LD}}}\right)K_1 T \quad (8\text{-}86)$$

or

$$\sum \$ = 3\left[(c_1 + c_2)(K_1 + K_2 T F'_{\text{LD}}) - (c_1^2 + c_2^2)(K_2 + K_1 T) + \frac{1}{4}(c_1^3 + c_2^3)\left(K_2 + \frac{K_1 T}{F'_{\text{LD}}}\right)\right] \quad (8\text{-}87)$$

where K_1 = a constant to convert energy loss savings to dollars, \$/kWh
K_2 = a constant to convert power loss savings to dollars, \$/kWh

Since the total capacitor-bank rating is equal to the sum of the ratings of the capacitor banks,

$$C_T = c_1 + c_2 \tag{8-88}$$

or

$$c_1 = C_T - c_2 \tag{8-89}$$

By substituting Eq. (8-89) into Eq. (8-87),

$$\sum \$ = 3\left[C_T(K_1 + K_2 T F'_{\text{LD}}) - (C_T^2 + 2c_2^2 - 2c_2 C_T)(K_1 T + K_2) + \frac{1}{4}(C_T^3 - 3c_2 C_T^2 + 3c_2^2 C_T)\left(K_2 + \frac{K_1 T}{F'_{\text{LD}}}\right)\right] \tag{8-90}$$

The optimum rating of the second fixed-capacitor bank as a function of total capacitor-bank rating can be found by differentiating Eq. (8-90) with respect to c_2, so that

$$\frac{\partial(\sum \$)}{\partial c_2} = -3(4c_2 - 2C_T)(K_2 + K_1 T) + \frac{3}{4}\left(-3C_T^2 + 6c_2 C_T\right)\left(K_2 + \frac{K_1 T}{F'_{\text{LD}}}\right) \tag{8-91}$$

and setting the resultant equation equal to zero,

$$2c_2 = C_T \tag{8-92}$$

and since

$$C_T = c_1 + c_2 \tag{8-93}$$

then

$$c_1 = c_2 \tag{8-94}$$

The result shows that if multiple fixed-shunt-capacitor banks are to be employed on a feeder with uniformly distributed loads, in order to receive the maximum savings all capacitor banks should have the same rating.

8-8-5 General Savings Equation for Any Number of Fixed Capacitors

From Eq. (8-64) and (8-79), the total savings equation in a three-phase primary feeder with a combination of concentrated and uniformly distributed loads can be found as

$$\sum \$ = 3K_1 \alpha c \sum_{i=1}^{n} x_i[(2 - x_i)F'_{\text{LD}} + x_i \lambda F'_{\text{LD}} - (2i - 1)c]T + 3K_2 \alpha c \sum_{i=1}^{n} x_i[(2 - x_i) + x_i \lambda - (2i - 1)c] - K_3 C_T \tag{8-95}$$

where K_1 = a constant to convert energy loss savings to dollars, \$/kWh
K_2 = a constant to convert power loss savings to dollars, \$/kWh
K_3 = a constant to convert total fixed-capacitor ratings to dollars, \$/kvar
x_i = ith capacitor location, pu length
n = total number of capacitor banks
F_{LD} = reactive load factor
C_T = total reactive compensation level
c = capacitor compensation ratio at each location
λ = ratio of reactive current at end of line segment to reactive load current at beginning of line segment

$$\alpha = \frac{1}{1 + \lambda + \lambda^2}$$

T = total time period during which fixed-shunt-capacitor banks are connected

By taking the first- and second-order partial derivatives of Eq. (8-95) with respect to x_i,

$$\frac{\partial(\sum \$)}{\partial x_i} = 3\alpha c[2x_i(K_2 + K_1 T F'_{LD})(\lambda - 1) + 2(K_2 + K_1 T F'_{LD}) - (2i - 1)c(K_2 + K_1 T)] \quad (8\text{-}96)$$

and

$$\frac{\partial^2(\sum \$)}{\partial x_i^2} = -6\alpha c(1 - \lambda)(K_2 + K_1 T F'_{LD}) < 0 \quad (8\text{-}97)$$

Setting Eq. (8-96) equal to zero, the optimum location for any fixed-capacitor bank with any rating can be found as

$$x_i = \frac{1}{1 - \lambda} - \frac{(2i - 1)c}{1 - \lambda} \frac{K_2 + K_1 T}{K_2 + K_1 T F'_{LD}} \quad (8\text{-}98)$$

where $0 \le x_i \le 1.0$ pu length. Setting the capacitor bank anywhere else on the feeder would decrease rather than increase the savings from loss reduction.

Some of the cardinal rules that can be derived for the application of capacitor banks include the following:

1. The location of fixed shunt capacitors should be based on the average reactive load.
2. There is only one location for each size of capacitor bank that produces maximum loss reduction.
3. One large capacitor bank can provide almost as much savings as two or more capacitor banks of equal size.
4. When multiple locations are used for fixed-shunt-capacitor banks, the banks should have the same rating to be economical.

5. For a feeder with a uniformly distributed load, a fixed-capacitor bank rated at two-thirds of the total reactive load and located at two-thirds of the distance out on the feeder from the source gives an 89 percent loss reduction.
6. The result of the two-thirds rule is particularly useful when the reactive load factor is high. It can be applied only when fixed shunt capacitors are used.
7. In general, particularly at low reactive load factors, some combination of fixed and switched capacitors gives the greatest energy loss reduction.
8. In actual situations, it may be difficult, if not impossible, to locate a capacitor bank at the optimum location; in such cases the permanent location of the capacitor bank ends up being suboptimum.

8-9 CAPACITOR TANK-RUPTURE CONSIDERATIONS

When the total energy input to the capacitor is larger than the strength of the tank's envelope to withstand such input, the tank of the capacitor ruptures. This energy input could happen under a wide range of current–time conditions. Through numerous testing procedures, capacitor manufacturers have generated tank-rupture curves as a function of fault current available. The resulting tank-rupture time–current characteristic curves with which fuse selection is coordinated have furnished comparatively good protection against tank rupture. Figure 8-42 shows the results of tank-rupture tests conducted on all-film capacitors. Figure 8-43 shows the capacitor reliability cycle during its lifetime. The longer it takes for the dielectric material to wear out due to the forces generated by the combination of electric stress and temperature, the greater is its reliability. In other words, the

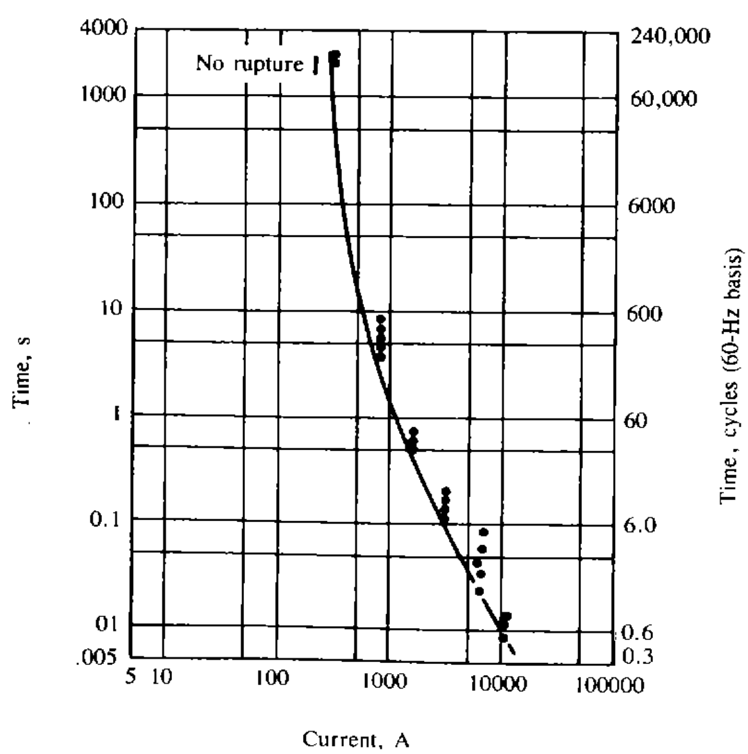


Figure 8-42 Time-to-rupture characteristics for 200-kvar 7.2-kV all-film capacitors. (*McGraw-Edison Company.*)

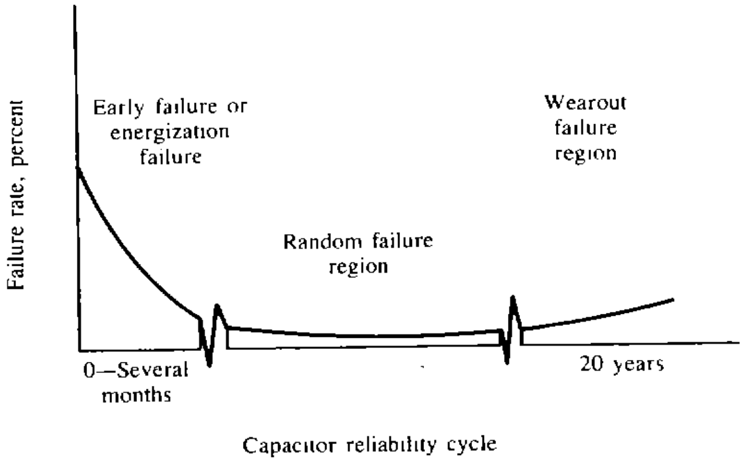


Figure 8-43 Capacitor reliability cycle. (*McGraw-Edison Company.*)

wear-out process or time to failure is a measure of life and reliability.

Currently, there are numerous methods that can be used to detect the capacitor tank ruptures. Burrage [19] categorizes them as:

1. Detection of sound produced by the rupture
2. Observation of smoke and/or vapor from the capacitor tank upon rupture
3. Observation of ultraviolet light generated by the arc getting outside the capacitor tank
4. Measurement of the change in arc voltage when the capacitor tank is breached
5. Detection of a sudden reduction in internal pressure
6. Measurement of the distortion generated by gas pressure within the capacitor tank

8-10 DYNAMIC BEHAVIOR OF DISTRIBUTION SYSTEMS

The characteristics of the distribution system dynamic behavior include (1) fault effects and transient recovery voltage, (2) switching and lightning surges, (3) inrush and cold-load current transients, (4) ferroresonance, and (5) harmonics.

In the event of a fault on a distribution system, there will be a substantial change in current magnitude on the faulted phase. It is also possible for the current to have a dc offset which is a function of the voltage wave at the time of the fault and the X/R ratio of the circuit. This may cause a voltage rise on the unfaulted phases due to neutral shift which results in saturation of transformers and increased load current magnitudes. On the other hand, it may cause a reduction in voltage and load current on the faulted phase. When a circuit recloser clears the fault at a current zero, a higher-frequency transient voltage is superimposed on the power-frequency recovery voltage. The resultant voltage is called the *transient recovery voltage* (TRV). It is possible to have its crest magnitudes be two or three times nominal system voltage. This may cause failure to clear or restrikes which may produce substantial switching surges.

Switching surges are generated when loads, station capacitor banks, or feeders are energized or deenergized; or when faults are initiated, cleared, and reinitiated. The factors affecting the magnitude and duration of the resultant voltage transients include (1) the system impedance characteristics, (2) the amount of capacitive kilovars connected at the time of switching, (3) the location of the capacitor bank on the system, (4) the type of breaker, and (5) the breaker pole-closing angles. In general, the switching surges on distribution systems have not been taken seriously so far. However, if the current trend toward higher distribution voltage levels and reduced insulation levels continues, this may change. But voltage surges resulting from lightning strokes to the distribution line will usually require the most severe design requirements. The factors affecting the lightning surge include (1) the system configuration and the system grounding and shielding, (2) the stroke characteristics and stroke location, (3) the sparkover of arresters remote from the converters, (4) the amount of the connected capacitive kilovars in the surge path, and (5) the loss mechanism (corona, skin effect) in the surge path.

The energization of motors, transformers, capacitors, feeders, and loads generate current transients. For example, when motors and other loads draw high starting currents, capacitors draw a high-frequency inrush based on the instantaneous voltage and the circuit inductance as well as the capacitance, whereas in a transformer the magnitude of this inrush depends upon the voltage wave at the time of energization and the residual flux in the core. It is important to recognize the fact that low voltage during inrush can harm equipment involved and stop the circuit from recovering without sectionalizing. Furthermore, protective devices may operate incorrectly or not operate due to the high-magnitude and high-frequency currents.

8-10-1 Ferroresonance

Ferroresonance is an oscillatory phenomenon caused by the interaction of system capacitance with the nonlinear inductance of a transformer. These capacitive and inductive elements make a series-resonant circuit that can generate high transient or sustained overvoltages which can damage system equipment. These overvoltages are more likely to take place where a considerable length of cable is connected to an overloaded three-phase transformer (or bank) and single-phase switching is done at a point remote from the transformer (e.g., riser pole). Serious damage to equipment may be prevented by recognizing the conditions which increase the probability of these overvoltages and taking appropriate preventive measures. The more serious overvoltages may be evidenced by (1) flashover or damage to lightning arresters, (2) transformer humming with only one phase closed, (3) damage to transformers and other equipment, (4) three-phase motor reversals, and (5) high secondary voltages.

Even though the ferroresonant phenomenon has been recognized for some time, until recently it has not been considered as a serious operating problem on electric distribution systems. Changes in the characteristics of distribution systems and in transformer design have resulted in the increased probability of ferroresonant overvoltages when switching three-phase transformer installations. For example, the capacitance of cable is much greater (i.e., capacitive reactance lower) than that of open wire, and present trends are toward a greater use of underground cables due to the esthetic considerations. Also, system operation at higher than nominal voltages and trends in transformer design have led to the operation of distribution transformer cores at higher saturation. Furthermore, the use of higher distribution voltages results in distribution transformers with greater magnetizing reactance. At the same time, underground system capacitance will be greater (capacitive reactance lower).

Consider the LC circuit shown in Fig. 8-44. Note that the resistance is neglected for the sake of simplicity. If the inductive reactance X_L of the inductor is equal in magnitude to the capacitive reactance X_C of the capacitor, the circuit is in resonance. Of course, the voltage E across the inductor is 180° out of phase with the voltage E_C across the capacitor. The voltages E_L and E_C can be expressed as

$$E_L = \frac{E}{jX_L - jX_C}(jX_L)$$

$$= \frac{E}{1 - X_C/X_L} \tag{8-99}$$

and

$$E_C = \frac{E}{jX_L - jX_C}(-jX_C)$$

$$= \frac{E}{1 - X_L/X_C} \tag{8-100}$$

For the purpose of illustration assume that $X_L/X_C = 0.9$, and therefore $X_C/X_L = 1.1111$. Thus the voltages E_L and E_C can be found as

$$E_L = \frac{E}{1 - X_C/X_L} = \frac{E}{1 - 1.1111} = -9E$$

Figure 8-44

and

$$E_C = \frac{E}{1 - X_L/X_C} = \frac{E}{1 - 0.9} = 10E$$

Therefore in this case the voltage across the capacitor is 10 times the source voltage. The nearer the circuit is to actual resonance the greater will be the overvoltage.

Although the above is a relatively simple example of a resonant circuit, the basic concept is very similar to ferroresonance with one notable exception. In a ferroresonant circuit the capacitor is in series with a nonlinear (iron-core) inductor. A plot of the voltampere or impedance characteristic of an iron-core reactor would have the same general shape as the *BH* curve of the iron core. If the iron-core reactor is operating at a point near saturation, a small increase in voltage can cause a large decrease in the effective inductive reactance of the reactor. Therefore the value of inductive reactance can vary widely and resonance can occur over a range of capacitance values. The effects of ferroresonance can be minimized by such measures as:

1. Using grounded-wye–grounded-wye transformer connection
2. Using open-wye–open-delta transformer connection
3. Using switches rather than fuses at the riser pole
4. Using single-pole devices only at the transformer location and three-pole devices for remote switching
5. Avoiding switching an unloaded transformer bank at a point remote from the transformers
6. Keeping X_C/X_M ratios high (10 or more)
7. Installing neutral resistance
8. Using dummy loads to suppress ferroresonant overvoltages
9. Assuring load is present during switching
10. Using larger transformers
11. Limiting the length of cable serving the three-phase installation
12. Using only three-phase switching and sectionalizing devices at the terminal pole
13. Temporarily grounding the neutral of a floating-wye primary during switching operations

8-10-2 Harmonics on Distribution Systems

The power industry has recognized the problem of power system harmonics since the 1920s when distorted voltage and current waveforms were observed on power lines. However, the levels of harmonics on distribution systems have generally been insignificant in the past. Today, it is obvious that the levels of harmonic voltages and currents on distribution systems are becoming a serious problem. Some of the most important power system operational problems caused by harmonics have been reported to include the following [25]:

1. Capacitor-bank failure from dielectric breakdown or reactive power overload
2. Interference with ripple control and power-line carrier systems, causing misoperation of systems which accomplish remote switching, load control, and metering
3. Excessive losses in—and heating of—induction and synchronous machines
4. Overvoltages and excessive currents on the system from resonance to harmonic voltages or currents on the network
5. Dielectric instability of insulated cables resulting from harmonic overvoltages on the system
6. Inductive interference with telecommunication systems
7. Errors in induction watthour meters
8. Signal interference and relay malfunction, particularly in solid-state and microprocessor-controlled systems
9. Interference with large motor controllers and power plant excitation systems (reported to cause motor problems as well as nonuniform output)

These effects depend, of course, on the harmonics source, its location on the power system, and the network characteristics that promote propagation of harmonics.

There are numerous sources of harmonics. In general, the harmonics sources can be classified as (1) previously known harmonics sources and (2) new harmonics sources. The previously known harmonics sources include.

1. Tooth ripples or ripples in the voltage waveform of rotating machines
2. Variations in air-gap reluctance over synchronous machine pole pitch.
3. Flux distortion in the synchronous machine from sudden load changes
4. Nonsinusoidal distribution of the flux in the air gap of synchronous machines
5. Transformer magnetizing currents
6. Network nonlinearities from loads such as rectifiers, inverters, welders, arc furnaces, voltage controllers, frequency converters, etc.

While the established sources of harmonics are still present on the system, the power network is also subjected to new harmonics sources:

1. Energy conservation measures, such as those for improved motor efficiency and load matching. which employ power semiconductor devices and switching for their operation. These devices often produce irregular voltage and current waveforms that are rich in harmonics.
2. Motor control devices such as speed controls for traction.
3. High-voltage dc power conversion and transmission.
4. Interconnection of wind and solar power converters with distribution systems.
5. Static-var compensators which have largely replaced synchronous condensors as continuously variable-var sources.
6. The development and potentially wide use of electric vehicles that require a significant amount of power rectification for battery charging.
7. The potential use of direct energy conversion devices, such as magnetohydrodynamics, storage batteries. and fuel cells, that require dc/ac power converters.

The presence of harmonics causes the distortion of the voltage or current waves. The distortions are measured in terms of the voltage or current harmonic factors. The IEEE Standard 519-1981 [29] defines the harmonic factors as the ratio of the root-mean-square value of all the harmonics to the root-mean-square value of the fundamental. Therefore, the voltage harmonic factor HF_V can be expressed as

$$HF_V = \frac{(E_3^2 + E_5^2 + E_7^2 + \cdots)^{1/2}}{E_1} \tag{8-101}$$

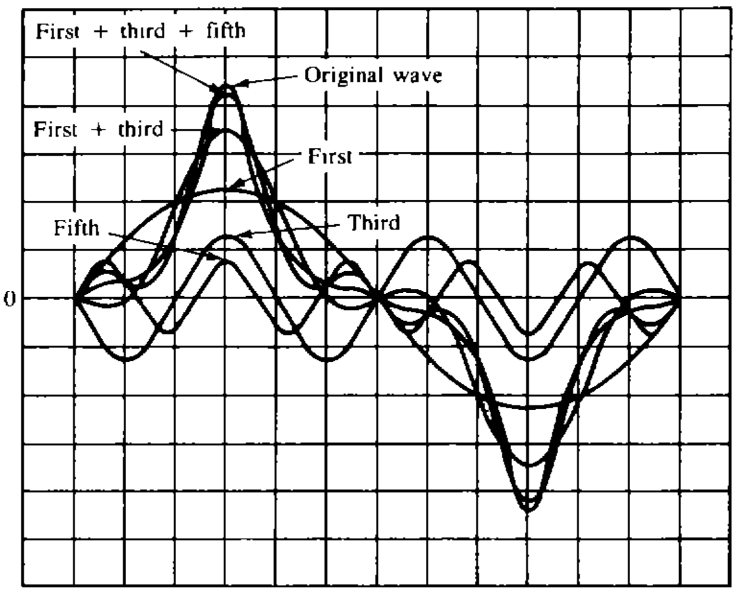


Figure 8-45 Harmonic analysis of peaked no-load current.

and the current harmonic factor HF_1 can be expressed as

$$HF_I = \frac{(I_3^2 + I_5^2 + I_7^2 + \cdots)^{1/2}}{I_1} \tag{8-102}$$

The presence of the voltage distortion results in harmonic currents. Figure 8-45 shows harmonic analysis of a peaked no-load current wave.

The characteristics of harmonics on a distribution system are functions of both the harmonic source and the system response. For example, utilities are presently installing more and larger transformers to meet ever-increasing power demands. Each transformer is a source of harmonics to the distribution system. Furthermore, these transformers are being operated closer to the saturation point. Transformer saturation results in a nonsinusoidal exciting current in the iron core when a sinusoidal voltage is applied. The level of transformer saturation is affected by the magnitude of the applied voltage. When the applied voltage is above the rated voltage, the harmonic components of the exciting current increase dramatically. Owen [26] has demonstrated this for a typical substation power transformer, as shown in Fig. 8-46. Also, some utility companies are overexciting distribution transformers as a matter of policy and practice, which compounds the harmonic problem.

The current harmonics of consequence which are produced by transformers are generally in the order of the third, fifth, and seventh. Table 8-5 gives a summary of the conditions obtaining third harmonics with different connections of double-wound three-phase transformers. The table is prepared for third harmonics in double-wound single-phase core- and shell-type transformers and in three-phase shell-type transformers for three-phase service. Figure 8-47 shows the shape of the resultant waves obtained when combining the fundamental and third harmonic along with different positions of the harmonic. Note that at harmonic frequencies the phase angles (due to the various harmonic impedances of each load) can be

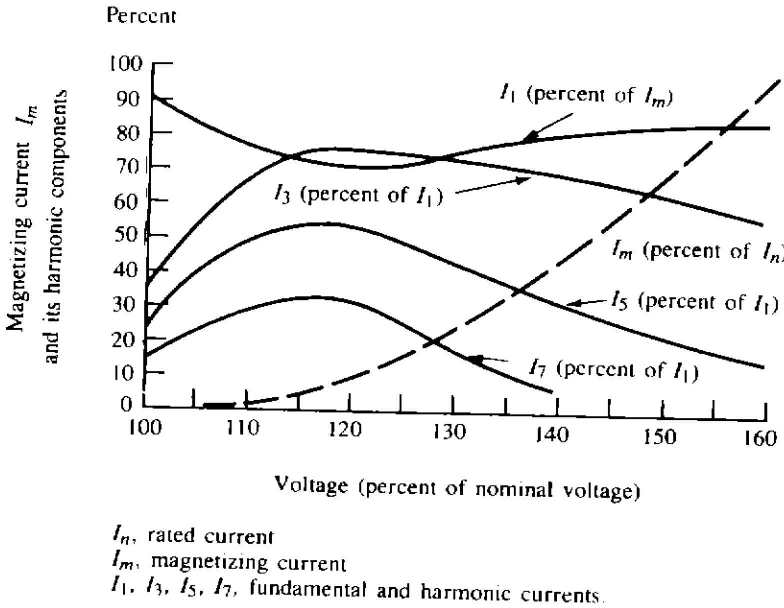


Figure 8-46 Harmonic components of transformer exciting current. (*From* [26].)

anything between zero and 360°. Also, as the harmonic order increases, the power-line impedance itself plays the role of a controlling factor, and therefore the harmonic current will have different phase angles at different locations.

The impact of harmonics on transformers are numerous. For example, voltage harmonics result in increased iron losses; current harmonics result in increased copper losses and stray flux losses. The losses may in turn cause the transformer to be overheated. The harmonics may also cause insulation stresses and resonances between transformer windings and line capacitances at the harmonic frequencies. The total eddy-current losses are proportional to the harmonic frequencies and can be expressed as

$$\text{TECL} = \text{ECL}_1 \sum_{h=1}^{\infty} \left(\frac{h \cdot I_h}{I_1}\right)^2 \tag{8-103}$$

where TECL = total eddy-current loss
ECL_1 = eddy-current loss at rated fundamental current
h = harmonic order
I_1 = rated fundamental current
I_h = harmonic current

Capacitor-bank sizes and locations are critical factors in a distribution system's response to harmonic sources. The combination of capacitors and the system reactance causes both series- and parallel-resonant frequencies for the circuit. The possibility of resonance between a shunt-capacitor bank and the rest of the system, at a harmonic frequency, may be determined by calculating equal order of harmonic h at which resonance may take place [28]. This equal order of harmonic is found from

$$h = \left(\frac{S_{sc}}{Q_c}\right)^{1/2} \tag{8-104}$$

Table 8-5 The influence of three-phase transformer connections on third harmonics

	Primary					Secondary			
	Currents		Voltages			Currents		Voltages	
Connections*	No-load	Line	Line	Phase	Flux	No-load	Line	Line	Phase
1. Wye I.N./wye I.N.	sine	sine	sine	Contains 3d h(P)	Contains 3d h(FT)		sine	sine	Contains 3d h(P)
2. Wye N. to G./wye I.N.	Contains 3d h(P)†	Contains 3d h(P)†	sine	Contains 3d h(P)†	Contains 3d h(FT)†		sine	sine	Contains 3d h(P)†
3. Wye I.N./wye, 4-wire	sine	sine	sine	Contains 3d h(P)†	Contains 3d h(FT)†	Contains 3d h(P)†	Contains 3d h(P)†	sine	Contains 3d h(P)†
4. Wye I.N. tertiary delta/wye I.N.	sine in star, 3d h in delta (P)	sine	sine	sine	sine		sine	sine	sine
5. Wye I.N./delta	sine	sine	sine	sine	sine	Contains 3d h(P)	sine	sine	sine
6. Wye N. to G./delta	Contains 3d h(P)†	Contains 3d h(P)†	sine	sine	sine	Contains 3d h(P)†	sine	sine	sine
7. Wye I.N./interconnected wye I.N.	sine	sine	sine	Contains 3d h(P)	Contains 3d h(FT)		sine	sine	sine
8. Wye I.N./interconnected wye, 4-wire	sine	sine	sine	Contains 3d h(P)	Contains 3d h(FT)		sine	sine	sine
9. Delta/wye I.N.	Contains 3d h(P)	sine	sine	sine	sine		sine	sine	sine
10. Delta/wye, 4-wire	Contains 3d h(P)	sine	sine	sine	sine	Contains 3d h(P)	Contains 3d h(P)	sine	sine
11. Delta/delta	Contains 3d h(P)	sine	sine	sine	sine	Contains 3d h(P)	sine	sine	sine

* I. N. means 'isolated neutral.' N. to G. means 'transformer primary neutral connected to generator neutral.' (P) means 'peaked wave.' (FT) means 'flattop wave.'

† In all these cases the third-harmonic component is less than it otherwise would be if (1) the circulating third-harmonic current flowed through a closed delta winding only or (2) the neutral point was isolated.

Source: From [27].

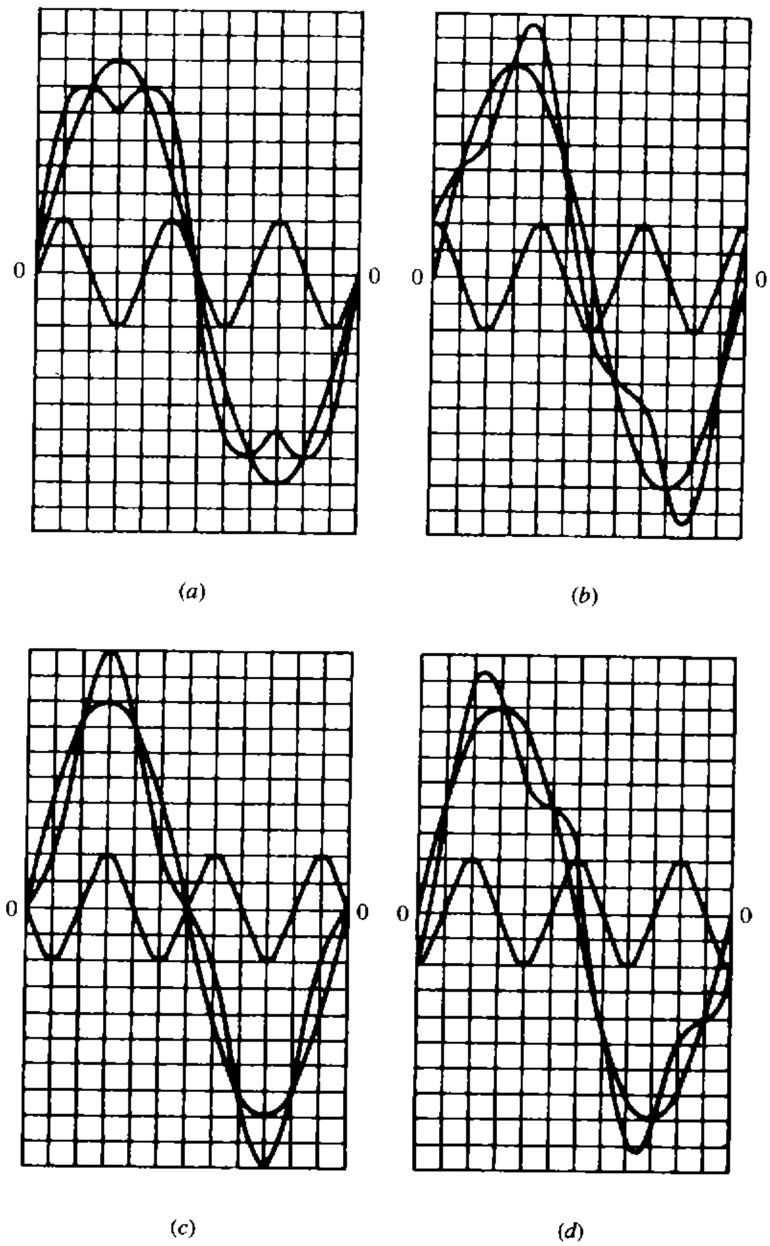


Figure 8-47 Combinations of fundamental and third-harmonic waves: (*a*) harmonic in phase, (*b*) harmonic 90° leading, (*c*) harmonic in opposition, and (*d*) harmonic 90° lagging. (*From* [27].)

where S_{sc} = short-circuit power of system at point of application, MVA
Q_c = capacitor-bank size, Mvar

The parallel-resonant frequency f_p can be expressed as

$$f_p = f_1 \cdot h \tag{8-105}$$

Substituting Eq. (8-104) into Eq. (8-105),

$$f_p = f_1 \left(\frac{S_{sc}}{Q_c}\right)^{1/2} \tag{8-106}$$

or

$$f_p = f_1 \left(\frac{X_c}{X_{sc}}\right)^{1/2} \tag{8-107}$$

or

$$f_p = \frac{1}{2\pi}\left(\frac{1}{L_{sc} \cdot C}\right)^{1/2} \tag{8-108}$$

where f_1 = fundamental frequency, Hz
X_c = reactance of capacitor bank, pu or Ω
X_{sc} = reactance of power system, pu or Ω,
L_{sc} = inductance of power system, H
C = capacitance of capacitor bank, F

The effects of the harmonics on the capacitor bank include (1) overheating of the capacitors, (2) overvoltage at the capacitor bank, (3) changed dielectric stress, and (4) losses in capacitors. According to Kimbark [28], the increase of losses in capacitors due to harmonics can be expressed as

$$\text{LCDH} = \sum C(\tan\delta)_h w_h V_h^2 \tag{8-109}$$

where LCDH = losses in capacitors due to harmonics
C = capacitance,
$(\tan\delta)_h$ = loss factor at frequency of hth harmonics
w_h = 2π times frequency of hth harmonic
V_h = rms voltage of hth harmonic

The harmonic control techniques include (1) locating the capacitor banks strategically, (2) selecting capacitor-bank sizes properly, (3) ungrounding or deleting the capacitor bank, (4) using shielded cables, (5) controlling grounds properly, and (6) using harmonic filters.

PROBLEMS

8-1 Assume that a feeder supplies an industrial consumer with a cumulative load of (1) induction motors totaling 300 hp which run at an average efficiency of 89 percent and a lagging average power factor of 0.85, (2) synchronous motors totaling 100 hp with an average efficiency of 86 percent, and (3) a heating load of 100 kW. The industrial consumer plans to use the synchronous motors to correct its overall power factor. Determine the required power factor of the synchronous motors to correct the overall power factor at peak load to:

(*a*) Unity.

(*b*) 0.96 lagging.

8-2 A 2.4/4.16-kV wye-connected feeder serves a peak load of 300 A at a lagging power factor of 0.7 connected at the end of the feeder. The minimum daily load is approximately 135 A at a power factor of 0.62. If the total impedance of the feeder is $0.50 + j1.35\ \Omega$, determine the following:

(*a*) The necessary kilovar rating of the shunt capacitors located at the load to improve the peak-load power factor to 0.96.

(*b*) The reduction in kilovoltamperes and line current due to the capacitors.

(*c*) The effects of the capacitors on the voltage regulation and voltage drop in the feeder.

(*d*) The power factor at minimum daily load level.

8-3 Assume that a locked-rotor starting current of 90 A at a lagging load factor of 0.30 is supplied to a motor which is operated discontinuously. A normal operating current of 15 A, at a lagging power factor of 0.80, is drawn by the motor from the 2.4/4.16-kV feeder of Prob. 8-2. Assume that a series capacitor is desired to be installed in the feeder to improve the voltage regulation and limit lamp flicker from the intermittent motor starting and determine the following:

(*a*) The voltage dip due to the motor starting, before the installation of the series capacitor.

(*b*) The necessary size of the capacitor to restrict the voltage dip at motor start to not more than 3 percent.

8-4 Assume that a three-phase distribution substation transformer has a nameplate rating of 7250 kVA and a thermal capability of 120 percent of the nameplate rating. If the connected load is 8816 kVA with a 0.85 lagging power factor, determine the following:

(*a*) The kilovar rating of the shunt-capacitor bank required to decrease the kilovoltampere load on the transformer to its capability level.

(*b*) The power factor of the corrected load.

(*c*) The kilovar rating of the shunt-capacitor bank required to correct the load power factor to unity.

(*d*) The corrected kilovoltampere load at this unity power factor.

8-5 Assume that the NP&NL Utility Company is presently operating at 90 percent power factor. It is desired to improve the power factor to 98 percent. To study the power-factor improvement, a number of load flow runs have been made and the results are summarized in the following table. Using the relevant additional information given in Example 8-5, repeat Example 8-5.

Table P8-5

Comment	At 90% PF	At 99% PF
Total loss reduction due to capacitors applied to substation busses, kW	496	488
Additional loss reduction due to capacitors applied to feeders, kW	84	72
Total demand reduction due to capacitors applied to substation buses and feeders, kVA	21,824	19,743
Total required capacitor additions at buses and feeders, kvar	9,512	2,785

8-6 Assume that a manufacturing plant has a three-phase in-plant generator to supply only three-phase induction motors totaling 1200 hp at 2.4 kV with a lagging power factor and efficiency of 0.82 and 0.93, respectively. Using the given information, determine the following:

(*a*) Find the required line current to serve the 1200-hp load, and the required capacity of the generator.

(*b*) Assume that 500 hp of the 1200-hp load is produced by an overexcited synchronous motor operating with a leading power factor and efficiency of 0.90 and 0.93, respectively. Find the required new total line current and the overall power factor.

(*c*) Find the required size of shunt capacitors to be installed to achieve the same overall power factor as found in part *b* by replacing the overexcited synchronous motor.

8-7 Verify that the loss reduction with two capacitor banks is

$$\Delta L = 3\alpha c\{x_1[(2 - x_1) + \lambda x_1 - 3c] + x_2[(2 - x_2) + \lambda x_2 - c]\}$$

8-8 Derive Eq. (8-65) from Eq. (8-64).

8-9 Verify that the optimum loss reduction is

$$\Delta L_{\text{opt}} = 3\alpha c \sum_{i=1}^{n} \left[\frac{1}{1-\lambda} - \frac{(2i-1)c}{1-\lambda} + \frac{i^2c^2}{1-\lambda} - \frac{c^2}{4(1-\lambda)} - \frac{ic^2}{1-\lambda}\right]$$

8-10 Derive Eq. (8-74) from Eq. (8-65).

8-11 Verify Eq. (8-75).

REFERENCES

1. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
2. McGraw-Edison Company: *The ABC of Capacitors*, Bulletin R230-90-1, 1968.
3. Hopkinson, R. H.: "Economic Power Factor-Key to kvar Supply," *Electr. Forum*, vol. 6, no. 3, 1980, pp. 20–22.
4. Zimmerman, R. A.: "Economic Merits of Secondary Capacitors," *AIEE Trans.*, vol. 72, 1953, pp. 694–97.
5. Wallace, R. L.: "Capacitors Reduce System Investment and Losses," *The Line*, vol. 76, no. 1, 1976, pp. 15–17.
6. Baum, W. U., and W. A. Frederick: "A Method of Applying Switched and Fixed Capacitors for Voltage Control." *IEEE Trans. Power Appar. Syst.*, vol. PAS-84, no. 1, January 1965, pp. 42–48.
7. Neagle, N. M., and D. R. Samson: "Loss Reduction from Capacitors Installed on Primary Feeders." *AIEE Trans.*, vol. 75, pt. III, October 1956, pp. 950–59.
8. Schmidt, R. A.: "DC Circuit Gives Easy Method of Determining Value of Capacitors in Reducing I^2R Losses," *AIEE Trans.*, vol. 75, pt. III, October 1956, pp. 840–48.
9. Maxwell, N.: "The Economic Application of Capacitors to Distribution Feeders," *AIEE Trans.*, vol. 79, pt. III, August 1960, pp. 353–59.
10. Cook, R. F.: "Optimizing the Application of Shunt Capacitors for Reactive Voltampere Control and Loss Reduction," *AIEE Trans.*, vol. 80, pt. III, August 1961, pp. 430–44.
11. Schmill, J. V.: "Optimum Size and Location of Shunt Capacitors on Distribution Systems," *IEEE Trans. Power Apper. Syst.*, vol. PAS-84, no. 9, September 1965, pp. 825–32.
12. Chang, N. E.: "Determination of Primary-Feeder Losses," *IEEE Trans. Power Appar. Syst.*, vol. Pas-87, no. 12, December 1968, pp. 1991–94.
13. Chang, N. E.: "Locating Shunt Capacitors on Primary Feeder for Voltage Control and Loss Reduction," *IEEE Trans Power Appar. Syst.*, vol. PAS-88, no. 10, October 1969, pp. 1574–77.
14. Chang, N. E.: "Generalized Equations on Loss Reduction with Shunt Capacitors," *IEEE Trans. Power Appar. Syst.*, vol. PAS-91, no. 5, September/October 1972, pp. 2189–95.
15. Bae, Y. G.: "Analytical Method of Capacitor Allocation on Distribution Primary Feeders," *IEEE Trans Power Appar. Syst.*, vol. PAS-97, no. 4, July/August 1978, pp. 1232–38.
16. Gönen, T., and F. Djavashi: "Optimum Loss Reduction from Capacitors Installed on Primary Feeders," *IEEE Midwest Power Symposium*, Purdue University, West Lafayette, Ind., Oct., 27–28, 1980.
17. Gönen, T., and F. Djavashi: "Optimum Shunt Capacitor Allocation on Primary Feeders," *IEEE MEXICON-80 International Conference*, Mexico City, Oct. 22–25 1980
18. Lapp, J.: "The Impact of Technical Developments on Power Capacitors, *The Line*, vol. 80, no. 2, July 1980, pp. 19–24.
19. Burrage, L. M.: "Capacitor Tank Ruptures Studied," *The Line*, vol. 76, no. 2, 1976, pp. 2–5.
20. Oklahoma Gas and Electric Company: *Engineering Guides*, Oklahoma City, January 1981.
21. Grainger, J. J., and S. H. Lee.: "Optimum Size and Location of Shunt Capacitors for Reduction of Losses on Distribution Feeders," *IEEE Trans. Power Appar. Syst.*, vol. PAS-100, March 1981, pp. 1105–1118.
22. Lee, S. H., and J. J. Grainger: "Optimum Placement of Fixed and Switched Capacitors on Primary Distribution Feeders," *IEEE Trans. Power Appar. Syst.*, vol. PAS-100, January 1981, pp. 345–351.
23. Grainger, J. J., A. A. El-Kib, and S. H. Lee: "Optimal Capacitor Placement on Three-Phase Primary Feeders: Load and Feeder Unbalance Effects." Paper 83 WM 160-9. IEEE PES Winter Meeting, New York, Jan. 30–Feb. 4, 1983.
24. Grainger, J. J., S. Civanlar, and S. H. Lee: "Optimal Design and Control Scheme for Continuous Capacitive Compensation of Distribution Feeders," Paper 83 WM 159-1. IEEE PES Winter Meeting. New York, Jan. 30–Feb. 4, 1983.

25. Mahmoud, A. A , R. E. Owen, and A. E. Emanuel: "Power System Harmonics: An Overview," *IEEE Trans. Power Appar. Syst.*, vol. PAS-102, no. 8, August 1983, pp. 2455–2460.
26. Owen, R. E.: "Distribution System Harmonics: Effects on Equipment and Operation," Pacific Coast Electrical Association Engineering and Operating Conference, Los Angeles, Calif., Mar. 15–16, 1979
27. Stigant, S. A., and A. C. Franklin: *The J&P Transformer Book*, Butterworth, London, 1973.
28. Kimbark, E. W.: *Direct Current Transmission*, Wiley, New York, 1971.
29. *IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters*, IEEE Std. 519-1981, 1981.
30. Electric Power Research Institute: *Study of Distribution System Surge and Harmonic Characteristics*, Final Report, EPRI EL-1627, Palo Alto, Calif., November 1980.
31. Owen, R. E., M. F. McGranaghan, and J. R. Vivirito: "Distribution System Harmonics: Controls for Large Power Converters," Paper 81 SM 482-9, IEEE PES Summer Meeting, Portland, Oreg., July 26–31, 1981.
32. McGranaghan, M. F., R. C. Dugan, and W. L. Sponsler: "Digital Simulation of Distribution System Frequency-Response Characteristics," Paper 80 SM 665-0, IEEE PES Summer Meeting, Minneapolis, Minn., July 13–18, 1980.
33. McGranaghan, M. F., J. H. Shaw, and R. E. Owen: "Measuring Voltage and Current Harmonics on Distribution Systems," Paper 81 WM 126-2, IEEE PES Winter Meeting, Atlanta, Ga., Feb. 1–6, 1981.
34. Szabados, B., E. J. Burgess, and W. A. Noble: "Harmonic Interference Corrected by Shunt Capacitors on Distribution Feeders," *IEEE Trans. Power Appar. Syst.*, vol. PAS-96, no. 1, January/February 1977, pp. 234–239.
35. Gönen, T., and A. A. Mahmoud: "Bibliography of Power System Harmonics, Part I," *IEEE Trans. Power Appar. Syst.*, vol. PAS-103, no. 9, September 1984, pp. 2460–2469.
36. Gönen, T., and A. A. Mahmoud: "Bibliography of Power System Harmonics, Part II," *IEEE Trans. Power Appar. Syst.*, vol. PAS-103, no. 9, September 1984, pp. 2470–2479.

CHAPTER

NINE

DISTRIBUTION SYSTEM VOLTAGE REGULATION

9-1 BASIC DEFINITIONS

Voltage regulation: the percent voltage drop of a line (e.g., a feeder) with respect to the receiving-end voltage. Therefore

$$\% \text{ regulation} = \frac{|V_s| - |V_r|}{|V_r|} \times 100 \tag{9-1}$$

Voltage drop: the difference between the sending-end and the receiving-end voltages of a line

Nominal voltage: the nominal value assigned to a line or apparatus or a system of a given voltage class

Rated voltage: the voltage at which performance and operating characteristics of apparatus are referred

Service voltage: the voltage measured at the ends of the service-entrance apparatus

Utilization voltage: the voltage measured at the ends of an apparatus

Base voltage: the reference voltage, usually 120 V

Maximum voltage: the largest 5-min average voltage

Minimum voltage: the smallest 5-min voltage

Voltage spread: the difference between the maximum and minimum voltages, without voltage dips due to motor starting

9-2 QUALITY OF SERVICE AND VOLTAGE STANDARDS

In general, performance of distribution systems and quality of the service provided are measured in terms of freedom from interruptions and maintenance of satisfactory voltage levels at the customer's premises that is within limits appropriate for this type of service. Due to economic considerations, an electric utility company cannot provide each customer with a constant voltage matching exactly the nameplate voltage on the customer's utilization apparatus. Therefore a common practice among the utilities is to stay with preferred voltage levels and ranges of variation for satisfactory operation of apparatus as set forth by the American National Standards Institute (ANSI) Standard [2]. In many states, the ANSI standard is the basis for the state regulatory commission rulings on setting forth voltage requirements and limits for various classes of electric service.

In general, based on experience, too-high steady-state voltage causes reduced light bulb life, reduced life of electronic devices, and premature failure of some types of apparatus. On the other hand, too-low steady-state voltage causes lowered illumination levels, shrinking of TV pictures, slow heating of heating devices, difficulties in motor starting, and overheating and/or burning out of motors. However, most equipment and appliances operate satisfactorily over some range of voltage so that a reasonable tolerance is allowable.

The nominal voltage standards for a majority of the electric utilities in the United States to serve residential and commercial customers are:

1. 120/240-V three-wire single-phase
2. 240/120-V four-wire three-phase delta
3. 208Y/120-V four-wire three-phase wye
4. 480Y/277-V four-wire three-phase wye

As shown in Fig. 9-1, the voltage on a distribution circuit varies from a maximum value at the customer nearest to the source (first customer) to a minimum value at the end of the circuit (last customer). For the purpose of illustration Table 9-1 gives typical secondary voltage standards applicable to residential and commercial customers. These voltage limits may be set by the state regulatory commission as a guide to be followed by the utility.

As can be observed in Table 9-1, for any given nominal voltage level, the actual operating values can vary over a large range. This range has been segmented into three zones, namely, (1) the *favorable zone* or *preferred zone*, (2) the *tolerable zone*, and (3) the *extreme zone*. The favorable zone includes the majority of the existing operating voltages and the voltages within this zone (i.e., range A) to produce satisfactory operation of the customer's equipment. The distribution engineer tries to keep the voltage of every customer on a given distribution circuit within the favorable zone. Figure 9-1 illustrates the results of such efforts on urban and rural circuits. The tolerable zone contains a band of operating voltages slightly above and below the favorable zone. The operating voltages in the tolerable zone (i.e., range B) are usually acceptable for most purposes. For example, in this zone the

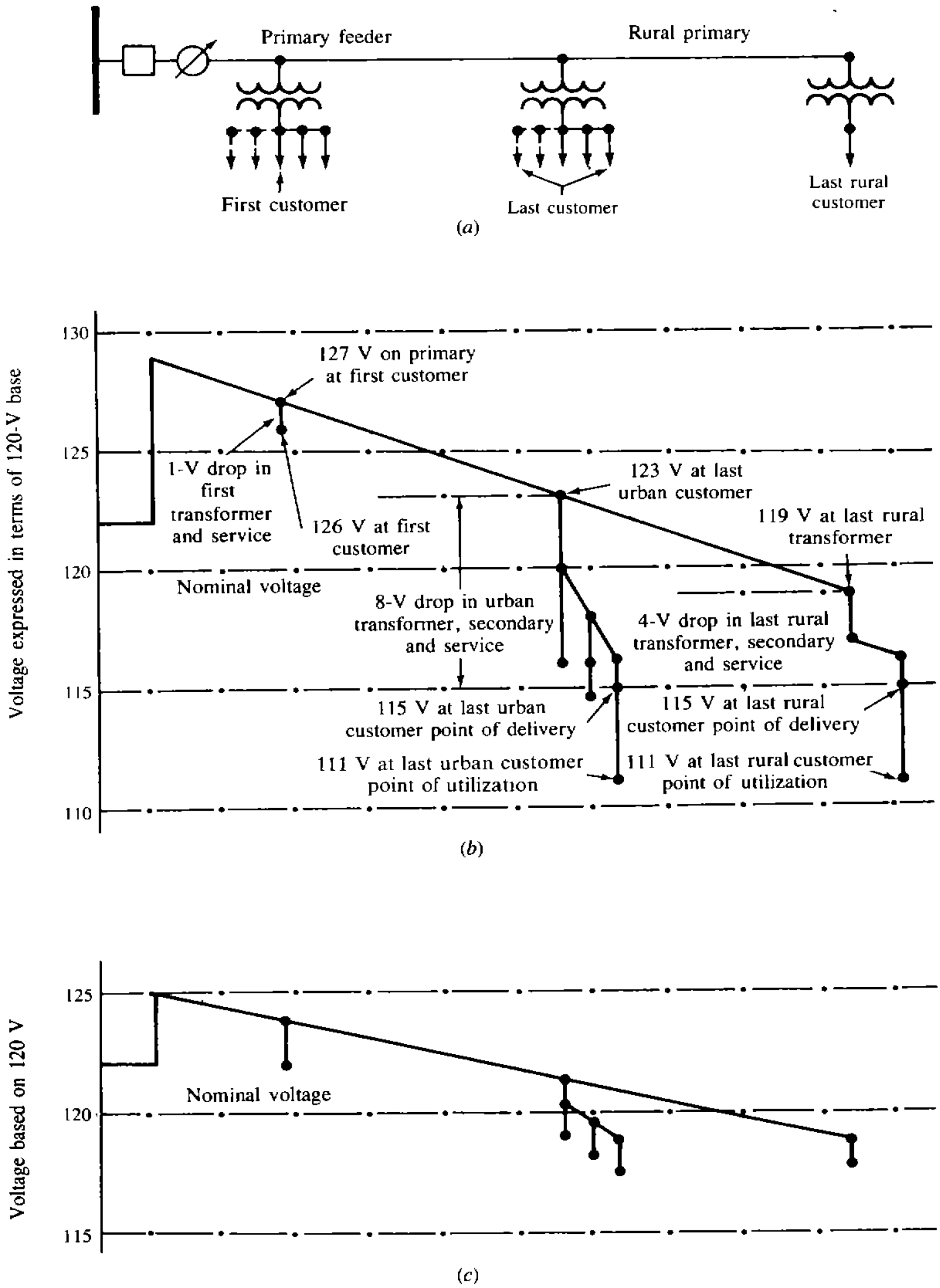


Figure 9-1 Illustration of voltage spread on a radial primary feeder: (*a*) one-line diagram of a feeder circuit, (*b*) voltage profile at peak-load conditions, and (*c*) voltage profile at light-load conditions.

Table 9-1 Typical secondary voltage standards applicable to residential and commercial customers

Nominal voltage class	Voltage limits		
	At point of delivery		At point of utilization
	Maximum	Minimum	Minimum
120/240-V 1ϕ and 240/120-V 3ϕ Δ:			
Favorable zone, range A	126/252	114/228	110/220
Tolerable zone, range B	127/254	110/220	106/212
Extreme zone, emergency	130/260	108/216	104/208
208Y/120-V 3ϕ:			
Favorable zone, range A	218Y/126	197Y/114	191Y/110
Tolerable zone, range B	220Y/127	191Y/110	184Y/106
Extreme zone, emergency	225Y/130	187Y/108	180Y/104
408Y/277-V 3ϕ:			
Favorable zone, range A	504Y/291	456Y/263	440Y/254
Tolerable zone, range B	508Y/293	440Y/254	424Y/245
Extreme zone, emergency	520Y/300	432Y/249	416Y/240

customer's apparatus may be expected to operate satisfactorily, although its performance may perhaps be less than warranted by the manufacturer. However, if the voltage in the tolerable zone results in unsatisfactory service of the customer's apparatus, the voltage should be improved. The extreme or emergency zone includes voltages on the fringes of the tolerable zone, usually within 2 or 3 percent above or below the tolerable zone. They may or may not be acceptable depending on the type of application. At times, the voltage that usually stays within the tolerable zone may infrequently exceed the limits because of some extraordinary conditions. For example, failure of the principal supply line, which necessitates the use of alternative routes or voltage regulators being out of service, can cause the voltages to reach the emergency limits. However, if the operating voltage is held within the extreme zone under these conditions, the customer's apparatus may still be expected to provide dependable operation, even though not the standard performance. However, voltages outside the extreme zone should not be tolerated under any conditions and should be improved right away. Usually, the maximum voltage drop in the customer's wiring between the point of delivery and the point of utilization is accepted as 4 V based on 120 V.

9-3 VOLTAGE CONTROL

To keep distribution-circuit voltages within permissible limits, means must be provided to control the voltage, i.e., to increase the circuit voltage when it is too low and to reduce it when it is too high. There are numerous ways to improve the distribution system's overall voltage regulation. The complete list is given by Lokay [1] as:

1. Use of generator voltage regulators
2. Application of voltage-regulating equipment in the distribution substations
3. Application of capacitors in the distribution substation
4. Balancing of the loads on the primary feeders
5. Increasing of feeder conductor size
6. Changing of feeder sections from single-phase to multiphase
7. Transferring of loads to new feeders
8. Installing of new substations and primary feeders
9. Increasing of primary voltage level
10. Application of voltage regulators out on the primary feeders
11. Application of shunt capacitors on the primary feeders
12. Application of series capacitors on the primary feeders

The selection of a technique or techniques depends upon the particular system requirement. However, automatic voltage regulation is always provided by (1) bus regulation at the substation, (2) individual feeder regulation in the substation, and (3) supplementary regulation along the main by regulators mounted on poles. Distribution substations are equipped with load-tap-changing (LTC) transformers that operate automatically under load or with separate voltage regulators that provide bus regulation.

Voltage-regulating apparatus are designed to maintain automatically a predetermined level of voltage that would otherwise vary with the load. As the load increases, the regulating apparatus boosts the voltage at the substation to compensate for the increased voltage drop in the distribution feeder. In cases where customers are located at long distances from the substation or where voltage drop along the primary circuit is excessive, additional regulators or capacitors, located at selected points on the feeder, provide supplementary regulation. Many utilities have experienced that the most economical way of regulating the voltage within the required limits is to apply both step voltage regulators and shunt capacitors. Capacitors are installed out on the feeders and on the substation bus in adequate quantities to accomplish the economic power factor. Many of these installations have sophisticated controls designed to perform automatic switching. Of course, a fixed capacitor is not a voltage regulator and cannot be directly compared to regulators, but, in some cases, automatically switched capacitors can replace conventional step-type voltage regulators for voltage control on distribution feeders.

9-4 FEEDER VOLTAGE REGULATORS

Feeder voltage regulators are used extensively to regulate the voltage of each feeder separately to maintain a reasonable constant voltage at the point of utilization. They are either the induction type or the step type. However, since today's

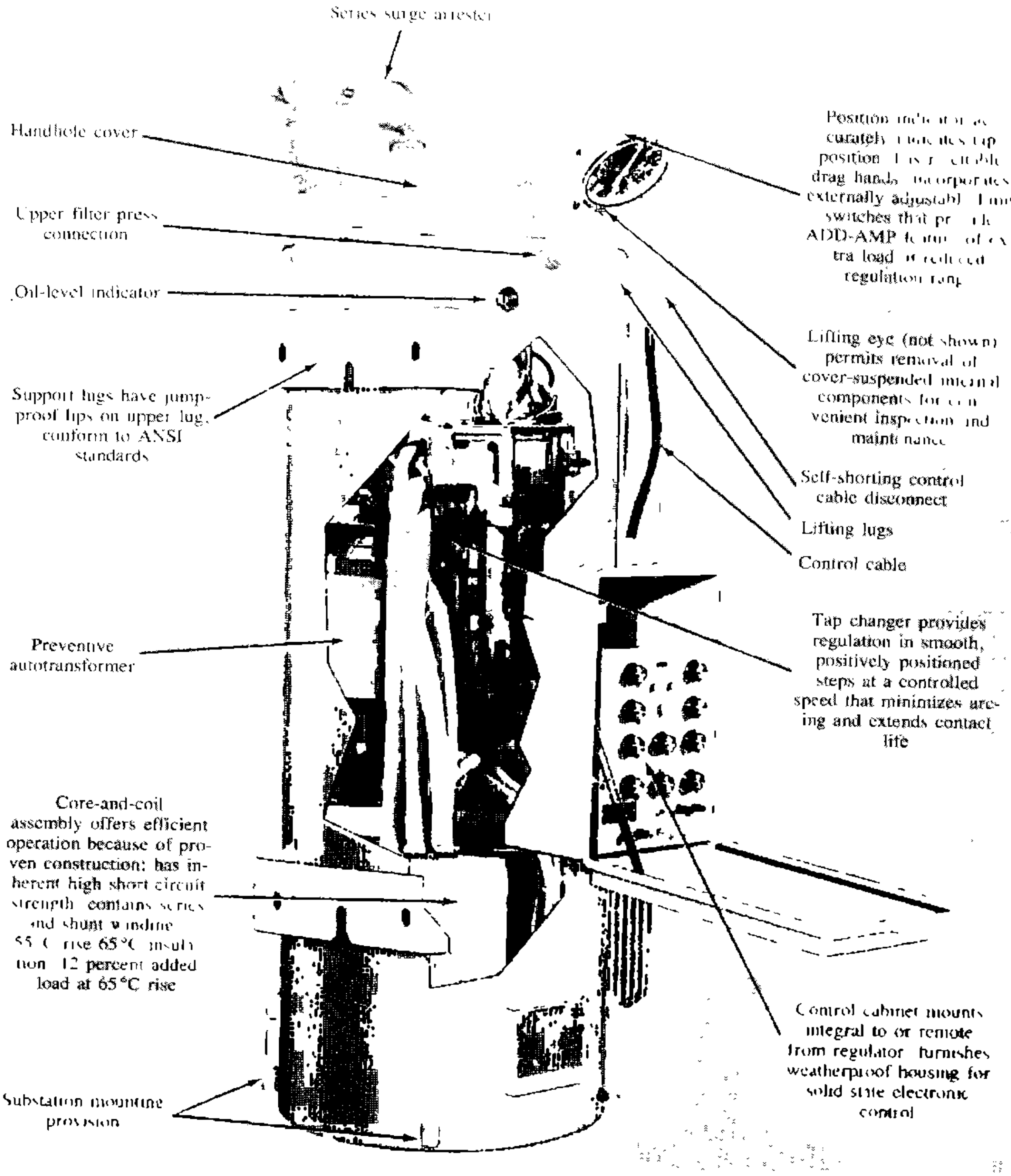


Figure 9-2 Typical single-phase 32-step pole-type voltage regulator used for 167 kVA or below. (*McGraw-Edison Company.*)

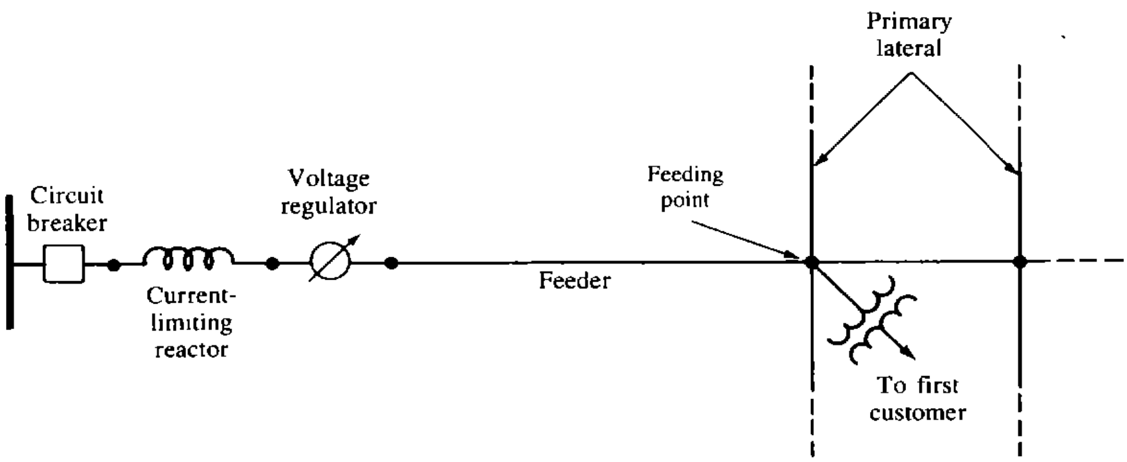


Figure 9-3 One-line diagram of a feeder, indicating the sequence of essential components.

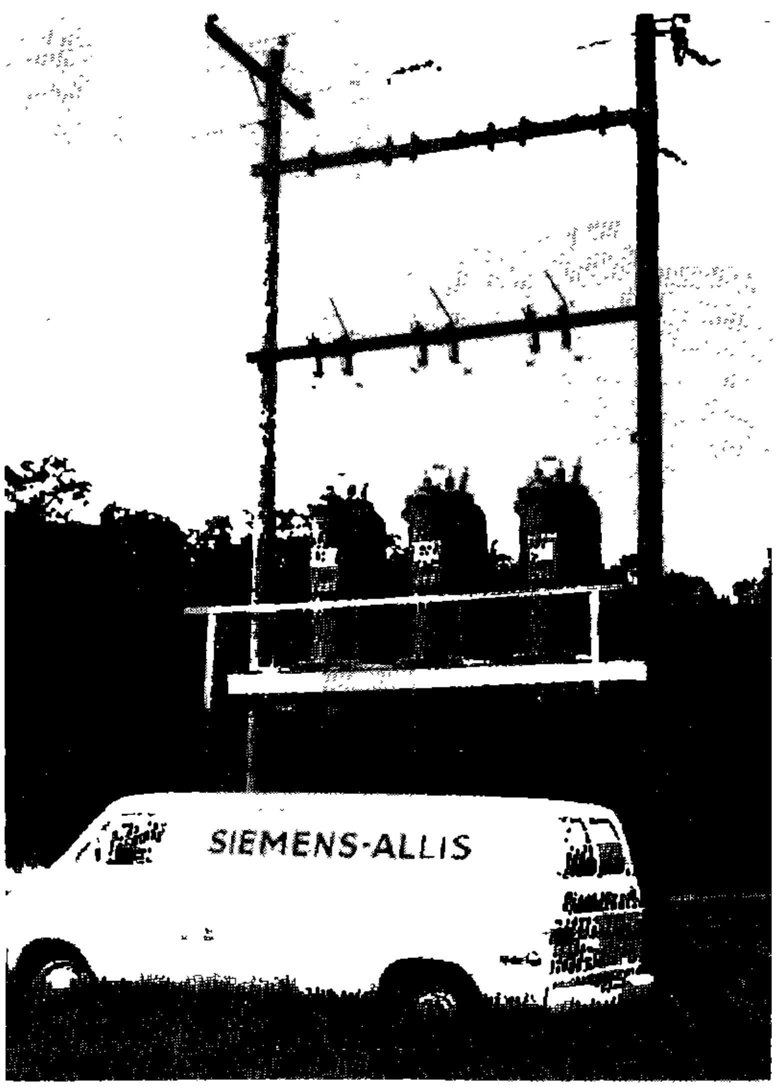


Figure 9-4 Typical platform-mounted voltage-regulators. (*Siemens-Allis Company.*)

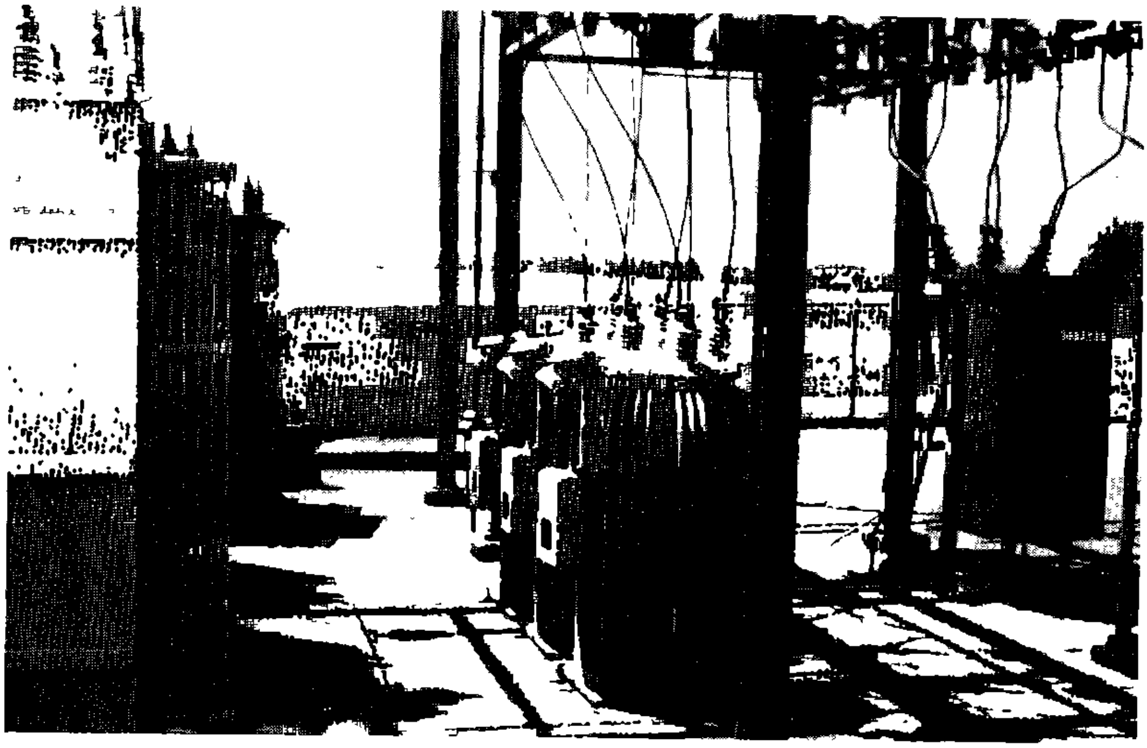


Figure 9-5 Individual feeder voltage regulation provided by a bank of distribution voltage regulators. (*Siemens-Allis Company.*)

modern step-type voltage regulators have practically replaced induction-type regulators, only step-type voltage regulators will be discussed in this chapter.

Step-type voltage regulators can be either (1) station-type, which can be single- or three-phase, and which can be used in substations for bus voltage regulation or individual feeder voltage regulation, or (2) distribution-type, which can be only single-phase and used pole-mounted out on overhead primary feeders. Single-phase step-type voltage regulators are available in sizes from 25 to 833 kVA, whereas three-phase step-type voltage regulators are available in sizes from 500 to 2000 kVA. For some units, the standard capacity ratings can be increased by 25 to 33 percent by forced-air cooling. Standard voltage ratings are available from 2400 to 19,920 V, allowing regulators to be used on distribution circuits from 2400 to 34,500 V grounded-wye/19,920 V multigrounded-wye. Station-type step voltage regulators for bus voltage regulation can be up to 69 kV.

A step-type voltage regulator is fundamentally an autotransformer with many taps (or steps) in the series winding. Most regulators are designed to correct the line voltage from 10 percent boost to 10 percent buck (that is, ± 10 percent) in 32 steps, with a $\frac{5}{8}$ percent voltage change per step. (Note that the full voltage regulation range is 20 percent, and therefore if the 20 percent regulation range is divided by the 32 steps, a $\frac{5}{8}$ percent regulation per step is found.) If two internal coils of a regulator are connected in series, the regulator can be used for ± 10 percent regulation; when they are connected in parallel, the current rating of the regulator would increase to 160 percent but the regulation range would decrease to ± 5 percent. Figure 9-2 shows a typical single-phase 32-step pole-type voltage

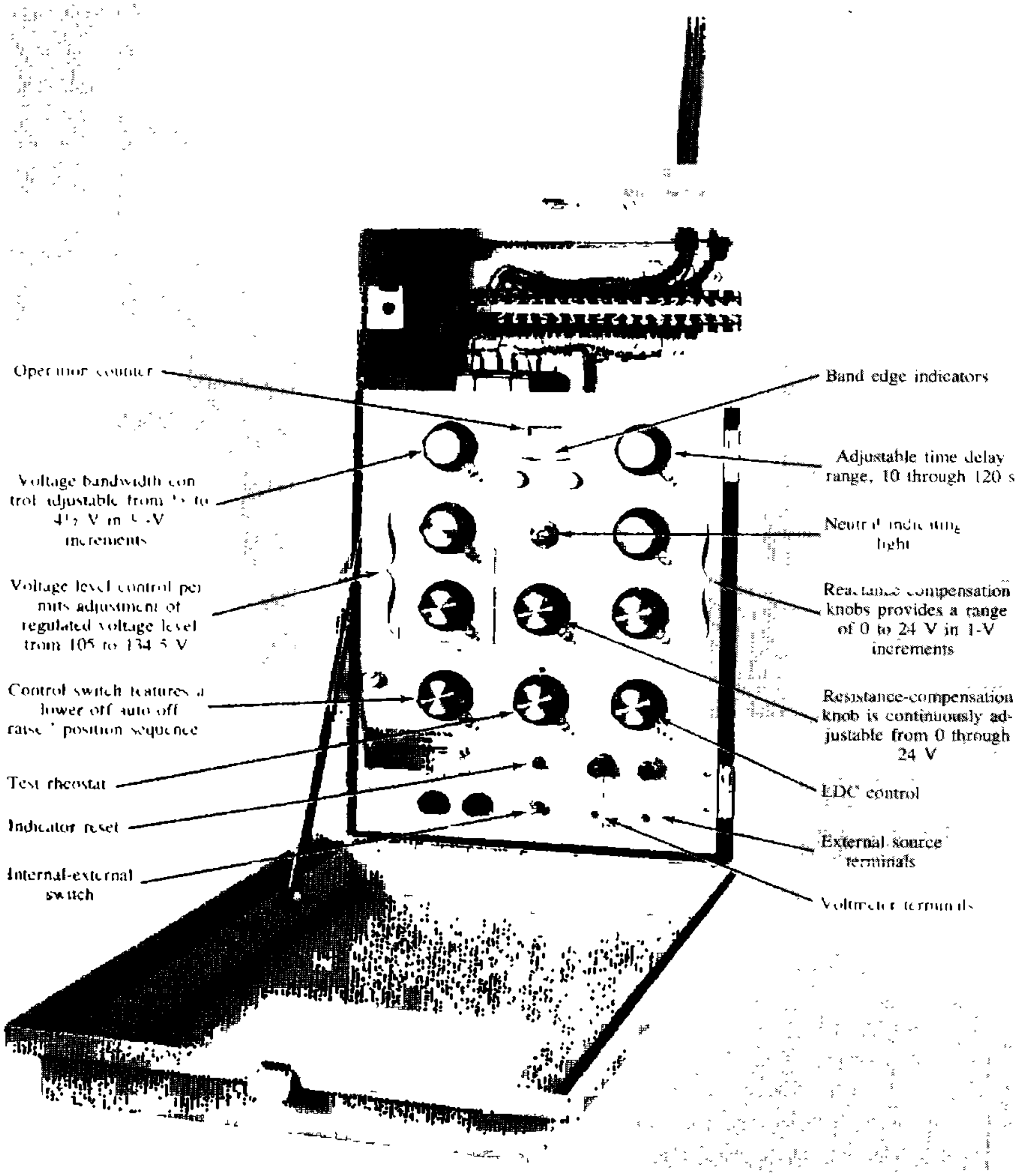


Figure 9-6 Features of the control mechanism of a single-phase 32-step voltage regulator. (*McGraw-Edison Company.*)

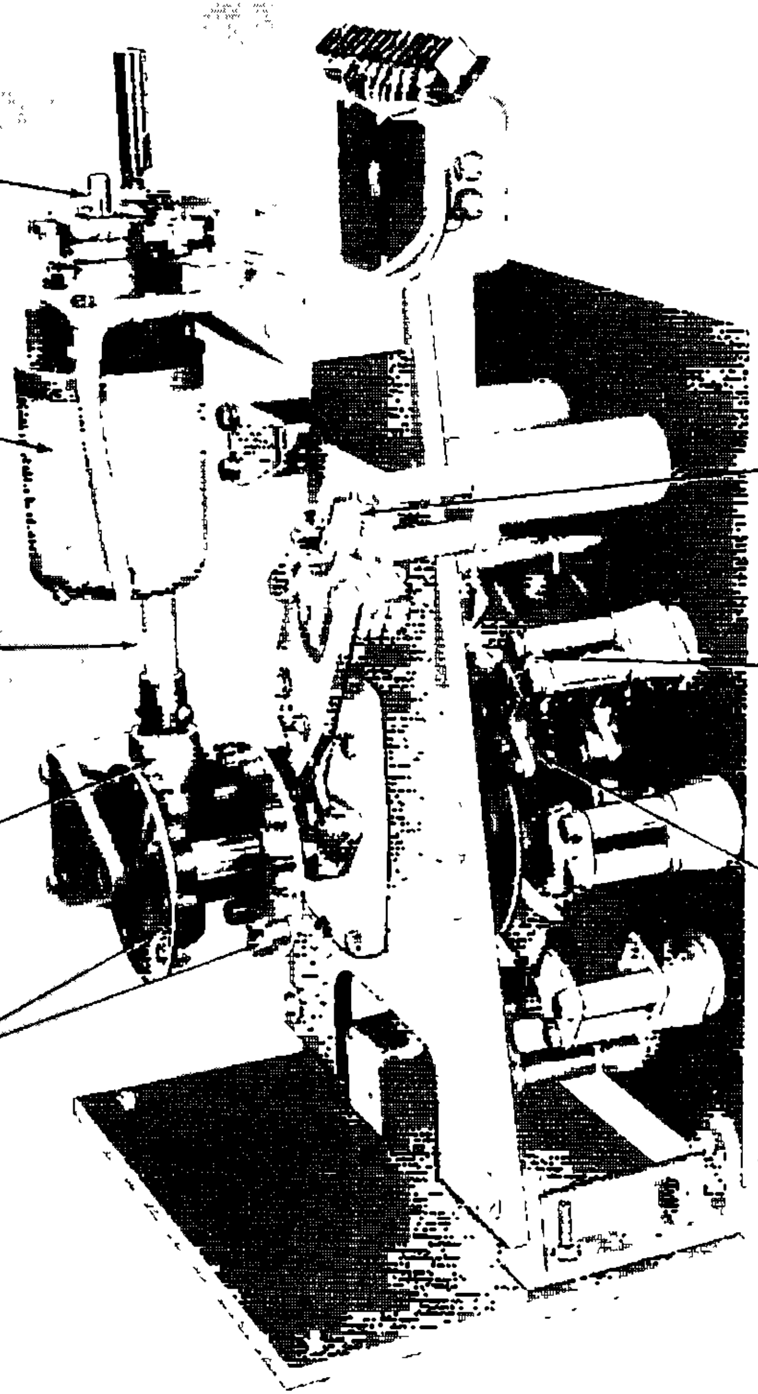
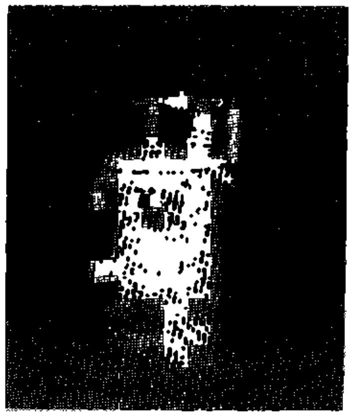
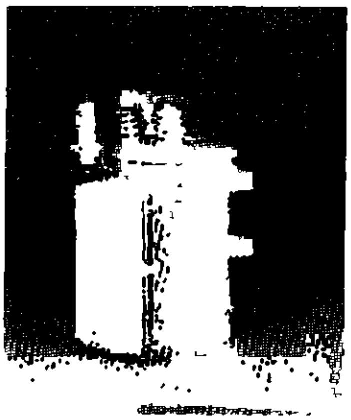


Figure 9-7 Standard direct drive tap changer used through 150 kV BIL, above 219 A. *(McGraw-Edison Company.)*



(a)



(b)

Figure 9-8 Four-step Auto-booster regulators: (*a*) 50-A unit and (*b*) 100-A unit. (*McGraw-Edison Company.*)

regulator; Fig. 9-3 shows its application on a feeder with essential components. Figure 9-4 shows typical platform-mounted voltage regulators. Individual feeder regulation for a large utility can be provided at the substation by a bank of distribution voltage regulators, as shown in Fig. 9-5.

In addition to its autotransformer component, a step-type regulator also has two other major components, namely, the tap-changing mechanism and the control mechanism, as shown in Fig. 9-2. Each voltage regulator ordinarily is equipped with the necessary controls and accessories so that the taps are changed automatically under load by a tap changer which responds to a voltage-sensing control to maintain a predetermined output voltage. By receiving its inputs from potential and current transformers, the control mechanism provides control of voltage level and bandwidth. Furthermore, it provides the ability to adjust line-drop compensation by selecting the resistance and reactance settings, as shown in Fig. 9-6. Figure 9-7 shows a standard direct-drive tap changer.

Figure 9-8 shows four-step Auto-booster regulators. Auto-boosters basically are single-phase regulating autotransformers which provide four-step feeder voltage regulation without the high degree of sophistication found in 32-step regulators. They can be used on circuits rated 2.4- to 12-kV delta and 2.4/4.16- to

19.92/34.5-kV multigrounded-wye. The Auto-booster unit can have a continuous-current rating of either 50 or 100 A. Each step represents either $1\frac{1}{2}$ or $2\frac{1}{2}$ percent voltage change depending on whether the unit has a 6 or 10 percent regulation range, respectively. They cost much less than the standard voltage regulators.

9-5 LINE-DROP COMPENSATION

Voltage regulators located in the substation or on a feeder are used to keep the voltage constant at a fictitious regulation or regulating point without regard to the magnitude or power factor of the load. The regulation point is usually selected to be somewhere between the regulator and the end of the feeder. This automatic voltage maintenance is achieved by dial settings of the adjustable resistance and reactance elements of a unit called the *line-drop compensator* (LDC) located on the control panel of the voltage regulator. Figure 9-9 shows a simple schematic diagram and phasor diagram of the control circuit and line-drop compensator circuit of a step or induction voltage regulator. Determination of the appropriate dial settings depends upon whether or not any load is tapped off the feeder between the regulator and the regulation point.

In the event that no load is tapped off the feeder between the regulator and the regulation point, the R dial setting of the line-drop compensator can be determined from

$$R_{\text{set}} = \frac{\text{CT}_P}{\text{PT}_N} \times R_{\text{eff}} \qquad \text{V} \tag{9-2}$$

where CT_P = rating of the current transformer's primary
PT_N = potential transformer's turns ratio = $V_{\text{pri}}/V_{\text{sec}}$

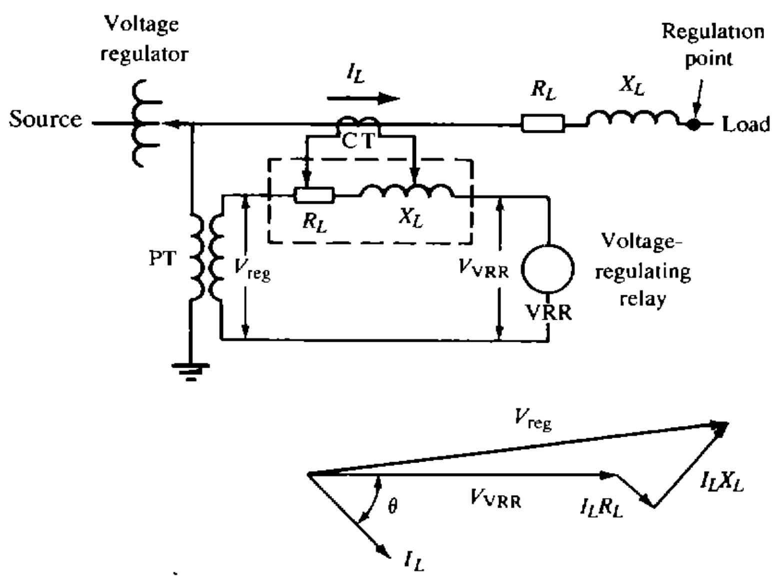


Figure 9-9 Simple schematic diagram and phasor diagram of the control circuit and line-drop compensator circuit of a step or induction voltage regulator. (*From* [*1*].)

R_{eff} = effective resistance of a feeder conductor from regulator station to regulation point, Ω

$$R_{\text{eff}} = r_a \times \frac{l - s_1}{2} \qquad \Omega \tag{9-3}$$

where r_a = resistance of a feeder conductor from regulator station to regulation point, Ω/mi per conductor

s_1 = length of three-phase feeder between regulator station and substation, mi (multiply length by 2 if feeder is in single-phase)

l = primary feeder length, mi

Also, the X dial setting of the line-drop compensator can be determined from

$$X_{\text{set}} = \frac{\text{CT}_P}{\text{PT}_N} \times X_{\text{eff}} \qquad \text{V} \tag{9-4}$$

where X_{eff} = effective reactance of a feeder conductor from regulator to regulation point, Ω

$$X_{\text{eff}} = x_L \times \frac{l - s_1}{2} \qquad \Omega \tag{9-5}$$

and

$$x_L = x_a + x_d \qquad \Omega/\text{mi} \tag{9-6}$$

where x_a = inductive reactance of individual phase conductor of feeder at 12-in spacing, Ω/mi

x_d = inductive-reactance spacing factor, Ω/mi

x_L = inductive reactance of feeder conductor, Ω/mi

Note that since the R and X settings are determined for the total connected load, rather than for a small group of customers, the resistance and reactance values of the transformers are not included in the effective resistance and reactance calculations.

On the other hand, in the event that load is tapped off the feeder between the regulator station and the regulation point, the R dial setting of the line-drop compensator can still be determined from Eq. (9-2), but the determination of the R_{eff} is somewhat more involved. Lokay [1] gives the following equations to calculate the effective resistance:

$$R_{\text{eff}} = \frac{\sum_{i=1}^{n} |\text{VD}_R|_i}{|I_L|} \qquad \Omega \tag{9-7}$$

and

$$\sum_{i=1}^{n} |\text{VD}_R|_i = |I_{L,1}| \times r_{a,1} \times l_1 + |I_{L,2}| \times r_{a,2} \times l_2 + \cdots + |I_{L,n}| \times r_{a,n} \times l_n \qquad \Omega \tag{9-8}$$

where $|\mathrm{VD}_R|_i$ = voltage drop due to line resistance of ith section of feeder between regulator station and regulation point, V/section

$\sum_{i=1}^{n} |\mathrm{VD}_R|_i$ = total voltage drop due to line resistance of feeder between regulator station and regulation point, V

$|I_L|$ = magnitude of load current at regulator location, A

$|I_{L,i}|$ = magnitude of load current in ith feeder section, A

$r_{a,i}$ = resistance of a feeder conductor in ith section of feeder, Ω/mi

l_i = length of ith feeder section, mi

Also, the X dial setting of the line-drop compensator can still be determined from Eq. (9-4), but the determination of the X_{eff} is again somewhat more involved. Lokay [1] gives the following equations to calculate the effective reactance:

$$X_{\text{eff}} = \frac{\sum_{i=1}^{n} |\mathrm{VD}_X|_i}{|I_L|} \qquad \Omega \tag{9-9}$$

and

$$\sum_{i=1}^{n} |\mathrm{VD}_X|_i = |I_{L,1}| \times X_{L,1} \times l_1 + |I_{L,2}| \times X_{L,2} \times l_2 + \cdots + |I_{L,n}| \times X_{L,N} \times l_n \qquad \mathrm{V} \tag{9-10}$$

where $|\mathrm{VD}_X|_i$ = voltage drop due to line reactance of ith section of feeder between regulator station and regulation point, V/section

$\sum_{i=1}^{n} |\mathrm{VD}_X|_i$ = total voltage drop due to line reactance of feeder between regulator station and regulation point, V

$X_{L,1}$ = inductive reactance [as defined in Eq. (9-6)] of ith section of feeder, Ω/mi

Since the methods just described to determine the effective R and X are rather involved, Lokay [1] suggests as an alternative and practical method to measure the current (I_L) and voltage at the regulator location and the voltage at the regulating point. The difference between the two voltage values is the total voltage drop between the regulator and the regulation point, which can also be defined as

$$\mathrm{VD} = |I_L| \times R_{\text{eff}} \times \cos\theta + |I_L| \times X_{\text{eff}} \times \sin\theta \qquad \mathrm{V} \tag{9-11}$$

from which the R_{eff} and X_{eff} values can be determined easily if the load power factor of the feeder and the average r/x ratio of the feeder conductors between the regulator and the regulating point are known.

Figure 9-10 gives an example for determining the voltage profiles for the peak and light loads. Note that the primary-feeder voltage values are based on a 120-V base.

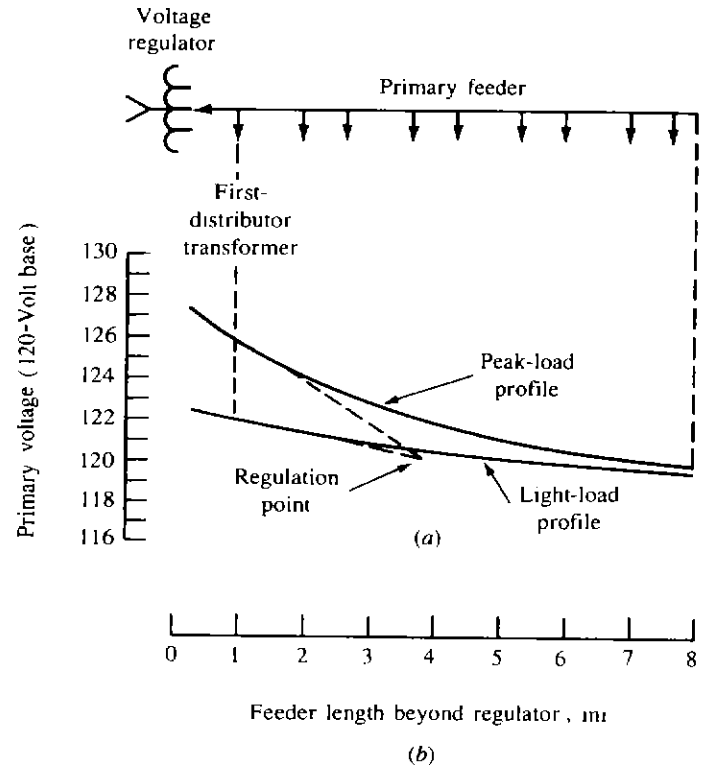


Figure 9-10 One-line diagram and voltage profiles of a feeder with distributed load beyond a voltage regulator location: (*a*) one-line diagram, and (*b*) peak- and light-load profile showing fictitious regulating point for line-drop compensator settings. It is assumed that the conductor size between regulator and first distribution transformer is #2/0 copper conductor with 44-in flat spacing with resistance and reactance of 0.481 and 0.718 Ω/mi, respectively. The PT and CT ratios of the voltage regulator are 7960 : 120 and 200 : 5, respectively. Distance to fictitious regulating point is 3.9 mi. Line-drop compensator settings are

$$R_{set} = 200 \times \tfrac{120}{7960} \times 0.481 \times 3.9 = 5.656$$

$$X_{set} = 200 \times \tfrac{120}{7960} \times 0.718 \times 3.9 = 8.4428$$

Voltage-regulating relay setting is 120.1 V. (*From* [*1*].)

Example 9-1 This example investigates the use of step-type voltage regulation (control) to improve the voltage profile of distribution systems. Figure 9-11 illustrates the elements of a distribution substation that is supplied from a subtransmission loop and feeds several radial primary feeders.

The substation LTC transformer can be used to regulate the primary distribution voltage (V_P) bus, holding V_P constant as both the subtransmission voltage (V_{ST}) and the IZ_T voltage drop in the substation transformer vary with load. If the typical primary-feeder main is voltage-drop-limited, it can be extended farther and/or loaded more heavily if a feeder voltage regulator bank is used wisely. In Fig. 9-11 the feeder voltage regulator, indicated with the symbol ∅ is located at the point $s = s_1$, and it varies its boost and buck automatically to hold a set voltage at the regulating point, that is, at $s = s_{RP}$.

Typical LTC and feeder regulator data The abbreviation VRR stands for voltage-regulating relay (or solid-state equivalent thereof), and it is adjustable within the approximate range from 110 to 125 V. The VRR measures the voltage at the regulating point, that is, V_{RP}, by means of the line-drop compensator (LDC).

The LDC has R and X settings which are both adjustable within the approximate range from 0 to 24 Ω [often called volts because the current transformers (CTs) used with regulators have 1-A secondaries].

The bandwidth (BW) of the VRR is adjustable within the approximate range from $\pm\frac{3}{4}$ to $\pm 1\frac{1}{2}$ V based on 120 V. The time delay (TD) is adjustable between about 10 and 120 s.

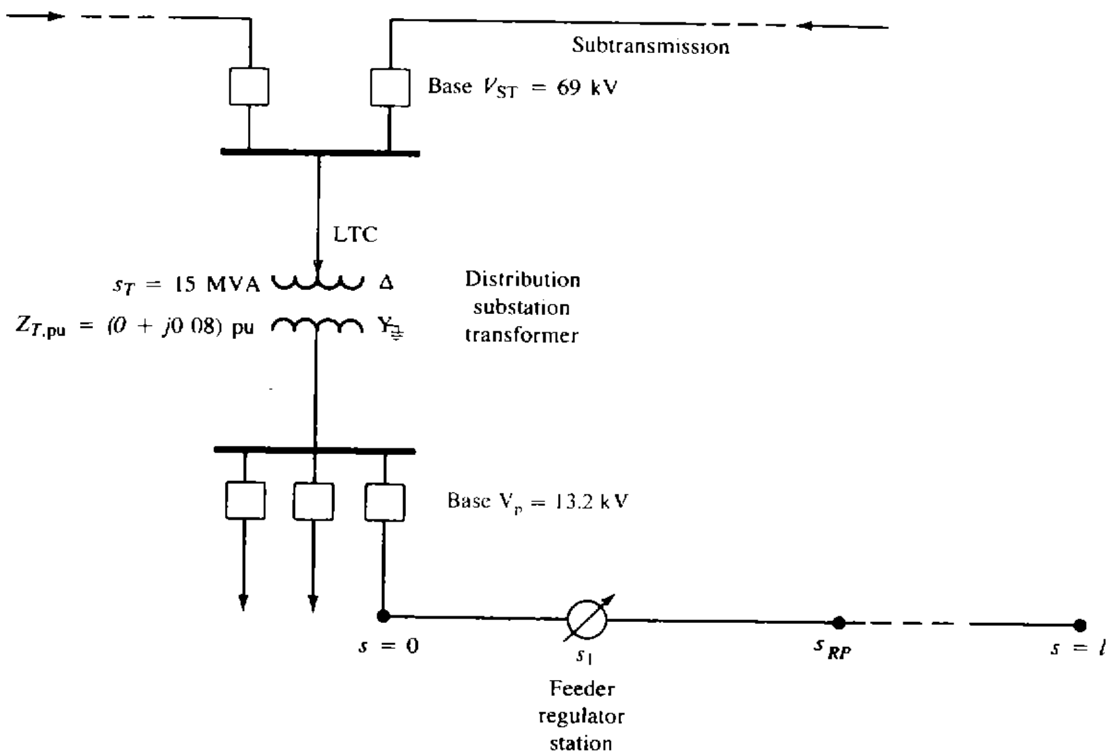


Figure 9-11

The location of the regulating point (RP) is controlled by the R and X settings of the LDC. If the R and X settings are set to be zero, the regulator regulates the voltage at its local terminal to the setting of the VRR $\pm$ BW. In this example, $s_{RP} = s_1$.

Overloading of step-type feeder regulators ANSI standards provide for regulator overload capacity as listed in Table 9-2 in case the full 10 percent range of regulation is not required. All modern regulators are provided with adjustments to reduce the range to which the motor can drive the tap-changer switching mechanism.

Table 9-2 Overloading of step-type feeder regulators

Reduced range of regulation, %	Percent of normal load current
±10.00	100
± 8.75	110
± 7.50	120
± 6.25	135
± 5.00	160

Good advantage sometimes can be taken of this designed "overload" type of limited-range operation. However, if load growth occurs, both a larger range of regulation and a larger regulator size (kilovoltamperes or current) can be expected to be needed. Table 9-3 gives some typical single-phase regulator sizes.

Substation data Make the following assumptions:

$$\text{Base MVA}_{3\phi} = 15 \text{ MVA}$$

$$\text{Subtransmission base } V_{L\text{-}L} = 69 \text{ kV}$$

$$\text{Primary base } V_{L\text{-}L} = 13.2 \text{ kV}$$

The substation transformer is rated 15 MVA, 69 to 7.62/13.2-kV grounded-wye and has a per unit impedance ($Z_{T,\text{pu}}$) of $0 + j0.08$ based on its ratings. Its three-phase LTC can regulate ± 10 percent voltage in 32 steps of $\frac{5}{8}$ percent each.

Load flow data Assume that the maximum subtransmission voltage (max V_{ST}) is 72.45 kV or 1.05 pu which occurs during the off-peak period at which the off-peak kilovoltamperage is 0.25 pu with a leading power factor of 0.95. The minimum subtransmission voltage (min V_{ST}) is 69 kV or 1.00 pu which occurs during the peak period at which the peak kilovoltamperage is 1.00 pu with a lagging power factor of 0.85.

Table 9-3 Some typical single-phase regulator sizes

Single-phase kVA	Volts	Amps	CT_P*	PT_N†
25	2500	100	100	20
⋮	⋮	⋮	⋮	⋮
125	2500	500	500	20
38.1	7620	50	50	63.5
57.2	7620	75	75	63.5
76.2	7620	100	100	63.5
114.3	7620	150	150	63.5
167	7620	219	250	63.5
250	7620	328	400	63.5

* Ratio of the current transformer contained within the regulator. (Here, the ratio is the high-voltage-side ampere rating because the low-voltage rating is 1.0 A.)

† Ratio of the potential transformer contained within the regulator. (All potential transformer secondaries are 120 V.)

Voltage data and voltage criteria Assume that the maximum secondary voltage is 125 V or 1.0417 pu V (based on 120 V) and the minimum secondary voltage is 116 V or 0.9667 pu V, and that the maximum voltage drop in secondaries is 0.035 pu V.

Assume that the maximum primary voltage (max V_P) is 1.0417 pu V at zero load and that, at annual peak load, the maximum primary voltage is 1.0767 pu V (1.0417 + 0.035) considering the nearest secondary to the regulator and the minimum primary voltage is 1.0017 pu V (0.9667 + 0.035) considering the most remote secondary.

Feeder data Assume that the annual peak load is 4000 kVA, at a lagging power factor of 0.85, and is distributed uniformly along the 10-mi-long feeder main. The main has 266.8-kcmil AACs (all-aluminum conductors) with 37 strands and 53-in geometric mean spacing. Use 3.88×10^{-6} pu VD/(kva · mi) at 0.85 lagging power factor as the K constant.

Assume that the substation transformer LTC is used for bus voltage regulation. Use a BW of ± 1.0 V or $\frac{1}{120} = 0.0083$ pu V. Also use rounded figures of 1.075 and 1.000 pu V for the maximum and the minimum primary voltages at peak load, respectively.

(*a*) Specify the setting of the VRR for the highest allowable primary voltage (V_P), bandwidth being considered, then round the setting to a convenient number.

(*b*) Find the maximum number of steps of buck and boost which will be required.

(*c*) Sketch voltage profiles of the feeder being considered for zero load and for the annual peak load. Label the significant voltage values on the curves.

SOLUTION

(*a*) Since the LDC of the regulator is not used,

$$R_{\text{set}} = 0 \qquad \text{and} \qquad X_{\text{set}} = 0$$

Therefore, the setting of the VRR for the highest allowable primary voltage, bandwidth being considered, occurs at the zero load and is

$$\begin{aligned} \text{VRR} &= (V_P)_{\max} - \text{BW} \\ &= 1.0417 - 0.0083 \\ &= 1.0334 \text{ pu V} \\ &\cong 1.035 \text{ pu V} \\ &= 124.2 \text{ V} \end{aligned}$$

(*b*) To find the maximum number of buck and boost which will be required, the highest allowable primary voltages at off-peak and on-peak have to be found. Therefore, at off-peak,

$$\bar{V}_{P,\text{pu}} = \bar{V}_{\text{ST},\text{pu}} - \bar{I}_{P,\text{pu}} \times \bar{Z}_{T,\text{pu}} \tag{9-12}$$

where $V_{\text{ST, pu}}$ = per unit subtransmission voltage at primary side of substation transformer

$= 1.05\angle 0^\circ$ pu V

$I_{P,\text{pu}}$ = per unit no-load primary current at substation (transformer)

$= 0.2381$ pu A

$Z_{T,\text{pu}}$ = per unit impedance of substation transformer

$= 0 + j0.08$ pu Ω

Therefore

$$\begin{aligned}\mathbf{V}_{P,\text{pu}} &= 1.05 - (0.2381)(\cos\theta + j\sin\theta)(0 + j0.08)\\ &= 1.05 - (0.2381)(0.95 + j0.3118)(0 + j0.08)\\ &= 1.0589 \text{ pu V}\end{aligned}$$

whereas, at on-peak,

$$\begin{aligned}V_{P,\text{pu}} &= 1.0 - (1.00)(0.85 - j0.53)(0 + j0.08)\\ &= 0.9602 \text{ pu V}\end{aligned}$$

Since the LTC of the substation can regulate ± 10 percent voltage in 32 steps of $\frac{5}{8}$ percent volts (or 0.00625 pu V) each, the maximum number of steps of buck required, at off-peak, is

$$\text{No. of steps} = \frac{V_{P,\text{pu}} - \text{VRR}_{\text{pu}}}{0.00625} \tag{9-13}$$

$$= \frac{1.0589 - 1.035}{0.00625}$$

$$\cong 3 \text{ or } 4 \text{ steps}$$

and the maximum number of steps of boost required, at peak, is

$$\text{No. of steps} = \frac{V_{P,\text{pu}} - \text{VRR}_{\text{pu}}}{0.00625} \tag{9-14}$$

$$= \frac{1.035 - 0.9602}{0.00625}$$

$$\cong 12 \text{ steps}$$

(*c*) To sketch voltage profiles of the primary feeder for the annual peak load, the total voltage drop of the feeder has to be known. Therefore

$$\sum \text{VD}_{\text{pu}} = K \times S \times \frac{l}{2} \tag{9-15}$$

$$= (3.88 \times 10^{-6})(4000 \text{ kVA})\left(\frac{10 \text{ mi}}{2}\right)$$

$$= 0.0776 \text{ pu V}$$

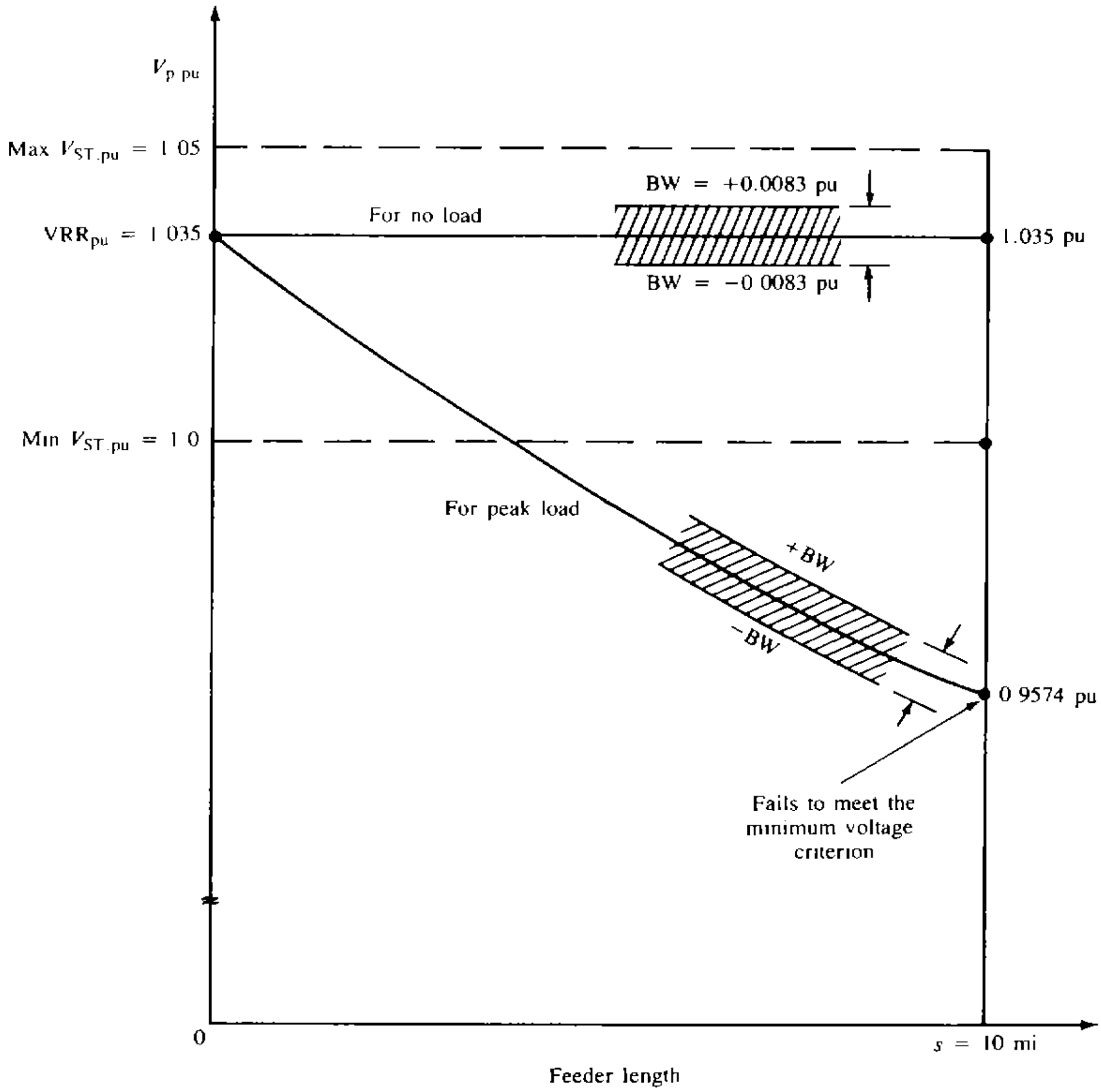


Figure 9-12 Feeder voltage profile.

and thus the minimum primary-feeder voltage at the end of the 10-mi feeder, as shown in Fig. 9-12, is

$$\begin{aligned}\text{Min } V_{P,\text{pu}} &= \text{VRR}_{\text{pu}} - \sum \text{VD}_{\text{pu}} \qquad (9\text{-}16)\\ &= 1.035 - 0.0776\\ &= 0.9574 \text{ pu V}\end{aligned}$$

At the annual peak load, the rounded voltage criteria are

$$\begin{aligned}\text{Max } V_{P,\text{pu}} &= 1.075 - \text{BW}\\ &= 1.075 - 0.0083\\ &= 1.0667 \text{ pu V}\end{aligned}$$

and

$$\begin{aligned}\text{Min } V_{P,\text{pu}} &= 1.00 + \text{BW}\\ &= 1.00 + 0.0083\\ &= 1.0083 \text{ pu V}\end{aligned}$$

At no-load, the rounded voltage criteria are

$$\begin{aligned}\text{Max } V_{P,\text{pu}} &= 1.0417 - \text{BW} \\ &= 1.0417 - 0.0083 \\ &\cong 1.035 \text{ pu V}\end{aligned}$$

and

$$\text{Min } V_{P,\text{pu}} = 1.0083 \text{ pu V}$$

As can be seen from Fig. 9-12, the minimum primary-feeder voltage at the end of the 10-mi feeder fails to meet the minimum voltage criterion at the annual peak load. Therefore a voltage regulator has to be used.

Example 9-2 Use the information and data given in Example 9-1 and locate the voltage regulator, that is, determine the s_1 distance at which the regulator must be located as shown in Fig. 9-11, for the following two cases, where the peak-load primary-feeder voltage ($V_{P,\text{pu}}$) at the input to the regulator is

(*a*) $V_{P,\text{pu}} = 1.010$ pu V
(*b*) $V_{P,\text{pu}} = 1.000$ pu V
(*c*) What is the advantage of part *a* over part *b*, or vice versa?

SOLUTION

(*a*) When $V_{P,\text{pu}} = 1.010$ pu V, the associated voltage drop at the distance s_1, as shown in Fig. 9-13, is

$$\begin{aligned}\text{VD}_{s_1} &= \text{VRR}_{\text{pu}} - V_{P,\text{pu}} \qquad (9\text{-}17) \\ &= 1.035 - 1.01 \\ &= 0.025 \text{ pu V}\end{aligned}$$

From Example 9-1, the total voltage drop of the feeder is

$$\sum \text{VD}_{\text{pu}} = 0.0776 \text{ pu V}$$

Therefore, the distance s_1 can be found from the following parabolic formula for the uniformly distributed load:

$$\frac{\text{VD}_{s_1}}{\sum \text{VD}_{\text{pu}}} = \frac{s_1}{l}\left(2 - \frac{s_1}{l}\right) \qquad (9\text{-}18)$$

or

$$\frac{0.025}{0.0776} = \frac{s_1}{10}\left(2 - \frac{s_1}{10}\right)$$

from which the following quadratic equation can be obtained,

$$s_1^2 - 20s_1 + 32.2165 = 0$$

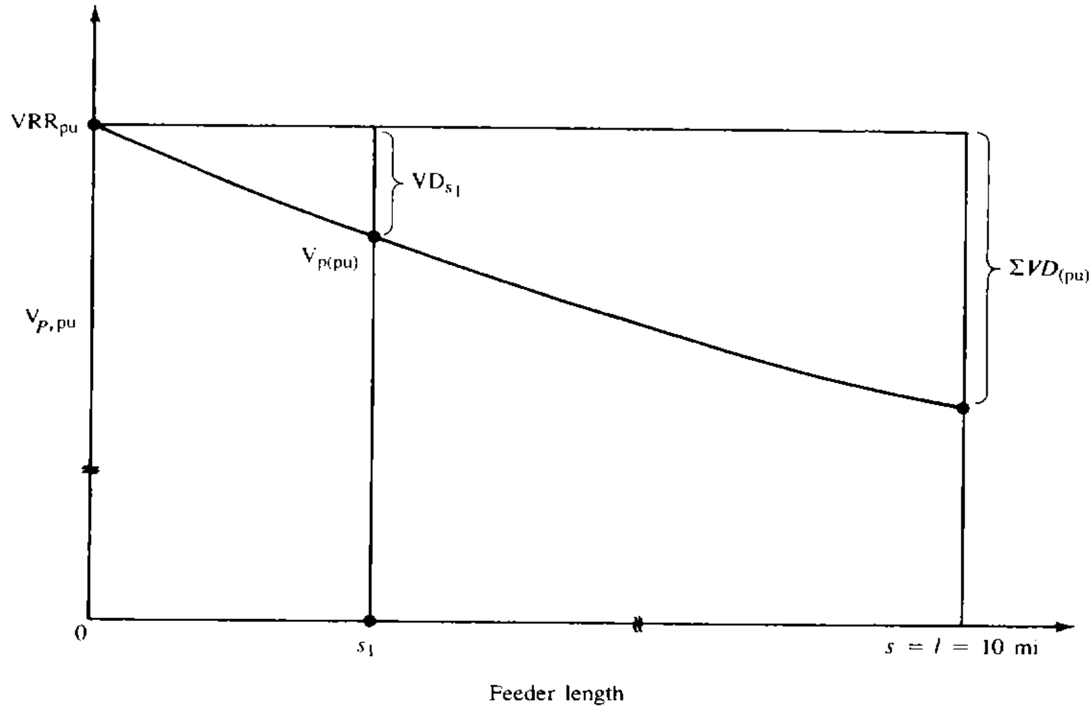


Figure 9-13

which has two solutions, namely, 1.75 and 18.23 mi. Therefore, the distance s_1, taking the acceptable answer, is 1.75 mi.

