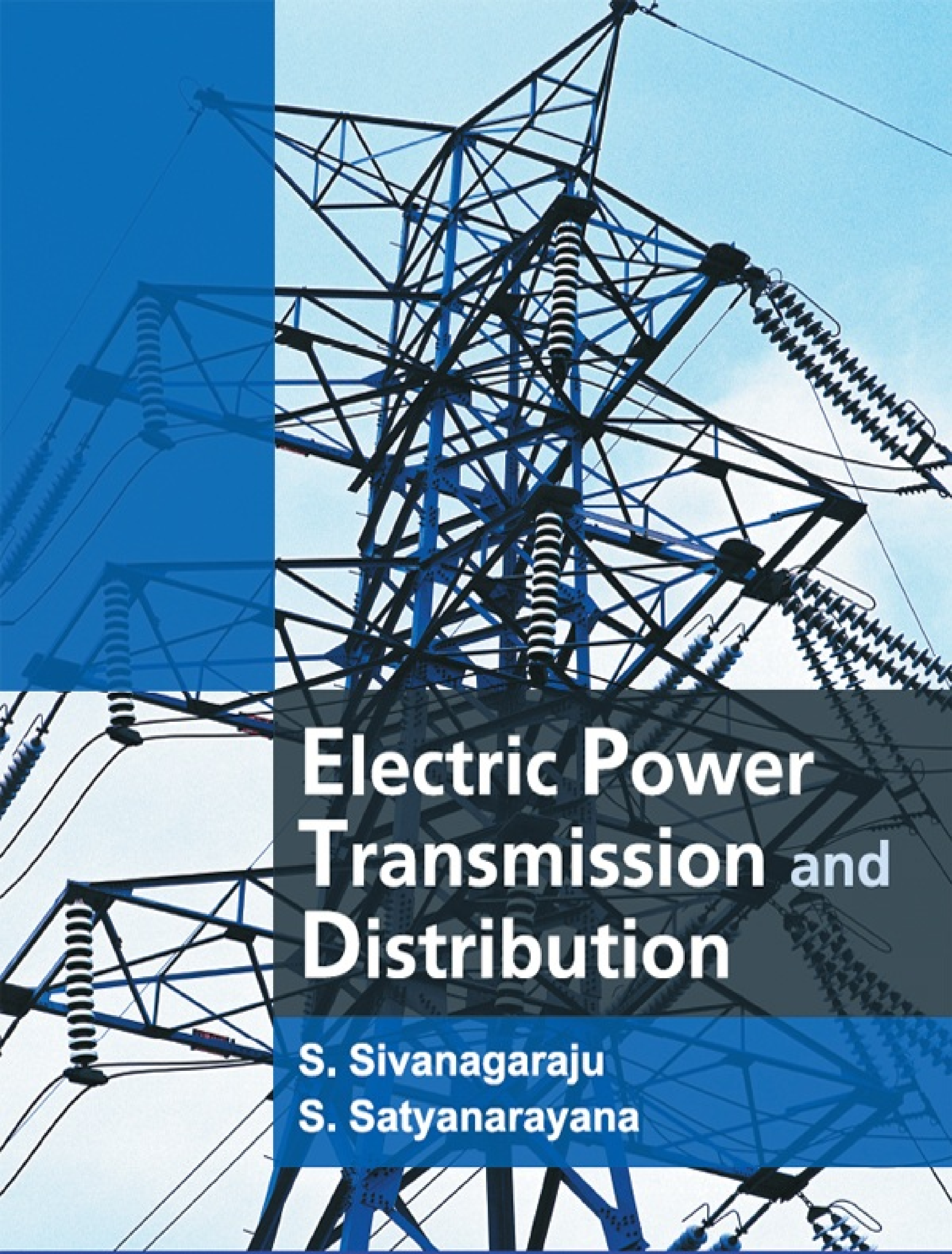

Electric Power Transmission and Distribution

S. Sivanagaraju
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ALWAYS LEARNING
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ELECTRIC POWER TRANSMISSION AND DISTRIBUTION

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TO MY PARENTS

Preface

Electric Power Transmission and Distribution has been designed for undergraduate courses in electrical and electronics engineering in Indian universities. Tailored to provide an elementary knowledge of power systems, a foundation in electric circuits and engineering is a pre-requisite for starting this course. The organization of the topics and the pace at which they unfold have been planned to enable students to proceed from the basic concepts to the difficult ones with ease. The text can ideally be taught at the sixth or seventh semester, and can also be used as a reference book by BE, BTech and AMIE students.

The contents of this book have been developed with emphasis on clarity, with equal stress on the basic concepts as well as advanced ideas, in detail over sixteen chapters.

Chapter 1 introduces the conventional sources of electrical energy, and explains about the load forecasting and its various aspects, the various levels of power transmission, and the need as well as the motivation for restructuring the power industry.

Chapter 2 explains the four transmission line parameters, namely, resistance and inductance in a series combination and a shunt combination of capacitance and conductance. Materials used for the manufacture of conductors and the various types of conductors are explained in detail. It is explained in detail about the current distortion effect and the effect of earth on transmission line capacitance to give a lucid understanding of the constituent elements of the transmission system.

Chapters 3 and 4 are devoted to the construction and performance analysis of overhead transmission lines. Classification of transmission lines, network constraints, long line equations, regulation concepts, surge impedance, Ferranti effect, and line loadability are dealt comprehensively. Power circle diagrams used to calculate the maximum power-transfer capacity, and synchronous phase-modifier capacity are also discussed.

Chapter 5 describes the power system transients due to surges. The travelling wave equation used to describe the phenomena of incident and reflected waves, the evaluation of surge impedance, its importance and the analysis of the travelling wave under different conditions, such as with open-circuited and short-circuited end lines, are highlighted in this chapter.

Chapter 6 is devoted to the corona phenomenon. The formation and effects of the corona, factors affecting corona loss, and corona interference with communication lines are examined.

Chapter 7 describes the mechanical design of overhead lines. Different line supports, sag calculations with reference to both equal and unequal supports, effect of ice and wind-loading on sag calculations, string charts, vibrations and dampers, and sag templates are all covered in this chapter. This chapter gives an idea of the different types of line supports and their design for overhead lines.

Chapter 8 is devoted to the description of overhead insulators. Insulating materials, types of insulators, stringing efficiency and methods for its improvement, arcing horns, and the various methods for testing insulators are discussed. This chapter also gives a selection of the ideal type of insulator for various given voltages.

Chapter 9 gives the general construction of underground cables, elaborates upon their various types, and explains their properties, advantages and disadvantages. The power factor, heating, testing and laying of cables are also discussed.

Chapter 10 is concerned with power factor improvement. The causes and demerits of lower power factor are followed by a discussion of the methods for their improvement. The most economical power factor is also expounded suitably.

Chapter 11 discusses the necessity of voltage control, the various sources of reactive power generation and absorption of reactive power, the methods of voltage control and rating of synchronous phase modifier. These concepts help to choose the rating of the capacitors for power factor improvement as well as for voltage control.

Chapter 12 discusses the economics of power-system designing. The choice of system frequency, system voltage and the advantage of high voltage transmission are elucidated here. The calculation of economical size of conductors using Kelvin's law, and the limitations of the applications of Kelvin's law are illustrated to give the student a holistic understanding of the methods of calculating the ideal size of conductors for different load levels.

Chapter 13 discusses the different types of substations, the substation equipment and types of bus-bar arrangements. Various types of neutral grounding systems are also discussed in this chapter. Based on the requirement and available budget, selection of bus-bar arrangements and suitable earthing methods, are described.

Chapter 14 gives a description of the different types of distribution systems, distribution-system losses, and their classification. We also look at voltage drop calculations of various types of DC and AC distributors in this chapter, to gain an insight into the designing of a suitable type of distributor for a given system.

Chapter 15 is designed to give an idea of the EHV and HVDC transmission systems including the need, advantages and disadvantages of EHV and HVDC transmission systems. The different types of HVDC systems, three-phase bridge converters, components of HVDC transmission systems, and harmonic filters form a part of this chapter.

Chapter 16, the concluding chapter of the book, introduces the flexible AC transmission systems technology and provides basic definitions of the various types of FACTS controllers, control of power systems and an overview of FACTS controller circuits. It also includes a brief discussion on voltage stability.

In-text Learning Aids

Each chapter opens with chapter objectives which are listed and an introduction. Worked-out solutions follow each section and serve to reinforce what the student has learnt. 'Test yourself' questions, designed to act as rapid review tests, are placed at strategic points within the chapters. Each chapter concludes with 'chapter at a glance', 'multiple choice questions' with answers, 'review questions' and practice problems, and are aimed to augment the student's understanding of the topics discussed.

An appendix containing datasheets, MATLAB programs as illustrative resolution of complex problems, and answers to odd questions added as a concluding section to the book, serve to further clarify and sum up the discussed concepts, enabling the student to solve problems with ease.

We appreciate the efforts of all those who provided inputs that helped us to develop and improve the text. We are committed to improving the text further in its future editions, and encourage you to contact us with your comments and suggestions.

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Acknowledgements

We would like to express our gratitude towards the people mentioned below, who have helped us in many ways to shape this book:

- To our college administration for providing us a conducive atmosphere to carry out this work.
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- For the services rendered by our students, especially D. Ravi Teja, for the successful outcome of this book.
- To all those who have, either directly or indirectly, helped us in translating our effort into reality.

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1

Transmission and Distribution: An Introduction

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Understand the various levels of power transmission
- Provide an overview of conventional sources of electrical energy
- Understand the purpose of load forecasting
- Analyse the various load models
- Understand the concept of deregulation

1.1 OVERVIEW

A power system consists of several generating stations and consumers interconnected by transmission and distribution networks. The objective of any power system is to generate electrical energy in sufficient quantities at the best suited locations, to transmit it to various load centres and then distribute it to various consumers, and to maintain the quality and reliability of transmission at an economic price. Here, quality implies that frequency and voltage are constantly maintained at a specified value. (In India, the frequency is maintained at 50 Hz.) Further, interruptions in supply of energy to consumers should be minimal.

The energy generated must be sufficient to meet the requirements of consumers. Because of the diverse nature of activities of the consumers (for example, domestic, industrial and agricultural) the load on the system varies from time to time. Hence, the generating station must be in a “state of readiness” to supply the load without any intimation from the consumer. This "variable load demand" has to be effectively tackled with a suitably designed power system. This necessitates a thorough understanding of the nature of the load to be supplied to the consumers. The behaviour and properties of the load can be easily studied from the load curve, load duration curve, etc. for estimating the load demand.

Electrical power is an essential infrastructure for the economic development of any developing country. In India, phenomenal expansion has taken place in the area of power systems in the last five decades. For example, power generation capacity has grown at an average rate of 9% and reached a level of more than 1,18,419 MW as in March 2005.

The total power generation capacity in India at the end of 1999 was 92,864.06 MW, comprising 22,438 MW hydro and 67,617.46 MW thermal, 968.12 MW wind and 840 MW nuclear. But total installed capacity at the end of March 2005 was 1,18,419 MW, comprising 30,936 MW hydro, 80,902 MW thermal, 3,811 MW wind and renewable energy and 2,770 MW nuclear systems. The total generation had increased by 21.58% from 1999 to 2005 in which, the hydel generation had increased by 27.47%, thermal by 16.4%, wind by 74.5% and nuclear by 69.67%.[1]

The development of transmission network has closely followed the growth of power generation. Starting with a meagre transmission network of 132 kV at the time of Independence, today the Indian power system has, besides high voltage direct current (HVDC) lines, 765 kV as the highest transmission voltage level, five operational regional grids and a national grid under formation, which will interconnect all regional grids.

In the subtransmission and distribution sector also, substantial expansion has taken place, and state electricity boards (SEB) have been more or less able to fulfil the objectives of the Government in terms of expanding the electricity network to rural areas. Out of 5,87,258 inhabited villages in the country as per the 1991 census, more than 86% villages have been electrified. Thirteen states have declared 100% electrification of selected villages. However, despite the impressive growth, the existing subtransmission and distribution network is not adequate to deliver power to the end-users with reliability. For the past many years, the expansion of the distribution system is not on par with the country’s power generating capacity and consumption needs. This could be attributed to low investments in the distribution sector in general.

1.2 VARIOUS LEVELS OF POWER TRANSMISSION

Electric power is normally generated at 11–21 kV in a power station. In order to transmit the power over long distances, the electric power so generated is then stepped up to 132 kV, 220 kV or 400 kV, as required. Power is carried through a transmission network of high-voltage lines that usually run into hundreds of kilometres and deliver the power into a common pool called the grid. The grid is connected to load centres (cities) through a subtransmission network of 132 kV or 33 kV lines that terminate at a 33 kV substation, where the voltage is stepped down to 11 kV for distribution of power to load points through a distribution network of lines at 11 kV and lower. This is illustrated in Fig. 1.1.

The power network, from the consumer’s perspective, is generally the distribution network of 11 kV lines or feeders, downstream to the 33 kV substations. Each 11 kV feeder, which emanates from the 33 kV substation, branches further into several subsidiary 11 kV feeders to carry power close to the load points (localities, industrial areas, villages, etc.). At these load points, a transformer further reduces the voltage from 11 kV to 400 V to provide the last-mile connection through 400 V feeders (also called low-tension feeders) to individual customers, either at 230 V (as single-phase supply) or at 400 V (as three-phase supply). A feeder could either be an overhead line or underground cable. In urban areas, owing to the density of consumers, the length of an 11 kV feeder is generally up to 3 km. On the other hand, in rural areas, the feeder length is much larger (up to 20 km). A 400 V feeder should normally be restricted to 0.5–1.0 km, as unduly long feeders lead to low voltage at the consumer end.

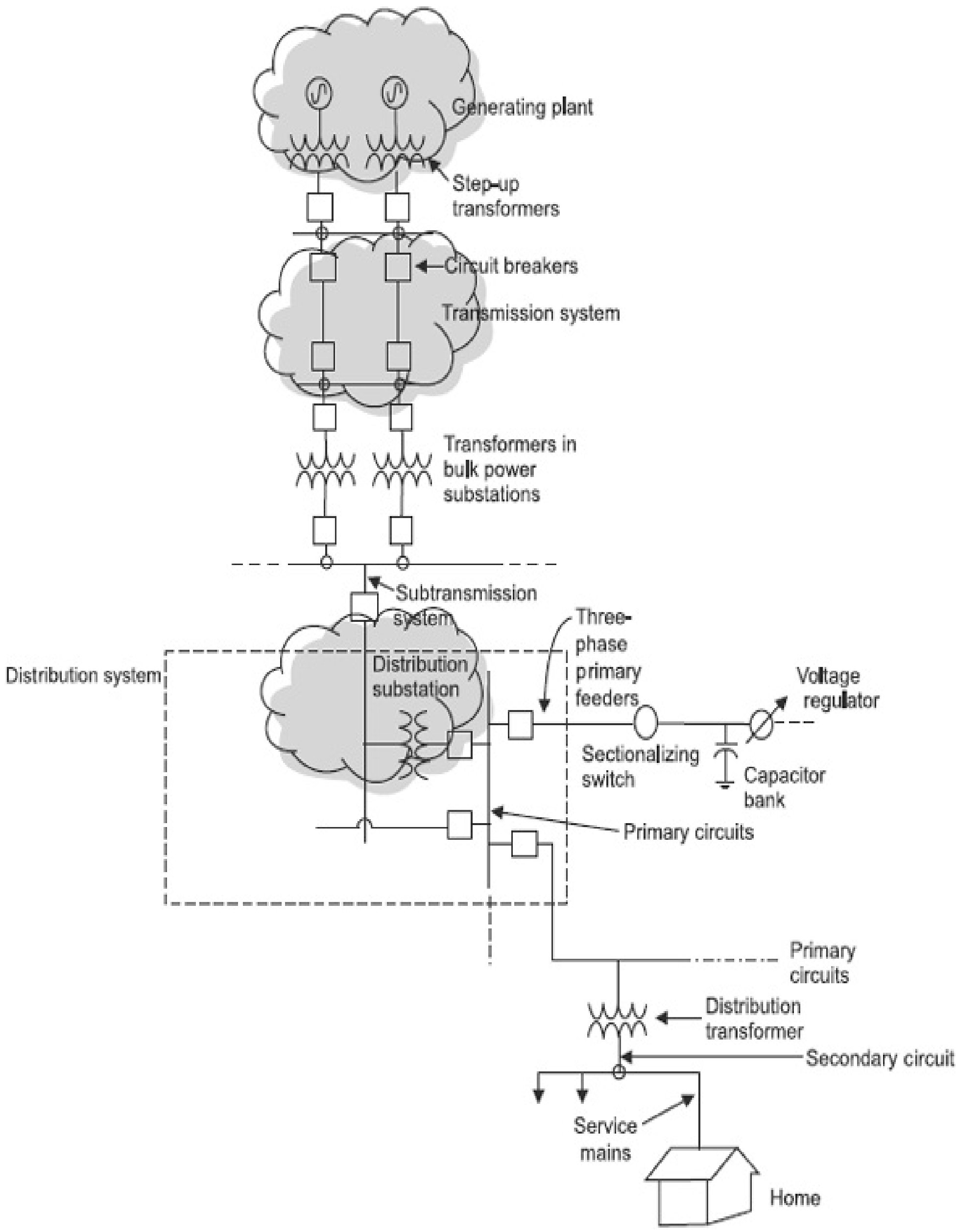

Fig. 1.1 Layout of a power system network

1.3 CONVENTIONAL SOURCES OF ELECTRICAL ENERGY

AC energy is generated by alternators, which require prime movers to drive them. The prime mover may be a turbine or a diesel engine. Depending upon the source from which the prime mover receives energy, power stations can be classified as follows:

- Hydro power stations
- Thermal power stations
- Nuclear power stations
- Diesel power stations

1.3.1 Hydro Power Stations

If huge volumes of water are stored at suitable locations and at calculated heights, the energy stored in the water can be used to generate electric power. This water, stored at higher altitudes, can be made to impinge on the blades of a hydraulic turbine through penstock. This leads to the sum of the potential and the kinetic energies stored in the water to be converted into mechanical energy. Since the turbine is mechanically coupled to the alternator, the mechanical energy imparted to the alternator is converted into electrical energy. Thus, for the functioning of a hydropower station, there must be an abundant quantity of water available throughout the year. It requires, therefore, a reservoir with a large catchment area in order to ensure the availability of water during the dry seasons. This restricts the possibilities for the location of hydroelectric plants to less populated hilly areas.

1.3.2 Thermal Power Stations

A thermal power station is one that uses a prime mover mechanically coupled to an alternator to derive energy from fuels burnt in boilers. The fuel burnt in the boilers may be solid fuel such as bituminous coal, brown coal or peat; or liquid fuel such as fuel oil, crude oil, petrol or paraffin oil; or gaseous fuel such as gasoline, kerosene, gas oil or diesel oil. Fuel oil is normally used for firing the boilers. The other gaseous fuels are used in internal combustion engines.

Power stations which require solid fuels are called thermal stations. Stations where liquid fuels are used are called diesel power stations. In a thermal station, steam turbines are used as the prime movers to drive the alternators. Water is converted to steam using the heat generated by burning the fuel in the boiler house. The steam thus produced is passed through the turbine to rotate its blades.

Thus, initially, heat energy is converted into mechanical energy. The mechanical energy imparted to the alternator is then converted into electrical energy. The overall efficiency of a thermal station generally ranges between 20% and 26%; however, it also depends upon plant capacity.

1.3.3 Nuclear Power Stations

Hydroelectric stations require backup from thermal stations because the operation of hydroelectric stations is completely dependent on the availability of water resources. Thermal stations require huge quantities of coal, the sources of which too are fast depleting. This has necessitated the identification of alternative sources. Nuclear energy is one among them. In India, at present, this source accounts for only 3% of the total power generated, with nuclear stations at Tarapur (Maharashtra), Kota (Rajasthan), Kalpakkam (Tamil Nadu), Narora (Uttar Pradesh) and Kakrapar (Gujarat).

As discussed in the previous section, steam required to drive the turbine is obtained by burning fuel in the boiler house. However, in nuclear power stations, there are no steam boilers. Instead, a nuclear reactor is used as the heat producing source. Heat can be generated by nuclear fission, i.e., by splitting the nucleus of a fissionable material such as Uranium (U^{235}), Thorium (Th^{232}) or Plutonium (Pu^{239}) (the superscripts 235, 232 and 239 refer to the atomic mass numbers of these elements). Uranium is most commonly used for the generation of nuclear energy. The energy thus produced heats some fluid like molten bismuth or sodium–potassium alloys. These, in turn, heat the feed water to form steam, which is used to drive the turbine.

These fissionable materials are highly expensive. Thus, if a nuclear power station is to be considered as an alternative, the availability of the fissionable materials and their high costs are to be given due consideration. However, the cost of running a nuclear power station is relatively minimal.

1.3.4 Diesel Power Stations

As mentioned in Section 1.3.2, liquid fuels are used in diesel power stations. A diesel engine acts as the prime mover. It derives its energy from a liquid fuel generally known as diesel oil, and converts it into mechanical energy. The alternator that is coupled to the diesel engine converts the mechanical energy into electrical energy.

A diesel engine is a “compression-ignition” type power source, since the compression of air in the engine cylinder provides the necessary heat for igniting the fuel charge, before it is introduced into the cylinder. Thus, in a diesel engine the chemical energy of combustion is released inside the cylinder to develop the mechanical power, which is subsequently converted into electrical power by the alternator.

The diesel engine, which forms a major component in a diesel-electric power station belongs to the general category of internal combustion engines. It may be a 2-stroke or a 4-stroke engine.

1.4 LOAD FORECASTING

Electrical energy cannot be stored. It has to be generated whenever there is a demand for it. It is, therefore, imperative for the electric power utilities to estimate the load on their systems in advance. This estimation of load is commonly known as “load forecasting” and is necessary for power system planning.[2]

Power system expansion planning starts with a forecast of anticipated future load requirements. Estimation of both demand and energy requirements is crucial to effective system planning. Demand predictions are used for determining the generation capacity, transmission and distribution system additions, etc. Load forecasts are used to establish procurement policies for construction of generating stations. Capital energy forecasts are required to determine the future fuel requirements. Thus, a good forecast reflecting the current and future trends is the key to all plans.

In general, the term “forecast” refers to projected load requirements determined using a systematic process of defining future load in sufficient quantitative detail. This permits the decision-making of important system expansion. Unfortunately, the consumer load is essentially uncontrollable, although, minor variations can be effected by frequency control and more drastically by load shedding. The variation of load does exhibit certain daily and yearly pattern repetitions and the analysis of these forms the basis of several load prediction techniques.

1.4.1 Purpose of Load Forecasting

Load forecasting is done to facilitate the following:

1. Proper planning of power system

2. Planning of transmission and distribution facilities
3. Power system operation
4. Financing
5. Manpower development
6. Grid formation
7. Electrical power sales

1. Proper Planning of Power System Proper planning of power system is required to determine the following:

1. The potential need for additional power-generating facilities.
2. The location of units.
3. Size of plants.
4. The year in which they are required.
5. Whether they should provide maximum primary capacity or energy or both.
6. Whether they should be constructed and owned by the Central Government, State Government, and electricity boards or by some autonomous corporation.

2. Planning of Transmission and Distribution Facilities The load forecasting is required for planning the transmission and distribution facilities so that the right amount of power is available at the right place and at the right time. Wastage of resources due to improper planning, like purchase of equipment not immediately required, can be avoided.

3. Power System Operation Load forecasts based on correct values of demand and diversity factors will prevent higher rating of conductor, as well as overloading of distribution transformers and feeders. Thus, it might help to correct voltage, power factor, and reduce the loss in the distribution system.

4. Financing Load forecasts help the State Electricity Boards to estimate future expenditure, earnings and returns and to schedule their financing programmes accordingly.

5. Manpower Development Accurate load forecasting reviewed annually will help the State Electricity Boards in manpower planning on a long-term basis. A realistic forecast will reduce unnecessary expenditure and put the Board's finances on a sound and profitable footing.

6. Grid Formation Interconnection between various state grids is now becoming more and more common and the aim is to have fully interconnected regional grids and ultimately even a super grid for the country as a whole. These expensive high-voltage interconnections must be based on reliable load data as otherwise, the generators connected to the grid may frequently fall out of step, causing power shut downs.

7. Electrical Power Sales In countries, where spinning reserves are more, proper planning and execution of an electrical power sales programme is aided by proper load forecasting.

1.4.2 Classification of Load Forecasting

Load forecasting can be classified as demand forecast and energy forecast.

Demand Forecast This is used to determine the reserve capacity of the generation, transmission and distribution systems. Future demand can be predicted on the basis of rate of growth of demand from past history and government policy. This will give the expected rate of growth of load.

Energy Forecast This is used to determine the type of facilities required, i.e., future demand requirements.

1.4.3 Forecasting Procedure

Depending on the time period of interest, a specific forecasting procedure may be adopted. This may be classified as:

- Short-term technique
- Medium-term (intermediate) technique
- Long-term technique

Short-term Forecast Short-term forecasting is done for day-to-day operations, to ensure enough generation capacity for a week and for maintaining the required spinning reserve. Hence, it is usually done 24 hours ahead when the weather forecast for the following day becomes available from the meteorological office. Mostly, this consists of estimating the seasonal power demand, taking into account any special event or festival anticipated.

The power supply authorities can construct seasonal load model of the system for this purpose or can refer some previous estimated table. The final estimate is arrived at after accounting for the transmission and distribution losses of the system. In addition to the prediction of hourly values, short-term load forecasting is also concerned with forecasting of daily peak system load, system load at certain hours of the day, hourly values of system energy, and daily and weekly system energy.

Applications of short-term load forecasting (STLF) are:

- To meet the short-term load demand with the most economic commitment of generation sources
- To access the power system security based on the information available to the dispatchers, to prepare the necessary corrective actions

Medium-term Forecast This forecasting is done for 5 or 6 years and plays a major role while planning the size of a power plant, and for the construction and installation of the equipment in power plants.

Applications of medium-term load forecasting (MTLF) are:

- For the estimation of fuel (coal, diesel, water, etc.)
- For the estimation of peak power and energy requirement for each month of the coming year

Long-term Forecast Long-term forecast may extend over a period of 20 years or even more, and in advance, in order to facilitate various plans like the preparation of maintenance schedules of the generating units, plan for future expansion of the generating capacity, enter into agreements for energy interchange with neighbouring utilities. Two approaches are available for this purpose:

Peak Load Approach In this approach, the simplest way is to extrapolate the trend curve, which is obtained by plotting the past values of annual peaks against years of operation. The following analytical functions can be used to determine the trend curve.

1. Straight line, $Y = a + bX$
2. Parabola, $Y = a + bX + cX^2$
3. S-curve, $Y = a + bX + cX^2 + dX^3$
4. Exponential, $Y = ce^{dX}$
5. Gompertz, $\log_e Y = a + ce^{dX}$

Let us derive the function for straight line by assuming that the load increases in an exponential fashion, i.e., to match exponential trend curve to the historical data. Let D be the power demand (MW), x be the year in which demand is considered, x_0 be the reference (or basic year). Under the assumption of an exponential growth of load demand, we have $D = e^{a + b(x-x_0)}$ where, a and b are the constants (to be determined).

Let us assume $x - x_0 = X$,

$$\therefore D = e^{a + bX}$$

Taking natural logarithms on both sides, we have

$$\ln D = a + bX, \text{ substitute } Y = \ln D$$

$\therefore Y = a + bX$, now Y and X are related to linear and hence varies in a straight line manner. The same procedure applies for the remaining.

In all these functions, *Y* represents the peak loads and *X* represents time in years. The most common method of finding co-efficient *a, b, c*, and *d* is the least squares curve-fitting technique.

The effect of weather conditions can be ignored on the basis that weather conditions as in the past are to be expected during the period under consideration. However, the effect of change in the economic condition should be accommodated by including an economic variable when extrapolating the trend curve. The economic variable may be, for example, the predicted national income, Gross Domestic Product (GDP).

Energy Approach In this approach, the annual energy sales is forecasted to different classes of customers like residential, commercial, industrial, etc., which is then converted to annual peak demand using the annual load factor. A detailed estimation of factors such as the rate of building a house, sale of electrical appliances, and growth in industrial and commercial activities are required in this method. Forecasting the annual load factor also contributes critically to the success of the method.

1.4.4 Load Characteristics

In general, the load can be divided into the following major categories:

Domestic Load This type of load mainly consists of domestic appliances such as lights, fans, heaters, refrigerators, air conditioners, mixers, ovens, heating ranges and small motors for pumping, and various other small household appliances. Peak real load refers to maximum real power load during a particular time. Peak reactive load refers to the maximum reactive power load during a particular time. The daily load curve (DLC) of a week day of domestic load in terms of peak real (P) and reactive (Q) loads as shown in Fig. 1.2.

Commercial Load Commercial load consists of lights, fans, air conditioners, heaters and other electrical appliances used in commercial establishments, such as shops, restaurants and market places. The DLC of a week day of this type of load in terms of peak load is shown in Fig. 1.3.

Industrial Load This type of load may be subdivided into small, medium and heavy depending on the required power range. For example, small-scale industries require loads up to 25 kW, medium-scale industries require between 25 kW and 100 kW, and heavy industries require loads of more than 500 kW. The chronological load curve for industrial load depends on the type of industry, one of the factors being most of the industries operate on shift basis. These loads are considered as base load that contain small weather-dependent variation. The DLC of a week day of this type of load in terms of peak load is shown in Fig. 1.4.

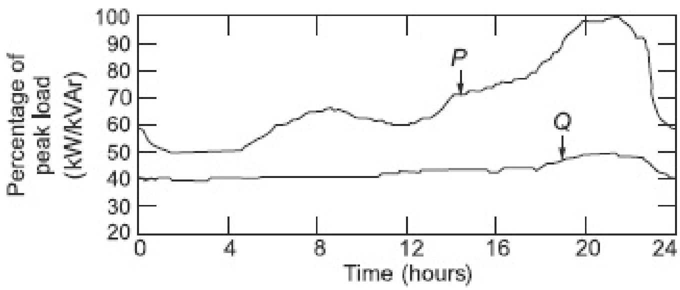

Fig. 1.2 DLC of a week day: Residential load curve in percentage of peak load

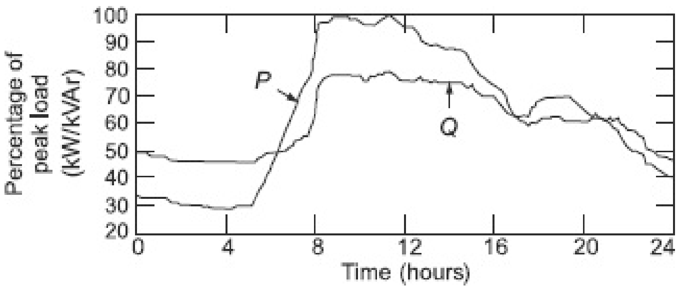

Fig. 1.3 DLC of a week day: Commercial load curve in percentage of peak load

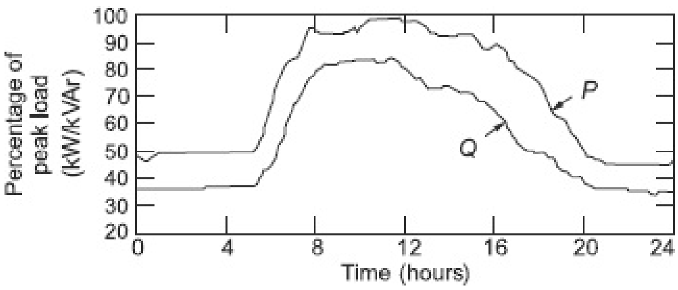

Fig. 1.4 DLC of a week day: Industrial load curve in percentage of peak load

1.5 LOAD MODELLING

Many electric appliances and devices have an electrical load that varies as the supply voltage is changed. Loads are grouped into three categories depending on how their demand varies as a function of voltage, viz., constant power (demand is constant regardless of voltage), as constant current (demand is proportional to voltage) or as constant impedance (power is proportional to square of voltage). The load at a particular point might be a mixture of some proportion of all these.

It is quite important in both planning and engineering to model the voltage sensitivities of load correctly. For example, incandescent lighting, resistive water heaters, shunt compensation and many other loads are constant impedance loads. Induction motors, controlled power supplies as well as tap-changing transformers in the power system are relatively constant power loads and thyristor application drives are relatively constant current loads.

In general, these models can be written as:

$$P = P^0 \left(\frac{V}{V^0} \right)^n$$

$$Q = Q^0 \left(\frac{V}{V^0} \right)^n$$

where, P^0, Q^0 and V^0 are the nominal real power, reactive power and voltages on per unit (p.u.) basis, respectively.

n = 0, for constant power

n = 1, for constant current

n = 2, for constant impedance

1.5.1 Characteristics of Load Models

The response of nearly all loads to changes in voltage can be represented by some combination of constant power, constant current, and constant impedance. Actually, the constant current model is unnecessary as it is nearly equivalent to 50% constant impedance load combined with 50% constant power load. It has been found convenient to retain the constant current model as it is easily comprehensible and is frequently used in the absence of a more complete data. Figures 1.5 and 1.6 show the relationship of load current and power with voltage for three simple load types.

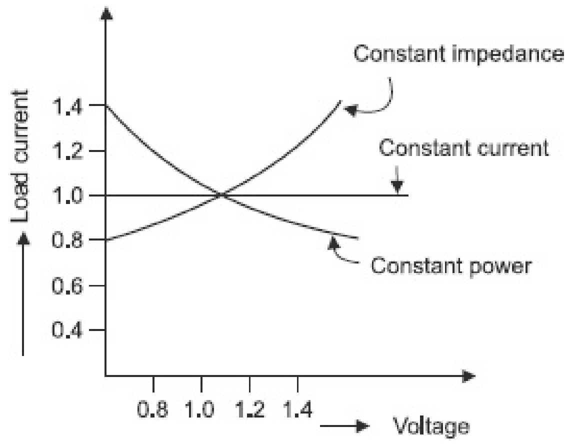

Fig. 1.5 Relationship between load current and node voltage for simple load types

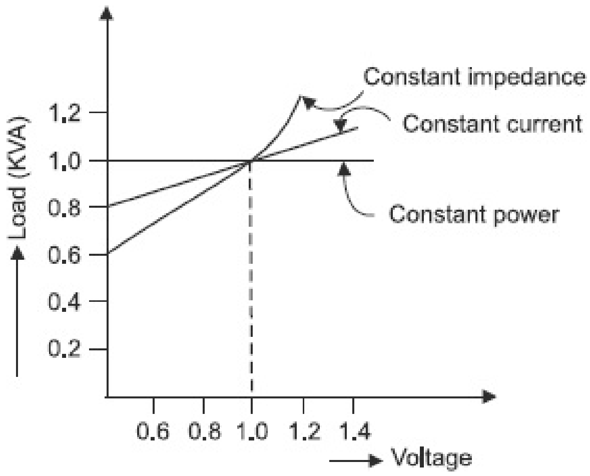

Fig. 1.6 Relationship between load (KVA) and node voltage for simple load types

The constant power type load representation is the most severe representation from the system stability point of view because of the effect in amplifying voltage oscillations. For example, a drop in voltage will cause an increase in load current resulting in a further voltage drop. Conversely, constant impedance load has a pre-decided damping effect on voltage oscillations.

All models are initially defined by a complex power and an assumed line-to-neutral voltage for star-connected load or an assumed line-to-line voltage for delta-connected load.

1.6 STAR-CONNECTED LOADS

The model of the star-connected load is shown in Fig. 1.7. The complex power and voltages are represented as:

Phase R: $|S_R|\angle\theta_R = (P_R + jQ_R)$ and $|V_{RN}|\angle\delta_R$

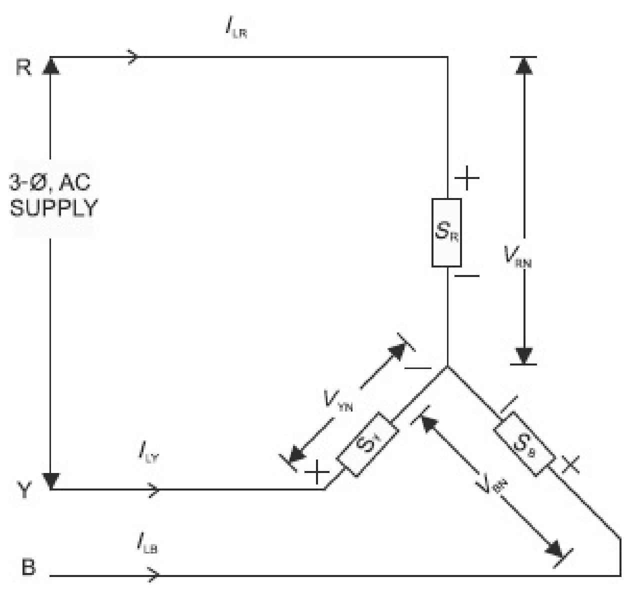

Fig. 1.7 Star-connected load

Phase Y : $|S_Y|\angle\theta_Y = (P_Y + jQ_Y)$ and $|V_{YN}|\angle\delta_Y$

Phase B : $|S_B|\angle\theta_B = (P_B + jQ_B)$ and $|V_{BN}|\angle\delta_B$

where, $|S_R|\angle\theta_R$, $|S_Y|\angle\theta_Y$ and $|S_B|\angle\theta_B$ are the apparent powers corresponding to the phase angle of phases R, Y and B, respectively. $(P_R + j\,Q_R)$, $(P_Y + j\,Q_Y)$ and $(P_B + j\,Q_B)$ are the real and reactive powers of the respective phases. $|V_{RN}|\angle\delta_R$, $|V_{YN}|\angle\delta_Y$ and $|V_{BN}|\angle\delta_B$ are the magnitudes of phase voltages and the corresponding phase angles of the respective phases.

1.6.1 Constant Power Model

The load power, $S = VI^*$

Current, $I = \left(\frac{S}{V}\right)^*$

The line currents of this load model are given by

$$
\begin{aligned}
I_{LR} &= \left(\frac{S_R}{V_{RN}}\right)^* = \frac{|S_{i}|}{|V_{RN}|}\angle(\delta_R - \theta_R) = |I_{LR}|\angle\alpha_R \\
I_{LY} &= \left(\frac{S_Y}{V_{YN}}\right)^* = \frac{|S_Y|}{|V_{YN}|}\angle(\delta_Y - \theta_Y) = |I_{LY}|\angle\alpha_Y \\
I_{LB} &= \left(\frac{S_B}{V_{BN}}\right)^* = \frac{|S_B|}{|V_{BN}|}\angle(\delta_B - \theta_B) = |I_{LB}|\angle\alpha_B
\end{aligned}
\tag{1.1}
$$

where, $|I_{LR}|\angle\alpha_R$, $|I_{LY}|\angle\alpha_Y$ and $|I_{LB}|\angle\alpha_B$ are the magnitudes of line currents and the corresponding phase angles of phases R, Y and B, respectively.

1.6.2 Constant Current Model

In this model, the magnitudes of currents are computed using Eq. (1.1) and are then held constant while the angle of voltage (δ) changes, resulting in a changed angle on the current so that the power factor of the load remains constant.

$$\begin{aligned}&\text{Line current in phase R, } I_{LR} = |I_{LR}|\angle(\delta_R - \theta_R)\\&\text{Line current in phase Y, } I_{LY} = |I_{LY}|\angle(\delta_Y - \theta_Y)\\&\text{Line current in phase B, } I_{LB} = |I_{LB}|\angle(\delta_B - \theta_B)\end{aligned} \quad (1.2)$$

1.6.3 Constant Impedance Model

This load model first determines constant load impedance from the specified complex power and assumed phase voltages.

$$\begin{aligned}&\text{Impedance in phase R,} \quad Z_R = \frac{|V_{RN}|^2}{S_R^*} = \frac{|V_{RN}|^2}{|S_R|}\angle\theta_R = Z_R\angle\theta_R\\&\text{Impedance in phase Y,} \quad Z_Y = \frac{|V_{YN}|^2}{S_Y^*} = \frac{|V_{YN}|^2}{|S_Y|}\angle\theta_Y = Z_Y\angle\theta_Y\\&\text{and, Impedance in phase B,} \quad Z_B = \frac{|V_{BN}|^2}{S_B^*} = \frac{|V_{BN}|^2}{|S_B|}\angle\theta_B = Z_B\angle\theta_B\end{aligned} \quad (1.3)$$

The load currents as a function of the constant load impedances are given by

$$I_{LR} = \frac{V_{RN}}{Z_R} = \frac{|V_{RN}|}{|Z_R|}\angle(\delta_R - \theta_R) = |I_{LR}|\angle\alpha_R$$

$$I_{LY} = \frac{V_{YN}}{Z_Y} = \frac{|V_{YN}|}{|Z_Y|}\angle(\delta_Y - \theta_Y) = |I_{LY}|\angle\alpha_Y \tag{1.4}$$

$$\text{and, } I_{LB} = \frac{V_{BN}}{Z_B} = \frac{|V_{BN}|}{|Z_B|}\angle(\delta_B - \theta_B) = |I_{LB}|\angle\alpha_B$$

Similarly, the load models are determined for the delta-connected loads by considering line-to-line voltage instead of phase-to-neutral voltage.

1.7 DEREGULATION

Deregulation of electric sector is nothing but its privatization. While the two words are different literally, "deregulation" often starts with the sale of state-owned utilities to the private sector. This is widely adopted to refer to the "introduction of competition". Deregulation often involves "unbundling", which refers to disaggregating an electric utility service into its basic components and offering each component separately for sale with separate rates for each. As shown in Fig. 1.8, generation, transmission and distribution could be unbundled and offered as discrete services.

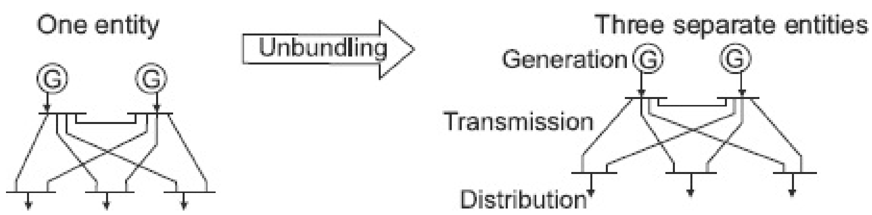

Fig. 1.8 Unbundling of utilities

The success of privatization of the airline and telecommunications industries has motivated the deregulation and restructuring of the electricity industry. In 1989, the UK became one of the pioneers in privatizing its vertically integrated electricity industry.

In many countries, a central independent body, usually called the independent system operator (ISO), is set up to cater to the demand, and the maintenance of system reliability and security. Sometimes the system operator is also responsible for matching the bids of generators with the demand bids to facilitate exchange.

The restructuring of the utility into separate generation, transmission and distribution companies has introduced competition in the generation and transmission of electricity. Several independent power producers and qualifying facilities produce electricity and the energy is traded on a real time basis to meet consumer requirements. The successful implementation of deregulation of electric utilities, in several developed countries, has motivated similar restructuring efforts in other developing countries. The effect of deregulation has had a great impact on the Indian power scenario, which has recently initiated deregulation and restructuring. There exist potential opportunities for the successful implementation of the principles of deregulation as there is an abundance of dispersed sources of generation, such as renewable energy sources, for supplying energy at remote locations or load centres.

There are several motives for deregulation and re-structuring of electric utility, important among them being the following factors, namely, (i) break up of entrenched bureaucracy, (ii) reduction of public sector debt, (iii) encouragement of private sector investment, (iv) lower electricity prices, (v) introduction of price competition, (vi) improvement of efficiency, and (vii) utilization of assets.

1.7.1 Need for Restructuring

Restructuring promises an alternative to vertically integrated monopoly and works on the basic principle that transmission services should accommodate consumer choice and supply competition. In the past, vertically integrated utilities have been monopolizing generation, transmission and distribution services. A restructuring of this monopoly was required to provide reliable power at a lower cost.

1.7.2 Motivation for Restructuring the Power Industry

A significant feature of restructuring the power industry is to allow for competition among generators and to create market conditions in the industry, which are seen as necessary for the reduction of the costs of energy production and distribution, the elimination of certain inefficiencies, the shedding of labour, and the increase of customer choice. Many factors such as technology advances, changes in political and ideological attitudes, regulatory failures, high tariffs, managerial inadequacy, global financial drives and the rise of environmentalism contribute to the worldwide trend towards restructuring.

There are two potential benefits resulting from deregulation. Firstly, the advance of technology makes low-cost power plants owned by independent power producers very efficient. These independent power producers would not have emerged without the reform. Secondly, unbundling the services may result in fairer tariffs being assigned to individual services. Restructuring was done with the view that private organizations could do a better job of running the power industry, and that higher operating inefficiencies and reduction in labour could be reached by privatization. Private utilities also refuse to subsidise rates and have a greater interest in eliminating power thefts and managerial or workplace inefficiencies. A competitive power industry will provide rewards to risk takers and encourage the use of new technologies and business approaches. The regulated monopoly scheme was unable to provide incentives for innovation, since the utility had little motivation to use new ideas and technologies to lower costs under a regulated rate of return framework.

1.8 DISTRIBUTION AUTOMATION

The customer demand for electrical energy with adequate reliability and quality, and the growing cost of investment required for distribution of energy have driven the utilities (Electricity Boards) towards automated distribution systems. The advent of microelectronics and advances in communication technology has promised a cost-effective automated control and operation of distribution system. Today, distribution automation means different things to different utilities. This situation has arisen because the automation of various functions of the distribution system such as distribution substation control and load management, are at different stages in the various utilities. The IEEE has defined Distribution Automation System (DAS) as a system that enables an electric utility to remotely monitor, coordinate and operate distribution components, in a real time mode.

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2

Transmission-Line Parameters

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Understand the various line parameters
- Provide an overview of materials used for transmission lines
- Calculate inductance and capacitance for various geometrical configurations of both single- and three-phase systems
- Understand the effect of ground on capacitance calculations

2.1 INTRODUCTION

An overhead transmission line has groups of conductors running parallel to each other, carried on line supports. An electric transmission line conductor has four parameters: which are series combination of resistance, inductance, shunt combination of capacitance and conductance. The parameters are symbolized as *R, L, C,* and *G*, respectively. These are uniformly distributed along the whole length of the line and representation of these parameters at any point on the line is not possible. These are usually expressed as resistance, inductance, capacitance and conductance per kilometre.

The first three parameters depend upon material used and physical dimensions of the conductor. Shunt conductance, which is mostly caused by leakage over the insulators, is always neglected in a power transmission line. The leakage loss in a cable is uniformly distributed over the length of the cable, whereas it is different in case of overhead lines. In overhead lines, it is limited only to the insulators and is very small under normal operating conditions, hence, it is neglected.

2.2 CONDUCTOR MATERIALS

The material used as conductor for power transmission and distribution lines must possess the following characteristics:

- Low specific resistance leading to less resistance and high conductivity.
- High tensile strength to withstand mechanical stresses.
- Low specific gravity in order to give low weight per unit volume.
- Low cost in order to use over long distances.

Copper and aluminium conductors are used for overhead transmission of electrical power. In case of high voltage transmission, aluminium with a steel core is generally used. Sometimes cadmium, copper, phosphor, bronze, copper weld and galvanized steel are also used as transmission conductors. The choice of the conductor used for transmission purely depends upon the cost, as well as required electrical and mechanical properties.

Copper: It is an ideal material for overhead lines having high electrical conductivity and greater tensile strength. Copper has high current density and its advantages are:

- Smaller cross-sectional area of conductor is sufficient.
- The area offered by the conductor to wind loads is low.

Aluminium: Due to non-availability and high cost of copper, it is replaced by aluminium conductors, which are low in cost and light in weight. For the same resistance, an aluminium conductor has a large diameter than the copper conductor. Larger diameter leads to lower voltage gradient on the conductor surface and less tendency to ionize the air around the conductor.

Steel-cored aluminium (ACSR): For transmission of high voltages, multi-stranded (composite) conductors, such as, stranded copper conductors, hollow copper conductors and aluminium conductor steel reinforced (ACSR) are used. An ACSR conductor has a central core of galvanized steel wire covered with successive layers of aluminium strands, which are electrically in parallel. Because of galvanized steel wire, the tensile strength of conductor is increased, so that the sag is reduced. It reduces the tower height and is useful for increasing the span length (refer Chapter 7 for detail). Another advantage of the ACSR is that the diameter of the conductor may be increased, so that the effect of the corona is less. The cross-sectional view of such a conductor is shown in Fig. 2.1. When compared with the equivalent copper conductor, the breaking strength of ACSR is high, whereas its weight is less.

For large span, the number of transmission line supports required will be reduced. Therefore, the cost of supports, foundation, insulators, erection, and at the same time, the cost of maintenance is reduced. 37/3 Panthar ACSR conductor is shown in Fig. 2.2.

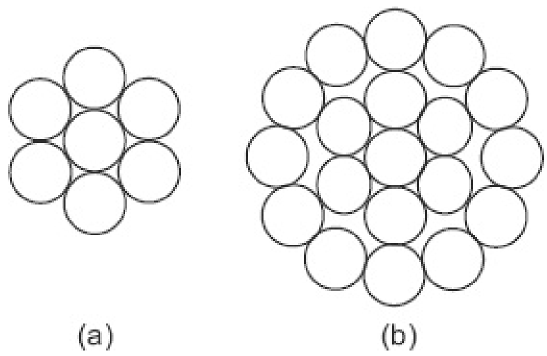

Fig. 2.1 Stranded conductors

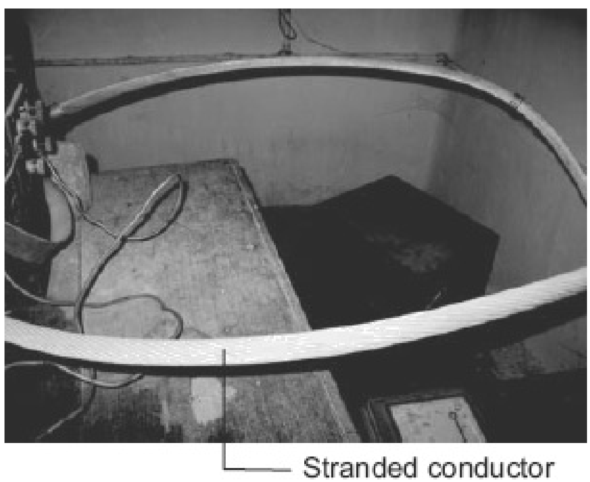

Fig. 2.2 37/3 panthar ACSR conductor

The various sizes of ACSR conductors commonly used for overhead lines are given in Table 2.1.

Table 2.1 Type of conductors used for different voltage levels

Sl. no	Transmission voltage (kV)	Size (SWG)	Diameter (mm)	Type of conductor
1.	400	54/3.53	27.57	ACSR
2.	220	37/4.27	29.89	ACSR
3.	132	37/3.35	23.45	ACSR
		37/3	21.00	ACSR
		37/2.8	19.60	ACSR
		37/2.59	18.13	ACSR
4.	66	7/4.09	12.27	ACSR
		7/3.65	10.95	ACSR
		7/3.175	9.525	ACSR stranded copper
5.	33	7/4.09	12.27	ACSR
		7/3.65	10.95	ACSR
6.	11	7/2.59	7.77	ACSR
		7/3.65	10.95	ACSR
		6	4.877	Solid copper

2.3 TYPES OF CONDUCTORS

In general, the types of conductors used for the transmission of electrical power are:

1. Solid conductors
2. Stranded conductors
3. Hollow conductors

(i) Solid conductors Solid copper conductors and copper-clad steel conductors of small cross-sectional area are used as solid conductors. Conductors of large size cannot be used because they are difficult to handle and transport. Further, when used for long spans, they tend to break at the points of support due to constant swing caused by the wind blows. Due to less flexibility solid aluminium conductors are not used.

(ii) Stranded conductors Generally, the stranded conductors (composite) are used in transmission lines for increasing the flexibility and overall diameter. These conductors normally have a central wire around which, there are successive layers of 6, 12, 18, etc., strands as shown in Fig. 2.1. The expression for the number of strands in a stranded conductor is given by $N = 3n\,(n + 1) + 1$, where n is the number of layers. The diameter of stranded conductor is $(D) = (2n + 1)d$, where d is the diameter of each strand.

In the process of construction, adjacent layers are spiralled in opposite direction, so that the layers are bound together. This type of construction is called *concentric lay*. Another type of construction used for conductors of large cross-section is *rope lay*.

(iii) Hollow conductors Hollow conductors have large diameter when compared to solid conductors. Corona loss is reduced or eliminated due to larger diameter. Skin effect (refer to Section 2.6.1) is also less when compared to stranded conductor. These conductors have low inductance and low voltage gradient when compared to solid conductors. A larger diameter of a conductor exposes a larger surface to wind pressure and large amount of ice accumulates over the surface of the conductor. This increases the weight of the conductor. This type of conductor is generally used for bay extension in 400 kV substations.

Test Yourself

1. Now-a-days, why are aluminium conductors used for power distribution?

2.4 BUNDLED CONDUCTORS

A bundled conductor is formed by two or more than two sub-conductors in each phase as shown in Fig. 2.3. These are used for transmitting huge units of power over long distances. In a bundled conductor, the sub-conductors are separated from each other by a constant distance, whereas in the case of composite conductors, the sub-conductors are placed in close proximity so that they touch each other. The overall diameter of bundled conductor increases due to filler material or air space in between the sub-conductors. Figure 2.4 shows a 750 kV bundle conductor consisting of four sub-conductors.

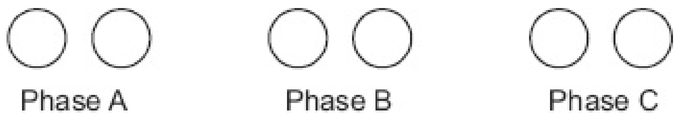

Fig. 2.3 Bundled conductor arrangements

The advantages of bundled conductors are:

- Reduced corona loss due to larger cross-sectional area
- Reduced interferences with communication circuits
- Reduced inductance per phase due to increased geometric mean radius (GMR), which in turn reduces the net series reactance
- Improved voltage regulation

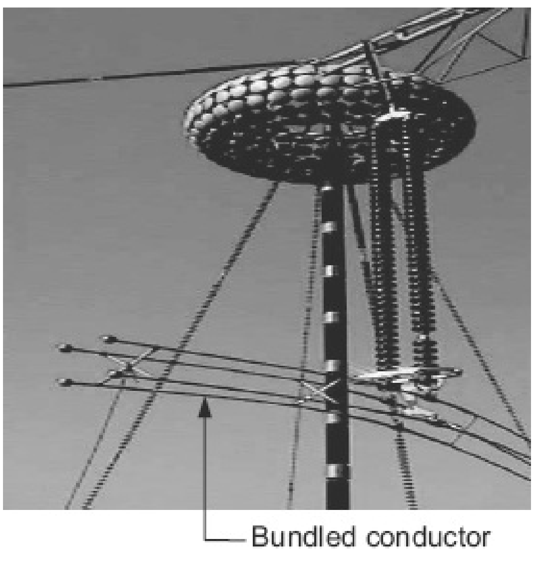

Fig. 2.4 Bundled conductor for 750 kV transmission line

- Improved stability margin
- Increased power transmission capacity with reduced power loss leading to increased efficiency of transmission.

Disadvantages of bundled conductors are:

- Increased ice and wind loading because of large cross-sectional area
- Increased clearance requirements at structures
- Increased charging kVA.

2.5 RESISTANCE

Resistance is defined as voltage per unit current at constant temperature. The resistance of transmission line conductors causes power loss in the transmission line. The term resistance means effective resistance. The effective resistance of a conductor is

$$R = \frac{\text{Power loss in conductor (in watts)}}{|I|^2}\ \Omega \tag{2.1}$$

where, I is the current (r.m.s.) in the conductor in amperes.

Resistance of a conductor is also determined by the formula

$$R = \frac{\rho L}{A}\Omega$$

where, ρ =	resistivity of the material used for conductor (Ω-m)
L =	length of the conductor (m)
A =	cross-sectional area of the conductor (m^2).

The value of the resistance *(R)* depends not only on the conductor material but also on its temperature. If R_1 and $R_2$are the values of resistance corresponding to the temperatures t_1 and t_2, then,

$$R_2 = R_1\left[1+\alpha t\right] \tag{2.2}$$

where, α is the temperature coefficient of resistance of the material (°C) and $t = (t_2 - t_1)$ is the change of temperature.

The resistance of the stranded conductor is slightly more than the solid conductor of equivalent cross-sectional area. Since a stranded conductor is spiralled, each strand is longer than the finished conductor.

Test Yourself

1. Why is the resistance in transmission lines low when compared to distribution systems?

2.6 CURRENT DISTORTION EFFECT

When a conductor is carrying a steady direct current (DC), it will distribute the current uniformly over the entire cross-section of the conductor. In practice, the alternating current (AC) does not distribute uniformly over the cross-section of the conductor, but is distorted due to:

1. Skin effect
2. Proximity effect
3. Spirality effect

The sum of these three effects is small at normal frequencies, but is a main factor in determining the conductor resistance under high frequency conditions.

2.6.1 Skin Effect

When DC flows in the conductor, the current is uniformly distributed over the whole cross-section of the conductor. But the flow of AC in the conductor is non-uniform, due to which the outer filaments of the conductor carry more current than the filaments closer to the centre. This results in a higher resistance to AC than to DC and is known as skin effect.

Due to skin effect, the effective area of cross-section of the conductor, through which the current flows, is reduced. Consequently, the resistance of the conductor is slightly increased when carrying an AC.

Cause of Skin Effect Consider a solid conductor consisting of a large number of filaments each carrying a small part of the current. The inductance of each filament will vary according to its position. Thus, the filaments near the centre are surrounded by a greater magnetic flux and hence have larger inductance than those near to the surface as shown in Fig. 2.5.

The high reactance of inner filaments causes the AC to flow near the surface of conductor. This crowding of current near the conductor surface is termed as the skin effect.

The skin effect depends upon the following factors:

1. Nature of material
2. Diameter of wire – increases with the diameter of wire
3. Frequency – increases with the increase in frequency
4. Shape of wire – less for stranded conductor than the solid conductor

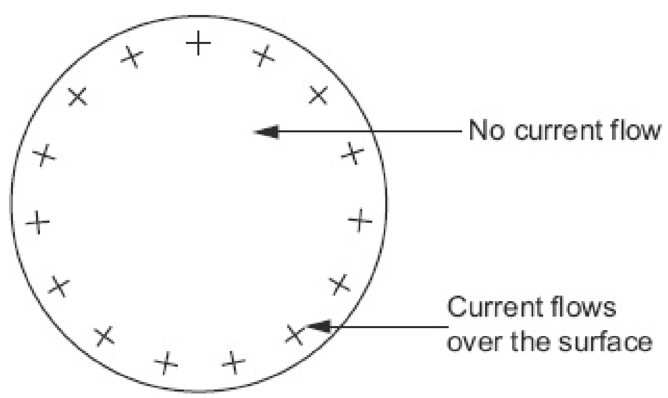

Fig. 2.5 Illustration of skin effect

5. Distance between conductors
6. Resistivity of material

7. Permeability of material

It may be noted that skin effect is negligible when the supply frequency is low and the diameter of the conductor is small.

2.6.2 Proximity Effect

Non-uniformity of current in the cross-section of the conductor is also observed in the case of proximity effect, and is similar to that of skin effect. The alternating magnetic flux in a conductor caused by the current flowing in a neighbouring conductor gives rise to circulating currents, which cause an apparent increase in the resistance of a conductor. This phenomenon is called the proximity effect.

Cause of Proximity Effect Let us assume that two conductors A and B are placed closer to each other. When current passes through conductor A, a flux is produced around it. A fraction or even the whole of this generated flux links with the conductor B, which is near conductor A.

The shaded portion of the conductor, shown in Fig. 2.6, is the part of the conductor which has more flux than the other part. In other words, the additional lines of flux in a two-wire system link elements otherwise situated far apart than the elements nearer each other. Therefore, the inductance of the elements farther apart is more as compared to the elements nearer to each other, and the current density is less in the elements farther apart, than in the elements situated near each other. The effective resistance is, therefore, increased due to non-uniform distribution of current.

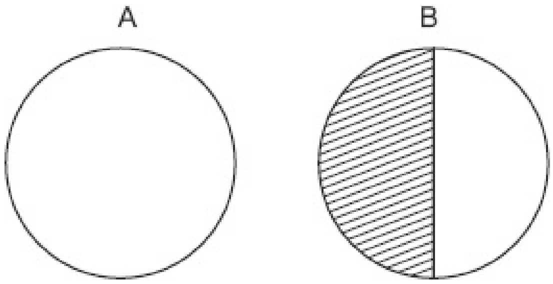

Fig. 2.6 Illustration of proximity effect

The proximity effect can be influenced by the following factors:

- Size of the conductor
- Frequency of supply
- Resistivity of material
- Permeability of material
- Distance between the conductors

The proximity effect is pronounced in case of cables where the distance between the conductors is small, whereas for overload lines with usual spacing the proximity effect is negligibly small.

2.6.3 Spirality Effect

This effect tends to increase both the resistance and internal reactance of a stranded conductor. The magnitude of the effect depends on the size and method of construction of the conductor (i.e., solid, stranded or hollow conductor). But this effect at normal supply frequencies is very less and can be ignored in non-magnetic conductors.

Test Yourself

1. Why are skin and proximity effects neglected in DC transmission?

2.7 INDUCTANCE

Inductance is a measure of the amount of magnetic flux produced for a given electric current. It is also defined as the flux linkages per unit ampere and is denoted by *L*.

$$\therefore \text{Inductance, } L = \frac{\text{Flux linkages}}{\text{Current}} = \frac{\psi}{I} \text{ (H)} \qquad (2.3)$$

where,

ψ = Flux linkage (Wb-T)

I = Current (A)

For a transmission line, the inductance depends upon the material used, and the dimensions of the conductor. It also depends on the configuration of lines, the spacing between them and the flux linkages of a particular conductor (due to the current in the conductor) and flux linkages between the conductors.

2.7.1 Inductance of A Conductor Due to Internal Flux

By Ampere's law, for a closed magnetic path the magneto motive force (m.m.f.) is equal to the current enclosed. Also by Maxwell's law, m.m.f. is equal to the line integral of magnetic field intensity i.e., m.m.f. = $\int H.dl$. Hence, line integral of field intensity is equal to the current enclosed.

Let us consider a conductor of radius *r* and the magnetic field H_x inside the conductor at a distance *x* from the center of the conductor as shown in Fig. 2.7. Since the field is symmetrical, and if I_x is the current enclosed by a circle of radius *x*, *Hx* is constant at any point on the circle.

According to the Ampere's law

$$\oint H_x \cdot dl = I_x \tag{2.4}$$

$$2\pi x H_x = I_x \tag{2.5}$$

If I_x is current enclosed by a circle of area πx^2, then I_x = current density × area enclosed

$$\therefore I_x = \frac{I}{\pi r^2} \pi x^2$$

where, I is the current enclosed within the conductor of radius r i.e., within the area πr^2.

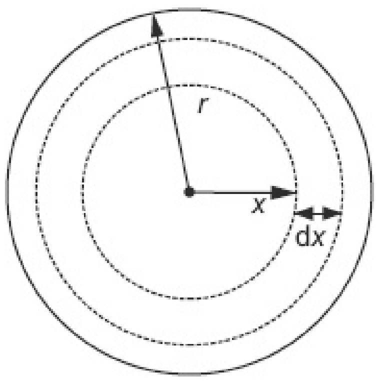

Fig. 2.7 Flux linkages due to internal flux

$$\text{Hence, } I_x = I\frac{x^2}{r^2} \tag{2.6}$$

Substitute the value of I_x from Eq. (2.6) in Eq. (2.5)

$$\therefore 2\pi x H_x = I \frac{x^2}{r^2}$$

$$\text{or } H_x = I \frac{x}{2\pi r^2} \text{ AT/m} \tag{2.7}$$

The flux density B_x over a circle of cross-section πx^2 is

$$B_x = \mu H_x = \mu I \frac{x}{2\pi r^2} \text{ Wb/m}^2 \tag{2.8}$$

Flux = flux density × area

Let us take an incremental area of the conductor whose width is dx and length is L m

Then, area = dx × L m^2.

∴ Flux = $B_x \cdot$ dx$\cdot$ L Wb

Flux enclosed by an element of thickness dx per axial length 1 m is

$$d\phi = \mu I \frac{x}{2\pi r^2} \cdot dx \cdot 1 \text{ Wb} \tag{2.9}$$

The flux dϕ, from Eq. (2.9) links with current $I_x = \frac{I}{\pi r^2} \pi x^2$

only.

Therefore, flux linkages per meter axial length of conductor is

$$d\psi = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu I x^3}{2\pi r^4} dx \text{ Wb-T/m} \qquad (2.10)$$

Therefore, the total internal flux link is

$$\therefore \psi_{int} = \int_0^r \frac{\mu I x^3}{2\pi r^4} dx = \frac{\mu I}{8\pi} \text{ Wb-T/m}$$

where,

μ	= permeability = $\mu_r \mu_o$
μ_r	= relative permeability
μ_o	= absolute permeability

The relative permittivity in dielectric medium i.e., air or free space, $\mu_r = 1$.

$$\therefore \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$
$$\text{Then, } \psi_{int} = 0.5 I \times 10^{-7} \text{ Wb-T/m}$$

Inductance of a conductor due to internal flux

linkages, $L_{int} = \frac{\psi_{int}}{I}$

$$\therefore L_{int} = 0.5 \times 10^{-7} \text{ H/m} \qquad (2.11)$$

2.7.2 Inductance of A Conductor Due to External Flux

To determine external flux linkages between two points 1 and 2, on circles of radius $R_1$and R_2, which are greater than the radius of conductor r (Fig. 2.8), let us consider a small elemental length dx between the points 1 and 2.

The field intensity at any point on the circle which has radius x greater than r is,

$$H_x = \frac{I}{2\pi x} \text{ AT/m}$$

And flux density is given by

$$B_x = \frac{\mu I}{2\pi x} \text{ Wb/m}^2$$

Now flux dϕ through a cylindrical shell of thickness dx is

$$d\phi = B_x dx = \frac{\mu I}{2\pi x} \cdot dx \cdot 1 \text{ Wb}$$

This flux dϕ links with the current I.

Therefore, the flux linkages dψ due to the flux dϕ is,

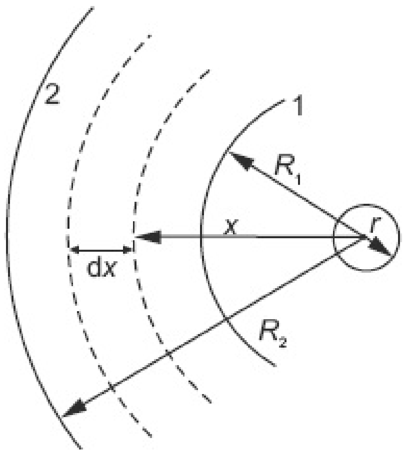

Fig. 2.8 Flux linkages between two external points

$$d\psi = d\phi$$

$$d\psi = \frac{\mu I}{2\pi x} dx \text{ Wb-T/m} \tag{2.12}$$

Thus, the flux linkages between the points 1 and 2 is,

$$\psi_{ext} = \psi_{12} = \int_{R_1}^{R_2} \frac{\mu I}{2\pi x} dx = \frac{\mu I}{2\pi} \ln \frac{R_2}{R_1} \text{ Wb-T/m}$$

The relative permeability in dielectric medium, $\mu_r = 1$

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\psi_{12} = \frac{4\pi \times 10^{-7} I}{2\pi} \ln \frac{R_2}{R_1} = 2I \ln \frac{R_2}{R_1} \times 10^{-7} \text{ Wb-T/m} \tag{2.13}$$

The inductance of conductor contributed by the flux including points 1 and 2 is

$$L_{12} = 2\times10^{-7}\ln\frac{R_2}{R_1}\ \text{H/m} \tag{2.14}$$

Let the external point be at a distance *D* from the centre of the conductor. The inductance of the conductor due to an external flux is obtained by substituting $R_1 = r$ and $R_2 = (D - r)$ in Eq. (2.14).

$$\therefore L_{12} = 2\times10^{-7}\ln\frac{D-r}{r} \simeq 2\times10^{-7}\ln\frac{D}{r}\ \text{H/m} \quad (\because D >> r) \tag{2.15}$$
$$= 2\times10^{-7}\ln\frac{D}{r}\ \text{H/m}$$

2.8 INDUCTANCE OF A SINGLE-PHASE TWO-WIRE SYSTEM

Consider a single-phase overhead line consisting of two circular conductors parallel to each other, each with a radius *r*, the spacing between the conductors (centre to centre) is *D* meters as shown in Fig. 2.9. One conductor is the return circuit to the other.

The internal flux linkage of conductor 1 due to current in the same conductor is

$$\psi_{\text{int}} = 0.5I\times10^{-7}\ \text{Wb-T/m} \qquad (\because \mu_r = 1)$$

The external flux linkage of conductor 1 due to current in the same conductor is

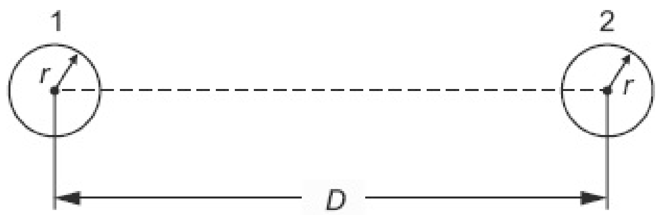

Fig. 2.9 Single-phase two-wire system

$$\therefore \psi_{ext} = 2\times10^{-7} I \ln\frac{D-r}{r} \text{ Wb-T/m}$$

$$\psi_{ext} \simeq 2\times10^{-7} I \ln\frac{D}{r} \text{ Wb-T/m} \qquad (\because D >> r)$$

Therefore, the flux linkages of the conductor 1 due to its own current is

$$\begin{aligned} \psi_{11} &= \psi_{int} + \psi_{ext} \\ &= I\left(\frac{1}{2}\times10^{-7} + 2\times10^{-7}\ln\frac{D}{r}\right) \\ &= 2\times10^{-7} I\left(\frac{1}{4} + \ln\frac{D}{r}\right) \\ &= 2\times10^{-7} I\left(\ln\frac{D}{re^{-1/4}}\right) \\ \therefore \psi_{11} &= 2\times10^{-7} I\left(\ln\frac{D}{r'}\right) \text{ Wb-T/m} \end{aligned}$$

where, $r' = re^{-1/4} = 0.7788r$ is the effective radius of the conductor (or GMR of the conductor).

Similarly, the flux linkage of conductor 1 due to current in the second conductor is

$$\therefore \psi_{12} = 2\times10^{-7} I\left(\ln\frac{r'}{D}\right) \text{ Wb-T/m}$$

Flux linkages of conductor 1 due to current in both the conductors,

$$\psi_1 = 2\times10^{-7} I\left(\ln\frac{D}{r'}\right) - 2\times10^{-7} I\left(\ln\frac{r'}{D}\right) \text{ Wb-T/m}$$

∵ current in one conductor is opposite to the other

$$\begin{aligned}\therefore \psi_1 &= 2\times10^{-7} I\left(\ln\frac{D}{r'}\right) + 2\times10^{-7} I\left(\ln\frac{D}{r'}\right) \text{ Wb-T/m} \\ &= 4\times10^{-7} I\left(\ln\frac{D}{r'}\right)\end{aligned} \tag{2.16}$$

The inductance of single-phase circuit is

$$L = 4\times10^{-7} \ln\frac{D}{r'} \text{ H/m} \tag{2.17}$$

Inductance per conductor $= \frac{L}{2}$

$$= 2\times10^{-7} \ln\left(\frac{D}{r'}\right) \text{H/m} \tag{2.18}$$

$$\text{Hence, } L = 0.2 \ln\left(\frac{D}{r'}\right) \text{mH/km} \tag{2.19}$$

The value of inductance of one conductor due to current in the same conductor is obtained from Eq. (2.19). Loop inductance means the inductance of a single-phase circuit. It is equal to double the inductance per conductor in a single-phase line.

EXAMPLE 2.1

Calculate the inductance of a single-phase two-wire system, if the distance between conductors is 2 m and radius of each conductor is 1.2 cm.

Solution:

Radius of each conductor, r = 1.2 cm

Effective radius, r' = 0.7788 × 1.2 = 0.93456 cm

Distance between conductors, D = 2 m = 200 cm

Loop inductance, = 4 x $10^{-7} \ln\left(\frac{D}{r'}\right)$

Inductance of a single phase circuit (both go and return),

$$L = 4\times10^{-7}\ln\left(\frac{200}{0.93456}\right)$$
$$= 21.46\times10^{-7} \text{ H/m}$$

Inductance of a single phase circuit (both go and return), L = 2.146 mH/km.

EXAMPLE 2.2

Calculate the loop inductance of a single-phase line with two parallel conductors spaced 3.5 m apart. The diameter of each conductor is 1.5 cm.

Solution:

Spacing between the conductors, D = 3.5 m

Diameter of the conductor, d = 1.5 cm

Radius of the conductor, r = 0.75 cm

Effective radius of conductor, r' =0.7788 × 0.75 cm = 0.7788 × 0.75 × 10^{-2} m

Inductance of a conductor, $L = 2\times10^{-7}\ln\frac{D}{r'}$ H/m

Inductance of both the conductors (loop inductance) $= 4\times10^{-7}\ln\frac{D}{r'}$ H/m

$$= 4\times10^{-7}\ln\frac{3.5}{0.7788\times0.75\times10^{-2}} \text{ H/m}$$
$$= 2.558 \text{ mH/km}$$

EXAMPLE 2.3

Calculate the inductance of a single-phase circuit comprising of two parallel conductors of 6 mm in diameter spaced 1 m apart. If the material of the conductor is (i) copper and (ii) steel with a relative permeability of 50.

Solution:

Spacing between the conductors, D = 1 m

Diameter of the conductor, d = 6 mm

Radius of the conductor, r = 3 mm = 3 × 10^{-3} m

Inductance of a conductor, $L = 2\times10^{-7}\ln\frac{D}{r'}$ H/m

Relative permeability of copper and steel are 1 and 50, respectively.

1. Inductance of any conductor due to the effect of internal and external fluxes, $L = 10^{-7}\left(\frac{\mu_r}{2}+2\times\ln\frac{D}{r}\right)$ H/m

   Inductance of copper conductor, $L_c = 10^{-7}\left(\frac{1}{2}+2\times\ln\frac{1}{3\times10^{-3}}\right)$ H/m

   Inductance of both the conductors (loop inductance)

   $$= 10^{-7}\left(1+4\times\ln\frac{1}{3\times10^{-3}}\right)\text{ H/m}$$
   $$= 2.424 \text{ mH/km}$$

2. Inductance of any conductor due to the effect of internal and external fluxes, $L = 10^{-7}\left(\frac{\mu_r}{2}+2\times\ln\frac{D}{r}\right)$ H/m

   Inductance of steel conductor, $L_s = 10^{-7}\left(\frac{50}{2}+2\times\ln\frac{1}{3\times10^{-3}}\right)$ H/m

   Inductance of both the conductors (loop inductance) $= 10^{-7}\left(50+4\times\ln\frac{1}{3\times10^{-3}}\right)$ H/m ($\because$ Loop inductance = Twice the conductor inductance)

   $= 7.324$ mH/km

EXAMPLE 2.4

Determine the inductive reactance of single-phase 25 Hz, 16 km long transmission line, which consists of a pair of conductors of diameter 1.2 cm and spaced (i) 1.2 m (ii) 60 cm apart.

Solution:

1. Spacing between the conductors, D = 1.2 m = 120 cm

   Diameter of the conductor, d = 1.2 cm

   Radius of the conductor, r = 0.6 cm

   Effective radius of conductor, r' = 0.7788 × r = 0.7788 × 0.6 cm

$$\text{Inductance of a conductor, } L = 2\times10^{-7}\ln\frac{D}{r'}\ \text{H/m}$$
$$= 2\times10^{-7}\ln\frac{120}{0.7788\times0.6}\ \text{H/m}$$
$$= 1.11\ \text{mH/km}$$

   Inductance of a single-phase system (both go and return) = 2 × 1.11 = 2.22 mH/km

   Total inductance for 16 km length = 2.22 × 16 = 35.52 mH

   Inductive reactance of the single-phase system

$$= \omega L = 2\pi fL = 2\pi\times25\times0.03552$$
$$= 5.579\ \Omega$$

2. Spacing between the conductors, D = 60 cm

   Diameter of the conductor, d =1.2 cm

   Radius of the conductor, r = 0.6 cm

   Effective radius of conductor, r' = 0.7788 × 0.6 cm

$$\text{Inductance of a conductor, } L = 2\times10^{-7}\ln\frac{D}{r'}\ \text{H/m}$$
$$= 2\times10^{-7}\ln\frac{60}{0.7788\times0.6}\ \text{H/m}$$
$$= 0.971\ \text{mH/km}$$

   Inductance of a single-phase system (both go and return) = 2 × 0.971 = 1.942 mH/km Total inductance for 16 km length = 1.942 × 16 = 31.07 mH

   Inductive reactance of the single-phase system

$$= \omega L = 2\pi fL = 2\pi\times25\times0.031073$$
$$= 4.88\ \Omega$$

2.9 FLUX LINKAGES WITH ONE SUB-CONDUCTOR OF A COMPOSITE CONDUCTOR

A composite conductor is a conductor which has n sub-conductors (strands) arranged electrically parallel to each other.

Consider a composite conductor, which has n sub-conductors (strands) as shown in Fig. 2.10. The currents carried by the strands 1, 2, 3, ... n are I_1, I_2, ... I_n, respectively. Then the total current $I_1 + I_2 + ... + I_n = 0$.

Let us consider a point P, which is far away from the composite conductor. Now take the distances from each strand to point P as D_{1p}, D_{2p}, ..., D_{np}.

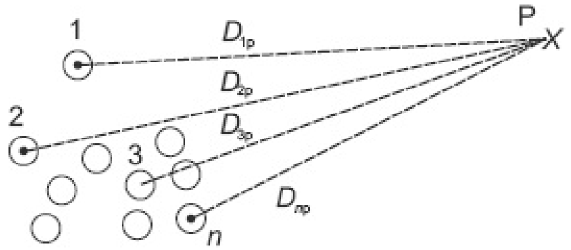

Fig 2.10 A composite conductor with n sub-conductors

If ψ_{1P_1} is the flux linkage between the strand 1 and point P due to current in strand 1 i.e., I_1 including internal flux linkages is

$$\psi_{1p1} = 2\times10^{-7} I_1 \ln \frac{D_{1p}}{r'} \text{ Wb-T/m} \tag{2.20}$$

And the flux linkage between the strand 1 and point P due to current I_2 in strand 2 is

$$\therefore \psi_{1p2} = 2\times10^{-7} I_2 \ln \frac{D_{2p}}{D_{12}} \text{ Wb-T/m} \tag{2.21}$$

Similarly, flux linkage between the strand 1 and point P due to current I_n in strand n is

$$\psi_{1pn} = 2\times10^{-7} I_n \ln\frac{D_{np}}{D_{1n}} \text{ Wb-T/m} \tag{2.22}$$

∴ Total flux linkage between the strand 1 and point P due to currents in all the n strands is

$$\begin{aligned}\psi_{1p} &= \psi_{1p1} + \psi_{1p2} + \cdots + \psi_{1pn} \\ &= 2\times10^{-7}\left[I_1 \ln\frac{D_{1p}}{r'} + I_2 \ln\frac{D_{2p}}{D_{12}} + I_3 \ln\frac{D_{3p}}{D_{13}} + \cdots + I_n \ln\frac{D_{np}}{D_{1n}}\right]\end{aligned} \tag{2.23}$$

Eq. (2.23) can be re-written as,

$$\psi_1 = 2\times10^{-7}\left[\begin{aligned}&I_1 \ln\frac{1}{r'} + I_2 \ln\frac{1}{D_{12}} + \cdots + I_n \ln\frac{1}{D_{1n}} + I_1 \ln D_{1p} \\ &+ I_2 \ln D_{2p} + \cdots + I_n \ln D_{np}\end{aligned}\right] \text{ Wb-T/m}$$

$$= 2\times10^{-7}\left[\begin{aligned}&I_1 \ln\frac{1}{r'} + I_2 \ln\frac{1}{D_{12}} + \cdots + I_n \ln\frac{1}{D_{1n}} + I_1 \ln D_{1p} + I_2 \ln D_{2p} \\ &+ \cdots + I_{n-1} \ln D_{(n-1)p} - (I_1 + I_2 + \cdots + I_{n-1}) \ln D_{np}\end{aligned}\right] \text{ Wb-T/m}$$

$$\psi_1 = 2\times10^{-7}\left[\begin{aligned}&I_1 \ln\frac{1}{r'} + I_2 \ln\frac{1}{D_{12}} + \cdots + I_n \ln\frac{1}{D_{1n}} + I_1 \ln\frac{D_{1p}}{D_{np}} + I_2 \ln\frac{D_{2p}}{D_{np}} \\ &+ \cdots + I_{n-1} \ln\frac{D_{(n-1)p}}{D_{np}}\end{aligned}\right] \text{ Wb-T/m} \tag{2.24}$$

Because the point P is very far away from the composite conductor, the values of

$$\frac{D_{1p}}{D_{np}} \cong \frac{D_{2p}}{D_{np}} \cong \ldots \cong \frac{D_{(n-1)p}}{D_{np}} \cong 1$$

$$\therefore \ln\frac{D_{1p}}{D_{np}} \cong \ln\frac{D_{2p}}{D_{np}} \cong \ldots \cong \ln\frac{D_{(n-1)p}}{D_{np}} \cong 0$$

Therefore, the flux linkages ψ_{1p} from Eq. (2.24) is

$$\psi_{1p} = 2\times10^{-7}\left(I_1\ln\frac{1}{r'} + I_2\ln\frac{1}{D_{12}} + \cdots + I_n\ln\frac{1}{D_{1n}}\right)\text{ Wb-T/m} \tag{2.25}$$

Moreover, the effective radius r' can be represented by D_{11} then Eq. (2.25) can be written as

$$\psi_1 = 2\times10^{-7}\left(I_1\ln\frac{1}{D_{11}} + I_2\ln\frac{1}{D_{12}} + \cdots + I_n\ln\frac{1}{D_{1n}}\right)\text{ Wb-T/m} \tag{2.26}$$

In a more compact form, $\psi_1 = 2\times10^{-7}\sum_{x=1}^{n} I_x \ln\frac{1}{D_{1x}}$

2.10 INDUCTANCE OF A SINGLE-PHASE SYSTEM (WITH COMPOSITE CONDUCTORS)

Consider a single-phase system consisting of two composite conductors A and B, each having *m* and *n* number of strands, respectively as shown in Fig. 2.11.

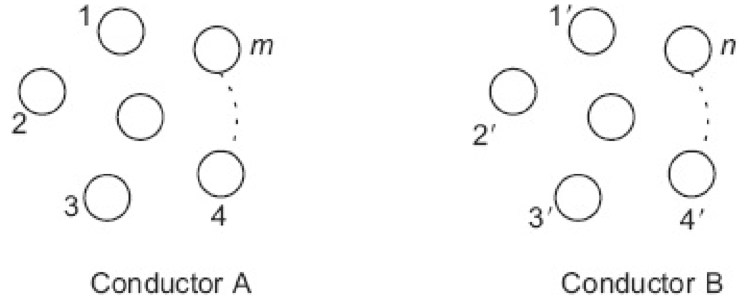

Fig. 2.11 Single-phase transmission line with composite conductors

The current is assumed to be equally divided amongst all the strands of 'A' conductor and is equal to $\frac{I}{m}$ (acts as 'go' conductor for the single-phase line) and current in all the strands of 'B' conductor is $\frac{-I}{n}$ (acts as 'return' conductor for the single-phase line).

Using Eq. (2.26), the flux linkages of strand 1 due to currents in all conductors is given by

$$\psi_1 = 2\times10^{-7}\frac{I}{m}\left(\ln\frac{1}{D_{11}}+\ln\frac{1}{D_{12}}+\cdots+\ln\frac{1}{D_{1m}}\right) - 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{D_{11'}}+\ln\frac{1}{D_{12'}}+\cdots+\ln\frac{1}{D_{1n}}\right) \quad (2.27)$$

$$\psi_1 = 2\times10^{-7}\frac{I}{m}\left(\ln\frac{1}{D_{11}D_{12}\ldots D_{1m}}\right) - 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{D_{11'}D_{12'}\ldots D_{1n}}\right)$$

$$\psi_1 = 2\times10^{-7}I\left(\ln\frac{1}{\sqrt[m]{D_{11}D_{12}\ldots D_{1m}}} - \ln\frac{1}{\sqrt[n]{D_{11'}D_{12'}\ldots D_{1n}}}\right)$$
$$= 2\times10^{-7}I\left[\ln\frac{\sqrt[n]{D_{11'}D_{12'}.D_{13'}\ldots D_{1n}}}{\sqrt[m]{D_{11}.D_{12}.D_{13}\ldots D_{1m}}}\right] \quad (2.28)$$

Inductance of a strand 1 of conductor A is

$$L_1 = \frac{\psi_1}{I/m} = 2m\times10^{-7}\ln\frac{\sqrt[n]{D_{11'}D_{12'}.D_{13'}\ldots D_{1n}}}{\sqrt[m]{D_{11}\cdot D_{12}\cdot D_{13}\ldots D_{1m}}}\ \text{H/m} \quad (2.29)$$

Inductance of strand 2 of conductor A is

$$L_2 = \frac{\psi_2}{I/m} = 2m \times 10^{-7} \left[\ln \frac{\sqrt[n]{D_{21'} D_{22'} D_{23'} \ldots D_{2n}}}{\sqrt[m]{D_{21} \cdot D_{22} \cdot D_{23} \ldots D_{2m}}} \right] \text{H/m} \tag{2.30}$$

Similarly, the inductance of strand *m* of conductor A is

$$\therefore L_m = \frac{\psi_m}{I/m} = 2m \times 10^{-7} \left[\ln \frac{\sqrt[n]{D_{m1'} . D_{m2'} . D_{m3'} \ldots D_{mn}}}{\sqrt[m]{D_{m1} . D_{m2} . D_{m3} \ldots D_{mm}}} \right] \text{H/m} \tag{2.31}$$

The average inductance of each strand in conductor A is

$$L_{av} = \frac{L_1 + L_2 + \cdots + L_m}{m} \tag{2.32}$$

Conductor A is composed of *m* strands connected electrically in parallel. If all the strands had the same inductance, the inductance of the conductor A would be 1/*m* times the inductance of one strand. Here, all the strands have different inductances, but the inductance of all of them in parallel (shown in Fig. 2.12) is 1/*m* times the average inductance. Thus the inductance of conductor A is

$$L_A = \frac{L_{av}}{m} = \frac{L_1 + L_2 + \cdots + L_m}{m^2} \tag{2.33}$$

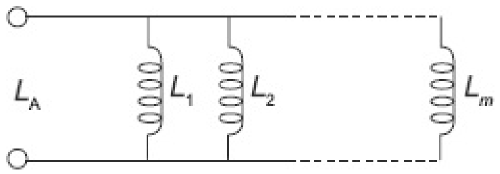

Fig. 2.12 *m*-Average inductances are in parallel

Substituting the logarithmic expression for inductance of each strand in Eq. (2.29) and combining terms, we get

$$L_A = 2\times10^{-7}\ln\left(\frac{\sqrt[mn]{(D_{11'}D_{12'}\ldots D_{1n})(D_{21'}D_{22'}\ldots D_{2n})\ldots(D_{n1'}D_{n2'}\ldots D_{nn})}}{\sqrt[m^2]{(D_{11}D_{12}\ldots D_{1m})(D_{21}D_{22}\ldots D_{2m})\ldots(D_{m1}D_{m2}\ldots D_{mn})}}\right) \quad (2.34)$$

In Eq. (2.34), the numerator is known as *Geometric Mean Distance* (GMD or mutual GMD) and is denoted as D_m and is equal to

$$D_m = \sqrt[mn]{(D_{11'}D_{12'}\ldots D_{1n})(D_{21'}D_{22'}\ldots D_{2n})\ldots(D_{n1'}D_{n2'}\ldots D_{nn})} \quad (2.35)$$

It is the *mn*th root of the product of the *mn* distances between *m* strands of conductor A and *n* strands of conductor B.

The denominator is known as GMR or self GMD, denoted as D_s and is equal to

$$D_s = \sqrt[m^2]{(D_{11}D_{12}\ldots D_{1m})(D_{21}D_{22}\ldots D_{2m})\ldots(D_{m1}D_{m2}\ldots D_{mn})}$$

It is the m^2 root of the product of m^2 distances within the conductor A. Now Eq. (2.34) can be written as

$$L_A = 2\times10^{-7} \ln\frac{D_m}{D_{sA}} \text{ H/m} \quad (2.36)$$

Similarly, the inductance of composite conductor B is

$$L_B = 2\times10^{-7} \ln\frac{D_m}{D_{sB}} \quad (2.37)$$

Therefore, the total inductance of a single-phase system of composite conductors is

$$L = L_A + L_B \quad (2.38)$$

If conductors A and B are identical i.e., $D_{sA} = D_{sB} = D_s$, then the inductance is

$$\therefore L = 4\times10^{-7} \ln\frac{D_m}{D_s} \text{ H/m}$$
$$\text{or } L = 0.4\ln\frac{D_m}{D_s} \text{ mH/km} \quad (2.39)$$

The GMR or self GMD of bundled conductors can be found in the same manner as that for stranded conductors.

Twin Conductor Bundle In this group, a two sub-conductor arrangement is shown in Fig. 2.13(a)

$$D_{sb} = \sqrt[4]{(r'D)^2} = \sqrt{(r'D)} \quad (2.40)$$

Triple Bundle A three sub-conductor arrangement is shown in Fig. 2.13(b)

$$D_{sb} = \sqrt[9]{(r'D^2)^3} = \sqrt[3]{(r'D^2)} \tag{2.41}$$

Quadruple Bundle A four sub-conductor arrangement is shown in Fig. 2.13(c)

$$D_{sb} = \sqrt[16]{(\sqrt{2}r'D^3)^4} = 1.09\sqrt[4]{(r'D^3)} \tag{2.42}$$

where, D_{sb} =	GMR of bundled conductor
D_s =	GMR of each sub-conductor of bundle
D =	Spacing between the sub-conductor of a bundle

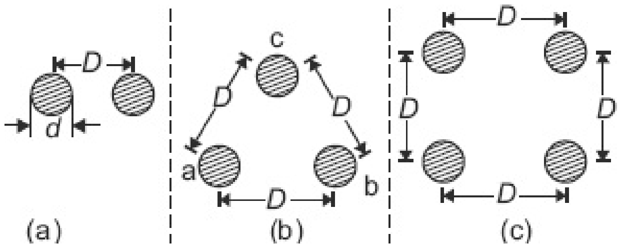

Fig. 2.13 Bundled conductors

The GMD of a bundled conductor line can be found by taking the root of the product of distances from each conductor of a bundle to every other conductor of the other bundles.

It can be observed from Eqs. (2.40) to (2.42), that the GMR of a bundled conductor increases. For the calculation of inductances, the GMR is a denominating factor. Therefore, the inductance of a bundled conductor line is less than the inductances of the line with one conductor per phase.

Test Yourself

1. Is the GMR less in bundled conductors? If yes, justify.

Example 2.5

Find GMR of a stranded conductor having seven identical strands each of radius *r* and is shown in Fig. 2.14. Also, find the ratio of GMR to overall conductor radius. Comment on the results.

Solution:

From Fig. 2.14, the distances from strand 1 to the other strands are

$$D_{12} = D_{16} = D_{17} = 2r; \quad D_{14} = 4r$$
$$D_{13} = \sqrt{D_{14}^2 - D_{34}^2} = \sqrt{(4r)^2 - (2r)^2} = 2\sqrt{3}r$$
$$D_{15} = D_{13}$$

The GMR of the seven-strand conductor is the 49th root of the product of 49 distances.

$$D_{s1} = D_{s2} = D_{s3} = D_{s4} = D_{s5} = D_{s6} = \sqrt[7]{D_{11} \cdot D_{12} \cdot D_{13} \cdot D_{14} \cdot D_{15} \cdot D_{16} \cdot D_{17}}$$
$$D_{s1} = D_{s2} = D_{s3} = D_{s4} = D_{s5} = D_{s6} = \sqrt[7]{r' \times (2r)^3 \times \left(2\sqrt{3}r\right)^2 \times 4r}$$
$$D_{s7} = \sqrt[7]{D_{71} \cdot D_{72} \cdot D_{73} \cdot D_{74} \cdot D_{75} \cdot D_{76} \cdot D_{77}}$$
$$D_{s7} = \sqrt[7]{r'(2r)^6}$$
$$D_s = \sqrt[7]{D_{s1} \cdot D_{s2} \cdot D_{s3} \cdot D_{s4} \cdot D_{s5} \cdot D_{s6} \cdot D_{s7}}$$
$$D_s = \sqrt[49]{r'^7 \times (2r)^{24} \times \left(2\sqrt{3}r\right)^{12} \times (4r)^6} = 2.177r$$

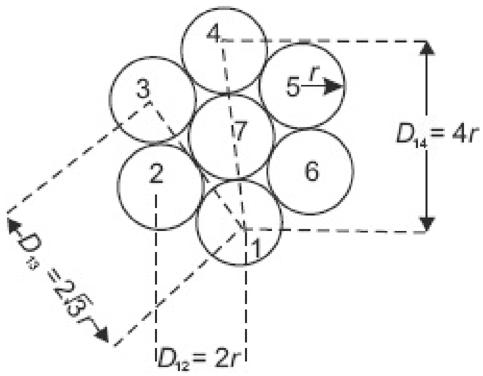

Fig. 2.14 Cross-sectional view of a seven-strand conductor

$$D_s = \sqrt[49]{(0.7788r)^7\left[(3r)^2/(4r)^2\times(5r)(3r)\right](3r)^6} = 2.177r$$

The overall conductor radius is 3*r*.

Therefore, the ratio of GMR to overall conductor radius

$$= \frac{2.177r}{3r} = 0.7257.$$

As the number of strands increases, the ratio approaches 0.7788, for a solid conductor. For three strands, the ratio is 0.678 and for 169 strands, the ratio is 0.777.

Example 2.6

Find the inductance of line consisting two ACSR conductors spaced between conductors of 1 m as shown in Fig. 2.15. The outer diameter of the single layer of aluminium strand is 50.4 mm and the radius of each strand is 8.4 mm. Neglect the effect of the central strand of steel on inductance.

Solution:

Diameter of steel strand = 50.4 – 4 × 8.4 = 16.8 mm = 1.68 cm

Radius of each strand, r = 8.4 mm = 0.84 cm

From Fig. 2.15, the distances from strand 1 to the other strands of conductor are

$$D_{11} = r' = 0.7788r,\ \ D_{12} = D_{16} = 2r;\ \ D_{14} = 4r$$
$$D_{13} = \sqrt{D_{14}^2 - D_{34}^2} = \sqrt{(4r)^2 - (2r)^2} = 2\sqrt{3}r$$
$$D_{15} = D_{13}$$

The GMR of the six strands (excluding steel; since it is given that the effect on inductance is neglected) conductor is the 36th root of the product of 36 distances.

$$D_{s1} = D_{s2} = D_{s3} = D_{s4} = D_{s5} = D_{s6} = \sqrt[6]{D_{11} \cdot D_{12} \cdot D_{13} \cdot D_{14} \cdot D_{15} \cdot D_{16}}$$
$$D_{s1} = D_{s2} = D_{s3} = D_{s4} = D_{s5} = D_{s6} = \sqrt[6]{r' \times (2r)^2 \times \left(2\sqrt{3}.r\right)^2 \times 4r}$$
$$= \sqrt[6]{0.7788r \times (2r)^2 \times \left(2\sqrt{3}.r\right)^2 \times 4r} = 2.304r$$
$$D_s = \sqrt[6]{(2.304r)^6} = 2.304 \times 0.84 = 1.935 \text{ cm}$$
$$D_m = 1 \text{ m} = 100 \text{ cm } (\because D >> r)$$

Inductance of each conductor, $L = 2 \times 10^{-7} \ln \frac{100}{1.935} = 789.012 \times 10^{-9}$ H/m
$= 0.789$ mH/km

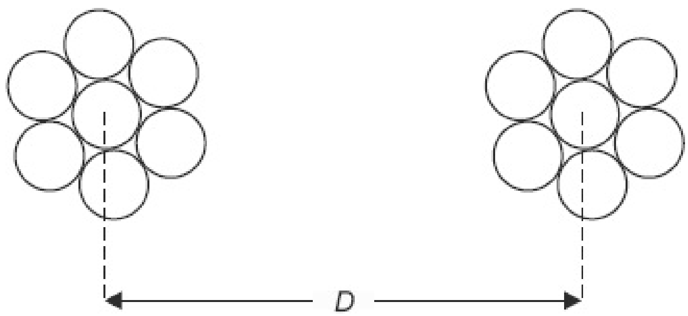

Fig. 2.15 Line composed of ACSR conductors

2.11 INDUCTANCE OF THREE-PHASE LINES

2.11.1 Equivalent (Symmetrical) Spacing

Figure 2.16 shows a three-phase line with conductors *a, b* and *c* spaced at the corners of an equilateral triangle, each side being *D* and the radius of each conductor being *r*.

The currents in conductor *a, b* and *c* are I_a, I_b, *and* I_c, respectively which satisfy the relationship $I_a + I_b + I_c = 0$. Flux linkages of conductor *a* due to currents I_a, I_b, and I_c is [by using the expression in Eq. (2.25)] given by

$$\psi_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right) \text{Wb-T/m} \tag{2.43}$$

since, $(I_b + I_c) = -I_a$

$$\because \psi_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D}\right) \text{Wb-T/m} \tag{2.44}$$

$$= 2\times10^{-7}\left(I_a \ln\frac{D}{r'}\right) \text{Wb-T/m}$$

Moreover, inductance of conductor *a* is

$$L_a = \frac{\psi_a}{I_a} = 2\times10^{-7} \ln\frac{D}{r'} \text{ H/m}$$

$$\text{or } L_a = 0.2 \ln\frac{D}{r'} \text{ mH/km} \tag{2.45}$$

Because of the symmetry, inductances of conductors *b* and *c* are also equal to inductance of conductor *a*.

$$\therefore \text{Inductance per phase, } L = 0.2 \ln\frac{D}{r'} \text{ mH/km} \tag{2.46}$$

For stranded conductors, replace *r*‘ by D_s (self GMD).

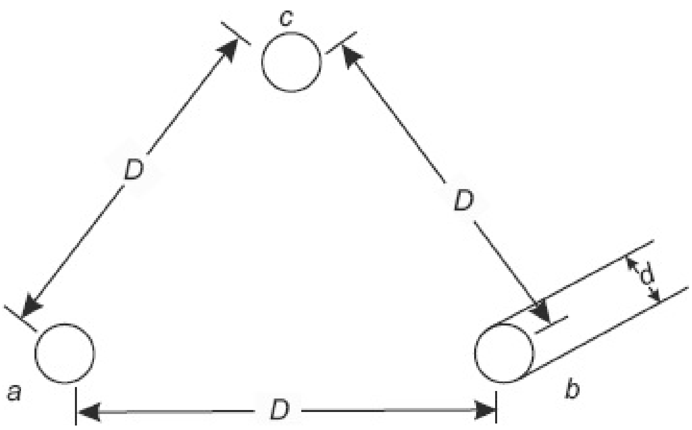

Fig. 2.16 Three-phase line with equilateral spacing

EXAMPLE 2.7

Calculate the inductance of a conductor (line-to-neutral) of a three-phase system as shown in Fig. 2.17, which has 1.2 cm diameter and conductors are placed at the corner of an equilateral triangle of sides 1.5 m. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Spacing between the conductors, D = 1.5 m = 150 cm

Diameter of the conductor, d = 1.2 cm

Radius of conductor, r = 0.6 cm

Effective radius of conductor, r' = 0.7788 × 0.6 cm

$$\text{Inductance of a conductor per phase, } L = 2\times10^{-7} \ln \frac{D}{r'} \text{ H/m}$$

$$= 2\times10^{-7} \ln \frac{150}{0.7788\times0.6} \text{ H/m}$$

$$= 1.154 \text{ mH/km}$$

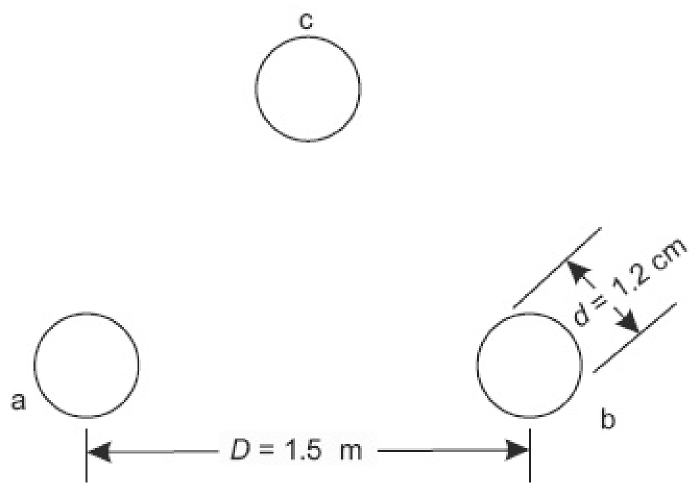

Fig. 2.17 Arrangement of conductors for Example 2.7

EXAMPLE 2.8

Calculate the inductance of a conductor per phase of a three-phase, three-wire system. The conductors are arranged at the corners of an equilateral triangle of 3.5 m sides and the diameter of each conductor is 2 cm.

Solution:

Spacing between the conductors, D = 3.5 m = 350 cm

Diameter of the conductor, d = 2 cm

Radius of conductor, r = 1 cm

Effective radius of conductor, r' = 0.7788 × 1 = 0.7788 cm

$$\text{Inductance of a conductor per phase, } L = 2\times 10^{-7} \ln \frac{D}{r'} \text{ H/m}$$
$$= 2\times 10^{-7} \ln \frac{350}{0.7788} \text{ H/m}$$

∴Inductance of a conductor per phase, L = 1.222 mH/km

2.11.2 Unsymmetrical Spacing (untransposed)

Consider a three-phase single circuit system as shown in Fig. 2.18(a) having three conductors *a*, *b* and *c*, carrying currents I_a, I_b and I_c, respectively. The three conductors are unsymmetrically placed i.e., $D_{12} \neq D_{23} \neq D_{31}$ and each conductor has a radius of *r* m. From Eq. (2.25), the flux linkages of conductor *a* due to I_a, I_b and I_c is

$$\psi_b = 2\times10^{-7}\left(I_b \ln\frac{1}{r'} + I_a \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right) \tag{2.47}$$

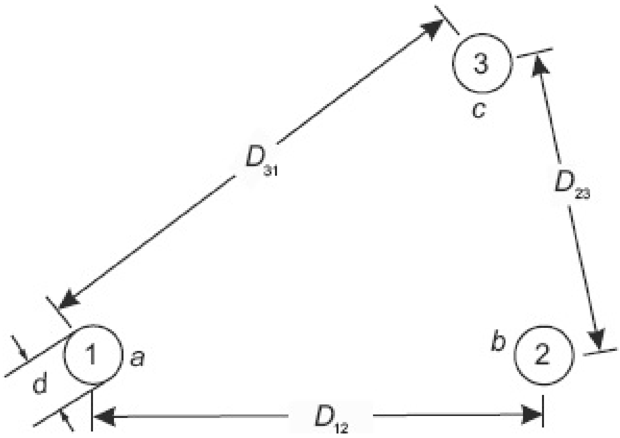

Fig. 2.18(a) Three-phase, untransposed line with unsymmetrical spacing

Similarly,

$$\psi_b = 2\times10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}}\right) \tag{2.48}$$

$$\psi_c = 2\times10^{-7}\left(I_a \ln\frac{1}{D_{31}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'}\right) \tag{2.49}$$

Now take I_a as reference phasor of unbalanced three-phase system as shown in Fig. 2.18(b)

From Fig. 2.18(b)

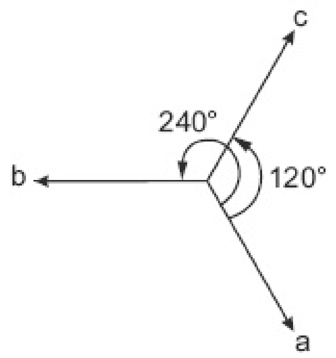

Fig. 2.18(b) Phasor diagram

$$I_b = \alpha^2 I_a \quad \text{and} \quad I_c = \alpha I_a$$

where,

$$\alpha = -0.5 + j0.886$$
$$\alpha^2 = -0.5 - j0.886$$

Substituting the values of I_b and I_c in the Eq. (2.47)

$$\psi_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_a(-0.5 - j0.866)\ln\frac{1}{D_{12}} + I_a(-0.5 + j0.866)\ln\frac{1}{D_{31}}\right)$$

The inductance of conductor *a* is

$$L_a = 2\times10^{-7}\left(\ln\frac{1}{r'} - \frac{1}{2}\ln\frac{1}{D_{12}D_{31}} - \frac{j\sqrt{3}}{2}\ln\frac{D_{31}}{D_{12}}\right)$$

$$\therefore L_a = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{12}D_{31}}}{r'} - \frac{j\sqrt{3}}{2}\ln\frac{D_{31}}{D_{12}}\right)\text{ H/m} \tag{2.50}$$

Similarly,

$$\psi_b = 2\times10^{-7}\left(I_b\ln\frac{1}{r'} + I_a\ln\frac{1}{D_{12}} + I_c\ln\frac{1}{D_{31}}\right)$$

$$L_b = 2\times10^{-7}\left(\ln\frac{1}{r'} + (-0.5 + j0.866)\ln\frac{1}{D_{12}} + (-0.5 - j0.866)\ln\frac{1}{D_{23}}\right)$$

$$\therefore L_b = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{12}D_{23}}}{r'} + \frac{j\sqrt{3}}{2}\ln\frac{D_{23}}{D_{12}}\right)\text{ H/m} \tag{2.51}$$

and

$$L_c = 2\times10^{-7}\left(\ln\frac{1}{r'} + (-0.5 - j0.866)\ln\frac{1}{D_{31}} + (-0.5 + j0.866)\ln\frac{1}{D_{23}}\right)$$

$$L_c = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{31}D_{23}}}{r'} - \frac{j\sqrt{3}}{2}\ln\frac{D_{23}}{D_{31}}\right)\text{ H/m} \tag{2.52}$$

From Eqs. (2.50), (2.51) and (2.52), it is clear that the inductance of conductors *a*, *b* and *c* are unequal and they contain imaginary values, which is due to the mutual inductance.

EXAMPLE 2.9

Calculate the inductance per phase of a three-phase transmission line as shown in Fig. 2.19. The radius of the conductor is 0.5 cm. The lines are untransposed. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Spacing between the conductors *a* and *b*, D_{ab} = 3.5 m

Spacing between the conductors *b* and *c*, D_{bc} =3.5 m

Spacing between the conductors *a* and *c*, D_{ac} = 7.0 m

Radius of each conductor, r = 0.5 cm =0.5×10^{-2} m

Effective radius, r' = $0.7788 \times 0.5 \times 10^{-2}$ = 0.003894 m

Now take I_a as reference for unbalanced three-phase system. From phasor diagram shown in Fig. 2.18(b),

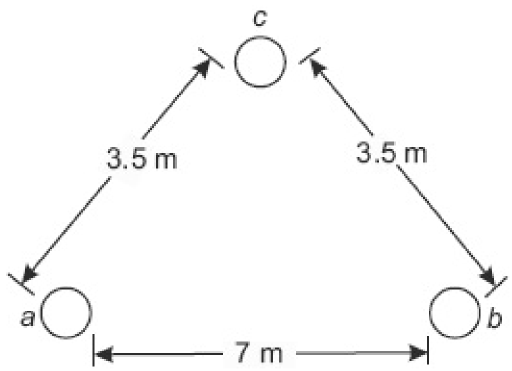

Fig. 2.19 Cross-sectional view of conductors′ location in triangle

$$I_b = \alpha^2 I_a = I_a \angle 240^\circ \quad \text{and} \quad I_c = \alpha I_a = I_a \angle 120^\circ$$

where,

$$\alpha = -0.5 + j0.886$$
$$\alpha^2 = -0.5 - j0.886$$

Inductance of conductor *a*, $L_a = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{12}D_{31}}}{r'} - \frac{j\sqrt{3}}{2}\ln\frac{D_{31}}{D_{12}}\right)$

$$L_a = 2\times10^{-7}\left(\ln\frac{\sqrt{3.5\times7}}{0.003894} - \frac{j\sqrt{3}}{2}\ln\frac{7}{3.5}\right)$$
$$= (1.4295 - j0.0963)\ \text{mH/km}$$

Similarly,

Inductance of conductor b, $L_b = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{12}D_{23}}}{r'} + \frac{j\sqrt{3}}{2}\ln\frac{D_{23}}{D_{12}}\right)$

$$= 2\times10^{-7}\left(\ln\frac{\sqrt{3.5\times3.5}}{0.003894} + \frac{j\sqrt{3}}{2}\ln\frac{3.5}{3.5}\right)$$
$$= (1.3602 + 0.0)\ \text{mH/km}$$

Inductance of conductor c, $L_c = 2\times10^{-7}\left(\ln\frac{\sqrt{D_{31}D_{23}}}{r'} - \frac{j\sqrt{3}}{2}\ln\frac{D_{23}}{D_{31}}\right)$

$$= 2\times10^{-7}\left(\ln\frac{\sqrt{7\times3.5}}{0.003894} - \frac{j\sqrt{3}}{2}\ln\frac{3.5}{7}\right)$$
$$= (1.4295 - j0.0309)\ \text{mH/km}$$

Note: It is clear from the results of the unsymmetrical untransposed transmission line that inductances of conductors *a*, *b* and *c* are unequal, and they contain imaginary term which is due to the mutual inductance.

2.11.3 Transposition of Overhead Lines

Transposition of overhead line conductors refers to exchanging the positions of power conductors at regular intervals along the line, so that each conductor occupies the original position of every other conductor over an equal distance.

When the conductors of a three-phase system are not spaced regularly, then the inductances and capacitances are not the same in the different phases. The important one is that there is an interchange of power between the phases so that there may be wide differences between the apparent resistances of the conductors. Thus, the line constants are affected by irregular spacing. Therefore, the general practice with such lines is to interchange or transpose the conductors at equal intervals along the lines, so that each of the three wires occupies all three positions relatively to the other conductors for one-third of the total length of the transmission line. This results in the elimination of effect of mutual inductance. Modern power lines are not transposed at regular intervals, but they interchange the positions of the conductors at switching stations in order to balance the inductance or capacitance of the phases more closely.

When a communication line such as a telephone line runs parallel along a high voltage overhead line, high voltages are induced in the communication line resulting in acoustic shock and noise. The induced voltages are due to electrostatic and electromagnetic induction, which is reduced considerably by transposing power lines as shown in Fig. 2.20(a).

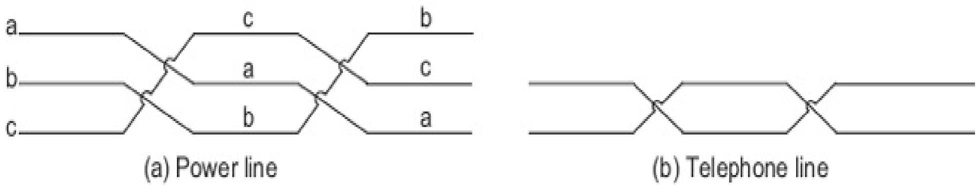

Fig. 2.20 Transposition of conductors

Effect of Transposition The purpose of transposition is to balance the capacitances of the line, so that the electrostatically induced voltages balance out in the length of a complete set of transposition, and this length is called a *barrel*.

Transposition also results in the decrease of electromagnetically induced e.m.f. on the wires, because of fluxes due to the positive and negative phase sequence, currents will add up to zero along the barrel.

This flux due to the zero phase sequence or longitudinal current is unaffected by transpositions of power systems because it flows along the three conductors in parallel. To reduce the e.m.f. in the telephone line as shown in Fig. 2.20(b) due to its zero phase sequence current, the telephone line is transposed by a proper co-ordination of transpositions of the power and communication lines. The induced voltage can be reduced to very small proportions under normal working conditions.

2.11.4 Unsymmetrical Spacing (Transposed)

When the conductors of a three-phase line are spaced at the corners of triangle, the inductances of three conductors are different because of different spaces between adjacent conductors. This results in unequal voltage drops in three-phases. Therefore, the voltages are not the same at the receiving-end. This problem can be overcome by the transposition of conductors.

Figure 2.21 shows the arrangement for a transposed line conductor *a* (say phase a) that will occupy all the position 1, 2 and 3 with 1/3rd of total length. Similarly, conductors *b* and *c* will occupy all three positions each for 1/3rd of total length.

Let us assume that the distance between positions 1 and 2, 2 and 3, 3 and 1 are D_{12}, D_{23} and D_{31}, respectively, and the currents carrying in conductors *a*, *b* and *c* are I_a, I_b and I_c, respectively.

By using Eq. (2.25), the flux linkages of conductor *a* due to the currents in all three conductors, when the conductor *a* is in position 1, *b* in position 2 and *c* in position 3 is

$$\psi_{a1} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right) \text{ Wb-T/m} \tag{2.53}$$

Similarly, ψ_{a2}, when *a* in position 2, *b* in position 3 and *c* in position 1

$$\psi_{a2} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right) \text{Wb-T/m} \tag{2.54}$$

And ψ_{a3}, when *a* in position 3, *b* in position 1 and *c* in position 2

$$\psi_{a3} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right) \text{Wb-T/m} \tag{2.55}$$

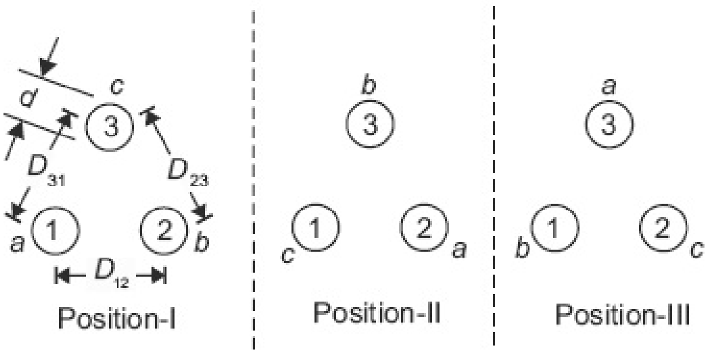

Fig. 2.21 Three-phase line with unsymmetrical spacing (transposed)

The average value of the flux linkages of *a* is

$$\psi_a = \frac{\psi_{a1} + \psi_{a2} + \psi_{a3}}{3}$$

$$= \frac{2\times10^{-7}}{3}\left[3I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{31}D_{12}D_{23}}\right] \quad (2.56)$$

$$\psi_a = \frac{2\times10^{-7}}{3}\left[3I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right] \quad [\because I_b + I_c = -I_a \text{ and } r' = D_s]$$

$$= 2\times10^{-7}\left[I_a \ln\frac{1}{D_s} - I_a \ln\left(\frac{1}{D_{12}D_{23}D_{31}}\right)^{1/3}\right]$$

$$= 2\times10^{-7} I_a \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s} \text{ Wb-T/m} \quad (2.57)$$

The average inductance of phase *a* is

$$L_a = 2\times10^{-7} \ln\frac{D_{eq}}{D_s} \text{ H/m} \quad (2.58)$$

where,

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

$$\therefore \text{ Inductance per phase, } L = 2\times10^{-7} \ln\frac{D_{eq}}{D_s} \text{ H/m}$$

$$\text{Inductance per phase, } L = 0.2\times\ln\frac{D_{eq}}{D_s} \text{ mH/km} \quad (2.59)$$

EXAMPLE 2.10

Calculate the inductance per phase of a 750 kV three-phase single circuit line that utilizes a bundled conductor arrangement as shown in Fig. 2.22. The space between the two phases is 15 m in a horizontal formation. The sub-conductors of a phase are at the corners of a square of sides 0.5 m, each sub-conductor having a diameter of 3 cm.

Solution:

Spacing between the conductors, $D_{ab} = D_{bc} = 15$ m and $D_{ca} = 30$ m

Distance between sub-conductors of a bundle, $d = 0.5$ m

Effective radius of conductor,

$$r' = 0.7788 \times \frac{3}{2} = 1.1682 \text{ cm} = 1.1682 \times 10^{-2} \text{ m}$$

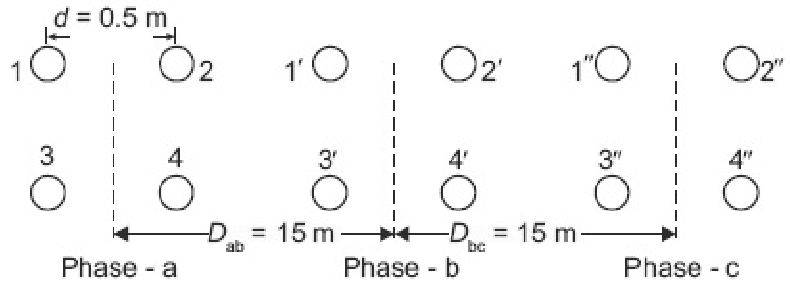

Fig. 2.22 Three-phase bundled conductor line with four sub-conductors per phase

Inductance per phase of the line, $L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s}$ H/m

where,

$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{15\times15\times30} = 18.894 \text{ m}$$

Self GMR of the bundled conductor

$$D_s = \sqrt[16]{(D_{11}D_{12}D_{13}D_{14})(D_{21}D_{22}D_{23}D_{24})(D_{31}D_{32}D_{33}D_{34})(D_{41}D_{42}D_{43}D_{44})}$$

where,

$$D_{11} = D_{22} = D_{33} = D_{44} = r'$$

$$D_{12} = D_{23} = D_{34} = D_{41} = d \text{ and } D_{14} = D_{23} = \sqrt{2}\ d$$

$$\therefore D_s = \sqrt[16]{4(r')^4 d^{12}} = 1.09\times\sqrt[4]{r'd^3} = 1.09\times\sqrt[4]{(1.1682\times10^{-2})(0.5)^3} = 0.213 \text{ m}$$

Inductance per phase of the line, $L = 2\times10^{-7} \ln\frac{18.894}{0.213} = 0.897\times10^{-6}$ H/m

$$= 0.897 \text{ mH/km}$$

EXAMPLE 2.11

Calculate the inductance and reactance of each phase of a three-phase 50 Hz overhead high-tension line (HTL) which has conductors of 2.5 cm diameter. The distance between the three-phases are (i) 5 cm between *a* and *b*, (ii) 4 m between *b* and *c* and (iii) 3 m between *c* and *a* as shown in Fig. 2.23.

Assume that the phase conductors are transposed regularly. (This question is also solved using MATLAB programs in the appendix.)

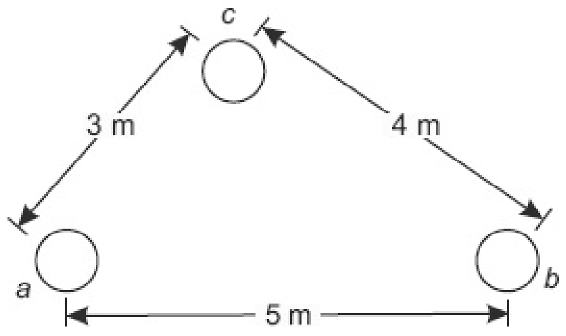

Fig. 2.23 Cross-sectional view of conductors′ location in triangle

Solution:

Diameter of each conductor, d = 2.5 cm

Radius of conductor, r = 1.25 cm = 0.0125 m

$$\text{Inductance per phase} = 2\times10^{-7}\ln\left(\frac{D_m}{D_s}\right)$$

$$\text{Mutual GMD } (D_m) \text{ or } D_{eq} = \sqrt[3]{D_{ab}\times D_{bc}\times D_{ca}}$$
$$= \sqrt[3]{5\times4\times3} = 3.9143 \text{ m}$$

$$\text{GMR or self GMD, } r' = 0.7788\, r$$
$$= 0.7788 \times 0.0125 = 0.009735 \text{ m}$$

$$\text{Inductance per phase} = 2\times10^{-7}\ln\frac{D_{eq}}{r'} \text{ H/m}$$
$$= 2\times10^{-7}\ln\frac{3.914}{0.009735} = 1.199 \text{ mH/km}$$

$$\text{Inductance per phase } X_L = 2\pi fL = 2\pi\times50\times1.199\times10^{-3} = 0.3767\ \Omega/\text{km}$$

EXAMPLE 2.12

Repeat the Example 2.9, when three-phase lines are fully transposed.

Solution:

Spacing between the conductors a and b, D_{ab} = 3.5 m

Spacing between the conductors *b* and *c*, D_{bc} = 3.5 m

Spacing between the conductors *a* and *c*, D_{ac} = 7.0 m

Radius of each conductor, r = 0.5 cm = 0.5×10^{-2} m

Effective radius of conductor, r' = 0.7788 × 0.5 cm = 0.3894×10^{-2} m

For transposed line:

$$\text{Inductance of each conductor, } L = 2\times10^{-7} \ln \frac{D_{eq}}{r'} \text{ H/m}$$

$$D_{eq} = 3\sqrt{D_{ab} D_{bc} D_{ca}}$$

$$\text{Mutual GMD } (D_m) \text{ or } (D_{eq}) = \sqrt[3]{3.5\times3.5\times7} = 4.409 \text{ m}$$

$$\text{Inductance of each conductor, } L = 2\times10^{-7} \ln \frac{4.409}{0.3894\times10^{-2}}$$

$$= 1.406\times10^{-6} \text{ H/m}$$

$\therefore$ Inductance per phase = 1.406 mH/m

EXAMPLE 2.13

A 300 kV, three-phase bundle conductor line with two sub-conductor per phase has a horizontal configuration as shown in Fig. 2.24. Find the inductance per phase, if the radius of each sub-conductor is 1.2 cm. (This question is also solved using MATLAB programs in the appendix.)

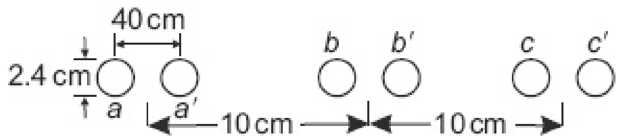

Fig. 2.24 Three-phase bundled conductor line with two sub-conductors per phase

Solution:

Radius of each conductor, r = 1.2 cm

Inductance per conductor of three-phase double circuit line,

$$L = 2\times10^{-7} \ln \frac{D_m}{D_s} \text{ H/m}$$

Self GMD in all three positions are same,

$$\therefore \text{ Self GMD, } D_s = \sqrt[4]{(D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})} \quad (\because D_{aa} = D_{a'a'} = r')$$
$$= \sqrt{(0.7788 \times 0.012 \times 0.4)} = 0.0611 \text{ m}$$

Mutual GMD in position 1 (D_{m1}) and position 3 (D_{m3}) are same.

$$D_{m1} = D_{m3} = \sqrt[8]{D_{ab} \times D_{ab'} \times D_{ac} \times D_{ac'} \times D_{a'b} \times D_{a'b'} \times D_{a'c} \times D_{a'c'}}$$
$$= \sqrt[8]{10 \times 10.4 \times 20 \times 20.4 \times 9.6 \times 10 \times 19.6 \times 20}$$
$$= \sqrt[8]{1596801024} = 14.1386 \text{ m}$$

Mutual GMD in position 2 (D_{m2})

$$\therefore D_{m2} = \sqrt[8]{D_{ab} \times D_{b'c} \times D_{bb'} \times D_{ba'} \times D_{bc'} \times D_{a'b'} \times D_{bc'} \times D_{b'c'}}$$
$$= \sqrt[8]{10 \times 10.4 \times 10 \times 9.6 \times 9.6 \times 10 \times 10.4 \times 10}$$
$$\therefore D_{m2} = \sqrt[8]{99680256} = 9.996 \text{ m}$$
$$\therefore D_m = \sqrt[3]{D_{m1} \times D_{m2} \times D_{m3}}$$
$$\therefore D_m = \sqrt[3]{14.1386 \times 9.996 \times 14.1386} = 12.595 \text{ m}$$

$$\text{Inductance per phase} = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \text{ H/m}$$
$$= 2 \times 10^{-7} \ln \frac{12.595}{0.0611} \text{ H/m}$$
$$= 1.0657 \text{ mH/km}$$

2.12 INDUCTANCE OF THREE-PHASE DOUBLE CIRCUIT LINE

In a three-phase double circuit transmission line, two conductors run parallel to each phase on the same tower. This leads to more reliability and increased transmission capacity due to reduced inductive reactance.

The aim of running the conductors in parallel is to have low inductance of the equivalent circuit. This can be achieved by reducing the spacing between phases (low value of D_m) and increasing the distance between the conductors in each phase (high value of D_s). The arrangement of the three-phase double circuit line in vertical spacing is shown in Figs. 2.25(a). 220 kV and 400 kV double circuit three-phase transmission lines are shown in Figs. 2.25(b) and (c).

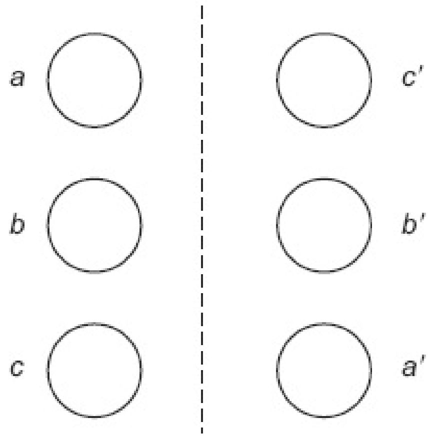

Fig. 2.25(a) Vertical arrangement of three-phase double circuit line

Fig. 2.25(b) 220 kV Three-phase double circuit line

Fig. 2.25(c) 400 kV double circuit bundled three-phase transmission line with two sub-conductors per phase

2.12.1 Inductance of Three-phase Double-Circuit Line with Symmetrical Spacing (Hexagonal)

In double-circuit line, it is necessary to arrange the phase conductors diagonally to reduce the inductance of each phase.

Conductors *a*, *b* and *c* belong to one circuit and *a′*, *b′* and *c′* to another circuit. Conductors *a* and *a′* are electrically parallel. Similarly, conductors' *b*, *b′* and *c*, *c′* are also parallel.

From Eq. (2.25), the flux linkages of conductor *a* is

$$\psi_a = 2\times10^{-7}\left[I_a\left(\ln\frac{1}{r_a'}+\ln\frac{1}{D_{aa'}}\right)+I_b\left(\ln\frac{1}{D_{ab}}+\ln\frac{1}{D_{ab'}}\right)+I_c\left(\ln\frac{1}{D_{ac}}+\ln\frac{1}{D_{ac'}}\right)\right] \text{ Wb-T/m} \qquad (2.60)$$

Since, the conductors are identical, the GMR of the conductors will be same. From Fig. 2.26, the distances between the conductors are

$$D_{aa'} = 2D,\ D_{ab'} = D_{ac} = \sqrt{3}D \text{ and } D_{ab} = D_{ac'} = D$$

$$\psi_a = 2\times10^{-7}\left[I_a \ln\left(\frac{1}{2r'D}\right) + I_b \ln\left(\frac{1}{\sqrt{3}D^2}\right) + I_c \ln\left(\frac{1}{\sqrt{3}D^2}\right)\right] \text{ Wb-T/m}$$

$$= 2\times10^{-7}\left[I_a \ln\left(\frac{1}{2r'D}\right) + (I_b + I_c)\ln\left(\frac{1}{\sqrt{3}D^2}\right)\right] \text{ Wb-T/m}$$

$$= 2\times10^{-7}\left[I_a \ln\left(\frac{1}{2Dr'}\right) - I_a \ln\left(\frac{1}{\sqrt{3}D^2}\right)\right] \text{ Wb-T/m} \quad (\because I_a + I_b + I_c = 0) \tag{2.61}$$

$$= 2\times10^{-7} I_a \ln\left(\frac{\sqrt{3}D^2}{2Dr'}\right)$$

$$= 2\times10^{-7} I_a \ln\frac{\sqrt{3}}{2}\frac{D}{r'} \text{ Wb-T/m} \tag{2.62}$$

Therefore, the inductance of conductor *a* is

$$L_a = \frac{\psi_a}{I_a} = 2\times10^{-7} \ln\frac{\sqrt{3}}{2}\frac{D}{r'} \text{ H/m} \tag{2.63}$$

Since, conductors are electrically in parallel, the inductance of each phase $= \frac{L}{2}$

$$\therefore \text{Inductance per phase, } L = 10^{-7} \ln\frac{\sqrt{3}}{2}\frac{D}{r'} \text{ H/m} \tag{2.64}$$

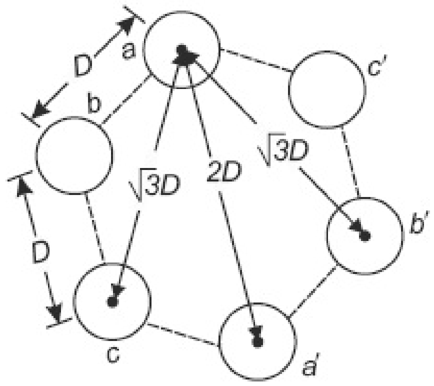

Fig. 2.26 Double-circuit line with symmetrical spacing

EXAMPLE 2.14

Calculate the inductance per phase of a three-phase double circuit line as shown in Fig. 2.27, if the conductors are spaced at the vertices of a hexagon of side 2 m each. The diameter of each conductor is 2.0 cm. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Each side of a hexagon, D = 2.0 m = 200 cm

Diameter of a conductor, d = 2.0 cm

Radius of a conductor, r = 1.0 cm

Effective radius of a conductor, r' = 0.7788 × 1.0 = 0.7788 cm

Inductance per phase, $L = 10^{-7} \ln\left[\frac{\sqrt{3}}{2} \times \frac{200}{0.7788}\right] = 0.54 \text{ mH/km}$

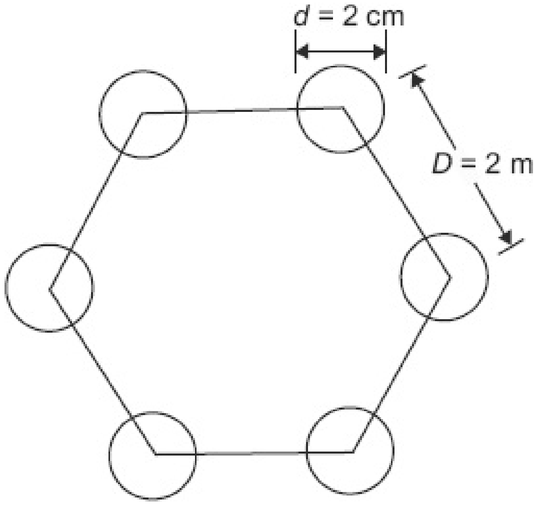

Fig. 2.27 Location of conductors for Example 2.14

2.12.2 Inductance of a Three-phase Transposed Double-Circuit Line with Unsymmetrical Spacing

The arrangement of three-phase double-circuit lines, which are vertically spaced is shown in Fig. 2.28. Since, the conductors are transposed at regular intervals, the transposition cycle and conductor positions are shown in Fig. 2.28 (a), (b), and (c).

Transposition cycle of the double-circuit three-phase line is given in Fig. 2.28(d)

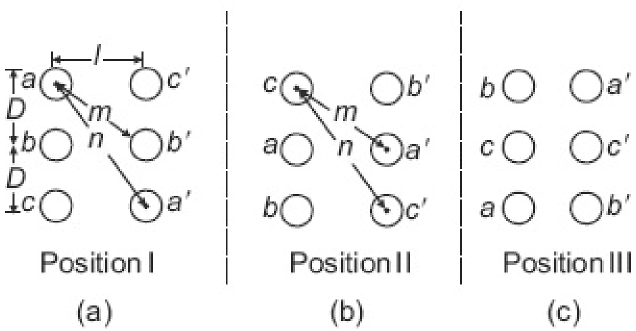

Fig. 2.28 (a), (b) and (c) Three-phase transposed double-circuit line

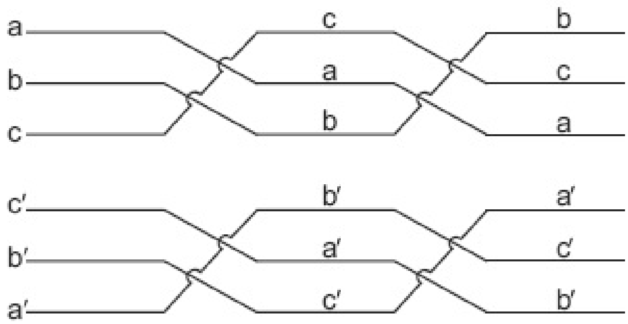

Fig. 2.28(d) Transposition cycle of double-circuit three-phase line

Now flux linkage of conductor *a* of phase 'a' in position 1 is

$$\psi_{a1} = 2\times10^{-7}\left[I_a\left(\ln\frac{1}{r'}+\ln\frac{1}{n}\right)+I_b\left(\ln\frac{1}{D}+\ln\frac{1}{m}\right)+I_c\left(\ln\frac{1}{2D}+\ln\frac{1}{l}\right)\right] \text{ Wb-T/m} \qquad (2.65)$$

Similarly, for position 2,

$$\psi_{a2} = 2\times10^{-7}\left[I_a\left(\ln\frac{1}{r'}+\ln\frac{1}{l}\right)+I_b\left(\ln\frac{1}{D}+\ln\frac{1}{m}\right)+I_c\left(\ln\frac{1}{D}+\ln\frac{1}{m}\right)\right] \text{ Wb-T/m} \qquad (2.66)$$

For position 3,

$$\psi_{a3} = 2\times10^{-7}\left[I_a\left(\ln\frac{1}{r'}+\ln\frac{1}{n}\right)+I_b\left(\ln\frac{1}{2D}+\ln\frac{1}{l}\right)+I_c\left(\ln\frac{1}{D}+\ln\frac{1}{m}\right)\right] \text{ Wb-T/m} \qquad (2.67)$$

The average value ψ_a; is

$$\psi_a = \frac{\psi_{a1} + \psi_{a2} + \psi_{a3}}{3} \text{ Wb-T/m} \tag{2.68}$$

Since for balanced three-phase system, $I_a + I_b + I_c = 0$

Substituting the Eqs. (2.65) to (2.67) in conditions in Eq. (2.68) and simplifying, we get,

$$\psi_a = \frac{2\times10^{-7}}{3}\left[I_a \ln\frac{1}{(r')^3 ln^2} + (I_b + I_c)\ln\frac{1}{2D^3 lm^2}\right] \text{ Wb-T/m}$$

$$\therefore \psi_a = \frac{2\times10^{-7}}{3}\left[I_a \ln\frac{1}{(r')^3 ln^2} - I_a \ln\frac{1}{2D^3 lm^2}\right] \text{ Wb-T/m} \quad (\because I_b + I_c = -I_a) \tag{2.69}$$

$$\psi_a = \frac{2\times10^{-7}}{3}\left[I_a \ln\frac{2D^3 lm^2}{(r')^3 ln^2}\right] \text{ Wb-T/m} \tag{2.70}$$

$$\psi_a = 2\times10^{-7} I_a \ln 2^{1/3}\left(\frac{D}{r'}\right)\left(\frac{m}{n}\right)^{2/3} \text{ Wb-T/m} \tag{2.71}$$

Therefore, the inductance of conductor *a* is

$$L_a = \frac{\psi}{I_a} = 2\times10^{-7} \ln 2^{1/3}\left(\frac{D}{r'}\right)\left(\frac{m}{n}\right)^{2/3} \text{ H/m} \tag{2.72}$$

Inductance of each phase, $L = \frac{1}{2}L_a$

$$\therefore L = 2\times10^{-7} \ln 2^{1/6}\left(\frac{D}{r'}\right)^{1/2}\left(\frac{m}{n}\right)^{1/3} \text{ H/m} \tag{2.73}$$

It can be observed from Fig. 2.28, that the conductor of a double circuit line is placed diagonally opposite rather than in the same horizontal plane in all three positions. By doing this the GMR of the conductors is increased and the GMD is reduced, thereby lowering the inductance/phase.

Test Yourself

1. What is the need of double circuit transmission lines?

EXAMPLE 2.15

Calculate the inductance of the single-phase double circuit line shown in Fig. 2.29. The diameter of each conductor is 2 cm.

Solution:

Diameter of each conductor, d =2.0 cm

Radius of each conductor, r =1.0 cm

Effective radius of each conductor, $r' = 0.7788 \times 1.0$ cm = 0.7788 cm

Inductance per conductor, $L = 2\times10^{-7}\ln\left(\frac{D_m}{D_s}\right)$

Geometric mean radius (Self GMD or GMR), $D_s = \sqrt[4]{(D_{aa}\times D_{aa'}\times D_{a'a'}\times D_{a'a})}$

$$= \sqrt[4]{0.7788\times100\times100\times0.7788} \quad (\because D_{aa} = D_{a'a'} = r')$$

$$= \sqrt{0.7788\times100} = 8.825 \text{ cm}$$

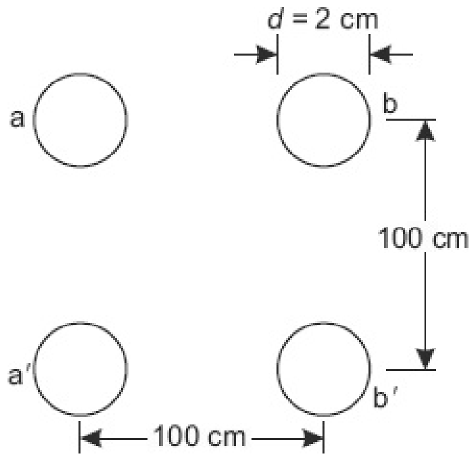

Fig. 2.29 Single-phase, double circuit line

$$\text{Mutual GMD, } D_m = \sqrt[4]{(D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})}$$
$$= \sqrt[4]{100 \times 141.42 \times 100 \times 141.42} = 118.921 \text{ cm}$$
$$\text{Inductance per conductor, } L = 2 \times 10^{-7} \ln \frac{118.921}{8.825} = 0.5201 \text{ mH/km}$$

∴ Inductance of a single-phase system = 0.5201 mH/km.

EXAMPLE 2.16

Calculate the inductance per phase of a three-phase, double circuit line as shown in Fig. 2.30. The diameter of each conductor is 1.5 cm. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Horizontal distance, l = 6 m

Vertical distance, D = 2.5 m

Diameter of conductor, d = 1.5 cm

Radius of conductor, r = 0.75 cm

Effective radius of conductor is

$$r' = 0.7788 \times 0.75 = 0.5841 \text{ cm}$$

$$m = \sqrt{6^2 + 2.5^2} = \sqrt{42.25} = 6.5 = 650 \text{ cm}$$

$$n = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.81 \text{ m} = 781 \text{ cm}$$

The inductance of conductor a is $L_a = 2\times10^{-7} \ln 2^{1/3} \left\{\left(\frac{D}{r'}\right)\left(\frac{m}{n}\right)^{2/3}\right\}$ H/m

Inductance of each phase, $L = \frac{1}{2} L_a$

$$\therefore \text{ Inductance per phase, } L = 2\times10^{-7} \ln \left\{2^{1/6}\left(\frac{D}{r'}\right)^{1/2}\left(\frac{m}{n}\right)^{1/3}\right\} \text{ H/m}$$

$$= 2\times10^{-7} \ln \left\{2^{1/6}\left(\frac{250}{0.5841}\right)^{1/2}\left(\frac{650}{781}\right)^{1/3}\right\} \text{ H/m}$$

$$= 0.617 \text{ mH/km}$$

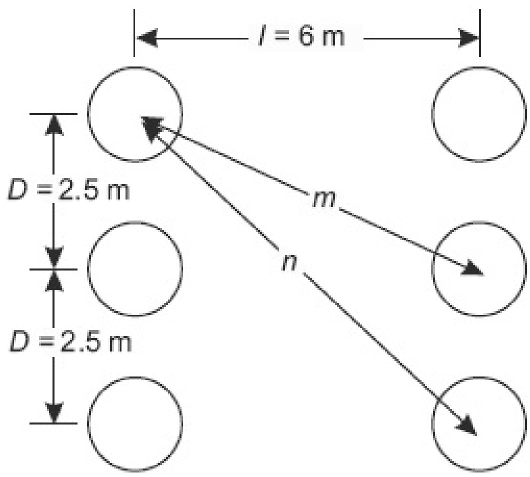

Fig. 2.30 Three-phase, double circuit line

EXAMPLE 2.17

Calculate the inductance per phase of a three-phase double circuit line as shown in Fig. 2.31. The diameter of the conductor is 2 cm. Assume that the line is completely transposed. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Diameter of conductor, d = 2 cm

Radius of conductor, r =1 cm

Effective radius of conductor, $r' = 1 \times 0.7788 = 0.7788$ cm

$$\text{Distance } a \text{ to } b,\ D_{ab} = \sqrt{3.5^2 + 1^2} = 3.64 \text{ m}$$
$$a \text{ to } b',\ D_{ab'} = \sqrt{3.5^2 + 7^2} = 7.826 \text{ m}$$
$$a \text{ to } a',\ D_{aa'} = \sqrt{7^2 + 6^2} = 9.22 \text{ m}$$
$$\text{Inductance per phase} = 0.2 \ln \frac{D_{eq}}{D_s} \text{ mH/km}$$

where,

D_S is the self $\text{GMD} = \sqrt[3]{D_{s1} D_{s2} D_{s3}}$ of one phase and suffixes 1, 2 and 3 denote the self GMD in positions 1, 2 and 3, respectively. Also D_S is the same for all the phases.

$$D_{s1} = \sqrt[4]{(D_{aa} D_{aa'} D_{a'a'} D_{a'a})} = \sqrt[4]{0.7788 \times 10^{-2} \times 9.22 \times 0.7788 \times 10^{-2} \times 9.22}$$
$$= 0.2680 \text{ m} = 26.8 \text{ cm}$$
$$D_{s2} = \sqrt[4]{(D_{aa} D_{aa'} D_{a'a'} D_{a'a'})} = \sqrt[4]{0.7788 \times 10^{-2} \times 8 \times 0.7788 \times 10^{-2} \times 8}$$
$$= \sqrt[4]{\left(4.67 \times 10^{-2}\right)^2 \times 8^2} = 0.2496 \text{ m} = 24.96 \text{ cm}$$
$$D_{s3} = D_{s1} = 26.8 \text{ cm} = 0.268 \text{ m}$$
$$D_s \text{ is the self GMD} = \sqrt[3]{D_{s1} D_{s2} D_{s3}}$$
$$= \sqrt[3]{(0.268)^2 \times 0.2496} = 0.2617 \text{ m} = 0.2617 \text{ cm}$$
$$D_m = \sqrt[3]{D_{m1} D_{m2} D_{m3}}$$

where, D_{m1}, D_{m2} and D_{m3} are the mutual GMDs in positions 1, 2 and 3, respectively.

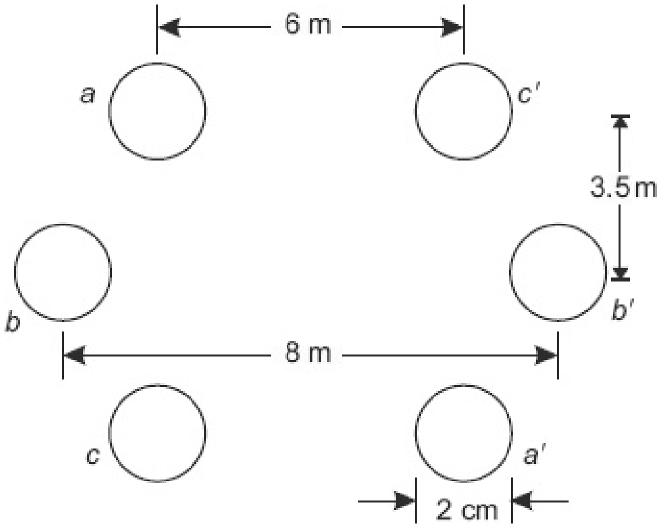

Fig. 2.31 Three-phase double circuit line

$$D_{m1} = \sqrt[8]{(D_{ab}D_{ac}D_{ab'}D_{ac'})(D_{a'b}D_{a'c}D_{a'b'}D_{a'c'})}$$

$$D_{m1} = D_{m3} = \sqrt[8]{3.64 \times 7 \times 7.826 \times 6 \times 7.826 \times 6 \times 3.64 \times 7} = 5.881 \text{ m}$$

$$D_{m2} = \sqrt[8]{(D_{ab}D_{ac}D_{ab'}D_{ac'})(D_{a'b}D_{a'c}D_{a'b'}D_{a'c'})}$$

$$= \sqrt[8]{3.64 \times 3.64 \times 7.826 \times 7.826 \times 7.826 \times 7.826 \times 3.64 \times 3.64} = 5.337 \text{ m}$$

$$\text{Mutual GMD, } D_m = \sqrt[3]{D_{m1}D_{m2}D_{m3}} = \sqrt[3]{5.881 \times 5.881 \times 5.337} = 5.694 \text{ m}$$

$$\therefore \text{ Inductance per phase, } L = 0.2 \ln \frac{D_m}{D_s} \text{ mH/km}$$

$$= 0.2 \ln \frac{5.694}{0.2617} = 0.616 \text{ mH/km}$$

EXAMPLE 2.18

In a single-phase double circuit transmission line as shown in Fig. 2.32, conductors *a* and *a′* are parallel to each other in one conductor, while conductors *b* and *b′* are parallel to each other in the other conductor. Each conductor radius is 2 cm. Determine the loop inductance of the line per kilometre assuming that the two parallel conductors equally share the current.

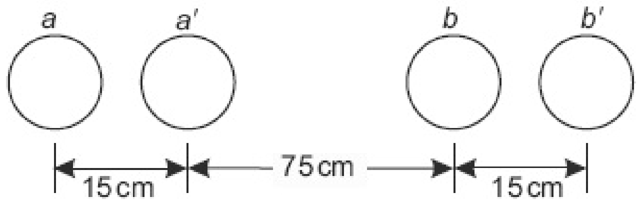

Fig. 2.32 Single-phase transmission line

Solution:

Loop inductance per kilometre is

$$L = 0.4\ \ln\frac{D_m}{D_s}\ \text{mH/km}$$

$$D_m = \sqrt[4]{D_{ab}D_{ab'}D_{a'b}D_{a'b'}}$$

$$= \sqrt[4]{90\times105\times75\times90} = 89.37\ \text{cm}$$

$$D_s = \sqrt[4]{D_{aa}D_{aa'}D_{a'a}D_{a'a}}$$

$$D_{aa} = D_{a'a'} = 0.7788\times2 = 1.5576\ \text{cm}$$

$$\text{and } D_{aa'} = D_{a'a} = 15\ \text{cm}$$

$$\therefore D_s = \sqrt[4]{(1.5576\times15)^2} = 4.83\ \text{cm}$$

$$L = 0.4\ln\left(\frac{89.37}{4.83}\right) = 1.167\ \text{mH/km}$$

2.13 CAPACITANCE

Considering magnetic field is important for the determination of inductance, whereas the electric field is important for capacitance.

Capacitance is the ability or capacity to store electric charges in conductors. It exists between any two insulated conductors and hence, electric energy is related to the capacitance of the conductors. A capacitor or condenser is made of two conductors separated by a dielectric or insulating medium. The capacitance of a capacitor is defined as the charge per unit potential difference. It is denoted by *C* and its units are faradays (F).

$$C = \frac{Q}{V} \text{ F} \tag{2.74}$$

where,

C is the capacitance in farads (F)

Q is the charge in coulombs (C)

V is the potential in volts (V)

2.14 POTENTIAL DIFFERENCE BETWEEN TWO POINTS DUE TO A CHARGE

The potential difference between two points is equal to the work in joules per coulomb necessary to move a coulomb of charge between the two points. The electric field intensity in volts per metre is equal to the force in Newton's per coulomb. According to the Gauss' theorem, the field intensity (E) at distance x on the circle is

$$E = \frac{Q}{2\pi\varepsilon x} \text{ V/m} \tag{2.75}$$

Between two points, the line integral of the force acting on a positive charge (Q) is the work done in moving the charge from the point of lower potential to the point of higher potential, and is numerically equal to the potential difference between the two points.

Consider a long straight wire carrying a positive charge as shown in Fig. 2.33. Points 1 and 2 are located at distances R_1 and R_2 metres from the centre of the wire. The potential difference between the points 1 and 2 on equipotential surfaces can be computed by integrating the electric field intensity over a radial path between the equipotential surfaces. Thus, the instantaneous potential difference between points 1 and 2 is

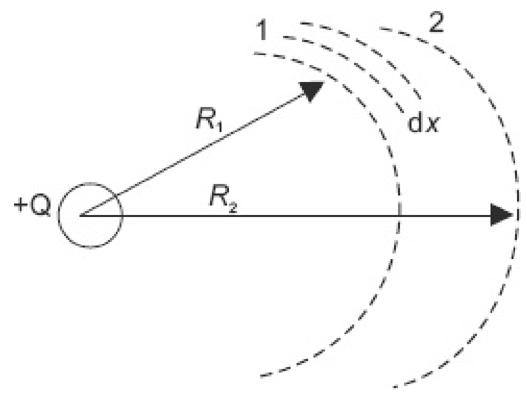

Fig. 2.33 Path of integration between two points in an electrical field

$$V_{12} = \int_{R_1}^{R_2} E\mathrm{d}x = \int_{R_1}^{R_2} \frac{Q}{2\pi x\varepsilon}\mathrm{d}x = \frac{Q}{2\pi\varepsilon}\ln\frac{R_2}{R_1} \text{ V} \qquad (2.76)$$

where, Q is the instantaneous charge on the wire in coulombs per metre length.

2.15 CAPACITANCE OF A SINGLE-PHASE LINE (TWO-WIRE LINE)

Figure 2.34(a) represents the cross-sectional view of single-phase two-wire system. Capacitance between the two conductors of a two-wire system is defined as, the charge on either conductor per unit potential difference between the conductors. Mathematically it can be written as,

$$C = \frac{Q}{V} \text{ F/m} \qquad (2.77)$$

where,

Q is the charge on the line in coulombs per metre length.

V is the potential difference between the conductors in volts.

Let us consider a single-phase system, which has conductors *a* and *b* with radius *r* and spaced *D* m apart. If the charge per metre length of conductor a is Q_a, then the charge on conductor *b* is Q_b.

Now, assume a point P, which is far away from the circuit as shown in Fig. 2.34(a). Then the voltage between the conductor *a* and point P (imaginary), V_{ap} (also equal to V_a) of the two-wire line can be written as per Eq. (2.76)

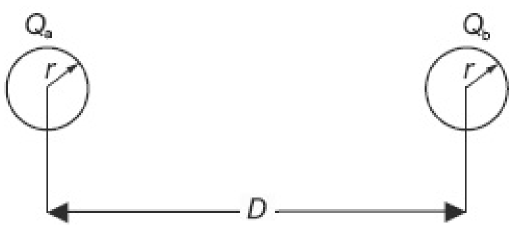

Fig. 2.34(a) Single-phase two-wire system

$$V_{ap} = V_a = \frac{Q_a}{2\pi\varepsilon} \ln \frac{D_{ap}}{r} \text{ V} \tag{2.78}$$

where,

r is the radius of each conductor

D is the distance between two conductors

Q_a is the charge on conductor *a*

D_{ap} is the distance between the conductor *a* and the point P

V_{ap} is the voltage between the conductor *a* and point P due to charge on conductor *a*

Similarly, the voltage V_{ap} is the voltage due to charge on conductor b between the conductor a and point P

$$V_{ap} = V_a = \frac{Q_b}{2\pi\varepsilon} \ln \frac{D_{bp}}{D} \text{ V} \tag{2.79}$$

By the principle of superposition, the voltage between the conductor a and the point P, due to the charges on both the conductors, is the sum total of the voltages caused by each charge alone

$$\therefore V_a = \frac{1}{2\pi\varepsilon}\left[Q_a \ln \frac{D_{ap}}{r} + Q_b \ln \frac{D}{D_{bp}}\right] \text{V} \tag{2.80}$$

where, Q_a and Q_b are the charges on both the conductors

$$V_a = \frac{1}{2\pi\varepsilon} Q_a \ln \frac{D}{r}\frac{D_{ap}}{D_{bp}} \quad (\because Q_a = -Q_b)$$
$$= \frac{1}{2\pi\varepsilon} Q_a \ln \frac{D}{r} \text{ V} \quad \left(\because \frac{D_{ap}}{D_{bp}} = 1\right) \tag{2.81}$$

The symbolic representation of line-to-line and line-to-neutral capacitances are shown in Figs. 2.34(b) and (c).

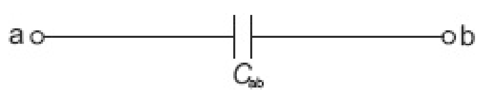

Fig. 2.34(b) Line-to-line capacitance

∴ Capacitance of conductor a (between line-to-neutral),

$$C_{an} = \frac{Q_a}{V_a} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{18\ln\frac{D}{r}} \text{ F/m} \quad (2.82)$$

The capacitance between the conductors *a* and *b* (line-to-line) is,

$$C_{ab} = \frac{\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{36\ln\frac{D}{r}} = \frac{1}{36\ln\frac{D}{r}} \ \mu\text{F/km} \quad (2.83)$$

Charging current due to capacitance The charging current for single-phase system is,

$$I_c = \frac{V}{\frac{-j}{\omega C_{ab}}} = j\omega C_{ab} V \text{ A}$$

where, V = voltage between the conductor in volts

$\omega = 2\pi f$ rad/sec

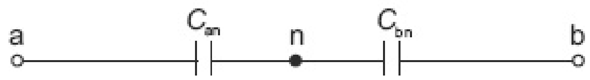

Fig. 2.34(c) Line-to-neutral capacitance

EXAMPLE 2.19

Calculate the capacitance of a conductor to neutral in a single-phase transmission line having two parallel conductors spaced 3 m apart. The diameter of each conductor is 1.2 cm.

Solution:

Spacing between the conductor, D = 3 m

Diameter of a conductor, d = 1.2 cm

Radius of conductor, r = 0.6 cm

∴Capacitance of conductor a to neutral,

$$C_a = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{18\ln\frac{300}{0.6}} = 8.94 \text{ nF/km}$$

The capacitance between the conductors,

$$C_{ab} = \frac{C_a}{2} = 4.47 \text{ nF/km}$$

EXAMPLE 2.20

Calculate the capacitance of a pair of parallel conductors spaced 40 cm apart in air. The diameter of the conductor is 12 mm.

Solution:

Spacing between the conductors, D = 40 cm

Diameter of the conductor, d = 12 mm

Effective radius of conductor, r = 6 mm = 0.6 cm

∴Capacitance between line-to-neutral,

$$C_1 = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{18\ln\frac{40}{0.6}} = 13.23 \text{ nF/km}$$

The capacitance between the two conductors (line-to-line),

$$C = \frac{C_1}{2} = 6.615 \text{ nF/km}$$

EXAMPLE 2.21

Calculate the capacitance of a single-phase overhead line consisting of a pair of parallel wires 12 mm in diameter and spaced uniformly 2.5 m apart. If the line is 30 km long and its one end is connected to 50 kV, 50 Hz system, what will be the charging current when the other end is open-circuited?

Solution:

Spacing between the conductors, D = 2.5 m

Diameter of the conductor, d = 12 mm

Effective radius of conductor, r = 6 mm = 6×10^{-3} m

$$\therefore \text{Capacitance of one conductor, } C_1 = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{18\ln\frac{2.5}{6\times10^{-3}}} = 9.21 \text{ nF/km}$$

$$\text{The capacitance between the two conductors, } C = \frac{C_1}{2} = 4.605 \text{ nF/km}$$

And the capacitance between the two conductors for 30 km, $C = 4.605 \times 10^{-3} \times 30\ \mu\text{F}$
$= 0.1381\ \mu\text{F}$

$$\text{The charging current for single-phase system, } I_c = \frac{V}{\frac{-j}{\omega C}} = j\omega CV \text{ A}$$

where,

V = voltage between the conductor in volts

$\omega = 2\pi f$ rad/sec

Charging current, $I_c = j\omega CV$ A

$= j2\times\pi\times50\times0.13815\times10^{-6}\times50000$

$= j2.17$ A

2.16 POTENTIAL DIFFERENCE BETWEEN TWO CONDUCTORS OF A GROUP OF CHARGED CONDUCTORS

Consider N number of parallel conductors arranged in a group, each conductor with a radius r (and assume all are in parallel). A group of parallel conductors are shown in Fig. 2.10.

Let us assume an infinite point P which is far away from the group of conductors and the potential of each is

$V_1, V_2, V_3,\ldots, V_N$ with respect to the point P due to the charges on all conductors in a group.

The potential of conductor 1 due to all charges, $V_1 = \frac{1}{2\pi\varepsilon_0}\left[Q_1 \ln\frac{1}{r} + Q_2 \ln\frac{1}{D_{12}} + \cdots + Q_N \ln\frac{1}{D_{1N}}\right]$

Similarly,

The potential of conductor 2, $V_2 = \frac{1}{2\pi\varepsilon_0}\left[Q_1 \ln\frac{1}{D_{21}} + Q_2 \ln\frac{1}{r} + \cdots + Q_N \ln\frac{1}{D_{2N}}\right]$

And the potential of conductor N, $V_N = \frac{1}{2\pi\varepsilon_0}\left[Q_1 \ln\frac{1}{D_{N1}} + Q_2 \ln\frac{1}{D_{N2}} + \cdots + Q_N \ln\frac{1}{r}\right]$

Then the potential difference between conductors 1 and 2 is

$$V_{12} = \frac{1}{2\pi\varepsilon_0}\left[Q_1 \ln\frac{D_{21}}{r} + Q_2 \ln\frac{r}{D_{12}} + \cdots + Q_N \ln\frac{D_{2N}}{D_{1N}}\right]$$

Similarly,

The potential difference between conductors 1 and N is

$$V_{1N} = \frac{1}{2\pi\varepsilon_0}\left[Q_1 \ln\frac{D_{N1}}{r} + Q_2 \ln\frac{D_{N2}}{D_{12}} + \cdots + Q_N \ln\frac{r}{D_{1N}}\right]$$

Here, $D_{12}, D_{13},\ldots, D_{1N}$ are the distances from the conductors 1 and 2, 1 and 3,..., 1 and N, respectively.

2.17 CAPACITANCE OF THREE-PHASE LINES

The capacitance of three-phase transmission lines can be determined by symmetrical and unsymmetrical spacing between the conductors which is discussed in the following sections.

2.17.1 Equilateral Spacing

Figure 2.35 shows the three-phase conductor *a*, *b* and *c* placed at the corners of an equilateral triangle and assumes each side to be *D* and the radius of each conductor to be *r*. If the conductors *a*, *b* and *c* have charges Q_a, Q_b and Q_c, respectively then the sum of all the charges is equal to zero.

Expression for the voltage between the conductor *a* and the point P which is far away from the circuit due to the charge on conductor *a* is

$$V_{apa} = \frac{1}{2\pi\varepsilon}\left(Q_a \ln\frac{1}{r}\right) \text{V} \tag{2.85}$$

Similarly expressions for the voltage between the conductor *a* and the point P due to the charges on conductors *b*, and *c*, are

$$V_{apb} = \frac{1}{2\pi\varepsilon}\left(Q_b \ln\frac{1}{D}\right) \text{V} \tag{2.86}$$

$$V_{apc} = \frac{1}{2\pi\varepsilon}\left(Q_c \ln\frac{1}{D}\right) \text{V} \tag{2.87}$$

The potential difference between the conductor *a* and the point P due to charges Q_a, Q_b and Q_c are

$$V_{ap} = \frac{1}{2\pi\varepsilon}\left(Q_a \ln\frac{1}{r} + Q_b \ln\frac{1}{D} + Q_c \ln\frac{1}{D}\right) \text{V} \tag{2.88}$$

$$= \frac{1}{2\pi\varepsilon}\left(Q_a \ln\frac{1}{r} + (Q_b + Q_c)\ln\frac{1}{D}\right) \text{V}$$

$$= \frac{1}{2\pi\varepsilon}\left(Q_a \ln\frac{1}{r} - Q_a \ln\frac{1}{D}\right) \text{V} \qquad (\because Q_b + Q_c = -Q_a)$$

$$\therefore V_{ap} = V_a = \frac{1}{2\pi\varepsilon} Q_a \ln\frac{D}{r} \text{ V}$$

$$\therefore \text{Capacitance of conductor, } C_a = \frac{Q_a}{V_a} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} = \frac{10^{-9}}{18\ln\left(\frac{D}{r}\right)} \text{ F/m} \tag{2.89}$$

Because of the symmetrical system the capacitance of all the conductors are same.