(*b*) When $V_{P,\text{pu}} = 1.00$ pu V, the associated voltage drop at the distance s_1 is

$$\begin{aligned} \text{VD}_{s_1} &= \text{VRR}_{\text{pu}} - V_{P,\text{pu}} \\ &= 1.035 - 1.00 \\ &= 0.035 \text{ pu V} \end{aligned}$$

Therefore, from Eq. (9-18),

$$\frac{0.035}{0.0776} = \frac{s_1}{10}\left(2 - \frac{s_1}{10}\right)$$

or

$$s_1^2 - 20s_1 + 45.1031 = 0$$

which has two solutions, namely, 2.6 and 17.4 mi. Thus, taking the acceptable answer, the distance s_1 is 2.6 mi.

(*c*) The advantage of part *a* over part *b* is that it can compensate for future growth. Otherwise, the $V_{P,\text{pu}}$ might be less than 1.00 pu V in the future.

Example 9-3 Assume that the peak-load primary-feeder voltage at the input to the regulator is 1.010 pu V as given in Example 9-2. Determine the necessary minimum kilovoltampere size of each of three single-phase feeder regulators.

SOLUTION From Example 9-2, the distance s_1 is found to be 1.75 mi. Previously, the annual peak load and the standard regulation range have been given as 4000 kVA and ± 10 percent, respectively.

The uniformly distributed three-phase load at s_1 is

$$S_{3\phi}\left(1 - \frac{s_1}{l}\right) = 4000\left(1 - \frac{1.75}{10.00}\right)$$

$$= 3300 \text{ kVA}$$

Therefore the single-phase load at s_1 is

$$\frac{3300 \text{ kVA}}{3} = 1100 \text{ kVA}$$

Since the single-phase regulator kilovoltampere rating is given by

$$S_{reg} = \frac{(\% R_{max})S_{ckt}}{100} \tag{9-19}$$

where S_{ckt} is the circuit kilovoltamperage, then

$$S_{reg} = \frac{10 \times 1100 \text{ kVA}}{100} = 110 \text{ kVA}$$

Thus, from Table 9-3, the corresponding minimum kilovoltampere size of the regulator size can be found as 114.3 kVA.

Example 9-4 Use the distance of $s_1 = 1.75$ mi found in Example 9-2 and assume that the distance of the regulating point is equal to s_1, that is, $s_{RP} = s_1$, or, in other words, the regulating point is located at the regulator station, and determine the following.

(*a*) Specify the best settings for the LDC's R and X, and for the VRR.
(*b*) Sketch voltage profiles for zero load and for the annual peak load. Label significant voltage values on the curves.
(*c*) Are the primary-feeder voltage ($V_{P,\text{pu}}$) criteria met?

SOLUTION

(*a*) The $X_{RP} = s_1$ means that the regulating point is located at the feeder regulator station. Therefore, the best settings for the LDC of the regulator are when settings for both R and X are zero and

$$\text{VRR}_{\text{pu}} = V_{\text{RP, pu}} = 1.035 \text{ pu V}$$

(*b*) The voltage drop occurring in the feeder portion between the regulating point and the end of the feeder is

$$\begin{aligned} \mathrm{VD}_{\mathrm{pu}} &= K \times S \times \frac{l}{2} \\ &= (3.88 \times 10^{-6})(3300)(\tfrac{8\cdot 25}{2}) \\ &= 0.0528 \text{ pu V} \end{aligned}$$

Thus the primary-feeder voltage at the end of the feeder for the annual peak load is

$$\begin{aligned} V_{P,\,10\,\mathrm{mi}} &= 1.035 - 0.0528 \\ &= 0.9809 \text{ pu V} \end{aligned}$$

Note that the $V_{P,\,\mathrm{pu}}$ used at the regulator point is the no-load value rather than the annual peak-load value. If, instead, the 1.0667 pu value is used, then, for example, television sets of those customers located at the vicinity of the regulating point might be damaged during the off-peak periods because of the too-high V_{RP} value.

As can be seen from Fig. 9-14, the peak-load voltage profile is not in linear but in parabolic shape. The voltage-drop value for any given point s between the substation and the regulator station can be calculated from

$$\mathrm{VD}_s = K\left(S_{3\phi} - \frac{S_{3\phi} \times s}{l}\right)s + K\left(\frac{S_{3\phi} \times s}{l}\right)\frac{s}{2} \qquad \text{pu V} \qquad (9\text{-}20)$$

where K = percent voltage drop per kilovoltampere-mile characteristic of feeder
$S_{3\phi}$ = uniformly distributed three-phase annual peak load, kVA
l = primary feeder length, mi
s = distance from substation, mi

Therefore, from Eq. (9-20),

$$\mathrm{VD}_s = 3.88 \times 10^{-6}\left(4000 - \frac{4000s}{10}\right)s + 3.88 \times 10^{-6}\left(\frac{4000s}{10}\right)\frac{s}{2} \qquad \text{pu V} \qquad (9\text{-}21)$$

For various values of s the associated values of the voltage drops and $V_{P,\,\mathrm{pu}}$ can be found, as given in Table 9-4.

The voltage-drop value for any given point s between the substation and the regulator station can be calculated from

$$\mathrm{VD}_s = I(r \cdot \cos\theta + \psi \cdot \sin\theta)s\left(1 - \frac{s}{2l}\right) \qquad \text{V} \qquad (9\text{-}22)$$

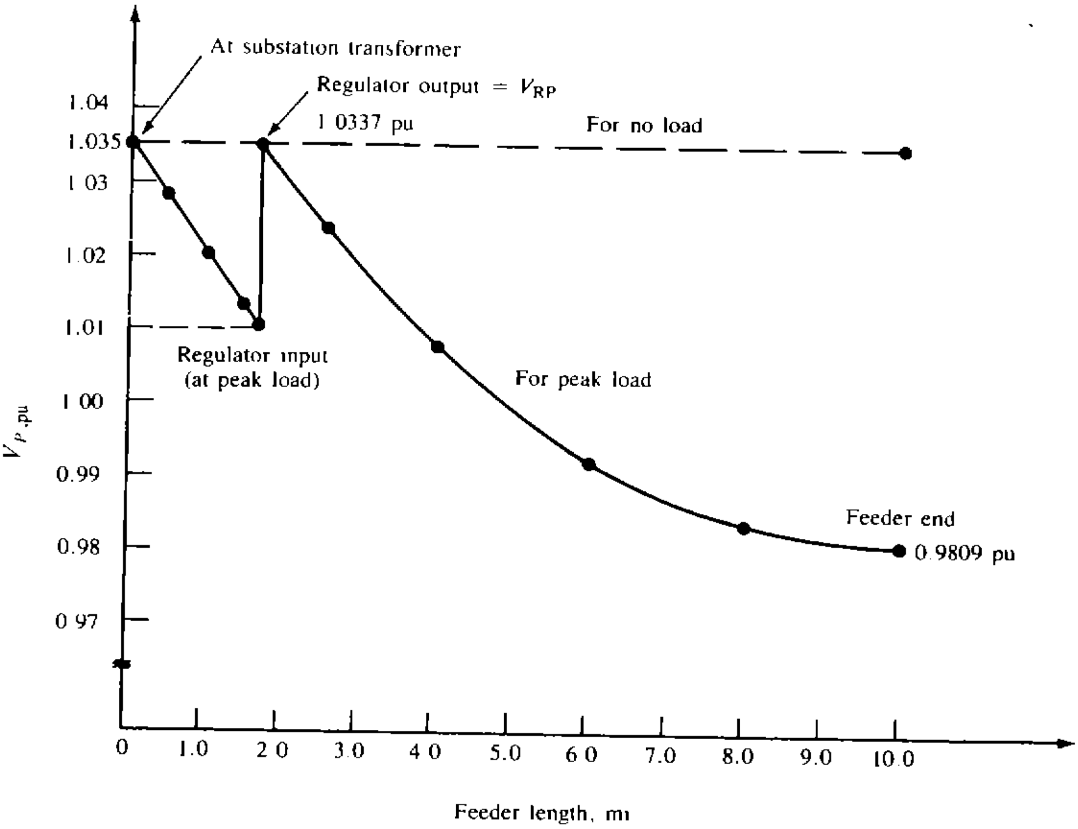


Figure 9-14 Feeder voltage profiles for zero load and for the annual peak load.

where

$$I_L = \text{load current in feeder at substation end}$$

$$= \frac{S_{3\phi}}{\sqrt{3} \times V_{L\text{-}L}} \quad \text{A} \tag{9-23}$$

r = resistance of feeder main, Ω/mi per phase
ψ = reactance of feeder main, Ω/mi per phase,

Therefore the voltage drop in per units can be found as

$$\text{VD}_s = \frac{\text{VD}_s}{V_{L\text{-}N}} \quad \text{pu V} \tag{9-24}$$

Table 9-4 For annual peak load

s, mi	VD_s, pu V	$V_{P,\text{pu}}$, pu V
0.0	0.0	1.035
0.5	0.0076	1.0274
1.0	0.0071	1.0203
1.5	0.0068	1.0135
1.75	0.025	1.010

The voltage-drop value for any given point s between the substation and the regulator station can be calculated from the following equation:

$$\mathrm{VD}_s = K\left(S'_{3\phi} - \frac{S'_{3\phi}s}{l - s}\right)s + K\left(\frac{S'_{3\phi}s}{l - s}\right)\frac{s}{2} \qquad \text{pu V} \qquad (9\text{-}25)$$

where $S'_{3\phi}$ = uniformly distributed three-phase annual peak load at distance s_1, kVA

$$= S_{3\phi}\left(1 - \frac{s_1}{l}\right) \qquad \text{kVA}$$

s_1 = distance of feeder regulator station from substation, mi

Therefore, from Eq. (9-25),

$$\mathrm{VD}_s = 3.88 \times 10^{-6}\left(3300 - \frac{3300s}{8.25}\right)s + 3.88 \times 10^{-6}\left(\frac{3300s}{8.25}\right)\frac{s}{2} \qquad \text{pu V} \qquad (9\text{-}26)$$

For various values of s the corresponding values of the voltage drops and $V_{P,\,\mathrm{pu}}$ can be found, as given in Table 9-5.

The voltage profiles for the annual peak load can be obtained by plotting the $V_{P,\,\mathrm{pu}}$ values from Tables 9-4 and 9-5. Of course, since there is no voltage drop at zero load, the $V_{P,\,\mathrm{pu}}$ remains constant at 1.035 pu. Therefore the voltage profile for the zero load is a horizontal line (with zero slope).

(c) The minimum $V_{P,\,\mathrm{pu}}$ criterion of 1.0083 pu V is not met even though the regulator voltage has been set as high as possible without exceeding the maximum voltage criterion of 1.035 pu V.

Example 9-5 Assume that the regulator station is located at the distance s_1 as found in part *a* of Example 9-2, but the RP has been moved to the end of the feeder so that $s_{\mathrm{RP}} = l = 10$ mi.

(a) Determine good settings for the values of VRR, R, and X so that all $V_{P,\,\mathrm{pu}}$ voltage criteria will be met, if possible.

(b) Sketch voltage profiles and label the values of significant voltages, in per unit volts.

Table 9-5 For annual peak load

s, mi	VD_s, pu V	$V_{P,\,\mathrm{pu}}$, pu V
0.00	0.00	1.0337
0.75	0.0092	1.0245
2.25	0.0157	1.0088
4.25	0.0155	0.9933
6.25	0.0093	0.9840
8.25	0.0031	0.9809

SOLUTION

(*a*) From Table A-4 of Appendix A, the resistance at 50°C and the reactance of the 266.8-kcmil AAC with 37 strands are 0.386 and 0.4809 Ω/mi, respectively. From Table A-10, the inductive-reactance spacing factor for the 53-in geometric mean spacing is 0.1802 Ω/mi. Therefore, from Eq. (9-6), the inductive reactance of the feeder conductor is

$$\begin{aligned} x_L &= x_a + x_d \\ &= 0.4809 + 0.1802 \\ &= 0.6611\ \Omega/\text{mile} \end{aligned}$$

From Eq. (9-3) and (9-5),

$$\begin{aligned} R_{\text{eff}} &= r_a \times \frac{l - s_1}{2} \\ &= 0.386 \times \frac{8.25}{2} \\ &= 1.5923\ \Omega \end{aligned}$$

and

$$\begin{aligned} X_{\text{eff}} &= X_L \times \frac{l - s_1}{2} \\ &= 0.6611 \times \frac{8.25}{2} \\ &= 2.7270\ \Omega \end{aligned}$$

From Table 9-3, for the regulator size of 114.3 kVA found in Example 9-3, the primary rating of the current transformer and the potential transformer ratio are 150 and 63.5, respectively. Therefore, from Eq. (9-2) and (9-4), the R and X dial settings can be found as

$$\begin{aligned} R_{\text{set}} &= \frac{\text{CT}_P}{\text{PT}_N} \times R_{\text{eff}} \\ &= \frac{150}{63.5} \times 1.5923 \\ &= 3.761\ \text{V} \qquad \text{or} \qquad 0.0313\ \text{pu V} \end{aligned}$$

based on 120 V and

$$\begin{aligned} X_{\text{set}} &= \frac{\text{CT}_P}{\text{PT}_N} \times X_{\text{eff}} \\ &= \frac{150}{63.5} \times 2.727 \\ &= 6.442\ \text{V} \qquad \text{or} \qquad 0.0537\ \text{pu V} \end{aligned}$$

Table 9-6

Voltage	Actual voltage pu V		Voltage criteria, pu V	
	At peak load	At zero load	At peak load	At zero load
Max $V_{P,\mathrm{pu}}$	1.0666	1.0138	1.0667	1.0337
Min $V_{P,\mathrm{pu}}$	1.0138	1.0138	1.0083	1.0083

Assume that the voltage at the regulating point (V_{RP}) is arbitrarily set to be 1.0138 pu V using the R and X settings of the LDC of the regulator so that the V_{RP} is always the same for zero load or for the annual peak load. Therefore the output voltage of the regulator for the annual peak load can be found from

$$V_{\mathrm{reg}} = V_{\mathrm{RP}} + \frac{S_{1\phi}/V_{L\text{-}N}(R_{\mathrm{set}} \times \cos\theta + X_{\mathrm{set}} \times \sin\theta)}{\mathrm{CT}_P \times V_B} \quad \text{pu V} \qquad (9\text{-}27)$$

$$= 1.0138 + \frac{\dfrac{1100}{7.62}(3.761 \times 0.85 + 6.442 \times 0.527)}{150 \times 120}$$

$$= 1.0666 \text{ pu V}$$

Here, note that the regulator regulates the regulator output voltage automatically according to the load at any given time in order to maintain the RP voltage at the predetermined voltage value.

Table 9-6 gives the $V_{P,\mathrm{pu}}$ values for the purpose of comparing the actual voltage values against the established voltage criteria for the annual peak and for zero load.

As can be observed from Table 9-6, the primary voltage criteria are met by using the R and X settings.

(*b*) The voltage profiles for the annual peak load and zero load can be obtained by plotting the $V_{P,\mathrm{pu}}$ values from Tables 9-6 and 9-7 [based on Eq. (9-26)], as shown in Fig. 9-15.

Table 9-7

s, mi	VD_s, pu V	$V_{P,\mathrm{pu}}$, pu V
0.00	0.00	1.0666
0.75	0.0092	1.0574
2.25	0.0157	1.0417
4.25	0.0155	1.0262
6.25	0.0093	1.0169
8.25	0.0031	1.0138

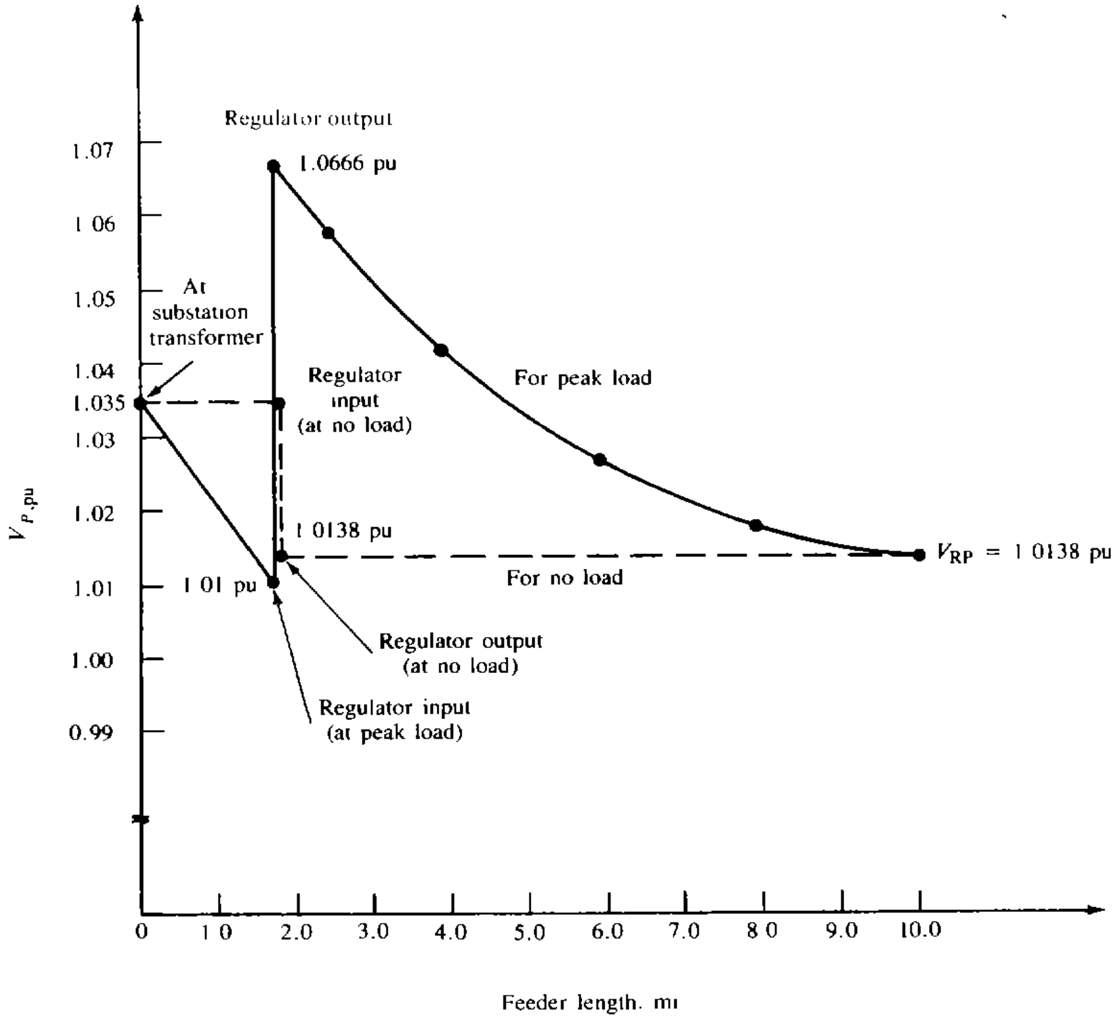


Figure 9-15 Voltage profiles.

Example 9-6 Consider the results of Examples 9-4 and 9-5 and determine the following:

(*a*) The number of steps of buck and boost the regulators will achieve in Example 9-4.

(*b*) The number of steps of buck and boost the regulators will achieve in Example 9-5.

SOLUTION

(*a*) For Example 9-4, the number of steps of buck is

$$\text{No. of steps} = \frac{1.035 - 1.0337}{0.00625}$$

$$= 0.208$$

thus it is either zero or one step. The number of boost is

$$\text{No. of steps} = \frac{1.0337 - 1.010}{0.00625}$$

$$= 3.79$$

therefore it is either three or four steps.

(*b*) For Example 9-5, the number of steps of buck is

$$\text{No. of steps} = \frac{1.035 - 1.0138}{0.00625}$$

$$= 3.39$$

hence it is either three or four steps. The number of steps of boost is

$$\text{No. of steps} = \frac{1.0666 - 1.010}{0.00625}$$

$$= 9.06$$

therefore it is either nine or ten steps.

Example 9-7 Consider the results of Examples 9-4 and 9-5 and answer the following:

(*a*) Can reduced range of regulation be used gainfully in Example 9-4? Explain.

(*b*) Can reduced range of regulation be used gainfully in Example 9-5? Explain.

SOLUTION

(*a*) Yes, the reduced range of regulation can be used gainfully in Example 9-4 since the next-smaller-size regulator, that is, 76.2 kVA, at ± 5 percent regulation range can be selected. This ± 5 percent regulation range would allow the capacity of the regulator to be increased to 160 percent (see Table 9-2) so that

$$1.6 \times 76.2 \text{ kVA} = 121.92 \text{ kVA}$$

which is much larger than the required capacity of 110 kVA. It would allow the use ± 8 steps of buck and boost, which is more than the required one step of buck and four steps of boost.

(*b*) No, the reduced range of regulation cannot be used gainfully in Example 9.5 since the required steps of buck and boost are four and ten, respectively. The reduced range of regulation at ± 6.25 percent would provide the ± 10 steps of buck and boost, but it would allow the capacity of the regulator to be increased only up to 135 percent (see Table 9-2) so that

$$1.35 \times 76.2 \text{ kVA} = 102.87 \text{ kVA}$$

which is smaller than the required capacity of 110 kVA.

Example 9-8 Figure 9-16 shows a one-line diagram of a primary feeder supplying an industrial customer. The nominal voltage at the utility substation low-voltage bus is 7.2/13.2-kV three-phase wye-grounded. The voltage regulator bank is made up of three single-phase step-type voltage regulators with a potential transformer ratio of 63.5 (7620:120).

The industrial customer's bus is located at the end of a 3-mi primary line with a resistance of 0.30 Ω/mi and an inductive reactance of 0.80 Ω/mi.

The customer's transformer is rated 5000 kVA in three-phase with a 12,800-V primary connected in delta (taps in use) and a 2400/4160-V secondary connected in grounded-wye. The transformer impedance is $0 + j0.05$ pu Ω based on the rated kilovoltamperes and tap voltages in use. Assume that the bases to be used are 5000 kVA, 2400/4160 V, and 7390/12,800 V.

Assume that the customer asks that the low-voltage bus be regulated to 2450/4244 V and determine the following:

(*a*) Find the necessary setting of the voltage-setting dial of the VRR of each single-phase regulator in use.

(*b*) Assume that the ratio of the current transformer in each regulator is 250:1 A and find the necessary *R* and *X* dial settings of LDCs.

Solution

(*a*) The voltage at the RP which is located at the customer's bus is

$$V_{RP} = \frac{2450 \text{ V}}{2400 \text{ V}}$$

$$= 1.02083 \text{ pu V}$$

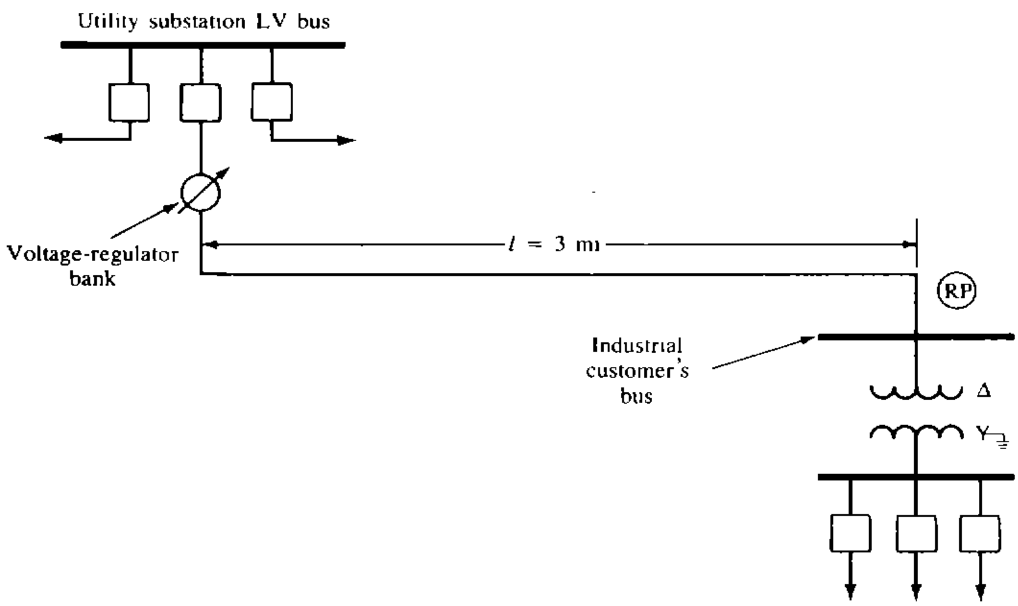


Figure 9-16

therefore

$$\text{VRR} = \frac{7390}{7620} \times 1.02083$$

$$= 0.99 \text{ pu V} \qquad \text{or} \qquad 7620 \text{ V}$$

Thus

$$\text{VRR}_{\text{set}} = \frac{7620}{\text{PT}_N} \times 0.99$$

$$= \frac{7620}{63.5} \times 0.99$$

$$= 120 \times 0.99$$

$$= 118.8 \text{ V}$$

or, alternatively,

$$\text{VRR}_{\text{set}} = V_{\text{RP}} \times V_{\text{B, sec}} \tag{9-28}$$

$$= 1.02083 \times \frac{12{,}800}{\sqrt{3} \times 63.5}$$

$$\cong 118.8 \text{ V}$$

(*b*) The applicable impedance base is

$$Z_B = \frac{(\text{kV}_{L\text{-}L})^2}{\text{MVA}}$$

$$= \frac{(12.8 \text{ kV})^2}{5 \text{ MVA}}$$

$$= 32.768 \ \Omega$$

therefore the transformer impedance is

$$\bar{Z}_T = \bar{Z}_{T,\text{pu}} \times Z_B \tag{9-29}$$

$$= (0 + j0.05) \times 32.768$$

$$= 0 + j1.6384 \ \Omega$$

Since here the R and X settings are determined for only one customer, the resistance and reactance values of the customer's transformer have to be included in the effective resistance and reactance calculations. Therefore

$$R_{\text{eff}} = r \times l + R_T \tag{9-30}$$

$$= 0.3 \times 3 + 0$$

$$= 0.9 \ \Omega$$

and

$$X_{\text{eff}} = x \times l + X_T \tag{9-31}$$
$$= 0.8 \times 3 + 1.6384$$
$$= 4.0384\ \Omega.$$

Thus the R dial setting of the LDC is

$$R_{\text{set}} = \frac{\text{CT}_P}{\text{PT}_N} \times R_{\text{eff}}$$
$$= \frac{250}{63.5} \times 0.9$$
$$= 3.5433\ \Omega$$

and the X dial setting is

$$X_{\text{set}} = \frac{250}{63.5} \times 4.0384$$
$$= 15.8992\ \Omega$$

Example 9-9 Consider the 10-mi feeder of Example 9-1. Assume that the substation has a bus-voltage regulator (BVR) with transformer LTC and that the primary feeder voltage (V_P) has been set on the VRR to be 1.035 pu V.

Assume that the main feeder is made up of 266.8-kcmil with 37 strands and 53-in geometric mean spacing. It has been found in Example 9-5 that the main feeder has an inductive reactance of 0.661 Ω/mi per conductor. Assume that the annual peak load is 4000 kVA at a lagging power factor of 0.85 and distributed uniformly along the main or, in other words, the uniformly distributed load is

$$S_{3\phi} = P_L + jQ_L = 3400 + j2100\ \text{kVA}$$

Assume that, as found in Example 9-1, the total voltage drop of the feeder is

$$\sum \text{VD}_{\text{pu}} = \text{VD}_{l,\,\text{pu}}$$
$$= 0.0776\ \text{pu V}$$

and the reactive load factor is 0.40. Use the given data and determine the following

(*a*) Design a fixed, i.e., nonswitched (NSW), capacitor bank for the maximum loss reduction.

(*b*) Sketch the voltage profiles when there is no capacitor (N/C) bank installed and when there is a fixed-capacitor bank, that is, Q_{NSW} installed.

(*c*) Add a switched-capacitor bank for voltage control on the feeder. Locate the switched-capacitor bank, that is, Q_{SW}, at the end of the feeder for the maximum voltage rise. Size Q_{SW} so that the new voltage at the end of the feeder at the annual peak load is 1.000 pu V. Sketch the associated voltage profiles.

SOLUTION

(*a*) From Fig. 8-31, the corrective ratio (CR) for the given reactive load factor of 0.40 is found to be 0.27. Therefore the required size of the nonswitched-capacitor bank is

$$\begin{aligned} Q_{\text{NSW}} &= \text{CR} \times Q_L \\ &= (0.27)(2100 \text{ kvar}) \\ &= 567 \text{ kvar per three phase} \end{aligned} \tag{9-32}$$

Thus two single-phase standard 100-kvar-size capacitor units are required to be used on each phase and located on the feeder at a distance of

$$\begin{aligned} s &= \tfrac{2}{3} \times l \\ &= \tfrac{2}{3} \times 10 \\ &\cong 6.67 \text{ mi} \end{aligned}$$

for the optimum result, as shown in Fig. 9-17.

Therefore the per unit voltage rise (VR_{pu}) due to the nonswitched-capacitor bank is

$$\begin{aligned} \text{VR}_{\text{pu}} &= \frac{Q_{\text{NSW}} \times \frac{2}{3} X_L}{1000(\text{kV}_{L\text{-}L})^2} \\ &= \frac{600 \times 4.41}{1000 \times 13.2^2} \\ &= 0.0152 \text{ pu V} \end{aligned} \tag{9-33}$$

(*b*) When there is no capacitor bank installed, the voltage drop for the uniformly distributed load at the distance of $s = \frac{2}{3}l$ can be found from Eq. (9-18) as

$$\frac{\text{VD}_{s,\text{pu}}}{\sum \text{VD}_{\text{pu}}} = \frac{s}{l}\left(2 - \frac{s}{l}\right) \tag{9-18}$$

or

$$\frac{\text{VD}_{s,\text{pu}}}{0.0776} = \frac{6.67}{10}\left(2 - \frac{6.67}{10}\right)$$

from which

$$\text{VD}_{s,\text{pu}} = 0.069 \text{ pu V}$$

Therefore the feeder voltage at the $\frac{2}{3}l$ distance is

$$\begin{aligned} V_{s,\text{pu}} &= V_{P,\text{pu}} - \text{VD}_{s,\text{pu}} \\ &= 1.035 - 0.069 = 0.966 \text{ pu V} \end{aligned} \tag{9-34}$$

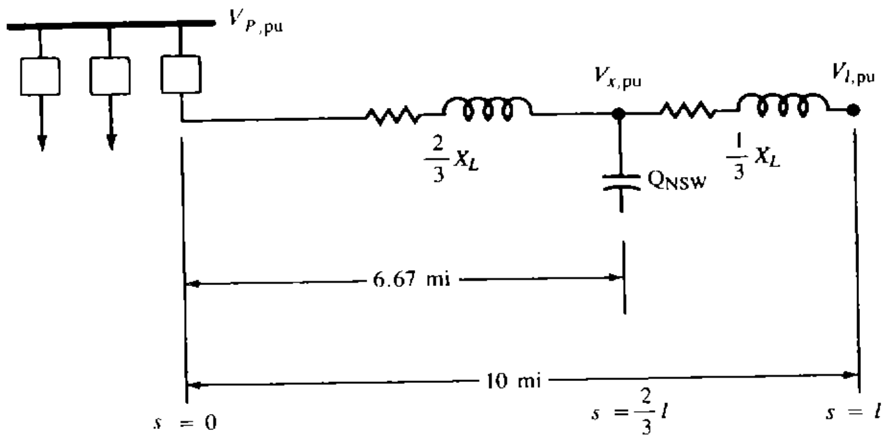


Figure 9-17

On the other hand, when there is a fixed-capacitor bank installed, the new voltage at the $\frac{2}{3}l$ distance due to the voltage rise is

$$\text{New } V_{s,\text{pu}} = V_{s,\text{pu}} + \text{VR}_{\text{pu}} \tag{9-35}$$
$$= 0.966 + 0.0152 = 0.9812 \text{ pu V}$$

When there is no capacitor bank installed, the voltage at the end of the feeder is

$$V_{l,\text{pu}} = V_{P,\text{pu}} - \sum \text{VD}_{\text{pu}} \tag{9-36}$$
$$= 1.035 - 0.0776$$
$$= 0.9574 \text{ pu V}$$

On the other hand, when there is a fixed-capacitor bank installed, the new voltage at the end of the feeder due to the voltage rise is

$$\text{New } V_{l,\text{pu}} = V_{l,\text{pu}} + \text{VR}_{\text{pu}} \tag{9-37}$$
$$= 0.9574 + 0.0152$$
$$= 0.9726 \text{ pu V}$$

The associated voltage profiles are shown in Fig. 9-18.

(*c*) Since the new voltage at the end of the feeder due to the Q_{SW} installation is 1.000 pu V at the annual peak load, the required voltage rise is

$$\text{VR}_{\text{pu}} = \text{new } V_{l,\text{pu}} - V_{l,\text{pu}} \tag{9-38}$$
$$= 1.000 - 0.9726$$
$$= 0.0274 \text{ pu V}$$

Therefore the required size of the switched-capacitor bank can be found from

$$\text{VR}_{\text{pu}} = \frac{Q_{3\phi,\text{SW}} \times X_L}{1000\,(\text{kV}_{L\text{-}L})^2} \text{ pu V} \tag{9-39}$$

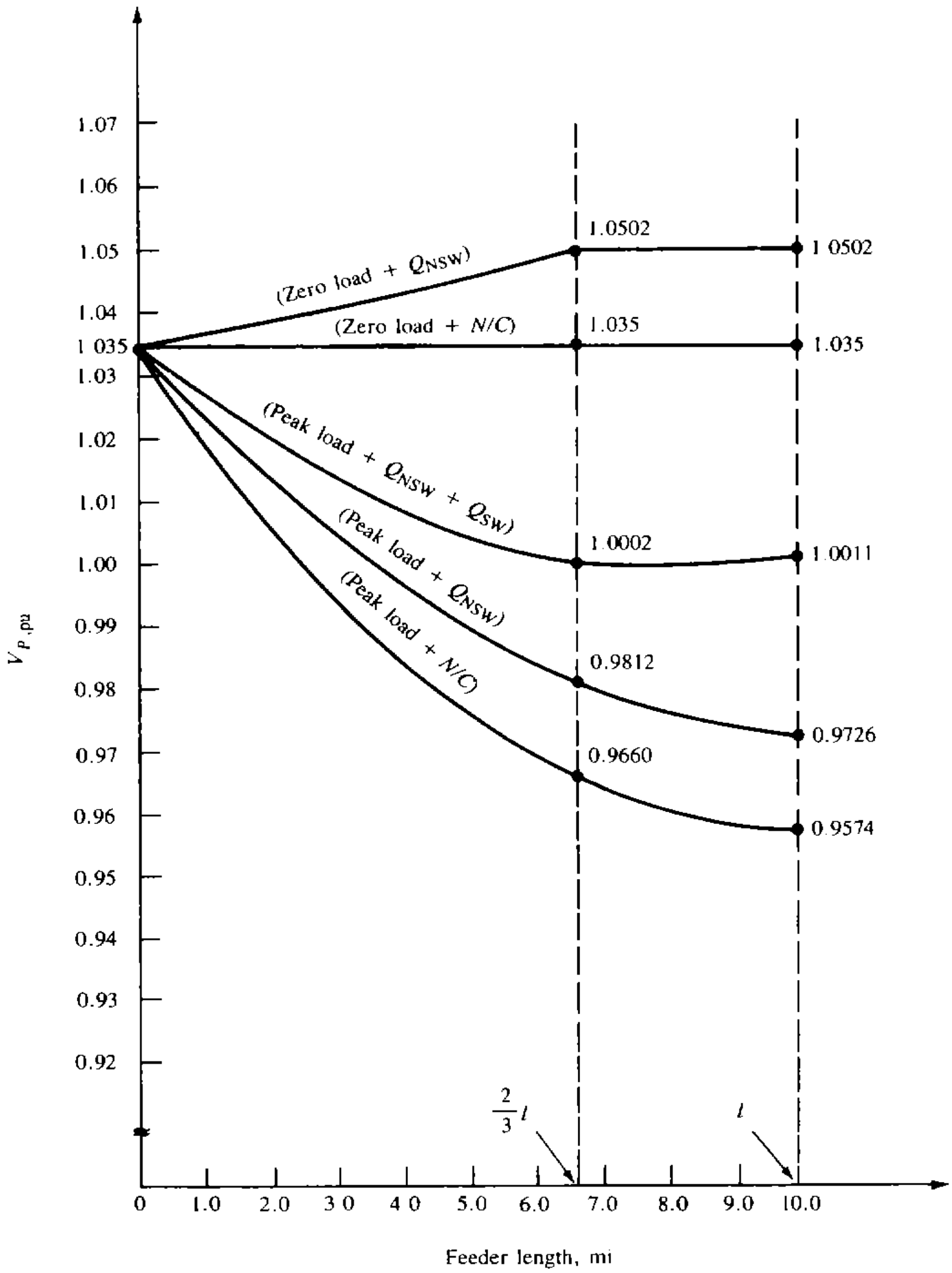


Figure 9-18 Voltage profiles.

or

$$Q_{3\phi,\text{SW}} = \frac{1000\,(\text{kV}_{L\text{-}L})^2\,\text{VR}_{\text{pu}}}{X_L} \tag{9-40}$$

$$= \frac{1000 \times 13.2^2 \times 0.0274}{6.611}$$

$$= 722.2 \text{ kvar}$$

Hence the possible combinations of the single-phase standard-size capacitor units to make up the capacitor bank are:

(i) Fifteen single-phase standard 50-kvar capacitor units, for a total of 750 kvar.
(ii) Six single-phase standard 100-kvar capacitor units, for a total of 600 kvar.
(iii) Nine single-phase standard 100-kvar capacitor units, for a total of 900 kvar.

For example, assume that the first combination, that is,

$$Q_{3\phi,\mathrm{SW}} = 750 \text{ kvar}$$

is selected. The resultant new voltage rises at the distance of l and $s = \frac{2}{3}l$ are

$$\mathrm{VR}_{l,\mathrm{pu}} = \mathrm{VR}_{\mathrm{pu}} \times \frac{\text{selected } Q_{\mathrm{SW}}}{\text{required } Q_{\mathrm{SW}}} \quad \text{pu V} \tag{9-41}$$

$$= 0.0274 \times \frac{750 \text{ kvar}}{721 \text{ kvar}}$$

$$= 0.0285 \text{ pu V}$$

and

$$\mathrm{VR}_{s,\mathrm{pu}} = \tfrac{2}{3}\mathrm{VR}_{l,\mathrm{pu}} \quad \text{pu V} \tag{9-42}$$

$$= \tfrac{2}{3} \times 0.0285$$

$$= 0.0190 \text{ pu V}$$

Therefore, at the peak load when both the nonswitched- and the switched-capacitor banks are on, the voltage at two-thirds of the line and at the end of the line are

$$V_{s,\mathrm{pu}} = \text{new } V_{s,\mathrm{pu}} + \mathrm{VR}_{s,\mathrm{pu}} \quad \text{pu V} \tag{9-43}$$

$$= 0.9812 + 0.0190$$

$$= 1.0002 \text{ pu V}$$

and

$$V_{l,\mathrm{pu}} = \text{new } V_{l,\mathrm{pu}} + \mathrm{VR}_{l,\mathrm{pu}} \quad \text{pu V} \tag{9-44}$$

$$= 0.9726 + 0.0285$$

$$= 1.0011 \text{ pu V}$$

respectively. At the zero load when there is no capacitor bank installed, the voltage at two-thirds of the line and at the end of the line are the same and equal to 1.035 pu V. The associated voltage profiles are shown in Fig. 9-18.

Example 9-10 Consider Example 9-8 and assume that the industrial load at the annual peak is 5000 kVA at 80 percent lagging power factor. Assume that the customer wishes to add some additional load, is currently paying a monthly power-factor penalty, and the single-phase voltage regulators are approaching full boost. Select a proper three-phase capacitor-bank size (in terms of the multiples of three-phase 150-kvar capacitor units) to be connected to the 4-kV bus that will (1) produce a voltage rise of at least 0.020 pu V on the 4-kV bus and (2) raise the on-peak power factor of the present load to at least 88 percent lagging power factor.

SOLUTION The presently existing load is

$$S_{3\phi} = 5000\underline{/36.87^\circ} \text{ kVA}$$

or

$$S_{3\phi} = 4000 + j3000 \text{ kVA}$$

at 80 percent lagging power factor. When a properly sized capacitor bank is connected to the bus to improve the on-peak power factor to 88 percent, the real power portion will be the same but the reactive power portion will be different. In other words,

$$|S|\underline{/28.36^\circ} = 4000 + jQ_{L,\text{new}}$$

from which

$$\tan 28.36^\circ = \frac{Q_{L,\text{new}}}{4000}$$

therefore

$$\begin{aligned} Q_{L,\text{new}} &= 4000 \times \tan 28.36^\circ \\ &= 4000 \times 0.5397 \\ &= 2158.97 \text{ kvar} \end{aligned}$$

and hence the magnitude of the new apparent power is

$$|S| = 4545.45 \text{ kVA}$$

Therefore the minimum size of the capacitor bank required to raise the load power factor to 0.88 is

$$\begin{aligned} Q_{3\phi} &= 3000 - 2158.97 \\ &= 841.03 \text{ kvar} \end{aligned}$$

Thus, if a 900-kvar-capacity bank is used, the resultant voltage rise from Eq. (9-39) is

$$\text{VR}_{\text{pu}} = \frac{Q_{3\phi} \times X_L}{1000\,(\text{kV}_{B,L\text{-}L})^2}$$

where

$$
\begin{aligned}
X_L &= X_{\text{line}} + X_T \\
&= 0.8 \times 3 + 1.6384 \\
&= 4.0384\ \Omega
\end{aligned}
$$

hence

$$
\begin{aligned}
\text{VR}_{\text{pu}} &= \frac{900 \times 4.0384}{1000 \times 12.8^2} \\
&= 0.0222 \text{ pu V}
\end{aligned}
$$

which is larger than the given voltage-rise criterion of 0.020 pu V. Therefore, it is proper to install six 150-kvar three-phase units as the capacitor bank to meet the criteria.

9-6 VOLTAGE FLUCTUATIONS

In general, voltage fluctuations and lamp flicker on distribution systems are caused by a customer's utilization apparatus. Lamp flicker can be defined as a sudden change in the intensity of illumination due to an associated abrupt change in the voltage across the lamp. Most flickers are caused by the starting of motors. The large momentary inrush of starting current creates a sudden dip in the illumination level provided by incandescent and/or fluorescent lamps since the illumination is a function of voltage.

Therefore a utility company tries not to endanger other customers, from the quality-of-service point of view, in the process of serving a new customer who could generate excessive flicker by the company's standards. Thus the distribution circuits are checked to determine whether or not the flicker caused by the new customer's load in addition to the existing flicker-generating loads will meet the company's voltage-fluctuating standards. The decision to serve such a customer is based on the load location, load type, service voltage, frequency of the motor starts, the motor's horsepower rating, and the motor's NEMA code considerations. Momentary, or pulsating, loads are considered for both their starting requirements and the change in power requirements per unit time. Often, more-severe flickers result due to the running operation of pulsating loads than to the starting loads. Usually, the pulsating loads, such as grinders, hammer mills, rock crushers, reciprocating pumps, and arc welders, require additional study.

The annoyance created by lamp flicker is a very subjective matter and differs from person to person. However, in certain cases, the flicker can be very objectionable and can create great discomfort. In general, the degree of objection to lamp flicker is a function of the frequency of its occurrence and the rate of change of the voltage dip. The voltage changes resulting in lamp flicker can be either cyclic or noncyclic in nature. Usually, cyclic flicker is more objectionable. Figure 9-19 shows a typical curve used by the utilities to determine the amount of voltage flicker to

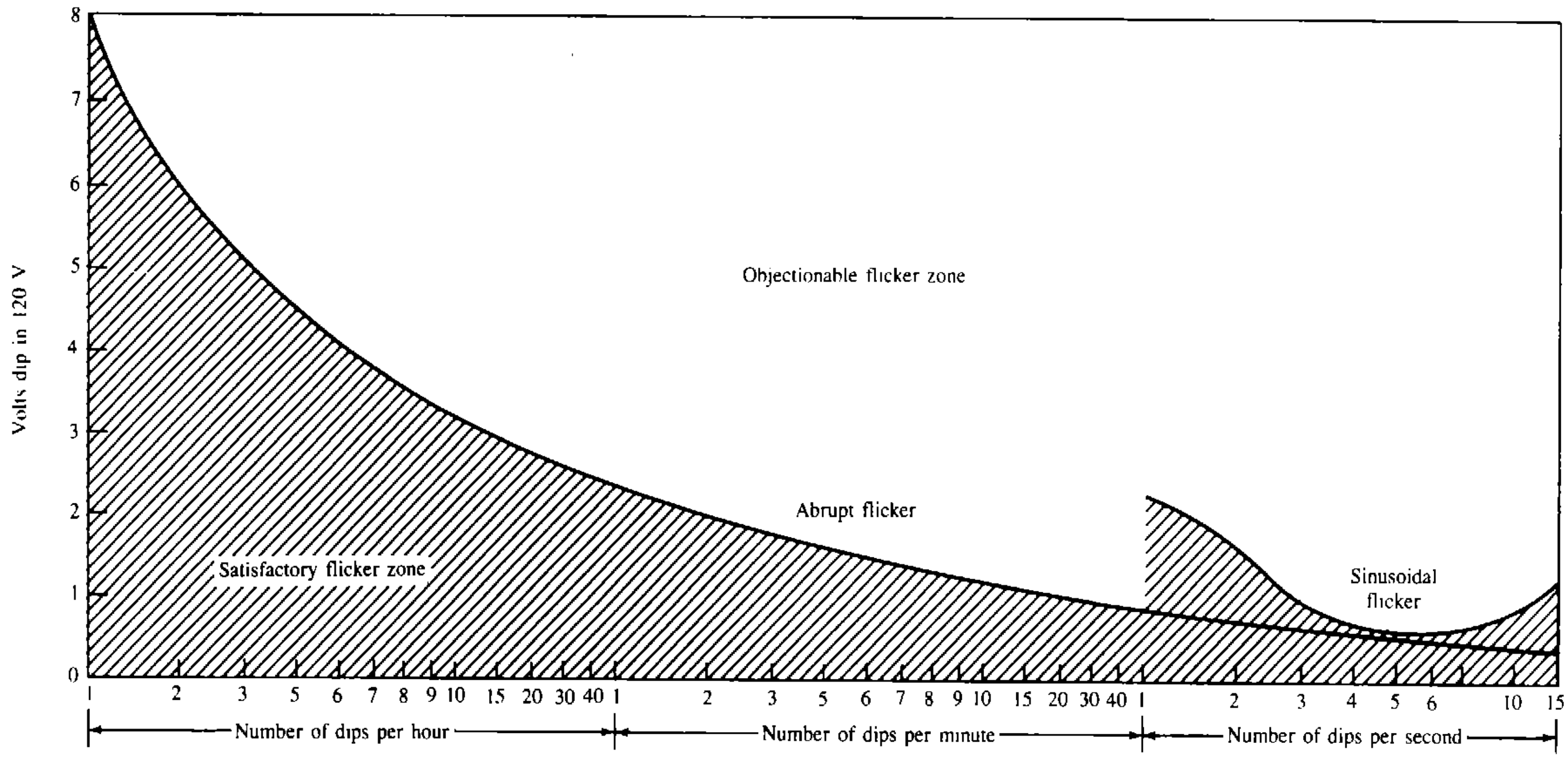


Figure 9-19 Permissible voltage-flicker-limit curve.

be allowed on their system. As indicated in the figure, flicker values located above the curve are likely to be objectionable to lighting customers. For example, from the figure it can be observed that 5-V dips, based on 120 V, are satisfactory to lighting customers as long as the number of dips does not exceed three per hour. Therefore more-frequent dips of this magnitude are in the objectionable-flicker zone; i.e., they are objectionable to the lighting customers. The curve for sinusoidal flicker should be used for the sinusoidal voltage change caused by pump compressors and equipment of similar characteristics. Each utility company develops its own voltage-flicker-limit curve based on its own experiences with customer complaints in the past. Distribution engineers strive to keep voltage flickers in the satisfactory zone by securing compliance with the company's flicker standards and requirements, and by designing new extensions and rebuilds that will provide service within the satisfactory-flicker zone. Flicker due to motor starting can be reduced by the following remedies:

1. Using a motor which requires less kilovoltamperes per horsepower to start.
2. Choosing a low-starting-torque motor if the motor starts under light load.
3. Replacing the large-size motor with a smaller-size motor or motors.
4. Employing motor starters to reduce the motor inrush current at the start.
5. Using shunt or series capacitors to correct the power factor.

As mentioned at the start of this section, distribution engineers try every reasonable means to satisfy the motor-start flicker requirement. After exhausting other alternatives, they may choose to satisfy the flicker condition by installing shunt or series capacitors. Shunt capacitors compensate for the low power factor of the motor during start. They are removed from the circuit when the motor reaches nominal running speed. At start and for a very short time, not to exceed 10 s, capacitors rated at line-to-neutral voltage are often connected line-to-line, i.e., at a voltage greater than their rating by a factor of $\sqrt{3}$. Thus the momentary effective kilovar rating of the capacitors becomes equal to three times the rated kilovars since $(V_{L\text{-}L}/V_{L\text{-}N})^2 = 3$.

On the other hand, if series capacitors are used, they should be installed between the substation transformer and the residential or lighting tap, as shown in Fig. 9-20. Installing the capacitor between the residential load and the fluctuating

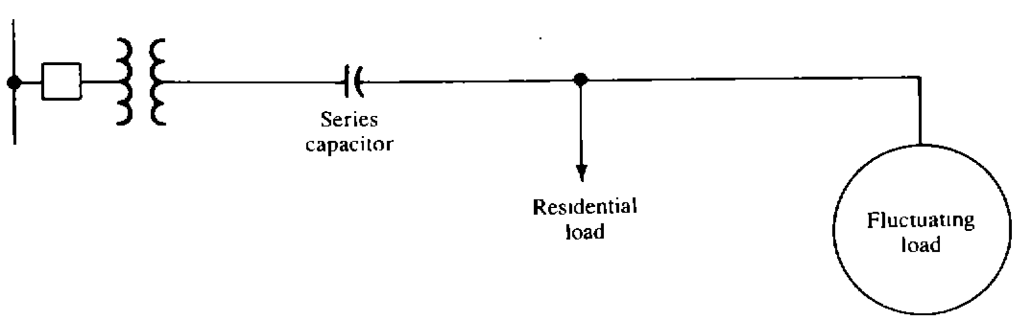


Figure 9-20 Installation of series capacitor to reduce the flicker voltage caused by a fluctuating load.

load would not reduce the flicker voltage since it would not reduce the impedance between the source and the lighting bus.

Since series capacitors are permanently installed in the primary feeder, they require special devices to protect them against overvoltage and resonance conditions. Therefore a typical series-capacitor installation costs three or four times as much as shunt-motor-start capacitors. Series capacitors correct the power factor of the system, not the motor. The position of the series capacitors in the circuit is especially important if there are other customers along the line.

9-6-1 A Shortcut Method to Calculate the Voltage Dips due to a Single-Phase Motor Start

If the starting kilovoltamperage of a single-phase motor is known, the motor's starting current can be found as,

$$I_{\text{start}} = \frac{S_{\text{start}}}{V_{L\text{-}N}} \quad \text{A} \tag{9-45}$$

and the voltage dip based on 120 V can be calculated from

$$\text{VDIP} = \frac{120 \times I_{\text{start}} \times Z_G}{V_{L\text{-}N}} \quad \text{V} \tag{9-46}$$

where

$$Z_G = \frac{V_{L\text{-}N}}{I_{f,\,L\text{-}G}} \quad \Omega \tag{9-47}$$

Substituting Eq. (9-45) and (9-47) into Eq. (9-46), the voltage dip can be expressed as

$$\text{VDIP} = \frac{120 \times S_{\text{start}}}{I_{f,\,L\text{-}G} \times V_{L\text{-}N}} \quad \text{V} \tag{9-48}$$

where VDIP = voltage dip due to single-phase motor start expressed in terms of 120-V base, V

S_{start} = starting kVA of single-phase motor, kVA

$I_{f,\,L\text{-}G}$ = line-to-ground fault current available at point of installation and obtained from fuse coordination, A

$V_{L\text{-}N}$ = line-to-neutral voltage, kV

Example 9-11 Assume that a 10-hp single-phase 7.2-kV motor with NEMA code letter "G" starting 15 times per hour is to be served at a certain location. If the starting kilovoltamperes per horsepower for this motor is given by the manufacturer as 6.3, and the line-to-ground fault at the installation location calculated to be 1438 A, determine the following:

(*a*) The voltage dip due to the motor start, in volts.

(*b*) Whether or not the resultant voltage dip is objectionable.

SOLUTION

(*a*) Since the starting kilovoltamperes per horsepower is given as 6.3, the starting kilovoltamperes can be found as

$$\begin{aligned} S_{\text{start}} &= (\text{kVA/hp})_{\text{start}} \times \text{hp}_{\text{motor}} \qquad \text{kVA} \qquad (9\text{-}49) \\ &= 6.3 \text{ kVA/hp} \times 10 \text{ hp} \\ &= 63 \text{ kVA} \end{aligned}$$

Therefore the voltage dip due to the motor start, from Eq. (9-48), can be calculated as

$$\begin{aligned} \text{VDIP} &= \frac{120 \times S_{\text{start}}}{I_{f,L\text{-}G} \times V_{L\text{-}N}} \\ &= \frac{120 \times 63 \text{ kVA}}{1438 \text{ A} \times 7.2 \text{ kV}} \\ &= 0.73 \text{ V} \end{aligned}$$

(*b*) From Fig. 9-19, it can be found that the voltage dip of 0.73 V with a frequency of 15 times per hour is in the satisfactory-flicker zone and therefore is not objectionable to the immediate customers.

9-6-2 A Shortcut Method to Calculate the Voltage Dips due to a Three-Phase Motor Start

If the starting kilovoltamperes of a three-phase motor is known, its starting current can be found as

$$I_{\text{start}} = \frac{S_{\text{start}}}{\sqrt{3} \times V_{L\text{-}L}} \qquad \text{A} \qquad (9\text{-}50)$$

and the voltage dip based on 120 V can be calculated from

$$\text{VDIP} = \frac{120 \times I_{\text{start}} \times Z_l}{V_{L\text{-}N}} \qquad \text{V} \qquad (9\text{-}51)$$

where

$$Z_l = \frac{V_{L\text{-}N}}{I_{3\phi}} \qquad \Omega \qquad (9\text{-}52)$$

Substituting Eqs. (9-50) and (9-52) into Eq. (9-51), the voltage dip can be expressed as

$$\text{VDIP} = \frac{69.36 \times S_{\text{start}}}{I_{3\phi} \times V_{L\text{-}L}} \qquad \text{V} \qquad (9\text{-}53)$$

where VDIP = voltage dip due to three-phase motor start expressed in terms of 120-V base, V

S_{start} = starting kVA of three-phase motor, kVA

$I_{3\phi}$ = three-phase fault current available at point of installation and obtained from fuse coordination, A

$V_{L\text{-}L}$ = line-to-line voltage, kV

Example 9-12 Assume that a 100-hp three-phase 12.47-kV motor with NEMA code letter "F" starting three times per hour is to be served at a certain location. If the starting kilovoltampere per horsepower for this motor is given by the manufacturer as 5.6, and the three-phase fault current at the installation location calculated to be 1765 A, determine the following:

(*a*) The voltage dip due to the motor start.

(*b*) Whether or not the resultant voltage dip is objectionable.

SOLUTION

(*a*) Since the starting kilovoltampere per horsepower is given as 5.6, the starting kilovoltampere can be found from Eq. (9-49) as

$$\begin{aligned} S_{\text{start}} &= (\text{kVA/hp})_{\text{start}} \times \text{hp}_{\text{motor}} \\ &= 5.6 \text{ kVA/hp} \times 100 \text{ hp} \\ &= 560 \text{ kVA} \end{aligned}$$

Therefore the voltage dip due to the motor start, from Eq. (9-53), can be calculated as

$$\begin{aligned} \text{VDIP} &= \frac{69.36 \times S_{\text{start}}}{I_{3\phi} \times V_{L\text{-}L}} \\ &= \frac{69.36 \times 560 \text{ kVA}}{1765 \text{ A} \times 12.47 \text{ kV}} \\ &= 1.76 \text{ V} \end{aligned}$$

(*b*) From Fig. 9-19, it can be found that the voltage dip of 1.72 V with a frequency of three times per hour is in the satisfactory-flicker zone and therefore is not objectionable to the immediate customers.

PROBLEMS

9-1 Derive, or prove, Eq. (9-18).

9-2 Derive, or prove, Eq. (9-20).

9-3 Repeat Example 9-1, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-4 Repeat Example 9-2, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-5 Repeat Example 9-3, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-6 Repeat Example 9-4, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-7 Repeat Example 9-5, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-8 Repeat Example 9-6, assuming 336.4-kcmil ACSR conductors and annual peak load of 5000 kVA at a lagging-load power factor of 0.90.

9-9 Assume that a subtransmission line is required to be designed to carry a contingency peak load of 2 × SIL. A 60 percent series compensation is to be used; i.e., the capacitive reactance (X_c) of the capacitor bank required to be installed is equal to 60 percent of the total series inductive reactance per phase of the transmission line. Assume that each phase of the series-capacitor bank is to be made up of series and parallel groups of two-bushing 12-kV 150-kvar shunt power-factor-correction capacitors. Assume that the three-phase SIL of the line is 416.5 MVA and its inductive line reactance is 117.6 Ω/phase. Specify the necessary series-parallel arrangement of capacitors for each phase.

9-10 In this problem design improvements of the designer choice to correct the undervoltage conditions are investigated on the radial system shown in Fig. P9-10.

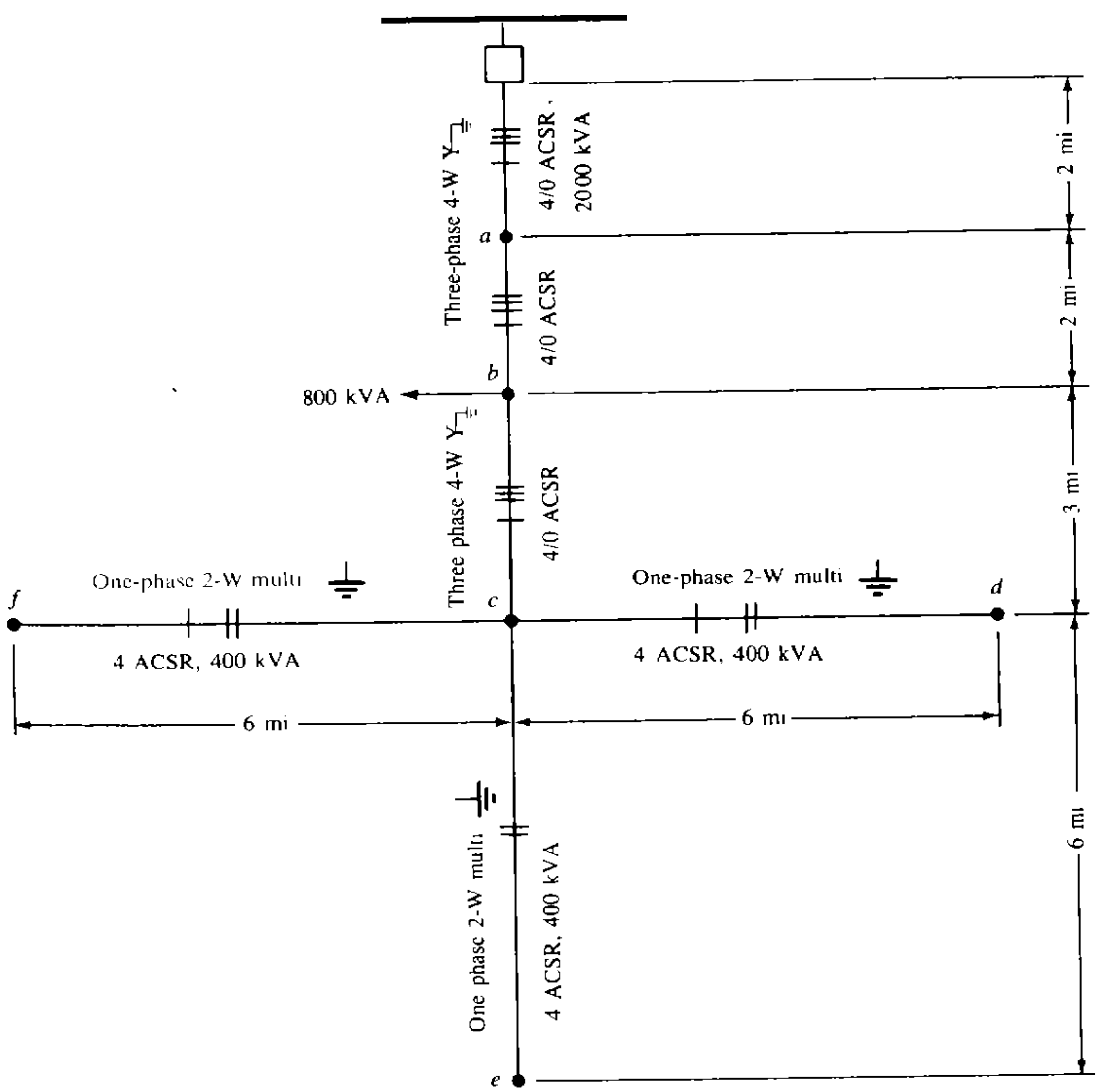


Figure P9-10

The voltage at the distribution substation low-voltage bus is kept at 1.04 pu V with bus-voltage regulation. The per unit voltages at annual peak-load values at the points a, b, c, d, e, and f are 1.0049, 0.9815, 0.9605, 0.8793, 0.8793, and 0.8793, respectively. Use the nominal operating voltage of 7200/12,470 V of the three-phase four-wire wye-grounded system as the base voltage. Assume that all given kilovoltampere loads are annual peak values at 85 percent lagging-load power factor. The load between the substation bus and point a is a uniformly distributed load of 2000 kVA. The loads on the laterals c-d, c-e, and c-f are also uniformly distributed, each with 400 kVA. There is a lumped load of 800 kVA at point b.

The line data for the #4/0 and 4 ACSR conductors are given in Table P9-10.

Table P9-10

Conductor size	R, Ω/(phase · mi)	X, Ω/(phase · mi)	K, pu VD/(kVA · mi)
4/0	0.592	0.761	5.85×10^{-6}
4	2.55	0.835	1.69×10^{-5}

To improve voltage conditions, consider any or all combinations of the following design remedies:

1. Installation of shunt capacitor bank(s)
2. Installation of 32-step voltage regulators with a maximum regulation range of ± 10 percent.
3. Addition of new phase conductors

Using these remedies attempt to meet the following primary-voltage criteria:

1. Maximum primary voltage must be 1.040 pu V at zero load.
2. Maximum primary voltage must be 1.07 pu V at peak load.
3. Minimum primary voltage must be 1.00 pu V at peak load.

If the installation of the capacitors alternative is chosen, determine the following:
(a) The rating of the capacitor bank(s) in three-phase kilovars.
(b) The location of the capacitor bank(s) on the given system.
(c) Whether or not voltage-controlled automatic switching is required.
If the installation of the voltage-regulators alternative is chosen, determine the following:
(a) The location of the regulator bank(s).
(b) The standard kilovoltampere rating of each single-phase regulator.
(c) The location of the regulating point (RP) on the system.
(d) The setting of the voltage-regulating relay (VRR).
(e) The R and X settings of the line-drop compensator (LDC).

9-11 Repeat Example 9-9. Assume that the annual peak load is 5000 kVA at a lagging power factor of 0.80 and that the reactive load factor is 0.60.

9-12 Figure P9-12 shows an open-wire primary line with many laterals and uniformly distributed load. The voltage at the distribution substation low-voltage bus is held at 1.03 pu V with bus voltage regulation. When there is no capacitor bank installed on the feeder, that is, $Q_{NSW} = 0$, the per unit voltage at the end of the line at annual peak load is 0.97. Use the nominal operating voltage of 7.97/13.8 kV of the three-phase four-wire wye-grounded system as the base voltage. Assume that the off-peak load of the system is about 25 percent of the on-peak load. Also assume that the line reactance is 0.80 Ω/(phase · mi) but the line resistance is not given and determine the following:

(a) When the shunt-capacitor bank is not used, find the V_X voltages at the times of peak load and off-peak load.

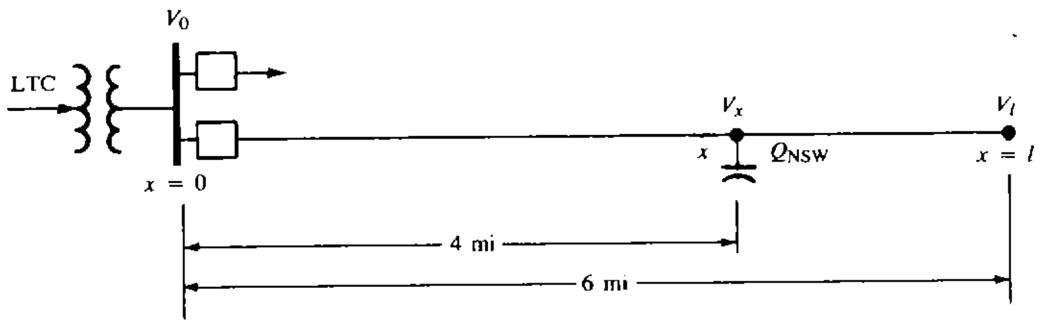


Figure P9-12

(*b*) Apply an unswitched-capacitor bank and locate it at the point of $X = 4$ mi on the line, and size the capacitor bank to yield the per unit voltage of 1.05 at point X at the time of zero load. Find the size of the capacitor (Q_{NSW}) in three-phase kilovars. Also find the per unit voltage of V_X and V_l at the time of peak load.

9-13 Figure P9-13 shows a system which has a load connected at the end of a 3.5 mi #4 ACSR open-wire primary line. The load belongs to an important scientific equipment installation, and it varies from nearly zero to 1000 kVA. The load requires a closely regulated voltage at the V_S bus. The consumer requests the voltage V_S to be equal to 1.000 $\pm$ 0.010 pu and offers to compensate properly the supplying utility company for such high-quality voltage-regulated service.

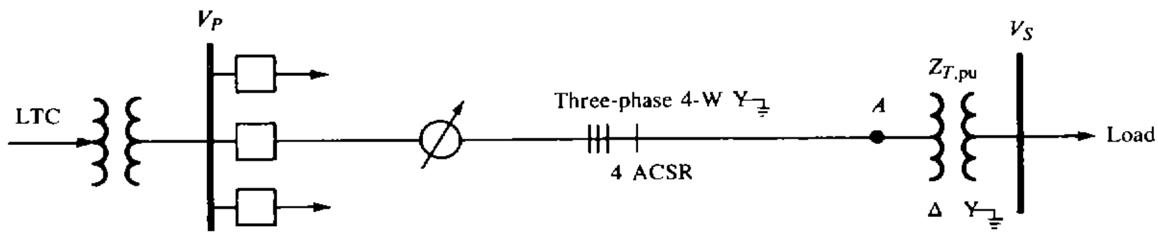


Figure P9-13

A junior engineer proposes to build the 3.5 mi #4 ACSR line to a nearby distribution substation and to place the feeder voltage regulators there in order to render the service requested. His wire size is generous for ampacity. He proposes bandwidth setting (BW) of ± 1.0 V based on 120 V.

There is bus voltage regulation (BVR) at the substation but at times the BVR equipment is disconnected and bypassed for maintenance and repair. Therefore, the substation bus voltage $V_{P,\text{pu}}$ is as follows. When BVR is in use,

$$V_{P,\text{pu}} = 1.030 \text{ pu V}$$

When BVR is out,

$$\text{Max } V_{P,\text{pu}} = 1.060 \text{ pu V}$$
$$\text{Min } V_{P,\text{pu}} = 0.970 \text{ pu V}$$

The nominal and base voltage at the distribution substation low-voltage bus is 7200/12,470 V for the three-phase four-wire wye-grounded system. The nominal and base voltage at the consumer's bus is 277/480 V for the three-phase four-wire wye-grounded service. Assume that the regulator bank is made up of three single-phase 32-step feeder voltage regulators with ± 10 percent regulation range. The feeder impedance is given as $2.55 + j0.835$ Ω/(mi $\cdot$ phase). Assume that the precalculated K constant of the line is 1.69×10^{-5} pu VD/(kVA $\cdot$ mi) at 85 percent power factor. The consumer's transformer is rated as 1000 kVA, three-phase, with 12,470-V high-voltage rating. It has an impedance of $0 + j0.055$ pu based on the transformer ratings.

Using the given information and data, determine and state whether or not the young engineer's proposed design will meet the consumer's requirements. (Check for both cases, i.e., when BVR is in use and BVR is not in use.)

9-14 Repeat Example 9-11, assuming 20 starts per hour and a line-to-ground current of 350 A.

9-15 Repeat Example 9-12, assuming 10 starts per hour and a three-phase fault current of 750 A.

REFERENCES

1. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
2. *Voltage Ratings for Electric Power Systems and Equipment*, American National Standards Institute, ANSI C84.1-1977.
3. McCrary, M. R.: "Regulating Voltage on a Major Power System," *The Line*, vol. 81, no. 1, 1981, pp.2–8.
4. Hopkinson, R. H.: "Recap S Computer Program Aids Voltage Regulation Studies," *Electr. Forum*, vol. 4, no. 4, 1978, pp.20–23.
5. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*, 11th ed., McGraw-Hill, New York, 1978.
6. Gönen, T., et al.: *Development of Advanced Methods for Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va.
7. Oklahoma Gas and Electric Company: *Engineering Guides*, Oklahoma City, January 1981.
8. Bovenizer, W. N.: "New 'Simplified' Regulator for Lower-Cost Distribution Voltage Regulation," *The Line*, vol. 75, no. 1, 1975, pp. 25–28.
9. Bovenizer, W. N.: "Paralleling Voltage Regulators," *The Line*, vol. 75, no. 2, 1975, pp.6–9.
10. Sealey, W. C.: "Increased Current Ratings for Step Regulators," *AIEE Trans.*, pt. III, August 1955, pp. 737–742.
11. Lokay, H. E., and D. N. Reps: "Distribution System Primary-Feeder Voltage Control: Part I—A New Approach Using the Digital Computer," *AIEE Trans.*, pt. III, October 1958, pp. 845–855.
12. Reps, D. N., and G. J. Kirk, Jr.: "Distribution System Primary-Feeder Voltage Control: Part II—Digital Computer Program," *AIEE Trans.*, pt. III, October 1958, pp. 856–865.
13. Amchin, H. K., R. J. Bentzel, and D. N. Reps: "Distribution System Primary-Feeder Voltage Control: Part III—Computer Program Application," *AIEE Trans.*, pt. III, October 1958, pp. 865–879.
14. Reps, D. N., and R. F. Cook: "Distribution System Primary-Feeder Voltage Control: Part IV—A Supplementary Computer Program for Main-Circuit Analysis," *AIEE Trans.*, pt. III, October 1958, pp. 904–913.
15. Chang, N. E.: "Locating Shunt Capacitors on Primary Feeder for Voltage Control and Loss Reduction," *IEEE Trans. Power Appar. Syst.*, vol. PAS-88, no. 10, October 1969, pp. 1574–77.
16. Gönen, T., and F. Djavashi: "Optimum Shunt Capacitor Allocation on Primary Feeders," *IEEE MEXICON-80 International Conference*, Mexico City, October 22–25, 1980.
17. Ku, W. S.: "Economic Comparison of Switched Capacitors and Voltage Regulators for System Voltage Control," *AIEE Trans.*, pt. III, December 1957, pp. 891–906.
18. Gönen, T., and F. Djavashi: "Optimum Loss Reduction from Capacitors Installed on Primary Feeders," *The Midwest Power Symposium*, Purdue University, West Lafayette, Ind., October 27–28, 1980.

CHAPTER

TEN

DISTRIBUTION SYSTEM PROTECTION

10-1 BASIC DEFINITIONS

Switch: a device for making, breaking, or changing the connection in an electric current

Disconnect switch: a switch designed to disconnect power devices at no-load conditions

Load-break switch: a switch designed to interrupt load currents but not (greater) fault currents

Circuit breaker: a switch designed to interrupt fault currents

Automatic circuit reclosers: an overcurrent protective device that trips and recloses a preset number of times to clear transient faults or to isolate permanent faults

Automatic line sectionalizer: an overcurrent protective device used only with backup circuit breakers or reclosers but not alone

Fuse: an overcurrent protective device with a circuit-opening fusible member directly heated and destroyed by the passage of overcurrent through it in the event of an overload or short-circuit condition

Relay: a device that responds to variations in the conditions in one electric circuit to affect the operation of other devices in the same or in another electric circuit

Lightning arrester: a device put on electric power equipment to reduce the voltage of a surge applied to its terminals.

10-2 OVERCURRENT PROTECTION DEVICES

The overcurrent protective devices applied to distribution systems include relay-controlled circuit breakers, automatic circuit reclosers, fuses, and automatic line sectionalizers.

10-2-1 Fuses

A fuse is an overcurrent device with a circuit-opening fusible member (i.e., fuse link) directly heated and destroyed by the passage of overcurrent through it in the event of an overload or short-circuit condition. Therefore the purpose of a fuse is to clear a permanent fault by removing the defective segment of a line or equipment from the system. A fuse is designed to blow within a specified time for a given value of fault current. The time–current characteristics of a fuse are represented by two curves: (1) the minimum-melt curve and (2) the total-clearing curve. The minimum-melt curve of a fuse is a plot of the minimum time vs. current required to melt the fuse link. The total-clearing curve is a plot of the maximum time vs. current required to melt the fuse link and extinguish the arc.

Fuses designed to be used above 600 V are categorized as *distribution cutouts* (also known as *fuse cutouts*) or *power fuses*. Figure 10-1 gives detailed classification for high-voltage fuses.

The liquid-filled (oil-filled) cutouts are mainly used in underground installations and contain the fusible elements in an oil-filled and sealed tank. The expulsion-type distribution cutouts are by far the most common type of protective device applied to overhead primary distribution systems. In these cutouts, the melting of the fuse link causes heating of the fiber fuse tube which, in turn, produces deionizing gases to extinguish the arc. Expulsion-type cutouts are classified according to their external appearance and operation methods as (1) enclosed-fuse cutouts, (2) open-fuse cutouts, and (3) open-link-fuse cutouts.

The ratings of the distribution fuse cutouts are based on continuous current-carrying capacity, nominal and maximum design voltages, and interrupting capacity. In general, the fuse cutouts are selected based upon the following data:

1. The type of system for which they are selected, e.g., overhead or underground, delta or grounded-wye system
2. The system voltage for which they are selected
3. The maximum available fault current at the point of application
4. The X/R ratio at the point of application
5. Other factors, e.g., safety, load growth, and changing duty requirements

The use of symmetrical ratings simplified the selection of cutouts as a simple comparison of the calculated system requirements with the available fuse cutout ratings. In spite of that, fuse cutouts still have to be able to interrupt asymmetrical currents which are, in turn, subject to the X/R ratios of the circuit. Therefore symmetrical cutout rating tables are prepared on the basis of assumed maximum

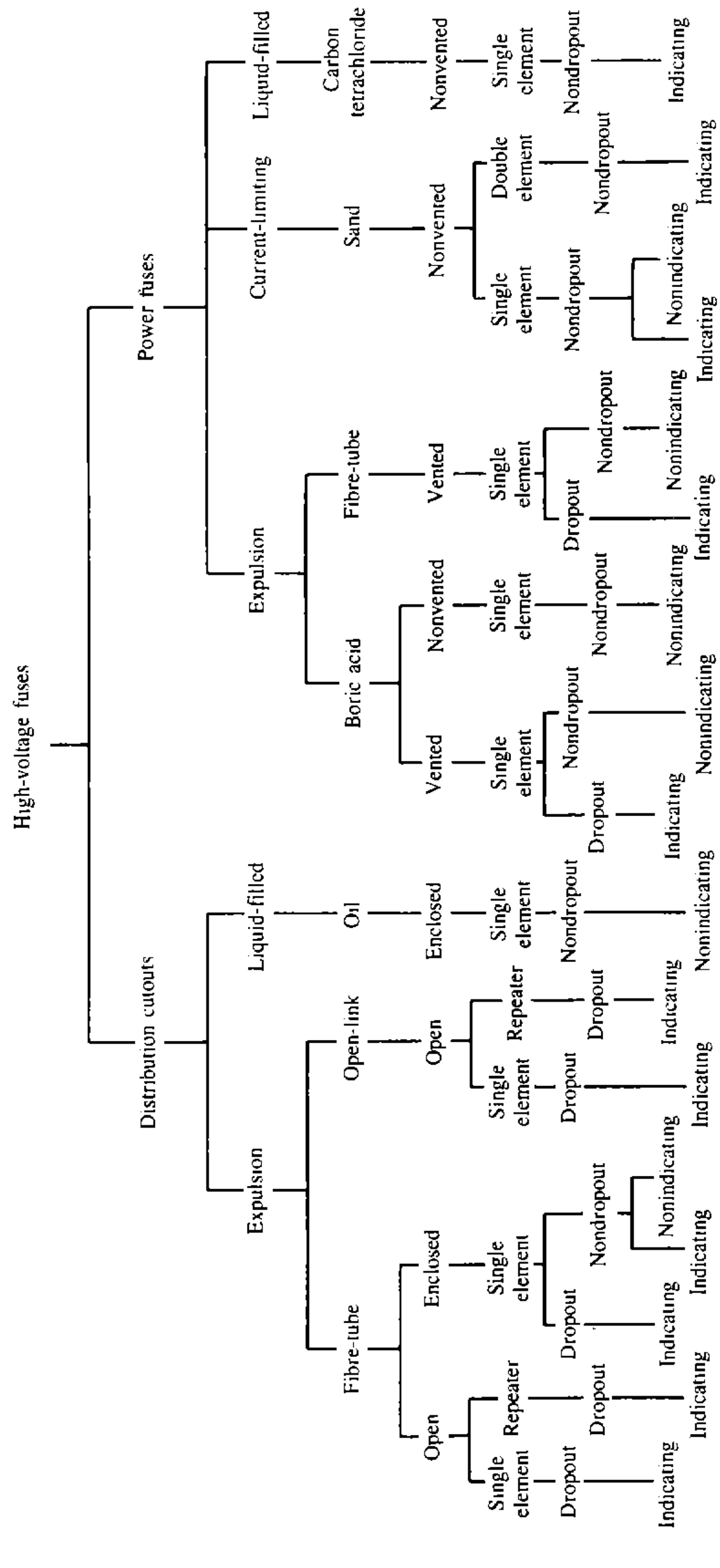


Figure 10-1 Classification of high-voltage fuses [1].

X/R ratios. Table 10-1 gives the interrupting ratings of open-fuse cutouts. Figure 10-2 shows a typical open-fuse cutout in pole-top style for 7.2/14.4-kV overhead distribution. Figure 10-3 shows a typical application of open-fuse cutouts in 7.2/14.4-kV overhead distribution.

In 1951, a joint study by the EEI and NEMA established standards specifying *preferred* and *nonpreferred* current ratings for fuse links of distribution fuse cutouts and their associated time–current characteristics in order to provide interchangeability for fuse links. The reason for stating certain ratings to be preferred or nonpreferred is based on the fact that the ordering sequence of the current ratings is set up such that a preferred-size fuse link will protect the next higher preferred size. This is also true for the nonpreferred sizes. The current ratings of fuse links for preferred sizes are given as 6, 10, 15, 25, 40, 65, 100, 140, and 200 A, and for nonpreferred sizes as 8, 12, 20, 30, 50, and 80 A.

Table 10-1 Interrupting ratings of open-fuse cutouts*

Rating of cutout			Interrupting rating in rms amperes at				
Continuous current, A	Nominal voltage, kV	Maximum design voltage, kV	5.2 kV	7.8 kV	15 kV	27 kV	Interrupting rating nomenclature
100	5.0	5.2	3000				Normal duty
100	5.0	5.2	5000				Heavy duty
100	5.0	5.2	10000				Extra heavy duty
200	5.0	5.2	4000				Normal duty
200	5.0	5.2	12000				Heavy duty
100	7.5	7.8		3000			Normal duty
100	7.5	7.8		5000			Heavy duty
100	7.5	7.8		10000			Extra heavy duty
200	7.5	7.8		4000			Normal duty
200	7.5	7.8		12000			Heavy duty
100	15	15			2000		Normal duty
100	15	15			4000		Heavy duty
100	15	15			8000		Extra heavy duty
200	15	15			4000		Normal duty
200	15	15			10000		Heavy duty
100	25	27				1200	Normal duty

* From [1].

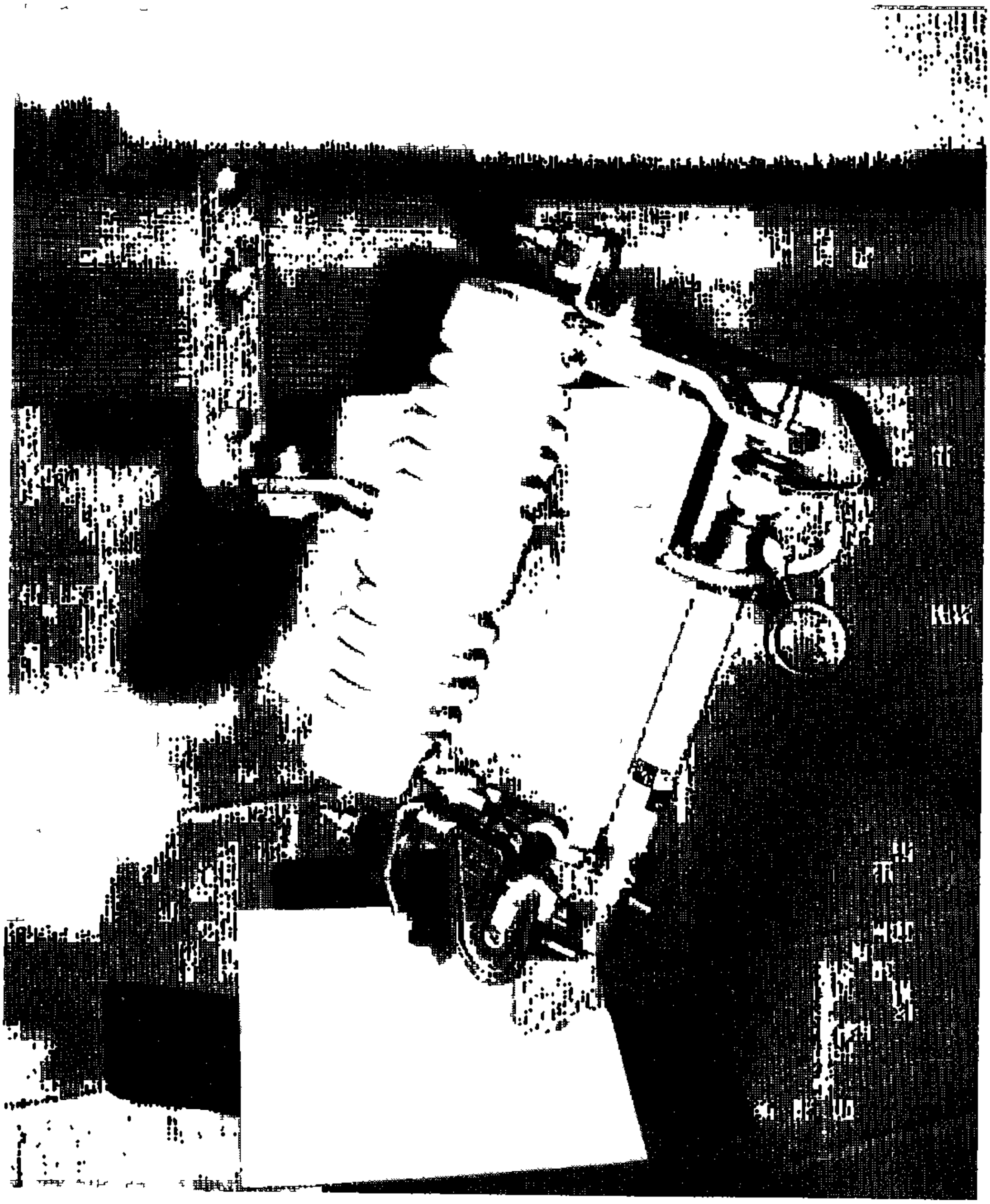


Figure 10-2 Typical open-fuse cutout in pole-top style for 7.2/14.4-kV overhead distribution. (*S&C Electric Company.*)

Figure 10-3 Typical application of open-fuse cutouts in 7.2/14.4-kV overhead distribution. (*S&C Electric Company.*)

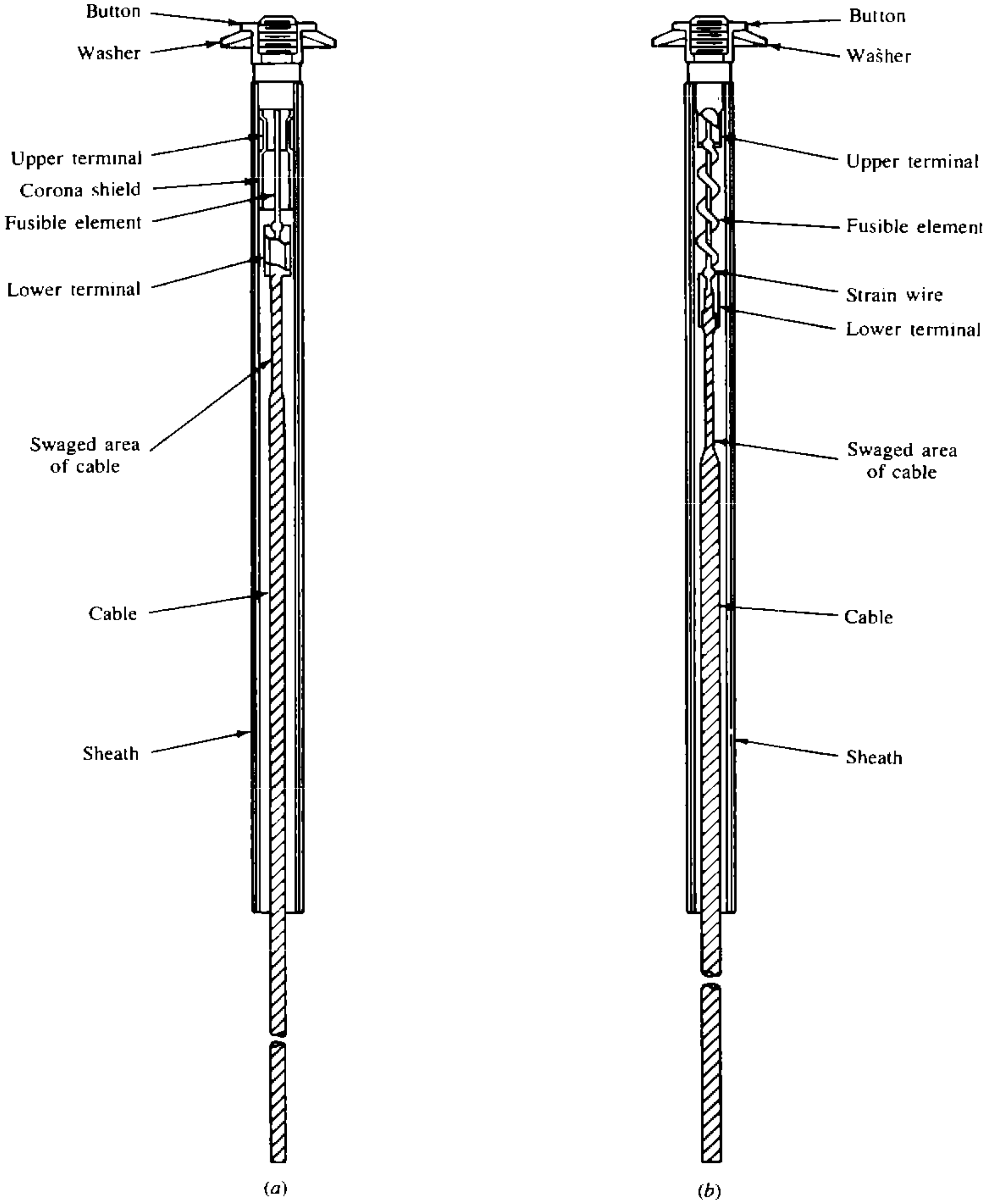


Figure 10-4 Typical fuse links used on outdoor distribution: (*a*) fuse link rated less than 10 A, and (*b*) fuse link rated 10 to 100 A. (*S&C Electric Company.*)

Furthermore, the standards also classify the fuse links as (1) type K (fast) and (2) type T (slow). The difference between these two fuse links is in the relative melting time which is defined by the speed ratio as

$$\text{Speed ratio} = \frac{\text{melting current at 0.1 s}}{\text{melting current at 300 or 600 s}}$$

Here, the 0.1 and 300 s are for fuse links rated 6 to 100 A, and the 0.1 and 600 s are for fuse links rated 140 to 200 A. Therefore the speed ratios for type K and type T fuse links are between 6 and 8, and 10 and 13, respectively. Figure 10-4 shows typical fuse links. Figure 10-5 shows minimum-melting-time-current characteristic curves for typical (fast) fuse links.

Power fuses are employed where the system voltage is 34.5 kV or higher and/or the interrupting requirements are greater than the available fuse cutout ratings. They are different from fuse cutouts in terms of (1) higher interrupting ratings, (2) larger range of continuous current ratings, (3) applicable not only for distribution but also for subtransmission and transmission systems, and (4) designed and built usually for substation mounting rather then pole and crossarm mounting. A power fuse is made of a fuse mounting and a fuse holder. Its fuse link

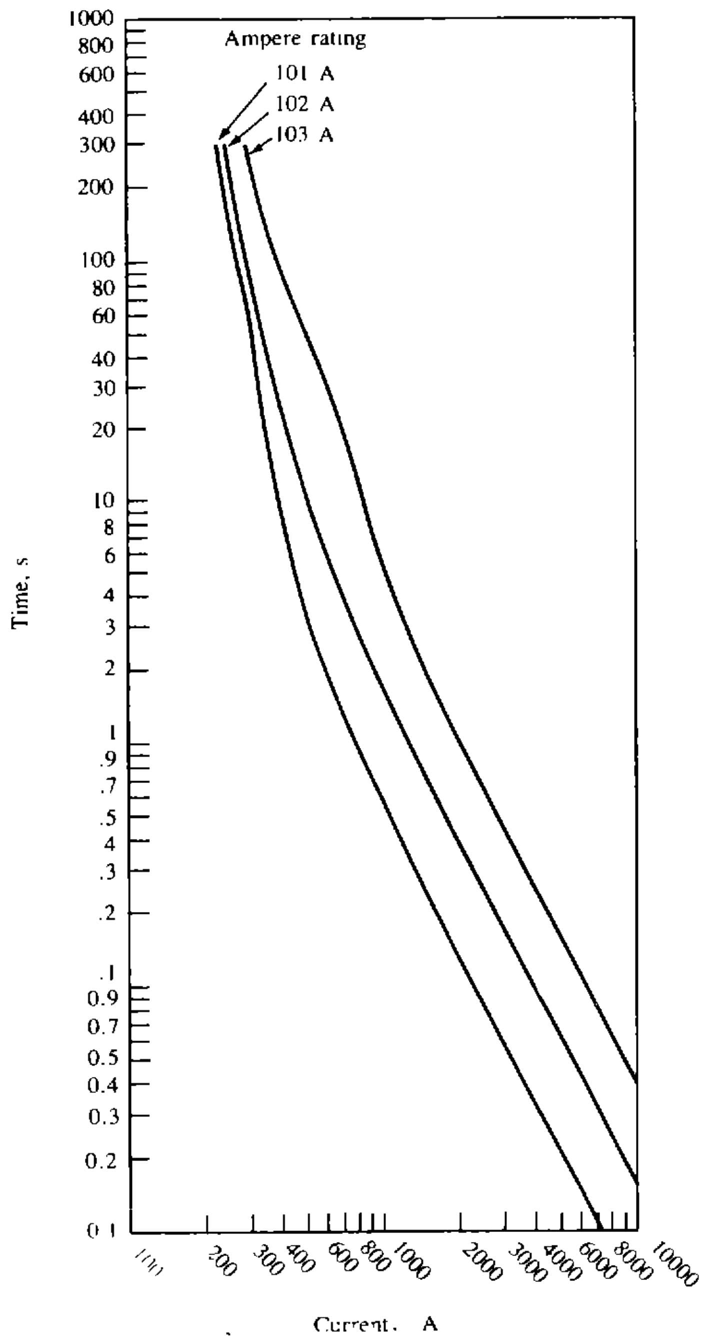


Figure 10-5 Minimum-melting-time-current characteristic curves for typical (fast) fuse links. Curves are plotted to minimum test points, so all variations should be +20 percent in current. (*S&C Electric Company*.)

Figure 10-6 Typical transformer protection application of 34.5-kV SM-type power fuses. (*S&C Electric Company.*)

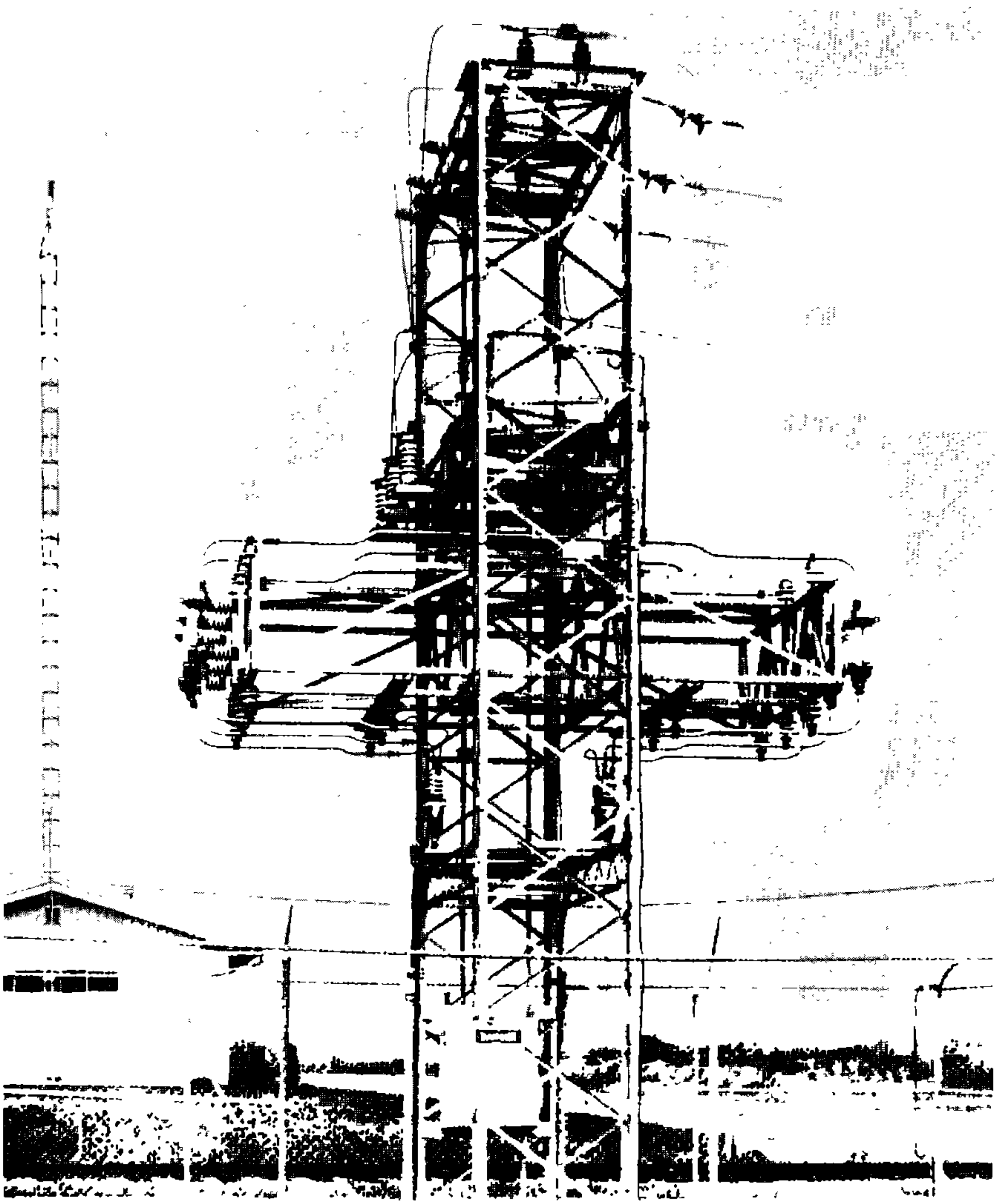


Figure 10-7 Feeder protection application of 34.5-kV SM-type power fuses. (*S&C Electric Company*.)

is called the *refill unit*. In general, they are designed and built as (1) expulsion (boric acid or other solid material [SM]) type, (2) current-limiting (silver-sand) type, or (3) liquid-filled type. Power fuses are identified by the letter "E" (for example, 200E or 300E) to specify that their time–current characteristics comply with the interchangeability requirements of the standard. Figure 10-6 shows a typical transformer protection application of 34.5-kV SM-type power fuses. Figure 10-7 shows a feeder protection application of 34.5-kV SM-type power fuses. Figure 10-8 shows a cutaway view of a typical 34.5-kV SM-type refill unit.

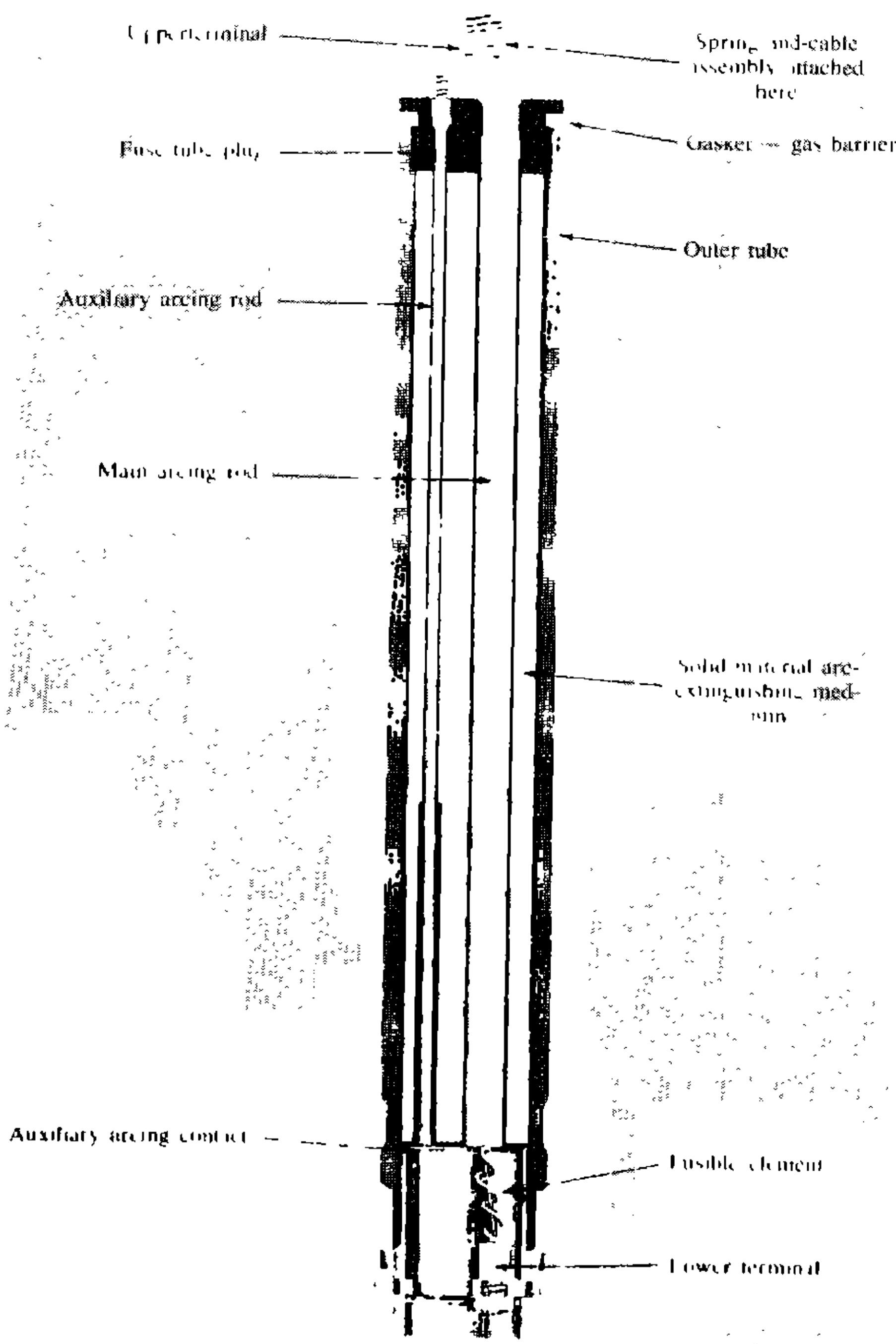


Figure 10-8 Cutaway view of typical 34.5-kV SM-type refill unit. (*S&C Electric Company.*)

10-2-2 Automatic Circuit Reclosers

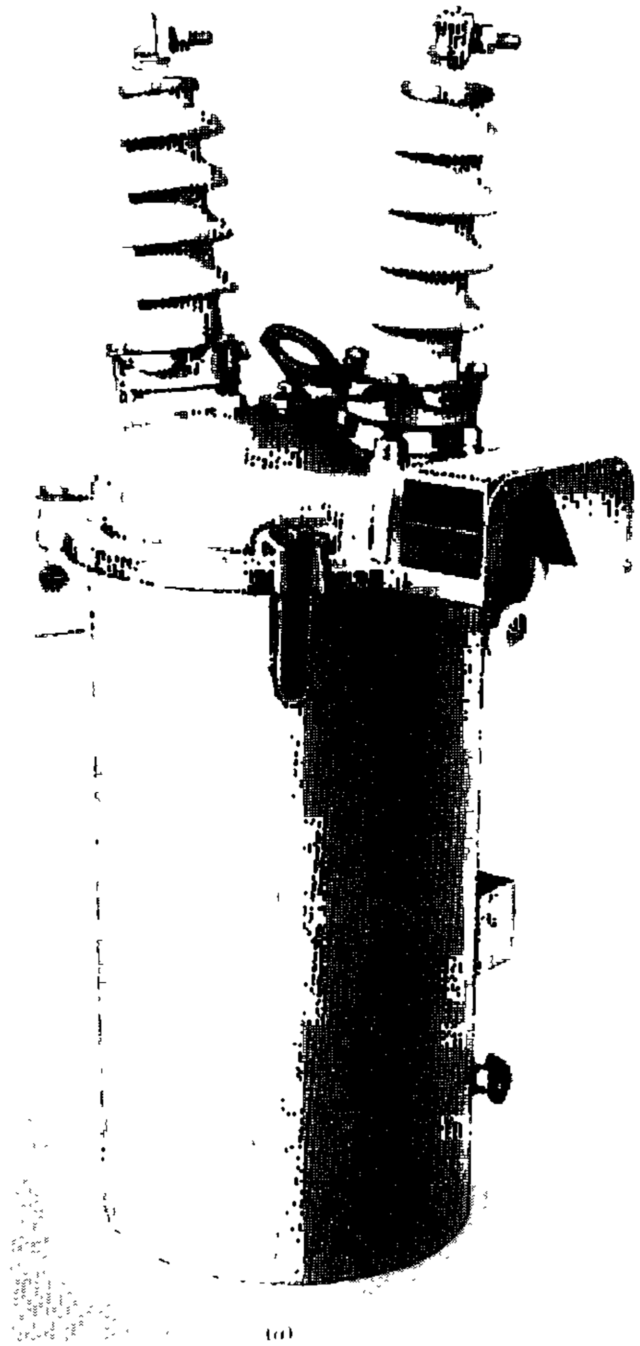
The automatic circuit recloser is an overcurrent protective device that automatically trips and recloses a preset number of times to clear temporary faults or isolate permanent faults. It also has provisions for manually opening and reclosing the circuit that is connected. Reclosers can be set for a number of different operation sequences such as (1) two instantaneous (trip and reclose) operations followed by two time-delay trip operations prior to lockout, (2) one instantaneous plus three time-delay operations, (3) three instantaneous plus one time-delay operations, (4) four instantaneous operations, or (5) four time-delay operations. The instantaneous and time-delay characteristics of a recloser are a function of its rating. Recloser ratings range from 5 to 1120 A for the ones with series coils and from 100 to 2240 A for the ones with nonseries coils. The minimum pickup for all ratings is usually set to trip instantaneously at two times the current rating. The reclosers must be able to interrupt asymmetrical fault currents related to their symmetrical rating. The rms asymmetrical current ratings can be determined by multiplying the symmetrical ratings by the asymmetrical factor, from Table 10-2, corresponding to the specified X/R circuit ratio. Note that the asymmetrical factors given in Table 10-2 are the ratios of the asymmetrical to the symmetrical rms fault currents at 0.5 cycle after fault initiation for different circuit X/R ratios.

A generally accepted rule of thumb is to assume that the X/R ratios on distribution feeders are not to surpass 5 and therefore the corresponding asymmetry factor is to be about 1.25. However, the asymmetry factor for other parts of the system is assumed to be approximately 1.6.

Line reclosers are often installed at points on the circuit to reduce the amount of exposure on the substation equipment. For example, a feeder circuit serving both urban and rural load would probably have reclosers on the main line serving the rural load. Therefore the installation of line reclosers will depend on the amount of exposure and operating experience. The maximum fault current available is always an important consideration in the application of line reclosers.