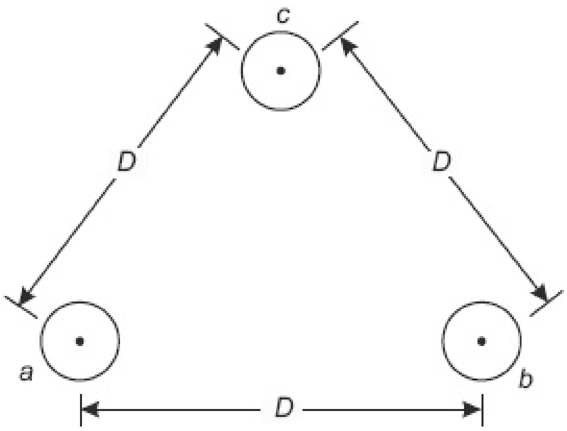

Fig. 2.35 Three-phase line with symmetrical spacing

$$\therefore \text{Capacitance per conductor, } C = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} = \frac{10^{-9}}{18\ln\left(\frac{D}{r}\right)} \text{ F/m} \tag{2.90}$$

Charging Current due to Capacitance The charging current per phase of a three-phase system is

$$I_c = j\omega C V_{ph} \text{ A} \tag{2.91}$$

where,

C is the capacitance per phase

V_{ph} is the voltage per phase

EXAMPLE 2.22

Calculate the capacitance of a conductor per phase of a three-phase 400 km long line, with the conductors spaced at the corners of an equilateral triangle of side 4 m and the diameter of each conductor being 2.5 cm. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Spacing between the conductors, D = 4 m = 400 cm

Diameter of the conductor, d = 2.5 cm

Radius of conductor, r = 1.25 cm = 1.25×10^{-2} m

$$\therefore \text{ Capacitance per conductor, } C = \frac{10^{-9}}{18\ln\left(\frac{D}{r}\right)} \text{ F/m}$$

$$= \frac{10^{-9}}{18\ln\left(\frac{400}{1.25}\right)} \text{F/m} = 9.631\times10^{-12} \text{ F/m}$$

$$= 9.631 \text{ nF/km}$$

$$\therefore \text{ Capacitance of a 400 km long conductor per phase, } C = 9.631\times10^{-12}\times400\times10^{3}$$

$$= 3.852\times10^{-6} \text{ F}$$

$$= 3.852\ \mu\text{F}$$

EXAMPLE 2.23

A three-phase, 50 Hz, 66 kV overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 3 m sides and the diameter of each conductor is 1.5 cm as shown in Fig. 2.36. Determine the inductance and capacitance per phase, if the length of line is 100 km and calculate the charging current.

Solution:

Spacing between the conductors, D =3.0 m = 300 cm

Diameter of each conductor, d =1.5 cm

Radius of each conductor, r = 0.75 cm

Effective radius of each conductor, r' = 0.7788 × 0.75 cm = 0.5841 cm

Operating voltage per phase, $V_{ph} = \frac{66}{\sqrt{3}} \times 1000 = 38105.1$ V

(a) Inductance per phase, $L = 2\times10^{-7} \ln\frac{D}{r'}$ H/m

$$L = 2\times10^{-7} \ln\frac{300}{0.5841} = 1.248\times10^{-6}\,\text{H/m} = 1.248\ \text{mH/km}$$

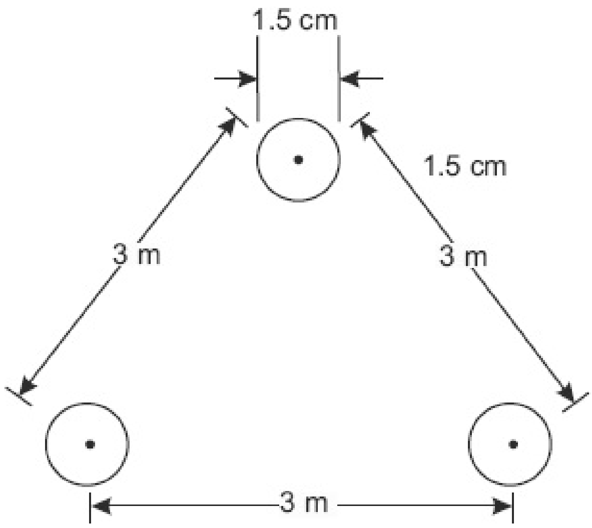

Fig. 2.36 Location of conductors for Example 2.23

Inductance of 100 km long conductor per phase = 1.248 × 10^{-3} × 100 = 0.1248 H

(b) Capacitance per phase of a three-phase circuit, $C = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$

$$= \frac{2\pi\varepsilon}{\ln\frac{300}{0.75}} = 9.28 \text{ pF/m} = 9.28 \text{ nF/km}$$

Capacitance for 100 km long line per phase = $9.28 \times 10^{-9} \times 100$ = 0.928 μF

Charging per phase, $I_c = j\omega C V_{ph}$ A

$$= j2\pi \times 50 \times 0.928 \times 10^{-6} \times 38105.1 = 11.11 \text{ A}$$

2.17.2 Capacitance of an Unsymmetrical Three-phase System (Transposed)

For a three-phase unsymmetrical system (untransposed), the capacitance between conductors to neutral of the three conductors are unequal, whereas, for transposed system, the average capacitance of each conductor to neutral is the same. The three positions of the conductor are shown in Fig. 2.37.

From Fig. 2.37, let us consider a point P, which is far away from the system. The potential difference between the conductor *a* and the point P due to the charges, Q_a, Q_b and Q_c in position 1 is

$$V_{a1} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_{1p}}{r} + Q_b \ln\frac{D_{2p}}{D_{12}} + Q_c \ln\frac{D_{3p}}{D_{31}}\right] \tag{2.92}$$

The potential difference between the conductor *a* and point P due to the charges Q_a, Q_b and Q_c in position 2 is

$$V_{a2} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_{2p}}{r} + Q_b \ln\frac{D_{3p}}{D_{23}} + Q_c \ln\frac{D_{1p}}{D_{12}}\right] \tag{2.93}$$

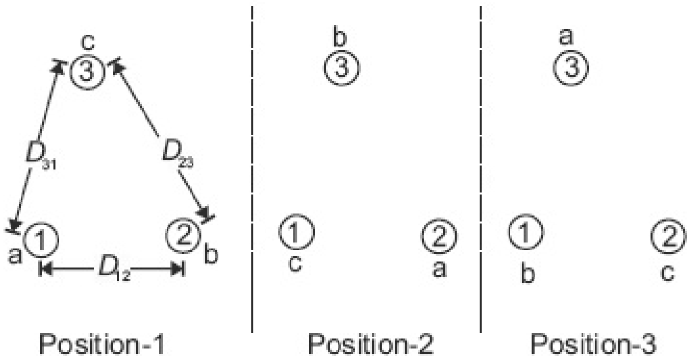

Fig. 2.37 Three-phase unsymmetrical spaced transmission line (transposed)

The potential difference between the conductor *a* and point P due to the charges Q_a, Q_b and Q_c in position 3 is

$$V_{a3} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_{3p}}{r} + Q_b \ln\frac{D_{1p}}{D_{31}} + Q_c \ln\frac{D_{2p}}{D_{23}}\right] \tag{2.94}$$

The average voltage of phase 'a' with respect to point P is

$$V_a = \frac{V_{a1} + V_{a2} + V_{a3}}{3} \tag{2.95}$$

Substituting the value of V_{a1}, V_{a2} and V_{a3} derived from Eqs. (2.92) to (2.94) in Eq. (2.95), we get

$$V_a = \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{D_1D_2D_3}{r^3} + Q_b \ln\frac{D_1D_2D_3}{D_{12}D_{23}D_{31}} + Q_c \ln\frac{D_1D_2D_3}{D_{12}D_{23}D_{31}}\right]$$

$$= \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{D_1D_2D_3}{r^3} - Q_a \ln\frac{D_1D_2D_3}{D_{12}D_{23}D_{31}}\right] \quad (\because Q_b + Q_c = -Q_a)$$

$$= \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3}\right]$$

$$= \frac{1}{2\pi\varepsilon} Q_a \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r} \tag{2.96}$$

$$\text{Capacitance of conductor 'a', } C_a = \frac{2\pi\varepsilon}{\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r}} \text{ F/m} \tag{2.97}$$

Similarly, the conductors *b* and *c* also occupy all the three positions as that of conductor *a*. The capacitance of the conductors *b* and *c* is also the same as conductor *a*.

Therefore, the capacitance per phase of the three-phase unsymmetrical spaced (transposed) single circuit system is

$$\text{Capacitance per phase, } C = \frac{2\pi\varepsilon}{\ln\frac{(D_{12}D_{23}D_{31})^{1/3}}{r}} \text{ F/m} \tag{2.98}$$

For the symmetrical spacing of the conductors,

i.e., $D_{12} = D_{23} = D_{31} = D$

$$\therefore \text{Capacitance per phase, } C = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} \tag{2.99}$$

EXAMPLE 2.24

Calculate the capacitance per phase of a three-phase three-wire transposed system as shown in Fig. 2.38 when the

conductors are arranged at the corners of a triangle with sides measuring 1.5 m, 2.0 m, and 2.5 m. Diameter of each conductor is 1.2 cm. (This question is also solved using MATLAB programs in the appendix.)

Solution:

Diameter of each conductor, d =1.2 cm

Spacing between the conductors a and b, D_{ab} = 1.5 m

Spacing between the conductors b and c, D_{bc} = 2.0 m

Spacing between the conductors c and a, D_{ca} = 2.5 m

Radius of each conductor, $r = \frac{1.2}{2} = 0.6 \text{ cm} = 0.6 \times 10^{-2} \text{ m}$

Capacitance of a conductor per phase, $C = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}} \text{ F/m} = \frac{10^{-9}}{18 \ln \frac{D_{eq}}{r}} \text{ F/m}$

Mutual GMD, D_m or $(D_{eq}) = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$

$$= (1.5 \times 2.0 \times 2.5)^{1/3} = 1.957 \text{ m} = 195.7 \text{ cm}$$

GMR, $r = 0.6$ cm.

$\therefore$ Capacitance of a conductor per phase $= \frac{10^{-9}}{18 \ln \frac{195.7}{0.6}} \text{ F/m} = 9.6 \text{ pF/m}$

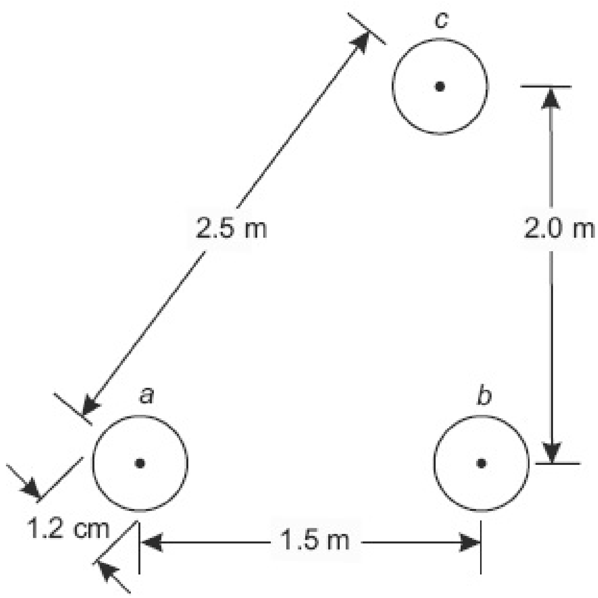

Fig. 2.38 Arrangement of conductors for Example 2.24

EXAMPLE 2.25

Calculate the capacitance of a three-phase three-wire system with triangular configuration with sides D_{12} = 3 m, D_{23} = 4 m and D_{31} = 5 m (See Fig. 2.39). The diameter of the conductor is 2.5 cm and the permittivity of the conductor material is 150.

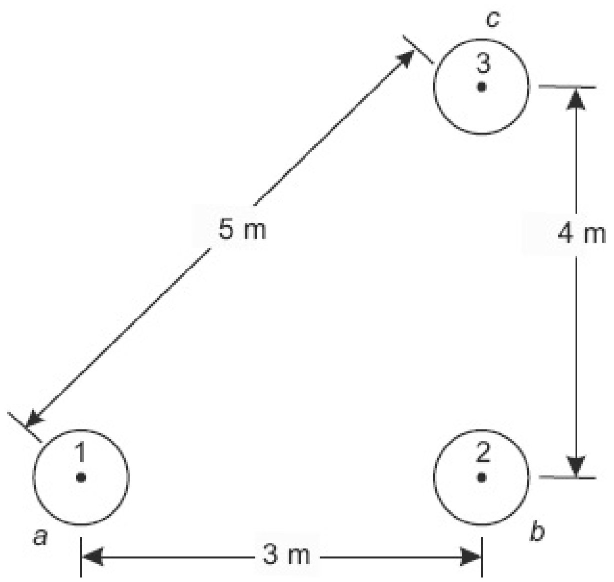

Fig. 2.39 Arrangement of conductors for example 2.25

Solution:

Diameter of each conductor, d = 2.5 cm

Spacing between the conductors a and b, D_{12} = 3 m

Spacing between the conductors b and c, D_{23} = 4 m

Spacing between the conductors c and a, D_{31} = 5 m

Radius of each conductor, $r = \frac{2.5}{2} = 1.25 \text{ cm} = 1.25 \times 10^{-2} \text{ m}$

Capacitance of a conductor per phase, $C = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}} \text{ F/m} = \frac{10^{-9}\varepsilon_r}{18 \ln \frac{D_{eq}}{r}} \text{ F/m}$

$$\text{Mutual GMD, } D_m \text{ or } (D_{eq}) = \sqrt[3]{D_{12}D_{23}D_{31}}$$
$$= (3 \times 4 \times 5)^{1/3} = 3.915 \text{ m} = 391.5 \text{ cm}$$

GMR, $r = 1.25$ cm.

$\therefore$ Capacitance of a conductor per phase $= \frac{10^{-9} \times 150}{18 \ln \frac{391.5}{1.25}} \text{ F/m} = 1.45\ \mu\text{F/km}$

EXAMPLE 2.26

A three-phase, three-wire 50 kV, 50 Hz overhead line conductor is placed in horizontal plane as shown in the Fig. 2.40. The conductor diameter is 1.5 cm. If the length of the line is 100 km, calculate (i) capacitance per phase per meter, (ii) charging current per phase.

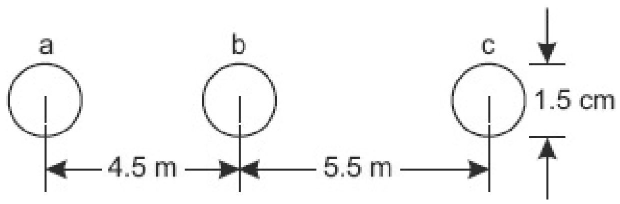

Fig. 2.40 Cross-sectional view of conductors in horizontal plane

Solution:

Diameter of each conductor, d = 1.5 cm

Radius of each conductor, r = 0.75 cm

$$\text{Voltage per phase, } V_{ph} = \frac{50\times10^3}{\sqrt{3}} = 28867.5 \text{ V}$$

$$\text{Mutual GMD, } D_m = \sqrt[3]{D_{ab}\times D_{bc}\times D_{ca}}$$

$$D_m = D_{eq} = \sqrt[3]{4.5\times5.5\times10} = 6.2785 \text{ m} = 627.85 \text{ cm}$$

$$\text{Capacitance of a conductor per phase, } C = \frac{2\pi\varepsilon}{\ln\frac{D_{eq}}{r}} \text{ F/m} = \frac{10^{-9}}{18\ln\frac{D_{eq}}{r}} \text{ F/m}$$

$$\text{Capacitance of a conductor per phase} = \frac{10^{-9}}{18\ln\frac{527.85}{0.75}} = 8.255 \text{ nF/km}$$

Capacitance for 100 km per phase = $8.255 \times 10^{-9} \times 100 = 0.8255$ μF

$$\text{Charging current per phase} = \omega C V_{ph} = 2\pi\times50\times0.8255\times10^{-6}\times28867.5 \text{ A}$$
$$= 7.49 \text{ A}$$

EXAMPLE 2.27

Calculate the capacitance per phase of a three-phase, three-wire system as shown in Fig. 2.41, when the conductors are arranged in a horizontal plane with spacing $D_{12} = D_{23} = 3.5$ m, and $D_{13} = 7$ m. The conductors are transposed and each has a diameter of 2.0 cm.

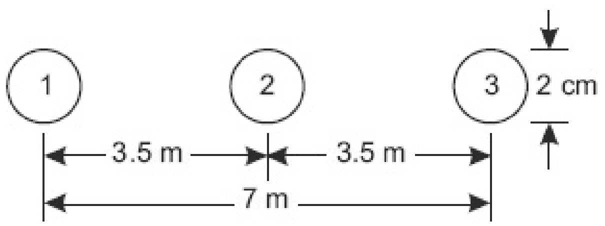

Fig. 2.41 Arrangement of conductor 1 for Example 2.27

Solution:

Diameter of each conductor, d = 2.0 cm

Radius of each conductor, r = 1.0 cm

$$\text{Mutual GMD, } D_m = \sqrt[3]{D_{12} \times D_{23} \times D_{31}}$$

$$D_m = D_{eq} = \sqrt[3]{3.5 \times 3.5 \times 7} = 4.41 \text{ m} = 441 \text{ cm}$$

$$\text{Capacitance of a conductor per phase, } C = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}} \text{ F/m} = \frac{10^{-9}}{18 \ln \frac{D_{eq}}{r}} \text{ F/m}$$

$$\text{Capacitance of a conductor per phase} = \frac{10^{-9}}{18 \ln \frac{441}{1.0}} = 9.124 \text{ pF/m} = 9.124 \text{ nF/km}$$

2.18 CAPACITANCE OF A THREE-PHASE DOUBLE-CIRCUIT LINE

Normally, two configurations of conductors are used in a double circuit line:

1. Hexagonal spacing
2. Flat vertical spacing

2.18.1 Hexagonal Spacing

Since the conductors of the same phase are connected in parallel, as shown in Fig. 2.42, the charge per unit length is the same. In addition, because of the symmetrical arrangement, the phases are balanced and the conductors of each phase are also balanced if the effect of group is neglected. Therefore, the transposition of conductors is not required.

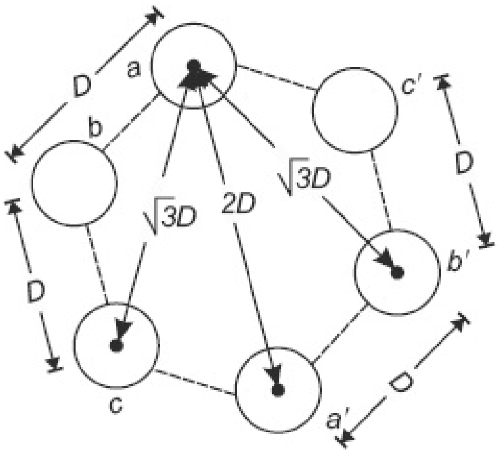

Fig. 2.42 Double-circuit three-phase line with hexagonal spacing

Assume a point P very far from the system of conductors, such that the distances of the conductors from P are almost same. It is to be noted here that point P corresponds to almost zero potential. The potential of conductor *a* with respect to point P due to the charge on the conductor itself and the charges on conductors *b*, *c*, *a*′, *b*′ and *c*′ is given by

$$V_a = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_a}{r} + Q_b \ln\frac{D_b}{D} + Q_c \ln\frac{D_c}{\sqrt{3}D} + Q_{a'} \ln\frac{D_{a'}}{2D} + Q_{b'} \ln\frac{D_{b'}}{\sqrt{3}D} + Q_{c'} \ln\frac{D_{c'}}{D}\right] \quad (2.100)$$

Since $Q_a = Q_{a'}$; $Q_b = Q_{b'}$; $Q_c = Q_{c'}$

$$\therefore V_a = \frac{1}{2\pi\varepsilon}\left[Q_a\left(\ln\frac{D_a}{r} + \ln\frac{D_{a'}}{2D}\right) + Q_b\left(\ln\frac{D_b}{D} + \ln\frac{D_{b'}}{\sqrt{3}D}\right) + Q_c\left(\ln\frac{D_c}{\sqrt{3}D} + \ln\frac{D_{c'}}{D}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a\left(\ln\frac{D_a D_{a'}}{2rD}\right) + Q_b\left(\ln\frac{D_b D_{b'}}{\sqrt{3}D^2}\right) + Q_c\left(\ln\frac{D_c D_{c'}}{\sqrt{3}D^2}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln D_a D_{a'} + Q_b \ln D_b D_{b'} + Q_c \ln D_c D_{c'} + Q_a \ln\frac{1}{2Dr} + Q_b \ln\frac{1}{\sqrt{3}D^2} + Q_c \ln\frac{1}{\sqrt{3}D^2}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln D_a D_{a'} + Q_b \ln D_b D_{b'} + Q_c \ln D_c D_{c'} + Q_a \ln\frac{1}{2Dr} + (Q_b + Q_c)\ln\frac{1}{\sqrt{3}D^2}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln D_a D_{a'} + Q_b \ln D_b D_{b'} - (Q_a + Q_b)\ln D_c D_{c'} + Q_a \ln\frac{1}{2Dr} - Q_a \ln\frac{1}{\sqrt{3}D^2}\right] \quad (\because Q_b + Q_c = -Q_a)$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_a D_{a'}}{D_c D_{c'}} + Q_b \ln\frac{D_b D_{b'}}{D_c D_{c'}} + Q_a \ln\frac{1}{2Dr} - Q_a \ln\frac{1}{\sqrt{3}D^2}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{\sqrt{3}D^2}{2Dr}\right] \quad \left(\because \frac{D_a D_{a'}}{D_c D_{c'}} = \frac{D_b D_{b'}}{D_c D_{c'}} = 1\right)$$

$$= \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{\sqrt{3}D^2}{2Dr}\right]$$

$$\text{Capacitance of conductor, } C_a = \frac{2\pi\varepsilon}{\ln\frac{\sqrt{3}D}{2r}} \text{ F/m} \quad (2.101)$$

Equation (2.101) represents an expression for the capacitance of conductor 'a' alone, whereas there are two conductors per phase, a and a'. Therefore, the capacitance of the system will be twice the capacitance of one conductor to neutral.

$$\text{Capacitance per phase, } C = 2C_a = \frac{4\pi\varepsilon}{\ln\left(\frac{\sqrt{3}D}{2r}\right)} = \frac{2\pi\varepsilon}{\ln\left(\frac{\sqrt{3}}{2}\right)^{1/2}\left(\frac{D}{r}\right)^{1/2}} \text{ F/m} \quad (2.102)$$

Since the conductors of different phases are symmetrically placed, the expression for capacitance for other phases will also be the same.

EXAMPLE 2.28

Calculate the capacitance (phase-to-neutral) of a three-phase 100 km long double circuit line shown in Fig. 2.43 with conductors of diameter 2.0 cm each arranged at the corners of an hexagon with sides measuring 2.1 m. (This question is also solved using MATLAB programs in the appendix.)

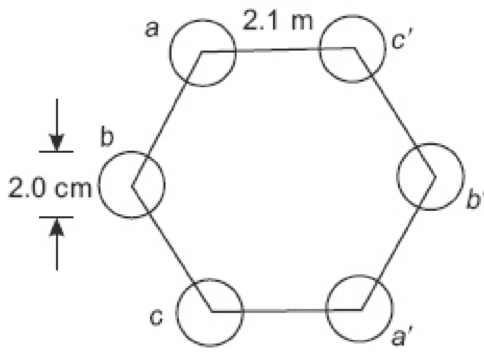

Fig. 2.43 Cross-sectional view of conductors in hexagonal spacing

Solution

Phase A is composed of conductors *a, a′*; phase B of *b* and *b′*; and phase C of *c* and *c′*. The two conductors of each phase are electrically parallel and have the same charge. Because of the symmetrical arrangement, the phases are balanced and the conductors of each individual phase are also balanced, if the effect of ground is neglected. Therefore, transposition of conductors is not necessary to balance the phases.

Diameter of each conductor, d = 2.0 cm

Radius of each conductor, r = 1.0 cm

Each side of a hexagon, D = 2.1 m = 210 cm

$$\text{Capacitance per phase, } C_n = 2C_{an} = \frac{2\times 2\pi\varepsilon}{\left(\ln\frac{\sqrt{3}D}{2r}\right)} \text{ F/m}$$

$$= \frac{2\pi\varepsilon}{\ln\left(\frac{\sqrt{3}D}{2r}\right)^{1/2}} = \frac{10^{-9}}{18\ln\left(\frac{\sqrt{3}D}{2r}\right)^{1/2}}$$

$$= \frac{10^{-9}}{18\ln\left(\frac{\sqrt{3}\times 210}{2\times 1}\right)^{1/2}} = 0.0213\ \mu\text{F/km}$$

Capacitance for 100 km length per phase, $C = 0.0213 \times 100 = 2.13\ \mu\text{F}$

2.18.2 Flat Vertical Spacing (Unsymmetrical Spacing)

Consider Fig. 2.44, the conductors of different phases are not symmetrically placed; therefore, the derivation of capacitance expression will require the transposition of conductors as shown in the figure.

The average voltage of conductor '*a*' is required to be calculated in the three different positions due to the charge on conductor '*a*' and the conductors *b*, *c*, *a*′, *b*′ and *c*′.

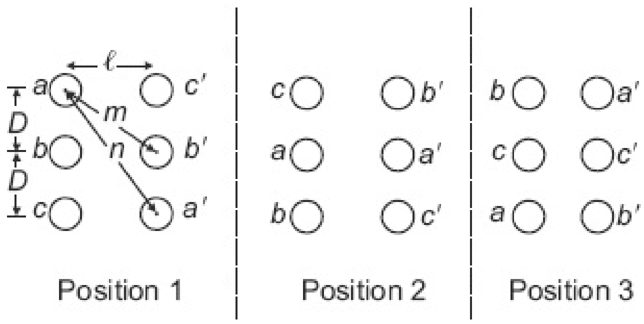

Fig. 2.44 Conductor arrangement of three-phase double-circuit transposed line

For this, we again assume a point P which is far away from the system of conductors such that $D_a \approx D_b \approx Dc \approx D_{a'} \approx D_{b'} \approx D_{c'}$.

Since point P is at a very large distance from the system of conductors, the potential of point P is approximately zero.

$$V_{a1} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_a}{r} + Q_b \ln\frac{D_b}{D} + Q_c \ln\frac{D_c}{2D} + Q_{a'} \ln\frac{D_{a'}}{n} + Q_{b'} \ln\frac{D_{b'}}{m} + Q_{c'} \ln\frac{D_{c'}}{l}\right] \quad (2.103)$$

Using the relations $D_a \cong D_b \cong D_c \cong D_{a'} \cong D_{b'} \cong D_{c'}$,

$$Q_a + Q_b + Q_c = 0 \text{ and } Q_a = Q_{a'}; Q_b = Q_{b'} \text{ and } Q_c = Q_{c'}$$

$$\therefore V_{a1} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{D_a D_{a'}}{nr} + Q_b \ln\frac{D_b D_{b'}}{mD} + Q_c \ln\frac{D_c D_{c'}}{2lD}\right]$$

$$V_{a1} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{1}{nr} + Q_b \ln\frac{1}{mD} + Q_c \ln\frac{1}{2lD}\right] \quad (2.104)$$

Similarly the potential of conductor a in position 2

$$V_{a2} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{1}{lr} + Q_b \ln\frac{1}{Dm} + Q_c \ln\frac{1}{Dm}\right] \quad (2.105)$$

and position 3

$$V_{a3} = \frac{1}{2\pi\varepsilon}\left[Q_a \ln\frac{1}{nr} + Q_b \ln\frac{1}{2lD} + Q_c \ln\frac{1}{Dm}\right] \quad (2.106)$$

The average potential of conductor a in all three positions is

$$V_a = \frac{V_{a1} + V_{a2} + V_{a3}}{3} \quad (2.107)$$

Substituting the Eqs. (2.104) to (2.106) in Eq. (2.107)

$$V_a = \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{1}{r^3 n^2 l} + Q_b \ln\frac{1}{2lm^2 D^3} + Q_c \ln\frac{1}{2lm^2 D^3}\right] \tag{2.108}$$

$$= \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{1}{r^3 n^2 l} - Q_a \ln\frac{1}{2lm^2 D^3}\right] \quad [\therefore Q_b + Q_c = -Q_a]$$

$$= \frac{1}{6\pi\varepsilon}\left[Q_a \ln\frac{2lm^2 D^3}{ln^2 r^3}\right]$$

$$V_a = \frac{1}{2\pi\varepsilon}\left[Q_a \ln 2^{1/3}\left(\frac{m}{n}\right)^{2/3}\frac{D}{r}\right] \tag{2.109}$$

$$\therefore \text{Capacitance per conductor, } C = \frac{Q_a}{V_a} = \frac{2\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \text{ F/m} \tag{2.110}$$

$$\text{The capacitance per phase, } C = \frac{4\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \text{ F/m} \tag{2.111}$$

$$= \frac{1}{9\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \ \mu\text{F/km} \tag{2.112}$$

EXAMPLE 2.29

Calculate the capacitance per phase of a three-phase, double-circuit line as shown in Fig. 2.45. The diameter of each conductor is 1.5 cm. (This question is also solved using MATLAB programs in the appendix.)

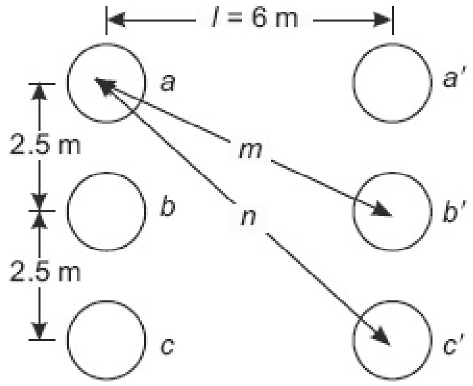

Fig. 2.45 Vertical configuration of three-phase, double-circuit line

Solution:

Horizontal distance, l = 6 m

Vertical distance, D = 2.5 m = 250 cm

Diameter of conductor, d = 1.5 cm

Radius of conductor, r = 0.75 cm

$$\text{And, } m = \sqrt{6^2 + 2.5^2} = \sqrt{42.25} = 6.5 = 650 \text{ cm}$$
$$n = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.81 \text{ m} = 781 \text{ cm}$$

$$\text{Capacitance of conductor } a,\ C_a = \frac{2\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} = \frac{2\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}}$$

$$\text{Capacitance per phase, } C = \frac{4\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \text{ F/m/phase}$$ ($\because$ Conductors per phase are connected in parallel)

$$C = \frac{1}{9\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \ \mu\text{F/km}$$

$$C = \frac{1}{9\ln\left\{2^{1/3}\frac{250}{0.75}\left(\frac{650}{781}\right)^{2/3}\right\}} \ \mu\text{F/km/phase} = 0.0188 \ \mu\text{F/km}$$

EXAMPLE 2.30

Calculate the capacitance per phase of a three-phase double-circuit line as shown in Fig. 2.46. The diameter of the conductor is 2 cm. Assume that the line is completely transposed. (This question is also solved using MATLAB programs in the appendix.)

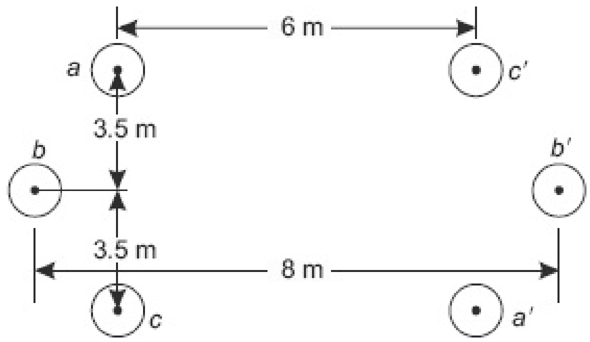

Fig. 2.46 Three-phase double circuit line

Solution:

Diameter of conductor, d = 2 cm

Radius of conductor, r = 1 cm

$$\text{Distance from } a \text{ to } b,\ D_{ab} = \sqrt{3.5^2 + 1^2} = 3.64 \text{ m}$$

$$a \text{ to } b',\ D_{ab'} = \sqrt{3.5^2 + 7^2} = 7.826 \text{ m}$$

$$a \text{ to } a',\ D_{aa'} = \sqrt{7^2 + 6^2} = 9.22 \text{ m}$$

$$\text{Capacitance per conductor, } C = \frac{1}{18 \ln \frac{D_m}{D_s}}\ \mu\text{F/km}$$

where, D_S is the self GMD = $\sqrt[3]{D_{s1} D_{s2} D_{s3}}$ of one phase and suffixes 1, 2 and 3 denote the self GMD in positions 1, 2 and 3, respectively. Also D_S is the same for all the phases.

$$
\begin{aligned}
D_{s1} &= \sqrt[4]{(D_{aa} D_{aa'} D_{a'a'} D_{a'a})} = \sqrt[4]{1\times10^{-2}\times9.22\times1\times10^{-2}\times9.22} \\
&= 0.3036 \text{ m} \\
D_{s2} &= \sqrt[4]{(D_{aa} D_{aa'} D_{a'a} D_{a'a'})} = \sqrt[4]{1\times10^{-2}\times8\times1\times10^{-2}\times8} \\
&= 0.2828 \text{ m}
\end{aligned}
$$

$$D_{s3} = D_{s1} = 0.3036 \text{ m}$$

$$D_s \text{ is the self GMD} = \sqrt[3]{D_{s1} D_{s2} D_{s3}}$$

$$= \sqrt[3]{(0.3036)^2 \times 0.2828} = 0.2965 \text{ m}$$

$$\text{Mutual GMD, } D_m = \sqrt[3]{D_{m1} D_{m2} D_{m3}}$$

where, D_{m1}, D_{m2} and D_{m3} are the mutual GMDs in positions 1, 2 and 3, respectively.

$$
\begin{aligned}
D_{m1} &= \sqrt[8]{(D_{ab} D_{ac} D_{ab'} D_{ac'})(D_{a'b} D_{a'c} D_{a'b'} D_{a'c'})} \\
&= \sqrt[8]{3.64\times7\times7.826\times6\times7.826\times6\times3.64\times7} = 5.881 \text{ m} = D_{m3} \\
D_{m2} &= \sqrt[8]{(D_{ab} D_{ac} D_{ab'} D_{ac'})(D_{a'b} D_{a'c} D_{a'b'} D_{a'c'})} \\
&= \sqrt[8]{3.64\times3.64\times7.826\times7.826\times7.826\times7.826\times3.64\times3.64} = 5.337 \text{ m}
\end{aligned}
$$

$$D_m = \sqrt[3]{D_{m1} D_{m2} D_{m3}} = \sqrt[3]{5.881\times5.881\times5.337} = 5.694 \text{ m}$$

$$\text{Capacitance per conductor, } C = \frac{1}{18 \ln \frac{D_m}{D_s}} \ \mu\text{F/km}$$

$$= \frac{1}{18 \ln \left[\frac{5.694}{0.2965}\right]} = 0.0188 \ \mu\text{F/km}$$

2.19 EFFECT OF EARTH ON TRANSMISSION LINE CAPACITANCE

Earth affects the capacitance of an overhead line, as its presence alters the electrical field of the line. The earth level is an equipotential surface, therefore the flux lines are forced to cut the surface of the earth orthogonally. The effect of the presence of earth can be accounted for by the method of image charges.

Consider a circuit, consisting of a single overhead conductor with a charge +*q* C/m of height *h* m above ground and imagine a fictitious conductor of charge –*q* C/m placed with a depth *h* m below the ground surface as shown in Fig. 2.47. This configuration, without the presence of the earth surface, will produce the same field distribution, which is produced by single charge, and the earth surface. Thus, for the purpose of calculation of capacitance, the earth may be replaced by an image charged conductor below the surface of the earth by a distance same as the height of the actual conductor above the earth. Such a conductor has a charge, equal in magnitude but opposite in sign to that of the original conductor and is known as *image conductor*.

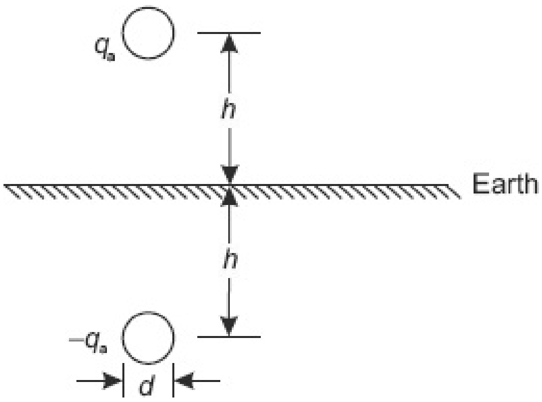

Fig. 2.47 One conductor line and its image

So far, the presence of earth was ignored while calculating the capacitance of transmission line. The effect of earth on capacitance can be conveniently taken into account by the method of images.

2.19.1 Capacitance of a Single Conductor

It is required to determine the capacitance of conductor to ground. The earth is replaced by a fictitious conductor as shown in Fig. 2.47. This means the single conductor with the earth is equivalent to a single-phase transmission line. The capacitance of a single-phase transmission line from Eq. (2.83) is given by

$$C = \frac{\pi\varepsilon_0}{\ln \frac{2h}{r}} \text{ F/m}$$

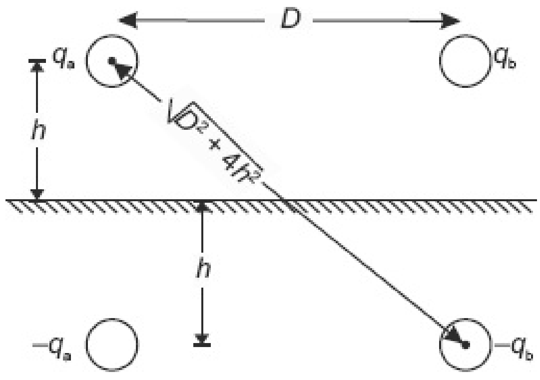

Fig. 2.48 Single-phase transmission line conductors and its images

The capacitance of the conductor with reference to ground is

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{2h}{r}} \text{ F/m} \tag{2.113}$$

2.19.2 Capacitance of a Single-Phase Transmission Line

Consider a single-phase transmission line as shown in Fig. 2.48. The earth surface is an equipotential surface. In order to include the effect of this surface on the capacitance of line *ab*, we imagine that there are two mirror image conductors below the earth surface with opposite charges.

Assume a point P very far from the system of conductors, such that the distances of the conductors from P are almost the same. It is to be noted here that point P corresponds to almost zero potential.

The expression for the voltage drop V_{ap} is calculated by the two charged conductors a and b and their images a' and b' can be written as

$$V_{ap} = V_a = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D} + q_{a'} \ln\frac{1}{2h} + q_{b'} \ln\frac{1}{\sqrt{D^2 + 4h^2}}\right)$$

$$V_a = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{1}{r} - q_a \ln\frac{1}{D} - q_a \ln\frac{1}{2h} + q_a \ln\frac{1}{\sqrt{D^2 + 4h^2}}\right)$$

$$\because q_{a'} = q_a,\ q_b = -q_a \text{ and } q_{b'} = q_b$$

$$V_a = \frac{1}{2\pi\varepsilon}\left\{q_a \ln\left(\frac{D}{r} \times \frac{2h}{\sqrt{D^2 + 4h^2}}\right)\right\}$$

$$C_a = \frac{q_a}{V_a}$$

$$C_n = 2C_a = \frac{2q_a}{V_a}$$

Capacitance of conductor ‘a’ is

$$C_a = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r} \cdot \frac{2h}{\sqrt{4h^2 + D^2}}\right)} \text{ F/m} \tag{2.114}$$

And the capacitance of single-phase system (between the conductors) is

$$C_{ab} = \frac{1}{36 \ln\left(\frac{D}{r} \cdot \frac{2h}{\sqrt{4h^2 + D^2}}\right)} \ \mu\text{F/km} \tag{2.115}$$

Comparing Eqs. (2.115) and (2.83), it is observed that due to earth effect, the capacitance of a single-phase transmission line is increased. However, when the distance between each conductor and the earth-level is large (when compared to distance between the conductors), the effect of earth on the capacitance is negligible.

EXAMPLE 2.31

Calculate the capacitance of a conductor to neutral in a single-phase transmission line having two parallel conductors spaced 3 m apart, by considering the earth effect on capacitance, when conductors are placed 5 m above the ground level as shown in the Fig. 2.49. The diameter of each conductor is 1.2 cm.

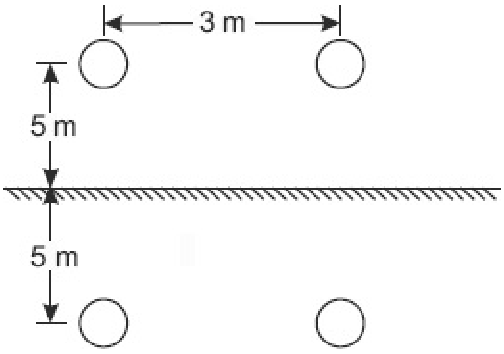

Fig. 2.49 Single-phase transmission line with two parallel conductors

Solution:

Spacing between the conductor, $D = 3$ m

Height of conductor above the ground, $h = 5$ m

Diameter of a conductor, $d = 1.2$ cm

Radius of conductor, $r = 0.6$ cm

$$\therefore \text{Capacitance of conductor } a,\ C_a = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\cdot\frac{2h}{\sqrt{4h^2+D^2}}\right)} \text{ F/m}$$

$$= \frac{10^{-9}}{18\ln\left(\frac{D}{r}\cdot\frac{2h}{\sqrt{4h^2+D^2}}\right)}$$

$$= \frac{10^{-9}}{18\ln\left(\frac{3}{0.006}\cdot\frac{2\times 5}{\sqrt{4\times 5^2+3^2}}\right)} \text{ F/m}$$

$$= 9.0 \text{ pF/m}$$

And the capacitance between the conductors is, $C_{ab} = \frac{C_a}{2} = 4.5$ nF/km

EXAMPLE 2.32

The conductors in a single-phase transmission line are 6 m above the ground as shown in the Fig. 2.50. Taking the effect of the earth into account, calculate the capacitance/kilometer. Each conductor is 2.0 cm in diameter and the conductors are spaced 3.5 m apart.

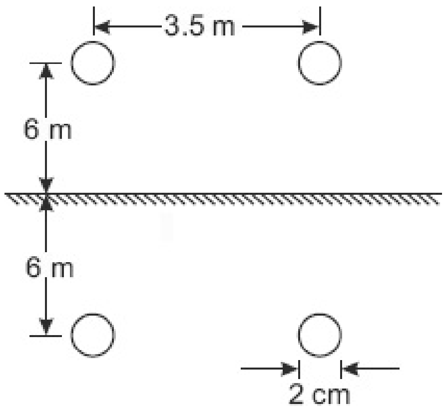

Fig. 2.50 Single-phase transmission line with two parallel conductors

Solution:

Diameter of a conductor, d = 2 cm

Radius of conductor, r = 1 cm

Spacing between the conductors, D = 3.5 m

$$\text{Capacitance of conductor a, } C_a = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\cdot\frac{2h}{\sqrt{4h^2+D^2}}\right)} \text{ F/m}$$

$$= \frac{10^{-9}}{18\ln\left(\frac{3.5}{0.01}\times\frac{2\times 6}{\sqrt{4\times 6^2+3.5^2}}\right)} \text{ F/m}$$

$$= 9.55 \text{ pF/m}$$

And the capacitance of the single-phase system (between the conductors) is

$$C_{ab} = \frac{C_a}{2} = 4.775 \text{ pF/m}$$

2.19.3 Capacitance of Three-Phase Line

The method of images can also be applied for the determination of capacitance of a three-phase line as shown in Fig. 2.51. The line is fully transposed. The conductors *a, b,* and *c* carry the charges q_a, q_b and q_c and occupy positions 1, 2 and 3, respectively in the first position of the transposed cycle. The earth effect is simulated by image conductors with charges q_a, q_b and q_c.

Assume a point P very far from the system of conductors such that the respective distances of the conductors from P are almost the same. It is to be noted here that point P corresponds to almost zero potential.

The expression for the voltage drop V_{ap} in three positions of the transposed line can be written by considering three charged conductors and their images.

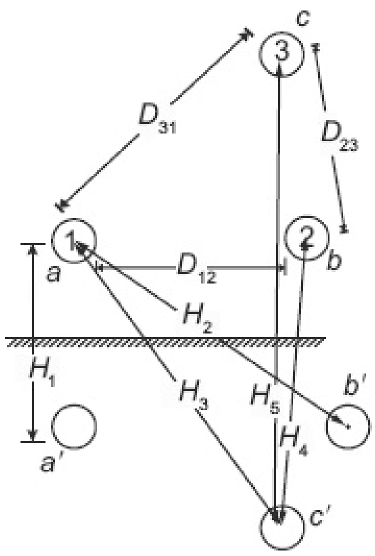

Fig. 2.51 Three-phase line conductors and its images

For position (1)

$$V_{a1} = \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{H_1}\right) + q_b\left(\ln\frac{1}{D_{12}} - \ln\frac{1}{H_2}\right) + q_c\left(\ln\frac{1}{D_{31}} - \ln\frac{1}{H_3}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{H_1}{r}\right) + q_b\left(\ln\frac{H_2}{D_{12}}\right) + q_c\left(\ln\frac{H_3}{D_{31}}\right)\right]$$

For position (2)

$$V_{a2} = \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{H_1}\right) + q_b\left(\ln\frac{1}{D_{23}} - \ln\frac{1}{H_4}\right) + q_c\left(\ln\frac{1}{D_{12}} - \ln\frac{1}{H_2}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{H_1}{r}\right) + q_b\left(\ln\frac{H_4}{D_{23}}\right) + q_c\left(\ln\frac{H_2}{D_{12}}\right)\right]$$

For position (3)

$$V_{a3} = \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{H_5}\right) + q_b\left(\ln\frac{1}{D_{31}} - \ln\frac{1}{H_3}\right) + q_c\left(\ln\frac{1}{D_{23}} - \ln\frac{1}{H_4}\right)\right]$$
$$= \frac{1}{2\pi\varepsilon}\left[q_a\left(\ln\frac{H_5}{r}\right) + q_b\left(\ln\frac{H_3}{D_{31}}\right) + q_c\left(\ln\frac{H_4}{D_{23}}\right)\right]$$

The average value of potential at a with respect to the point P is

$$\text{i.e., } V_{aP} = V_a = \frac{V_{a1} + V_{a2} + V_{a3}}{3}$$

$$V_a = \frac{1}{6\pi\varepsilon}\left[q_a\left(\ln\frac{H_1^2 H_5}{r^3}\right) + q_b\left(\ln\frac{H_2 H_3 H_4}{D_{12} D_{23} D_{31}}\right) + q_c\left(\ln\frac{H_2 H_3 H_4}{D_{12} D_{23} D_{31}}\right)\right]$$

$$= \frac{1}{6\pi\varepsilon}\left[q_a\left(\ln\frac{H_1^2 H_5}{r^3}\right) + (q_b + q_c)\left(\ln\frac{H_2 H_3 H_4}{D_{12} D_{23} D_{31}}\right)\right]$$
$$= \frac{1}{6\pi\varepsilon}\left[q_a\left(\ln\frac{H_1^2 H_5}{r^3}\right) - q_a\left(\ln\frac{H_2 H_3 H_4}{D_{12} D_{23} D_{31}}\right)\right] \qquad (\because q_b + q_c = -q_a)$$
$$= \frac{1}{6\pi\varepsilon}\left[q_a\left(\ln\left(\frac{D_{12} D_{23} D_{31}}{r^3}\,\frac{H_1^2 H_5}{H_2 H_3 H_4}\right)\right)\right]$$
$$= \frac{1}{2\pi\varepsilon} q_a \ln\left\{\frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{r}\left(\frac{H_1^2 H_5}{H_2 H_3 H_4}\right)^{1/3}\right\}$$

Capacitance of conductor *a* is

$$C_a = \frac{q_a}{V_a} = \frac{2\pi\varepsilon}{\ln\left\{\frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{r}\left(\frac{H_1^2 H_5}{H_2 H_3 H_4}\right)^{1/3}\right\}}$$

$$C_a = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r} H_{eq}\right)} \qquad (2.116)$$

$$\text{where, } D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

$$H_{eq} = \left(\frac{H_1^2 H_5}{H_2 H_3 H_4}\right)^{1/3}$$

Comparing Eqs. (2.116) and (2.98), it is observed that due to earth effect, the capacitance of a three-phase transmission line is increased. But when the distance between conductors and the earth-level is large as compared to the distance between the conductors, the effect of earth on the capacitance of a three-phase line is negligible.

Test Yourself

1. Do you require to consider the earth effect while calculating capacitance? If yes, justify.

EXAMPLE 2.33

Calculate the capacitance per phase of a three-phase, three-wire system by considering earth effect, when the conductors are arranged in a horizontal plane with spacing $D_{12} = D_{23}$ = 3.5 m, and D_{31} = 7 m as shown in the Fig. 2.52. The conductors are transposed and each has a diameter of 2.0 cm. Assume that the transmission line is 4 m above the ground level. (This question is also solved using MATLAB programs in the appendix.)

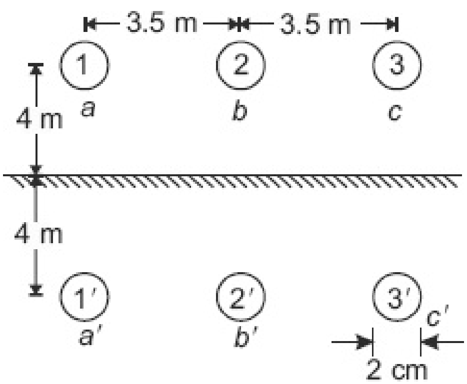

Fig. 2.52 Horizontal configuration of three-phase transmission line

Solution:

Diameter of conductor, d = 2.0 cm

Radius of conductor, r = 1.0 cm

$$\text{Capacitance per phase, } C = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\cdot H_{eq}\right)} = \frac{10^{-9}}{18\ln\left(\frac{D_{eq}}{r}\cdot H_{eq}\right)} \text{ F/m}$$

where,

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$
$$H_{eq} = \frac{H_1}{\left(H_2^2 H_3\right)^{1/3}}$$

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$
$$= \sqrt[3]{7\times3.5\times3.5} = 4.41 \text{ m} = 441 \text{ cm}$$

H_1 is the distance between conductor a and its image a' = 8 m

H_2 is the distance between conductor a and its image b' =

$$\sqrt{8^2 + 3.5^2} = 8.732 \text{ m}$$

H_2 is the distance between conductor a and its image c' =

$$\sqrt{8^2 + 7^2} = 10.63 \text{ m}$$

$$H_{eq} = \frac{H_1}{\left(H_2^2 H_3\right)^{1/3}} = \frac{8}{\left(8.732^2\times10.63\right)^{1/3}} = 0.858 \text{ m}$$

$$\text{Capacitance per phase, } C = \frac{10^{-9}}{18\ln\left[\frac{4.41}{0.01}\times0.858\right]} \text{ F/m} = 9.36 \text{ pF/m}$$

CHAPTER AT A GLANCE

1. Transmission line constants or parameters are resistance, inductance, capacitance and shunt conductance.

2. Shunt conductance is caused due to leakage in current (through insulators, etc.) which is very minimal and hence, neglected.
3. Cadmium, copper, phosphor, bronze, copper weld and galvanized steel are used as transmission line conductors.
4. **Resistance:** DC resistance of a conductor is given by, $R_{DC} = \rho \frac{l}{A}$. The AC resistance is more than DC resistance due to skin and proximity effects

   $\therefore R_{AC} = 1.1\ R_{DC}$ or $1.2\ R_{DC}$

   Normally, $R_{AC} = 1.5\ R_{DC}$
5. A bundled conductor is a conductor, which is formed by two or more than two sub-conductors in each phase. The use of bundled conductors increases the GMR. Therefore, the inductance of the bundled conductor line is less than the inductance of the line with one conductor per phase.
6. Inductance is defined as the flux linkages per unit current.
7. **Inductance calculations:**
   1. Inductance due to internal flux linkage, $L_{int} = 0.5 \times 10^{-7}$ H/m
   2. External inductance, $L_{ext} = 2 \times 10 \ln \frac{D}{r}$ H/m
   3. Inductance of single-phase two-wire line (loop inductance):

      $$L = L_{int} + L_{ext} = 4 \times 10^{-7} \ln \frac{D}{r'} \text{ H/m}$$

      $$\text{Inductance per conductor} = 2 \times 10^{-7} \ln \left(\frac{D}{r'} \right) \text{H/m}$$

      where, $r' = re^{-1/4} = 0.7788r$ GMR of the conductor.
   4. Inductance of a single-phase system with composite conductors

      $$L = 4 \times 10^{-7} \ln \frac{D_m}{D_s} \text{ H/m.}$$

5. Inductance of three-phase lines with equivalent spacing between the conductors.

   Inductance per phase, $L = 2 \times 10^{-7} \ln \dfrac{D}{r'}$ H/m

6. Inductance of three-phase lines with unequivalent spacing between the conductors (transposed),

   Inductance per phase, $L = 2 \times 10^{-7} \ln \dfrac{D_{eq}}{D_s}$ H/m

   where, $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

7. Inductance of three-phase double circuit line with symmetrical spacing (hexagonal), Inductance per phase, $L = 10^{-7} \ln \left(\dfrac{\sqrt{3}}{2} \dfrac{D}{r'} \right)$ H/m

8. Inductance of a three-phase double circuit line with unsymmetrical spacing (transposed), inductance per phase is

$$L = 2 \times 10^{-7} \ln \left\{ 2^{1/6} \left(\frac{D}{r'} \right)^{1/2} \left(\frac{m}{n} \right)^{1/3} \right\} \text{ H/m}$$

8. **Transposition:** Transposition of overhead line conductors refers to the exchanging of the positions of power conductors at regular intervals. By doing this, the average distance between conductors is made equal, which in turn will help the system to have equal inductances per phase. This method also reduces radio interference.
9. **Capacitance:** It is defined as charge per unit of potential difference.
10. **Capacitance calculations:**
    1. Capacitance of a single-phase (or two-wire) line

$$C = \frac{\pi\varepsilon}{\ln\frac{D}{r}} = \frac{10^{-9}}{36\ln\frac{D}{r}} \text{ F/m}$$

and charging current is

$$I_c = j\omega C V_{ph} \text{ A}$$

2. Capacitance of three-phase transmission lines with equivalent spacing between the conductors, capacitance per conductor is

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} = \frac{10^{-9}}{18\ln\left(\frac{D}{r}\right)} \text{ F/m}$$

and charging current per phase is

$$I_c = j\omega C V_{ph} \text{ A}$$

3. Capacitance of a unsymmetrical three-phase system (transposed)

   The capacitance per phase of three-phase unsymmetrical spaced (transposed) single circuit system is Capacitance per phase =

$$\frac{2\pi\varepsilon}{\ln\left[\frac{(D_{12}D_{23}D_{31})^{1/3}}{r}\right]} \text{ F/m}$$

   For the symmetrical spacing of the conductors, i.e., $D_{12} = D_{23} = D_{31} = D$

   $\therefore$Capacitance per phase, $C = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$

4. Capacitance of a three-phase double circuit line with symmetrical spacing (hexagonal), Capacitance per phase, $C = \frac{2\pi\varepsilon}{\ln\left\{\left(\frac{\sqrt{3}}{2}\right)^{1/2}\left(\frac{D}{r}\right)^{1/2}\right\}}$ F/m

5. Capacitance of a three-phase double circuit line with flat vertical spacing (unsymmetrical spacing),

   The capacitance per phase,

$$C = \frac{4\pi\varepsilon}{\ln\left\{2^{1/3}\frac{D}{r}\left(\frac{m}{n}\right)^{2/3}\right\}} \text{ F/m}$$

6. Effect of earth on transmission line capacitance: The line capacitance increases marginally due to the effect of earth on capacitance
   1. Capacitance of single conductor:

      The capacitance of the conductor with reference to ground is

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{r}} \text{ F/m}$$

   2. Capacitance of single-phase transmission line:

      Capacitance of single-phase system (between the conductors) is

$$C = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\cdot\frac{2h}{\sqrt{4h^2+D^2}}\right)} \text{ F/m}$$

   3. Capacitance of three-phase line (transposed), capacitance of each conductor is

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}H_{eq}\right)} \text{ F/m}$$

Where, $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

$$H_{eq} = \left(\frac{H_1^2 H_5}{H_2 H_3 H_4}\right)^{\frac{1}{3}}$$

SHORT ANSWER QUESTIONS

1. Define resistance.
2. Define inductance.
3. Define capacitance.
4. Define skin effect.
5. Write the expression of capacitance to neutral.
6. What are the factors affecting the skin effect?
7. What is meant by proximity effect?
8. What are the causes of proximity effect?
9. What is meant by loop inductance?
10. What are the properties of conducting material?
11. What are the advantages due to increase in voltage transmission?
12. What are the advantages of bundled conductor?
13. What is the use of double circuit transmission lines?

MULTIPLE CHOICE QUESTIONS

1. If the conductor diameter decreases, inductance of the line
   1. increases
   2. decreases
   3. remains same
   4. none of these
2. A conductor carries more current on the surface as compared to its core, this phenomenon is known as
   1. permeability
   2. corona
   3. skin effect
   4. unsymmetrical faults
3. The inductance of a power transmission line increases with
   1. decrease in line length
   2. increase in diameter of conductor
   3. increased spacing between conductor
   4. increase in load current carried by the conductors
4. The inductance of the line is minimum when
   1. GMD is high
   2. GMR is high
   3. GMD and GMR are high
   4. GMD is low and GMR is high
5. The self inductance of a long cylindrical conductor due to its internal flux linkages is 1 mH/km. If the diameter of the conductor is doubled, then the self-inductance of the conductor due to its internal flux linkages would be
   1. 0.5 mH/km

2. 1 mH/km
3. 1.414 mH/km
4. 4 mH/km

6. In DC transmission, full cross-section of the conductor is utilized because of no
   1. inductance
   2. capacitance
   3. phase displacement
   4. skin effect
7. Skin effect depends on
   1. supply frequency
   2. size of conductor
   3. nature of material
   4. all of these
8. A three-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2 m side. Calculate the capacitance of each line conductor per kilometer. Given that diameter of each conductor is 1 cm
   1. $\dfrac{2\pi\varepsilon_0}{\ln\dfrac{200}{0.5}}$ F/m
   2. $\dfrac{2\pi\varepsilon_0}{\ln\dfrac{2}{0.5}}$ F/m
   3. $\dfrac{2\pi\varepsilon_0}{\ln\dfrac{200}{1}}$ F/m
   4. $\dfrac{2\pi\varepsilon_0}{\ln\dfrac{2}{1}}$ F/m
9. The DC resistance of conductor is less than AC resistance due to
   1. skin effect
   2. proximity effect
   3. both are correct
   4. none of these
10. For increasing the capacity of a transmission line to transmit power, which of the following must be decreased
    1. voltage
    2. line inductance
    3. capacitance
    4. all of these
11. The following effects are associated with transmission lines i) skin effect ii) corona effect iii) proximity effect The effect resistance of a conductor is increased by

1. i and ii only
2. ii and iii only
3. i and iii only
4. i, ii and iii

12. Inductance is equal to
    1. Flux/current
    2. Flux linkages/current
    3. EMF/flux
    4. EMF/current
13. In which of the following supply system, is skin effect ignored
    1. AC
    2. DC
    3. both
    4. none
14. Use of bundle conductor increases
    1. GMR
    2. GMD
    3. potential gradient
    4. radius of the conductor
15. Which of the following is not a standard transmission voltage
    1. 400 kV
    2. 11 kV
    3. 190 kV
    4. 750 kV
16. The overall diameter (D)of stranded conductor is given by
    1. $D = 2(n - 1)d$
    2. $D = (2n - 1)/d$
    3. $D = (2n - 1)d$
    4. $D = (n - 1)d$

    Where, d = diameter of each strand, n = number of layers
17. Shunt conductance in transmission lines is caused by the
    1. leakage current
    2. shunt capacitance
    3. shunt inductance
    4. series resistance loss
18. Stranded conductors are easier to
    1. handle
    2. transport
    3. handle and transport
    4. none of these
19. ACSR conductor consists of a central core of
    1. copper
    2. steel
    3. stainless steel
    4. cadmium
20. The inductance of a single-phase two-wire power transmission line per kilometre gets doubled when the
    1. distance between the wires is doubled
    2. distance between the wires is increased four-fold
    3. distance between the wires increases as square of the

original distance
4. radius of the wire is doubled
21. Capacitance of a transmission line_______________ with increase in its length.
1. increases
2. decreases
3. remains same
4. none of these
22. The capacitance between the conductor and neutral of the single-phase line is
1. equal to the capacitance between the lines
2. half the capacitance between lines
3. double the capacitance between lines
4. none of these
23. ACSR conductors have
1. all conductors made of aluminium
2. outer conductors made of aluminium
3. inner conductors made of aluminium
4. no conductors made of aluminium
24. The number of conductors is increased in a bundle, when the self GMD
1. decreases
2. increases
3. remains same
4. none of these
25. Both skin and proximity effects depends upon
1. conductor size
2. frequency
3. distance between conductors and permeability
4. all of these
26. The self GMD of the conductor with three strands, each of radius *r* and touching each other is
1. $r(0.7788 \times 2 \times 2)^{\frac{1}{3}}$
2. $r(0.7788 \times 2 \times 2 \times 2)$
3. $r(0.7788 \times 2 \times 2 \times 2)^3$
4. $r(0.7788 \times 2 \times 2)^3$
27. The following are regarded as line parameters
1. shunt reactance and conductance
2. series reactance and resistance
3. a or b
4. a and b
28. The skin effect does not depend on
1. nature of material
2. size of wire
3. supply frequency
4. ambient temperature
29. For the overhead transmission lines, the self GMD method is used to evaluate.
1. capacitive

2. inductive
3. both a and b
4. none of these

30. If the supply frequency increases, the skin effect is
    1. decreased
    2. increased
    3. remains same
    4. none of these
31. The power loss in an overhead transmission line mainly depends on
    1. inductance of each conductor
    2. capacitance of each conductor
    3. resistance of each conductor
    4. none of these
32. In which of the following conductors is spirality effect ignored
    1. magnetic
    2. non-magnetic
    3. both
    4. none of these
33. The inductance of a bundle conductor line is________ than that of the line with one conductor per phase
    1. less
    2. greater
    3. same
    4. none of these

34. Highest transmission voltage in India is
    1. 500 kV
    2. 450 kV
    3. 750 kV
    4. 400 kV
35. If the height of transmission tower is increased
    1. the line capacitance will increase but line inductance will decrease
    2. the line capacitance and inductance will not change
    3. the line capacitance will decrease and line inductance will increase
    4. the line capacitance will decrease but line inductance will remain unchanged.
36. Bundle conductors in a transmission line
    1. increase radio interference
    2. decrease radio interference
    3. no effect
    4. increase the resistance
37. Distribution of AC current over the cross-section of a conductor is distorted due to
    1. skin effect
    2. proximity effect
    3. spirality effect
    4. skin, proximity and spirality effect
38. The capacitance of an overhead line increases with (i) increase in

mutual GMD (ii) increase in height of conductors above ground

1. both are true
2. both are false
3. only (i) is correct
4. none of these

39. Bundled conductors are used in EHV lines primarily for
    1. reducing cost of the line
    2. reducing corona loss and radio interference
    3. increasing stability limit
    4. none of these
40. Expanded ACSR conductors are used
    1. to increase the tensile strength of the line
    2. to reduce corona loss
    3. to reduce $I^2 R$ loss
    4. to reduce the voltage drop
41. A conductor with 19 strands, each of same diameter and each having an inductance of L H is used for a transmission line. The total inductance of the conductor will be
    1. $L/19$
    2. $L/361$
    3. $19L$
    4. $38L$
42. The line-to-neutral capacitance of single-phase line with conductors of radius 1 cm and spaced 1 m apart is equal to
    1. $10^{-9}/72$ F/m
    2. $10^{-9}/36\pi$F/m
    3. $2\pi^2 10^{-7}$F/m
    4. $\pi^2 10^{-7}$ F/m
43. In a double-circuit line with hexagonal spacing,
    1. The phases are balanced, but the conductors of each individual phase are not balanced
    2. The conductors of each individual phase are balanced, but the phases are not balanced
    3. The phases, and the conductors of each individual phase are both balanced
    4. None of these
44. Bundle conductors are preferred in EHV transmission lines because
    1. It is easy to fabricate thin conductors and combine them to make a bundle
    2. Inductance of the line is reduced, and the corona loss, and radio and TV interference is minimized
    3. Tower height is reduced and hence transmission cost is low
    4. None of these
45. Inductive interference between power and communication lines can be minimized by
    1. Increasing the spacing of power line conductors
    2. Transposing power line conductors
    3. Transposing communication line conductors
    4. Either b or c

46. Which one of the following statements is true?
    1. Resistance of a conductor decreases and the internal inductance increases as the frequency is increased
    2. Resistance and internal inductance of a conductor both increase with increase in frequency
    3. Resistance of a conductor increases and the internal inductance decreases as the frequency is increased
    4. None of these
47. Which one of the following statements is not true?
    1. The GMD method of finding inductance does not apply to ACSR conductors
    2. Current density in ACSR conductors is uniform
    3. The GMD between two circular areas, each of different diameters, is equal to the distance between their centres
    4. None of these
48. As the temperature increases, the temperature of coefficient of resistance
    1. increases
    2. decreases
    3. does not change
    4. doubles
49. Crowding of current towards the surface of conductor is known as
    1. skin effect
    2. proximity effect
    3. spirality effect
    4. Ferranti effect
50. AC resistance of a conductor is more than DC resistance due to
    1. proximity effect
    2. skin effect
    3. corona effect
    4. sagging effect

Answers:

1. a	2. c	3. c	4. d	5. b
6. d	7. d	8. a	9. c	10. b
11. c	12. b	13. b	14. a	15. c
16. c	17. a	18. c	19. b	20. c
21. a	22. c	23. b	24. b	25. d
26. c	27. d	28. d	29. b	30. b
31 c	32. b	33. a	34. d	35. d
36. b	37. d	38. b	39. b	40. b
41. a	42. a	43. c	44. b	45. d
46. c	47. b	48. b	49. a	50. b

REVIEW QUESTIONS

1. What are skin and proximity effects?

2. Clearly explain what you understand by GMR and GMD of a transmission line.
3. What are ACSR conductors? Explain the advantages of ACSR conductors, when used for overhead lines.
4. What are bundled conductors? Discuss the advantages of bundled conductors, when used for overhead lines.
5. Distinguish between AC and DC resistance of a conductor. Do the two differ? Explain.
6. Prove that the inductance of a group of parallel wires carrying current can be represented in terms of their geometric distance.
7. A conductor is composed of seven identical copper strands each having a radius r. Find the GMR of the conductor.
8. Derive an expression for the flux linkages of one conductor in a group of n conductors carrying currents whose sum is zero. Hence, derive an expression for inductance of composite conductors of a single-phase line consisting of m strands in one conductor and n strands in the other conductor.
9. What do you understand by transposition of lines? What are its effects on the performance of the line?
10. Derive the expression for inductance of a three-phase unsymmetrically spaced transmission line.
11. Develop an expression for the inductance of a single-phase transmission line taking into account the internal flux linkages.
12. Derive the expression for inductance per kilometer of a three-phase line with diagonal spacing between the conductors.
13. Derive an expression for the capacitance of a single-phase overhead transmission line.
14. Derive an expression for the capacitance of a three-phase transmission line with unequal spacing assuming uniform transposition.
15. What do you understand by electric potential? Derive an expression for electrical potential and deduce the formula for capacitance for the following cases:
    1. Single-phase two conductor line
    2. Three-phase unsymmetrical spacing but transposed.

PROBLEMS

1. Calculate the inductance of a conductor (line-to-neutral) of a three-phase system, which has a 1.8 cm diameter and conductors placed at the corner of an equilateral triangle of sides 3.5 m.
2. Calculate the inductance (line-to-neutral) of a three-phase three-wire system when the conductors each of diameter 1.5 cm are placed at the corners of a triangle with sides 4 m, 3.8 m and 5.3 m as shown in Fig. 2.53. Assume that the conductors are transposed at regular intervals and that the load is balanced.

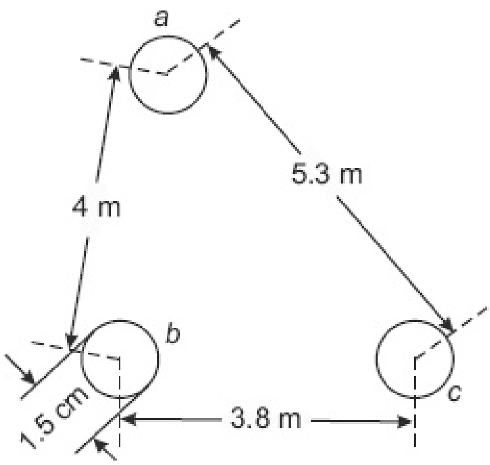

Fig. 2.53 Conductors' location in triangle

3. Calculate the inductance and reactance of each line of a three-phase 50 Hz overhead high-tension line which has conductors of diameter 2 cm. The distance between three-phase shown as in Fig. 2.54 are: between A and B is 3 m, B and C is 4.6 m and C and A is 2.1 m. Assume the lines are transposed regularly.

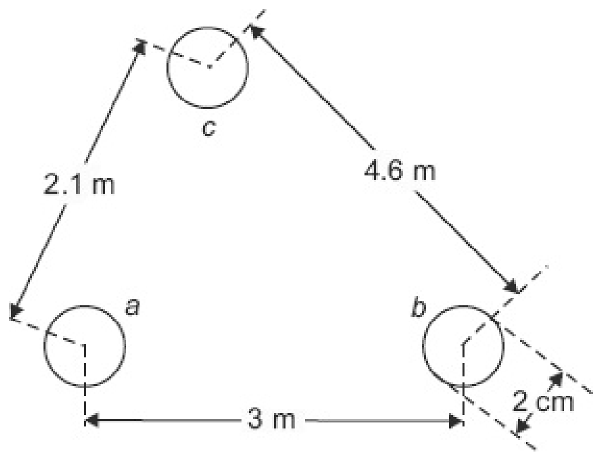

Fig. 2.54 Conductors' location in triangle

4. A 300 kV, three-phase bundle conductor line with two sub-conductors per phase has a horizontal configuration as shown in Fig. 2.55. Find the inductance per phase, if the radius of each sub conductor is 1.5 cm.

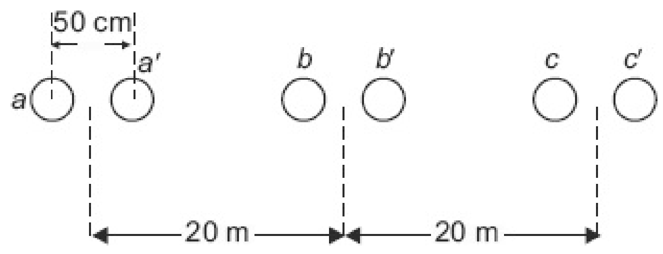

Fig. 2.55 Conductors' location in horizontal configuration

5. Calculate the inductance per phase of a three-phase transmission line with horizontal spacing as shown in Fig. 2.56. The radius of the conductor is 0.9 cm. The line is untransposed.

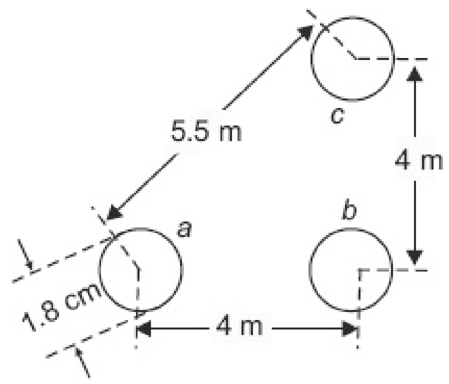

Fig. 2.56 Conductors' location in triangle

6. Calculate the inductance per phase of a three-phase double circuit line if the conductors are spaced at the vertices of a hexagon with sides 1.8 m each. The diameter of each conductor is 1.5 cm.
7. Calculate the inductance per phase of a three-phase, double circuit line as shown in Fig. 2.57. The diameter of each conductor is 2.1 cm.

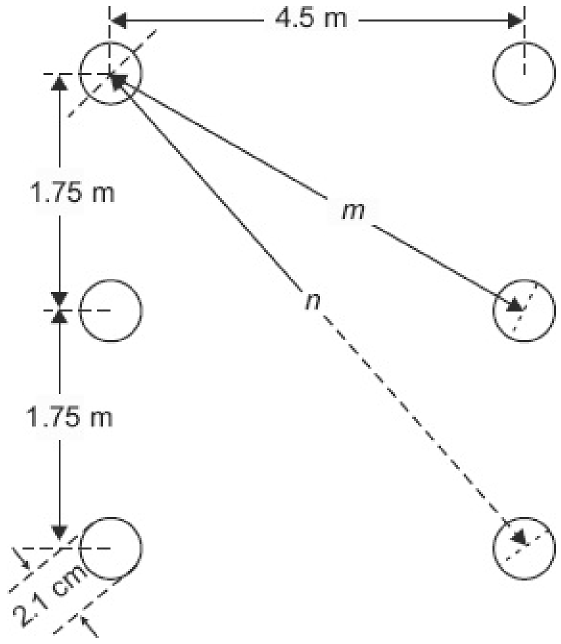

Fig. 2.57 Three-phase, double circuit line

8. Calculate the capacitance of a conductor per phase of a three-phase 450 km long line with the conductors spaced at the corners of an equilateral triangle of side 3.5 m and the diameter of each conductor is 2.8 cm.
9. Calculate the capacitance per phase of a three-phase three-wire transposed system when the conductors are arranged at the corners of a triangle with sides of 1.6 m, 1.8 m, and 2.3 m. The diameter of each conductor is 1.8 cm.
10. Calculate the capacitance (phase-to-neutral) of a three-phase 150 km double circuit line as shown in Fig. 2.58 has a conductor of diameter 1.8 cm and the conductors are arranged at the corners of an hexagon with side 2.8 m.

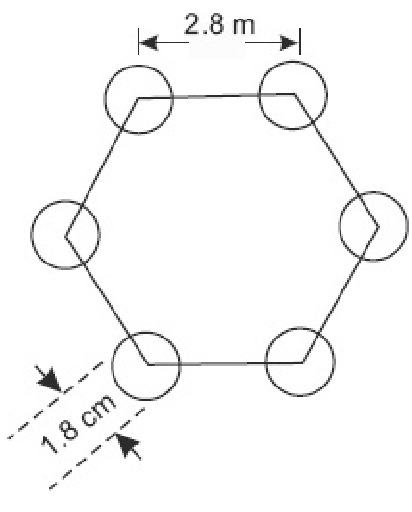

Fig. 2.58 Conductor' location in hexagonal spacing

11. Calculate the capacitance per phase of a three-phase, double circuit line as shown in Fig. 2.59. The diameter of each conductor is 1.8 cm.

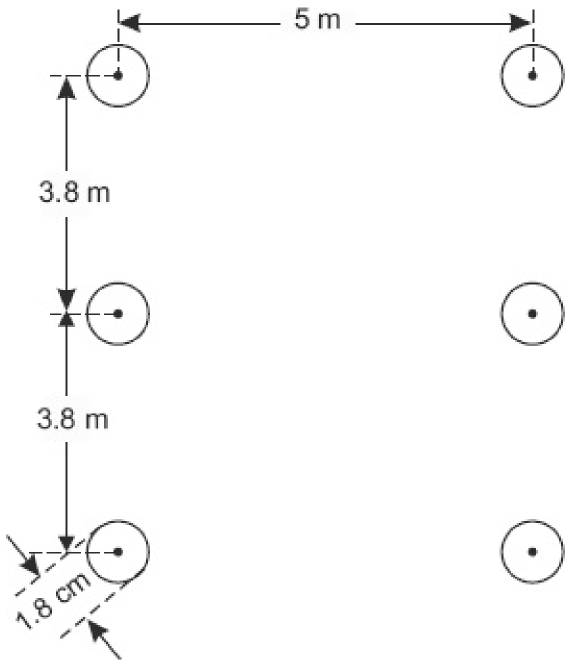

Fig. 2.59 Vertical configuration of three-phase, double circuit line

12. Calculate the capacitance per phase of a three-phase double circuit line as shown in Fig. 2.60. The diameter of the conductor is 1.8 cm. Assume the line is completely transposed.

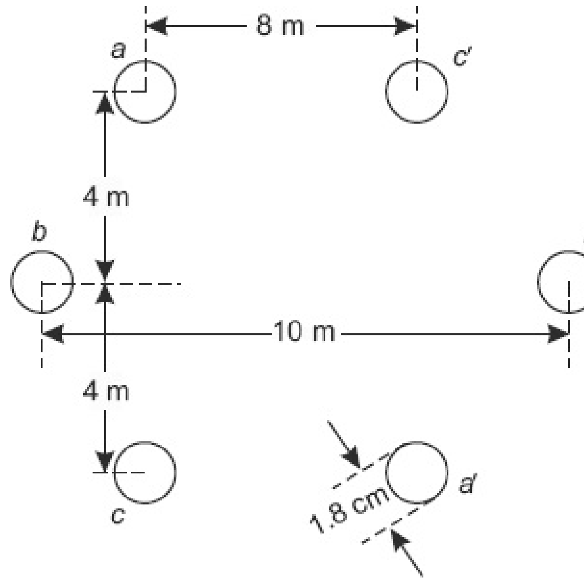

Fig. 2.60 Three-phase, double circuit line

13. Calculate the capacitance per phase of a three-phase, three-wire system by considering earth effect, when the conductors are arranged in a horizontal plane with spacing $D_{12} = D_{23} = 5$ m, and $D_{31} = 10$m.The conductors are transposed and each has a diameter of 1.9 cm. Assume that the transmission line is 6.5 m above the ground level.

3

Performance of Short and Medium Transmission Lines

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Develop mathematical models for short and medium transmission lines
- Analyse performance of short and medium lines
- Calculate A, B, C, D parameters from mathematical models

3.1 INTRODUCTION

Generally, power is generated at generating stations and is then supplied to the various categories of consumers through transmission lines. They act, therefore, as pathways for the flow of power from its source to the consumption points in a power system. A transmission line therefore, has both sending and receiving ends and its own parameters namely, series resistance and inductance, and shunt capacitance and conductance. The performance and design of a transmission line is purely dependent upon its parameters that are uniformly distributed, along its length. Performance of a transmission line then, refers to the calculation of efficiency, regulation, and power flows of transmission lines as affected by these.

Usually, the values of voltage, current, and power factor are known at the sending end. These values are prerequisites of system planning. With power systems expanding rapidly, it has become difficult to manually compute these values at the receiving end. At present, digital computers are being used extensively for these calculations. However, it is necessary to know the formulae and methods for such calculation.

Efficiency of a transmission line is defined as the ratio of receiving-end power to the sending-end power.

$$\% \text{ Efficiency} = \frac{\text{Receiving-end power}}{\text{Sending-end power}} \times 100$$

$$= \frac{\text{Power delivered at the receiving end}}{(\text{Power delivered at the receiving end} + \text{losses})} \times 100$$

Mathematically, efficiency of the transmission line can be represented by

$$\eta = \frac{V_r I_r \cos\phi_r}{V_r I_r \cos\phi_r + I^2 R} \times 100 \qquad (\text{Per phase basis})$$

or

$$\eta = \frac{V_r I_r \cos\phi_r}{V_s I_s \cos\phi_s} \times 100$$

where,

V_r, I_r are the voltage and current at the receiving end

$\cos\phi_r$ is the power factor at the receiving end

V_s, I_s are the voltage and current at the sending end

$\cos\phi_s$ is the power factor at the sending end

When a transmission line carries current, there is a voltage drop in the line due to the resistance and the inductance of the line. This results in reduced receiving-end voltage as compared to the sending-end voltage.

Regulation is defined as the change in voltage at the receiving-end when full load is thrown off; with constant sending-end voltage, expressed as a percentage of receiving-end voltage at full load.

$$\% \text{ Regulation } = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

where,

V_{NL} = receiving-end voltage at no load

V_{FL} = receiving-end voltage at full load.

3.2 REPRESENTATION OF LINES

Figure 3.1(a) shows a three-phase star load connected to the generator through a three-phase overhead transmission system. The sum of all the currents in a balanced polyphase network is zero. Therefore, the current passing through the wire connected between the star points of load and neutral of the system is zero. This means that the star point of load and neutral of the system have the same potential.

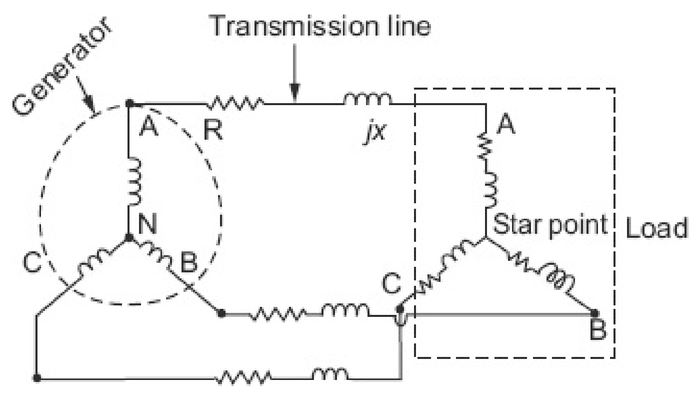

Fig. 3.1(a) Three-phase power system

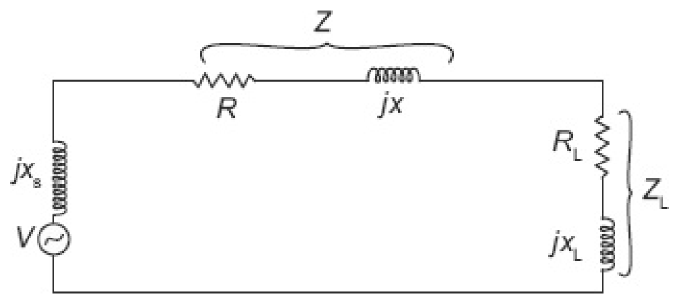

Fig. 3.1(b) Equivalent single-phase representation of Fig 3.1(a)

A three-phase can, therefore, be analyzed on single-phase basis where in the neutral, the wire is of zero impedance. The equivalent single-phase representation of a three-phase system is shown in Fig. 3.1(b).

Test Yourself

1. Why are the three-phase lines represented in single-phase base?

3.3 CLASSIFICATION OF TRANSMISSION LINES

Transmission lines are classified into three types based on the length of the transmission line and the operating voltage. They are:

1. Short transmission lines
2. Medium transmission lines
3. Long transmission lines

In case of short and medium transmission lines, the total resistance, inductance and capacitance (line parameters) are assumed to be lumped at one place but actually, they are distributed along the line. For long transmission lines, they are considered as uniformly distributed parameters.

3.4 SHORT TRANSMISSION LINE

When the length of an overhead transmission line is less than 80 km with an operating voltage upto 20 kV, it is considered as short transmission line as shown in Fig. 3.2(b). Due to smaller length and low operating voltage, the charging current is low. So, the effect of capacitance on performance of short transmission lines is extremely small and therefore, can be neglected, as in the case of distribution lines (a distribution line is that which connects distribution substations to the consumer point).

So these lines have resistance (*R*) and inductive reactance (*X*) as shown in Fig. 3.2(a).

From the equivalent circuit of Fig. 3.2(a),

$$V_r = V_s - IZ \qquad (\text{since } I = I_r = I_s) \tag{3.1}$$

where, *IZ* = voltage drop along the line.

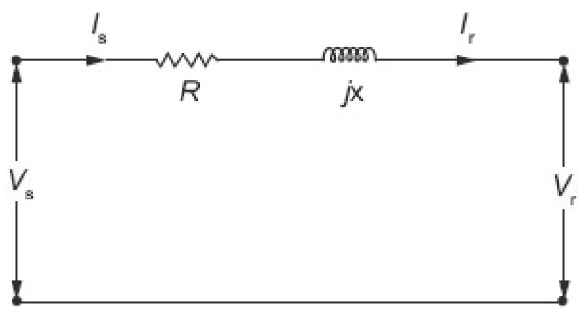

Fig. 3.2(a) Short transmission-line model

Fig. 3.2(b) View of a 11 kV distribution line

The phasor diagram of a short line for lagging power factor is shown in Fig. 3.2(c). The horizontal line *OA* is the per phase receiving-end voltage (V_r) (considered as a reference phasor), *OD* is the load current (*I*) which is lagging behind the V_r by an angle ϕ_r. To the voltage V_r, the resistance drop and the inductance drop are added in correct phase to get V_s as shown in Fig. 3.2(c). The angle ϕ_s is the angle between the sending-end voltage V_s and the sending-end current, *I*. The angle between V_s and V_r is known as load angle (or torque angle) and is denoted by δ.

From Fig. 3.2(c),

$$
\begin{aligned}
OC^2 &= OE^2 + EC^2 \\
V_s^2 &= (V_r \cos\phi_r + IR)^2 + (V_r \sin\phi_r + IX)^2 \\
&= V_r^2 + 2IRV_r \cos\phi_r + 2IXV_r \sin\phi_r + I^2(R^2 + X^2) \\
&= V_r^2 + 2IRV_r \cos\phi_r + 2IXV_r \sin\phi_r + I^2 Z^2 \qquad (3.2)
\end{aligned}
$$

Real power received, $P_r = V_r\, I_r \cos\phi_r$

Real power received, $Q_r = V_r I_r \sin\phi_r$

$$\text{then, } I = \frac{P_r}{V_r \cos\phi_r}$$

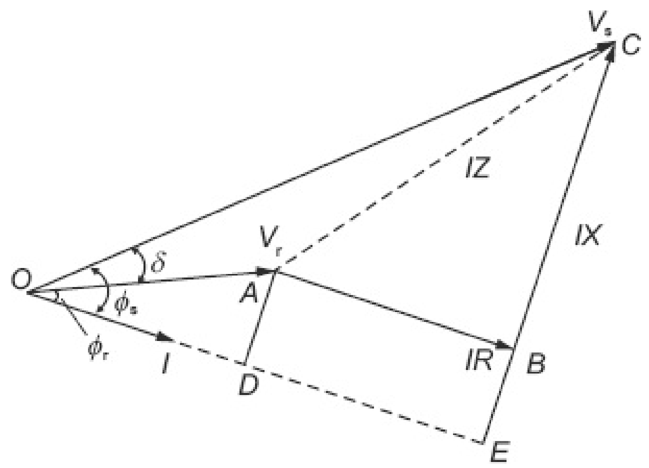

Fig. 3.2(c) Phasor diagram

Substituting these values in Eq. (3.2), we get

$$V_s^2 = V_r^2 + 2(P_r R + Q_r X) + \frac{P_r^2 Z^2}{V_r^2 \cos^2\phi_r}$$

$$\text{or } V_r^2 = V_s^2 - 2(P_r R + Q_r X) - \frac{P_r^2 Z^2}{V_r^2 \cos^2\phi_r} \quad (3.3)$$

Multiplying the Eq. (3.3) by $V_r^2\cos^2\phi_r$ on both sides and re-arranging, we get

$$V_r^4 \cos^2 \phi_r = V_r^2 V_s^2 \cos^2 \phi_r - 2V_r^2 (P_r R + Q_r X)\cos^2 \phi_r - P_r^2 Z^2$$

$$V_r^4 = V_r^2 \left[V_s^2 - 2\left(P_r R + Q_r X\right)\right] - \frac{P_r^2 Z^2}{\cos^2 \phi_r}$$

$$V_r^4 - V_r^2 \left[V_s^2 - 2\left(PR + QX\right)\right] + \frac{P^2 Z^2}{\cos^2 \phi_r} = 0 \qquad (3.4)$$

From Eq. (3.4), we can determine any one quantity by knowing the others.

Alternative Method

From Fig. 3.2(c), we can write

$$V_s = OC = \sqrt{(OE)^2 + (EC)^2}$$

$$= \sqrt{(OD + DE)^2 + (EB + BC)^2}$$

$$= \sqrt{(V_r \cos\phi_r + IR)^2 + (V_r \sin\phi_r + IX)^2}$$

$$= \sqrt{V_r^2 + 2V_r IR \cos\phi_r + 2V_r IX \sin\phi_r + I^2 \left(R^2 + X^2\right)}$$

$$= V_r \sqrt{1 + \frac{2IR}{V_r}\cos\phi_r + \frac{2IX}{V_r}\sin\phi_r + \frac{I^2}{V_r^2}\left(R^2 + X^2\right)}$$

$$\cong V_r \sqrt{1 + \frac{2\left(IR \cos\phi_r + IX \sin\phi_r\right)}{V_r}}$$

because $\frac{I^2(R^2 + X^2)}{V_r^2}$ is very small when compared with the other terms

$$\therefore V_s = V_r \left(1 + 2\frac{(IR \cos\phi_r + IX \sin\phi_r + \text{higher order terms})}{2V_r}\right) \qquad (3.5)$$

$$\text{Note}: (1+x)^{1/2} = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \ldots\ldots$$

In practice, the value of higher order terms of Eq. (3.5) are small and can be neglected, and therefore, we get the approximate formula for V_s as

$$V_s \cong V_r + IR\cos\phi_r + IX\sin\phi_r \tag{3.6}$$

From Fig. 3.2(c), the power factor at sending end is given by

$$\cos\phi_s = \frac{OE}{OC} = \frac{V_r\cos\phi_r + IR}{V_s}$$

Regulation:

$$\begin{aligned}\%\text{ Regulation} &= \frac{V_s - V_r}{V_r}\times 100\\ &= \frac{V_r + IR\cos\phi_r + IX\sin\phi_r - V_r}{V_r}\times 100\\ &= \frac{IR\cos\phi_r + IX\sin\phi_r}{V_r}\times 100 \qquad (3.7)\end{aligned}$$

Efficiency:

In short transmission lines, power delivered, $P_r = V_r I \cos\phi_r$

Line losses per phase = I^2R

Power supplied per phase, $P_s = V_r I\cos\phi_r + I^2R$

The efficiency of a short transmission line,

$$\eta,\% = \frac{\text{Power delivered}}{\text{Power supplied}}\times 100$$

$$= \frac{P_r}{P_s} \times 100 \qquad (3.8)$$

Test Yourself

1. Why is line capacitance neglected in short transmission lines?

Example 3.1

A short transmission line has an impedance of (0.2 + *j*0.45) Ω per phase [see Fig. 3.2(d)]. The sending-end voltage being 3.3 kV (L-L) and the load at the receiving end being 250 kW per phase at a 0.8 p.f. lagging, calculate (i) the receiving-end voltage (ii) the line current, and (iii) efficiency.

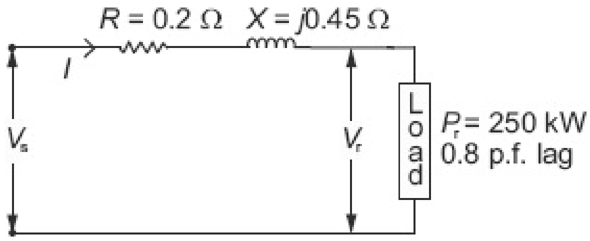

Fig. 3.2(d) Circuit model for Example 3.1

Solution:

Impedance of the line, $Z = (0.2 + j0.45)$ Ω/phase

Therefore, resistance per phase, $R = 0.2$ Ω

Reactance per phase, $X = 0.45$ Ω

Sending-end voltage, $V_s = 3.3\text{kV}(L-L) = \frac{3.3}{\sqrt{3}}\text{kV} = 1905.26$ V/phase

Load at receiving end per phase, $P_r = 250$ kW

Power factor at receiving end, $\cos\phi_r = 0.8$ lag

Reactive power at receiving end, $Q_r = \frac{250}{0.8} \times 0.6 = 187.5$ kVAr

1. Voltage at the receiving end, $V_r^4 = V_r^2[V_s^2 - 2(P_rR + Q_rX)] - \dfrac{P_r^2 Z^2}{\cos^2\phi_r}$

$$V_r^4 = V_r^2\left[1905.26^2 - 2\left(250\times10^3\times0.2 + 187.5\times10^3\times0.45\right)\right] - \frac{250^2\times10^6\times0.492^2}{0.8^2}$$

$$V_r^4 - 3361265.7\,V_r^2 + 2.3639\times10^{10} = 0$$

$$V_r^2 = \frac{3361265.7 \pm \sqrt{1.120355\times10^{13}}}{2}$$

$$= \frac{3361265.7 \pm 3347170.6}{2} = 3354218.2$$

$$V_r = \sqrt{3354218.2}$$

$$V_r = 1831.45 \text{ V/phase}$$

$$= 1831.45\times\sqrt{3} = 3172.17 \text{ V(L} - \text{L)}$$

$$= 3.17 \text{ kV}(L - L)$$

2. Load current, $I = \dfrac{250\times10^3}{1831.45\times0.8} = 170.63 \text{ A}$

Transmission loss $= \dfrac{3\times170.63^2\times0.2}{10^3} = 17.47 \text{ kW}$

3. Transmission efficiency,

$$\eta = \frac{\text{Output power}}{\text{Input power}}\frac{3\times250}{3\times250 + 17.47}\times100 = 97.72\%.$$

Example 3.2

A 220 kV, 50 Hz, three-phase transmission line is 50 km long. The resistance per phase is 0.15 Ω/km, the inductance per phase is 1.33 mH/km and the shunt capacitance is negligible. Use the short line model to determine (i) the voltage and power at the sending end, (ii) voltage regulation and efficiency when the line is supplying a three-phase load of 400 MVA, 220 kV at 0.8 p.f. lagging.

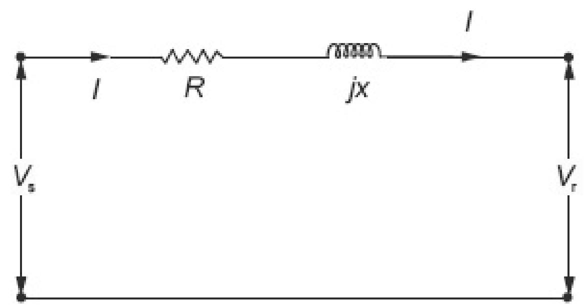

Fig. 3.3 Circuit model for Example 3.2

Solution:

Receiving end voltage, $V_r = \frac{220}{\sqrt{3}} \times 10^3 = 127017$ V/phase

Impedance, $Z_{ph} = (0.15 + j314 \times 1.33 \times 10^{-3}) \times 50$

$$= 7.5 + j20.881 = 22.187\angle 70.24°\ \Omega$$

Receiving end power factor, $\cos\phi_r$ = 0.8 lag

Load current,
$$I = \frac{400 \times 10^3}{\sqrt{3} \times 220} = 1049.73\text{ A}$$
$$= 1049.73\angle -36.86°$$
$$= 839.8 - j629.84\text{ A}$$

From the circuit diagram shown in Fig. 3.3,

1. Sending-end voltage is, $V_S = V_r + IZ$

$$= 127017 + 1049.73\angle -36.86° \times 22.187\angle 70.24°$$
$$= 127017 + 23290.36\angle 33.37°$$
$$= 127017 + j0 + 19450.62 + j12810.71$$
$$= (146467.62 + j12810.71)\text{ V}$$
$$= 147026.8\angle 5°\text{ V}$$

Sending-end line voltage, $V_s = \sqrt{3} \times 147026.8 = 254657.89$ V
$$= 254.66\text{ kV (L} - \text{L)}$$

Sending-end power, $P_s = \sqrt{3}\, V_s I \cos\phi_s$
$$= \sqrt{3} \times 254.66 \times 1049.73 \times \cos[5° - (-36.86°)]$$
$$= 344.846\text{ MW}$$

2. Regulation $= \frac{V_s - V_r}{V_r} \times 100$

$$= \frac{254657.89 - 220000}{220000} \times 100$$
$$= 15.75\%$$
$$\text{Line losses} = 3I^2R$$
$$= 3 \times (1049.73)^2 \times 7.5$$
$$= 24.79 \text{ MW}$$
$$\text{Efficiency, } \%\eta = \frac{400 \times 0.8}{400 \times 0.8 + 24.79} \times 100$$
$$= 92.81\%$$

Example 3.3

A three-phase short transmission line has resistance and reactance per phase 4 Ω and 6 Ω, respectively. The sending-end voltage and receiving-end voltage are 110 kV and 100 kV respectively, for some receiving-end load at a 0.8 p.f. lag. Calculate (i) output power (ii) sending-end power factor and (iii) efficiency.

Solution:

Resistance of each conductor, R = 4Ω

Reactance of each conductor, X_L = 6Ω

Load power factor, $\cos\phi_r$ = 0.8 lag

Sending-end voltage per phase, $V_s = \frac{110 \times 10^3}{\sqrt{3}} = 63508.5$ V

Receiving-end voltage per phase, $V_r = \frac{100 \times 10^3}{\sqrt{3}} = 57735$ V

Let I be the load current, the approximate expression for V_S is

$$V_s = V_r + IR\cos\phi_r + IX_L\sin\phi_r$$

Substituting the values in the above equation

$$\therefore\ 63508.5 = 57735 + I \times 4 \times 0.8 + I \times 6 \times 0.6$$
$$\text{or } 6.8\, I = 5773.5$$
$$\text{or } I = \frac{5773.5}{6.8} = 849.044 \text{ A}$$

1. Output power = $3 V_r I \cos\phi_r$

$$= 3 \times 57735 \times 849.044 \times 0.8 = 117.647 \text{ MW}$$

2. In phase component of voltages, $V_S \cos\phi_S = V_r + IR \cos\phi_r$

Therefore, sending end power factor, $\cos\phi_s = \dfrac{V_r \cos\phi_r + IR}{V_s}$

$$= \frac{57735 \times 0.8 + 849.044 \times 4}{63508.5}$$
$$= 0.78 \text{ lag}$$

Input power, $P_s = \sqrt{3} \times V_s I \cos\phi_s$

$$= \sqrt{3} \times 110 \times 849.044 \times 0.78$$
$$= 126.176 \text{ MW}$$

3. Transmission efficiency,

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{117.647}{126.176} \times 100 = 93.24\%.$$

Example 3.4

The following data refers to a 50 Hz, three-phase transmission line: length 10 km; sending-end voltage = 11 kV; load delivered at receiving end 100 kW at 0.8 p.f. lagging; resistance of each conductor = 0.4 Ω/km; reactance per phase = 0.45 Ω/km. Find (i) receiving-end voltage, (ii) line current and (iii) efficiency of transmission.

Solution:

Resistance of each conductor, R = 0.4Ω/km × 10 = 10Ω

Reactance of each conductor, X_L = 0.45Ω/km × 10 = 4.5Ω

Load power factor, $\cos\phi_r$ = 0.8 (lag)

Sending end phase voltage, $V_s = \dfrac{11000}{\sqrt{3}} = 6350.85 \text{ V}$

Let V_r be the phase voltage at receiving end.

$$\text{Line current, } I = \frac{\text{Power delivered}}{3 \times V_r \times \cos\phi_r} = \frac{100 \times 10^3}{3 \times V_r \times 0.8} = \frac{41.667 \times 10^3}{V_r}$$

1. The approximate expression for V_s is

$$V_s = V_r + IR\cos\phi_r + IX_L\sin\phi_r$$
$$6350.85 = V_r + \frac{41.667 \times 10^3}{V_r} \times 4 \times 0.8 + \frac{41.667 \times 10^3}{V_r} \times 4.5 \times 0.6$$
$$6350.85 V_r = V_r^2 + 245835.3$$
$$V_r^2 - 6350.85 V_r + 245835.3 = 0$$

On solving this equation, we get (consider only positive value of V_r)

V_r = 6311.9 V

$$\text{Line voltage at receiving end} = \sqrt{3} \times 6311.9 \text{ V} = 10.932 \text{ kV}$$

2. Line current, $I = \frac{41.667 \times 10^3}{10.932 \times 10^3} = 3.811 \text{ A}$

Line losses = $3 I^2 R = 3 \times 3.811^2 \times 4 = 174.33$W

3. Transmission efficiency, $\eta = \frac{100 \times 10^3}{100 \times 10^3 + 174.33} \times 100 = 99.83\%.$

3.4.1 Effect of Power Factor on Regulation and Efficiency

A change in the power factor of the load affects the value of the sending-end voltage. The phasor diagram for the short line is drawn for the lagging power factor as shown in Fig. 3.2(c).

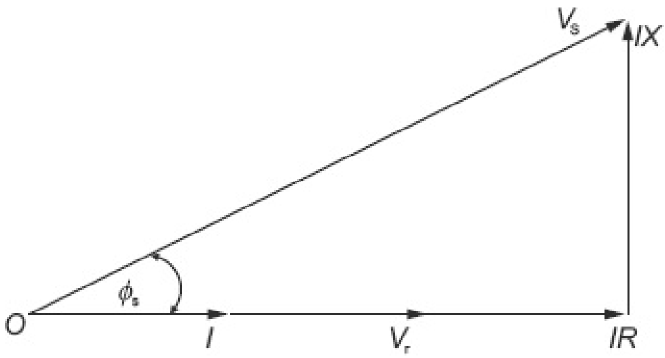

Fig. 3.4(a) Phasor diagram for unity power factor

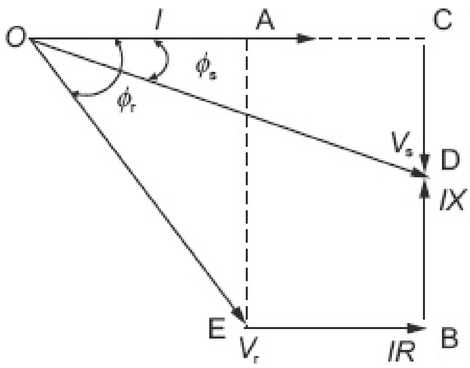

Fig. 3.4(b) Phasor diagram for leading power factor

The phasor diagram for unity power factor is shown in Fig. 3.4(a).

From Fig. 3.4(a), $V_S = [(V_r + IR)^2 + (IX)^2]^{1/2}$

The phasor diagram for leading power factor is shown in Fig. 3.4(b).

$$(OD)^2 = (OC)^2 + (CD)^2$$
$$= (OA + AC)^2 + (BC - BD)^2$$
$$V_s = \sqrt{(V_r \cos\phi_r + IR)^2 + (V_r \sin\phi_r - IX)^2}$$

The regulation will depend upon the power factor of the load. If the power factor is lagging, the voltage at the sending end is more than that at the receiving end. Hence, voltage regulation is positive. On the other hand, if the power factor is leading, the voltage at the sending end will be somewhat less than that at the receiving end. In that case, the regulation is negative.

The current is inversely proportional to the power factor of the load, since $I_L = \frac{P}{\sqrt{3}V_L \cos\phi}$. When the current increases, the power loss in the line increases with fall in the power factor of the load. Thus, efficiency of the line decreases with the fall in power factor and vice versa.

Test Yourself

1. Does regulation vary with the power factor? If so, draw a power factor vs. regulation curve.
2. Does the efficiency vary with power factor ? If yes, why?

Example 3.5

A single-phase, 11 kV line with a length of 15 km is to transmit 500 kVA. The inductive reactance of the line is 0.6 Ω/km and the resistance is 0.25 Ω/km. Calculate the efficiency and regulation for a power factor of (i) 0.75 lagging, (ii) 0.75 leading and (iii) unity.

Solution:

Receiving-end voltage, V_r = 11000 V

Length of the line, L =15 km

Power to be transmitted, S_r = 500 kVA = 500 × 10^3 VA

Resistance of the line = 0.25 Ω/km

Inductive reactance of the line = 0.6 Ω/km

Total resistance of the line, R = 0.25 × 15 = 3.75 Ω

Total inductive reactance of the line, X = 0.6 × 15 = 9.0 Ω

$$\text{Line current, } I = \frac{S_r}{V_r} = \frac{500 \times 10^3}{11000} = 45.45\text{A}$$

1. When power factor at receiving end, $\cos\phi_r = 0.75$ lag and $\sin\phi_r = \sin(\cos^{-1} 0.75) = 0.6614$

$$\text{Sending-end voltage, } V_s = \sqrt{(V_r \cos\phi_r + IR)^2 + (V_r \sin\phi_r + IX)^2}$$
$$= \sqrt{(11000 \times 0.75 + 45.45 \times 3.75)^2 + (11000 \times 0.6614 + 45.45 \times 9)^2}$$
$$= 11399.76 \text{ V}$$

$$\text{Percentage regulation} = \frac{V_s - V_r}{V_r} \times 100$$
$$= \frac{11399.76 - 11000}{11000} \times 100$$
$$= 3.634\%$$

Receiving-end power, $P_r = 500 \times 0.75 = 375$ kW

Losses in the line $= I^2R = (45.45)^2 \times 3.75 = 7.75$ kW

Sending-end power $= 375 + 7.75 = 382.75$ kW.

$$\text{Efficiency of the line, } \eta = \frac{\text{Receiving-end power}}{\text{Sending-end power}} \times 100 = \frac{375}{382.75} \times 100 = 97.98\%$$

2. When power factor at receiving end, $\cos\phi_r = 0.75$ lead and $\sin\phi_r = \sin(\cos^{-1} 0.75) = 0.6614$

$$\text{Sending-end voltage, } V_s = \sqrt{(V_r \cos\phi_r + IR)^2 + (V_r \sin\phi_r - IX_L)^2}$$
$$= \sqrt{(11000 \times 0.75 + 45.45 \times 3.75)^2 + (11000 \times 0.6614 - 45.45 \times 9)^2}$$
$$= 10865.1 \text{ V}$$

$$\text{Percentage regulation} = \frac{V_s - V_r}{V_r} \times 100$$
$$= \frac{10865.1 - 11000}{11000} \times 100$$
$$= -1.23\%$$

Receiving-end power, $P_r = 500 \times 0.75 = 375$ kW

Losses in the line $= I^2R = (45.45)^2 \times 3.75 = 7.75$ kW

Sending-end power $= 375 + 7.75 = 382.75$ kW

$$\text{Efficiency of the line, } \eta = \frac{\text{Receiving-end power}}{\text{Sending-end power}} \times 100 = \frac{375}{382.75} \times 100 = 97.98\%$$

3. When the power factor at receiving end is unity i.e., $\cos\phi_r$ = unity and $\sin\phi_r = 0$

$$\text{Sending-end voltage, } V_s = \sqrt{(V_r\cos\phi_r + IR)^2 + (IX_L)^2}$$
$$= \sqrt{(11000\times 1 + 45.45\times 3.75)^2 + (45.45\times 9)^2}$$
$$= 11177.925 \text{ V}$$

$$\text{Percentage regulation} = \frac{V_s - V_r}{V_r}\times 100$$
$$= \frac{11177.925 - 11000}{11000}\times 100$$
$$= 1.617\%$$

Receiving-end power, $P_r = 500$ kW

Losses in the line $= I^2R = (45.45)^2 \times 3.75 = 7.75$ kW

Sending-end power $= 500 + 7.75 = 507.75$ kW

$$\text{Efficiency of the line, } \eta = \frac{500}{507.75}\times 100 = 98.47\%.$$

3.5 GENERALISED NETWORK CONSTANTS

We can represent a three-phase overhead line as a two-port (four-terminal) network with one-port terminals as input (where power enters) and other port terminals as output (where power leaves). Further, the current entering in one port (sending-end) equals the current leaving the other port (receiving-end). Also such a circuit is passive, linear and bilateral.

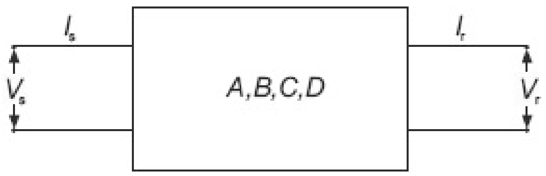

Fig. 3.5 Two-port representation of transmission line

The three-phase transmission line can be represented as a two-port network as shown in Fig. 3.5.

The steady-state voltages at the sending and receiving ends are expressed in terms of voltage and current at the receiving end.

They are expressed as

$$V_s = AV_r + BI_r \quad (3.9)$$

$$I_s = CV_r + DI_r \quad (3.10)$$

where, *A, B, C* and *D* are called the generalised transmission network constants. They are also known as transmission-line constants or auxiliary network constants, etc. They are complex numbers. The constants *A* and *D* is dimensionless; the constant *B* has the dimension of impedance (ohms) and the constant *C* has the dimension of admittance (mhos).

Equations (3.9) and (3.10) can be written in matrix form as:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \quad (3.11)$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the *transfer matrix* or the transmission-line matrix of the network.

The relation of a two-terminal pair network expressed in terms of general network constants *A, B, C* and *D* are useful in the analysis of power systems, viz., for the combination of the network, for drawing circle diagrams, etc.

The Eqs. (3.9) and Eqs. (3.10) can be re-written for computing receiving-end voltage and receiving-end current in terms of voltage and current at the sending end.

From Eqs. (3.9), $I_r = \dfrac{(V_s - AV_r)}{B}$

Substituting the expression of I_r in Eq. (3.10), we get

$$\begin{aligned}
I_s &= CV_r + \frac{D(V_s - AV_r)}{B} \\
&= V_r\left(C - \frac{AD}{B}\right) + \frac{D}{B}V_s \\
&= -V_r\frac{(AD - BC)}{B} + \frac{D}{B}V_s \\
\therefore V_r &= DV_s - BI_s
\end{aligned} \tag{3.12}$$

The condition for any passive network is $AD - BC = 1$.

From Eq. (3.10), we have

$$DI_r = I_s - CV_r$$

Substituting the value of V_r from Eq. (3.12) in the expression, we get

$$\begin{aligned}
DI_r &= I_s - C(DV_s - BI_s) \\
&= I_s - CDV_s + BCI_s \\
\text{or } I_r &= -CV_s + \frac{I_s(1 + BC)}{D}
\end{aligned}$$

Substituting $AD - BC$ for 1, we get

$$\begin{aligned}
I_r &= -CV_s + \frac{I_s(AD - BC + BC)}{D} \\
\therefore I_r &= -CV_s + AI_s
\end{aligned} \tag{3.13}$$

3.6 A, B, C, D CONSTANTS FOR SHORT TRANSMISSION LINES

A short transmission line can be considered as a circuit, having a series impedance. Its capacitance is negligible. Therefore, the shunt admittance, *Y*, is zero.

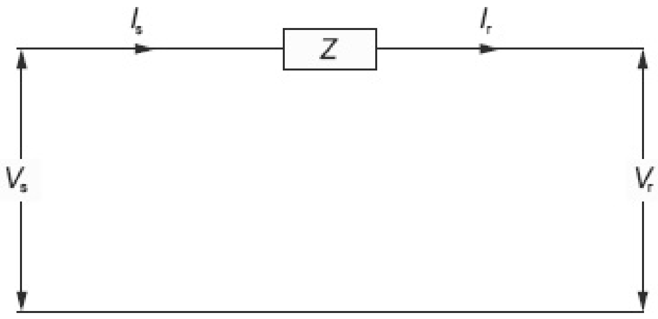

Fig. 3.6 Short transmission-line model

For the network shown in Fig. 3.6, we may write

$$V_s = V_r + ZI_r \qquad (3.14)$$

$$\text{and } I_s = I_r \qquad (3.15)$$

For generalized transmission circuit constants, comparing Eqs. (3.14) and (3.15) with the Eqs. (3.9) and (3.10), we get

$$\begin{aligned} A &= 1, B = Z \\ C &= 0, D = 1 \end{aligned} \qquad (3.16)$$

The transfer matrix for the network is $\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

We can verify the relation $AD - BC = 1 \times 1$ for this network

$$\text{i.e., } AB - BC = 1 \times 1 - Z \times 0 = 1.$$

3.7 MEDIUM TRANSMISSION LINE

When the length of an overhead transmission line is between 100 km and 250 km with an operating voltage ranging from 20 kV to 100 kV, it is considered as a medium transmission line. In medium lines, the series impedance and shunt admittance (pure capacitance) lumped at a few pre-determined locations are considered for calculation. These lines can be analyzed by using load end capacitance, nominal-T and nominal-π methods.

Subtransmission lines are examples of medium transmission lines. The subtransmission line is connected between transmission substation and distribution substation as shown in Fig. 3.7.

Fig. 3.7 View of a 132 kV transmission line

3.7.1 Load End Capacitance Method

In this method, the entire line capacitance is assumed to be concentrated at the receiving end as shown in Fig. 3.8(a).

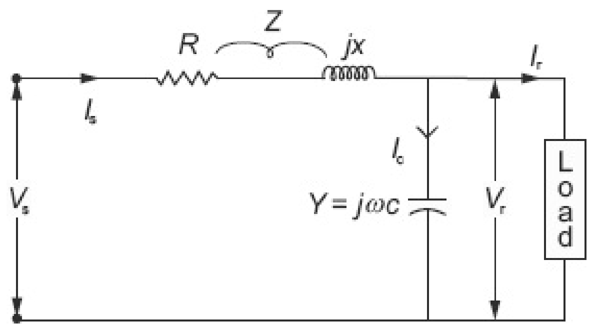

Fig. 3.8(a) Circuit diagram

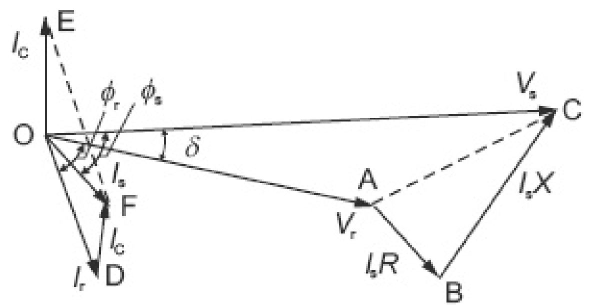

Fig. 3.8(b) Phasor diagram

Phasor Diagram The phasor diagram for the load end capacitance circuit is shown in Fig. 3.8(b).

The horizontal line *OA* is the per phase receiving-end voltage (V_r) (considered as a reference phasor). *OD* is the load current (I_r) which is lagging behind the V_r by an angle ϕ_r. *OE* = *DF* is the current passing through the shunt capacitance connected at load end (I_c) which leads the receiving-end voltage V_r by 90°. *OF* is the sending-end current (I_S) passing through the line and is equal to the phasor sum of receiving-end current (I_r) and capacitive current (I_c). *AB* is the resistive voltage drop

($I_S R$) which is in phase with I_S BC is the reactive voltage drop $I_S X$ that leads the I_S by 90°. OC is the sending-end voltage (V_S) and is equal to the phasor sum of V_r, $I_S R$ and $I_S X$ (or phasor sum of V_r and $I_S Z$).

The angle ϕ_S is the angle between the sending-end voltage V_S and sending-end current I_S and the angle between V_S and V_r is known as load angle (or torque angle) and is denoted by δ.

From the Fig. 3.8(a), the sending-end current is

$$I_s = I_r + I_c = I_r + YV_r$$
$$\therefore I_s = YV_r + I_r \quad (3.17)$$

The sending-end voltage is

$$V_s = V_r + I_s Z$$

By substituting I_S from Eq. (3.17) in the equation, we get

$$V_s = V_r + (YV_r + I_r)Z$$
$$\therefore V_s = (1 + YZ)V_r + I_r Z \quad (3.18)$$

Regulation To calculate regulation, it is required to calculate the no-load receiving-end voltage at (V_r') keeping the sending-end voltage (V_S) as constant. From Fig. 3.8(a), the voltage at the receiving end (V_r') at no load is

$$V_r' = \frac{V_s\left(\frac{-j}{\omega C}\right)}{R + jX - \frac{j}{\omega C}}$$

$$\text{Regulation, \%} = \frac{V_r' - V_r}{V_r} \times 100 \tag{3.19}$$

Efficiency

$$\text{The efficiency, } \eta = \frac{P_r}{P_r + 3I_s^2 R} \tag{3.20}$$

For generalized transmission circuit constants, comparing Eqs. (3.18) and (3.17) with Eqs. (3.9) and (3.10), we get

A = 1, *YZ*, *B* = Z

C = *Y*, *D* = 1.

$$\text{The transfer matrix for the network is } \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix} \tag{3.21}$$

The *disadvantages* of this method are:

1. There is a considerable error in calculations because the distributed capacitance has been assumed to be lumped.
2. This method overestimates the effect of line capacitance.

Example 3.6

A three-phase transmission line of 160 km long has the following constants: Resistance per km (*r*/km) = 0.15 Ω, inductive reactance per kilometre (*x*/km) = 0.6 Ω, charging admittance per kilometre (*y*/km) = 8 × 10^{-6} ℧ and the receiving-end voltage = 66 kV. Using load-end capacitance method, calculate (i) sending-end current (ii) the sending-end voltage and (iii) power factor at the sending end, when the line is delivering 15 MW at 0.8 p.f. lag.

Solution:

$$\text{Receiving-end voltage, } V_r = 66 \text{ kV(L-L)} = \frac{66000}{\sqrt{3}} = 38105 \text{ V/phase}$$

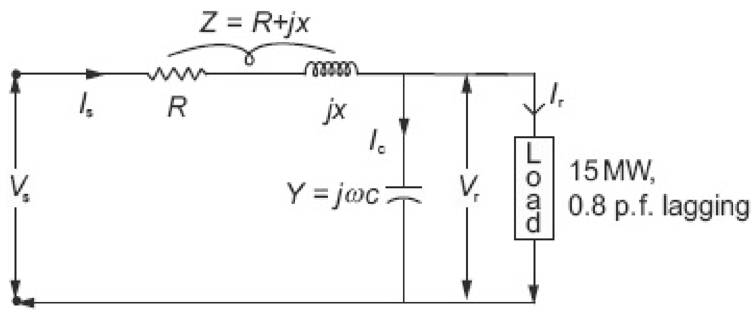

Fig. 3.9 Circuit diagram

Power factor of load, p.f. = 0.8 lag

Resistance per phase, $R = 0.150 \times 160 = 24\ \Omega$

Reactance per phase, $X = 0.60 \times 160 = 96\ \Omega$

Impedance, $Z = 24 + j96 = 98.95\angle 75.96°\Omega$

Admittance per phase, $Y = 8 \times 10^{-6} \times 160 = 12.8 \times 10^{-4}$ S

$$\text{Receiving-end current, } I_r = \frac{15 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0.8} = 164.02 \text{ A}$$

1. Current at the sending end, $I_S = V_r Y + I_r$

$$\begin{aligned} &= 38105 \times j12.8 \times 10^{-4} + 164.02\angle -36.87° \\ &= j48.7744 + 131.22 - j98.412 \text{ A} \\ &= 131.22 - j49.64 \text{ A} \\ &= 140.3\angle -20.72° \text{ A} \end{aligned}$$

2. Voltage at the sending end, $V_S = V_r + I_S Z$

$$\begin{aligned} \therefore V_s &= 38105 + 140.3\angle -20.72 \times 98.95\angle 75.96 \\ &= 38105 + 13882.685\angle 55.24° \\ &= 38105 + 7915.08 + j11405.28 \\ &= (46020.08 + j11405.28) \text{ V/phase} \\ &= 47412.32\angle 13.92° \text{ V/phase} \end{aligned}$$

3. Sending-end power factor angle, $\phi_S = 13.92° + 20.72° = 34.64°$

$$\therefore \text{power factor} = \cos\phi_s = 0.823 \text{ lag.}$$

Example 3.7

A single-phase transmission line 100 km long has the following constants: resistance per kilometre is 0.4 Ω, reactance per kilometre is 0.6 Ω, admittance per kilometre is 14 $\mu℧$, and receiving-end voltage is 33 kV. Assuming that the total capacitance of line is localized at the receiving-end, determine (i) sending-end current, (ii) sending-end voltage, (iii) regulation, (iv) efficiency, and (v) supply power factor if the line is delivering 5 MW at p.f. 0.8 lagging.

Solution:

Resistance of line per phase, $R = 100 \times 0.4 = 40\ \Omega$

Reactance of line per phase, $X = 100 \times 0.6 = 60\ \Omega$

Admittance per phase, $Y = 100 \times 14 \times 10^{-6} = 14 \times 10^{-4}$ S

Recieving-end voltage, $V_r = 33$ kV $= 33000$ V

$$\text{Load current, } I_r = \frac{5 \times 10^6}{33000 \times 0.8} = 189.4 \text{ A lag} = 189.4\angle -36.87°\text{A}$$

Line impedance, $Z = (R + jX) = (40 + j60)\ \Omega = 72.11\angle 56.31°\Omega$

Using load-end capacitance method and taking receiving-end voltage as reference vector

Voltage of reference phasor, $V_r = (33000 + j0)$ V

Power factor, $\cos\phi = 0.8$

$$\sin\phi = \sqrt{1 - 0.8^2} = 0.6$$

Charging current, $I_c = YV_r = j14 \times 10^{-4} \times 33000 = j46.2$ A

1. From Fig. 3.10, sending-end current, $I_S = I_r + I_c$

$$= 189.4\angle -36.87^\circ + j46.2$$
$$= 151.52 - j113.64 + j46.2$$
$$= 151.52 - j67.44 = 165.85\angle -24^\circ \text{ A}$$

Sending-end voltage, $V_s = V_r + ZI_s = 33000 + 165.85\angle -24^\circ \times 72.11\angle 56.31^\circ$
$$= 33000 + 11959.44\angle 32.31^\circ$$
$$= 33000 + 10107.74 + j6392.32$$
$$= 43107.74 + j6392.32 = 43579.11\angle 8.43^\circ \text{ V}$$

2. Sending-end voltage, V_S = 43579.11 V

Rise in voltage at no load, $V_r' = \dfrac{V_s\left(\dfrac{-j}{\omega C}\right)}{R + jX - \dfrac{j}{\omega C}}$

$$= \frac{43579.11\left(\dfrac{-j}{14\times 10^{-4}}\right)}{40 + j60 - \dfrac{j}{14\times 10^{-4}}} = \frac{-j31127935.71}{40 - j654.3}$$

$$= \frac{31127935.71\angle -90^\circ}{655.52\angle -86.5} = 47485.86\angle -3.5^\circ \text{ V}$$

3. Voltage regulation $= \dfrac{V_r' - V_r}{V_s} \times 100 = \dfrac{47485.86 - 33000}{33000} \times 100 = 43.9\%$

4. Line efficiency, $\eta = \dfrac{V_r I_r \cos\phi_r}{V_s I_s \cos\phi_s} \times 100$

$$= \frac{33000 \times 189.4 \times 0.8}{43579.11 \times 165.85 \times \cos(8.43 + 24)} \times 100 = 81.96\%$$

5. Supply power factor, $\cos\phi_s = \cos(8.43 + 24) = 0.844$ lag.

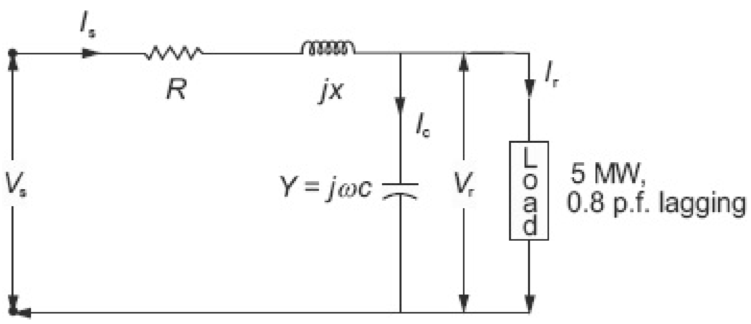

Fig. 3.10 Circuit diagram for Example 3.7

3.7.2 Nominal-T Method

In this method, the capacitance of each conductor is assumed to be concentrated at the mid-point of the line with half the series impedance on either side of it, as shown in Fig. 3.11(a). The charging current, therefore, flows through half the length of the line and the drop of volts in the impedance of this half-length of line will be approximately equal to the drop in the actual line. Here, Y represents the shunt admittance.

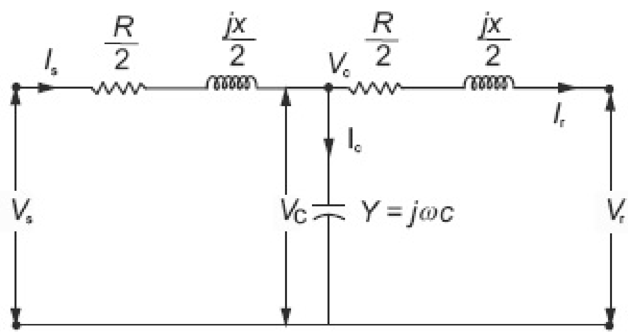

Fig. 3.11(a) T-equivalent circuit of medium transmission line

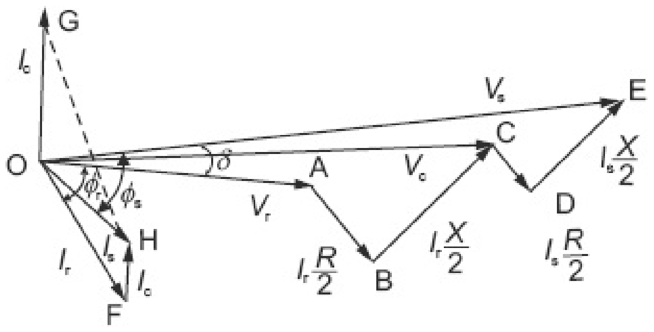

Fig. 3.11(b) Phasor diagram

Phasor Diagram The phasor diagram of the nominal-T circuit is shown in Fig. 3.11(b).

The horizontal line *OA* is the per phase receiving-end voltage (V_r) (considered as a reference phasor). *OF* is the load current (I_r) passing through the right half of the series impedance and lags behind the V_r

by an angle ϕ_r. *AB* is the resistive voltage drop $\left(\frac{I_r R}{2}\right)$ in

the right half of the series impedance and is in phase

with I_r. *BC* is the reactive voltage drop $\left(\frac{I_r X}{2}\right)$ in the right

half of the series impedance and leads with I_r by 90°. *OC* is the voltage across the capacitor (V_c) and is equal to the

phasor sum of V_r, $\frac{I_r R}{2}$ and $\frac{I_r X}{2}$ $\left(\text{or phasor sum of } V_r \text{ and } \frac{I_r Z}{2}\right)$.

OG is the charging current (I_c) which passes through shunt admittance and this current leads the capacitor voltage V_c by 90°.

OH is the sending-end current (I_s) passing through the left half of the series impedance and is equal to the phasor sum of the receiving-end current (I_r) and the charging current (I_c). *CD* is the resistive voltage drop

$\left(\frac{I_s R}{2}\right)$ in the left half of the series impedance which is in phase with I_S. *DE* is the reactive voltage drop $\left(\frac{I_s X}{2}\right)$ in the left half of the series impedance and this voltage drop leads the sending-end current (I_S) by 90°.

OE is the sending-end voltage (V_S) and is equal to the phasor sum of V_r, $\frac{I_s R}{2}$ and $\frac{I_s X}{2}$ $\left(\text{or phasor sum of } V_c \text{ and } \frac{I_s Z}{2}\right)$.

The angle ϕ_S is the angle between the sending-end voltage V_S and the sending-end current I_S, and the angle between V_S and V_r is known as load angle (or torque angle) and is denoted by δ.

From Fig. 3.11 (a)

$$V_c = V_r + \frac{Z}{2} I_r \qquad (3.22)$$

The current through the shunt admittance is

$$I_c = V_c Y$$
$$= \left(V_r + \frac{Z}{2} I_r\right) Y$$
$$I_c = YV_r + \frac{YZ}{2} I_r \qquad (3.23)$$

Now the sending-end current is

$$
\begin{aligned}
I_s &= I_r + I_c \\
&= I_r + YV_r + \frac{YZ}{2} I_r
\end{aligned}
$$

$$
I_s = YV_r + \left(1 + \frac{YZ}{2}\right) I_r \tag{3.24}
$$

and the sending-end voltage is

$$
\begin{aligned}
V_s &= V_s + \frac{Z}{2} I_s \\
&= V_r + \frac{Z}{2} I_r + \frac{Z}{2}\left[YV_r + \left(1 + \frac{YZ}{2}\right) I_r \right] \\
&= \left(1 + \frac{YZ}{2}\right) V_r + \left(\frac{Z}{2} + \frac{Z}{2} + \frac{YZ^2}{4}\right) I_r
\end{aligned}
$$

$$
\therefore V_s = \left(1 + \frac{YZ}{2}\right) V_r + Z\left(1 + \frac{YZ}{4}\right) I_r \tag{3.25}
$$

For general transmission circuit constants, comparing the Eqs. (3.24) and (3.25) with the Eqs. (3.10) and (3.9), respectively, we get

$$
A = 1 + \frac{YZ}{2} = D; \quad B = Z\left(1 + \frac{YZ}{4}\right), \quad C = Y
$$

The transfer matrix for the network is

$$
\begin{bmatrix} 1 + \frac{YZ}{2} & Z\left(1 + \frac{YZ}{4}\right) \\ Y & 1 + \frac{YZ}{2} \end{bmatrix} \tag{3.26}
$$

Regulation Under no load condition, the equivalent circuit of Fig. 3.11 (a) is shown in Fig. 3.11 (c).

At no load, the voltage at the receiving end (V_r') of the transmission line is same as the voltage (V_c) across the admittance, which is located at midpoint of the transmission line.

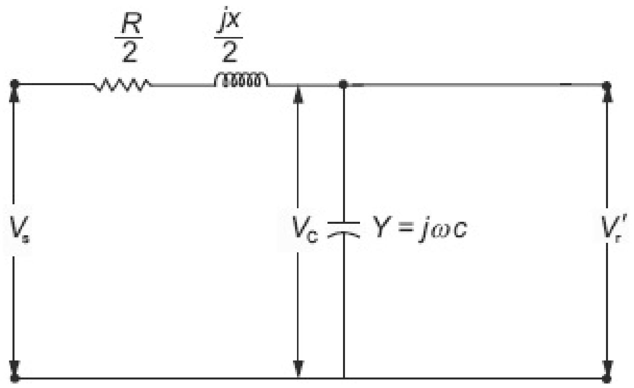

Fig. 3.11(c)

From Fig. 3.11 (c), the voltage across the capacitor by using voltage divider rule is

$$V_c = \frac{V_s\left(\frac{-j}{\omega C}\right)}{\frac{R}{2}+j\frac{X}{2}-\frac{j}{\omega C}}$$

When the receiving end is on no load, the no-load voltage, $V_r' = V_c$

$$\therefore V_r' = V_c = \frac{V_s\left(\frac{-j}{\omega C}\right)}{\frac{R}{2}+j\frac{X}{2}-\frac{j}{\omega C}} \tag{3.27}$$

$$\%\ \text{Regulation} = \frac{V_r' - V_r}{V_r}\times 100 \tag{3.28}$$

Efficiency

$$\text{The efficiency, } \eta = \frac{\text{Power delivered at the receiving end } (P_r)}{\text{Power delivered at the receiving end } (P_r) + 3\frac{R}{2}\left(I_r^2 + I_s^2\right)} \tag{3.29}$$

Example 3.8

A 50 Hz, three-phase transmission line is 250 km long. It has a total series impedance of (40 + *j*100) Ω and a shunt admittance of 914 × 10^{-6} Ω. It delivers 50 MW at 220 kV with a p.f. of 0.9 lag. Find the (i) sending-end voltage, (ii) voltage regulation and (iii) transmission efficiency by nominal-T method.

Solution:

Length of the line, l = 250 km

Impedance per phase, $Z = (40 + j100)\Omega$

Admittance per phase, $Y = j\,914 \times 10^{-6}$ ℧

Power at receiving end, P_r = 50 MW

Voltage at receiving end, V_r = 220 kV (L-L)

Power factor, $\cos\phi_r$ = 0.9 lag

Impedance of each series branch, $\frac{Z}{2} = (20 + j50)\,\Omega = 52.2\angle 73.3°\,\Omega$

Voltage per phase, $V_r = \frac{220 \times 10^3}{\sqrt{3}} = 127017\,\text{V}$

Receiving current, $I_r = \frac{P_r}{\sqrt{3}\,V_r \cos\phi_r} = \frac{50 \times 10^6}{\sqrt{3} \times 220 \times 10^3 \times 0.9} = 145.8\text{ A lag}$

$$\therefore I_r = 145.8\angle -25.84°\,\text{A}$$

From Fig. 3.12,

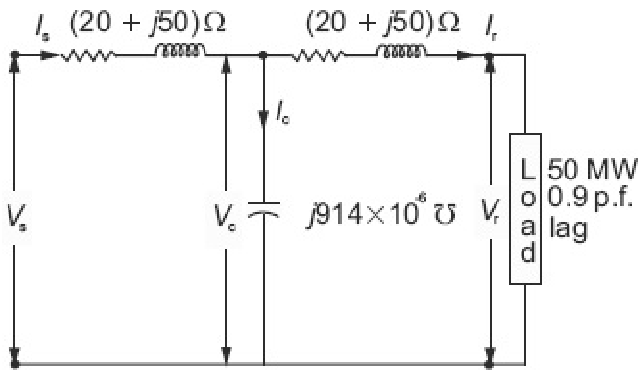

Fig. 3.12 Circuit diagram for Example 3.8

$$
\begin{aligned}
V_c &= V_r + I_r(Z/2)\\
&= 127017\angle 0^\circ + 145.86\angle{-25.84^\circ} \times 53.85\angle 68.2^\circ \text{V}\\
&= 127017 + 7854.8\angle 42.36^\circ\\
&= 132926.64\angle 2.28^\circ \text{ V.}\\
I_c &= jV_cY\\
&= (132926.64\angle\, 2.28)\,(914 \times 10^{-6}\angle\, 90^\circ)\\
&= 121.5\angle\, 92.28\\
&= (-4.83 + j121.4)\text{ A}\\
I_s &= I_r + I_c\\
&= 131.27 - j63.57 - 4.83 + j121.4\\
&= 126.45 + j57.83 = 139\angle\, 24.59^\circ \text{ A}
\end{aligned}
$$

1. $V_s = V_c + I_s \dfrac{Z}{2}$

$$
\begin{aligned}
&= 132926.64\angle 2.28^\circ + (139\angle 24.59^\circ)(53.85\angle 68.2^\circ)\\
&= 133070.68\angle 5.51^\circ \text{ V/phase}
\end{aligned}
$$

Sending-end voltage, V_S = 230.48 kV (L-L)

2. Efficiency, $\eta = \dfrac{P_r}{P_s} = \dfrac{\sqrt{3}V_rI_r\cos\phi_r}{\sqrt{3}V_sI_s\cos\phi_s}$

$$= \frac{50 \times 10^3}{\sqrt{3} \times 230.48 \times 139 \times \cos(5.51° - 24.59°)} = 95.3\%$$

Receiving-end voltage at no load, $= \dfrac{V_s \times \dfrac{-j}{\omega C}}{\dfrac{R + jX}{2} - \dfrac{j}{\omega C}}$

$$= \frac{133070.68\angle 5.51° \times \dfrac{-j}{914 \times 10^{-6}}}{\dfrac{40 + j100}{2} + \dfrac{-j}{914 \times 10^{-6}}} = 137537.81\angle 3.35°\ \text{V}$$

$$= 137.54\ \text{kV}.$$

3. Voltage regulation $= \dfrac{V_r' - V_r}{V_r} \times 100 = \dfrac{137.54 - 127}{127} \times 100 = 8.23\%$.

3.7.3 Nominal-Π Method

In this method, the capacitance of each conductor is assumed to be divided into two halves, one half being shunted between conductor and neutral at the receiving-end and the other half at the sending-end. The total impedance is placed in between them as shown in Fig. 3.13 (a).

The disadvantage of nominal-T and π methods is error in the calculation of sending-end voltage in the order of 10% at 50 Hz frequency since parameters are assumed to be lumped for medium transmission lines.

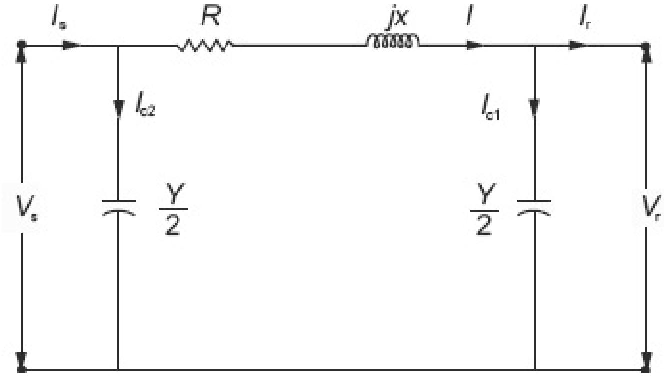

Fig. 3.13(a) π-equivalent circuit of medium transmission line

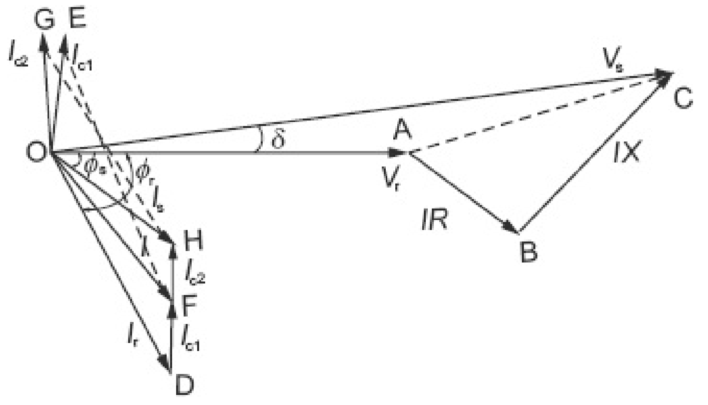

Fig. 3.13(b) Phasor diagram

Phasor Diagram The phasor diagram of the nominal-π circuit is shown in Fig. 3.13 (b). The horizontal line *OA* is the per phase receiving-end voltage (V_r) (considered as a reference phasor). *OD* is the load current (I_r) and this current lags behind the V_r by an angle ϕ_r. *OE* is the charging current (I_{c1}) of the right half of the shunt admittance and is equal to $\frac{V_r Y}{2}$. *OF* is the current passing through the series impedance (I) and is equal to the phasor sum of I_r and I_{c1}. *AB* is the resistive voltage drop (*IR*) in series impedance and is in phase with *I*. *BC* is the reactive voltage drop (*IX*) in series impedance and this voltage leads the current *I* by 90°. *OC* is the voltage at the sending end or the voltage across the shunt admittance connected at the sending end (V_s) and is equal to the phasor sum of V_r, *IR* and *IX* (or phasor sum of V_r and *IZ*). *OG* is the charging current (I_{c2}) which passes through shunt admittance at the sending end and this current leads the voltage V_s by 90°. *OH* is the sending-end current (I_s) and is equal to the phasor sum of the current (I) and charging current (I_{c2}).

The angle ϕ_s is the angle between the sending-end voltage (V_s) and the sending end current (I_s), and the angle between V_s and V_r is known as load angle (or torque angle) and is denoted by δ.

From Fig. 3.13 (a)

$$V_{c1} = V_r$$
$$I_{c1} = V_{c1}\frac{Y}{2} = \frac{Y}{2}V_r \qquad (3.30)$$

$$I = I_r + I_{c1} = I_r + \frac{Y}{2}V_r \qquad (3.31)$$
$$V_{c2} = V_{c1} + IZ = V_r + \left(I_r + \frac{Y}{2}V_r\right)Z$$
$$V_{c2} = \left(1+\frac{YZ}{2}\right)V_r + ZI_r$$

V_{c2} is also equal to V_s

$$\therefore V_s = \left(1+\frac{YZ}{2}\right)V_r + ZI_r \qquad (3.32)$$

The charging current, $I_{c2} = V_s\dfrac{Y}{2}$

The sending-end current, $I_s = I + I_{c2}$

$$= I_r + \frac{Y}{2}V_r + \left[\left(1+\frac{YZ}{2}\right)V_r + ZI_r\right]\frac{Y}{2}$$
$$= \left(\frac{Y}{2}+\frac{Y}{2}+\frac{Y^2Z}{4}\right)V_r + \left(1+\frac{YZ}{2}\right)I_r \qquad (3.33)$$
$$= Y\left(1+\frac{YZ}{4}\right)V_r + \left(1+\frac{YZ}{2}\right)I_r$$

For general network constants, comparing the Eqs. (3.32) and (3.33) with general transmission circuit constants of Eqs. (3.9) and (3.10)

$$A = D = 1 + \frac{YZ}{2}; \quad B = Z$$
$$C = Y\left(1 + \frac{YZ}{4}\right)$$

$$\text{The transfer matrix for the network is } \begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2} \end{bmatrix} \tag{3.34}$$

Regulation To calculate regulation, it is required to calculate the no load receiving-end voltage (V_r') keeping V_S as constant. The voltage at the receiving end at no load V_r' is equivalent to V_{C1}.

From Fig. 3.13 (a), the voltage at the receiving end under no load is

$$V_r' = \frac{V_s\left(-\frac{2j}{\omega C}\right)}{R + jX - \frac{2j}{\omega C}} \tag{3.35}$$

$$\%\ \text{Regulation} = \frac{V_r' - V_r}{V_r} \times 100 \tag{3.36}$$

Efficiency

$$\text{The efficiency, } \eta = \frac{\text{Power delivered at the receiving end } (P_r)}{\text{Power delivered at the receiving end } (P_r) + 3I^2R}$$

Test Yourself

1. Will the sending-end voltage be equal to the receiving-end voltage at unity power factor load for medium lines? If yes, why?
2. What is the order of error for the methods usually employed for medium lines? Justify your answers.
3. What is the effect of line capacitance for lagging load?

Example 3.9

A three-phase, 50 Hz transmission line, 100 km long delivers 25 MW at 110 kV and a 0.85 p.f. lagging (see Fig. 3.14). The resistance and reactance of the lines per phase per kilometre are 0.3 Ω and 0.5 Ω respectively, while capacitive admittance is 2.5×10^{-6} Ω/km/ph. Calculate the efficiency of transmission. Use nominal-π and T methods.

Solution:

Nominal-π method:

Admittance per kilometre per phase, $Y = 2.5 \times 10^{-6}$ ℧

Total admittance per phase, $Y = 2.5 \times 10^{-6} \times 100 = 2.5 \times 10^{-4}$ ℧

Impedance per kilometre per phase, $Z = (0.3 + j\,0.5)$ Ω

Total impedance, $Z = (0.3 + j\,0.5) \times 100 = (30 + j\,50) = 58.31\angle 59°$ Ω

Receiving-end voltage, $V_r = 110$ kV (L-L)

$$= \frac{110}{\sqrt{3}} \times 10^3 = 63508.5 \text{ V/phase}$$

$$\text{Load current, } I_r = \frac{25 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0.85} = 154.37 \text{ A lag}$$

$$= 154.37\angle -31.790 \text{ A}$$

$$\therefore \text{Charging current, } I_{c1} = V_r \frac{Y}{2} (\because V_{c1} = V_r)$$

$$= 63508.5\angle 0 \times 1.25 \times 10^{-4}\angle 90°$$

$$= 7.94\angle 90° \text{ A}$$

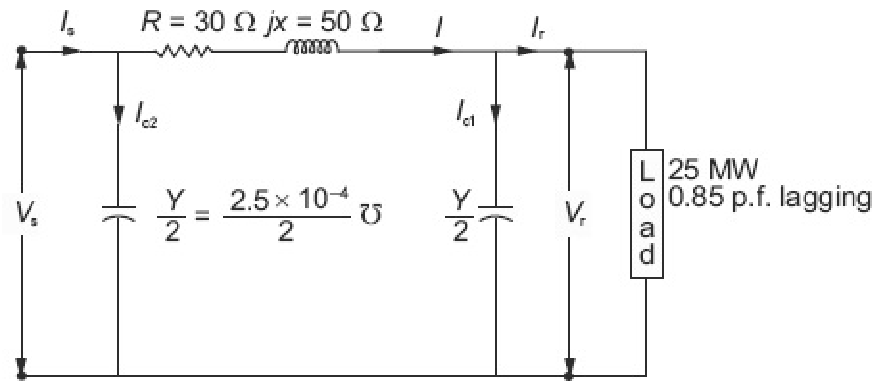

Fig. 3.14 Circuit diagram

$$\therefore I = I_r + I_{c1} = 154.37\angle{-31.79^\circ} + 7.94\angle 90^\circ$$
$$= 131.21 - j81.32 + 0 + j7.94 = 131.21 - j73.38 = 150.34\angle{-29.22^\circ}\text{ A}$$
$$\therefore V_s = V_{c2} = V_{c1} + IZ = 63508.5\angle 0 + 150.34\angle{-29.22^\circ} \times 58.31\angle 59^\circ$$
$$= 63508.5 + 8766.32\angle 29.78^\circ$$
$$= 71250.29\angle 3.5^\circ\text{ V}$$
$$\therefore \text{Sending-end voltage, } V_s = 123.41\text{ kV (L-L)}$$
$$\text{Charger current, } I_{c2} = V_s\frac{Y}{2} = 71250.29\angle 3.15^\circ \times 1.25 \times 10^{-4}\angle 90^\circ$$
$$= 8.91\angle 93.5^\circ\text{ A}$$
$$\therefore \text{Sending-end current, } I_s = I + I_{c2}$$
$$= 150.34\angle{-29.22^\circ} + 8.91\angle 93.5^\circ$$
$$= 145.72\angle{-26.27^\circ}\text{ A}$$
$$\therefore \text{Power factor at sending end} = \cos\phi_s = \cos(3.5^\circ + 26.24^\circ) = 0.87\text{ p.f. lagging.}$$
$$\text{Sending-end power, } P_s = \sqrt{3}V_s I_s \cos\phi_s$$
$$= \sqrt{3} \times 123.41 \times 145.72 \times 0.87$$
$$= 27.037\text{ kW}$$
$$\text{Transmission-line efficiency, } \eta = \frac{P_r}{P_s} \times 100$$
$$= \frac{25}{27.037} \times 100$$
$$= 92.46\ \%.$$

Nominal-T method:

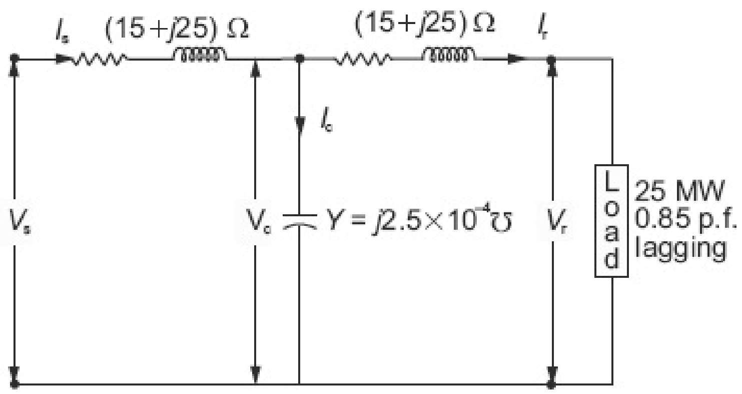

Fig. 3.15 Circuit diagram

As seen in Fig. 3.15,

Total admittance per phase, $Y = 2.5 \times 10^{-4}$ S

Total impedance, $Z = (30 + j50)\ \Omega$/phase

Receiving-end voltage, V_r = 110 kV (L-L)

$$= \frac{110}{\sqrt{3}} \times 10^3 = 63508.5 \text{ V/phase}$$

$$\text{Load current, } I_r = \frac{25\times10^6}{\sqrt{3}\times110\times10^3\times0.85} = 154.37 \text{ A}$$

$$= 154.37\angle -31.79° \text{ A}$$

Voltage across condenser, $V_c = V_r + I_r \frac{Z}{2}$

$$= 63508.5 + 154.37\angle{-31.79°} \times 29.15\angle 59°$$
$$= 63508.5 + 4499.89\angle 27.21°$$
$$= 63508.5 + 4001.92 + j2057.59$$
$$= 67510.42 + j2057.59$$
$$= 67541.81\angle 1.746° \text{ V}$$

Charging current, $I_c = V_c Y$

$$= 67541.81\angle 1.746° \times 2.5 \times 10^{-4}\angle 90°$$
$$= 16.88\angle 91.746° \text{ A}$$

Sending-end current, $I_s = I_r + I_c$

$$= 154.37\angle{-31.79°} + 16.88\angle 91.746°$$
$$= 131.21 - j81.32 - 0.51 + j16.87$$
$$= 130.7 - j64.45$$
$$= 145.73\angle{-26.25°}$$

Sending-end phase voltage $V_s = V_c + I_s \frac{Z}{2}$

$$= 67541.81\angle 1.746° + 145.73\angle{-26.25°} \times 29.15\angle 59°$$
$$= 67541.81\angle 1.746° + 4248.03\angle 32.75$$
$$= 67541.81 + j0.00148 + 3572.70 + j2298$$
$$= 71083.2 + j4355.98$$
$$= 71216.55\angle 3.51°$$

Sending-end line voltage, $V_s = \sqrt{3} \times 71216.55 = 123.35 \text{ kV}$

Total line loss $= 3\frac{R}{2}(I_s^2 + I_r^2) = 3 \times 15\left(145.5^2 + 154.37^2\right)$

$$= 2028029.84 \text{ W} = 2.03 \text{ MW}$$

Transmission efficiency, $\eta = \frac{25}{25 + 2.03} \times 100 = 92.48\%$.

Example 3.10

A three-phase 50 Hz transmission line is 150 km long and delivers 25 MW at 110 kV and a 0.85 p.f. lagging (see Fig. 3.16). The resistance and reactance of the line per conductor per kilometre are 0.3 Ω and 0.9 Ω, respectively. The line charging admittance is 0.3 × 10^{-6} ℧/km/ph. Compute the voltage regulation and transmission efficiency by applying the nominal-π method.

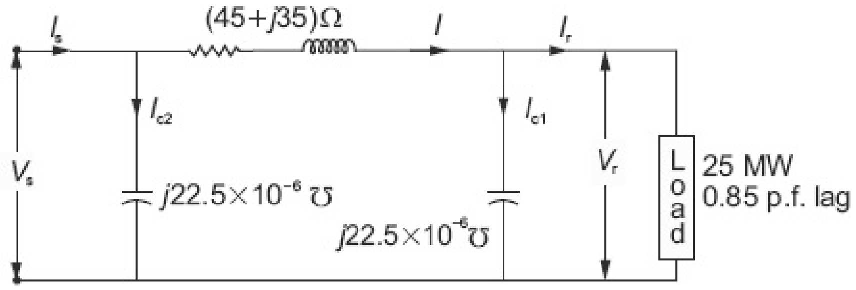

Fig. 3.16 Circuit diagram for Example 3.10

Solution:

Total resistance phase, $R = 0.3 \times 150 = 45\ \Omega$

Total reactance phase, $X = 0.9 \times 150 = 135\ \Omega$

Capacitive admittance per phase; $Y = 0.3 \times 10^{-6} \times 150 = 45 \times 10^{-6}$ S

Receiving-end voltage phase, $V_r = \dfrac{110\times10^3}{\sqrt{3}} = 63.508$ kV

Load current, $I_r = \dfrac{25\times10^6}{\sqrt{3}\times110\times10^3\times0.85} = 154.37$ A lag

$\cos\phi_r = 0.85$, $\sin\phi_r = 0.53$

$\therefore I_r = 154.37\,(0.85 - j0.53) = (131.21 - j81.81)$ A

$$I_{c1} = V_r \frac{jY}{2}$$

$$= 63.5\times10^3 \times j\frac{45\times10^{-6}}{2} = j1.43 \text{ A}$$

$$I = I_r + I_{c1}$$
$$= 131.21 - j81.81 + j1.43$$
$$= (131.21 - j80.38) \text{ A}$$

Sending-end voltage, $V_s = V_r + IZ$

$$= 63508 + (131.21 - j80.38)(45 + j135)$$
$$= 63508 + (153.87\angle -31.49^\circ)(142.3\angle 71.56^\circ)$$
$$= 63508 + 21896\angle 40.07^\circ$$
$$= 63508 + 16756.1 + j14094.95$$
$$= 80263.75 + j14096.76$$
$$= 81.49\angle 9.96^\circ \text{ kV/phase}$$

Line-to-line sending-end voltage, $(V_{s(L\text{-}L)}) = 81.49\times\sqrt{3}$

$$= 141.14 \text{ kV}$$

$\therefore$ Receiving-end voltage at no load, $V_r' = \dfrac{V_s\left(-\dfrac{2j}{\omega C}\right)}{R + jX - \dfrac{j}{\omega C/2}}$

$$= \frac{81.49\angle 9.96^\circ \times \dfrac{-j2}{45\times10^{-6}}}{45 + j135 - \dfrac{j2}{45\times10^{-6}}}$$

$$= \frac{3621777.8\angle -80.04^\circ}{45 - j44309.4^\circ}$$

$$= \frac{3621777.8\angle -80.04^\circ}{44309.42\angle -89.94^\circ}$$

$$= 81.74\angle 9.9$$

$$\therefore V_r' = 81.74\angle 9.9^\circ \text{ kV}$$

Percentage of regulation $= \dfrac{V_r' - V_r}{V_r}\times100$

$$= \frac{81.74 - 63.5}{63.5}\times100$$
$$= 28.7\%$$

$$I_{C2} = jV_s \frac{Y}{2}$$

$$= (80263.87 + j14094.76) \times \frac{j45 \times 10^{-6}}{2}$$

$$= j1.81 - 0.32$$

$$= (-0.32 + j1.81)\ \text{A}$$

$\therefore$ Sending-end current, $I_s = I + I_{C2}$

$$= 131.21 - j80.38 - 0.32 + j1.81$$

$$= 130.89 - j78.57$$

$$I_s = 152.66\angle{-30.97°}\ \text{A}$$

Sending-end phase angle, $\phi_s = 9.96° + 30.97° = 40.93°$

Sending-end power, $P_s = 3V_sI_s\cos\phi_s$

$$= 3 \times 81490 \times 152.66 \times \cos 40.93°$$

$$= 28.19\ \text{MW}$$

$\therefore$ Receiving-end power, $P_r = 25$ MW (given)

$\therefore$ Transmission efficiency $= \frac{P_r}{P_s} \times 100$

$$= \frac{25}{28.18} \times 100 = 88.72\%.$$

Example 3.11

A three-phase, 50 Hz transmission line as seen in Fig. 3.17, has resistance, inductance and capacitance per phase of 1 Ω, 0.3 H and 0.01 μF, respectively and delivers a load of 25 MW at 110 kV and 0.8 p.f. lagging. Determine the efficiency and regulation of the line using nominal-π method.

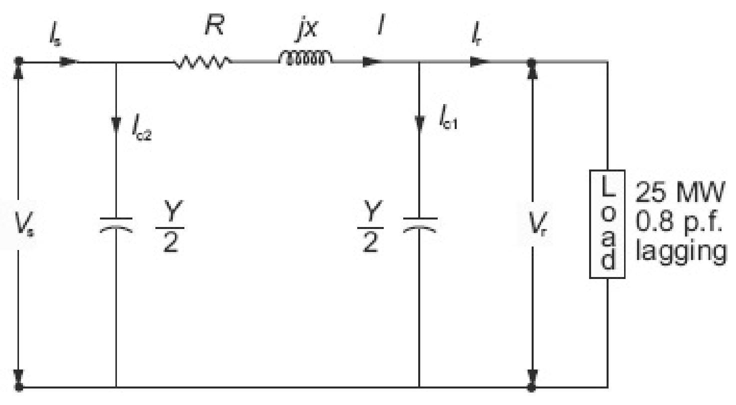

Fig. 3.17 Circuit diagram for Example 3.11

Solution:

Resistance $R = 1\ \Omega$

Inductance, $L = 0.3$ H

Inductance reactance, $X = 2\pi fL$

$$= 2\pi \times 50 \times 0.3 = 94.25\ \Omega$$

Series impedance, $Z = 1 + j\,94.25 = 94.26\angle 89.39°\ \Omega$

Capacitance, $C = 0.01\ \mu F$

Shunt admittance, $Y = j2\pi fC$

$$= j2 \times \pi \times 50 \times 0.01 \times 10^{-6} = j3.142 \times 10^{-6}\ S$$

Voltage at receiving end, $V_r = 110$ kV

Power factor, $\cos\phi = 0.8$ lag

Receiving-end current, $I_r = \frac{P_r}{\sqrt{3} V_r \cos\phi_r} = \frac{25 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0.8} = 164.02\angle -36.87°$ A

$$V_r = V_{c1} = \frac{110}{\sqrt{3}} \times 1000 = 63508.5 \text{ V}$$

$$I_{c1} = \frac{Y}{2} \times V_{c1} = \frac{j3.142 \times 10^{-6}}{2} \times 63508.5$$

$= 0.1\angle 90°$ A

Then, $I = I_r + I_{c1} = 164.02\angle -36.87° + 0.1\angle 90°$

$= 131.22 - j98.41 + 0 + j0.1$

$= 131.22 - j98.31$

$= 163.96\angle -36.84°$ A

Sending-end voltage, $V_s = V_{c2} = V_{c1} + IZ$

$= 63508.5\angle 0° + 163.96\angle -36.84° \times 94.26\angle 89.39°$

$= 63508.5\angle 0° + 15454.87\angle 52.55°$

$= 63508.5 + j0.0 + 9397.62 + j12269.4$

$= 72906.2 + j12269.4$

$= 73931.32\angle 9.6°$ V

Sending-end line voltage, $V_s = 73931.32 \times \sqrt{3} = 128.05$ kV

Charging current, $I_{c2} = \frac{Y}{2} V_{c2} = 73931.32\angle 9.6° \times \frac{j3.142}{2} \times 10^{-6} \quad (\because V_{c2} = V_s)$

$= 0.116\angle 99.6°$ A

Sending end current, $I_s = I + I_{c2} = 131.22 - j98.31 + 0.116\angle 99.6°$

$= 131.22 - j98.31 - 0.019 + j0.114 = 131.201 - j98.196$

$= 163.88\angle -36.81°$ A

$\therefore$ Sending-end power factor, $\cos(\phi_s) = \cos(9.6° + 36.81°) = 0.6895$

$\therefore$ Power at sending end, $P_s = 3 \times V_{sph} \times I_s \times \cos\phi_s$

$= 3 \times V_s \times I_s \times \cos\phi_s$

$= 3 \times 73931.32 \times 163.88 \times 0.6895$

$= 25.06$ MW

$\therefore$ Efficiency, $\eta = \frac{P_r}{P_s} \times 100 = \frac{25}{25.06} \times 100 = 99.76\%$

Receiving-end voltage at no load, $V_r' = \frac{V_s}{(R + jX) - \frac{2j}{\omega C}} \left(\frac{-2j}{\omega C} \right)$

$$= \frac{73931.32\angle 9.6° \times 636537.24\angle -90°}{1 + j94.25 + 0 - j636537.24}$$

$$= \frac{4.706 \times 10^{10}\angle -80.04°}{636442.99\angle -90} = 73942.38\angle 9.96° \text{V}$$

$$\%\text{Regulation} = \frac{V_r' - V_r}{V_r} \times 100$$

$$= \frac{73942.38 - 76210.2}{76210.2} \times 100 = -2.98\%$$

Example 3.12

A three phase, 50 Hz, 100 km long transmission line (see Fig. 3.18) delivers a load of 20,000 kW at 110 kV at 0.9 p.f. lagging. The copper conductors of the line are 1.25 cm in diameter and are spaced at the corners of the equilateral triangle with 2.5 m sides. Using nominal-π method, calculate (i) sending-end voltage, (ii) current, (iii) power factor, (iv) regulation, and (v) efficiency of the line. Neglect the leakage.

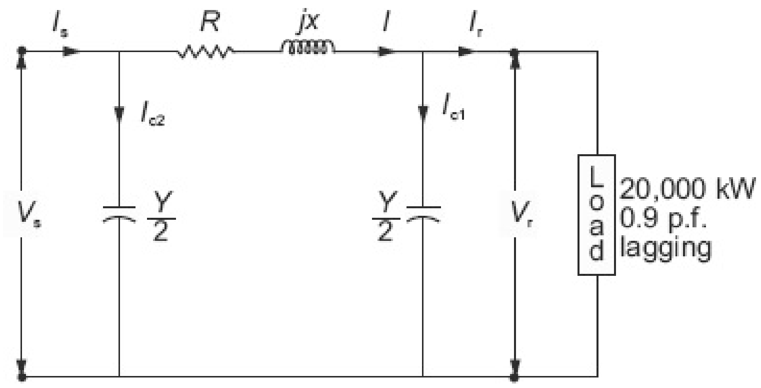

Fig. 3.18 Circuit diagram for Example 3.12

Solution:

Diameter of copper conductors, d = 1.25 cm

Radius of copper conductors, $r = \frac{1.25}{2} = 0.625\,\text{cm}$

Effective radius of conductor, $r' = 0.7788 \times 0.625 = 0.48675$ cm

Distance between conductors, $D = 2.5$ m = 250 cm

$\therefore$ The inductance of the line, $L = 2\times10^{-7}\ln\frac{D}{r'}$ H/m

$$= 2\times10^{-7}\ln\frac{250}{0.48675}\text{H/m}$$
$$= 2.12\text{ mH/km}$$

$\therefore$ For 100 km, inductance $(L) = 2.12 \times 10^{-3} \times 100 = 0.212$ H

Inductive reactance, $X_L = 2\pi fL$

$$= 2 \times 3.14 \times 50 \times 0.212 = 66.57\,\Omega$$

The capacitance of the line, $C = \frac{2\pi\varepsilon_r}{\ln\frac{d}{r}}$

$$= \frac{2\times3.14\times8.854\times10^{-12}}{\ln\frac{250}{0.625}} = 9.28\times10^{-12}\text{ F/m}$$

$\therefore$ For 100 km, capacitance, $C = 9.28\times10^{-12}\times100\times10^{3} = 9.28\times10^{-7}$ F

Total admittance, $Y = j2\pi fC = 2\pi\times50\times9.28\times10^{-7} = j2.916\times10^{-4}$ S

Receiving-end voltage per phase, $V_r = \frac{110\times10^{3}}{\sqrt{3}}$ V

$$= 63508.5\text{ V}$$

Current at receiving end, $I_r = \frac{20\times10^{3}}{\sqrt{3}\times110\times0.9}$ A = 116.64 A lag

$$= 116.64\angle-25.84^\circ\text{ A}$$

$$I_{c1} = \frac{Y}{2}V_r$$
$$= j\frac{2.916}{2}\times10^{-4}\times63508.5 = j9.26\text{ A}$$

$$I = I_r + I_{c1}$$
$$= 105 - j50.84 + j9.26$$
$$= (105 - j41.58)\text{ A}$$

1. Sending-end voltage, $V_S = V_r + IZ$

$$= 63508.5 + (105 - j41.58)(j66.57) \quad (\because R = 0)$$
$$= 66644.1\angle6.02^\circ\text{ V}$$

2. The sending-end current, $I_S = I + V_S Y/2$

$$= (105 - j41.58) + (66644.1\angle 6.02°) \times \frac{j2.916 \times 10^{-4}}{2}$$
$$= 108.79\angle -17.06° \text{ A}$$

3. Sending-end power factor, $\cos\phi_S = \cos(6.02° + 17.06°) = 0.919$ lag

$$\text{Receiving-end voltage at no load, } V_r' = \frac{66644.1\angle 6.02° \times \dfrac{-2j}{2.916 \times 10^{-4}}}{j66.67 + \dfrac{-2j}{2.916 \times 10^{-4}}}$$
$$= 67298.27\angle 6.02 \text{ V}$$

4. Percentage of regulation $= \dfrac{67298.27 - 63508.5}{63508.5} \times 100 = 5.96\%$

$$\text{The line current} = I^2 R$$
$$= (105)^2 \times 0 \quad (\text{since } R = 0)$$
$$= 0$$

5. Efficiency = 100%.

Example 3.13

A balanced three-phase load of 30 MW is supplied at 132 kV, 50 Hz and 0.85 p.f. lag by means of a transmission line. The series impedance of a single conductor is (20 + *j*40) Ω and the total phase-neutral admittance is 315 × 10^{-6} ℧. Use nominal-π and T methods to determine (i) *A, B, C, D* constants of the line, (ii) V_S, and (iii) regulation of the line.