Table 10-2 Asymmetrical factors as function of X/R ratios

$\frac{X}{R}$	Asymmetrical factor
2	1.06
4	1.20
8	1.39
10	1.44
12	1.48
14	1.51
25	1.60

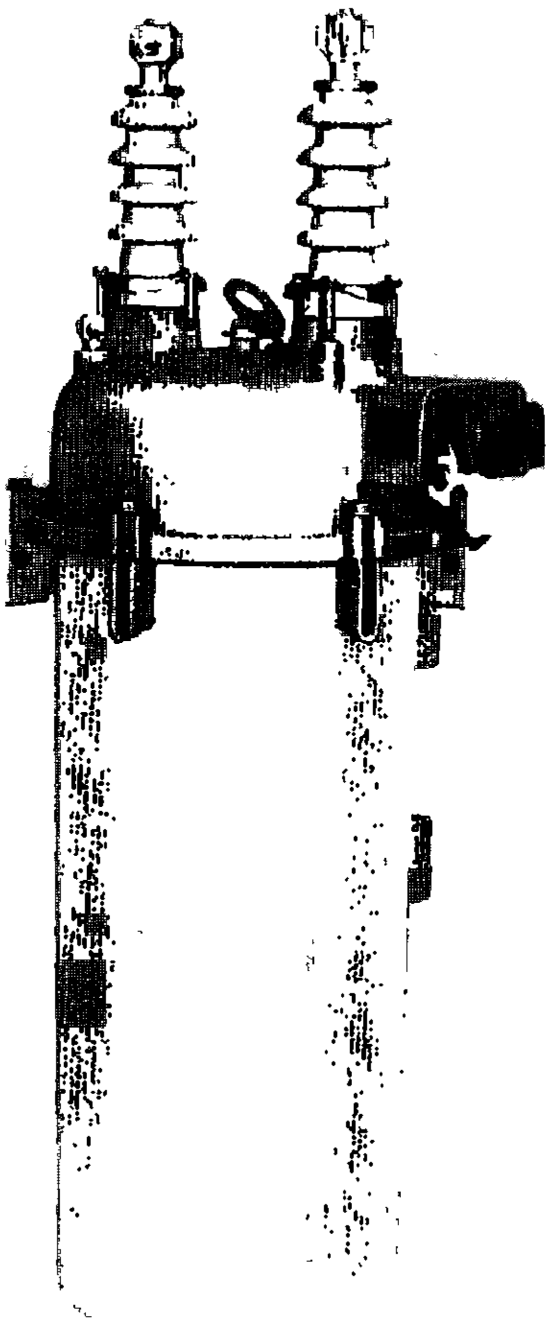


(a)

Figure 10-9 (*a*) Typical single-phase hydraulically controlled automatic circuit recloser: type H, 4H, V4H, or L.

In a sense, a recloser fulfills the same task as the combination of a circuit breaker, overcurrent relay, and reclosing relay. Fundamentally, a recloser is made of an interrupting chamber and the related main contacts which operate in oil, a control mechanism to trigger tripping and reclosing, an operator integrator, and a lockout mechanism.

Reclosers are designed and built in either single-phase or three-phase units. Single-phase reclosers inherently result in better service reliability as compared to three-phase reclosers. If the three-phase primary circuit is wye-connected, either a three-phase recloser or three single-phase reclosers are used. However, if the



(b)

Figure 10-9 (*b*) Type D, E, 4E, or DV. (*McGraw-Edison Company.*)

three-phase primary circuit is delta-connected, the use of two single-phase reclosers is adequate for protecting the circuit against either single- or three-phase faults. Figure 10-9 shows a typical single-phase hydraulically controlled automatic circuit recloser. Figures 10-10 and 10-11 show typical three-phase hydraulically controlled and electronically controlled automatic circuit reclosers, respectively. Single-phase reclosers inherently result in better service reliability as compared to three-phase reclosers.

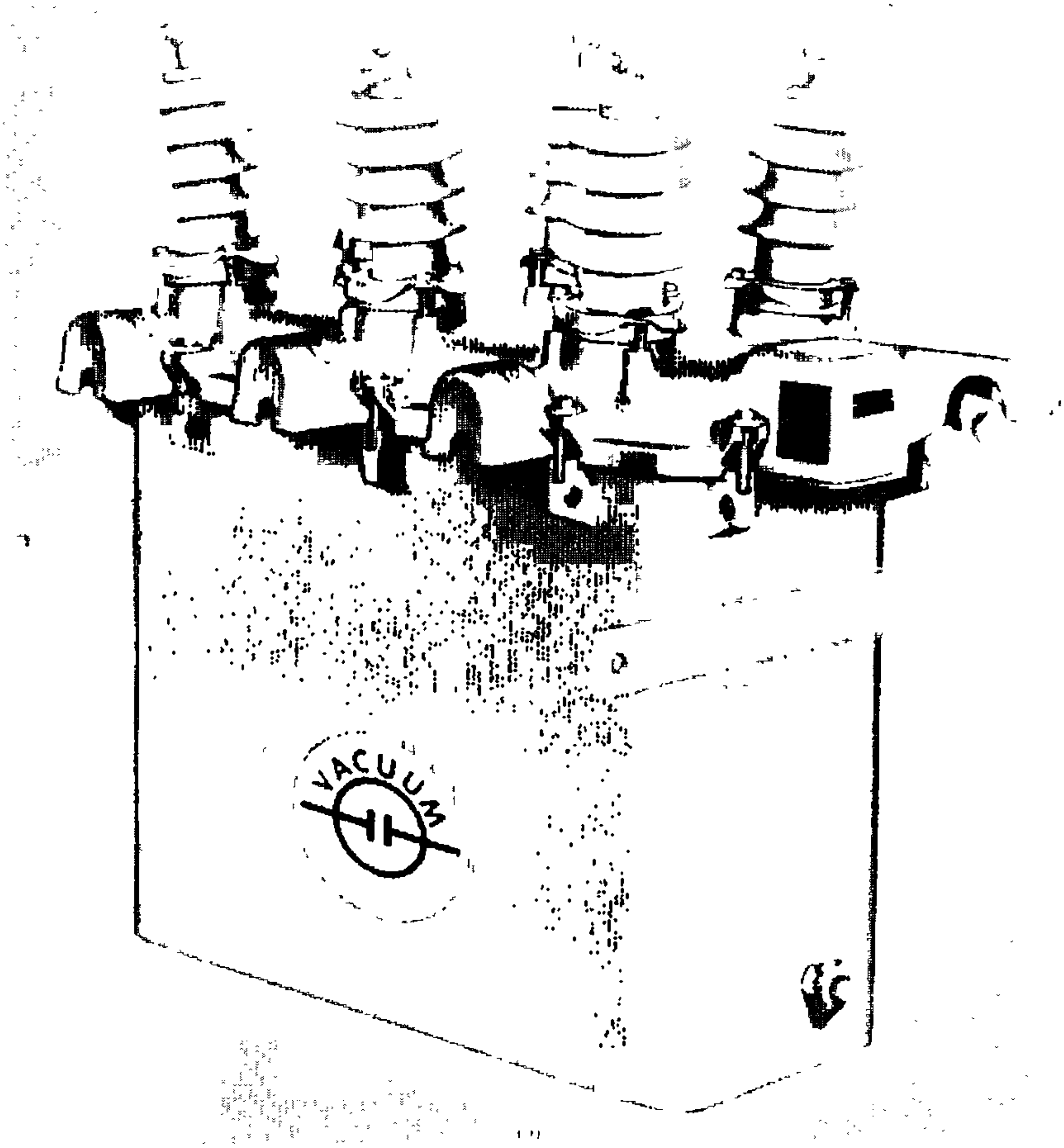
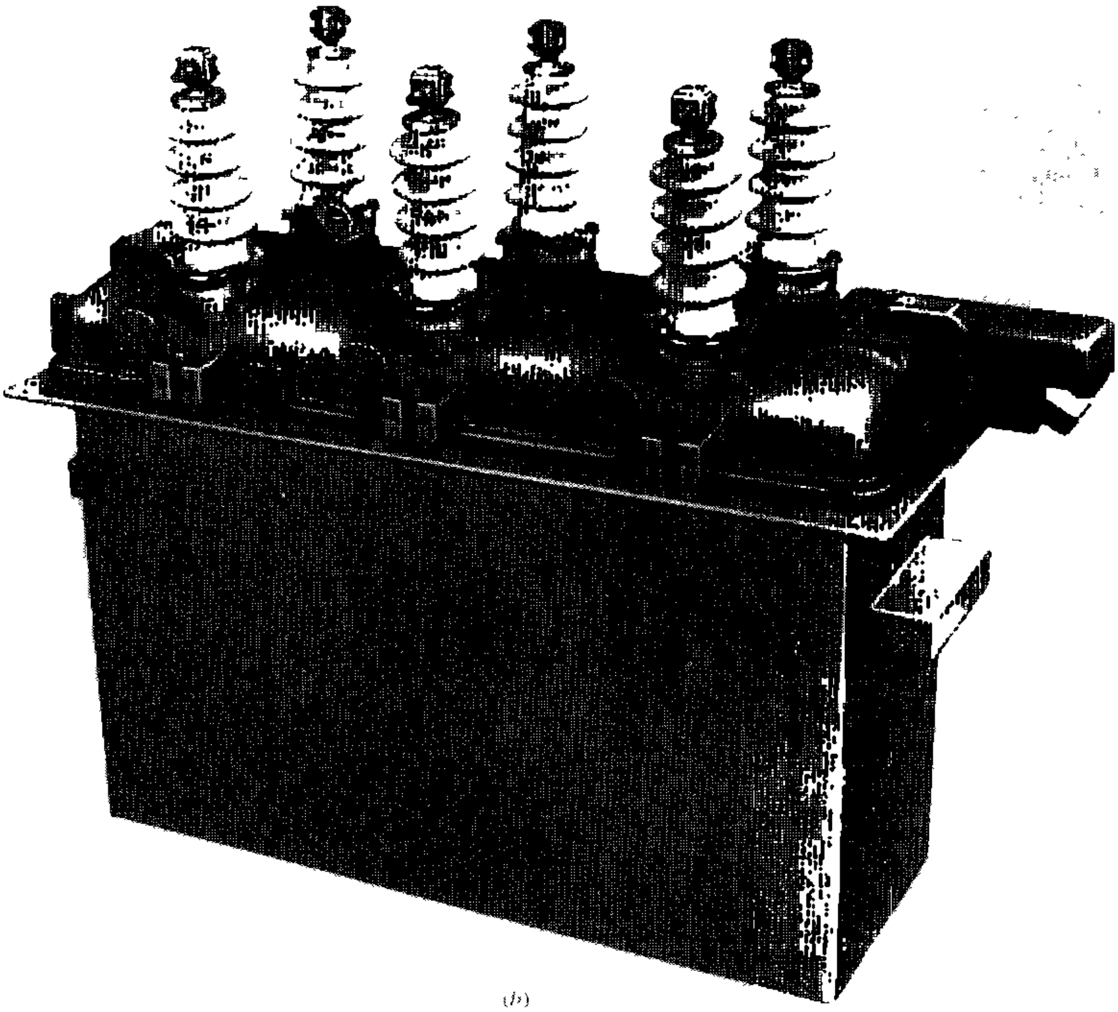


Figure 10-10 Typical three-phase hydraulically controlled automatic circuit reclosers: (*a*) type 6H or V6H.

10-2-3 Automatic Line Sectionalizers

The automatic line sectionalizer is an overcurrent protective device installed only with backup circuit breakers or reclosers. It counts the number of interruptions caused by a backup automatic interrupting device and opens during dead circuit time after a preset number (usually two or three) of tripping operations of the backup device.



(b)

Figure 10-10 (*b*) Type RV, RVE, RX, RXE, etc. (*McGraw-Edison Company.*)

Zimmerman [1] summarizes the operation modes of a sectionalizer as follows:

1. If the fault is cleared while the reclosing device is open, the sectionalizer counter will reset to its normal position after the circuit is reclosed.
2. If the fault persists when the circuit is reclosed, the fault-current counter in the sectionalizer will again prepare to count the next opening of the reclosing device.
3. If the reclosing device is set to go to lockout on the fourth trip operation, the sectionalizer will be set to trip during the open-circuit time following the third tripping operation of the reclosing device.

Contrary to expulsion-type fuses, a sectionalizer provides coordination (without inserting an additional time–current coordination) with the backup devices associated with very high fault currents and consequently provides an additional sectionalizing point on the circuit. On overhead distribution systems, they are usually installed on poles or crossarms. The application of sectionalizers entails certain requirements:

1. They have to be used in series with other protective devices but not between two reclosers.

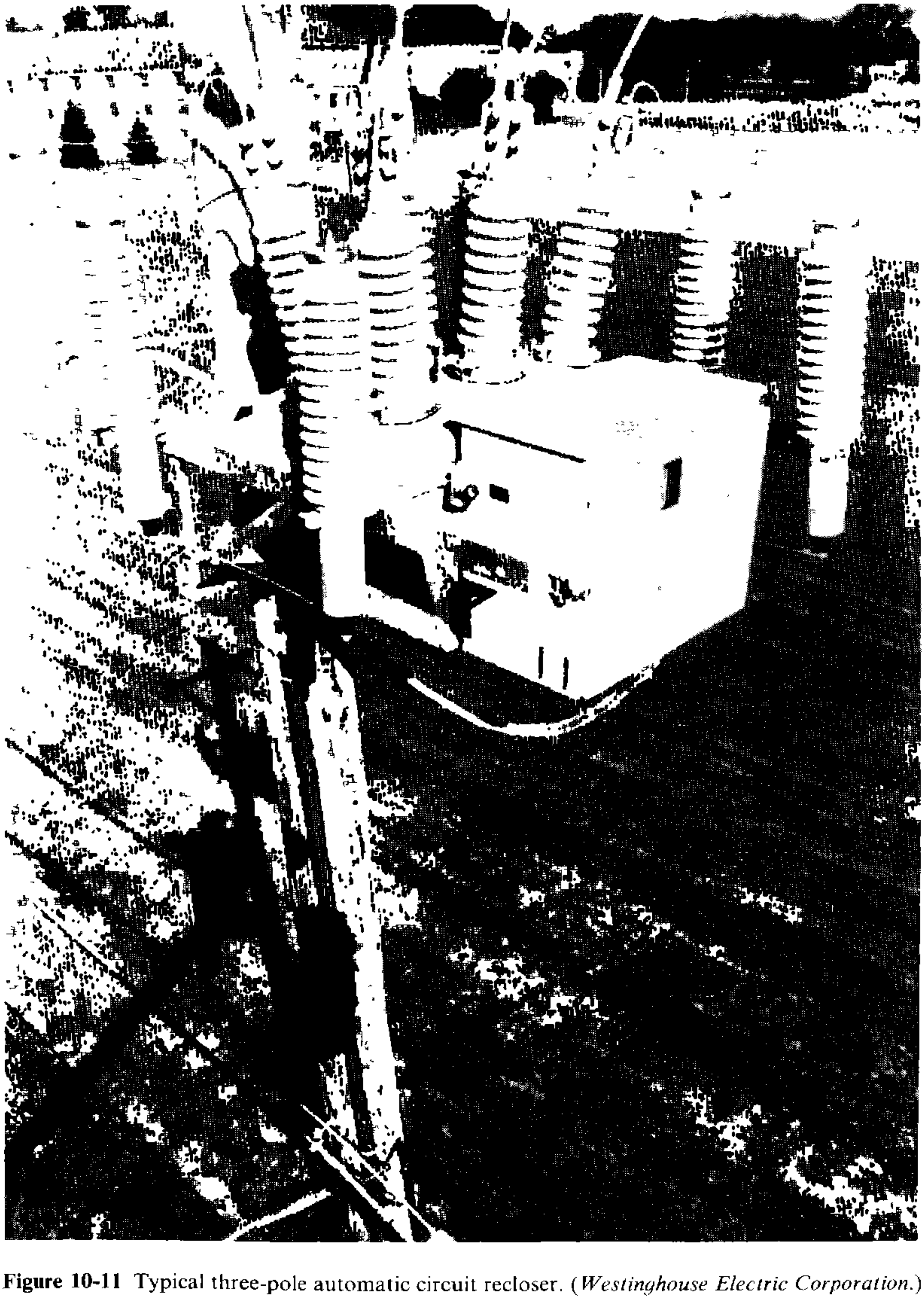


Figure 10-11 Typical three-pole automatic circuit recloser. (*Westinghouse Electric Corporation.*)

2. The backup protective device has to be able to sense the minimum fault current at the end of the sectionalizer's protective zone.
3. The minimum fault current has to be greater than the minimum actuating current of the sectionalizer.
4. Under no circumstances should the sectionalizer's momentary and short-time ratings be exceeded.
5. If there are two backup protective devices connected in series with each other and located ahead of a sectionalizer toward the source, the first and second backup devices should be set for four and three tripping operations, respectively, and the sectionalizer should be set to open during the second dead circuit time for a fault beyond the sectionalizer.
6. If there are two sectionalizers connected in series with each other and located after a backup protective device that is close to the source, the backup device should be set to lockout after the fourth operation, and the first and second sectionalizers should be set to open following the third and second counting operations, respectively.

The standard continuous current ratings for the line sectionalizers range from 10 to 600 A. Figure 10-12 shows typical single- and three-phase automatic line sectionalizers.

The advantages of using automatic line sectionalizers are:

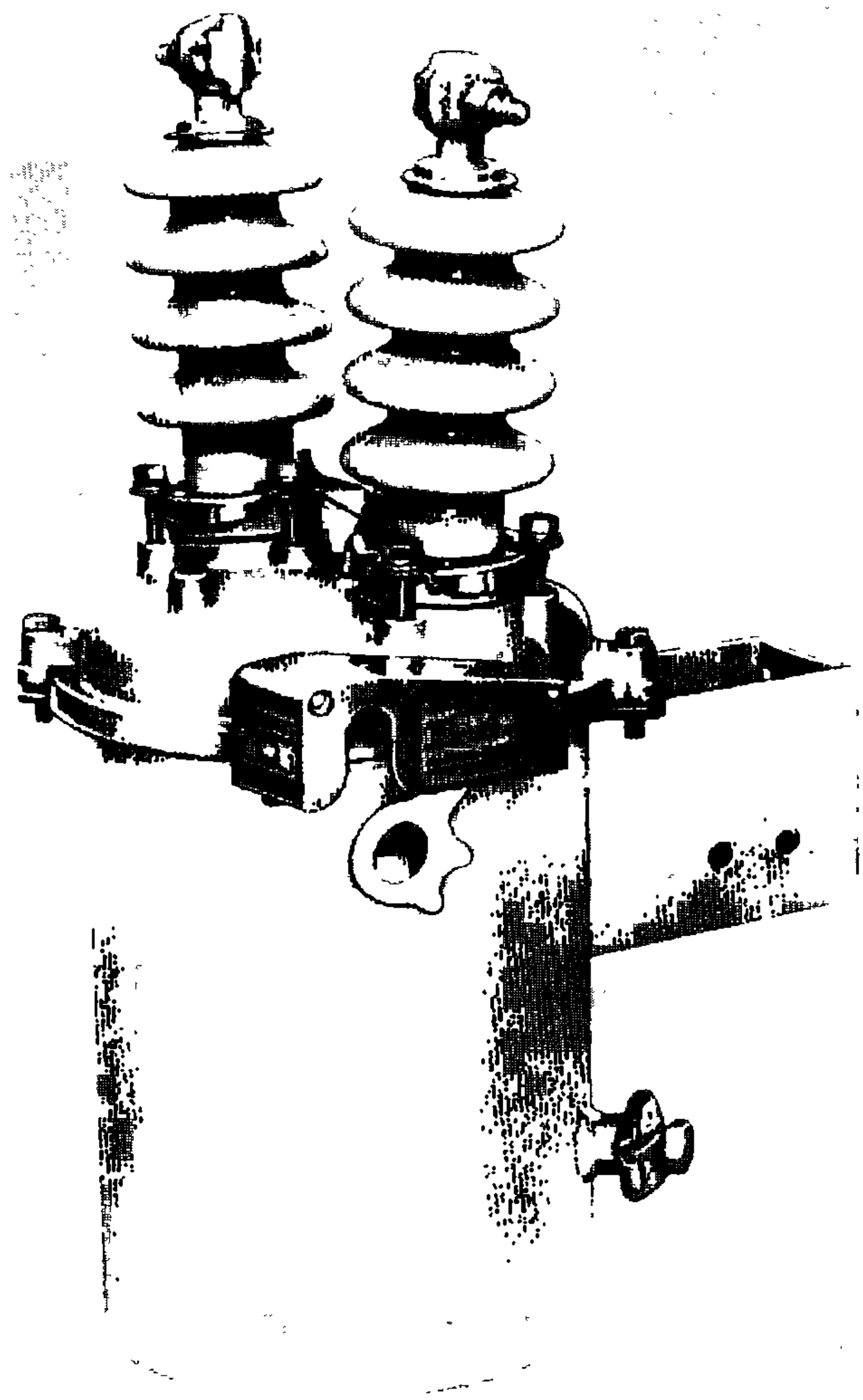
1. When employed as a substitute for reclosers, they have a lower initial cost and demand less maintenance.
2. When employed as a substitute for fused cutouts, they do not show the possible coordination difficulties experienced with fused cutouts due to improperly sized replacement fuses.
3. They may be employed for interrupting or switching loads within their ratings.

On the other hand, the disadvantages of using automatic line sectionalizers are:

1. When employed as a substitute for fused cutouts, they are more costly initially and demand more maintenance.
2. In general, in the past, their failure rate has been greater than that of fused cutouts.

10-2-4 Automatic Circuit Breakers

Circuit breakers are automatic interrupting devices which are capable of breaking and reclosing a circuit under all conditions, i.e., faulted or normal operating conditions. The primary task of a circuit breaker is to extinguish the arc that develops due to separation of its contacts in an arc-extinguishing medium, for example, in air, as is the case for air circuit breakers, in oil, as is the case for oil circuit breakers (OCBs), in SF_6 (sulphur hexafluoride), or in vacuum. In some types, the arc is



(a)

Figure 10-12 Typical single- and three-phase automatic line sectionalizers: (*a*) type GH.

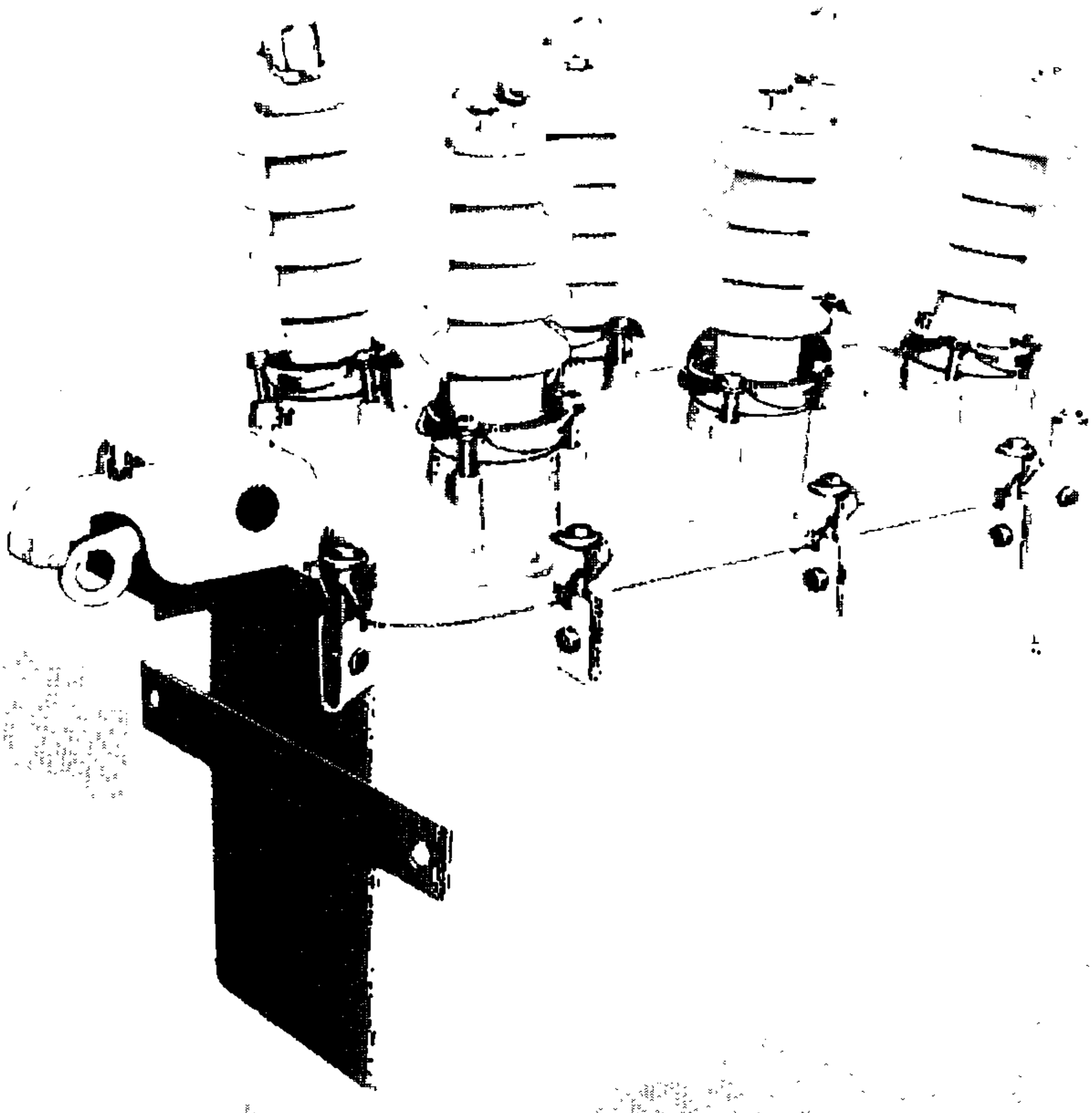
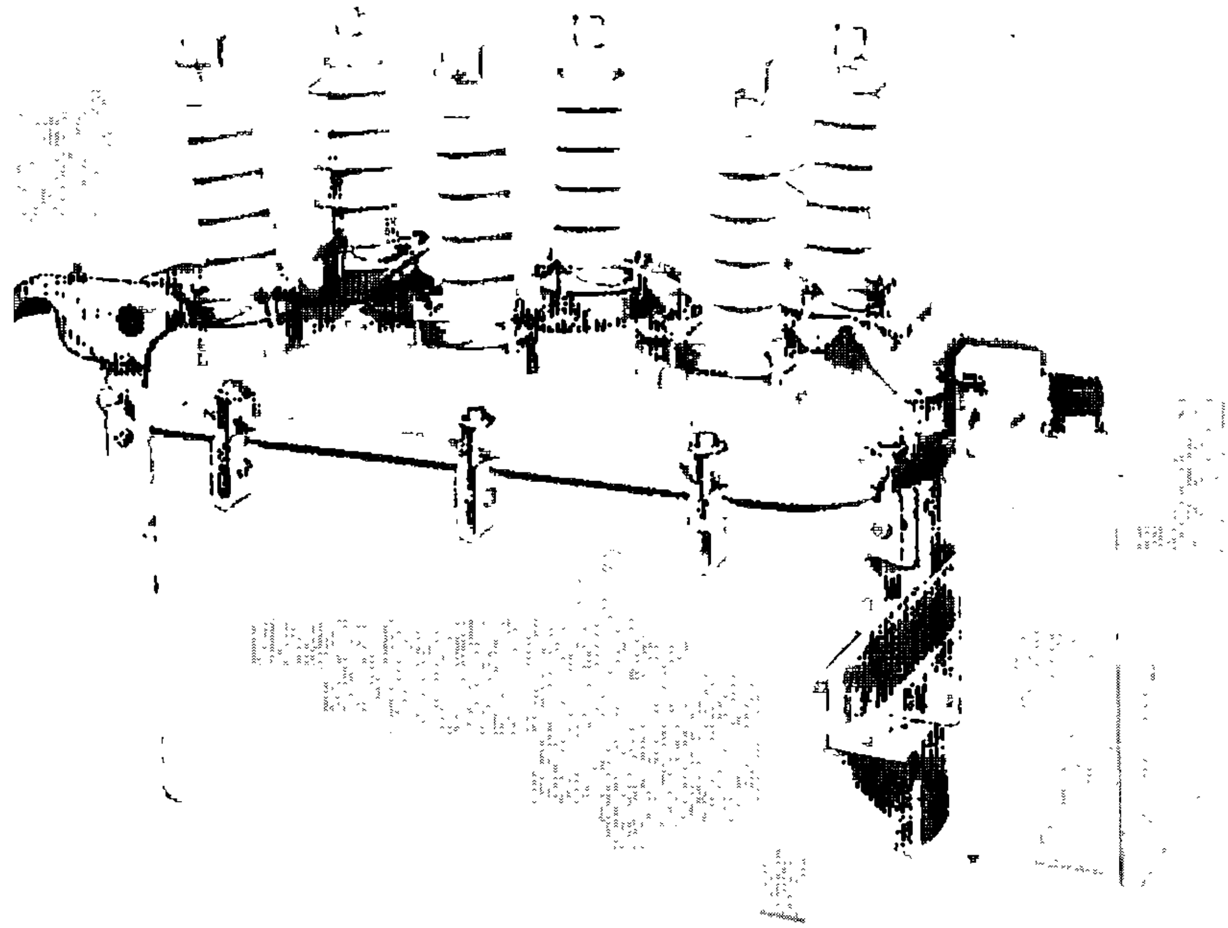


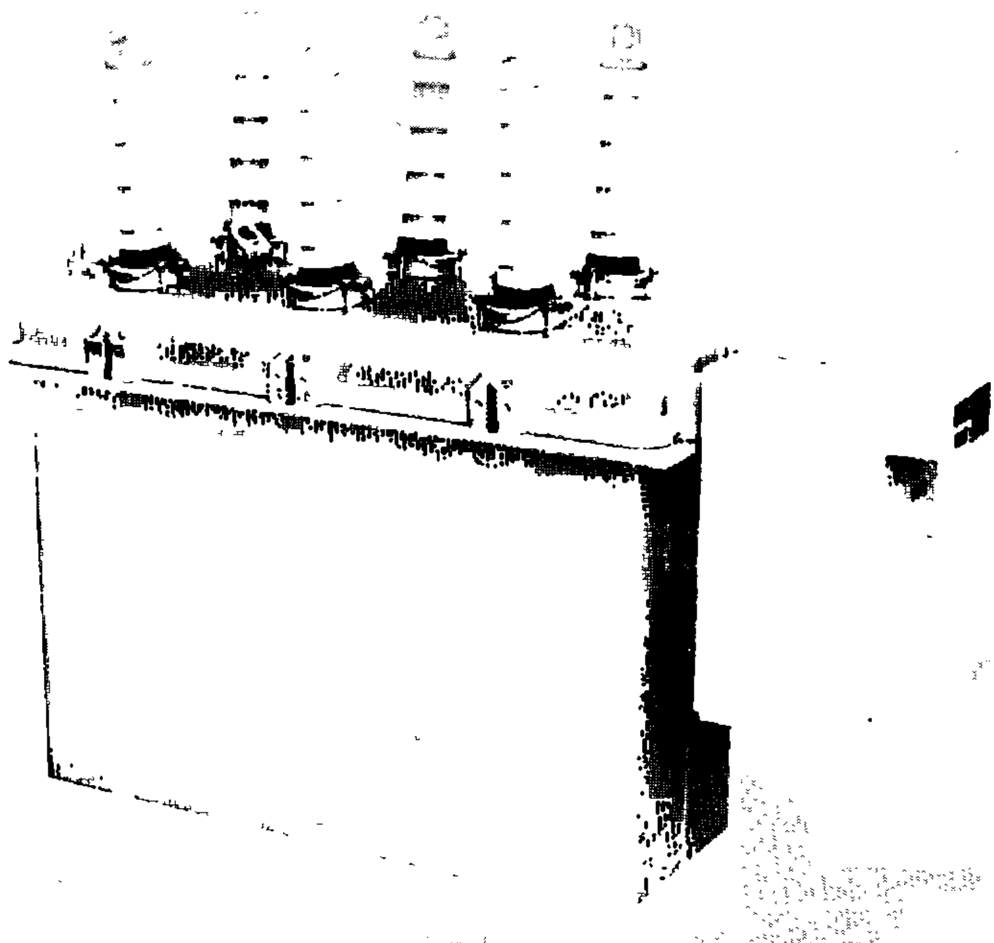
Figure 10-12 (*b*) type GN3.



(c)

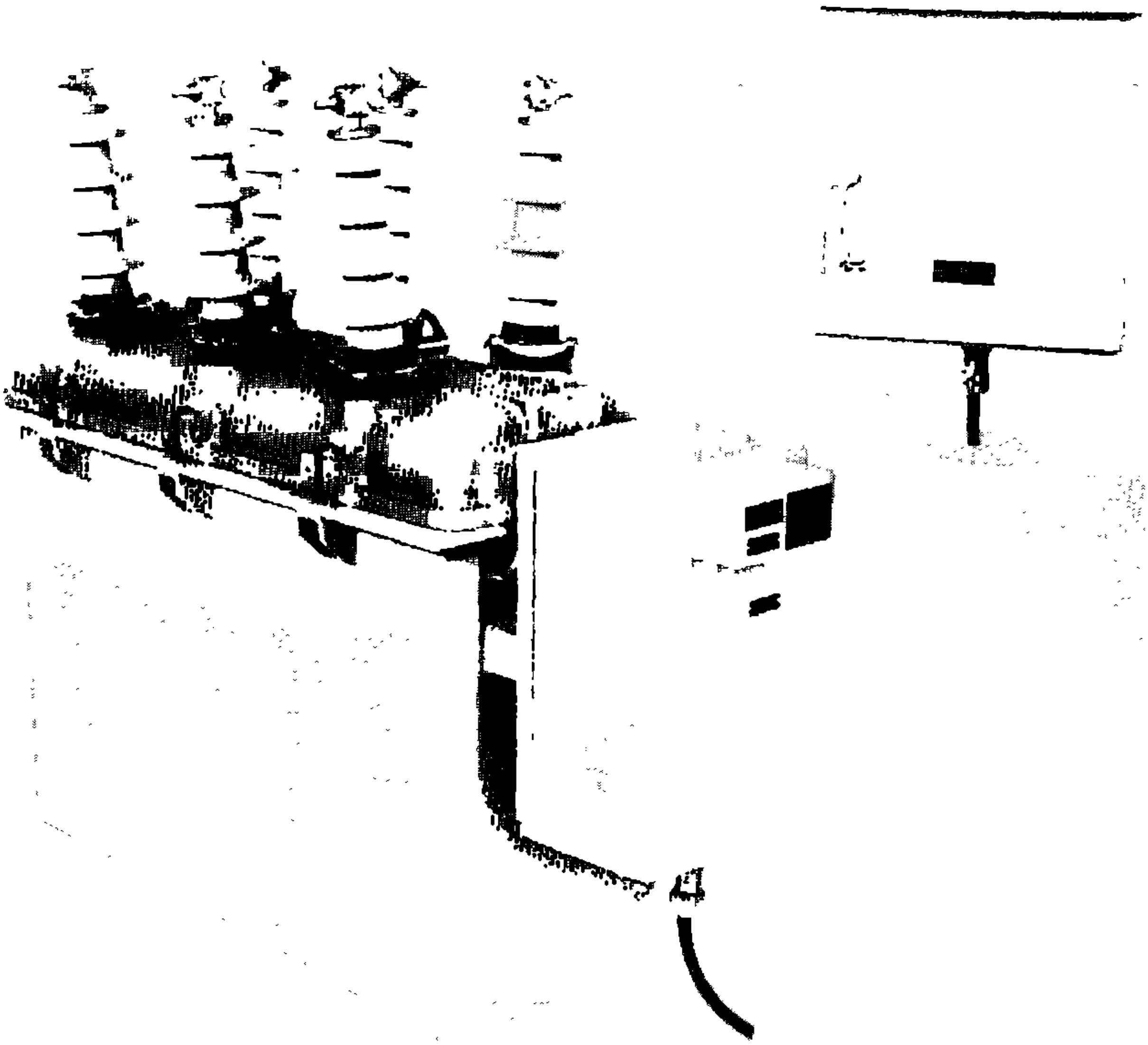
Figure 10-12 (c) type GN3E,

Figure 10-12 (*d*) type GV,



(e)

Figure 10-12 (e) type GW,



(f)

Figure 10-12 (*f*) type GWC. (*McGraw-Edison Company.*)

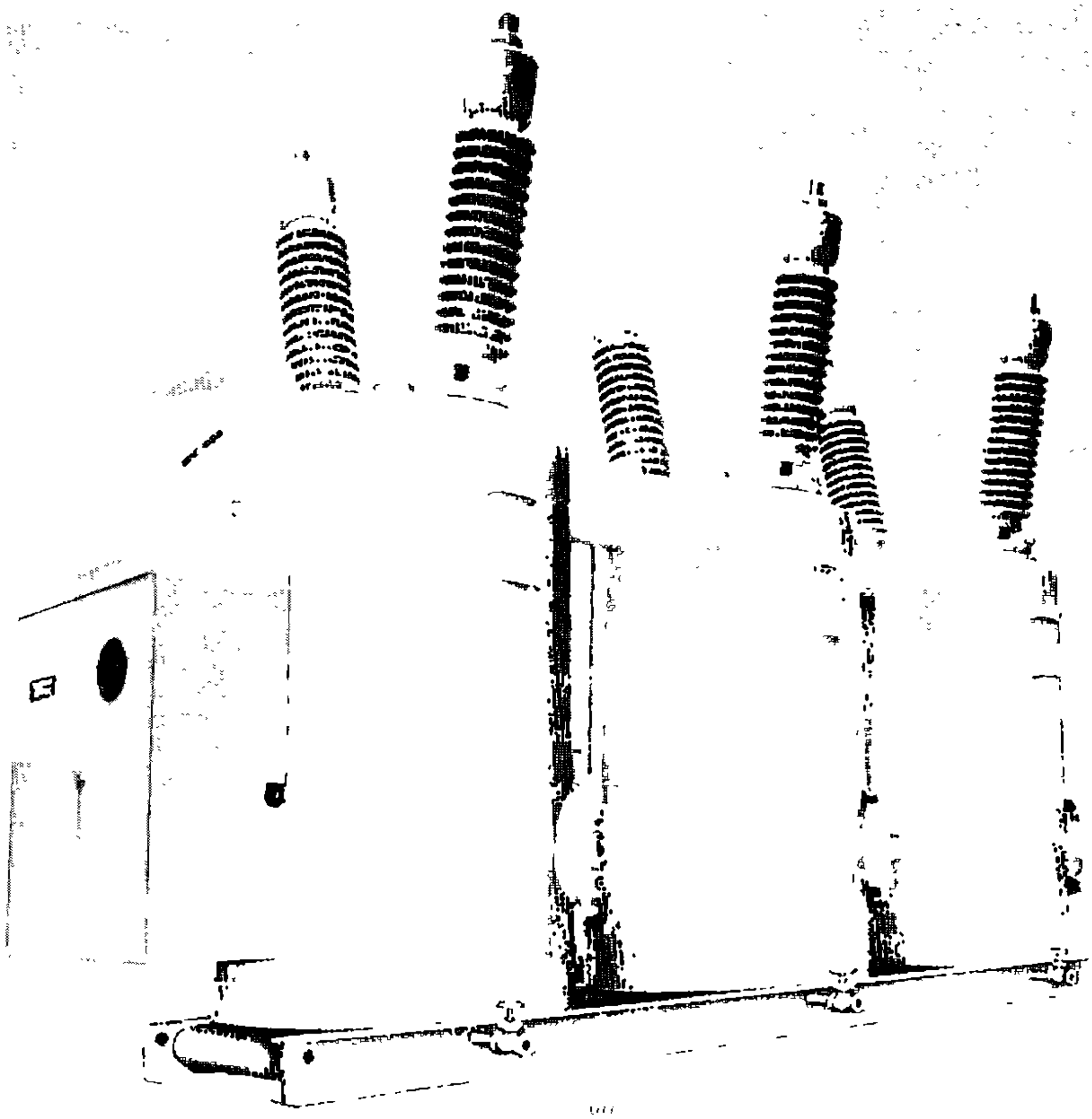


Figure 10-13 Typical oil circuit breakers. (*McGraw-Edison Company.*)

extinguished by a blast of compressed air, as is the case for magnetic blow-out circuit breakers. The circuit breakers used at distribution system voltages are of the air circuit breaker or oil circuit breaker type. For low-voltage applications molded-case circuit breakers are available.

Oil circuit breakers controlled by protective relays are usually installed at the source substations to provide protection against faults on distribution feeders. Figures 10-13 and 10-14 show typical oil and vacuum circuit breakers, respectively.

Currently, circuit breakers are rated on the basis of rms symmetrical current. Usually, circuit breakers used in the distribution systems have minimum operating times of 5 cycles. In general, relay-controlled circuit breakers are preferred to

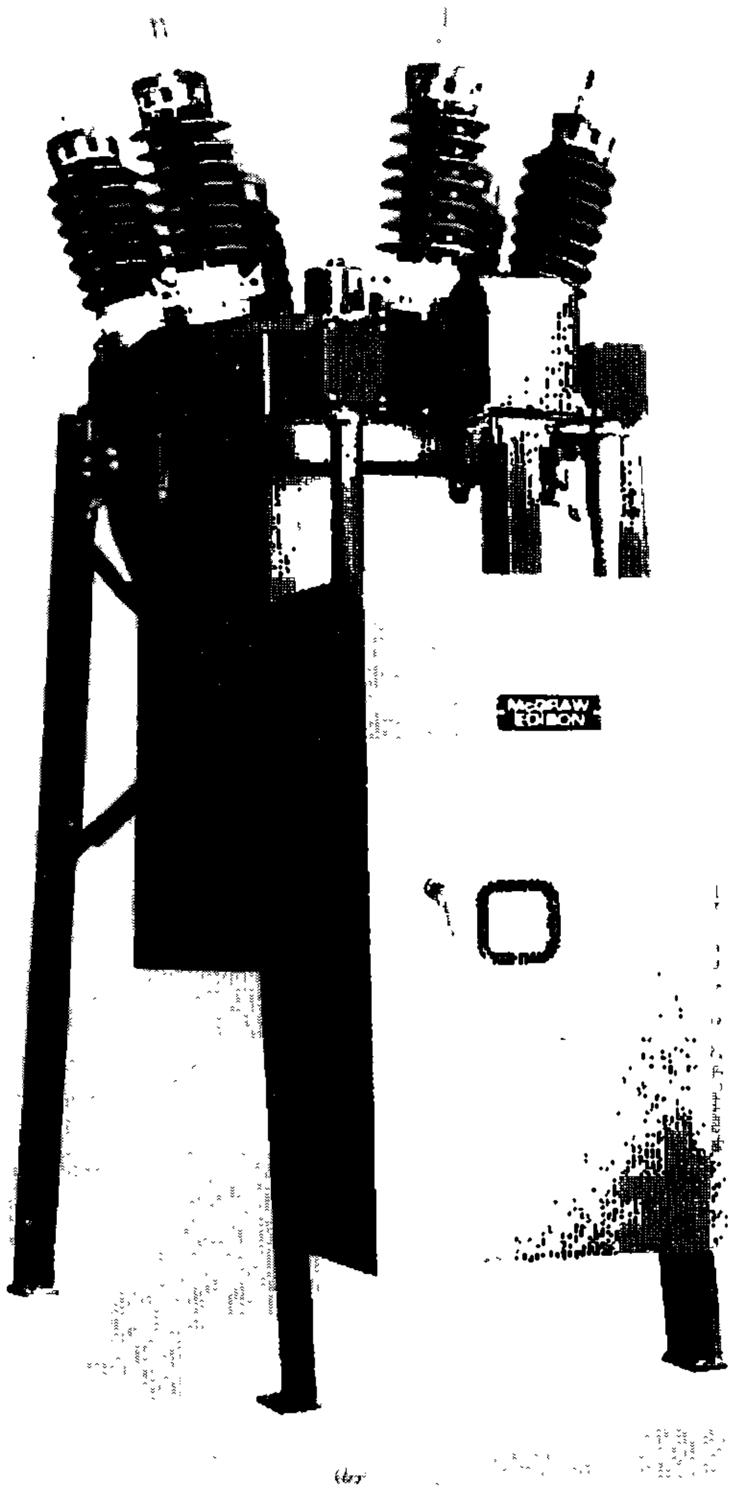


Figure 10-13 (*b*).

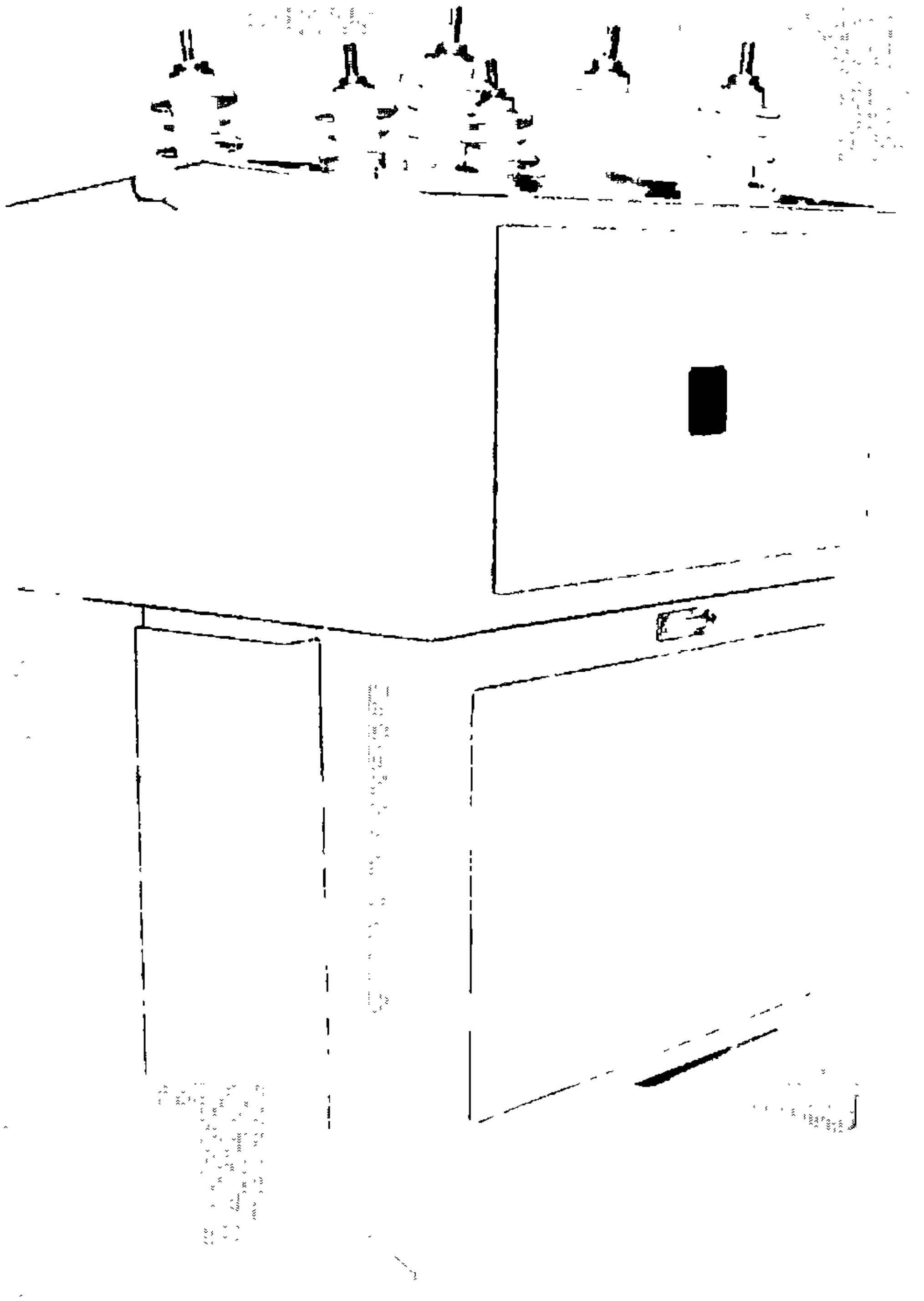


Figure 10-14 Typical vacuum circuit breaker. (*McGraw-Edison Company.*)

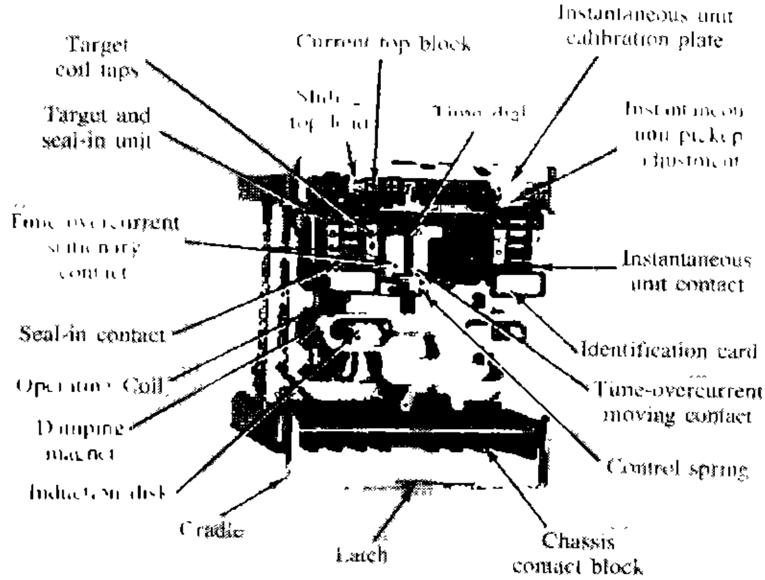


Figure 10-15 Typical IAC time-overcurrent relay (*General Electric Company.*)

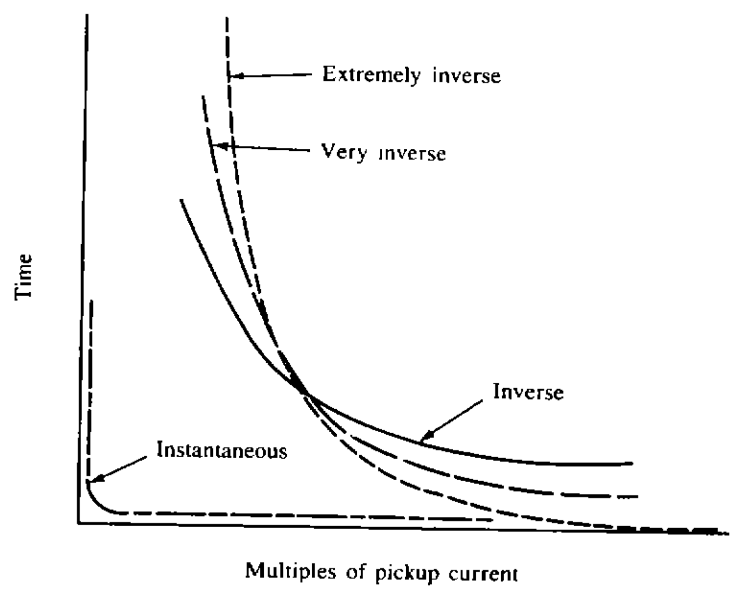


Figure 10-16 Time–current characteristic of overcurrent relays. (*From* [5].)

reclosers due to their greater flexibility, accuracy, design margins, and esthetics. However, they are much more expensive than reclosers.

The relay, or fault-sensing device, that opens the circuit breaker is generally an overcurrent induction type with inverse, very inverse, or extremely inverse time–current characteristics, e.g., the CO relays by Westinghouse or the IAC relays by General Electric. Figure 10-15 shows a typical IAC single-phase overcurrent-relay unit. Figure 10-16 shows typical time–current characteristics of overcurrent relays. Figure 10-17 shows time–current curves of typical overcurrent relays with inverse characteristics.

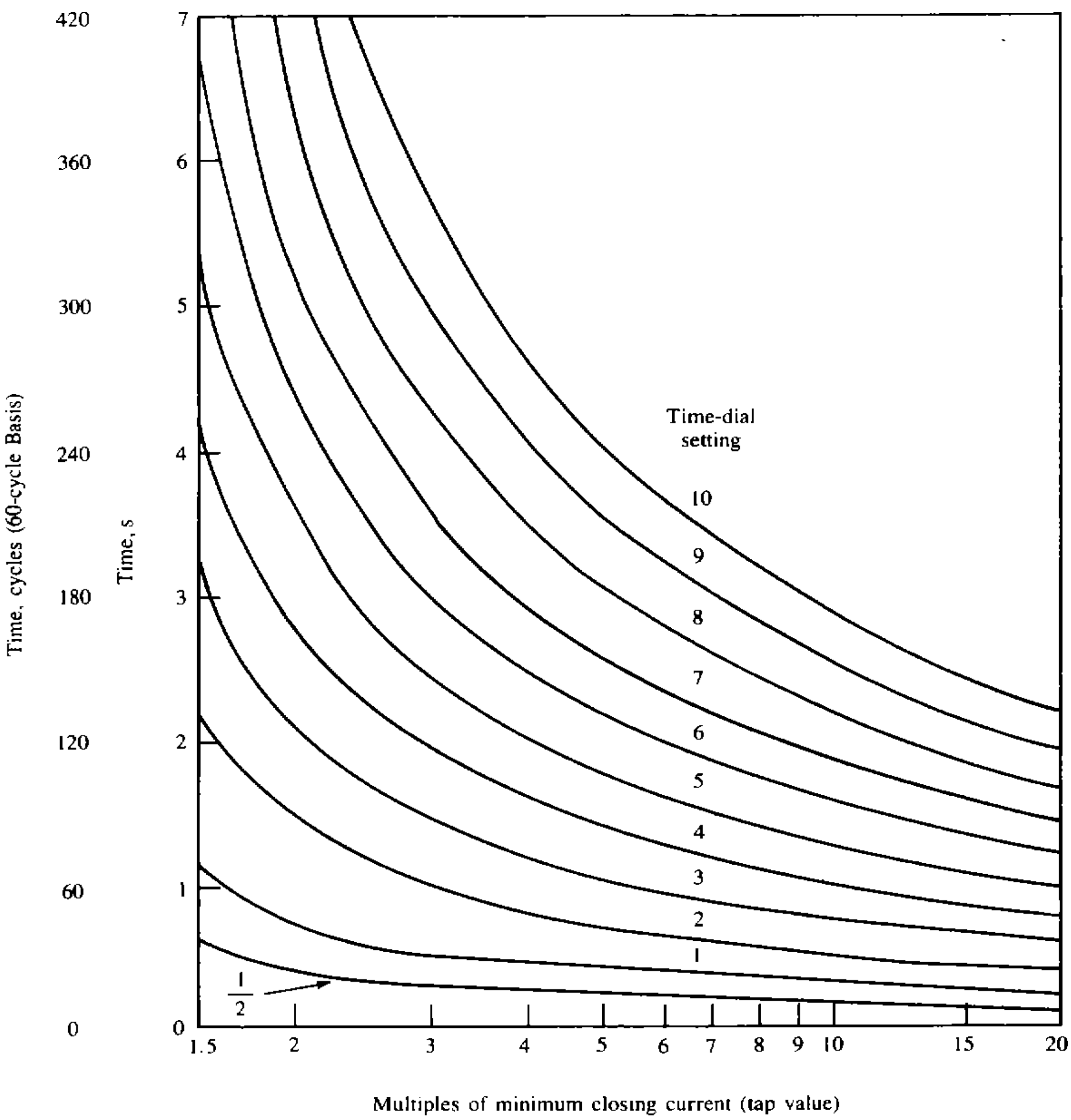


Figure 10-17 Time-current curves of IAC overcurrent relays with inverse characteristics. (*General Electric Company.*)

10-3 OBJECTIVE OF DISTRIBUTION SYSTEM PROTECTION

The main objectives of distribution system protection are (1) to minimize the duration of a fault and (2) to minimize the number of consumers affected by the fault.

The secondary objectives of distribution system protection are (1) to eliminate safety hazards as fast as possible, (2) to limit service outages to the smallest possible segment of the system, (3) to protect the consumers' apparatus, (4) to protect the system from unnecessary service interruptions and disturbances, and (5) to disconnect faulted lines, transformers, or other apparatus.

Overhead distribution systems are subject to two types of electrical faults, namely, transient (or temporary) faults and permanent faults.

Depending on the nature of the system involved, approximately 75 to 90 percent of the total number of faults are temporary in nature [2]. Usually, *transient faults* occur when phase conductors electrically contact other phase conductors or ground momentarily due to trees, birds or other animals, high winds, lightning, flashovers, etc. Transient faults are cleared by a service interruption of sufficient length of time to extinguish the power arc. Here, the fault duration is minimized and unnecessary fuse blowing is prevented by using instantaneous or high-speed tripping and automatic reclosing of a relay-controlled power circuit breaker or the automatic tripping and reclosing of a circuit recloser. The breaker speed, relay settings, and recloser characteristics are selected in a manner to interrupt the fault current before a series fuse (i.e., the nearest source-side fuse) is blown, which would cause the transient fault to become permanent.

Permanent faults are those which require repairs by a repair crew in terms of (1) replacing burned-down conductors, blown fuses, or any other damaged apparatus, (2) removing tree limbs from the line, and (3) manually reclosing a circuit breaker or recloser to restore service. Here, the number of customers affected by a fault is minimized by properly selecting and locating the protective apparatus on the feeder main, at the tap point of each branch, and at critical locations on branch circuits. Permanent faults on overhead distribution systems are usually sectionalized by means of fuses. For example, permanent faults are cleared by fuse cutouts installed at submain and lateral tap points. This practice limits the number of customers affected by a permanent fault and helps locate the fault point by reducing the area involved. In general, the only part of the distribution circuit not protected by fuses is the main feeder and feeder tie line. The substation is protected from faults on feeder and tie lines by circuit breakers and/or reclosers located inside the substation.

On the other hand, most of the faults are permanent on an underground distribution system, thereby requiring a different protection approach. Even though the number of faults occurring on an underground system is relatively much less than that on the overhead systems, they are usually permanent and can affect a larger number of customers. Faults occurring in the URD systems are cleared by the blowing of the nearest sectionalizing fuse or fuses. Faults occurring on the feeder are cleared by tripping and lockout of the feeder breaker.

Figure 10-18 shows a protection scheme of a distribution feeder circuit. As shown in the figure, each distribution transformer has a fuse which is located either externally, i.e., in a fuse cutout next to the transformer, or internally, i.e., inside the transformer tank as is the case for a completely self-protected (CSP) transformer.

As shown in Fig. 10-18, it is a common practice to install a fuse at the head of each lateral (or branch). The fuse must carry the expected load, and it must coordinate with load-side transformer fuses or other devices. It is customary to select the rating of each lateral fuse adequately large so that it is protected from damage by the transformer fuses on the lateral. Furthermore, the lateral fuse is

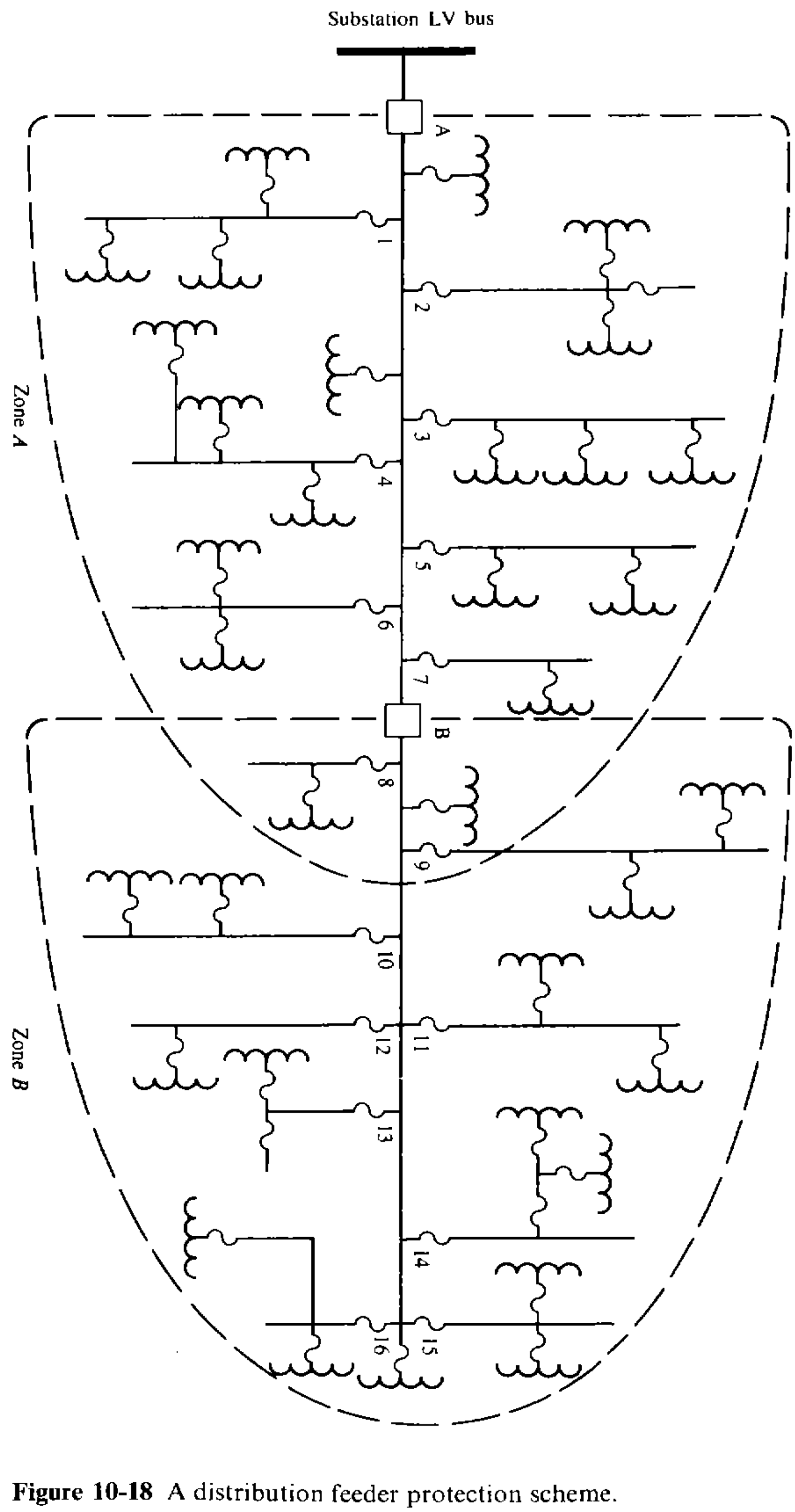


Figure 10-18 A distribution feeder protection scheme.

usually expected to clear faults occurring at the ends of the lateral. If the fuse does not clear the faults, then one or more additional fuses may be installed on the lateral.

As shown in the figure, a recloser, or circuit breaker A with reclosing relays, is located at the substation to provide a backup protection. It clears the temporary faults in its protective zone. At the limit of the protective zone, the minimum available fault current, determined by calculation, is equal to the smallest value of current (called *minimum pickup current*) which will trigger the recloser, or circuit breaker, to operate. However, a fault beyond the limit of this protection zone may not trigger the recloser, or circuit breaker, to operate. Therefore this situation may require that a second recloser, with a lower pickup current rating be installed at location B, as shown in the figure. The major factors which play a role in making a decision to choose a recloser over a circuit breaker are (1) the costs of equipment and installation and (2) the reliability. Usually, a comparable recloser can be installed for approximately one-third less than a relay-controlled oil circuit breaker. Even though a circuit breaker provides a greater interrupting capability, this excess capacity is not always required. Also, some distribution engineers prefer reclosers because of their flexibility, due to the many extras that are available with reclosers but not with circuit breakers.

10-4 COORDINATION OF PROTECTIVE DEVICES

The process of selecting overcurrent protection devices with certain time–current settings and their appropriate arrangement in series along a distribution circuit in order to clear faults from the lines and apparatus according to a preset sequence of operation is known as *coordination*. When two protective apparatus installed in series have characteristics which provide a specified operating sequence, they are said to be *coordinated* or *selective*. Here, the device which is set to operate first to isolate the fault (or interrupt the fault current) is defined as the *protecting device*. It is usually the apparatus closer to the fault. The apparatus which furnishes backup protection but operates only when the protecting device fails to operate to clear the fault is defined as the *protected device*. Properly coordinated protective devices help (1) to eliminate service interruptions due to temporary faults, (2) to minimize the extent of faults in order to reduce the number of customers affected, and (3) to locate the fault, thereby minimizing the duration of service outages.

Since coordination is primarily the selection of protective devices and their settings to develop zones that provide temporary fault protection and limit an outage area to the minimum size possible if a fault is permanent, to coordinate protective devices, in general, the distribution engineer must assemble the following data:

1. Scaled feeder-circuit configuration diagram (map).
2. Locations of the existing protective devices
3. Time–current characteristics (TCC) curves of protective devices

4. Load currents (under normal and emergency conditions)
5. Fault currents or megavoltamperes (under minimum and maximum generation conditions) at every point where a protective apparatus might be located

Usually, these data are not readily available and therefore must be brought together from numerous sources. For example, the TCCs of protective devices are gathered from the manufacturers, the values of the load currents and fault currents are usually taken from computer runs called the *load flow studies* and *fault studies*, respectively.

In general, manual techniques for coordination are still employed by most utilities, especially where distribution systems are relatively small or simple and therefore only a small number of protective devices are used in series. However, some utilities have established standard procedures, tables, or other means to aid the distribution engineer and field personnel in coordination studies. Some utilities employ semiautomated, computerized coordination programs developed either by the protective device manufacturers or by the company's own staff.

A general coordination procedure, regardless of whether it is manual or computerized, can be summarized as [3, 4]:

1. Gather the required and aforementioned data.
2. Select initial locations on the given distribution circuit for protective (i.e., sectionalizing) devices.
3. Determine the maximum and minimum values of fault currents (specifically for three-phase, line-to-line, and line-to-ground faults) at each of the selected locations and at the end of the feeder main, branches, and laterals.
4. Pick out the necessary protective devices located at the distribution substation in order to protect the substation transformer properly from any fault that might occur in the distribution circuit.
5. Coordinate the protective devices from the substation outward or from the end of the distribution circuit back to the substation.
6. Reconsider and change, if necessary, the initial locations of the protective devices.
7. Reexamine the chosen protective devices for current-carrying capacity, interrupting capacity, and minimum pickup rating.
8. Draw a composite TCC curve showing the coordination of all protective devices employed, with curves drawn for a common base voltage (this step is optional).
9. Draw a circuit diagram which shows the circuit configuration, the maximum and minimum values of the fault currents, and the ratings of the protective devices employed, etc.

There are also some additional factors that need to be considered in the coordination of protective devices (i.e., fuses, reclosers, and relays) such as (1) the differences in the TCCs and related manufacturing tolerances, (2) preloading conditions of the apparatus, (3) ambient temperature, and (4) effect of reclosing cycles. These factors affect the adequate margin for selectivity under adverse conditions.

10-5 FUSE-TO-FUSE COORDINATION

The selection of a fuse rating to provide adequate protection to the circuit beyond its location is based upon several factors. First of all, the selected fuse must be able to carry the expanded load current, and, at the same time, it must be sufficiently selective with other protective apparatus in series. Furthermore, it must have an adequate reach; i.e., it must have the capability to clear a minimum fault current within its zone in a predetermined time duration.

A fuse is designed to blow within a specified time for a given value of fault current. The TCCs of a fuse are represented by two curves; the minimum-melting curve and the total-clearing curve, as shown in Fig. 10-19. The minimum-melting curve of a fuse represents the minimum time, and therefore it is the plot† of the minimum time vs. current required to melt the fuse. The total-clearing (time) curve represents the total time, and therefore it is the plot of the maximum time vs. current required to melt the fuse and extinguish the arc, plus manufacturing tolerance. It is

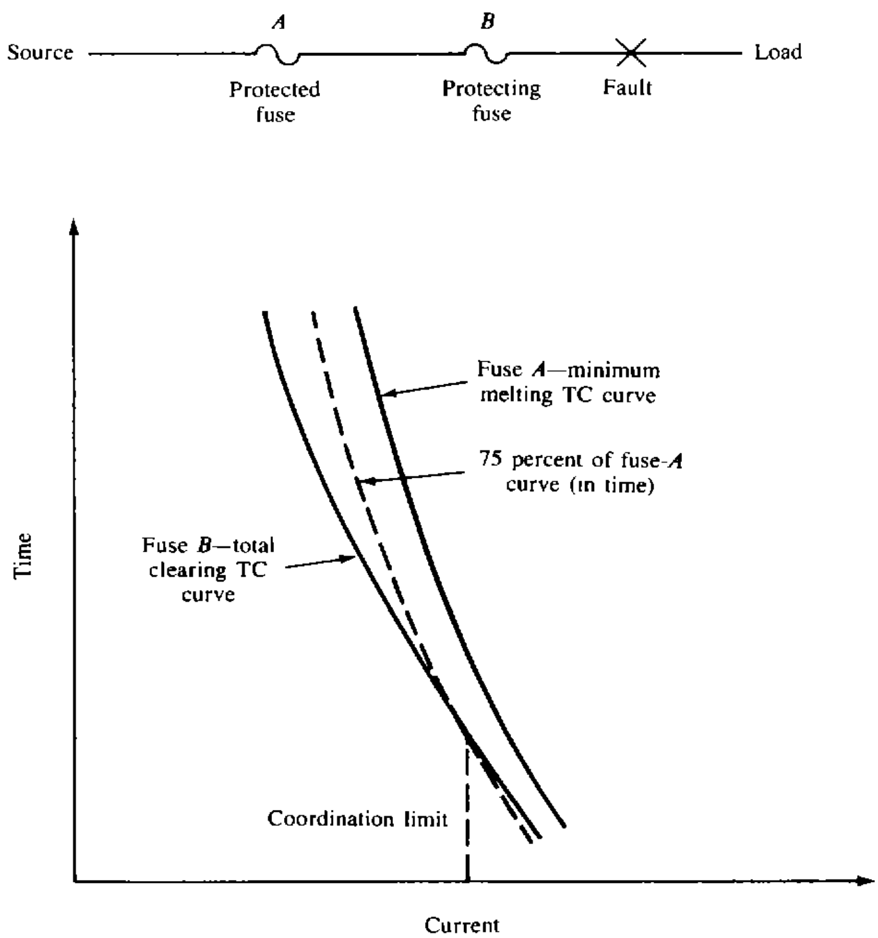
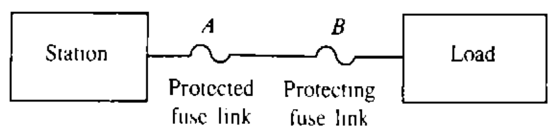


Figure 10-19 Coordinating fuses in series using TCC curves of the fuses connected in series.

† TCC curves of overcurrent protective devices are plotted on log-log coordinate paper. The use of this standard-size transparent paper allows the comparison of curves by superimposing one sheet over another.

Table 10-3 Coordination table for G.E. type "K" (fast) fuse links used in G.E. 50-, 100-, or 200-A expulsion fuse cutouts and connected in series

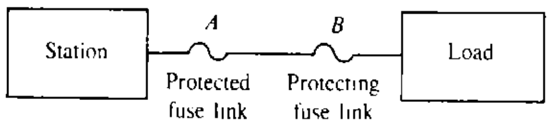


When the protecting fuse link is used in a G-E single-element fuse cutout

Type "K" ratings of protecting fuse links (B in diagram), A	Type "K" ratings of protected fuse links (A in diagram), A																		
	6K	8K	10K	12K	15K	20K	25K	30K	40K	50K	65K	80K	52*	100K	101*	140K	200K	102*	103*
	Maximum short-circuit rms amperes to which fuse links will be protected																		
1K	135	215	300	395	530	660	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
2K	110	195	300	395	530	660	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
3K	80	165	290	395	530	660	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
5-A series Hi-surge	14	133	270	395	530	660	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
6K		37	145	270	460	620	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
8K			133	170	390	560	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
10-A series Hi-surge		16	24	260	530	660	820	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
10K				38	285	470	720	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
12K					140	360	660	1100	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
15K						95	410	960	1370	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
20K							70	700	1200	1720	2200	2750	3250	3600	5800	6000	9700	9500	16000
25K								140	580	1300	2200	2750	3250	3600	5800	6000	9700	9500	16000
30K									215	700	1800	2750	3250	3600	5800	6000	9700	9500	16000
40K										170	1200	2750	3250	3600	5800	6000	9700	9500	16000
50K											195	1600	3250	3600	5800	6000	9700	9500	16000
65K												330		2300	5800	6000	9700	9500	16000
52*														290	5500	6000	9700	9500	16000
80K														580	5800	6000	9700	9500	16000
100K															300	4300	9700	9500	16000
101*																385	7500		16000
140K																	2800		16000
102*																	1250		

* G. E. coordinating fuse links.
Source: From [4].

Table 10-4 Coordination table for G.E. type "T" (slow) fuse links used in G.E. 50-, 100-, or 200-A expulsion fuse cutouts and connected in series



When the protecting fuse link is used in a G-E single-element fuse cutout.

Type "T" ratings of protecting fuse links (B in diagram), A	Type "T" ratings of protected fuse links (A in diagram), A															
	6T	8T	10T	12T	15T	20T	25T	30T	40T	50T	65T	80T	100T	140T	200T	103†
	Maximum short-circuit rms amperes to which fuse links will be protected															
1N*	250	395	540	710	950	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
2N*	250	395	540	710	950	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
3N*	250	395	540	710	950	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
6T		33	365	650	950	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
8T			125	480	850	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
10-A series Hi-surge		19	540	710	950	1220	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
10T				74	620	1130	1500	1930	2500	3100	3950	4950	6300	9600	15000	16000
12T					135	770	1400	1930	2500	3100	3950	4950	6300	9600	15000	16000
15T						100	880	1750	2500	3100	3950	4950	6300	9600	15000	16000
20T							105	1150	2300	3100	3950	4950	6300	9600	15000	16000
25T								190	1500	3100	3950	4950	6300	9600	15000	16000
30T									115	1900	3950	4950	6300	9600	15000	16000
40T										310	2350	4950	6300	9600	15000	16000
50T											150	3400	6300	9600	15000	16000
65T												270	4300	9600	15000	16000
80T													660	9200	15000	16000
100T														6000	15000	16000
140T															6600	

* The 1N, 2N, and 3N ampere ratings of the G.E. 5-A series Hi-surge fuse links have time–current characteristics closely approaching those established by the American Standards for 1T, 2T, and 3T ampere ratings respectively. Hence, they are recommended for applications requiring 1T, 2T, or 3T fuse links.

† G. E. coordinating fuse links.

Source: From [4].

also a standard procedure to develop "damaging" time curves from the minimum-melting-time curves by using a safety factor of 25 percent. Therefore the damaging curve (due to the partial melting) is developed by taking 75 percent of the minimum melting time of a specific-size fuse at various current values. The time unit used in these curves is seconds.

Fuse-to-fuse coordination, i.e., the coordination between fuses connected in series, can be achieved by two methods:

1. Using the TCC curves of the fuses
2. Using the coordination tables prepared by the fuse manufacturers

Furthermore, some utilities employ certain rules of thumb as a third type of fuse-to-fuse coordination method.

In the first method, the coordination of the two fuses connected in series, as shown in Fig. 10-19, is achieved by comparing the total-clearing-time–current curve of the "protecting fuse," i.e., fuse B, with the damaging-time curve of the "protected fuse," i.e., fuse A. Here, it is necessary that the total clearing time of the protecting fuse not exceed 75 percent of the minimum melting time of the protected fuse. The 25 percent margin has been selected to take into account some of the operating variables, such as preloading, ambient temperature, and the partial melting of a fuse link due to a fault current of short duration. Of course, if there is no intersection between the aforementioned curves, a complete coordination in terms of selectivity is achieved. However, if there is an intersection of the two curves, the associated current value at the point of the intersection gives the coordination limit for the partial coordination achieved.

In the second method of fuse-to-fuse coordination, coordination is established by using the fuse sizes from coordination tables developed by the fuse link manufacturers. Tables 10-3 and 10-4 are such tables developed by the General Electric Company for fast and slow fuse links, respectively. These tables give the maximum fault currents to achieve coordination between various fuse sizes and are based upon the 25 percent margin described in the first method. Here, the determination of the total clearing curve is not necessary since the maximum value of fault current to which each combination of series fuses can be subjected with guaranteed coordination is given in the tables, depending upon the type of fuse link selected.

10-6 RECLOSER-TO-RECLOSER COORDINATION

The need for recloser-to-recloser coordination may arise due to any of the following situations that may exist in a given distribution system:

1. Having two three-phase reclosers
2. Having two single-phase reclosers
3. Having a three-phase recloser at the substation and a single-phase recloser on one of the branches of a given feeder

The required coordination between the reclosers can be achieved by using one of the following remedies:

1. Employing different recloser types and some mixture of coil sizes and operating sequences
2. Employing the same recloser type and operating sequence but using different coil sizes.
3. Employing the same recloser type and coil sizes but using different operating sequences.

In general, the utility industry prefers to use the first remedy over the other two. However, there may be some circumstances, e.g., having two single-phase reclosers of the same type, where the second remedy can be applied. When the TCC curves of the two reclosers are less than 12 cycles separate from each other, the reclosers may do their instantaneous or fast operations at the same time. To achieve coordination between the delayed tripping curves of two reclosers, at least a minimum time margin of 25 percent must be applied.

10-7 RECLOSER-TO-FUSE COORDINATION

In Figure 10-20, curves represent the instantaneous, time-delay, and extended time-delay (as an alternative) tripping characteristics of a conventional automatic circuit recloser. Here, curves A and B symbolize the first and second openings, and the third and fourth openings of the recloser, respectively.

To provide protection against permanent faults, fuse cutouts (or power fuses) are installed on overhead feeder taps and laterals. The use of an automatic reclosing device as a backup protection against temporary faults eliminates many unnecessary outages that occur when using fuses only. Here, the backup recloser can

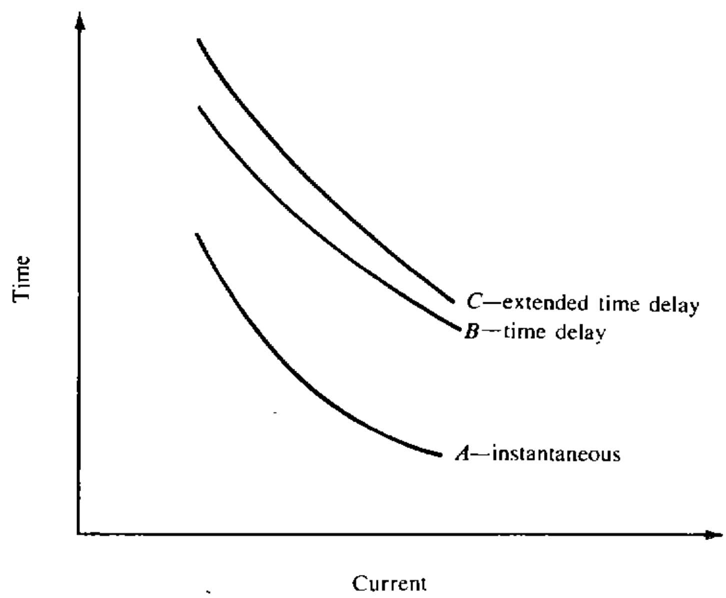


Figure 10-20 Typical recloser tripping characteristics.

be either the substation feeder recloser, usually with an operating sequence of one fast- and two delayed-tripping operations, or a branch feeder recloser, with two fast- and two delayed-tripping operations. The recloser is set to trip for a temporary fault before any of the fuses can blow, and then reclose the circuit. However, if the fault is a permanent one, it is cleared by the correct fuse before the recloser can go on time-delay operation following one or two instantaneous operations.

Figure 10-21 shows a portion of a distribution system where a recloser is installed ahead of a fuse. The figure also shows the superposition of the TCC curve of the fuse *C* on the fast and delayed TCC curves of the recloser *R*. If the fault beyond fuse *C* is temporary, the instantaneous tripping operations of the recloser protect the fuse from any damage. This can be observed from the figure by the fact that the instantaneous recloser curve *A* lies below the fuse TCC for currents less than that associated with the intersection point *b*. However, if the fault beyond fuse *C* is a permanent one, the fuse clears the fault as the recloser goes through a delayed operation *B*. This can be observed from the figure by the fact that the time-delay curve *B* of the recloser lies above the total-clearing-curve portion of the fuse TCC for currents greater than that associated with the intersection point *a*. The distance between the intersection points *a* and *b* gives the coordination range for the fuse and recloser.

Therefore a proper coordination of the trip operations of the recloser and the total clearing time of the fuse prevents the fuse link from being damaged during instantaneous trip operations of the recloser. The required coordination between the recloser and the fuse can be achieved by comparing the respective time-current curves and taking into account other factors, e.g., preloading, ambient temperature, curve tolerances, and accumulated heating and cooling of the fuse link during the fast-trip operations of the recloser.

Figure 10-22 illustrates the temperature cycle of a fuse link during recloser operations. As can be observed from the figure, each of the first two (instantaneous)

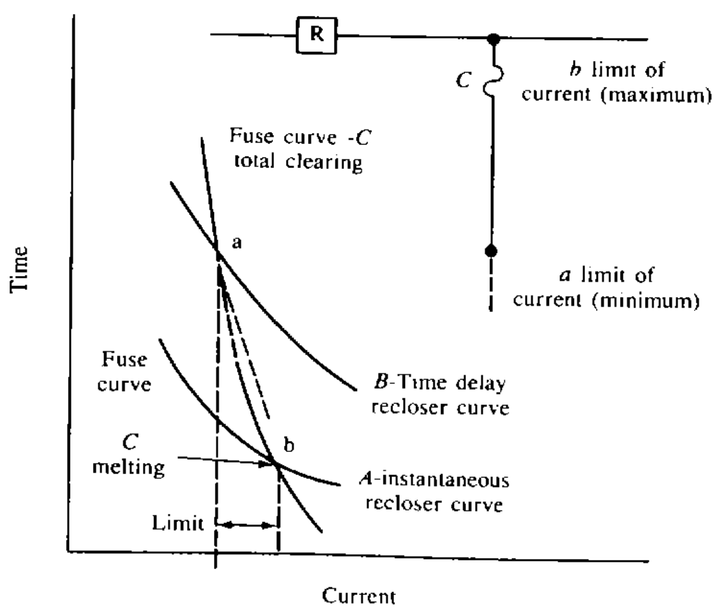


Figure 10-21 Recloser TCC curves superimposed on fuse TCC curves. (*From* [5].)

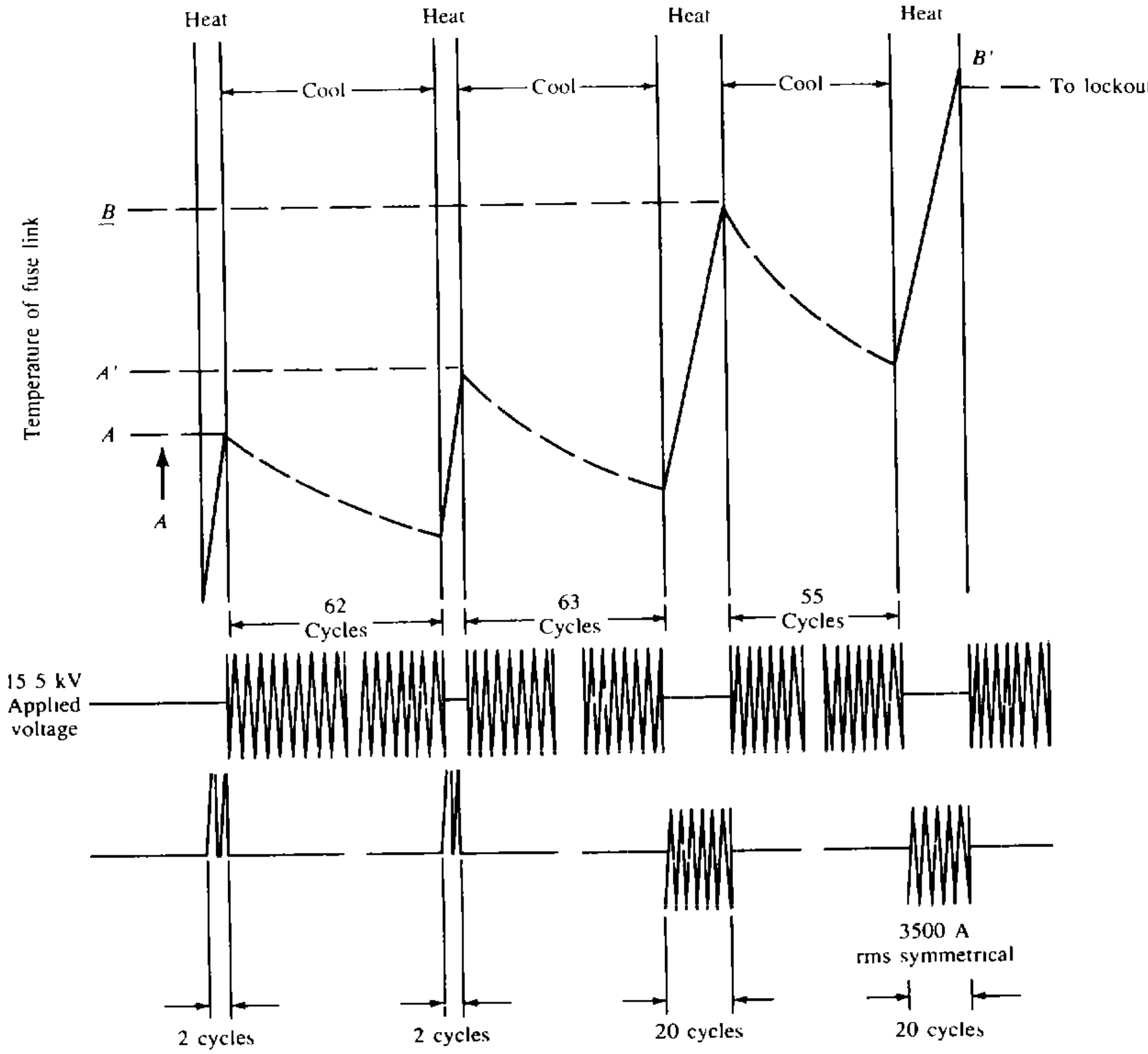


Figure 10-22 Temperature cycle of fuse link during recloser operation. (*From* [5].)

operations takes only 2 cycles, but each of the last two (delayed) operations last 20 cycles. After the fourth operation the recloser locks itself open.

Therefore the recloser-to-fuse coordination method illustrated in Fig. 10-21 is an approximate one since it does not take into account the effect of the accumulated heating and cooling of the fuse link during recloser operation. Thus, it becomes necessary to compute the heat input to the fuse during, for example, two instantaneous recloser operations if the fuse is to be protected from melting during these two openings.

Figure 10-23 illustrates a practical yet sufficiently accurate method of coordination. Here, the maximum coördinating current is found by the intersection (at point b') of two curves, the fuse-damage curve (which is defined as 75 percent of the minimum-melting-time curve of the fuse) and the maximum-clearing-time curve of the recloser's fast-trip operation (which is equal to $2 \times A$ "in time", since there are two fast trips). Similarly, point a' is found from the intersection of the fuse

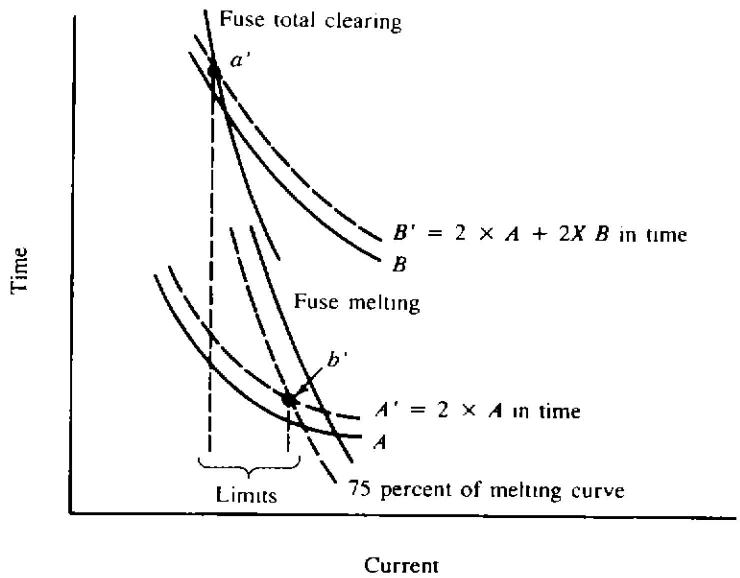


Figure 10-23 Recloser-to-fuse coordination (corrected for heating and cooling cycle. (*From* [5].)

total clearing curve with the shifted curve B' (which is equal to $2 \times A + 2 \times B$ "in time," since in addition to the two fast trips there are two delayed trips.

Some distribution engineers use rule-of-thumb methods, based upon experience, to allow extra margin in the coordination scheme.

As shown in Table 10-5, there are also coordination tables developed by the manufacturers to coordinate reclosers with fuse links in a simpler way.

Table 10-5 Automatic recloser and fuse ratings

Recloser rating, rms A (continuous)	Fuse link ratings, rms A	Ratings of G.E. type T fuse links, A								
		2N*	3N*	6T	8T	10T	12T	15T	20T	25T
		Range of coordination, rms A								
5	Min	14	17.5	68						
	Max	55	55	123						
10	Min			31	45	75	200			
	Max†			110	152	220	300, 250			
15	Min			30	34	59	84	200	380	
	Max†			105	145	210	280	375	450‡	
25	Min			50	50	50	68	105	145	300
	Max			89	130	190	265	360	480	610

* The 1N, 2N, and 3N ampere ratings of the G.E.5-A series Hi-surge fuse links have time-current characteristics closely approaching those established by the EEI-NEMA Standards for 1T, 2T, and 3T ampere ratings, respectively. Hence they are recommended for applications requiring 1T, 2T, or 3T fuse links.

† Where maximum lines have two values, the smaller value is for the 50-A frame, single-phase recloser. The larger value is for all others: 50-A frame, three-phase; 140-A frame, single-phase, and three-phase.

‡ Coordination with 50-A frame size single-phase recloser not possible since maximum interrupting capacity is *less* than minimum value.

Source: From [2]. Used with permission.

10-8 RECLOSER-TO-SUBSTATION TRANSFORMER HIGH-SIDE FUSE COORDINATION

Usually, a power fuse, located at the primary side of a delta-wye-connected substation transformer, provides protection for the transformer against the faults in the transformer or at the transformer terminals and also provides backup protection for feeder faults. These fuses have to be coordinated with the reclosers or reclosing circuit breakers located on the secondary side of the transformer to prevent the fuse from any damage during the sequential tripping operations. The effects of the accumulated heating and cooling of the fuse element can be taken into account by adjusting the delayed-tripping time of the recloser.

To achieve a coordination, the adjusted tripping time is compared to the minimum melting time of the fuse element, which is plotted for a phase-to-phase fault that might occur on the secondary side of the transformer. If the minimum melting time of the backup fuse is greater than the adjusted tripping time of the recloser, a coordination between the fuse and recloser is achieved. The coordination of a substation circuit breaker with substation transformer primary fuses dictates that the total clearing time of the circuit breaker (i.e., relay time plus breaker interrupting time) be less than 75 to 90 percent of the minimum melting time of the fuses at all values of current up to the maximum fault current. The selected fuse must be able to carry 200 percent of the transformer full-load current continuously in any emergency in order to be able to carry the transformer "magnetizing" inrush current (which is usually 12 to 15 times the transformer full-load current) for 0.1s [5].

10-9 FUSE–TO–CIRCUIT-BREAKER COORDINATION

The fuse–to–circuit-breaker (overcurrent relay) coordination is somewhat similar to the fuse-to-recloser coordination. In general, the reclosing time intervals of a circuit breaker are greater than those of a recloser. For example, 5 s is usually the minimum reclosing time interval for a circuit breaker, whereas the minimum reclosing time interval for a recloser can be as small as $\frac{1}{2}$ s. Therefore, when a fuse is used as the backup or protected device, there is no need for heating and cooling adjustments. Thus, to achieve a coordination between a fuse and circuit breaker, the minimum-melting-time curve of the fuse is plotted for a phase-to-phase fault on the secondary side. If the minimum melting time of the fuse is approximately 135 percent of the combined time of the circuit breaker and related relays, the coordination is achieved. However, when the fuse is used as the protecting device, the coordination is achieved if the relay operating time is 150 percent of the total clearing time of the fuse.

In summary, when the circuit breaker is tripped instantaneously, it has to clear the fault before the fuse is blown. On the other hand, the fuse has to clear the fault before the ciruit breaker trips on time-delay operations. Therefore it is necessary that the relay characteristic curve, at all values of current up to the maximum current available at the fuse location, lie above the total clearing characteristic curve of the fuse. Thus it is usually customary to leave a margin between the relay and fuse characteristic curves to include a safety factor of 0.1 to 0.3 + 0.1 s for relay overtravel time.

A sectionalizing fuse installed at the riser pole to protect underground cables does not have to coordinate with the instantaneous trips since underground lines are usually not subject to transient faults. On looped circuits the fuse size selected is usually the minimum size required to serve the entire load of the loop, whereas on lateral circuits the fuse size selected is usually the minimum size required to serve the load and coordinate with the transformer fuses, keeping in mind the cold pickup load.

10-10 RECLOSER–TO–CIRCUIT-BREAKER COORDINATION

The reclosing relay recloses its associated feeder circuit breaker at predetermined intervals (for example, 15-, 30-, or 45-s cycles) after the breaker has been tripped by overcurrent relays. If desired, the reclosing relay can provide an instantaneous initial reclosure plus three time-delay reclosures. However, if the fault is permanent, the reclosing relay recloses the breaker the predetermined number of times and then goes to the lockout position. Usually, the initial reclosing is so fast that customers may not even realize that service has been interrupted.

The crucial factor in coordinating the operation of a recloser and a circuit breaker (better yet, the relay that trips the breaker) is the reset time of the overcurrent relays during the tripping and reclosing sequence. If the relay used is of an electromechanical type, rather than a solid-state-type, it starts to travel in the trip direction during the operation of the recloser. If the reset time of the relay is not adjusted properly, the relay can accumulate enough movement (or travel) in the trip direction, during successive recloser operations, to trigger a false tripping.

Example 10-1 Figure 10-24 gives an example† for proper recloser-to-relay coordination. In the figure, curves A and B represent, respectively the instantaneous and time-delay TCCs of the 35-A reclosers. Curve C represents the TCC of the extremely inverse type IAC overcurrent relay set on the number 1.0 time-dial adjustment and 4-A tap (160-A primary with 200:5 current transformer). Assume a permanent fault current of 700 A located at point X in the figure. Determine the necessary relay and recloser coordination.

Solution From Fig. 10-24, the operating time of the relay and recloser can be found as the following:

For recloser: Instantaneous (from curve A) = 0.03 s

Time delay (from curve B) = 0.17 s

† For further information, see Ref. 5.

For relay: Pickup (from curve C) = 0.42 s

$$\text{Reset} = \frac{1.0}{10} \times 60 = 6.0 \text{ s}$$

assuming a 60-s reset time for the relay with a number 10 time-dial setting [5].

Using the signs (+) for trip direction and (−) for reset direction, the percent of total relay travel, during the operation of the recloser, can be calculated in the following manner. During the instantaneous operation (from curve A) of the recloser,

$$\text{Relay-closing travel} = \frac{\text{recloser-instantaneous time}}{\text{relay-pickup time}} \tag{10-1}$$

$$= \frac{0.03}{0.42}$$

$$= 0.0714 \qquad \text{or} \qquad 7.14 \text{ percent}$$

Assuming that the recloser is open for 1 s,

$$\text{Relay-reset travel} = \frac{(-)\text{ recloser-open time}}{\text{relay-reset time}} \tag{10-2}$$

$$= \frac{-1.0}{6.0}$$

$$= -0.1667 \qquad \text{or} \qquad -16.67 \text{ percent}$$

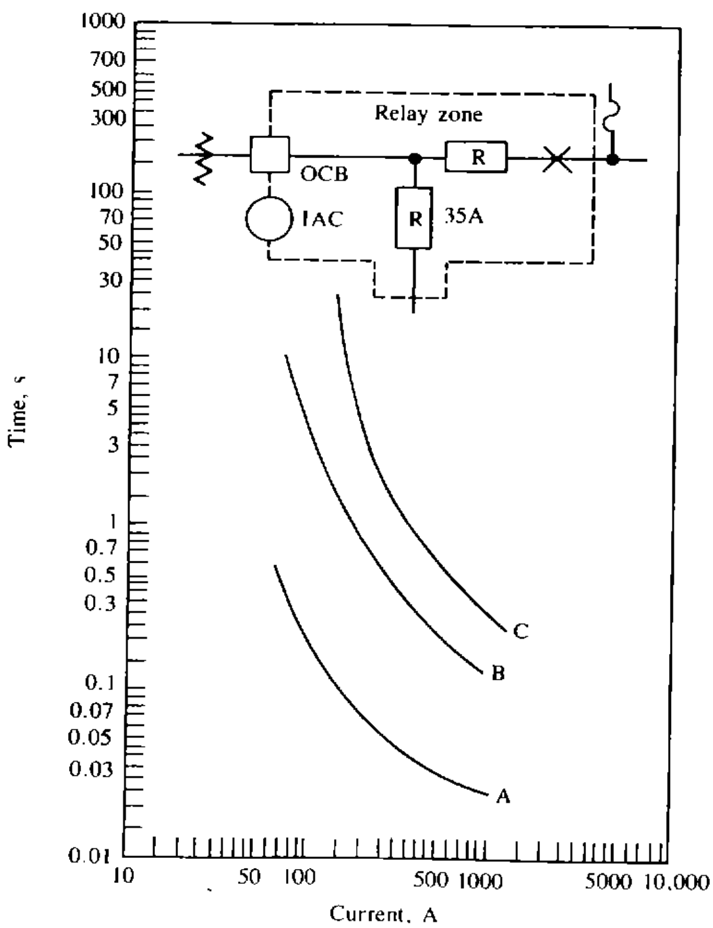


Figure 10-24 An example of recloser-to-relay coordination. Curve A represents TCCs of one instantaneous recloser opening. Curve B represents TCCs of one extended time-delay recloser opening. Curve C represents TCCs of the IAC relay. (*From* [41].)

From the results it can be seen that

$$|\text{Relay-closing travel}| < |\text{relay-reset travel}|$$

or

$$|7.14\%| < |16.67\%|$$

and therefore the relay will completely reset during the time that the recloser is open following each instantaneous opening.

Similarly, the travel percentages during the delayed tripping operations can be calculated in the following manner. During the first time-delay trip operation (from curve *B*) of the recloser,

$$\text{Relay-closing travel} = \frac{\text{recloser time-delay}}{\text{recloser-pickup time}} \tag{10-3}$$

$$= \frac{0.17}{0.42}$$

$$\cong 0.405 \qquad \text{or} \qquad 40.5 \text{ percent}$$

Assuming that the recloser opens for 1 s,

$$\text{Relay-reset travel} = (-)\,\frac{\text{recloser-open time}}{\text{relay-reset time}}$$

$$= -\frac{1.0}{6.0}$$

$$= -16.67 \text{ percent}$$

During the second time-delay trip of the recloser,

$$\text{Relay-closing travel} = 40.5 \text{ percent}$$

Therefore the net total relay travel is 64.3 percent

$$(= +40.5\% - 16.67\% + 40.5\%).$$

Since this net total relay travel is less than 100 percent, the desired recloser-to-relay coordination is accomplished. In general, a 0.15- to 0.20-s safety margin is considered to be adequate for any possible errors that might be involved in terms of curve readings, etc.

Some distribution engineers use a rule-of-thumb method to determine whether the recloser-to-relay coordination is achieved or not. For example, if the operating time of the relay at any given fault current value is less than twice the delayed tripping time of the recloser, assuming a recloser operation sequence which includes two time-delay trips, there will be a possible lack of coordination. Whenever there is a lack of coordination, either the time dial or pickup settings of the relay must be increased or the recloser has to be relocated until the coordination is achieved.

In general, the reclosers are located at the end of the relay reach. The rating of each recloser must be such that it will carry the load current, have sufficient interrupting capacity for that location, and coordinate both with the relay and load-side apparatus. If there is a lack of coordination with the load-side apparatus, then the recloser rating has to be increased. After the proper recloser ratings are determined, each recloser has to be checked for reach. If the reach is insufficient, additional series reclosers may be installed on the primary main.

10-11 FAULT-CURRENT CALCULATIONS†

There are four possible fault types that might occur in a given distribution system:

1. Three-phase grounded or ungrounded fault (3ϕ).
2. Phase-to-phase (or line-to-line) ungrounded fault (L-L).
3. Phase-to-phase (or double line-to-ground) grounded fault (2LG).
4. Phase-to-ground (or single line-to-ground) fault (SLG).

The first type of fault can take place only on three-phase circuits, and the second and third on three-phase or two-phase (i.e., vee or open-delta) circuits. However, even on these circuits usually only SLG faults will take place due to the multigrounded construction. The relative numbers of the occurrences of different fault types depend upon various factors, e.g., circuit configuration, the height of ground wires, voltage class, method of grounding, relative insulation levels to ground and between phases, speed of fault clearing, number of stormy days per year, and atmospheric conditions. Based on Ref. 6, the probabilities of prevalence of the various types of faults are‡

$$\begin{aligned} \text{SLG faults} &= 0.70 \\ \text{L-L faults} &= 0.15 \\ \text{2LG faults} &= 0.10 \\ 3\phi \text{ faults} &= 0.05 \\ \hline \text{Total} &= 1.00 \end{aligned}$$

The actual fault current is usually less than the bolted three-phase value. (Here, the term *bolted* means that there is no fault impedance (or fault resistance) resulting from fault arc, that is, $Z_f = 0$.) However, the SLG fault often produces a greater fault current that the 3ϕ fault especially (1) where the associated generators

† More rigorous and detailed treatment of the subject is given by Anderson [3, 8].

‡ One should keep in mind that these probabilities may differ substantially from one system to another in practice.

have solidly grounded neutrals or low-impedance neutral impedances, and (2) on the wye-grounded side of delta-wye grounded transformer banks [7]. Therefore, for a given system, each fault at each fault location must be calculated based on actual circuit conditions. When this is done, according to Anderson [8], it is usually the case that the SLG fault is the most severe, with the 3ϕ, 2LG, and L-L following in that order. In general, since the 2LG fault value is always somewhere in between the maximum and minimum, it is usually neglected in the distribution system fault calculations [3].

In general, the maximum and minimum fault currents are both calculated for a given distribution system. The maximum fault current is calculated based on the following assumptions:

1. All generators are connected, i.e., in service.
2. The fault is a bolted one; i.e., the fault impedance is zero.
3. The load is maximum, i.e., on-peak load.

whereas the minimum current is calculated based on the following assumptions:

1. The number of generators connected is minimum.
2. The fault is not a bolted one, i.e., the fault impedance is not zero but has a value somewhere between 30 and 40 Ω.
3. The load is minimum, i.e., off-peak load.

On 4-kV systems, the value of the minimum fault current available may be taken as 60 to 70 percent of the calculated maximum line-to-ground fault current.

In general, these fault currents are calculated for each sectionalizing point, including the substation, and for the ends of the longest sections. The calculated maximum fault-current values are used in determining the required interrupting capacities (i.e., ratings) of the fuses, circuit breakers, or other fault-clearing apparatus; the calculated minimum fault-current values are used in coordinating the operations of fuses, reclosers, and relays.

To calculate the fault currents one has to determine the zero-, positive-, and negative-sequence Thevenin impedances of the system† at the high-voltage side of the distribution substation transformer looking into the system. These impedances are usually readily available from transmission system fault studies. Therefore, for any given fault on a radial distribution circuit, one can simply add the appropriate impedances to the Thevenin impedances as the fault is moved away from the substation along the circuit. The most common types of distribution substation transformer connections are (1) delta-wye solidly grounded and (2) delta-delta.

† Anderson [3] recommends letting the positive-sequence Thevenin impedance of the system, i.e. $Z_{1,\text{sys}}$, be equal to zero, if there is no exact system information available and, at the same time, the substation transformer is small. Of course, by using this assumption the system is treated as an infinitely large system.

10-11-1 Three-Phase Faults

Since this fault type is completely balanced, there are no zero- or negative-sequence currents. Therefore, when there is no fault impedance,

$$I_{f,3\phi} = I_{f,a} = I_{f,b} = I_{f,c}$$
$$= \left|\frac{\bar{V}_{L\text{-}N}}{\bar{Z}_1}\right| \quad \text{A} \tag{10-4}$$

and when there is a fault impedance,

$$I_{f,3\phi} = \left|\frac{\bar{V}_{L\text{-}N}}{\bar{Z}_1 + \bar{Z}_f}\right| \quad \text{A} \tag{10-5}$$

where $I_{f,3\phi}$ = three-phase fault current, A
$\bar{V}_{L\text{-}N}$ = line-to-neutral distribution voltage, V
$\bar{Z}_1$ = total positive-sequence impedance, Ω
$\bar{Z}_f$ = fault impedance, Ω

$I_{f,a}, I_{f,b}\ I_{f,c}$ = fault currents in a, b, and c phases

Since the total positive-sequence impedance can be expressed as

$$\bar{Z}_1 = \bar{Z}_{1,\text{sys}} + \bar{Z}_{1,T} + \bar{Z}_{1,\text{ckt}} \tag{10-6}$$

where $\bar{Z}_{1,\text{sys}}$ = positive-sequence Thevenin-equivalent impedance of the system (or source) referred to distribution voltage,† Ω
$\bar{Z}_{1,T}$ = positive-sequence transformer impedance referred to distribution voltage,‡ Ω
$\bar{Z}_{1,\text{ckt}}$ = positive-sequence impedance of faulted segment of distribution circuit, Ω

Substituting Eq. (10-6) into Eqs. (10-4) and (10-5), the three-phase fault current can be expressed as

$$I_{f,3\phi} = \left|\frac{\bar{V}_{L\text{-}N}}{\bar{Z}_{1,\text{sys}} + \bar{Z}_{1,T} + \bar{Z}_{1,\text{ckt}}}\right| \quad \text{A} \tag{10-7}$$

and

$$I_{f,3\phi} = \left|\frac{\bar{V}_{L\text{-}N}}{\bar{Z}_{1,\text{sys}} + \bar{Z}_{1,T} + \bar{Z}_{1,\text{ckt}} + \bar{Z}_f}\right| \quad \text{A} \tag{10-8}$$

Equations (10-7) and (10-8) are applicable whether the source connection is wye-grounded or delta. At times, it might be necessary to reflect a three-phase

† Remember that an impedance can be converted from one base voltage to another by using $Z_2 = Z_1(V_2/V_1)^2$, where Z_1 = impedance on V_1 base and Z_2 = impedance on V_2 base

‡ Note that there has been a shift in notation and the symbol $Z_{1,T}$ stands for Z_T.

fault on the distribution system as a three-phase fault on the subtransmission system. This can be accomplished by using

$$I_{F,3\phi} = \frac{V_{L\text{-}L}}{V_{\mathrm{ST},L\text{-}L}} \times I_{f,3\phi} \qquad \text{A} \tag{10-9}$$

where $I_{F,3\phi}$ = three-phase fault current referred to subtransmission voltage, A
$I_{f,3\phi}$ = three-phase fault current based on distribution voltage, A
$V_{L\text{-}L}$ = line-to-line distribution voltage, V
$V_{\mathrm{ST},L\text{-}L}$ = line-to-line subtransmission voltage, V

10-11-2 Line-to-Line Faults

Assume that a line-to-line fault exists between phases b and c. Therefore, if there is no fault impedance,

$$I_{f,a} = 0$$

$$I_{f,L\text{-}L} = I_{f,c} = -I_{f,b}$$

$$= \left| \frac{j\sqrt{3} \times \overline{V}_{L\text{-}N}}{\overline{Z}_1 + \overline{Z}_2} \right| \qquad \text{A} \tag{10-10}$$

where $I_{f,L\text{-}L}$ = line-to-line fault current, A
$\overline{Z}_2$ = total negative-sequence impedance, Ω

However,

$$\overline{Z}_1 = \overline{Z}_2$$

thus

$$I_{f,L\text{-}L} = \left| \frac{j\sqrt{3} \times \overline{V}_{L\text{-}N}}{2\overline{Z}_1} \right| \qquad \text{A} \tag{10-11}$$

or substituting Eq. (10-6) into Eq. (10-11),

$$I_{f,L\text{-}L} = \left| \frac{j\sqrt{3} \times \overline{V}_{L\text{-}N}}{2(\overline{Z}_{1,\mathrm{sys}} + \overline{Z}_{1,T} + \overline{Z}_{1,\mathrm{ckt}})} \right| \qquad \text{A} \tag{10-12}$$

However, if there is a fault impedance,

$$I_{f,L\text{-}L} = \left| \frac{j\sqrt{3} \times V_{L\text{-}N}}{2(\overline{Z}_{1,\mathrm{sys}} + \overline{Z}_{1,T} + \overline{Z}_{1,\mathrm{ckt}}) + \overline{Z}_f} \right| \qquad \text{A} \tag{10-13}$$

By comparing Eq. (10-11) with Eq. (10-4), one can determine a relationship between the three-phase fault and line-to-line fault currents as

$$I_{f,L\text{-}L} = \frac{\sqrt{3}}{2} \times I_{f,3\phi}$$

$$= 0.866 \times I_{f,3\phi} \tag{10-14}$$

which is applicable to any point on the distribution system. The equations derived in this section are applicable whether the source connection is wye-grounded or delta.

10-11-3 Single Line-to-Ground Faults

Assume that a single line-to-ground fault exists on phase a. If there is no fault impedance,

$$I_{f,L\text{-}G} = \left| \frac{\bar{V}_{L\text{-}N}}{\bar{Z}_G} \right| \quad \text{A} \tag{10-15}$$

where $I_{f,L\text{-}G}$ = line-to-ground fault current, A
$\bar{Z}_G$ = impedance to ground, Ω
$\bar{V}_{L\text{-}N}$ = line-to-neutral distribution voltage, V

However,

$$\bar{Z}_G = \frac{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_0}{3} \tag{10-16}$$

or

$$\bar{Z}_G = \frac{2\bar{Z}_1 + Z_0}{3} \tag{10-17}$$

since

$$\bar{Z}_1 = \bar{Z}_2$$

Therefore, by substituting Eq. (10-17) into Eq. (10-15),

$$I_{f,L\text{-}G} = \left| \frac{\bar{V}_{L\text{-}N}}{\frac{1}{3}[2\bar{Z}_1 + \bar{Z}_0]} \right| \quad \text{A} \tag{10-18}$$

However, if there is a fault impedance,

$$I_{f,L\text{-}G} = \left| \frac{\bar{V}_{L\text{-}N}}{\frac{1}{3}[2\bar{Z}_1 + \bar{Z}_0] + \bar{Z}_f} \right| \quad \text{A} \tag{10-19}$$

where $\bar{Z}_0$ = total zero-sequence impedance, Ω.

Equations (10-18) and (10-19) are only applicable if the source connection is wye-grounded. If the source connection is delta, they are not applicable since the fault current would be equal to zero due to the zero-sequence impedance being infinite.

If the primary distribution feeders are supplied by a delta-wye solidly grounded substation transformer, a single line-to-ground fault on the distribution system is reflected as a line-to-line fault on the subtransmission system. Therefore, the low-voltage-side fault current may be referred to the high-voltage side by using the equation

$$I_{F,L\text{-}L} = \frac{V_{L\text{-}L}}{\sqrt{3} \times V_{\text{ST},L\text{-}L}} \times I_{f,L\text{-}G} \quad \text{A} \tag{10-20}$$

where $I_{f,\,L\text{-}G}$ = single line-to-ground fault current based on distribution voltage, A
$I_{F,\,L\text{-}L}$ = single line-to-ground fault current reflected as a line-to-line fault current on the subtransmission system, A
$V_{L\text{-}L}$ = line-to-line distribution voltage, V
$V_{\mathrm{ST},\,L\text{-}L}$ = line-to-line subtransmission voltage, V

In general, the zero-sequence impedance Z_0 of a distribution circuit with multigrounded neutral is very hard to determine precisely, but it is usually larger than its positive-sequence impedance $\bar{Z}_1$. However, some empirical approaches are possible. For example, Anderson [3] gives the following relationship between the zero- and positive-sequence impedances of a distribution circuit with multigrounded neutral:

$$\bar{Z}_0 = K_0 \cdot \bar{Z}_1 \tag{10-21}$$

where Z_0 = zero-sequence impedance of distribution circuit, Ω
Z_1 = positive-sequence impedance of distribution circuit, Ω
K_0 = a constant

Table 10-6 gives various possible values for the constant K_0. If the earth has a very bad conducting characteristic, the constant K_0 is totally established by the neutral-wire impedance. Anderson [3] suggests using an average value of 4 where exact conditions are not known.

10-11-4 Components of the Associated Impedance to the Fault

Impedance of the source If the associated fault duty S given in megavoltamperes at the substation bus is available from transmission system fault studies, the system impedance, i.e., "backup" impedance, can be calculated as

$$\begin{aligned} Z_{1,\,\mathrm{sys}} &= \frac{V_{L\text{-}N}}{I_L} \\ &= \frac{V_{L\text{-}L}}{\sqrt{3} \times I_L} \end{aligned} \tag{10-22}$$

Table 10-6 Estimated values of the K_0 constant for various conditions*

Condition	K_0
Perfectly conducting earth (e.g., a system with multiple water-pipe grounds)	1.0
Ground wire same size as phase wire	4.0
Ground wire one size smaller	4.6
Ground wire two sizes smaller	4.9
Finite earth impedance	3.8–4.2

* From [3].

but

$$I_L = \frac{S}{\sqrt{3} \times V_{L\text{-}L}} \tag{10-23}$$

therefore

$$Z_{1,\text{sys}} = \frac{V_{L\text{-}L}^2}{S} \tag{10-24}$$

If the system impedance is given at the transmission substation bus rather than at the distribution substation bus, then the subtransmission line impedance has to be involved in the calculations so that the total impedance (i.e., the sum of the system impedance and the subtransmission line impedance) represents the impedance up to the high side of the distribution substation transformer.

If the maximum three-phase fault current on the high-voltage side of the distribution substation transformer is known, then

$$Z_{1,\text{sys}} + Z_{1,\text{ST}} = \frac{V_{\text{ST},L\text{-}L}}{\sqrt{3}(I_{F,3\phi})_{\max}} \tag{10-25}$$

where $Z_{1,\text{sys}}$ = positive-sequence impedance of system, Ω
$Z_{1,\text{ST}}$ = positive-sequence impedance of subtransmission line, Ω
$V_{\text{ST},L\text{-}L}$ = line-to-line subtransmission voltage, V
$(I_{F,3\phi})_{\max}$ = maximum three-phase fault current referred to subtransmission voltage, A

Note that the impedances found from Eq. (10-24) and (10-25) can be referred to the base voltage by using Eq. (10-9).

Impedance of the substation transformer If the percent impedance of the substation transformer is known, the transformer impedance† can be expressed as

$$Z_{1,T} = \frac{(\%\,Z_T)(V_{L\text{-}L}^2)10}{S_{T,3\phi}} \tag{10-26}$$

where $\%\,Z_T$ = percent transformer impedance
$V_{L\text{-}L}$ = line-to-line base voltage, kV
$S_{\text{T},3\phi}$ = three-phase transformer rating, kVA

Impedance of the distribution circuits The impedance values for the distribution circuits depend on the pole-top conductor configurations and can be calculated by means of symmetrical components. For example, Fig. 10-25 shows a typical pole-top overhead distribution circuit configuration. The equivalent spacing (i.e., mutual geometric mean distance) of phase wires and the equivalent spacing between phase wires and neutral wire can be determined from

$$dp = D_{\text{eq}} = D_m \triangleq (D_{ab} \cdot D_{bc} \cdot D_{ca})^{1/3} \tag{10-27}$$

† Usually the resistance and reactance values of a substation transformer are approximately equal to 2 and 98 percent of its impedance, respectively.

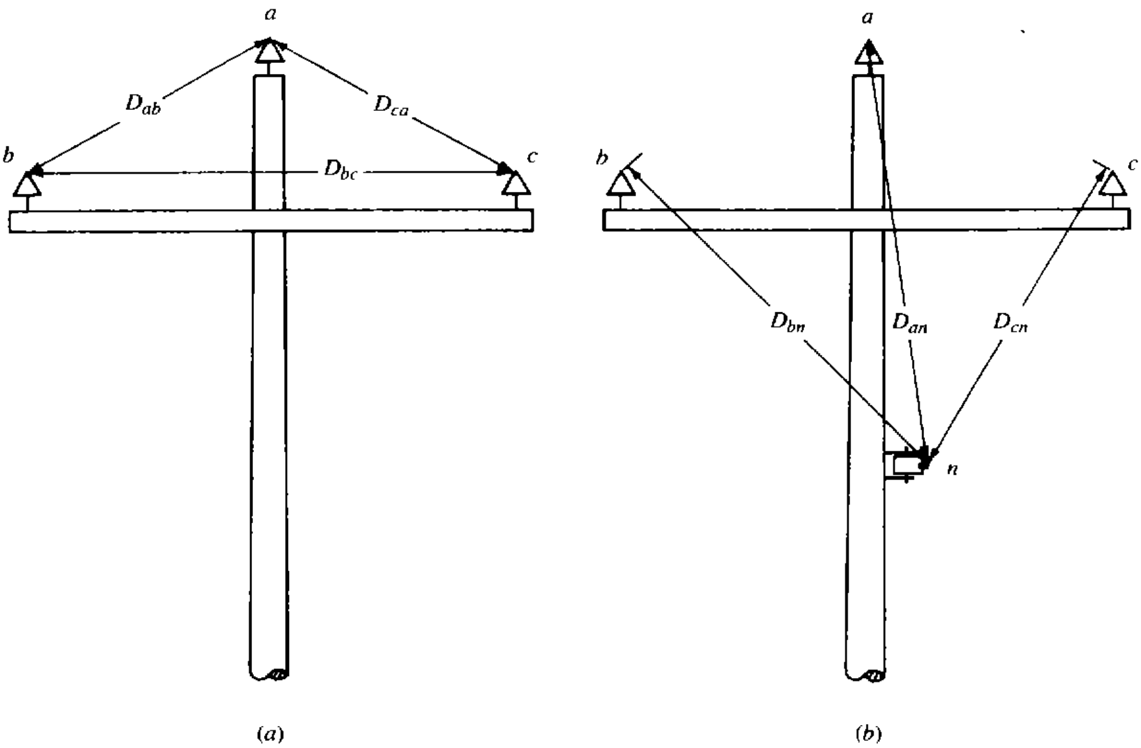


Figure 10-25 Typical pole-top overhead distribution circuit configuration.

and

$$dn \triangleq (D_{an} \cdot D_{bn} \cdot D_{cn})^{1/3} \tag{10-28}$$

Similarly, the mutual reactances (spacing factors) for phase wires, and between phase wires and neutral wire (due to equivalent spacings), can be determined as

$$x_{dp} = 0.05292 \log_{10} dp \qquad \Omega/1000 \text{ ft} \tag{10-29}$$

and

$$x_{dn} = 0.05292 \log_{10} dn \qquad \Omega/1000 \text{ ft} \tag{10-30}$$

1. If the distribution circuit is a three-phase circuit, the positive- and negative-sequence impedances are

$$\bar{z}_1 = \bar{z}_2 = r_{ap} + j(x_{ap} + x_{dp}) \qquad \Omega/1000 \text{ ft} \tag{10-31}$$

and the zero-sequence impedance is

$$\bar{z}_0 = \bar{z}_{0,a} - \frac{\bar{z}_{0,ag}^2}{\bar{z}_{0,g}} \qquad \Omega/1000 \text{ ft} \tag{10-32}$$

with

$$\bar{z}_{0,a} = r_{ap} + r_e + j(x_{ap} + x_e - 2x_{dp}) \qquad \Omega/1000 \text{ ft} \tag{10-33}$$

$$\bar{z}_{0,ag} = r_e + j(x_e - 3x_{dn}) \qquad \Omega/1000 \text{ ft} \tag{10-34}$$

$$\bar{z}_{0,g} = 3r_{an} + r_e + j(3x_{an} + x_e) \qquad \Omega/1000 \text{ ft} \tag{10-35}$$

where r_e = resistance of earth = 0.0542 Ω/1000 ft
x_e = reactance of earth = 0.4676 Ω/1000 ft
x_{dn} = spacing factor between phase wires and neutral wire, Ω/1000 ft
x_{dp} = spacing factor for phase wires, Ω/1000 ft
r_{ap} = resistance of phase wires, Ω/1000 ft
r_{an} = resistance of neutral wires, Ω/1000 ft
x_{ap} = reactance of phase wire with 1-ft spacing, Ω/1000 ft
x_{an} = reactance of neutral wire with 1-ft spacing, Ω/1000 ft
$z_{0,a}$ = zero-sequence self-impedance of phase circuit, Ω/1000 ft
$z_{0,g}$ = zero-sequence self-impedance of one ground wire, Ω/1000 ft
$z_{0,aq}$ = zero-sequence mutual impedance between the phase circuit as one group of conductors and the ground wire as other conductor group, Ω/1000 ft

2. If the distribution circuit is an open-wye and single-phase delta circuit, the positive- and negative-sequence impedances are

$$\bar{z}_1 = \bar{z}_2 = r_{ap} + j(x_{ap} + x_{dp}) \qquad \Omega/1000\text{ ft} \tag{10-36}$$

and the zero-sequence impedance is

$$\bar{z}_0 = \bar{z}_{0,a} - \frac{\bar{z}_{0,ag}^2}{\bar{z}_{0,g}} \qquad \Omega/1000\text{ ft} \tag{10-37}$$

where

$$\bar{z}_{0,a} = r_{ap} + \frac{2r_e}{3} + j\left(x_{ap} + \frac{2x_e}{3} - x_{dp}\right) \qquad \Omega/1000\text{ ft} \tag{10-38}$$

$$\bar{z}_{0,ag} = \frac{2r_e}{3} + j\left(\frac{2x_e}{3} - 2x_{dn}\right) \qquad \Omega/1000\text{ ft} \tag{10-39}$$

$$\bar{z}_{0,g} = 2r_{an} + \frac{2r_e}{3} + j\left(2x_{an} + \frac{2x_e}{3}\right) \qquad \Omega/1000\text{ ft} \tag{10-40}$$

3. If the distribution circuit is a single-phase multigrounded circuit, its impedance is

$$\bar{z}_{1\phi} = \bar{z}_{0,a} - \frac{\bar{z}_{0,ag}^2}{\bar{z}_{0,g}} \qquad \Omega/1000\text{ ft} \tag{10-41}$$

where

$$\bar{z}_{0,a} = \bar{z}_{0,g} = r_{ap} + \frac{r_e}{3} + j\left(x_{ap} + \frac{x_e}{3}\right) \qquad \Omega/1000\text{ ft} \tag{10-42}$$

$$\bar{z}_{0,ag} = \frac{r_e}{3} + j\left(\frac{x_e}{3} - x_{dn}\right) \qquad \Omega/1000\text{ ft} \tag{10-43}$$

10-11-5 Sequence-Impedance Tables for the Application of Symmetrical Components

The zero-sequence impedance equation (given as Eq. (10-32), (10-37), or (10-41) in Sec. 10-11-4), that is,

$$\bar{z}_0 = \bar{z}_{0,a} - \frac{\bar{z}_{0,ag}^2}{\bar{z}_{0,g}} \qquad \Omega/1000 \text{ ft}$$

can be expressed as

$$\bar{z}_0 = \bar{z}_{0,a} + \bar{z}_0' \qquad \Omega/1000 \text{ ft} \tag{10-44}$$

or

$$\bar{z}_0 = \bar{z}_{0,a} + \bar{z}_0'' \qquad \Omega/1000 \text{ ft} \tag{10-45}$$

where $z_{0,a}$ = zero-sequence self-impedance of phase circuit, Ω/1000 ft

z_0' = equivalent zero-sequence impedance due to combined effects of zero-sequence self-impedance of one ground wire, and zero-sequence mutual impedance between the phase circuit as one group of conductors and the ground wire as another conductor group, assuming a specific vertical distance between the ground wire and phase wires, for example, 38 in

$\bar{z}_0''$ = Same as z_0', except the vertical distance is a different one, for example, 62 in

Therefore it is possible to develop precalculated sequence-impedance tables for the application of symmetrical components. For example, Figs. 10-26 to 10-30 show various overhead pole-top conductor configurations with and without ground wire. The corresponding sequence-impedance values at 60 Hz and 50°C are given in Tables 10-7 to 10-11.

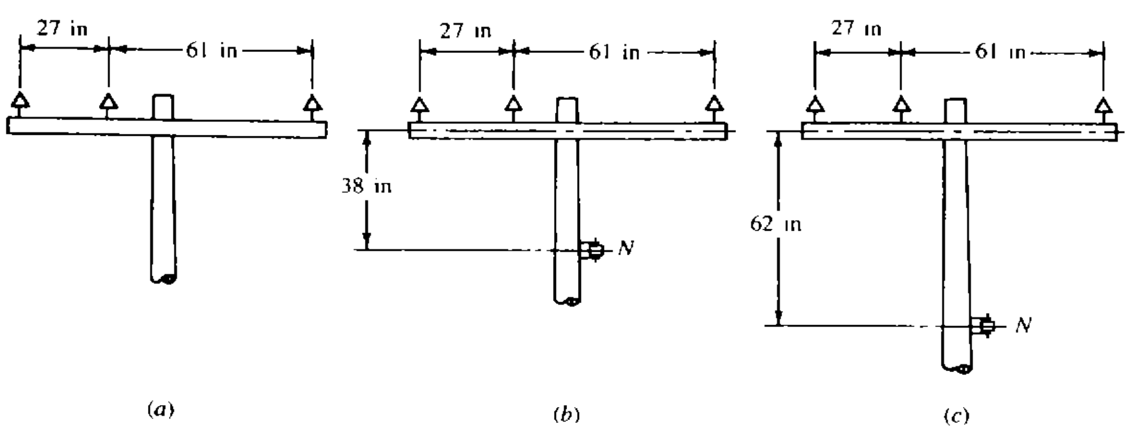


Figure 10-26 Various overhead pole-top conductor configurations: (*a*) without ground wire, $z_0 = z_{0,a}$; (*b*) with ground wire, $z_0 = z_{0,a} + z_0'$; (*c*) with ground wire, $z_0 = z_{0,a} + z_0''$.

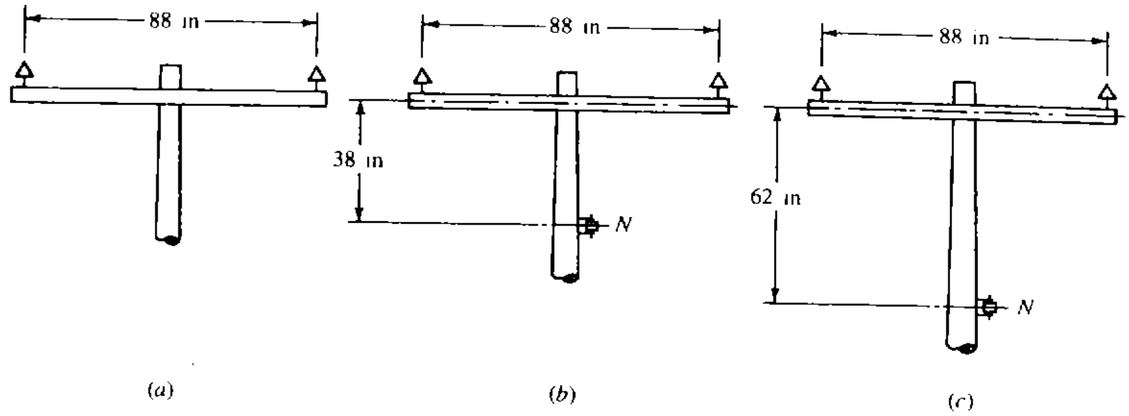


Figure 10-27 Various overhead pole-top conductor configurations: (*a*) without ground wire, $z_0 = z_{0(a)}$; (*b*) with ground wire, $z_0 = z_{0,a} + z_0'$; (*c*) with ground wire, $z_{0,a} + z_0''$.

Example 10-2 Assume that a rural substation has a 3750-kVA 69/12.47-kV LTC transformer feeding a three-phase four-wire 12.47-kV circuit protected by 140-A type L reclosers and 125-A series fuses. It is required to calculate the bolted fault current at point 10, 2 mi from the substation on circuit 456319. Assume that the sizes of the phase conductors are 336AS37 (that is, 336-kcmil bare aluminum steel conductors with 37 strands) and that neutral conductor is OAS7, spaced 62 in. If the system impedance to the regulated 12.47-kV bus and the system impedance to ground are given as $0.7199 + j3.4619\ \Omega$ and $0.6191 + j3.3397$ ohms, respectively, determine the following:

(*a*) The zero- and positive-sequence impedances of the line to point 10.
(*b*) The impedance to ground of the line to point 10.
(*c*) The total positive-sequence impedance to point 10 including system impedance to the regulated 12.47-kV bus.
(*d*) The total impedance to ground to point 10 including system impedance of the regulated 12.47-kV bus.
(*e*) The three-phase fault current at point 10.
(*f*) The line-to-line fault current at point 10.
(*g*) The line-to-ground fault current at point 10.

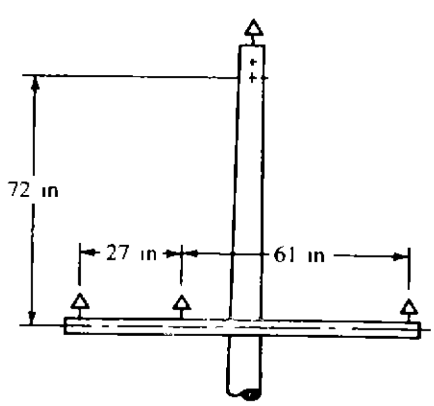


Figure 10-28 Various overhead pole-top conductor configurations with ground wire, $z_0 = z_{0,a} + z_0'$.

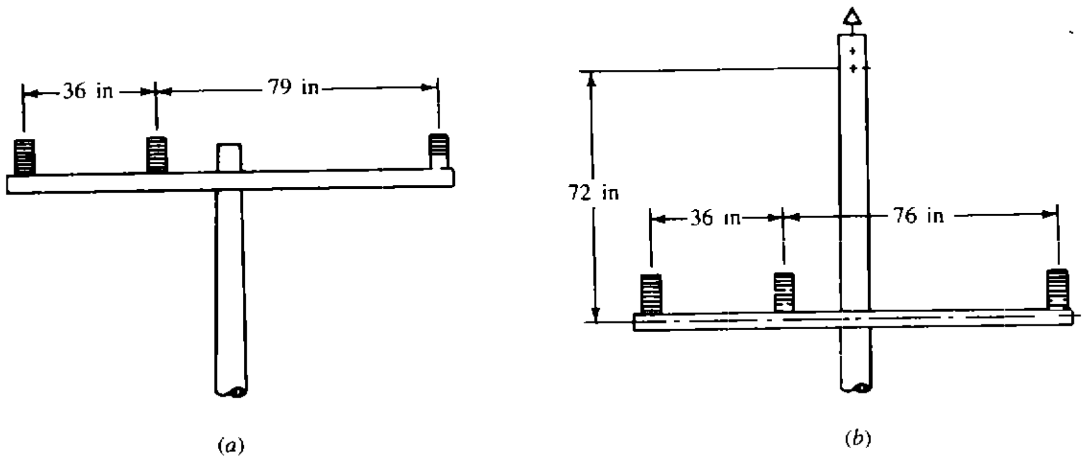


Figure 10-29 Various overhead pole-top conductor configurations: (*a*) without ground wire, $z_0 = z_{0,a}$; (*b*) with ground wire, $z_0 = z_{0,a} + z_0'$.

SOLUTION

(*a*) The zero-sequence impedance of the line to point 10 can be found by using Table 10-7 as

$$\begin{aligned}\bar{Z}_{0,\text{ckt}} &= 2(z_{0,a} + z_0'')5.28 \\ &= 2[(0.1122 + j0.4789) + (-0.0385 - j0.0996)]5.28 \\ &= 0.7783 + j4.0054\ \Omega\end{aligned}$$

Similarly, the positive-sequence impedance of the line to point 10 can be found as

$$\begin{aligned}\bar{Z}_{1,\text{ckt}} &= 2(0.0580 + j0.1208)5.28 \\ &= 0.6125 + j1.2756\ \Omega\end{aligned}$$

(*b*) From Eq. (10-17), the impedance to ground of the line to point 10 is

$$\begin{aligned}\bar{Z}_G &= \frac{2\bar{z}_1 + \bar{z}_0}{3} \\ &= \frac{2(0.6125 + j1.2756) + (0.7783 + j4.0054)}{3} \\ &= 2.0033 + j2.1855\ \Omega\end{aligned}$$

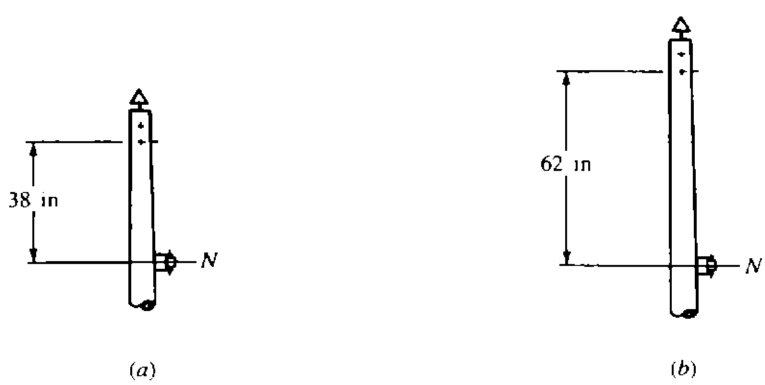


Figure 10-30 Single-phase overhead pole-top configurations with ground wires: (*a*) $z_{1\phi} = z_{1\phi}'$ and (*b*) $z_{1\phi} = z_{1\phi}''$.

Table 10-7 Sequence-impedance values associated with Fig. 10-26, Ω/1000 ft

Conductor size & code	$z_1 = z_2$	$z_{0,a}$	z_0'	z_0''
		Bare-aluminum steel (AS)		
4AS7	$0.4867 + j0.1613$	$0.5409 + j0.5195$	$0.0518 - j0.0543$	$0.0454 - j0.0493$
4AS8	$0.4830 + j0.1605$	$0.5372 + j0.5187$	$0.0520 - j0.0548$	$0.0456 - j0.0497$
3AS7	$0.3920 + j0.1617$	$0.4462 + j0.5198$	$0.0541 - j0.0685$	$0.0472 - j0.0620$
2AS7	$0.3202 + j0.1624$	$0.3743 + j0.5206$	$0.0535 - j0.0827$	$0.0465 - j0.0747$
2AS8	$0.3125 + j0.1581$	$0.3667 + j0.5162$	$0.0543 - j0.0846$	$0.0471 - j0.0764$
1AS8	$0.2614 + j0.1802$	$0.3156 + j0.5384$	$0.0459 - j0.0954$	$0.0395 - j0.0859$
1AS7	$0.2614 + j0.1624$	$0.3156 + j0.5206$	$0.0504 - j0.0970$	$0.0435 - j0.0874$
0AS7	$0.2121 + j0.1607$	$0.2663 + j0.5189$	$0.0451 - j0.1108$	$0.0385 - j0.0996$
000AS7	$0.1377 + j0.1541$	$0.1919 + j0.5123$	$0.0295 - j0.1346$	$0.0242 - j0.1206$
267AS33	$0.0729 + j0.1245$	$0.1271 + j0.4827$	$0.0092 - j0.1663$	$0.0056 - j0.1486$
336AS37	$0.0580 + j0.1208$	$0.1122 + j0.4789$	$0.0008 - j0.1722$	$-0.0020 - j0.1537$
477AS33	$0.0409 + j0.1168$	$0.0951 + j0.4789$	$-0.0101 - j0.1779$	$-0.0119 - j0.1587$
636AS33	$0.0306 + j0.1145$	$0.0848 + j0.4727$	$-0.0175 - j0.1807$	$-0.0184 - j0.1610$
795AS33	$0.0244 + j0.1120$	$0.0786 + j0.4702$	$-0.0221 - j0.1830$	$-0.0226 - j0.1630$
		Bare hard-drawn copper (X)		
8X1	$0.7194 + j0.1624$	$0.7739 + j0.5206$	$0.0432 - j0.0340$	$0.0380 - j0.0310$
6X1	$0.4527 + j0.1571$	$0.5069 + j0.5153$	$0.0535 - j0.0588$	$0.0468 - j0.0533$
4X1	$0.2847 + j0.1518$	$0.3389 + j0.5100$	$0.0547 - j0.0916$	$0.0474 - j0.0826$
4X3	$0.2875 + j0.1499$	$0.3417 + j0.5081$	$0.0554 - j0.0910$	$0.0480 - j0.0821$
3X3	$0.2280 + j0.1473$	$0.2822 + j0.5054$	$0.0513 - j0.1080$	$0.0441 - j0.0972$
2X1	$0.1790 + j0.1465$	$0.2332 + j0.5047$	$0.0432 - j0.1237$	$0.0366 - j0.1111$
1X1	$0.1420 + j0.1437$	$0.1962 + j0.5018$	$0.0342 - j0.1366$	$0.0283 - j0.1225$
1X3	$0.1432 + j0.1420$	$0.1976 + j0.5001$	$0.0352 - j0.1367$	$0.0292 - j0.1226$
0X7	$0.1150 + j0.1398$	$0.1692 + j0.4981$	$0.0255 - j0.1466$	$0.0205 - j0.1313$
00X7	$0.0911 + j0.1372$	$0.1453 + j0.4954$	$0.0156 - j0.1547$	$0.0115 - j0.1384$
000X7	$0.0723 + j0.1346$	$0.1265 + j0.4928$	$0.0065 - j0.1608$	$0.0033 - j0.1436$
0000X7	$0.0574 + j0.1317$	$0.1116 + j0.4899$	$-0.0016 - j0.1654$	$-0.0041 - j0.1476$
300X12	$0.0407 + j0.1255$	$0.0949 + j0.4837$	$-0.0114 - j0.1719$	$-0.0129 - j0.1532$
		Bare hard-drawn aluminum (AL)		
0AL7	$0.1843 + j0.1394$	$0.2385 + j0.4976$	$0.0468 - j0.1235$	$0.0398 - j0.1110$
000AL7	$0.1161 + j0.1345$	$0.1703 + j0.4927$	$0.0277 - j0.1485$	$0.0224 - j0.1330$
267AL7	$0.0731 + j0.1292$	$0.1273 + j0.4874$	$0.0082 - j0.1636$	$0.0047 - j0.1462$
477AL19	$0.0413 + j0.1212$	$0.0955 + j0.4794$	$-0.0105 - j0.1747$	$-0.0121 - j0.1558$

Table 10-8 Sequence-impedance values associated with Fig. 10-27, Ω/1000 ft

Conductor size & code	$z_1 = z_2$	$z_{0,a}$	z_0'	z_0''
		Bare-aluminum steel (AS)		
4AS7	0.4867 + *j*0.1706	0.5229 + *j*0.3907	0.0338 − *j*0.0357	0.0279 − *j*0.0310
4AS8	0.4830 + *j*0.1698	0.5191 + *j*0.3900	0.0340 − *j*0.0360	0.0280 − *j*0.0313
3AS7	0.3920 + *j*0.1710	0.4282 + *j*0.3911	0.0353 − *j*0.0449	0.0289 − *j*0.0389
2AS7	0.3201 + *j*0.1717	0.3562 + *j*0.3919	0.0349 − *j*0.0542	0.0284 − *j*0.0468
2AS8	0.3125 + *j*0.1674	0.3486 + *j*0.3875	0.0354 − *j*0.0554	0.0288 − *j*0.0479
1AS8	0.2614 + *j*0.1895	0.2975 + *j*0.4097	0.0299 − *j*0.0625	0.0240 − *j*0.0537
1AS7	0.2614 + *j*0.1717	0.2975 + *j*0.3919	0.0328 − *j*0.0636	0.0265 − *j*0.0547
0AS7	0.2121 + *j*0.1700	0.2483 + *j*0.3902	0.0293 − *j*0.0726	0.0233 − *j*0.0623
000AS7	0.1377 + *j*0.1634	0.1738 + *j*0.3836	0.0191 − *j*0.0882	0.0143 − *j*0.0753
267AS33	0.0729 + *j*0.1339	0.1090 + *j*0.3540	0.0057 − *j*0.1089	0.0024 − *j*0.0926
336AS33	0.0580 + *j*0.1301	0.0941 + *j*0.3502	0.0002 − *j*0.1127	−0.0023 − *j*0.0957
477AS33	0.0409 + *j*0.1261	0.0770 + *j*0.3462	−0.0070 − *j*0.1164	−0.0085 − *j*0.0987
636AS33	0.0306 + *j*0.1238	0.0668 + *j*0.3440	−0.0118 − *j*0.1182	−0.0126 − *j*0.1001
795AS33	0.0244 + *j*0.1214	0.0605 + *j*0.3415	−0.0148 − *j*0.1197	−0.0152 − *j*0.1013
		Bare hard-drawn copper (X)		
8X1	0.7197 + *j*0.1717	0.7558 + *j*0.3919	0.0282 − *j*0.0223	0.0234 − *j*0.0196
6X1	0.4527 + *j*0.1664	0.4888 + *j*0.3866	0.0349 − *j*0.0386	0.0288 − *j*0.0335
4X1	0.2847 + *j*0.1611	0.3208 + *j*0.3813	0.0357 − *j*0.0601	0.0289 − *j*0.0518
4X3	0.2875 + *j*0.1592	0.3236 + *j*0.3794	0.0361 − *j*0.0596	0.0293 − *j*0.0514
3X3	0.2280 + *j*0.1566	0.2642 + *j*0.3767	0.0334 − *j*0.0708	0.0268 − *j*0.0608
2X1	0.1790 + *j*0.1558	0.2151 + *j*0.3760	0.0281 − *j*0.0811	0.0220 − *j*0.0695
1X1	0.1420 + *j*0.1530	0.1782 + *j*0.3731	0.0221 − *j*0.0895	0.0168 − *j*0.0765
1X3	0.1434 + *j*0.1513	0.1795 + *j*0.3714	0.0228 − *j*0.0896	0.0173 − *j*0.0766
0X7	0.1150 + *j*0.1492	0.1511 + *j*0.3693	0.0164 − *j*0.0960	0.0118 − *j*0.0819
00X7	0.0911 + *j*0.1466	0.1272 + *j*0.3667	0.0099 − *j*0.1013	0.0062 − *j*0.0862
000X7	0.0723 + *j*0.1439	0.1085 + *j*0.3640	0.0040 − *j*0.1052	0.0010 − *j*0.0894
0000X7	0.0574 + *j*0.1411	0.0935 + *j*0.3612	−0.0014 − *j*0.1083	−0.0036 − *j*0.0919
300X12	0.0407 + *j*0.1348	0.0769 + *j*0.3550	−0.0078 − *j*0.1125	−0.0091 − *j*0.0953
		Bare hard-drawn aluminum (AL)		
0AL7	0.1843 + *j*0.1487	0.2204 + *j*0.3689	0.0304 − *j*0.0809	0.0240 − *j*0.0694
000AL7	0.1161 + *j*0.1438	0.1522 + *j*0.3640	0.0179 − *j*0.0972	0.0130 − *j*0.0830
267AL7	0.0731 + *j*0.1385	0.1092 + *j*0.3586	0.0050 − *j*0.1071	0.0019 − *j*0.0911
477AL19	0.0413 + *j*0.1306	0.0774 + *j*0.3507	−0.0072 − *j*0.1143	−0.0086 − *j*0.0969

Table 10-9 Sequence-impedance values for bare-aluminum steel (AS) associated with Fig. 10-28, Ω/1000 ft

Conductor size & code	$z_1 = z_2$	$z_{0,a}$	z'_0
4AS8	$0.4830 + j0.1605$	$0.5372 + j0.5187$	$0.0439 - j0.0484$
0AS7	$0.2121 + j0.1607$	$0.2663 + j0.5189$	$0.0368 - j0.0967$
000AS7	$0.1377 + j0.1541$	$0.1919 + j0.5123$	$0.0229 - j0.1169$
267AS33	$0.0729 + j0.1245$	$0.1271 + j0.4827$	$0.0047 - j0.1440$
336AS37	$0.0580 + j0.1208$	$0.1122 + j0.4789$	$-0.0027 - j0.1489$
477AS33	$0.0409 + j0.1168$	$0.0951 + j0.4749$	$-0.0123 - j0.1536$
636AS33	$0.0306 + j0.1145$	$0.0848 + j0.4727$	$-0.0187 - j0.1558$
795AS33	$0.0244 + j0.1120$	$0.0786 + j0.4702$	$-0.0227 - j0.1577$

(*c*) The total positive-sequence impedance is

$$\begin{aligned} \bar{Z}_1 &= \bar{Z}_{1,\text{sys}} + \bar{Z}_{1,\text{ckt}} \\ &= (0.7199 + j3.4619) + (0.6125 + j1.2756) \\ &= 1.3324 + j4.7375\ \Omega \end{aligned}$$

(*d*) The total impedance to ground is

$$\begin{aligned} \bar{Z}_G &= \bar{Z}_{G,\text{sys}} + \bar{Z}_{G,\text{ckt}} \\ &= (0.6191 + j3.3397) + (2.0033 + j2.1855) \\ &= 2.6224 + j5.5252\ \Omega \end{aligned}$$

(*e*) From Eq. (10-7), the three-phase fault at point 10 is

$$\begin{aligned} I_{f,3\phi} &= \left|\frac{\bar{V}_{L\text{-}N}}{\bar{Z}_1}\right| \\ &= \frac{7200}{4.9213} \\ &= 1463\ \text{A} \end{aligned}$$

Table 10-10 Sequence-impedance values for bare-aluminum steel (AS) associated with Fig. 10-29, Ω/1000 ft

Conductor size & code	$z_1 = z_2$	$z_{0,a}$	z'_0
4AS8	$0.4830 + j0.1660$	$0.5372 + j0.5077$	$0.0406 - j0.0458$
0AS7	$0.2121 + j0.1662$	$0.2663 + j0.5079$	$0.0334 - j0.0909$
000AS7	$0.1377 + j0.1596$	$0.1919 + j0.5012$	$0.0202 - j0.1097$
267AS33	$0.0729 + j0.1301$	$0.1271 + j0.4717$	$0.0029 - j0.1349$
336AS37	$0.0580 + j0.1263$	$0.1122 + j0.4679$	$-0.0040 - j0.1394$
477AS33	$0.0409 + j0.1223$	$0.0951 + j0.4639$	$-0.0131 - j0.1437$
636AS33	$0.0306 + j0.1200$	$0.0808 + j0.4617$	$-0.0191 - j0.1456$
795AS33	$0.0244 + j0.1176$	$0.0786 + j0.4592$	$-0.0229 - j0.1475$

Table 10-11 Impedance values associated with Fig. 10-30, Ω/1000 ft

Conductor size & code	$z'_{1\phi}$	$z''_{1\phi}$
	Bare aluminum-steel (AS)	
4AS7	0.5230 + j0.2618	0.5202 + j0.2640
4AS8	0.5193 + j0.2609	0.5165 + j0.2631
3AS7	0.4292 + j0.2572	0.4262 + j0.2601
2AS7	0.3570 + j0.2531	0.3540 + j0.2565
2AS8	0.3497 + j0.2481	0.3466 + j0.2516
1AS8	0.2957 + j0.2664	0.2929 + j0.2705
1AS7	0.2973 + j0.2481	0.2942 + j0.2522
0AS7	0.2462 + j0.2415	0.2433 + j0.2464
000AS7	0.1664 + j0.2265	0.1641 + j0.2326
267AS33	0.0946 + j0.1859	0.0930 + j0.1936
336AS37	0.0767 + j0.1800	0.0755 + j0.1880
477AS33	0.0559 + j0.1740	0.0551 + j0.1824
636AS33	0.0431 + j0.1708	0.0426 + j0.1793
795AS33	0.0352 + j0.1675	0.0349 + j0.1762
	Bare hard-drawn copper (X)	
8X1	0.7529 + j0.2701	0.7507 + j0.2713
6X1	0.4895 + j0.2561	0.4866 + j0.2585
4X1	0.3221 + j0.2393	0.3189 + j0.2432
4X3	0.3251 + j0.2377	0.3219 + j0.2415
3X3	0.2643 + j0.2291	0.2611 + j0.2338
2X1	0.2124 + j0.2228	0.2096 + j0.2283
1X1	0.1724 + j0.2154	0.1698 + j0.2216
1X3	0.1740 + j0.2137	0.1715 + j0.2198
0X7	0.1423 + j0.2081	0.1401 + j0.2148
00X7	0.1150 + j0.2026	0.1132 + j0.2097
000X7	0.0931 + j0.1978	0.0916 + j0.2053
0000X7	0.0753 + j0.1934	0.0742 + j0.2011
300X12	0.0552 + j0.1848	0.0546 + j0.1929
	Bare hard-drawn aluminum (AL)	
0AL7	0.2190 + j0.2158	0.2159 + j0.2212
000AL7	0.1442 + j0.2021	0.1419 + j0.2089
267AL7	0.0944 + j0.1915	0.0929 + j0.1990
477AL19	0.0557 + j0.1796	0.0554 + j0.1878

(*f*) From Eq. (10-14), the line-to-line fault at point 10 is

$$\begin{aligned} I_{f,L\text{-}L} &= 0.866 \times I_{f,3\phi} \\ &= 0.866 \times 1463 \\ &= 1267 \text{ A} \end{aligned}$$

(*g*) From Eq. (10-15), the single line-to-ground fault at point 10 is

$$\begin{aligned} I_{f,L\text{-}G} &= \left| \frac{\bar{V}_{L-N}}{\bar{Z}_G} \right| \\ &= \frac{7200}{6.1159} \\ &= 1177.3 \text{ A} \end{aligned}$$

Note that the fault currents are calculated on the basis of a bolted fault. Therefore, they are accurate for faults caused by a low-impedance object making solid contact with the grounds. However, usually the object causing the fault has either a high impedance or does not make solid contact with the conductors. Of course, this introduces an additional impedance into the circuit, which reduces the fault current to some value below the calculated value. Therefore, to be sure that the high-impedance faults will be cleared, it is crucial that all bolted faults clear within 3 s.

10-12 FAULT-CURRENT CALCULATIONS IN PER UNITS

Fault currents can also be determined by using per unit values rather than actual system values, of course. For example, Anderson [3] gives fault-current formulas which use per unit values, as shown in Table 10-12.

Table 10-12 Fault-current formulas in per units

Fault type	Fault-current formula	Source connection
3ϕ	$\bar{I}_{f,a} = \dfrac{\bar{V}_f}{\bar{Z}_{sys} + \bar{Z}_T + \bar{Z}_{ckt} + \bar{Z}_f}$	Delta or grounded-wye
L-L	$\bar{I}_{f,a} = -\bar{I}_{f,c} = -\dfrac{j\sqrt{3} \times \bar{V}_f}{2(\bar{Z}_{sys} + \bar{Z}_T + \bar{Z}_{ckt}) + \bar{Z}_f}$	Delta or grounded-wye
	$I_{f,a} = 0$	Delta
L-G	$\bar{I}_{f,a} = \dfrac{3\bar{V}_f}{2\bar{Z}_{sys} + 3\bar{Z}_T + 6\bar{Z}_{ckt} + 3\bar{Z}_f}$	Grounded-wye*

* Using $K_0 = 4$, from Table 10-6.
Source: From [3].

Example 10-3 Assume that a distribution substation, shown in Fig. 10-31, has a 5000-kVA 69/12.47-kV LTC transformer feeding a three-phase four-wire 12.47-kV distribution system. The transformer has a reactance of 0.065 pu Ω. Assume that the faults are bolted with zero fault impedance and that the maximum and minimum power generations of the system are 600 and 360 MVA, respectively. Use 1 MVA as the three-phase power base:

(*a*) Under the maximum (system) power generation conditions, determine the available three-phase, line-to-line, and single line-to-ground fault currents at buses 1 and 2 in per units, in amperes, and in megavoltamperes.

(*b*) Under the minimum (system) power generation conditions, determine the available three-phase, line-to-line, and single line-to-ground fault currents at buses 1 and 2 in per units, in amperes, and in megavoltamperes.

(*c*) Tabulate the results obtained in parts *a* and *b*.

SOLUTION

(*a*) Selecting 69 kV as the voltage base, the impedance base can be determined as

$$Z_B = \frac{V^2_{B,\,L\text{-}L}}{S_{B,\,3\phi}}$$

$$= \frac{(69 \times 10^3)^2}{1 \times 10^6}$$

$$= 4761\ \Omega$$

Therefore, under the maximum (system) power generation conditions, the system impedance is

$$Z_{\text{sys}} = \frac{(69 \times 10^3)^2}{600 \times 10^6}$$

$$= 7.935\ \Omega$$

or

$$Z_{\text{sys}} = \frac{7.935}{4761}$$

$$= 0.0017 \text{ pu } \Omega$$

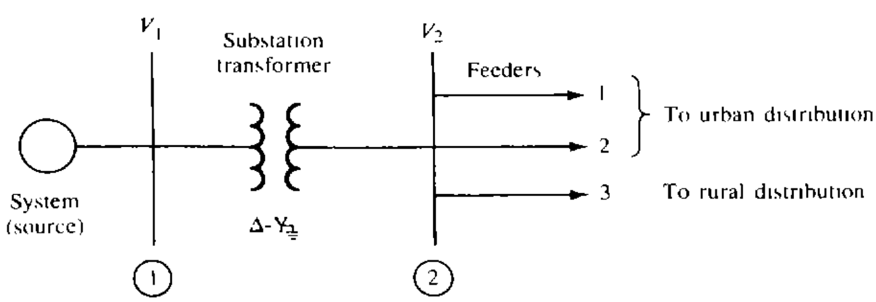


Figure 10-31

Similarly, the three-phase current base can be found as

$$I_B = \frac{S_{B,\,3\phi}}{\sqrt{3} \times V_{B,\,L\text{-}L}}$$

$$= \frac{1 \times 10^6}{\sqrt{3}(69 \times 10^3)}$$

$$= 8.3674 \text{ A}$$

(i) At bus 1, from Table 10-12, the three-phase fault current can be calculated as

$$I_{f,\,3\phi} = \left| \frac{\bar{V}_f}{\bar{Z}_{\text{sys}} + \bar{Z}_T + \bar{Z}_{\text{ckt}} + \bar{Z}_f} \right|$$

$$= \left| \frac{1.0\dagger}{j0.0017 + 0 + 0 + 0} \right|$$

$$\cong 588.2 \text{ pu A}$$

or

$$I_{f,\,3\phi} = (588.2 \text{ pu A})(8.3674 \text{ A})$$

$$= 4922 \text{ A}$$

or

$$S_{f,\,3\phi} = \sqrt{3}(69 \text{ kV})(4922) \times (10^{-3})$$

$$\cong 588.2 \text{ MVA}$$

The line-to-line fault current can be calculated by using the appropriate equation from Table 10-12 or from

$$I_{f,\,L\text{-}L} = 0.866\, I_{f,\,3\phi}$$

$$= 0.866(588.2 \text{ pu A})$$

$$\cong 509.38 \text{ pu A}$$

$$= 4262.5 \text{ A}$$

or

$$S_{f,\,L\text{-}L} = (69 \text{ kV})(4267.2 \text{ A})(\times 10^{-3})$$

$$= 294.1 \text{ MVA}$$

† Assuming that the voltage is 1.0 pu V at the fault point.

From Table 10-12, the single line-to-ground fault current can be calculated as

$$I_{f,L\text{-}G} = \left| \frac{3\bar{V}_f}{2\bar{Z}_{\text{sys}} + 3\bar{Z}_T + 6\bar{Z}_{\text{ckt}} + 3\bar{Z}_f} \right|$$

$$= \left| \frac{3(1.0)}{2(j0.0017) + 0 + 0 + 0} \right|$$

$$= 882.35 \text{ pu A}$$

$$= 7383 \text{ A}$$

or

$$S_{f,L\text{-}G} = \left(\frac{69 \text{ kV}}{\sqrt{3}}\right)(7391.7 \text{ A}) \times (10^{-3})$$

$$= 294.1 \text{ MVA}$$

(ii) At bus 2, since the given transformer reactance of 0.065 pu Ω value is based on 5 MVA, it has to be converted to the new base of 1 MVA. Therefore

$$Z_{T,\text{new}} = Z_{T,\text{old}}\left(\frac{V_{B,L\text{-}L,\text{old}}}{V_{B,L\text{-}L,\text{new}}}\right)^2 \frac{S_{B,3\phi,\text{new}}}{S_{B,3\phi,\text{old}}}$$

$$= j0.065\left(\frac{69 \text{ kV}}{69 \text{ kV}}\right)^2 \frac{1 \text{ MVA}}{5 \text{ MVA}}$$

$$= j0.013 \text{ pu } \Omega$$

$$Z_B = \frac{(12.47 \times 10^3)^2}{1 \times 10^6}$$

$$= 155.5 \, \Omega$$

$$I_B = \frac{1 \times 10^6}{\sqrt{3}(12.47 \times 10^3)}$$

$$= 46.2991 \text{ A}$$

Thus

$$I_{f,3\phi} = \left| \frac{1.0}{j0.0017 + j0.013 + 0 + 0} \right|$$

$$= 68.0272 \text{ pu A}$$

or

$$I_{f,3\phi} = (68.0272 \text{ pu A})(46.2991 \text{ A})$$

$$\cong 3149.6 \text{ A}$$

or

$$S_{f,3\phi} = \sqrt{3}(12.47\text{ kV})(3149.6)(10^{-3})$$
$$= 68.0272\text{ MVA}$$

$$I_{f,L\text{-}L} = 0.866(I_{f,3\phi})$$
$$= 0.866(68.0272\text{ pu A})$$
$$= 58.9116\text{ pu A}$$
$$= 2727.6\text{ A}$$

or

$$S_{f,L\text{-}L} = (12.47\text{ kV})(2727.6\text{ A})(10^{-3})$$
$$= 34.01\text{ MVA}$$

$$I_{f,L\text{-}G} = \left|\frac{3(1.0)}{2(j0.0017) + 3(j0.013) + 0 + 0}\right|$$
$$= 70.7547\text{ pu A}$$
$$\cong 3275.9\text{ A}$$

or

$$S_{f,L\text{-}G} = \left(\frac{12.47\text{ kV}}{\sqrt{3}}\right)(3275.9\text{ A})(10^{-3})$$
$$\cong 23.58\text{ MVA}$$

(*b*) Under the minimum (system) power generation conditions, the system impedance becomes

$$Z_{\text{sys}} = \frac{(69 \times 10^3)^2}{360 \times 10^6}$$
$$= 13.2250\ \Omega$$

or

$$Z_{\text{sys}} = \frac{13.2250}{4761}$$
$$= j0.0028\text{ pu }\Omega$$

(i) At bus 1,

$$I_{f,3\phi} = \left|\frac{1.0}{j0.0028 + 0 + 0 + 0}\right|$$
$$= 360\text{ pu A}$$
$$= 3012.3\text{ A}$$

or

$$S_{f,3\phi} = \sqrt{3}(69 \text{ kV})(3012.3 \text{ A})(10^{-3})$$
$$= 360 \text{ MVA}$$

$$I_{f,L\text{-}L} = 0.866 \times I_{f,3\phi}$$
$$= 311.76 \text{ pu A}$$
$$\cong 2608.7 \text{ A}$$

or

$$S_{f,L\text{-}L} = (69 \text{ kV})(2608.7 \text{ A})(10^{-3})$$
$$\cong 180.2 \text{ MVA}$$

$$I_{f,L\text{-}G} = \left|\frac{3(1.0)}{2(j0.0028) + 0 + 0 + 0}\right|$$
$$= 535.7 \text{ pu A}$$
$$= 4482.5 \text{ A}$$

or

$$S_{f,L\text{-}G} = \left(\frac{69 \text{ kV}}{\sqrt{3}}\right)(4487.8 \text{ A})(10^{-3})$$
$$\cong 178.6 \text{ MVA}$$

(ii) At bus 2,

$$I_{f,3\phi} = \left|\frac{1.0}{j0.0028 + j0.013 + 0 + 0}\right|$$
$$= 63.2911 \text{ pu A}$$
$$\cong 2930.3 \text{ A}$$

or

$$S_{f,3\phi} \sqrt{3}(12.47 \text{ kV})(2933.8 \text{ A})(10^{-3})$$
$$\cong 63.29 \text{ MVA}$$

$$I_{f,L\text{-}L} = 0.866 \times I_{f,3\phi}$$
$$= 54.81 \text{ pu A}$$
$$\cong 2537.7 \text{ A}$$

or

$$S_{f,L\text{-}L} = (12.47 \text{ kV})(2540.7 \text{ A})(10^{-3})$$
$$\cong 36.6 \text{ MVA}$$

$$I_{f,L\text{-}G} = \left|\frac{3(1.0)}{2(j0.0028) + 3(j0.013) + 0 + 0}\right|$$
$$= 67.2646 \text{ pu A}$$
$$\cong 3114.3 \text{ A}$$

Table 10-13

		Maximum generation		Minimum generation	
Bus	Fault	A	MVA	A	MVA
1	3ϕ	4922	588.2	3012.3	360
	$L-L$	4266.5	294.1	2608.7	180.2
	$L-G$	7383	294.1	4482.5	178.6
2	3ϕ	3149.6	68.0	2930.3	63.29
	$L-L$	2727.6	34.0	2537.7	36.6
	$L-G$	3275.9	23.6	3114.3	23.42

or

$$S_{f,L\text{-}G} = \left(\frac{12.47 \text{ kV}}{\sqrt{3}}\right)(3114.3 \text{ A})(10^{-3})$$

$$\cong 22.42 \text{ MVA}$$

(*c*) The results are given in Table 10-13.

10.13 SECONDARY-SYSTEM FAULT-CURRENT CALCULATIONS

10-13-1 Single-Phase 120/240-V Three-Wire Secondary Service

As shown in Fig. 10-32, a line-to-ground fault may involve line l_1 and neutral or line l_2 and neutral. Therefore the maximum line-to-ground fault current can be calculated as

$$\bar{I}_{f,L\text{-}G} = \frac{\bar{V}_{L\text{-}N}}{\bar{Z}_{eq}} \quad \text{A} \tag{10-46}$$

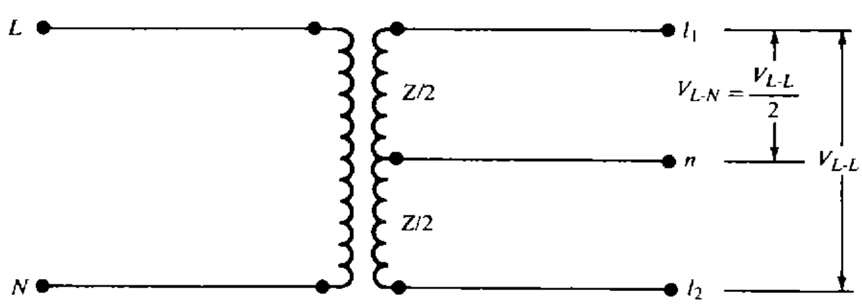


Figure 10-32

or

$$\bar{I}_{f,L\text{-}G} = \frac{0.5\bar{V}_{L\text{-}L}}{\bar{Z}_{\text{eq}}} \quad \text{A} \tag{10-47}$$

where

$$\begin{aligned}\bar{Z}_{\text{eq}} &= \bar{Z}_T + n^2\bar{Z}_G + \bar{Z}_{G.\text{SL}} \\ &= \text{equivalent impedance to fault, } \Omega\end{aligned} \tag{10-48}$$

$$\bar{Z}_T = \text{equivalent impedance of distribution transformer,† } \Omega$$

$$= \bar{Z}/2 = \text{half the secondary winding impedance, } \Omega$$

$$= 1.5R_T + j1.2X_T \tag{10-49}$$

$$\bar{Z}_G = \text{line-to-ground impedance of primary system, } \Omega$$

$$\bar{Z}_{G,\text{SL}} = \text{line-to-ground impedance of secondary line, } \Omega$$

$$n = \text{primary-to-secondary-impedance transfer ratio‡}$$

$$= \frac{\text{sec } V_{L\text{-}N}}{\text{pri } V_{L\text{-}N}} \tag{10-50}$$

Also, maximum line-to-line fault may occur between lines l_1 and l_2. Therefore

$$\bar{I}_{f,L\text{-}L} = \frac{\bar{V}_{L\text{-}L}}{\bar{Z}_{\text{eq}}} \tag{10-51}$$

where

$$\bar{Z}_{\text{eq}} = \bar{Z}_T + n^2\bar{Z}_G + \bar{Z}_{1,\text{SL}} \tag{10-52}$$

$$\begin{aligned}\bar{Z}_T &= \text{equivalent impedance of distribution transformer, } \Omega \\ &= Z\end{aligned} \tag{10-53}$$

$$\bar{Z}_{1,\text{SL}} = \text{positive-sequence impedance of secondary line, } \Omega$$

10-13-2 Three-Phase 240/120- or 480/240-V Wye-Delta or Delta-Delta Four-Wire Secondary Service

For a delta-connected secondary, if the primary is connected in wye, its impedance must be converted to its equivalent delta impedance, as indicated in Fig. 10-33. Therefore

$$\bar{Z}_\Delta = \frac{\bar{Z}_1\bar{Z}_1 + \bar{Z}_1\bar{Z}_1 + \bar{Z}_1\bar{Z}_1}{\bar{Z}_1} = 3\bar{Z}_1 \quad \Omega \tag{10-54}$$

† Note that there has been a shift in notation and the symbol Z_T stands for $Z_{1,T}$.

‡ Note that there has been a shift in notation and the symbol n stands for primary-to-secondary-impedance transfer ratio. (It is the inverse of the transformer turns ratio.)

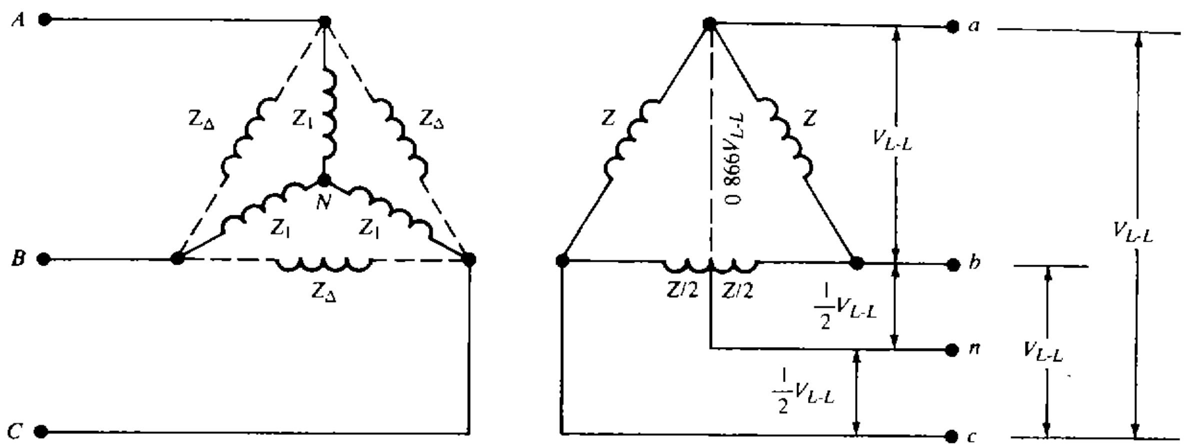


Figure 10-33

where $\bar{Z}_1$ = positive-sequence impedance, Ω
$\bar{Z}_\Delta$ = equivalent delta impedance, Ω
If the primary is already connected in delta, then

$$\bar{Z}_\Delta = \bar{Z}_1 \qquad \Omega \tag{10-55}$$

As shown in Fig. 10-33, a line-to-ground fault may involve phase a and neutral. The resultant maximum line-to-ground fault current can be expressed as

$$\bar{I}_{f,L\text{-}G} = \frac{0.866 \times \bar{V}_{L\text{-}L}}{\bar{Z}_{\text{eq}}} \qquad \text{A} \tag{10-56}$$

where

$$\bar{Z}_{\text{eq}} = \bar{Z}_T + n^2\bar{Z}_\Delta + \bar{Z}_{G,\text{SL}} \qquad \Omega \tag{10-57}$$

$$\bar{Z}_T = \frac{(\bar{Z} + \bar{Z}/2)(\bar{Z} + \bar{Z}/2)}{2(\bar{Z} + \bar{Z}/2)} = \frac{3\bar{Z}}{4} \qquad \Omega \tag{10-58}$$

If the line-to-ground fault involves phase b and neutral or phase c and neutral, then the maximum available line-to-ground fault current can be calculated as

$$I_{f,L\text{-}G} = \frac{0.5 \times V_{L\text{-}L}}{Z_{\text{eq}}} \qquad \text{A} \tag{10-59}$$

where $\bar{Z}_{\text{eq}}$ is found from Eq. (10-57) and

$$\bar{Z}_T = \frac{\bar{Z}/2(2\bar{Z} + \bar{Z}/2)}{2(\bar{Z} + \bar{Z}/2)} = \frac{5\bar{Z}}{12} \qquad \Omega \tag{10-60}$$

A line-to-line fault may involve phases a and b, or b and c, or c and a. In any case, the maximum line-to-line fault current can be calculated from Eq. (10-51) where

$$Z_{\text{eq}} = \bar{Z}_T + n^2\bar{Z}_\Delta + \bar{Z}_{1,\text{SL}} \qquad \Omega \tag{10-61}$$

and

$$\bar{Z}_T = \frac{(2\bar{Z})(\bar{Z})}{2\bar{Z} + \bar{Z}} = \frac{2\bar{Z}}{3} \qquad \Omega \tag{10-62}$$

A three-phase fault, of course, involves all three phases. Therefore

$$\bar{I}_{f,3\phi} = \frac{\bar{V}_{L\text{-}L}}{\sqrt{3} \times \bar{Z}_{eq}} \qquad \text{A} \tag{10-63}$$

where

$$\bar{Z}_{eq} = \bar{Z}_T + n^2\bar{Z}_\Delta + \bar{Z}_{1,\text{cable}} \qquad \Omega \tag{10-64}$$

$$\bar{Z}_T = \bar{Z} \qquad \Omega \tag{10-65}$$

10-13-3 Three-Phase 240/120- or 480/240-V Open Wye Primary and Four-Wire Open-Delta Secondary Service

Figure 10-34 shows a three-phase open-wye primary and four-wire open-delta secondary connection. If a line-to-ground fault involves phase b and neutral or phase c and neutral, the maximum available fault current can be calculated as

$$\bar{I}_{f,L\text{-}G} = \frac{0.5 \times \bar{V}_{L\text{-}L}}{\bar{Z}_{eq}} \qquad \text{A} \tag{10-66}$$

where $\bar{Z}_{eq}$ is found from Eq. (10-48) and n is the transfer ratio found from Fig. 10-35a.

If the line-to-ground fault involves phase a and neutral, the maximum available fault current can be expressed as

$$\bar{I}_{f,L\text{-}G} = \frac{0.866 \times \bar{V}_{L\text{-}L}}{\bar{Z}_{eq}} \qquad \text{A} \tag{10-67}$$

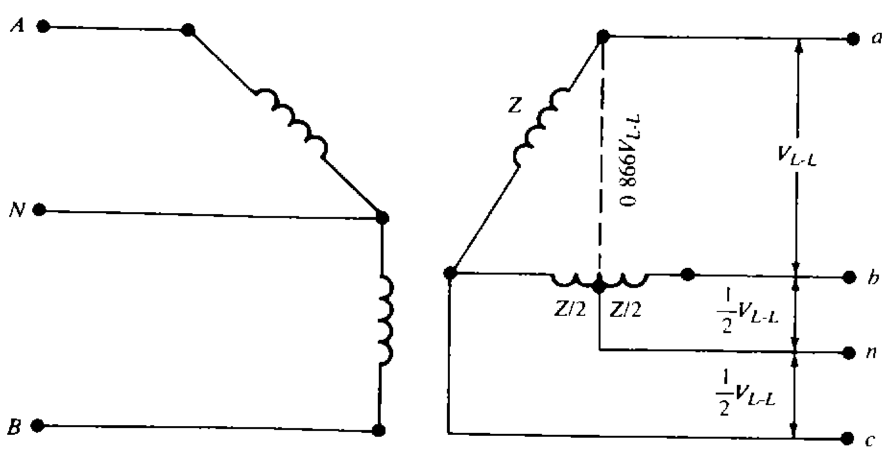


Figure 10-34

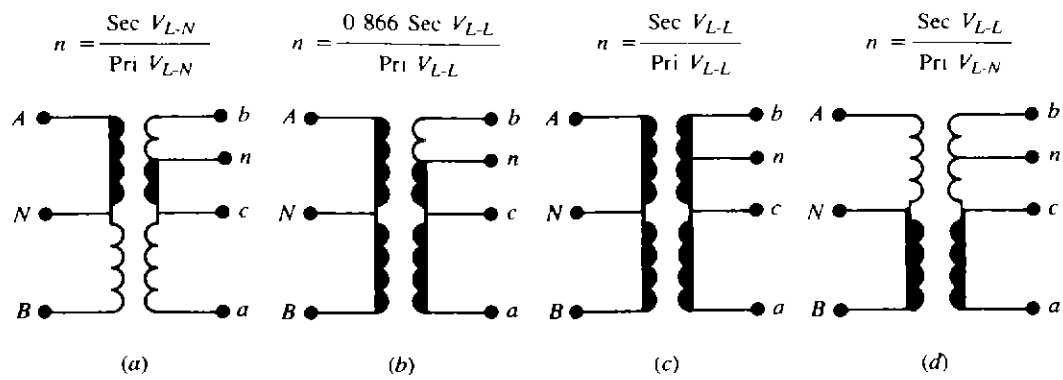


Figure 10-35 Various fault-current paths in the transformer and associated impedance transfer ratios. (The shaded path determines the primary- and secondary-transformer fault impedances.)

where

$$\bar{Z}_{\text{eq}} = \bar{Z}_T + n^2\bar{Z}_1 + \bar{Z}_{G,\text{SL}} \quad \Omega \tag{10-68}$$

$$n = \text{transfer ratio from Fig. 10-35}b$$

$$\bar{Z}_T = \bar{Z} + \bar{Z}/2$$

$$= (R_T + 1.5R_T) + j(X_T + 1.2X_T) \quad \Omega \tag{10-69}$$

If a line-to-line fault involves phases a and b, the available fault current can be calculated from Eq. (10-51), where

$$\bar{Z}_{\text{eq}} = \bar{Z}_T + n^2\bar{Z}_1 + \bar{Z}_{1,\text{SL}} \quad \Omega \tag{10-70}$$

$$n = \text{transfer ratio from Fig. 10-35}c$$

$$\bar{Z}_T = 2\bar{Z} \quad \Omega \tag{10-71}$$

If a line-to-line fault involves phases a and c or phases b and c, the available fault current can be calculated by using Eqs. (10-51) and (10-52), where

$$n = \text{transfer ratio from Fig. 10-35}d$$

$$\bar{Z}_T = \bar{Z} \quad \Omega$$

Since a three-phase fault involves all three phases, the maximum three-phase fault current can be determined from

$$\bar{I}_{f,3\phi} = \frac{\bar{V}_{L\text{-}L}}{\bar{Z}_{\text{eq}}} \quad \text{A} \tag{10-72}$$

where $\bar{Z}_{\text{eq}}$ is found from Eq. (10-70) and

$$\bar{Z}_T = \frac{(3\bar{Z})(3\bar{Z})}{6\bar{Z}} = \frac{3\bar{Z}}{2} \quad \Omega \tag{10-73}$$

10-13-4 Three-Phase 208Y/120-V, 480Y/277-V, or 832Y/480-V Four-Wire Wye-Wye Secondary Service

Figure 10-36 shows a three-phase wye-wye-connected four-wire secondary connection. A line-to-ground fault may involve any one of the three phases and neutral. The maximum line-to-ground fault current can be calculated as

$$\bar{I}_{f,\,L\text{-}G} = \frac{\bar{V}_{L\text{-}N}}{\bar{Z}_{\text{eq}}} \qquad \text{A} \tag{10-74}$$

where $\bar{Z}_{\text{eq}}$ is found from Eq. (10-48) and

$$\bar{Z}_T = \bar{Z} \qquad \Omega$$

A line-to-line fault may involve phases a and b, or b and c, or c and a. The available line-to-line fault current is

$$\bar{I}_{f,\,L\text{-}L} = \frac{\bar{V}_{L\text{-}L}}{2\bar{Z}_{\text{eq}}} \qquad \text{A} \tag{10-75}$$

where $\bar{Z}_{\text{eq}}$ is determined from Eq. (10-70) and

$$\bar{Z}_T = \bar{Z} \qquad \Omega$$

A three-phase fault may involve all three phases; therefore the available three-phase fault current can be expressed as

$$\bar{I}_{f,\,3\phi} = \frac{\bar{V}_{L\text{-}L}}{\sqrt{3} \times \bar{Z}_{\text{eq}}} = \frac{\bar{V}_{L\text{-}N}}{\bar{Z}_{\text{eq}}} \qquad \text{A} \tag{10-76}$$

where $\bar{Z}_{\text{eq}}$ is also determined from Eq. (10-70).

Example 10-4 Assume that there is a single-phase line-to-line secondary fault on a 120/240-V three-wire service, as shown in Fig. 10-37, and that the subtransmission system is taken as an infinite bus. The substation transformer is three-phase 7500 kVA with 7 percent impedance and 1 percent resistance. The 12.47-kV primary feeder has three phase conductors of 336AS37 with a

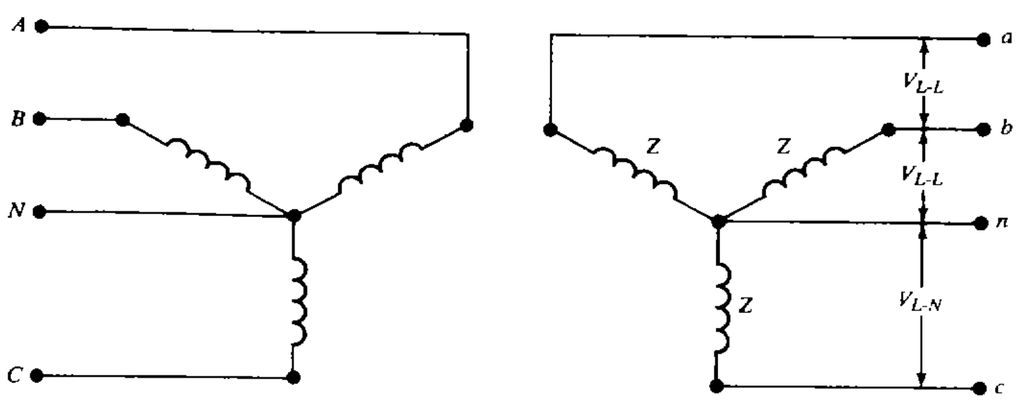


Figure 10-36

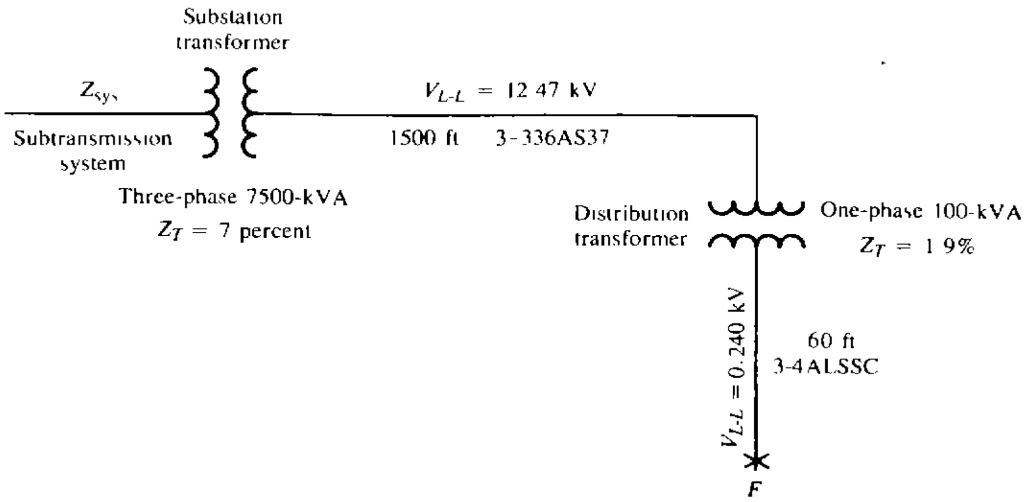


Figure 10-37

neutral conductor of OAS7 at 62-in spacing. The secondary transformer (i.e., distribution transformer) has 100 kVA capacity with 1.9 percent impedance and 1 percent resistance. The 60-ft-long self-supporting service cable (SSC) with ACSR neutral has three wires and is given to be 3-4ALSSC. Assume that it has a resistance of 0.4660 Ω/1000 ft and a reactance of 0.0293 Ω/1000 ft. Use Table 10-7 to determine the necessary sequence-impedance values for the primary line and determine the following:

(*a*) The impedance of the substation transformer in ohms.
(*b*) The positive- and zero-sequence impedance of the line in ohms.
(*c*) The line-to-ground impedance in the primary system in ohms.
(*d*) The total impedance through the primary in ohms.
(*e*) The total primary impedance referred to secondary in ohms.
(*f*) The distribution transformer impedance in ohms.
(*g*) The impedance of the secondary cable in ohms.
(*h*) The total impedance to the fault in ohms.
(*i*) The single-phase line-to-line fault for the 120/240-V three-wire service in amperes.

SOLUTION

(*a*) Since the impedance of the substation transformer can be expressed as

$$\bar{Z}_T = R_T + jX_T$$

where its reactance is

$$X_T = (Z_T^2 - R_T^2)^{1/2}$$
$$= (7^2 - 1^2)^{1/2} = 6.9282\,\%\,\Omega$$

and

$$\bar{Z}_T = 1 + j6.928\,\%\,\Omega$$

therefore

$$\bar{Z}_T = \frac{(1 + j6.9282)(12.47)^2(10)}{7500} = 0.2073 + j1.4365\ \Omega$$

(*b*) From Table 10-7, the positive-sequence impedance of the primary line is

$$\bar{Z}_1 = 1.5(0.0580 + j0.1208) = 0.0870 + j0.1812\ \Omega$$

and similarly the zero-sequence impedance is

$$\bar{Z}_0 = 0.1653 + j0.4878\ \Omega$$

(*c*) From Eq. (10-17),

$$\bar{Z}_G = \frac{2Z_1 + \bar{Z}_0}{3}$$

$$= \frac{2(0.0870 + j0.1812) + (0.1653 + j0.4878)}{3} = 0.1131 + j0.2834\ \Omega$$

(*d*) Since the subtransmission system is assumed to be an infinite bus,

$$\bar{Z}_{sys} = 0 + j0\ \Omega$$

therefore the total impedance through the primary is

$$\begin{aligned}\hat{Z}_{eq} &= \bar{Z}_{sys} + \bar{Z}_T + \bar{Z}_G \\ &= (0 + j0) + (0.2073 + j1.4365) + (0.1131 + j0.2834) \\ &= 0.3204 + j1.7199\ \Omega\end{aligned}$$

(*e*) From Eq. (10-50), the total primary impedance referred to secondary is

$$n^2\hat{Z}_{eq} = \hat{Z}_{eq}\left(\frac{\sec V_{L-L}^2}{\text{pri } V_{L-N}}\right)$$

$$= (0.3204 + j1.7199)\left(\frac{0.240}{7.2}\right)^2 = 0.0004 + j0.0019\ \Omega$$

(*f*) The secondary (i.e., distribution) transformer reactance can be determined as

$$\begin{aligned}X_T &= (Z_T^2 - R_T^2)^{1/2} \\ &= (1.9^2 - 1^2)^{1/2} = 1.6155\%\ \Omega\end{aligned}$$

Therefore its impedance can be expressed as

$$\bar{Z}_T = 1 + j1.6155\%\ \Omega$$

or

$$\bar{Z}_T = \frac{(1 + j1.6155)(0.240)^2(10)}{100} = 0.0058 + j0.0093\ \Omega$$

(*g*) Since the impedance of the secondary cable is given in ohms per thousand feet, for a 60-ft length,

$$\bar{Z}_{1,\mathrm{SL}} = \tfrac{60}{1000}(0.4660 + j0.0293) = 0.0280 + j0.0018\ \Omega$$

(*h*) Therefore, the total impedance to the fault can be found as

$$\begin{aligned}\bar{Z}_{\mathrm{eq}} &= n^2\hat{Z}_{\mathrm{eq}} + \bar{Z}_T + \bar{Z}_{1,\mathrm{SL}} \\ &= (0.004 + j0.0019) + (0.0058 + j0.0093) + (0.0280 + j0.0018) \\ &= 0.0342 + j0.0130\ \Omega\end{aligned}$$

(*i*) Thus, from Eq. (10-51), the fault current at the secondary fault point F is

$$\begin{aligned}I_{f,L\text{-}L} &= \frac{V_{L\text{-}L}}{Z_{\mathrm{eq}}} \\ &= \frac{240}{0.0366} = 6559.63\ \mathrm{A}\end{aligned}$$

PROBLEMS

10-1 Repeat Example 10-1, assuming that the fault current is 1000 A.

10-2 Repeat Example 10-1, assuming that the fault current is 500 A.

10-3 In Prob. 10-2, determine the lacking relay travel that is necessary for the relay to close its contacts and trip its breaker:

(*a*) In percent

(*b*) In seconds

10-4 Assume that an inverse-time overcurrent relay is installed at a location on a feeder. It is desired that the substation oil circuit breaker trip on a sustained current of approximately 400 A, and trip in 2 s on a short-circuit current of 4000 A. Assuming that current transformers of 60 :1 ratio are used, determine the following:

(*a*) The current-tap setting of the relay.

(*b*) The time setting of the relay.

10-5 Repeat Example 10-2, assuming that the transformer is rated 3750 kVA 69/4.16 kV feeding a three-phase four-wire 4.16-kV circuit and that the sizes of the phase conductors and neutral conductor are 267AS33 and 0AS7, respectively.

10-6 Repeat Example 10-3, assuming that the faults are bolted and that the fault impedance is 40 Ω.

10-7 Assume that there is a bolted fault at a certain point F on a distribution circuit, as indicated in Fig. P10-7. Also assume that the maximum power generation of the system is 600 MVA. Determine the following:

(*a*) Maximum values of the available three-phase, line-to-line, and single line-to-ground fault currents at the fault point F, using actual system values.

(*b*) Minimum value of the available single line-to-ground fault, assuming that it is equal to 60 percent of its maximum value found in part *a*.

10-8 Repeat Example 10-4, assuming that the substation transformer's impedance is 7.5 percent and that the distribution transformer has a capacity of 75 kVA with 2 percent impedance. Also assume that the primary line is made of three 477AS33 conductors and a neutral conductor of 0AS7 at 62-in spacing, and that the lengths of the primary line and secondary cable are 1000 and 50 ft, respectively. Assume that the impedance of the three-wire 0ALSSC secondary cable is $0.1843 + j0.0273\ \Omega/1000$ ft.

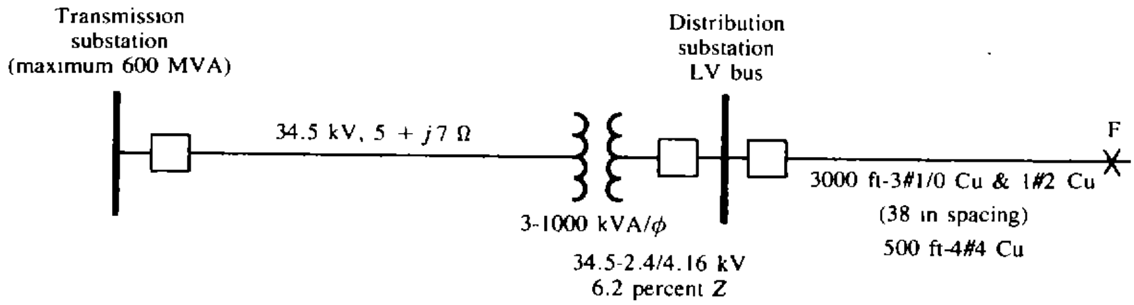


Figure P10-7

REFERENCES

1. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
2. Fink, D. G., and H. W. Beaty: *Standard Handbook for Electrical Engineers*, 11th ed., McGraw-Hill, New York, 1978.
3. Anderson, P. M.: *Elements of Power System Protection*, Cyclone Copy Center, Ames, Iowa, 1975.
4. General Electric Company: *Overcurrent Protection for Distribution Systems*, Application Manual GET-1751A, 1962.
5. General Electric Company: *Distribution System Feeder Overcurrent Protection*, Application Manual GET-6450, 1979.
6. Westinghouse Electric Corporation: *Westinghouse Transmission and Distribution Reference Book*, East Pittsburgh, Pa., 1964.
7. *Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems*, IEEE Standard 242-1975, 1975.
8. Anderson, P. M.: *Analysis of Faulted Power Systems*, Iowa State University Press, Ames, 1973.
9. Rural Electrification Administration: *Guide for Making a Sectionalizing Study on Rural Electric Systems*, REA Bulletin 61-2, March, 1958.
10. Wagner, C. F., and R. D. Evans: *Symmetrical Components*, McGraw-Hill, New York, 1933.
11. Stevenson, W. D.: *Elements of Power System Analysis*, 3d ed., McGraw-Hill, New York, 1975.
12. Gross, C. A.: *Power System Analysis*, Wiley, New York, 1979.
13. Carson, J. R.: "Wave Propagation in Overhead Wires with Ground Return," *Bell Syst. Techn. J.*, vol. 5, October 1926, pp. 539–555.

CHAPTER

ELEVEN

DISTRIBUTION SYSTEM RELIABILITY

11-1 BASIC DEFINITIONS

Most of the following definitions of terms for reporting and analyzing outages of electrical distribution facilities and interruptions are taken from Ref. 1 and included here by permission of the Institute of Electrical and Electronics Engineers, Inc.

Outage. Describes the state of a component when it is not available to perform its intended function due to some event *directly associated* with that component. An outage may or may not cause an interruption of service to consumers depending on system configuration.

Forced outage. An outage caused by emergency conditions directly associated with a component that require the component to be taken out of service immediately, either automatically or as soon as switching operations can be performed, or an outage caused by improper operation of equipment or human error.

Scheduled outage. An outage that results when a component is deliberately taken out of service at a selected time, usually for purposes of construction, preventive maintenance, or repair. The key test to determine if an outage should be classified as forced or scheduled is as follows. If it is possible to defer the outage when such deferment is desirable, the outage is a scheduled outage; otherwise, the outage is a forced outage. Deferring an outage may be desirable, for example, to prevent overload of facilities or an interruption of service to consumers.

Partial outage. "Describes a component state where the capacity of the component to perform its function is reduced but not completely eliminated" [2].

Transient forced outage. A component outage whose cause is immediately self-clearing so that the affected component can be restored to service either automatically or as soon as a switch or circuit breaker can be reclosed or a fuse replaced. An example of a transient forced outage is a lightning flashover which does not permanently disable the flashed component.

Persistent forced outage. A component outage whose cause is *not* immediately self-clearing but must be corrected by eliminating the hazard or by repairing or replacing the affected component before it can be returned to service. An example of a persistent forced outage is a lightning flashover which shatters an insulator, thereby disabling the component until repair or replacement can be made.

Interruption. The loss of service to one or more consumers or other facilities and is the result of one or more component outages, depending on system configuration.

Forced interruption. An interruption caused by a forced outage.

Scheduled interruption. An interruption caused by a scheduled outage.

Momentary interruption. It has a duration limited to the period required to restore service by automatic or supervisor-controlled switching operations or by manual switching at locations where an operator is immediately available. Such switching operations are typically completed in a few minutes.

Temporary interruption. "It has a duration limited to the period required to restore service by manual switching at locations where an operator is not immediately available. Such switching operations are typically completed within 1–2 hours" [2].

Sustained interruption. "It is any interruption not classified as momentary or temporary" [2].

At the present time, there are no industrywide standard outage reporting procedures. More or less, each electric utility company has its own standards for each type of customer and its own methods of outage reporting and compilation of statistics. A unified scheme for the reporting of outages and the computation of reliability indices would be very useful but is not generally practical due to the differences in service areas, load characteristics, number of customers, and expected service quality

System interruption frequency index. "The average number of interruptions per customer served per time unit. It is estimated by dividing the accumulated number of customer interruptions in a year by the number of customers served" [3].

Customer interruption frequency index. "The average number of interruptions experienced per customer affected per time unit. It is estimated by dividing the number of customer interruptions observed in a year by the number of customers affected" [3].

Load interruption index. "The average kVA of connected load interrupted per unit time per unit of connected load served. It is formed by dividing the annual load interruption by the connected load" [3].

Customer curtailment index. "The kVA-minutes of connected load interrupted per affected customer per year. It is the ratio of the total annual curtailment to the number of customers affected per year" [3].

Customer interruption duration index. "The interruption duration for customers interrupted during a specific time period. It is determined by dividing the sum of all customer-sustained interruption durations during the specified period by the number of sustained customer interruptions during that period" [3].

According to an IEEE committee report [4], the following basic information should be included in an equipment outage report:

1. Type, design, manufacturer, and other descriptions for classification purposes
2. Date of installation, location on system, length in the case of a line
3. Mode of failure (short-circuit, false operation, etc.)
4. Cause of failure (lightning, tree, etc.)
5. Times (both out of service and back in service, rather than outage duration alone), date, meteorological conditions when the failure occurred
6. Type of outage, forced or scheduled, transient or permanent

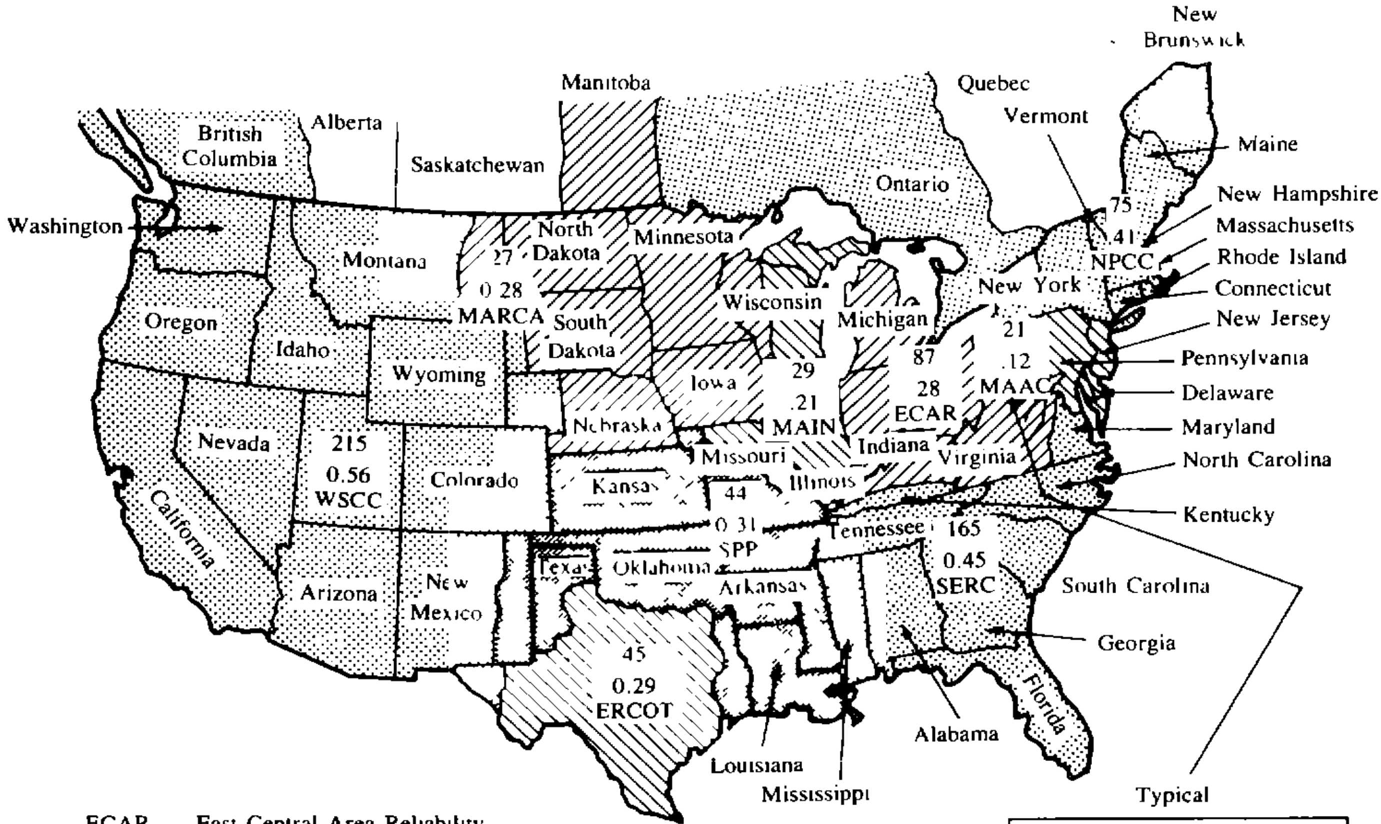
Furthermore, the committee has suggested that the total number of similar components in service should also be reported in order to determine outage rate per component per service year. It is also suggested that every component failure, regardless of service interruption, i.e., whether it caused a service interruption to a customer or not, should be reported in order to determine component failure rates properly [4]. Failure reports provide very valuable information for preventive maintenance programs and equipment replacements.

There are various types of probabilistic modeling of components to predict component-failure rates which include (1) fitting a modified time-varying Weibull distribution to component-failure cases and (2) component survival rate studies. However, in general, there may be some differences between the predicted failure rates and observed failure rates due to the following factors [5]:

1. Definition of failure
2. Actual environment compared with prediction environment
3. Maintainability, support, testing equipment, and special personnel
4. Composition of components and component-failure rates assumed in making the prediction
5. Manufacturing processes including inspection and quality control
6. Distributions of times to failure
7. Independence of component failures

11-2 NATIONAL ELECTRIC RELIABILITY COUNCIL

In 1968, a national organization, the National Electric Reliability Council (NERC), was established to increase the reliability and adequacy of bulk power supply in the electric utility systems of North America. It is a form of nine regional reliability councils and covers all the power systems of the United States and some of the power systems in Canada, including Ontario, British Columbia, Manitoba, New Brunswick, and Alberta, as shown in Fig. 11-1.



ECAR	East Central Area Reliability Coordination Agreement		
MAIN	MID-AMERICA Interpool Network	SERC	Southeastern Electric Reliable Council
MAAC	Mid-Atlantic Council	SPP	Southeastern Power Pool
MARCA	Mid-Continent Area Reliability Coordination Agreement	ERCOT	Electric Reliability Council of Texas
NPCC	Northeast Power Coordinating Council	WSCC	Western Systems Coordinating Council

Number bulk outage incidents
Ratio $= \dfrac{\text{Outage incidents}}{\text{Electric sales}}$

Figure 11-1 Regional Electric Reliability Councils. (*From* [6].)

Here, the terms of reliability and adequacy define two separate but interdependent concepts. The term *reliability* describes the security of the system and the avoidance of power outages, whereas the term *adequacy* refers to having sufficient system capacity to supply the electric energy requirements of the customers.

In general, regional and nationwide annual load forecasts and capability reports are prepared by the NERC. Guidelines to member utilities for system planning and operations are prepared by the regional reliability councils to improve reliability and reduce costs.

Also shown in Fig. 11-1 are the total number of bulk power outages reported and the ratio of the number of bulk outages to electric sales for each regional electric reliability council area to provide a meaningful comparison.

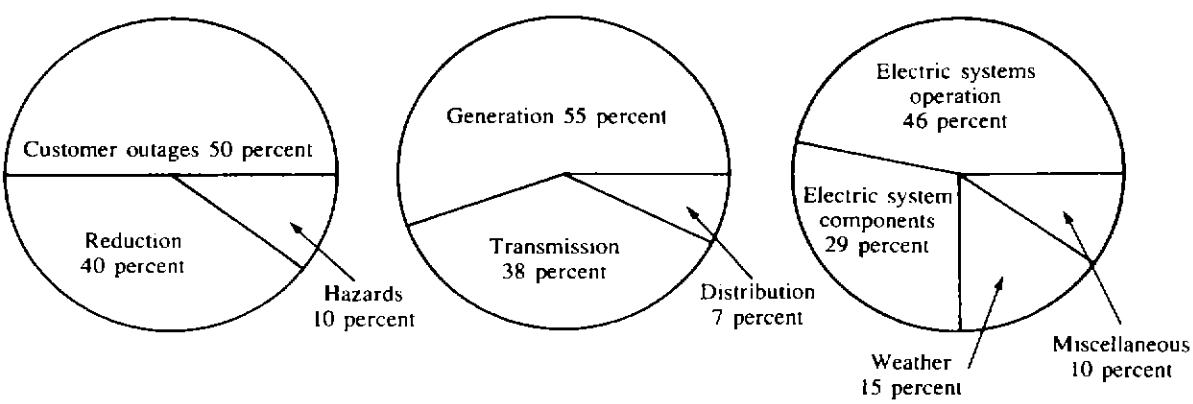
Table 11-1 gives the generic and specific causes for outages based upon the *National Electric Reliability Study* [6]. Figure 11-2 shows three different classifications of the reported outage events by (*a*) types of events, (*b*) generic subsystems,

Table 11-1 Classification of generic and specific causes of outages*

Weather	Miscellaneous	System components	System operation

Blizzard/snow
Cold
Flood
Heat
Hurricane
Ice
Lightning
Rain
Tornado
Wind
Other
	Airplane/helicopter
	Animal/bird/snake
	Vehicle:
	  Automobile/truck
	  Crane
	Dig-in
	Fire/explosion
	Sabotage/vandalism
	Tree
	Unknown
	Other
		Electric and mechanical:
		  Fuel supply
		  Generating unit failure
		  Transformer failure
		  Switchgear failure
		  Conductor failure
		  Tower, pole attachment
		  Insulation failure:
		    Transmission line
		    Substation
		    Surge arrestor
		    Cable failure
		  Voltage control equipment:
		    Voltage regulator
		    Automatic tap changer
		    Capacitor
		    Reactor
		Protection and control:
		  Relay failure
		  Communication signal error
		  Supervisory control error
			System conditions:
			  Stability
			  High/low voltage
			  High/low frequency
			  Line overload/transformer overload
			  Unbalanced load
			  Neighboring power system
			  Public appeal:
			    Commercial and industrial
			    All customers
			  Voltage reduction:
			    0–2% voltage reduction
			    Greater than 2–8% voltage reduction
			  Rotating blackout
			Utility personnel:
			  System operator error
			  Powerplant operator error
			  Field operator error
			  Maintenance error
			  Other

* From [6].



(a) (b) (c)

Figure 11-2 Classification of reported outage events in the National Electric Reliability Study for the period July 1970 to June 1979: (*a*) types of events, (*b*) generic subsystems, and (*c*) generic causes. (*From* [6].)

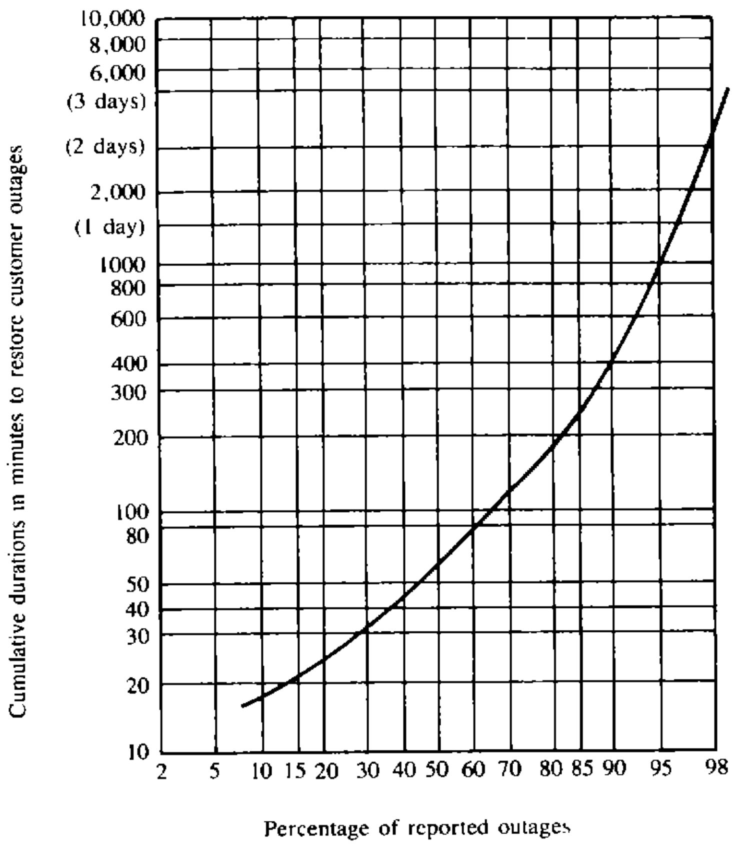


Figure 11-3 Cumulative duration in minutes to restore reported customer outages. (*From* [6].)

and (*c*) generic causes. The cumulative duration to restore customer outages is shown in Fig. 11-3, which indicates that 50 percent of the reported bulk power system customer outages are restored in 60 min or less and 90 percent of the bulk outages are restored in 7 h or less.

A casual glance at Fig. 11-2*b* may be misleading. Because, in general, utilities do not report their distribution system outages, the 7 percent figure for the distribution system outages is not realistic. According to *The National Electric Reliability Study* [7], approximately 80 percent of all interruptions occur due to failures in the distribution system.

The National Electric Reliability Study [7] gives the following conclusions:

1. Although there are adequate methods for evaluating distribution system reliability, there is insufficient data on reliability performance to identify the most cost-effective distribution investments.
2. Most distribution interruptions are initiated by severe weather-related interruptions with a major contributor being inadequate maintenance.
3. Distribution system reliability can be improved by the timely identification and response to failures.

11-3 APPROPRIATE LEVELS OF DISTRIBUTION RELIABILITY

The electric utilities are expected to provide continuous and quality electric service to their customers at a reasonable rate by making economical use of available system and apparatus. Here, the term *continuous electric service* has customarily

Table 11-2 Detailed industrial service interruption cost example*

Industry	Overlapped duration, h	Downtime, h	Normal prod., h/yr	Fraction of annual prod. loss	Value added lost*	Payroll lost*	Cleanup and spoil prod.*	Standby power cost*	Interruption cost			
									Lower*	Upper*	Lower, $/kWh	Upper, $/kWh
Food	4	6.00	2016	0.00298	4,260	1,812	279	0.00	2,091	4,539	2.38	5.17
Tobacco	4	6.00	2016	0.00298	0	0	0	0.00	0	0	0.00	0.00
Textiles	4	76.00	8544	0.00890	10,172	5,262	150	0.00	5,413	10,323	10.35	19.74
Apparel	4	6.00	2016	0.00298	25,309	1,358	83	0.00	1,441	25,391	5.54	97.69
Lumber	4	5.25	2016	0.00260	1,133	617	248	0.08	865	1,381	3.83	6.12
Furniture	4	6.00	2016	0.00298	1,074	527	52	0.00	579	1,127	3.51	6.83
Paper	4	14.00	8544	0.00164	3,006	1,363	144	0.57	1,508	3,151	0.98	2.05
Printing	4	6.00	8544	0.00070	1,146	569	127	0.00	696	1,273	1.74	3.19
Chemicals	4	24.00	8544	0.00281	3,899	1,102	27	0.16	1,129	3,925	2.63	9.13
Petroleum refining	4	6.00	8544	0.00070	888	439	32	0.00	471	919	6.48	6.74
Rubber and plastics	4	6.00	8544	0.00070	592	325	38	0.00	363	630	2.11	4.12
Leather	4	5.25	2016	0.00260	1,765	757	563	0.19	1,321	2,328	3.05	5.29
Stone, clay, glass	4	7.75	8544	0.00091	925	562	380	0.20	942	1,306	2.58	4.54
Primary metal	4	5.25	2016	0.00061	1,731	818	688	0.23	1,507	2,419	1.71	2.37
Nonelectric machinery	4	5.25	4864	0.00108	4,851	2,192	944	0.32	3,137	5,795	2.41	3.86
Electric machinery	4	6.00	8544	0.00070	2,322	1,069	246	0.29	1,315	2,568	3.65	6.75
Transportation equipment	4	5.25	8544	0.00061	1,739	1,005	858	0.88	1,864	2,598	1.68	3.20
Measuring equipment	4	6.00	4864	0.00123	2,565	1,112	104	0.00	1,215	2,669	2.34	3.26
Miscellaneous manufacturing	4	6.00	4864	0.00123	1,817	794	97	0.11	891	1,914	3.72	8.17
Agriculture									21	21	2.90	7.79
					69,293	21,779	5,059	3.07	26,861	74,375	2.81	7.79

* In thousands of dollars. *Source*: From [6].

meant meeting the customers' electric energy requirements as demanded, consistent with the safety of personnel and equipment. On the other hand, *quality electric service* involves meeting the customer demand within specified voltage and frequency limits.

To maintain reliable service to customers, a utility has to have adequate redundancy in its system to prevent a component outage becoming a service interruption to the customers, causing loss of goods, services, or benefits. To calculate the cost of reliability, the cost of an outage must be determined. Table 11-2 gives an example for calculating industrial service interruption cost. Presently, there is at least one public utility commission which requires utilities to pay for damages caused by service interruptions [6].

Reliability costs are used for rate reviews and requests for rate increases. The economic analysis of system reliability can also be a very useful planning tool in determining the capital expenditures required to improve service reliability by providing the real value of additional (and incremental) investments into the system.

As the *National Electric Reliability Study* [6] points out, "it is neither possible nor desirable to avoid all component failures or combinations of component failures that result in service interruptions. The level of reliability can be considered to be 'appropriate' when the cost of avoiding additional interruptions exceeds the consequences of those interruptions to consumers. Thus the appropriate level of reliability from the consumer perspective may be defined as that level of reliability when the sum of the supply costs plus the cost of interruptions which occur are at a minimum". Figure 11-4 illustrates this theoretical concept. Note that the system's reliability improvement and investment are not linearly related, and that the

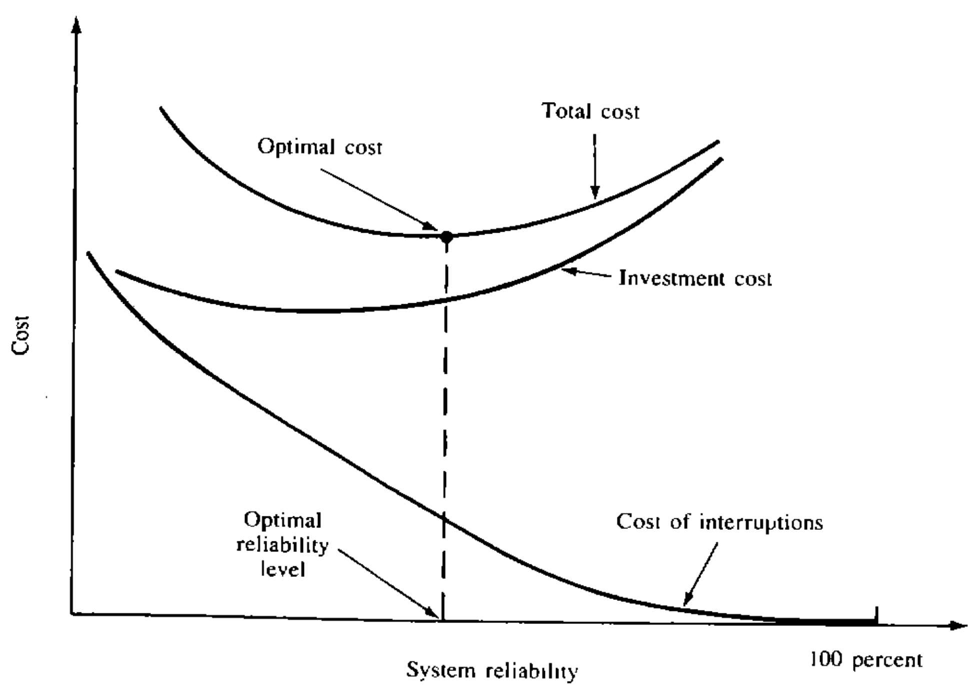


Figure 11-4 Cost vs. system reliability.