Solution:

Series impedance of the line = (20 + *j* 40) Ω/phase

Shunt admittance of the line = *j* 13 × 10^{-6}℧/phase

Nominal-*π* method

1. $A = D = 1 + \frac{YZ}{2} = 1 + \frac{(20 + j40)\,j315 \times 10^{-6}}{2}$

$$= 1 + \frac{44.72\angle 63.43° \times 315 \times 10^{-6}\angle 90°}{2}$$

$$= 1 + 0.00704\angle 153.435° = 0.9937 + j0.00315$$

$\therefore A = D = 0.9937\angle 0.1816°$

$B = Z = 20 + j40 = 44.72\angle 63.43°\ \Omega$

$C = Y\left[1 + \frac{YZ}{4}\right]$

$= 315 \times 10^{-6}\angle 90°\ [1 + 0.00352\angle 153.435°]$

$= 315 \times 10^{-6}\angle 90°\ [0.99685 + j0.001575]$

$= 315 \times 10^{-6}\angle 90°[0.99685\angle 0.0905°]$

$\therefore C = 314.01 \times 10^{-6}\angle 90.0905°$ S

Receiving-end voltage, $V_r = \frac{132}{\sqrt{3}}$ kV/phase = $76210\angle 0$ V/phase

Receiving-end current, $I_r = \frac{30 \times 10^6}{\sqrt{3} \times 132 \times 1000 \times 0.85} = 154.37\angle -31.79°$ A

2. $V_S = AV_r + BI_r$

$= 0.9937\angle 0.1816° \times 76210\angle 0 + 44.72\angle 63.43° \times 154.37\angle -31.79°$

$= 75729.87\angle 0.1816° + 6903.64\angle 31.645°$

$= 81606.67 + j3862.047$

$= 81698\angle 2.71°$

$V_s = 81698.0$ V/phase

Sending-end line voltage = $\sqrt{3} \times 81698.0 = 141.505$ kV

3. Voltage at receiving end under no load, $V_r' = \dfrac{V_s\left(\dfrac{-2j}{\omega C}\right)}{R + jX - \dfrac{2j}{\omega C}}$

$$= \frac{81698\angle 2.71° \times \left(\frac{-j2}{135\times 10^{-6}}\right)}{(20+j40) - \left(\frac{+j2}{135\times 10^{-6}}\right)}$$

$$= \frac{1210340741\angle -87.29°}{14774.83\angle -89.92°}$$

$$= 81919.098\angle 2.63° \text{ V}$$

$$\% \text{ Regulation} = \frac{V_r' - V_r}{V_r}\times 100 = \frac{81919.098 - 76210}{76210}\times 100$$

$$= 7.49\%.$$

Nominal-T method:

1. $A = D = 0.9937 < 0.1816°$

$$B = Z\left[1 + \frac{YZ}{4}\right]$$

$$= 44.72\angle 63.43°[1 + 0.00352\angle 153.435°]$$
$$= 44.72\angle 63.43°[0.99685 + j0.001575]$$
$$= 44.72\angle 63.43° [0.99685\angle 0.0905°]$$
$$= 44.579\angle 63.521° \ \Omega$$
$$C = Y = 315 \times 10^{-6}\angle 90° \text{ S}$$

2. $V_S = AV_r + BI_r$

$$= 0.9937\angle 0.1816° \times 76210\angle 0° + 44.72\angle 63.43° \times 154.37\angle -31.79°$$
$$= 75729.88\angle 0.1816° + 6881.68\angle -31.79°$$
$$= 81673.81\angle 2.71°$$
$$= 81673.81 \text{ V/Phase}$$

$$\text{Sending-end line voltage } = \sqrt{3}\times 81529.06 = 141.463 \text{ kV}$$

3. Voltage at receiving end under no load,

$$V_r' = \frac{V_s\left(\frac{-j}{\omega C}\right)}{\left(\frac{R}{2} + j\frac{X}{2}\right) - \frac{j}{\omega C}} = \frac{81673.81\angle 2.71° \times \frac{-j}{135\times 10^{-6}}}{\left(\frac{20+j40}{2}\right)\left[\frac{-j}{135\times 10^{-6}}\right]\omega}$$

$$= \frac{81673.81\angle 2.71° \times 7407.41\angle -90°}{(10 + j20) - j7407.41}$$

$$= \frac{604991396.9\angle -87.29°}{7387.414\angle -89.92°}$$

$$= 81894.882\angle 2.63°$$

$$\therefore V_r' = 81894.882 \text{ V}$$

$$\% \text{ Regulation } = \frac{V_r' - V_r}{V_r} \times 100 = \frac{81894.882 - 76210}{76210} \times 100$$

$$= 7.46\%.$$

Example 3.14

The *A, B, C, D* constants of a lossless three phase 400 kV transmission line are *A* = *D* = 0.86 + *j*0, *B* =0 + *j*130.2 Ω, *C* = *j* 0.002 ℧. Obtain the sending-end voltage and the voltage regulation when line delivers 120 MVA at 400 kV and 0.8 p.f. lagging.

Solution:

$A = D = 0.86 + j\,0$, $B = 0 + j130.2\ \Omega$ $C = j0.002$ S

Receiving-end phase voltage, $V_r = \dfrac{400\,\text{kV}}{\sqrt{3}} = 230940.1\ \text{V}$

Receiving-end current, $I_r = \dfrac{120\times10^6}{\sqrt{3}\times400\times10^3} = 173.205\ \text{A}$

$= 173.205\angle -36.87°\ \text{A}$

Sending end voltage, $V_s = A\,V_r + B\,I_r$

$$= 0.86 \times 230940.1 + j130.2 \times 173.205\angle{-36.87°}$$
$$= 198608.486 + 22551.291\angle 53.13°$$
$$= 198608.486 + 13530.81 + j18041.009$$
$$= 212139.3 + j18041.009$$
$$= 212905\angle 4.86°\ \text{V}$$

At no load, $I_r = 0$

$V_s = AV_{r0}$

where, V_{r0} = voltage at receiving end at no load

$$V_{r0} = \frac{V_s}{A}$$

$$\%\ \text{Regulation} = \frac{\frac{V_s}{A} - V_r}{V_r}\times100$$

$$= \frac{\frac{212905}{0.86} - 230940.1}{230940.1}\times100$$

$= 7.2\%$.

CHAPTER AT A GLANCE

1. Performance of the line refers to the determination of efficiency and regulation of lines. For better performance, efficiency must be high and regulation must below.
2. Efficiency of a transmission line is defined as the ratio of receiving-end power to the sending-end power.

$$\%\ \text{Efficiency} = \frac{\text{Receiving-end power}}{\text{Sending-end power}}\times100$$

3. Regulation is defined as the change in voltage at the receiving-end when full load is thrown off; the sending-end voltage remains same, expressed as percentage of receiving-end voltage at full load.

$$\% \text{ Regulation } = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

However, if losses are low, then efficiency is high and if the drop in voltage is less, then regulation will be good.

4. Transmission lines can be classified into three types, i.e., short, medium and long transmission lines.
5. **Short transmission lines:** The lines of length below 80 km, with an operating voltage up to 20 kV, are called short lines. This line is represented by the lumped parameters *R* and L.
    1. Voltage at the sending end can be approximately calculated as

    $$V_s = V_r + IR\cos\phi_r + IX\sin\phi_r$$

    2. % Regulation $= \frac{IR\cos\phi_r + IX\sin\phi_r}{V_r} \times 100$
    3. If the power factor is lagging, the voltage at the sending end is more than that at the receiving end. Hence, voltage regulation is positive. In case the power factor is leading, the V_S will be somewhat less than V_r. In this case, the regulation is negative.
    4. Voltage regulation is zero when $\tan\phi_r = \frac{R}{X}$. This is

       possible only at leading power factor.
    5. The generalized transmission-circuit constants for short lines are

       $A = 1$, $B = Z$, $C = 0$ and $D = 1$.
6. **Medium transmission line:** When the length of the line is between 80 km and 160 km, with an operating voltage from 20 kV to 100 kV, it is called a medium line. These lines can be analyzed by using three different ways, i.e., load-end capacitance, nominal-T and nominal-π methods.
    1. **Load-end capacitance:** This method assumes that the entire line capacitance is concentrated at the

receiving end.

The sending-end current, $I_s = YV_r + I_r$

The sending-end voltage, $I_s = (1 + YZ)\, V_r + I_r Z$

The generalized transmission circuit constants are

$A = 1 + YZ$, $B = Z$, $C = Y$ and $D = 1$.

2. **Nominal-T method:** In this method, the capacitance of each conductor is assumed to be concentrated at the mid point of the line with half the series impedance on either side.

   The sending-end current, $I_s = YV_r + \left(1 + \frac{YZ}{2}\right) I_r$

   The sending-end voltage,

   $$V_s = \left(1 + \frac{YZ}{2}\right) V_r + Z\left(1 + \frac{YZ}{4}\right) I_r$$

   The generalized transmission circuit constants are

   $$A = 1 + \frac{YZ}{2} = D; \quad B = Z\left(1 + \frac{YZ}{4}\right), \quad C = Y$$

3. **Nominal-π method:** In this method, the capacitance of each conductor is assumed to be divided into two halves, one half being shunted between conductor and neutral at the receiving end and the other half shunted at the sending end and the total impedance being placed between them.

   The sending-end voltage, $V_s = \left(1 + \frac{YZ}{2}\right) V_r + ZI_r$

   The sending-end current,

   $$I_s = Y\left(1 + \frac{YZ}{4}\right) V_r + \left(1 + \frac{YZ}{2}\right) I_r$$

   The generalised transmission circuit constants are

$$A = D = 1 + \frac{YZ}{2}; B = Z, C = Y\left(1 + \frac{YZ}{4}\right)$$

SHORT ANSWER QUESTIONS

1. What are the different types of overhead transmission lines?
2. Define voltage regulation.
3. Define the efficiency of transmission line.
4. Write the expression for transmission efficiency for short transmission lines.
5. What are the methods used for solution of medium transmission lines?
6. Write *A*, *B*, *C*, and *D* constants for medium transmission lines using nominal-T and π methods.
7. Define lumped parameters.
8. Define distributed parameter.
9. How do you classify transmission lines as short, medium and long lines?
10. What are the disadvantages of load-end capacitance method?
11. How are capacitance considered for medium lines?
12. What are the limitations of nominal-T and π methods?
13. What is the order of loss for the methods usually employed for medium lines?
14. What is the effect of line capacitance for lagging load?
15. Define the nominal-T network.

MULTIPLE CHOICE QUESTIONS

1. The length of a short transmission line is up to about ____________ km.
   1. 80
   2. 120
   3. 150
   4. 300
2. Regulation is defined as the change in voltage at the receiving end when full load is thrown off, the ___________ end voltage remaining the same.
   1. receiving
   2. receiving node
   3. sending
   4. either a or c
3. The formulae for percentage regulation is
   1. $\frac{V_s - V_r}{V_r} \times 100$

2. $\frac{V_s - V_r}{V_r}$

3. $\frac{V_r - V_s}{V_r}$

4. $\frac{V_r - V_s}{V_s}$

4. Efficiency of a transmission line is given by
   1. $\frac{\text{receiving end power}}{\text{sending end power}}$
   2. $\frac{\text{sending-end power}}{\text{receiving-end power}}$
   3. $\frac{\text{sending-end power}}{\text{receiving-end power loss}} \times 100$
   4. $\frac{\text{receiving-end power}}{\text{sending-end power}} \times 100$
5. The formula for regulation of short transmission lines is given by
   1. $\frac{V_{\text{NO LOAD}} - V_{\text{LOAD}}}{V_{\text{NO LOAD}}}$
   2. $\frac{V_{\text{NOLOAD}} - V_{\text{LOAD}}}{V_{\text{LOAD}}} \times 100$
   3. $\frac{V_{\text{NOLOAD}} - V}{V_{\text{LOAD}}} \times 100$
   4. none of these
6. A single-phase line is transmitting 1100 kW power to a factory at 33 kV at 0.9 p.f. lagging. It has a total resistance of 10 Ω, and a total inductive reactance of 15 Ω. The voltage at the sending end is
   1. 33577.26 V
   2. 33708.33 V
   3. 33608.9 V
   4. 44505 V
7. A single-phase line is transmitting 1100 kW power to a factory at 33 kV at 0.9 p.f. lagging. It has a total resistance of 10 Ω, and a

total inductive reactance of 15 Ω. The percentage regulation is

1. 3%
2. 4%
3. 1.75%
4. 2.16%

8. A single-phase line is transmitting 1100 kW power to a factory at 33 kV at 0.9 p.f. lagging. It has a total resistance of 10 Ω, and a loop reactance of 15 Ω. The transmission efficiency is
   1. 90%
   2. 98%
   3. 98.45%
   4. 98.77%
9. The capacitance of a transmission line is a _________ element.
   1. series
   2. shunt
   3. both (a) and (b)
   4. none of these
10. A single-phase transmission line is delivering 500 kVA load at 2 kV, resistance is 0.2 Ω, inductive reactance is 0.4 Ω, the voltage regulation if the power factor is 0.707 lag is
    1. −5.3%
    2. 5.3%
    3. 5%
    4. 5.2%
11. A single-phase transmission line is delivering 500 kVA load at 2 kV, resistance is 0.2 Ω, inductive reactance is 0.4 Ω, the voltage regulation if the power factor is 0.707 lead is
    1. 1.64%
    2. −1.64%
    3. 2%
    4. 3%
12. The most important cause of power loss in the transmission line is
    1. resistance
    2. reactance
    3. capacitance
    4. admittance
13. In any transmission line, $AD - BC =$
    1. 1
    2. 0
    3. −1
    4. 2
14. The line constants of a transmission line are
    1. uniformly distributed
    2. lumped
    3. non-uniformly distributed
    4. non-linear
15. If the power factor of the load decreases, the line losses
    1. increase
    2. decrease
    3. remain constant

4. none of these
16. The generalized constants of *A* and *D* of the transmission line have
    1. no dimension
    2. dimension of ohm
    3. dimension of mho
    4. none of these
17. The sending-end voltage of a transmission line will be equal to the receiving-end voltage on load when the power factor of the load is
    1. leading
    2. leading
    3. a and b
    4. none of these
18. Voltage regulation of a short transmission line is
    1. always positive
    2. always negative
    3. either positive, negative, or zero
    4. zero
19. The percentage regulation of an overhead transmission line can be zero when the load power factor is
    1. lagging
    2. unity
    3. leading
    4. none of these
20. Transmission lines having length between 80 km and 160 km are known as ___________ lines.
    1. short
    2. medium
    3. long
    4. all
21. For a short transmission line, the transfer matrix is given by
    1. $\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$
    2. $\begin{bmatrix} 1 & 0 \\ 1 & Z \end{bmatrix}$
    3. $\begin{bmatrix} 1 & Z \\ 0 & 0 \end{bmatrix}$
    4. $\begin{bmatrix} 1 & 1 \\ Z & 1 \end{bmatrix}$
22. The performance of a medium transmission line can be obtained by
    1. nominal-T method

2. nominal-π method
3. a or b
4. a and b

23. For a short line with r/x ratio 1.0, the regulation will be zero when the load power factor is
    1. unity
    2. 0.707 lead
    3. 0.707 lag
    4. z power factor lead
24. While finding out the relation between V_s and V_r, shunt capacitance is neglected, in
    1. short transmission line
    2. medium transmission
    3. long transmission line
    4. none of these
25. Which of the following relationships is not valid for short transmission lines
    1. $I_s = I_r$
    2. $B = Z = C$
    3. $A = D = 1$
    4. none of these

Answers:

1. a	2. c	3. a	4. a	5. b
6. a	7. c	8. d	9. b	10. b
11. b	12. a	13. a	14. a	15. a
16. b	17. a	18. c	19. c	20. b
21. a	22. c	23. b	24. a	25. b

REVIEW QUESTIONS

1. How do you classify transmission lines?
2. Write a brief note on short transmission lines. Derive their performance characteristics.
3. What do you understand by the terms “nominal-T” and “nominal-π” circuits? Derive the expressions for the *A*, *B*, *C* and *D* constants for the nominal-π circuit of a medium transmission line.
4. Draw the vector diagrams of nominal-T and nominal-π models of a medium transmission line. Derive the expression for voltage regulation of both the models.
5. Show that for a transmission line the receiving-end voltage and current in terms of the sending-end voltage and current and auxiliary constraints are given by

$$V_r = DV_s - BI_s \quad \text{and} \quad I_r = -CV_s + AI_s$$

6. Explain the effect of power factor on regulation and efficiency.
7. What are the various parameters of a transmission line and how are they considered for different lines?
8. Draw the phasor diagram of a short transmission line and derive an expression for voltage regulation.
9. Draw the vector diagram of nominal-T and nominal-π models.
10. Explain the physical significance of the generalized constants *A, B, C* and *D*.
11. What is a nominal-circuit representation? Find *A, B, C* and *D* constants for nominal-T circuit of a transmission line.

PROBLEMS

1. A three-phase line delivers 3600 kW at a 0.85 p.f. lagging to a load. The resistance and reactance of each conductor is 5 Ω and 8 Ω, respectively. If the sending-end voltage is 33 kV, determine the following
   1. Receiving-end voltage,
   2. Line current, and
   3. Transmission efficiency.
2. An overhead single-phase delivers a load of 1.5 kW at 33 kV at 0.9 p.f. lagging. The total resistance and inductance of the overhead transmission line is 8 Ω and 15 Ω, respectively. Determine the following:
   1. Percentage of voltage regulation
   2. Sending-end power factor
   3. Transmission efficiency.
3. An overhead three-phase transmission line delivers a load of 5 MW at 22 kV at 0.85 p.f. lagging. The resistance and reactance of each conductor of overhead line is 3 Ω and 4 Ω, respectively. Determine the following:
   1. Sending-end voltage
   2. Voltage regulation,%
   3. Transmission efficiency
4. A three-phase overhead line conductor has the following data:

   Resistance = 15 Ω/phase;

   Inductive reactance = 35 Ω/phase

   Capacitive suceptance = 0.0004 ℧/phase

   The receiving-end load is 12500 kVA at 66 kV, 0.85 p.f. lagging. Determine the regulation and efficiency of the transmission line using nominal-T method.

5. A three-phase, 50 Hz, 80 km transmission line is designed to deliver a load of 20 MVA at 0.86 p.f. lagging at 6.6 kV to a balanced load. The conductors are of copper, each having a resistance of 0.105 Ω/km and an outside diameter of 1.8 cm, and

are spaced equilaterally 2 m apart. Determine the sending-end voltage and current using nominal-π method.

6. A balanced three-phase load of 35 MW is supplied at 110 kV, 50 Hz and 0.8 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (15 + *j*35) Ω and the total phase-neutral admittance is 300×10^{6} ℧. Use nominal-π and T methods to determine (i) *A*, *B*, *C* and *D* constants of the line, (ii) V_s (iii) regulation of the line and (iv) efficiency.

4

Performance of Long Transmission Lines

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Develop mathematical models for long transmission lines
- Understand the performance analysis of long lines with the help of analytical and circle diagrams
- Understand the effect of surge impedance and Ferranti effect on long lines

4.1 INTRODUCTION

It is a known fact that the resistance (R), inductance (L), capacitance (C), and conductance (G) are the parameters of overhead transmission lines distributed uniformly over the whole length of the line. The localized capacitance methods (for both short and medium lines) have already been discussed in Chapter 3, but adopting these methods to long transmission lines will result in considerable error. Hence, the rigorous solution approach is adopted, which assumes that line parameters are distributed uniformly along the length of the line. Lines of length above 250 km and operating voltage above 100 kV would be categorized as long transmission lines, the parameters of these lines are not lumped but, rather, are distributed uniformly throughout their length. A 400 kV transmission line is shown in Fig. 4.1(a).

The performance of a long line is calculated by considering uniformly distributed parameters along the length of the line for obtaining the results accurately.

Any three-phase balanced system can be represented on a single-phase basis. The equivalent single line diagram of a long transmission line is shown in Fig. 4.1(b).

where, r =	resistance per unit length of the line
x_L =	inductive reactance per unit length of the line
b_c =	capacitive susceptence per unit length of the line
g =	conductance per unit length of the line

Fig. 4.1(a) View of 400 kV transmission line

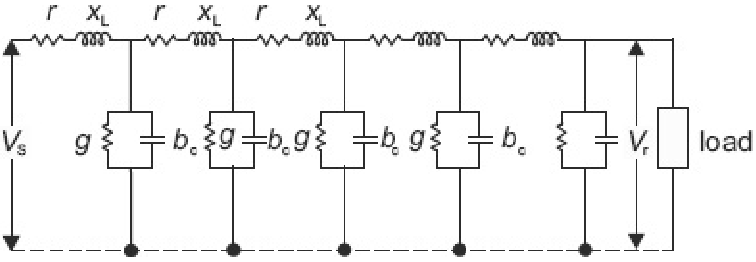

Fig. 4.1(b) Schematic diagram of single-phase circuit model for long transmission line

The following points should be observed:

- The line parameters r, x_L, b_c and g are uniformly distributed over the whole length of the line
- The series elements of line are r and x_L
- The shunt elements of line are b_c and g

For the analysis of long transmission lines, shunt conductance is neglected.

4.2 RIGOROUS SOLUTION

The rigorous mathematical treatment is required for getting a fair degree of accuracy in results of long overhead transmission lines. For the performance calculation of long transmission lines, the exact equivalent circuit representation of a long transmission line along with distributed parameters is shown in Fig. 4.1(c).

Let z = series impedance per unit length

y = shunt admittance per unit length

l = length of the line

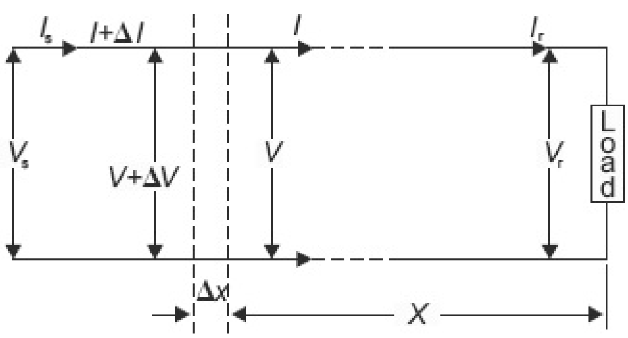

Fig. 4.1(c) Equivalent single-phase representation of a long transmission line

Then,

$Z = zl$ = total series impedance of the line

$Y = yl$ = total shunt admittance of the line

Consider a very small element of length Δx at a distance of x from the receiving-end of the line.

The voltage and current at distance x from the receiving-end are V, I and at distance $x + \Delta x$ are $V + \Delta x$ and $I + \Delta x$, respectively.

So, the change of voltage, $\Delta V = Iz\Delta x$ (4.1)

where, $z\Delta x$ is the impedance of the element considered

$$\frac{\Delta V}{\Delta x} = Iz$$

$$\underset{\Delta x \to 0}{Lt} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = Iz \quad (4.2)$$

Similarly, the change of current, $\Delta I = Vy\Delta x$ (4.3)

where, $y\,\Delta x$ is the admittance of element considered

$$\frac{\Delta I}{\Delta x} = Vy$$

$$\underset{\Delta x \to 0}{Lt} \frac{\Delta I}{\Delta x} = \frac{dI}{dx} = Vy \quad (4.4)$$

Differentiating the Eq. (4.2) with respect to x, we get

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} \tag{4.5}$$

Substituting the value of $\frac{dI}{dx}$ from Eq. (4.4) in Eq. (4.5), we get

$$\frac{d^2V}{dx^2} = zyV \tag{4.6}$$

Equation (4.6) is a second order differential equation. The solution of this equation is

$$V = Ae^{\sqrt{yz}x} + Be^{-\sqrt{yz}x} \tag{4.7}$$

Differentiating Eq. (4.7) with respect to *x*, we get

$$\frac{dv}{dx} = A\sqrt{yz}\ e^{\sqrt{yz}x} - B\sqrt{yz}\ e^{-\sqrt{yz}x} \tag{4.8}$$

From Eqs. (4.2) and (4.8)

$$Iz = A\sqrt{yz}\ e^{\sqrt{yz}x} - B\sqrt{yz}\ e^{-\sqrt{yz}x}$$

$$I = A\sqrt{\frac{y}{z}}\ e^{\sqrt{yz}x} - B\sqrt{\frac{y}{z}}\ e^{-\sqrt{yz}x} \tag{4.9}$$

From Eq. (4.7)

$$V = Ae^{\gamma x} + Be^{-\gamma x} \tag{4.10}$$

From Eq. (4.9)

$$I = \frac{A}{Z_c} e^{\gamma x} + \frac{B}{Z_c} e^{-\gamma x} \tag{4.11}$$

where, $Z_c = \sqrt{\frac{z}{y}}$ is known as characteristic impedance or surge impedance and $\gamma = \sqrt{yz}$ is known as propagation constant.

The constants A and B can be evaluated by using the conditions at the receiving end of the line.

The conditions are,

at $x = 0$, $V = V_r$ and $I = I_r$

Substitute the above conditions in Eqs. (4.10) and (4.11), we get

$$\therefore V_r = A + B \tag{4.12}$$

$$\text{and } I_r = \frac{1}{Z_c}(A - B) \tag{4.13}$$

Solving Eqs. (4.12) and (4.13), we get

$$A = \frac{V_r + I_r Z_c}{2} \quad \text{and} \quad B = \frac{V_r - I_r Z_c}{2}$$

Now substituting the values of *A* and *B* in Eqs. (4.10) and (4.11), *V* can be expressed as:

$$V = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r - I_r Z_c}{2} e^{-\gamma x} \tag{4.14a}$$

$$= V_r \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right] + I_r Z_c \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right]$$

$$\therefore V = V_r \cosh \gamma x + I_r Z_c \sinh \gamma x \tag{4.14b}$$

$$I = \frac{1}{Z_c} \left[\frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x} \right] \tag{4.15a}$$

$$= \frac{1}{Z_c} V_r \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_r \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right)$$

$$= \frac{V_r}{Z_c} \sinh \gamma x + I_r \cosh \gamma x \tag{4.15b}$$

where, *V* and *I* are the voltages and currents at any distance *x* from the receiving end.

At $x = l$, $V = V_S$ and $I = I_S$

Putting these values in Eqs. (4.14b) and (4.15b), V_S and I_S is expressed as

$$V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l \tag{4.16a}$$

$$I_s = \frac{V_r}{Z_c}\sinh \gamma l + I_r \cosh \gamma l \qquad (4.16b)$$

By comparison of Eqs. (4.16a) and (4.16b) with general transmission circuit constant of Eqs. (3.9) and (3.10) yields

$$A = D = \cosh \gamma l,$$
$$B = Z_c \sinh \gamma l \text{ and}$$
$$C = \frac{1}{Z_c}\sinh \gamma l$$

4.3 INTERPRETATION OF THE LONG LINE EQUATIONS

Consider Eq. (4.14a) as

$$V = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r - I_r Z_c}{2} e^{-\gamma x}$$

This equation can be modified as

$$V = \left|\frac{V_r + I_r Z_c}{2}\right| e^{\alpha x} e^{j(\beta x + \psi 1)} + \left|\frac{V_r - I_r Z_c}{2}\right| e^{-\alpha x} e^{j(-\beta x + \psi 2)} \qquad (4.17)$$

where, $\gamma = \alpha + j\beta$

α = attenuation constant

β = phase constant

$\psi_1 = \angle(V_r + I_r Z_c)$

$\psi_2 = \angle(V_r - I_r Z_c)$

The instantaneous voltage V can be expressed from Eq. (4.17) as

$$V = \ln\left\{\sqrt{2}\left[\left(\left|\frac{V_r + I_r Z_c}{2}\right|\right)e^{\alpha x}e^{j(\omega t+\beta x+\psi_1)} + \left(\left|\frac{V_r - I_r Z_c}{2}\right|\right)e^{-\alpha x}e^{j(\omega t-\beta x+\psi_2)}\right]\right\}$$

$$= \sqrt{2}\left(\left|\frac{V_r + I_r Z_c}{2}\right|\right)e^{\alpha x}\sin(\omega t+\beta x+\psi_1) + \sqrt{2}\left(\left|\frac{V_r - I_r Z_r}{2}\right|\right)e^{-\alpha x}\sin(\omega t-\beta t+\psi_2) \tag{4.18}$$

Similarly, from Eq. (4.15a), the instantaneous current *i* can be written as

$$i = \sqrt{2}\left(\left|\frac{V_r + I_r Z_c}{2Z_c}\right|\right)e^{\alpha x}\sin(\omega t+\beta x+\psi_1-\theta) - \sqrt{2}\left(\left|\frac{V_r - I_r Z_c}{2Z_c}\right|\right)e^{-\alpha x}\sin(\omega t-\beta t+\psi_2-\theta) \tag{4.19}$$

where, $\theta = \angle Z_c$

The Eq. (4.18) or (4.19) is composed of two traveling waves. The first term in Eq. (4.18), increases in magnitude and advances in phase with increase in distance *x* from the receiving end. Conversely, if the line from the sending end towards the receiving end is considered, the term diminishes in magnitude and is retarded in phase. This is the characteristic of a traveling wave. The first term in Eq. (4.18) is called the *incident voltage wave* and its magnitude decreases with increasing distance from the sending end.

The second term in Eq. (4.18) diminishes in magnitude and is retarded in phase as the distance increases from the receiving end. It is known as *reflected voltage wave*. At any point along the line, the voltage is the sum of two components (the incident and the reflected) of voltages at that point. These two components travel as wave along the line and is called the *velocity of propagation* of wave.

Since Eq. (4.19) for current and Eq. (4.18) for voltage are similar, the current may be considered as being composed of incident and reflected current waves. Therefore, the voltage and current at a point along the line consists of incident and reflected waves travelling in opposite directions.

4.3.1 Propagation Constant

The magnitude and phase of a travelling wave is governed by the complex quantity known as propagation constant γ. The real part of the propagation constant is α, which determines the change in magnitude per unit length of the line of the wave, which is called *attenuation constant*. It is expressed in Nepers per unit length. The imaginary part of the propagation constant is β. It determines the change in phase of the wave per unit length of the line and is called as *phase constant* or *wave length constant*. It is expressed in radians per unit length.

We know, $\gamma = \alpha + j\beta = \sqrt{yz} = \sqrt{j\omega C\,(r + j\omega L)}$

Since shunt conductance, $g = 0$.

$$\therefore \alpha + j\beta = \sqrt{j^2\omega^2 LC}\sqrt{\frac{r}{j\omega C} + 1}$$

$$= j\omega\sqrt{LC}\sqrt{1 - \frac{jr}{\omega L}}$$

$$\cong j\omega\sqrt{LC}\left(1 - \frac{jr}{2\omega L}\right)$$

$$= \frac{r}{2}\sqrt{\frac{C}{L}} - j\omega\sqrt{LC} \qquad (4.20)$$

The real part of Eq. (4.20) is

$$\alpha = \frac{r}{2}\sqrt{\frac{C}{L}} \text{ Np/m}$$

and the imaginary part is

$$\beta = \omega\sqrt{LC} \text{ rad/m.}$$

4.3.2 Wave Length and Velocity of Propagation

At any moment, the voltage and current vary harmonically along the line with respect to distance *x*. A complete voltage or current cycles along the line corresponds to a change of 2π rad in the angular argument β*x*. The corresponding line length is called the *wave length*.

$$\lambda = \frac{2\pi}{\beta} \text{ m}$$

The velocity of propagation of a wave in m/sec is the product of the wave-length in meters and the frequency in hertz.

i.e., Velocity of wave propagation, $v = \lambda f$ m/s.

$$\upsilon = f\frac{2\pi}{\beta} = \frac{1}{\sqrt{LC}} \qquad (\because\ \beta = \omega\sqrt{LC}).$$

4.4 EVALUATION OF TRANSMISSION LINE CONSTANTS

The transmission line constants can be written from Eq. (4.16) as

$$A = D = \cosh \gamma l = \cosh \sqrt{yz}\, l = \cosh(\alpha + j\beta)l \qquad (4.21a)$$

$$B=\sqrt{\frac{z}{y}}\sinh\gamma l=Z_c\sinh\sqrt{yz}\,l=Z_c\sinh(\alpha+j\beta)l \tag{4.21b}$$

$$C=\sqrt{\frac{y}{z}}\sinh\gamma l=\frac{1}{Z_c}\sinh\sqrt{yz}\,l=\frac{1}{Z_c}\sinh(\alpha+j\beta)l \tag{4.21c}$$

From Eqs. [4.21(a), (b), and (c)], it is observed that γ is a complex quantity and it is not easy to obtain the hyperbolic function of it directly from tables or calculators.

The following two methods are generally used to compute the transmission line constants.

Convergent Series of Real Angle Method In this method, the hyperbolic sines and cosines of Eqs. [4.21 (a), (b), and (c)] are expanded by the trigonometrical formulae. From the tables of trigonometric functions and hyperbolic functions of real numbers, the transmission line constants can be calculated.

$$\text{i.e., } \sqrt{yz}=\alpha+j\beta$$

$$\begin{aligned}\therefore A = D &= \cosh\sqrt{yz}\,l = \cosh(\alpha+j\beta)\,l = \cosh\alpha l\cosh j\beta l+\sinh\alpha\sinh j\beta l\\ &= \cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta l \qquad (4.22a)\\ &\quad[\text{Since, } \cosh j\beta l=\cos\beta l \text{ and } \sinh j\beta l=\sinh\beta l]\end{aligned}$$

$$\begin{aligned}B &= Z_c\sinh\sqrt{yz}\,l = Z_c(\sinh\alpha l\cosh j\beta l+\cosh\alpha l\sinh j\beta l)\\ &= Z_c(\sinh\alpha l\cos\beta l + j\cosh\alpha l\sin\beta l) \qquad (4.22b)\end{aligned}$$

and similarly

$$C = \frac{1}{Z_c}\sinh\sqrt{yz}l = [\sinh \alpha l \cos \beta l + j\cosh \alpha l \sin\beta l] \quad (4.22c)$$

Convergent Series of Complex Angle Method By using the well known Maclaurin's series (convergent series), the hyperbolic sines and cosines are expanded.

According to this series

$$\cosh \theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \ldots$$

$$\sinh \theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$

where, θ may be real or complex

Now, $\gamma = \sqrt{yz}$ and $\gamma l = \sqrt{yz}l$

$$A = D = \cosh\sqrt{yz}l = 1 + \frac{(\sqrt{yz}l)^2}{2!} + \frac{(\sqrt{yz}l)^4}{4!} + \frac{(\sqrt{yz}l)^6}{6!} + \cdots$$

$$= 1 + \frac{(\sqrt{yz})^2 l^2}{2} + \frac{(\sqrt{yz})^4 l^4}{24} + \frac{(\sqrt{yz})^6 l^6}{720} + \cdots$$

$$= 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \frac{Y^3Z^3}{720} + \cdots$$

$$\cong 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} \quad \text{(Neglecting higher order terms)}$$

$$B = Z_c \sinh\sqrt{yz}l = Z_c\left[\frac{\sqrt{yz}l}{1} + \frac{(\sqrt{yz}l)^3}{3!} + \frac{(\sqrt{yz}l)^5}{5!} + \cdots\right]$$

$$= \sqrt{\frac{z}{y}}\left[\frac{\sqrt{yz}\,l}{1} + \frac{(\sqrt{yz})^3 l^3}{6} + \frac{(\sqrt{yz})^5 l^5}{120} + \cdots\right]$$

$$= \frac{Z}{1} + \frac{YZ^2}{6} + \frac{Y^2Z^3}{120} + \cdots$$

$$= Z\ (1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \cdots)$$

$$\cong Z\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right] \quad \text{(Neglecting higher order terms)} \tag{4.23}$$

$$C = \frac{1}{Z_c}\sinh\sqrt{yz}l$$

$$= \sqrt{\frac{y}{z}}\left[\frac{\sqrt{yz}l}{1} + \frac{(\sqrt{yz}l)^3}{6} + \frac{(\sqrt{yz}l)^5}{120} + \cdots\right]$$

$$= \frac{Y}{1} + \frac{Y^2Z}{6} + \frac{Y^3Z^2}{120} + \cdots$$

$$\cong Y\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right] \quad \text{(Neglecting higher order terms)} \tag{4.24}$$

Note: This method is not very useful beyond the seventh power.

The A, B, C, D constants for transmission line modes are given in Table 4.1.

Table 4.1 Overhead transmission line *ABCD* constants

Sl. no.	Type of line model	Parameter
1	Short line	A = 1, B = Z, C = 0, D = 1
2	Medium line:	

	Load-end capacitance	A = 1 + YZ, B = Z, C = Y, D = 1
	Nominal-T method	$A = 1 + \frac{YZ}{2} = D; \quad B = Z\left(1 + \frac{YZ}{4}\right); \quad C = Y$
	Nominal-π method	$A = D = 1 + \frac{YZ}{2}; \quad B = Z; \quad C = Y\left(1 + \frac{YZ}{4}\right)$
3	Long line:	
	Real angle method	$A = D = \cosh \alpha l \cos\beta l + j\sinh\alpha l \sin\beta l$ $B = Z_c(\sinh \alpha l \cos\beta l + j\cosh \alpha l \sin \beta l)$ $C = \frac{1}{Z_c}(\sinh \alpha l \cos \beta l + j\cosh \alpha l \sin\beta l)$
	Complex angle method	$A = D \cong 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24}$ $B \cong Z\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right]$ $C \cong Y\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right]$

4.5 REGULATION

The voltage regulation of a transmission line refers to the rise in voltage at the receiving-end. It is expressed in percent of full-load voltage, when full load at a specified power factor is removed with constant sending-end voltage.

$$\text{Percentage regulation} = \frac{V_{r,NL} - V_{r,FL}}{V_{r,FL}} \times 100$$

At no load the receiving-end voltage, $V_{r, NL}$ can be computed from Eq. (3.9).

$$\therefore V_{r,NL} = \frac{V_s}{A}, \text{ at no load}$$

And if $V_{r, FL}$ is the receiving-end voltage at full load for a given sending-end voltage of V_s, the above equation becomes

$$\text{Percentage of regulation} = \frac{\frac{V_s}{A} - V_{r,FL}}{V_{r,FL}} \times 100 \tag{4.25}$$

EXAMPLE 4.1

A three-phase, 200 km long transmission line has the following constants. Resistance/ph/km = 0.15 Ω, reactance/ph/km = 0.20 Ω, shunt admittance/ph/km = 1.2 × 10^{-6} ℧. Calculate by rigorous method, the sending-end voltage and current when the line is delivering a load of 20 MW at 0.8 p.f. lagging. The receiving-end voltage is kept constant at 110 kV.

Solution:

Resistance per phase, R/ph = 0.15 × 200 = 30 Ω

Reactance per phase, X_L/ph = 0.20 × 200 = 40 Ω

Shunt admittance per phase, $Y/\text{ph} = j1.2 \times 10^{-6} \times 200 = 0.00024\angle 90^\circ$

Impedance, $Z/\text{ph} = R + jX_L = 30 + j40 = 50\angle 53.13^\circ \Omega$

Sending-end voltage, $V_s = V_r \cosh\sqrt{YZ} + I_r\sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY}$

Now, $\sqrt{ZY} = \sqrt{50\angle 53.13^\circ \times 0.00024\angle 90^\circ} = 0.1095\angle 71.56^\circ$

$$ZY = 0.012\angle 143.13^\circ$$

$$Z^2Y^2 = 0.000144\angle 286.26^\circ$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{50\angle 53.13^\circ}{0.00024\angle 90^\circ}} = 456.43\angle -18.43^\circ$$

$$\sqrt{\frac{Y}{Z}} = 0.00219\angle 18.43^\circ$$

$$\begin{aligned}\cosh\sqrt{YZ} &= 1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} = 1 + \frac{0.012}{2}\angle 143.13^\circ + \frac{0.000144}{24}\angle 286.26^\circ \\ &= 1 + 0.006\angle 143.13^\circ + 0.000006\angle 286.26^\circ \\ &= 1 - 0.0048 + j0.0036 + 0.00000168 - j0.00000576 \\ &= 0.99519 + j0.0036 \\ &= 0.9952\angle 0.207^\circ\end{aligned}$$

$$\begin{aligned}\sinh\sqrt{YZ} &= \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} = 0.1095\angle 71.56^\circ + \frac{0.001313\angle 214.68^\circ}{6} \\ &= 0.1095\angle 71.56^\circ + 0.0002188\angle 214.68^\circ \\ &= 0.03464 + j0.1039 - 0.0001799 - j0.0001245 \\ &= 0.03446 + j0.103776 \\ &= 0.10935\angle 71.63^\circ\end{aligned}$$

Receiving-end current, $I_r = \frac{20\times 10^6}{\sqrt{3}\times 110\times 10^3 \times 0.8} = 131.22\angle -36.87^\circ$ A

Receiving-end voltage/phase, $V_r = \frac{110\times 10^3}{\sqrt{3}} = 63508.53$ V

$$\begin{aligned}\therefore \text{Sending-end voltage, } V_S &= 63508.53\angle 0^\circ \times 0.9952\angle 0.207^\circ \\ &\quad + 131.22\angle -36.87^\circ \times 456.43\angle -18.43^\circ \times 0.10935\angle 71.63^\circ \\ &= 63203.68\angle 0.207^\circ + 6549.27\angle 16.33^\circ \\ &= 69488.3282 + j2069.796 \\ &= 69519.147\angle 1.706^\circ \text{ V} \\ V_{S_{LL}} &= \sqrt{3}\times 69519.147 \\ &= 120.410 \text{ kV}\end{aligned}$$

$$\begin{aligned}\text{Sending-end current, } I_S &= V_r\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_r\cosh\sqrt{YZ} \\ &= 63508.53\angle 0^\circ \times 0.00219\angle 18.43^\circ \times 0.10935\angle 71.63^\circ \\ &\quad + 131.22\angle -36.87^\circ \times 0.9952\angle 0.207^\circ \\ &= 15.21\angle 90.06^\circ + 130.59\angle -36.66^\circ \\ &= 104.742 + j62.760 \\ &= 122.106\angle -30.929^\circ \text{ A}\end{aligned}$$

$\therefore$ Sending-end current $= 122.106$ A.

EXAMPLE 4.2

A three-phase, 50 Hz, 180 km long transmission line has three conductors each of 0.6 cm radius, spaced at the corners of an equilateral triangle of side 3 m. The resistance of each conductor is 0.2 Ω/km and the line delivers 20 MVA at 110 kV and at a lagging p.f. of 0.9. Determine (i) *ABCD* constants (both real and complex angle methods) (ii) sending-end voltage and current (iii) efficiency and regulation.

Solution:

Radius of each conductor, r = 0.6 cm

Spacing between the conductors, D = 3 m = 300 cm

Supply frequency, f = 50 Hz

Length of the line, l = 180 km

Power to be transmitted, S = 20 MVA

Receiving-end line voltage, V_r = 110 kV

Load power fraction, $\cos\phi_r$ = 0.9 lag

Receiving-end power, $P_r = S \cos \phi_r = 20 \times 0.9 = 18$ MW

Resistance per conductor per kilometre = 0.2 Ω

Total resistance of the line, $R = 0.2 \times 180 = 36$ Ω

Geometrical mean distance, $D_m = \sqrt[3]{3\times 3\times 3} = 3$ cm

Geometrical mean radius, $D_s = 0.7788 \times 0.6 = 0.46728$ cm

Inductance of each conductor (per phase), $L = 2\times 10^{-7} \ln \frac{D_m}{D_s} \times l$ H

$$= 2\times 10^{-7} \ln\left(\frac{300}{0.7788\times 0.6}\right)\times 180000 = 0.2327 \text{ H}$$

Inductive reactance per phase, $X_L = 2\pi fL = 2\pi \times 50 \times 0.2327 = 73.1$ Ω

Impedance of the line, $Z = 36 + j73.1$ Ω $= 81.48\angle 63.78^\circ$ Ω

$$\text{Capacitance of each conductor (per phase), } C = \frac{10^{-9}}{18\ln\frac{D_m}{r}} \times l\,\text{F}$$

$$= \frac{10^{-9}}{18\ln\frac{300}{0.6}} \times 180000 = 1.609\,\mu\text{F}$$

Capacitive suceptance per phase, $Y_c = 2\pi fC = 2\pi \times 50 \times 1.609 \times 10^{-6} =$ 5.055×10^{-4} ℧

$$\gamma l = \sqrt{5.055\times10^{-4}\angle 90^\circ \times 81.48\angle 63.78^\circ} = 0.203\angle 76.89^\circ = 0.046 + j0.197$$
$$= \alpha l + j\beta l$$

$$\text{Line current, } I = \frac{S}{\sqrt{3}V_r} = \frac{20\times10^6}{\sqrt{3}\times110000} = 105\text{ A}$$

$$\text{Receiving-end phase voltage, } V_r = \frac{110000}{\sqrt{3}} = 63508.5\text{ V}$$

(i) *ABCD* constants (real angle method)

$$A = D = \cosh\gamma l = \cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta l$$
$$= \cosh(0.046 + j0.197)$$
$$= (\cosh 0.046)\cos 0.197 + j(\sinh 0.046)\sin 0.197$$
$$= 0.9816 + j9.007\times10^{-3} = 0.9816\angle 0.525^\circ$$

$$\sinh\gamma l = \sinh\alpha l\cos\beta l + j\cosh\alpha l\sin\beta l$$
$$= (\sinh 0.046)\cos 0.197 + j(\cosh 0.046)\sin 0.197$$
$$= 0.045 + j0.196 = 0.2\angle 77.07^\circ$$

$$B = Z_c\sinh\gamma l = \sqrt{\frac{z}{y}}\times\sinh\gamma l = \sqrt{\frac{81.48\angle 63.78^\circ}{5.055\times10^{-4}\angle 90^\circ}}\times 0.2\angle 77.07^\circ$$
$$= 401.48\angle -13.13^\circ \times 0.2\angle 77.07^\circ = 80.296\angle 63.94^\circ\,\Omega$$

$$C = \frac{1}{Z_c}\sinh\gamma l = \sqrt{\frac{y}{z}}\times\sinh\gamma l = 2.49\times10^{-3}\angle 13.13^\circ \times 0.2\angle 77.07^\circ$$
$$= 4.98\times10^{-4}\angle 90.2^\circ\ \text{℧}$$

ABCD constants (complex angle method)

$$A = D = 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24}$$

$$= 1 + \frac{5.055\times10^{-4}\angle90^\circ\times81.48\angle63.78^\circ}{2} + \frac{(5.055\times10^{-4}\angle90^\circ)^2\times(81.48\angle63.78^\circ)^2}{24}$$

$$= 1 + 0.0207\angle153.78^\circ + 7.06\times10^{-5}\angle307.56^\circ$$

$$= 0.9815\angle0.53^\circ$$

$$B = Z\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right)$$

$$= 81.48\angle63.78^\circ\left(1 + \frac{5.055\times10^{-4}\angle90^\circ\times81.48\angle63.78^\circ}{6} + \frac{(5.055\times10^{-4}\angle90^\circ\times81.48\angle63.78^\circ)^2}{120}\right)$$

$$= 81.48\angle63.78^\circ(0.99441\angle0.17^\circ)$$

$$= 81.0245\angle63.95^\circ\ \Omega$$

$$C = Y\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right)$$

$$= 5.055\times10^{-4}\angle90^\circ[0.99441\angle0.17^\circ]$$

$$= 5.0267\times10^{-4}\angle90.17^\circ\ \mho$$

(ii) Sending-end voltage, $V_S = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l$

$$= 63508.5\angle0^\circ\times0.9816\angle0.525^\circ + 105\angle-25.84^\circ\times401.48\angle-13.13^\circ\times0.2\angle77.07^\circ$$

$$= 69213.26\angle4.78^\circ\ \text{V}$$

Sending-end current, $I_s = \dfrac{V_r \sinh\gamma l}{Z_c} + I_r \cosh\gamma l$

$$= \frac{63508.5\angle0^\circ\times0.2\angle77.07^\circ}{401.48\angle-13.13^\circ} + 105\angle-25.84^\circ\times0.9816\angle0.525^\circ$$

$$= 93.897\angle-7.59^\circ\ \text{A}$$

(iii) Percentage of regulation $= \dfrac{\frac{V_s}{A} - V_r}{V_r}\times100$

$$= \frac{\frac{69213.26}{0.9816} - 63508.5}{63508.5}\times100 = 11.02\%$$

Sending-end power, $P_s = 3V_sI_s\cos\theta_s$

$$= 3\times69213.26\times93.897\times\cos(4.78^\circ+7.59^\circ)$$

$$= 19.044\ \text{MW}$$

Transmission efficiency, $\eta = \dfrac{\text{Receiving-end power}}{\text{Sending-end power}}\times100 = \dfrac{18}{19.044}\times100 = 94.52\%$.

4.6 EQUIVALENT CIRCUIT REPRESENTATION OF LONG LINES

In the third chapter, we have seen that a transmission line can be represented by nominal-π or nominal-T. But they do not represent it accurately, since the parameters of the line are not assumed to be uniformly distributed along the length of the line. The discrepancy becomes larger as the length of the line increases. However, it is possible to find the equivalent circuit of a long line and to represent the line accurately, insofar as the measurements at the ends of the line are concerned by a network of lumped parameters. They are called equivalent-π and equivalent-T networks.

4.6.1 REPRESENTATION OF A LONG LINE BY EQUIVALENT- Π MODEL

To represent the long lines by π-equivalent, draw the π-network and replace *Z* by *Z′* and *Y* by *Y′* as indicated in Fig. 4.2.

Now the generalized equations from the π-equivalent network are [from Eqs. (3.32) and (3.33)]

$$V_s = \left(1 + \frac{Y'Z'}{2}\right)V_r + Z'I_r \tag{4.26}$$

$$I_s = Y'\left(1 + \frac{Y'Z'}{4}\right)V_r + \left(1 + \frac{Y'Z'}{2}\right)I_r \tag{4.27}$$

Compare the Eqs. (4.26) and (4.27) with generalized transmission circuit constant of Eqs. (3.9) and (3.10)

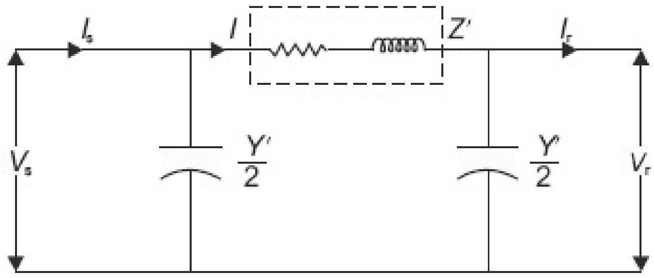

Fig. 4.2 π-equivalent circuit representation of a long line

$$A = 1 + \frac{Y'Z'}{2}$$
$$B = Z'$$
$$C = Y'\left(1 + \frac{Y'Z'}{4}\right)$$
$$D = 1 + \frac{Y'Z'}{2}$$

Now comparing Eq. (4.26) with Eq. (4.16a) yields

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \tag{4.28}$$

$$\text{and } Z' = Z_c \sinh \gamma l \tag{4.29}$$

Considering Eq. (4.29) as;

$$
\begin{aligned}
Z' &= Z_c \sinh \gamma l \\
&= \sqrt{\frac{z}{y}} \frac{\sinh \gamma l}{\sqrt{yz}\, l} \sqrt{yz}\, l \\
&= zl \frac{\sinh \gamma l}{\gamma l} \\
&= Z \frac{\sinh \gamma l}{\gamma l}
\end{aligned}
$$

From the above results, the equivalent series impedance of the circuit shown in Fig. 4.2 is equal to the product of the series impedance of the circuit of Fig. 3.12(a) and sinh $\gamma l/\gamma l$.

Again, by comparing Eq. (4.27) and Eq. (4.16b), we get

$$\left(1+\frac{Y' Z'}{2}\right)=\cosh \gamma l \tag{4.30}$$

$$\text{And} \left(1+\frac{Y' Z'}{2}\right)=\cosh \gamma l \tag{4.31}$$

Considering Eq. (4.31) after substituting $Z' = Z_C$ sinh γl from Eq. (4.29), it can be written as;

$$1+\frac{Y'}{2}Z_c \sinh\gamma l = \cosh\gamma l$$

$$\frac{Y'}{2}Z_c \sinh\gamma l = \cosh\gamma l - 1$$

$$\frac{Y'}{2}Z_c \sinh\gamma l = \cosh^2\frac{\gamma l}{2} + \sinh^2\frac{\gamma l}{2} - \cosh^2\frac{\gamma l}{2} + \sinh^2\frac{\gamma l}{2}$$

$$\text{or } 2\,\frac{Y'}{2}Z_c \sinh\frac{\gamma l}{2}\cosh\frac{\gamma l}{2} = 2\sinh^2\frac{\gamma l}{2}$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\tanh\frac{\gamma l}{2} = \sqrt{\frac{y}{z}}.\frac{\sqrt{yz}\,l}{2}.\frac{\tanh\left(\gamma l/2\right)}{\sqrt{yz}\,l/2}$$

$$= \frac{Y}{2}.\frac{\tanh\left(\gamma l/2\right)}{\gamma l/2} \qquad (4.32)$$

where, Y' = total shunt admittance.

Resulting view of Eq. (4.32), the equivalent shunt admittance of the circuit of Fig. 4.2 is equal to the product of the shunt admittance of the circuit of Fig. 3.13(a) and (tanh $\gamma l/\gamma l$.

The equivalent-π circuit can be represented as shown in Fig. 4.3.

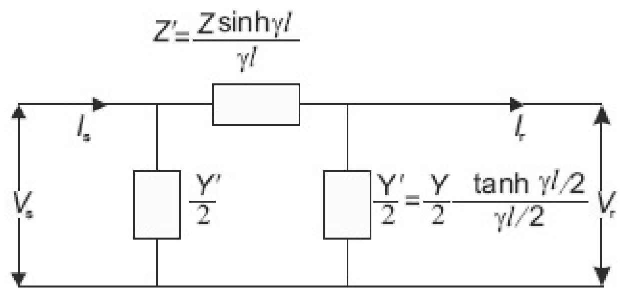

Fig. 4.3 Equivalent π-network

Test Yourself

1. Is the equivalent-π model of a long transmission line accurately represented in all conditions?

4.6.2 Representation of a Long Line by Equivalent-T Model

To represent long lines by T-equivalent, draw the T-network and replace Z by Z', Y by Y' as indicated in Fig. 4.4.

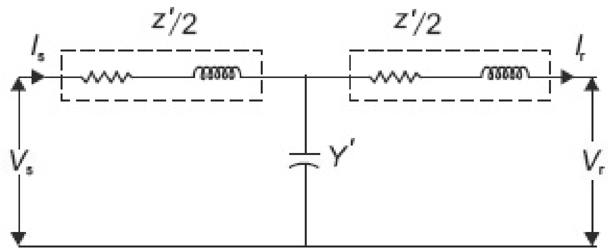

Fig. 4.4 T-equivalent circuit representation of a long line

Now the generalized equations from the T-network [from Eqs. (3.24) and (3.25)] are as follows:

$$V_S = \left(1 + \frac{Y'Z'}{2}\right)V_r + Z'\left(1 + \frac{Y'Z'}{4}\right)I_r \qquad (4.33)$$

$$I_S = Y'V_r + \left(1 + \frac{Y'Z'}{2}\right)I_r \qquad (4.34)$$

By comparison of the Eqs. (4.33) and (4.34) with generalized transmission circuit constant vide Eqs. (3.9) and (3.10), we can write;

$$A = \left(1 + \frac{Y'Z'}{2}\right)$$
$$B = Z'\left(1 + \frac{Y'Z'}{4}\right)$$
$$C = Y'$$

$$D = \left(1 + \frac{Y'Z'}{2}\right)$$

Now compare Eqs. (4.33) and (4.34) with Eqs. (4.16a) and (4.16b), to obtain;

$$Z'\left(1 + \frac{Y'Z'}{4}\right) = Z_c \sinh \gamma l \tag{4.35}$$

$$Y' = \frac{\sinh \gamma l}{Z_c} \tag{4.36}$$

$$\text{and} \left(1 + \frac{Y'Z'}{2}\right) = \cosh \gamma l \tag{4.37}$$

To determine the shunt branch of the equivalent-T circuit, consider the Eq. (4.36) as;

$$\begin{aligned} Y' &= \frac{1}{Z_c} \sinh \gamma l = \sqrt{\frac{y}{z}} \sqrt{yzl} \frac{\sinh \left(\gamma l/2\right)}{\sqrt{yzl}} \\ &= yl \frac{\sinh \gamma l}{\gamma l} \\ &= Y \frac{\sinh \gamma l}{\gamma l} \end{aligned} \tag{4.38}$$

From the above result, the equivalent shunt branch admittance of circuit vide Fig. 4.4 is equal to the product of the shunt branch admittance of circuit of Fig. 3.10(a) and (sinh $\gamma l/\gamma l$).

To get the series impedance of the equivalent-T, substitute Y' from Eq. (4.38) in Eq. (4.37) to obtain;

$$1+\frac{Z'}{2}\frac{\sinh\gamma l}{Z_c}=\cosh\gamma l \tag{4.39}$$

$$\frac{Z'}{2}.2\frac{\sinh\frac{\gamma l}{2}.\cosh\frac{\gamma l}{2}}{Z_c}=\cosh^2\frac{\gamma l}{2}+\sinh^2\frac{\gamma l}{2}-\cosh^2\frac{\gamma l}{2}+\sinh^2\frac{\gamma l}{2}$$

$$\frac{Z'}{2}.2\frac{\sinh\frac{\gamma l}{2}.\cosh\frac{\gamma l}{2}}{Z_c}=2\sinh^2\frac{\gamma l}{2}$$

$$\frac{Z'}{2}=Z_c\tanh\frac{\gamma l}{2}$$

$$=\sqrt{\frac{y}{z}}.\frac{\sqrt{yz}.l}{2}.\frac{\tanh\left(\gamma l/2\right)}{\frac{\sqrt{yz}}{2}}$$

$$\therefore\frac{Z'}{2}=\frac{Z}{2}\frac{\tanh\left(\frac{\gamma l}{2}\right)}{\gamma l/2} \tag{4.40}$$

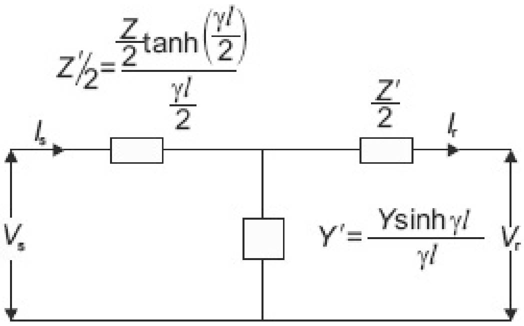

Fig. 4.5 Equivalent T-network

This means, in order to get the series branch of the equivalent-T circuit, the series branch of the nominal-T (lumped series impedance) should be multiplied by the factor $\dfrac{\tanh\left(\frac{\gamma l}{2}\right)}{\gamma l/2}$

The equivalent-T circuit can be represented as shown in Fig. 4.5.

EXAMPLE 4.3

A three-phase, 50 Hz, 160 km long transmission line has three conductors each of 0.75 cm radius spaced at the corners of triangle of sides 2.5 m, 3 m and 3.5 m. The resistance of each conductor is 0.3 Ω/km and the line delivers 30 MVA at 132 kV and at a lagging p.f. of 0.95. Determine *ABCD* constants as (i) medium line (both T and π methods) (ii) long line (both real and complex angle methods) and (iii) parameters of equivalent T and π representations of long lines.

Solution:

Radius of each conductor, r =0.75 cm

Spacing between the conductors, D_{12} = 2.5 m, D_{23} = 3 m and D_{31} = 3.5 m

Supply frequency, f = 50 Hz

Length of the line, l =160 km

Power to be transmitted, S = 30 MVA

Receiving-end line voltage, V_r = 132 kV

Load p.f., $\cos\phi_r$ = 0.95 lag

Resistance per conductor per kilometre = 0.3 Ω

Total resistance of the line, R = 0.3 × 160 = 48 Ω

$$\text{Geometrical mean distance, } D_m = \sqrt[3]{D_{12}\times D_{23}\times D_{31}}$$

$$= \sqrt[3]{2.5\times 3\times 3.5} = 2.972 \text{ m} = 297.2 \text{ cm}$$

Geometrical mean radius, D_S = 0.7788×0.75 = 0.5841 cm

Inductive reactance per phase, $X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

Impedance of the line, $Z = 48 + j62.83 = 79.067\angle 52.62^{\circ}\ \Omega$

$$\text{Capacitance of each conductor (per phase), } C = \frac{10^{-9}}{18\ln\frac{D_m}{r}} \times l \text{ F}$$

$$= \frac{10^{-9}}{18\ln\frac{297.2}{0.75}} \times 160000 = 1.486\mu\text{F}$$

$$\text{Capacitive suceptance per phase, } Y_c = 2\pi fC = 2\pi \times 50 \times 1.486 = 10^{-6} = 4.6684 \times 10^{-4}\ \mho$$

$$\gamma l = \sqrt{4.6684\times10^{-4}\angle 90^{\circ}\times 79.067\angle 52.62^{\circ}} = 0.192\angle 70.33^{\circ} = 0.065 + j0.18$$

$$= \alpha l + j\beta l.$$

(i) Medium lines

Nominal-T method

$$A = D = 1 + \frac{YZ}{2}$$

$$= 1 + \frac{4.6684\times10^{-4}\angle 90^{\circ}\times 79.067\angle 52.62^{\circ}}{2}$$

$$= 1 + 0.01846\angle 142.62^{\circ}$$

$$= 1 - 0.01274 + j0.01336 = 0.98726 + j0.01336 = 0.9874\angle 0.65^{\circ}$$

$$B = Z\left(1 + \frac{YZ}{4}\right)$$

$$= 79.067\angle 52.62^{\circ}(1 - 0.006368 + j0.006682)$$

$$= 79.067\angle 58.47^{\circ}(0.9936 + j0.006682)$$

$$= 79.067\angle 58.47^{\circ} \times 0.99362\angle 0.428^{\circ} = 78.5626\angle 52.95^{\circ}\ \Omega$$

$$C = Y = 4.6684\times10^{-4}\angle 90^{\circ}\text{S}$$

Nominal-π method

$$A = D = 1 + \frac{YZ}{2} = 0.9874\angle 0.65°$$

$$B = Z = 79.067\angle 52.62°\ \Omega$$

$$C = Y\left(1 + \frac{YZ}{4}\right)$$

$$= 4.668410^{-4}\angle 90° \times 0.99362\angle 0.428° = 4.6386\angle 90.428°\ \text{S}$$

$$= 0.0145 + j0.03392.$$

(ii) Long lines

***ABCD* constants (real angle method)**

$$A = D = \cosh\gamma l = \cosh\alpha l \cos\beta l + j\sinh\alpha l \sin\beta l$$
$$= \cosh(0.065 + j0.18)$$
$$= \cosh(0.065)\cos 0.18 + j(\sinh 0.065)\sin 0.18$$
$$= 0.968 + j0.01164$$
$$= 0.986\angle 0.676°$$
$$\sinh\gamma l = \sinh(0.065 + j0.18)$$
$$= (\sinh 0.065)\cos 0.18 + j(\cosh 0.065)\sin 0.18$$
$$= 0.064 + j0.1794$$
$$= 0.064\angle 70.36°$$

$$B = Z_c \sinh\gamma l = \sqrt{\frac{z}{y}} \times \sinh\gamma l = \sqrt{\frac{79.067\angle 52.62°}{4.6684\times 10^{-4}\angle 90°}} \times 0.064\angle 70.36°$$

$$= 26.3386\angle 51.70°\ \Omega$$

$$C = \frac{1}{Z_c}\sinh\gamma l = \sqrt{\frac{4.6684\times 10^{-4}\angle 90°}{79.067\angle 52.62°}} \times 0.064\angle 70.36°$$

$$= 1.55\times 10^{-4}\angle 89.025°\ \mho$$

***ABCD* constants (complex angle method)**

$$A = D = 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24}$$

$$= 1 + \frac{4.6684 \times 10^{-4} \angle 90° \times 79.067 \angle 52.62°}{2} + \frac{\left(4.6684 \times 10^{-4} \angle 90°\right)^2 \times \left(79.067 \angle 52.62°\right)^2}{24}$$

$$= 1 + 0.0185 \angle 148.47° + 5.677 \times 10^{-5} \angle 296.94°$$

$$= 1 - 0.01276 + j0.01339 + 3.1123 \times 10^{-5} - j4.7478 \times 10^{-5}$$

$$= 0.9873 + j0.01334 = 0.9874 \angle 0.65°$$

$$B = Z\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right)$$

$$= 79.067 \angle 52.62° \left(1 + \frac{4.6684 \times 10^{-4} \angle 90° \times 79.067 \angle 52.62°}{6} + \frac{\left(4.6684 \times 10^{-4} \angle 90°\right)^2 \times \left(79.067 \angle 52.62°\right)^2}{120}\right)$$

$$= 79.067 \angle 52.62° \left(1 + 6.1519 \times 10^{-3} \angle 142.62° + 1.1354 \times 10^{-5} \angle 285.24°\right)$$

$$= 79.067 \angle 52.62° \left(0.99512 \angle 0.214°\right)$$
$$= 78.681 \angle 52.83° \ \Omega$$

$$C = Y\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120}\right)$$
$$= 4.6684 \times 10^{-4} \angle 90° \left(0.99512 \angle 0.214°\right)$$
$$= 4.64 \times 10^{-4} \angle 90.214° \ \mho$$

(iii) Equivalent-T representation

Shunt admittance, $Y_C = 4.6684 \times 10^{-4} \angle 90° \ \mho$

Series impedance, $Z = 79.067 \angle 52.62° \ \Omega$

$$\gamma l = 0.192\angle 70.335°$$

$$\frac{\gamma l}{2} = 0.096\angle 70.335° = 0.032 + j0.09$$

Then,

$$Y_C = 4.6684 \times 10^{-4}\angle 90°\ \Omega$$

$$\frac{Z'}{2} = \frac{Z}{2}\frac{\tanh \gamma l/2}{\gamma l/2}$$

$$Y' = Y\frac{\sinh \gamma l}{\gamma l}$$

$$\tanh(\gamma l/2) = \frac{\sinh(\gamma l/2)}{\cosh(\gamma l/2)}$$

$$\begin{aligned}\sinh \gamma l/2 &= \sinh(0.032 + j0.09)\\ &= (\sinh 0.032)\cos 0.09 + j(\cosh 0.032)\sin 0.09\\ &= 0.03187 + j0.0899 = 0.09538\angle 70.48°\end{aligned}$$

$$\begin{aligned}\cosh \gamma l/2 &= \cosh(0.032 + j0.09)\\ &= (\cosh 0.032)\cos 0.09 + j(\sinh 0.032)\sin 0.09\\ &= 0.9965 + j2.92 \times 10^{-3} = 0.9965\angle 0.168°\end{aligned}$$

$$\tanh(\gamma l/2) = \frac{\sinh(\gamma l/2)}{\cosh(\gamma l/2)} = \frac{0.09538\angle 70.48°}{0.9965\angle 0.168°} = 0.0957\angle 70.31°$$

$$\frac{Z'}{2} = \frac{Z}{2}\frac{\tanh(\gamma l/2)}{\gamma l/2} = \frac{79.067\angle 52.62°}{2} \times \frac{0.0957\angle 70.31°}{0.096\angle 70.335°} = 39.416\angle 52.59°\ \Omega$$

$$Y' = Y\frac{\sinh \gamma l}{\gamma l} = 4.6684 \times 10^{-4}\angle 90° \times \frac{0.064\angle 70.36°}{0.192\angle 70.335°} = 1.556 \times 10^{-4}\angle 90.03°\ \mho$$

Equivalent-π representation

$$Z' = Z\frac{\sinh \gamma l}{\gamma l} = 79.067\angle 52.62° \times \frac{0.064\angle 70.36°}{0.192\angle 70.335°} = 26.356\angle 52.64°\ \Omega$$

$$\frac{Y'}{2} = \frac{Y}{2}\frac{\tanh(\gamma l/2)}{\gamma l/2} = \frac{4.6684 \times 10^{-4}\angle 90°}{2} \times \frac{0.0957\angle 70.31°}{0.096\angle 70.335°} = 4.6538 \times 10^{-4}\angle 89.96°\ \Omega.$$

EXAMPLE 4.4

Derive the *ABCD* constants when two transmission lines are connected in cascade.

Solution:

Two transmission lines are said to be connected in cascade when the output of one transmission line is connected to the input of the other.

Let the constants A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 be two transmission lines which are connected in cascade as shown in Fig. 4.6.

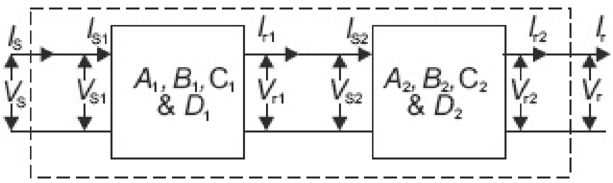

Fig. 4.6 Cascade connection of two transmission lines

From transmission line 1, the sending-end voltage and the current are expressed in terms of line constants as

$$V_{s1} = A_1 V_{r1} + B_1 I_{r1} \tag{4.41}$$

$$I_{s1} = C_1 V_{r1} + D_1 I_{r1} \tag{4.42}$$

Similarly, from transmission line 2

$$V_{s2} = A_2 V_{r2} + B_2 I_{r2} \tag{4.43}$$

$$I_{s2} = C_2 V_{r2} + D_2 I_{r2} \tag{4.44}$$

Conditions for combined transmission lines are

$$V_{s1} = V_{s,} I_{s1} = I_s;\ V_{r2} = V_r, I_{r2} = I_r;\ V_{r1} = V_{s2} \text{ and } I_{r1} = I_{s2} \tag{4.45}$$

After consideration of conditions from Eq. (4.45), Eqs. (4.41) to (4.44) becomes

$$V_s = A_1 V_{s2} + B_1 I_{s2} \tag{4.46}$$

$$I_s = C_1 V_{s2} + D_1 I_{s2} \tag{4.47}$$

$$V_{s2} = A_2 V_r + B_2 I_r \tag{4.48}$$

$$\text{and } I_{s2} = C_2 V_r + D_2 I_r \tag{4.49}$$

Substitute the values of V_{S2} and *Is2* from Eqs. (4.48) and (4.49) in Eqs. (4.46) and (4.47), we get

$$\begin{aligned} V_s &= A_1 (A_2 V_r + B_2 I_r) + B_1 (C_2 V_r + D_2 I_r) \\ &= (A_1 A_2 + B_1 C_2) V_r + (A_1 B_2 + B_1 D_2) I_r \end{aligned} \tag{4.50}$$

$$\begin{aligned} I_s &= C_1 (A_2 V_r + B_2 I_r) + D_1 (C_2 V_r + D_2 I_r) \\ &= (C_1 A_2 + D_1 C_2) V_r + (C_1 B_2 + D_1 D_2) I_r \end{aligned} \tag{4.51}$$

Compare the Eqs. (4.50) and (4.51) with generalized transmission circuit constant of Eqs. (3.9) and (3.10)

$$\left.\begin{aligned} A &= A_1 A_2 + B_1 C_2 \\ B &= A_1 B_2 + B_1 D_2 \\ C &= A_2 C_1 + C_2 D_1 \\ D &= B_2 C_1 + D_1 D_2 \end{aligned}\right\} \tag{4.52}$$

The relation is given in matrix form as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \tag{4.53}$$

Note: If transmission line 2 is connected at the sending end and 1 at the receiving end, the overall constants for the combined transmission line can be obtained by interchanging the subscripts in Eq. (4.53).

EXAMPLE 4.5

Derive the *ABCD* constants for two transmission lines connected in parallel. Solution

Solution

In case two transmission lines are connected in parallel as shown in Fig. 4.7, the constants for the overall transmission line can be obtained as follows:

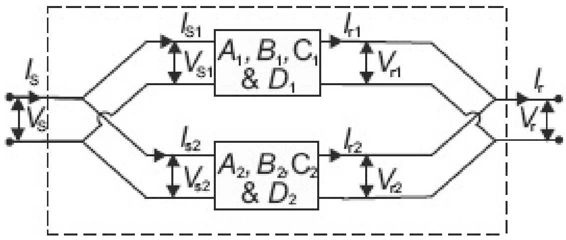

Fig. 4.7 Parallel connection of two transmission lines

From transmission line 1, the sending-end voltage and the current are expressed in terms of line constants as

$$V_{s1} = A_1 V_{r1} + B_1 I_{r1} \tag{4.54}$$

$$I_{s1} = C_1 V_{r1} + D_1 I_{r1} \tag{4.55}$$

Similarly, from transmission line 2

$$V_{s2} = A_2 V_{r2} + B_2 I_{r2} \tag{4.56}$$

$$I_{s2} = C_2 V_{r2} + D_2 I_{r2} \tag{4.57}$$

Conditions for combined transmission lines are

$$V_s = V_{s1} = V_{s2};\ V_r = V_{r1} = V_{r2};\ I_s = I_{s1} + I_{s2}$$
$$\text{and } I_r = I_{r1} + I_{r2} \tag{4.58}$$

On applying the conditions from Eq. (4.58) to Eq. (4.54) to (4.57), we get

$$V_s = A_1 V_r + B_1 I_{r1} \tag{4.59}$$

$$I_{s1} = C_1 V_r + D_1 I_{r1} \tag{4.60}$$

$$V_s = A_2 V_r + B_2 I_{r2} \tag{4.61}$$

$$I_{s2} = C_2 V_r + D_2 I_{r2} \tag{4.62}$$

Multiply Eqs. (4.59) and (4.61) by B_2 and B_1, respectively and add, we get

$$V_s = \frac{A_1 B_2 + B_1 A_2}{B_1 + B_2} V_r + \frac{B_1 B_2}{B_1 + B_2} I_r \tag{4.63}$$

From Eq. (4.58)

$$I_r = I_{r1} + I_{r2}$$
$$I_{r2} = I_r - I_{r1} \tag{4.64}$$

From Eqs. (4.59) and (4.61)

$$A_1V_r + B_1I_{r1} = A_2V_r + B_2I_{r2}$$
$$(A_1 - A_2)\,V_r = B_2I_r - I_{r1}(B_1 + B_2)$$
$$\therefore I_{r1} = \frac{B_2I_r - (A_1 - A_2)V_r}{B_1 + B_2} \tag{4.65}$$

Adding Eqs. (4.60) and (4.62), we get

$$\begin{aligned} I_s &= I_{s1} + I_{s2} \\ &= C_1V_r + D_1I_{r1} + C_2V_r + D_2I_{r2} \\ &= (C_1 + C_2)V_r + D_2I_r + I_{r1}(D_1 - D_2) \\ &= (C_1 + C_2)V_r + D_2I_r + \frac{B_2I_r - (A_1 - A_2)V_r}{B_1 + B_2}(D_1 - D_2) \\ &= \left[C_1 + C_2 - \frac{(A_1 - A_2)(D_1 - D_2)}{B_1 + B_2}\right]V_r + \left[\frac{B_1D_2 + D_1B_2}{B_1 + B_2}\right]I_r \end{aligned} \tag{4.66}$$

Compare the Eqs. (4.63) and (4.66) with generalized transmission circuit constant of Eqs. (3.9) and (3.10)

$$\left.\begin{aligned} A &= \frac{A_1B_2 + B_1A_2}{B_1 + B_2} \\ B &= \frac{B_1B_2}{B_1 + B_2} \\ C &= C_1 + C_2 - \frac{(A_1 - A_2)(D_1 - D_2)}{B_1 + B_2} \\ D &= \frac{B_1D_2 + D_1B_2}{B_1 + B_2} \end{aligned}\right\} \tag{4.67}$$

EXAMPLE 4.6

Derive the *ABCD* constants of a transmission line with series impedance at the receiving end.

Solution:

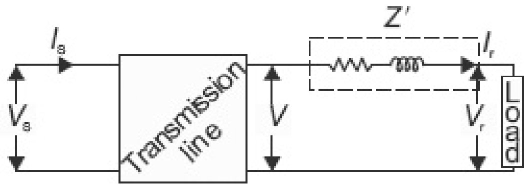

Fig. 4.8 Transmission line with series impedance at the receiving end

From Fig. 4.8,

$$V = V_r + I_r Z \qquad (4.68)$$
$$\text{and } V_s = AV + BI_r$$

Substituting the value of V from Eq. (4.68) in the above V_S expression

$$V_s = A(V_r + I_r Z) + BI_r \qquad (4.69)$$
$$\therefore V_s = AV_r + (B + AB)I_r$$
$$\text{Also } I_s = CV + DI_r$$

Substituting the value of V from Eq. (4.68) in the above I_S expression

$$I_s = C(V_r + I_r Z) + DI_r$$
$$\therefore I_s = CV_r + (D + CZ)I_r \qquad (4.70)$$

If the new generalized constants for the whole combination are A_1, B_1, C_1, and D_1, then from Eqs.(4.69) and (4.70), we get

$$A_1 = A, B_1 = B + AZ$$
$$C_1 = C, D_1 = D + CZ = A + CZ \qquad (\text{since } A = D).$$

EXAMPLE 4.7

Derive the *ABCD* constants of a transmission line with series impedance at the sending end.

Solution:

From Fig. 4.9,

$$V = AV_r + BI_r$$
$$I_s = CV_r + DI_r \tag{4.71}$$

and $V_s = V + I_sZ$
$$= AV_r + BI_r + I_sZ$$
$$= AV_r + BI_r + (CV_r + DI_r)Z$$
$$\therefore V_s = (A + CZ)V_r + (B + DZ)I_r \tag{4.72}$$

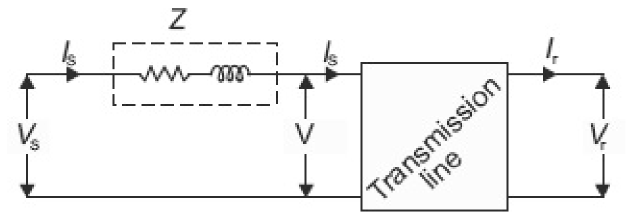

Fig. 4.9 Transmission line with series impedance at the sending end

If the generalized constants for the whole combination are A_1, B_1, C_1, and D_1 then from Eqs. (4.71) and (4.72), we get

$$A_1 = A + CZ, B_1 = B + DZ$$
$$C_1 = C, D_1 = D = A \qquad (\because A = D).$$

EXAMPLE 4.8

Derive the *ABCD* constants of a transmission line with transformers (series impedance) at both ends.

Solution:

A step-up transformer is connected at the sending-end of the transmission line and a step-down transformer is connected at the receiving end of the line, which is equivalent to series impedances connected at both the ends of the transmission line.

Let the impedance of the transformer at the sending and receiving ends be Z_1 and Z_2, respectively and the resultant equivalent circuit of a transmission line is shown in Fig. 4.10.

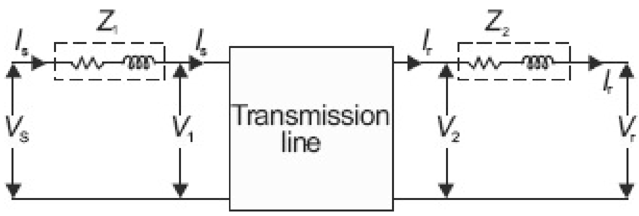

Fig. 4.10 Transmission line with series impedance at the sending and receiving ends

From Fig. 4.10,

$$V_2 = V_r + I_r Z_2$$
$$\text{and } V_1 = AV_2 + BI_r$$
$$= A(V_r + Z_2 I_r) + BI_r$$
$$\therefore V_1 = AV_r + (Z_2 A + B)I_r \quad (4.73)$$

$$I_s = CV_2 + DI_r$$
$$= (V_r + I_r Z_2)C + DI_r$$
$$\therefore I_s = CV_r + (D + Z_2 C)I_r \quad (4.74)$$

$V_S = V_1 + Z_1 I_S$

Substituting V_1 and I_S from Eqs. (4.73) and (4.74) in the above equation, we get

$$V_s = AV_r + (Z_2 A + B)I_r + Z_1[CV_r + (D + Z_2 C)I_r]$$
$$= (A + CZ_1)V_r + (Z_2 A + B + DZ_1 + CZ_1 Z_2)I_r$$

Since the line is symmetrical, so $D = A$

$$\therefore V_s = (A + CZ_1)V_r + [B + A(Z_1 + Z_2) + CZ_1 Z_2]I_r \quad (4.75)$$

If the generalized constants for the whole combination are A_1, B_1, C_1 and *D1* then from Eqs. (4.74) and (4.75), we get

$$A_1 = A + CZ_1, B_1 = B + A(Z_1 + Z_2) + CZ_1Z_2$$
$$C_1 = C, D_1 = D + CZ_2 = A + CZ_2 \qquad \text{(since } A = D\text{)}$$

4.7 TUNED TRANSMISSION LINES

In a long transmission line, if the sending-end voltage and current are numerically equal to the receiving- end voltage and current then the line is called a *tuned line*.

i.e., $|V_s| = |V_r|$

and $|I_s| = |I_r|$

$$\text{If, } \omega l\sqrt{LC} = n\pi, n = 1, 2, 3 \ldots$$
$$l = \frac{n\pi}{\omega\sqrt{LC}} = \frac{n\pi}{2\pi f\sqrt{LC}} \qquad (4.76)$$

$$\text{Since } \frac{1}{\sqrt{LC}} = v = \text{velocity of light}$$
$$\cong 3 \times 10^8 \text{ m/s.}$$

For tuned performance, the length of line

$$l = \frac{n}{2f\sqrt{LC}} = \frac{n}{100} \times 3 \times 10^8 \text{ m} \qquad (\because f = 50 \text{ Hz})$$
$$= 3 \times 10^6 \text{ m, for } n = 1$$
$$= 6 \times 10^6 \text{ m, for } n = 2$$

Therefore, the length of the tuned lines can be 3000 km, 6000 km, 9000 km, etc. These lengths are too long for power transmission from the point of view of cost and efficiency. In practice, such lines would have to be at EHV-levels. In long AC lines, the length is subdivided into sections of 250–300 km and compensation is provided at such intervals.

Test Yourself

1. Is it practically possible to maintain $|V_s| = |V_r|$ and $|I_s| = |I_r|$ in a transmission system?

4.8 CHARACTERISTIC IMPEDANCE

Characteristic impedance is defined as the square root of the ratio of line impedance *z* to shunt admittance *y*.

i.e., $Z_c = \sqrt{\frac{z}{y}}$

where,

$\sqrt{\frac{z}{y}}$ is a complex number, when *y* and/or *z* are in complex.

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L}{g + j\omega C}}$$

From this equation, Z_c depends upon the characteristic of the line per unit length. It is, therefore, called *characteristic impedance* of the line. It also depends upon the length of the line, radius and spacing between the conductors. For a lossless line, $r = g = 0$, the characteristic impedance becomes

$$Z_c = \sqrt{\frac{L}{C}} \tag{4.77}$$

The characteristic impedance is also called the surge or natural impedance of the line.

Surge impedance is the impedance offered to the propagation of a voltage or current wave during its travel along the line. The approximate value of surge impedance for overhead lines is 400 Ω, for under ground cables it is 40 Ω and the transformers have several thousand ohms.

Test Yourself

1. Why is surge impedance low in underground systems as compared to overhead systems? Justify your answer.

4.9 SURGE IMPEDANCE LOADING (SIL)

The surge impedance loading (SIL) of a transmission line is the power (MW) loading of a transmission line when the line is lossless.

A transmission line produces reactive power (MVAr) due to their natural capacitance. The amount of MVAr produced is dependent on the transmission line's capacitive reactance (X_c) and the voltage (kV) at which the line is energized.

Now the MVAr produced is,

$$\text{MVAr} = \frac{(\text{kV})^2}{X_c} \tag{4.78}$$

Transmission lines also utilize reactive power to support their magnetic fields. The magnetic field strength is dependent on the magnitude of the current flow in the line (kA) and the natural inductive reactance (X_L)of the line. The amount of MVAr used by a transmission line is a function of the current flow and inductive reactance.

$$\text{The MVAr used by a transmission line} = I^2X_L \tag{4.79}$$

Where, *I* is in kA,

Transmission line SIL is simply the MW loading (at a unity power factor) at which the line MVAr usage is equal to the line MVAr production. From the above statement, the SIL occurs when:

$$I^2X_L = \frac{(\text{kV})^2}{X_c}$$

$$X_L X_c = \frac{(\text{kV})^2}{I^2} \tag{4.80}$$

And the Eq. (4.80), can be rewritten as

$$\sqrt{\frac{V^2}{I^2}} = \frac{2\pi fL}{2\pi fC} \qquad \text{(Since for lossless line, } R \approx 0\text{)}$$

$$\text{or } \frac{V}{I} = \sqrt{\frac{L}{C}} = \text{surge impedance} \tag{4.81}$$

The term $\sqrt{\frac{L}{C}}$ in the Eq. (4.77) / (4.81) is the surge

impedance.

The theoretical significance of the surge impedance is that, if a purely reactive load that is equal to the surge impedance were connected to the end of a transmission line with no resistance, a voltage surge introduced at the sending end of the line would be absorbed completely at the receiving end. The voltage at the receiving end would have the same magnitude as that at the sending end, and would have a phase angle that is lagging with respect to the sending-end by an amount equal to the time required for the volt age wave to travel across the line from the sending to the receiving end.

The concept of surge impedance is more readily applied to telecommunication systems than to power systems. However, we can extend the concept to the power transferred across a transmission line. The surge impedance loading (power transmitted at this condition) or SIL (in MW) is equal to the ratio of voltage squared (in kV) to surge impedance (in ohms).

$$\therefore \text{SIL (MW)} = \frac{V_{\text{L\_L}}^2}{\text{Surge impedance}} \tag{4.82}$$

Note: From Eq. (4.82), it is observed that the SIL is dependent on the voltage (kV) of the line to which it is energized and the line surge impedance. The line length is not a factor in the SIL or surge impedance calculations. Therefore, the SIL is not a measure of a transmission line power transfer capability, neither does it take into account the line length, nor does it consider the strength of the local power system.

For loading much higher than SIL, shunt capacitors may be needed for improving the voltage profile along the line and for light load conditions, i.e, load much less than the SIL, shunt inductors may be needed to reduce the line charging current.

4.10 FERRANTI EFFECT

When medium or long transmission lines are operated at no-load or light-load, the receiving-end voltage becomes more than the sending-end voltage. The phenomenon of rise in voltage at the receiving-end of a transmission line during no load or light load condition is called the Ferranti effect. The charging current produces a voltage drop in the series reactance of the line. This voltage drop is in phase opposition to the receiving-end voltage, and hence the sending-end voltage becomes smaller than the receiving-end voltage.

In order to determine the magnitude of voltage rise, one-half of the total line capacitance will be assumed to be concentrated at the receiving-end as shown in Fig. 4.11(a). The phasor diagram is shown in Fig. 4.11(b).

Taking receiving-end voltage as reference phasor, we have

$$V_r = V_r \angle 0^\circ \tag{4.83}$$

And is represented by phasor *OA*

Charging current,

$$I_c = YV_r \tag{4.84}$$

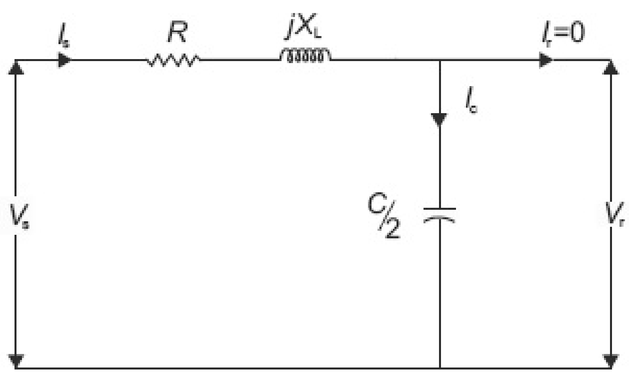

Fig. 4.11(a) Circuit diagram

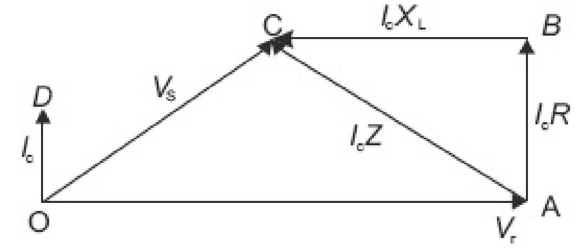

Fig. 4.11(b) Phasor diagram

This is represented by phasor *OD*

The sending-end voltage,

$$\begin{aligned} V_s &= V_r + I_c(R + jX_L) \\ &= V_r + jYV_r(R + jX_L) \\ &= V_r - YX_LV_r + jYRV_r \end{aligned} \qquad (4.85)$$

The vector *OC* represents the sending-end voltage under no load condition and is less than the receiving-end voltage. The line resistance is usually small as compared to the line inductive reactance. Hence, the resistance is neglected.

Neglecting resistive drop of the line, I_cR, we get

$$\begin{aligned} \text{Rise in voltage, } V' &= OC - OA \\ &= V_s - V_r \\ &= -YX_LV_r \end{aligned} \qquad (4.86)$$

The negative sign in Eq (4.86) indicates that V_r is more than V_s.

If C_0 and L_0 are the capacitance and inductance of the transmission line per meter length, respectively and *l* is the length of the line in meters, then Eq. (4.86) becomes,

$$V' = -\frac{\omega C_0 l}{2} \times \omega L_0 l \times V_r$$

$$V' = -\frac{\omega^2 C_0 L_0 l^2 V_r}{2} \tag{4.87}$$

In case of short lines, the effect is negligible, but it increases rapidly with the increase in length of the line. Therefore, this phenomenon is observable only in medium and long lines.

For long high voltage and EHV transmission lines, shunt reactors are provided to absorb a part of the charging current or shunt capacitive VAr of the transmission line under no load or light load conditions, in order to prevent the over voltage on the line.

Test Yourself

1. What compensation is required to avoid Ferranti effect on transmission lines? Justify your answer.

EXAMPLE 4.9

Find the no load sending-end voltage and the voltage rise from the sending-end to the receiving-end for a 50 Hz, 300 km long line if the receiving-end voltage is 220 kV.

Solution:

The receiving-end voltage, $V_r = \frac{220}{\sqrt{3}} = 127.017\,\text{kV}$

Rise in voltage, $V' = -\frac{\omega^2 C_0 L_0 l^2 V_r}{2}$

The quantity $\frac{1}{\sqrt{L_0 C_0}}$ is constant for all transmission lines and it is equal

to the speed of light in km/sec

i.e., 3 × 105 km/sec.

$$\text{Therefore, rise in voltage, } V' = -\frac{\omega^2 l^2 V_r}{2\times(3\times10^5)^2} = -\frac{\omega^2 l^2 V_r \times 10^{-10}}{18}$$

$$V_s - V_r = -\left[\frac{(2\pi\times50)^2\times300\times300\times10^{-10}\times127.017}{18}\right] = -6.268 \text{ kV}$$

As, $V_s - V_r = -6.2689$ kV

The sending-end voltage,

$V_s = V_r - 6.268$

$\therefore V_s = 127.017 - 6.268 = 120.749$ kV/phase

$= \sqrt{3}\times120.749 = 209.14 \text{ kV (L} - \text{L)}.$

4.11 CONSTANT VOLTAGE TRANSMISSION

In case of constant voltage transmission, a constant voltage drop is maintained along the line by installing a synchronous condenser (a synchronous motor running without a mechanical load) at the receiving-end. When the loads are changed, the power factor of the system is also changed by the synchronous condenser (motor). Therefore, the voltage drop along the line is constant even when the loads are changed.

The main advantage is the availability of constant voltage at the load-end for all loads.

Disadvantages are:

- Short circuit current of the system is increased.
- Risk of interruption of supply becomes more in case of synchronous phase modifiers falling out off synchronism.

4.12 CHARGING CURRENT IN LINES

The effect of capacitance of an overhead transmission line above 100 km long is taken into consideration for all calculations. The effect of the line capacitance is to produce a current usually called the charging current. This current will be in quadrate with the applied voltage. It flows through the line even though the receiving-end is open-circuited. It has its maximum value at the sending-end of the circuit and decreases with increased length from the sending-end and finally reaches zero at the receiving end.

The charging current of the open circuit line is referred as the amount of current flowing into the line from the sending-end to the receiving-end when there is no load. In many cases, the total charging current of the line is determined by multiplying the total admittance of the line by the receiving-end voltage. This would be correct if the entire length of the line had the same voltage as at the receiving-end. However, this method of finding charging current is sufficiently accurate for most lines.

Therefore, assuming voltage-equality along the line, the actual value of the charging current will decrease uniformly from its maximum value at the sending-end to zero at the receiving-end. Due to charging current, there will be power loss in the line even if the line is open-circuited.

4.12.1 Power Loss Due to Charging Current (or Open-Circuited Line)

Consider an open-ended single line diagram of a transmission line of length l with a charging current I_c per unit length and the resistance per unit length r ohms.

Let us assume a small section dx at a distance x km from the sending-end of the line as shown in Fig. 4.12.

The current flowing through a small section dx from sending-end = $i(l–x)$ A

Resistance of a section dx = rdx

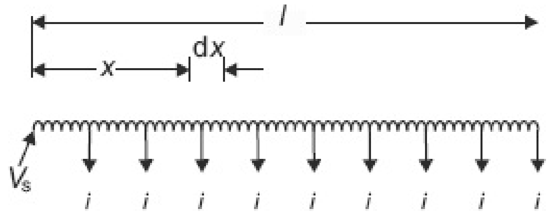

Fig. 4.12 Equivalent phase diagram of open-circuited transmission line

The total charging current, $I_c = il$ A

∴ Loss in the element, d$x = i^2(1 - x)^2\, r\mathrm{d}x$

$$\text{Total loss in the entire line } = \int_0^l i^2 (l-x)^2\, r\mathrm{d}x$$

$$= \int_0^l i^2 r \left(l^2 - 2xl + x^2\right)\mathrm{d}x$$

$$= i^2 r \left(l^2 x - x^2 l + \frac{x^3}{3}\right)_0^l$$

$$= i^2 r \frac{l^3}{3} = \frac{I_c^2 R}{3} \tag{4.88}$$

From the Eq. (4.88), the total power loss in the line is equal to 1/3 of the maximum power loss due to charging current (no load current).

Test Yourself

1. Why should an open-circuited line encounter power loss?

4.13 LINE LOADABILITY

In practice, power lines are not operated to deliver their theoretical maximum power, which is based on rated terminal voltage and load angle, $\delta = 90°$ across the line

i.e., $P_{\max} = \dfrac{V_s V_r}{X}$. Figure 4.13 shows a practical line

loadability curve plotted below the theoretical steady state stability limit. This curve is based on the voltage-drop limit ($V_r \geq 0.95$) V_s and on a maximum load angle of 30°–35°, in order to maintain stability during transient disturbances.

Note that for the short lines of length less than 100 km, loadability is limited by the maximum current carrying capacity (thermal rating) of the conductors or by terminal equipment rating, and not by voltage drop or stability considerations.

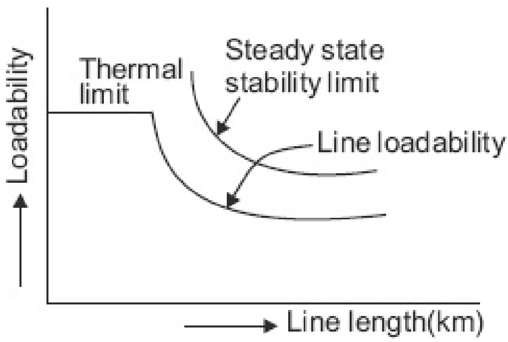

Fig. 4.13 Practical line loadability curve

4.14 POWER FLOW THROUGH A TRANSMISSION LINE

Generally, loads are expressed in terms of active and reactive powers. Consider a single-line diagram of a three-phase transmission line with a two-bus system as shown in Fig. 4.14.

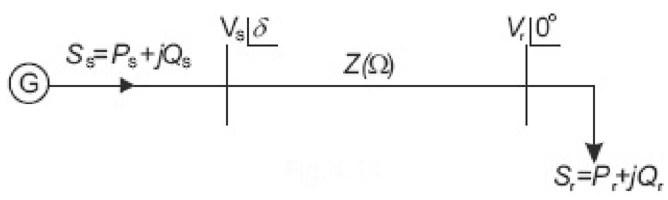

Fig. 4.14 Equivalent single-phase representation of a three-phase transmission line

Let us take the voltage at the receiving-end as a reference i.e., $V_r \angle 0°$ and then the sending-end voltage would be $V_s \angle \delta$, here δ is the torque angle (δ is the angle between V_s and V_r).

Now, the complex power per phase at the sending end is

$$S_s = P_s + jQ_s = V_s I_s^* \tag{4.89}$$

And the complex power per phase at the receiving end is

$$S_r = P_r + jQ_r = V_r I_r^* \tag{4.90}$$

where, I_s^* and I_r^* are complex conjugate of I_s and I_r, respectively.

From the Eqs. (3.9) and (3.12), we can write

$$I_r = \frac{1}{B}V_s - \frac{A}{B}V_r$$

$$\text{and } I_s = \frac{D}{B}V_s - \frac{1}{B}V_r \tag{4.91}$$

where, A, B and D are the transmission line constants and are expressed as, $A = |A| \angle \alpha$, $B = |B| \angle \beta$ and $D = |D| \angle \alpha$. ($\because A = D$)

Now the expressions from Eq. (4.91) can be written as

$$I_r = \frac{1}{|B|}V_s\angle(\delta-\beta) - \left|\frac{A}{B}\right|V_r\angle(\alpha-\beta) \tag{4.92}$$

and $$I_s = \left|\frac{D}{B}\right|V_s\angle(\alpha+\delta-\beta) - \frac{1}{|B|}V_r\angle-\beta \tag{4.93}$$

Substituting I_r from Eq. (4.92) in Eq. (4.90)

$$\begin{aligned} S_r &= |V_r|\angle 0\left[\frac{1}{|B|}V_s\angle(\beta-\delta) - \left|\frac{A}{B}\right||V_r|\angle(\beta-\alpha)\right] \\ &= \frac{|V_s||V_r|}{|B|}\angle(\beta-\delta) - \left|\frac{A}{B}\right||V_r^2|\angle(\beta-\alpha) \end{aligned}$$

Similarly, power supplied from the sending end per phase is

$$S_s = \left|\frac{D}{B}\right|V_s^2\angle(\beta-\alpha) - \frac{|V_s||V_r|}{|B|}\angle(\beta+\delta)$$

Expressing the S_r and S_s in real and imaging parts, and writing the active and reactive powers at the receiving end, we get

$$P_r = \frac{|V_s V_r|}{|B|}\cos(\beta-\delta) - \left|\frac{A}{B}\right||V_r^2|\cos(\beta-\alpha) \tag{4.94}$$

$$Q_r = \frac{|V_s V_r|}{|B|}\sin(\beta - \delta) - \left|\frac{A}{B}\right||V_r^2|\sin(\beta - \alpha) \tag{4.95}$$

Similarly, the active and reactive powers at the sending end are

$$P_s = \frac{|D|}{|B|}|V_s|^2 \cos(\beta - \alpha) - \frac{|V_s V_r|}{|B|}\cos(\beta + \delta) \tag{4.96}$$

$$Q_s = \frac{|D|}{|B|}|V_s|^2 \sin(\beta - \alpha) - \frac{|V_s V_r|}{|B|}\sin(\beta + \delta) \tag{4.97}$$

The receiving power P_r will be maximum when $\delta = \beta$, so that from Eq. (4.94), we get

$$P_r\ (\max) = \frac{|V_s V_r|}{|B|} - \left|\frac{A}{B}\right|\cos(\beta - \alpha) \tag{4.98}$$

And corresponding $Q_r = -\left|\frac{A}{B}\right||V_r|^2 \sin(\beta - \alpha)$ (4.99)

Now, consider a special case of a short transmission line with a series impedance Z i.e., $A = D = 1\angle 0$

$B = Z = Z\angle\theta$, where, θ = impedance angle.

Substituting the above quantities in Eqs. (4.94) and (4.95), the receiving-end powers are

$$P_r = \frac{|V_s V_r|}{|Z|}\cos(\theta - \delta) - \frac{|V_r|^2}{|Z|}\cos\theta \tag{4.100}$$

where, $\alpha = 0$, $\beta = \theta$ for short lines

$$\text{and } Q_r = \frac{|V_s V_r|}{|Z|}\sin(\theta - \delta) - \frac{|V_r|^2}{|Z|}\sin\theta \tag{4.101}$$

Similarly, sending-end powers are

$$P_s = \frac{|V_s|^2}{|Z|}\cos\theta - \frac{|V_s V_r|}{|Z|}\cos(\theta + \delta) \tag{4.102}$$

$$Q_s = \frac{|V_s|^2}{|Z|}\sin\theta - \frac{|V_s V_r|}{|Z|}\sin(\theta + \delta) \tag{4.103}$$

The receiving-end power is maximum, when $\delta = \theta$.

From Eq. (4.100)

$$\begin{aligned} P_r\,(\max) &= \frac{|V_s V_r|}{|Z|} - \frac{|V_r|^2}{|Z|}\cos\theta \\ &= \frac{|V_s V_r|}{|Z|} - \frac{|V_r|^2}{|Z|^2}R, \qquad \left(\text{since } \cos\theta = \frac{R}{Z}\right) \end{aligned} \tag{4.104}$$

The resistance of a transmission line is negligible when compared to its reactance i.e., $R \approx 0$, then $Z = X$

$$\therefore \theta = \tan^{-1}\left(\frac{X}{R}\right) \cong 90°$$

∴ From Eq. (4.100), the receiving-end power is

$$P_r = \frac{|V_s V_r|}{X}\sin\delta \tag{4.105}$$

And from Eq. (4.101), the reactive power at receiving end is

$$Q_r = \frac{|V_s V_r|}{X}\cos\delta - \frac{|V_s|^2}{X} \tag{4.106}$$

The load angle δ is assumed to be small for the consideration of power system stability. So the Eq. (4.106) can be further modified as

$$Q_r = \frac{|V_r|}{X}(|V_s| - |V_r|)$$
$$\text{or } Q_r = \frac{|V_r|}{X}|\Delta V| \tag{4.107}$$

where, $|\Delta V| = (|V_S| - |V_S|)$ = magnitude of voltage drop across the transmission line.

From these equations it is observed that:

1. When the resistance of a transmission line is negligibly small, i.e., zero, the active power transmitted through the transmission line is proportional to $\sin\delta \approx \delta$ (for small values of δ) and the reactive power is proportional to the voltage drop across the line i.e., ΔV.
2. The active power received is maximum when $\delta = 90°$ and has a value

$\frac{|V_s V_r|}{X}$. However, the value of δ is always less than 90° for stability considerations.

3. Maximum active power transfer over the line can be obtained by raising the excitation of the system or by reducing transfer reactance of the line.
4. From Eq. (4.107), the reactive power transfer over the transmission line is proportional to the line voltage drop and is independent of δ. Therefore, for maintaining a desired voltage profile at the receiving-end, it is necessary to control the reactive power transfer.

EXAMPLE 4.10

A typical line has the following parameters:

$\boldsymbol{A = D = 1\angle 1.0^\circ,\ B = 98\angle 80^\circ}$

1. **If the line supplies a load of 30 MW at 0.86 p.f. lagging, 105 kV, determine the sending-end voltage and hence the regulation of the line.**
2. **For a load of 45 MW at 0.86 p.f. lagging, 105 kV, find the reactive power supplied by the line and by synchronous capacitors if the sending-end voltage is 120 kV. Also determine the power factor of the line at the receiving end.**
3. **Find the maximum power that can be transmitted if the sending-end and receiving-end voltages are as in (a).**
4. **Find the power and power factor of the load if the voltages at the two ends are 105kV and with a phase difference of 25°.**

Solution:

1.

$$\text{Receiving-end voltage per phase, } V_r = \frac{105\times 1000}{\sqrt{3}} = 60621.8 \text{ V}$$

$$\text{Receiving-end current, } I_r = \frac{\text{load in MW}\times 10^6}{\sqrt{3}V_{rL}\cos\phi_r} = \frac{30\times 10^6}{\sqrt{3}\times 105\times 10^3\times 0.86} = 191.81\text{A}$$

$$\text{Receiving-end current, } I_r = 191.81\angle -30.86^\circ \text{ A}$$

$$\text{Sending-end phase voltage, } V_s = AV_r + BI_r$$

$$= 1\angle 1.0^\circ \times 60621.8\angle 0^\circ + 98\angle 80^\circ \times 191.81\angle -30.86^\circ$$

$$= 60621.8\angle 1.0^\circ + 18797.38\angle 49.14^\circ = 74492.8\angle 11.83^\circ \text{ V}$$

$$\text{Sending-end line voltage } V_{SL} = \sqrt{3}\times 74492.8 = 129025.3 \text{ V or } 129.025 \text{ kV}$$

$$\text{Voltage regulation } = \frac{V_{SL} - V_{rL}}{V_{rL}}\times 100 = \frac{129.02 - 105}{105}\times 100 = 22.8\%$$

2.

Receiving-end true power, $P_r = \frac{V_{SL}V_{rL}}{B}\cos(\beta - \delta) - \frac{A}{B}V_{rL}^2\cos(\beta - \alpha)$

$$= \frac{120 \times 105}{98}\cos(\beta - \delta) - \frac{1}{98} \times 105^2 \cos(80° - 1°) = 45\,\text{MW}$$

or $\cos(\beta - \delta) = 0.516$

$\therefore (\beta - \delta) = 58.87°$

Receiving-end reactive power, $Q_r = \frac{V_{SL}V_{rL}}{B}\sin(\beta - \delta) - \frac{A}{B}V_{rL}^2\sin(\beta - \alpha)$

$$= \frac{120 \times 105}{98}\sin(58.87°) - \frac{1}{98} \times 105^2 \sin(80° - 1°)$$

$$= 110.06 - 110.43 = -0.373\ \text{MVAr}$$

Reactive power of load = $45\sin(\cos^{-1}0.86) = 22.96$ MVAr

$\therefore$ The synchronous capacitors must deliver = 22.96 + 0.373 = 23.33 MVAr.

Power factor of the line at receiving end, $\cos\phi_r = \cos\left(\tan^{-1}\frac{Q_r}{P_r}\right)$

$$= \cos\left(\tan^{-1}\frac{-0.373}{45}\right) = 0.9999\ \text{lead}$$

3.

Maximum power transmitted, $P_{r\,max} = \frac{V_{SL}V_{rL}}{B} - \frac{A}{B}V_{rL}^2\cos(\beta - \alpha)$

$$= \frac{120 \times 105}{98} - \frac{1}{98}\cos(79°) = 128.569\ \text{MW}$$

4.

Real power transmitted, $P_r = \frac{V_{SL}V_{rL}}{B}\cos(\beta - \alpha) - \frac{A}{B}V_{rL}^2\cos(\beta - \alpha)$

$$= \frac{105 \times 105}{98}\cos(80° - 25°) - \frac{1}{98} \times 105^2\cos(80° - 1°)$$

$$= 112.5\,(0.5736 - 0.1908) = 43.0639\ \text{MW}$$

Reactive power transmitted, $Q_r = \frac{V_{SL}V_{rL}}{B}\sin(\beta - \alpha) - \frac{A}{B}V_{rL}^2\sin(\beta - \alpha)$

$$= \frac{(105)^2}{98}\sin(80° - 25°) - \sin(79°)$$

$$= -18.278\ \text{mVAr}$$

$\therefore$ Power factor of the load, $\cos\phi_r = \cos\left(\tan^{-1}\frac{-18.278}{43.0639}\right) = 0.999$ lead

EXAMPLE 4.11

A 200 km, three-phase, 50 Hz transmission line has the following data: $A = D = 0.98\angle 1.2^o$; $B = 131.2\angle 72.3^o$ Ω/phase;

C = $0.0015\angle 90^o$ S/phase. The sending-end voltage is 225 kV.

Determine (i) the receiving-end voltage when the load is disconnected, (ii) the line charging current and (iii) the maximum power that can be transmitted at a receiving end voltage of 220 kV, and the corresponding load reactive power required at the receiving end.

Solution:

Sending-end phase voltage, $V_s = \dfrac{225 \times 1000}{\sqrt{3}} = 129903.8$ V

When the load is disconnected, $I_r = 0$

Sending-end phase voltage, $V_S = AV_r + BI_r = AV_r + B \times 0$

Or receiving-end phase voltage, $V_r = \dfrac{V_S}{A} = \dfrac{129903.8\angle 0°}{0.98\angle 1.2°} = 132554.8\angle -1.2°$ V

1. Receiving-end line voltage,

   $V_{rL} = \sqrt{3} \times 132554.8 = 229591.8$ V or 229.59 kV

2. Line charging current, $I_C = CV_r = 0.0015\angle 90^o \times 132554.8\angle -1.2^o =$

   $198.83\angle 88.8^o$ A

3. Maximum power that can be transmitted for V_{rL} = 220 kV and V_S = 225 kV

$$P_{rMAX} = \frac{V_{SL}V_{rL}}{B} - \frac{A}{B}V_{rL}^2 \cos(\beta - \alpha) \text{ MW}$$
$$= \frac{225 \times 220}{131.2} - \frac{0.98}{131.2} \times 225^2 \cos(72.3° - 1.2°)$$
$$= 377.29 - 122.49 = 254.8 \text{ MW}$$

Corresponding reactive power required at the receiving-end, $Q_r = -\dfrac{A}{B}V_{rL}^2 \sin(\beta - \alpha) \qquad (\because \beta = \delta)$

$$= -\frac{0.98}{131.2} \times 225^2 \times \sin(72.3° - 1.2°)$$
$$= -357.76 \text{ MVAr}$$

4.15 CIRCLE DIAGRAM

The performance of a transmission line can be studied either by analytical methods or graphical methods. Analytical methods are often found to be cumbersome and laborious. In such cases, a graphical method of analysing by means of a circle diagram approves to be a convenient method. These diagrams are useful in system stability studies, reactive volt-amps calculations and in the system design and operation. They are easy to construct and use, and the accuracy is fairly high.

4.15.1 Receiving-end Phasor Diagram

Consider Eqs. (3.9) and (3.10)

i.e., and

$$\text{i.e.,}\quad V_s = AV_r + BI_r \tag{4.108}$$

$$\text{and}\quad I_s = AV_r + DI_r \tag{4.109}$$

The problem is to determine the value of V_s, I_s and steady the system behavior for known values of V_r, I_r and the power factor at the receiving end. The best method is to draw curves relating to these variables.

In Eq. (4.108), the complex constants A, B and D may be written in the form of their magnitudes and arguments as

$$A = |A|\angle\alpha,\ B = |B|\angle\beta,\ \text{and}\ D = |D|\angle\Delta$$

Assume the receiving-end voltage (V_r) as a reference vector and the receiving-end current (Ir) lags behind the V_r at a phase angle of ϕ_r.

Now, $V_s = AV_r + BI_r$

The sending-end voltage, V_s is obtained by adding AV_r and BI_r. The vector diagram for V_s is drawn as shown in Fig. 4.15.

V_s leads V_r by an angle δ called torque angle or load angle.

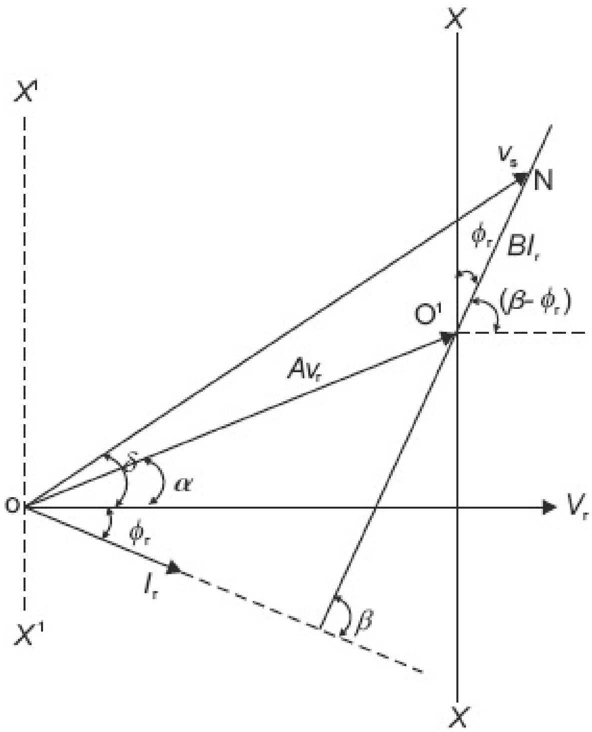

Fig. 4.15 Receiving-end phasor diagram

Construction of phasor diagram is as follows:

1. Draw a line, which is equal to V_r (reference phasor) from O.
2. Draw another line equal to AV_r and make a leading angle α with V_r phasor.
3. Draw BI_r phasor from AV_r making an angle $(\beta-\phi_r)$ with the horizontal axis and then join the points O and N. This ON phasor is the sending-end voltage, V_s.

   From the phasor diagram shown in Fig. 4.15, the angle between V_r and I_r is ϕ_r; V_r and AV_r is α; V_r and BI_r is $\beta-\alpha-\phi_r$; V_r and V_s is δ; and the angle I_r and BI_r is β. Also draw XX line in parallel with line $X'X'$ and indicate the angle ϕ_r and phasor BI_r.

$$\text{Complex power, } S_r = V_r \cdot I_r^*$$
$$= V_r \angle 0^\circ I_r \angle \phi_r$$
$$= V_r I_r \cos\phi_r + jV_r I_r \sin\phi_r$$
$$= P_r + jQ_r \quad (4.110)$$

4.15.2 Receiving-end Power Circle Diagram

It is the circle diagram drawn with receiving-end real and reactive powers as horizontal and vertical coordinates, respectively. The receiving-end power circle diagram is developed from the voltage phasor diagram shown in Fig. 4.15.

Consider the expression for V_s from Eq. (4.108) and its phasor diagram shown in Fig. 4.15. In this diagram, all the phasors except I_r are voltage phasors. These phasors can be converted into power

vectors by multiplying the entire voltage phasors by

current phasor, which is equal to $\left(\frac{V_r}{B}\right)$.

Let us examine the resultant phasors of a new diagram after multiplying voltage phasors by current. The resultant phasors are

$$V_r\left(\frac{V_r}{B}\right) = \left|\frac{V_r^2}{B}\right| \angle -\beta$$
$$AV_r\left(\frac{V_r}{B}\right) = \left|\frac{AV_r^2}{B}\right| \angle \alpha - \beta$$
$$BI_r\left(\frac{V_r}{B}\right) = \left|V_r I_r\right| \angle -\phi_r$$
$$V_s\left(\frac{V_r}{B}\right) = \left|\frac{V_s V_r}{B}\right| \angle \delta - \beta$$

Due to multiplication of all the voltage phasors of Fig. 4.15 by current $\left(\frac{V_r}{B}\right)$, the phasors in Fig. 4.15 are rotated by an angle-β and the resultant diagram obtained is shown in Fig. 4.16.

Let the line *XO'X* be taken as the horizontal axis, and line *YO'Y* as the vertical axis. In Fig. 4.16, the product V, I, gives the total volt-amps at the receiving end. The horizontal projection of V_rI_r is $V_rI_r \cos\phi_r$ (active power). The vertical projection of V_rI_r is $V_rI_r \sin\phi_r$ (reactive power). Hence, *XO'X* and *YO'Y* become active and reactive power axes.

Under the sign convention adapted in general, the inductive load draws positive reactive volt-amps. So, the diagram shown in Fig. 4.16 is to be rotated 180° about the horizontal axis *XO'X*. This diagram shown in Fig. 4.17, gives the receiving-end power circle diagram.

The important part of the diagram is the triangle *OO'N*. For constant values of V_s and V_r, and variable receiving current I_r, it will be seen that the only degree of freedom in the triangle *OO'N* is for point *N* to move in a circle whose radius is equal to $\left(\frac{V_sV_r}{B}\right)$ and the centre *O*.

Since the locus of the operating point is a circle with the axis of reference as real power and reactive volt-amps at the receiving end, the diagram thus obtained is called the receiving-end power circle diagram.

The co-ordinates of the centre of the receiving-end power circle represents the (–*O'K*, '*KO*)in Fig. 4.17.

$$-O'K = -\frac{A}{B}V_r^2\cos(\beta-\alpha)\,\text{W}$$

$$-KO = -\frac{A}{B}V_r^2\sin(\beta-\alpha)\ \text{VARs.} \quad (4.111)$$

Radius of receiving end power circle $= MN = \left(\frac{V_s V_r}{B}\right)$ VA

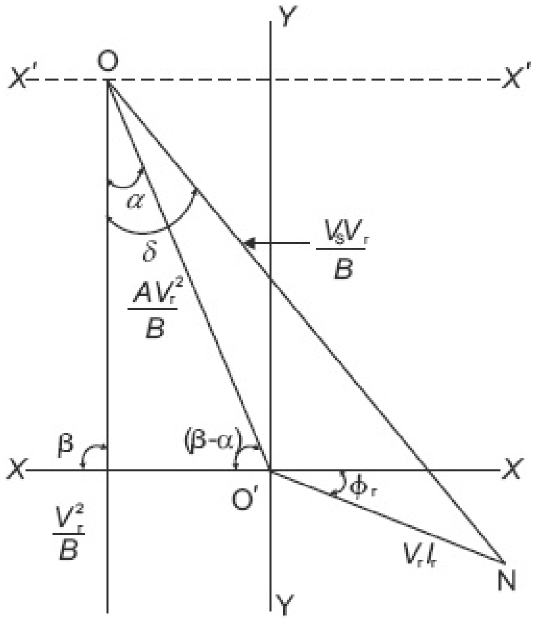

Fig. 4.16 Receiving-end phasor diagram rotated by an angle-β

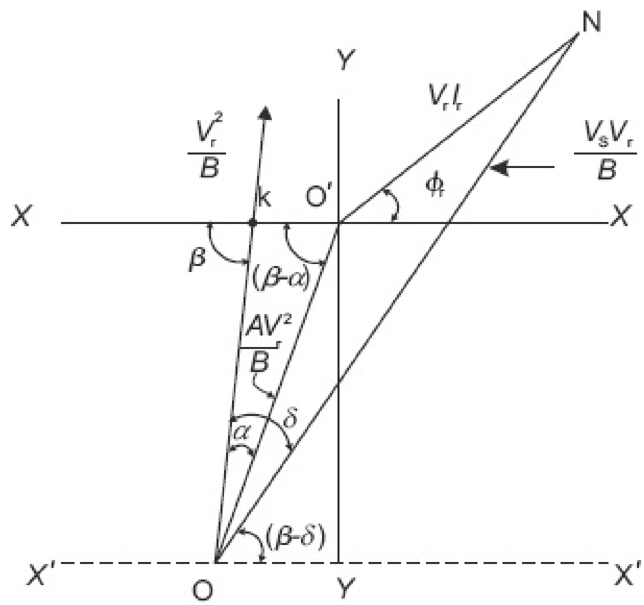

Fig. 4.17 Receiving-end power circle diagram

The distance of the centre *O* from the origin *O'* is,

$$OO' = \frac{AV_r^2}{B}$$

The equation of the power circle referred to P_r and Q_r as referred axes (–*O'K*, –*KO*) and radius *ON* may be written as

$$(P_r + O'K)^2 + (Q_r + KO)^2$$

Substituting the values obtained in Eq. (4.111) in the above equation, we get

$$\left(P_r + \frac{A}{B}V_r^2 \cos(\beta - \alpha)\right)^2 + \left(Q_r + \frac{A}{B}V_r^2 \sin(\beta - \alpha)\right)^2 = \left(\frac{V_sV_r}{B}\right)^2 \quad (4.112)$$

It is to be noted that $\left(\frac{V_r^2}{B}\right)$ is marked as reference line from which torque angle δ is measured. This line corresponds to phasor of Fig. 4.15. δ is the angle between the line $\left(\frac{V_r^2}{B}\right)$ and *ON*. The reference line and torque angles are important in correlating a receiving-end power diagram with a sending-end power diagram. We shall consider the following two cases.

Case 1: Receiving-end voltage is kept constant and sending-end voltage is changed

In this case, centre of receiving-end circle is fixed (independent of sending-end voltage) i.e., the point *O* is same, but the radius of circles vary for different values of V_S. Such concentric circles are shown in Fig. 4.18.

Case 2: Sending-end voltage is fixed and receiving-end voltage is changed

In this case, the centres of the circles are at different points for each value of γ but all centres lie on line *OO'*. The radii of circles vary with changes in V_r. and are shown in Fig. 4.19.

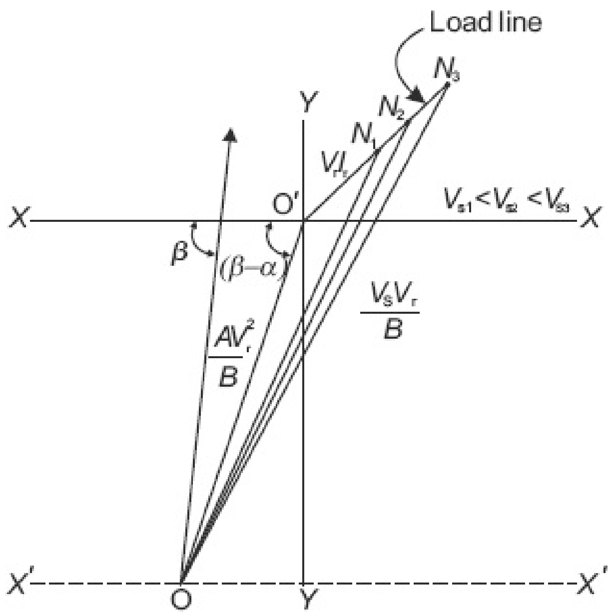

Fig. 4.18 Receiving-end power circle diagram for different values of V_s

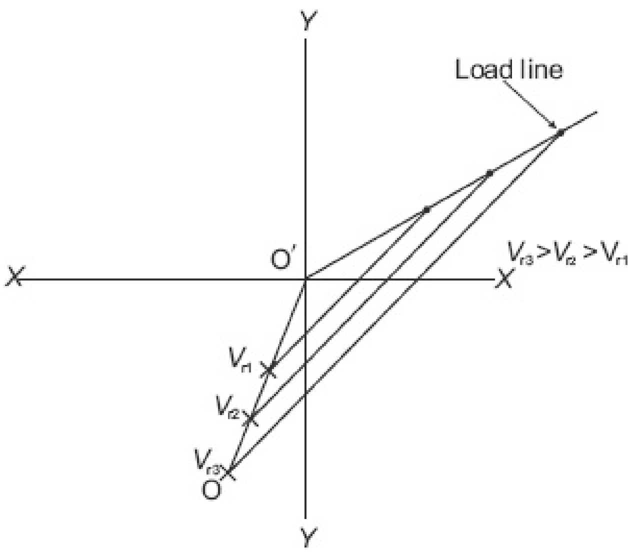

Fig. 4.19 Receiving-end power circle diagram for different values of V_r

4.15.3 Analytical Method for Receiving-end Power Circle Diagram

The receiving-end power circle diagram can be derived analytically by considering the phasor volt-amps at the receiving-end.

From the equation, $V_s = AV_r + BI_r$

$$\therefore I_r = \frac{V_s}{B} - \frac{A}{B}V_r$$

$$= \frac{V_s\angle\delta}{B\angle\beta} - \frac{A\angle\alpha}{B\angle\beta}V_r\angle 0$$

$$= \frac{V_s}{B}\angle(\delta - \beta) - \frac{A}{B}V_r\angle(\alpha - \beta)$$

$$I_r^* = \frac{V_s}{B}\angle(\beta - \delta) - \frac{A}{B}V_r\angle(\beta - \alpha)$$

Volt-ampere at the receiving end will be

$$S_r = P_r + jQ_r = V_r I_r^*$$

$$= \frac{V_r V_s}{B}\angle(\beta - \delta) - \frac{A}{B}V_r^2\angle(\beta - \alpha)$$

$$= \frac{V_r V_s}{B}\left[\cos(\beta - \delta) + j\sin(\beta - \delta)\right] - \frac{A}{B}V_r^2\left[\cos(\beta - \alpha) + j\sin(\beta - \alpha)\right]$$

$$S_r = \left[\frac{V_r V_s}{B}\cos(\beta - \delta) - \frac{A}{B}V_r^2\cos(\beta - \alpha)\right] + j\left[\frac{V_r V_s}{B}\sin(\beta - \delta) - \frac{A}{B}V_r^2\sin(\beta - \alpha)\right] \qquad (4.113)$$

By separating real and imaginary parts, we have

$$P_r = \frac{V_r V_s}{B}\cos(\beta - \delta) - \frac{A}{B}V_r^2\cos(\beta - \alpha) \qquad (4.114)$$

$$Q_r = \frac{V_r V_s}{B}\sin(\beta - \delta) - \frac{A}{B}V_r^2\sin(\beta - \alpha) \qquad (4.115)$$

$$\text{or } P_r + \frac{A}{B}V_r^2\cos(\beta - \alpha) = \frac{V_r V_s}{B}\cos(\beta - \delta) \qquad (4.116)$$

$$Q_r + \frac{A}{B}V_r^2 \sin(\beta - \alpha) = \frac{V_r V_s}{B}\sin(\beta - \delta) \tag{4.117}$$

Squaring and adding Eqs. (4.116) and (4.117), we get

$$\left[P_r + \frac{A}{B}V_r^2 \cos(\beta - \alpha)\right]^2 + \left[Q_r + \frac{A}{B}V_r^2 \sin(\beta - \alpha)\right]^2 = \left(\frac{V_s V_r}{B}\right)^2 \tag{4.118}$$

This is the equation of the circle in rectangular coordinates.

The co-ordinates of centre of the circle are:

$$\text{Horizontal coordinate } = -\frac{AV_r^2}{B}\cos(\beta - \alpha)\,\text{W}$$

$$\text{Vertical coordinate } = -\frac{AV_r^2}{B}\sin(\beta - \alpha)\,\text{VARs} \tag{4.119}$$

$$\text{Radius} = \frac{V_s V_r}{B}\,\text{VA}$$

(Note that both the co-ordinates of the circle are negative, and therefore, it is in the third quadrant in the *X*-*Y* plane. Other results obtained analytically are the same as that of the one obtained graphically.)

4.15.4 Sending-end Power Circle Diagram

It is the circle diagram drawn with the sending-end real and reactive powers as the horizontal and the vertical coordinates, respectively. The process of construction of a power circle diagram is the same as that of the receiving-end diagram.

First, the phasor diagram of voltage is drawn and is shown in Fig. 4.20. The basic equation for the sending-end power circle diagram is $V_r = DV_s - BI_s$

Taking V_s as the reference vector

$$V_s = V_s\angle 0;\ D = D\angle\Delta;\ I_s = I_s\angle -\phi_s$$
$$V_r = V_r\angle -\delta$$
$$\therefore\ V_r\angle -\delta = D\angle\Delta\,.\,V_s\angle 0 - B\angle\beta.\,I_s\angle -\phi_s$$
$$\text{i.e., } V_r\angle -\delta = DV_s\angle\Delta - BI_s\angle(\beta-\phi_s) \quad (4.120)$$

Once the sending-end phasor diagram is drawn (see Fig. 4.20), then draw *XX* line parallel to line *X'X'*, also indicate the angle ϕ_s and vector BI_s.

Now, the sending-end circle diagram is drawn from the sending-end phasor diagram by following a method similar to that of a receiving-end circle diagram.

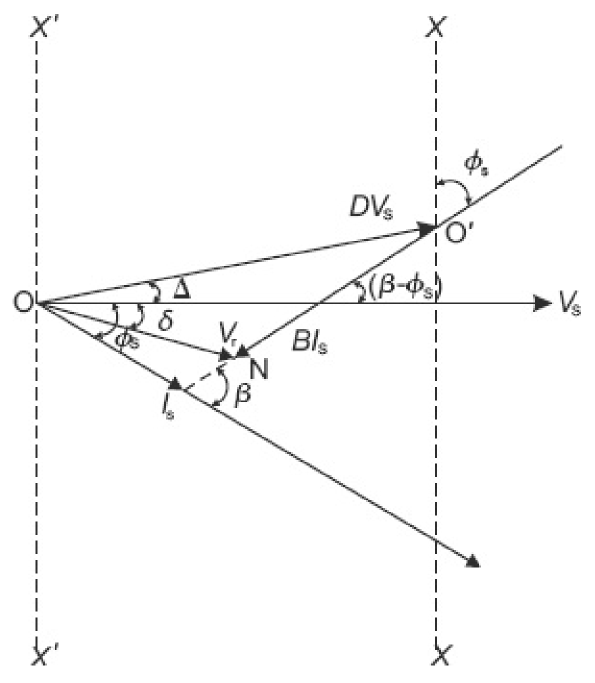

Fig. 4.20 Sending-end phasor diagram

Each phasor of Fig. 4.20 is multiplied by $\frac{-V_s}{B}$

$$OO^1 \times \left(\frac{-V_s}{B}\right) = DV_s \times \left(\frac{-V_s}{B}\right)$$

$$= \frac{D\angle\Delta V_s\angle 0}{B\angle\beta} \times V_s\angle 180^\circ.$$

$$= \frac{DV_s^2}{B}\angle(180^\circ + \Delta - \beta)$$

$$O^1N \times \left(\frac{-V_s}{B}\right) = (-BI_s) \times \left(\frac{-V_s}{B}\right) = V_s I_s$$

$$ON \times \left(\frac{-V_s}{B}\right) = \frac{V_r\angle -\delta \quad V_s\angle 180^\circ}{B\angle\beta} = \frac{V_s V_r}{B}\angle(180^\circ - \delta - \beta)$$

By such multiplication it is found that all the voltage phasors are converted into volt-amps and Fig. 4.20 is rotated to an angle 180°.

The modified diagram is shown in Fig. 4.21. In this phasor, $V_S I_S$ represents the volt-amps at the sending-end and is inclined at an angle ϕ_S with the horizontal axis *XO'X*. The projection of V_S I_S on the horizontal axis *X–X* gives the real power in watts and projection on the vertical axis *Y–Y* gives the reactive volt-amperes.

The diagram in Fig. 4.21 is rotated 180° about the horizontal axis to conform to the convention of the sign of reactive volt-amps and is shown in Fig. 4.22.

If V_s and V_r are held constant as the power delivered to the network is varied, the location of the point *O* remains fixed and the distance from the point *O* to point *N* remains constant. The location of point *N,* however, varies with the changes in the load delivered to the network and *N* is constrained to move in a circle, since, it must remain at a constant distance from the fixed point *O*. Thus, the locus operating point *N* is a circle with centre *O* and radius *ON*. Since the locus of the operating point is a circle with the axes of the reference as active power and reactive volt-amps at the sending-end, the diagram so obtained is called the sending-end power circle diagram.

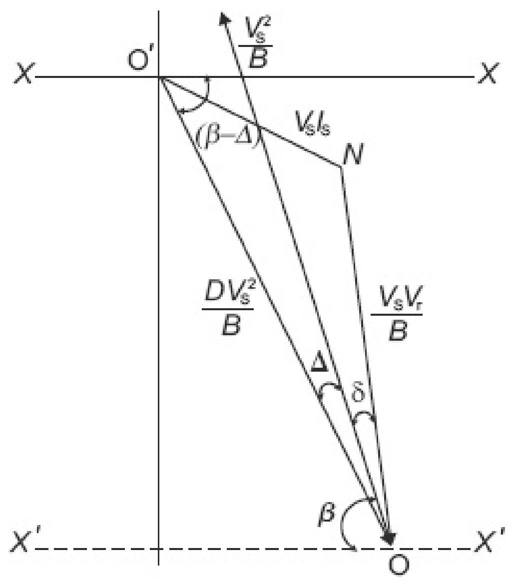

Fig. 4.21 Sending-end phasor diagram rotated through an angle (180°)

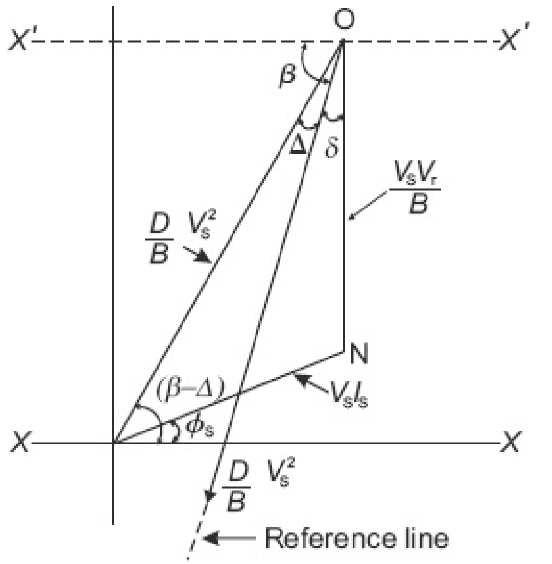

Fig. 4.22 Sending-end phasor diagram rotated 180° about horizontal axis

$$\text{Horizontal coordinate } = \frac{DV_s^2}{B}\cos(\beta - \Delta) \text{ W}$$

$$\text{Vertical coordinate } = \frac{DV_s^2}{B}\sin(\beta - \Delta) \text{ VARs} \tag{4.121}$$

$$\text{Radius} = \frac{V_s V_r}{B} \text{ VA}$$

The equation of the sending-end power circle is

$$\left[P_s - \frac{D}{B}V_s^2\cos(\beta - \Delta)\right]^2 + \left[Q_s - \frac{D}{B}V_s^2\sin(\beta - \Delta)\right]^2 = \left(\frac{V_s V_r}{B}\right)^2 \tag{4.122}$$

where,

P_s and Q_s are the real and reactive powers at the sending-end.

$\left(\frac{V_s^2}{B}\right)$ is the reference line from which torque angle δ is measured.

If V_s is maintained constant, we get a family of concentric circles for various values of V_r. If V_r is kept constant, we get a family of circles lying on the same line making an angle $(\beta - \Delta)$ with the horizontal axis.

4.15.5 Analytical Method for Sending-end Power Circle Diagram

The sending-end power circle diagram can also be drawn from the consideration of phasor volt–amps at the sending-end.

From the equation,

$$V_r = DV_s - BI_s$$
$$I_s = \frac{DV_s}{B} - \frac{V_r}{B}$$

Taking V_s as the reference vector

$$I_s = \frac{D\angle\Delta . V_s\angle 0}{B\angle\beta} - \frac{V_r\angle -\delta}{B\angle\beta}$$
$$= \frac{DV_s}{B}\angle(\Delta - \beta) - \frac{V_s}{B}\angle -(\delta + \beta)$$
$$I^* = \frac{DV_s}{B}\angle(\beta - \Delta) - \frac{V_s}{B}\angle(\delta + \beta)$$

Phasor volt-amps at the sending-end

$$S_s = V_s I_s^*$$

$$\text{i.e., } P_s + jQ_s = \frac{DV_s^2}{B}\angle(\beta - \Delta) - \frac{V_s V_r}{B}\angle(\delta + \beta)$$

$$\therefore P_s + jQ_s = \frac{DV_s^2}{B}\left[\cos(\beta - \Delta) + j\sin(\beta - \Delta)\right] - \frac{V_s V_r}{B}\left[\cos(\delta + \beta) + j\sin(\delta + \beta)\right]$$

Separating the real and imaginary parts, we get

$$\left.\begin{aligned} P_s &= \frac{V_s V_r}{B}\cos(\delta + \beta) + \frac{DV_s^2}{B}\cos(\beta - \Delta) \\ Q_s &= \frac{V_s V_r}{B}\sin(\delta + \beta) + \frac{DV_s^2}{B}\sin(\beta - \Delta) \end{aligned}\right\} \quad (4.123)$$

From Eq. (4.123) we can write

$$\left[P_s - \frac{D}{B}V_s^2\cos(\beta - \Delta)\right]^2 + \left[Q_s - \frac{D}{B}V_s^2\sin(\beta - \Delta)\right]^2 = \left(\frac{V_s V_r}{B}\right)^2 \quad (4.124)$$

This equation is similar to the Eq. (4.122) obtained earlier in the graphical method.

When V_s and V_r are phase voltages, their co-ordinates of the sending-end circle will be in watts and VARs per phase. The readings will be in volt–amps per phase. However, if the voltages are line to line in kV, then the co-ordinates of the center will be MW and MVAr for all the three phases and the radius of the circle will be total MVA.

Test Yourself

1. Why are the receiving-end power circle diagrams for different values of V_r, for a given value of V_s not concentric?

2. Is it necessary to draw the circle diagram using per phase values only? If so, why?

Example 4.12

The generalized circuit constants of a transmission line are as follows

$$A = D = 0.895\angle 1.4°, B = 182.5\angle 78.6°$$

1. **If the line supplies a load of 50 MW at 0.85 p.f. and 220 kV, find the sending-end voltage.**
2. **For a load of 80 MW at 0.85 p.f. lagging, at 220 kV, determine the reactive power supplied by the line and by the synchronous condenser, if the sending-end voltage is 240 kV. Also determine the power factor of the line at the receiving end.**
3. **Determine the maximum power transmitted if the sending-end and receiving-end voltages areas in (ii)**

Solution:

The generalized circuit constants of a transmission line

$A = D = 0.895\angle 1.4°, B = 182.5\angle 78.6°$ ohms

load, P = 50 MW

Receiving-end p.f. =0.85

Receiving-end voltage, V_r = 220 kV

$$\text{Current at receiving-end, } I_r = \frac{50\times 10^3}{\sqrt{3}\times 220\times 0.85} = 154.37\text{ A}$$

and $(\beta - \alpha) = 78.6° - 1.4° = 77.2°$

Scale

1 cm = 20 MW (on x-axis) and 1 cm = 20 MVAr (on y-axis)

Co-ordinates of receiving-end power circle diagram

$$X\text{-co-ordinate} = -\frac{AV_r^2}{B}\cos(\beta - \alpha)$$

$$= -\frac{0.895 \times 220^2}{182.5} \times \cos 77.2°$$

$$= -52.59 \text{ MW}$$

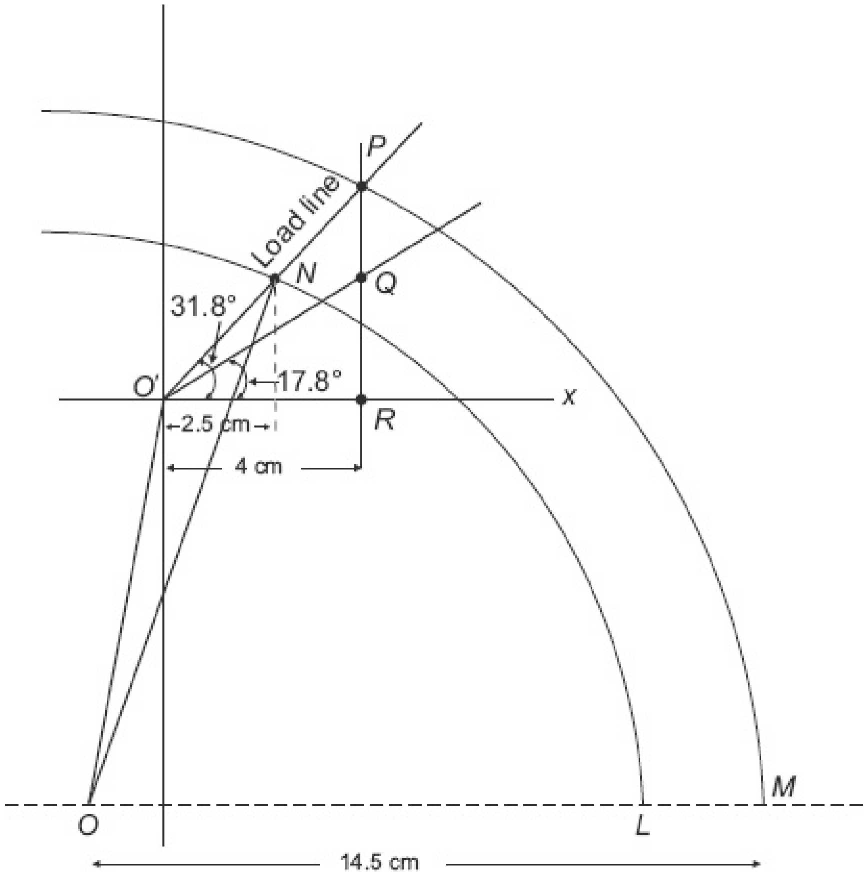

Fig. 4.23 Receiving-end power circle diagram

To scale, it is equal to 2.63 cm.

$$Y\text{-co-ordinate} = -\frac{AV_r^2}{B}\sin(\beta - \alpha) = -\frac{0.895 \times 220^2}{182.5} \times \sin 77.2° = -231.46 \text{ MVAr}$$

To scale, it is equal to 11.57 cm.

Construction of circle diagram

1. Locate point O at a co-ordinate of (–2.63, –11.57)
2. Draw load line at angle $\phi_r = \cos^{-1} 0.85 = 31.8°$ to the x-axis and

cut a point *N* corresponding to 50 MW (2.5 cm) on *x*-axis and add the points *O* and *N*.

1. From the graph shown in Fig. 4.23,

$$ON = 14.25 \text{ cm} = 285 \text{ MVA it is also equal to } \frac{V_s V_r}{B}$$

$$\therefore \text{Sending-end voltage}, V_s = \frac{285 \times 182.5}{220} = 236.42 \text{ kV}$$

$$\% \text{ Regulation} = \frac{\frac{V_s}{A} - V_r}{V_r} \times 100 = \frac{\frac{236.42}{0.895} - 220}{220} \times 100 = 20.07$$

2. When load is 80 MW (4 cm) at p.f. of 0.85, receiving-end voltage is 220 kV and sending-end voltage is240 kV

$$\text{Radius of new circle} = \frac{V_s V_r}{B} = \frac{240 \times 220}{182.5} = 289.31 \text{ MW} = 14.5 \text{ cm}$$

PR is the total reactive power (MVAr) required by the load. Out of total *PR, QR* is supplied by the line and *PQ* is supplied by the synchronous capacitor.

$$\therefore \; PQ = 1.2 \text{ cm} = 24 \text{ MVAr}$$
$$QR = 1.2 \text{ cm} = 24 \text{ MVAr}$$

Receiving-end p.f. = $\cos\phi_r$ = cos 17.8° = 0.952

3. The maximum power that can be transmitted corresponds to *LM,* which is 11.9 cm and therefore the maximum power that can be transmitted is 238 MW.

Example 4.13

A three-phase, 300 km long transmission line has *R* = 0.1 Ω/km, *X*= 0.4 Ω/km and *Y* = 5 X 10–^{6} ∠90° ℧/km. By drawing the receiving and sending-end power circle diagrams, determine the sending-end voltage, current and p.f. when the line is delivering a load of 200 MW at a p.f. of 0.8 lag and 250 kV. Assume T-configuration.

Solution

Resistance of line per phase, $R = 0.1 \times 300 = 30\ \Omega$

Reactance of line per phase, $X = 0.4 \times 300 = 120\ \Omega$

Impedance of line per phase, $Z = 30 + j120\ \Omega$

Admittance of line per phase, $Y = 5 \times 10^{-6} \angle 90° \times 300 = 15 \times 10^{-4} \angle$

$90°$ ℧

Line constants

$$A = D = 1 + \frac{YZ}{2} = 1 + \frac{15\times10^{-4}\times(30+j120)}{2}$$
$$= 1 + \frac{15\times10^{-4}\angle 90°\times 123.7\angle 75.96°}{2} = 1 + \frac{0.18555\angle 165.96°}{2} = 1 + 0.0938\angle 165.96°$$
$$= 1 - 0.091 + j0.023 = 0.909 + j0.023 = 0.9093\angle 1.45°$$
$$B = 30 + j120 = 123.7\angle 75.96°\ \Omega$$

Load p.f. angle, $\phi_r = \cos^{-1} 0.8 = 36.86°$

Receiving-end power circle diagram

$\beta - \alpha = 75.96° - 1.45° = 74.51°$

Scale

1 cm = 50 MW (on x-axis) and 1 cm = 50 MVAr (on y-axis)

Co-ordinates of receiving-end power circle diagram

$$X\text{-co-ordinate} = -\frac{AV_r^2}{B}\cos(\beta-\alpha) = -\frac{0.9093\times250^2}{123.7}\times\cos 74.51° = -122.67\ \text{MW}$$

To scale, it is equal to 2.45 cm

$$Y\text{-co-ordinate} = -\frac{AV_r^2}{B}\sin(\beta-\alpha) = -\frac{0.9093\times250^2}{123.7}\times\sin 74.51° = -442.74\ \text{MVAr}$$

To scale, it is equal to 8.85 cm

Construction of circle diagram

1. Locate point O at co-ordinates of (–2.45, –8.85)
2. Draw load line at angle, $\phi_r = \cos^{-1} 0.8 = 36.9°$ to the x-axis and cut a point N corresponding to 200 MW (4 cm) and add the points O and N.

1. From the graph shown in Fig. 4.24(a),

ON = 14.2 cm = 710 MVA it is also equal to $\frac{V_s V_r}{B}$

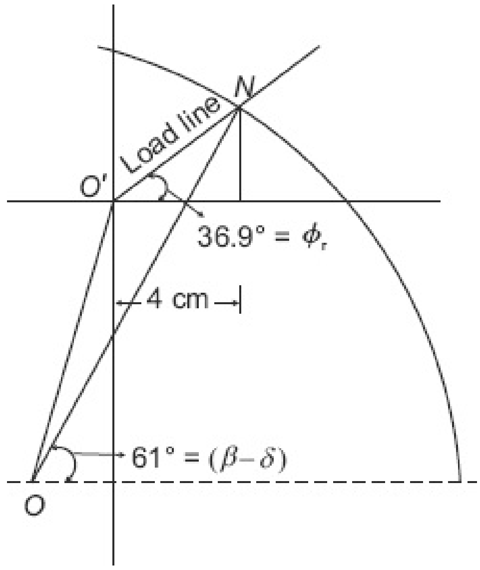

Fig. 4.24(a) Receiving-end power circle diagram

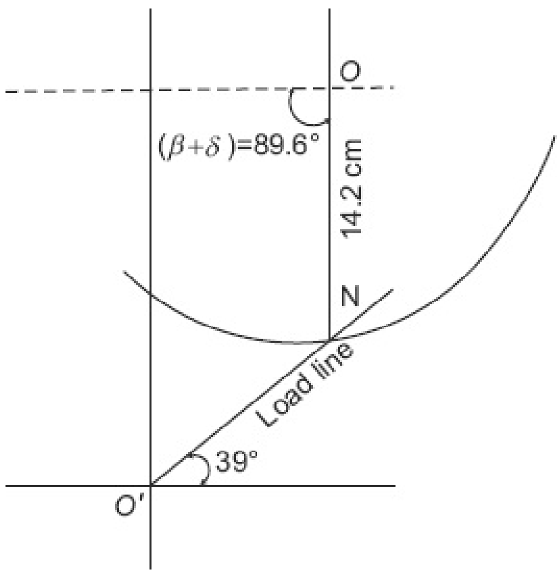

Fig. 4.24(b) Sending-end power circle diagram

$$\therefore \text{Sending-end voltage}, V_s = \frac{710 \times 123.7}{250} = 351.31 \text{ kV}$$

And $(\beta - \delta) = 61°$

$\therefore \delta = \beta - 61° = 75.96° - 61° = 14.96°$

$\therefore \beta + \delta = 75.96° + 14.96° = 90.92°$

$\beta - \Delta = 75.96° - 1.45° = 74.51°$

Construction of sending-end power circle diagram

$$X\text{-co-ordinate} = \frac{DV_s^2}{B}\cos(\beta - \Delta) = \frac{0.9093 \times 351.31^2}{123.7} \times \cos 74.51° = 242.29 \text{MW}$$

To scale, it is equal to 4.85 cm

$$Y\text{-co-ordinate} = \frac{DV_s^2}{B}\cos(\beta - \Delta) = \frac{0.9093 \times 351.31^2}{123.7} \times \sin 74.51° = 874.27 \text{MW}$$

To scale, it is equal to 17.48 cm

$$\text{Radius of circle} = \frac{V_s V_r}{B} = \frac{351.31 \times 250}{123.7} = 710 \text{MVA}$$

To scale, it is equal to 14.2 cm

Construction of sending-end power circle diagram

1. Locate point *O* at co-ordinates of (4.85, 17.84)
2. Draw a circle with *O* as center and 14.2 cm as radius. Draw line *ON* inclined at angle to the *x*-axis, where *N*is the point on the sending-end circle diagram as shown in Fig. 4.24(b).
3. Join points *O'* and *N*.

From diagram shown in Fig. 4.24(b)

The line *O'N* is the load line and is equal to 7.1 cm. It is equal to 355 MVA

$$\sqrt{3}V_s I_s = 355$$

$$\text{Sending-end current}, \ I_s = \frac{355}{\sqrt{3}V_s} = \frac{355}{\sqrt{3} \times 351.31} = 583.41 \text{A}$$

Sending-end power factor angle, $\phi_S = 39°$

Power factor at sending-end = cos 39 = 0.777.

Example 4.14

A three-phase transmission line transmits a load of 100 MW at a p.f. of 0.85 lag. The line voltage at the receiving-end is 220 kV. The constants of a nominal-π network are as follows:

$$A = D = 0.9693\angle 1.42°,\ B = 123.7\angle 75.36°\,\Omega\ ,\ C = 0.000503\angle 90.1°\,\mho.$$

Calculate

1. **Sending-end voltage, current, power factor, regulation and efficiency of the transmission line.**
2. **Load in kW at 0.85 power factor lag that could be carried at 10% regulation.**
3. **Voltage drop if the load is 125 MW at the same power factor and the MVAr leading required for 125 MW load at 10% regulation.**
4. **Rating and power factor of the synchronous phase modifier connected in parallel with the load so that the voltage transmitted at the sending end is same as at the receiving end.**
5. **What is the maximum power transmitted in case (iv). Also calculate the corresponding power factor by constructing receiving-end and sending-end power circle diagrams.**

Solution:

Receiving-end power circle diagram

Load power factor angle, $\phi_r = \cos^{-1} 0.85 = 31.78°$

$\beta - \alpha = 75.36 - 1.42 = 73.94°$

Scale

1 cm = 50 MW (on x-axis) and 1 cm = 50 MVAr (on y-axis)

Co-ordinates of receiving-end power circle diagram

$$X\text{-co-ordinate} = -\frac{AV_r^2}{B}\cos(\beta-\alpha) = -\frac{0.9693\times 220^2}{123.7}\times\cos 73.94° = -104.92\,\text{MW}$$

To scale, it is equal to 2.1 cm

$$Y\text{- co-ordinate} = -\frac{AV_r^2}{B}\sin(\beta-\alpha) = -\frac{0.9693\times 220^2}{123.7}\times\sin 73.94° = -364.5\,\text{MVAr}$$

To scale, it is equal to 7.29 cm.

Construction of circle diagram

1. Locate point *O* at a co-ordinate of (−2.1, −7.29)
2. Draw load line at angle, $\phi_r = \cos^{-1} 0.85 = 31.7°$ to the *x*-axis and cut a point *N* corresponding to 100 MW(2 cm on *x*-axis) and add the points *O* and *N*.

(i) From the graph shown in Fig. 4.25(a), *ON* = 9.6 cm = 480 MVA, it is also equal to $\frac{V_s V_r}{B}$

$$\therefore \text{Sending-end voltage, } V_s = \frac{480 \times 123.7}{220} = 269.89\,\text{kV}$$

$$(\beta - \delta) = 63.5°$$

$$\therefore \delta = \beta - 63.5° = 75.36 - 63.5 = 11.86°$$

$$\beta + \delta = 75.36 + 11.86° = 87.22°$$

$$\beta + \Delta = 75.36 - 1.42 = 73.94°$$

Construction of sending-end power circle diagram

$$X\text{-co-ordinate} = \frac{DV_s^2}{B}\cos(\beta - \Delta) = \frac{0.9693 \times 269.89^2}{123.7} \times \cos 73.94° = 157.9\,\text{MW}$$

To scale, it is equal to 3.16 cm

$$Y\text{-co-ordinate} = \frac{DV_s^2}{B}\sin(\beta - \Delta) = \frac{0.9693 \times 269.89^2}{123.7} \times \sin 73.94° = 548.5\,\text{MVAr}$$

To scale, it is equal to 10.97 cm

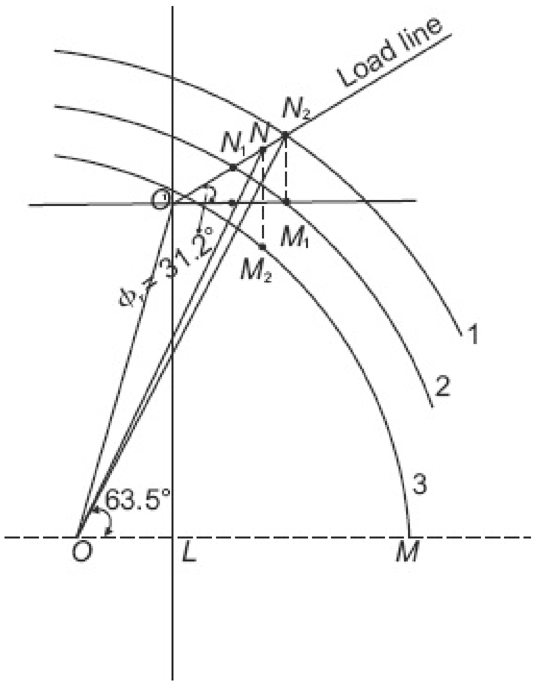

Fig. 4.25(a) Receiving-end power circle diagram

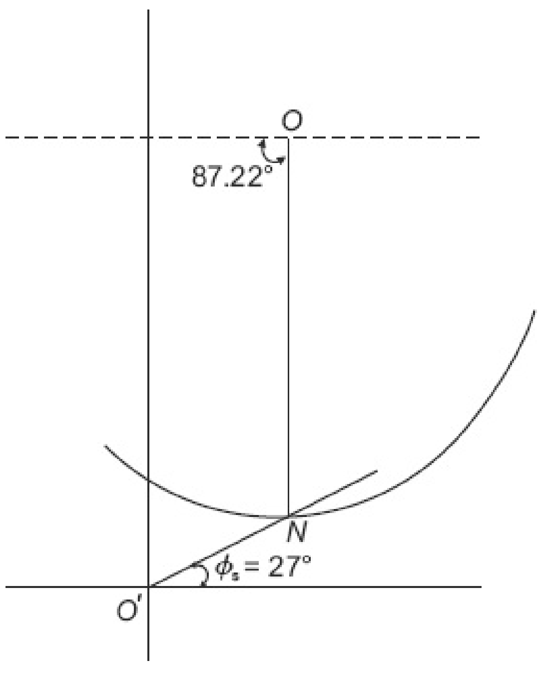

Fig. 4.25(b) Sending-end power circle diagram

$$\text{Radius of circle} = \frac{V_s V_r}{B} = \frac{269,89 \times 220}{123.7} = 480\,\text{MVA}$$

To scale, it is equal to 9.6 cm

Construction of sending-end power circle diagram

1. Locate point *O* at co-ordinates of (3.16, 10.97).
2. Draw a circle with *O* as center and radius 9.6 cm. Draw line *ON* inclined at angle $\beta + \delta$ = 87.22° to the *x*-axis. Where, *N*is the point on the sending-end circle diagram as shown in Fig. 4.25(b).
3. Join points *O'* and N.

From Fig. 4.25(b), the line *ON* is the load line and is equal to 3.15 cm. It is equal to 157.5 MVA.SV_S *I,* =157.5

$$\sqrt{3}\, V_s I_s = 157.5$$

$$\text{Sending-end current, } I_s = \frac{157.5}{\sqrt{3}\, V_s} = \frac{157.5}{\sqrt{3} \times 269.89} = 336.92\,\text{A}$$

Sending-end p.f. angle, ϕ_s = 27°

Power factor at sending-end = cos 27° = 0.891

$$\text{Regulation, } \% = \frac{V_s - V_r}{V_r} = \frac{269.89 - 220}{220} = 22.68$$

$$\text{Power at sending end, } P_s = \sqrt{3}\, V_s I_s \cos\phi_s$$

$$= \sqrt{3} \times 269.89 \times 336.92 \times 0.892 = 140.33\,\text{MW}$$

$$\%\ \text{Efficiency} = \frac{P_s}{P_r} \times 100 = \frac{100}{140.33} \times 100 = 71.26\,\text{A}$$

(ii) At a regulation of 10%

$$\frac{V_s - V_r}{V_r} = 10\%$$

$$V_s = 1.1\, V_r = 1.1 \times 220 = 242\,\text{kV}$$

$$\therefore \text{Radius of circle at 10\% regulation} = \frac{V_s V_r}{B} = \frac{242 \times 220}{123.7} = 430.4\ \text{MVA}$$

To scale, it is equal to 8.6 cm.

Draw the receiving-end circle diagram (2) [Fig. 4.25(a)] with radius 8.6 cm, it cuts the load line at N_1. The point corresponding to $O'N_1$on the x-axis is the load carried by the line at 10% regulation.

$$\therefore \text{The load carried by the line} = 1.1 \text{ cm} = 55 \text{ MW} = \frac{V_s V_r}{B}$$

(iii) At power, P = 125 MW

The new point N_2 is marked on the load line corresponding to a load of 125 MW

$$\text{The length } ON_2 = 10.05 \text{ cm} = 502.5 \text{ MVA} = \frac{V_s V_r}{B}$$

$$\therefore V_s = \frac{502.5 \times 123.7}{220} = 282.5 \text{ kV}$$

$$\therefore \text{Voltage drop} = V_s - V_r = 282.5 - 220 = 62.52 \text{ kV}$$

Since the required regulation is 10%, the sending-end voltage is 1.1 V_r.

For increased load, therefore, N_2 must move vertically downwards to cut circle diagram (2) at M_1.

∴ Leading MVAr required to maintain10% of voltage regulation with a

load of 125 MW is N_2M_1 = 0.6 cm = 30 MVAr

(iv) When Vs = Vr

$$\text{Radius of receiving-end circle} = \frac{220 \times 220}{123.7} = 391.27 \text{ MVA}$$

To scale, it is equal to 7.82 cm.

Draw the circle diagram (3) with a radius of 7.82 cm. From the operating point N, draw a vertical line downward on circle (3) and this point is M_2

∴ Leading MVAr = NM_2 = 2 cm = 100

(v) Maximum power under condition (iv) can be found by drawing a horizontal line through O and cutting the circle at M.

Maximum power = LM = 5.7 cm = 285 MW.

CHAPTER AT A GLANCE

1. **Long transmission lines:** The lines having length above 250 km, with operating voltage above 100 kV, falls under this category. Line parameters are distributed uniformly along the length of the line.

   The sending-end voltage, $V_S = V_r \cosh \gamma l + I_r Z_C \sinh \gamma l$

   $$\text{The sending-end current, } I_s = \frac{V_r}{Z_c} \sinh \gamma l + I_r \cosh \gamma l$$

   The generalized transmission circuit constants are $A = D = \cosh \gamma l$; $B = Z_c \sinh \gamma l$ and $C = \frac{1}{Z_c} \sinh \gamma l$

2. Characteristic impedance, $Z_C = \sqrt{\frac{z}{y}}$, for lossless line

   $Z_C = \sqrt{\frac{L}{C}}$ is known as surge impedance.