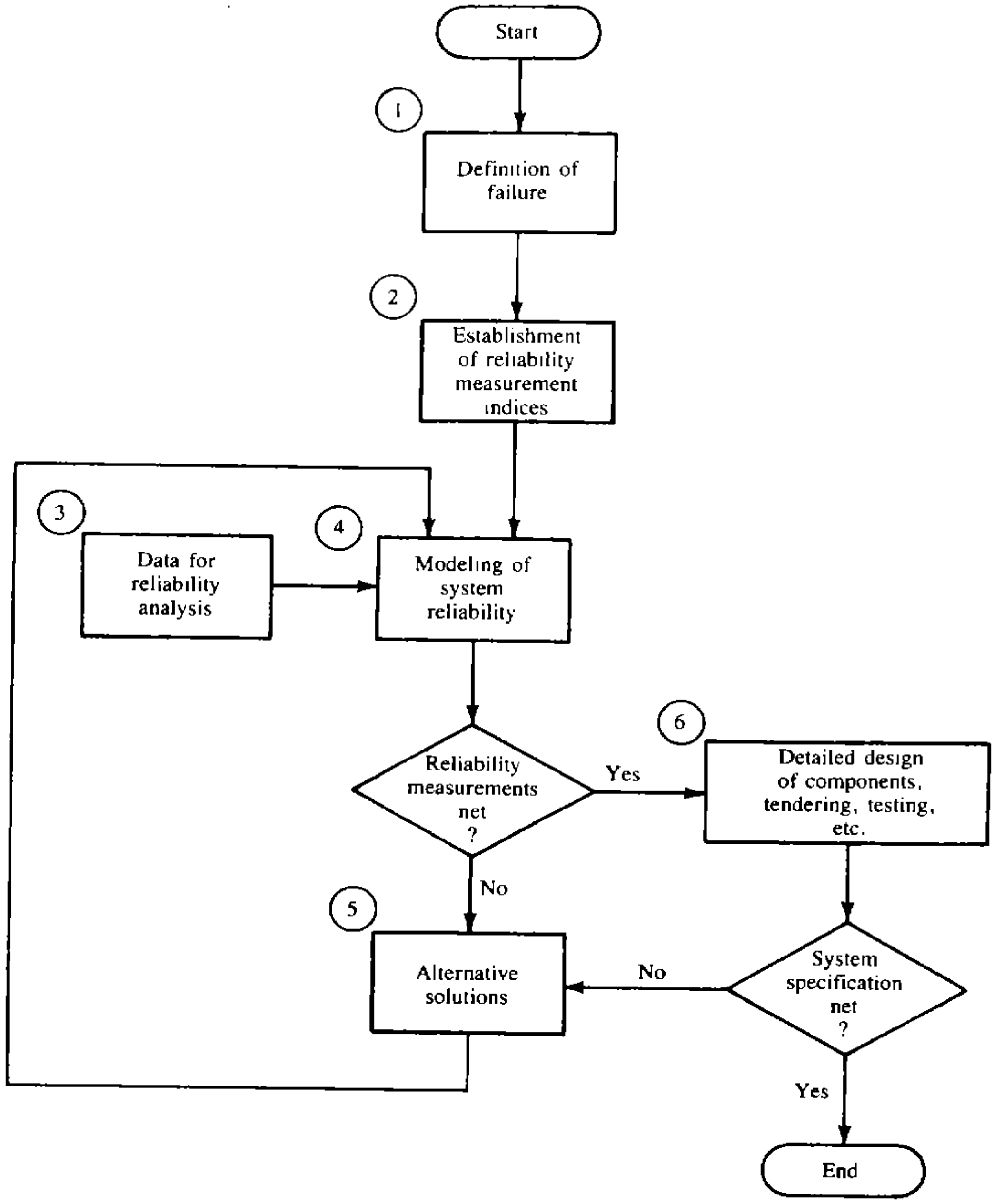


Figure 11-5 A reliability planning procedure. (*From* [5, 9].)

optimal (or appropriate) reliability level of the system corresponds to the optimal cost, i.e., the minimum total cost. However, Billinton [8] points out that "the most improper parameter is perhaps not the actual level of reliability though this cannot be ignored but the incremental reliability cost. What is the increase in reliability per dollar invested? Where should the next dollar be placed within the system to achieve the maximum reliability benefit?"

In general, other than "for possible sectionalizing or reconfiguration to minimize either the number of customers affected by an equipment failure or the interruption duration, the only operating option available to the utility to enhance reliability is to minimize the duration of the interruption by the timely repair of the failed equipment(s)" [6].

Experience indicates that most distribution system service interruptions are the result of damage from natural elements, such as lightning, wind, rain, ice, and animals. Other interruptions are attributable to defective materials, equipment failures, and human actions such as vehicles hitting poles, cranes contacting overhead wires, felling of trees, vandalism, and excavation equipment damaging buried cable or apparatus. Some of the most damaging and extensive service interruptions on distribution systems result from snow or ice storms that cause breaking of overhanging trees which in turn damage distribution circuits. Hurricanes also cause widespread damage, and tornadoes are even more intensely destructive, though usually very localized. In such severe cases, restoration of service is hindered by the conditions causing the damage, and most utilities do not have a sufficient number of crews with mobile and mechanized equipment to quickly restore all service when a large geographic area is involved.

The coordination of preventive maintenance scheduling with reliability analysis can be very effective. Most utilities design their systems to a specific contingency level, e.g., single contingency, so that, due to existing sufficient redundancy and switching alternatives, the failure of a single component will not cause any customer outages. Therefore, contingency analysis helps to determine the weakest spots of the distribution system. The special form of contingency analysis in which the probability of a given contingency is clearly and precisely expressed is known as the *risk analysis*. The risk analysis is performed only for important segments of the system and/or customers. The resultant information is used in determining whether to build the system to a specific contingency level or to risk a service interruption. Figure 11-5 shows the flowchart of a reliability planning procedure.

11-4 BASIC RELIABILITY CONCEPTS AND MATHEMATICS

Endrenyi [2] gives the classical definition of *reliability* as "the probability of a device or system performing its function adequately, for the period of time intended, under the operating conditions intended." In this sense, not only the probability of failure but also its magnitude, duration, and frequency are important.

11-4-1 The General Reliability Function

It is possible to define the probability of failure of a given component (or system) as a function of time as

$$P(T \leq t) = F(t) \qquad t \geq 0 \tag{11-1}$$

where T = a random variable representing the failure time

$F(t)$ = probability that component will fail by time t

Here, $F(t)$ is the failure distribution function which is also known as the *unreliability function*. Therefore, the probability that the component will not fail in performing its intended function at a given time t is defined as the *reliability of the component*. Thus the *reliability function* can be expressed as

$$R(t) = 1 - F(t)$$
$$= P(T > t) \tag{11-2}$$

where $R(t)$ = reliability function
$F(t)$ = unreliability function

Note that the $R(t)$ reliability function represents the probability that the component will survive at time t.

On the other hand, if the time-to-failure random variable T has a density function $f(t)$, from Eq. (11-2),

$$R(t) = 1 - F(t)$$
$$= 1 - \int_0^t f(t)\,dt$$
$$= \int_t^\infty f(t)\,dt \tag{11-3}$$

Therefore the probability of failure of a given system in a particular time interval (t_1, t_2) can be given either in terms of the unreliability function, as

$$\int_{t_1}^{t_2} f(t)\,dt = \int_{-\infty}^{t_2} f(t)\,dt - \int_{-\infty}^{t_1} f(t)\,dt$$
$$= F(T_2) - F(t_1) \tag{11-4}$$

or in terms of the reliability function, as

$$\int_{t_1}^{t_2} f(t)\,dt = \int_{t_1}^{\infty} f(t)\,dt - \int_{t_2}^{\infty} f(t)\,dt$$
$$= R(t_1) - R(t_2) \tag{11-5}$$

Here, the rate at which failures happen in a given time interval (t_1, t_2) is defined as the *hazard rate*, or *failure rate*, during that interval. It is the probability that a failure per unit time happens in the interval, provided that a failure has not happened before the time t_1, that is, at the beginning of the time interval. Therefore

$$h(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} \tag{11-6}$$

If the time interval is redefined so that

$$t_1 = t$$
$$t_2 = t + \Delta t$$

or

$$\Delta t = t_2 - t_1$$

then since the hazard rate is the instantaneous failure rate, it can be defined as

$$h(t) = \lim_{\Delta t \to 0} \frac{P\{\text{ a component of age } t \text{ will fail in } \Delta t \mid \text{it has survived up to } t\}}{\Delta t} \tag{11-7}$$

or

$$\begin{aligned} h(t) &= \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t \cdot R(t)} \\ &= \frac{1}{R(t)} \left[-\frac{d}{dt} R(t) \right] \\ &= \frac{f(t)}{R(t)} \end{aligned} \tag{11-8}$$

where $f(t)$ = probability density function

$$= -\frac{dR(t)}{dt}$$

Also, by substituting Eq. (11-3) into Eq. (11-8),

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{11-9}$$

Therefore

$$h(t)\,dt = \frac{dF(t)}{1 - F(t)} \tag{11-10}$$

or

$$\int_0^t h(t)\,dt = -\ln\left[1 - F(t)\right] \int_0^t \tag{11-11}$$

Hence

$$\ln \frac{1 - F(t)}{1 - F(0)} = -\int_0^t h(t)\,dt \tag{11-12}$$

or

$$1 - F(t) = \exp\left[-\int_0^t h(t)\,dt\right] \tag{11-13}$$

Taking derivatives of Eq. (11-13) or substituting Eq. (11-13) into Eq. (11-9),

$$f(t) = h(t) \exp\left[-\int_0^t h(t)\,dt\right] \tag{11-14}$$

Also, substituting Eq. (11-3) into Eq. (11-13),

$$R(t) = \exp\left[-\int_0^t h(t)\,dt\right] \qquad (11\text{-}15)$$

where

$$\exp[\;] = e^{[\;]} \qquad (11\text{-}16)$$

Let

$$\lambda(t) = h(t) \qquad (11\text{-}17)$$

hence Eq. (11-16) becomes

$$R(t) = \exp\left[-\int_0^t \lambda(t)\,dt\right] \qquad (11\text{-}18)$$

Equation (11-18) is known as the *general reliability function*. Note that in Eq. (11-18) both the reliability function and the hazard (or failure) rate are functions of time.

Assume that the hazard or failure function is independent of time, that is,

$$h(t) = \lambda \qquad \text{failures/unit time}$$

From Eq. (11-14), the failure density function is

$$f(t) = \lambda e^{-\lambda t} \qquad (11\text{-}19)$$

Therefore, from Eq. (11-8), the reliability function can be expressed as

$$R(t) = \frac{f(t)}{h(t)}$$

$$= e^{-\lambda t} \qquad (11\text{-}20)$$

which is independent of time. Thus a constant failure rate causes the time-to-failure random variable to be an exponential density function.

Figure 11-6 shows a typical hazard function known as the *bathtub curve*. The curve illustrates that the failure rate is a function of time. The first period represents the *infant mortality* period, which is the period of decreasing failure rate. This initial period is also known as the *debugging period*, *break-in period*, *burn-in period*, or *early life period*. In general, during this period, failures occur due to design or manufacturing errors

The second period is known as the *useful life period*, or *normal operating period*. The failure rates of this period are constant, and the failures are known as *chance failures*, *random failures*, or *catastrophic failures* since they occur randomly and unpredictably.

The third period is known as the *wear-out period*. Here, the hazard rate increases as equipment deteriorates because of aging or wear as the components approach their "rated lives." Of course, if the time t_2 could be predicted with certainty, then equipment could be replaced before this wear-out phase begins.

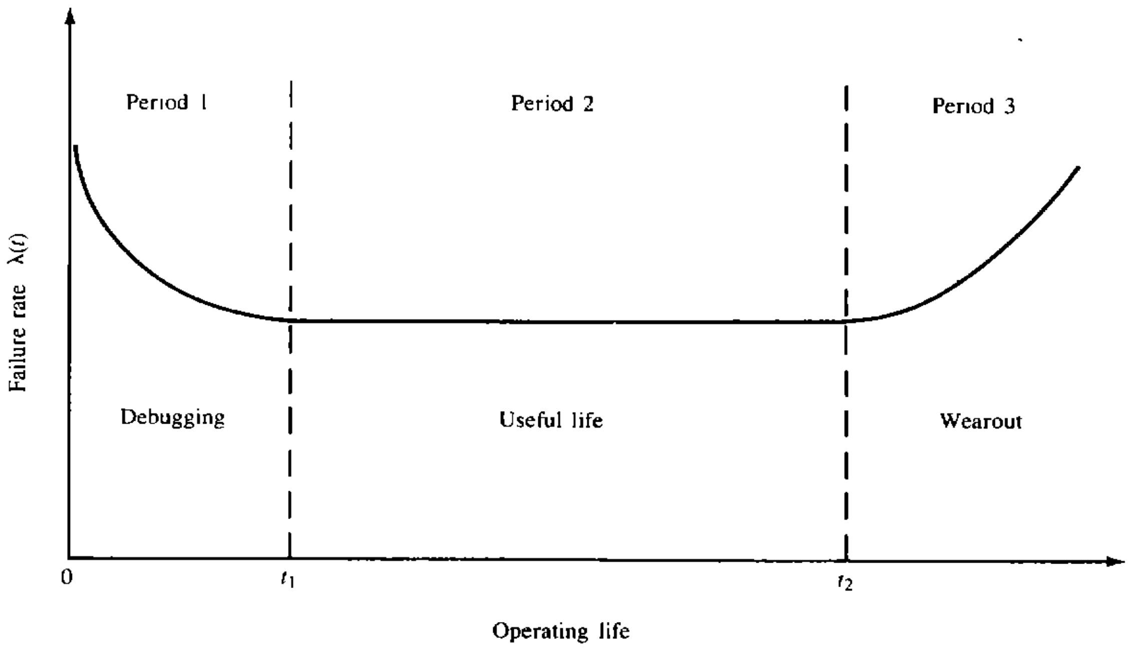


Figure 11-6 The *bathtub* hazard function.

In summary, since the probability density function is given as

$$f(t) = -\frac{dR(t)}{dt} \tag{11-21}$$

it can be shown that

$$f(t)\,dt = -dR(t) \tag{11-22}$$

and by integrating Eq. (11-22),

$$\begin{aligned}\int_0^t f(t)\,dt &= -\int_1^{R(t)} dR(t)\\ &= -[R(t) - 1]\\ &= 1 - R(t)\end{aligned} \tag{11-23}$$

However,

$$\int_0^t f(t)\,dt + \int_t^\infty f(t)\,dt = \int_0^\infty f(t)\,dt \triangleq 1 \tag{11-24}$$

From Eq. (11-24),

$$\int_0^t f(t)\,dt = 1 - \int_t^\infty f(t)\,dt \tag{11-25}$$

Therefore, from Eqs. (11-23) and (11-25), reliability can be expressed as

$$R(t) = \int_t^\infty f(t)\,dt \tag{11-26}$$

However,

$$R(t) + Q(t) \triangleq 1 \tag{11-27}$$

Thus the unreliability can be expressed as

$$\begin{aligned} Q(t) &= 1 - R(t) \\ &= 1 - \int_t^\infty f(t)\, dt \\ &= \int_0^t f(t)\, dt \end{aligned} \tag{11-28}$$

Therefore the relationship between reliability and unreliability can be illustrated graphically, as shown in Fig. 11-7.

11-4-2 Basic Single-Component Concepts

Theoretically, the expected life, i.e., the expected time during which a component will survive and perform successfully, can be expressed as

$$E(T) = \int_0^\infty t f(t)\, dt \tag{11-29}$$

Substituting Eq. (11-21) into Eq. (11-29),

$$E(T) = -\int_0^\infty t \frac{dR(t)}{dt}\, dt \tag{11-30}$$

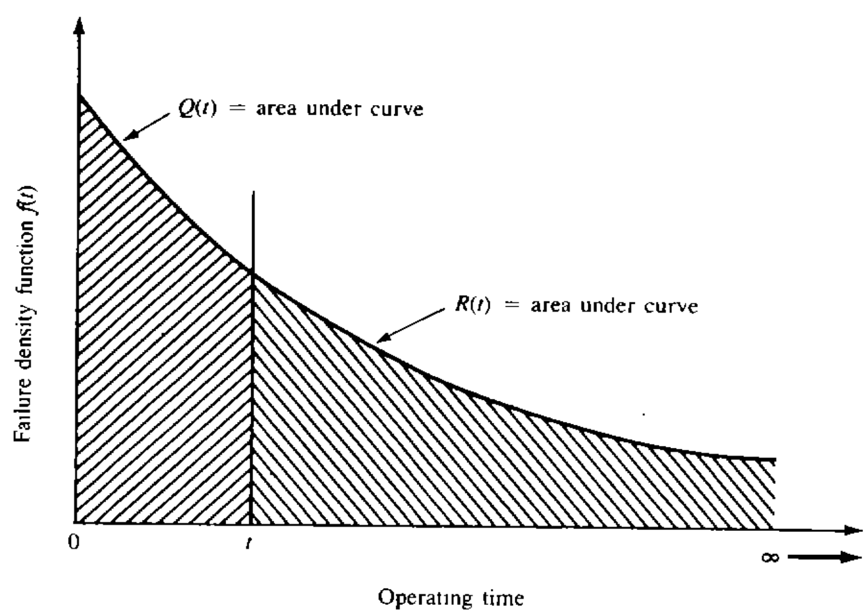


Figure 11-7 Relationship between reliability and unreliability.

Integrating by parts,

$$E(T) = -tR(t)\Big|_0^{\infty} + \int_0^{\infty} R(t)\,dt \tag{11-31}$$

since

$$R(t = 0) = 1 \tag{11-32}$$

and

$$R(t = \infty) = 0 \tag{11-33}$$

the first term of Eq. (11-31) equals zero, and therefore the expected life can be expressed as

$$E(T) = \int_0^{\infty} R(t)\,dt \tag{11-34a}$$

or

$$E(T) = \int_0^{\infty} \left\{ \exp\left[- \int_0^{t} \lambda(t)\,dt \right] \right\} dt \tag{11-34b}$$

Of course, the special case of useful life can be expressed, when there is a constant failure rate, by substituting Eq. (11-20) into Eq. (11-34a), as

$$E(T) = \int_0^{\infty} e^{-\lambda t}\,dt = \frac{1}{\lambda} \tag{11-35}$$

Note that if the system in question is not renewed through maintenance and repairs but simply replaced by a good system, then the $E(T)$ useful life is also defined as the *mean time to failure* and denoted as

$$\text{MTTF} = \bar{m} = \frac{1}{\lambda} \tag{11-36}$$

where λ = constant failure rate

Similarly, if the system in question is renewed through maintenance and repairs, then the $E(T)$ useful life is also defined as the *mean time between failures* and denoted as

$$\text{MTBF} = \bar{T} = \bar{m} + \bar{r} \tag{11-37}$$

where $\bar{T}$ = mean cycle time
$\bar{m}$ = mean time to failure
$\bar{r}$ = mean time to repair

Note that the *mean time to repair* is defined as the reciprocal of the average (or mean) repair rate and denoted as

$$\text{MTTR} = \bar{r} = \frac{1}{\mu} \tag{11-38}$$

where μ = mean repair rate

Consider the two-state model shown in Fig. 11-8*a*. Assume that the system is either in the up (or in) state or in the down (or out) state at a given time, as shown in Fig. 11-8*b*. Therefore the mean time to failure can be reasonably estimated as

$$\text{MTTF} = \bar{m} = \frac{\sum_{i=1}^{n} m_i}{n} \tag{11-39}$$

where $\bar{m}$ = mean time to failure
m_i = observed time to failure for ith cycle
n = total number of cycles

Similarly, the mean time to repair can be reasonably estimated as

$$\text{MTTR} = \bar{r} = \frac{\sum_{i=1}^{n} r_i}{n} \tag{11-40}$$

where $\bar{r}$ = mean time to repair
r_i = observed time to repair for ith cycle
n = total number of cycles

Therefore Eq. (11-37) can be reexpressed as

$$\text{MTBF} = \text{MTTF} + \text{MTTR} \tag{11-41}$$

The assumption that the behaviors of a repaired system and a new system are identical from a failure standpoint constitutes the base for much of renewal theory. In general, however, perfect renewal is not possible, and in such cases, terms such as the *mean time to the first failure* or the *mean time to the second failure* become appropriate.

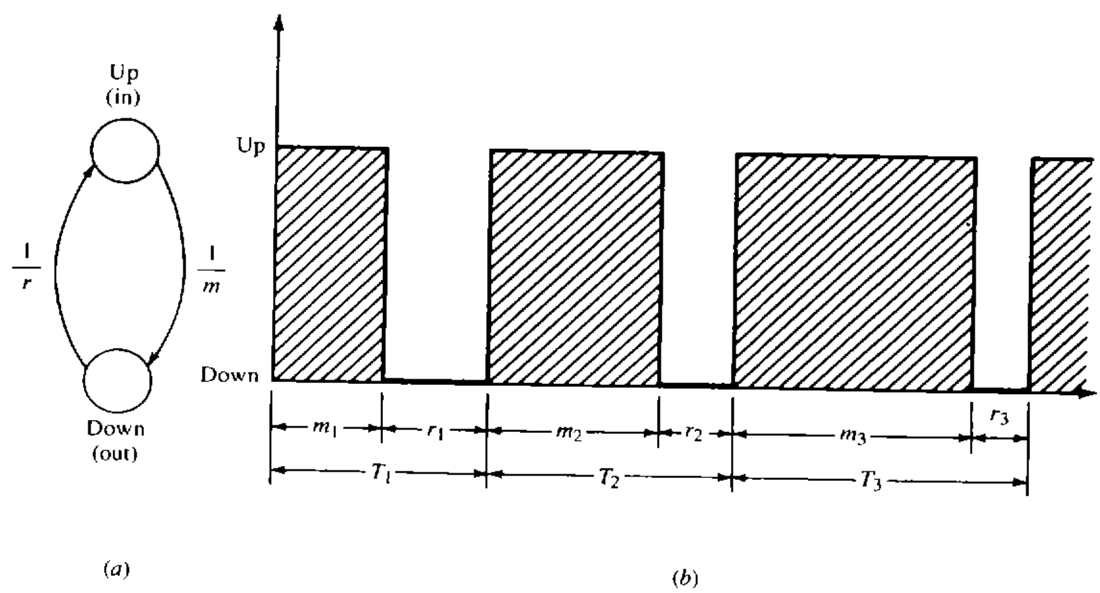


Figure 11-8 Two-state model.

Note that the term *mean cycle time* defines the average time that it takes for the component to complete one cycle of operation, i.e., failure, repair and restart. Therefore

$$\bar{T} = \bar{m} + \bar{r} \tag{11-42}$$

Substituting Eqs. (11-36) and (11-38) into Eq. (11-42),

$$\bar{T} = \frac{1}{\lambda} + \frac{1}{\mu}$$

$$= \frac{\lambda + \mu}{\lambda\mu} \tag{11-43}$$

The reciprocal of the mean cycle time is defined as the *mean failure frequency* and denoted as

$$\bar{f} = \frac{1}{\bar{T}}$$

$$= \frac{\lambda\mu}{\lambda + \mu} \tag{11-44}$$

When the states of a given component, over a period to time, can be characterized by the two-state model, as shown in Fig. 11-8, then it can be assumed that the component is either up (i.e., available for service) or down (i.e., unavailable for service). Therefore it can be shown that

$$A + U = 1 \tag{11-45}$$

where A = availability of component, i.e., the fraction of time component is up
$U = \bar{A}$ = unavailability of component, i.e., the fraction of time component is down

Therefore, on the average, as time t goes to infinity, it can be shown that the availability is

$$A \triangleq \frac{\bar{m}}{\bar{T}} = \frac{\text{MTTF}}{\text{MTBF}} \tag{11-46}$$

or

$$A = \frac{\bar{m}}{\bar{m} + \bar{r}} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \tag{11-47}$$

or

$$A = \frac{\mu}{\mu + \lambda} \tag{11-48}$$

Thus the unavailability can be expressed as

$$U \triangleq 1 - A \tag{11-49}$$

Substituting Eq. (11-46) into Eq. (11-49),

$$
\begin{aligned}
U &= 1 - \frac{\bar{m}}{\bar{T}} \\
&= \frac{\bar{T} - \bar{m}}{\bar{T}} \\
&= \frac{(\bar{m} + \bar{r}) - \bar{m}}{\bar{T}} \\
&= \frac{\bar{r}}{\bar{T}} = \frac{\text{MTTR}}{\text{MTBF}} \qquad (11\text{-}50)
\end{aligned}
$$

or

$$U = \frac{\bar{r}}{\bar{r} + \bar{m}} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}} \qquad (11\text{-}51)$$

or

$$U = \frac{\lambda}{\lambda + \mu} \qquad (11\text{-}52)$$

Consider Eq. (11-47) for a given system's availability, that is,

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \qquad (11\text{-}47)$$

when the total number of components involved in the system is quite large and

$$\text{MTTF} \gg \text{MTTR}$$

then the division process becomes considerably tedious. However, it is possible to use an approximation form. Therefore, from Eq. (11-47),

$$\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = 1 - \frac{\text{MTTR}}{\text{MTTF}} + \cdots (-1)^n \frac{(\text{MTTR})^n}{(\text{MTTF})^n} \qquad (11\text{-}53)$$

or

$$\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = \sum_{n=0}^{\infty} (-1)^n \frac{(\text{MTTR})^n}{(\text{MTTF})^n} \qquad (11\text{-}54)$$

or, approximately,

$$\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \cong 1 - \frac{\text{MTTR}}{\text{MTTF}} \qquad (11\text{-}55)$$

It is somewhat unfortunate, but it has become customary in certain applications, e.g., nuclear power plant reliability studies, to employ the MTBF for both nonrepairable components and repairable equipment and systems. In any event, however, it represents the same statistical concept of the mean time at which failures occur. Therefore, using this concept, for example, the availability is

$$A = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{11-56}$$

and the unavailability is

$$U = \frac{\text{MTTR}}{\text{MTTR} + \text{MTBF}} \tag{11-57}$$

11-5 SERIES SYSTEMS

11-5-1 Unrepairable Components in Series

Figure 11-9 shows a block diagram for a series system which has two components connected in series. Assume that the two components are independent. Therefore, to have the system operate and perform its designated function, both components (or subsystems) must operate successfully. Thus

$$R_{\text{sys}} = P[E_1 \cap E_2] \tag{11-58}$$

and since it is assumed that the components are independent,

$$R_{\text{sys}} = P(E_1)P(E_2) \tag{11-59}$$

or

$$R_{\text{sys}} = R_1 \cdot R_2$$

or

$$R_{\text{sys}} = \prod_{i=1}^{2} R_i \tag{11-60}$$

where E_i = event that component i (or subsystem i) operates successfully
$R_i = P(E_i)$ = reliability of component i (or subsystem i)
R_{sys} = reliability of system (or system reliability index)

To generalize this concept, consider a series system with n independent components, as shown in Fig. 11-10. Therefore the system reliability can be expressed as

$$R_{\text{sys}} = P[E_1 \cap E_2 \cap E_3 \cap \cdots \cap E_n] \tag{11-61}$$

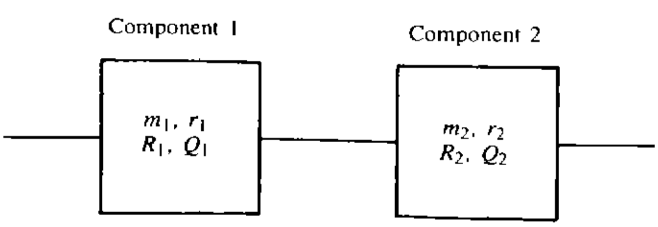


Figure 11-9 Block diagram of a series system with two components.

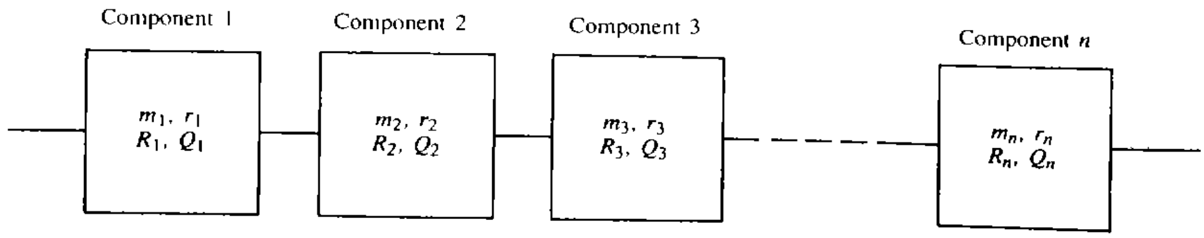


Figure 11-10 Block diagram of a series system with n components.

and since the n components are independent,

$$R_{sys} = P(E_1)P(E_2)P(E_3)\cdots P(E_n) \tag{11-62}$$

or

$$R_{sys} = R_1 \cdot R_2 \cdot R_3 \cdots R_n$$

or

$$R_{sys} = \prod_{i=1}^{n} R_i \tag{11-63}$$

Note that Eq. (11-63) is known as the *product rule* or the *chain rule of reliability.* System reliability will always be less than or equal to the least-reliable component, that is,

$$R_{sys} \le \min_i \{R_i\} \tag{11-64}$$

Therefore the system reliability, due to the characteristic of the series system, is the function of the number of series components and the component reliability level. Thus the reliability of a series system can be improved by (1) decreasing the number of series components or (2) increasing the component reliabilities. This concept has been illustrated in Fig. 11-11.

Assume that the probability that a component will fail is q and it is the same for all components for a given series system. Therefore the system reliability can be expressed as

$$R_{sys} = (1 - q)^n \tag{11-65}$$

or, according to the binomial theorem,

$$R_{sys} = 1 + n(-q)^1 + \frac{n(n-1)}{2}(-q)^2 + \cdots + (-q)^n \tag{11-66}$$

If the probability of the component failure q is small, an approximate form, for the system reliability, from Eq. (11-66), can be expressed as

$$R_{sys} \cong 1 - nq \tag{11-67}$$

where n = total number of components connected in series in the system
q = probability of component failure

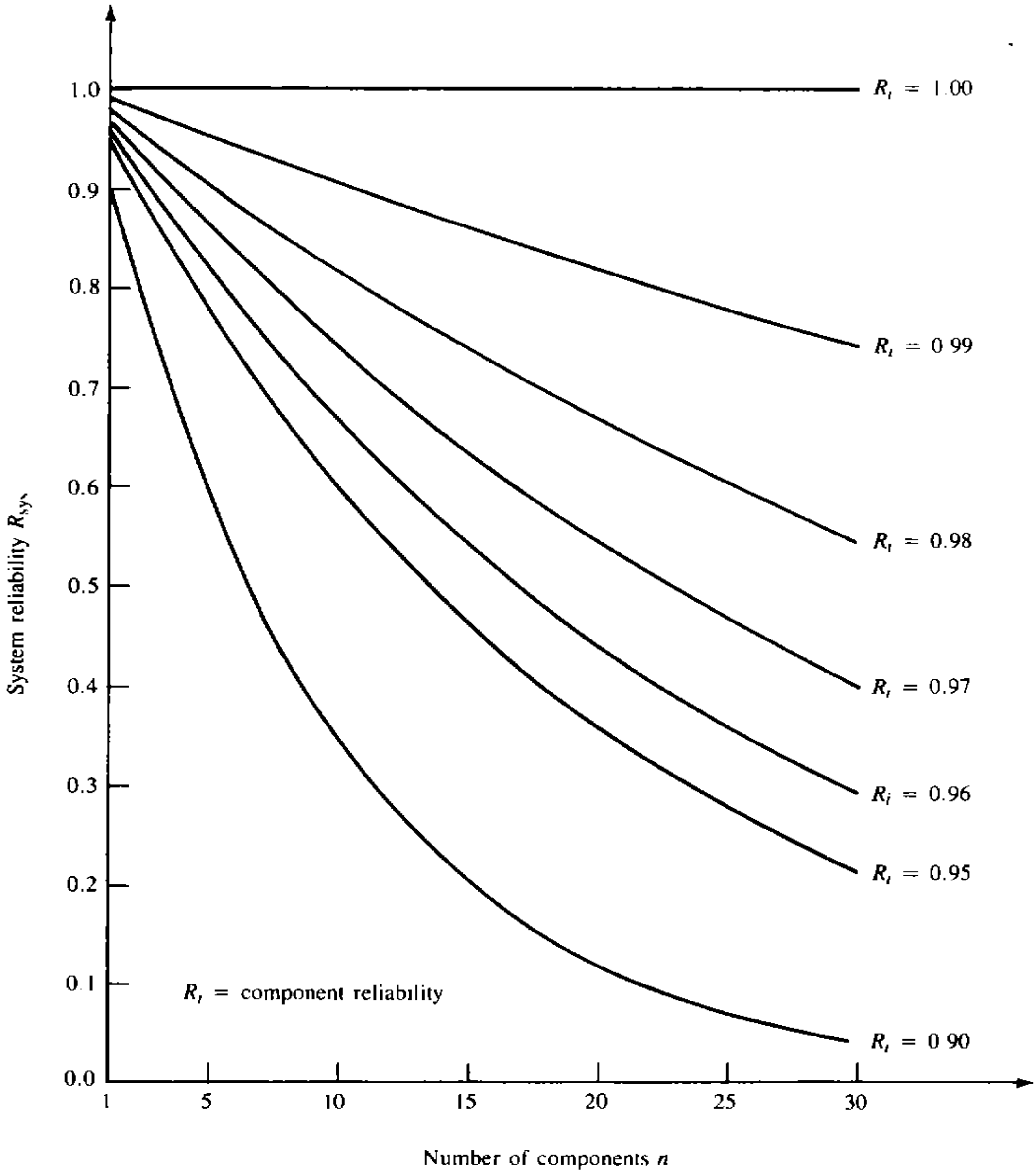


Figure 11-11 The reliability of a series system (structure) of n identical components.

On the other hand, if the probabilities of component failures, that is, q_i's, are different for each component, then the approximate form of the system reliability can be expressed as

$$R_{sys} \cong 1 - \sum_{i=1}^{n} q_i \tag{11-68}$$

Example 11-1 Assume that 15 identical components are going to be connected in series in a given system. If the minimum acceptable system reliability is 0.99, determine the approximate value of the component reliability.

SOLUTION From Eq. (11-67),

$$R_{sys} \cong 1 - nq$$

$$0.99 = 1 - 15(q)$$

and

$$q = 0.0007$$

Therefore the approximate value of the component reliability required to meet the particular system reliability can be found as

$$R_i \cong 0.9993$$

11-5-2 Repairable Components in Series†

Consider a series system with two components, as shown in Fig. 11-9. Assume that the components are independent and repairable. Therefore the availability or the steady-state probability of success (i.e., operation) of the system can be expressed as

$$A_{\text{sys}} = A_1 \cdot A_2 \tag{11-69}$$

where A_{sys} = availability of system
A_1 = availability of component 1
A_2 = availability of component 2

Since

$$A_1 = \frac{\overline{m}_1}{\overline{m}_1 + \bar{r}_1} \tag{11-70}$$

and

$$A_2 = \frac{\overline{m}_2}{\overline{m}_2 + \bar{r}_2} \tag{11-71}$$

substituting Eqs. (11-70) and (11-71) into Eq. (11-69) gives

$$A_{\text{sys}} = \frac{\overline{m}_1}{\overline{m}_1 + \bar{r}_1} \frac{\overline{m}_2}{\overline{m}_2 + \bar{r}_2} \tag{11-72}$$

or

$$A_{\text{sys}} = \frac{\overline{m}_{\text{sys}}}{\overline{m}_{\text{sys}} + \bar{r}_{\text{sys}}} \tag{11-73}$$

where $\overline{m}_1$ = mean time to failure of component 1
$\overline{m}_2$ = mean time to failure of component 2
$\overline{m}_{\text{sys}}$ = mean time to failure of system
$\bar{r}_1$ = mean time to repair of component 1
$\bar{r}_2$ = mean time to repair of component 2
$\bar{r}_{\text{sys}}$ = mean time to repair of system

† The technique presented in this section is primarily based on Ref. 10, by Billinton, Ringlee, and Wood.

The average frequency of the system failure is the sum of the average frequency of component 1 failing, given that component 2 is operable, plus the average frequency of component 2 failing while component 1 is operable. Thus

$$\bar{f}_{sys} = A_2 \cdot \bar{f}_1 + A_1 \cdot \bar{f}_2 \tag{11-74}$$

where $\bar{f}_{sys}$ = average frequency of system failure
$\bar{f}_i$ = average frequency of failure of component i
A_i = availability of component i

Since

$$\bar{f}_i = \frac{1}{\bar{m}_i + \bar{r}_i} \tag{11-75}$$

and

$$A_i = \frac{\bar{m}_i}{\bar{m}_i + \bar{r}_i} \tag{11-76}$$

substituting Eqs. (11-75) and (11-76) into Eq. (11-74) gives

$$\bar{f}_{sys} = \frac{1}{\bar{m}_1 + \bar{r}_1} \frac{\bar{m}_2}{\bar{m}_2 + \bar{r}_2} + \frac{1}{\bar{m}_2 + \bar{r}_2} \frac{\bar{m}_1}{\bar{m}_1 + \bar{r}_1} \tag{11-77}$$

Note that Eq. (11-73) can be expressed as

$$A_{sys} = \bar{m}_{sys} \cdot \bar{f}_{sys} \tag{11-78}$$

Thus the mean time to failure for a given series system with two components can be expressed as

$$\bar{m}_s = \frac{1}{1/\bar{m}_1 + 1/\bar{m}_2} \tag{11-79}$$

Hence the mean time to failure of a given series system with n components can be expressed as

$$\bar{m}_s = \frac{1}{1/\bar{m}_1 + 1/\bar{m}_2 + 1/\bar{m}_3 + \cdots + 1/\bar{m}_n} \tag{11-80}$$

Since the reciprocal of the mean time to failure is defined as the failure rate, for the two-component system,

$$\lambda_{sys} = \lambda_1 + \lambda_2 \tag{11-81}$$

and for the n-component system,

$$\lambda_{sys} = \lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n \tag{11-82}$$

Similarly, it can be shown that the mean time to repair for the given two-component series system is

$$\bar{r}_{sys} = \frac{\lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2 + (\lambda_1 \bar{r}_1)(\lambda_2 \bar{r}_2)}{\lambda_{sys}} \tag{11-83}$$

or, approximately,†

$$\bar{r}_{sys} = \frac{\lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2}{\lambda_{sys}} \tag{11-84}$$

Therefore the mean time to repair for an n-component series system is

$$\bar{r}_{sys} \cong \frac{\lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2 + \lambda_3 \bar{r}_3 + \cdots + \lambda_n \bar{r}_n}{\lambda_{sys}} \tag{11-85}$$

11-6 PARALLEL SYSTEMS

11-6-1 Unrepairable Components in Parallel

Figure 11-12 shows a block diagram for a system which has two components connected in parallel. Assume that the two components are independent. Therefore to have the system fail and not be able to perform its designated function, both components must fail simultaneously. Thus the system unreliability is

$$Q_{sys} = P[\bar{E}_1 \cap \bar{E}_2] \tag{11-86}$$

and since it is assumed that the components are independent,

$$Q_{sys} = P(\bar{E}_1)P(\bar{E}_2) \tag{11-87}$$

or

$$Q_{sys} = \prod_{i=1}^{2} (1 - R_i) \tag{11-88}$$

where $\bar{E}_i$ = event that component i fails
$Q_i = P(\bar{E}_i)$ = unreliability of component i
Q_{sys} = unreliability of system (or system unreliability index)

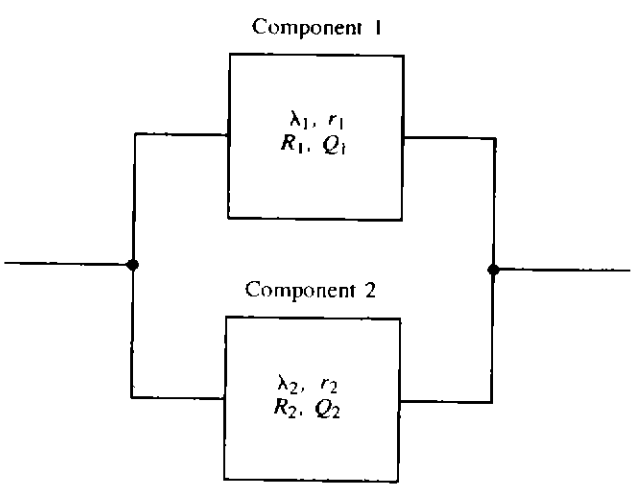


Figure 11-12 Block diagram of a parallel system with two components.

† Note that Eq. (11-84) gives an exact value if there is a dependency between the components; that is, one component must not fail while the other component is on repair.

Then the system reliability is given by the complementary probability as

$$R_{sys} = 1 - \prod_{i=1}^{2}(1 - R_i) \tag{11-89}$$

for this two-unit redundant system.

To generalize this concept, consider a parallel system with m independent components, as shown in Fig. 11-13. Therefore the system unreliability can be expressed as

$$Q_{sys} = P[\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \cdots \cap \bar{E}_m] \tag{11-90}$$

and since the m components are independent,

$$Q_{sys} = P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3) \cdots P(\bar{E}_m) \tag{11-91}$$

or

$$Q_{sys} = Q_1 \cdot Q_2 \cdot Q_3 \cdots Q_m \tag{11-92}$$

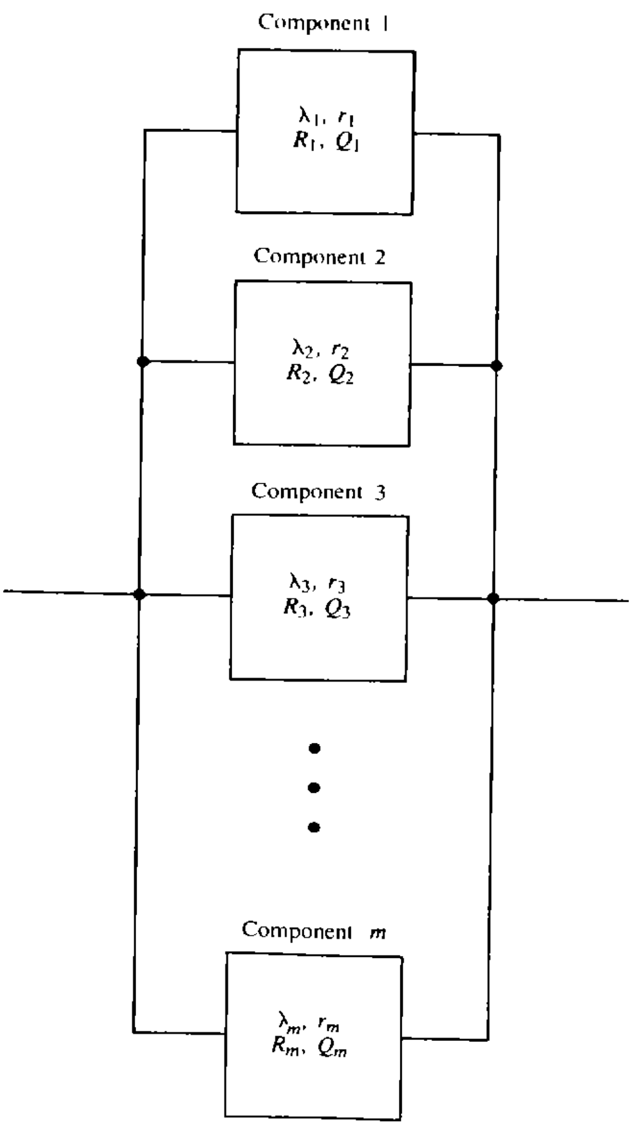


Figure 11-13 Block diagram of a parallel system with m components.

Therefore the system reliability is

$$
\begin{aligned}
R_{\text{sys}} &= 1 - Q_{\text{sys}} \\
&= 1 - [Q_1 \cdot Q_2 \cdot Q_3 \cdots Q_m] \\
&= 1 - [(1 - R_1)(1 - R_2)(1 - R_3) \cdots (1 - R_m)] \\
&= 1 - \prod_{i=1}^{m} Q_i \\
&= 1 - \prod_{i=1}^{m} (1 - R_i)
\end{aligned}
\tag{11-93}
$$

Note that there is an implied assumption that all units are operating simultaneously and that failures do not influence the reliability of the surviving subsystems.

The instantaneous failure rate of a parallel system is a variable function of the operating time, even though the failure rates and mean times between failures of the particular components are constant. Therefore the system reliability is the joint function of the mean time between failures of each path and the number of parallel paths. As can be seen in Fig. 11-14, for a given component reliability the marginal gain in the system reliability due to the addition of parallel paths decreases rapidly. Thus the greatest gain in system reliability occurs when a second path is added to a single path. The reliability of a parallel system is not a simple exponential but a sum of exponentials. Therefore the system reliability for a two-component parallel system is

$$
\begin{aligned}
R_{\text{sys}}(t) &= 1 - (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t}) \\
&= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}
\end{aligned}
\tag{11-94}
$$

where λ_1 = failure rate of component 1
λ_2 = failure rate of component 2

11-6-2 Repairable Components in Parallel†

Consider a parallel system with two components as shown in Fig. 11-12. Assume that the components are independent and repairable. Therefore the unavailability or the steady-state probability of failure of the system can be expressed as

$$U_{\text{sys}} = U_1 \cdot U_2 \tag{11-95}$$

where U_{sys} = unavailability of system
U_1 = unavailability of component 1
U_2 = unavailability of component 2

† The technique presented in this section is primarily based on Ref. 10, by Billinton, Ringlee, and Wood.

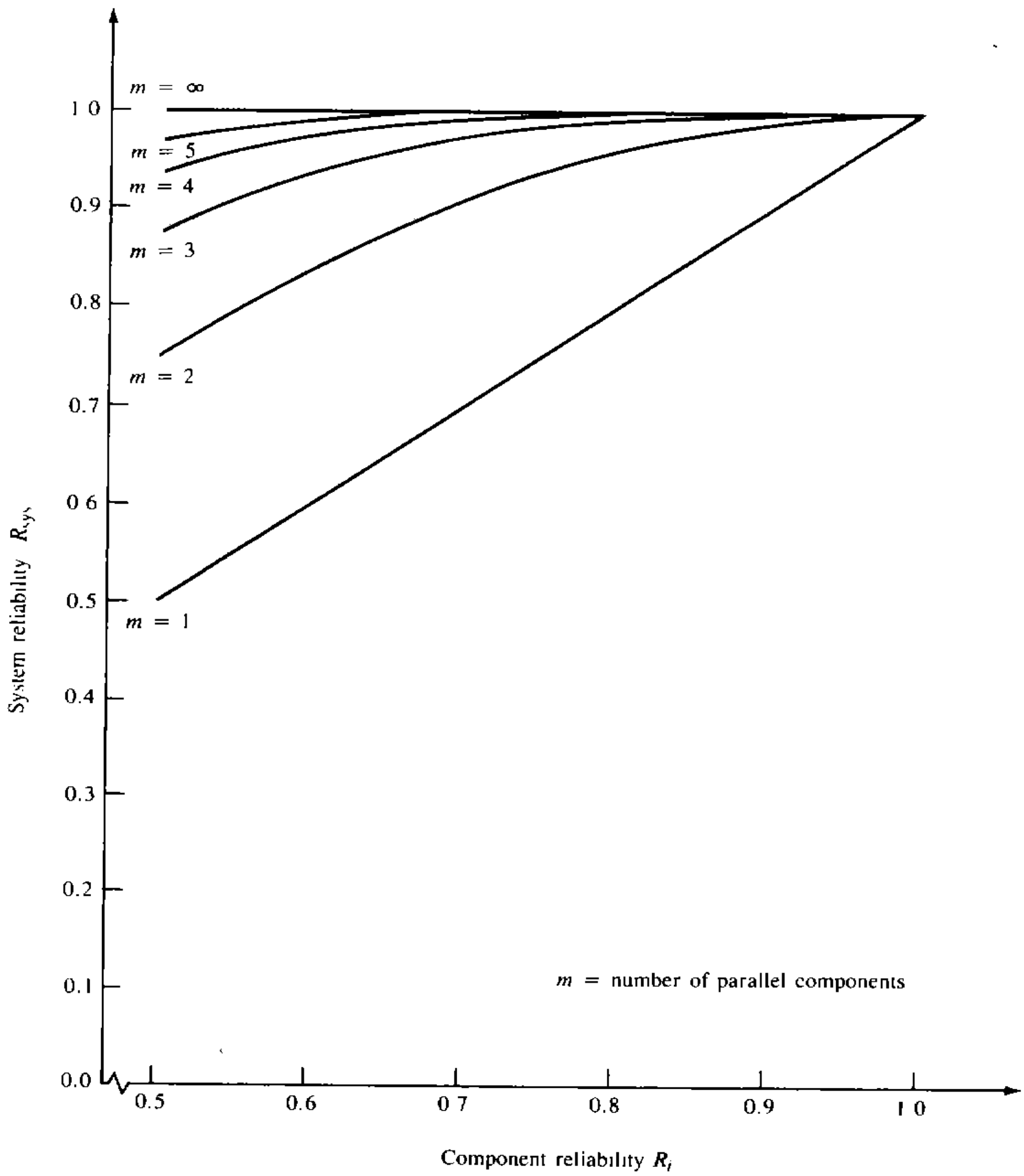


Figure 11-14 The reliability of a parallel system (structure) of n parallel components.

Since

$$U_1 = 1 - A_1$$

$$= \frac{\lambda_1 \bar{r}_1}{1 + \lambda_1 \bar{r}_1} \tag{11-96}$$

and

$$U_2 = 1 - A_2$$

$$= \frac{\lambda_2 \bar{r}_2}{1 + \lambda_2 \bar{r}_2} \tag{11-97}$$

substituting Eqs. (11-96) and (11-97) into Eq. (11-95) gives

$$U_{sys} = \frac{\lambda_1 \bar{r}_1}{1 + \lambda_1 \bar{r}_1} \frac{\lambda_2 \bar{r}_2}{1 + \lambda_2 \bar{r}_2} \tag{11-98}$$

However, the average frequency of the system failure is

$$\bar{f}_{sys} = U_2 \bar{f}_1 + U_1 \bar{f}_2 \tag{11-99}$$

where $\bar{f}_{sys}$ = average frequency of system failure
$\bar{f}_i$ = average frequency of failure of component i
U_i = unavailability of component i

Since

$$\bar{f}_1 = \frac{\lambda_1}{1 + \lambda_1 \bar{r}_1} \tag{11-100}$$

and

$$\bar{f}_2 = \frac{\lambda_2}{1 + \lambda_2 \bar{r}_2} \tag{11-101}$$

substituting equation sets (11-96), (11-97) and (11-100), (11-101) into Eq. (11-99) and simplifying gives

$$\bar{f}_{sys} = \frac{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)}{(1 + \lambda_1 \bar{r}_1)(1 + \lambda_2 \bar{r}_2)} \tag{11-102}$$

From Eq. (11-50), the system unavailability can be expressed as

$$U_{sys} \triangleq \frac{\bar{r}_{sys}}{\bar{T}_{sys}} \tag{11-103}$$

or

$$U_{sys} = \bar{r}_{sys} \cdot \bar{f}_{sys} \tag{11-104}$$

so that

$$\bar{r}_{sys} = \frac{U_{sys}}{\bar{f}_{sys}} \tag{11-105}$$

Therefore, substituting Eqs. (11-98) and (11-102) into Eq. (11-105), the average repair time (or downtime) of the two-component parallel system can be expressed as†

$$\bar{r}_{sys} = \frac{\bar{r}_1 \cdot \bar{r}_2}{\bar{r}_1 + \bar{r}_2} \tag{11-106}$$

† Notice the analogy between the total repair time and total (or equivalent) resistance value of a parallel connection of two resistors.

or

$$\frac{1}{\bar{r}_{\text{sys}}} = \frac{1}{\bar{r}_1} + \frac{1}{\bar{r}_2} \tag{11-107}$$

Similarly, from Eq. (11-51), the system unavailability can be expressed as

$$U_{\text{sys}} \triangleq \frac{\bar{r}_{\text{sys}}}{\bar{r}_{\text{sys}} + \bar{m}_{\text{sys}}} \tag{11-108}$$

from which

$$\bar{m}_{\text{sys}} = \frac{\bar{r}_{\text{sys}}(1 - U_{\text{sys}})}{U_{\text{sys}}} \tag{11-109}$$

Substituting Eqs. (11-98) and (11-106) into Eq. (11-109), the average time to failure (or operation time, or uptime) of the parallel system can be expressed as

$$\bar{m}_{\text{sys}} = \frac{1 + \lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2}{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)} \tag{11-110}$$

Of course, the failure rate of the parallel system is

$$\lambda_{\text{sys}} \triangleq \frac{1}{\bar{m}_{\text{sys}}} \tag{11-111}$$

or

$$\lambda_{\text{sys}} = \frac{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)}{1 + \lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2} \tag{11-112}$$

When more than two identical units are in parallel and/or when the system is not purely redundant, i.e., parallel, the probabilities of the states or modes of the system can be calculated by using the binomial distribution or conditional probabilities.

Example 11-2 Figure 11-15 shows a 4-mi-long distribution express feeder which is used to provide electric energy to a load center located in the downtown area of Ghost City from the Ghost River Substation. Approximately 1 mi of the feeder has been built underground due to aesthetic considerations in the vicinity of the downtown area, while the rest of the feeder is overhead. The underground feeder has two termination points. On the average, two faults per circuit-mile for the overhead section and one fault per circuit-mile for the underground section of the feeder have been recorded in the last 10 years.

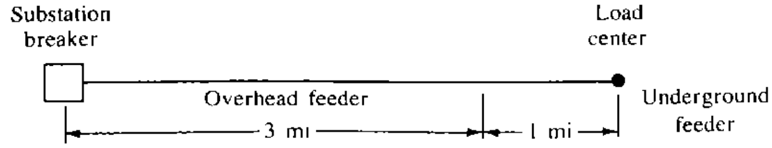


Figure 11-15

The annual cable termination fault rate is given as 0.3 percent per cable termination. Furthermore, based on past experience, it is known that, on the average, the repair times for the overhead section, underground section, and each cable termination are 3, 28, and 3 h, respectively. Using the given information, determine the following:

(*a*) Total annual fault rate of the feeder.
(*b*) Average annual fault restoration time of the feeder in hours.
(*c*) Unavailability of the feeder.
(*d*) Availability of the feeder.

SOLUTION

(*a*) Total annual fault rate of the feeder is

$$\lambda_{\text{FDR}} = \sum_{i=1}^{3} \lambda_i = \lambda_{\text{OH}} + \lambda_{\text{UG}} + 2\lambda_{\text{CT}}$$

where λ_{OH} = total annual fault rate of overhead section of feeder
λ_{UG} = total annual fault rate of underground section of feeder
λ_{CT} = total annual fault rate of cable terminations

Therefore

$$\begin{aligned}\lambda_{\text{FDR}} &= 3\left(\tfrac{2}{10}\right) + 1\left(\tfrac{1}{10}\right) + 2(0.003)\\ &= 0.706 \text{ faults/yr}\end{aligned}$$

(*b*) Average fault restoration time of the feeder per fault is

$$\bar{r}_{\text{FDR}} = \sum_{i=1}^{3} \bar{r}_i = \bar{r}_{\text{OH}} + \bar{r}_{\text{UG}} + 2\bar{r}_{\text{CT}}$$

where $\bar{r}_{\text{OH}}$ = average repair time for overhead section of feeder, h
$\bar{r}_{\text{UG}}$ = average repair time for underground section of feeder, h
$\bar{r}_{\text{CT}}$ = average repair time per cable termination, h

Thus

$$\begin{aligned}\bar{r}_{\text{FDR}} &= 3 + 28 + 2(3)\\ &= 37\ h\end{aligned}$$

However, the average annual fault restoration time of the feeder is

$$\bar{r}_{\text{FDR}} = \frac{\sum_{i=1}^{3} \lambda_i \times \bar{r}_i}{\sum_{i=1}^{3} \lambda_i}$$

or

$$\bar{r}_{FDR} = \frac{(l_{OH} \times \lambda_{OH})(\bar{r}_{OH}) + (l_{UG} \times \lambda_{UG}(\bar{r}_{UG}) + (2\lambda_{CT})(\bar{r}_{CT})}{\lambda_{FDR}}$$

$$= \frac{(3 \times 0.2)(3) + (1 \times 0.1)(28) + (2 \times 0.003)(3)}{0.706}$$

$$= \frac{4.618}{0.706}$$

$$= 6.54 \text{ h}$$

(*c*) Unavailability of the feeder is

$$U_{FDR} = \frac{\bar{r}_{FDR}}{\bar{r}_{FDR} + \bar{m}_{FDR}}$$

where

$$\begin{aligned}\bar{m}_{FDR} &= \text{annual mean time to failure}\\ &= 8760 - \bar{r}_{FDR}\\ &= 8760 - 6.54\\ &= 8753.46 \text{ h/yr}\end{aligned}$$

Therefore

$$\begin{aligned}U_{FDR} &= \frac{6.54}{6.54 + 8753.46}\\ &= 0.0007 \qquad \text{or} \qquad 0.07\%\end{aligned}$$

(*d*) Availability of the feeder is

$$\begin{aligned}A_{FDR} &= 1 - U_{FDR}\\ &= 1 - 0.0007\\ &= 0.9993 \qquad \text{or} \qquad 99.93\%\end{aligned}$$

Example 11-3 Assume that the primary main feeder shown in Fig. 11-16 is manually sectionalized and that presently only the first three feeder sections exist and serve customers *A*, *B*, and *C*. The annual average fault rates for primary main and laterals are 0.08 and 0.2 fault/(circuit · mile), respectively. The average repair times for each primary main section and for each primary lateral are 3.5 and 1.5 h, respectively. The average time for manual sectionalizing of each feeder section is 0.75 h. Assume that at the time of having one of the feeder sections in fault, the other feeder section(s) are sectionalized manually

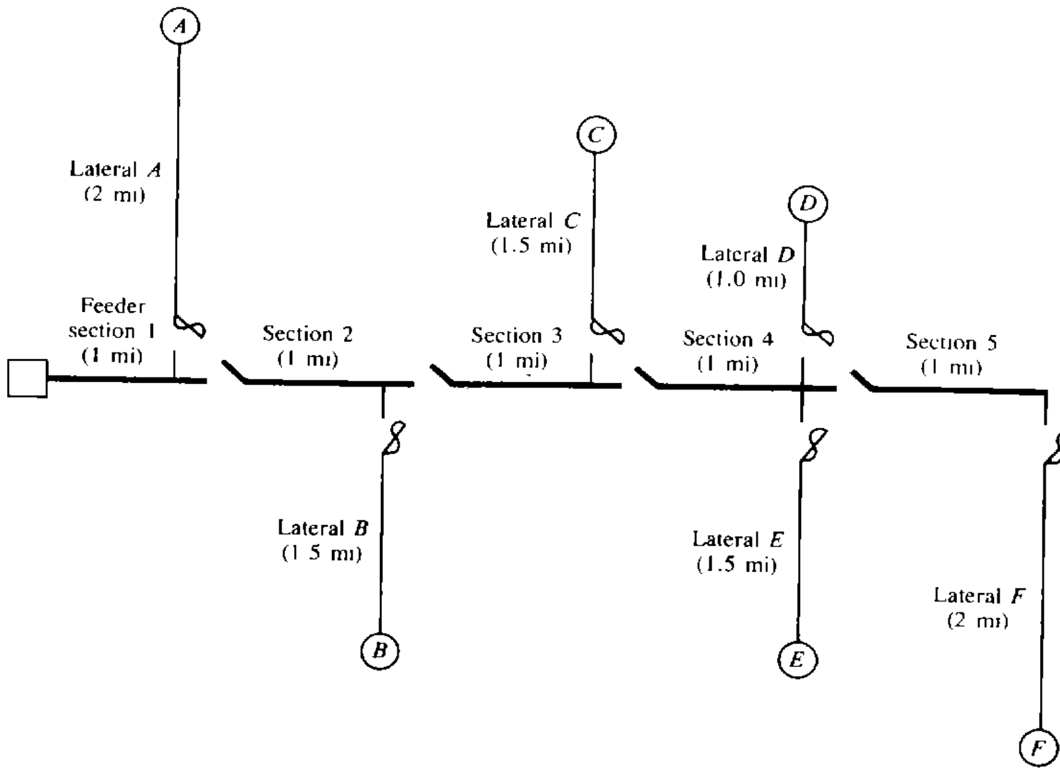


Figure 11-16

as long as they are not in the mainstream of the fault current, i.e., not in between the faulted section and the circuit breaker. Otherwise, they have to be repaired also. Based on the given information, prepare an interruption analysis study for the first contingency only, i.e., ignore the possibility of simultaneous outages, and determine the following:

(*a*) The total annual sustained interruption rates for customers A, B and C.

(*b*) The average annual repair times, i.e., downtimes, for customers A, B and C.

SOLUTION

(*a*) Total annual sustained interruption rates for customers A, B and C are

$$\lambda_A = \sum_{i=1}^{4} \lambda_i = \lambda_{\text{sec. 1}} + \lambda_{\text{sec. 2}} + \lambda_{\text{sec. 3}} + \lambda_{\text{lat. } A}$$

$$= (1 \text{ mi})(0.08) + (1 \text{ mi})(0.08) + (1 \text{ mi})(0.08) + (2 \text{ mi})(0.2)$$

$$= 0.64 \text{ fault/yr}$$

$$\lambda_B = \sum_{i=1}^{4} \lambda_i = \lambda_{\text{sec. 1}} + \lambda_{\text{sec. 2}} + \lambda_{\text{sec. 3}} + \lambda_{\text{lat. } B}$$

$$= (1 \text{ mi})(0.08) + (1 \text{ mi})(0.08) + (1 \text{ mi})(0.08) + (1.5 \text{ mi})(0.2)$$

$$= 0.54 \text{ fault/yr}$$

$$\lambda_C = \lambda_B = 0.54 \text{ fault/yr}$$

(*b*) Average annual repair time, i.e., downtime (or restoration time), for customer A is

$$\bar{r}_A = \frac{\lambda_{\text{sec. 1}} \times \bar{r}_{\text{fault}} + \lambda_{\text{sec. 2}} \times \bar{r}_{\text{MS}} + \lambda_{\text{sec. 3}} \times \bar{r}_{\text{MS}} + \lambda_{\text{lat. }A} \times \bar{r}_{\text{lat. fault}}}{\lambda_A}$$

where $\bar{r}_A$ = average repair time for customer A
$\lambda_{\text{sec. }i}$ = total fault rate for feeder section i per year
$\bar{r}_{\text{fault}}$ = average repair time for faulted primary main section
$\bar{r}_{\text{lat. fault}}$ = average repair time for faulted primary lateral
$\bar{r}_{\text{MS}}$ = average time for manual sectionalizing per section
$\lambda_{\text{lat. }A}$ = total fault rate for lateral A per year

therefore

$$\bar{r}_A = \frac{(0.08)(3.5) + (0.08)(0.75) + (0.08)(0.75) + (2 \times 0.2)(1.5)}{0.64}$$

$$= \frac{1.00}{0.64}$$

$$= 1.56 \text{ h}$$

Similarly, for customer B,

$$\bar{r}_B = \frac{\lambda_{\text{sec. 1}} \times \bar{r}_{\text{fault}} + \lambda_{\text{sec. 2}} \times \bar{r}_{\text{fault}} + \lambda_{\text{sec. 3}} \times \bar{r}_{\text{MS}} + \lambda_{\text{lat. }B} \times \bar{r}_{\text{lat. fault}}}{\lambda_B}$$

$$= \frac{(0.08)(3.5) + (0.08)(3.5) + (0.08)(0.75) + (1.5 \times 0.2)(1.5)}{0.54}$$

$$= \frac{1.07}{0.54}$$

$$= 1.98 \text{ h}$$

and for customer C,

$$\bar{r}_C = \frac{\lambda_{\text{sec. 1}} \times \bar{r}_{\text{fault}} + \lambda_{\text{sec. 2}} \times \bar{r}_{\text{fault}} + \lambda_{\text{sec. 3}} \times \bar{r}_{\text{fault}} + \lambda_{\text{lat. }C} \times \bar{r}_{\text{la . fault}}}{\lambda_C}$$

$$= \frac{(0.08)(3.5) + (0.08)(3.5) + (0.08)(3.5) + (1.5 \times 0.2)(1.5)}{0.54}$$

$$= \frac{1.29}{0.54}$$

$$= 2.39 \text{ h}$$

11-7 SERIES AND PARALLEL COMBINATIONS

Simple combinations of series and parallel subsystems (or components) can be analyzed by successively reducing subsystems into equivalent parallel or series components.

Figure 11-17 shows a parallel-series system which has a high-level redundancy. The equivalent reliability of the system with m parallel paths of n components each can be expressed as

$$R_{\text{sys}} = 1 - (1 - R^n)^m \tag{11-113}$$

where R_{sys} = equivalent reliability of system
R^n = equivalent reliability of a path
R = reliability of a component
n = total number of components in a path
m = total number of paths

Figure 11-18 shows a series-parallel system which has a low-level redundancy. The equivalent reliability of the system of n series units (or banks) with m parallel components in each unit (or bank) can be expressed as

$$R_{\text{sys}} = [1 - (1 - R)^m]^n \tag{11-114}$$

where R_{sys} = equivalent reliability of system
$1 - (1 - R)^m$ = equivalent reliability of a parallel unit (or bank)
R = reliability of a component
m = total number of components in a parallel unit (or bank)
n = total number of units (or banks)

The comparison of the two systems shows that the series-parallel configuration provides higher system reliability than the equivalent parallel-series configuration for a given system. Therefore it can be concluded that the lower the system level at which redundancy is applied, the larger the effective system reliability. Of course, the difference between parallel-series and series-parallel systems is not as pronounced if components have high reliabilities.

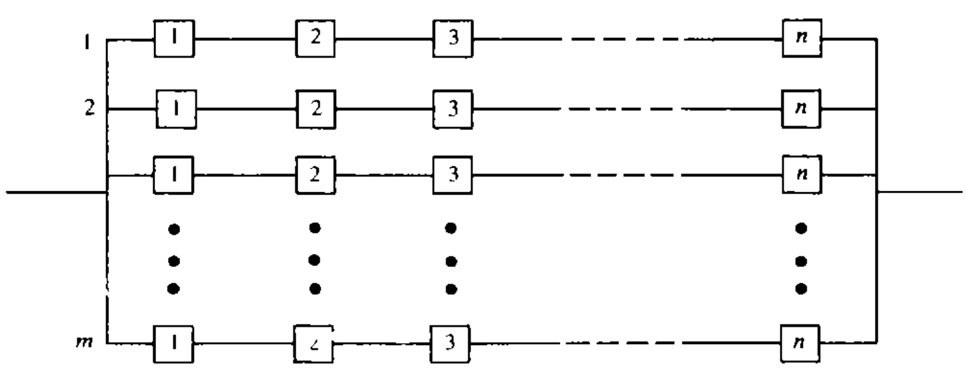


Figure 11-17 A parallel-series system.

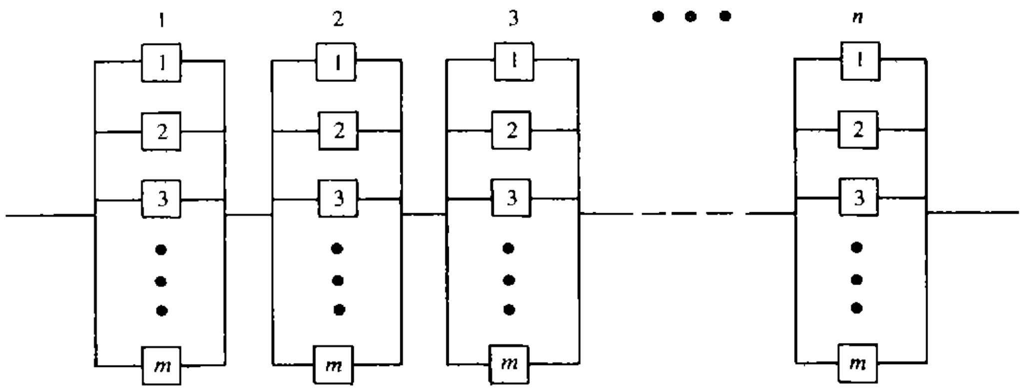


Figure 11-18 A series-parallel system.

Example 11-4 Consider the various combinations of the reliability block diagrams shown in Fig. 11-19. Assume that they are based on the logic diagrams of each subsystem and that the reliability of each component is 0.85. Determine the equivalent system reliability of each configuration.

SOLUTION

(*a*) From Eq. (11-63), the equivalent system reliability for the series system is

$$R_{eq} = R_{sys} = \prod_{i=1}^{4} R_i$$
$$= (0.85)^4$$
$$= 0.5220$$

(*b*) For the parallel-series system from Eq. (11-113),

$$R_{eq} = 1 - (1 - R^4)^2$$
$$= 1 - [1 - (0.85)^4]^2$$
$$= 0.7715$$

(c) For the mixed-parallel system,

$$R_{eq} = [1 - (1 - R^2)^2][1 - (1 - R^2)^2]$$
$$= [1 - (1 - 0.85^2)^2][1 - (1 - 0.85^2)^2]$$
$$= 0.8519$$

(*d*) For the mixed-parallel system,

$$R_{eq} = [1 - (1 - R)^2][1 - (1 - R^3)^2]$$
$$= [1 - (1 - 0.85)^2][1 - (1 - 0.85^3)^2]$$
$$= 0.8320$$

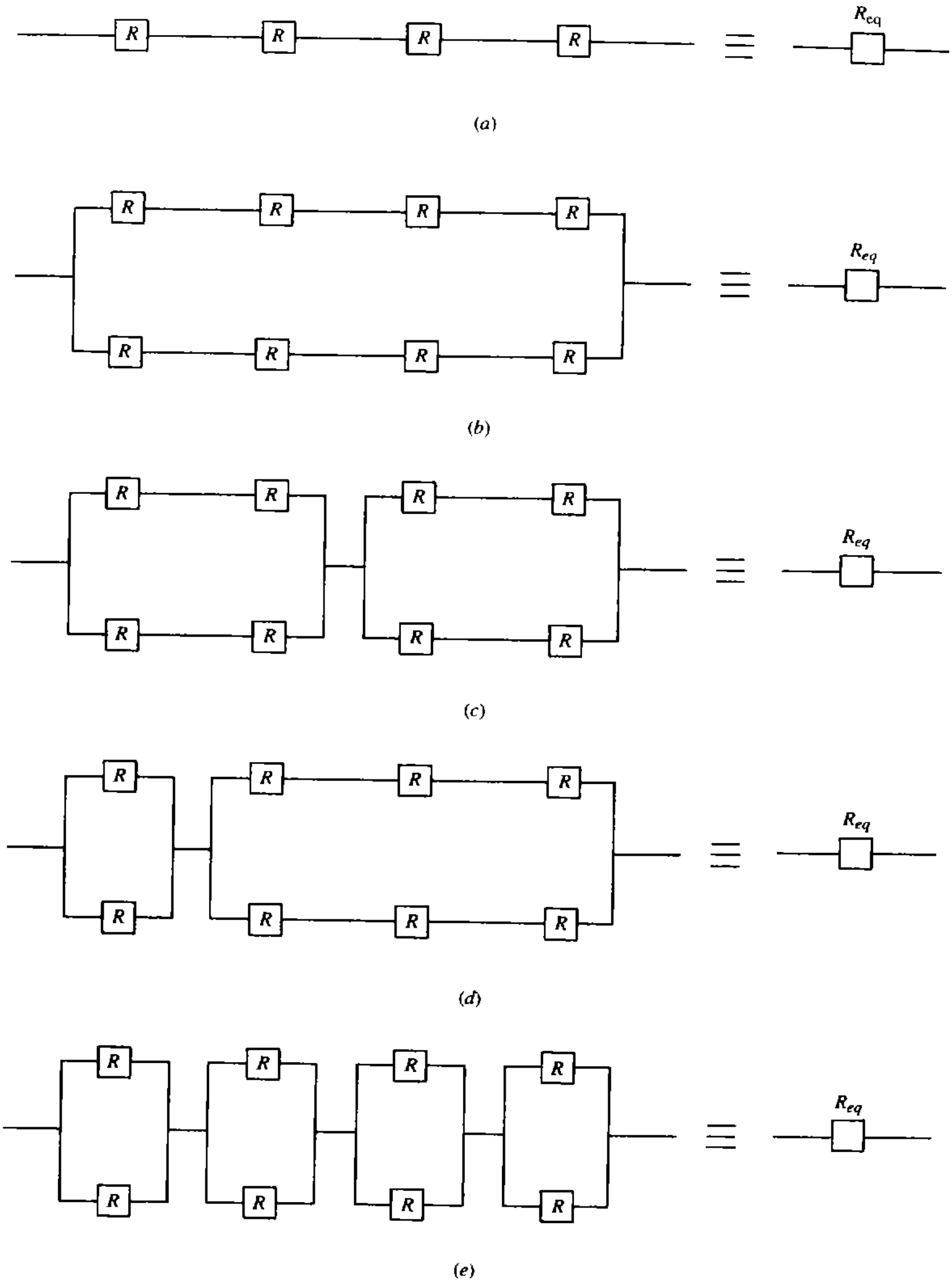


Figure 11-19 Various combinations of block diagrams: (*a*) series, (*b*) parallel-series, (*c*) mixed-parallel, (*d*) mixed-parallel, and (*e*) series-parallel.

(*e*) For the series-parallel system from Eq. (11-114),

$$\begin{aligned} R_{\text{eq}} &= [1 - (1 - R)^2]^4 \\ &= [1 - (1 - 0.85)^2]^4 \\ &= 0.9130 \end{aligned}$$

Example 11-5 Assume that a system has five components, namely, A, B, C, D, and E, as shown in Fig. 11-20, and that each component has different reliability as indicated in the figure. Determine the following:

(*a*) The equivalent system reliability.

(*b*) If the equivalent system reliability is desired to be at least 0.80, or 80 percent, design a system configuration to meet this system requirement by using each of the five components at least once.

SOLUTION

(*a*) From Eq. (11-63), the equivalent system reliability is

$$\begin{aligned} R_{\text{eq}} &= \prod_{i=1}^{5} R_i \\ &= (0.80)(0.95)(0.99)(0.90)(0.65) \\ &= 0.4402 \qquad \text{or} \qquad 44.02 \text{ percent} \end{aligned}$$

(*b*) In general, the best way of improving the overall system reliability is to back the less-reliable components by parallel components. Therefore, since the relatively less reliable components are A and E, they can be backed by parallel redundancy as shown in Fig. 11-21. Therefore the new equivalent system reliability becomes

$$\begin{aligned} R_{\text{sys}} &= \prod_{i=1}^{5} \text{R}_i \\ &= [1 - (1 - 0.80)^2](0.95)(0.99)(0.90)[1 - (1 - 0.65)^4] \\ &= 0.8004 \qquad \text{or} \qquad 80.04 \text{ percent} \end{aligned}$$

Example 11-6 Assume that a three-phase transformer bank consists of three single-phase transformers identified as A, B, and C for the sake of convenience. Assume that (1) transformer A is an old unit and therefore has a reliability of 0.90, (2) transformer B has been in operation for the last 20 years and therefore has been estimated to have a reliability of 0.95, and (3) transformer C is a brand-new one with a reliability of 0.99. Based on the given information and assumption of independence, determine the following:

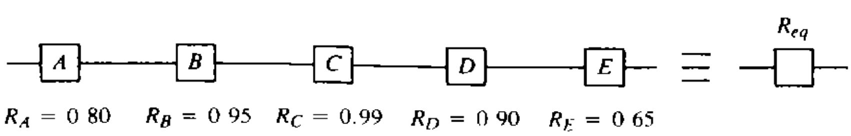


Figure 11-20 System configuration.

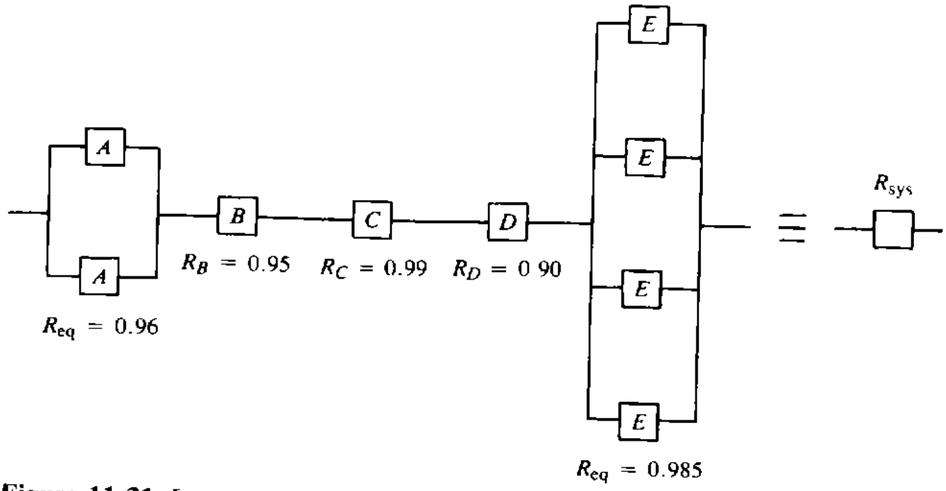


Figure 11-21 Imposed system configuration.

(*a*) The probability of having no failing transformer at any given time.
(*b*) If one out of the three transformers fails at any given time, what are the probabilities for that unit being the transformer A, or B, or C?
(*c*) If two out of the three transformers fail at any given time, what are the probabilities for those units being the transformers A and B, or B and C, or C and A?
(*d*) What is the probability of having all three transformers out of service at any given time?

SOLUTION

(*a*) The probability of having no failing transformer at any given time is

$$\begin{aligned} P[A \cap B \cap C] &= P(A)P(B)P(C) \\ &= (0.90)(0.95)(0.99) \\ &= 0.84645 \end{aligned}$$

(*b*) If one out of the three transformers fails at any given time, the probabilities for that unit being the transformer A, or B, or C are

$$\begin{aligned} P[\bar{A} \cap B \cap C] &= P(\bar{A})P(B)P(C) \\ &= (0.10)(0.95)(0.99) \\ &= 0.09405 \end{aligned}$$

$$\begin{aligned} P[A \cap \bar{B} \cap C] &= P(A)P(\bar{B})P(C) \\ &= (0.90)(0.05)(0.99) \\ &= 0.04455 \end{aligned}$$

$$\begin{aligned} P[A \cap B \cap \bar{C}] &= P(A)P(B)P(\bar{C}) \\ &= (0.90)(0.95)(0.01) \\ &= 0.00855 \end{aligned}$$

(c) If two out of the three transformers fail at any given time, the probabilities for those units being the transformers A and B, or B and C, or C and A are

$$P[\bar{A} \cap \bar{B} \cap C] = P(\bar{A})P(\bar{B})P(C)$$
$$= (0.10)(0.05)(0.99)$$
$$= 0.00495$$
$$P[A \cap \bar{B} \cap \bar{C}] = P(A)P(\bar{B})P(\bar{C})$$
$$= (0.90)(0.05)(0.01)$$
$$= 0.00045$$
$$P[\bar{A} \cap B \cap \bar{C}] = P(\bar{A})P(B)P(\bar{C})$$
$$= (0.10)(0.95)(0.01)$$
$$= 0.00095$$

(d) The probability of having all three transformers out of service at any given time† is

$$P[\bar{A} \cap \bar{B} \cap \bar{C}] = P(\bar{A})P(\bar{B})P(\bar{C})$$
$$= (0.10)(0.05)(0.01)$$
$$= 0.00005$$

Therefore the aforementioned reliability calculations can be summarized as given in Table 11-3.

Table 11-3

Number of failed transformers	System modes	Probability
0	$A \cap B \cap C$	0.84645
1	$\bar{A} \cap B \cap C$	0.09405
	$A \cap \bar{B} \cap C$	0.04455
	$A \cap B \cap \bar{C}$	0.00855
2	$\bar{A} \cap \bar{B} \cap C$	0.00495
	$A \cap \bar{B} \cap \bar{C}$	0.00045
	$\bar{A} \cap B \cap \bar{C}$	0.00095
3	$\bar{A} \cap \bar{B} \cap \bar{C}$	0.00005
		$\sum = 1.00000$

† Note that as time goes to infinity the reliability goes to zero by definition.

11-8 MARKOV PROCESSES‡

A *stochastic process*, $\{X(t); t \in T\}$, is a family of random variables such that for each t contained in the index set T, $X(t)$ is a random variable. Often T is taken to be the set of nonnegative integers.

In reliability studies, the variable t represents time, and $X(t)$ describes the *state* of the system at time t. The states at a given time t_n actually represent the (exhaustive and mutually exclusive) outcomes of the system at that time. Therefore the number of possible states may be finite or infinite. For instance, the Poisson distribution

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \qquad n = 0, 1, 2, \ldots \tag{11-115}$$

represents a stochastic process with an infinite number of states. If the system starts at time 0, the random variable n represents the number of occurrences between 0 and t. Therefore the states of the system at any time t are given by $n = 0, 1, 2 \ldots.$

A *Markov process* is a stochastic system for which the occurrence of a future state depends on the immediately preceding state and only on it. Because of this reason, the markovian process is characterized by a lack of memory. Therefore a discrete parameter stochastic process, $\{X(t); t = 0, 1, 2, \ldots\}$, or a continuous parameter stochastic process, $\{X(t); t \geq 0\}$, is a Markov process if it has the following *markovian property*:

$$\begin{aligned} P\{X(t_n) \geq x_n \mid X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{n-1}) = x_{n-1}\} \\ = P\{X(t_n) \leq x_n \mid X(t_{n-1}) = x_{n-1}\} \end{aligned} \tag{11-116}$$

for any set of n time points, $t_1 < t_2 < \cdots < t_n$ in the index set of the process, and any real numbers $x_1, x_2, \ldots, x_n$. The probability of

$$p_{x_{n-1}, x_n} = P\{x(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}\} \tag{11-117}$$

is called the *transition probability* and represents the *conditional probability* of the system being in x_n at t_n, given it was x_{n-1} at t_{n-1}. It is also called the *one-step* transition probability due to the fact that it represents the system between t_{n-1} and t_n. Of course, one can define a *k-step* transition probability as

$$p_{x_n, x_{n+k}} = P\{X(t_{n+k}) = x_{n+k} \mid X(t_n) = x_n\} \tag{11-118}$$

or as

$$p_{x_{n-k}, x_n} = P\{X(t_n) = x_n \mid X(t_{n-k}) = x_{n-k}\} \tag{11-119}$$

‡ The fundamental methodology given here was developed by the Russian mathematician A. A. Markov of the University of Kazan around the beginning of the twentieth century.

A *Markov chain* is defined by a sequence of discrete-valued random variables, $\{X(t_n)\}$, where t_n is discrete-valued or continuous. Therefore one can also define the Markov chain as the Markov process with a discrete state space. Define

$$p_{ij} = P\{X(t_n) = j \mid X(t_{n-1}) = i\} \tag{11-120}$$

as the *one-step transition probability* of going from state i at t_{n-1} to state j at t_n and assume that these probabilities do not change over time. The term used to describe this assumption is *stationarity*. If the transition probability depends only on the time difference, then the Markov chain is defined to be stationary in time. Therefore a Markov chain is completely defined by its transition probabilities, of going from state i to state j, given in a matrix form:

$$\mathbf{P} = \text{From state } i \left\{ \begin{array}{c|cccccc} & \multicolumn{6}{c}{\overbrace{\hspace{6cm}}^{\text{To state } j}} \\ & 0 & 1 & 2 & 3 & \cdots & n \\ 0 & p_{00} & p_{01} & p_{02} & p_{03} & \cdots & p_{0n} \\ 1 & p_{10} & p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ 2 & p_{20} & p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ 3 & p_{30} & p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n & p_{n0} & p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{array} \right. \tag{11-121}$$

The matrix **P** is called a *one-step transition matrix* (or *stochastic matrix*) since all the transition probabilities p_{ij}'s are fixed and independent of time. The matrix **P** is also called just the *transition matrix* when there is no possibility of confusion. Since the p_{ij}'s are conditional probabilities, they must satisfy the conditions

$$\sum_{j}^{n} p_{ij} = 1 \qquad \text{for all } i \tag{11-122}$$

and

$$p_{ij} \geq 0 \qquad \text{for all } ij \tag{11-123}$$

where $i = 0, 1, 2, \ldots, n$
$\quad\quad\; j = 0, 1, 2, \ldots, n$

Note that when the number of transitions (or states) is not too large, the information in a given transition matrix **P** can be represented by a transition diagram. The transition diagram is a pictorial map of the process in which states are represented by nodes and transitions by arrows. Here, the focus is not on time but on the structure of allowable transitions. The arrow from node i to node j is labeled as p_{ij}. Since row i of the matrix **P** corresponds to the set of arrows leaving node i, the sum of their probabilities must be equal to unity.

Assume that a given system has two states, namely, state 1 and state 2. For example, here, states 1 and 2 may represent the system being up and down, respectively. Therefore the associated transition probabilities can be defined as

p_{11} = probability of being in state 1 at time t, given that it was in state 1 at time zero

p_{22} = probability of being in state 2 at time t, given that it was in state 2 at time zero

p_{12} = probability of being in state 2 at time t, given that it was in state 1 at time zero

p_{21} = probability of being in state 1 at time t, given that it was in state 2 at time zero

Therefore the associated transition matrix can be expressed as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{11-124}$$

Figure 11-22*a* shows the associated transition diagram.

By the same token, if the given system has three states, its transition matrix can be expressed as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \tag{11-125}$$

and its transition diagram can be drawn as shown in Fig. 11-22*b*.

Example 11-7 Based on past history, a distribution engineer of the NL&NP Company has gathered the following information on the operation of the distribution transformers served by the Riverside Substation. The records

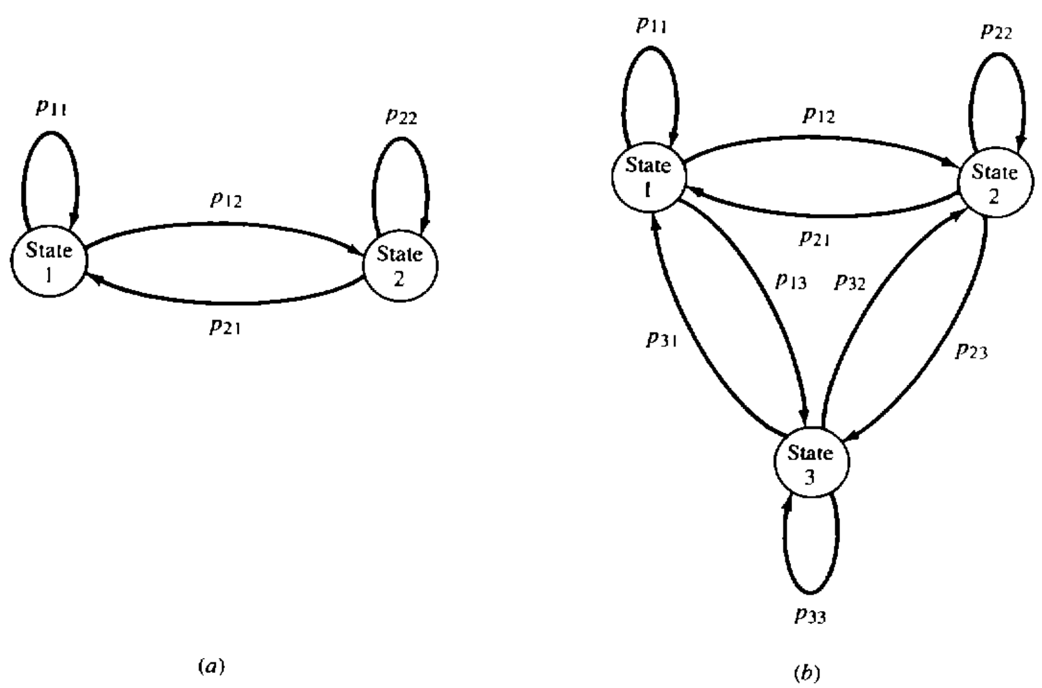


Figure 11-22 Transition system (*a*) for a two-state system and (*b*) for a three-state system.

indicate that only 2 percent of the transformers which are presently down and therefore being repaired now will be down and therefore will need repair next time. The records also show that 5 percent of those transformers which are currently up and therefore in service now will be down and therefore will need repair next time. Assuming that the process is discrete, markovian, and has stationary transition probabilities, determine the following:

(*a*) The conditional probabilities.
(*b*) The transition matrix.
(*c*) The transition diagram.

SOLUTION

(*a*) Let t and $t + 1$ represent the present time (i.e., now) and the next time, respectively. Therefore the associated conditional probabilities are

$$P\{X_{t+1} = \text{down} \mid X_t = \text{down}\} = 0.02$$
$$P\{X_{t+1} = \text{up} \mid X_t = \text{down}\} = 0.98$$
$$P\{X_{t+1} = \text{down} \mid X_t = \text{up}\} = 0.05$$
$$P\{X_{t+1} = \text{up} \mid X_t = \text{up}\} = 0.95$$

(*b*) Let numbers 1 and 2 represent the states of down and up, respectively. Therefore, from Eq. (11-120) and part *a*,

$$p_{11} = 0.02 \qquad p_{12} = 0.98$$
$$p_{21} = 0.05 \qquad p_{22} = 0.95$$

or, from Eq. (11-121),

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} \;1 & \;\;2 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \end{array}$$
$$= \begin{bmatrix} 0.02 & 0.98 \\ 0.05 & 0.95 \end{bmatrix}$$

(*c*) Therefore the transition diagram can be drawn as shown in Fig. 11-23.

Example 11-8 Assume that a distribution engineer of the NL&NP Company has studied the feeder outage statistics of the troublesome Riverside Substation and found out (1) that there is a markovian relationship between the feeder outages occurring at the present time and the next time and (2) that the relationship is a stationary one. Assume that the engineer has summarized the findings as shown in Table 11-4. For example, the table shows that if the presently outaged feeder is number 1, then the chances for the next outaged

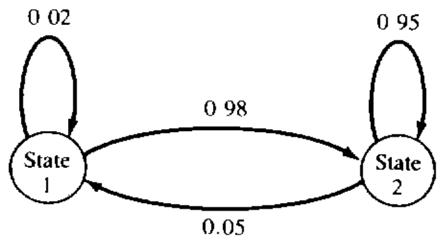


Figure 11-23 Transition diagram.

feeder being feeder 1, 2, or 3 are 40, 30, and 30 percent, respectively. Using the given data, determine the following:
(*a*) The conditional outage probabilities.
(*b*) The transition matrix.
(*c*) The transition diagram.

SOLUTION
(*a*) Let t and $t + 1$ represent the present time and the next time, respectively. Therefore the probability of the next outaged feeder being number 1, given it is number 1 now, can be expressed as

$$p_{11} = P\{X_{t+1} = 1 | X_t = 1\} = 0.40$$

where X_{t+1} = outaged feeder at next time
X_t = outaged feeder at present time
Similarly,

$$p_{12} = P\{X_{t+1} = 2 | X_t = 1\} = 0.30$$
$$p_{13} = P\{X_{t+1} = 3 | X_t = 1\} = 0.30$$
$$p_{21} = P\{X_{t+1} = 1 | X_t = 2\} = 0.20$$
$$p_{22} = P\{X_{t+1} = 2 | X_t = 2\} = 0.50$$
$$p_{23} = P\{X_{t+1} = 3 | X_t = 2\} = 0.30$$
$$p_{31} = P\{X_{t+1} = 1 | X_t = 3\} = 0.25$$
$$p_{32} = P\{X_{t+1} = 2 | X_t = 3\} = 0.25$$
$$p_{33} = P\{X_{t+1} = 3 | X_t = 3\} = 0.50$$

Table 11-4 Feeder outage data

Presently outaged feeder	Chances, in percent, for the next outaged feeder being		
	1	2	3
1	40	30	30
2	20	50	30
3	25	25	50

(*b*) Therefore the transition matrix is

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 0.40 & 0.30 & 0.30 \\ 0.20 & 0.50 & 0.30 \\ 0.25 & 0.25 & 0.50 \end{bmatrix}$$

(*c*) The associated transition diagram is shown in Fig. 11-24.

11-8-1 Chapman-Kolmogorov Equations

Assume that S_j represents the exhaustive and mutually exclusive states (outcomes) of a given system at any time, where $j = 0, 1, 9, \ldots$. Also assume that the system is markovian and that $p_j^{(0)}$ represents the absolute probability that the system is in state S_j at t_0. Therefore, if $p_j^{(0)}$ and the transition matrix $\mathbf{P}$ of a given Markov chain are known, one can easily determine the absolute probabilities of the system after n-step transitions. By definition, the one-step transition probabilities are

$$p_{ij} = p_{ij}^{(1)} = P\{X(t_1) = j \mid X(t_0) = i\} \tag{11-126}$$

Therefore the n-step transition probabilities can be defined by induction as

$$p_{ij}^{(n)} = P\{X(t_n) = j \mid X(t_0) = i\} \tag{11-127}$$

In other words, $p_{ij}^{(n)}$ is the probability (absolute probability) that the process is in state j at time t_n, given that it was in state i at time t_0. Of course, it can be observed from this definition that $p_{ij}^{(0)}$ must be 1 if $i = j$, and 0 otherwise.

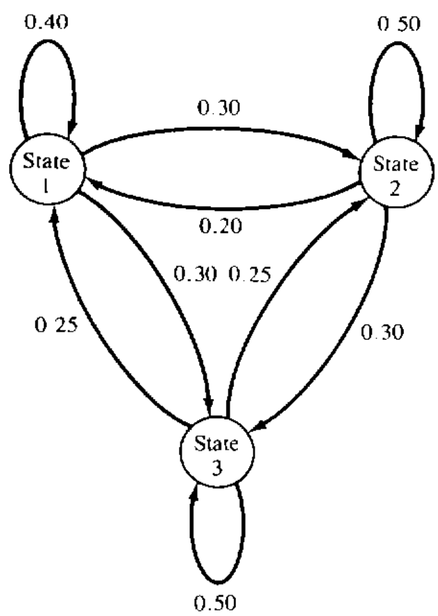


Figure 11-24 Transition diagram.

The Chapman-Kolmogorov equations provide a method for computing these n-step transition probabilities. In general form, these equations are given as

$$p_{ij}^{(n)} = \sum_k p_{ik}^{(n-m)} \cdot p_{kj}^{(m)} \qquad \forall_{ij} \tag{11-128}$$

for any m between zero and n. Note that Eq. (11-128) can be represented in matrix form by

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n-m)}\mathbf{P}^{(m)} \tag{11-129}$$

Therefore the elements of a higher-order transition matrix, that is, $\|p_{ij}^{(n)}\|$, can be obtained directly by matrix multiplication. Hence

$$\|p_{ij}^{(n)}\| = \mathbf{P}^{(n-m)}\mathbf{P}^{(m)} = \mathbf{P}^{(n)} = \mathbf{P}^n \tag{11-130}$$

Note that a special case of Eq. (11-128) is

$$p_{ij}^{(n)} = \sum_k p_{ik}^{(n-1)} \cdot p_{kj} \qquad \forall_{ij} \tag{11-131}$$

and therefore the special cases of Eqs. (11-129) and (11-130) are

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n-1)}\mathbf{P} \tag{11-132}$$

and

$$\|p_{ij}^{(n)}\| = \mathbf{P}^{(n-1)}\mathbf{P} = \mathbf{P}^{(n)} = \mathbf{P}^n \tag{11-133}$$

respectively.

The unconditional probabilities such as

$$p_j^{(n)} = P\{X(t_n) = j\} \tag{11-134}$$

are called the *absolute probabilities* or *state probabilities*. To determine the state probabilities, the initial conditions must be known. Therefore

$$\begin{aligned} p_j^{(n)} &= P\{X(t_n) = j\} \\ &= P\sum_i \{X(t_n) = j \mid X(t_0) = i\}P\{X(t_0) = i\} \\ &= \sum_i p_i^{(0)} p_{ij}^{(n)} \end{aligned} \tag{11-135}$$

Note that Eq. (11-135) can be represented in matrix form by

$$\mathbf{p}^{(n)} = \mathbf{p}^{(0)}\mathbf{P}^{(n)} \tag{11-136}$$

where $\mathbf{p}^{(n)}$ = vector of state probabilities at time t_n
$\mathbf{p}^{(0)}$ = vector of initial state probabilities at time t_0
$\mathbf{P}^{(n)}$ = n-step transition matrix

Of course, the state probabilities or absolute probabilities are defined in vector form as

$$\mathbf{p}^{(n)} = [p_1^{(n)} p_2^{(n)} p_3^{(n)} \cdots p_k^{(n)}] \tag{11-137}$$

and

$$\mathbf{p}^{(0)} = [p_1^{(0)} p_2^{(0)} p_3^{(0)} \cdots p_k^{(0)}] \tag{11-138}$$

Example 11-9 Consider a Markov chain, with two states, having the one-step transition matrix of

$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$$

and the initial state probability vector of

$$\mathbf{p}^{(0)} = [0.8 \quad 0.2]$$

and determine the following:
(*a*) The vector of state probabilities at time t_1.
(*b*) The vector of state probabilities at time t_4.
(*c*) The vector of state probabilities at time t_8.

SOLUTION
(*a*) From Eq. (11-136),

$$\begin{aligned}\mathbf{p}^{(1)} &= \mathbf{p}^{(0)}\mathbf{P}^{(1)} \\ &= [0.8 \quad 0.2]\begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} \\ &= [0.54 \quad 0.46]\end{aligned}$$

(*b*) From Eq. (11-136),

$$\mathbf{p}^{(4)} = \mathbf{p}^{(0)}\mathbf{P}^{(4)}$$

where

$$\begin{aligned}\mathbf{P}^{(2)} &= \mathbf{P}^{(1)}\mathbf{P}^{(1)} \\ &= \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} \\ &= \begin{bmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{bmatrix}\end{aligned}$$

and thus

$$\begin{aligned}\mathbf{P}^{(4)} &= \mathbf{P}^{(2)}\mathbf{P}^{(2)} \\ &= \begin{bmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{bmatrix}\begin{bmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{bmatrix} \\ &= \begin{bmatrix} 0.4332 & 0.5668 \\ 0.4251 & 0.5749 \end{bmatrix}\end{aligned}$$

Therefore

$$\mathbf{p}^{(4)} = [0.8 \quad 0.2] \begin{bmatrix} 0.4332 & 0.5668 \\ 0.4251 & 0.5749 \end{bmatrix}$$

$$= [0.4316 \quad 0.5684]$$

(*c*) From Eq. (11-136),

$$\mathbf{p}^{(8)} = \mathbf{p}^{(0)}\mathbf{P}^{(8)}$$

where

$$\mathbf{P}^{(8)} = \mathbf{P}^{(4)}\mathbf{P}^{(4)}$$

$$= \begin{bmatrix} 0.4332 & 0.5668 \\ 0.4251 & 0.5749 \end{bmatrix} \begin{bmatrix} 0.4332 & 0.5668 \\ 0.4251 & 0.5749 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4286 & 0.5714 \\ 0.4285 & 0.5715 \end{bmatrix}$$

Therefore

$$\mathbf{p}^{(8)} = [0.8 \quad 0.2] \begin{bmatrix} 0.4286 & 0.5714 \\ 0.4285 & 0.5715 \end{bmatrix}$$

$$= [0.4286 \quad 0.5714]$$

Here, it is interesting to observe that the rows of the transition matrix $\mathbf{P}^{(8)}$ tend to be the same. Furthermore, the state probability vector $\mathbf{p}^{(8)}$ tends to be the same with the rows of the transition matrix $\mathbf{P}^{(8)}$. These results show that the long-run absolute probabilities are independent of the initial state probabilities, that is, $\mathbf{p}^{(0)}$. Therefore, the resulting probabilities are called the *steady-state probabilities* and defined as the set of π_j, where

$$\pi_j = \lim_{n \to \infty} p_j^{(n)} = \lim_{n \to \infty} P\{X(t_n) = j\} \tag{11-139}$$

In general, the initial state tends to be less important to the n-step transition probability as n increases, such that

$$\lim_{n \to \infty} P\{X(t_n) = j \mid X(t_0) = i\} = \lim_{n \to \infty} P\{X(t_n) = j\} = \Pi_j \tag{11-140}$$

so that one can get the unconditional steady-state probability distribution from the n-step transition probabilities by taking n to infinity without taking the initial states into account. Therefore

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n-1)}\mathbf{P} \tag{11-141}$$

or

$$\lim_{n \to \infty} \mathbf{P}^{(n)} = \lim_{n \to \infty} \mathbf{P}^{(n-1)}\mathbf{P} \tag{11-142}$$

and thus,

$$\mathbf{\Pi} = \mathbf{\Pi P} \tag{11-143}$$

where

$$\mathbf{\Pi} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \cdots & \pi_k \\ \pi_1 & \pi_2 & \pi_3 & \cdots & \pi_k \\ \pi_1 & \pi_2 & \pi_3 & \cdots & \pi_k \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \pi_1 & \pi_2 & \pi_3 & \cdots & \pi_k \end{bmatrix} \tag{11-144}$$

Note that the matrix $\mathbf{\Pi}$ has identical rows so that each row is a row vector of

$$\mathbf{\Pi} = [\pi_1 \quad \pi_2 \quad \pi_3 \quad \cdots \quad \pi_k] \tag{11-145}$$

Since the transpose of a row vector $\mathbf{\Pi}$ is a column vector $\mathbf{\Pi}^t$, Eq. (11-143) can also be expressed as

$$\mathbf{\Pi}^t = \mathbf{P}^{(t)}\mathbf{\Pi}^{(t)} \tag{11-146}$$

which is a set of linear equations.

To be able to solve equation sets (11-143) or (11-146) for individual π_i's, one additional equation is required. This equation is called the *normalizing equation* and can be expressed as

$$\sum_{\text{all } i} \pi_i = 1 \tag{11-147}$$

11-8-2 Classification of States in Markov Chains

Two states i and j are said to *communicate*, denoted as $i \sim j$, if each is accessible (reachable) from the other, i.e., if there exists some sequence of possible transitions which would take the process from state i to state j.

A *closed set* of states is a set such that if the system, once in one of the states of this set, will stay in the set indefinitely; i.e., once a closed set is entered, it cannot be left. Therefore an *ergodic set* of states is a set in which all states communicate and which cannot be left once it is entered. Of course, an *ergodic state* is an element of an ergodic set. A state is called *transient* if it is not ergodic. If a single state forms a closed set, the state is called an *absorbing state*. Thus a state is an absorbing state if and only if $p_{ii} = 1$.

11-9 DEVELOPMENT OF THE STATE-TRANSITION MODEL TO DETERMINE THE STEADY-STATE PROBABILITIES

The Markov technique can be used to determine the steady-state probabilities. The model given in this section is based on the zone-branch technique developed by Koval and Billinton [11,12].

Assuming the process given in a markovian model is irreducible and all states are ergodic, one can derive a set of linear equations to determine the steady-state probabilities as

$$\pi_j = \lim_{t \to \infty} p_{ij}(t) \tag{11-148}$$

Therefore, for example, the system differential equations can be expressed in the matrix form for a single-component state as [13]

$$\begin{bmatrix} P'_0(t) \\ P'_0(t) \\ P'_2(t) \\ P'_3(t) \end{bmatrix} = \begin{bmatrix} -(\lambda + \hat{n}) & \hat{m} & \mu & 0 \\ \hat{n} & -(\hat{m} + \lambda') & 0 & \mu' \\ \lambda & 0 & -(\mu + \hat{n}) & \hat{m} \\ 0 & \lambda' & \hat{n} & -(\mu' + \hat{m}) \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} \tag{11-149}$$

where λ = normal weather failure rate of component
μ = normal weather repair rate of component
λ' = adverse weather failure rate of component
μ' = adverse weather repair rate of component

Also,

$$\hat{n} = \frac{1}{N} \tag{11-150}$$

and

$$\hat{m} = \frac{1}{S} \tag{11-151}$$

where N = expected duration of normal weather period
S = expected duration of adverse weather period

Equation (11-148) can be expressed in the matrix form as

$$\left[\frac{dP(t)}{dt}\right] = \mathbf{P}(t)\mathbf{\Lambda} \tag{11-152}$$

where $\left[\frac{dP(t)}{dt}\right]$ = matrix whose (i, j)th element is $\frac{dp_{ij}(t)}{dt}$
$\mathbf{P}(t)$ = matrix whose (i, j)th element is $p_{ij}(t)$
$\mathbf{\Lambda}$ = matrix whose (i, j)th element is λ_{ij}

Also, each element in matrix equation (11-152) can be expressed as

$$\frac{dp_{ij}(t)}{dt} = \sum_k p_{ik}(t)\lambda_{kj} \tag{11-153}$$

or

$$\lim_{t \to \infty} \frac{dp_{ij}(t)}{dt} = \lim_{t \to \infty} \sum_k p_{ik}(t)\lambda_{kj} \tag{11-154}$$

since

$$\lim_{t \to \infty} \frac{dp_{ij}(t)}{dt} = \frac{d}{dt} \lim_{t \to \infty} p_{ij}(t) \tag{11-155}$$

$$\frac{d}{dt} \lim_{t \to \infty} p_{ij}(t) = \sum_k \lim_{t \to \infty} p_{ik}(t)\lambda_{kj} \tag{11-156}$$

or

$$\frac{d\pi_j}{dt} = \sum_k \pi_k \lambda_{kj} \tag{11-157}$$

However, since the differentiation of a constant is zero, that is,

$$\frac{d\pi_j}{dt} = 0 \tag{11-158}$$

Eq. (11-157) becomes

$$0 = \sum_k \pi_k \lambda_{kj} \tag{11-159}$$

or, in the matrix form,

$$\mathbf{0} = \mathbf{\Pi\Lambda} \tag{11-160}$$

where $\mathbf{0}$ = row vector of zeros
$\mathbf{\Lambda}$ = matrix of transition rates
$\mathbf{\Pi}$ = row vector of steady-state probabilities

Since the equations in the matrix equation (11-160) are dependent, introduction of an additional equation is necessary, that is,

$$\sum \pi_i = 1 \tag{11-161}$$

which is called the *normalizing equation.*

The matrix $\mathbf{\Lambda}$ can be expressed as

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \tag{11-162}$$

where $\lambda_{ij} = -d_i$ for $i = j$, called the *rate of departure from state i*
$\lambda_{ij} = e_{ij}$ for $i \neq j$, called the *rate of entry from state i to state j*

Therefore matrix equation (11-162) can be reexpressed as

$$\mathbf{\Lambda} = \begin{bmatrix} -d_1 & e_{12} & \cdots & e_{1n} \\ e_{21} & -d_2 & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & -d_n \end{bmatrix} \tag{11-163}$$

Likewise,

$$\Pi = [p_1 \quad p_2 \quad \cdots \quad p_n] \tag{11-164}$$

Therefore, substituting Eqs. (11-163) and (11-164) into Eq. (11-160),

$$[0 \quad 0 \quad \cdots \quad 0] = [p_1 \quad p_2 \quad \cdots \quad p_n] \begin{bmatrix} -d_1 & e_2 & \cdots & e_{1n} \\ e_{21} & -d_2 & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & -d_n \end{bmatrix} \tag{11-165}$$

or

$$\begin{aligned} 0 &= -p_1 d_1 + p_2 e_{21} + \cdots + p_n e_{n1} \\ 0 &= p_1 e_{12} - p_2 d_2 + \cdots + p_n e_{n2} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 &= p_1 e_{1n} + p_2 e_{2n} + \cdots - p_n d_n \end{aligned} \tag{11-166}$$

Therefore

$$0 = -p_i \sum d_i + \sum p_j e_{ij} \tag{11-167}$$

or

$$p_i \sum d_i = \sum p_j e_{ij} \tag{11-168}$$

Also,

$$p_1 + p_2 + p_3 + \cdots + p_n = 1 \tag{11-169}$$

or

$$p_1\left(1 + \frac{p_2}{p_1} + \frac{p_3}{p_1} + \cdots + \frac{p_n}{p_1}\right) = 1 \tag{11-170}$$

As Koval and Billinton [11, 12] suggested, once the long-term or steady-state probabilities of each state are computed from Eqs. (11-168) and (11-170), one can readily calculate the total failure rate and the average repair rate of the particular zone i and branch j. These rates also take into consideration the effects of interruptions on other parts of the system. The total failure rate of zone i branch j is given by Koval and Billinton [11] as

$$\lambda_{ij} = \lambda_s + \sum \mathrm{RIA}(ij, k) \times \lambda_I \tag{11-171}$$

where λ_{ij} = total failure rate of zone branch ij

λ_s = failure rate of supply, i.e., feeding substation

RIA(ij, k) = recognition and isolation array coefficients,

I = failed zone-branch array coefficient = FZB(k)

Likewise, the average downtime, i.e., repair time, for each zone i branch j is given as

$$r_{ij} = \frac{\sum \mathrm{DTA}(ij, k) \times \lambda_I}{\lambda_{ijT}} \tag{11-172}$$

or

$$r_{ij} = \frac{\text{total annual outage time of zone branch } i, j}{\text{total failure rate of Zone Branch } i, j} \tag{11-173}$$

where r_{ij} = average repair time for each zone i branch j
$\sum \mathrm{DTA}(ij, k)$ = downtime array coefficients
I = failed zone-branch array coefficient, = $\mathrm{FZB}(k)$

11-10 THE APPLICATION OF THE ZONE-BRANCH TECHNIQUE

The aforementioned zone-branch technique has been applied [14] to a selected portion of the service area of the Oklahoma Gas and Electric Company's distribution system at the city of Muskogee, as shown in Fig. 11-25. Figure 11-26 shows the developed circuit reliability zone-branch single-line diagram of the configuration shown in Fig. 11-25.

The sum of the failure rates of all components in zone i branch j is denoted as λ_{ij} in Fig. 11-26. For example, λ_{32} is the failure rate of branch 2 in zone 3. The symbol S_{ij} denotes an isolating device, e.g., fuse, automatic or manual switch, sectionalizer, recloser, or circuit breaker, located on or related to zone i branch j. The associated state-transition diagram of the configuration is shown in Fig. 11-27. It is based upon two assumptions: (1) failure rates have exponential distribution, and (2) only single-contingency outage can occur; i.e., only one component can fail at a time.

As stated by Koval and Billinton [11], "the inclusion of the recognition and isolating characteristics of the protective coordination equipment in the state transition model has a significant impact on the load point reliability indices. Its significance, in many cases, can outweigh the consideration of overlapping outages of independent system components."

The definitions of the symbols and the transition rates are given in Tables 11-5 and 11-6, respectively. The p_{ij} and q_{ij} are the long-term probabilities of being in each of 102 states. They can be computed by using Eqs. (11-135) and (11-137). By using Eq. (11-138), the recognition and isolation array coefficients can be calculated, as shown in Table 11-7. Also, by employing Eq. (11-172), the average downtime, i.e., repair time, coefficients for branch j in zone i can be calculated, as shown in Tables 11-8 and 11-9.

The numerical input data for equipment failure rates and average repair times for both normal and disastrous weather are given in Table 11-10.

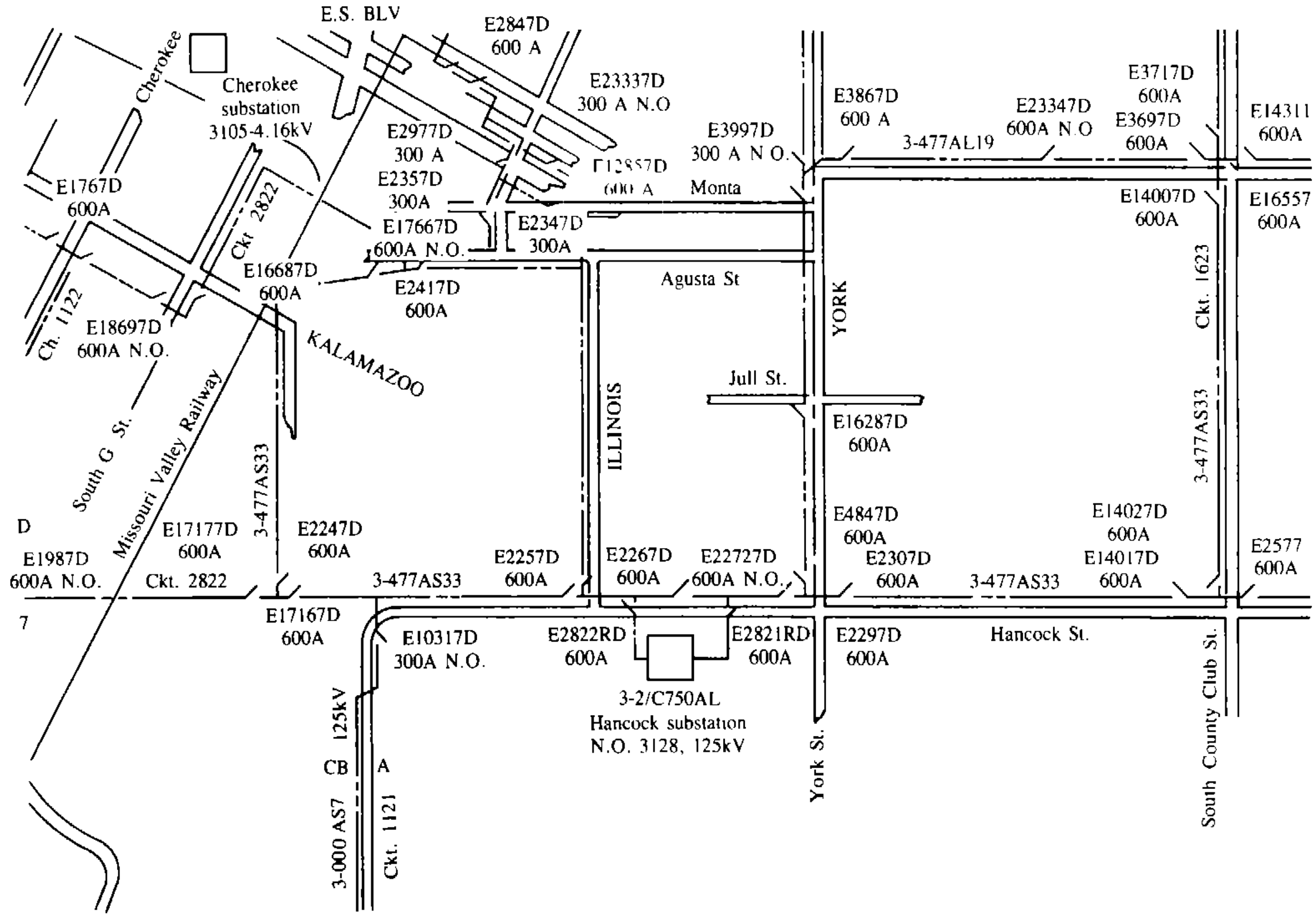


Figure 11-25 A section of the Oklahoma Gas and Electric Company's distribution system at the city of Muskogee. (*The Oklahoma Gas and Electric Company.*)

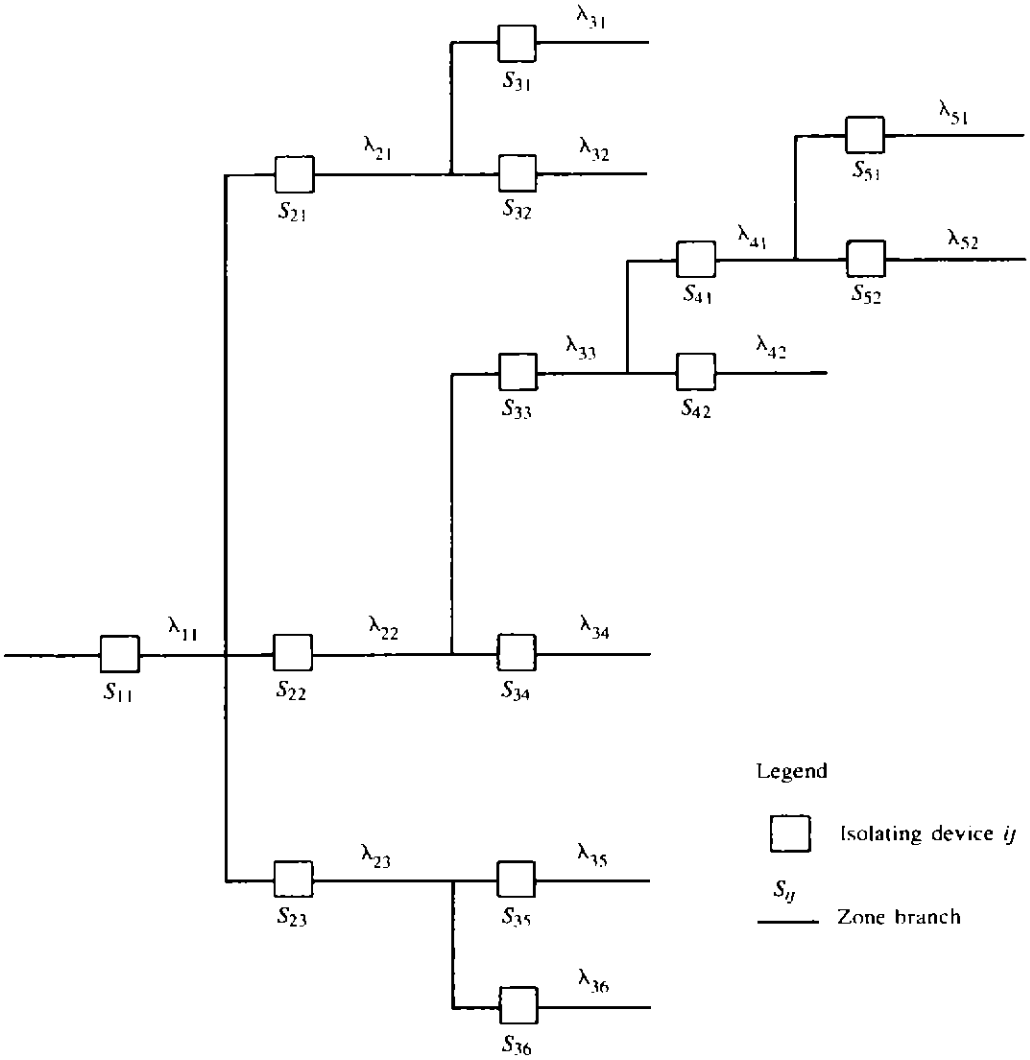


Figure 11-26 Single-line zone-branch reliability diagram for Fig. 11-25.

Table 11-11 gives the calculated long-term probabilities of being in each of 102 different states of the system shown in Figs. 11-1 and 11-2.

For the purpose of illustration, the failure rates for various zone branches of the given system for both normal and disastrous weather can be calculated by employing Eq. (11-171) and using the probabilities of protective devices not recognizing and therefore not isolating the fault as $q = 0.001$, $q = 0.01$, $q = 0.1$, and $q = 1.0$ (for manually operated switches). The results for both normal and disastrous weather are given in Table 11-12 and plotted on Fig. 11-28*a* and *b*, respectively. Furthermore, the total annual outage durations for various zone branches of the given system for both normal and disastrous weather can be calculated as given in Table 11-13 and plotted as shown on Fig. 11-29*a* and *b*, respectively. Again, the probabilities of protective devices not recognizing and therefore not isolating the fault can be given as $q = 0.001$, $q = 0.01$, $q = 0.1$, and $q = 1.0$ as before. Of course, the assigned value of q upon the degree of the protection coordination, i.e., the lower the value of q, the higher the degree of

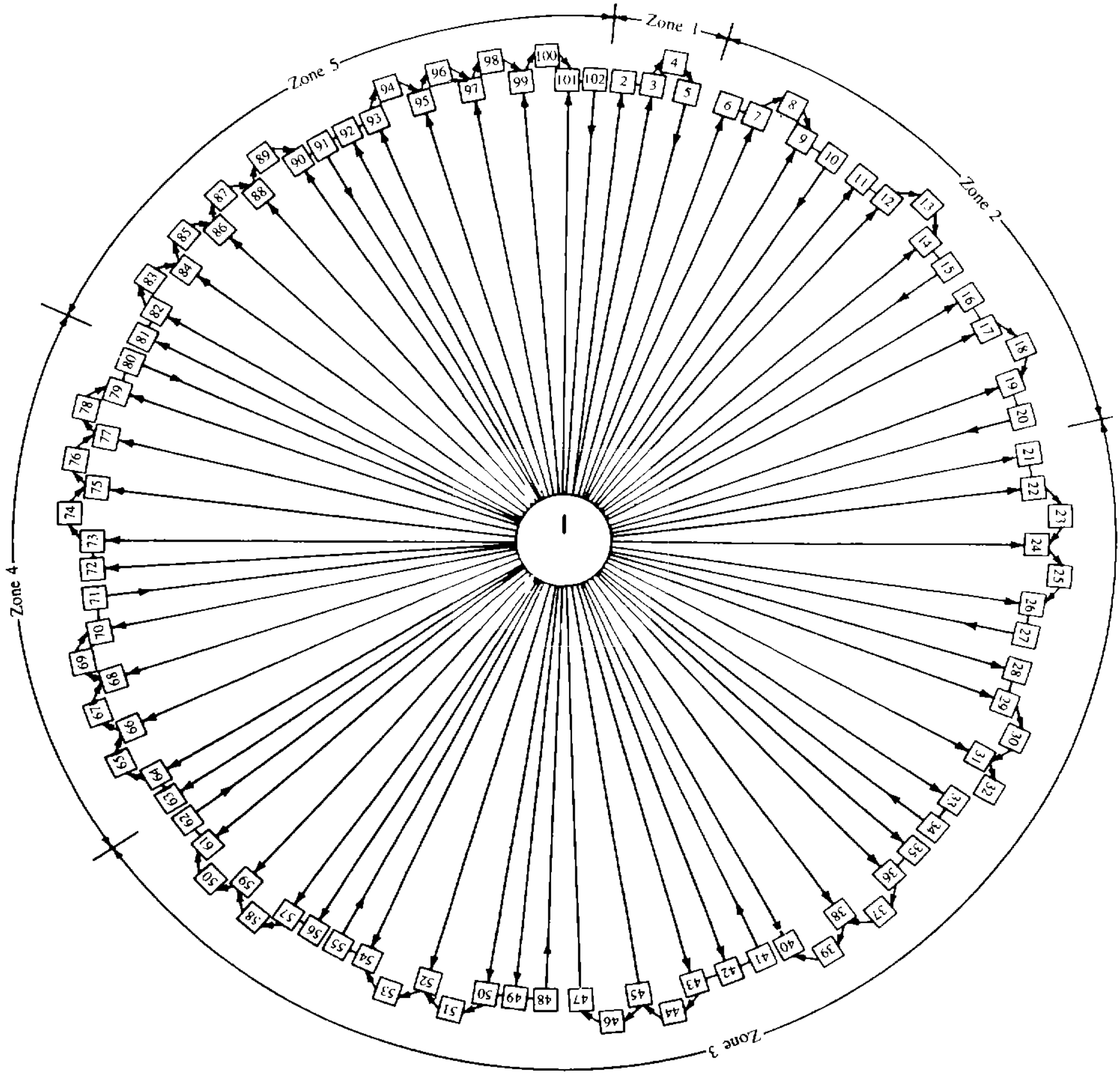


Figure 11-27 State-transition diagram.

Table 11-5 Symbols*

Symbol	Description
p_{ij}	Probability that an isolating equipment will recognize the permanent fault in zone i branch j or in the higher zone branches and isolate it very fast.
q_{ij}	Probability that an isolating equipment will fail to recognize the permanent fault in zone i branch j or in the higher zone branches and not isolate it.
λ_{ij}	Total failure rate of the whole zone i branch j, which is the sum of failure rate due to stuck breakers, maintenance, and weather as $$\lambda_{ij} = \lambda_{ijB} + \lambda_{ijM} + \lambda_{ijW}$$ where λ_{ijB} = failure rate of breaker in zone i branch j; λ_{ijM} = failure rate due to preventive maintenance in zone i branch j; λ_{ijW} = failure rate due to weather in zone i branch j, which includes effects of normal, stormy, and disastrous weather
λ_s	Failure rate of supply, i.e., substation feeding the system
μ_s	Restoration rate of supply
μ_{ij}	Repair rate of zone i branch j
μ_{SijC}	Reclosing rate of reclosing equipment S located in zone i branch j.
μ_{SijO}	Isolation rate of isolating equipment S located in zone i branch j. (For example, μ_{S12O} is the isolation rate of isolating equipment S_{12} located in zone 1 and branch 2.)

* Based upon [11].

Table 11-6 Definition of the transition rates

States		Transition rate	States		Transition rate
1	2	$q_{11}\lambda_{11}$	16	17	μ_{S11O}
2	3	μ_{S11O}	1	17	$p_{11}q_{23}\lambda_{23}$
1	3	$p_{11}\lambda_{11}$	17	18	μ_{S23O}
3	4	μ_{11}	18	19	μ_{S11C}
4	1	μ_{S11C}	1	19	$p_{23}\lambda_{23}$
1	5	λ_S	19	20	μ_{23}
5	1	μ_S	20	1	$\mu_{S23}C$
1	6	$q_{11}q_{21}\lambda_{21}$	1	21	$q_{11}q_{21}q_{31}\lambda_{31}$
6	7	μ_{S11O}	21	22	μ_{S11O}
1	7	$p_{11}q_{21}\lambda_{21}$	1	22	$p_{11}q_{21}q_{31}\lambda_{31}$
7	8	μ_{S21O}	22	23	μ_{S11O}
8	9	μ_{S11C}	23	24	μ_{S11C}
1	9	$p_{21}\lambda_{21}$	1	24	$p_{21}q_{31}\lambda_{31}$
9	10	μ_{21}	24	25	μ_{S31O}
10	1	μ_{S21C}	25	26	μ_{S21C}
1	11	$q_{11}q_{22}\lambda_{22}$	1	26	$p_{31}\lambda_{31}$
11	12	μ_{S11O}	26	27	μ_{31}
1	12	$p_{11}q_{22}\lambda_{22}$	27	1	μ_{S31C}
12	13	μ_{S22O}	1	28	$q_{11}q_{21}q_{32}\lambda_{32}$
13	14	μ_{S11C}	28	29	μ_{S11O}
1	14	$p_{22}\lambda_{22}$	1	29	$p_{11}q_{21}q_{32}\lambda_{32}$
14	15	μ_{22}	29	30	μ_{S21O}
15	1	μ_{S22C}	30	31	μ_{S11C}
1	16	$q_{11}q_{23}\lambda_{23}$	1	31	$p_{21}q_{32}\lambda_{32}$

31	32	μ_{S32O}
32	33	μ_{S21C}
1	33	$p_{32}\lambda_{32}$
33	34	μ_{32}
34	1	μ_{S32C}
1	35	$q_{11}q_{22}q_{33}\lambda_{33}$
35	36	μ_{S11O}
1	36	$p_{11}q_{22}q_{33}\lambda_{33}$
36	37	μ_{S22O}
37	38	μ_{S11C}
1	38	$p_{22}q_{33}\lambda_{33}$
38	39	μ_{S33O}
39	40	μ_{S22C}
1	40	$p_{33}\lambda_{33}$
40	41	μ_{33}
41	1	μ_{S33C}
1	42	$q_{11}p_{22}q_{34}\lambda_{34}$
42	43	μ_{S11O}
1	43	$p_{11}q_{22}q_{34}\lambda_{34}$
43	44	μ_{S22O}
44	45	μ_{S11C}
1	45	$p_{22}q_{34}\lambda_{34}$
45	46	μ_{S34O}
46	47	μ_{S22C}
1	47	$p_{34}\lambda_{34}$
47	48	μ_{34}
48	1	μ_{S34C}
1	49	$q_{11}q_{33}q_{35}\lambda_{35}$
49	50	μ_{S11O}
1	50	$p_{11}q_{23}q_{35}\lambda_{35}$
50	51	μ_{S23O}
51	52	μ_{S11C}
1	52	$p_{23}q_{35}\lambda_{35}$
52	53	μ_{S35O}
53	54	μ_{S23C}
1	54	$p_{35}\lambda_{35}$
54	55	μ_{35}
55	1	μ_{S35C}
1	56	$q_{11}q_{23}q_{36}\lambda_{36}$
56	57	μ_{S11O}
1	57	$p_{11}q_{23}q_{36}\lambda_{36}$
57	58	μ_{S23O}
58	59	μ_{S11C}
1	59	$p_{23}q_{36}\lambda_{36}$
59	60	μ_{S36O}
60	61	μ_{S23C}
1	61	$p_{36}\lambda_{36}$
61	62	μ_{36}
62	1	μ_{S36C}
1	63	$q_{11}q_{22}q_{33}q_{41}\lambda_{41}$
63	64	μ_{S11O}
1	64	$p_{11}q_{22}q_{33}q_{41}\lambda_{41}$
64	65	μ_{S22O}
65	66	μ_{S11C}
1	66	$p_{22}q_{33}q_{41}\lambda_{41}$
66	67	μ_{S33O}
67	68	μ_{S22C}
1	68	$p_{33}q_{41}\lambda_{41}$
68	69	μ_{S41O}
69	70	μ_{S33C}
1	70	$p_{41}\lambda_{41}$
70	71	μ_{41}
71	1	μ_{S41C}
1	72	$q_{41}q_{22}q_{33}q_{42}\lambda_{42}$
72	73	μ_{S11O}
1	73	$p_{11}q_{22}q_{33}q_{42}\lambda_{42}$
73	74	μ_{S22O}
74	75	μ_{S11C}
1	75	$p_{22}q_{33}q_{42}\lambda_{42}$
1	75	$p_{22}q_{33}q_{42}\lambda_{42}$
75	76	μ_{S33O}
76	77	μ_{S22C}
1	77	$p_{33}q_{42}\lambda_{42}$
77	78	μ_{S42O}
78	79	μ_{S33C}
1	79	$p_{42}\lambda_{42}$
79	80	μ_{42}
80	1	μ_{S42C}
1	81	$q_{11}q_{22}q_{33}q_{41}q_{51}\lambda_{51}$
81	82	μ_{S11O}
1	82	$p_{11}q_{22}q_{33}q_{41}q_{51}\lambda_{51}$
82	83	μ_{S22O}
83	84	μ_{S11C}
1	84	$p_{22}q_{33}q_{41}q_{51}\lambda_{51}$
84	85	μ_{S33O}
85	86	μ_{S22C}
1	86	$p_{33}q_{41}q_{51}\lambda_{51}$
86	87	μ_{S41O}
87	88	μ_{S33C}
1	88	$p_{41}q_{51}\lambda_{51}$
88	89	μ_{S51O}
89	90	μ_{S41C}
1	90	$p_{51}\lambda_{51}$
90	91	μ_{51}
91	1	μ_{S51C}
1	92	$q_{11}q_{22}q_{33}q_{41}q_{51}\lambda_{52}$
92	93	μ_{S11O}
1	93	$p_{11}q_{22}q_{33}q_{41}q_{51}\lambda_{52}$
93	94	μ_{S22O}
94	95	μ_{S11C}
1	95	$p_{22}q_{33}q_{41}q_{52}\lambda_{52}$
95	96	μ_{S33O}
96	97	μ_{S22C}
1	97	$p_{33}q_{41}q_{52}\lambda_{52}$
97	98	μ_{S41O}
98	99	μ_{S33C}
1	99	$p_{41}q_{52}\lambda_{52}$
99	100	μ_{S52O}
100	101	μ_{S41C}
1	101	$p_{52}\lambda_{52}$
101	102	μ_{52}
102	1	μ_{S51C}

Table 11-7 Recognition and isolation array (RIA) coefficients

Zone branch i, j	Failed zone-branch array FZB(k)													
	11	21	22	23	31	32	33	34	35	36	41	42	51	52
	k													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
11	1	q_{21}	q_{22}	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	$q_{22}q_{33}$	$q_{22}q_{34}$	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
21	1	1	q_{22}	q_{23}	q_{31}	q_{32}	$q_{22}q_{33}$	$q_{22}q_{34}$	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
22	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	q_{33}	q_{34}	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{33}q_{41}$	$q_{33}q_{42}$	$q_{33}q_{41}q_{51}$	$q_{33}q_{41}q_{52}$
23	1	q_{21}	q_{22}	1	$q_{21}q_{31}$	$q_{21}q_{32}$	$q_{22}a_{33}$	$a_{22}a_{34}$	q_{35}	q_{36}	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
31	1	1	q_{22}	q_{23}	1	q_{32}	$q_{22}q_{33}$	$q_{22}q_{34}$	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$

32	1	1	q_{22}	q_{23}	q_{31}	1	$q_{22}q_{33}$	$q_{22}q_{34}$	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
33	1	1	q_{22}	q_{23}	q_{31}	$q_{21}q_{32}$	1	$q_{22}q_{34}$	$q_{23}q_{35}$	$q_{23}q_{36}$	q_{41}	q_{42}	$q_{41}q_{51}$	$q_{41}q_{52}$
34	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	q_{31}	1	$q_{23}q_{35}$	$q_{23}q_{36}$	$q_{33}q_{41}$	$q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
35	1	q_{21}	q_{22}	1	$q_{21}q_{31}$	$q_{21}q_{32}$	$q_{22}q_{33}$	$q_{22}q_{34}$	1	q_{36}	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
36	1	q_{21}	q_{22}	1	$q_{21}q_{31}$	$q_{21}q_{32}$	$q_{22}q_{33}$	$q_{22}q_{34}$	q_{35}	1	$q_{22}q_{33}q_{41}$	$q_{22}q_{33}q_{42}$	$q_{22}q_{33}q_{41}q_{51}$	$q_{22}q_{33}q_{41}q_{52}$
41	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	1	q_{34}	$q_{23}q_{35}$	$q_{23}q_{36}$	1	q_{42}	q_{51}	q_{52}
42	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	1	q_{34}	$q_{23}q_{35}$	$q_{23}q_{36}$	q_{41}	1	$q_{41}q_{51}$	$q_{41}q_{51}$
51	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	1	q_{34}	$q_{23}q_{35}$	$q_{22}q_{36}$	1	q_{42}	1	q_{52}
52	1	q_{21}	1	q_{23}	$q_{21}q_{31}$	$q_{21}q_{32}$	1	q_{34}	$q_{23}q_{35}$	$q_{23}q_{36}$	1	q_{42}	q_{51}	1

Table 11-8 Downtime array (DTA) coefficients

	11	21	22	23	31	32	33	34
11	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
21	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O}) + r_{21} + r_{S21C}$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O}) + q_{31}(r_{S31O} + r_{S21C})$	$q_{21}q_{32}(r_A + r_{S21O}) + q_{32}(r_{S32O} + r_{S21C})$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + q_{S22O})$
22	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22O}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O})$ $q_{33}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$
23	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O}) + r_{23} + r_{S23C}$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
31	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O}) + r_{21} + r_{S21C}$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O}) + q_{31}(r_{S31O} + r_{S21O}) + r_{31} + r_{S31C}$	$q_{21}q_{32}(r_A + r_{S21O}) + q_{32}(r_{S32O} + r_{S21C})$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
32	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O}) + r_{21} + r_{S21C}$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O}) + q_{31}(r_{S31O} + r_{S21C})$	$q_{21}q_{32}(r_A + r_{S21O}) + q_{32}(r_{S32O} + r_{S21C}) + r_{32} + r_{S32C}$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
33	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22C}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{S33O} + r_{S22C}) + r_{33} + r_{S33C}$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$

34	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22C}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{S33O} + r_{S22C})$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C}) + r_{34} + r_{S34C}$
35	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S21O}) + r_{23} + r_{S23C}$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{23}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
36	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O})$	$q_{23}(r_A + r_{S23O}) + r_{23} + r_{S23C}$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O})$	$q_{22}q_{34}(r_A + r_{S22O})$
41	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22C}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{S33O} + r_{S22C}) + r_{33} + r_{S33C}$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$
42	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22C}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{S33O} + r_{S22C}) + r_{33} + r_{S33C}$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$
51	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22C}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{33O} + r_{S22C}) + r_{33} + r_{S33C}$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$
52	$r_{11} + r_A$	$q_{21}(r_A + r_{S21O})$	$q_{22}(r_A + r_{S22O}) + r_{22} + r_{S22O}$	$q_{23}(r_A + r_{S23O})$	$q_{21}q_{31}(r_A + r_{S21O})$	$q_{21}q_{32}(r_A + r_{S21O})$	$q_{22}q_{33}(r_A + r_{S22O}) + q_{33}(r_{S33O} + r_{S22C}) + r_{33} + r_{S33C}$	$q_{22}q_{34}(r_A + r_{S22O}) + q_{34}(r_{S34O} + r_{S22C})$

Note: $r_A = q_{11}r_{S11O} + r_{S11C}$

Table 11-9 Downtime array (DTA) coefficients

	35	36	41	42	51	52
11	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}$ $\times (r_A + r_{S22O})$
21	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}$ $\times (r_A + r_{S22O})$
22	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}q_{41}q_{52}$ $\times (r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$
23	$q_{23}q_{35}(r_A + r_{S23O})$ $+ q_{35}(r_{S35O} + r_{S23C})$	$q_{23}q_{36}(r_A + r_{S23O})$ $+ q_{36}(r_{S36O} + r_{S23C})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}$ $\times (r_A + r_{S22O})$
31	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$
32	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$
33	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$ $+ q_{41}(r_{S41O} + r_{S33C})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$ $+ q_{42}(r_{S42O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{51}$ $\times (r_{S41O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{52}(r_{S41O} + r_{S33C})$
34	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$	$q_{22}q_{33}q_{41}q_{51}$ $\times (r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}$ $\times (r_{S33O} + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22C})$

35	$q_{23}q_{35}(r_A + r_{S23O})$ $+ q_{35}(r_{S35O} + r_{S23O})$ $+ r_{35} + r_{S35C}$	$q_{23}q_{36}(r_A + r_{S23O})$ $+ q_{36}(r_{S36O} + r_{S23C})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$
36	$q_{23}q_{35}(r_A + r_{S23O})$ $+ q_{35}(r_{S35O} + r_{S23C})$	$q_{23}q_{36}(r_A + r_{S23O})$ $+ q_{36}(r_{S36O} + r_{S23C})$ $+ r_{36} + r_{S36O}$	$q_{22}q_{33}q_{41}(r_A + r_{S22}\ )$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{51}$ $\times(r_A + r_{S22O})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$
41	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$ $+ q_{41}(r_{S41O} + r_{S33C})$ $+ r_{41} + r_{S41C}$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$ $+ q_{42}(r_{S42O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{51}$ $\times(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}$ $\times(r_{S33O} + r_{S22C})$ $+ q_{41}q_{51}(r_{S41O} + r_{S33C})$ $+ q_{51}(r_{S51O} + r_{S41C})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{52}(r_{S41O} + r_{S33C})$ $+ 52(r_{S52O} + r_{S41C})$
42	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{22C})$ $+ q_{41}(r_{S41O} + r_{S33C})$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$ $+ q_{42}(r_{S41O} + r_{S33C})$ $+ q_{42} + r_{S42C}$	$q_{22}q_{33}q_{41}q_{51}$ $\times(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{51}(r_{S41O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22O})$ $+ q_{41}q_{52}(r_{S41O} + r_{S33O})$
51	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + q_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$ $+ q_{41}(r_{S41O} + r_{S33C})$ $+ q_{41} + r_{S41C}$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S22C})$ $+ q_{42}(r_{S42O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{51}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{51}(r_{S41O} + r_{S33C})$ $+ q_{51}(r_{51O} + r_{S41C})$ $+ r_{51} + r_{S51C}$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{52}(r_{S41O} + r_{S33C})$ $+ q_{52}(r_{S52O} + r_{S41C})$
52	$q_{23}q_{35}(r_A + r_{S23O})$	$q_{23}q_{36}(r_A + r_{S23O})$	$q_{22}q_{33}q_{41}(r_A + r_{S22O})$ $+ q_{33}q_{41}(r_{S33O} + r_{S22C})$ $+ q_{41}(r_{S41O} + r_{S33C})$ $+ r_{41} + r_{S41C}$	$q_{22}q_{33}q_{42}(r_A + r_{S22O})$ $+ q_{33}q_{42}(r_{S33O} + r_{S22C})$ $+ q_{42}(r_{S42O} + r_{S33C})$	$q_{22}q_{33}q_{41}q_{51}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{51}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{51}(r_{S41O} + r_{S33C})$ $+ q_{51}(r_{S51O} + r_{S41C})$	$q_{22}q_{33}q_{41}q_{52}(r_A + r_{S22O})$ $+ q_{33}q_{41}q_{52}(r_{S33O} + r_{S22C})$ $+ q_{41}q_{52}(r_{S41O} + r_{S33C})$ $+ q_{52}(r_{S52O} + r_{S41C})$ $+ r_{52} + r_{S52C}$

Note: $r_A = q_{11}r_{S11O} + r_{S11C}$

Table 11-10 Annual equipment failure rates and average repair times for normal and disastrous weather

Component	Normal weather failure rate, failure/(yr·equip)	Average repair time, h/failure	Disastrous weather failure rate, failure/(yr·equip)
Feeder circuit breaker	0.09	5.6	0.333
Distribution transformer	0.06	3.0	0.166
Three-phase switch	0.103	1.5	0.25
Fuse	0.00232	1.0	0.00232
Three-phase switch on single-phase lateral	0.103	1.5	0.25

Table 11-11 Steady-state probability of being in each state

State	Probability	State	Probability	State	Probability
P_1	0.66647117	P_{35}	1.439×10^{-13}	P_{69}	1.416×10^{-11}
P_2	2.266×10^{-11}	P_{36}	1.79947×10^{-12}	P_{70}	2.266×10^{-6}
P_3	2.266×10^{-6}	P_{37}	1.133×10^{-12}	P_{71}	1.732×10^{-10}
P_4	2.266×10^{-9}	P_{38}	2.266×10^{-11}	P_{72}	1.1529×10^{-14}
P_5	0.333235	P_{39}	1.41625×10^{-11}	P_{73}	1.446×10^{-13}
P_6	1.79947×10^{-12}	P_{40}	2.266×10^{-5}	P_{74}	0.906×10^{-13}
P_7	2.2660×10^{-11}	P_{41}	1.732×10^{-10}	P_{75}	1.832×10^{-12}
P_8	1.4129×10^{-11}	P_{42}	1.439×10^{-13}	P_{76}	1.146×10^{-12}
P_9	2.2660×10^{-5}	P_{43}	1.799×10^{-12}	P_{77}	2.266×10^{-11}
P_{10}	1.66617×10^{-10}	P_{44}	1.133×10^{-12}	P_{78}	1.416×10^{-11}
P_{11}	1.79947×10^{-12}	P_{45}	2.266×10^{-11}	P_{79}	2.266×10^{-5}
P_{12}	2.2660×10^{-11}	P_{46}	1.4162×10^{-11}	P_{80}	1.732×10^{-10}
P_{13}	1.399580×10^{-11}	P_{47}	2.266×10^{-5}	P_{81}	0.933×10^{-15}
P_{14}	2.266×10^{-5}	P_{48}	1.732×10^{-10}	P_{82}	1.1529×10^{-14}
P_{15}	1.66617×10^{-10}	P_{49}	1.4395×10^{-13}	P_{83}	0.706×10^{-14}
P_{16}	1.79947×10^{-12}	P_{50}	1.7994×10^{-12}	P_{84}	1.446×10^{-13}
P_{17}	2.2660×10^{-11}	P_{51}	1.133×10^{-12}	P_{85}	0.9064×10^{-13}
P_{18}	1.339589×10^{-11}	P_{52}	2.266×10^{-11}	P_{86}	1.799×10^{-12}
P_{19}	2.266×10^{-5}	P_{53}	1.416×10^{-11}	P_{87}	1.133×10^{-12}
P_{20}	1.66617×10^{-10}	P_{54}	2.266×10^{-5}	P_{89}	1.416×10^{-11}
P_{21}	1.43957×10^{-13}	P_{55}	1.7328×10^{-10}	P_{90}	2.199×10^{-5}
P_{22}	1.79947×10^{-12}	P_{56}	1.4395×10^{-13}	P_{91}	1.666×10^{-10}
P_{23}	1.133×10^{-12}	P_{57}	1.7994×10^{-12}	P_{92}	0.993×10^{-15}
P_{24}	2.266×10^{-11}	P_{58}	1.133×10^{-12}	P_{93}	1.15239×10^{-14}
P_{25}	1.41625×10^{-11}	P_{59}	2.266×10^{-11}	P_{94}	0.706×10^{-14}
P_{26}	2.266×10^{-5}	P_{60}	1.4162×10^{-11}	P_{95}	1.446×10^{-13}
P_{27}	1.7328×10^{-10}	P_{61}	2.266×10^{-5}	P_{96}	0.906×10^{-13}
P_{28}	1.43957×10^{-13}	P_{62}	1.732×10^{-10}	P_{97}	1.799×10^{-12}
P_{29}	1.79947×10^{-12}	P_{63}	1.152×10^{-14}	P_{98}	1.133×10^{-12}
P_{30}	1.133×10^{-12}	P_{64}	1.446×10^{-13}	P_{99}	2.266×10^{-11}
P_{31}	2.266×10^{-11}	P_{65}	0.906×10^{-13}	P_{100}	1.416×10^{-11}
P_{32}	1.41625×10^{-11}	P_{66}	1.832×10^{-12}	P_{101}	2.199×10^{-5}
P_{33}	2.266×10^{-5}	P_{67}	1.1463×10^{-12}	P_{102}	1.666×10^{-10}
P_{34}	1.73282×10^{-10}	P_{68}	2.266×10^{-11}		

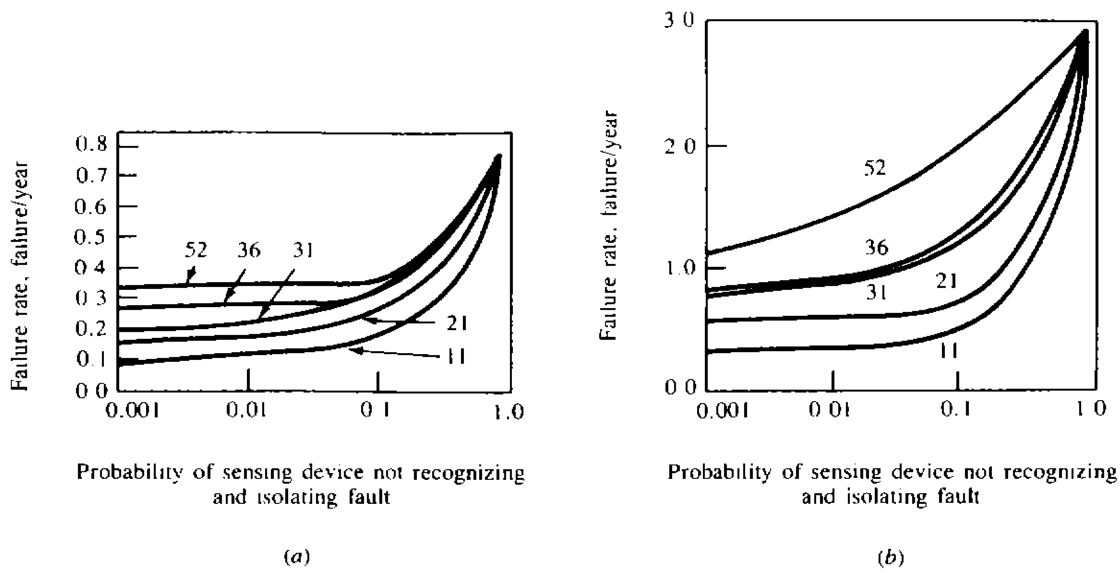


Figure 11-28 Failure rates of various zone branches for (*a*) normal weather and (*b*) disastrous weather.

protection. The results show that the failure rates and average annual outage durations of customers located on zone branches heavily depend on the characteristics of the protective devices used and their degree of coordination.

As Koval and Billinton [11] suggested, the use of overall average customer reliability indices for a circuit is misleading since every branch in a given circuit can, with some probability, affect the reliability indices of a customer connected to a given branch.

The comparison of the results for normal and disastrous weather (e.g., tornado) can help the planning engineer design a more reliable system. Of course, the

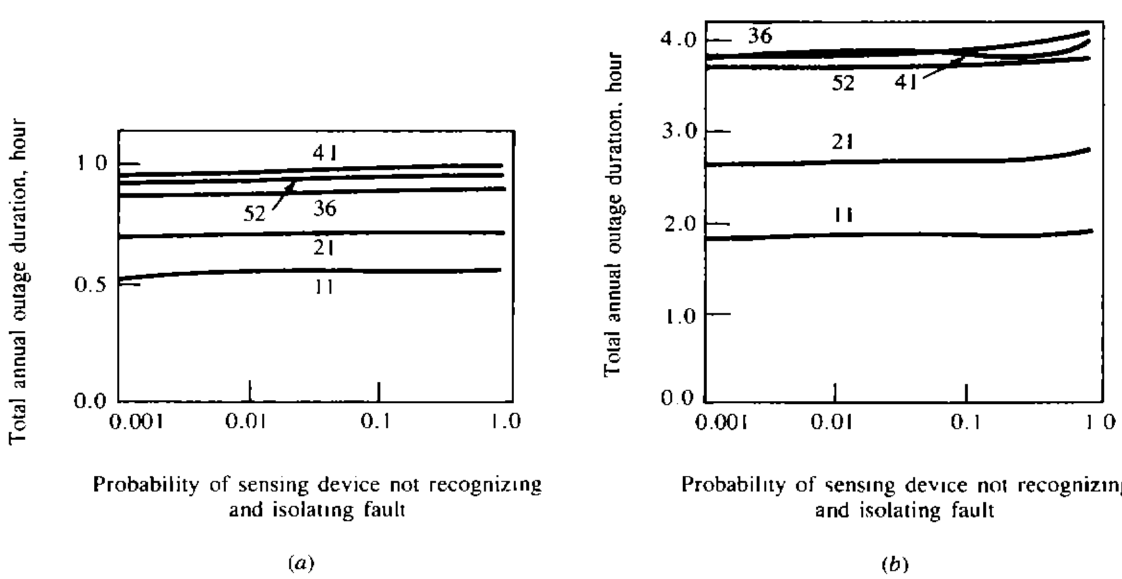


Figure 11-29 Total annual outage durations for various zone branches for (*a*) normal weather and (*b*) disastrous weather.

Table 11-12 Failure rates for various zone branches

Zone	branch	$q = 0.001$	$q = 0.01$	$q = 0.10$	$q = 1.0$
For normal weather					
1	1	0.09	0.1044	0.1502	0.790
2	1	0.15	0.1660	0.2417	0.790
2	2	0.14	0.1592	0.2509	0.790
2	3	0.13	0.1460	0.2310	0.790
3	1	0.19	0.2096	0.3110	0.790
3	2	0.175	0.1944	0.2615	0.790
3	3	0.18	0.2240	0.2655	0.790
3	4	0.17	0.2044	0.3235	0.790
3	5	0.25	0.2896	0.2900	0.790
3	6	0.27	0.2844	0.3032	0.790
4	1	0.26	0.2640	0.3632	0.790
4	2	0.25	0.2792	0.3087	0.790
5	1	0.29	0.3450	0.3460	0.790
5	2	0.33	0.3340	0.3442	0.790
For disastrous weather					
1	1	0.3334	0.3334	0.4723	2.913
2	1	0.5832	0.5926	0.7276	2.913
2	2	0.5839	0.6361	0.8926	2.913
2	3	0.4992	0.8193	1.1947	2.913
3	1	0.7496	0.9050	1.2804	2.913
3	2	0.8330	0.9897	1.3651	2.913
3	3	0.7489	1.1490	1.5244	2.913
3	4	0.6657	0.9894	1.3648	2.913
3	5	0.7493	1.0734	1.4488	2.913
3	6	0.8321	0.9050	1.2936	2.913
4	1	1.000	1.4790	1.8454	2.913
4	2	0.9152	1.2352	1.6206	2.913
5	1	0.9156	1.5650	2.2347	2.913
5	2	1.0830	1.4850	2.0872	2.913

Table 11-13 Total annual outage durations for various zone branches

Zone	branch	$q = 0.001$	$q = 0.01$	$q = 0.10$	$q = 1.0$
For normal weather					
1	1	0.5040	0.5068	0.5078	0.5018
2	1	0.6840	0.6867	0.6789	0.6898
2	2	0.6845	0.6870	0.6880	0.6897
2	3	0.8640	0.6892	0.6920	0.6892
3	1	0.8720	0.8767	0.8796	0.8923
3	2	0.8640	0.8690	0.8820	0.8972
3	3	0.8040	0.8067	0.8161	0.8253
3	4	0.7920	0.8010	0.8212	0.8615
3	5	0.8340	0.8372	0.8472	0.8516
3	6	0.8640	0.8731	0.9012	0.9131
4	1	0.9840	0.9868	0.9882	0.9910
4	2	0.8142	0.8202	0.8327	0.8451
5	1	0.9249	0.9268	0.9312	0.9517
5	2	0.9540	0.9568	0.9602	0.9703
For disastrous weather					
1	1	1.8648	1.8651	1.8772	1.9848
2	1	2.6148	2.6152	2.6272	2.7525
2	2	2.6138	2.6172	2.6242	2.7925
2	3	2.6172	2.6180	2.6721	2.7557
3	1	2.6156	2.6194	2.6524	2.7591
3	2	3.1128	3.1131	3.1254	3.2505
3	3	3.2090	3.2131	3.2252	3.3060
3	4	3.1075	3.1182	3.1252	3.2772
3	5	3.1124	3.2050	3.2741	3.3505
3	6	3.8630	3.8640	3.8/52	4.000
4	1	3.8620	3.8652	3.8754	4.0983
4	2	3.8628	3.8671	3.8752	4.0628
5	1	3.6108	3.8111	3.6232	4.0894
5	2	3.7011	3.7217	3.8271	3.8572

customer's reliability indices can be improved by providing alternate feeding paths. However, the inverse relationship between reliability and its cost has to be considered.

PROBLEMS

11-1 Assume that the given experiment is tossing a coin three times and that a single outcome is defined as a certain succession of heads (H) and tails (T), for example, (HHT).

(*a*) How many possible outcomes are there? Name them.

(*b*) What is the probability of tossing three heads, that is, (HHH)?

(*c*) What is the probability of getting heads on the first two tosses?

(*d*) What is the probability of getting heads on any two tosses?

11-2 Two cards are drawn from a shuffled deck. What is the probability that both cards will be aces?

11-3 Two cards are drawn from a shuffled deck.

(*a*) What is the probability that two cards will be the same suit?

(*b*) What is the probability if the first card is replaced in the deck before the second one is drawn?

11-4 Assume that a substation transformer has a constant hazard rate of 0.005 per day.

(*a*) What is the probability that it will fail during the next 5 years?

(*b*) What is the probability that it will not fail?

11-5 Consider the substation transformer in Prob. 11-4 and determine the probability that it will fail during year 6, given that it survives 5 years without any failure.

11-6 What is the MTTF for the substation transformer of Prob. 11-4?

11-7 Determine the following for a parallel connection of three components:

(*a*) The reliability.

(*b*) The availability.

(*c*) The MTTF.

(*d*) The frequency.

(*e*) The hazard rate.

11-8 A large factory of the International Zubits Company has 10 identical loads which switch on and off intermittently and independently with a probability p of being "on." Testing of the loads over a long period has shown that, on the average, each load is on for a period of 12 min/h. Suppose that when switched on each load draws some X kVA from the Ghost River Substation which is rated $7X$ kVA. Find the probability that the substation will experience an overload. (*Hint*: Apply the binomial expansion.)

11-9 Verify Eq. (11-79).

11-10 Verify Eq. (11-83).

11-11 Using Eq. (11-78), derive and prove that the mean time to repair of a two-component system is

$$\bar{r}_{sys} = \frac{(\bar{m}_1 + \bar{r}_1)(\bar{m}_2 + \bar{r}_2) - \bar{m}_1\bar{m}_2}{\bar{m}_1 + \bar{m}_2}$$

11-12 Calculate the equivalent reliability of each of the system configurations in Fig. P11-12, assuming that each component has the indicated reliability.

11-13 Calculate the equivalent reliability of each of the system configurations in Fig. P11-13, assuming that each component has the indicated reliability.

11-14 Determine the equivalent reliability of the system in Fig. P11-14.

11-15 Using the results of Example 11-6, determine the following:

(*a*) The probability of having any one of the three transformers out of service at any given time.

(*b*) The probability of having any two of the three transformers out of service at any given time.

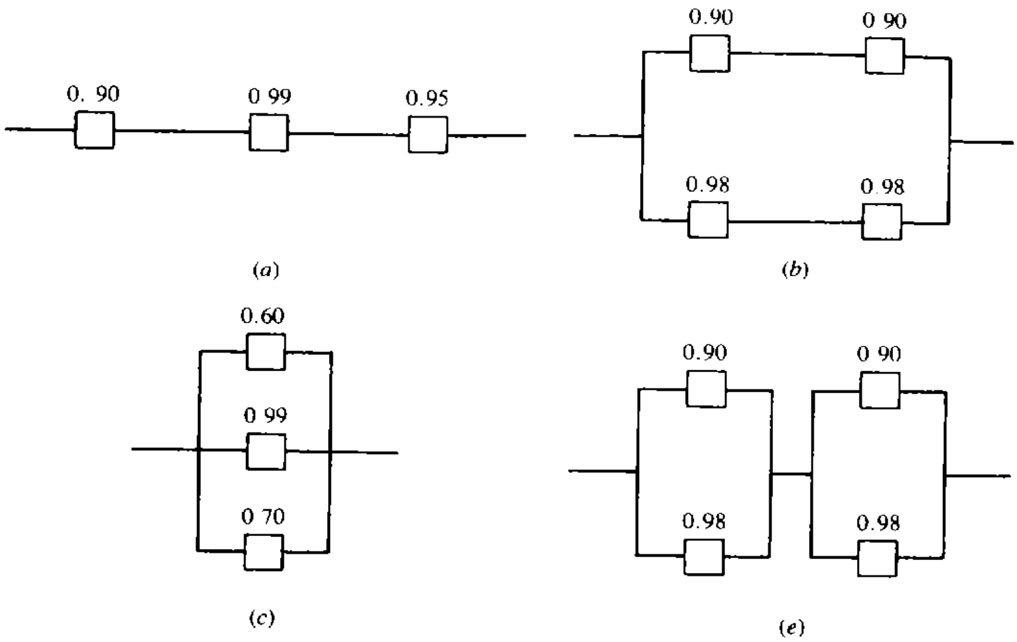


Figure P11-12

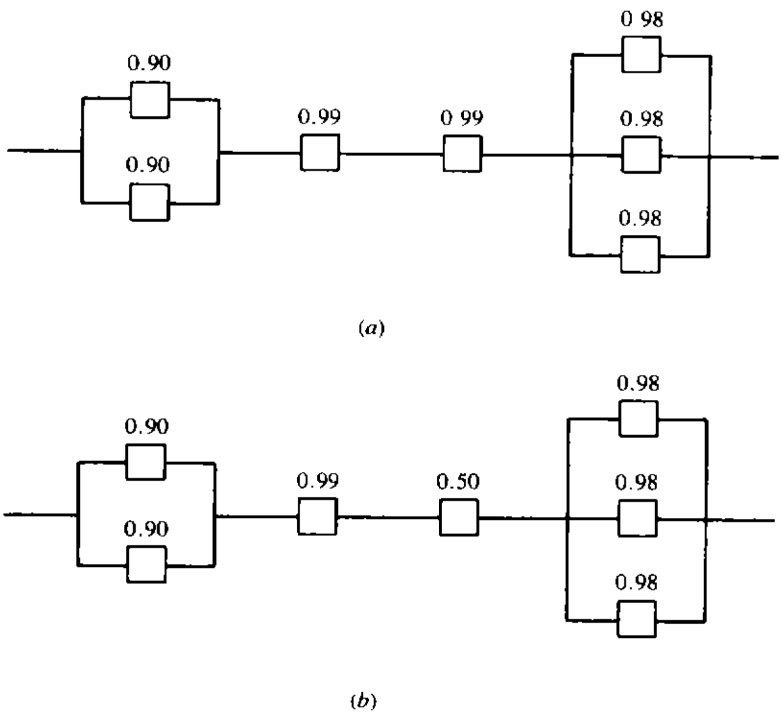


Figure P11-13

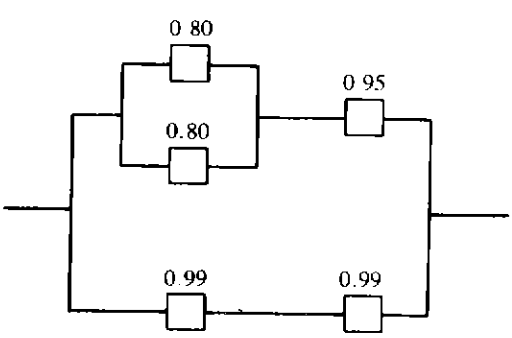


Figure P11-14

11-16 Using the results of Example 11-6, determine the following:

(*a*) The probability of having at least one of the three transformers out of service at any given time.

(*b*) The probability of having at least two of the three transformers out of service at any given time.

11-17 Repeat Example 11-2, assuming that the underground section of the feeder has been increased another mile due to growth in the downtown area and that on the average, the annual fault rate of the underground section has increased to 0.3 due to the growth and aging.

11-18 Repeat Example 11-3 for customers D to F, assuming that they all exist as shown in Fig. 11-16.

11-19 Repeat Prob. 11-18 but assume that during emergency the end of the existing feeder can be connected to and supplied by a second feeder over a normally open tie breaker.

11-20 Verify Eq. (11-172) for a two-component system.

11-21 Verify Eq. (11-172) for an n-component system.

11-22 Derive Eq. (11-131) based upon the definition of n-step transition probabilities of a Markov chain.

11-23 Use the data given in Example 11-8 and assume that feeder 1 has just had an outage. Using the joint probability concept of the classical probability theory techniques and the system's probability tree diagram, determine the probability that there will be an outage on feeder 2 at time after next outage.

11-24 Repeat Prob. 11-23 by using the Markov chains concept rather than the classical probability theory techniques.

11-25 Use the data given in Example 11-8 and the Markov chains concept. Assuming that there is an outage on feeder 3 at the present time, determine the following:

(*a*) The probabilities of being in each of the respective states at time t_1.

(*b*) The probabilities of being in each of the respective states at time t_2.

11-26 Use the data given in Example 11-8 and the Markov chains concept. Assume that there is an outage on feeder 2 at the present time and determine the probabilities associated with this outage at time t_4.

11-27 Use the data given in Example 11-8 and the Markov chains concept. Determine the complete outage probabilities at time t_4.

REFERENCES

1. IEEE Committee Reports: "Proposed Definitions of Terms for Reporting and Analyzing Outages of Electrical Transmission and Distribution Facilities and Interruptions," *IEEE Trans. Power Appar. Syst.*, vol. 87, no. 5, May 1968, pp. 1318–23.
2. Endrenyi, J.: *Reliability Modeling in Electric Power Systems*, Wiley, New York, 1978.
3. IEEE Committee Report: "Definitions of Customer and Load Reliability Indices for Evaluating Electric Power Performance," Paper A75 588-4, presented at the IEEE PES Summer Meeting, San Francisco, Calif., July 20–25, 1975.
4. IEEE Committee Report: "List of Transmission and Distribution Components for Use in Outage Reporting and Reliability Calculations," *IEEE Trans. Power Appar. Syst.*, vol. PAS-95, no. 4, July/August 1976, pp. 1210–15.
5. Smith, C. O.: *Introduction to Reliability in Design*, McGraw-Hill, New York, 1976.
6. *The National Electric Reliability Study: Technical Study Reports*, U.S. Department of Energy DOE/EP-0005, April 1981.
7. *The National Electric Reliability Study: Executive Summary*, U.S. Department of Energy, DOE/EP-0003, April 1981.
8. Billinton, R.: *Power System Reliability Evaluation*, Gordon and Breach, New York, 1978.
9. Albrect, P. F.: "Overview of Power System Reliability," *Workshop Proceedings: Power System Reliability—Research Needs and Priorities*, EPRI Report WS-77-60, Palo Alto, Calif., October 1978.
10. Billinton, R., R. J. Ringlee, and A. J. Wood: *Power-System Reliability Calculations*, M.I.T., Cambridge, Mass., 1973.

11. Koval, D. O., and R. Billinton: "Evaluation of Distribution Circuit Reliability," Paper F77 067-2, IEEE PES Winter Meeting, New York, N.Y., January–February, 1977.
12. Koval, D. O., and R. Billinton: "Evaluation of Elements of Distribution Circuit Outage Durations," Paper A77 685-1, IEEE PES Summer Meeting, Mexico City, Mexico, July 17–22, 1977.
13. Billinton, R , and M. S. Grover: "Quantitative Evaluation of Permanent Outages in Distribution Systems," *IEEE Trans. Power Appar. Syst.*, vol. PAS-94, May/June 1975, pp. 733–741.
14. Gönen, T., and M. Tahani: "Distribution System Reliability Analysis," *Proceedings of the IEEE MEXICON-80 International Conference*, Mexico City, Mexico, Oct. 22–25, 1980.
15. *Standard Definitions in Power Operations Terminology Including Terms for Reporting and Analyzing Outages of Electrical Transmission and Distribution Facilities and Interruptions to Customer Service*, IEEE Standard 346-1973, 1973.
16. Heising, C. R.: "Reliability of Electrical Power Transmission and Distribution Equipment," *Proceedings of the Reliability Engineering Conference for the Electrical Power Industry*, February 1974, Seattle, Wash.
17. Electric Power Research Institute: *Analysis of Distribution R&D Planning*, EPRI Report 329, Palo Alto, Calif., October 1975.
18. Howard, R. A.: *Dynamic Probabilistic Systems*, vol. I: *Markov Models*, Wiley, New York, 1971.
19. Markov, A., "Extension of the Limit Theorems of Probability Theory to a Sum of Variables Connected in a Chain," *Izv. Akad. Nauk St. Petersburg* (translated as *Notes of the Imperial Academy of Sciences of St. Petersburg*), Dec. 5, 1907.
20. Gönen, T., and M. Tahani: "Distribution System Reliability Performance," *IEEE Midwest Power Symposium*, Purdue University, West Lafayette, Ind., Oct. 27–28, 1980.
21. Gönen, T., et al.: *Development of Advanced Methods for Planning Electric Energy Distribution Systems*, U.S. Department of Energy, October 1979. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va.

APPENDIX

A

IMPEDANCE TABLES FOR LINES, TRANSFORMERS, AND UNDERGROUND CABLES

Table A-1 Characteristics of copper conductors, hard-drawn, 97.3 percent conductivity

Size of Conductor		Number of Strands	Diameter of Individual Strands Inches	Outside Diameter Inches	Breaking Strength Pounds	Weight Pounds per Mile	Approx Current Carrying Capacity* Amps	Geometric Mean Radius at 60 Cycles Feet	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor Per Mile At 1 Ft Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor Per Mile At 1 Ft Spacing		
									25°C (77°F)				50°C (122°F)									
Circular Mils	A.W.G. or B. & S.								d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
1 000 000		37	0.1644	1.151	43 830	16 300	1 300	0.0368	0.0585	0.0594	0.0620	0.0634	0.0640	0.0648	0.0672	0.0685	0.1666	0.333	0.400	0.216	0.1081	0.0901
900 000		37	0.1560	1.092	39 510	14 670	1 220	0.0349	0.0650	0.0658	0.0682	0.0695	0.0711	0.0718	0.0740	0.0752	0.1693	0.339	0.406	0.220	0.1100	0.0916
800 000		37	0.1470	1.029	35 120	13 040	1 130	0.0329	0.0731	0.0739	0.0760	0.0772	0.0800	0.0806	0.0826	0.0837	0.1722	0.344	0.413	0.224	0.1121	0.0934
750 000		37	0.1424	0.997	33 400	12 230	1 090	0.0319	0.0780	0.0787	0.0807	0.0818	0.0853	0.0859	0.0878	0.0888	0.1739	0.348	0.417	0.226	0.1132	0.0943
700 000		37	0.1375	0.963	31 170	11 410	1 040	0.0308	0.0836	0.0842	0.0861	0.0871	0.0914	0.0920	0.0937	0.0947	0.1759	0.352	0.422	0.229	0.1145	0.0954
600 000		37	0.1273	0.891	27 020	9 781	940	0.0285	0.0975	0.0981	0.0997	0.1006	0.1066	0.1071	0.1086	0.1095	0.1799	0.360	0.432	0.235	0.1173	0.0977
500 000		37	0.1162	0.814	22 510	8 151	840	0.0260	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1845	0.369	0.443	0.241	0.1205	0.1004
500 000		19	0.1622	0.811	21 590	8 151	840	0.0256	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1853	0.371	0.445	0.241	0.1206	0.1005
450 000		19	0.1539	0.770	19 750	7 336	780	0.0243	0.1300	0.1304	0.1316	0.1323	0.1422	0.1426	0.1437	0.1443	0.1879	0.376	0.451	0.245	0.1224	0.1020
400 000		19	0.1451	0.726	17 560	6 521	730	0.0229	0.1462	0.1466	0.1477	0.1484	0.1600	0.1603	0.1613	0.1619	0.1909	0.382	0.458	0.249	0.1245	0.1038
350 000		19	0.1357	0.679	15 590	5 706	670	0.0214	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1943	0.389	0.466	0.254	0.1269	0.1058
350 000		12	0.1708	0.710	16 140	5 706	690	0.0225	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1918	0.384	0.460	0.251	0.1253	0.1044
300 000		19	0.1257	0.629	13 510	4 891	610	0.01987	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1982	0.396	0.476	0.259	0.1296	0.1080
300 000		12	0.1581	0.657	13 170	4 891	610	0.0208	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1957	0.392	0.470	0.256	0.1281	0.1068
250 000		19	0.1147	0.574	11 360	4 076	540	0.01813	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.203	0.406	0.487	0.266	0.1329	0.1108
250 000		12	0.1443	0.600	11 130	4 076	540	0.01902	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.200	0.401	0.481	0.263	0.1313	0.1094
211 600	4/0	19	0.1055	0.528	9 617	3 450	480	0.01668	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.207	0.414	0.497	0.272	0.1359	0.1132
211 600	4/0	12	0.1328	0.552	9 483	3 450	490	0.01750	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.204	0.409	0.491	0.269	0.1343	0.1119
211 600	4/0	7	0.1739	0.522	9 154	3 450	480	0.01579	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.210	0.420	0.503	0.273	0.1363	0.1136
167 800	3/0	12	0.1183	0.492	7 556	2 736	420	0.01569	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.210	0.421	0.505	0.277	0.1384	0.1153
167 800	3/0	7	0.1548	0.464	7 366	2 736	420	0.01404	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.216	0.431	0.518	0.281	0.1405	0.1171
133 100	2/0	7	0.1379	0.414	5 926	2 170	360	0.01252	0.440	0.440	0.440	0.440	0.481	0.481	0.481	0.481	0.222	0.443	0.532	0.289	0.1445	0.1205
105 500	1/0	7	0.1228	0.368	4 752	1 720	310	0.01113	0.555	0.555	0.555	0.555	0.606	0.607	0.607	0.607	0.227	0.455	0.546	0.298	0.1488	0.1240
83 690	1	7	0.1093	0.328	3 804	1 364	270	0.00992	0.699	0.699	0.699	0.699	0.765				0.233	0.467	0.560	0.306	0.1528	0.1274
83 690	1	3	0.1670	0.360	3 620	1 351	270	0.01016	0.692	0.692	0.692	0.692	0.757				0.233	0.464	0.557	0.299	0.1495	0.1246
66 370	2	7	0.0974	0.292	3 045	1 082	230	0.00883	0.881	0.881	0.882	0.882	0.964				0.239	0.478	0.574	0.314	0.1570	0.1308
66 370	2	3	0.1487	0.320	2 913	1 071	240	0.00903	0.873				0.955				0.238	0.476	0.571	0.307	0.1537	0.1281
66 370	2	1		0.258	3 003	1 061	220	0.00836	0.864				0.945				0.242	0.484	0.581	0.323	0.1614	0.1345
52 630	3	7	0.0867	0.260	2 433	858	200	0.00787	1.112				1.216				0.245	0.490	0.588	0.322	0.1611	0.1343
52 630	3	3	0.1325	0.285	2 359	850	200	0.00805	1.101	Same as d-c			1.204	Same as d-c			0.244	0.488	0.585	0.316	0.1579	0.1316
52 630	3	1		0.229	2 439	841	190	0.00745	1.090				1.192				0.248	0.496	0.595	0.331	0.1656	0.1380
41 740	4	3	0.1180	0.254	1 879	674	180	0.00717	1.388				1.518				0.250	0.499	0.599	0.324	0.1619	0.1349
41 740	4	1		0.204	1 970	667	170	0.00663	1.374				1.503				0.254	0.507	0.609	0.339	0.1696	0.1415
33 100	5	3	0.1050	0.226	1 505	534	150	0.00638	1.750				1.914				0.256	0.511	0.613	0.332	0.1661	0.1384
33 100	5	1		0.1819	1 591	529	140	0.00590	1.733				1.895				0.260	0.519	0.623	0.348	0.1738	0.1449
26 250	6	3	0.0935	0.201	1 205	424	130	0.00568	2.21				2.41				0.262	0.523	0.628	0.341	0.1703	0.1419
26 250	6	1		0.1620	1 280	420	120	0.00526	2.18				2.39				0.265	0.531	0.637	0.356	0.1779	0.1483
20 820	7	1		0.1443	1 030	333	110	0.00468	2.75				3.01				0.271	0.542	0.651	0.364	0.1821	0.1517
16 510	8	1		0.1285	826	264	90	0.00417	3.47				3.80				0.277	0.554	0.665	0.372	0.1862	0.1552

* For conductor at 75°C, air at 25°C, wind 1.4 mi/h (2 ft/s), frequency = 60 cycles.
Source: [1, 2].

Table A-2 Characteristics of Anaconda hollow copper conductors

Design Number	Size of Conductor Circular Mils or A W G	Wires		Outside Diameter Inches	Breaking Strain Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps‡	r_a Resistance Ohms per Conductor per Mile				x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft Spacing		
									25°C (77°F)		50°C (122°F)							
		Number	Diameter Inches						d-c 25 cycles	50 cycles 60 cycles	d-c 25 cycles	50 cycles 60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
966	890 500	28	0.1610	1.650	36 000	15 085	0.0612	1395	0.0671	0.0676	0.0734	0.0739	0.1412	0.282	0.339	0.1907	0.0953	0.0794
96R1	750 000	42	0.1296	1.155	34 200	12 345	0.0408	1160	0.0786	0.0791	0.0860	0.0865	0.1617	0.323	0.388	0.216	0.1080	0.0900
939	650 000	50	0.1097	1.126	29 500	10 761	0.0406	1060	0.0909	0.0915	0.0994	0.1001	0.1621	0.324	0.389	0.218	0.1089	0.0908
360R1	600 000	50	0.1053	1.007	27 500	9 905	0.0387	1020	0.0984	0.0991	0.1077	0.1084	0.1644	0.329	0.395	0.221	0.1105	0.0921
938	550 000	50	0.1009	1.036	25 200	9 103	0.0373	960	0.1076	0.1081	0.1177	0.1183	0.1663	0.333	0.399	0.224	0.1119	0.0932
4R5	510 000	50	0.0970	1.000	22 700	8 485	0.0360	910	0.1173	0.1178	0.1283	0.1289	0.1681	0.336	0.404	0.226	0.1131	0.0943
892R3	500 000	18	0.1558	1.080	21 400	8 263	0.0394	900	0.1178	0.1184	0.1289	0.1296	0.1630	0.326	0.391	0.221	0.1104	0.0920
933	450 000	21	0.1353	1.074	19 300	7 476	0.0398	850	0.1319	0.1324	0.1443	0.1448	0.1630	0.326	0.391	0.221	0.1106	0.0922
924	400 000	21	0.1227	1.014	17 200	6 642	0.0376	810	0.1485	0.1491	0.1624	0.1631	0.1658	0.332	0.398	0.225	0.1126	0.0939
925R1	380 500	22	0.1211	1.003	16 300	6 331	0.0373	780	0.1565	0.1572	0.1712	0.1719	0.1663	0.333	0.399	0.226	0.1130	0.0942
565R1	350 000	21	0.1196	0.950	15 100	5 813	0.0353	750	0.1695	0.1700	0.1854	0.1860	0.1691	0.338	0.406	0.230	0.1150	0.0958
936	350 000	15	0.1444	0.860	15 400	5 776	0.0311	740	0.1690	0.1695	0.1849	0.1854	0.1754	0.351	0.421	0.237	0.1185	0.0988
378R1	350 000	30	0.1059	0.736	16 100	5 739	0.0253	700	0.1685	0.1690	0.1843	0.1849	0.1860	0.372	0.446	0.248	0.1241	0.1034
954	321 000	22	0.1113	0.920	13 850	5 343	0.0340	700	0.1851	0.1856	0.202	0.203	0.1710	0.342	0.410	0.232	0.1161	0.0968
935	300 000	18	0.1205	0.839	13 100	4 984	0.0307	670	0.1980	0.1985	0.216	0.217	0.1761	0.352	0.423	0.239	0.1194	0.0995
903R1	300 000	15	0.1338	0.797	13 200	4 953	0.0289	660	0.1969	0.1975	0.215	0.216	0.1793	0.359	0.430	0.242	0.1212	0.1010
178R2	300 000	12	0.1507	0.750	13 050	4 937	0.0266	650	0.1964	0.1969	0.215	0.216	0.1833	0.367	0.440	0.247	0.1234	0.1028
926	250 000	18	0.1100	0.766	10 950	4 155	0.0279	600	0.238	0.239	0.260	0.261	0.1810	0.362	0.434	0.245	0.1226	0.1022
915R1	250 000	15	0.1214	0.725	11 000	4 148	0.0266	590	0.237	0.238	0.259	0.260	0.1834	0.367	0.440	0.249	0.1246	0.1038
24R1	250 000	12	0.1368	0.683	11 000	4 133	0.0245	580	0.237	0.238	0.259	0.260	0.1876	0.375	0.450	0.253	0.1267	0.1066
923	4/0	18	0.1005	0.700	9 300	3 521	0.0255	530	0.281	0.282	0.307	0.308	0.1855	0.371	0.445	0.252	0.1258	0.1049
922	4/0	15	0.1109	0.663	9 300	3 510	0.0238	520	0.281	0.282	0.307	0.308	0.1889	0.378	0.453	0.256	0.1278	0.1065
50R2	4/0	14	0.1152	0.650	9 300	3 510	0.0234	520	0.280	0.281	0.306	0.307	0.1898	0.380	0.455	0.257	0.1285	0.1071
158R1	3/0	16	0.0961	0.606	7 500	2 785	0.0221	460	0.354	0.355	0.387	0.388	0.1928	0.386	0.463	0.262	0.1310	0.1091
495R1	3/0	15	0.0996	0.595	7 600	2 785	0.0214	460	0.353	0.354	0.386	0.387	0.1943	0.389	0.466	0.263	0.1316	0.1097
570R2	3/0	12	0.1123	0.560	7 600	2 772	0.0201	450	0.352	0.353	0.385	0.386	0.1976	0.395	0.474	0.268	0.1338	0.1115
909R2	2/0	15	0.0880	0.530	5 950	2 213	0.0191	370	0.446	0.446	0.487	0.487	0.200	0.400	0.481	0.271	0.1357	0.1131
412R2	2/0	14	0.0913	0.515	6 000	2 207	0.0184	370	0.446	0.446	0.487	0.487	0.202	0.404	0.485	0.274	0.1368	0.1140
937	2/0	13	0.0950	0.505	6 000	2 203	0.0181	370	0.446	0.446	0.487	0.487	0.203	0.406	0.487	0.275	0.1375	0.1146
930	125 600	14	0.0885	0.500	5 650	2 083	0.0180	360	0.473	0.473	0.517	0.517	0.203	0.406	0.487	0.276	0.1378	0.1149
934	121 300	15	0.0836	0.500	5 400	2 015	0.0179	350	0.491	0.491	0.537	0.537	0.203	0.407	0.488	0.276	0.1378	0.1149
901	119 400	12	0.0936	0.470	5 300	1 979	0.0165	340	0.507	0.507	0.555	0.555	0.207	0.415	0.498	0.280	0.1400	0.1167

‡ For conductor at 75°C, air at 25°C, wind 1.4 mi/h (2 ft/s), frequency = 60 cycles, average tarnished surface.

Source: [1, 2].

Table A-3 Characteristics of General Cable type HH hollow copper conductors

Conductor Size Circular Mils or A.W.G.	Outside[1] Diameter Inches	Wall Thickness Inches	Weight Pounds per Mile	Breaking Strength Pounds	Geometric Mean Radius Feet	Approx. Current Carrying Capacity[2] Amps	r_a Resistance Ohms per Conductor per Mile, 25°C (77°F.)				50°C. (122°F.)				x_a Inductive Reactance Ohms per Conductor per Mile at 1 Foot Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Foot Spacing		
							d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
1 000 000	2.103	0.150*	16 160	43 190	0.0833	1620	0.0576	0.0576	0.0577	0.0577	0.0630	0.0630	0.0631	0.0631	0.1257	0.251	0.302	0.1734	0.0867	0.0722
950 000	2.035	0.147*	15 350	41 030	0.0805	1565	0.0606	0.0606	0.0607	0.0607	0.0663	0.0664	0.0664	0.0664	0.1274	0.255	0.306	0.1757	0.0879	0.0732
900 000	1.966	0.144*	14 540	38 870	0.0778	1505	0.0640	0.0640	0.0641	0.0641	0.0700	0.0701	0.0701	0.0701	0.1291	0.258	0.310	0.1782	0.0891	0.0742
850 000	1.901	0.140*	13 730	36 710	0.0751	1450	0.0677	0.0678	0.0678	0.0678	0.0741	0.0742	0.0742	0.0742	0.1309	0.262	0.314	0.1805	0.0903	0.0752
800 000	1.820	0.137*	12 920	34 550	0.0722	1390	0.0720	0.0720	0.0720	0.0721	0.0788	0.0788	0.0788	0.0788	0.1329	0.266	0.319	0.1833	0.0917	0.0764
790 000	1.650	0.131†	12 760	34 120	0.0646	1335	0.0729	0.0729	0.0730	0.0730	0.0797	0.0798	0.0799	0.0799	0.1385	0.277	0.332	0.1906	0.0953	0.0794
750 000	1.750	0.133*	12 120	32 390	0.0691	1325	0.0768	0.0768	0.0768	0.0769	0.0840	0.0840	0.0841	0.0841	0.1351	0.270	0.324	0.1864	0.0932	0.0777
700 000	1.686	0.130*	11 310	30 230	0.0665	1265	0.0822	0.0823	0.0823	0.0823	0.0900	0.0900	0.0901	0.0901	0.1370	0.274	0.329	0.1891	0.0945	0.0788
650 000	1.610	0.126*	10 500	28 070	0.0635	1200	0.0886	0.0886	0.0886	0.0887	0.0969	0.0970	0.0970	0.0970	0.1394	0.279	0.335	0.1924	0.0962	0.0802
600 000	1.558	0.123*	9 692	25 910	0.0615	1140	0.0959	0.0960	0.0900	0.0960	0.1050	0.1051	0.1051	0.1051	0.1410	0.282	0.338	0.1947	0.0974	0.0811
550 000	1.478	0.119*	8 884	23 750	0.0583	1075	0.1047	0.1048	0.1048	0.1048	0.1146	0.1146	0.1147	0.1147	0.1437	0.287	0.345	0.1985	0.0992	0.0827
512 000	1.400	0.115*	8 270	22 110	0.0551	1020	0.1124	0.1125	0.1125	0.1125	0.1230	0.1230	0.1231	0.1231	0.1466	0.293	0.352	0.202	0.1012	0.0843
500 000	1.390	0.115*	8 076	21 590	0.0547	1005	0.1151	0.1151	0.1152	0.1152	0.1259	0.1260	0.1260	0.1260	0.1469	0.294	0.353	0.203	0.1014	0.0845
500 000	1.268	0.109†	8 074	21 590	0.0494	978	0.1151	0.1152	0.1152	0.1152	0.1259	0.1260	0.1260	0.1261	0.1521	0.304	0.365	0.209	0.1047	0.0872
500 000	1.100	0.130†	8 068	21 590	0.0420	937	0.1150	0.1151	0.1152	0.1153	0.1258	0.1259	0.1260	0.1260	0.1603	0.321	0.385	0.219	0.1098	0.0915
500 000	1.020	0.144†	8 063	21 590	0.0384	915	0.1150	0.1150	0.1152	0.1152	0.1258	0.1259	0.1260	0.1261	0.1648	0.330	0.396	0.225	0.1124	0.0937
450 000	1.317	0.111*	7 268	19 430	0.0518	939	0.1279	0.1280	0.1280	0.1280	0.1400	0.1401	0.1401	0.1401	0.1496	0.299	0.359	0.207	0.1033	0.0861
450 000	1.188	0.105†	7 266	19 430	0.0462	910	0.1278	0.1279	0.1279	0.1280	0.1399	0.1400	0.1400	0.1401	0.1554	0.311	0.373	0.214	0.1070	0.0892
400 000	1.218	0.106*	6 460	17 270	0.0478	864	0.1439	0.1440	0.1440	0.1440	0.1575	0.1576	0.1576	0.1576	0.1537	0.307	0.369	0.212	0.1061	0.0884
400 000	1.103	0.100†	6 458	17 270	0.0428	838	0.1438	0.1439	0.1439	0.1440	0.1574	0.1575	0.1575	0.1576	0.1593	0.319	0.382	0.219	0.1097	0.0914
350 000	1.128	0.102*	5 653	15 110	0.0443	790	0.1644	0.1645	0.1645	0.1645	0.1799	0.1800	0.1800	0.1800	0.1576	0.315	0.378	0.218	0.1089	0.0907
350 000	1.014	0.096†	5 650	15 110	0.0393	764	0.1644	0.1645	0.1645	0.1646	0.1799	0.1800	0.1800	0.1801	0.1637	0.328	0.393	0.225	0.1127	0.0939
300 000	1.020	0.096*	4 845	12 950	0.0399	709	0.1918	0.1919	0.1919	0.1919	0.210	0.210	0.210	0.210	0.1628	0.326	0.391	0.225	0.1124	0.0937
300 000	0.919	0.091†	4 843	12 950	0.0355	687	0.1917	0.1918	0.1918	0.1919	0.210	0.210	0.210	0.210	0.1688	0.338	0.405	0.232	0.1162	0.0968
250 000	0.914	0.091*	4 037	10 790	0.0357	626	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1685	0.337	0.404	0.233	0.1163	0.0970
250 000	0.818	0.086†	4 036	10 790	0.0315	606	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1748	0.350	0.420	0.241	0.1203	0.1002
250 000	0.766	0.094†	4 034	10 790	0.0292	594	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1787	0.357	0.429	0.245	0.1226	0.1022
214 500	0.650	0.098†	3 459	9 265	0.0243	524	0.268	0.268	0.268	0.268	0.293	0.293	0.293	0.294	0.1879	0.376	0.451	0.257	0.1285	0.1071
4/0	0.733	0.082†	3 415	9 140	0.0281	539	0.272	0.272	0.272	0.272	0.297	0.297	0.298	0.298	0.1806	0.361	0.433	0.248	0.1242	0.1035
3/0	0.608	0.080†	2 707	7 240	0.0230	454	0.343	0.343	0.343	0.343	0.375	0.375	0.375	0.375	0.1907	0.381	0.458	0.262	0.1309	0.1091
2/0	0.500	0.080†	2 146	5 750	0.0186	382	0.432	0.432	0.432	0.432	0.472	0.473	0.473	0.473	0.201	0.403	0.483	0.276	0.1378	0.1149

(1) Conductors of smaller diameter for given cross-sectional area also available; in the naught sizes, some additional diameter expansion is possible.
(2) For conductor at 75°C, air at 25°C, wind 1.4 mi/h (2 ft/s), frequency = 60 cycles.
* Thickness at edges of interlocked segments.
† Thickness uniform throughout.
Source: [1, 2].

Table A-4 Characteristics of Alcoa aluminum conductors, hard-drawn, 61 percent conductivity

Size of Conductor: Circular Mils or AWG	No. of Strands	Diameter of Individual Strands, Inches	Outside Diameter, Inches	Ultimate Strength, Pounds	Weight, Pounds per Mile	Geometric Mean Radius at 60 Cycles, Feet	Approx. Current-Carrying Capacity*, Amps	r_a Resistance, Ohms per Conductor per Mile, 25°C (77°F): d-c	25°C: 25 cycles	25°C: 50 cycles	25°C: 60 cycles	60°C (122°F): d-c	60°C: 25 cycles	60°C: 50 cycles	60°C: 60 cycles	x_a Inductive Reactance, Ohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	x_a: 50 cycles	x_a: 60 cycles	x'_a Shunt Capacitive Reactance, Megohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	x'_a: 50 cycles	x'_a: 60 cycles
6	7	0.0612	0.184	528	130	[illegible]	100	3.56	3.56	3.56	3.56	3.91	3.91	3.91	3.91	0.2626	0.5251	0.6301	0.3468	0.1734	0.1445
4	7	0.0772	0.232	826	207	[illegible]	134	2.24	2.24	2.24	2.24	2.46	2.46	2.46	2.46	0.2506	0.5017	[illegible]	0.3307	0.1651	[illegible]
3	7	0.0867	0.260	1022	261	[illegible]	165	1.77	1.77	1.77	1.77	1.95	1.95	1.95	1.95	[illegible]	[illegible]	0.5879	0.3221	0.1610	0.1342
2	7	0.0974	0.292	1266	329	0.00883	180	1.41	1.41	1.41	1.41	1.55	1.55	1.55	1.55	0.2191	[illegible]	[illegible]	0.3139	[illegible]	0.1308
1	7	0.1094	0.328	1537	414	0.00992	209	1.12	1.12	1.12	1.12	1.23	1.23	1.23	1.23	[illegible]	0.4665	0.5598	0.3055	0.1528	0.1273
1/0	7	0.1228	0.368	1865	523	0.0111[illegible]	[illegible]	0.885	[illegible]	0.8853	0.885	0.973	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.5434	0.2976	0.1488	0.1240
1/0	19	0.0745	0.373	2090	523	0.011[illegible]	[illegible]	0.885	[illegible]	0.8853	0.885	0.973	[illegible]	[illegible]	0.973	[illegible]	[illegible]	0.5391	[illegible]	0.1482	0.1235
2/0	7	0.1379	0.414	2350	659	0.01251	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.771	[illegible]	[illegible]	[illegible]	[illegible]	0.4431	0.531[illegible]	[illegible]	0.1446	0.1204
2/0	19	0.0837	0.419	2586	659	0.01321	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1441	0.1201
3/0	7	0.1548	0.464	2845	832	0.01404	327	[illegible]	[illegible]	[illegible]	0.558	0.612	[illegible]	0.6124	0.613	[illegible]	0.4314	0.5177	0.2810	0.1405	0.1171
3/0	19	0.0940	0.470	3200	832	0.01483	328	0.557	[illegible]	[illegible]	0.558	0.612	[illegible]	0.6124	0.613	0.2179	[illegible]	[illegible]	[illegible]	0.1400	0.1167
4/0	7	[illegible]	0.522	3590	1049	0.0157	380	0.441	0.4411	0.4415	0.442	0.485	0.4851	0.4855	0.486	[illegible]	0.4196	0.5036	[illegible]	0.1363	0.1136
4/0	19	[illegible]	0.528	3890	1049	0.01668	381	0.441	0.4411	0.4415	0.442	0.485	0.4851	0.4855	0.486	0.2071	0.4141	0.4969	[illegible]	0.1358	0.1132
250 000	37	[illegible]	0.575	4860	1239	0.01841	423	0.374	0.3741	0.3746	0.375	0.411	0.4111	0.4115	0.412	[illegible]	0.4040	0.4848	[illegible]	0.1329	0.1107
266 800	[illegible]	[illegible]	0.586	4525	[illegible]	0.01771	441	0.350	0.3502	0.3506	0.351	0.385	[illegible]	0.3855	0.386	0.2040	[illegible]	0.4895	0.2642	0.1321	0.1101
[illegible]	37	[illegible]	0.594	5180	[illegible]	0.01902	443	0.350	0.3502	0.3506	0.351	0.385	0.3852	[illegible]	[illegible]	[illegible]	[illegible]	0.4806	[illegible]	[illegible]	0.1097
300 000	19	0.1257	0.629	5300	1487	0.01983	478	0.311	0.3112	0.3117	0.312	0.342	[illegible]	0.3425	0.343	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1080
300 000	37	0.0900	0.630	5830	1487	0.02017	478	0.311	0.3112	0.3117	0.312	0.342	[illegible]	0.3426	0.343	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1080
336 400	19	0.1331	0.666	5940	1667	0.02100	514	[illegible]	[illegible]	0.2788	[illegible]	0.306	[illegible]	[illegible]	0.307	0.1953	0.3907	[illegible]	0.2551	[illegible]	[illegible]
336 400	37	0.0954	0.668	6400	1667	0.02135	514	0.278	[illegible]	[illegible]	0.279	0.306	0.3062	[illegible]	0.307	0.1945	[illegible]	[illegible]	0.2549	[illegible]	[illegible]
350 000	37	[illegible]	0.681	6680	1735	[illegible]	528	0.267	[illegible]	[illegible]	0.268	0.294	[illegible]	[illegible]	0.295	0.1935	0.3870	0.4644	0.233	[illegible]	0.1057
[illegible]	19	[illegible]	0.724	6880	[illegible]	[illegible]	576	0.235	[illegible]	[illegible]	[illegible]	0.258	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.2491	[illegible]	0.1038
[illegible]	19	[illegible]	0.793	8090	2384	0.02501	646	0.196	0.1963	[illegible]	0.198	0.215	0.2153	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1012
[illegible]	19	[illegible]	0.812	8476	[illegible]	[illegible]	694	[illegible]	[illegible]	0.1882	0.189	0.206	0.2062	[illegible]	[illegible]	0.1853	[illegible]	0.4448	0.2412	[illegible]	[illegible]
[illegible]	37	[illegible]	0.813	9010	2478	[illegible]	694	0.187	0.1873	[illegible]	0.189	0.206	0.2062	[illegible]	[illegible]	0.1845	[illegible]	0.442	0.2410	[illegible]	[illegible]
556 500	19	[illegible]	0.856	9440	[illegible]	[illegible]	710	0.168	0.1693	[illegible]	0.170	0.185	[illegible]	[illegible]	0.187	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	37	0.1311	0.918	11240	[illegible]	[illegible]	[illegible]	[illegible]	0.1474	[illegible]	[illegible]	0.162	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.4283	[illegible]	0.1182	[illegible]
[illegible]	37	0.1391	[illegible]	[illegible]	[illegible]	[illegible]	817	[illegible]	[illegible]	[illegible]	0.133	0.144	0.1444	[illegible]	0.146	[illegible]	[illegible]	0.4210	[illegible]	0.1141	[illegible]
[illegible]	37	0.1424	0.997	[illegible]	[illegible]	[illegible]	864	0.125	0.1254	[illegible]	0.127	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	61	0.1109	0.998	13510	[illegible]	[illegible]	864	0.125	0.1254	[illegible]	0.127	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	37	[illegible]	1.026	13770	3940	[illegible]	897	[illegible]	0.1175	[illegible]	0.120	[illegible]	0.1204	[illegible]	[illegible]	[illegible]	[illegible]	0.4145	[illegible]	0.1122	[illegible]
[illegible]	37	[illegible]	1.077	[illegible]	4334	[illegible]	949	[illegible]	0.1075	[illegible]	0.110	0.118	0.1185	[illegible]	[illegible]	[illegible]	[illegible]	0.4088	[illegible]	0.1106	[illegible]
[illegible]	37	[illegible]	[illegible]	[illegible]	4728	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.100	0.108	0.1085	[illegible]	[illegible]	[illegible]	[illegible]	0.4036	[illegible]	[illegible]	[illegible]
[illegible]	61	[illegible]	1.152	[illegible]	4956	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.103	0.1035	[illegible]	[illegible]	[illegible]	[illegible]	0.3998	[illegible]	[illegible]	[illegible]
[illegible]	91	[illegible]	1.153	[illegible]	4956	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.103	0.1035	[illegible]	[illegible]	[illegible]	[illegible]	0.3994	[illegible]	[illegible]	[illegible]
[illegible]	37	[illegible]	1.170	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	61	0.1351	1.216	[illegible]	[illegible]	[illegible]	1110	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1 192 500	61	0.1398	1.258	[illegible]	[illegible]	[illegible]	1160	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3892	[illegible]	[illegible]	[illegible]
[illegible]	91	0.1145	[illegible]	[illegible]	[illegible]	[illegible]	1160	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3888	[illegible]	[illegible]	[illegible]
[illegible]	61	0.1444	[illegible]	[illegible]	[illegible]	[illegible]	1210	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3211	0.3851	[illegible]	[illegible]	[illegible]
[illegible]	61	[illegible]	1.340	[illegible]	6700	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3818	[illegible]	[illegible]	[illegible]
[illegible]	61	[illegible]	[illegible]	[illegible]	7091	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0759	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3123	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	61	[illegible]	1.454	[illegible]	7883	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	91	[illegible]	1.454	[illegible]	7883	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3713	[illegible]	[illegible]	[illegible]

* For conductor at 75°C, wind 1.4 mi/h (2 ft/s), frequency = 60 cycles.

Source: [1, 2].

Table A-5 Characteristics of Alcoa aluminum cable, steel-reinforced

Circular Mils or A W G Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent* Circular Mils or A W G	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity† Amps	r_a Resistance, Ohms per Conductor per Mile, 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a, 50°C (122°F) Current Approx. 75% Capacity‡: d-c	25 cycles	50 cycles	60 cycles	x_a Inductive Reactance, Ohms per Conductor per Mile at 1 Ft. Spacing, All Currents: 25 cycles	50 cycles	60 cycles	x_a' Shunt Capacitive Reactance, Megohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	50 cycles	60 cycles
1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.1495	0.299	0.359	0.1953	0.0977	0.0814
1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.1508	0.302	0.362	0.1971	0.0986	0.0821
1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.1522	0.304	0.365	0.1991	0.0996	0.0830
1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.1536	0.307	0.369	0.201	0.1006	0.0838
1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.1551	0.310	0.372	0.203	0.1016	0.0847
1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.1568	0.314	0.376	0.206	0.1028	0.0857
1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.1585	0.317	0.380	0.208	0.1040	0.0867
1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.1603	0.321	0.385	0.211	0.1053	0.0878
954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.1624	0.325	0.390	0.214	0.1068	0.0890
900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.1639	0.328	0.393	0.216	0.1078	0.0898
874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.1646	0.329	0.395	0.217	0.1083	0.0903
795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.1670	0.334	0.401	0.220	0.1100	0.0917
795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1660	0.332	0.399	0.219	0.1095	0.0912
795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1637	0.327	0.393	0.217	0.1085	0.0904
715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.1697	0.339	0.407	0.224	0.1119	0.0932
715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1687	0.337	0.405	0.223	0.1114	0.0928
715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1664	0.333	0.399	0.221	0.1104	0.0920
666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.1715	0.343	0.412	0.226	0.1132	0.0943
636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.1726	0.345	0.414	0.228	0.1140	0.0950
636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1718	0.344	0.412	0.227	0.1135	0.0946
636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1693	0.339	0.406	0.225	0.1125	0.0937
605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.1739	0.348	0.417	0.230	0.1149	0.0957
605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.1730	0.346	0.415	0.229	0.1144	0.0953
556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1751	0.350	0.420	0.232	0.1159	0.0965
556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1728	0.346	0.415	0.230	0.1149	0.0957
500 000	30	2	0.1291	7	0.1291	0.904	314 500	24 400	4 122	0.0311	690	0.187	0.187	0.187	0.187	0.206				0.1754	0.351	0.421	0.234	0.1167	0.0973
477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.1790	0.358	0.430	0.237	0.1186	0.0988
477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.1766	0.353	0.424	0.235	0.1176	0.0980
397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.1834	0.367	0.441	0.244	0.1219	0.1015
397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as d-c		0.259		Same as d-c		0.1812	0.362	0.435	0.242	0.1208	0.1006
336 400	26	2	0.1138	7	0.0885	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.1872	0.376	0.451	0.250	0.1248	0.1039
336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.1855	0.371	0.445	0.248	0.1240	0.1032
300 000	26	2	0.1074	7	0.0835	0.680	188 700	13 850	2 178	0.0230	490	0.311				0.342				0.1908	0.382	0.458	0.254	0.1269	0.1056
300 000	30	2	0.1000	7	0.1000	0.700	188 700	16 190	2 473	0.0241	500	0.311				0.342				0.1883	0.377	0.452	0.252	0.1258	0.1049
266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.1934	0.387	0.465	0.256	0.1281	0.1068

Single Layer Conductors:

Circular Mils or A W G Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent* Circular Mils or A W G	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet, For Current Approx. 75% Capacity‡	Approx. Current Carrying Capacity† Amps	r_a 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a 50°C (122°F) Current Approx. 75% Capacity‡: d-c	25 cycles	50 cycles	60 cycles	x_a Small Currents: 25 cycles	50 cycles	60 cycles	x_a Current Approx. 75% Capacity‡: 25 cycles	50 cycles	60 cycles	x_a': 25 cycles	50 cycles	60 cycles
266 800	6	1	0.2109	7	0.0703	0.633	3/0	9 645	1 802	0.00684	460	0.351	0.351	0.351	0.352	0.386	0.430	0.510	0.552	0.194	0.388	0.466	0.252	0.504	0.605	0.259	0.1294	0.1079
4/0	6	1	0.1878	1	0.1878	0.563	2/0	8 420	1 542	0.00814	340	0.441	0.442	0.444	0.445	0.485	0.514	0.567	0.592	0.218	0.437	0.524	0.242	0.484	0.581	0.267	0.1336	0.1113
3/0	6	1	0.1672	1	0.1672	0.502	1/0	6 675	1 223	0.00600	300	0.556	0.557	0.559	0.560	0.612	0.642	0.697	0.723	0.225	0.450	0.540	0.259	0.517	0.621	0.273	0.1377	0.1147
2/0	6	1	0.1490	1	0.1490	0.447	1	5 345	970	0.00510	270	0.702	0.702	0.704	0.706	0.773	0.806	0.866	0.895	0.231	0.462	0.554	0.267	0.534	0.641	0.284	0.1418	0.1182
1/0	6	1	0.1327	1	0.1327	0.398	2	4 280	769	0.00446	230	0.885	0.885	0.887	0.888	0.974	1.01	1.08	1.12	0.237	0.473	0.568	0.273	0.547	0.656	0.292	0.1460	0.1216
1	6	1	0.1182	1	0.1182	0.355	3	3 480	610	0.00418	200	1.12	1.12	1.12	1.12	1.23	1.27	1.34	1.38	0.242	0.483	0.580	0.277	0.554	0.665	0.300	0.1500	0.1250
2	6	1	0.1052	1	0.1052	0.316	4	2 790	484	0.00418	180	1.41	1.41	1.41	1.41	1.55	1.59	1.66	1.69	0.247	0.493	0.592	0.277	0.554	0.665	0.306	0.1542	0.1285
2	7	1	0.0974	1	0.1299	0.325	4	3 525	566	0.00504	180	1.41	1.41	1.41	1.41	1.55	1.59	1.62	1.65	0.247	0.493	0.592	0.267	0.535	0.642	0.306	0.1532	0.1276
3	6	1	0.0937	1	0.0937	0.281	5	2 250	384	0.00430	160	1.78	1.78	1.78	1.78	1.95	1.95	2.04	2.07	0.252	0.503	0.604	0.275	0.551	0.661	0.317	0.1583	0.1320
4	6	1	0.0834	1	0.0834	0.250	6	1 830	304	0.00437	140	2.24	2.24	2.24	2.24	2.47	2.50	2.54	2.57	0.257	0.514	0.611	0.274	0.549	0.659	0.325	0.1627	0.1355
4	7	1	0.0772	1	0.1029	0.257	6	2 288	356	0.00452	140	2.24	2.24	2.24	2.24	2.47	2.50	2.53	2.55	0.257	0.515	0.616	0.273	0.545	0.654	0.323	0.1615	0.1346
5	6	1	0.0743	1	0.0743	0.223	7	1 460	241	0.00416	120	2.82	2.82	2.82	2.82	3.10	3.12	3.16	3.18	0.262	0.525	0.630	0.279	0.557	0.668	0.334	0.1666	0.1388
6	6	1	0.0661	1	0.0661	0.198	8	1 170	191	0.00394	100	3.56	3.56	3.56	3.56	3.92	3.94	3.97	3.98	0.266	0.534	0.643	0.281	0.561	0.673	0.342	0.1708	0.1423

* Based on copper 97 percent, aluminum 61 percent conductivity.

† For conductor at 75°C, air at 25°C wind 1.4 mi/h (2 ft/s), frequency = 60 cycles.

‡ "Current approx. 75% capacity" is 75% of the "Approx. current carrying capacity in amps." and is approximately the current which will produce 60°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 mi/h.

Source: [1].

Table A-6 Characteristics of Alcoa "Expanded" aluminum cable, steel-reinforced

Circular Mils or A W G Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Filler Section Aluminum Strands	Filler Section Aluminum Strand Dia. Inches	Filler Section Paper Strands	Filler Section Paper Layers	Outside Diameter Inches	Copper Equivalent Circular Mils or A W G	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps	r_a Resistance, Ohms per Conductor per Mile, 25°C (77°F) Small Currents: d-c, 25, 50, 60 cycles	r_a 50°C (122°F) Current Approx. 75% Capacity: d-c, 25, 50, 60 cycles	x_a Inductive Reactance, Ohms per Conductor per Mile at 1 Ft. Spacing, All Currents: 25 cycles	50 cycles	60 cycles	x_a' Shunt Capacitive Reactance, Megohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	50 cycles	60 cycles
850 000	54	2	0.1255	19	0.0834	4	0.1182	23	2	1.240	534 000	35 371	7 200	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
1 150 000	54	2	0.1409	19	0.0921	4	0.1353	24	2	1.385	724 000	41 900	9 070										
1 338 000	66	2	0.1350	19	0.100	4	[illegible]	19	2	1.750	840 000	49 278	11 340										

(1) Electrical characteristics not available until laboratory measurements are completed.

Source: [1].

Table A-7 Characteristics of Copperweld copper conductors

Nominal Designation	Size of Conductor: Number and Diameter of Wires: Copperweld	Copper	Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Rated Breaking Load Lbs	Weight Lbs per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current carrying capacity at 60 Cycles Amps*	r_a Resistance Ohms per Conductor per Mile at 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a Resistance Ohms per Conductor per Mile at 50°C (122°F) Currents Approx. 75% of Capacity**: d-c	25 cycles	50 cycles	60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile One ft Spacing Average Currents: 25 cycles	50 cycles	60 cycles	x_a' Capacitive Reactance Megohms per Conductor per Mile One ft Spacing: 25 cycles	50 cycles	60 cycles
350 E	7x.1576"	12x.1576"	0.788	350 000	32 420	7 409	0.0220	660	0.1658	0.1728	0.1789	0.1812	0.1812	0.1915	0.201	0.204	0.1929	0.386	0.463	0.243	0.1216	0.1014
350 EK	4x.1470"	15x.1470"	0.735	350 000	43 850	6 536	0.0245	660	0.1658	0.1682	0.1700	0.1705	0.1812	0.1845	0.1873	0.1882	0.1875	0.375	0.450	0.248	0.1241	0.1034
350 V	3x.1751"	9x.1893"	0.754	350 000	31 060	6 573	0.0226	650	0.1655	0.1725	0.1800	0.1828	0.1809	0.1910	0.202	0.206	0.1915	0.383	0.460	0.246	0.1232	0.1027
300 E	7x.1459"	12x.1459"	0.729	300 000	27 770	6 351	0.0204	600	0.1934	0.200	0.207	0.209	0.211	0.222	0.232	0.235	0.1969	0.394	0.473	0.249	0.1244	0.1037
300 EK	4x.1361"	15x.1361"	0.680	300 000	30 960	5 602	0.0227	610	0.1934	0.1958	0.1976	0.198	0.211	0.215	0.218	0.219	0.1914	0.383	0.460	0.254	0.1269	0.1067
300 V	3x.1621"	9x.1752"	0.698	300 000	20 730	5 639	0.0209	590	0.1930	0.200	0.208	0.210	0.211	0.222	0.233	0.237	0.1954	0.391	0.469	0.252	0.1259	0.1060
250 E	7x.1332"	12x.1332"	0.666	250 000	23 920	5 292	0.01856	540	0.232	0.239	0.245	0.248	0.254	0.265	0.275	0.279	0.202	0.403	0.484	0.256	0.1276	0.1064
250 EK	4x.1242"	15x.1242"	0.621	250 000	27 840	4 669	0.0207	540	0.232	0.235	0.236	0.237	0.254	0.258	0.261	0.261	0.1960	0.392	0.471	0.260	0.1301	0.1084
250 V	3x.1480"	9x.1600"	0.637	250 000	22 420	4 699	0.01911	530	0.232	0.239	0.246	0.249	0.253	0.264	0.276	0.281	0.200	0.400	0.480	0.258	0.1292	0.1077
4/0 E	7x.1225"	12x.1225"	0.613	4/0	20 730	4 479	0.01711	480	0.274	0.281	0.287	0.290	0.300	0.312	0.323	0.326	0.206	0.411	0.493	0.261	0.1306	0.1088
4/0 G	2x.1944"	5x.1944"	0.583	4/0	16 640	4 168	0.01409	460	0.273	0.284	0.294	0.298	0.299	0.318	0.336	0.342	0.215	0.431	0.517	0.265	0.1324	0.1103
4/0 EK	4x.1143"	15x.1143"	0.571	4/0	15 370	3 951	0.01903	490	0.274	0.277	0.278	0.279	0.300	0.304	0.307	0.308	0.200	0.401	0.481	0.266	0.1331	0.1109
4/0 V	3x.1361"	9x.1472"	0.586	4/0	15 000	3 977	0.01758	470	0.274	0.281	0.286	0.291	0.299	0.311	0.323	0.328	0.204	0.409	0.490	0.264	0.1322	0.1101
4/0 F	1x.1833"	6x.1833"	0.550	4/0	11 290	3 750	0.01558	470	0.273	0.280	0.285	0.287	0.299	0.309	0.318	0.322	0.210	0.421	0.505	0.269	0.1344	0.1220
3/0 E	7x.1091"	12x.1091"	0.545	3/0	16 800	3 552	0.01521	420	0.346	0.353	0.359	0.361	0.378	0.391	0.402	0.407	0.212	0.423	0.508	0.270	0.1348	0.1123
3/0 J	3x.1851"	4x.1851"	0.555	3/0	18 170	3 732	0.01156	410	0.344	0.356	0.367	0.372	0.377	0.398	0.419	0.428	0.225	0.451	0.541	0.268	0.1341	0.1118
3/0 G	2x.1731"	2x.1731"	0.519	3/0	13 860	3 305	0.01254	400	0.344	0.355	0.365	0.369	0.377	0.397	0.416	0.423	0.221	0.443	0.531	0.273	0.1365	0.1137
3/0 EK	4x.1018"	4x.1018"	0.509	3/0	12 370	3 134	0.01697	420	0.346	0.348	0.360	0.361	0.378	0.382	0.386	0.386	0.206	0.412	0.495	0.274	0.1372	0.1143
3/0 V	3x.1311"	9x.1311"	0.522	3/0	12 220	3 154	0.01566	410	0.345	0.352	0.360	0.362	0.377	0.390	0.403	0.408	0.210	0.420	0.504	0.273	0.1363	0.1136
3/0 F	1x.1632"	6x.1632"	0.490	3/0	9 980	2 974	0.01388	410	0.344	0.351	0.356	0.358	0.377	0.388	0.397	0.401	0.216	0.432	0.519	0.277	0.1385	0.1155
2/0 K	4x.1780"	3x.1780"	0.534	2/0	17 600	3 411	0.00912	360	0.434	0.447	0.459	0.466	0.476	0.499	0.524	0.535	0.237	0.475	0.570	0.271	0.1355	0.1129
2/0 J	3x.1648"	4x.1648"	0.494	2/0	13 430	2 960	0.01029	350	0.434	0.446	0.457	0.462	0.475	0.498	0.520	0.530	0.231	0.463	0.555	0.277	0.1383	0.1152
2/0 G	2x.1542"	5x.1542"	0.463	2/0	10 510	2 623	0.01119	350	0.434	0.445	0.456	0.459	0.475	0.497	0.518	0.526	0.227	0.454	0.545	0.281	0.1406	0.1171
2/0 V	3x.1080"	9x.1167"	0.465	2/0	9 846	2 502	0.01395	360	0.435	0.442	0.450	0.452	0.476	0.489	0.504	0.509	0.216	0.432	0.518	0.281	0.1404	0.1170
2/0 F	1x.1454"	6x.1454"	0.436	2/0	8 094	2 356	0.01235	350	0.434	0.441	0.446	0.448	0.475	0.487	0.497	0.501	0.222	0.444	0.533	0.286	0.1430	0.1189
1/0 K	4x.1585"	3x.1585"	0.475	1/0	14 490	2 703	0.00812	310	0.548	0.560	0.573	0.579	0.599	0.625	0.652	0.664	0.243	0.487	0.584	0.279	0.1397	0.1164
1/0 J	3x.1467"	4x.1467"	0.440	1/0	10 970	2 348	0.00917	310	0.548	0.559	0.570	0.576	0.600	0.624	0.648	0.659	0.237	0.474	0.569	0.285	0.1423	0.1186
1/0 G	2x.1373"	5x.1373"	0.412	1/0	8 563	2 079	0.00996	310	0.548	0.559	0.568	0.573	0.599	0.623	0.645	0.654	0.233	0.466	0.559	0.289	0.1447	0.1206
1/0 F	1x.1294"	6x.1294"	0.388	1/0	6 536	1 876	0.01099	310	0.548	0.554	0.559	0.562	0.599	0.612	0.622	0.627	0.228	0.456	0.547	0.294	0.1469	0.1224
1 N	5x.1546"	2x.1546"	0.464	1	15 410	2 541	0.00638	280	0.691	0.705	0.719	0.726	0.755	0.787	0.818	0.832	0.256	0.512	0.614	0.281	0.1405	0.1171
1 K	4x.1412"	3x.1412"	0.423	1	11 900	2 144	0.00723	270	0.691	0.704	0.716	0.722	0.756	0.784	0.813	0.825	0.249	0.498	0.598	0.288	0.1438	0.1198
1 J	3x.1307"	4x.1307"	0.392	1	9 000	1 861	0.00817	270	0.691	0.703	0.714	0.719	0.755	0.783	0.808	0.820	0.243	0.486	0.583	0.293	0.1465	0.1221
1 G	2x.1222"	5x.1222"	0.367	1	6 956	1 649	0.00887	260	0.691	0.702	0.712	0.716	0.755	0.781	0.805	0.815	0.239	0.478	0.573	0.298	0.1488	0.1240
1 F	1x.1153"	6x.1153"	0.346	1	5 266	1 488	0.00980	270	0.691	0.698	0.704	0.705	0.756	0.769	0.781	0.786	0.234	0.468	0.561	0.302	0.1509	0.1258
2 P	6x.1540"	1x.1540"	0.462	2	16 870	2 487	0.00501	250	0.871	0.886	0.901	0.909	0.952	0.988	1.024	1.040	0.268	0.536	0.643	0.281	0.1406	0.1172
2 N	5x.1377"	2x.1377"	0.413	2	12 880	2 015	0.00568	240	0.871	0.885	0.899	0.906	0.952	0.986	1.020	1.035	0.261	0.523	0.627	0.289	0.1446	0.1208
2 K	4x.1257"	3x.1257"	0.377	2	9 730	1 701	0.00644	240	0.871	0.884	0.896	0.902	0.952	0.983	1.014	1.028	0.255	0.510	0.612	0.296	0.1479	0.1232
2 J	3x.1164"	4x.1164"	0.349	2	7 322	1 476	0.00727	230	0.871	0.883	0.894	0.899	0.952	0.982	1.010	1.022	0.249	0.498	0.598	0.301	0.1506	0.1255
2 A	1x.1699"	2x.1699"	0.366	2	6 876	1 356	0.00763	240	0.869	0.875	0.880	0.882	0.950	0.962	0.973	0.979	0.247	0.493	0.592	0.298	0.1489	0.1241
2 G	2x.1089"	5x.1089"	0.327	2	5 626	1 307	0.00790	230	0.871	0.882	0.892	0.896	0.952	0.980	1.006	1.016	0.245	0.489	0.587	0.306	0.1529	0.1276
2 F	1x.1026"	6x.1026"	0.308	2	4 233	1 176	0.00873	230	0.871	0.878	0.884	0.886	0.952	0.965	0.979	0.985	0.230	0.479	0.576	0.310	0.1551	0.1292
3 P	6x.1371"	1x.1371"	0.411	3	13 910	1 973	0.00445	220	1.098	1.113	1.127	1.136	1.200	1.239	1.273	1.296	0.274	0.547	0.657	0.290	0.1448	0.1207
3 N	5x.1226"	2x.1226"	0.368	3	10 390	1 598	0.00506	210	1.098	1.112	1.126	1.133	1.200	1.237	1.273	1.289	0.267	0.534	0.641	0.298	0.1487	0.1239
3 K	4x.1120"	3x.1120"	0.336	3	7 910	1 349	0.00574	210	1.098	1.111	1.123	1.129	1.200	1.233	1.267	1.281	0.261	0.522	0.626	0.304	0.1520	0.1266
3 J	3x.1036"	4x.1036"	0.311	3	5 955	1 171	0.00648	200	1.098	1.110	1.121	1.126	1.200	1.232	1.262	1.275	0.255	0.509	0.611	0.309	0.1547	0.1289
3 A	1x.1513"	2x.1513"	0.326	3	4 810	1 075	0.00679	210	1.096	1.102	1.107	1.109	1.198	1.211	1.225	1.229	0.252	0.505	0.606	0.306	0.1531	0.1275
4 P	6x.1221"	1x.1221"	0.366	4	11 420	1 564	0.0039	190	1.385	1.400	1.414	1.423	1.514	1.555	1.598	1.616	0.280	0.559	0.671	0.298	0.1489	0.1241
4 N	5x.1092"	2x.1092"	0.328	4	8 460	1 267	0.00451	180	1.385	1.399	1.413	1.420	1.514	1.554	1.593	1.610	0.273	0.546	0.655	0.306	0.1528	0.1274
4 D	2x.1615"	1x.1615"	0.348	4	7 340	1 191	0.00566	190	1.382	1.389	1.396	1.399	1.511	1.529	1.544	1.542	0.262	0.523	0.628	0.301	0.1507	0.1256
4 A	1x.1347"	2x.1347"	0.290	4	3 938	853	0.00604	180	1.382	1.388	1.393	1.395	1.511	1.525	1.540	1.545	0.258	0.517	0.620	0.314	0.1572	0.1310
5 P	6x.1087"	1x.1087"	0.326	5	9 311	1 240	0.00353	160	1.747	1.762	1.776	1.785	1.909	1.954	2.00	2.02	0.285	0.571	0.685	0.306	0.1531	0.1275
5 D	2x.1438"	1x.1438"	0.310	5	6 035	944	0.00504	160	1.742	1.749	1.756	1.759	1.905	1.924	1.941	1.939	0.268	0.536	0.643	0.310	0.1548	0.1290
5 A	1x.1200"	2x.1200"	0.258	5	3 193	676	0.00538	160	1.742	1.748	1.753	1.755	1.905	1.920	1.938	1.941	0.264	0.528	0.634	0.323	0.1614	0.1345
6 D	2x.1281"	1x.1281"	0.276	6	4 942	749	0.00449	140	2.20	2.21	2.21	2.22	2.40	2.42	2.44	2.44	0.273	0.547	0.656	0.318	0.1590	0.1325
6 A	1x.1068"	2x.1068"	0.230	6	2 585	536	0.00479	140	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.270	0.540	0.648	0.331	0.1655	0.1379
6 C	1x.1046"	2x.1046"	0.225	6	2 143	514	0.00469	130	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.271	0.542	0.651	0.333	0.1663	0.1386
7 D	2x.1141"	1x.1141"	0.246	7	4 022	594	0.00400	120	2.77	2.78	2.79	2.79	3.03	3.05	3.07	3.07	0.279	0.558	0.670	0.326	0.1631	0.1359
7 A	1x.1266"	2x.0895"	0.223	7	2 754	495	0.00441	120	2.77	2.78	2.78	2.78	3.03	3.05	3.07	3.07	0.274	0.548	0.658	0.333	0.1666	0.1388
8 D	2x.1016"	1x.1016"	0.219	8	3 256	471	0.00356	110	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.285	0.570	0.684	0.334	0.1672	0.1393
8 A	1x.1127"	2x.0797"	0.199	8	2 233	392	0.00394	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.87	0.280	0.560	0.672	0.341	0.1706	0.1422
8 C	1x.0808"	2x.0834"	0.179	8	1 362	320	0.00373	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.283	0.565	0.679	0.349	0.1744	0.1453
9½ D	2x.0808"	1x.0808"	0.174	9½	1 743	298	0.00283	85	4.91	4.92	4.92	4.93	5.37	5.39	5.42	5.42	0.297	0.593	0.712	0.351	0.1754	0.1462

* Based on a conductor temperature of 75°C and an ambient of 25°C wind 1.4 mi/h (2ft/s), (frequency = 60 cycles, average tarnished surface.