   1. The approximate value of surge impedance for overhead lines is 400 Ω and that for underground cables is 40 Ω.
   2. The phase angle of Z_C for transmission line is usually between 0° and –15°.

3. Propagation constant, $\gamma = \sqrt{yz}$

   $$\gamma = \alpha + j\beta$$

   $$\alpha = \text{attenuation constant} = \frac{r}{2}\sqrt{\frac{C}{L}} \text{ Np/m}$$

   $$\beta = \text{ phase constant} = \omega\sqrt{LC} \text{ rad/m}$$

   $$\text{and wave length, } l = \frac{2\Pi}{\beta} \text{ m}$$

   $$\text{Velocity of wave propagation, } v = \frac{1}{\sqrt{LC}}$$

4. **Tuned transmission lines:** If the sending-end voltage and current is numerically equal to the receiving-end voltage

and current respectively, then the line is called a tuned line.

5. **Surge impedance loading (SIL):** It is the MW loading at which the line's MVAr usage is equal to the line's MVAr production.

$$\text{SIL (MW)} = \frac{(kV_{L\_L})^2}{\text{Surge impedance}}$$

6. **Ferranti effect:** The phenomenon of rise in voltage at the receiving end of the open-circuited or lightly loaded line is called the Ferranti effect.

$$\therefore \text{ Rise in voltage, } V' = -\frac{\omega^2 C_0 L_0 l^2 V_r}{2}$$

7. **Constant voltage transmission:** In this case, a constant voltage drop is always maintained along the line by installing specially designed synchronous motors at the receiving end.
8. **Charging current in lines:** The effect of the line capacitance is to produce a current usually called the charging current.

Power loss due to charging current = $\frac{I_c^2 R}{3}$

SHORT ANSWER QUESTIONS

1. Write the expression for sending-end voltage (V_S) for long transmission lines.
2. What is meant by Ferranti effect?
3. Define the surge impedance.
4. Define the surge-impedance loading.
5. Write the expression for sending-end current for a long transmission line by rigorous method.
6. What is meant by constant-voltage transmission?
7. What are the advantages of constant-voltage transmission?
8. Define the charging current in transmission lines.
9. What is the order of power loss due to charging current?
10. What are the disadvantages of constant-voltage transmission?
11. Why is the rigorous solution method required for long lines?
12. Write down the *A, B, C, D* constants for a long line.
13. What are the methods used for computing the hyperbolic

functions in the solution of long lines?
14. Define the propagation constant.
15. What are the parameters neglected while considering a lossless line?
16. Where are the circle diagrams useful?
17. What is meant by tuned transmission lines?
18. Define the nominal-T network

MULTIPLE CHOICE QUESTIONS

1. The surge impedance of a 110 kV, three-phase transmission line is 440 Ω. The surge impedance loading of the line is
    1. $\frac{\sqrt{3}(110)^2}{440}$ MW
    2. $\frac{(110)^2}{440}$ MW
    3. $\frac{(110)^2}{\sqrt{3}\times 440}$ MW
    4. $\frac{(100)^2}{440}$ MW
2. The capacitance and inductance per unit length of a three-phase line, operating at 110 kV are 0.01 μF and 2.5 mH. The surge impedance of the line is
    1. 50 Ω
    2. 500 Ω
    3. 250 Ω
    4. 100
3. A long transmission line is energized at the sending end and is kept open-circuited at the receiving end. The magnitudes of the sending-end voltage V_S and of the receiving-end voltage V_r satisfy the following relationship
    1. $V_S = V_r$
    2. V_S is greater than V_r
    3. V_S is less than V_r
    4. none
4. Shunt compensation for long EHV lines is primarily resorted to
    1. improve voltage profile
    2. improve stability
    3. reduce fault currents
    4. increase the current
5. Series compensation is primarily resorted to
    1. improve voltage profile
    2. improve stability

3. reduce fault currents
4. to increase the current

6. Which one of the following statements is true?
   1. skin effect at 50 Hz is negligible for larger diameter conductors but becomes appreciable for smaller conductors.
   2. skin effect at 50 Hz is negligible whatever be the diameter of the conductor
   3. skin effect at 50 Hz is negligible for the smaller diameter conductors but becomes appreciable for the larger conductors
   4. none of these
7. The surge impedance of a double-circuit power transmission line is
   1. 40 Ω
   2. 200 Ω
   3. 400 Ω
   4. 800 Ω
8. The surge impedance of a telephone line is
   1. 50 Ω
   2. 75 Ω
   3. 200 Ω
   4. 400 Ω
9. Which one of the following statements is true?
   1. skin effect increases the resistance of a conductor, but proximity effect decreases the resistance
   2. both skin effect and proximity effect increase the resistance of a conductor
   3. both skin effect and proximity effect increase the internal inductance of a conductor
   4. none of these
10. A transmission line having parameters A_1, B_1, C_1, D_1, is in parallel with another having parameters A_2, B_2, C_2, D_2. The overall A parameter of the combination is
    1. $A_1A_2 + B_1C_2$
    2. $(A_1B_2 + A_2B_1) / (B_1 + B_2)$
    3. $C_1 + C_2 + (A_1 - A_2)(D_2 - D_1)/(B_1 + B_2)$
    4. $(B_1 + B_2) / (A_1B_2 + A_2B_1)$
11. The transmission lines having length above 160 km, and line voltage above 100 kV are known as
    1. short lines
    2. medium lines
    3. long lines
    4. none of these
12. If the receiving-end voltage is greater than the sending-end

voltage, regulation is

1. positive
2. negative
3. zero
4. none of these

13. The sending-end voltage of a transmission line will be equal to the receiving-end voltage on load when the p.f. of the load is
    1. leading
    2. lagging
    3. unity
    4. none of these
14. The charging current in the transmission line
    1. lags the voltage by 90°
    2. leads the voltage by 45°
    3. leads the voltage by 90°
    4. leads the voltage by 180°
15. The coefficient of reflection for current for an open-ended line is
    1. 1.0
    2. 0.5
    3. −1.0
    4. 2
16. Ferranti effect on long overhead lines is experienced when it is
    1. lightly loaded
    2. on full load at upf
    3. on full load at 0.8 p.f. lag
    4. none of these
17. The power loss due to charging current is
    1. $I_c^2 R$
    2. $\frac{I_c^2 R}{2}$
    3. $\frac{I_c^2 R}{2}$
    4. $\frac{I_c^2 R}{4}$
18. The value of charging current at the receiving end of the line is
    1. less
    2. more
    3. none of these
19. The surge impedance of a 400 km long overhead transmission line is 400 Ω. For a 200 km length of the same line, the surge impedance will be
    1. 200 Ω
    2. 800 Ω
    3. 400 Ω

4. 100 Ω

20. For a good voltage profile under no load conditions a long line needs
    1. shunt capacitors at the receiving end
    2. shunt reactor at the receiving end
    3. shunt resistance at the receiving end
    4. series capacitors at the receiving end
21. For a good voltage profile under load conditions a long line needs
    1. shunt capacitors at the receiving end
    2. shunt reactor at the receiving end
    3. shunt resistance at the receiving end
    4. series capacitors at the receiving end
22. The nature of line constants in the rigorous solution of transmission lines is
    1. distributed parameters
    2. lumped
    3. both a and b
    4. none of these
23. If the loading of the line corresponds to the surge impedance loading, the voltage at the receiving end is
    1. greater than the sending end
    2. less than the sending end
    3. equals to the sending end
    4. none of these
24. The square root of the ratio of line impedance to shunt admittance is called the
    1. regulation of the line
    2. surge impedance of the line
    3. conductance of the line
    4. surge admittance of the line
25. The coefficient of reflection of voltage for the short circuit line is
    1. 1.0
    2. −1.0
    3. 0
    4. 2.0

Answers

1. b	2. b	3. c	4. a	5. b
6. c	7. b	8. b	9. b	10. b
11. c	12. b	13. a	14. c	15. c
16. a	17. c	18. a	19. c	20. b
21. a	22. a	23. c	24. b	25. b

REVIEW QUESTIONS

1. Discuss why the receiving-end voltage of an unloaded long line may be more than its sending-end voltage.

2. Starting from the first principles, deduce expressions for *ABCD* constants of a long line in terms of its parameters.
3. What is Ferranti effect? Deduce a simple expression for the voltage rise of an unloaded line.
4. What is constant voltage system of transmission? Discuss its advantages and disadvantages.
5. Define the characteristic impedance and propagation constant of a transmission line.
6. Explain surge impedance loading.
7. Show how the receiving-end power circle diagram of a transmission line based on generalized constants can be drawn. Also, show that power at the receiving end can be calculated for any torque angle from such a diagram.
8. Show how the sending-end power circle diagram of a transmission line based on generalized constants can be drawn. Also, show that the power at the sending end can be calculated for any torque angle from such diagram.
9. What is the maximum load that can be delivered at the receiving end? Show it graphically and verify by the analytical method.
10. What is an equivalent-T circuit of a long line? Derive an expression for parameters of this circuit in terms of line parameters.
11. What is an equivalent π circuit of a long line? Derive an expression for parameters of this circuit in terms of line parameters.
12. What is meant by Nominal-π method of solution for the performance of long transmission lines? Draw a phasor diagram with the receiving-end voltage as reference.
13. Derive equivalent parameters of two transmission lines when they are connected in (i) series, and (ii) parallel.

PROBLEMS

1. A three-phase transmission line is 400 km long and delivers a load of 350 MVA, 0.85 p.f. lag at 400 kV. The *ABCD* constants of the line are $A = D = 0.918\angle 1.5°$; $B = 175\angle 85°$; $C = 0.0019\angle 90°$

   Ʊ Determine the following under full load and no load conditions

   1. Sending-end line-to-neutral voltage,
   2. The sending-end current, and
   3. The percent voltage regulation.

2. The line constants of a three-phase long line are: $A = 0.85\angle 2.3°$; $B = 180\angle 75°$; $C = 0.0014\angle 90°$.Determine the sending-end voltage, the current and power factor when the open-circuit voltage at receiving-end of the line is 220 kV.
3. The line constants of a three-phase long line are: $A = D = 0.931 +$

$j0.01$; $B = 35 + j130\ \Omega$; $C = (-6 + j900)\ 10^{-6}$ ℧. The load at the receiving-end of the line is 100 MW at 220 kV with a p.f. of 0.8 lag. Determine the sending-end voltage and the regulation of line.

4. A three-phase, 50 Hz and 250 km long line whose resistance per km is 0.015 Ω and inductance per kilometre is 0.8 mH and capacitance per kilometre is 0.01 μF. Determine the network constants of along transmission line while neglecting the conductance of the line.
5. A three-phase, 50 Hz, 1000 kilometre long transmission line has the following line constants per phase per kilometre uniformly distributed $r = 0.35\ \Omega$; $x = 0.58\ \Omega$; $g = 4 \times 10^{-6}$ and $b = 2.53 \times 10^{-6}$. Find the auxiliary constants (i) by using convergent series of complex angles (ii) by using convergent series of real angles.
6. A three-phase overhead transmission line has series impedance per phase of $250 \angle 80°\ \Omega$ and a total shunt admittance of 0.0019 $\angle 80°$ S/per phase. The line delivers a load of 100 MW at 0.8 p.f. lagging and 200 kV between the lines. Calculate the sending-end voltage and current by the rigorous method.

5

Transmission Line Transients

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Provide an analysis of travelling waves on transmission lines
- Derive a wave equation
- Understand the effect of travelling wave phenomena when line is terminated through resistance, inductance and capacitance
- Draw the Bewley Lattice Diagram

5.1 INTRODUCTION

When a transmission line is connected to a voltage source, the whole of the line is not instantly energized. Some time elapses between the initial and the final steady states. This is due to the distributed parameters of the transmission lines. The process is similar to launching a voltage wave, which travels along the length of the line at a certain velocity. The travelling voltage wave also called surge, may be caused by switching or lightning. The voltage wave is always accompanied by a current wave. The surge reaches the terminal approach such as cable boxes, transformers and switch gears, and may damage them if they are not properly protected. As the waves travel along the line their wave shapes and magnitudes are also modified. This is called distortion. The study of travelling waves helps in knowing the voltages and currents at all points in a power system. It helps in the design of insulators, protective equipment, and the insulation of the terminal equipment and overall insulation coordination.

Generally, a power system operates under a steady-state condition. However, transients are initiated due to disturbances like switching, occurrence of short-circuit faults or lightning discharge which may result in current and voltages higher in magnitude as compared to those in steady-state conditions.

5.2 TYPES OF SYSTEM TRANSIENTS

Depending upon the speed of the transients, these can be classified as:

- Surge phenomena (extremely fast transients)
- Short circuit phenomena (medium fast transients)
- Transient stability (slow transients)

Lightning and switching causes transient or surge phenomena. These transients (surges) travel along the transmission line with a velocity of light (3×10^8 m/s), i.e., in one millisecond, it travels 300 km along a transmission line. Thus, the transient phenomenon associated with these travelling waves occurs during the first few milliseconds after their initiation. The present-line losses cause fast attenuation of these waves, which die out after a few reflections.

The reflection of surges at open line ends, or at transformers which presents high inductance, causes maximum voltage buildup which may eventually damage the insulation of high-voltage equipment with consequent short circuit. Lightning on 33/11 kV substation is shown in Fig. 5.1.

Fig. 5.1 View of lightning on 33/11 kV substation

5.3 TRAVELLING WAVES ON A TRANSMISSION LINE

A transmission line is a circuit with distributed parameters. A typical characteristic of a circuit is to support travelling waves of voltage and current, in addition to which, it also has a finite velocity of electromagnetic field propagation. In such a circuit, the changes in voltage and current due to switching or lightning are spread out in all parts of the circuit in the form of travelling waves or surges.

Connected to a source of electrical energy due to sudden lightening or switching, the transmission line encounters a travelling wave of voltage and current passing through it at a finite velocity.

For explanation of the travelling wave phenomenon, consider a lossless transmission line which has series inductance ($L\Delta x$) of length Δx and shunt capacitance ($C\Delta x$) of length Δx as shown in Fig. 5.2. After the switch S is closed, the applied voltage will not appear instantly at the load end because of inductance and capacitance of the lossless transmission line. When the switch S is closed, the current passing through the first inductance is zero because it acts as an open circuit, and the voltage across the first capacitor is zero because it acts as a short circuit at the same time. At this instant, the next sections cannot be charged because the voltage across the first capacitor is zero. When the first capacitance is charged through first inductance, the capacitance of the next section starts charging and so on. It is therefore clear from the discussion that the voltage at the successive sections builds up gradually and finally the voltage wave reaches the other end of the line.

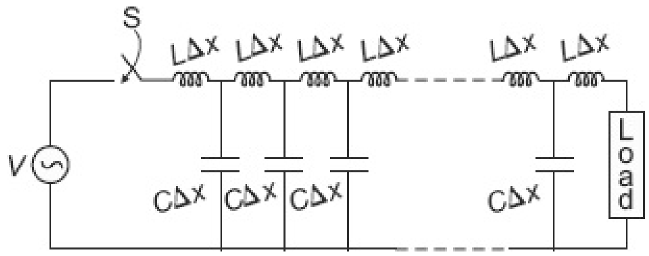

Fig. 5.2 Equivalent circuit of a long transmission line

5.4 THE WAVE EQUATION

Assume a transmission line with distributed parameters as shown in Fig. 5.3.

Let R =	Resistance of line per unit length
L =	Inductance of line per unit length
C =	Capacitance of line per unit length
G =	Shunt conductance of line per unit length

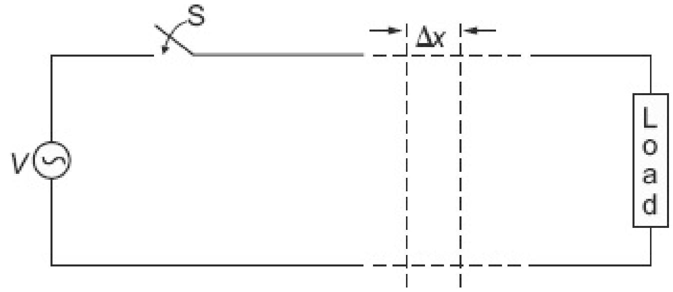

Fig. 5.3 Transmission line representation with distributed parameters

For a small section of the line Δx, the resistance, inductance, capacitance, and conductance are $R\Delta x$, $L\Delta x$, $C\Delta x$ and $G\Delta x$, respectively.

The voltage at distance $(x + \Delta x)$ from the sending-end is $V(x + \Delta x)$.

By Taylor theorem,

$$v(x+\Delta x) = v(x) + \frac{\partial v}{\partial x}\Delta x \tag{5.1}$$

The difference in voltages between the distances x and $(x + \Delta x)$ due to the resistance and inductance from Eq. (5.1) is

$$\begin{aligned} v(x) - v(x+\Delta x) &= v(x) - \left[v(x) + \frac{\partial v}{\partial x}\Delta x\right] \\ &= (R\Delta x)i + (L\Delta x)\frac{\partial i}{\partial t} \\ \therefore \frac{-\partial v}{\partial x}\Delta x &= (R\Delta x)i + (L\Delta x)\frac{\partial i}{\partial t} \\ \text{or} \quad -\frac{\partial v}{\partial x} &= Ri + L\frac{\partial i}{\partial t} \end{aligned} \tag{5.2}$$

Similarly, the difference in current can be written as:

$$\begin{aligned} i(x) - \left[i(x) + \frac{\partial i}{\partial x}\Delta x\right] &= (G\Delta x)v + (C\Delta x)\frac{\partial v}{\partial t} \\ \text{or} \quad \frac{-\partial i}{\partial x} &= Gv + C\frac{\partial v}{\partial t} \end{aligned} \tag{5.3}$$

When we consider a lossless transmission line, i.e., $R = 0$, $G = 0$, then the (5.3) become

$$\frac{\partial v}{\partial x} = -L\frac{\partial i}{\partial t} \tag{5.4}$$

and $$\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t} \tag{5.5}$$

Differentiating Eq. (5.4) partially with reference to distance x and Eq. (5.5) with reference to time t

$$\frac{\partial^2 v}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \tag{5.6}$$

and $$\frac{\partial^2 i}{\partial x \partial t} = -C\frac{\partial^2 v}{\partial t^2} \tag{5.7}$$

Also, $$\frac{\partial^2 i}{\partial t \partial x} = \frac{\partial^2 i}{\partial x \partial t}$$

From Eqs. (5.6) and (5.7)

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2} \tag{5.8}$$

Similarly differentiating Eq. (5.4) with reference to t and Eq. (5.5) with reference to x

$$\frac{\partial^2 v}{\partial x \partial t} = -L\frac{\partial^2 i}{\partial t^2} \tag{5.9}$$

and $$\frac{\partial^2 i}{\partial x^2} = -C\frac{\partial^2 v}{\partial x \partial t} \tag{5.10}$$

From Eqs. (5.9) and (5.10), we get

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \tag{5.11}$$

The Eqs. (5.8) and (5.11) are identical in form and give similar solutions. They are called the travelling wave equations in a lossless transmission line.

They represent the distribution of voltage and current along the line in terms of time and distance.

Solutions of Eqs. (5.8) and (5.11) represent the voltage and current waves that can travel in either direction, i.e., in the forward or backward direction without change in shape or magnitude with a velocity equal to $\frac{1}{\sqrt{LC}}$.

If the wave travelling in the forward or positive *x* direction can be expressed as a function of $(\sqrt{LC}x - t)$ then the function, $v = f(\sqrt{LC}x - t)$.

Similarly, the wave travelling in the backward or negative *x* direction can be expressed as another function, $v = \phi(\sqrt{LC}x + t)$. It can be proved that $v = f(\sqrt{LC}x - t)$ is a solution of Eq. (5.10). To do this, let us write

$$\left(\sqrt{LC}x - t\right) = s \tag{5.12}$$

One solution is $v = f(s)$ (5.13)

Differentiating Eq. (5.13) with reference to *x*

$$\frac{\partial v}{\partial x} = \frac{\partial f}{\partial s} \cdot \frac{\partial s}{\partial x}$$

But from Eq. (5.12), $= \frac{\partial s}{\partial x} = \sqrt{LC}$

$$\frac{\partial v}{\partial x} = \sqrt{LC} \cdot \frac{\partial f}{\partial s}$$

Taking the second derivative with reference to *x,* we obtain

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 f}{\partial s^2} \qquad (5.14)$$

In a similar way, it can be shown that $\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 f}{\partial s^2}$ (5.15)

From Eqs. (5.14) and (5.15), we get

$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ which is same as Eq. (5.8).

This equation satisfies the function $f(\sqrt{LC}x - t)$ and also

$v = \phi(\sqrt{LC}x + t)$.

Hence, the complete solution can be written as:

$$v = f\left(\sqrt{LC}x - t\right) + \phi\left(\sqrt{LC}x + t\right) \qquad (5.16)$$

This may be written in the form:

$$v = v' + v'' \tag{5.17}$$

Where $v' = f\left(\sqrt{LC}x - t\right)$ represents the incident wave, i.e., the wave travelling in *x* increasing direction and $v'' = \phi\left(\sqrt{LC}x + t\right)$ represents the reflected wave, i.e., the wave travelling in *x* decreasing direction.

Thus, the transient voltage wave is seen to have two voltage wave components v' travelling in the forward direction (i.e., when *x* is increasing) and v'' travelling in the backward direction (i.e., when *x* is decreasing).

The wave travelling in the forward direction is called the 'Forward travelling wave' and the wave travelling in the backward direction is called the 'Backward travelling wave'.

The solution for the current may be written similarly.

From Eqs. (5.5) and (5.16), we get

$$\frac{\partial i}{\partial x} = \frac{C}{\sqrt{LC}}\left[\frac{\partial f\left(\sqrt{LC}x - t\right)}{\partial x} - \frac{\partial \phi\left(\sqrt{LC}x + t\right)}{\partial x}\right] \tag{5.18}$$

After integrating Eq. (5.18), we get

$$i = \frac{1}{\sqrt{L/C}}\left[f\left(\sqrt{LC}x - t\right) - \phi\left(\sqrt{LC}x + t\right)\right] \tag{5.19}$$

The quantity $\sqrt{L/C}$ is denned as the 'characteristic impedance' of the line and is denoted by symbol Z_0.The characteristic impedance of a lossless line is a real quantity. It has the characteristics of resistance and the dimensions of ohm. Therefore, it is also called characteristic resistance or surge resistance. The surge resistance is denoted by R_0.

$$\therefore Z_0 = R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\text{Inductance of the line/km/phase}}{\text{Capacitance of the line/km/phase}}} \text{ for a lossless line.}$$

Substituting Z_0 from the above equation in Eq. (5.19), we get

$$i = \frac{1}{Z_0}\left[f\left(\sqrt{LC}x - t\right) - \phi\left(\sqrt{LC}x + t\right)\right]$$
$$i = \frac{1}{Z_0}[v' - v'']$$
$$i = \frac{v'}{Z_0} - \frac{v''}{Z_0} \tag{5.20}$$

This may be written as,

$$i = i' + i'' \tag{5.21}$$

where, $i' = \left(\frac{v'}{Z_0} \right)$ and represents the incident current wave,

$i'' = \left(\frac{-v''}{Z_0} \right)$ and represents the reflected

current wave.

Consider a voltage wave which propagates in the forward direction and a wave which propagates in the backward direction, as illustrated in Fig. 5.4(a). The backward wave is called the reflected wave. The positive direction of current is taken as the direction of propagation of the wave itself. In case of a forward wave, the direction of current and voltage are the same. But for a backward wave, the direction of propagation of current is opposite to that of voltage, so it is taken as negative. Figs. 5.4(b) and (c) represent the wave shapes for forward and reflected waves for voltage and current, and their resultant at any instant. The mathematical relation between them is given as:

$$v' = i' Z_0$$
$$v'' = i'' Z_0$$

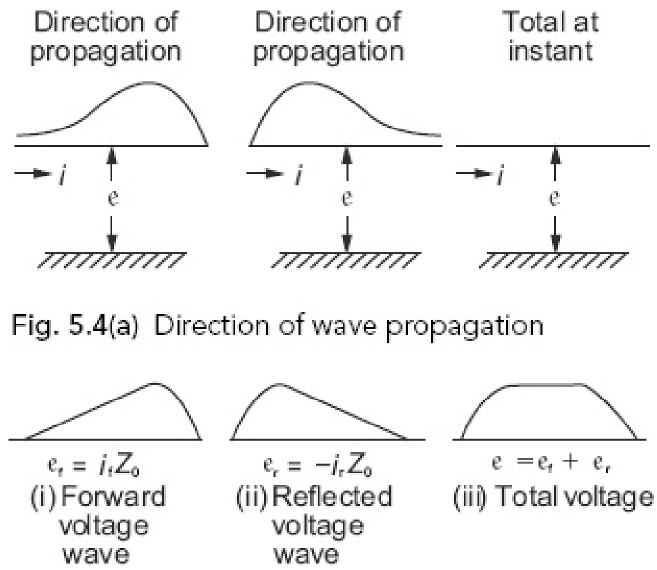

Fig. 5.4(a) Direction of wave propagation

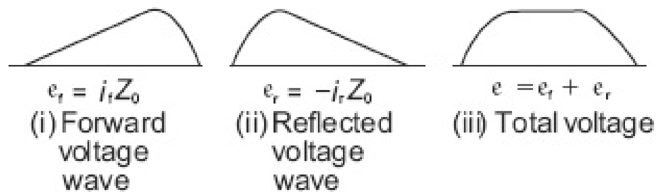

Fig. 5.4(b) Wave shapes of voltage

A function of the form $f\left(\sqrt{LC}x \pm t\right)$ represents a travelling wave because, for any value of *t*, a corresponding value of *x* can be found such that $\left(\sqrt{LC}x \pm t\right)$ has a constant value and, therefore, defines a fixed point on $f\left(\sqrt{LC}x \pm t\right)$. Corresponding values of *x* and *t* which define the same points on a wave are given by $\left(\sqrt{LC}x - t\right)$ and $\left(\sqrt{LC}x + t\right)$.

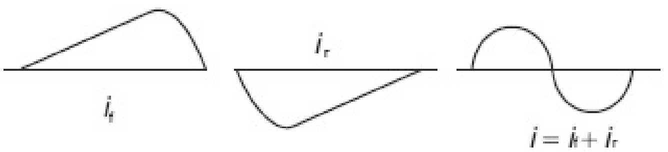

Fig. 5.4(c) Wave shapes of current

Test Yourself

1. Why is characteristic impedance also called characteristic resistance?

5.5 EVALUATION OF SURGE IMPEDANCE

In this section, we will be calculating the surge impedance for overhead transmission lines and underground cables.

1. **Overhead transmission line**

$$L = 2\times10^{-7}\ln(D/r)\ \text{H/phase/m}$$

$$C = \frac{2\pi\varepsilon}{\ln(D/r)} = \frac{2\pi\times(10^{-9}/36\pi)}{\ln(D/r)} = \frac{10^{-9}}{18\ln(D/r)}\ \text{F/phase/m}$$

where, D is the distance between the centres of the conductors and r is the radius of the conductor and $D >> r$.

$$Z_0 = \sqrt{(L/C)} = \sqrt{\frac{2\times10^{-7}\ln(D/r)}{10^{-9}/18\ln(D/r)}} = 60\ln(D/r)\ \Omega$$

2. **Cable**

$$L = 2\times10^{-7}\ln(R/r)\ \text{H/phase/m}$$

$$C = \frac{2\pi\varepsilon}{\ln(R/r)}\ \text{F/phase/m}$$

$$= \frac{10^{-9}\varepsilon_r}{18\ln(R/r)}$$

where, *R* is the radius of the cable and *r* is the radius of the conductor.

Assuming a dielectric having a relative dielectric constant of ε_r

$$Z_0 = \sqrt{\left(\frac{L}{C}\right)} = \frac{60\ln\left(\frac{R}{r}\right)}{\sqrt{\varepsilon_r}}\ \Omega$$

A value of 500 Ω is usually assumed for the surge impedance of an overhead line while a value of 50 Ω is assumed for the surge impedance of a cable.

Test Yourself

1. Why is the surge impedance in overhead lines more than in underground cables?

5.6 IMPORTANCE OF SURGE IMPEDANCE

Knowledge of surge impedance is extremely useful as it enables the calculation of the transient voltages and currents which may occur in a circuit. For example, if a line carrying a current *I* has been interrupted suddenly, the maximum value of the oscillating voltage produced and is equal to IZ_0, which may reach a dangerous value if Z_0 is high.

Power transmission systems are always complex in character, i.e., they consist of sections or elements, such as generators, transformers, transmission lines, and loads with different electrical constants. In such a compound circuit, oscillations harmful in one port of the circuit may reach a dangerous state in another port, due to variations in Z_0. For example, consider the case of a long transmission line connected to an underground cable. An oscillation current in the cable of lower impedance of its own, may give rise to dangerously high oscillating voltages when it enters into the overhead part of the line owing to the far higher natural impedance of the latter. Similarly, if a transformer is connected across the end of an overhead line, the natural impedance of such a transformer may be between 2000 Ω and 4000 Ω, which is very much higher than that of the line itself. Consequently, an oscillating current which only gives rise to moderate oscillating voltages in the line may produce destructive voltages in the winding of the transformer.

Therefore, a consideration of Z_0 of different apparatus gives considerable information regarding the relative danger and the preventive action to be taken.

5.7 TRAVELLING WAVE

A lightning discharge or sudden switching in or out results in the impression of an electric energy suddenly in a transmission line. This moves along the line at a speed of light (approximately) as a travelling wave or impulse until it has been diverted from the wave front and has, by piling up the voltage locally in the windings of reactive apparatus, had destructive consequences. The travelling wave is also called impulse wave. This is shown in Fig. 5.5.

At point ‘a’ the voltage of the wave is zero. As the wave moves along the line, the voltage at ‘a’ will rise from zero to the peak value at ‘b’ and again fall to zero on moving further. The left portion of the peak value is called the wave front and the right portion is called the “tail” of the wave. Usually, the impulse wave is designated by time T_1

taken to attain maximum value and the time T_2 taken for the tail to fall 50% of the peak value. Thus, suppose T_1 is 1 μs and T_2 is 50 μs, then the wave is designated as 1/50 wave.

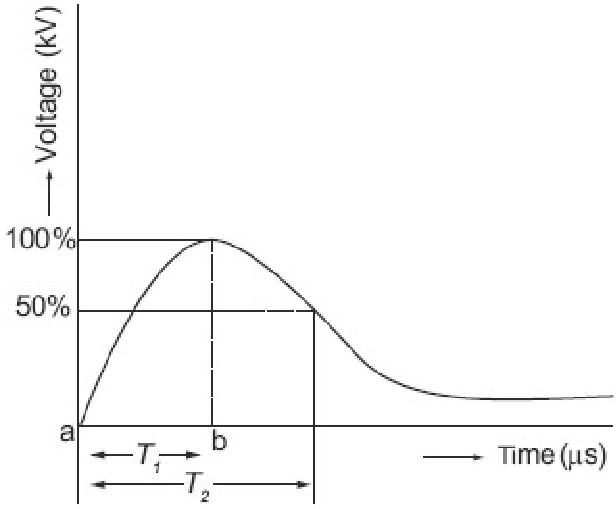

Fig. 5.5 Travelling wave shape

The equation of an impulse wave is of the form:

$$V = V_0 \left(e^{-\alpha t} - e^{-\beta t}\right) \tag{5.22}$$

where, V_0 represents a factor that depends on the peak value.

α and β are constants which control the wave front and wave tail times, respectively. Their values are $\alpha = -0.01436$ and $\beta = -2.467$ for 1.2/50 μs, impulse wave.

5.8 EVALUATION OF VELOCITY OF WAVE PROPAGATION

Consider the Eq. (5.12),

$$s=\left(\sqrt{LC}x-t\right)$$

Differentiating this expression with respect to time *t*, we get

$$0=\sqrt{LC}\frac{\partial x}{\partial t}-1$$
$$\frac{\partial x}{\partial t}=\frac{1}{\sqrt{LC}}$$

where, $\frac{\partial x}{\partial t}$ is the rate of change of distance, which is equivalent to velocity.

$$\text{Velocity of wave propagation, } \upsilon=\frac{1}{\sqrt{LC}}\text{ m/s} \tag{5.23}$$

1. **Overhead transmission line**

$$L=2\times10^{-7}\ln\left(D/R\right)\text{ H/phase/m}$$
$$C=\frac{2\pi\varepsilon}{\ln\left(D/R\right)}=\frac{2\pi\times\left(10^{-9}/36\pi\right)}{\ln\left(D/R\right)}=\frac{10^{-9}}{18\ln\left(D/R\right)}\text{ F/phase/m}$$

Where, *D* is the distance between the centres of the conductors and *R* is the radius of the conductor and *D* > *R*.

$$\upsilon=\frac{1}{\sqrt{(LC)}}=\frac{1}{\sqrt{2\times10^{-7}\ln\left(D/R\right)\times10^{-9}/18\ln\left(D/R\right)}}=3\times10^{8}\text{ m/s}$$

2. **Cable**

$$L = 2\times10^{-7}\ln(R/r)\,\text{H/phase/m}$$

$$C = \frac{2\pi\varepsilon}{\ln(R/r)}\,\text{F/phase/m}$$

$$= \frac{10^{-9}\varepsilon_r}{18\ln(R/r)}$$

where, *R* is the radius of the cable and *r* is the radius of the conductor.

Assuming a dielectric having a relative dielectric constant of ε_r

$$\upsilon = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{2\times10^{-7}\ln(R/r)\times10^{-9}\varepsilon_r/18\ln(R/r)}}$$

$$= \frac{3\times10^8}{\sqrt{\varepsilon_r}}\ \text{m/s} \qquad (5.24)$$

Example 5.1

A cable has a conductor of radius 0.75 cm and a sheath of inner radius 2.5 cm. Find (i) the inductance per meter length (ii) capacitance per meter length (iii) surge impedance and (iv) velocity of propagation, if the permittivity of insulation is 4.

Solution:

Radius of conductor, r = 0.75 cm

Inner radius of sheath, R = 2.5 cm

Permittivity of insulation, ε_r = 4

1. The inductance per metre length of the cable is,

$$L = 2\times10^{-7}\ln\frac{R}{r}\ \text{H/m}$$

$$= 2\times10^{-7}\times\ln\frac{2.5}{0.75}$$

$$= 2.41\times10^{-7}\ \text{H/m}$$

2. The capacitance per metre length of the cable is,

$$
\begin{aligned}
C &= \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{R}{r}} \text{ F/m} \\
&= \frac{4\times10^{-9}}{18\times\ln\frac{2.5}{0.75}} \\
&= 0.1846\times10^{-9} \text{ F/m}
\end{aligned}
$$

3. Surge impedance of the cable is,

$$
\begin{aligned}
Z_C &= \sqrt{\frac{L}{C}} \\
&= \sqrt{\frac{2.41\times10^{-7}}{0.1846\times10^{-9}}} \\
&= 36.13\ \Omega
\end{aligned}
$$

4. The velocity of wave propagation is,

$$
\begin{aligned}
\upsilon &= \frac{1}{\sqrt{LC}} \\
&= \frac{1}{\sqrt{2.41\times10^{-7}\times0.1846\times10^{-9}}} \\
&= 1.5\times10^{8} \text{ m/s}
\end{aligned}
$$

5.9 REFLECTION AND REFRACTION COEFFICIENT (LINE TERMINATED THROUGH A RESISTANCE)

Consider a lossless transmission line which has a surge impedance of Z_0 terminated through a resistance R as shown in Fig. 5.6. When the wave travels along the line and absorbs any change (line end, change of series or shunt impedance), then it is partly or totally reflected.

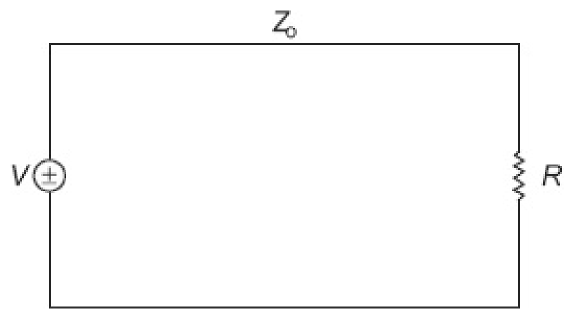

Fig. 5.6 Line terminated through a resistance

The expression for reflected current is:

$$i'' = \left(\frac{-v''}{Z_0}\right)$$

where, v″ and i″ are the reflected voltage and current waves, respectively.

Let v and i be the transmitted voltage and current waves and v' and i' be the incident waves.

From Fig. 5.6,

Incident current, $i' = \frac{v'}{Z_0}$

Reflected current, $i'' = -\frac{v''}{Z_0}$

and, transmitted current, $i = \frac{v}{R}$

Since $i = i' + i''$ and $v = v' + v''$

$$\therefore \frac{v}{R} = \frac{v'}{Z_0} - \frac{v''}{Z_0} \tag{5.25}$$

$$= \frac{v'}{Z_0} - \frac{v - v'}{Z_0} \qquad (\because\ v'' = v - v')$$

$$= \frac{2v'}{Z_0} - \frac{v}{Z_0}$$

Therefore, the transmitted voltage, $v = \frac{2R}{Z_0 + R} v'$ (5.26)

and, transmitted current, $i = \frac{v}{R} = \frac{2}{Z_0 + R} v'$

$$= \frac{v'}{Z_0} \times \frac{2Z_0}{Z_0 + R}$$

$$= i' \times \frac{2Z_0}{Z_0 + R} \tag{5.27}$$

From Eq. (5.27), the coefficient of transmitted or refraction current waves is $\frac{2Z_0}{Z_0 + R}$ (5.28)

and, transmitted coefficient for voltage waves $= \frac{2R}{Z_0 + R}$ (5.29)

Similarly, substituting for *v* in terms of v′ + v″, the Eq. (5.25), becomes

$$\frac{v' + v''}{R} = \frac{v'}{Z_0} - \frac{v''}{Z_0}$$

$$v'' = v' \times \frac{R - Z_0}{R + Z_0} \tag{5.30}$$

$$\text{and } i'' = -\frac{v''}{Z_0} = -\frac{v'}{Z_0} \times \frac{R - Z_0}{R + Z_0} \tag{5.31}$$

$$\text{Coefficient of reflection for current waves} = -\frac{R - Z_0}{R + Z_0}$$

$$\text{and, reflected coefficient for voltage waves} = +\frac{R - Z_0}{R + Z_0}$$

Example 5.2

A rectangular wave travels along a 500 km line terminated with a resistance of 1000 Ω. The line has a resistance of 0.3 Ω/km and a surge impedance of 400 Ω. If the voltage at the termination point after two successive reflections is 200 kV, determine the amplitude of the incoming surge.

Solution:

Length of the line = 500 km

Terminated resistance, R =1000 Ω

Line resistance = 0.3 Ω/km

Surge impedance, Z_C = 400 Ω

Termination voltage = 200 kV

The line resistance for 500 km,

=0.3 × 500 = 150 Ω/km

$$\text{The amplitude of the incoming surge, } v = \frac{2Rv'}{R + Z_C}$$

$$= \frac{2 \times 1000 \times 200 \times 10^3}{1000 + 150 + 400}$$

$$= 258 \text{ kV}$$

Example 5.3

A voltage having a crest value of 3000 kV is travelling on a 750 kV line. The protective level is 1700 kV and the surge impedance of the line is 300 Ω. Calculate (i) the current in the line before reaching the arrester, (ii) current through the arrester, (iii) the value of arrester resistant for this condition and (iv) reflect voltage. Verify the reflection and refraction coefficients.

Solution:

$Z_c = 300\ \Omega$, $v' = 3000$ kV, $v_a = 1700$ kV

1. $i' = \frac{v'}{Z_c} = \frac{3000\times 10^3}{300} = 10^4$ A

2. The voltage equation is

$$2v' = Z_c i_a + v_a$$
$$2 \times 3000 \times 10^3 = 300 i_a + 1700 \times 10^3$$
$$\therefore i_a = \frac{6000\times 10^3 - 1700\times 10^3}{300} = 14333 \text{ A}$$

3. Resistance of arrester, $R = \frac{v_a}{i_a} = \frac{1700\times 10^3}{14333} = 118.61\ \Omega$

4. Reflected voltage,

$$\text{Reflected voltage, } v = v' + v''$$
$$1700 = 3000 + v''$$
$$\text{or } v' = -1300 \text{ kV}$$
$$\text{Reflection coefficients} = -\frac{1300}{3000} = -0.433$$
$$\text{Refraction coefficients} = \frac{1700}{3000} = 0.567$$
$$\text{Reflection coefficients} = \frac{R - Z_c}{R + Z_c} = \frac{(118.61 - 300)}{(118.61 + 300)} = -0.433$$
$$\text{Refraction coefficients} = \frac{2R}{R + Z_c} = \frac{2\times 118.61}{118.61 + 300} = 0.567$$

5.9.1 Line Open-Circuited At The Receiving End

Consider Fig. 5.6, when the receiving end is open-circuited, i.e., $R = \infty$, the equivalent circuit is shown in Fig. 5.7.

Consider the transmitted coefficient of voltage wave

$$= \frac{2R}{Z_0 + R}$$

When $R = \infty$,

$$\text{the transmitted coefficient of voltage wave} = \frac{2}{1 + Z_0/R} = \frac{2}{1 + Z_0/\infty} = 2 \tag{5.32}$$

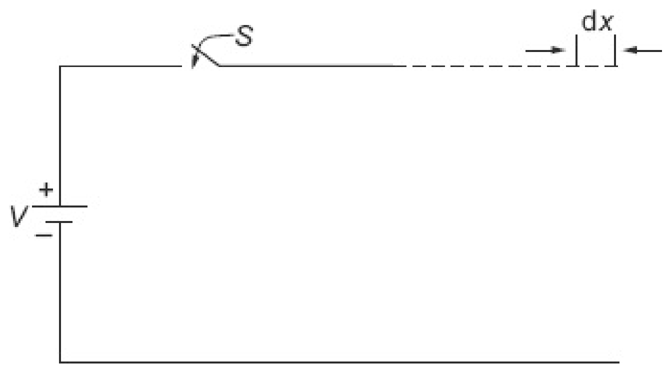

Fig. 5.7 Case of an open-circuit ended line

$$\text{and transmitted coefficient of current wave} = \frac{2Z_0}{\infty + R} = 0 \tag{5.33}$$

$$\text{and reflection coefficient of voltage wave} = +\frac{R - Z_0}{R + Z_0} = 1 \tag{5.34}$$

$$\text{Similarly, reflection coefficient of current wave} = -\frac{R - Z_0}{R + Z_0} = -\frac{1 - Z_0/R}{1 + Z_0/R} = -1 \tag{5.35}$$

From Eq. (5.32), the transmitted coefficient is two, i.e., the voltage at the open-ended line is 2v′. This means that the voltage of the open-ended line is raised by v' due to reflection.

Transmitted wave = incident wave + reflected wave

For an open-ended line, a travelling voltage wave is reflected back with a positive sign and the coefficient of reflection is unity [from Eq. (5.34)]. From Eq. (5.33), the transmission coefficient of current is zero, i.e., the current at the open-ended line is zero. This means a current wave of magnitude i' travels back with a negative sign and the coefficient of reflection is unity [from Eq. (5.35)]. The above cycle is repeated for voltage and current waves. This cycle occupies the time taken for a wave to travel four times the length of line and is explained through Fig. 5.8.

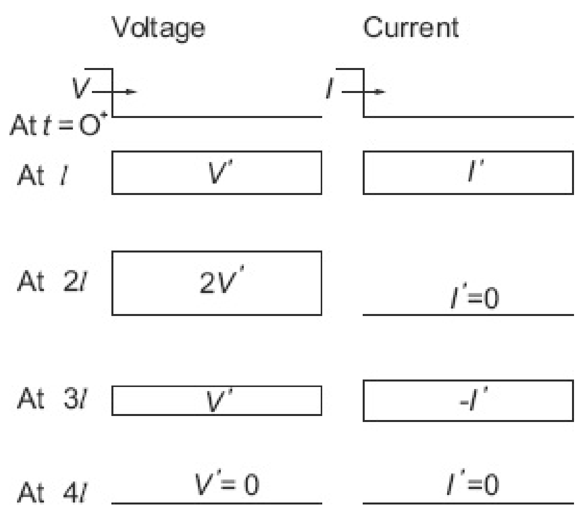

Fig. 5.8 Variation of voltage and current in an open-circuit ended line

5.9.2 Line Short-Circuited At The Receiving End

Consider Fig. 5.6. when the receiving end is short-circuited, i.e., $R = 0$, the equivalent circuit is shown in Fig. 5.9 (a).

Consider the transmitted coefficient of voltage

$$\text{wave} = \frac{2R}{Z_0 + R}$$

wave $R = 0$,

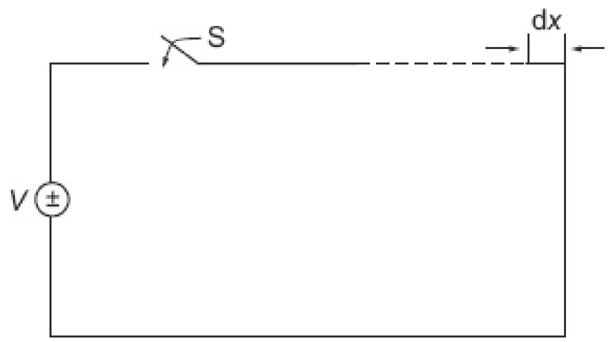

Fig. 5.9(a) Case of a short-circuit ended line

$$\text{the transmitted coefficient of voltage wave} = \frac{2R}{Z_0 + R} = 0 \tag{5.36}$$

$$\text{and transmitted coefficient of current wave} = \frac{2Z_0}{Z_0 + R} = 2 \tag{5.37}$$

$$\text{and reflection coefficient of voltage wave} = +\frac{R - Z_0}{R + Z_0} = -1 \tag{5.38}$$

$$\text{Similarly, reflection coefficient of current wave} = -\frac{R - Z_0}{R + Z_0} = 1 \tag{5.39}$$

From Eq. (5.36), the transmitted coefficient is zero, i.e., the voltage at the short-circuit ended line is zero. This means that a voltage wave of magnitude V' travels back with a negative sign and the coefficient of reflection is unity [from Eq. (5.38)].

Transmitted wave = incident wave + reflected wave

From Eq. (5.37), the transmission coefficient of current is two, i.e., the current at the short-circuit ended line is $2i'$as seen in Fig. 5.9(b). This means that the current of the short-circuit ended line is raised by i' due to reflection.

From this discussion, we can conclude that the line voltage is periodically reduced to zero but at each reflection of either end, the current is increased by the incident current, $\left(i' = \frac{v'}{Z_0}\right)$. Theoretically, the current will become infinite for infinite reflections, but practically the current will be limited by the resistance of the line in an actual system and its final value will be, $i = \frac{v}{R}$.

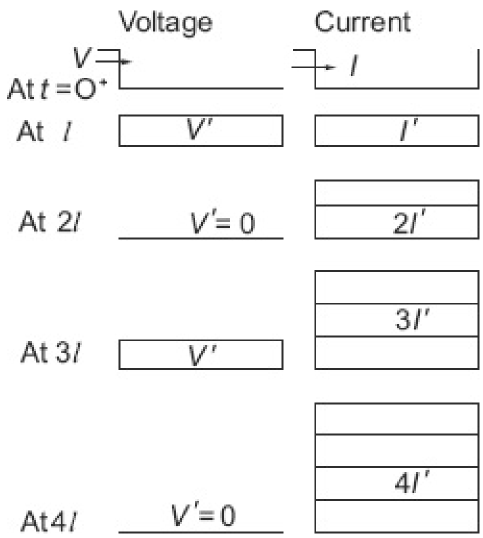

Fig. 5.9(b) Variation of voltage and current in short-circuit ended line

Test Yourself

Is the reflection coefficient of current wave in an open-circuit condition 1? If yes, justify.

5.10 LINE CONNECTED TO A CABLE

When a wave travels towards the cable from the line (see Fig. 5.10), because of the difference in impedances at the junction, part of the wave is reflected and the rest is transmitted.

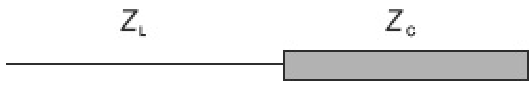

Fig. 5.10 Line connected to a cable

The transmitted voltage wave [from Eq. (5.26)] is given by

$$v = v' \times \frac{2Z_c}{Z_L + Z_c}$$

and, the transmitted current wave [from Eq. (5.27)] is given by

$$i = i' \times \frac{2Z_c}{Z_L + Z_c}$$

Similarly, reflected voltage and current waves are

$$v'' = v' \times \frac{Z_c - Z_L}{Z_c + Z_L}$$

$$\text{and } i'' = -i' \times \frac{Z_c - Z_L}{Z_c + Z_L}$$

Example 5.4

An overhead line with inductance and capacitance per km length of 1.3 mH and 0.09 μF, respectively is connected in series with an ungrounded cable (see Fig. 5.11) having inductance and capacitance of 0.2 mH/km and 0.3 μF/km, respectively. Calculate the values of reflected and refracted (transmitted) waves of voltage and current at the junction due to a voltage surge of 100 kV travelling to the junction (i) along the line towards the cable and (ii) along the cable towards the line.

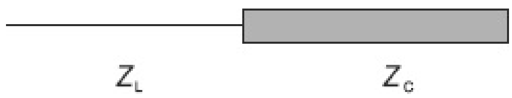

Fig. 5.11 Circuit diagram for Example 5.4

Solution:

The natural impedance of overhead line, $Z_L = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.3\times10^{-3}}{0.09\times10^{-6}}} = 120.18\ \Omega$

The natural impedance of cable, $Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.2\times10^{-3}}{0.3\times10^{-6}}} = 25.82\ \Omega$

1. The voltage wave of magnitude 100 kV which is initiated in an overhead line is partly reflected and partly transmitted on the cable at the junction of the line and the cable.

Therefore, transmitted (refracted) voltage, $v = \frac{2Z_c v'}{Z_c + Z_L}$

$$= \frac{2\times25.82\times100}{25.82+120.18} = 35.37\ \text{kV}$$

$$\text{and, reflected voltage, } v'' = \frac{Z_c - Z_L}{Z_c + Z_L} \times v'$$

$$= \frac{25.82 - 120.18}{25.82 + 120.18} \times 100 = -64.63 \text{ kV}$$

$$\text{Transmitted current, } i = i' \times \frac{2Z_L}{Z_c' + Z_L}$$

$$= \frac{v'}{Z_L} \times \frac{2Z_L}{Z_c + Z_L}$$

$$= \frac{100}{120.18} \times \frac{2 \times 120.18}{25.82 + 120.18} = 1.37 \text{ kA}$$

$$\text{and, reflected current, } i'' = -\frac{v''}{Z_L}$$

$$= -\frac{64.63}{120.188} = -537.74 \text{ A}$$

2. The voltage wave of magnitude 100 kV which is initiated in the cable is partly reflected and partly transmitted on the overhead transmission line at the junction of the cable and the line.

$$\text{Therefore, transmitted voltage, } v = \frac{2Z_L v'}{Z_L + Z_c}$$

$$= \frac{2 \times 120.18 \times 100}{120.18 + 25.82} = 164.63 \text{ kV}$$

$$\text{and, reflected voltage, } v'' = \frac{Z_L - Z_c}{Z_L + Z_c} \times v'$$

$$= \frac{120.18 - 25.82}{120.18 + 25.82} \times 100 = 64.63 \text{ kV}$$

$$\text{Transmitted current, } i = i' \times \frac{2Z_c}{Z_L + Z_c}$$

$$= \frac{v'}{Z_c} \times \frac{2Z_c}{Z_L + Z_c}$$

$$= \frac{100}{25.82} \times \frac{2 \times 25.82}{120.18 + 25.82} = 1.37 \text{ kA}$$

$$\text{and, reflected current, } i'' = -\frac{v''}{Z_c}$$

$$= -\frac{64.63}{25.82} = -2.503 \text{ kA}$$

Example 5.5

Two stations are connected together by an underground cable having a surge impedance of 50 Ω joined to an overhead line with a surge impedance of 400 Ω. If a surge having a maximum value of 110 kV travels along the cable towards the

junction with the overhead line, determine the value of the reflected and the transmitted wave of voltage and current at the junction.

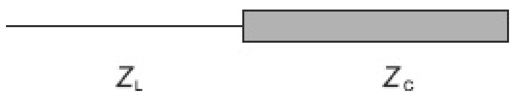

Fig. 5.12 Circuit diagram for Example 5.5

Solution:

Surge impedance of the cable, Z_c = 50Ω

Surge impedance of the overhead line, Z_L = 400 Ω

The voltage wave (magnitude of 110 kV) initiated in cable is partly reflected and partly transmitted on the overhead transmission line at the junction of cable and line.

Therefore, transmitted voltage, $v = \dfrac{2Z_L v'}{Z_L + Z_c}$

$$= \frac{2 \times 400 \times 110}{400 + 50} = 195.56 \text{ kV}$$

and, reflected voltage, $v'' = \dfrac{Z_L - Z_c}{Z_L + Z_c} \times v'$

$$= \frac{400 - 50}{400 + 50} \times 110 = 85.56 \text{ kV}$$

Transmitted current, $i = i' \times \dfrac{2Z_c}{Z_L + Z_c}$

$$= \frac{v'}{Z_c} \times \frac{2Z_c}{Z_L + Z_c}$$

$$= \frac{110}{50} \times \frac{2 \times 50}{400 + 50} = 488.89 \text{ A}$$

and, reflected current, $i'' = \dfrac{-v''}{Z_c}$

$$= \frac{-85.56}{50} = -1711.2 \text{ A}$$

Example 5.6

The ends of two long transmission lines, A and C are connected by a cable B of length 1 km. The surge impedances of *A*, *B* and *C* are 400, 50 and 500 Ω, respectively. A rectangular voltage wave of 25 kV magnitude and of infinite length is initiated in A and travels to C. Determine the first and second voltages impressed on C.

Solution:

Referring to Fig. 5.13, the voltage wave of magnitude 25 kV is initiated in line A and is partly reflected and partly refracted onto cable B when reaching the junction J_1.

The transmitted wave,

$$v_2 = \frac{2Z_B v'}{Z_A + Z_B} = \frac{2\times 50\times 25}{400+50} = 5.56 \text{ kV}$$

This transmitted wave, when reaching the junction J_2, again observes that a part of it is reflected and another refracted onto line C. This transmitted voltage wave thus is calculated as:

$$v_4 = \frac{2Z_c v_2}{Z_B + Z_c} = \frac{2\times 500\times 5.56}{50+500} = 10.11 \text{ kV}$$

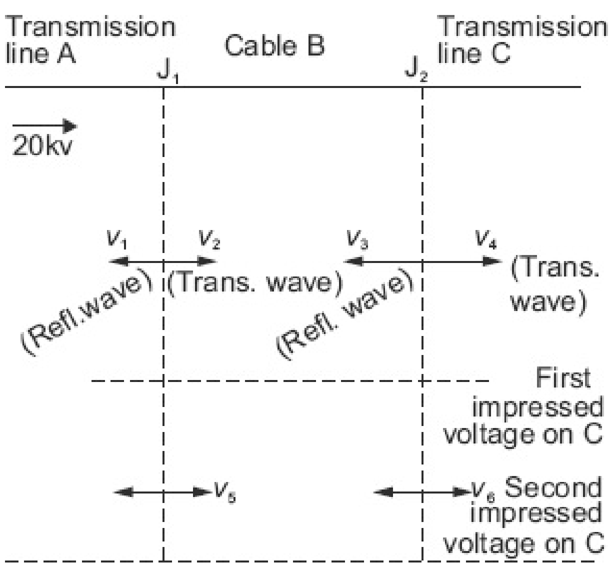

Fig. 5.13 Circuit diagram for Example 5.6

v_4 = 10.11 kV is the first voltage impressed on C.

The reflected wave v_3 at junction J_2 is,

$$v_3 = \frac{Z_c - Z_B}{Z_c + Z_B} \times v_2$$
$$= \frac{500 - 50}{500 + 50} \times 5.56 = 4.55 \text{ kV}$$

v_3 is transmitted and has reached junction J_1. From here, it is partially reflected and partially transmitted onto A. Let v_5 be the reflected wave at junction J_1. Then,

$$v_5 = \frac{Z_A - Z_B}{Z_A + Z_B} \times v_3$$
$$= \frac{400 - 50}{400 + 50} \times 4.55 = 3.54 \text{ kV}$$

However, v_5 on reaching the junction J_2, gets partially transmitted onto line C. Let this be v_6.

Then,

$$v_6 = \frac{2Z_c v_5}{Z_B + Z_c}$$
$$= \frac{2 \times 500 \times 3.54}{50 + 500} = 6.44 \text{ kV}$$

Then, second impressed voltage =	$v_4 + v_6$ = 10.11 + 6.44 = 16.55 kV

5.11 REFLECTION AND REFRACTION AT A T-JUNCTION

A voltage wave v is travelling over the lossless transmission line with natural impedance Z_l towards the junction as shown in Fig. 5.14. At the junction, due to the change of impedance, part of the wave is reflected back and the other is transmitted over the parallel lines which have natural impedances Z_2 and Z_3, respectively. Let v_2, $i_2$and v_3, i_3 be the voltages and currents in parallel branches.

As far as the voltage wave is concerned, the reflected portion will be the same for both branches, i.e., $v_2 = v_3 = v$, since they are parallel to each other. The following relations hold good at the transition point.

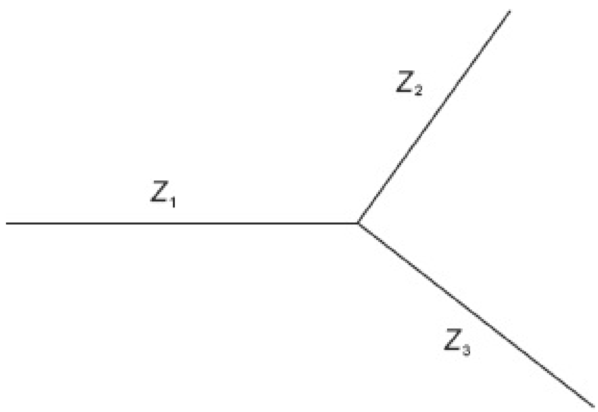

Fig. 5.14 T-Junction

$$v = v' + v''$$
$$i' = \frac{v'}{Z_1}$$
$$i'' = -\frac{v''}{Z_1}$$
$$i_2 = \frac{v}{Z_2},\quad i_3 = \frac{v}{Z_3}$$

and

$$i_2 + i_3 = i' + i'' \tag{5.37}$$

Substituting the values of currents in Eq. (5.37)

$$\frac{v}{Z_2}+\frac{v}{Z_3}=\frac{v'}{Z_1}-\frac{v''}{Z_1}$$

Substituting for v′ = $v - v'$

$$\frac{v}{Z_2}+\frac{v}{Z_3}=\frac{v'}{Z_1}-\frac{v-v'}{Z_1}$$
$$v\left[\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right]=\frac{2v'}{Z_1}$$

The transmitted voltage is

$$v=\frac{2v'/Z_1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}} \tag{5.38}$$

and, the reflected voltage is

$$v''=\frac{\frac{1}{Z_1}-\frac{1}{Z_2}-\frac{1}{Z_3}}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}}\times v' \tag{5.39}$$

Example 5.7

A 220 kV surge travels on a line of 400 Ω surge impedance and reaches a junction where two branch lines of surge impedances 550 Ω and 350 Ω, respectively are connected

with the transmission line (see Fig. 5.15). Find the surge voltage and current transmitted into each branch line. Also find the reflected voltage and current.

Solution:

$$\text{Surge voltage, } v = \frac{2v'}{Z_c + \dfrac{Z_1 \times Z_2}{Z_1 + Z_2}} \times \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{2 \times 220}{400 + \dfrac{550 \times 350}{550 + 350}} \left\{ \frac{550 \times 350}{550 + 350} \right\}$$
$$= 153.3 \text{ kV}$$

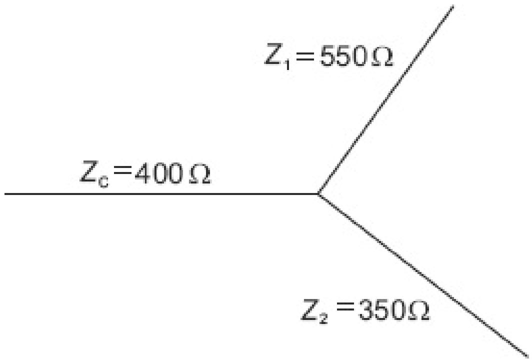

Fig. 5.15 Circuit diagram for Example 5.7

$$i_1 = \frac{v}{z_1} = \frac{153.3 \times 10^3}{550} = 278.7 \text{ A}$$

$$i_2 = \frac{v}{z_2} = \frac{153.3 \times 10^3}{350} = 438 \text{ A}$$

$$\text{Reflected voltage, } v'' = v - v' = 153.3 - 220 = -66.7 \text{ kV}$$

$$\text{Reflected current, } i'' = i_1 + i_2 - i' = 278.7 + 438 - \frac{220 \times 10^3}{400} = 166.7 \text{ A}$$

Example 5.8

A surge of 110 kV travels on a line of surge impedance 500 Ω and reaches the junction of the line with two branch lines as in Fig. 5.16. The surge impedances of the branch lines are 450 Ω and 50 Ω, respectively. Find the transmitted voltage and currents. Also find the reflected voltage and current.

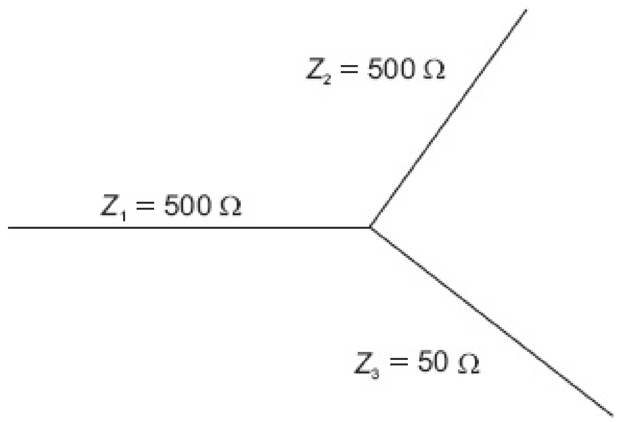

Fig. 5.16 Circuit diagram for Example 5.8

Solution:

The various impedances are

Z_1 = 500 Ω, Z_2 =450 Ω, and Z_3 = 50 Ω

The surge voltage (magnitude), v' = 110 kV

The surge reaches the junction and experiences reflection due to change in impedance and here the two lines (Z_2 and Z_3) are parallel. Therefore, the transmitted voltage will have the same magnitude and is given by,

$$v = \frac{2v'/Z_1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}}$$

$$= \frac{2\times 110/500}{\frac{1}{500}+\frac{1}{450}+\frac{1}{50}} = 18.156\text{ kV}$$

The transmitted current in branch line 1,

$$i_1 = \frac{v}{Z_2} = \frac{18.165\times 10^3}{450} = 40.37\text{ A}$$

The transmitted current in branch line 2,

$$i_2 = \frac{v}{Z_3} = \frac{18.165 \times 10^3}{50} = 363.3 \text{ A}$$

The reflected voltage,

$$v'' = \frac{\frac{1}{Z_1} - \frac{1}{Z_2} - \frac{1}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \times v'$$

$$= \frac{\frac{1}{500} - \frac{1}{450} - \frac{1}{50}}{\frac{1}{500} + \frac{1}{450} + \frac{1}{50}} \times 110$$

$$= 91.91 \text{ kV}$$

The reflected current, $i'' = \frac{-v''}{Z_1} = \frac{91.91 \times 10^3}{500} = 183.82 \text{ A}$

Example 5.9

An overhead line has a surge impedance of 450 Ω. A surge voltage $V = 250(e^{-0.05t} - e^{-t})$ kV, where t is in μs, travels along the line. The termination of the line is connected to two parallel overhead line transformer feeders. The surge impedance of the feeder is 350 Ω. These two transformers are protected by surge diverters each of surge impedance being 40 Ω. Determine the maximum voltage which would initially appear across the feeder-end windings of each transformer due to the surge. Assume the transformer to have infinite surge impedance.

Solution:

Figure 5.17 shows the circuit. Since AB and AC are parallel to each other, the voltage transmitted in them will be the same. The transmitted voltage in AB or AC is given by

$$v = 2v' \frac{\frac{1}{z_1}}{\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}} = 2v' \frac{\frac{1}{450}}{\frac{1}{450}+\frac{1}{350}+\frac{1}{40}} = 0.14776v'$$

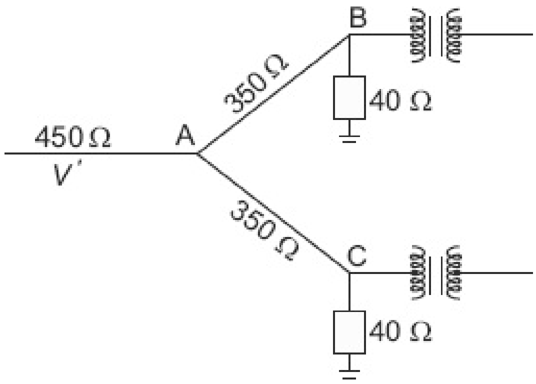

Fig. 5.17 Circuit diagram for Example 5.9

The conditions at junctions B and C are identical. The voltage transmitted at B or C

$$v_1 = v \times \frac{2z_4}{z_2 + z_4}$$

where,

$$z_4 = \text{surge impedance of a diverter} = 40\ \Omega$$
$$v_1 = 0.14776v' \times \frac{2 \times 40}{350 + 40} = 0.03031v'$$

where,

$$v' = 250\ (e^{-0.05t} - e^{-t})$$
$$\therefore\ v_1 = 0.03031 \times 250\ (e^{-0.05t} - e^{-t})$$

For maximum voltage to appear $\frac{dv_1}{dt}$ is zero, i.e.,

$$\begin{aligned}\frac{d}{dt}\left[7.5775\left(e^{-0.05t}-e^{-t}\right)\right]&=0\\ -0.05e^{-0.05t}+e^{-t}&=0\\ e^{-t}&=0.05e^{-0.05t}\\ e^{0.95t}&=20\\ t&=3.1534\ \mu s\end{aligned}$$

Substituting the value of t in the expression for v_1 gives the maximum transmitted voltage. Therefore, the maximum voltage appearing across the feeder end winding of each transformer is

$$\begin{aligned}v_1&=7.5775\left(e^{-0.05\times 3.1534}-e^{-3.1534}\right)\\ &=7.5775\times(0.854-0.0427)\\ &=6.1485\ kV\end{aligned}$$

5.12 REACTANCE TERMINATION

In this section, we consider the line terminated through capacitive and inductive reactance.

5.12.1 Line Terminated Through Capacitance

Assume that a line is terminated through a capacitor C as shown in Fig. 5.18(a). When the wave is travelling along the line with natural impedance Z_0 and terminated through C, then the transmitted voltage is determined from Eq. (5.26).

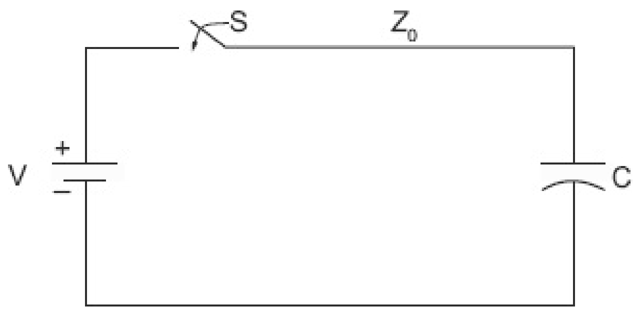

Fig. 5.18(a) Line terminated through a capacitor

The transmitted voltage is,

$$v(s) = \frac{2\times\frac{1}{Cs}}{Z_0+\frac{1}{Cs}}\times\frac{v'}{s} = \frac{2v'}{Z_0Cs\left(s+\frac{1}{Z_0C}\right)}$$
$$= 2v'\left(\frac{1}{s}-\frac{1}{s+\frac{1}{Z_0C}}\right)$$

Taking the inverse Laplace transform on both sides of the above equation

$$v(t) = 2v'\left(1-e^{-\frac{t}{Z_0C}}\right) \qquad (5.40)$$

The shape of the voltage wave is as shown in Fig. 5.18(b). There is more practical importance for a case with terminal capacitance.

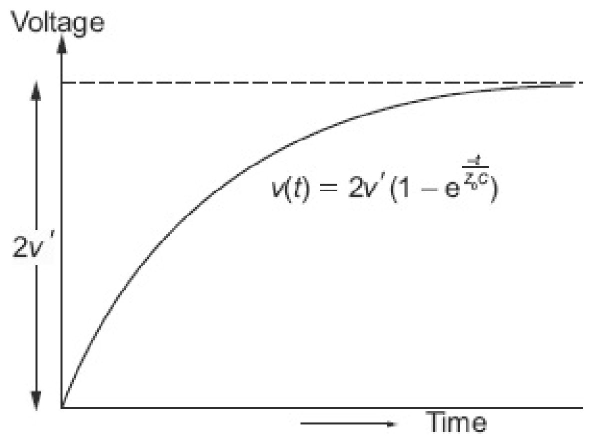

Fig. 5.18(b) Variation of voltage across the capacitor

The final value of the voltage at its terminals is 2v′.The effect of capacitance is to cause the voltage at the terminal to rise to the full value gradually, instead of abruptly, so it flattens the wave front. Flattening the wave front is beneficial because it reduces the stress on the line-end windings of a transformer connected to the line.

Test Yourself

What is the benefit achieved by flattening the wave front of the incident wave in a transformer?

Example 5.10

A 210 kV, 2.5 μβ rectangular surge travels on a line of surge impedance of 400 Ω. The line is terminated in a capacitance of 2500 p.f. Find the voltage across the capacitance.

Solution:

Voltage wave magnitude V' = 210 kV

Time duration, t = 2.5 μs

Surge impedence, Z_0 = 400 Ω

Terminated capacitance, C = 2500 p.F.

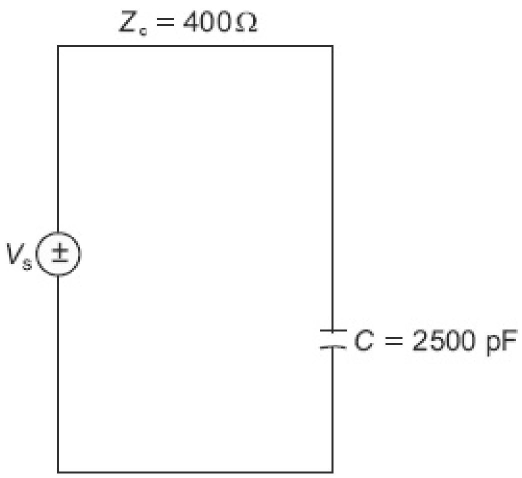

Fig. 5.19 Circuit diagram for Example 5.10

$$v(t) = 2v'(1 - e^{-t/CZ_0})$$
$$= 2 \times 210 \times \left(1 - e^{-\frac{2.5\times10^{-6}}{2500\times10^{-12}\times400}}\right)$$
$$= 385.5 \text{ kV}$$

Example 5.11

A 500 kV, 2.5 μs duration rectangular surge passes through a line having surge impedance of 400 Ω and approaches a station at which the concentrated earth capacitance is 3 × 10^3 p.f. Calculate the maximum value of surge transmitted to the second line.

Solution:

Voltage wave magnitude, v = 500 kV

Time duration, t = 2.5 μs

Surge impedance, Z = 400 Ω

Earth capacitance, $C = 3\ x10^3$ p.F.

The maximum value of surge transmitted to the second line is given by,

$$
\begin{aligned}
v(t) &= 2v'(1 - e^{-t/ZC}) \\
&= 2 \times 500\left(1 - e^{-2.5\times10^{-6}/400\times3\times10^{3}\times10^{-12}}\right) \\
&= 875.49 \text{ kV}
\end{aligned}
$$

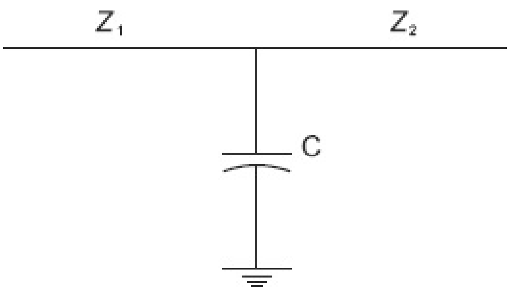

Fig. 5.20 Capacitor connected at T

Capacitor Connection at T From Fig. 5.20, the transmitted voltage across the capacitor of the circuit is calculated from Eq. (5.26).

The transmitted voltage is

$$
v(s) = \frac{\dfrac{2v'}{Z_1 s}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + Cs} = \frac{2v'Z_2}{s} \times \frac{\dfrac{1}{Z_1 Z_2 C}}{\dfrac{Z_1 + Z_2}{Z_1 Z_2 C} + s}
$$

$$
= \frac{2v'}{sZ_1 C} \times \frac{1}{s + \dfrac{Z_1 + Z_2}{Z_1 Z_2 C}}
$$

Let,

$$
\frac{Z_1 + Z_2}{Z_1 Z_2 C} = \alpha
$$

Then

$$v(s) = \frac{2v'}{s} \times \frac{\frac{1}{Z_1 C}}{s+\alpha}$$

$$v(s) = \frac{2v'}{s} \times \frac{Z_2}{Z_1+Z_2} \times \frac{Z_1 + \frac{Z_2}{Z_1 Z_2 C}}{s+\alpha}$$

$$= \frac{2v'}{s} \times \frac{Z_2}{Z_1+Z_2} \times \frac{\frac{\alpha}{Z_1 Z_2 C}}{s+\alpha}$$

$$= \frac{2v' Z_2}{Z_1+Z_2} \left[\frac{1}{s} - \frac{1}{s+\alpha} \right]$$

Taking the inverse Laplace transform on both sides of the above equation,

$$v(t) = \frac{2v' Z_2}{Z_1+Z_2} \left(1 - e^{-\left(\frac{Z_1+Z_2}{Z_1 Z_2 C}t\right)} \right)$$

5.12.2 Line Terminated Through Inductance

From Fig. 5.21, the transmitted voltage across the inductor of the circuit is calculated from Eq. (5.26).

$$\text{The transmitted voltage, } v(s) = \frac{2 \times Ls}{Z_0 + Ls} \times \frac{v'}{s}$$
$$= \frac{2v'}{(s + Z_0/L)}$$

Take inverse Laplace transform on both sides of the above equation,

$$v(t) = 2v' e^{-\frac{Z_0}{L}t} \tag{5.42}$$

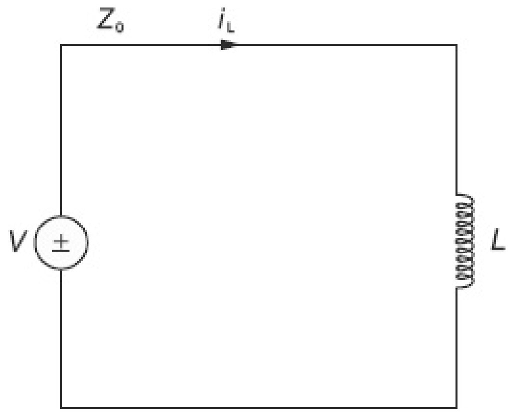

Fig. 5.21 Transmission line terminated by inductance

$$\text{Reflected voltage, } v''(t) = v'(2e^{-\frac{Z_0}{L}t} - 1) \tag{5.43}$$

Example 5.12

A step wave of 110 kV travels through a line having a surge impedance of 350 Ω. The line is terminated by an inductance of 5000μH. Find the voltage across the inductance and reflected voltage wave.

Solution:

Voltage wave magnitude, v = 110 kV

Surge impedance, Z_0= 350 Ω

Inductance connected, L = 5000 μH

Let, time duration = t μs

Then, the voltage across the inductance is,

$$
\begin{aligned}
v(t) &= 2v'e^{\left(-\frac{Z_0}{L}t\right)} \\
&= 2\times110\times e^{\left(-\frac{350}{5000\times10^{-6}}\times t\times10^{-6}\right)} \\
&= 220\times e^{-0.07t}\ \text{kV}
\end{aligned}
$$

$$
\begin{aligned}
\text{Reflected voltage, } v''(t) &= v'\left(2e^{-\frac{Z_0}{L}t}-1\right) \\
&= 110\left(2e^{\left(-\frac{350}{5000\times10^{-6}}\times t\times10^{-6}\right)}-1\right) \\
&= 110\left(2e^{-0.07t}-1\right)\ \text{kV}
\end{aligned}
$$

Example 5.13

A rectangular surge of 2.5 μs duration and magnitude 120 kV travels along a line of surge impedance 400 Ω. The latter is connected to another line of equal impedance through an inductor of 500μH. Calculate the maximum value of the surge transmitted to the second line.

Solution:

Voltage wave magnitude, V = 120 kV

Time duration, t = 2.5 μs

Surge impedance, Z_0 = 400 Ω

Inductance connected, L = 500 μH

The maximum value of surge transmitted to the second line is given by,

$$
\begin{aligned}
v(t) &= 2v'e^{\left(-\frac{Z_0}{L}t\right)} \\
&= 2\times120\times e^{\left(-\frac{400}{500\times10^{-6}}\times2.5\times10^{-6}\right)} \\
&= 240\times e^{-2} \\
&= 240\times0.1353 \\
&= 32.48\ \text{kV}
\end{aligned}
$$

5.13 BEWLEY’S LATTICE DIAGRAM

This is a graphical representation of the time-space relation, which shows the position and direction of motion at any instant of incident, reflected and transmitted current or voltage surges. In a Lattice diagram, the horizontal axes represent the distance travelled along the system and vertical axes represent the time taken to travel. At each instant of change in impedance, the reflected and transmitted values (current or voltages) can be calculated by multiplying incident wave values with reflected and transmitted coefficients.

Case-1: Receiving End is Open-circuited

Consider a line connected to a source of constant voltage v at one end and open circuited at the other, as shown in Fig. 5.22(a). Let α_s and α_L be the reflection coefficients at the sending end and the load end respectively, and t, the time taken by the wave to travel from one end to the other end. When time t = 0 s, the voltage v is connected to the source end (s) and starts travelling along the line reaching the load end in time t s with the same magnitude. Since the load end is an open circuit, the wave reflected back with a magnitude of $\alpha_L v = v$(because $\alpha_L = 1$ for an open-ended line) at time t^+ s, reaches source end in time 2t with a magnitude of v. The reflected wave is reflected back once again with a magnitude of $\alpha_s v$ from the source end after reaching the source end at time $2t^+$ and this process is continued indefinitely. The same procedure can be implemented for current waves also. This procedure is illustrated in the Lattice diagram shown in Fig. 5.22(b).

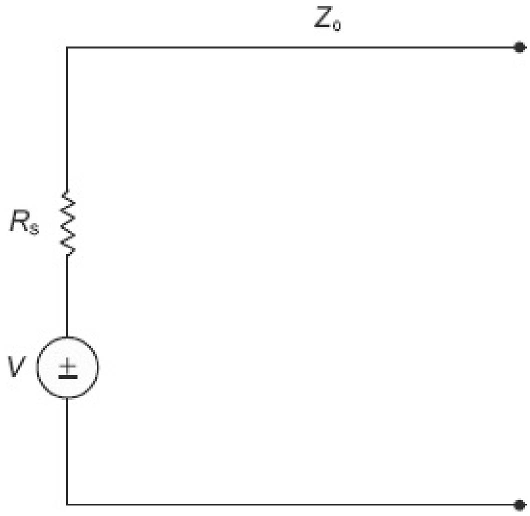

Fig. 5.22(a) Circuit diagram

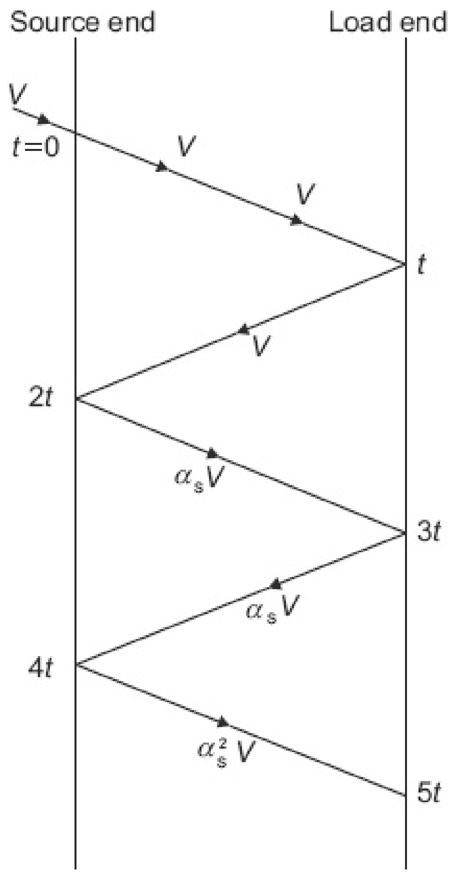

Fig. 5.22(b) Lattice diagram

Case-2: Receiving End is Connected with Resistance R

Consider a line connected to a source of constant *V* at one end and the other end is connected by a resistance *R* as shown in Fig. 5.23(a). Let α_s and α_L be the reflection coefficients at the sending-end and the load-end respectively, and the time taken by the wave to travel from one end to the other in it.