** Resistances at 50°C total temperature, based on an ambient of 25°C plus 25°C rise due to heating effect of current The approximate magnitude of current necessary to produce the 25°C rise is 75% of the "Approximate current carrying capacity of 60 Cycles."

Source: [1, 2].

Table A-8 Characteristics of Copperweld conductors

Nominal Conductor Size	Number and Size of Wires	Outside Diameter Inches	Area of Conductor Circular Mils	Rated Breaking Load Pounds: High Strength	Rated Breaking Load Pounds: Extra High Strength	Weight Pounds per Mile	Geometric Mean Radius at 60 cycles and Average Currents Feet	Approx. Current carrying Capacity* Amps at 60 Cycles
30% Conductivity								
7/8"	19 No. 5	0.910	628 900	[illegible]	66 910	9 344	0.00758	620
13/16"	19 No. 6	0.810	498 800	45 830	55 830	7 410	[illegible]	540
23/32"	19 No. 7	0.721	395 600	[illegible]	45 850	5 877	0.00601	[illegible]
21/32"	19 No. 8	0.642	313 [illegible]	31 040	[illegible]	4 660	0.00535	410
9/16"	19 No. 9	0.572	248 800	[illegible]	30 610	3 696	0.00477	360
5/8"	7 No. 4	0.613	292 200	[illegible]	[illegible]	4 324	0.00511	410
9/16"	7 No. 5	0.546	231 700	[illegible]	[illegible]	3 429	0.00455	360
1/2"	7 No. 6	0.486	183 800	16 890	20 460	2 719	0.00405	310
7/16"	7 No. 7	0.433	145 700	13 910	16 890	2 157	0.00361	[illegible]
3/8"	7 No. 8	0.385	115 600	11 440	13 890	1 710	0.00321	230
11/32"	7 No. 9	0.343	91 650	9 393	11 280	1 356	0.00286	200
5/16"	7 No. 10	0.306	72 680	7 758	9 196	1 076	0.00255	170
3 No. 5	3 No. 5	0.392	99 310	9 262	11 860	1 467	0.00457	220
3 No. 6	3 No. 6	0.349	78 750	7 639	9 754	1 163	0.00407	190
3 No. 7	3 No. 7	0.311	62 450	6 291	7 922	922.4	0.00363	140
3 No. 8	3 No. 8	0.277	49 530	5 174	6 282	731.5	0.00323	140
3 No. 9	3 No. 9	0.247	39 280	4 250	5 129	580.1	0.00288	120
3 No. 10	3 No. 10	0.220	31 150	3 509	4 160	460.0	[illegible]	110
40% Conductivity								
7/8"	19 No. 5	0.910	628 900	50 240		9 344	0.01175	690
13/16"	19 No. 6	0.810	498 800	41 900		7 410	0.01046	610
23/32"	19 No. 7	0.721	395 600	34 390		5 877	0.00931	530
21/32"	19 No. 8	0.642	313 [illegible]	28 380		4 660	[illegible]	470
9/16"	19 No. 9	0.572	248 800	23 390		3 696	[illegible]	410
5/8"	7 No. 4	0.613	292 200	22 310		4 324	[illegible]	410
9/16"	7 No. 5	0.546	231 700	18 810		3 429	[illegible]	410
1/2"	7 No. 6	0.486	183 800	15 330		2 719	[illegible]	350
7/16"	7 No. 7	0.433	145 700	12 670		[illegible]	[illegible]	310
3/8"	7 No. 8	0.385	115 600	10 460		1 710	[illegible]	270
11/32"	7 No. 9	0.343	91 650	8 616		1 356	0.00443	230
5/16"	7 No. 10	0.306	72 680	7 121		1 076	0.00395	200
3 No. 5	3 No. 5	0.392	99 310	8 373		1 467	[illegible]	250
3 No. 6	3 No. 6	0.349	78 750	6 934		1 163	[illegible]	220
3 No. 7	3 No. 7	0.311	62 450	5 732		922.4	[illegible]	190
3 No. 8	3 No. 8	0.277	49 530	[illegible]		731.5	[illegible]	160
3 No. 9	3 No. 9	0.247	39 280	3 698		580.1	[illegible]	140
3 No. 10	3 No. 10	1.220	31 150	3 221		460.0	0.00348	120
3 No. 12	3 No. 12	0.174	19 590	2 236		289.3	[illegible]	90

Nominal Conductor Size	r_a Resistance Ohms per Conductor per Mile at 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a Resistance Ohms per Conductor per Mile at 75°C (167°F)** Current Approx. 75% of Capacity*: d-c	25 cycles	50 cycles	60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile One Ft Spacing Average Currents: 25 cycles	50 cycles	60 cycles	x_a' Capacitive Reactance Megohms per Conductor per Mile One Ft Spacing: 25 cycles	50 cycles	60 cycles
30% Conductivity														
7/8"	0.306	0.315	0.326	0.331	0.363	0.419	0.476	0.499	[illegible]	[illegible]	0.592	0.233	0.1165	0.0971
13/16"	0.386	0.396	0.406	0.411	0.458	0.518	0.580	0.605	[illegible]	0.508	0.606	0.241	[illegible]	0.1005
23/32"	0.486	0.496	0.506	0.511	0.577	0.643	0.710	[illegible]	[illegible]	[illegible]	0.521	0.250	0.1248	0.1040
21/32"	0.613	0.623	0.633	0.638	0.728	0.799	0.872	[illegible]	[illegible]	[illegible]	0.635	[illegible]	[illegible]	0.1074
9/16"	0.773	0.783	0.793	0.798	0.917	0.995	[illegible]	1.106	0.285	0.541	0.649	[illegible]	0.1330	0.1109
5/8"	0.656	0.664	[illegible]	[illegible]	[illegible]	0.824	[illegible]	[illegible]	0.281	0.533	0.640	[illegible]	0.1306	0.1088
9/16"	0.827	0.835	0.843	[illegible]	0.981	1.033	1.080	[illegible]	[illegible]	0.545	0.554	[illegible]	0.1347	0.1122
1/2"	1.043	[illegible]	[illegible]	[illegible]	[illegible]	1.290	[illegible]	[illegible]	0.293	0.557	0.668	[illegible]	0.1388	0.1157
7/16"	1.315	1.323	[illegible]	1.336	[illegible]	1.617	1.675	[illegible]	0.299	0.569	0.683	[illegible]	0.1429	0.1191
3/8"	1.658	1.666	[illegible]	1.678	1.967	2.03	2.09	2.12	[illegible]	0.561	[illegible]	0.294	0.1471	0.1226
11/32"	2.09	2.10	2.11	2.11	2.48	2.55	2.61	2.64	0.311	[illegible]	0.711	0.303	0.1512	0.1260
5/16"	2.64	2.64	2.65	2.66	3.13	3.20	3.27	3.30	0.316	[illegible]	0.726	0.311	0.1553	0.1294
3 No. 5	1.926	1.931	1.936	1.938	2.29	2.31	2.34	2.36	0.289	0.545	0.654	0.293	0.1465	0.1221
3 No. 6	2.43	2.43	2.44	2.44	2.88	2.91	2.94	2.96	[illegible]	0.556	0.668	0.301	0.1506	[illegible]
3 No. 7	3.06	3.07	3.07	3.07	3.63	3.66	3.70	[illegible]	[illegible]	0.566	0.682	0.310	[illegible]	0.1289
3 No. 8	3.86	3.87	3.87	3.87	4.58	4.61	4.65	4.66	0.307	0.580	0.696	0.318	0.1589	0.1324
3 No. 9	4.87	4.87	4.88	4.88	5.76	5.81	5.85	5.86	0.315	0.591	0.710	0.326	0.1629	0.1356
3 No. 10	6.14	6.14	6.15	6.16	7.28	7.32	7.36	7.38	0.319	0.603	0.724	0.334	0.1671	0.1392
40% Conductivity														
7/8"	[illegible]	0.239	0.249	0.254	[illegible]	0.321	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.233	0.1165	0.0971
13/16"	0.289	0.299	0.309	0.314	0.343	0.396	0.450	[illegible]	[illegible]	0.481	0.553	0.241	[illegible]	[illegible]
23/32"	0.365	[illegible]	0.385	0.390	0.433	[illegible]	0.549	[illegible]	[illegible]	[illegible]	[illegible]	0.250	0.1248	[illegible]
21/32"	0.460	[illegible]	0.480	0.486	0.546	[illegible]	[illegible]	0.698	0.253	0.483	0.582	0.258	0.1269	0.1074
9/16"	[illegible]	0.590	0.600	0.606	0.688	[illegible]	0.826	[illegible]	0.259	0.496	0.595	0.266	0.1330	0.1109
5/8"	0.492	0.500	0.504	0.512	0.594	[illegible]	0.664	0.680	0.255	0.489	[illegible]	[illegible]	0.1306	0.1088
9/16"	0.620	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.843	0.840	0.281	0.501	0.601	0.269	[illegible]	[illegible]
1/2"	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.021	1.040	[illegible]	[illegible]	0.615	[illegible]	0.1388	[illegible]
7/16"	[illegible]	[illegible]	[illegible]	1.006	[illegible]	[illegible]	[illegible]	1.291	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1191
3/8"	1.244	1.252	[illegible]	1.264	1.476	[illegible]	1.584	1.606	[illegible]	[illegible]	[illegible]	0.294	[illegible]	[illegible]
11/32"	[illegible]	[illegible]	[illegible]	[illegible]	1.861	1.919	1.978	2.00	[illegible]	[illegible]	[illegible]	0.303	0.1512	0.1260
5/16"	1.978	1.986	[illegible]	[illegible]	[illegible]	2.41	2.47	2.50	[illegible]	[illegible]	[illegible]	0.311	[illegible]	[illegible]
3 No. 5	1.445	[illegible]	[illegible]	[illegible]	1.714	1.738	1.762	[illegible]	[illegible]	[illegible]	[illegible]	0.293	[illegible]	[illegible]
3 No. 6	1.821	[illegible]	[illegible]	[illegible]	2.16	2.19	2.21	[illegible]	0.275	[illegible]	[illegible]	0.301	[illegible]	[illegible]
3 No. 7	2.30	[illegible]	[illegible]	[illegible]	2.73	2.75	2.78	[illegible]	[illegible]	0.537	[illegible]	0.310	[illegible]	[illegible]
3 No. 8	2.90	2.90	2.91	2.91	3.44	3.47	3.50	[illegible]	0.286	0.549	0.659	0.318	0.1589	0.1324
3 No. 9	3.65	3.66	3.66	3.66	4.33	4.37	4.40	4.41	[illegible]	[illegible]	[illegible]	[illegible]	0.1629	[illegible]
3 No. 10	4.61	4.61	4.62	4.62	5.46	5.50	5.53	[illegible]	[illegible]	0.572	0.687	0.334	0.1671	0.1392
3 No. 12	7.32	7.33	7.33	7.34	8.69	8.73	8.77	8.78	[illegible]	0.598	0.718	0.361	0.1754	0.1462

* Based on conductor temperature of 125°C and an ambient of 25°C.

** Resistance at 75°C total temperature, based on an ambient of 25°C plus 50°C rise due to heating effect of current. The approximate magnitude of current necessary to produce the 50°C rise is 75% of the "Approximate current carrying capacity at 60 Cycles."

Source: [1, 2].

Table A-9 Electrical characteristics of overhead ground wires

Part A: Alumoweld strand

Strand (AWG)	Resistance, Ω/mi: Small currents 25°C oc	Small currents 25°C 60 Hz	75% of cap. 75°C oc	75% of cap. 75°C 60 Hz	60-Hz reactance for 1-ft radius: Inductive, Ω/mi	Capacitive, MΩ · mi	60-Hz geometric mean radius, ft
7 NO. 5	1.217	1.240	1.432	1.669	0.707	0.1122	0.002958
7 NO. 6	1.507	1.536	1.773	2.010	0.721	0.1157	0.002633
7 NO. 7	1.900	1.937	2.240	2.470	0.735	0.1191	0.002345
7 NO. 8	2.400	2.440	2.820	3.060	0.749	0.1226	0.002085
7 NO. 9	3.020	3.080	3.560	3.800	0.763	0.1260	0.001858
7 NO. 10	3.810	3.880	4.480	4.730	0.777	0.1294	0.001658
3 NO. 5	2.780	2.780	3.270	3.560	0.707	0.1221	0.002940
3 NO. 6	3.510	3.510	4.130	4.410	0.721	0.1255	0.002618
3 NO. 7	4.420	4.420	5.210	5.470	0.735	0.1289	0.002333
3 NO. 8	5.580	5.580	6.570	6.820	0.749	0.1324	0.002078
3 NO. 9	7.040	7.040	8.280	8.520	0.763	0.1358	0.001853
3 NO. 10	8.870	8.870	10.440	10.670	0.777	0.1392	0.001650

Part B: Single-layer ACSR

Code	Resistance, Ω/mi: 25°C dc	60 Hz, 75°C: $I = 0$ A	$I = 100$ A	$I = 200$ A	60-Hz reactance for 1-ft radius: Inductive, Ω/mi at 75°C: $I = 0$ A	$I = 100$ A	$I = 200$ A	Capacitive, MΩ · mi
Brahma	0.394	0.470	0.510	0.565	0.500	0.520	0.545	0.1043
Cochin	0.400	0.480	0.520	0.590	0.505	0.515	0.550	0.1065
Dorking	0.443	0.535	0.575	0.650	0.515	0.530	0.565	0.1079
Dotterel	0.479	0.565	0.620	0.705	0.515	0.530	0.575	0.1091
Guinea	0.531	0.630	0.685	0.780	0.520	0.545	0.590	0.1106
Leghorn	0.630	0.760	0.810	0.930	0.530	0.550	0.605	0.1131
Minorca	0.765	0.915	0.980	1.130	0.540	0.570	0.640	0.1160
Petrel	0.830	1.000	1.065	1.220	0.550	0.580	0.655	0.1172
Grouse	1.080	1.295	1.420	1.520	0.570	0.640	0.675	0.1240

Part C: Steel conductors

Grade (7-strand)	Dia., in	Resistance, Ω/mi, at 60 Hz			60-Hz reactance for 1-ft radius			
					Inductive, Ω/mi			Capacitive, MΩ · mi
		$I = 0$ A	$I = 30$ A	$I = 60$ A	$I = 0$ A	$I = 30$ A	$I = 60$ A	
Ordinary	1/4	9.5	11.4	11.3	1.3970	3.7431	3.4379	0.1354
Ordinary	9/32	7.1	9.2	9.0	1.2027	3.0734	2.5146	0.1319
Ordinary	5/16	5.4	7.5	7.8	0.8382	2.5146	2.0409	0.1288
Ordinary	3/8	4.3	6.5	6.6	0.8382	2.2352	1.9687	0.1234
Ordinary	1/2	2.3	4.3	5.0	0.7049	1.6893	1.4236	0.1148
E.B.	1/4	8.0	12.0	10.1	1.2027	4.4704	3.1565	0.1354
E.B.	9/32	6.0	10.0	8.7	1.1305	3.7783	2.6255	0.1319
E.B.	5/16	4.9	8.0	7.0	0.9843	2.9401	2.5146	0.1288
E.B.	3/8	3.7	7.0	6.3	0.8382	2.5997	2.4303	0.1234
E.B.	1/2	2.1	4.9	5.0	0.7049	1.8715	1.7616	0.1148
E.B.B.	1/4	7.0	12.8	10.9	1.6764	5.1401	3.9482	0.1354
E.B.B.	9/32	5.4	10.9	8.7	1.1305	4.4833	3.7783	0.1319
E.B.B.	5/16	4.0	9.0	6.8	0.9843	3.6322	3.0734	0.1288
E.B.B.	3/8	3.5	7.9	6.0	0.8382	3.1168	2.7940	0.1234
E.B.B.	1/2	2.0	5.7	4.7	0.7049	2.3461	2.2352	0.1148

Source: Reprinted by permission from *Analysis of Faulted Power Systems* by Paul M. Anderson, © 1973 by The Iowa State University Press, Ames, Iowa 50010.

Table A-10 Inductive reactance spacing factor x_d, Ω/(conductor · mi), at 60 Hz

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		−0.2794	−0.1953	−0.1461	−0.1112	−0.0841	−0.0620	−0.0433	−0.0271	−0.0128
1	0.0	0.0116	0.0221	0.0318	0.0408	0.0492	0.0570	0.0644	0.0713	0.0779
2	0.0841	0.0900	0.0957	0.1011	0.1062	0.1112	0.1159	0.1205	0.1249	0.1292
3	0.1333	0.1373	0.1411	0.1449	0.1485	0.1520	0.1554	0.1588	0.1620	0.1651
4	0.1682	0.1712	0.1741	0.1770	0.1798	0.1825	0.1852	0.1878	0.1903	0.1928
5	0.1953	0.1977	0.2001	0.2024	0.2046	0.2069	0.2090	0.2112	0.2133	0.2154
6	0.2174	0.2194	0.2214	0.2233	0.2252	0.2271	0.2290	0.2308	0.2326	0.2344
7	0.2361	0.2378	0.2395	0.2412	0.2429	0.2445	0.2461	0.2477	0.2493	0.2508
8	0.2523	0.2538	0.2553	0.2568	0.2582	0.2597	0.2611	0.2625	0.2639	0.2653
9	0.2666	0.2680	0.2693	0.2706	0.2719	0.2732	0.2744	0.2757	0.2769	0.2782
10	0.2794	0.2806	0.2818	0.2830	0.2842	0.2853	0.2865	0.2876	0.2887	0.2899
11	0.2910	0.2921	0.2932	0.2942	0.2953	0.2964	0.2974	0.2985	0.2995	0.3005
12	0.3015	0.3025	0.3035	0.3045	0.3055	0.3065	0.3074	0.3084	0.3094	0.3103
13	0.3112	0.3122	0.3131	0.3140	0.3149	0.3158	0.3167	0.3176	0.3185	0.3194
14	0.3202	0.3211	0.3219	0.3228	0.3236	0.3245	0.3253	0.3261	0.3270	0.3278
15	0.3286	0.3294	0.3302	0.3310	0.3318	0.3326	0.3334	0.3341	0.3349	0.3357
16	0.3364	0.3372	0.3379	0.3387	0.3394	0.3402	0.3409	0.3416	0.3424	0.3431
17	0.3438	0.3445	0.3452	0.3459	0.3466	0.3473	0.3480	0.3487	0.3494	0.3500
18	0.3507	0.3514	0.3521	0.3527	0.3534	0.3540	0.3547	0.3554	0.3560	0.3566
19	0.3573	0.3579	0.3586	0.3592	0.3598	0.3604	0.3611	0.3617	0.3623	0.3629
20	0.3635	0.3641	0.3647	0.3653	0.3659	0.3665	0.3671	0.3677	0.3683	0.3688
21	0.3694	0.3700	0.3706	0.3711	0.3717	0.3723	0.3728	0.3734	0.3740	0.3745
22	0.3751	0.3756	0.3762	0.3767	0.3773	0.3778	0.3783	0.3789	0.3794	0.3799
23	0.3805	0.3810	0.3815	0.3820	0.3826	0.3831	0.3836	0.3841	0.3846	0.3851
24	0.3856	0.3861	0.3866	0.3871	0.3876	0.3881	0.3886	0.3891	0.3896	0.3901
25	0.3906	0.3911	0.3916	0.3920	0.3925	0.3930	0.3935	0.3939	0.3944	0.3949
26	0.3953	0.3958	0.3963	0.3967	0.3972	0.3977	0.3981	0.3986	0.3990	0.3995
27	0.3999	0.4004	0.4008	0.4013	0.4017	0.4021	0.4026	0.4030	0.4035	0.4039
28	0.4043	0.4048	0.4052	0.4056	0.4061	0.4065	0.4069	0.4073	0.4078	0.4082
29	0.4086	0.4090	0.4094	0.4098	0.4103	0.4107	0.4111	0.4115	0.4119	0.4123
30	0.4127	0.4131	0.4135	0.4139	0.4143	0.4147	0.4151	0.4155	0.4159	0.4163

31	0.4167	0.4171	0.4175	0.4179	0.4182	0.4186	0.4190	0.4194	0.4198	0.4202
32	0.4205	0.4209	0.4213	0.4217	0.4220	0.4224	0.4228	0.4232	0.4235	0.4239
33	0.4243	0.4246	0.4250	0.4254	0.4257	0.4261	0.4265	0.4268	0.4272	0.4275
34	0.4279	0.4283	0.4286	0.4290	0.4293	0.4297	0.4300	0.4304	0.4307	0.4311
35	0.4314	0.4318	0.4321	0.4324	0.4328	0.4331	0.4335	0.4338	0.4342	0.4345
36	0.4348	0.4352	0.4355	0.4358	0.4362	0.4365	0.4368	0.4372	0.4375	0.4378
37	0.4382	0.4385	0.4388	0.4391	0.4395	0.4398	0.4401	0.4404	0.4408	0.4411
38	0.4414	0.4417	0.4420	0.4423	0.4427	0.4430	0.4433	0.4436	0.4439	0.4442
39	0.4445	0.4449	0.4452	0.4455	0.4458	0.4461	0.4464	0.4467	0.4470	0.4473
40	0.4476	0.4479	0.4492	0.4485	0.4488	0.4491	0.4494	0.4497	0.4500	0.4503
41	0.4506	0.4509	0.4512	0.4515	0.4518	0.4521	0.4524	0.4527	0.4530	0.4532
42	0.4535	0.4538	0.4541	0.4544	0.4547	0.4550	0.4553	0.4555	0.4558	0.4561
43	0.4564	0.4567	0.4570	0.4572	0.4575	0.4578	0.4581	0.4584	0.4586	0.4589
44	0.4592	0.4595	0.4597	0.4600	0.4603	0.4606	0.4608	0.4611	0.4614	0.4616
45	0.4619	0.4622	0.4624	0.4627	0.4630	0.4632	0.4635	0.4638	0.4640	0.4643
46	0.4646	0.4648	0.4651	0.4654	0.4656	0.4659	0.4661	0.4664	0.4667	0.4669
47	0.4672	0.4674	0.4677	0.4680	0.4682	0.4685	0.4687	0.4690	0.4692	0.4695
48	0.4697	0.4700	0.4702	0.4705	0.4707	0.4710	0.4712	0.4715	0.4717	0.4720
49	0.4722	0.4725	0.4727	0.4730	0.4732	0.4735	0.4737	0.4740	0.4742	0.4744
50	0.4747	0.4749	0.4752	0.4754	0.4757	0.4759	0.4761	0.4764	0.4766	0.4769
51	0.4771	0.4773	0.4776	0.4778	0.4780	0.4783	0.4785	0.4787	0.4790	0.4792
52	0.4795	0.4797	0.4799	0.4801	0.4804	0.4806	0.4808	0.4811	0.4813	0.4815
53	0.4818	0.4820	0.4822	0.4824	0.4827	0.4829	0.4831	0.4834	0.4836	0.4838
54	0.4840	0.4843	0.4845	0.4847	0.4849	0.4851	0.4854	0.4856	0.4858	0.4860
55	0.4863	0.4865	0.4867	0.4869	0.4871	0.4874	0.4876	0.4878	0.4880	0.4882
56	0.4884	0.4887	0.4889	0.4891	0.4893	0.4895	0.4897	0.4900	0.4902	0.4904
57	0.4906	0.4908	0.4910	0.4912	0.4914	0.4917	0.4919	0.4921	0.4923	0.4925
58	0.4927	0.4929	0.4931	0.4933	0.4935	0.4937	0.4940	0.4942	0.4944	0.4946
59	0.4948	0.4950	0.4952	0.4954	0.4956	0.4958	0.4960	0.4962	0.4964	0.4966
60	0.4968	0.4970	0.4972	0.4974	0.4976	0.4978	0.4980	0.4982	0.4984	0.4986

Table A-10 (Continued)

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
61	0.4988	0.4990	0.4992	0.4994	0.4996	0.4998	0.5000	0.5002	0.5004	0.5006
62	0.5008	0.5010	0.5012	0.5014	0.5016	0.5018	0.5020	0.5022	0.5023	0.5025
63	0.5027	0.5029	0.5031	0.5033	0.5035	0.5037	0.5039	0.5041	0.5043	0.5045
64	0.5046	0.5048	0.5050	0.5052	0.5054	0.5056	0.5058	0.5060	0.5062	0.5063
65	0.5065	0.5067	0.5069	0.5071	0.5073	0.5075	0.5076	0.5078	0.5080	0.5082
66	0.5084	0.5086	0.5087	0.5089	0.5091	0.5093	0.5095	0.5097	0.5098	0.5100
67	0.5102	0.5104	0.5106	0.5107	0.5109	0.5111	0.5113	0.5115	0.5116	0.5118
68	0.5120	0.5122	0.5124	0.5125	0.5127	0.5129	0.5131	0.5132	0.5134	0.5136
69	0.5138	0.5139	0.5141	0.5143	0.5145	0.5147	0.5148	0.5150	0.5152	0.5153
70	0.5155	0.5157	0.5159	0.5160	0.5162	0.5164	0.5166	0.5167	0.5169	0.5171
71	0.5172	0.5174	0.5176	0.5178	0.5179	0.5181	0.5183	0.5184	0.5186	0.5188
72	0.5189	0.5191	0.5193	0.5194	0.5196	0.5198	0.5199	0.5201	0.5203	0.5204
73	0.5206	0.5208	0.5209	0.5211	0.5213	0.5214	0.5216	0.5218	0.5219	0.5221
74	0.5223	0.5224	0.5226	0.5228	0.5229	0.5231	0.5232	0.5234	0.5236	0.5237
75	0.5239	0.5241	0.5242	0.5244	0.5245	0.5247	0.5249	0.5250	0.5252	0.5253
76	0.5255	0.5257	0.5258	0.5260	0.5261	0.5263	0.5265	0.5266	0.5268	0.5269
77	0.5271	0.5272	0.5274	0.5276	0.5277	0.5279	0.5280	0.5282	0.5283	0.5285
78	0.5287	0.5288	0.5290	0.5291	0.5293	0.5294	0.5296	0.5297	0.5299	0.5300
79	0.5302	0.5304	0.5305	0.5307	0.5308	0.5310	0.5311	0.5313	0.5314	0.5316
80	0.5317	0.5319	0.5320	0.5322	0.5323	0.5325	0.5326	0.5328	0.5329	0.5331

81	0.5332	0.5334	0.5335	0.5337	0.5338	0.5340	0.5341	0.5343	0.5344	0.5346
82	0.5347	0.5349	0.5350	0.5352	0.5353	0.5355	0.5356	0.5358	0.5359	0.5360
83	0.5362	0.5363	0.5365	0.5366	0.5368	0.5369	0.5371	0.5372	0.5374	0.5375
84	0.5376	0.5378	0.5379	0.5381	0.5382	0.5384	0.5385	0.5387	0.5388	0.5389
85	0.5391	0.5392	0.5394	0.5395	0.5396	0.5398	0.5399	0.5401	0.5402	0.5404
86	0.5405	0.5406	0.5408	0.5409	0.5411	0.5412	0.5413	0.5415	0.5416	0.5418
87	0.5419	0.5420	0.5422	0.5423	0.5425	0.5426	0.5427	0.5429	0.5430	0.5432
88	0.5433	0.5434	0.5436	0.5437	0.5438	0.5440	0.5441	0.5442	0.5444	0.5445
89	0.5447	0.5448	0.5449	0.5451	0.5452	0.5453	0.5455	0.5456	0.5457	0.5459
90	0.5460	0.5461	0.5463	0.5464	0.5466	0.5467	0.5468	0.5470	0.5471	0.5472
91	0.5474	0.5475	0.5476	0.5478	0.5479	0.5480	0.5482	0.5483	0.5484	0.5486
92	0.5487	0.5488	0.5489	0.5491	0.5492	0.5493	0.5495	0.5496	0.5497	0.5499
93	0.5500	0.5501	0.5503	0.5504	0.5505	0.5506	0.5508	0.5509	0.5510	0.5512
94	0.5513	0.5514	0.5515	0.5517	0.5518	0.5519	0.5521	0.5522	0.5523	0.5524
95	0.5526	0.5527	0.5528	0.5530	0.5531	0.5532	0.5533	0.5535	0.5536	0.5537
96	0.5538	0.5540	0.5541	0.5542	0.5544	0.5545	0.5546	0.5547	0.5549	0.5550
97	0.5551	0.5552	0.5554	0.5555	0.5556	0.5557	0.5559	0.5560	0.5561	0.5562
98	0.5563	0.5565	0.5566	0.5567	0.5568	0.5570	0.5571	0.5572	0.5573	0.5575
99	0.5576	0.5577	0.5578	0.5579	0.5581	0.5582	0.5583	0.5584	0.5586	0.5587
100	0.5588	0.5589	0.5590	0.5592	0.5593	0.5594	0.5595	0.5596	0.5598	0.5599

Table A-10 (Continued)
Zero-sequence resistive and inductive factors r_e^*, x_e^*, Ω/(conductor · mi)

	ρ, $\Omega \cdot$ m	r_e, x_e (f = 60 Hz)
r_e	All	0.2860
	1	2.050
	5	2.343
	10	2.469
x_e	50	2.762
	100†	2.888†
	500	3.181
	1000	3.307
	5000	3.600
	10,000	3.726

* From formulas:

$$r_e = 0.004764f$$

$$x_e = 0.006985f \log_{10} 4{,}665{,}600 \frac{\rho}{f}$$

where f = frequency
ρ = resistivity, $\Omega \cdot$ m

† This is an average value which may be used in the absence of definite information.

Fundamental equations:

$$z_1 = z_2 = r_a + j(x_a + x_d)$$

$$z_0 = r_a + r_e + j(x_a + x_e - 2x_d)$$

where $x_d = \omega k \ln d$
d = separation, ft

Source: Reprinted by permission from *Analysis of Faulted Power Systems* by Paul M. Anderson, © 1973 by The Iowa State University Press, Ames, Iowa 50010.

Table A-11 Shunt capacitive reactance spacing factor x'_d, MΩ/(conductor · mi), at 60 Hz

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		−0.0683	−0.0477	−0.0357	−0.0272	−0.0206	−0.0152	−0.0106	−0.0066	−0.0031
1	0.0000	0.0028	0.0054	0.0078	0.0100	0.0120	0.0139	0.0157	0.0174	0.0190
2	0.0206	0.0220	0.0234	0.0247	0.0260	0.0272	0.0283	0.0295	0.0305	0.0316
3	0.0326	0.0336	0.0345	0.0354	0.0363	0.0372	0.0380	0.0388	0.0396	0.0404
4	0.0411	0.0419	0.0426	0.0433	0.0440	0.0446	0.0453	0.0459	0.0465	0.0471
5	0.0477	0.0483	0.0489	0.0495	0.0500	0.0506	0.0511	0.0516	0.0521	0.0527
6	0.0532	0.0536	0.0541	0.0546	0.0551	0.0555	0.0560	0.0564	0.0569	0.0573
7	0.0577	0.0581	0.0586	0.0590	0.0594	0.0598	0.0602	0.0606	0.0609	0.0613
8	0.0617	0.0621	0.0624	0.0628	0.0631	0.0635	0.0638	0.0642	0.0645	0.0649
9	0.0652	0.0655	0.0658	0.0662	0.0665	0.0668	0.0671	0.0674	0.0677	0.0680
10	0.0683	0.0686	0.0689	0.0692	0.0695	0.0698	0.0700	0.0703	0.0706	0.0709
11	0.0711	0.0714	0.0717	0.0719	0.0722	0.0725	0.0727	0.0730	0.0732	0.0735
12	0.0737	0.0740	0.0742	0.0745	0.0747	0.0749	0.0752	0.0754	0.0756	0.0759
13	0.0761	0.0763	0.0765	0.0768	0.0770	0.0772	0.0774	0.0776	0.0779	0.0781
14	0.0783	0.0785	0.0787	0.0789	0.0791	0.0793	0.0795	0.0797	0.0799	0.0801
15	0.0803	0.0805	0.0807	0.0809	0.0811	0.0813	0.0815	0.0817	0.0819	0.0821
16	0.0823	0.0824	0.0826	0.0828	0.0830	0.0832	0.0833	0.0835	0.0837	0.0839
17	0.0841	0.0842	0.0844	0.0846	0.0847	0.0849	0.0851	0.0852	0.0854	0.0856
18	0.0857	0.0859	0.0861	0.0862	0.0864	0.0866	0.0867	0.0869	0.0870	0.0872
19	0.0874	0.0875	0.0877	0.0878	0.0880	0.0881	0.0883	0.0884	0.0886	0.0887
20	0.0889	0.0890	0.0892	0.0893	0.0895	0.0896	0.0898	0.0899	0.0900	0.0902
21	0.0903	0.0905	0.0906	0.0907	0.0909	0.0910	0.0912	0.0913	0.0914	0.0916
22	0.0917	0.0918	0.0920	0.0921	0.0922	0.0924	0.0925	0.0926	0.0928	0.0929
23	0.0930	0.0931	0.0933	0.0934	0.0935	0.0937	0.0938	0.0939	0.0940	0.0942
24	0.0943	0.0944	0.0945	0.0947	0.0948	0.0949	0.0950	0.0951	0.0953	0.0954
25	0.0955	0.0956	0.0957	0.0958	0.0960	0.0961	0.0962	0.0963	0.0964	0.0965
26	0.0967	0.0968	0.0969	0.0970	0.0971	0.0972	0.0973	0.0974	0.0976	0.0977
27	0.0978	0.0979	0.0980	0.0981	0.0982	0.0983	0.0984	0.0985	0.0986	0.0987
28	0.0989	0.0990	0.0991	0.0992	0.0993	0.0994	0.0995	0.0996	0.0997	0.0998
29	0.0999	0.1000	0.1001	0.1002	0.1003	0.1004	0.1005	0.1006	0.1007	0.1008
30	0.1009	0.1010	0.1011	0.1012	0.1013	0.1014	0.1015	0.1016	0.1017	0.1018

Table A-11 (Continued)

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
31	0.1019	0.1020	0.1021	0.1022	0.1023	0.1023	0.1024	0.1025	0.1026	0.1027
32	0.1028	0.1029	0.1030	0.1031	0.1032	0.1033	0.1034	0.1035	0.1035	0.1036
33	0.1037	0.1038	0.1039	0.1040	0.1041	0.1042	0.1043	0.1044	0.1044	0.1045
34	0.1046	0.1047	0.1048	0.1049	0.1050	0.1050	0.1051	0.1052	0.1053	0.1054
35	0.1055	0.1056	0.1056	0.1057	0.1058	0.1059	0.1060	0.1061	0.1061	0.1062
36	0.1063	0.1064	0.1065	0.1066	0.1066	0.1067	0.1068	0.1069	0.1070	0.1070
37	0.1071	0.1072	0.1073	0.1074	0.1074	0.1075	0.1076	0.1077	0.1078	0.1078
38	0.1079	0.1080	0.1081	0.1081	0.1082	0.1083	0.1084	0.1085	0.1085	0.1086
39	0.1087	0.1088	0.1088	0.1089	0.1090	0.1091	0.1091	0.1092	0.1093	0.1094
40	0.1094	0.1095	0.1096	0.1097	0.1097	0.1098	0.1099	0.1100	0.1100	0.1101
41	0.1102	0.1102	0.1103	0.1104	0.1105	0.1105	0.1106	0.1107	0.1107	0.1108
42	0.1109	0.1110	0.1110	0.1111	0.1112	0.1112	0.1113	0.1114	0.1114	0.1115
43	0.1116	0.1117	0.1117	0.1118	0.1119	0.1119	0.1120	0.1121	0.1121	0.1122
44	0.1123	0.1123	0.1124	0.1125	0.1125	0.1126	0.1127	0.1127	0.1128	0.1129
45	0.1129	0.1130	0.1131	0.1131	0.1132	0.1133	0.1133	0.1134	0.1135	0.1135
46	0.1136	0.1136	0.1137	0.1138	0.1138	0.1139	0.1140	0.1140	0.1141	0.1142
47	0.1142	0.1143	0.1143	0.1144	0.1145	0.1145	0.1146	0.1147	0.1147	0.1148
48	0.1148	0.1149	0.1150	0.1150	0.1151	0.1152	0.1152	0.1153	0.1153	0.1154
49	0.1155	0.1155	0.1156	0.1156	0.1157	0.1158	0.1158	0.1159	0.1159	0.1160
50	0.1161	0.1161	0.1162	0.1162	0.1163	0.1164	0.1164	0.1165	0.1165	0.1166
51	0.1166	0.1167	0.1168	0.1168	0.1169	0.1169	0.1170	0.1170	0.1171	0.1172
52	0.1172	0.1173	0.1173	0.1174	0.1174	0.1175	0.1176	0.1176	0.1177	0.1177
53	0.1178	0.1178	0.1179	0.1180	0.1180	0.1181	0.1181	0.1182	0.1182	0.1183
54	0.1183	0.1184	0.1184	0.1185	0.1186	0.1186	0.1187	0.1187	0.1188	0.1188
55	0.1189	0.1189	0.1190	0.1190	0.1191	0.1192	0.1192	0.1193	0.1193	0.1194
56	0.1194	0.1195	0.1195	0.1196	0.1196	0.1197	0.1197	0.1198	0.1198	0.1199
57	0.1199	0.1200	0.1200	0.1201	0.1202	0.1202	0.1203	0.1203	0.1204	0.1204
58	0.1205	0.1205	0.1206	0.1206	0.1207	0.1207	0.1208	0.1208	0.1209	0.1209
59	0.1210	0.1210	0.1211	0.1211	0.1212	0.1212	0.1213	0.1213	0.1214	0.1214
60	0.1215	0.1215	0.1216	0.1216	0.1217	0.1217	0.1218	0.1218	0.1219	0.1219

61	0.1220	0.1220	0.1221	0.1221	0.1221	0.1222	0.1222	0.1223	0.1223	0.1224
62	0.1224	0.1225	0.1225	0.1226	0.1226	0.1227	0.1227	0.1228	0.1228	0.1229
63	0.1229	0.1230	0.1230	0.1231	0.1231	0.1231	0.1232	0.1232	0.1233	0.1233
64	0.1234	0.1234	0.1235	0.1235	0.1236	0.1236	0.1237	0.1237	0.1237	0.1238
65	0.1238	0.1239	0.1239	0.1240	0.1240	0.1241	0.1241	0.1242	0.1242	0.1242
66	0.1243	0.1243	0.1244	0.1244	0.1245	0.1245	0.1246	0.1246	0.1247	0.1247
67	0.1247	0.1248	0.1248	0.1249	0.1249	0.1250	0.1250	0.1250	0.1251	0.1251
68	0.1252	0.1252	0.1253	0.1253	0.1254	0.1254	0.1254	0.1255	0.1255	0.1256
69	0.1256	0.1257	0.1257	0.1257	0.1258	0.1258	0.1259	0.1259	0.1260	0.1260
70	0.1260	0.1261	0.1261	0.1262	0.1262	0.1262	0.1263	0.1263	0.1264	0.1264
71	0.1265	0.1265	0.1265	0.1266	0.1266	0.1267	0.1267	0.1268	0.1268	0.1268
72	0.1269	0.1269	0.1270	0.1270	0.1270	0.1271	0.1271	0.1272	0.1272	0.1272
73	0.1273	0.1273	0.1274	0.1274	0.1274	0.1275	0.1275	0.1276	0.1276	0.1276
74	0.1277	0.1277	0.1278	0.1278	0.1278	0.1279	0.1279	0.1280	0.1280	0.1280
75	0.1281	0.1281	0.1282	0.1282	0.1282	0.1283	0.1283	0.1284	0.1284	0.1284
76	0.1285	0.1285	0.1286	0.1286	0.1286	0.1287	0.1287	0.1288	0.1288	0.1288
77	0.1289	0.1289	0.1289	0.1290	0.1290	0.1291	0.1291	0.1291	0.1292	0.1292
78	0.1292	0.1293	0.1293	0.1294	0.1294	0.1294	0.1295	0.1295	0.1296	0.1296
79	0.1296	0.1297	0.1297	0.1297	0.1298	0.1298	0.1299	0.1299	0.1299	0.1300
80	0.1300	0.1300	0.1301	0.1301	0.1301	0.1302	0.1302	0.1303	0.1303	0.1303

Table A-11 (Continued)

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
81	0.1304	0.1304	0.1304	0.1305	0.1305	0.1306	0.1306	0.1306	0.1307	0.1307
82	0.1307	0.1308	0.1308	0.1308	0.1309	0.1309	0.1309	0.1310	0.1310	0.1311
83	0.1311	0.1311	0.1312	0.1312	0.1312	0.1313	0.1313	0.1313	0.1314	0.1314
84	0.1314	0.1315	0.1315	0.1316	0.1316	0.1316	0.1317	0.1317	0.1317	0.1318
85	0.1318	0.1318	0.1319	0.1319	0.1319	0.1320	0.1320	0.1320	0.1321	0.1321
86	0.1321	0.1322	0.1322	0.1322	0.1323	0.1323	0.1324	0.1324	0.1324	0.1325
87	0.1325	0.1325	0.1326	0.1326	0.1326	0.1327	0.1327	0.1327	0.1328	0.1328
88	0.1328	0.1329	0.1329	0.1329	0.1330	0.1330	0.1330	0.1331	0.1331	0.1331
89	0.1332	0.1332	0.1332	0.1333	0.1333	0.1333	0.1334	0.1334	0.1334	0.1335
90	0.1335	0.1335	0.1336	0.1336	0.1336	0.1337	0.1337	0.1337	0.1338	0.1338
91	0.1338	0.1339	0.1339	0.1339	0.1340	0.1340	0.1340	0.1340	0.1341	0.1341
92	0.1341	0.1342	0.1342	0.1342	0.1343	0.1343	0.1343	0.1344	0.1344	0.1344
93	0.1345	0.1345	0.1345	0.1346	0.1346	0.1346	0.1347	0.1347	0.1347	0.1348
94	0.1348	0.1348	0.1348	0.1349	0.1349	0.1349	0.1350	0.1350	0.1350	0.1351
95	0.1351	0.1351	0.1352	0.1352	0.1352	0.1353	0.1353	0.1353	0.1353	0.1354
96	0.1354	0.1354	0.1355	0.1355	0.1355	0.1356	0.1356	0.1356	0.1357	0.1357
97	0.1357	0.1357	0.1358	0.1358	0.1358	0.1359	0.1359	0.1359	0.1360	0.1360
98	0.1360	0.1361	0.1361	0.1361	0.1361	0.1362	0.1362	0.1362	0.1363	0.1363
99	0.1363	0.1364	0.1364	0.1364	0.1364	0.1365	0.1365	0.1365	0.1366	0.1366
100	0.1366	0.1366	0.1367	0.1367	0.1367	0.1368	0.1368	0.1368	0.1369	0.1369

Table A-11 (Continued)
Zero-sequence shunt capacitive reactance factor x_0', $M\Omega$/(conductor · mi)

Conductor height above ground, ft	x_0' (f = 60 Hz)
10	0.267
15	0.303
20	0.328
25	0.318
30	0.364
40	0.390
50	0.410
60	0.426
70	0.440
80	0.452
90	0.462
100	0.472

$$x_0' = \frac{12.30}{f} \log_{10} 2h$$

where h = height above ground
f = frequency
Fundamental equations:

$$x_1' = x_2' = x_a' = x_d'$$

$$x_0' = x_a' + x_c' - 2x_d'$$

where $x_d' = (1/\omega k') \ln d$
d = separation, ft

Source: Reprinted by permission from *Analysis of Faulted Power Systems* by Paul *M*. Anderson, © 1973 by The Iowa State University Press, Ames, Iowa 50010.

Table A-12 Standard impedances of distribution transformers*

kVA rating	Rating of transformer primary winding																	
	2.4 kV		4.8 kV		7.2 kV		12 kV		24.9/14.4 Gnd Y		23 kV		34.5 kV		46 kV		69 kV	
	% R	% Z	% R	% Z	% R	% Z	% R	% Z	% R	% Z	% R	% Z	% R	% Z	% R	% Z	% R	% Z
	Single-phase																	
3	1.9	2.3	2.1	2.3	2.5	2.8			3.0	3.5								
10	1.7	2.1	1.8	2.1	1.9	2.3	2.1	2.6	2.2	2.9								
25	1.5	2.3	1.6	2.3	1.6	2.2	1.6	2.3	1.7	2.6	2.0	5.2	2.2	5.2				
50	1.2	2.3	1.4	2.2	1.3	2.2	1.4	2.4	1.5	2.8	1.7	5.2	1.7	5.2	1.8	5.7		
100	1.2	2.7	1.3	2.6	1.2	3.2	1.3	3.2			1.4	5.2	1.5	5.2	1.5	5.7	1.4	6.5
333	1.1	4.8	1.1	4.8	1.0	4.9	1.0	5.1			1.0	5.2	1.1	5.2	1.1	5.7	1.1	6.5
500	1.0	4.8	1.0	4.8	1.0	5.1	1.0	5.0			0.9	5.2	1.0	5.2	1.0	5.7	1.0	6.5
	Three-phase																	
9	2.0	2.4	2.1	2.5	2.4	2.7												
30	1.6	2.5	1.8	2.5	1.9	2.6	2.1	3.1										
75	1.5	3.2	1.6	3.1	1.6	3.2	1.6	3.3										
150	1.2	4.2	1.4	4.3	1.4	4.3	1.4	4.2			1.6	5.5						
300	1.3	4.9	1.3	4.9	1.3	4.9	1.3	5.0			1.3	5.5	1.4	5.5	1.4	6.2		
500	1.2	4.9	1.2	4.9	1.1	5.0	1.1	5.1			1.2	5.5	1.2	5.5	1.3	6.3	1.2	6.7

* From [5].

Table A-13 Standard impedances for power transformers 10,000 kVA and below*

Highest-voltage winding, BIL kV	Low-voltage winding, BIL kV (For intermediate BIL, use value for next higher BIL listed)	At kVA base equal to 55°C rating of largest capacity winding Self-cooled (OA), self-cooled rating of self-cooled/forced-air cooled (OA/FA) Standard impedance, percent	
		Ungrounded neutral operation	Grounded neutral operation
110 and below	45 60, 75, 95, 110	5.75 5.5	
150	45 60, 75, 95, 110	5.75 5.5	
200	45 60, 75, 95, 110 150	6.25 6.0 6.5	
250	45 60, 150 200	6.75 6.5 7.0	
350	200 250	7.0 7.5	
450	200 250 350	7.5 8.0 8.5	7.00 7.50 8.00
550	200 350 450	8.0 9.0 10.0	7.50 8.25 9.25
650	200 350 550	8.5 9.5 10.5	8.00 8.50 9.50
750	250 450 650	9.0 10.0 11.0	8.50 9.50 10.25

* From [5].

Table A-14 Standard impedance limits for power transformers above 10,000 kVA*

Highest-voltage winding, BIL kV	Low-voltage winding, BIL kV (For intermediate BIL, use value for next higher BIL listed)	At kVA base equal to 55°C rating of largest capacity winding							
		Self-cooled (OA), self-cooled rating of self-cooled/forced-air cooled (OA/FA), self-cooled rating of self-cooled/forced-air, forced-oil cooled (OA/FOA) Standard impedance, percent				Forced-oil cooled (FOA and FOW) Standard impedance, percent			
		Ungrounded neutral operation		Grounded neutral operation		Ungrounded neutral operation		Grounded neutral operation	
		Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
110 and below	110 and below	5.0	6.25			8.25	10.5		
150	110	5.0	6.25			8.25	10.5		
200	110	5.5	7.0			9.0	12.0		
	150	5.75	7.5			9.75	12.75		
250	150	5.75	7.5			9.5	12.75		
	200	6.25	8.5			10.5	14.25		
350	200	6.25	8.5			10.25	14.25		
	250	6.75	9.5			11.25	15.75		
450	200	6.75	9.5	6.0	8.75	11.25	15.75	10.5	14.5
	250	7.25	10.75	6.75	9.5	12.0	17.25	11.25	16.0
	350	7.75	11.75	7.0	10.25	12.75	18.0	12.0	17.25

550	200	7.25	10.75	6.5	9.75	12.0	18.0	10.75	16.5
	350	8.25	13.0	7.25	10.75	13.25	21.0	12.0	18.0
	450	8.5	13.5	7.75	11.75	14.0	22.5	12.75	19.5
650	200	7.75	11.75	7.0	10.75	12.75	19.5	11.75	18.0
	350	8.5	13.5	7.75	12.0	14.0	22.5	12.75	19.5
	450	9.25	14.0	8.5	13.5	15.25	24.5	14.0	22.5
750	250	8.0	12.75	7.5	11.5	13.5	21.25	12.5	19.25
	450	9.0	13.75	8.25	13.0	15.0	24.0	13.75	21.5
	650	10.25	15.0	9.25	14.0	16.5	25.0	15.0	24.0
825	250	8.5	13.5	7.75	12.0	14.25	22.5	13.0	20.0
	450	9.5	14.25	8.75	13.5	15.75	24.0	14.5	22.25
	650	10.75	15.75	9.75	15.0	17.25	26.25	15.75	24.0
900	250			8.25	12.5			13.75	21.0
	450			9.25	14.0			15.25	23.5
	750			10.25	15.0			16.5	25.5
1050	250			8.75	13.5			14.75	22.0
	550			10.0	15.0			16.75	25.0
	825			11.0	16.5			18.25	27.5
1175	250			9.25	14.0			15.5	23.0
	550			10.5	15.75			17.5	25.5
	900			12.0	17.5			19.5	29.0
1300	250			9.75	14.5			16.25	24.0
	550			11.25	17.0			18.75	27.0
	1050			12.5	18.25			20.75	30.5

* From [5].

Table A-15 60-Hz characteristics of three-conductor belted paper-insulated cables

Voltage Class	Insulation Thickness Mils: Conductor	Insulation Thickness Mils: Belt	Circular Mils or AWG (B. & S.)	Type of (6) Conductor	Weight per 1000 Feet	Diameter or Sector Depth (5) Inches	Resistance, Ohms Per Mile (1)	GMR of One Conductor—inches (2)	Positive & Negative-Seq: Series Reactance Ohms per Mile	Positive & Negative-Seq: Shunt Capacitive Reactance Ohms per Mile (3)	GMR - Three Conductors	Zero-Sequence: Series Resistance Ohms per Mile (4)	Zero-Sequence: Series Reactance Ohms per Mile (4)	Zero-Sequence: Shunt Capacitive Reactance Ohms per Mile (3)	Sheath: Thickness Mils	Sheath: Resistance Ohms per Mile at 50°C
1 Kv	60	35	6	SR	1 500	0 184	2 50	0 067	0 185	6300	0 184	10 66	0 315	11 600	65	2 69
	60	35	4	SR	1 910	0 232	1 58	0 084	0 175	5400	0 218	8 39	0 293	10 200	90	2 27
	60	35	2	SR	2 390	0 292	0 987	0 106	0 165	4700	0 262	6 99	0 273	9 000	90	2.00
	60	35	1	SR	2 820	0 332	0 786	0 126	0 155	4300	0 295	6 07	0 256	8 400	95	1.76
	60	35	0	SR	3 210	0 373	0 622	0 14	0 15	4000	0 326	5 54	0 246	7 900	95	1 64
	60	35	00	CS	3 160	0 323	0 495	0 151	0 138	2800	0 290	5 96	0 250	5 400	95	1 82
	60	35	000	CS	3 650	0 364	0 392	0 171	0 134	2300	0 320	5 46	0 241	4 500	95	1 69
	60	35	0000	CS	4 390	0 417	0 310	0 191	0 131	2000	0 355	4 72	0 237	4 000	100	1.47
	60	35	250 000	CS	4 900	0 455	0 263	0 210	0 129	1800	0 387	4 46	[illegible]	3 600	100	1 40
	60	35	300 000	CS	5 600	0 497	0 220	0 230	0 128	1700	0 415	3 97	[illegible]	3 400	105	1 25
	60	35	350 000	CS	6 310	0 539	0 190	0 249	0 126	1500	0 446	3 73	0 218	3 100	105	1 18
	60	35	400 000	CS	7 080	0 572	0 166	0 265	0 124	1400	0 467	3 41	0 214	2 900	110	1.08
	60	35	500 000	CS	8 310	0 642	0 134	0 297	0 123	1300	0 517	3 11	0 208	2 600	110	0 993
	65	40	600 000	CS	9 800	0 700	0 113	0 327	0 122	1200	[illegible]	2 74	0 197	2 400	115	0 877
	65	40	750 000	CS	11 800	0 780	0 091	0 366	0 121	1100	0 623	2 40	0 194	2 100	120	0 771
3 Kv	70	40	6	SR	1 690	0 184	2 50	0 067	0 192	8700	0 192	9 67	0 322	12 500	90	2 39
	70	40	4	SR	2 030	0 232	1 58	0 084	0 181	5800	0 227	8 06	0 298	11 [illegible]	90	2 16
	70	40	2	SR	2 600	0 292	0 987	0 106	0 171	5100	0 271	6 39	0 278	9 800	95	1.80
	70	40	1	SR	2 930	0 332	0 786	0 126	0 161	4700	0 304	5 83	0 263	9 200	95	1.68
	70	40	0	SR	3 440	0.373	0 622	0 142	0 156	4400	[illegible]	5 06	0 256	9 000	100	1 48
	70	40	00	CS	3 300	0.323	0 495	0.151	0 142	3500	0 297	5 69	0 259	6 700	95	1.73
	70	40	000	CS	3 890	0.364	0 392	0.171	0 138	2700	0 329	5 28	0 246	5 100	95	1 63
	70	40	0000	CS	4 530	0.417	0 310	0.191	0 135	2400	0 307	4 57	0 237	4 600	100	1.42
	70	40	250 000	CS	5 160	0 455	0 263	0 210	0 132	2100	0 396	4 07	0 231	4 200	105	1 27
	70	40	300 000	CS	5 810	0.497	0 220	0 230	0 130	1900	0 424	3 82	0 228	3 800	105	1.20
	70	40	350 000	CS	6 470	0 539	0 190	0 249	0 129	1800	0 455	3 61	0 219	3 700	105	1.14
	70	40	400 000	CS	7 240	0 572	0 166	0 265	0 128	1700	0 478	3 32	0 218	3 400	110	1 05
	70	40	500 000	CS	8 660	0 642	0 134	0 297	0 126	1500	[illegible]	2 89	0 214	3 000	115	0 916
	75	40	600 000	CS	9 910	0 700	0 113	0 327	0 125	1400	[illegible]	2 68	0 210	2 800	115	0 855
	75	40	750 000	CS	11 920	0 780	0 091	0 366	0 121	1300	0 633	2 37	0 204	2 500	120	0 758
5 Kv	105	55	6	SR	2 150	0 184	2 50	0 067	0 215	8000	0 218	8 14	0 342	15 000	95	1 88
	100	55	4	SR	2 470	0 232	1 58	0 084	0 199	7000	0 250	6 86	0 317	13 600	95	1 76
	95	50	2	SR	2 900	0 292	0 987	0 106	0 184	6100	0 291	5 88	0 290	11 300	95	1 63
	90	45	1	SR	3 280	0 332	0 786	0 126	0 171	5400	0 321	5 23	0 270	10 200	100	1 48
	90	45	0	SR	3 660	0 373	0 622	0 142	0 165	5000	0 352	4 79	0 259	9 600	100	1 39
	85	45	00	CS	3 460	0 323	0 495	0 151	0 148	3600	0 317	5 42	0 263	9 300	95	1 64
	85	45	000	CS	4 060	0 364	0 392	0 171	0 143	3200	0 343	4 74	0 254	8 700	100	1 45
	85	45	0000	CS	4 720	0.417	0 310	0 191	0 141	2600	0 380	4 33	0 245	8 300	100	1 34
	85	45	250 000	CS	5 370	0.455	0 263	0 210	0 138	2600	0 410	3 89	0 237	7 800	105	1 21
	85	45	300 000	CS	6 050	0 497	0 220	0 230	0 135	2400	0 436	3 67	0 231	7 400	105	1 15
	85	45	350 000	CS	6 830	0 539	0 190	0 249	0 133	2200	0 470	3 31	0 225	7 000	110	1 04
	85	45	400 000	CS	7 480	0 572	0 166	0.265	0 131	2000	0 493	3 17	0 221	6 700	110	1 00
	85	45	500 000	CS	8 890	0 642	0 134	0 297	0 129	1800	0 547	2 79	0 216	6 200	115	0 885
	85	45	600 000	CS	10 300	0 700	0 113	0 327	0 128	1600	0 587	2 51	0 210	5 800	120	0 798
	85	45	750 000	CS	12 340	0 780	0 091	0 366	0 125	1500	0 643	2 21	0 206	5 400	125	0 707
8 Kv	130	65	6	SR	2 450	0 184	2 50	0 067	0 230	9600	0 236	7 57	0 353	16 300	95	1 69
	125	65	4	SR	2 900	0 232	1 58	0 084	0 212	8300	0 269	6 08	0 329	14 500	100	1 50
	115	60	2	SR	3 280	0 292	0 987	0 106	0 193	6800	0 307	5 25	0 302	12 500	100	1 42
	110	55	1	SR	3 660	0 332	0 786	0 126	0 179	6100	0 338	4 90	0 280	11 400	100	1.37
	110	55	0	SR	4 090	0 373	0 622	0 142	0 174	5700	0 308	4 31	0 272	10 700	105	1 23
	105	55	00	CS	3 870	0 323	0 495	0 151	0 156	4300	0 330	4 79	0 273	8 300	100	1 43
	105	55	000	CS	4 390	0 364	0 392	0 171	0 151	3800	0 362	4 41	0 263	7 400	100	1 34
	105	55	0000	CS	5 150	0 417	0 310	0 191	0 147	3500	0 399	3 88	0 254	6 600	105	1 19
	105	55	250 000	CS	5 830	0 455	0 263	0 210	0 144	3200	0 428	3 50	0 246	6 200	110	1 08
	105	55	300 000	CS	6 500	0 497	0 220	0 230	0 141	2900	0 454	3 31	0 239	5 600	110	1 03
	105	55	350 000	CS	7 160	0 539	0 190	0 249	0 139	2700	0 489	3 12	0 233	5 200	110	0 978
	105	55	400 000	CS	7 980	0 572	0 166	0 265	0 137	2500	0 513	2 86	0 230	4 900	115	0 899
	105	55	500 000	CS	9 430	0 642	0 134	0 297	0 135	2200	0 563	2 53	0 224	4 300	120	0 800
	105	55	600 000	CS	10 680	0 700	0 113	0 327	0 132	2000	0 606	2 39	0 218	3 900	120	0 758
	105	55	750 000	CS	12 740	0 780	0 091	0 366	0 129	1800	0 663	2 11	0 211	3 500	125	0 673
15 Kv	170	85	2	SR	4 350	0 292	0 987	0 106	0 217	8600	0 349	4 20	0 323	15 000	110	1 07
	165	80	1	SR	4 640	0 332	0 786	0 126	0 202	7800	0 381	3 88	0 305	13 800	110	1 03
	160	75	0	SR	4 990	0 373	0 622	0 142	0 193	7100	0 409	3 62	0 288	12 800	110	1 00
	155	75	00	SR	5 600	0 419	0 495	0 159	0 185	6500	0 439	3 25	0 280	12 000	115	0 918
	155	75	000	SR	6 230	0 470	0 392	0 178	0 180	6000	0 476	2 99	0 272	11 300	115	0 867
	155	75	0000	SR	7 180	0 528	0 310	0 200	0 174	5600	0 520	2 64	0 263	10 600	120	0 778
	155	75	250 000	SR	7 840	0 575	0 263	0 218	0 168	5300	0 555	2 50	0 256	10 200	120	0 744
	155	75	300 000	CS	7 480	0 497	0 220	0 230	0 155	5400	0 507	2 79	0 254	7 900	115	0 855
	155	75	350 000	CS	8 340	0 539	0 190	0 249	0 152	5100	0 536	2 54	0 250	7 200	120	0 784
	155	75	400 000	CS	9 030	0 572	0 166	0 265	0 149	4900	0 561	2 44	0 245	6 900	120	0 758
	155	75	500 000	CS	10 550	0 642	0 134	0 297	0 145	4600	0 611	2 26	0 239	6 200	125	0 680
	155	75	600 000	CS	12 030	0 700	0 113	0 327	0 142	4300	0 656	1 97	0 231	5 700	130	0 620
	155	75	750 000	CS	14 190	0 780	0 091	0 366	0 139	4000	0 712	1 77	0 226	5 100	135	0 568

[1] Ac resistance based upon 100% conductivity at 65°C including 2% allowance for stranding.

[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.

[3] For dielectric constant = 3.7.

[4] Based upon all return current in the sheath; none in ground.

[5] See Fig. 7, pp. 67, of Ref. 1.

[6] The following symbols are used to designate the cable types; SR—stranded round; CS—compact sector.

Source: [1].

Table A-16 60-Hz characteristics of three-conductor shielded paper-insulated cables

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Type of Conductor (6)	Weight per 1000 Feet	Diameter or Sector Depth (2) inches	Resistance—Ohms per Mile (1)	GMR of one Conductor (2) inches	Positive & Negative Sequence: Series Reactance Ohms per Mile	Positive & Negative Sequence: Shunt-Capacitive Reactance—Ohms per Mile (3)	GMR—Three Conductors	Zero-Sequence: Series Resistance Ohms per Mile (4)	Zero-Sequence: Series Reactance Ohms per Mile (4)	Zero-Sequence: Shunt-Capacitive Reactance—Ohms per Mile (3)	Sheath: Thickness Mils	Sheath: Resistance Ohms per Mile at 50°C
15 Kv	[illegible]	4	SR	[illegible]	[illegible]	1.58	0.084	[illegible]	[illegible]	0.328	5.15	0.325	[illegible]	105	1.19
	190	2	SR	[illegible]	[illegible]	0.987	0.106	0.226	[illegible]	0.365	4.44	0.298	[illegible]	105	1.15
	185	1	SR	[illegible]	[illegible]	0.786	0.126	0.210	[illegible]	0.398	3.91	0.285	[illegible]	110	1.04
	180	0	SR	[illegible]	0.373	[illegible]	0.141	0.201	6400	0.425	3.65	0.275	[illegible]	110	1.01
	175	00	CS	[illegible]	0.323	[illegible]	0.151	0.178	[illegible]	0.397	3.95	0.268	[illegible]	105	1.15
	175	000	CS	5 510	0.364	[illegible]	0.171	0.170	4800	0.432	3.48	0.256	4800	110	1.03
	175	0000	CS	6 180	0.417	[illegible]	0.191	[illegible]	4400	0.468	3.24	0.249	4400	110	0.975
	175	250 000	CS	6 910	0.455	[illegible]	0.210	[illegible]	4100	0.498	2.95	[illegible]	4100	115	0.897
	175	300 000	CS	7 610	0.497	[illegible]	0.230	[illegible]	3800	0.530	2.80	[illegible]	3800	115	0.860
	175	350 000	CS	8 480	0.539	[illegible]	0.249	0.153	3600	0.561	2.53	[illegible]	3600	120	[illegible]
	175	400 000	CS	9 170	0.572	[illegible]	0.265	0.151	3400	0.585	2.45	[illegible]	3400	120	[illegible]
	175	[illegible]	CS	10 710	0.642	0.134	0.297	[illegible]	3100	0.636	2.19	[illegible]	[illegible]	[illegible]	[illegible]
	175	[illegible]	CS	12 230	0.700	0.113	0.327	0.143	2900	0.681	1.98	0.215	[illegible]	130	[illegible]
	175	[illegible]	CS	14 380	0.780	[illegible]	0.366	[illegible]	[illegible]	0.737	1.78	0.211	[illegible]	[illegible]	[illegible]
23 Kv	265	2	SR	5 590	0.292	[illegible]	0.106	[illegible]	[illegible]	0.418	3.60	0.317	8300	115	0.870
	250	1	SR	5 860	0.332	[illegible]	0.126	[illegible]	[illegible]	0.450	3.26	[illegible]	[illegible]	[illegible]	0.851
	250	[illegible]	SR	[illegible]	0.373	[illegible]	0.141	[illegible]	6900	0.477	2.99	[illegible]	[illegible]	[illegible]	0.788
	240	[illegible]	CS	[illegible]	0.323	[illegible]	0.151	0.196	[illegible]	0.446	3.18	[illegible]	[illegible]	[illegible]	0.890
	240	[illegible]	CS	[illegible]	0.364	[illegible]	0.171	[illegible]	[illegible]	0.480	2.95	[illegible]	[illegible]	115	0.851
	240	[illegible]	CS	[illegible]	0.410	[illegible]	0.191	0.181	[illegible]	0.515	2.64	[illegible]	[illegible]	120	0.775
	240	[illegible]	CS	[illegible]	0.447	[illegible]	0.210	0.177	5200	0.545	2.50	[illegible]	[illegible]	120	0.747
	240	[illegible]	CS	[illegible]	0.490	[illegible]	0.230	0.171	[illegible]	0.579	2.29	[illegible]	[illegible]	125	0.690
	240	[illegible]	CS	9 720	0.532	[illegible]	0.249	0.167	[illegible]	0.610	2.10	[illegible]	[illegible]	125	0.665
	240	[illegible]	CS	10 650	0.566	[illegible]	0.265	0.165	4400	0.633	2.03	[illegible]	[illegible]	130	0.620
	240	[illegible]	CS	12 280	0.635	0.134	0.297	0.159	3900	0.687	1.82	[illegible]	[illegible]	135	0.562
	240	600 000	CS	13 610	0.690	0.113	0.327	0.154	3700	0.730	1.73	[illegible]	[illegible]	135	0.540
	240	[illegible]	CS	15 830	0.767	[illegible]	0.366	0.151	3400	0.787	1.56	[illegible]	[illegible]	140	0.488
35 Kv	355	[illegible]	SR	8 520	0.288	[illegible]	0.141	0.239	9900	0.523	2.40	0.330	[illegible]	130	0.594
	345	[illegible]	SR	9 180	0.323	[illegible]	0.159	0.226	9100	0.548	2.17	0.322	[illegible]	135	0.559
	345	[illegible]	SR	9 900	0.364	[illegible]	0.178	0.217	8500	0.585	2.01	0.312	[illegible]	135	0.538
	345	[illegible]	CS	9 830	0.410	[illegible]	0.191	0.204	7200	0.594	2.00	0.290	[illegible]	135	0.563
	345	[illegible]	[illegible]	10 470	0.447	[illegible]	0.210	0.197	6800	0.628	1.90	0.280	[illegible]	135	0.545
	345	[illegible]	[illegible]	11 290	0.490	[illegible]	0.230	0.191	6400	0.663	1.80	0.273	[illegible]	135	0.527
	345	[illegible]	[illegible]	12 280	0.532	0.190	0.249	0.187	6000	0.693	1.66	0.270	[illegible]	140	0.491
	345	[illegible]	[illegible]	13 030	0.566	[illegible]	0.265	0.183	5700	0.721	1.61	0.265	[illegible]	140	0.480
	345	[illegible]	[illegible]	14 760	0.635	0.134	[illegible]	0.177	5200	0.773	1.46	0.257	[illegible]	145	0.441
	345	[illegible]	[illegible]	16 420	0.690	0.113	[illegible]	0.171	4900	0.819	1.35	0.248	[illegible]	150	0.412
	345	750 000	CS	18 860	0.767	0.091	[illegible]	0.165	4500	0.879	1.22	0.243	[illegible]	155	0.377

[1] Ac resistance based on 100% conductivity at 65°C including 2% allowance for stranding.

[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.

[3] For dielectric constant = 3.7.

[4] Based on all return current in the sheath; none in ground.

[5] See Fig. 7, pp. 67, of Ref. 1.

[6] The following symbols are used to designate conductor types: SR—stranded round; CS—compact sector.

Source: [1].

Table A-17 60-Hz characteristics of three-conductor oil-filled paper-insulated cables

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B. & S.)	Type of Conductor (6)	Weight per 1000 Feet	Diameter or Sector Depth (5)—inches	Resistance-Ohms Per Mile (1)	GMR of One Conductor (2)—inches	POSITIVE & NEGATIVE SEQ: Series Reactance Ohms Per Mile	POSITIVE & NEGATIVE SEQ: Shunt Capacitive Reactance-Ohms Per Mile (3)	GMR—Three Conductors	ZERO-SEQUENCE: Series Resistance Ohms Per Mile (4)	ZERO-SEQUENCE: Series Reactance Ohms Per Mile (4)	ZERO-SEQUENCE: Shunt Capacitive Reactance-Ohms Per Mile (3)	SHEATH: Thickness Mils	SHEATH: Resistance-Ohms Per Mile at 50°C
35 Kv	190	[illegible]	CS	[illegible]	0.323	[illegible]	0.151	[illegible]	[illegible]	[illegible]	[illegible]	0.265	[illegible]	115	1.02
		[illegible]	CS	[illegible]	0.364	[illegible]	0.171	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	115	0.970
		[illegible]	CS	[illegible]	0.117	[illegible]	0.191	[illegible]	[illegible]	[illegible]	[illegible]	0.243	[illegible]	115	0.918
		[illegible]	CS	[illegible]	0.455	[illegible]	0.210	[illegible]	[illegible]	[illegible]	[illegible]	0.238	[illegible]	125	0.820
		[illegible]	CS	[illegible]	0.497	0.220	0.230	[illegible]	[illegible]	0.539	2.58	[illegible]	[illegible]	125	0.788
		[illegible]	CS	9 190	0.539	0.190	0.249	0.160	[illegible]	0.570	2.44	[illegible]	[illegible]	125	0.752
		[illegible]	CS	9 900	0.572	0.166	0.265	0.157	[illegible]	0.595	2.35	[illegible]	[illegible]	125	0.729
		[illegible]	CS	11 550	0.642	0.134	0.297	0.153	[illegible]	0.646	2.04	[illegible]	[illegible]	135	0.636
		[illegible]	CS	12 900	0.700	[illegible]	0.327	0.150	[illegible]	0.691	1.94	0.210	[illegible]	135	0.608
		[illegible]	CS	15 660	0.780	[illegible]	0.366	0.148	[illegible]	0.763	1.73	[illegible]	[illegible]	140	0.548
46 Kv	225	[illegible]	CS	6 360	0.323	[illegible]	0.151	0.195	[illegible]	0.436	3.28	[illegible]	[illegible]	[illegible]	0.928
		[illegible]	CS	6 940	0.364	[illegible]	0.171	0.188	[illegible]	0.468	2.87	[illegible]	[illegible]	[illegible]	0.826
		[illegible]	CS	7 660	0.410	[illegible]	0.191	0.180	5520	0.503	2.67	0.256	[illegible]	[illegible]	0.788
		[illegible]	CS	8 280	0.447	[illegible]	0.210	0.177	5180	0.533	2.55	0.247	[illegible]	[illegible]	0.761
		[illegible]	CS	9 690	0.490	[illegible]	0.230	0.172	4820	0.566	2.41	0.241	[illegible]	[illegible]	0.729
		[illegible]	CS	10 100	0.532	[illegible]	0.249	0.168	4490	0.596	2.16	0.237	[illegible]	[illegible]	0.658
		[illegible]	CS	10 820	0.566	[illegible]	0.265	0.165	4220	0.623	2.08	0.232	4220	[illegible]	0.639
		[illegible]	CS	12 220	0.635	[illegible]	0.297	0.160	3870	0.672	1.94	0.226	3870	[illegible]	0.603
		[illegible]	CS	13 930	0.690	0.110	0.327	0.156	3670	0.718	1.74	0.219	3670	140	0.542
		[illegible]	CS	16 040	0.767	0.091	0.366	0.151	3350	0.773	1.62	0.213	3350	140	0.510
		[illegible]	CS												
69 Kv	315	[illegible]	CR	8 240	0.376	[illegible]	0.147	[illegible]	[illegible]	0.532	2.41	[illegible]	8330	135	0.639
		[illegible]	CS	8 830	0.364	[illegible]	0.171	[illegible]	[illegible]	0.538	2.32	[illegible]	[illegible]	135	0.642
		[illegible]	CS	9 660	0.410	[illegible]	0.191	[illegible]	[illegible]	0.573	2.16	[illegible]	6840	135	0.618
		[illegible]	CS	[illegible]	0.447	[illegible]	0.210	[illegible]	[illegible]	0.607	2.06	[illegible]	6500	[illegible]	0.597
		[illegible]	CS	[illegible]	0.490	[illegible]	0.230	[illegible]	[illegible]	0.640	1.85	[illegible]	6030	140	0.543
		[illegible]	CS	[illegible]	0.532	[illegible]	0.249	0.185	[illegible]	0.672	1.77	[illegible]	5700	140	0.527
		[illegible]	CS	[illegible]	0.566	[illegible]	0.265	0.181	[illegible]	0.700	1.55	[illegible]	5440	140	0.513
		[illegible]	CS	14 890	0.635	[illegible]	0.297	0.176	[illegible]	0.750	1.51	[illegible]	5050	150	0.460
		[illegible]	CS	[illegible]	0.690	[illegible]	0.327	0.171	[illegible]	0.797	1.44	[illegible]	4740	150	0.442
		[illegible]	CS	18 980	0.767	0.091	0.366	0.165	4360	0.834	1.29	0.230	4360	155	0.399

[1] Ac resistance based on 100% conductivity at 65°C, including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant = 3.5.
[4] Based on all return current in sheath, none in ground.
[5] See Fig. 7, pp. 67, of Ref. 1.
[6] The following symbols are used to designate the cable types: CR—Compact Round; CS—Compact Sector.

Source: [1].

Table A-18 60-Hz characteristics of single-conductor concentric-strand paper-insulated cables

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight Per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[1]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[2]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C	Shunt Capacitive Reactance Ohms Per Phase Per Mile[3]	Lead Sheath Thickness—Mils
1 Kv	60	6	560	0.184	[illegible]	0.628	0.489	2.50	[illegible]	[illegible]	[illegible]
	60	4	670	[illegible]	[illegible]	0.602	0.475	1.58	[illegible]	[illegible]	[illegible]
	60	2	880	[illegible]	[illegible]	[illegible]	0.458	0.987	[illegible]	[illegible]	[illegible]
	60	1	990	[illegible]	[illegible]	0.552	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	60	0	1 110	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.61	[illegible]	[illegible]
	60	00	1 270	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	90
	60	000	1 510	0.470	[illegible]	0.512	[illegible]	[illegible]	[illegible]	1940	95
	60	0000	[illegible]	[illegible]	[illegible]	0.496	[illegible]	[illegible]	[illegible]	1450	85
	60	[illegible]	1 930	0.575	[illegible]	[illegible]	0.409	[illegible]	[illegible]	[illegible]	[illegible]
	60	[illegible]	[illegible]	0.681	[illegible]	0.484	0.392	0.190	[illegible]	[illegible]	90
	60	[illegible]	[illegible]	0.814	[illegible]	0.442	0.378	0.134	[illegible]	[illegible]	[illegible]
	60	[illegible]	4 380	[illegible]	[illegible]	0.417	0.358	0.091	[illegible]	[illegible]	[illegible]
	60	[illegible]	5 560	1.15	[illegible]	0.400	0.344	0.070	1.40	800	100
	60	[illegible]	8 000	1.41	[illegible]	[illegible]	0.319	0.050	1.05	645	110
	60	[illegible]	10 190	1.63	[illegible]	0.356	0.305	0.041	[illegible]	[illegible]	115
3 Kv	[illegible]	6	600	[illegible]	[illegible]	[illegible]	0.481	2.50	5.80	[illegible]	[illegible]
	[illegible]	4	720	0.23	[illegible]	0.602	0.487	1.58	5.23	[illegible]	75
	[illegible]	2	[illegible]	0.292	[illegible]	[illegible]	0.453	[illegible]	4.31	[illegible]	[illegible]
	75	1	1 040	[illegible]	[illegible]	0.552	0.445	[illegible]	4.03	[illegible]	80
	75	0	1 170	0.373	[illegible]	[illegible]	0.434	[illegible]	[illegible]	[illegible]	80
	75	00	1 320	0.419	0.159	[illegible]	[illegible]	[illegible]	[illegible]	2150	80
	[illegible]	000	[illegible]	0.470	[illegible]	0.512	[illegible]	[illegible]	[illegible]	[illegible]	85
	[illegible]	0000	1 600	0.528	0.200	0.498	0.412	0.310	[illegible]	[illegible]	85
	[illegible]	[illegible]	1 930	0.575	0.221	0.484	0.403	[illegible]	[illegible]	[illegible]	85
	[illegible]	[illegible]	[illegible]	0.681	0.262	0.484	0.389	0.190	[illegible]	1510	90
	75	[illegible]	[illegible]	0.814	0.313	0.442	0.375	0.134	[illegible]	[illegible]	95
	[illegible]	[illegible]	[illegible]	[illegible]	0.385	0.417	0.352	0.091	1.63	1060	100
	[illegible]	[illegible]	[illegible]	1.152	0.445	0.400	0.341	0.070	1.37	980	100
	[illegible]	[illegible]	[illegible]	[illegible]	0.543	[illegible]	0.316	0.050	[illegible]	805	110
	[illegible]	[illegible]	[illegible]	1.63	[illegible]	0.356	[illegible]	[illegible]	[illegible]	685	115
5 Kv	120	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.456	[illegible]	[illegible]	[illegible]	80
	115	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	110	2	[illegible]	[illegible]	[illegible]	[illegible]	0.439	[illegible]	[illegible]	[illegible]	80
	110	1	[illegible]	[illegible]	[illegible]	0.552	0.431	[illegible]	[illegible]	4100	80
	105	0	[illegible]	[illegible]	[illegible]	0.509	0.425	[illegible]	[illegible]	[illegible]	80
	100	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.09	[illegible]	85
	100	[illegible]	[illegible]	[illegible]	[illegible]	0.512	0.412	0.392	[illegible]	[illegible]	85
	95	[illegible]	[illegible]	0.525	[illegible]	0.498	0.408	0.310	[illegible]	[illegible]	85
	90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.400	[illegible]	[illegible]	[illegible]	85
	90	[illegible]	[illegible]	0.681	[illegible]	0.464	[illegible]	[illegible]	[illegible]	[illegible]	90
	90	[illegible]	3 410	0.814	[illegible]	0.442	0.369	[illegible]	1.65	[illegible]	95
	90	[illegible]	4 950	0.998	[illegible]	0.417	0.350	[illegible]	[illegible]	1350	100
	90	[illegible]	5 950	[illegible]	[illegible]	0.400	0.339	[illegible]	[illegible]	1140	105
	90	[illegible]	8 160	[illegible]	[illegible]	[illegible]	0.318	[illegible]	[illegible]	950	110
	90	[illegible]	10 370	1.63	[illegible]	[illegible]	[illegible]	0.041	[illegible]	[illegible]	115
8 Kv	150	[illegible]	890	[illegible]	[illegible]	[illegible]	0.431	[illegible]	[illegible]	[illegible]	[illegible]
	150	4	1 010	[illegible]	[illegible]	0.602	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	140	2	1 150	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	140	1	1 330	[illegible]	[illegible]	0.552	0.411	[illegible]	[illegible]	[illegible]	[illegible]
	135	0	1 450	[illegible]	[illegible]	0.539	0.408	0.622	[illegible]	[illegible]	85
	130	00	1 590	0.418	[illegible]	[illegible]	0.403	0.495	[illegible]	[illegible]	[illegible]
	[illegible]	000	[illegible]	0.470	[illegible]	0.512	[illegible]	0.392	[illegible]	[illegible]	[illegible]
	120	0000	1 980	0.528	[illegible]	0.496	0.389	[illegible]	2.29	[illegible]	90
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.484	0.383	[illegible]	[illegible]	[illegible]	90
	115	[illegible]	[illegible]	0.681	[illegible]	0.464	[illegible]	[illegible]	[illegible]	[illegible]	95
	115	[illegible]	[illegible]	0.814	[illegible]	0.442	0.361	[illegible]	1.69	[illegible]	95
	115	[illegible]	[illegible]	0.998	[illegible]	0.417	0.341	[illegible]	[illegible]	[illegible]	100
	115	[illegible]	[illegible]	[illegible]	[illegible]	0.400	0.330	[illegible]	[illegible]	[illegible]	105
	115	1 500 000	8 250	1.412	0.543	0.374	[illegible]	[illegible]	[illegible]	[illegible]	110
	115	[illegible]	10 480	1.632	0.663	0.356	[illegible]	[illegible]	[illegible]	1055	120
15 Kv	220	4	1 340	0.232	0.084	[illegible]	0.412	1.58	[illegible]	[illegible]	85
	215	2	1 500	[illegible]	0.106	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	85
	210	1	1 610	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	85
	200	0	1 710	0.373	[illegible]	[illegible]	[illegible]	0.622	[illegible]	[illegible]	85
	195	00	1 940	0.419	[illegible]	[illegible]	[illegible]	0.495	[illegible]	[illegible]	90
	185	000	2 100	0.470	[illegible]	[illegible]	[illegible]	0.392	[illegible]	[illegible]	90
	180	0000	2 300	0.528	[illegible]	[illegible]	[illegible]	0.310	2.14	[illegible]	90
	175	250 000	2 500	0.575	[illegible]	[illegible]	[illegible]	[illegible]	2.06	[illegible]	90
	175	350 000	3 110	[illegible]	[illegible]	[illegible]	0.366	0.190	1.98	[illegible]	95
	175	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.352	0.134	1.51	[illegible]	100
	175	[illegible]	5 240	0.998	[illegible]	[illegible]	0.336	0.091	[illegible]	[illegible]	105
	175	1 000 000	[illegible]	1.15	[illegible]	[illegible]	0.325	0.070	1.15	[illegible]	108
	175	1 500 000	[illegible]	[illegible]	[illegible]	[illegible]	0.305	0.050	0.90	1790	118
	175	[illegible]	[illegible]	1.63	[illegible]	[illegible]	0.294	0.041	0.772	1570	120
23 Kv	295	2	[illegible]	[illegible]	0.106	[illegible]	0.383	0.987	[illegible]	[illegible]	90
	285	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.786	[illegible]	[illegible]	90
	275	0	[illegible]	[illegible]	0.141	[illegible]	[illegible]	0.622	[illegible]	[illegible]	90
	265	00	[illegible]	0.418	0.159	[illegible]	0.375	0.495	[illegible]	[illegible]	90
	260	000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.392	[illegible]	[illegible]	95
	250	0000	[illegible]	[illegible]	0.200	[illegible]	0.366	0.310	1.78	5350	95
	245	[illegible]	[illegible]	[illegible]	0.221	[illegible]	0.361	0.263	1.72	4950	95
	240	[illegible]	[illegible]	[illegible]	0.262	0.464	0.352	0.190	[illegible]	4310	100
	240	[illegible]	[illegible]	[illegible]	0.313	0.442	0.341	[illegible]	[illegible]	[illegible]	100
	240	750 000	[illegible]	[illegible]	0.385	0.417	0.325	[illegible]	[illegible]	[illegible]	105
	240	1 000 000	6 910	[illegible]	0.445	0.400	0.313	[illegible]	[illegible]	[illegible]	110
	240	1 500 000	9 460	1.412	0.546	0.374	0.296	0.050	[illegible]	[illegible]	120
	240	2 000 000	[illegible]	1.632	0.633	[illegible]	[illegible]	0.041	[illegible]	[illegible]	[illegible]
35 Kv	395	0	2 900	0.373	0.141	[illegible]	0.352	[illegible]	[illegible]	[illegible]	100
	385	00	3 040	0.418	0.159	[illegible]	0.350	0.495	[illegible]	[illegible]	100
	370	000	3 190	0.470	0.179	[illegible]	0.347	0.392	[illegible]	[illegible]	100
	355	0000	3 380	0.528	0.200	[illegible]	0.344	0.310	[illegible]	[illegible]	100
	350	[illegible]	3 590	0.575	0.221	[illegible]	0.342	0.263	1.39	6410	100
	345	[illegible]	4 230	0.681	[illegible]	[illegible]	[illegible]	0.190	1.24	5640	105
	345	500 000	[illegible]	[illegible]	[illegible]	[illegible]	0.375	0.134	1.15	4940	105
	345	[illegible]	6 430	[illegible]	0.385	[illegible]	[illegible]	0.091	0.975	4250	110
	345	1 000 000	[illegible]	[illegible]	[illegible]	[illegible]	0.302	0.070	0.866	3780	115
	345	1 500 000	10 470	[illegible]	[illegible]	[illegible]	[illegible]	0.050	0.700	3210	125
	345	2 000 000	[illegible]	[illegible]	[illegible]	0.366	[illegible]	0.041	0.611	2830	130
46 Kv	475	000	[illegible]	[illegible]	[illegible]	[illegible]	0.331	0.392	1.20	8890	105
	460	0000	4 080	0.528	0.200	[illegible]	[illegible]	0.310	1.19	8100	105
	[illegible]	250 000	4 290	0.575	0.221	[illegible]	[illegible]	0.263	1.16	7570	105
	445	350 000	4 890	0.681	[illegible]	0.464	0.319	0.190	1.05	6720	110
	445	600 000	5 820	0.814	[illegible]	[illegible]	[illegible]	0.134	0.930	5950	115
	445	[illegible]	[illegible]	0.998	0.385	0.417	0.298	0.091	0.807	5130	120
	445	1 000 000	9 680	1.152	0.445	0.400	0.280	0.070	0.752	4810	120
	445	[illegible]	11 420	1.412	0.546	0.374	0.275	0.050	0.615	3930	130
	445	2 000 000	13 910	1.632	0.633	[illegible]	0.264	[illegible]	[illegible]	[illegible]	135
69 Kv	650	350 000	6 720	0.681	0.262	0.464	0.292	0.190	[illegible]	[illegible]	120
	650	500 000	7 810	0.814	0.313	0.442	0.284	[illegible]	[illegible]	[illegible]	125
	650	750 000	9 420	0.998	0.385	0.417	0.275	0.091	[illegible]	[illegible]	130
	650	1 000 000	10 940	1.152	0.445	0.400	0.267	[illegible]	[illegible]	[illegible]	135
	650	1 500 000	13 680	1.412	0.546	0.374	0.266	[illegible]	[illegible]	[illegible]	140
	650	2 000 000	16 320	1.632	0.633	0.356	0.246	0.041	0.437	4710	145

[1] Conductors are standard concentric-stranded, not compact round.
[2] Ac Resistance based on 100% conductivity at 65°C including 2% allowance for stranding.
[3] For dielectric constant = 3.7.
Source: [1].

Table A-19 60-Hz characteristics of single-conductor oil-filled (hollow-core) paper-insulated cables

INSIDE DIAMETER OF SPRING CORE = 0.5 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor—inches	x_a Reactance at 12 Inches, Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[1]	r_s Resistance of Sheath Per Phase Per Mile, °C	Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	00	3 980	0.736	0.345	0.431	0.331	0.445	[illegible]	[illegible]	110
		000	4 090	0.768	0.356	0.427	0.331	[illegible]	1.15	[illegible]	110
		0000	4 320	0.807	0.373	0.421	0.328	0.310	[illegible]	[illegible]	110
		250 000	4 650	0.837	0.381	0.418	0.325	0.263	1.05	[illegible]	115
		350 000	5 180	0.918	0.408	0.410	0.320	0.188	[illegible]	[illegible]	115
		500 000	6 100	1.028	0.448	0.399	0.312	0.133	[illegible]	4070	[illegible]
		750 000	7 310	1.180	0.505	0.384	0.302	0.089	[illegible]	[illegible]	[illegible]
		1 000 000	8 630	1.310	0.550	0.374	0.294	0.068	[illegible]	[illegible]	[illegible]
		[illegible]	11 090	1.547	0.639	0.356	0.281	0.049	[illegible]	[illegible]	130
		2 000 000	[illegible]	1.760	0.716	0.342	0.270	0.039	0.550	[illegible]	140
115 Kv	490	[illegible]	[illegible]	[illegible]	[illegible]	0.421	0.305	0.310	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	0.381	0.418	0.303	0.263	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	0.410	0.298	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	0.399	0.291	0.133	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	0.384	0.283	0.089	[illegible]	[illegible]	130
		[illegible]	[illegible]	[illegible]	550	0.374	0.276	[illegible]	[illegible]	[illegible]	135
		[illegible]	[illegible]	[illegible]	[illegible]	0.356	0.265	[illegible]	[illegible]	[illegible]	140
		[illegible]	[illegible]	[illegible]	[illegible]	0.342	0.255	0.039	[illegible]	[illegible]	[illegible]
138 Kv	560	0000	[illegible]	[illegible]	[illegible]	0.421	0.295	0.310	[illegible]	[illegible]	[illegible]
		250 000	[illegible]	[illegible]	0.381	0.418	0.293	[illegible]	[illegible]	[illegible]	[illegible]
		350 000	[illegible]	[illegible]	0.408	0.410	0.288	[illegible]	[illegible]	[illegible]	130
		500 000	[illegible]	[illegible]	[illegible]	0.399	0.282	[illegible]	[illegible]	[illegible]	130
		750 000	[illegible]	1.180	0.505	0.384	0.274	[illegible]	[illegible]	[illegible]	135
		1 000 000	[illegible]	[illegible]	0.550	0.374	0.268	[illegible]	0.541	[illegible]	140
		1 500 000	13 720	1.547	0.639	0.356	0.257	0.048	[illegible]	[illegible]	[illegible]
		2 000 000	16 080	1.760	0.716	0.342	0.248	0.039	[illegible]	[illegible]	150
161 Kv	650	[illegible]	600	0.837	0.381	[illegible]	[illegible]	[illegible]	0.860	[illegible]	[illegible]
		[illegible]	[illegible]	0.918	0.408	0.410	[illegible]	[illegible]	0.811	[illegible]	[illegible]
		[illegible]	[illegible]	1.028	[illegible]	0.399	[illegible]	[illegible]	[illegible]	[illegible]	135
		[illegible]	[illegible]	1.180	0.505	[illegible]	[illegible]	0.089	0.532	[illegible]	[illegible]
		[illegible]	[illegible]	1.310	0.550	[illegible]	[illegible]	0.068	0.483	5880	[illegible]
		[illegible]	15 090	1.547	0.639	0.356	0.246	0.048	0.433	5190	[illegible]
		[illegible]	[illegible]	1.760	0.716	0.342	0.241	0.039	0.391	[illegible]	[illegible]

INSIDE DIAMETER OF SPRING CORE = 0.69 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[3]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile	r_s Resistance of Sheath—Ohms Per Phase Per Mile at 50°C	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	[illegible]	[illegible]	[illegible]	0.439	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	0.450	[illegible]	[illegible]	0.310	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	0.460	0.396	0.315	0.263	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	1.050	0.483	0.390	0.310	0.188	0.897	4000	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	120
		[illegible]	[illegible]	[illegible]	0.550	[illegible]	[illegible]	[illegible]	0.759	3410	125
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.688	3140	130
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.047	0.601	2750	135
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	038	0.533	2510	140
115 Kv	480	[illegible]	[illegible]	0.956	[illegible]	[illegible]	[illegible]	310	0.760	5950	125
		[illegible]	6 800	0.983	[illegible]	[illegible]	[illegible]	[illegible]	0.752	5790	125
		[illegible]	[illegible]	1.050	[illegible]	[illegible]	[illegible]	[illegible]	0.729	5540	125
		[illegible]	[illegible]	[illegible]	[illegible]	0.382	[illegible]	[illegible]	0.669	5150	130
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.606	4770	135
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.573	4430	135
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.490	3920	145
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	150
138 Kv	560	[illegible]	[illegible]	0.958	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	130
		[illegible]	7 810	0.983	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6480	130
		[illegible]	[illegible]	1.050	[illegible]	[illegible]	0.281	[illegible]	[illegible]	6180	130
		[illegible]	[illegible]	[illegible]	0.516	[illegible]	0.276	[illegible]	[illegible]	[illegible]	135
		[illegible]	10 680	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	140
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.067	[illegible]	4940	140
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.253	[illegible]	[illegible]	[illegible]	145
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.245	0.038	0.404	4060	[illegible]
161 Kv	650	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.275	0.263	[illegible]	7210	135
		350 000	[illegible]	1.050	[illegible]	0.390	0.272	[illegible]	0.580	[illegible]	135
		600 000	10 280	[illegible]	[illegible]	[illegible]	0.267	[illegible]	[illegible]	6430	[illegible]
		750 000	11 770	[illegible]	[illegible]	[illegible]	0.261	0.089	0.492	[illegible]	145
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.067	[illegible]	5540	145
		[illegible]	[illegible]	[illegible]	0.692	[illegible]	[illegible]	[illegible]	0.421	[illegible]	150
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.238	[illegible]	[illegible]	4600	[illegible]
[illegible]	925	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7610	[illegible]
		1 000 000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.355	7140	[illegible]
		2 000 000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	170

[1] Ac Resistance based on 100% conductivity at 65°C including 2% allowance for stranding.
[2] For dielectric constant = 3.5.
[3] Calculated for circular tube.
Source: [1].

Table A-20 Current-carrying capacity of three-conductor belted paper-insulated cables

Conductor Size AWG or 1000 CM	Conductor type[1]	ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	60	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
4500 Volts														Copper Temperature 85°C							
6	H	82	80	78	75	81	78	73	68	79	74	68	63	78	72	65	58	76	69	61	54
4	SR	109	106	103	98	108	102	96	89	104	97	89	81	102	94	84	74	100	90	79	69
2	SR	143	139	134	128	139	133	124	115	136	127	[illegible]	104	133	121	109	95	130	117	101	89
1	SR	164	161	153	146	159	152	141	130	156	145	130	118	152	138	[illegible]	108	148	133	115	100
0	CS	189	184	177	168	184	175	162	149	180	166	149	134	175	159	140	122	170	152	130	114
00	CS	218	211	203	192	211	201	185	170	208	190	170	152	201	181	158	138	195	173	148	128
000	CS	250	242	232	219	242	229	211	193	237	217	193	172	229	208	[illegible]	156	223	197	167	145
0000	CS	286	276	264	249	276	260	240	218	270	246	218	194	261	234	202	176	254	223	189	163
250	CS	318	305	291	273	305	288	263	239	297	271	239	212	288	258	[illegible]	192	278	244	206	177
300	CS	354	340	324	304	340	321	292	264	332	301	264	234	321	285	245	211	310	271	227	195
350	CS	392	378	357	334	375	353	[illegible]	288	366	330	288	255	351	311	[illegible]	229	341	296	218	211
400	CS	424	406	385	358	408	380	344	309	395	355	309	272	380	334	[illegible]	244	367	317	264	224
500	CS	487	[illegible]	439	408	465	433	390	348	451	403	348	305	433	378	320	273	417	[illegible]	[illegible]	251
600	CS	544	[illegible]	487	[illegible]	517	480	430	383	501	444	383	334	480	416	[illegible]	298	462	393	313	273
750	CS	619	[illegible]	550	505	565	541	482	427	566	500	427	371	541	466	390	331	519	439	359	302
		(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.81 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]			
7500 Volts														Copper Temperature 80°C							
6	S	81	80	77	74	79	76	72	67	78	74	67	62	77	71	64	57	[illegible]	69	60	53
4	SR	107	105	101	97	104	100	94	87	103	96	87	79	100	92	[illegible]	73	98	89	77	68
2	SR	140	137	132	126	136	131	122	113	134	125	113	102	130	119	[illegible]	93	[illegible]	114	99	87
1	SR	161	156	150	143	156	149	138	128	153	142	128	115	149	136	120	105	145	130	112	98
0	CS	186	180	174	165	180	172	156	146	177	163	148	131	172	155	136	120	167	149	128	111
00	CS	214	206	198	188	206	198	181	166	202	186	168	148	196	177	155	135	191	169	145	125
000	CS	243	238	228	214	238	224	206	188	230	211	188	168	223	200	174	152	217	192	163	141
0000	CS	280	270	258	241	270	255	235	214	264	241	213	190	255	229	198	172	247	218	184	159
250	CS	311	300	287	264	300	283	259	235	293	266	235	208	282	252	217	188	273	240	202	174
300	CS	349	[illegible]	[illegible]	[illegible]	335	316	288	260	326	296	259	230	315	279	240	207	304	265	213	190
350	CS	385	[illegible]	[illegible]	[illegible]	[illegible]	346	315	283	359	323	282	249	345	305	261	224	333	289	242	206
400	CS	[illegible]	[illegible]	378	353	398	373	338	303	388	348	303	267	371	317	279	239	360	309	257	220
500	CS	478	454	479	399	454	423	381	341	440	[illegible]	340	298	422	369	312	267	406	348	288	245
600	CS	534	508	479	441	517	471	422	378	491	438	375	327	469	408	343	291	451	384	315	267
750	CS	607	571	540	497	573	532	473	419	555	489	418	363	529	455	381	323	507	428	350	295
		(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]			
15 000 Volts														Copper Temperature 75°C							
6	S	78	77	74	71	78	74	69	64	75	70	64	59	73	68	61	54	72	65	57	50
4	SR	102	99	96	92	98	95	89	83	97	91	83	75	95	87	78	69	93	[illegible]	73	64
2	SR	132	129	125	119	129	123	115	106	126	117	108	96	123	112	99	88	120	108	93	82
1	SR	151	147	142	135	146	140	131	120	144	133	120	109	140	128	112	99	136	[illegible]	107	92
0	CS	175	170	163	155	169	161	150	138	166	153	117	123	161	116	128	112	156	139	120	104
00	CS	200	194	187	177	194	184	170	156	189	175	156	139	183	166	145	127	178	158	135	117
000	CS	230	[illegible]	214	202	222	211	195	178	217	199	177	158	210	189	165	143	203	180	153	132
0000	CS	268	257	245	232	253	242	222	202	249	228	201	179	240	215	187	158	233	205	173	149
250	CS	295	284	271	255	281	268	245	221	276	251	220	196	208	239	204	177	257	225	189	163
300	CS	330	317	301	283	316	297	271	245	307	278	244	215	[illegible]	264	245	194	285	248	208	178
350	CS	365	349	332	310	348	327	297	267	339	305	266	235	324	289	[illegible]	211	313	271	227	193
400	CS	394	377	357	333	375	352	319	286	365	327	285	251	340	307	262	224	336	300	241	206
500	CS	449	[illegible]	406	377	428	399	359	321	414	306	319	280	396	346	293	250	379	326	269	229
600	CS	502	479	450	417	476	443	396	352	459	409	351	308	438	380	319	273	420	358	294	249
750	CS	572	543	510	468	540	499	444	393	520	458	391	341	494	425	358	302	471	399	326	275
		(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types: S—solid copper, SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions:

a. Ambient earth temperature = 20°C.

b. 60-cycle alternating current.

c. Ratings include dielectric loss, and all induced ac losses.

d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

Source: [1].

Table A-21 Current-carrying capacity of three-conductor shielded paper-insulated cables

Conductor Size AWG or 1000 CM	Conductor Type[1]	Number of Equally Loaded Cables in Duct Bank																			
		ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
	15 000 Volts													Copper Temperature 81°C							
6	S	94	[illegible]	88	83	91	[illegible]	[illegible]	[illegible]	89	83	74	66	87	78	69	60	[illegible]	[illegible]	[illegible]	[illegible]
4	SR	[illegible]	[illegible]	[illegible]	107	119	[illegible]	[illegible]	95	116	108	95	85	113	102	89	[illegible]	[illegible]	[illegible]	83	[illegible]
2	SR	[illegible]	[illegible]	146	137	153	144	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	107	144	129	112	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	SR	[illegible]	[illegible]	166	156	[illegible]	[illegible]	[illegible]	136	169	[illegible]	[illegible]	121	162	145	125	109	[illegible]	138	[illegible]	[illegible]
0	CS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	154	[illegible]	[illegible]	154	137	183	164	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
00	CS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	218	[illegible]	174	156	211	187	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
000	CS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	249	[illegible]	148	174	241	[illegible]	[illegible]	[illegible]	232	[illegible]	[illegible]	144
0000	CS	308	[illegible]	281	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	196	275	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
250	CS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	305	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	193	291	[illegible]	[illegible]	[illegible]
300	CS	380	[illegible]	344	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
350	CS	417	[illegible]	375	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	255	[illegible]	318	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
400	CS	[illegible]	[illegible]	403	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	313	[illegible]	[illegible]	340	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
500	CS	[illegible]	[illegible]	450	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
600	CS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	437	385	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
750	CS	[illegible]	[illegible]	[illegible]	514	[illegible]	[illegible]	485	426	576	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	519	[illegible]	[illegible]	293
		(1.08 at [illegible] 0.91 at 30°C 0.8[illegible] at 40°C 0.71 at 50°C)[3]				([illegible] at 10°C 0.91 at 30°C 0.8[illegible] at 40°C 0.71 at 50°C)[3]				(1.08 at 10°C 0.91 at 30°C 0.82 at 40°C 0.71 at 50°C)[3]				(1.08 at 10°C 0.91 at 30°C 0.82 at 40°C 0.71 at 50°C)[3]				(1.08 at [illegible] [illegible] at 30°C [illegible] at 40 [illegible] at 50°C)[3]			
	23 000 Volts													Copper Temperature 77°C							
2	SR	[illegible]	[illegible]	143	134	[illegible]	141	130	117	145	132	117	105	140	125	107	84	134	119	100	86
1	SR	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	160	145	133	164	149	132	[illegible]	[illegible]	140	121	105	154	133	112	97
0	CS	200	[illegible]	183	[illegible]	[illegible]	182	166	149	186	169	147	[illegible]	[illegible]	[illegible]	136	118	173	149	126	109
00	CS	227	[illegible]	[illegible]	[illegible]	[illegible]	208	189	[illegible]	[illegible]	193	168	[illegible]	[illegible]	[illegible]	156	134	196	172	144	123
000	CS	[illegible]	[illegible]	[illegible]	220	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	220	191	169	[illegible]	[illegible]	175	150	222	195	162	139
0000	CS	301	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	246	[illegible]	[illegible]	250	215	190	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	182	157
[illegible]	CS	334	[illegible]	298	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	275	236	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	170
[illegible]	CS	373	349	328	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	302	259	[illegible]	[illegible]	283	[illegible]	[illegible]	[illegible]	[illegible]	217	186
[illegible]	CS	405	379	358	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	327	280	243	347	305	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	199
[illegible]	CS	434	409	386	[illegible]	[illegible]	[illegible]	340	[illegible]	[illegible]	348	298	260	374	325	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	CS	492	465	436	401	461	427	[illegible]	[illegible]	443	391	333	288	424	363	[illegible]	[illegible]	400	336	[illegible]	230
[illegible]	CS	543	516	484	440	512	470	[illegible]	[illegible]	[illegible]	428	365	313	464	396	[illegible]	[illegible]	441	[illegible]	[illegible]	[illegible]
750	CS	616	583	541	495	577	528	[illegible]	[illegible]	[illegible]	479	402	347	520	439	364	[illegible]	490	408	[illegible]	[illegible]
		(1.09 at 10°C, 0.90 at 30°C 0.80 at 40°C 0.67 at 50°C)[3]				(1.09 at 10°C 0.90 at [illegible] 0.80 at 40°C, 0.67 at [illegible])				([illegible] at 10°C 0.90 at 30°C, [illegible] at 40°C 0.67 at 50°C)[3]				(1.09 at 10°C 0.90 at [illegible] 0.79 at 40°C, 0.66 at [illegible])				([illegible] at 10 [illegible] at 40 [illegible])			
	34 500 Volts													Copper Temperature 70°C							
0	CS	193	[illegible]	[illegible]	165	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	161	140	124	171	149	129	111	164	142	119	103
00	CS	219	209	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	182	[illegible]	[illegible]	[illegible]	[illegible]	145	126	185	161	134	115
000	CS	250	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	179	[illegible]	[illegible]	[illegible]	165	141	209	182	152	128
0000	CS	288	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	263	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	166	[illegible]	[illegible]	205	170	144
250	CS	316	[illegible]	[illegible]	266	[illegible]	[illegible]	[illegible]	[illegible]	289	258	[illegible]	196	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	222	187	157
300	CS	352	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	284	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	298	244	203	171
350	CS	384	[illegible]	[illegible]	318	[illegible]	[illegible]	[illegible]	[illegible]	346	306	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	311	263	217	184
400	CS	413	[illegible]	[illegible]	341	384	[illegible]	321	[illegible]	[illegible]	[illegible]	[illegible]	244	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	282	232	195
500	CS	468	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	358	317	418	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	313	[illegible]	[illegible]
600	CS	514	[illegible]	[illegible]	416	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	401	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	340	[illegible]	[illegible]
750	CS	584	[illegible]	[illegible]	466	541	496	435	[illegible]	515	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	377	[illegible]	[illegible]
		(1.10 at 10°C 0.89 at 30°C 0.76 at 40°C 0.61 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at [illegible] 0.76 at 40 [illegible])				[illegible]				([illegible] at 10°C [illegible] at 40°C [illegible])			

[1] The following symbols are used here to designate conductor types: S - solid copper. SR standard round concentric-stranded, CS - compact- sector stranded.

[2] Current ratings are based on the following conditions:

a. Ambient earth temperature = 20°C
b. 60-cycle alternating current.
c. Ratings include dielectric loss, and all induced ac losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

Source: [1].

Table A-22 Current-carrying capacity of single-conductor solid paper-insulated cables

Conductor Size AWG or MCM	Number of Equally Loaded Cables in Duct Bank															
	THREE				SIX				NINE				TWELVE			
	Per Cent Load Factor															
	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
	AMPERES PER CONDUCTOR†															
7500 Volts													Copper Temperature, 85°C			
6	116	113	109	103	115	110	103	96	113	107	98	90	111	104	94	85
4	154	149	142	135	152	144	134	125	149	140	128	118	147	136	122	110
2	202	196	186	175	199	189	175	162	196	183	[illegible]	151	192	178	159	142
1	234	226	214	201	230	218	201	185	226	210	[illegible]	172	222	204	181	162
0	270	262	245	232	266	251	231	212	261	242	[illegible]	196	256	234	208	184
00	311	300	283	262	309	290	270	241	303	278	[illegible]	224	295	268	236	208
000	356	344	324	300	356	333	303	275	348	319	[illegible]	255	340	308	270	236
0000	412	395	371	345	408	380	347	314	398	364	[illegible]	[illegible]	390	352	307	269
250	[illegible]	436	409	379	449	418	379	344	437	400	[illegible]	[illegible]	427	386	336	294
300	[illegible]	491	459	423	499	464	420	380	486	442	[illegible]	[illegible]	474	428	371	325
350	[illegible]	[illegible]	500	[illegible]	546	507	457	403	[illegible]	[illegible]	[illegible]	[illegible]	518	466	403	352
400	[illegible]	[illegible]	540	[illegible]	593	548	493	445	576	[illegible]	[illegible]	[illegible]	560	502	434	378
500	[illegible]	[illegible]	611	[illegible]	679	626	560	504	[illegible]	[illegible]	[illegible]	[illegible]	641	571	490	427
600	[illegible]	[illegible]	[illegible]	[illegible]	757	696	621	557	[illegible]	[illegible]	[illegible]	[illegible]	714	632	542	470
700	[illegible]	[illegible]	741	[illegible]	827	758	674	604	[illegible]	[illegible]	[illegible]	[illegible]	779	688	587	508
750	881	[illegible]	771	[illegible]	860	789	700	627	[illegible]	[illegible]	[illegible]	[illegible]	810	714	609	526
800	[illegible]	[illegible]	797	[illegible]	[illegible]	817	725	648	[illegible]	[illegible]	[illegible]	[illegible]	840	[illegible]	630	544
1000	[illegible]	980	898	816	[illegible]	922	815	725	[illegible]	874	[illegible]	[illegible]	950	832	705	606
1250	[illegible]	1108	1012	[illegible]	[illegible]	1039	914	809	1104	981	[illegible]	730	1068	[illegible]	784	673
1500	[illegible]	1224	[illegible]	[illegible]	[illegible]	1146	1000	884	1220	1078	[illegible]	[illegible]	1178	[illegible]	855	[illegible]
1750	[illegible]	1332	[illegible]	[illegible]	[illegible]	1240	[illegible]	949	1342	1168	[illegible]	851	1280	[illegible]	919	783
2000	[illegible]	1442	[illegible]	[illegible]	[illegible]	1343	[illegible]	1019	1442	1260	[illegible]	914	1385	[illegible]	[illegible]	839
	(1.07 at 10°C, 0.9[illegible] at 3[illegible], 0.83 at 40°C, 0.73 at 5[illegible])				(1.07 at 10°C, 0.9[illegible] at [illegible], 0.83 at 40°C, 0.73 at [illegible])				(1.07 at 10°C, 0.92 at 3[illegible], 0.83 at 40°C, 0.73 at 5[illegible])				(1.07 at 10°C, 0.92 at 30[illegible], 0.83 at 40[illegible], [illegible]3 at 5[illegible])			
15 000 Volts													Copper Temperature, 81°C			
6	113	110	105	100	112	107	100	93	110	104	96	87	108	101	92	83
4	149	145	138	131	147	140	131	117	144	136	125	114	142	132	119	107
2	[illegible]	[illegible]	180	170	193	183	170	157	189	[illegible]	161	146	186	172	154	137
1	226	[illegible]	208	195	222	211	195	179	218	204	185	167	214	197	175	157
0	[illegible]	[illegible]	234	220	252	239	220	203	247	[illegible]	[illegible]	188	242	223	198	177
00	[illegible]	[illegible]	271	254	295	278	253	232	287	[illegible]	[illegible]	[illegible]	283	257	226	202
000	344	[illegible]	312	290	341	320	293	267	333	306	[illegible]	[illegible]	327	296	260	230
0000	[illegible]	[illegible]	361	335	392	367	335	305	383	[illegible]	[illegible]	[illegible]	374	340	298	263
250	440	[illegible]	396	367	432	404	367	334	422	387	345	[illegible]	412	372	325	286
300	[illegible]	[illegible]	439	406	481	449	406	369	470	[illegible]	382	[illegible]	457	413	359	316
350	[illegible]	[illegible]	481	444	527	491	443	401	514	468	416	[illegible]	[illegible]	450	391	342
400	[illegible]	[illegible]	522	480	572	530	478	432	556	[illegible]	447	[illegible]	542	485	419	366
500	[illegible]	[illegible]	592	543	655	605	542	468	630	[illegible]	507	[illegible]	[illegible]	551	474	412
600	[illegible]	[illegible]	656	601	727	668	598	537	705	637	[illegible]	[illegible]	[illegible]	[illegible]	521	452
700	[illegible]	[illegible]	712	652	790	726	647	581	766	691	[illegible]	[illegible]	[illegible]	[illegible]	564	488
750	[illegible]	[illegible]	736	674	[illegible]	753	672	602	795	716	[illegible]	[illegible]	[illegible]	[illegible]	584	505
800	[illegible]	[illegible]	[illegible]	[illegible]	850	780	695	622	823	741	[illegible]	[illegible]	[illegible]	[illegible]	604	522
1000	[illegible]	[illegible]	864	[illegible]	968	882	782	697	933	832	[illegible]	[illegible]	[illegible]	794	675	581
1250	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1000	883	784	1063	941	816	[illegible]	[illegible]	[illegible]	759	650
1500	[illegible]	[illegible]	1072	[illegible]	[illegible]	1105	972	856	1175	1037	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	707
1750	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1198	1042	919	1278	1124	[illegible]	824	[illegible]	[illegible]	886	[illegible]
2000	[illegible]	[illegible]	1233	[illegible]	[illegible]	1274	1105	970	1360	1192	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	795
	(1.06 at [illegible], 0.9[illegible] at 30°C, 0.[illegible] at 4[illegible], [illegible] at 50°C)†				([illegible] at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)†				(1.08 at 10°C, 0.92 at 3[illegible], 0.82 at 40°C, 0.71 at 60°C)†				(1.08 at 10[illegible], 0.92 at 30[illegible], 0.82 at 40[illegible], 0.71 at 50°C)†			
23 000 Volts													Copper Temperature, 77°C			
2	186	181	[illegible]	162	184	175	162	150	180	169	154	[illegible]	176	164	147	132
1	214	[illegible]	[illegible]	186	211	200	185	171	206	193	176	[illegible]	[illegible]	187	167	150
0	[illegible]	[illegible]	[illegible]	213	244	230	213	196	239	222	197	[illegible]	[illegible]	[illegible]	192	171
00	[illegible]	[illegible]	[illegible]	242	278	263	243	221	275	253	225	[illegible]	[illegible]	[illegible]	217	193
000	[illegible]	[illegible]	[illegible]	277	320	302	276	252	315	290	[illegible]	[illegible]	[illegible]	[illegible]	247	220
0000	[illegible]	[illegible]	[illegible]	317	367	345	315	288	360	332	297	[illegible]	[illegible]	[illegible]	281	250
250	[illegible]	396	[illegible]	348	405	380	348	316	396	365	[illegible]	[illegible]	[illegible]	[illegible]	307	272
300	[illegible]	[illegible]	[illegible]	386	450	422	382	349	438	404	[illegible]	[illegible]	[illegible]	[illegible]	340	301
350	[illegible]	[illegible]	466	422	493	461	418	380	481	442	[illegible]	[illegible]	[illegible]	[illegible]	369	326
400	[illegible]	[illegible]	491	454	536	498	451	409	521	478	[illegible]	[illegible]	[illegible]	[illegible]	398	349
500	[illegible]	[illegible]	559	514	615	570	514	464	597	546	[illegible]	421	[illegible]	[illegible]	450	392
600	[illegible]	[illegible]	[illegible]	[illegible]	684	632	568	511	663	603	529	[illegible]	[illegible]	[illegible]	496	431
700	[illegible]	[illegible]	[illegible]	[illegible]	744	689	617	554	725	656	574	[illegible]	[illegible]	[illegible]	538	467
750	[illegible]	[illegible]	702	[illegible]	779	717	641	574	754	681	596	[illegible]	[illegible]	[illegible]	558	483
800	[illegible]	[illegible]	[illegible]	[illegible]	808	743	663	595	782	706	617	[illegible]	[illegible]	[illegible]	576	500
1000	[illegible]	805	[illegible]	[illegible]	921	[illegible]	747	667	[illegible]	797	692	[illegible]	[illegible]	759	646	560
1250	1080	[illegible]	935	848	1052	[illegible]	845	751	1014	904	781	676	[illegible]	858	725	630
1500	[illegible]	[illegible]	1025	[illegible]	[illegible]	[illegible]	926	818	1118	993	[illegible]	[illegible]	[illegible]	940	[illegible]	[illegible]
1750	[illegible]	[illegible]	1106	994	[illegible]	[illegible]	991	875	1206	1087	[illegible]	[illegible]	[illegible]	1007	[illegible]	[illegible]
2000	[illegible]	[illegible]	1180	1058	[illegible]	[illegible]	1053	928	1293	1137	[illegible]	[illegible]	[illegible]	1073	[illegible]	[illegible]
	(1.09 at [illegible], 0.90 at 30°C, 0.80 at [illegible], 0.68 at 50°C)†				(1.09 at 10[illegible], 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)†				(1.09 at 10°C, 0.90 at 3[illegible], 0.80 at 40°C, 0.68 at 60[illegible])				(1.09 at 10°C, 0.90 at 3[illegible], 0.80 at 40°C, 0.62 at 60[illegible])			

Continued

Table A-22 (Continued)

Conductor Size AWG or MCM	Number of Equally Loaded Cables in Duct Bank: THREE				SIX				NINE				TWELVE			
	Per Cent Load Factor															
	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
	AMPERES PER CONDUCTOR[1]															
34 500 Volts													Copper Temperature, 70°C			
0	227	221	209	197	225	213	197	182	220	205	187	160	215	199	177	158
00	260	251	239	224	255	242	224	205	249	234	211	190	245	226	200	179
000	299	[illegible]	273	256	295	278	258	235	288	268	242	217	282	259	230	204
0000	341	330	312	291	336	317	291	267	[illegible]	304	274	246	321	293	259	230
250	380	367	345	322	374	352	321	294	364	337	303	270	356	324	286	253
300	422	408	382	355	416	390	356	324	405	374	334	298	395	359	315	278
350	464	446	419	389	455	426	388	353	443	408	364	324	432	392	343	302
400	502	484	451	419	491	460	417	379	478	440	390	347	466	421	368	323
500	575	551	514	476	562	524	474	429	547	500	442	392	532	479	416	364
600	644	616	573	528	629	584	526	475	610	556	491	433	593	532	459	401
700	710	675	626	577	690	639	574	517	669	608	535	470	649	580	500	435
750	736	702	651	598	718	664	595	535	698	631	554	486	675	602	518	450
800	765	730	676	620	747	690	617	555	723	654	571	503	700	624	535	465
1000	875	832	766	701	852	783	698	624	823	741	646	564	708	706	601	520
1250	994	941	864	786	967	882	782	696	930	833	[illegible]	628	898	790	670	577
1500	1098	1036	949	859	1068	972	856	760	1025	914	788	682	988	865	730	626
1750	1192	1123	1023	925	1156	1048	919	814	1109	984	845	730	1066	929	780	668
2000	[illegible]	1197	1088	981	1234	1115	975	860	1182	1043	893	770	1135	985	824	704
2500	1418	1324	1196	1072	1307	[illegible]	1064	930	1305	1144	973	834	1248	1075	893	760
	(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[2]			
46 000 Volts													Copper Temperature, 65°C			
000	279	270	256	240	274	259	239	221	268	249	226	204	262	241	214	191
0000	322	312	294	276	317	299	274	251	309	287	259	[illegible]	302	276	244	217
250	352	340	321	300	346	326	299	274	336	313	282	[illegible]	329	301	266	236
300	394	380	358	334	385	364	332	304	377	349	313	280	367	335	295	260
350	433	417	392	365	425	398	364	331	413	382	341	304	401	366	321	283
400	460	451	423	393	459	430	391	356	446	411	367	326	433	394	344	307
500	534	512	482	444	522	487	441	400	506	464	412	365	492	444	386	339
600	602	[illegible]	538	496	589	540	494	447	570	520	460	400	553	497	430	377
700	663	633	589	542	645	518	538	486	628	569	502	441	615	542	468	408
750	689	656	611	561	672	602	559	504	650	590	530	457	630	562	485	422
800	717	683	638	583	698	645	578	522	674	612	548	472	652	582	501	436
1000	816	776	718	657	784	731	653	585	768	691	604	528	740	657	562	487
1250	927	879	810	738	900	825	732	654	865	777	675	589	814	738	626	541
1500	1020	968	887	805	992	904	799	703	951	850	735	638	914	802	679	585
1750	1110	1047	959	867	1074	976	859	762	1028	915	766	682	[illegible]	872	728	623
2000	1184	1115	1016	918	1144	1035	909	805	1094	970	833	718	1048	913	766	656
2500	1314	1232	1115	1002	1265	1138	974	875	1205	1062	905	778	1151	1016	830	708
	(1.11 at 10°C, 0.87 at 30°C, 0.73 at 40°C, 0.54 at 50°C)[2]				(1.11 at 10°C, 0.87 at 30°C, 0.72 at 40°C, 0.53 at 50°C)[2]				(1.11 at 10°C, 0.87 at 30°C, 0.72 at 40°C, 0.52 at 50°C)[2]				(1.12 at 10°C, 0.87 at 30°C, 0.70 at 40°C, 0.51 at 50°C)[2]			
69 000 Volts													Copper Temperature, 60°C			
350	395	382	360	336	387	364	333	305	375	348	312	279	365	332	293	259
400	428	413	389	362	418	393	358	328	405	375	335	300	394	358	315	278
500	489	470	441	409	477	446	406	370	461	425	379	337	447	405	354	312
600	545	524	490	454	532	496	450	409	513	471	419	371	497	448	391	343
700	599	573	536	495	582	543	490	444	561	514	455	403	542	489	425	372
750	623	597	556	514	605	562	508	460	583	533	472	417	563	506	439	384
800	644	617	577	531	626	582	525	475	603	554	487	430	582	[illegible]	453	396
1000	736	702	652	599	711	660	592	533	685	622	547	481	606	569	508	442
1250	832	792	734	672	800	742	664	595	772	698	610	515	741	[illegible]	564	489
1500	918	872	804	733	888	814	724	647	848	783	664	560	812	718	611	529
1750	994	942	865	766	957	876	770	692	913	818	711	618	873	770	653	563
2000	1066	1008	924	840	1020	931	832	712	972	866	750	651	927	814	688	592
2500	1163	1096	1001	900	1115	1013	892	791	1060	942	811	700	1007	880	741	635
	(1.13 at 10°C, 0.85 at 30°C, 0.67 at 40°C, 0.41 at 50°C)[2]				(1.13 at 10°C, 0.85 at 30°C, 0.66 at 40°C, 0.40 at 50°C)[2]				(1.13 at 10°C, 0.84 at 30°C, 0.65 at 40°C, 0.36 at 50°C)[2]				(1.14 at 10°C, 0.84 at 30°C, 0.64 at 40°C, 0.32 at 50°C)[2]			

[1] Current ratings are based on the following conditions:

a. Ambient earth temperature = 20°C.
b. 60-cycle alternating current.
c. Sheaths bonded and grounded at one point only (open-circuited sheaths).
d. Standard concentric stranded conductors.
e. Ratings include dielectric loss and skin effect.
f. One cable per duct, all cables equally loaded and in outside ducts only.

[2] Multiply tabulated values by these factors when earth temperature is other than 20°C.

Source: [1].

Table A-23 60-Hz characteristics of self-supporting rubber-insulated neoprene-jacketed aerial cable

Voltage Class	Conductor Size	Stranding	Insulation Thickness	Shielding	Jacket Thickness	Diameter	Messenger Used with Copper Conductors	Wt. Per 1000 Ft. Messenger and Copper	Messenger Used with Aluminum Conductors	Wt. Per 1000 Ft. Messenger and Aluminum	Positive Sequence 60~ AC Ohms/Mi: Resistance (1) Copper	Positive Sequence: Resistance (1) Aluminum	Positive Sequence: Reactance Series Inductive	Positive Sequence: Reactance Shunt Capacitive(2)	Zero Sequence(3) 60~ AC Ohms/Mi: Resistance (1) Copper	Zero Sequence: Resistance (1) Aluminum	Zero Sequence: Reactance Series Inductive Copper	Zero Sequence: Reactance Series Inductive Aluminum	Zero Sequence: Reactance Shunt Capacitive(2)
3-kv Ungrounded Neutral 5-kv Grounded Neutral	6	7	11/64	No	4/64	0.59	5/16" 30% CCS	1020	1/4" 30% CCS	854	2.52	4.13	0.258	...	3.592	5.082	3.712	3.712	....
	4	7	10/64	No	4/64	0.67	5/16" 30% CCS	1230	1/4" 30% CCS	958	1.58	2.58	0.246	...	2.632	3.572	3.662	3.662	....
	2	7	11/64	No	4/64	0.73	5/16" 30% CCS	1530	1/4" 30% CCS	1100	1.00	1.64	0.229	...	2.025	2.605	3.615	3.615	....
	1	19	11/64	No	4/64	0.77	5/16" 30% CCS	1780	1/4" 30% CCS	1250	0.701	1.29	0.211		1.815	2.275	3.582	3.582	....
	1/0	19	11/64	No	4/64	0.81	5/16" 30% CCS	2070	1/4" 30% CCS	1390	0.635	1.03	0.207	...	1.644	2.015	3.555	3.555	....
	2/0	19	11/64	No	4/64	0.85	5/16" 30% CCS	2510	1/4" 30% CCS	1530	0.501	0.816	0.200	...	1.622	1.803	3.162	3.526	....
	3/0	19	11/64	No	4/64	0.91	5/16" 30% CCS	2900	1/4" 30% CCS	1610	0.402	0.644	0.194	.	1.517	1.637	3.135	3.499	....
	4/0	19	11/64	No	4/64	0.99	3/8" 30% CCS	3570	1/4" 30% CCS	1900	0.318	0.516	0.191		1.401	1.508	2.665	3.459	....
	250	37	11/64	No	4/64	1.08	3/8" 30% CCS	4040	1/4" 30% CCS	2180	0.289	0.437	0.189		1.351	1.430	2.635	3.429	....
	300	37	11/64	No	4/64	1.13	3/8" 30% CCS	4620	5/16" 30% CCS	2500	0.228	0.366	0.184		1.308	1.465	2.612	3.042	....
	350	37	11/64	No	4/64	1.18	3/8" 30% CCS	5190	1/4" 30% CCS	2780	0.197	0.316	0.180		1.277	1.415	2.591	3.021	....
	400	37	11/64	No	4/64	1.23	3/8" 30% CCS	5600	3/8" 30% CCS	3040	0.172	0.276	0.176		1.252	1.377	2.576	3.006	....
	500	37	11/64	No	4/64	1.32	3/8" 30% CCS	6660	3/8" 30% CCS	3650	0.141	0.223	0.172		1.219	1.290	2.543	2.543	....
5-kv Ungrounded Neutral	6	7	10/64	Yes	4/64	0.74	1/4" 30% CCS	1310	1/4" 30% CCS	1140	2.52	4.13	0.292	4070			...	...	...
	4	7	10/64	Yes	4/64	0.79	1/4" 30% CCS	1540	1/4" 30% CCS	1270	1.58	2.58	0.272	4320		...		...	...
	2	7	10/64	Yes	4/64	0.88	1/4" 30% CCS	1950	1/4" 30% CCS	1530	1.00	1.64	0.257	3830				...	...
	1	19	10/64	Yes	4/64	0.92	1/4" 30% CCS	2180	1/4" 30% CCS	1640	0.701	1.29	0.241	3330			...	...	...
	1/0	19	10/64	Yes	4/64	0.96	5/16" 30% CCS	2450	1/4" 30% CCS	1770	0.655	1.03	0.233	[illegible]			...	...	...
	2/0	19	10/64	Yes	4/64	1.00	5/16" 30% CCS	2810	1/4" 30% CCS	1930	0.501	0.816	0.223	[illegible]		...		...	...
	3/0	19	10/64	Yes	4/64	1.06	5/16" 30% CCS	3320	1/4" 30% CCS	2120	0.402	0.644	0.215	[illegible]		...		...	...
	4/0	19	10/64	Yes	4/64	1.11	5/16" 30% CCS	4030	1/4" 30% CCS	2350	0.318	0.518	0.207	2380		...		...	...
	250	37	11/64	Yes	4/64	1.20	3/8" 30% CCS	4570	5/16" 30% CCS	2770	0.269	0.437	0.206	[illegible]			...	...	...
	300	37	11/64	Yes	4/64	1.29	3/8" 30% CCS	5260	1/4" 30% CCS	3140	0.228	0.366	0.203	[illegible]			...	...	...
	350	37	11/64	Yes	4/64	1.34	3/8" 30% CCS	5840	5/16" 30% CCS	3340	0.197	0.316	0.199	2090		...	...	...	...
	400	37	11/64	Yes	4/64	1.39	3/8" 30% CCS	6380	5/16" 30% CCS	3610	0.172	0.276	0.194	1890			...	...	...
	500	37	11/64	Yes	4/64	1.47	3/8" 30% CCS	7470	3/8" 30% CCS	4240	0.141	0.223	0.187	1740			...	...	.....
15-kv Grounded Neutral	6	19	19/64	Yes	4/64	1.05	1/4" 30% CCS	2090	1/4" 30% CCS	1920	2.51	4.13	0.326	7150	3.446	5.316	3.396	3.396	7150
	4	19	19/64	Yes	4/64	1.10	1/4" 30% CCS	2440	1/4" 30% CCS	2060	1.58	2.58	0.302	6260	2.901	3.831	3.364	3.364	6260
	2	19	19/64	Yes	4/64	1.18	1/4" 30% CCS	2680	1/4" 30% CCS	2430	1.00	1.64	0.279	5460	2.470	3.039	2.851	2.851	5460
	1	19	19/64	Yes	4/64	1.20	1/4" 30% CCS	3170	1/4" 30% CCS	2580	0.701	1.29	0.268	5110	2.239	2.701	2.837	2.837	5110
	1/0	19	19/64	Yes	4/64	1.27	1/4" 30% CCS	3560	1/4" 30% CCS	2880	0.655	1.03	0.260	4720	2.052	2.426	2.825	2.825	4720
	2/0	19	19/64	Yes	4/64	1.32	5/16" 30% CCS	4120	1/4" 30% CCS	3070	0.501	0.816	0.249	4370	1.896	2.214	2.251	2.801	4370
	3/0	19	19/64	Yes	4/64	1.37	5/16" 30% CCS	4580	5/16" 30% CCS	3510	0.402	0.644	0.241	4120	[illegible]	2.008	2.240	2.240	4120
	4/0	19	19/64	Yes	4/64	1.43	3/8" 30% CCS	5150	5/16" 30% CCS	3790	0.318	0.516	0.231	3770	1.681	1.864	2.235	2.235	3770
	250	37	19/64	Yes	4/64	1.47	3/8" 30% CCS	5590	3/8" 30% CCS	3980	0.269	0.437	0.223	3570	1.630	1.782	2.227	2.227	3570
	300	37	19/64	Yes	4/64	1.53	3/8" 30% CCS	6260	3/8" 30% CCS	4330	0.228	0.366	0.217	3330	1.577	1.701	2.226	2.226	3330
	350	37	19/64	Yes	4/64	1.59	3/8" 30% CCS	6870	3/8" 30% CCS	4600	0.197	0.316	0.212	[illegible]	1.536	1.640	2.226	2.226	3130
	400	37	19/64	Yes	4/64	1.63	3/8" 30% CCS	7450	3/8" 30% CCS	4860	0.172	0.276	0.208	[illegible]	1.500	1.592	2.216	2.216	2980
	500	37	19/64	Yes	4/64	1.75	3/8" 30% CCS	8070	3/8" 30% CCS	5560	0.141	0.223	0.204	2830	1.454	1.524	2.198	2.198	2830

(1) Ac resistance based on 65°C with allowance for stranding, skin effect, and proximity effect.

(2) Dielectric constant assumed 6.0.

(3) Zero-sequence impedance based on return current both in the messenger and in 100 mΩ earth.

Source: [1].

REFERENCES

1. Westinghouse Electric Corporation: *Electrical Transmission and Distribution Reference Book*, East Pittsburgh, Pa., 1964.
2. Westinghouse Electric Corporation: *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, Pa., 1965.
3. Edison Electric Institute: *Transmission Line Reference Book*, New York, 1968.
4. Anderson, P. M.: *Analysis of Faulted Power Systems*, Iowa State University Press, Ames, 1973.
5. Westinghouse Electric Corporation: *Applied Protective Relaying*, Newark, N.J., 1970.
6. Insulated Power Cable Engineers Association: *Current Carrying Capacity of Impregnated Paper, Rubber, and Varnished Cambric Insulated Cables*, 1st ed., Publication P-29-226.

APPENDIX

B

GRAPHIC SYMBOLS USED IN DISTRIBUTION SYSTEM DESIGN

Some of the most commonly used graphic symbols for distribution systems, both in this book and in general usage, are given on the following pages.

Table B-1 Graphic symbols used in distribution system design

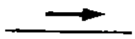
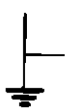
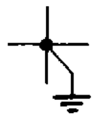
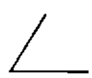
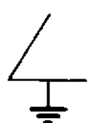
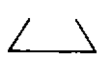
Symbol	Usage
 or  or 	Polarity Markings. current transformer with instantaneous polarity markings
 or  or 	potential transformer with instantaneous polarity markings
 or	Power Flow Direction. one-way
or	either way (not simultaneously)
or	both ways (simultaneously)
	Connection Symbols: 2-phase 3-wire, ungrounded
	2-phase 3-wire, grounded
	2-phase 4-wire

Table B-1 (Continued)

Symbol	Usage
	2-phase 5-wire, grounded
	3-phase 3-wire, delta or mesh
	3-phase 3-wire, delta, grounded
	3-phase 4-wire, delta, ungrounded
	3-phase 4-wire, delta, grounded
	3-phase, open-delta
	3-phase, open-delta, grounded at middle point of one winding
	3-phase, broken-delta

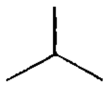
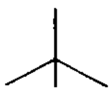
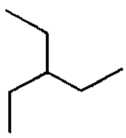
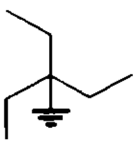
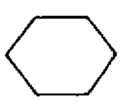
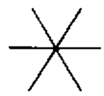
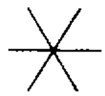
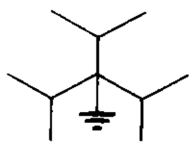
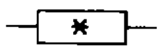
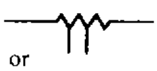
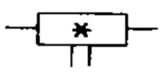
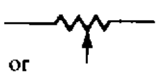
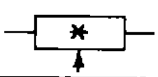
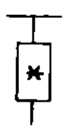
Symbol	Usage
	3-phase, wye or star, ungrounded
	3-phase, wye, grounded neutral
	3-phase 4-wire, ungrounded
	3-phase, zigzag, ungrounded
	3-phase, zigzag, grounded
	3-phase, Scott or T
	6-phase, double-delta
	6-phase, hexagonal (or chordal)

Table B-1 (Continued)

Symbol	Usage
	6-phase, star (or diametrical)
	6-phase, star, with grounded neutral
	6-phase, double zigzag with neutral brought out and grounded
 or 	Resistor: resistor (general)
 or 	tapped resistor
 or 	resistor with adjustable contact
 or 	shunt resistor
	series resistor and path open

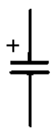
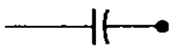
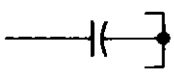
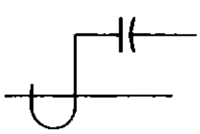
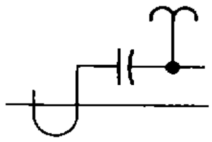
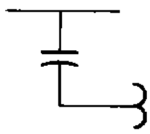
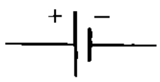
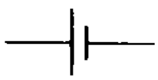
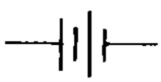
Symbol	Usage
	series resistor and path short-circuited
 or 	Capacitor: capacitor (general)
 or 	polarized capacitor
 or 	variable capacitor
	series capacitor and path open
	series capacitor and path short-circuited
	shunt capacitor
	capacitor bushing for circuit breaker or transformer

Table B-1 (Continued)

Symbol	Usage
	capacitor-bushing potential device
	coupling capacitor potential device
	Battery: battery (general)
	battery with one cell
	battery with multicell
 	Transmission Path (conductor, cable wire): bus bar, with connection
	conductor or path
or	2 conductors or paths

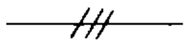
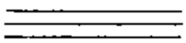
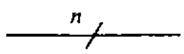
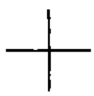
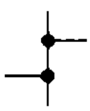
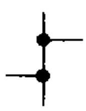
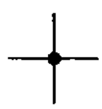
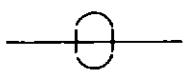
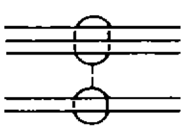
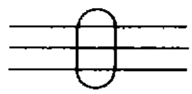
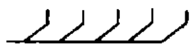
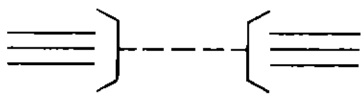
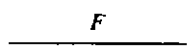
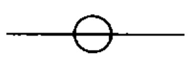
Symbol	Usage
 or 	3 conductors or paths
 or (draw individual paths)	n conductors or paths
	crossing of two conductors or paths, not connected
	junction
 or  or 	junction of connected paths
 or 	shielded single conductor
	shielded 5-conductor cable
	shielded 2-conductor cable with conductors separated on the diagram for convenience

Table B.1 (Continued)

Symbol	Usage
	3-conductor cable
 or 	Grouping of Leads: general
	interrupted
	Transmission and Distribution Lines: telephone line
 or	cable (or line) underground
	submarine line
	overhead line
	loaded line

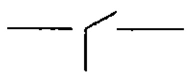
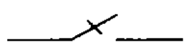
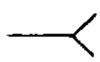
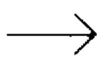
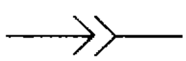
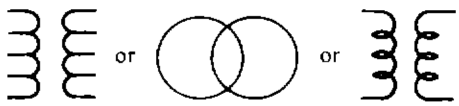
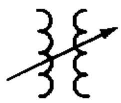
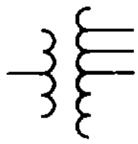
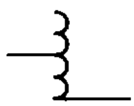
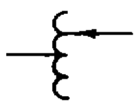
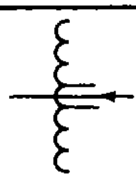
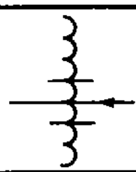
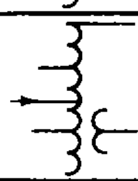
Symbol	Usage
	Ground: ground (general)
	Switch. single-throw switch (disconnect switch)
	double-throw switch
	knife switch
	Connector: female contact
	male contact
	separable connectors
 or 	Operating Coil: operating coil (general) e g., reactor

Table B-1 (Continued)

Symbol	Usage
	Transformer: transformer (general)
	adjustable mutual inductor (constant-current transformer)
	single-phase transformer with taps
	single-phase autotransformer
	adjustable
	step-voltage regulator or load-ratio control autotransformer
	step-voltage regulator
	load-ratio control autotransformer

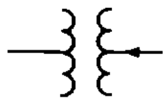
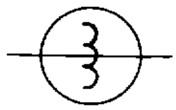
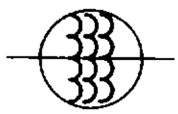
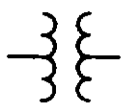
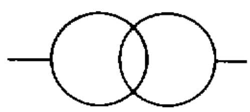
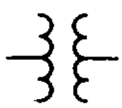
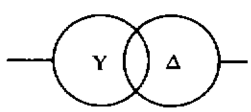
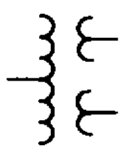
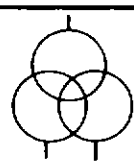
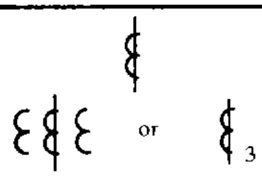
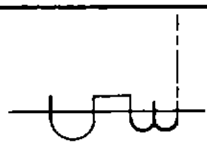
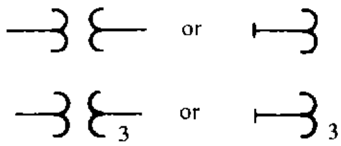
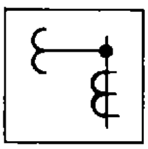
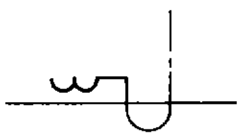
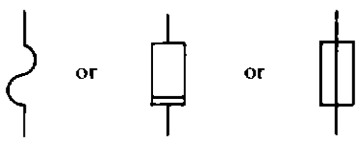
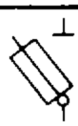
Symbol	Usage
	load-ratio control transformer with taps
 or 	single-phase induction voltage regulator
	Triplex induction voltage regulator
	3-phase induction voltage regulator
 or 	1-phase, 2-winding transformer
 Y−Δ or 	3-phase bank of 1-phase, 2-winding transformers with wye-delta connections
 or 	Polyphase Transformer: polyphase transformer (general)
 or 	1-phase, 3-winding transformer

Table B-1 (Continued)

Symbol	Usage
	Current Transformer: current transformer (general)
	bushing-type current transformer
	Potential Transformers: potential transformer (general)
	outdoor metering device
	linear coupler
	Fuse: fuse (general)
	fuse (supply side indicated by a thick line)
	isolating fuse-switch (HV primary fuse cutout) dry

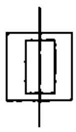
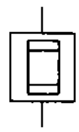
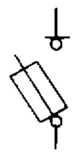
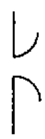
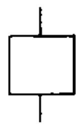
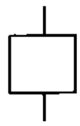
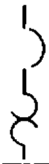
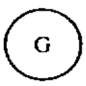
Symbol	Usage
 or 	HV primary fuse cutout, oil
	isolating fuse-switch for on-load switching
	current limiter (for power cable)
	lightning arrester
	horn gap
	multigap (general)
	Circuit Breaker: circuit breaker, air (for dc or ac rated at 1.5kV or less)
	network protector

Table B-1 (Continued)

Symbol	Usage
 or  CB	HV circuit breaker (for ac rated at above 1.5kV)
 or 	circuit breaker with thermal-overload device
	circuit breaker with magnetic thermal-overload device
	circuit breaker, drawout type
 or 	Rotating Machine: generator (general)
	generator, dc
	generator, ac
	generator, synchronous

Symbol	Usage
M or MOT	Motor (general)
$\underline{\mathrm{M}}$	motor, dc
$\underset{\sim}{\mathrm{M}}$	motor, ac
MS	motor, synchronous

APPENDIX

C

GLOSSARY FOR DISTRIBUTION SYSTEM TERMINOLOGY

Some of the most commonly used terms, both in this book and in general usage, are defined on the following pages. Most of the definitions given in this glossary are based on Refs. 1 to 8.

AAAC: Abbreviation for all-aluminum-alloy conductors. Aluminum-alloy conductors have higher strength than those of the ordinary electric-conductor grade of aluminum.

AAC: Abbreviation for all-aluminum conductors.

ACAR: Abbreviation for aluminum conductor alloy-reinforced. It has a central core of higher-strength aluminum surrounded by layers of electric-conductor-grade aluminum.

ACL cable: A cable with a lead sheath over the cable insulation that is suitable for wet locations. It is used in buildings at low voltage.

ACSR: An abbreviation for aluminum conductor, steel-reinforced. It consists of a central core of steel strands surrounded by layers of aluminum strands.

Admittance: The ratio of the phasor equivalent of the steady-state sine-wave current to the phasor equivalent of the corresponding voltage.

Adverse weather: Weather conditions which cause an abnormally high rate of forced outages for exposed components during the periods such conditions persist, but which do not qualify as major storm disasters. Adverse weather conditions can be defined for a particular system by selecting the proper values and combinations of conditions reported by the Weather

Bureau: thunderstorms, tornadoes, wind velocities, precipitation, temperature, etc.

Aerial cable: An assembly of insulated conductors installed on a pole line or similar overhead structures; it may be self-supporting or installed on a supporting messenger cable.

Air-blast transformer: A transformer cooled by forced circulation of air through its core and coils.

Air circuit breaker: A circuit breaker in which the interruption occurs in air.

Air switch: A switch in which the interruptions of the circuit occur in air.

Al: Symbol for aluminum.

Ampacity: Current rating in amperes, as of a conductor.

ANSI: Abbreviation for American National Standards Institute.

Apparent sag (at any point): The departure of the wire at the particular point in the span from the straight line between the two points of the span, at 60°F, with no wind loading.

Arcing time of fuse: The time elapsing from the severance of the fuse link to the final interruption of the circuit under specified conditions.

Arc-over of insulator: A discharge of power current in the form of an arc following a surface discharge over an insulator.

Armored cable: A cable provided with a wrapping of metal, usually steel wires, primarily for the purpose of mechanical protection.

Askarel: A generic term for a group of nonflammable synthetic chlorinated hydrocarbons used as electrical insulating media. Askarels of various compositional types are used. Under arcing conditions the gases produced, while consisting predominantly of noncombustible hydrogen chloride, can include varying amounts of combustible gases depending upon the askarel type. Because of environmental concerns, it is not used in new installations anymore.

Automatic substations: Those in which switching operations are so controlled by relays that transformers or converting equipment are brought into or taken out of service as variations in load may require, and feeder circuit breakers are closed and reclosed after being opened by overload relays.

Autotransformer: A transformer in which at least two windings have a common section.

AWG: Abbreviation for American Wire Gauge. It is also sometimes called the Brown and Sharpe Wire Gauge.

Base load: The minimum load over a given period of time.

Benchboard: A switchboard with a horizontal section for control switches, indicating lamps, and instrument switches; may also have a vertical instrument section.

BIL: Abbreviation for basic impulse insulation levels, which are reference levels expressed in impulse-crest voltage with a standard wave not longer than $1.5 \times 50\,\mu s$. The impulse waves are defined by a combination of two numbers. The first number is the time from the start of the wave to the instant crest value; the second number is the time from the start to the instant of half-crest value on the tail of the wave.

Billing demand: The demand used to determine the demand charges in accordance with the provisions of a rate schedule or contract.

Branch circuit: A set of conductors that extend beyond the last overcurrent device in the low-voltage system of a given building. A branch circuit usually supplies a small portion of the total load.

Breakdown: Also termed *puncture*, denoting a disruptive discharge through insulation.

Breaker, primary-feeder: A breaker located at the supply end of a primary feeder which opens on a primary-feeder fault if the fault current is of sufficient magnitude.

Breaker-and-a-half scheme: A scheme which provides the facilities of a double main bus at a reduction in equipment cost by using three circuit breakers for each two circuits.

Bus: A conductor or group of conductors that serves as a common connection for two or more circuits in a switchgear assembly.

Bus, transfer: A bus to which one circuit at a time can be transferred from the main bus.

Bushing: An insulating structure including a through conductor, or providing a passageway for such a conductor, with provision for mounting on a barrier, conductor or otherwise, for the purpose of insulating the conductor from the barrier and conducting from one side of the barrier to the other.

BVR: Abbreviation for bus voltage regulator or regulation.

BW: Abbreviation for bandwidth.

BX cable: A cable with galvanized interlocked steel spiral armor. It is known as ac cable and used in a damp or wet location in buildings at low voltage.

Cable: Either a standard conductor (single-conductor cable) or a combination of conductors insulated from one another (multiple-conductor cable).

Cable fault: A partial or total load failure in the insulation or continuity of the conductor.

Capability: The maximum load-carrying ability expressed in kilovoltamperes or kilowatts of generating equipment or other electric apparatus under specified conditions for a given time interval.

Capability, net: The maximum generation expressed in kilowatthours per hour which a generating unit, station, power source, or system can be expected to supply under optimum operating conditions.

Capacitor bank: An assembly at one location of capacitors and all necessary accessories (switching equipment, protective equipment, controls, etc.) required for a complete operating installation.

Capacity: The rated load-carrying ability expressed in kilovoltamperes or kilowatts of generating equipment or other electric apparatus.

Capacity factor: The ratio of the average load on a machine or equipment for the period of time considered to the capacity of the machine or equipment.

Charge: The amount paid for a service rendered or facilities used or made available for use.

Circuit, earth (ground) return: An electric circuit in which the earth serves to complete a path for current.

Circuit breaker: A device that interrupts a circuit without injury to itself so that it can be reset and reused over again.

Circuit-breaker mounting: Supporting structure for a circuit breaker.

Circular mil: A unit of area equal to $\pi/4$ of a square mil ($=0.7854$ square mil). The cross-sectional area of a circle in circular mils is therefore equal to the square of its diameter in mils. A circular inch is equal to 1 million circular mils. A mil is one one-thousandth of an inch. There are 1974 circular mils in a square millimeter. Abbreviated cmil.

CL: Abbreviation for current-limiting (fuse).

cmil: Abbreviation for circular mil.

Coincidence factor: The ratio of the maximum coincident total demand of a group of consumers to the sum of the maximum power demands of individual consumers comprising the group both taken at the same point of supply for the same time.

Coincident demand: Any demand that occurs simultaneously with any other demand; also the sum of any set of coincident demands.

Component: A piece of equipment, a line, a section of a line, or a group of items which is viewed as an entity.

Condenser: Also termed *capacitor*; a device whose primary purpose is to introduce capacitance into an electric circuit. The term *condenser* is deprecated.

Conductor: A substance which has free electrons or other charge carriers that permit charge flow when an emf is applied across the substance.

Conductor tension, final unloaded: The longitudinal tension in a conductor after the conductor has been stretched by the application for an appreciable period, with subsequent release, of the loadings of ice and wind, at the temperature decrease assumed for the loading district in which the conductor is strung (or equivalent loading).

Conduit: A structure containing one or more ducts; commonly formed from iron pipe or electrical metallic tubing, used in buildings at low voltage.

Connection charge: The amount paid by a customer for connecting the customer's facilities to the supplier's facilities.

Contactor: An electric power switch, not operated manually and designed for frequent operation.

Contract demand: The demand that the supplier of electric service agrees to have available for delivery.

CT: Abbreviation for current transformers.

Cu: Symbol for copper.

Customer charge: The amount paid periodically by a customer without regard to demand or energy consumption.

Demand: The load at the receiving terminals averaged over a specified interval of time.

Demand charge: That portion of the charge for electric service based upon a customer's demand.

Demand factor: The ratio of the maximum coincident demand of a system, or part of a system, to the total connected load of the system, or part of the system, under consideration.

Demand, instantaneous: The load at any instant.

Demand, integrated: The demand integrated over a specified period.

Demand interval: The period of time during which the electric energy flow is integrated in determining demand.

Depreciation: The component which represents an approximation of the value of the portion of plant consumed or "used up" in a given period by a utility.

Disconnecting or isolating switch: A mechanical switching device used for changing the connections in a circuit or for isolating a circuit or equipment from the source of power.

Disconnector: A switch that is intended to open a circuit only after the load has been thrown off by other means. Manual switches designed for opening loaded circuits are usually installed in a circuit with disconnectors to provide a safe means for opening the circuit under load.

Distribution center: A point of installation for automatic overload protective devices connected to buses where an electric supply is subdivided into feeders and/or branch circuits.

Distribution switchboard: A power switchboard used for the distribution of electric energy at the voltages common for such distribution within a building.

Distribution system: That portion of an electric system which delivers electric energy from transformation points in the transmission, or bulk power system, to the consumers.

Distribution transformer: A transformer for transferring electric energy from a primary distribution circuit to a secondary distribution circuit or consumer's service circuit; it is usually rated in the order of 5 to 500 kVA.

Diversity factor: The ratio of the sum of the individual maximum demands of the various subdivisions of a system to the maximum demand of the whole system.

Duplex cable: A cable composed of two insulated stranded conductors twisted together. They may or may not have a common insulating covering.

Effectively grounded: Grounded by means of a ground connection of sufficiently low impedance that fault grounds which may occur cannot build up voltages dangerous to connected equipment.

EHV: Abbreviation for extra high voltage.

Electric rate schedule: A statement of an electric rate and the terms and conditions governing its application.

Electric system loss: Total electric energy loss in the electric system. It consists of transmission, transformation, and distribution losses between sources of supply and points of delivery.

Electrical reserve: The capability in excess of that required to carry the system load.

Emergency rating: Capability of installed equipment for a short time interval.

EMT: Abbreviation for electrical metallic tubing. A raceway which has a thin wall that does not permit threading. Connectors and couplings are secured either by compression rings or setscrews. It is used in buildings at low voltage.

Energy: That which does work or is capable of doing work. As used by electric utilities, it is generally a reference to electric energy and is measured in kilowatt-hours.

Energy charge: That portion of the charge for electric service based upon the electric energy consumed or billed.

Energy loss: The difference between energy input and output as a result of transfer of energy between two points.

Express feeder: A feeder which serves the most distant networks and which must traverse the systems closest to the bulk power source.

Extra high voltage: A term applied to voltage levels higher than 230 kV. Abbreviated EHV.

Facilities charge: The amount paid by the customer as a lump sum, or periodically, as reimbursement for facilities furnished. The charge may include operation and maintenance as well as fixed costs.

FCN: Abbreviation for full-capacity neutral.

Feeder: A set of conductors originating at a main distribution center and supplying one or more secondary distribution centers, one or more branch-circuit distribution centers, or any combination of these two types of load.

Feeder, multiple: Two or more feeders connected in parallel.

Feeder, tie: A feeder that connects two or more independent sources of power and has no tapped load between the terminals. The source of power may be a generating system, substation, or feeding point.

First-contingency outage: The outage of one primary feeder.

Fixed-capacitor bank: A capacitor bank with fixed, not switchable, capacitors.

Forced interruption: An interruption caused by a forced outage.

Forced outage: An outage that results from emergency conditions directly associated with a component, requiring that it be taken out of service immediately, either automatically or as soon as switching operations can be performed; or an outage caused by improper operation of equipment or by human error.

Fuel adjustment clause: A clause in a rate schedule that provides for adjustment of the amount of the bill as the cost of fuel varies from a specified base amount per unit.

Fuse: An overcurrent protective device with a circuit-opening fusible part that is heated and severed by the passage of overcurrent through it.

Fuse cutout: An assembly consisting of a fuse support and holder; it may also include a fuse link.

Ground: Also termed *earth*; a conductor connected between a circuit and the soil; an accidental ground occurs due to cable insulation faults, an insulator defect, etc.

Ground wire: A conductor having grounding connections at intervals that is suspended usually above but not necessarily over the line conductor to provide a degree of protection against lightning discharges

HV: Abbreviation for high voltage.

HMWPE: Abbreviation for high-molecular-weight polyethylene (cable insulation).

Impedance: The ratio of the phasor equivalent of a steady-state sine-wave voltage to the phasor equivalent of a steady-state sine-wave current.

Incremental energy costs: The additional cost of producing or transmitting electric energy above some base cost.

Index of reliability: A ratio of cumulative customer minutes that service was available during a year to total customer minutes demanded; can be used by the utility for feeder reliability comparisons.

Indoor transformer: A transformer that must be protected from the weather.

Installed reserve: The reserve capability installed on a system.

Interruptible load: A load which can be interrupted as defined by contract.

Interruption: The loss of service to one or more consumers. An interruption is the result of one or more component outages.

Interruption duration: The period from the initiation of an interruption to a consumer until service has been restored to that consumer.

Investment-related charges Those certain charges incurred by a utility which are directly related to the capital investment of the utility.

kcmil: Abbreviation for a thousand circular mils.

Lag: Denotes that a given sine wave passes through its peak at a later time than a reference time wave.

Lambda: The incremental operating cost at the load center, commonly expressed in mils per kilowatthour.

Lateral conductor: A wire or cable extending in a general horizontal direction or at an angle to the general direction of the line; service wires either overhead or underground are considered laterals from the street mains.

LDC: Abbreviation for line-drop compensator.

Lightning arrestor: A device that reduces the voltage of a surge applied to its terminals and restores itself to its original operating condition.

L-L: Abbreviation for line to line.

Limit switch: A switch that is operated by a moving part at the end of its travel—typically to stop or reverse the motion.

Limiter: A device in which some characteristic of the output is automatically prevented from exceeding a predetermined value.

Line: A component part of a system extending between adjacent stations or from a station to an adjacent interconnection point. A line may consist of one or more circuits.

Line-drop compensator: A device which causes the voltage-regulating relay to increase the output voltage by an amount that compensates for the impedance drop in the circuit between the regulator and a predetermined location at the circuit.

Line loss: Energy loss on a transmission or distribution line.

L-N: Abbreviation for line to neutral.

Load, interruptible: A load which can be interrupted as defined by contract.

Load center: A point at which the load of a given area is assumed to be concentrated.

Load diversity: The difference between the sum of the maxima of two or more individual loads and the coincident or combined maximum load, usually measured in kilowatts over a specified period of time.

Load duration curve: A curve of loads, plotted in descending order of magnitude, against time intervals for a specified period.

Load factor: The ratio of the average load over a designated period of time to the peak load occurring in that period.

Load-interrupter switch: An interrupter switch designed to interrupt currents not in excess of the continuous-current rating of the switch.

Load losses, transformer: Those losses which are incident to the carrying of a specified load. They include I^2R loss in the winding due to load and eddy currents, stray loss due to leakage fluxes in the windings, etc., and the loss due to circulating currents in parallel windings.

Load tap changer: A selector switch device applied to power transformers to maintain a constant low-side or secondary voltage with a variable primary voltage supply, or to hold a constant voltage out along the feeders on the low-voltage side for varying load conditions on the low-voltage side. Abbreviated LTC.

Load-tap-changing transformer: A transformer used to vary the voltage, or phase angle, or both, of a regulated circuit in steps by means of a device that connects different taps of tapped winding(s) without interrupting the load.

Loop feeder: A number of tie feeders in series, forming a closed loop. There are two routes by which any point on a loop feeder can receive electric energy, so that the flow can be in either direction.

Loop service: Two services of substantially the same capacity and characteristics, supplied from adjacent sections of a loop feeder. The two sections of the loop feeder are normally tied together on the consumer's bus through switching devices.

Loss factor: The ratio of the average power loss to the peak-load power loss during a specified period of time.

LTC: Abbreviation for load tap changer.

LV: Abbreviation for low voltage.

Main distribution center: A distribution center supplied directly by mains.

Maintenance expenses: The expense required to keep the system or plant in proper operating repair.

Maximum demand: The largest of a particular type of demand occurring within a specified period.

MC: Abbreviation for metal-clad (cable).

Messenger cable: A galvanized steel or copperweld cable used in construction to support a suspended current-carrying cable.

Metal-clad switchgear, outdoor: A switchgear that can be mounted in suitable weatherproof enclosures for outdoor installations. The base units are the same for both indoor and outdoor applications. The weatherproof housing is constructed integrally with the basic structure and is not merely a steel enclosure. The basic structure, including the mounting details and withdrawal mechanisms for the circuit breakers, bus compartments, transformer compartments, etc., is the same as that of indoor metal-clad switchgear.

Metal-enclosed switchgear: Primarily indoor-type switchgear. It can, however, be furnished in weatherproof houses suitable for outdoor operation. The switchgear is suitable for 600 V maximum service.

Minimum demand: The smallest of a particular type of demand occurring within a specified period.

Momentary interruption: An interruption of duration limited to the period required to restore service by automatic or supervisory-controlled switching operations or by manual switching at locations where an operator is immediately available.

Monthly peak duration curve: A curve showing the total number of days within the month during which the net 60-min clock-hour integrated peak demand equals or exceeds the percent of monthly peak values shown.

N.C.: Abbreviation for normally closed.

NEC: Abbreviation for National Electric Code.

NESC: Abbreviation for National Electrical Safety Code.

Net system energy: Energy requirements of a system, including losses, defined as (1) net generation of the system, plus (2) energy received from others, less (3) energy delivered to other systems.

Network distribution system: A distribution system which has more than one simultaneous path of power flow to the load.

Network protector: An electrically operated low-voltage air circuit breaker with self-contained relays for controlling its operation. It provides automatic isolation of faults in the primary feeders or network transformers. Abbreviated NP.

N.O.: Abbreviation for normally open.

No-load current: The current demand of a transformer primary when no current demand is made on the secondary.

No-load loss: Energy losses in an electric facility when energized at rated voltage and frequency but not carrying load.

Noncoincident demand: The sum of the individual maximum demands regardless of time of occurrence within a specified period.

Normal rating: Capacity of installed equipment.

Normal weather: All weather not designated as adverse or major storm disaster.

Normally closed: Denotes the automatic closure of contacts in a relay when deenergized. Abbreviated N.C.

Normally open: Denotes the automatic opening of contacts in a relay when deenergized. Abbreviated N.O.

NP: Abbreviation for network protector.

NSW: Abbreviation for nonswitched.

NX: Abbreviation for nonexpulsion (fuse).

Off-peak energy: Energy supplied during designated periods of relatively low system demands.

On-peak energy: Energy supplied during designated periods of relatively high system demands.

OH: Abbreviation for overhead.

Operating expenses: The labor and material costs for operating the plant involved.

Outage: The state of a component when it is not available to perform its intended function due to some event directly associated with that component. An outage may or may not cause an interruption of service to consumers depending upon the system configuration.

Outage duration: The period from the initiation of an outage until the affected component or its replacement once again becomes available to perform its intended function.

Outage rate: For a particular classification of outage and type of component, the mean number of outages per unit exposure time per component.

Overhead expenses: The costs which in addition to direct labor and material are incurred by all utilities.

Overload: Loading in excess of normal rating of equipment.

Overload protection: Interruption or reduction of current under conditions of excessive demand, provided by a protective device.

Pad-mounted: A general term describing equipment positioned on a surface-mounted pad located outdoors. The equipment is usually enclosed with all exposed surfaces at ground potential.

Pad-mounted transformer: A transformer utilized as part of an underground distribution system, with enclosed compartment(s) for high-voltage and low-voltage cables entering from below, and mounted on a foundation pad.

Panelboard: A distribution point where an incoming set of wires branches into various other circuits.

PE: An abbreviation used for polyethylene (cable insulation).

Peak current: The maximum value (crest value) of an alternating current

Peak voltage: The maximum value (crest value) of an alternating voltage

Peaking station: A generating station which is normally operated to provide power only during maximum load periods.

Peak-to-peak value: The value of an ac waveform from its positive peak to its negative peak. In the case of a sine wave, the peak-to-peak value is double the peak value.

Pedestal: A bottom support or base of a pillar, statue, etc.

Percent regulation: See Percent voltage drop.

Percent voltage drop: The ratio of voltage drop in a circuit to voltage delivered by the circuit, multiplied by 100 to convert to percent.

Permanent forced outage: An outage whose cause is not immediately self-clearing but must be corrected by eliminating the hazard or by repairing or replacing the component before it can be returned to service. An example of a permanent forced outage is a lightning flashover which shatters an insulator, thereby disabling the component until repair or replacement can be made.

Permanent forced outage duration: The period from the initiation of the outage until the component is replaced or repaired.

Phase: The time of occurrence of the peak value of an ac waveform with respect to the time of occurrence of the peak value of a reference waveform.

Phase angle: An angular expression of phase difference.

Pole: A column of wood or steel, or some other material, supporting overhead conductors, usually by means of arms or brackets.

Pole fixture: A structure installed in lieu of a single pole to increase the strength of a pole line or to provide better support for attachments than would be provided by a single pole. Examples are A fixtures, H fixtures.

Primary disconnecting devices: Self-coupling separable contacts provided to connect and disconnect the main circuits between the removable element and the housing.

Primary distribution feeder: A feeder operating at primary voltage supplying a distribution circuit.

Primary distribution mains: The conductors that feed from the center of distribution to direct primary loads or to transformers that feed secondary circuits.

Primary distribution network: A network consisting of primary distribution mains.

Primary distribution system: A system of ac distribution for supplying the primaries of distribution transformers from the generating station or substation distribution buses.

Primary distribution trunk line: A line acting as a main source of supply to a distribution system.

Primary feeder: That portion of the primary conductors between the substation or point of supply and the center of distribution.

Primary lateral: That portion of a primary distribution feeder that is supplied by a main feeder or other laterals and extends through the load area with connections to distribution transformers or primary loads.

Primary main feeder: The higher-capacity portion of a primary distribution feeder that acts as a main source of supply to primary laterals or direct-connected distribution transformers and primary loads.

Primary network: A network supplying the primaries of transformers whose secondaries may be independent or connected to a secondary network.

Primary open-loop service: A service which consists of a single distribution transformer with dual primary switching, supplied from a single primary circuit which is arranged in an open-loop configuration.

Primary selective service: A service which consists of a single distribution transformer with primary throw-over switching, supplied by two independent primary circuits.

Primary transmission feeder: A feeder connected to a primary transmission circuit.

Primary unit substation: A unit substation in which the low-voltage section is rated above 1000 V.

Protective relay: A device whose function is to detect defective lines or apparatus or other power-system conditions of an abnormal or dangerous nature and to initiate appropriate control circuit action.

Power: The rate (in kilowatts) of generating, transferring, or using energy.

Power, active: The product of the rms value of the voltage and the rms value of the in-phase component of the current.

Power, apparent: The product of the rms value of the voltage and the rms value of the current.

Power, instantaneous: The product of the instantaneous voltage multiplied by the instantaneous current.

Power, reactive: The product of the rms value of the voltage and the rms value of the quadrature component of the current.

Power factor: The ratio of active power to apparent power.

Power-factor adjustment clause: A clause in a rate schedule that provides for an adjustment in the billing if the customer's power factor varies from a specified reference.

Power pool: A group of power systems operating as an interconnected system and pooling their resources.

Power transformer: A transformer which transfers electric energy in any part of the circuit between the generator and the distribution primary circuits.

PT: Abbreviation for potential transformers.

pu: Abbreviation for per unit.

Raceway: A channel for holding wires, cables, or busbars. The channel may be in the form of a conduit, electrical metallic tubing, or a square sheet-metal duct. It is used in buildings at low voltage.

Radial distribution system: A distribution system which has a single simultaneous path of power flow to the load.

Radial service: A service which consists of a single distribution transformer supplied by a single primary circuit.

Radial system, complete: A radial system which consists of a radial subtransmission circuit, a single substation, and a radial primary feeder with several distribution transformers each supplying radial secondaries; has the lowest degrees of service continuity.

Ratchet demand: The maximum past or present demands which are taken into account to establish billings for previous or subsequent periods.

Ratchet demand clause: A clause in a rate schedule which provides that maximum past or present demands be taken into account to establish billings for previous or subsequent periods.

Rate base: The net plant investment or valuation base specified by a regulatory authority upon which a utility is permitted to earn a specified rate of return.

RCN: Abbreviation for reduced-capacity neutral.

Recloser: A dual-timing device which can be set to operate quickly to prevent downline fuses from blowing.

Reclosing device: A control device which initiates the reclosing of a circuit after it has been opened by a protective relay.

Reclosing fuse: A combination of two or more fuse holders, fuse units, or fuse links mounted on a fuse support(s), mechanically or electrically interlocked, so that one fuse can be connected into the circuit at a time and the functioning of that fuse automatically connects the next fuse into the circuit, thereby permitting one or more service restorations without replacement of fuse links, refill units, or fuse units.

Reclosing relay: A programming relay whose function is to initiate the automatic reclosing of a circuit breaker.

Reclosure: The automatic closing of a circuit-interrupting device following automatic tripping. Reclosing may be programmed for any combination of instantaneous, time-delay, single-shot, multiple-shot, synchronism-check, dead-line–live-bus, or dead-bus–live-line operation.

Required reserve: The system planned reserve capability needed to ensure a specified standard of service.

Resistance: The real part of impedance.

Return on capital: The requirement which is necessary to pay for the cost of investment funds used by the utility.

RP: Abbreviation for regulating point.

Sag: The distance measured vertically from a conductor to the straight line joining its two points of support. Unless otherwise stated, the sag referred to is the sag at the midpoint of the span.

Sag, final unloaded: The sag of a conductor after it has been subjected for an appreciable period to the loading prescribed for the loading district in which it is situated, or equivalent loading, and the loading removed. Final unloaded sag includes the effect of inelastic deformation.

Sag, initial unloaded: The sag of a conductor prior to the application of any external load.

SAG of a conductor (at any point in a span): The distance measured vertically from the particular point in the conductor to a straight line between its two points of support.

Sag section: The section of line between snub structures. More than one sag section may be required to properly sag the actual length of conductor which has been strung.

Sag span: A span selected within a sag section and used as a control to determine the proper sag of the conductor, thus establishing the proper conductor level and tension. A minimum of two, but normally three, sag spans are required within a sag section to sag properly. In mountainous terrain or where span lengths vary radically, more than three sag spans could be required within a sag section.

Scheduled interruption: An interruption caused by a scheduled outage.

Scheduled outage: An outage that results when a component is deliberately taken out of service at a selected time, usually for purposes of construction, preventive maintenance, or repair.

Scheduled outage duration: The period from the initiation of the outage until construction, preventive maintenance, or repair work is completed.

Scheduled maintenance (generation): Capability which has been scheduled to be out of service for maintenance.

SCV: Abbreviation for steam-cured (cable insulation).

Seasonal diversity: Load diversity between two (or more) electric systems which occurs when their peak loads are in different seasons of the year.

Secondary, radial: A secondary supplied from either a conventional or completely self-protected (type CSP) distribution transformer.

Secondary current rating: The secondary current existing when the transformer is delivering rated kilovoltamperes at rated secondary voltage.

Secondary disconnecting devices: Self-coupling separable contacts provided to connect and disconnect the auxiliary and control circuits between the removable element and the housing.

Secondary distributed network: A service consisting of a number of network transformer units at a number of locations in an urban load area connected to an extensive secondary cable grid system.

Secondary distribution feeder: A feeder operating at secondary voltage supplying a distribution circuit.

Secondary distribution mains: The conductors connected to the secondaries of distribution transformers from which consumers' services are supplied.

Secondary distribution network: A network consisting of secondary distribution mains.

Secondary distribution system: A low-voltage ac system that connects the secondaries of distribution transformers to the consumers' services.

Secondary distribution trunk line: A line acting as a main source of supply to a secondary distribution system.

Secondary fuse: A fuse used on the secondary-side circuits, restricted for use on a low-voltage secondary distribution system that connects the secondaries of distribution transformers to consumers' services.

Secondary mains: Those which operate at utilization voltage and serve as the local distribution main. In radial systems secondary mains that supply general lighting and small power are usually separate from mains that supply three-phase power because of the dip in voltage caused by starting motors. This dip in voltage if sufficiently large, causes an objectionable lamp flicker.

Secondary network: It consists of two or more network transformer units connected to a common secondary system and operating continuously in parallel.

Secondary network service: A service which consists of two or more network transformer units connected to a common secondary system and operating continuously in parallel.

Secondary selective service: A service which consists of two distribution transformers, each supplied by an independent primary circuit, and with secondary main and tie breakers.

Secondary spot network: A network which consists of at least two and as many as six network-transformer units located in the same vault and connected to a common secondary service bus. Each transformer is supplied by an independent primary circuit.

Secondary system, banked: A system which consists of several transformers supplied from a single primary feeder, with the low-voltage terminals connected together through the secondary mains.

Secondary unit substation: A unit substation whose low-voltage section is rated 1000 V and below.

Secondary voltage regulation: A voltage drop caused by the secondary system, it includes the drop in the transformer and in the secondary and service cables.

Second-contingency outage: The outage of a secondary primary feeder in addition to the first one.

Sectionalizer: A device which resembles an oil circuit recloser but lacks the interrupting capability.

Service area: Territory in which a utility system is required or has the right to supply or make available electric service to ultimate consumers.

Service availability index: See Index of reliability.

Service drop: The overhead conductors, through which electric service is supplied, between the last utility company pole and the point of their connection to the service facilities located at the building or other support used for the purpose.

Service entrance: All components between the point of termination of the overhead service drop or underground service lateral and the building main disconnecting device, with the exception of the utility company's metering equipment.

Service entrance conductors: The conductors between the point of termination of the overhead service drop or underground service lateral and the main disconnecting device in the building.

Service entrance equipment: Equipment located at the service entrance of a given building that provides overcurrent protection to the feeder and service conductors, provides a means of disconnecting the feeders from energized service conductors, and provides a means of measuring the energy used by the use of metering equipment

Service lateral: The underground conductors, through which electric service is supplied, between the utility company's distribution facilities and the first point of their connection to the building or area service facilities located at the building or other support used for the purpose.

SF_6: Formula for sulfur hexafluoride (gas).

St: Abbreviation for steel.

Strand: One of the wires, or groups of wires, of any stranded conductor.

Stranded conductor: A conductor composed of a group of wires, or of any combination of groups of wires. Usually, the wires are twisted together.

Submarine cable: A cable designed for service under water. It is usually a lead-covered cable with a steel armor applied between layers of jute.

Submersible transformer: A transformer so constructed as to be successfully operable when submerged in water under predetermined conditions of pressure and time.

Substation: An assemblage of equipment for purposes other than generation or utilization, through which electric energy in bulk is passed for the purpose of switching or modifying its characteristics.

Substation voltage regulation: The regulation of the substation voltage by means of the voltage regulation equipment which can be LTC (load-tap-changing) mechanisms in the substation transformer, a separate regulator between the transformer and low-voltage bus, switched capacitors at the low-voltage bus, or separate regulators located in each individual feeder in the substation.

Subtransmission: That part of the distribution system between bulk power source(s) (generating stations or power substations) and the distribution substation.

Susceptance: The imaginary part of admittance.

Switch: A device for opening and closing or for changing connections in a circuit.

Switch, isolating: An auxiliary switch for isolating an electric circuit from its source of power; it is operated only after the circuit has been opened by other means.

Switch, limit: A switch that is operated by some part or motion of a power-driven machine or equipment to alter the electric circuit associated with the machine or equipment.

Switchboard: A large single panel, frame, or assembly of panels on which are mounted (on the face, or back, or both) switches, fuses, buses, and usually instruments.

Switched-capacitor bank: A capacitor bank with switchable capacitors.

Switchgear: A general term covering switching or interrupting devices and their combination with associated control, instrumentation, metering, protective, and regulating devices; also assemblies of these devices with associated interconnections, accessories, and supporting structures.

Switching time: The period from the time a switching operation is required due to a forced outage until that switching operation is performed.

System: A group of components connected together in some fashion to provide flow of power from one point or points to another point or points.

System interruption duration index: The ratio of the sum of all customer interruption durations per year to the number of customers served. It gives the number of minutes out per customer per year.

Underground distribution system: That portion of a primary or secondary distribution system which is constructed below the earth's surface. Transformers and equipment enclosures for such a system may be located either above or below the surface as long as the served and serving conductors are located underground.

Unit substation: A substation consisting primarily of one or more transformers which are mechanically and electrically connected to and coordinated in design with one or more switchgear or motor control assemblies or combinations thereof.

URD: Abbreviation for underground residential distribution.

Utilization factor: The ratio of the maximum demand of a system to the rated capacity of the system.

VD: Abbreviation for voltage drop.

VDIP Abbreviation for voltage dip.

Voltage, base: A reference value which is a common denominator to the nominal voltage ratings of transmission and distribution lines, transmission and distribution equipment, and utilization equipment.

Voltage, maximum: The greatest 5-min average or mean voltage.

Voltage, minimum: The least 5-min average or mean voltage.

Voltage, nominal: A nominal value assigned to a circuit or system of a given voltage class for the purpose of convenient designation.

Voltage, rated: The voltage at which operating and performance characteristics of equipment are referred.

Voltage, service: Voltage measured at the terminals of the service entrance equipment.

Voltage, utilization: Voltage measured at the terminals of the machine or device.

Voltage dip: A voltage change resulting from a motor starting.

Voltage drop: The difference between the voltage at the transmitting and receiving ends of a feeder, main or service.

Voltage flicker: Voltage fluctuation caused by utilization equipment resulting in lamp flicker, i.e., in a lamp illumination change.

Voltage regulation: The percent voltage drop of a line with reference to the receiving-end voltage.

$$\% \text{ regulation} = \frac{|E_s| - |E_r|}{|E_r|} \times 100$$

where $|E_s|$ is the magnitude of the end voltage and $|E_r|$ is the magnitude of the receiving-end voltage.

Voltage regulator: An induction device having one or more windings in shunt with, and excited from, the primary circuit, and having one or more windings in series between the primary circuit and the regulated circuit, all suitably adapted and arranged for the control of the voltage, or of the phase angle, or of both, of the regulated circuit.

Voltage spread: The difference between maximum and minimum voltages.

VRR: Abbreviation for voltage-regulating relay.

XLPE: Abbreviation for cross-linked polyethylene (cable insulation).

REFERENCES

1. IEEE Committee Report: "Proposed Definitions of Terms for Reporting and Analyzing Outages of Electrical Transmission and Distribution Facilities and Interruptions," *IEEE Trans. Power Appar. Syst.*, vol. PAS-87, no. 5, May 1968, pp. 1318–23.
2. IEEE Committee Report: "Guidelines for Use in Developing a Specific Underground Distribution System Design Standard," *IEEE Trans. Power Appar. Syst.*, vol. PAS-97, no. 3, May/June 1978, pp. 810–27.
3. *IEEE Standard Definitions in Power Operations Terminology*, IEEE Standard 346-1973, Nov. 2, 1973.
4. *Proposed Standard Definitions of General Electrical and Electronics Terms*, IEEE Standard 270, 1966.
5. Pender, H., and W. A. Del Mar. *Electrical Engineers' Handbook-Electrical Power*, 4th ed., Wiley, New York, 1962.
6. *National Electrical Safety Code*, 1977 ed., ANSI C2, IEEE, New York, November, 1977.
7. Fink, D. G., and J. M. Carroll (eds.): *Standard Handbook for Electrical Engineers*, 10th ed., McGraw-Hill, New York, 1969.
8. *IEEE Standard Dictionary of Electrical and Electronics Terms*, IEEE, New York, 1972.

NOTATION

Capital English letters

A	component availability (Chap. 11)
A	levelized annual cost, \$ (Chap. 7)
A	weighted average Btu/kWh net generation (Chap. 2)
A, B, C	phase designation
A, B, C, D	general line (circuit) constants (Chap. 5)
A_{FDR}	feeder availability (Chap. 11)
A_i	availability of component i (Chap. 11)
A_n	area served by one of n substation feeders, mi^2
A_{SD}	service-drop conductor size, cmil (Chap. 6)
A_{SL}	secondary-line conductor size, cmil (Chap. 6)
A_{sys}	system availability (Chap. 11)
AEC	annual equivalent of energy cost, \$
AEIC$_c$	annual equivalent of total installed capacitor bank cost, \$ (Chap. 8)
AIC	annual equivalent of feeder investment cost, \$
B	average fuel cost, \$/MBtu (Chap. 2)
BEC	original (base) annual kWh energy consumption
BVR	bus voltage regulator
BW	bandwidth of voltage-regulating relay
C	capacitance, F
C	common winding (Chap. 3)
C_F	installed feeder cost, \$/kVA (Chap. 8)
C_G	generation system cost, \$/kVA

C_S	distribution substation cost, \$/kVA
C_T	transmission system cost, \$/kVA
C_T	transmission cost, \$/kVA (Eq. 8-26)
C_T	total reactive compensation (= cn) (Eq. 8-85)
CR	corrective ratio (Chap. 9)
CT_P	primary-side rating of current transformer (Chap. 9)
CTR	current transformer ratio
D	distance or separation, ft
D	load density, kVA/mi^2
D	ratio of kWh losses to net system input (Chap. 2)
D_g	coincident maximum group demand, W
D_i	demand of load i, W
DF_i	demand factor of load group i
DTA(ij, k)	downtime array coefficients (Chap. 11)
E	source emf; voltage
E_i	event that component i operates successfully (Chap. 11)
EC	energy cost, \$/kWh
$\mathrm{EC}_{\mathrm{off}}$	incremental cost of off-peak electric energy, \$/kWh
$\mathrm{EC}_{\mathrm{on}}$	incremental cost of on-peak electric energy, \$/kWh (Chap. 6)
ECL_1	eddy-current loss at rated fundamental current
$E(T)$	expected time during which a component will survive
F	fault point
F'_{LD}	reactive load factor (= Q/S)
F_c	coincident factor
F_D	diversity factor
F_{LD}	load factor
F_{LL}	load-location factor (Chap. 7)
F_{LS}	loss factor
F_{LSA}	loss-allowance factor (Chap. 7)
F_{PR}	peak-responsibility factor
F_R	reserve factor
F_u	utilization factor
FCAF	fuel cost adjustment factor, \$/kWh
FDR	feeder (Chap. 11)
$F(t)$	unreliability function (Chap. 11)
H	transformer higher-voltage-side winding
HF_I	current harmonic factor
HF_V	voltage harmonic factor
I	failed zone-branch array coefficient (Chap. 11)
I	rms phasor current, A
$\mathbf{I}$	current matrix
I_{AB}	current in higher-voltage-side winding between phases A and B, A
I_{ab}	current in lower-voltage-side winding between phases a and b, A
$I_{a,3\phi}$	current in phase a due to single-phase load, A
I_B	base current, A

I_C	current in common winding (Chap. 3)
I_c	core-loss component of excitation current (Chap. 3)
I_e	excitation current (Chap. 3)
I_{exc}	per unit excitation current (Chap. 6)
$I_{f,a}, I_{f,b}, I_{f,c}$	fault currents in phases a, b, and c
$I_{f,3\phi}$	three-phase fault current, A
$I_{F,3\phi}$	three-phase fault current referred to subtransmission voltage, A (Chap. 10)
$(I_{F,3\phi})_{\max}$	maximum three-phase fault current, A (Chap. 10)
$I_{F,\text{HV}}$	fault current in transformer high-voltage side, A
$I_{f,L\text{–}G}$	line-to-ground fault current, A
$I_{f,\text{LV}}$	fault current in transformer low-voltage side, A
$I_{f,\text{L–L}}$	line-to-line fault current, A
$I_{\phi 1}$	current due to single-phase load, A
I_h	harmonic current
I_L	line current; load current, A
I_m	magnetizing current component of excitation current (Chap. 3)
I_m	current in feeder main at substation, A
I_N	current in primary neutral, A
I_n	current in secondary neutral, A
I_{op}	operating current, A
$I_{P,\text{pu}}$	no-load primary current at substation transformer, pu
I_{ra}	rated current, A
I_S	current in series winding (Chap. 3)
IC_c	installed cost of capacitor bank, \$/kvar (Chap. 8)
IC_{cap}	total installed cost of shunt capacitors, \$
IC_F	installed feeder cost, \$ (Chap. 7)
IC_{PH}	annual installed cost of pole and its hardware, \$ (Chap. 6)
IC_{SD}	annual installed cost of service drop, \$ (Chap. 6)
IC_{SL}	annual installed cost of secondary line, \$ (Chap. 6)
IC_{sys}	average investment cost of power system upstream, \$/kVA
IC_T	annual installed cost of distribution transformer, \$
K	percent voltage drop per kilovoltampere-mile
$\tilde{K}$	per unit voltage drop per 10,000 A·ft
$\hat{K}$	constant (Eq. 5-63)
K_h	watthour meter constant (Chap. 2)
K_R	conversion factor for resistance (Chap. 7)
K_r	number of watthour meter disk revolutions (Chap. 2)
K_X	conversion factor for reactance (Chap. 7)
K_1	a constant to convert energy loss savings to dollars, \$/kWh (Eq. 8-87)
K_2	a constant to convert power loss savings to dollars, \$/kWh (Eq. 8-87)
K_3	a constant to convert total fixed capacitor size to dollars, \$/kWh (Eq. 8-95)

L_{sc}	system inductance, H (Eq. 8-108)
LCDH	losses in capacitors due to harmonics
LD	load diversity, W
LD	load (Chaps. 2 and 5)
LDC	line-drop compensator
LS	loss
LTC	load tap changer
LV	low voltage
MTTR	mean time to repair ($=\bar{r}$) (Chap. 11)
MTBF	mean time between failures ($=\overline{T}$) (Chap. 11)
MTTF	mean time to failure ($=\overline{m}$) (Chap. 11)
N	expected duration of normal weather (Chap. 11)
N	neutral primary terminal
$\mathbf{O}$	row vector of zeros (Chap. 11)
$\mathrm{OC}_{\mathrm{exc}}$	annual operating cost of transformer excitation current, \$ (Chap. 6)
$\mathrm{OC}_{\mathrm{SD,Cu}}$	annual operating cost of service-drop cable due to copper losses, \$ (Chap. 6)
$\mathrm{OC}_{\mathrm{SL,Cu}}$	annual operating cost of secondary line due to copper losses, \$ (Chap. 6)
$\mathrm{OC}_{T,\mathrm{Cu}}$	annual operating cost of transformer due to copper losses, \$ (Chap. 6)
$\mathrm{OC}_{T,\mathrm{Fe}}$	annual operating cost of transformer due to core losses, \$
P	average power, W
$\mathbf{P}$	transition (or stochastic) matrix (Chap. 11)
P'_{LS}	power loss after capacitor bank addition, W (Eq. 8-46)
P_{av}	average power, W
$P_{\mathrm{LS,av}}$	average power loss, W
P_i	peak load i, W
P_{LD}	average power of load, W
P_{LS}	average power loss, W
$P_{\mathrm{LS},i}$	peak loss at peak load i, W
$\mathrm{P}_{\mathrm{LS,max}}$	maximum power loss, W
$P_{\mathrm{LS},1\phi}$	single-phase power loss, W (Chap. 7)
$P_{LS,3}$	three-phase power loss, W (Chap. 7)
P_n	load at year n, W (Chap. 2)
P_0	initial load, W
P_r	receiving-end average power, VA
$P_{T,\mathrm{Cu}}$	transformer copper loss, W (Chap. 6)
$P_{T,\mathrm{Fe}}$	transformer core loss, W (Chap. 6)
$P_{\mathrm{SL,Cu}}$	power loss of secondary line due to copper losses, W
PF	power factor
PTR	potential transformer ratio
PT_N	turns ratio of potential transformer (Chap. 9)
Q	average reactive power, var
Q_c	reactive power due to corrective capacitors, var (Chap. 8)

$Q_{c,3\phi}$	three-phase reactive power due to corrective capacitors, var (Eq. 8-30)
Q_i	unreliability of component i (Chap. 11)
Q_r	receiving-end average reactive power, VA
Q_{sys}	system unreliability (Chap. 11)
R	resistance, Ω
R_{eff}	effective resistance, Ω (Chap. 9)
R_L	resistance of load impedance, Ω
R_{set}	R dial setting of line-drop compensator (Chap. 9)
R_{sys}	system reliability (Chap. 11)
RIA(ij, k)	recognition and isolation array coefficients (Chap. 11)
RP	regulating point
S	apparent power, VA
S	$= P + jQ$, complex apparent power, VA
S	expected duration of adverse weather (Chap. 11)
S	series winding (Chap. 3)
S_B	base apparent power, VA
S_{sc}	short-circuit apparent power, VA
S_{ckt}	circuit capacity, VA (Chap. 9)
S_G	generation capacity, VA (Chap. 8)
S_L	load apparent power, VA
S_{Li}	apparent power of load i, VA
$S_{\angle-\angle}$	apparent power rating of an open-delta bank
S_{lump}	apparent power of lumped load, VA
$S_{L,3\phi}$	three-phase apparent power of load, VA (Chap. 8)
S_m	total kVA load served by one feeder main
S_n	kVA load served by one of n substation feeders
S_{PK}	feeder apparent power at peak load, VA
S_{reg}	regulator capacity, VA (Chap. 9)
S_S	substation capacity, VA (Eq. 8-27)
S_T	transformer apparent power, VA
S_T	transmission capacity, VA (Eq. 8-24)
$S_{T,ab}$	apparent power rating of single-phase transformer connected between phases a and b, VA
S_{T_i}	apparent power rating of transformer i, VA
$S_{T,3\phi}$	three-phase transformer apparent power, VA
$S_{1\phi}$	single-phase VA rating
$S_{3\phi}$	three-phase VA rating
$S_{\Delta-\Delta}$	apparent power rating of a delta-delta bank
SD	service drop (Chap. 6)
SW	switchable capacitors (Chap. 8)
T	a random variable representing failure time (Chap. 11)
T	time
T	transformer
TA_n	total area served by all n feeders, mi^2

TAC	total annual cost, $
$\mathrm{TAEL}_{\mathrm{Cu}}$	total annual energy loss due to copper losses, W
TCD_i	total connected group demand i, W
TD	time delay
TECL	total eddy-current loss (Chap. 8)
TS_n	total kVA load served by a substation with n feeders
U	component unavailability (Chap. 11)
U_{FDR}	feeder unavailability (Chap. 11)
UG	underground
URD	underground residential distribution
V	volt, unit symbol abbreviation for voltage
$\mathbf{V}$	voltage matrix
$V_{ab,\,\mathrm{pu}}$	voltage between phases a and b, pu
$V_{B,\,\phi}$	single-phase base voltage, V
$V_{B,\,3\phi}$	three-phase base voltage, V
V_C	voltage across common winding (Chap. 3)
V_H	higher-voltage-side voltage, V (Chap. 3)
V_h	rms voltage of hth harmonic
V_{L-L}	line-to-line distribution voltage, V (Chap. 10)
V_{L-L}	line-to-line voltage, V
V_{L-N}	line-to-neutral voltage, V
$V_{l,\,\mathrm{pu}}$	per unit voltage at feeder end (Chap. 9)
V_P	primary distribution voltage, V (Chap. 9)
$V_{P,\,\max}$	maximum primary distribution voltage, V
V_r	receiving-end voltage
V_{reg}	output voltage of regulator, V
V_{RP}	voltage at regulating point, V
V_S	voltage across series winding (Chap. 3)
V_s	sending-end voltage
V_{ST}	subtransmission voltage, V (Chap. 9)
$V_{\mathrm{ST},\,L-L}$	line-to-line subtransmission voltage, V (Chap. 10)
V_X	lower-voltage-side voltage, V (Chap. 3)
VD	voltage drop, V
$\mathrm{VD}_{\mathrm{pu}}$	per unit voltage drop
$\mathrm{VD}_{\mathrm{pu},\,1\phi}$	single-phase voltage drop, pu
$\mathrm{VD}_{\mathrm{pu},\,3\phi}$	three-phase voltage drop, pu
$\mathrm{VD}_{\mathrm{SD}}$	voltage drop in service-drop cable, V
$\mathrm{VD}_{\mathrm{SL}}$	voltage drop in secondary line, V
VD_T	voltage drop in transformer, V
$\%\,\mathrm{VD}_{ab}$	percent voltage drop between a and b
$\%\,\mathrm{VD}_m$	percent voltage drop in feeder main
VDIP	voltage dip, V
$\mathrm{VDIP}_{\mathrm{SD}}$	voltage dip in service-drop cable, V
$\mathrm{VDIP}_{\mathrm{SL}}$	voltage dip in secondary line, V
VDIP_T	voltage dip in transformer, V

$\mathrm{VR}_{l,\,\mathrm{pu}}$	per unit voltage rise at distance l (Chap. 9)
$\mathrm{VR}_{\mathrm{pu}}$	per unit voltage regulation
$\%\,\mathrm{VR}$	percent voltage regulation
$\%\,\mathrm{VR}$	percent voltage rise (Chap. 8)
$\%\,\mathrm{VR}_{\mathrm{NSW}}$	percent voltage rise due to nonswitchable capacitors (Chap. 8)
$\%\,\mathrm{VR}_{\mathrm{SW}}$	percent voltage rise due to switchable capacitors (Chap. 8)
VRR	voltage-regulating relay
$\mathrm{VRR}_{\mathrm{pu}}$	per unit setting of voltage-regulating relay
W	wire (in transformer connections) (Chap. 3)
X	reactance, Ω; transformer lower-voltage-side winding
X_c	capacitive reactance
X_L	reactance of load impedance, Ω
X_{sc}	system reactance, Ω (Eq. 8-107)
X_{set}	X dial setting of line-drop compensator (Chap. 9)
$X(t_n)$	sequence of discrete-valued random variables (Chap. 11)
Y	admittance, Ω; wye connection
$\mathbf{Y}$	admittance matrix
Z	impedance, Ω
Z	secondary-winding impedance, Ω (Eq. 10-71)
$\mathbf{Z}$	impedance matrix
Z_{eq}	equivalent (total) impedance to fault, Ω (Chap. 10)
Z_f	fault impedance, Ω
Z_G	impedance to ground, Ω
$Z_{G,\,\mathrm{ckt}}$	impedance to ground of circuit, Ω
Z_{LD}	load impedance, Ω
Z_M	impedance of secondary main, Ω
Z_T	transformer impedance, Ω
Z_T	equivalent impedance of distribution transformer, Ω
$Z_{T,\,\mathrm{pu}}$	per unit transformer impedance
Z_Δ	equivalent delta impedance, Ω (Eq. 10-54)
Z_0	zero-sequence impedance, Ω
$Z_{0,\,\mathrm{ckt}}$	zero-sequence impedance of circuit, Ω
Z_1	positive-sequence impedance, Ω
$Z_{1,\,\mathrm{ckt}}$	positive-sequence impedance of circuit, Ω
$Z_{1,\,\mathrm{SL}}$	positive-sequence impedance of secondary line, Ω (Chap. 10)
$Z_{1,\,\mathrm{ST}}$	positive-sequence impedance of subtransmission line, Ω
$Z_{1\ \mathrm{sys}}$	positive-sequence impedance of system, Ω
$Z_{1,\,T}'$	positive-sequence impedance of transformer, Ω
Z_2	negative-sequence impedance, Ω

Lowercase English letters

a, b, c	phase designation
c	capacitor compensation ratio (Chap. 8)
c_i	contribution factor of load i
dn	mutual geometric mean distance of phase and neutral wires, ft

dp	mutual geometric mean distance between phase wires, ft
$\bar{f}$	mean failure frequency (Chap. 11)
f_p	parallel resonant frequency, Hz
f_1	fundamental frequency, Hz
$\bar{f}_{sys}$	average failure frequency of system (Chap. 11)
$f(t)$	probability density function
h	harmonic order
$h(t)$	hazard rate (Chap. 11)
i	investment fixed charge rate (Chap. 6)
i_c	annual fixed charge rate for capacitors
i_F	annual fixed rate for feeder
i_G	annual fixed charge rate for generation system
i_S	annual fixed charge rate for distribution substation
i_T	annual fixed charge rate for transmission system
k	constant used in computing loss factor (Chap. 2)
l	inductance per unit length; leakage inductance
l	feeder length, mi
l_n	linear dimension of primary-feeder service area, mi
m_i	observed time to failure for cycle i (Chap. 11)
$\overline{m}_s$	mean time to failure of series system (Chap. 11)
n	total number of cycles (Chap. 11)
n	transfer ratio (inverse of turns ratio) (Chap. 10)
n	$= n_1/n_2$, turns ratio; neutral secondary terminal; number of feeders emanating from a substation.
n_1	number of turns in primary winding
n_2	number of turns in secondary winding
$\mathbf{p}^{(n)}$	vector of state probabilities at time t_n (Chap. 11)
p_{ij}	transition probabilities (Chap. 11)
p_{ij}	probability of proper operation of isolating equipment in zone branch ij (Chap. 11)
q	probability of component failure (Chap. 11)
q_{ij}	probability of failure of isolating equipment in zone branch ij (Chap. 11)
r	receiving end
r	radius; internal (source) resistance; resistance per unit length
r_{ap}	resistance of phase wires, Ω/1000 ft
r_e	earth resistance, Ω/1000 ft
r_{eq}	transformer equivalent resistance, Ω
r_i	observed time to repair for cycle i (Chap. 11)
r_l	lateral resistance per unit length
r_m	resistance of feeder main, Ω/mi
$\bar{r}_s$	mean time to repair of series system (Chap. 11)
s	sending end; effective feeder (main) length, mi (Chap. 4)
s	series system (Chap. 11)
t	time

x	line reactance per unit length; internal (source) reactance
x_a	self-inductive reactance of a phase conductor, Ω/mi
x_{ap}	reactance of phase wire with 1-ft spacing, Ω/1000 ft
x_{an}	reactance of neutral wire with 1-ft spacing, Ω/1000 ft
x_d	inductive reactance spacing factor, Ω/mi
x_{dn}	mutual reactance between phase and neutral wires, Ω/1000 ft
x_{dp}	mutual reactance of phase wires, Ω/1000 ft
x_e	earth reactance, Ω/1000 ft
x_{eq}	transformer equivalent reactance, Ω
$x_{i,\,\mathrm{opt}}$	optimum location of capacitor bank i in per unit length
x_L	inductive line reactance (Chap. 5 and 8)
x_l	lateral reactance per unit length
x_m	reactance of feeder main, Ω/mi
x_{RP}	regulating point distance from substation, mi (Chap. 9)
x_T	transformer reactance, % Ω (Chap. 8)
z	impedance per unit length
z_l	lateral impedance per unit length
z_m	impedance of feeder main, Ω/mi
$z_{0,a}$	zero-sequence self-impedance of phase circuit, Ω/1000 ft
$z_{0,ag}$	zero-sequence mutual impedance between phase and ground wires, Ω/1000 ft
$z_{0,g}$	zero-sequence self-impedance of ground wire, Ω/1000 ft

Capital Greek letters

Δ	delta connection; determinant
Δ	difference; increment; savings; benefits
$\Delta\mathrm{ACE}$	annual conserved energy, Wh (Chap. 8)
$\Delta\mathrm{BEC}$	additional energy consumption increase, %
$\Delta\mathrm{EL}$	energy-loss reduction
ΔP_{LS}	additional decrease in power loss, W (Chap. 8)
$\Delta P_{\mathrm{LS,\,opt}}$	optimum loss reduction, W (Chap. 8)
ΔQ_c	required additional capacitor size, var (Chap. 8)
ΔS_F	released feeder capacity, VA (Chap. 8)
ΔS_G	released generation capacity, VA (Chap. 8)
ΔS_S	released substation capacity, VA (Eq. 8-29)
ΔS_T	released transmission capacity, VA (Eq. 8-24)
ΔS_{sys}	released system capacity, W (Chap. 8)
$\Delta \$_{\mathrm{ACE}}$	annual benefits due to conserved energy, \$ (Chap. 8)
$\Delta \$_F$	annual benefits due to released feeder capacity, \$ (Eq. 8-36)
$\Delta \$_G$	annual benefits due to released generation capacity, \$ (Chap. 8)
$\Delta \$_S$	annual benefits due to released substation capacity, \$ (Eq. 8-29)
$\Delta \$_T$	annual benefits due to released transmission capacity, \$ (Eq. 8-26)
$\mathbf{\Lambda}$	transition rate matrix (Chap. 11)
$\mathbf{\Pi}$	unconditional steady-state probability matrix (Chap. 11)
Σ	total savings due to capacitor installation, \$ (Eq. 8-86)

Lowercase Greek letters

α	a constant $[=(1+\lambda+\lambda^2)^{-1}]$(Chap. 11)
δ	power angle
θ	power-factor angle
θ_{max}	power-factor angle at maximum voltage drop
λ	ratio of reactive current at line end to reactive current at line beginning
λ	failure rate (Chap. 11)
$\bar{\lambda}$	complex flux linkages, (Wb · T)/m
λ_{CT}	annual fault rate of cable terminations (Chap. 11)
λ_{FDR}	annual feeder fault rate (Chap. 11)
λ_{ij}	total failure rate of zone branch *ij* (Chap. 11)
λ_{ijB}	breaker failure rate in zone *i* branch *j* (Chap. 11)
λ_{ijM}	zone branch *ij* failure rate due to preventive maintenance (Chap. 11)
λ_{ijW}	zone branch *ij* failure rate due to adverse weather (Chap. 11)
λ_{OH}	annual fault rate of overhead feeder section (Chap. 11)
λ_s	failure rate of supply (substation) (Chap. 11)
λ_{UG}	annual fault rate of underground feeder section (Chap. 11)
μ	mean repair rate (Chap. 11)
μ_{ij}	zone branch *ij* repair rate (Chap. 11)
μ_{SijC}	reclosing rate of reclosing equipment in zone branch *ij* (Chap. 11)
μ_{SijO}	isolation rate of isolating equipment in zone-branch *ij* (Chap. 11)
ϕ	$=\tan^{-1}(X/R)$, impedance angle
ϕ	magnetic flux; phase angle
ω	radian frequency

Subscripts

A	phase *a*
a	phase *a*
B	phase *b*
b	phase *b*
B	base quantity
C	phase *C*; common winding (Chap. 3)
c	phase *c*
c	capacity; capacitive; coincident (Chap. 3)
cap	shunt capacitor
ckt	circuit
CT	cable termination (Chap. 11)
Cu	copper
eff	effective
eq	equivalent circuit quantity
exc	excitation
D	diversity

F	feeder; fault point; referring to fault
f	referring to fault
FDR	feeder
Fe	iron
H	high-voltage side (HV)
L	inductive (reactance); load (Chap. 3)
L	line; load
l	lateral; inductive (reactance); length
LD	load
L–G	line-to-ground
L–L	line-to-line
LL	load location (Chap. 7)
L–N	line-to-neutral
LS	loss (Chap. 2 and 5)
LSA	loss allowance
M	secondary main
m	feeder main
max	maximum
min	minimum
N	turns ratio
N	primary neutral
n	number of feeders emanating from a substation
n	neutral
NSW	nonswitchable (fixed) capacitors (Chap. 9)
off	off-peak
OH	overhead (Chap. 11)
op	operating
opt	optimum
on	on-peak
P	primary
PK	peak
PR	peak responsibility (Chap. 7)
pu	per unit
r	receiving end
ra	rated
reg	regulator
S	substation; series winding (Chap. 3)
s	sending end
sc	short circuit
SD	service drop
set	dial setting (line-drop compensator)
SL	secondary line (Chap. 6)
ST	subtransmission
SW	switchable (capacitors)
sys	power system

T	transformer
T_i	transformer i
X	low-voltage side (LV)
Y	wye connection
1ϕ, 3ϕ	single-phase, three-phase
0, 1, 2	zero-, positive-, negative-sequence quantity
Δ	delta connection
$\angle$	open-delta connection

ANSWERS TO SELECTED PROBLEMS

Chapter 2

2-1 (*a*) 1112.5 kW; (*b*) 10.08 kW; (*c*) 88,300.8 kWh
2-2 0.62
2-3 (*a*) 1.0; (*b*) 0.50; (*c*) 0.60; (*d*) 0.44
2-5 0.64
2-6 (*a*) 1.0; (*b*) 0.55; (*c*) 0.65; (*d*) 0.48
2-7 0.46
2-8 0.75 and 0.33
2-9 (*a*) 131,400 kWh; (*b*) $3285
2-11 (*a*) 0.40; (*b*) 0.50
2-13 (*a*) $370.40; (*b*) 0.11; (*c*) 0.34; (*d*) justifiable
2-15 (*a*) 0.41 and 0.29; (*b*) 30 kVA and 60 kVA; (*c*) $196 and $262; (*d*) 4.937 kvar; (*e*) not justifiable

Chapter 3

3-1 (*a*) $13.2\underline{/60^\circ}$ kV; (*b*) $7.62\underline{/60^\circ}$ kV
3-2 (*a*) 27,745.67 VA; (*b*) 13,872.83 VA; (*c*) 41,618.5 VA; (*d*) 24 kVA; (*e*) 24 kVA; (*f*) 48 kVA; (*g*) 1.078
3-4 (*a*) $100\underline{/-30^\circ}$ A, $100\underline{/30^\circ}$ A, and $100\underline{/90^\circ}$ A
(*b*) $\bar{I}_A = 11.54\underline{/0^\circ}$ A, $\bar{I}_B = 11.53\underline{/-120^\circ}$ A, and $\bar{I}_C = 11.53\underline{/120^\circ}$ A
3-5 (*a*) 0 var and 83,040 W
(*b*) 24 kVA, and 13.8 kVA and 27.6 kVA
(*c*) 1.075
3-9 $113.93 and $117.65
3-10 (*a*) 24.305 A, 0.7 pu A, and 69.444 A
(*b*) 7.2576 and 0.8891 Ω
(*c*) 218.2526 and 623.5792 A
(*d*) 0.042 pu V and 302.4 V

Chapter 4

4-4 (*a*) $3.5935 \times 10^{-5}\%$ VD/(kVA · mi)

4-5 $4.7524 \times 10^{-5}\%$ VD/(kVA · mi)
4-6 $5.5656 \times 10^{-5}\%$ VD/(kVA · mi)
4-7 $7.7686 \times 10^{-5}\%$ VD/(kVA · mi)
4-8 0.02% VD/(kVA · mi) and 10% VD
4-9 5%
4-10 6.667%
4-11 (*a*) 2.0; (*b*) 1.5; (*c*) 1.33
4-14 2.7 mi, 5.4 mi, and 14,670 kVA

Chapter 5

5-2 1.39%
5-3 (*a*) 2.98%; (*b*) it meets the VD criterion; (*c*) $\Delta X_d = 0.0436\ \Omega$/mi
5-4 (*a*) 2.31%; (*b*) yes; (*c*) the same
5-5 (*a*) The same; (*b*) 1.49%; (*c*) 4.66%

Chapter 6

6-1 (*a*) 1/0 AWG or 105.5 kcmil
(*b*) Not applicable
(*c*) 25 kVA
(*d*) $920.55/block/year
(*e*) $935.47/block/year
(*f*) $950.25/block/year
(*g*) $2.3163/customer/month
(*h*) $0.9318/customer/month
6-4 (*a*) 1/0 AWG; (*b*) not applicable; (*c*) $935.47/block/year
6-5 (*a*) $113.3 - j5.09$ A and $-78.6 + j1.5$ A
(*b*) $35.03\angle{-6.45^\circ}$ A
(*c*) $113.4\angle{-2.6^\circ}$ V and $117.9\angle{-1.1^\circ}$ V
(*d*) $231.3\angle{-1.8^\circ}$ V
6-7 (*a*) $112.1\angle{-35.4^\circ}$ A and $116.3\angle{175.9^\circ}$ A
(*b*) $24.6 + j56.6$ A
(*c*) $112.1\angle{1.5^\circ}$ V and $116.3\angle{-4.1^\circ}$ V
(*d*) $228.1\angle{-1.3^\circ}$ V

Chapter 7

7-1 (*a*) 8.94 pu V; (*b*) 24.24 kW; (*c*) 12.342 kvar; (*d*) 81.6 kVA and 0.891 lagging
7-4 (*a*) 0.0106 pu V; (*b*) 0.0135 pu V; (*c*) not applicable
7-5 (*a*) 0.72 pu A; (*b*) 0.3038 pu V

Chapter 8

8-1 (*a*) 0.4863 leading; (*b*) 0.95 leading
8-2 (*a*) 1200 kvar and 0.9753 lagging
(*b*) 84.7 A and 610 kVA
(*c*) 225 V or 9.38% V
(*d*) 0.81
8-4 (*a*) 228.5 kvar; (*b*) 0.86; (*c*) 4644.1 kvar; (*d*) 7493.6 kVA

8-5 (*a*) 8 kW
(*b*) 12 kW
(*c*) 20 kW
(*d*) 2081 kVA
(*e*) 6727 kvar
(*f*) \$940/yr
(*g*) \$11,237.40/yr
(*h*) \$1340.30/yr
(*i*) \$6390.70/yr
(*j*) \$7127/yr
(*k*) No

8-6 (*a*) 283 A; (*b*) $235\underline{/11.4^\circ}$ A; (*c*) 164.5 kvar/phase

8-8 $X_i = \dfrac{1}{1-\lambda} - \dfrac{(2i-1)C}{2(1-\lambda)}$

8-10 $X_1 = \dfrac{3}{3(1-\lambda)}$

Chapter 9

9-3 (*a*) 124.2 V
(*b*) 4 and 12 steps
(*c*) At peak load, 1.0667 pu V and 1.0083 pu V
At no load, 1.035 pu V and 1.0083 pu V

9-4 (*a*) 1.74 mi; (*b*) 2.55 mi; (*c*) it takes into account the future growth.

9-5 167 kVA

9-9 12.5 A

9-10 (*a*) Bus *a*; (*b*) 76.2 kVA; (*c*) bus *c*; (*d*) 0.0455 puV; (*e*) 0.0314 puV and 0.0404 puV

9-11 (*a*) 0.0304 pu V; (*b*) 0.9878 pu V; (*c*) 1.0011 pu V

9-13 No

9-14 (*a*) 3 V; (*b*) objectionable

Chapter 10

10-3 It lacks 30%

10-4 (*a*) 7-A tap; (*b*) 8.93

10-5 (*a*) $1.3422 + j3.5281$ and $0.7698 + j1.3147\ \Omega$
(*b*) $0.9696 + j2.0525\ \Omega$
(*c*) $1.4897 + j4.7766\ \Omega$
(*d*) $1.5797 + j5.3922\ \Omega$
(*e*) 479.66 A
(*f*) 415.4 A
(*g*) 427.14 A

10-7 (*a*) 2020 A, 1750 A, and 1450 A
(*b*) 870 A

10-8 (*a*) $0.2083 + j1.5485\ \Omega$
(*b*) $0.0409 + j0.1168$ and $0.1336 + j0.3753\ \Omega$
(*c*) $0.0718 + j0.2030\ \Omega$
(*d*) $0.2801 + j1.7515\ \Omega$
(*e*) $0.0003112 + j0.0019461\ \Omega$
(*f*) $0.00768 + j0.0133025\ \Omega$
(*g*) $0.009215 + j0.001365\ \Omega$
(*h*) $0.0172062 + j0.0166136\ \Omega$
(*i*) 10,068 A

Chapter 11

11-2 0.0045
11-3 (*a*) 0.0588 (*b*) 0.0625
11-5 0.8347
11-6 200 days
11-8 0.0000779264
11-12 (*a*) 0.8465; (*b*) 0.9925; (*c*) 0.9988; (*d*) 0.996
11-13 (*a*) 0.9703; (*b*) 0.49
11-14 0.912
11-15 (*a*) 0.14715; (*b*) 0.00635
11-16 (*a*) 0.15355; (*b*) 0.00640
11-17 (*a*) 1.206 faults/yr; (*b*) 15.44 h; (*c*) 0.18%; (*d*) 99.93%
11-19 (*a*) 0.6, 0.7, and 0.8 faults/yr
(*b*) 0.916, 0.8071, and 0.725 h
11-23 0.345
11-24 0.345

INDEX

	Gconen,Turan.
Title:	**Electric power distribution system engineering**
Publication:	McGraw Hill, New York, US, 1986 Out of print
ISBN:	0070237077
Est. Lib. Price ($A):	
Aust RRP or List:	
Binding:	Hardback (737p; 250mm; illustrations, diagrams)
Edition:	
Date entered:	21 DEC 1985
Description:	
Dewey:	621.3121
Topical Subject:	Electrical & electronic eng
Kit Subject:	
Readership Level:	
Lib. Of Congress:	TK3001.G58