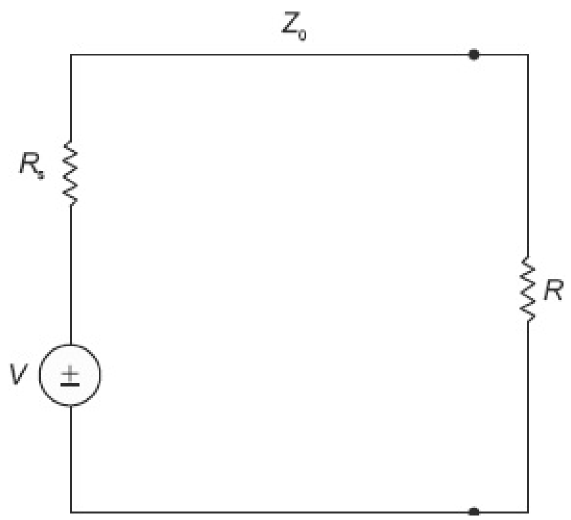

Fig. 5.23(a) Circuit diagram

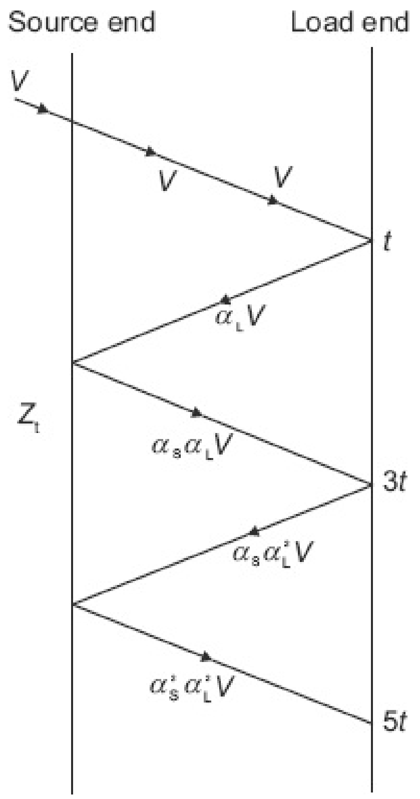

Fig. 5.23(b) Lattice diagram

When time $t = 0$ s, the voltage V is connected to the source end (s) and starts travelling along the line, and reaching the load end in time t s with the same magnitude. After reaching the junction, the wave is split into two parts, one part of the wave is transmitted and the other is reflected back with a magnitude of $\alpha_L V$ at time t^+ it travels towards source and reaches the source end in time $2t$ with a magnitude of $\alpha_L V$. The wave, which has reached the source end, splits into two parts once again. One part is transmitted and the other is reflected back with a magnitude of $\alpha_s \alpha_L V$ from source end at time $2t^+$. This process is continued indefinitely. The same procedure can be implemented for current waves also. This procedure is illustrated in the Lattice diagram shown in Fig. 5.23(b).

Example 5.14

Construct a Bewley lattice diagram when a pulse source of magnitude *v* volts with a resistance of 150 Ω, is applied across a loss-free line with surge impedance of 400 Ω terminated with a resistance of 200 Ω (see Fig. 5.24). Assume the line to be of 10 km length.

Solution:

$$\alpha_s = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{150 - 400}{150 + 400} = -0.4545$$

$$\alpha_L = \frac{R - Z_0}{R + Z_0} = \frac{200 - 400}{200 + 400} = -0.33$$

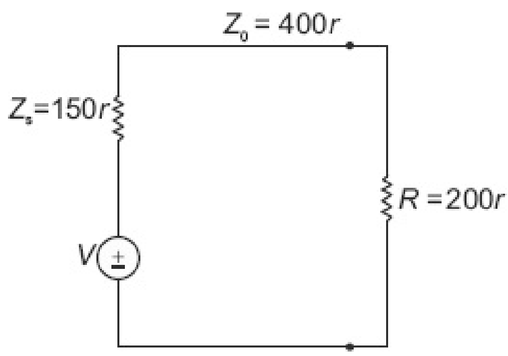

Fig. 5.24 Circuit diagram for Example 5.14

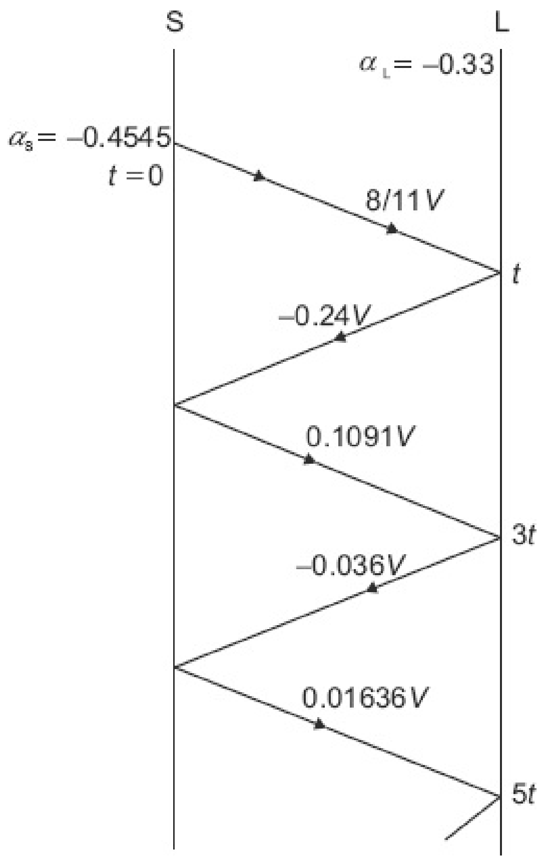

Fig. 5.25 Lattice diagram for Example 5.14

At the input of the line, the impressed voltage on line

$$= \frac{Z_0}{Z_s + Z_0} \times \mathrm{V} = \frac{400}{400 + 150} \mathrm{V} = \frac{8}{11} \mathrm{V}$$

The reflected voltage at load end.

$$= \alpha_L \times \frac{8}{11} \mathrm{V} = -0.33 \times \frac{8}{11} \mathrm{V} = -0.24 \mathrm{~V}$$

This −0.24 V backward pulse reaches the source after *t* s and gets reflection. The reflection voltage at source = α_S × (0.24 V) = 0.4545 × 0.24 V = 0.10908 V.

This reflected voltage 0.10908 V reaches the load-end and is again reflected back. This processes is continued and is shown in Fig. 5.25.

5.14 ATTENUATION OF TRAVELLING WAVES

So far we have studied the lossless overhead transmission lines, so there is no attenuation. It is not true for practical systems. The analysis is more difficult due to the presence of losses. However, these losses are very much attactive because the energy of waves is dissipated through these losses. These losses are due to the presence of resistance R and conductance G of overhead lines.

Consider *r*, *L*, *C* and *g* as the parameters per unit length of an overhead transmission line and V_0 and I_0 as the voltage and current waves at $x = 0$ as shown in Fig. 5.26. The aim is to determine the voltage (V) and current (I) waves after traveling a distance *x* with time *t* s.

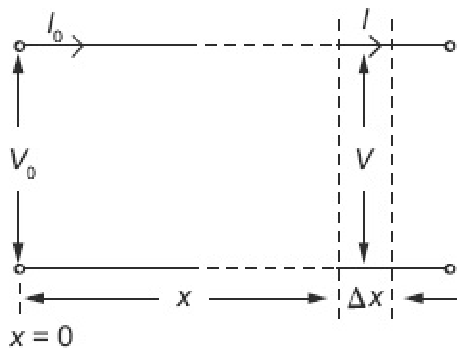

Fig. 5.26 Wave travelling on a lossy line

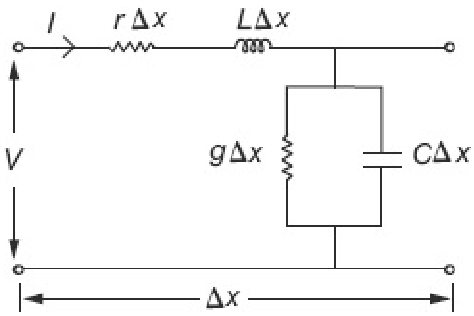

Fig. 5.27 Equivalent circuit of differential element of overhead line

Let us consider a small distance dx traveled by the wave in time d*t*. The differential length, Δx, of the overhead line is shown in Fig. 5.27

$$\text{Power loss, } P = I^2 r + V^2 g = \frac{V^2}{Z_0^2} r + V^2 g \tag{5.44}$$

On differentiation of Eq. (5.44) with respect to *x*, we get

$$dP = \frac{V^2}{Z_0^2} r dx + V^2 r dx \tag{5.45}$$

$$\text{Power at a distance x, } P = VI = -\frac{V^2}{Z_0} \tag{5.46}$$

Negative sign indicates there is reduction in power as the wave travels with time.

Differentiation of Eq. (5.46) with respect to *V* is

$$dP = -\frac{2V}{Z_0} dV \tag{5.47}$$

From Eqs. (5.45) and (5.47)

$$-\frac{2V}{Z_0}dV = \frac{V^2}{Z_0^2}rdx + V^2 rdx$$

$$\frac{dv}{dx} = -\frac{V}{2Z_0}\left(r + gZ_0^2\right)$$

$$\frac{dv}{V} = -\frac{r + gZ_0^2}{2Z_0}dx$$

$$\int \frac{dv}{V} = -\frac{r + gZ_0^2}{2Z_0}\int dx$$

$$\ln V = -\frac{r + gZ_0^2}{2Z_0}x + K$$

At $x = 0$, $V = V_0$

$\therefore \mathrm{K} = \ln V_0$

$$\ln V = -\frac{r + gZ_0^2}{2Z_0}x + \ln V_0$$

$$\ln\frac{V}{V_0} = -\frac{r + gZ_0^2}{2Z_0}x$$

$$\frac{V}{V_0} = \mathrm{e}^{-\left(\frac{r+gZ_0^2}{2Z_0}\right)x} = \mathrm{e}^{-\alpha x}$$

where

$$\alpha = -\frac{r + gZ_0^2}{2Z_0}$$

$$\therefore V = V_0\mathrm{e}^{-\alpha x} \qquad (5.48)$$

Similarly we can derive the expression for current, $I = I_0\mathrm{e}^{-\alpha x}$ (5.49)

From eqs. (5.48) and (5.49) the voltage and current waves are attenuated exponentially as they travel over the transmission line and the magnitude of attenuation depends upon the overhead line parameters.

From the empirical formula of Foust and Menger, voltage and current at any point of the overhad line after traveling x distance can be calculated as

$$V = \frac{V_0}{1 + KxV_0} \text{ kV}$$

where K = attenuation constant
= 0.00037 for chopped waves
= 0.00019 for short waves
= 0.0001 for long waves

CHAPTER AT A GLANCE

1. A lightning discharge or a sudden switching in or out in a power system suddenly impresses electrical energy in a transmission line, which moves along the line at nearly the speed of light and is called a **travelling wave.**
2. **Types of system transients:** Depending upon the speed of the transients, these can be classified as: surge phenomena, short circuit phenomena and transient stability.
3. **Wave equations:** $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ or $\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2}$
4. **Incident wave:** $v' = f\left(\sqrt{LC}x - t\right)$ or $i' = (v'/Z_0)$
5. **Reflected wave:** $v'' = \phi\left(\sqrt{LC}x + t\right)$ or $i'' = (-v''/Z_0)$
6. **Evaluation of surge resistance:**

   Overhead transmission line, $Z_0 = 60\ln(D/R)\ \Omega$

   Underground cables, $Z_0 = \frac{60\ln\left(R/r\right)}{\sqrt{\varepsilon_r}}\ \Omega$
7. **Importance of surge impedance:** Knowledge of surge impedance is extremely useful as it enables the calculation of the transient voltages and currents which may occur in a circuit.
8. **Evaluation of velocity of wave propagation**
   1. Overhead transmission line, $v = 3 \times 10^8$ m/s

2. Underground cables, $\upsilon = \frac{3\times10^8}{\sqrt{\varepsilon_r}}$ m/s

SHORT ANSWER QUESTIONS

1. What is a travelling wave?
2. What are the factors that cause a travelling wave?
3. What are the values of characteristic impedance for transmission lines, cables and transformers?
4. What is the velocity of propagation of a surge in overhead lines and cables?
5. Write the equation of an impulse wave explaining the significance of each term.
6. What is the effect when the reflected wave meets (i) a short-circuited line (ii) an open-circuited line, and (iii) a resistance equal to characteristic impedance of the line?
7. What are the expressions for the voltage and current when a line is terminated by an (i) inductance (ii) a capacitance?
8. Why is the velocity of propagation same for all overhead lines?
9. What is meant by crest of a wave?
10. What is meant by ‘wave front’?
11. A transmission line of surge impedance $Z_0$is terminated through a resistance R.Give the coefficients of refraction and reflection.
12. What is the effect of shunt capacitance at the terminal of a transmission line?
13. What is the application of Bewley’s diagram?

MULTIPLE CHOICE QUESTIONS

1. For lossless line terminated by its surge impedance, the natural reactive power loading is
   1. V^2/X
   2. V^2/Z
   3. V^2/Z_c
   4. V^2/R
2. A lossless line terminated with its surge impedance has
   1. flat voltage profile
   2. transmission line angle greater than actual length of line
   3. transmission line angle less than actual length of line
   4. a and b
3. An overhead transmission line having a surge impedance of 400 Ω is connected in series with an underground cable having a surge impedance of 100 Ω. If a source of 50 kV travels from the line end towards the line-cable junctions, the value of the transmitted-voltage wave at the junction is
   1. 30 kV

2. 20 kV
3. 80 kV
4. −30 kV

4. When a transmission line is energized through________, propagate on it.
   1. voltage wave
   2. current wave
   3. both voltage and current
   4. power wave
5. The coefficient of reflection for current at an open-ended line is
   1. 1.0
   2. 0.5
   3. −1.0
   4. zero
6. The reflection between travelling voltage and current waves is given as
   1. $vi = \sqrt{\frac{L}{C}}$
   2. $\frac{v}{i} = \sqrt{\frac{L}{C}}$
   3. $vi = \sqrt{LC}$
   4. $\frac{v}{i} = \sqrt{LC}$
7. Travelling voltage and current waves have the same waveforms and travel together along the transmission line at a
   1. velocity of sound
   2. velocity of light
   3. slightly lesser than light
   4. more than the light
8. For an open-circuited line, the resulting current will be
   1. zero
   2. infinity
   3. equal to the incident voltage
   4. twice the incident voltage
9. For a short-circuited line, the resulting voltage will be
   1. infinity
   2. zero
   3. equal to the incident voltage
   4. twice the incident voltage
10. Steepness of the travelling wave is attenuated by
    1. line resistance
    2. line inductance
    3. line capacitance
    4. both b and c

11. The steepness of the wave front can be reduced by connecting
    1. an inductor in series with the line
    2. a capacitor between line and earth
    3. both a and b
    4. an inductor between line and earth or a capacitor in series with the line
12. The reflection coefficient of the voltage wave in overhead lines is given by
    1. $\frac{R_0}{R_0 - R_1}$
    2. $\frac{R_1}{R_0 - R_1}$
    3. $\frac{R_1 - R_o}{R_0 - R_1}$
    4. $\frac{R_1 + R_o}{R_0 - R_1}$
13. The reflection coefficient of a short circuit line is
    1. −1
    2. 1
    3. 0.5
    4. 0
14. The propagation constant of a transmission line is $(0.15 \times 10^{-3} + j1.5 - 10^{-3})$ and the wave length of the travelling wave is
    1. $\frac{1.5 \times 10^{-3}}{2\pi}$
    2. $\frac{2\pi}{1.5 \times 10^{-3}}$
    3. $\frac{1.5 \times 10^{-3}}{2\pi}$
    4. $\frac{\pi}{1.5 \times 10^{-3}}$
15. The reflection coefficient at the load end of a short-circuited line is
    1. 0
    2. $1\angle 0^0$

3. $1\angle 90^0$

4. $1\angle 180^0$

16. The reflection coefficient of the wave when load connected to a transmission line of surge impedance equals the load surge impedance is
    1. 1
    2. – 1
    3. 0
    4. infinity
17. A surge voltage of 1000 kV is applied to an overhead line with its receiving end open. If the surge impedance of the line is 500 Ω, then the total surge power in the line is
    1. 2000 MW
    2. 500 MW
    3. 2 MW
    4. 0.5 MW
18. A surge of 260 kV travelling in a line of neutral impedance of 500 Ω arrives at the junction with two lines of neutral impedances of 250 Ω and 50 Ω, respectively. The voltage transmitted in the branch line is
    1. 400 kV
    2. 260 kV
    3. 80 kV
    4. 40 kV
19. The coefficient of reflection for current for an open-ended line is
    1. 1.0
    2. -1.0
    3. 0.5
    4. 0
20. The real part of the propagation constant of a transmission line is
    1. attenuation constant
    2. phase constant
    3. reliability factor
    4. line constant
21. Two transmission lines each having an impedance of 200 Ω is separated by a cable. For zero reflection the impedance of the cable should be
    1. 100 Ω
    2. 200 Ω
    3. 400 Ω
    4. 600 Ω
22. An overhead line with surge impedance of 400 Ω is terminated through a resistance *R*.A surge travelling over the line will not suffer any reflection at the junction, if the value of *R* is
    1. 100 Ω
    2. 400 Ω
    3. 200 Ω
    4. 600 Ω

Answers:

1. d,	2. a,	3. b,	4. c,	5. c,
6. b,	7. c,	8. a,	9. b,	10. a,
11. c,	12. c,	13. a,	14. b,	15. d,
16. a,	17. a,	18. d,	19. b,	20. a,
21. b,	22. c			

REVIEW QUESTIONS

1. Develop an equivalent circuit at the transition points of transmission lines for analyzing the behaviour of travelling waves.
2. Discuss the phenomena of wave reflection and refraction. Derive an expression for the reflection and refraction coefficients.
3. Describe the construction and the working principle of a zinc oxide gapless arrester with a neat sketch.
4. Starting from the first principles, show that surges behave as travelling waves. Derive expressions for surge impedance and wave velocity.
5. Explain Bewley's Lattice diagram and give its uses.
6. Define surge impedance of a line. Obtain the expressions for voltage and current waves at a junction or at a transition point.

PROBLEMS

1. A voltage having a crest value of 2000 kV is travelling on a 400 kV line. The protective level is 1200 kV. The surge impedance of the line is 200 Ω. Calculate (a) the current in the line before reaching the arrester, (b) the current through the arrester and (c) the value of arrester resistance for this condition (d) the reflected voltage. Verify the reflection and refraction coefficients.
2. A 500 kV surge travels on an overhead line of surge impedance 400 Ω towards its junction with a cable which has a surge impedance of 4 Ω. Find (a) transmitted voltage and current (b) reflected voltage and current.
3. A 200 kV surge travels on a transmission line of400 Ω surge impedance and reaches a junction where two branch lines of surge impedances of 500 Ω and 300 Ω respectively are connected with the transmission line. Find the surge voltage and current transmitted into each branch line. Also, find the reflected voltage and current.
4. A transmission line has an inductance of 0.93 H/km and a capacitance of0.0078 μF/km. This overhead line is connected to an underground cable having an inductance of 0.155 mH/km and a capacitance of 0.187 μF/km. If a surge of crest 100 kV travels in the cable towards its junction with the line, find the surge

transmitted along the line.

5. A 200 kV, 3 μs, rectangular surge travels on a line of surge impedance of400 Ω. The line is terminated in a capacitance of 3000 p.f. Find an expression for voltage across the capacitance.
6. An inductance of 700 μH connects two sections of a transmission line each having a surge impedance of 350 Ω. A 400 kV, 1 μs rectangular surge travels along the line towards the inductance. Find the maximum value of the transmitted wave.

6

Corona

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Define the physical phenomenon of corona
- Understand the effect of corona on transmission lines
- Discuss the effect of corona on radio interference, audible noise and power loss
- Discuss the methods for reducing corona
- Discuss the interference with communication lines

6.1 INTRODUCTION

Corona is a phenomenon whereby, the air around the conductors is ionized. Consider a single-phase two-wire system, whose spacing is large as compared to the diameter of the wires. When a low alternating voltage is applied across two conductors, there is no change in the condition of the air around. If the applied voltage is gradually increased, it will result in a gradual increase of the voltage gradient. The moment it reaches the maximum value i.e., about 30 kV/cm (peak) or 21.1 kV/cm (r.m.s.), the air surrounding the conductor starts conducting with a hissing sound. In dark, a faint violet glow occurs around the conductor and ozone is produced, and this effect is called Corona.

This corona glow around the conductor is uniform throughout the length if the conductors are polished and smooth, and if not, the glow appears only at the rough points. In case of HVDC transmission lines, there is a difference in the appearance of glow on the two conductors. The positive conductor will have a uniform glow, and the negative conductor will have a spotty one.

6.2 THEORY OF CORONA FORMATION (CORONA DISCHARGE)

Due to atmospheric radiation and the presence of ultraviolet rays, the air naturally contains some ionic particles. Along with the neutral atoms, these ionized particles surround the conductor under normal conditions. When a voltage gradient is set up between the two conductors, the free electrons attain greater momentum. Intense electric fields as high as 30 kV/cm in the air, may occur at the surface of the conductors, resulting in the ionization of the neutral molecules as the high momentum free electrons dislodge more electrons from them, thus making the process of ionization cumulative. This aggravation of ions causes the electrical breakdown of the air surrounding the conductor leading to what is known as corona discharge. The presence of even small protrusions on the conductor surface, such as water drops, snowflakes, insects, or the raised edges of nicks in the metal, produce strong local enhancements of the field. The corona activity consequently varies markedly with variations in surface and atmospheric conditions.

The effects of corona are as follows:

1. A faint violet glow is observed around the conductor.
2. A hissing sound and ozone gas is produced.
3. The glow is maximum over the rough and dirty surfaces of the conductor.
4. Corona is accompanied with a power loss.
5. The charging current under corona condition increases because the corona introduces harmonic currents.
6. When the corona is present, the effective capacitance of the conductor increases.
7. It works as a safety valve for surges.

Test Yourself

1. Is the effect of corona neglected in a distribution system? Why?
2. Why is the corona effect less in HVDC transmission lines?

6.3 ELECTRIC STRESS

Consider a single-phase system consisting of two conductors A and B. Each conductor has a radius of *r* and the spacing between conductors A and B is *D*.

Let q_a be the charge on conductor A and q_b be the charge on conductor B. The fields due to both charges are in the same direction, so $q_a = -q_b = q$.

If the point P is in between the conductors A and B and is located at distance *x* from A as shown in Fig. 6.1, then the field intensity (stress) at distance *x* due to both the charges is

$$E_x = \frac{1}{2\pi\varepsilon}\left(\frac{q}{x} + \frac{q}{D-x}\right) \text{ V/m}$$

The potential difference between the conductors is

$$V = \int_r^{D-r} E_x \mathrm{d}x = \frac{q}{2\pi\varepsilon}\int_r^{D-r}\left(\frac{1}{x} + \frac{1}{D-x}\right)\mathrm{d}x \text{ V/m}$$

$$= \frac{q}{2\pi\varepsilon}\left|\ln x - \ln(D-x)\right|_r^{D-r} \text{ V/m}$$

$$= \frac{q}{2\pi\varepsilon}\left[\ln(D-r) - \ln(D-D+r) - \ln r + \ln(D-r)\right] = \frac{q}{2\pi\varepsilon}\left[-2\ln r + 2\ln(D-r)\right]$$

$$= \frac{q}{2\pi\varepsilon}\cdot 2\ln\frac{D-r}{r} = \frac{q}{\pi\varepsilon}\ln\frac{D}{r} \text{ V/m} \qquad (\because D \gg r)$$

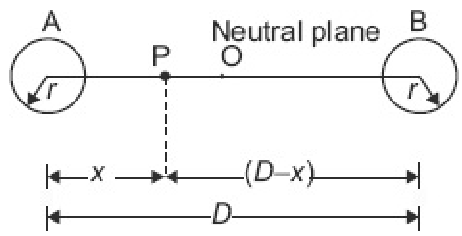

Fig. 6.1 Two-wire transmission line

$$\therefore \text{Charge, } q = \frac{\pi\varepsilon V}{\ln\frac{D}{r}} \tag{6.1}$$

The field intensity or stress is also denoted by *g*

The value of *g* will be minimum at $x = D/2$ and maximum at $x = r$

$$\therefore g_{max} = \frac{q}{2\pi\varepsilon}\left(\frac{1}{r} + \frac{1}{D-r}\right)$$

$$\text{or } g_{max} = \frac{q}{2\pi\varepsilon}\left(\frac{D-r+r}{r(D-r)}\right)$$

$$g_{max} = \frac{q}{2\pi\varepsilon}\left(\frac{D}{r(D-r)}\right)$$

$$g_{max} = \frac{q}{2\pi\varepsilon}\left(\frac{1}{r}\right) \qquad (\because D \gg r)$$

Substituting the value of *q* from Eq. (6.1) in the above equation, we get

$$g_{max} = \frac{V}{2r\ln(D/r)}$$

where, V is the voltage between conductors, so for a single-phase line it is $V/2$(or V_0) and for three-phase line, it is the voltage between phase and neutral.

$$\therefore \text{Potential gradient, } g_{max} = \frac{V_0}{r \ln \frac{D}{r}}$$

$$\therefore V_0 = r g_{max} \ln \frac{D}{r} \text{ volts}$$

6.4 CRITICAL DISRUPTIVE VOLTAGE

The minimum voltage at which the ionization takes place is called the critical disruptive voltage. This voltage corresponds to the gradient at the surface, which is equal to the breakdown strength of air. The dielectric strength (breakdown strength) is normally denoted by g_0 and is equal to 30 kV/cm (peak) or 21.21 kV/cm (r.m.s.) at normal temperature and pressure (NTP).

$$\text{Critical disruptive voltage, } V_{d0} = r g_0 \ln \frac{D}{r} \tag{6.2}$$

This is true for a smooth polished conductor and at the atmospheric pressure of 76 cm of mercury, at 25° C.

From the experimental results obtained by Peek, the critical disruptive voltage for rough and dirty conductors is lower than that of smooth conductors. It is calculated by the formula: $m_0\ \delta V_{do}$, where δ is the air density factor and m_0 is the irregularity factor. Under normal weather conditions, the value of δ being 1, the critical disruptive voltage is equal to $m_0 V_{do}$ and the value of m_0 varies from 0.8 to 1 during unfair weather conditions. The value of δ

is calculated by the formula: $\frac{3.92b}{273+t} = \delta$

where, *b* is the barometric pressure in centimetre of mercury and *t*, the temperature in degree centigrade.

Now, the critical disruptive voltage, $V_{d0} = g_0 m_0 \delta r \ln \frac{D}{r}$ V/phase (r.m.s.) (6.3)

The value of disruptive critical voltage is considerably reduced during bad atmospheric conditions such as fog, sleet, rain, and snowstorms. The same relationship holds true for a three-phase symmetrically spaced systems.

Example 6.1

A three-phase line has conductors each of radius 1.0 cm, spaced at the corners of an equilateral triangle of side 2.5 m each. If the dielectric strength of air is 30 kV/cm, determine the disruptive critical voltage at which corona will occur. Take relative air density factor δ = 0.96 and irregularity factor m_0 = 0.94.

Solution:

Radius of conductors, r = 1.0 cm

Spacing of conductors, D = 2.5 m = 250 cm

Dielectric strength of air, $g_0 = 30/\sqrt{2}$ kV/cm (r.m.s.)

Irregularity factor, m_0 = 0.94

Air density factor, δ = 0.96

Disruptive critical voltage to neutral, $V_{d0} = g_0 \delta \, m_0 r \ln \frac{D}{r}$

$$= \frac{30}{\sqrt{2}} \times 0.96 \times 0.94 \times 1.0 \ln \frac{250}{1.0}$$
$$= 21.21 \times 0.96 \times 0.94 \times 5.52146$$
$$= 105.7 \text{ kV}$$

Disruptive critical voltage (line to line) $= \sqrt{3} \times 105.7$
$$= 183.077 \text{ kV}$$

Example 6.2

Find the disruptive critical voltage for a 3-phase transmission line having the following values:

1. **Conductor spacing = 1.5 m**
2. **Conductor (stranded) radius = 1.2 cm**
3. **Barometric pressure = 76 cm of Hg**
4. **Temperature = 35° C**
5. **Air breakdown potential gradient (at 76 cm of Hg at 25° C) = 21.1 kV (r.m.s.)/cm and it has an irregularity factor 0.85**.

Solution:

Conductor spacing, D = 1.5 m = 150 cm

Conductor radius, r = 1.2 cm

Barometric pressure, b = 76 cm of Hg

Temperature, t = 35° C

Irregularity factor, m_0 = 0.85

Air breakdown potential gradient, g_0 = 21.1 kV (r.m.s.)/cm

Air density factor, $\delta = \frac{3.92b}{273+t}$
$$= \frac{3.92 \times 76}{273+35}$$
$$= 0.9673$$

Disruptive critical voltage to neutral, $V_{d0} = g_0 m_0 \delta \, r \ln \frac{D}{r}$
$$= 21.1 \times 0.85 \times 0.9673 \times 1.2 \times \ln \frac{150}{1.2}$$
$$= 21.1 \times 0.85 \times 0.9673 \times 1.2 \times 4.8283$$
$$= 100.52 \text{ kV}$$

Disruptive critical voltage (line to line) $= \sqrt{3} \times 100.52$
$$= 174.1 \text{ kV (r.m.s.)}$$

Example 6.3

A 132 kV overhead line conductor of radius 1 cm is built so that corona takes place when the line voltage is 210 kV (r.m.s.). If the value of voltage gradient at which ionization occurs can be taken as 21.21 kV (r.m.s.)/cm, determine the spacing between the conductors.

Solution:

Conductor radius, r = 1 cm.

Dielectric strength of air, g_0 = 21.21 kV/cm (r.m.s.)

Disruptive critical voltage to neutral, $V_{d0} = \frac{210}{\sqrt{3}} = 70\sqrt{3}$ kV/phase

Assuming smooth conductors i.e., irregularity factor m_0 = 1, standard pressure and temperature for which air density factor, δ = 1 and spacing of conductors D cm, we have

$$V_{d0} = g_0 m_0 \delta\, r \ln \frac{D}{r}$$

$$70\sqrt{3} = 21.21 \times 1 \times 1 \times 1 \times \ln \frac{D}{1}$$

$$\ln D/r = 5.716$$

$$D/r = 303.69$$

Therefore spacing between conductors, D = 303.69 cm
= 3.03 m.

Example 6.4

In a three-phase overhead line, the conductors have an overall diameter of 3.0 cm each and are arranged in delta formation. Assuming a critical disruptive voltage of 250 kV between the lines, an air density factor of 0.90 and m_0= 0.95, find the minimum spacing between conductors that is allowable. Assume fair weather conditions.

Solution:

Diameter of the conductor, d = 3.0 cm

Radius of conductor, r = 1.5 cm

Irregularity factor, m_o = 0.95

Air density factor, δ = 0.9

Disruptive critical voltage to neutral, V_{do} = 300 kV (L–L)

Spacing of conductors, D = ?

Assume dielectric strength of air, $g_0 = 30/\sqrt{2}$ kV/cm (r.m.s.)

Disruptive critical voltage to neutral, $V_{d0} = m_0 \delta \, g_0 r \ln\left(\frac{D}{r}\right)$

$$\therefore \frac{250}{\sqrt{3}} = 0.95 \times 0.9 \times \frac{30}{\sqrt{2}} \times 1.5 \ln\left(\frac{D}{r}\right)$$

$$\ln\left(\frac{D}{r}\right) = 5.3054$$

$$\frac{D}{r} = 201.42 \text{ cm}$$

$$D = 1.5 \times 201.42 \text{ cm}$$
$$= 302.12 \text{ cm} = 3.02 \text{ m}$$

∴ Spacing between conductor, D = 3.02 m.

Example 6.5

A single-phase overhead line has two conductors of diameter 1.5 cm each with a spacing of 1.2 m between their centres. If the dielectric strength of air is 21 kV/cm, determine the voltage for which corona will commence on the line. Derive the formula used.

Solution:

Diameter of the conductor, d = 1.5 cm

Radius of conductor, r = 0.75 cm

Spacing between conductor, D = 1.2 m = 120 cm

Dielectric strength, g_o = 21 kV/cm

Assume a smooth surface, m_o = 1 and normal pressure and temperature, δ = 1

Voltage for which the corona will commence is, $V_{do} = g_0 \delta m_0 r \ln\left(\frac{D}{r}\right)$

$$= 21 \times 1 \times 1 \times 0.75 \times \ln\left(\frac{120}{0.75}\right)$$

$$= 79.93 \text{ kV}$$

(*Refer to* Section 6.4 *for derivation.*)

6.5 VISUAL CRITICAL VOLTAGE

The minimum voltage at which the corona becomes visible is called the visual critical voltage.

In case of parallel conductors, it has been observed that corona occurs at a voltage higher than the disruptive critical voltage (V_{do}), which may be referred in this text as the visual critical voltage and be denoted by V_v.

Thus, the visual critical voltage, $V_v = g_0 \delta m_v r \left(1 + 0.3/\sqrt{\delta r}\right) \ln\frac{D}{r}$ kV to neutral (r.m.s.) (6.4)

The value of roughness factor (m_v) is different from the irregularity factor (m_o). It may be taken as unity for smooth conductors, 0.93–0.98 for rough conductors exposed to atmospheric severities and 0.72 for stranded conductors.

Since the surface of the conductor is irregular, the coronal losses do not start simultaneously on the whole surface and it is called as local corona. For this m_v = 0.72 and for general corona, m_v = 0.82.

Example 6.6

Determine the disruptive critical voltage and the visual critical voltage for the local and the general corona on a three-phase overhead transmission line consisting of three stranded copper conductors spaced 2.5 m apart at the corners of an equilateral triangle. Air temperature and pressure are 21° C and 73.5 cm of Hg, respectively. Conductor

diameter is 1.8 cm, irregularity factor (m_0) 0.85 and surface factors (m_v) 0.7 for local and general corona 0.7 and 0.8, respectively. Breakdown strength of air is 21.1 kV (r.m.s.)/cm.

Solution:

Conductor spacing, D = 2.5 m = 250 cm

Diameter of conductor, d = 1.8 cm

Radius of conductor, r = 0.0.9 cm

Air temperature, t = 21° C

Pressure, b = 73.5 cm

Irregularity factor, m_0 = 0.85

Surface factor for local, m_v = 0.7

Surface factor for general, m_v = 0.8

Breakdown strength of air, g_0 = 21.1 kV

$$\text{Air density factor, } \delta = \frac{3.92b}{273+t} = \frac{3.92\times 73.5}{273+21} = 0.98$$

$$\text{Disruptive critical voltage to neutral, } V_{d0} = g_0 m_0 \delta r \ln\frac{D}{r}$$

$$= 21.1\times 0.85\times 0.98\times 0.9\times \ln\frac{250}{0.9}$$

$$= 89 \text{ kV/phase}$$

$$V_{d0} = \sqrt{3}\times 89 = 154.15 \text{ kV (L–L)}$$

For local corona,

$$\text{Visual critical voltage, } V_v = 21.1 m_v \delta r \left(1+\frac{0.3}{\sqrt{\delta r}}\right)\ln\frac{D}{r} \text{ kV}$$

$$= 21.1\times 0.7\times 0.98\times 0.9\left(1+\frac{0.3}{\sqrt{0.98\times 0.9}}\right)\ln\frac{250}{0.9}$$

$$= 21.1\times 0.7\times 0.98\times 0.9\times 1.3194\times 5.6268$$

$$= 96.71 \text{kV}$$

$$= \sqrt{3}\times 96.71 = 1167.5 \text{ kV (L–L)}$$

$$\text{For general corona, } V_v = 21.1\times 0.8\times 0.98\times 0.52\left(1+\frac{0.3}{\sqrt{0.98\times 0.52}}\right)\ln\left[\frac{244}{0.52}\right]$$

$$= 75.15 \text{ kV/phase}$$

$$V_v = \sqrt{3}\times 75.15 = 130.16 \text{ kV (L–L).}$$

6.6 POWER LOSS DUE TO CORONA

There is a power loss due to the formation of corona that affects the transmission efficiency of the lines. Its effect on regulation, however, is less. This power loss is affected by the atmospheric and line conditions too. Power loss due to corona under fair weather condition is given by

$$P = \frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{ph} - V_{d0}\right)^2 \times 10^{-5} \text{ kW/km/phase (Peek's formula)} \tag{6.5}$$

where, V_{ph} = voltage to neutral in kV
V_{do} = disruptive critical voltage to neutral in kV
f = supply frequency in Hz

Under abnormal weather conditions, V_{do} is 0.8 times its value, under fair weather conditions and power loss due to corona is given by

$$P = \frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{ph} - 0.8V_{d0}\right)^2 \times 10^{-5} \text{ kW/km/phase} \tag{6.6}$$

When power loss due to corona is low and the ratio V_{ph}/V_{do} is less than 1.8, Peterson's formula is used to determine corona losses and is given as

$$P = \frac{21\times 10^{-6} f V_{ph}^2}{\left(\log_{10}\frac{D}{r}\right)^2} \times K \text{ kW/km/phase} \tag{6.7}$$

where, K is a factor, which varies with the ratio of V_{ph}/V_{do}.

Example 6.7

A three-phase, 220 kV, 50 Hz transmission line has equilateral triangular spacing of side 2 m. The conductor diameter is 3.0 cm. The air density factor and the irregularity factor is 0.95 and 0.83, respectively. Find the disruptive critical voltage and corona loss per kilometre. Assume any data required.

Solution:

Line voltage = 220 kV(L–L)

Frequency, f = 50 Hz

Spacing between conductors, D = 2 m (equilateral spacing)

Radius of conductor, $r = d/2 = 3.0/2 = 1.5$ cm

Irregularity factor, $m_0 = 0.83$

Air density factor, $\delta = 0.95$

Assume dielectric strength of air, $g_0 = 21.2$ kV/cm

$$\text{Disruptive critical voltage } V_d = m_0 g_0 \delta r \ln\left(\frac{D}{r}\right) \text{kV}$$

$$= 21.2 \times 0.83 \times 0.95 \times 1.5 \ln\left(\frac{200}{1.5}\right)$$

$$= 122.685 \text{ kV}$$

$$\text{Corona loss, } P_c = 244\left(\frac{f+25}{\delta}\right)\sqrt{\frac{r}{d}}(V - V_{d0})^2 \times 10^{-5} \text{ kW/km/phase}$$

$$= 244\left(\frac{50+25}{0.95}\right)\sqrt{\frac{1.5}{200}}(127 - 122.685)^2 \times 10^{-5}$$

$$= 0.3106 \text{ kW/km/phase}$$

$$\text{Total corona loss } = 3 \times 0.3106$$

$$= 0.9318 \text{ kW/km}$$

Example 6.8

An overhead transmission line operates at 220 kV between phases at 50 Hz. The conductors are arranged in a 5 m delta formation. What is the maximum diameter of the conductor

that can be used for no corona loss under fair weather conditions? Assume an air density factor of 0.95 and an irregularity factor of 0.85. The critical voltage is 230 kV. Also, find the power loss under stormy weather conditions.

Solution:

Given,

The line voltage, V = 220 kV

Disruptive critical voltage, V_{do} = 230 kV (L–L)

Air density factor, δ = 0.95

Irregularity factor, m_o = 0.85

Frequency, f = 50 Hz

Conductor spacing, D = 5 m = 500 cm

The dielectric strength of air, $g_0 = 30/\sqrt{2}$ kV/cm (max)

For no corona loss under fair weather conditions, the normal working voltage *(V)* should not exceed critical disruptive voltage, V_{do}.

$$\text{Disruptive critical voltage, } V_{do} = g_0 \delta m_0 r \log_e \frac{D}{r}$$

$$\frac{230}{\sqrt{3}} = \frac{30}{\sqrt{2}} \times 0.95 \times 0.85 \times r \ln \frac{500}{r}$$

$$r \ln \frac{500}{r} = 7.752$$

$$r\left[(\ln 500) - (\ln r)\right] = 7.752$$

By trial and error method, r = 1.303 cm.

Therefore, diameter of the conductor, d = 2.606 cm.

Under stormy weather conditions the disruptive critical voltage $= 0.8 V_{do}$

$$= 0.8 \times \frac{220}{\sqrt{3}} = 106.23 \text{ kV/phase.}$$

Operating voltage per phase, $V = \frac{220}{\sqrt{3}} = 127.017$ kV

The power loss under stormy weather conditions, $P = \frac{244}{\delta}(f + 25)\sqrt{\frac{r}{D}}\left(V_{ph} - 0.8 V_{do}\right)^2 \times 10^{-5}$ kW/km/phase

$$P = \frac{244}{0.95}(50 + 25)\sqrt{\frac{1.303}{500}}(127.02 - 106.23)^2 \times 10^{-5} = 4.25 \text{ kW/km/phase}$$

Total power loss $= 3 \times 4.25 = 12.75$ kW/km

Example 6.9

A 110 kV, 50 Hz, 175 km long three-phase transmission line consists of three 1.2 cm diameter stranded copper conductors spaced in a 2 m delta arrangement. Assume that temperature is 25° C and barometric pressure, 74 cm. Assume surface irregularity factor m = 0.85 (roughness factor), m_v for local corona = 0.72 and m_v for general corona = 0.82. Find

1. **Disruptive critical voltage**
2. **Visual corona voltage for local corona**
3. **Visual corona voltage for general corona, and**
4. **Power loss due to corona using Peek's formula under fair weather and wet weather conditions.**

Solution:

Conductor diameter, d = 1.2 cm

Conductor radius, r = 0.6 cm

Spacing of conductors, D = 2 m = 200 cm

Dielectric strength of air, $g_0 = \frac{30}{\sqrt{2}}$ kV/cm (r.m.s. assumed)

Barometric pressure, b = 74 cm

Temperature, t = 25° C

Irregularity factor, m_0 = 0.85

Roughness factor, m_v = 0.72 for local corona

Roughness factor, m'_v = 0.82 for general corona

$$\text{Operating voltage, } V_{ph} = \frac{110}{\sqrt{3}} = 63.51 \text{ kV}$$

$$\text{Air density factor, } \delta = \frac{3.92b}{273+t} = \frac{3.92 \times 74}{273+25} = 0.973$$

1.

$$\text{Critical disruptive voltage to neutral, } V_{d0} = g_0 m_0 \delta r \ln \frac{D}{r}$$

$$= \frac{30}{\sqrt{2}} \times 0.973 \times 0.85 \times 0.6 \ln \frac{200}{0.6}$$

$$= 61.15 \text{ kV(r.m.s.)}$$

2.

$$\text{Visual critical voltage to neutral for local corona, } V_v = g_0 m_v \delta r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \ln \frac{D}{r}$$

$$= \frac{30}{\sqrt{2}} \times 0.72 \times 0.973 \times 0.6 \times \left(1 + \frac{0.3}{\sqrt{0.973 \times 0.6}}\right) \ln \frac{200}{0.6}$$

$$= 72.13 \text{ kV(r.m.s.)}$$

3.

$$\text{Visual critical voltage to neutral for general corona, } V'_v = 72.13 \times \frac{0.82}{0.72}$$

$$= 82.14 \text{ kV(r.m.s.)}$$

4. Power loss due to corona for fair weather conditions:
According to the Peek's formula,

$$\text{Power loss, } P_C = \frac{244}{\delta}(f + 25)\sqrt{\frac{r}{D}}\left(V_{ph} - V_{d0}\right)^2 \times 10^{-5} \text{ kW/km/phase}$$

$$= \frac{244}{0.973}(50 + 25)\sqrt{\frac{0.6}{200}}(63.51 - 61.14)^2 \times 10^{-5}$$

$$= 57.86 \text{ W/km/phase}$$

$$\text{Total corona loss for 175 km long, three-phase line} = 175 \times 3 \times 57.86 \text{ W}$$

$$= 30.38 \text{ kW}$$

Power loss due to corona for stormy weather conditions:

$$\text{Disruptive critical voltage, } V'_{d0} = 0.8\ V_{d0}$$

$$= 0.8 \times 61.14$$

$$= 48.912 \text{ kV}$$

Total corona loss for 175 km, three-phase line according to the Peek's formula,

$$
\begin{aligned}
P_c &= 175\times 3\times \frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{ph}-V'_{d0}\right)^2\times 10^{-5}\ \text{kW}\\
&= 175\times 3\times \frac{244}{0.973}(50+25)\sqrt{\frac{0.6}{200}}(63.51-48.912)^2\times 10^{-5}\\
&= 115.25\ \text{kW.}
\end{aligned}
$$

Example 6.10

A three-phase equilaterally spaced transmission line has a total corona loss of 55 kW at 110 kV and a loss of 110 kW at 120 kV. What is the disruptive critical voltage between lines? What is the corona loss at 125 kV?

Solution:

Total corona loss of 55 kW at 110 kV

Total corona loss of 110 kW at 120 kV

Power loss due to corona for three-phase,

$$
\begin{aligned}
&P = 3\times \frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{ph}-V_{d0}\right)^2\times 10^{-3}\ \text{kW/km}\\
&P \propto \left(V_{ph}-V_{d0}\right)^2
\end{aligned}
$$

Taking air density factor δ, supply frequency f radius of conductor r and spacing of conductors D as a constant.

$$
\begin{aligned}
&55 \propto \left(\frac{110}{\sqrt{3}}-V_{d0}\right)^2 \propto \left(63.51-V_{d0}\right)^2\\
&\text{and } 110 \propto \left(\frac{120}{\sqrt{3}}-V_{d0}\right)^2 \propto \left(69.28-V_{d0}\right)^2\\
&\frac{110}{55} = \frac{\left(69.28-V_{d0}\right)^2}{\left(63.51-V_{d0}\right)^2}\\
&\frac{\left(69.28-V_{d0}\right)^2}{\left(63.51-V_{d0}\right)^2} = 2\\
&\frac{\left(69.28-V_{d0}\right)}{\left(63.51-V_{d0}\right)} = 1.414\\
&69.28-V_{d0} = 89.803-1.414V_{d0}\\
&0.414V_{d0} = 20.523\\
&V_{d0} = 49.57\ \text{kV}
\end{aligned}
$$

Disruptive critical voltage between lines = $\sqrt{3} \times 49.57 = 85.86$ kV Total corona loss at 125 kV

$$P \propto \left(\frac{125}{\sqrt{3}} - V_{d0}\right)^2 \propto (72.169 - V_{d0})^2$$

$$\frac{P}{55} = \frac{(72.169 - V_{d0})^2}{(63.51 - V_{d0})^2} = \frac{(72.169 - 49.57)^2}{(63.51 - 49.57)^2} = 2.628$$

Power loss due to corona, $P = 2.628 \times 55 = 144.54$ kW

6.7 FACTORS AFFECTING CORONA LOSS

The following are the factors affecting corona loss in overhead transmission lines:

1. Electrical factors
2. Atmospheric factors
3. Factors connected with the conductors

6.7.1 Electrical Factors

Effect of Frequency From Eq. (6.5) to Eq. (6.7), the corona loss is observed to be a function of frequency. Losses, therefore, increase with increased frequency. Since there is no frequency in DC transmission, corona loss is less compared to AC transmission.

Line Voltage Transmission line operating voltage largely affects corona loss. Corona losses increase with increased operating voltage when it exceeds the disruptive critical voltage.

Effect of Load Current Temperature of the conductor increases with increased load current. Therefore, it prevents deposition of snow on the surface of the conductor. This reduces the corona loss. The heating of the conductor has no effect on corona loss during rain.

6.7.2 Atmospheric Factors

Pressure and Temperature Affect Corona loss is a function of the air density factor δ and is inversely proportional to δ. The corona loss also depends upon the critical disruptive voltage.

i.e., loss α $(V - V_{do})^2$.

From the above equation, we see that the critical disruptive voltage (V_{do}) depends upon the air density factor (δ). Therefore, the corona loss increases with decreased value of δ. For lower values of δ, the pressure should be low and the temperature, high. Therefore, the corona loss is more on hilly areas than on plain areas.

Dust, Rain, Snow, and Hail Affect The critical disruptive voltage decreases due to dust particles deposited on the conductor and bad atmospheric conditions such as rain, snow, and hailstorm. This causes increased corona loss.

6.7.3 Factors Related to the Conductors

Conductor Size Consider the expressions given below

$$\text{Power loss} \propto \sqrt{\frac{r}{D}} \tag{6.8}$$

$$\text{Power loss} \propto \left(V_{\text{ph}} - V_{\text{d0}}\right)^2 \tag{6.9}$$

From Eq. (6.8), we can conclude that power losses are directly proportional to the square root of the radius of the conductor. In addition, from Eq. (6.9), we can conclude that the losses will be reduced due to increased diameter, because as is shown in Eq. (6.3), V_{do} is proportional to the radius of the conductor. The change of losses due to Eq. (6.9) is high when compared to Eq. (6.8). Therefore, the corona losses decreases with an increase in the diameter of the conductor.

Spacing Between Conductors From the loss formula, it is seen that the loss is inversely proportional to the square root of the distance between the conductors. Therefore, higher the spacing of conductors, lower will be the losses. There may not be any corona effect, if the spacing is made very large compared to their diameters.

6.8 METHODS FOR REDUCING CORONA LOSS

From the Eqs. (6.3) and (6.4), it is seen that disruptive critical and the visual critical voltages are dependent on the size of the conductors and the spacing between the conductors in a transmission line. The effect of corona can be reduced by either increasing the size of the conductor or the space between the conductors.

The corona loss can be reduced by using:

- **Conductors with large diameters:** The voltage at which the corona occurs can be increased by increasing the size of the conductor and hence, the corona loss can be reduced.
- **Hollow conductors:** These are used to increase the effective diameter of the conductor without using any additional material. Since, corona loss is inversely proportional to the diameter of the conductor, corona loss decreases with an increase in the diameter.
- **Bundled conductors:** These are made up of two or more sub-conductors and is used as a single-phase conductor. When using two or more sub-conductors as one conductor, the effective diameter of the conductor increases, resulting in reduced corona loss.

6.9 ADVANTAGES AND DISADVANTAGES OF CORONA

Advantages

- It acts as a safety valve by reducing the magnitude of high-voltage steep-fronted waves that may be caused by lightning or power switching.
- With the formation of corona, the air surrounding the conductor becomes conductive and there is a virtual increase in the effective diameter of the conductor. Due to increased diameter, the maximum voltage gradient between the conductors is reduced.

Disadvantages

- Transmission efficiency is affected due to corona loss. Even under fair weather conditions some loss is encountered.
- Inductive interference to neighboring communication lines due to the

non-sinusoidal voltage drop that occurs in the line.

- With the appearance of the corona glow, the charging current increases because the corona introduces harmonics.
- Due to the formation of the corona, ozone gas is generated which chemically react with the conductor and causes corrosion.

Test Yourself

1. How can you reduce the corona in transmission lines by using bundled conductors?
2. Why is corona effect less in regulation?
3. What is the advantage of corona? Justify your answer.

6.10 EFFECT OF CORONA ON LINE DESIGN

In order to improve transmission-line efficiency, it is desirable to avoid corona loss on lines under fair weather conditions. Bad weather conditions such as rain and sleet greatly increase the corona loss and lower the disruptive critical voltage of the line. Because of the latter effect, it is not feasible to design high voltage lines, which will be corona free at all times. If the lines are designed so that they do not encounter any corona formation, then even during bad weather conditions, the height of the towers, the spacing and size of the conductors will be more and hence, uneconomical. On the other hand, bad weather conditions in any particular region occur only for a very short duration in the year and hence, the average corona loss throughout the year will be less.

6.11 RADIO INTERFERENCE

The corona discharges emit radiations, which may introduce a noise or signal called radio interference (RI). When RI propagates along the line, it causes interruptions to radio broadcasts in the range of 0.5 MHz. It also interferes with telephone communication. If the field strength of RI at the antennas used for receiving broadcast sound and television services is too high, it can cause degradation of the output, and picture in the case of television.

The RI originates from the same source as the corona and has an important bearing in the design of EHV lines. Conductors should be selected so that they give the least RI at the lowest possible cost. Since RI is associated with corona, it mainly depends on the potential gradients at the conductors. It is also influenced by all those factors, which influence corona, such as air density, humidity, wind contaminants, imperfections, and precipitations. The RI in bad weather is about 10–25 dB higher than that during fair weather. RI levels are the highest at lower frequencies and decreases with increased frequency, being the least or neglected at higher frequencies.

The radio noise (RN) is measured adjacent to a transmission line by an antenna equipped with a radio noise metre. The standard noise metre operates at 1 MHz (in the standard AM broadcast band) with a bandwidth of 5 kHz. For measurements in the RI range, a rod antenna usually determines the electric field, E and a loop antenna usually determines the magnetic field component H.

6.12 AUDIO NOISE

When corona is present on the conductors, the EHV lines generate an audible noise, which is especially high during polluted weather. The noise is broadband, which extends from a very low frequency to about 20 kHz. Corona discharges generate positive and negative ions, which are alternately attracted and repelled by the periodic reversal of polarity of the AC excitations. Their movement gives rise to sound–pressure waves at frequencies twice the power frequency and its multiples, in addition to the broadband spectrum, which is the result of random motions of the ions. An audible noise can become a serious problem from the point of view of “psychoacoustics”, leading to insanity due to loss of sleep at night to inhabitants residing close to an EHV line. AN meter range from 20 dB to 140 dB is shown in Fig. 6.2.

Fig. 6.2 AN meter range from 20 dB to 140 dB.

6.13 INTERFERENCE WITH COMMUNICATION LINES

When a communication line such as a telephone line runs parallel along a high voltage overhead line, high voltages are induced in the communication line resulting in the production of acoustic shock and noise. The induced voltages are due to electrostatic and electromagnetic induction and these are reduced considerably by transposing power lines (these are discussed in detail in Section 2.11.3).

6.13.1 Electromagnetic Effect

Consider a three-phase overhead transmission system consisting of three conductors R, Y and B spaced at the corners of a triangle and two telephone conductors P and

Q below the power line conductors running on the same supports as shown in Fig. 6.3(a).

Let us assume that the radius of each conductor is r and consider the loop formed by the conductors R and P. Now the distances between R and P, and R and Q are D_{RP} and D_{RQ}, respectively.

Flux linkages of conductor P due to currents in all conductors of power line from Eq. 2.26, are given by;

$$\psi_P = 2\times10^{-7}\left(I_R \ln\frac{1}{D_{RP}} + I_Y \ln\frac{1}{D_{YP}} + I_B \ln\frac{1}{D_{BP}}\right)$$

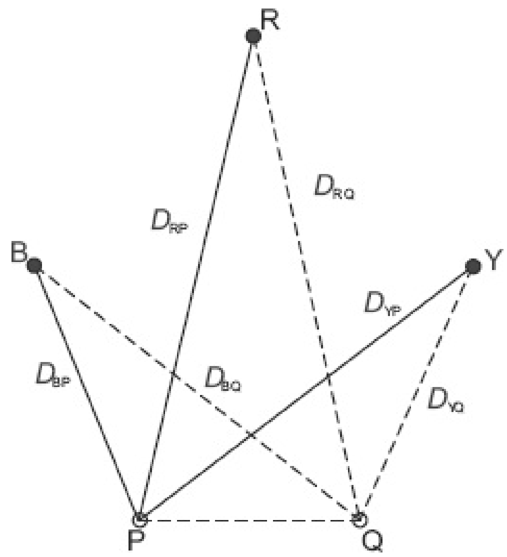

Fig. 6.3(a) Three-phase power line and communication line

Flux linkages of conductor Q due to currents in all conductors of power line,

$$\psi_Q = 2\times10^{-7}\left(I_R \ln\frac{1}{D_{RQ}} + I_Y \ln\frac{1}{D_{YQ}} + I_B \ln\frac{1}{D_{BQ}}\right)$$

Total flux linkages of communication lines,

$$\psi = \psi_p - \psi_Q = 2\times10^{-7}\left[\left(I_R \ln\frac{1}{D_{RP}} + I_Y \ln\frac{1}{D_{YP}} + I_B \ln\frac{1}{D_{BP}}\right) - \left(I_R \ln\frac{1}{D_{RQ}} + I_Y \ln\frac{1}{D_{YQ}} + I_B \ln\frac{1}{D_{BQ}}\right)\right]$$

$$= 2\times10^{-7}\left(I_R \ln\frac{D_{RQ}}{D_{RP}} + I_Y \ln\frac{D_{YQ}}{D_{YP}} + I_B \ln\frac{D_{BQ}}{D_{BP}}\right)$$

Therefore, the induced voltage in the communication line, $V = 2\pi f\psi$ V/m

Here, the voltage induced and flux linkages of communication line depends upon the values of I_R, I_Y, and I_B

Case-I: If the currents I_R, I_Y, and I_B are balanced and power lines are transposed, then flux linkages with the communication lines are zero. Therefore, voltage induced is also zero.

Case-II: If the currents I_R, I_Y, and I_B are balanced and power lines are untransposed, then flux linkages with the communication lines are small. Therefore, voltage induced is also small.

Case-III: If the currents I_R, I_Y, and I_B are unbalanced, then there is flux linkage with the communication lines and therefore, voltage is induced.

The induced voltage can be reduced by increasing the distance between the power lines and the communication lines or even by transposing them. EMI test receiver from 9 kHz to 30 MHz is shown in Fig. 6.3(b).

Fig. 6.3(b) EMI test receiver from 9 kHz to 30 MHz

6.13.2 Electrostatic Effect

Consider a three-phase system consisting of three conductors R, Y, and B, which are placed at the corners of a triangle and two telecommunication lines P, Q connected parallel to the three-phase system as shown in Fig. 6.4.

The potential distribution between the three-phase system and plane (earth) is the same as the potential distribution between the image of the three-phase system and the plane.

Consider a conductor R of a three-phase system.

Let H_R be the height of conductor R from the ground.

q is the charge per metre length of conductor R and –q is the charge on image of conductor R′.

The electric-field intensity at a distance x from the centre of conductor R,

$$E_x = \frac{q}{2\pi\varepsilon x} + \frac{q}{2\pi\varepsilon\,(2H_R - x)}$$

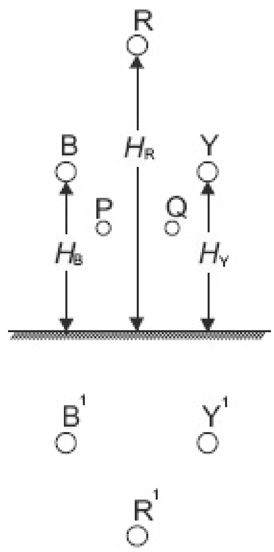

Fig. 6.4 Three-phase power line, communication line and image of power line

The potential of conductor R with respect to earth,

$$V_R = \int_r^{H_R} E_x \mathrm{d}x = \frac{q}{2\pi\varepsilon}\int_r^{H_R}\left(\frac{1}{x} + \frac{1}{(2H_R - x)}\right)\mathrm{d}x$$

$$V_R = \frac{q}{2\pi\varepsilon}\left(-\ln\frac{1}{r} + \ln\frac{1}{2H_R - r}\right) = \frac{q}{2\pi\varepsilon}\ln\frac{2H_R - r}{r}$$

$$= \frac{q}{2\pi\varepsilon}\ln\frac{2H_R}{r} \quad (\because 2H_R >> r)$$

where, r is the radius of conductor R

Suppose P is placed at a distance D_{PR} from the conductor R, then the potential of conductor P with respect to earth, $V_{PR} = \int_{D_{PR}}^{H_R} E_x \mathrm{d}x = \frac{q}{2\pi\varepsilon} \int_{D_{PR}}^{H_R} \left(\frac{1}{x} + \frac{1}{(2H_R - x)} \right) \mathrm{d}x$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{2H_R - D_{PR}}{D_{PR}}$$

$$= V_R \frac{\ln\left(\frac{2H_R - D_{PR}}{D_{PR}} \right)}{\ln \frac{2H_R}{r}}$$

Similarly, the potential at P due to the charge on conductors Y and B i.e., V_{PY} and V_{PB} can be calculated. In addition, the resultant potential of P with respect to earth due to charge on conductors R, Y, and B is $V_P = V_{PR} + V_{PY} + V_{PB}$ (vector addition).

In a similar way, the resultant potential of conductor Q with respect to the earth due to charge on conductors R, Y, and B can be calculated.

Example 6.11

A single-phase, 50 Hz power transmission line spaced 2.5 m apart is supported on a horizontal cross arm. A telephone line is run on the same support and spaced 50 cm apart at the ends of the cross arm which is placed 2 m below the power line. Determine the mutual inductance between the circuits and the voltage per kilometre induced in the telephone line if the radius of the power conductor is 0.8 cm and the current in the power line is 100 A.

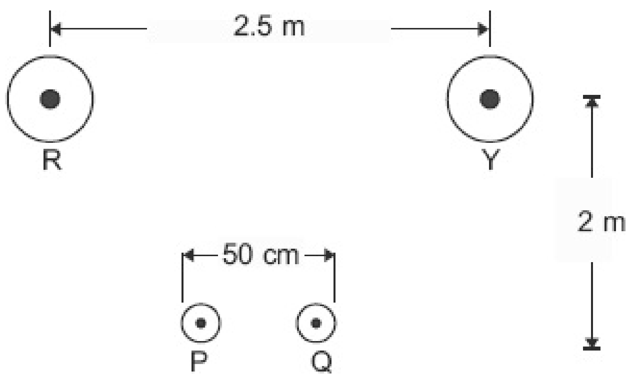

Fig. 6.5 Single-phase power line and communication line

Solution:

$$D_{RP} = \sqrt{2^2 + 1^2} = 2.236\,\text{m} = D_{YQ}$$
$$D_{RQ} = \sqrt{2^2 + 1.5^2} = 2.5\,\text{m} = D_{YP}$$

Flux linkages of communication-line conductor P due to current I_R and I_Y of power lines

$$\psi_P = 2\times10^{-7}\left(I_R \ln\frac{1}{D_{RP}} + I_Y \ln\frac{1}{D_{YP}}\right) = 2\times10^{-7} I \ln\frac{D_{YP}}{D_{RP}} \quad (\because I_R = -I_Y = I)$$

Similarly,

$$\psi_Q = 2\times10^{-7} I \ln\frac{D_{YQ}}{D_{RQ}}$$

Total flux linkages of communication line,

$$\psi = \psi_P - \psi_Q = 2\times10^{-7} I\left(\ln\frac{D_{YP}}{D_{RP}} - \ln\frac{D_{YQ}}{D_{RQ}}\right) = 2\times10^{-7} I \ln\left(\frac{D_{YP}D_{RQ}}{D_{RP}D_{YQ}}\right) \text{Wb/m}$$

Total flux linkages of communication line, $\psi = 2\times10^{-7} I \ln\left(\frac{2.5\times2.5}{2.236\times2.236}\right)$

$$\psi = 0.446\times10^{-7} I \text{ Wb/m}$$

Voltage induced, $V = 2\pi f\psi = 2\pi\times50\times0.446\times10^{-7}\times100 = 1.4\,\text{V/km}.$

Example 6.12

A three-phase, 132 kV, 50 Hz, 160 km long transmission line delivers a total load of 30 MW at 0.8 p.f. lag. Conductors of diameter 20 mm each are placed at the corners of an

equilateral triangle of sides 5 m. If the telephone line is run on the same support as shown in Fig. 6.6, calculate the voltage induced in the telephone circuit due to electromagnetic effect. Also determine the potential of telephone conductor P above earth due to electrostatic effect.

Solution:

From Fig. 6.6

$$D_{RO} = \sqrt{4^2 - 2^2} = 3.464 \text{ m}$$
$$D_{RP} = 6.5 + 3.464 = 9.694 \text{ m}$$
$$D_{RQ} = 10.464 \text{ m}$$
$$D_{YP} = \sqrt{2^2 + 6.5^2} = 6.8 \text{m} = D_{BP}$$
$$D_{YQ} = \sqrt{2^2 + 7^2} = 7.28 \text{m} = D_{BQ}$$

$$\text{Line current, } I = \frac{30\times10^6}{\sqrt{3}\times132\times10^3\times0.8} = 164.02 \text{ A}$$

Total flux linkages of communication lines,

$$= 2\times10^{-7}\left(I_R \ln\frac{D_{RQ}}{D_{RP}} + I_Y \ln\frac{D_{YQ}}{D_{YP}} + I_B \ln\frac{D_{BQ}}{D_{BP}}\right)$$
$$= 2\times10^{-7}\left(164.02\angle0°\left(\ln\frac{10.464}{9.964}\right) + 164.02\angle-120°\left(\ln\frac{7.28}{6.8}\right) + 164.02\angle-240°\left(\ln\frac{7.28}{6.8}\right)\right)$$
$$= 2\times10^{-7}\left(8.03\angle0° + 11.19\angle-120° + 11.19\angle-240°\right)$$
$$= 2\times10^{-7}\left(8.03 + j0 - 5.595 - j9.69 - 5.595 + j9.69\right)$$
$$= -6.32\times10^{-7} \text{ Wb/m}$$

Therefore, the induced voltage in the communication line,

$$V = 2\pi f\psi \text{ V/m}$$
$$= 2\pi\times50\times6.32\times10^{-7} = 0.1985 \text{ V/km}$$
$$= 0.1985\times160 = 31.768 \text{ V}$$

The potential of conductor P with respect to earth is,

$$V_{RP} = V_R \frac{\ln\left(\dfrac{2H_R - D_{PR}}{D_{PR}}\right)}{\ln\dfrac{2H_R}{r}}$$

$$= V_R \frac{\ln\left(\dfrac{36.928 - 9.964}{9.964}\right)}{\ln\dfrac{36.928}{0.01}} = 0.121$$

Similarly, $$V_{YP} = V_Y \frac{\ln\left(\dfrac{2H_R - D_{PY}}{D_{PY}}\right)}{\ln\dfrac{2H_R}{r}} = V_Y \frac{\left(\ln\dfrac{30-6.8}{6.8}\right)}{\ln\dfrac{30}{0.01}} = 0.1533\ V_Y$$

And $$V_{BP} = V_B \frac{\ln\left(\dfrac{2H_B - D_{PB}}{D_{PB}}\right)}{\ln\dfrac{2H_B}{r}} = V_B \frac{\ln\left(\dfrac{30-6.8}{6.8}\right)}{\ln\dfrac{30}{0.01}} = 0.1533\ V_B$$

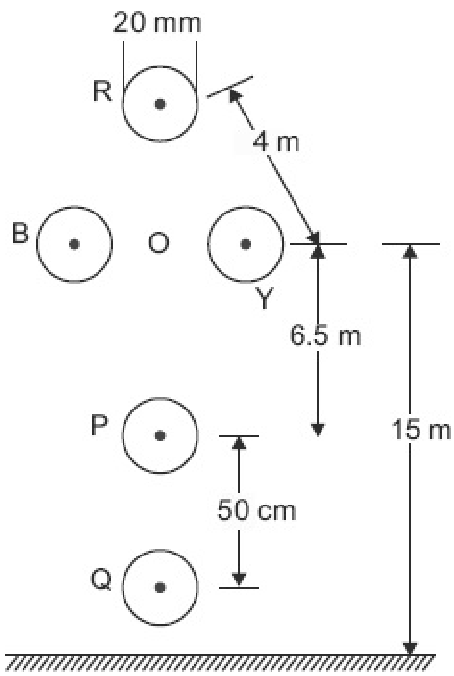

Fig. 6.6 Three-phase power line and communication line

The phase voltages,

$$V_R = \frac{132 \times 10^3}{\sqrt{3}} \angle 0 = 76210\angle 0°,\ V_Y = 76210\angle -120°,\ V_B = 76210\angle 120°$$

Therefore, the resultant potential of P with respect to earth due to charge on conductors R, Y and B is V_P

$$\begin{aligned}\therefore V_P &= V_{PR} + V_{PY} + V_{PB} \\ &= 0.121V_R + 0.1533V_Y + 0.1533V_B \\ &= 76210(0.121\angle 0° + 0.1533\angle -120° + 0.1533\angle 120°) \\ &= 76210(0.121 - 0.1533) = 2461.6\text{V}.\end{aligned}$$

6.14 CORONA PHENOMENA IN HVDC LINES

Corona on AC and DC overhead transmission lines is due to ionization of the air surrounding the conductor. But the direction of the movement of charged particles around the conductor in AC and DC is different.

In case of DC, the corona phenomenon is related with field distribution rather than surface stress. Therefore, the importance for the consideration of the corona phenomenon while designing HVDC lines is less serious than while designing AC lines. In addition, corona loss is less due to the absence of factor $\sqrt{2}$ in HVDC voltage waveform.

Corona losses on DC lines is an important factor and should be considered while calculating the line losses for designing the HVDC lines. Losses in HVDC monopolar lines are less when compared to those in HVDC bipolar lines.

Critical disruptive HVDC voltage in monopolar line is given by

$$V_{do} = V_{cr}.m.r.\log\frac{2h}{r}$$

where,

	V_{cr} = critical stress kV/cm = 14 kV/cm for DC voltages m = surface irregularity factor 0.8–0.89 r = radius of conductor (cm) h = height of conductor above earth (cm) V_{do} = critical disruptive voltage of pole to earth (kV DC)

Whereas, in bipolar line is

$$V_{do} = 2.V_{cr}.m.r \frac{1}{\sqrt{1+\left(\frac{D}{2h}\right)^2}}.\ln\frac{D}{r} \text{ V}$$

where,

D = distance between pole to pole.

CHAPTER AT A GLANCE

1. The phenomenon whereby the conductors on the transmission line are ionized, a violet glow is formed, a hissing noise is emitted and ozone gas is produced, is known as corona.
2. The minimum voltage at which the ionization takes place is called the critical disruptive voltage. The maximum potential gradient occurring at the conductor surface is given by

$$g_{max} = \frac{V}{r\ln\frac{D}{r}}$$

3. The minimum voltage at which the corona becomes visible is called the visual critical voltage and is given by

$$V_v = g_0 \delta m_v r \left(1 + 0.3/\sqrt{\delta r}\right) \ln \frac{D}{r} \text{ kV to neutral (r.m.s.)}$$

4. There is a power loss due to the formation of corona. When corona occurs, power is dissipated due to the generation of heat, glow and sound. These results affect the transmission efficiency, but its effect on regulation is less.

   Power loss due to corona is given by

$$P = \frac{244}{\delta}(f + 25)\sqrt{\frac{r}{D}}\left(V_{ph} - V_{d0}\right)^2 \times 10^{-5} \text{ kW/km/phase (Peek's formula)}$$

5. Corona loss depends on three factors, namely, electrical factors, atmospheric factors, and factors connected with the conductors.
6. The corona loss can be reduced by using large diameter conductors, hollow conductors and bundled conductors.
7. DC corona loss is less as compared with AC corona loss.

SHORT ANSWER QUESTIONS

1. What is meant by corona?
2. Define critical disruptive voltage.
3. Define visual critical voltage.
4. What is a corona loss?
5. What are the factors affecting corona loss?
6. What are the methods for the reduction of corona effect?
7. What are the advantages of corona?
8. What are the disadvantages of corona?
9. Write Peek's formula for corona loss.
10. Write Petersons' formula for corona loss.
11. What are the affects of corona?

MULTIPLE CHOICE QUESTIONS

1. The phenomenon of corona is accompanied by
   1. hissing sound
   2. production of ozone
   3. power loss
   4. all of them
2. The corona loss is affected by___________ of the conductor.
   1. size
   2. shape

3. surface condition
4. all of them

3. Corona loss is less when the shape of the conductor is
    1. circular
    2. flat
    3. oval
    4. independent of shape
4. Corona loss increases with
    1. increase in supply frequency and conductor size
    2. increase in supply frequency but reduction in conductor size
    3. decrease in supply frequency and conductor size
    4. decrease in supply frequency but increase in conductor size.
5. Bad weather conditions such as rain and sleet
    1. increase the corona loss
    2. decrease the corona loss
    3. does not affect the corona los
    4. none of these
6. The ionization of air surrounding the power conductor is known as_________________ phenomena.
    1. corona
    2. sag
    3. Ferranti effect
    4. none of these
7. ____________is the minimum voltage at which the corona just starts
    1. corona
    2. critical disruptive voltage
    3. critical visual voltage
    4. none of these
8. The formula for the air density correction factor δ is given by _________ kV.
    1. $\frac{3.92b}{(273+t)}$
    2. $\frac{3.92t}{(273+b)}$
    3. $\frac{5.92b}{(273+t)}$
    4. $\frac{3.92g}{(273+t)}$
9. The formula for the critical disruptive voltage is _________ kV/phase.

1. $g_0 \delta m_0 r^2 \ln \frac{D}{r}$

2. $g_0 \delta m_0 r \ln \frac{r}{D}$

3. $g_0 \delta m_0 r \ln \frac{D}{r}$

4. $g_0 \delta m_0 r \ln \frac{D}{r'}$

10. For higher voltages __________ conductors are used.
    1. ACSR
    2. copper
    3. both
    4. silver
11. For polished wires, the irregularity factor *m* value is ______________
    1. 1.0
    2. 0.98
    3. 0.72
    4. 1.5
12. The formula for the visual disruptive voltage is __________ kV/phase.

    1. $g_0 \delta m_v r \left(1 + 0.3 / \sqrt{\delta r}\right) \ln \frac{D}{r}$

    2. $g_0 \delta m_v r \left(1 + 0.3 / \sqrt{\delta r}\right) \ln \frac{D^2}{r}$

    3. $g_0 \delta m_v r \left(1 + 0.3 / \sqrt{\delta r}\right) \ln \frac{D}{r'}$

    4. $g_0 \delta m_v r \left(1 + 0.3 / \sqrt{\delta r}\right) \ln \frac{r}{D}$

13. The formula for the corona loss is _________ kV/ph/km.

    1. $\frac{244}{\delta}(f + 50)\sqrt{\frac{r}{D}}\left(V_{\text{ph}} - V_{d0}\right)^2 \times 10^{-5}$

    2. $\frac{244}{\delta}(f + 25)\sqrt{\frac{r}{D}}\left(V_{\text{ph}} - V_{d0}\right)^2 \times 10^{-5}$

3. $\frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{\text{ph}}-V_{d0}\right)^4\times10^{-5}$

4. $\frac{244}{\delta}(f+25)\sqrt{\frac{r}{D}}\left(V_{\text{ph}}-V_{d0}\right)^2\times10^{-8}$

14. The frequency of supply being high, corona losses are
    1. higher
    2. lower
    3. none of these
15. The DC corona loss is _________ as compared with the A.C corona loss.
    1. less
    2. more
    3. none of these
16. The corona loss on the middle conductor is ________ as compared with the two outer conductors.
    1. more
    2. less
    3. none of these
17. The corona loss is _______ on hilly areas than on plain areas.
    1. more
    2. less
    3. none of these
18. The bad atmospheric conditions such as rains, snow, and hailstorm reduce the critical disruptive voltage and hence _________ in corona loss.
    1. increase
    2. decrease
    3. none of these
19. The corona loss is proportional to the square root of the
    1. size of the conductor
    2. frequency of the conductor
    3. both
    4. none of these
20. Larger the diameter of the conductor, the ________ the corona loss will be.
    1. large
    2. smaller
    3. none of these
21. The size of the conductor on modern EHV lines is obtained based on
    1. voltage drop
    2. current density
    3. corona
    4. power
22. The insulation of the modern EHV lines is designed, based on
    1. the lighting voltage
    2. the switching voltage

3. corona
4. switching power

23. The corona loss will be reduced by using _______ conductors.
    1. large diameter
    2. hollow
    3. bundled
    4. all of these
24. The disadvantage of corona is
    1. hissing noise
    2. ozone gas
    3. power loss
    4. all of these
25. The advantage of corona is
    1. increase effect of transients
    2. corrosion of conductor
    3. increase effective conductor size
    4. increase effect of subtransients

Answers:

1. d	2. d	3. a	4. b	5. a
6. a	7. b	8. a	9. c	10. a
11. a	12. a	13. b	14. a	15. a
16. a	17. a	18. a	19. d	20. b
21. c	22. b	23. d	24. d	25. c

REVIEW QUESTIONS

1. Explain "Corona".
2. Describe the phenomenon of corona. Explain the factors affecting corona.
3. Explain the losses of corona on HV line.
4. What are the advantages and disadvantages of corona? Why are these different in different weather conditions?
5. Derive a formula for disruptive critical voltage between two smooth circular wires assuming the breakdown strength of air to be 30 kV (peak)/cm.
6. Explain about corona loss and discuss the methods for the reduction of corona loss.
7. Explain the phenomena of corona and derive an expression for disruptive critical voltage.
8. What are the disadvantages of corona? Explain how the corona considerations effect the design of a line?

PROBLEMS

1. An overhead 132 kV line conductor of 2.5 cm diameter is built so

that corona takes place if the line voltage is 220 kV (r.m.s.). Determine the spacing between the conductors, if the value of potential gradient at which ionization occurs can be taken as 30 kV/cm (peak).

2. A single-phase transmission line consists of two conductors of diameter 1.2 cm each, with 1.5 m spacing between centres. Determine the value of the line voltage at which corona commences, if the disruptive critical voltage for air is 21.21 kV/cm.
3. A three-phase, 220 kV line consists of 20 mm diameter conductors spaced in a 6 m delta configuration. Determine the disruptive critical voltage and visual corona voltage (local corona as well as general corona) for the following data: temperature 25° C, pressure 73 cm Hg, surface factor 0.84, irregularity factor for local corona 0.72, irregularity factor for general corona 0.82 m.
4. A grid line operating at 132 kV consists of 2 cm diameter conductors spaced 4 m apart. Determine the disruptive critical voltage and visual corona voltage for the following data: temperature 44° C, barometric pressure 73.7 cm Hg, conductor surface factor 0.84, fine weather 0.8 and rough weather 0.66.
5. A three-phase, 220 kV, 50 Hz, overhead line consists of 2.5 cm diameter conductors spaced 3 m apart in equilateral triangle formation. Determine the corona loss per kilometre of the line at 20° C and atmospheric pressure 75 cm Hg. Take irregularity factor as 0.8.
6. A 110 kV, three-phase, 50 Hz, 150 km long overhead line consists of three 1.8 cm diameter stranded copper conductors spaced in 2.5 m delta arrangement. Assume surface irregularity factor m = 0.89 (roughness factor), m_v for local corona = 0.74 and m_v for general corona is 0.84. Determine the following at 290° C and barometric pressure of 76 cm Hg.
    1. Disruptive voltage
    2. Visual corona voltage for local corona
    3. Visual corona voltage for general corona
    4. Power loss due to corona, using Peek's formula, under fair weather and in wet conditions
    5. Power loss due to corona, using Peterson's formula, under fair weather and in wet conditions
7. A three-phase, 220 kV, 250 km long line consists of ratio of $\frac{D}{r}$ is

    680. Using Peterson's formula, determine the total corona loss. Assume factor K as 0.04.

7

Mechanical Design of Transmission Line

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Understand the overview of transmission line supports
- Understand the calculation of sag
- Discuss the effect of tower structure and environmental effects on sag calculation

7.1 INTRODUCTION

The design of a transmission line has to satisfy electrical as well as mechanical considerations. An overhead line comprises mainly of conductors, line supports, insulators, and pole fittings. These should have sufficient mechanical strength to withstand the worst probable weather conditions and other external interferences. This calls for the use of proper mechanical factors of safety in order to ensure the continuity of operation in the line.

7.2 FACTORS AFFECTING MECHANICAL DESIGN

The factors affecting the mechanical design of an overhead transmission line are as follows:

Selection of Line Route The selection of the route depends upon the distance, geographical conditions, transportation facility, etc.

Types of Towers/Poles The selection of the type of towers/poles depends upon the line span. The conductor weight, line operating voltage and cost, local conditions and soil conditions also help to ascertain what type of tower/pole should be used.

Ground and Conductor Clearance The clearance of conductors is decided not only by the electrical consideration of working voltage, but also by mechanical factors such as length of the span, weight of conductors, prevalent wind direction, ice loading on the conductors, etc.

Tower Spacing and Span Length The tower spacing and span length depends on the weight of the conductors, prevalent wind direction and ice loading on the conductors, soil conditions, local conditions, load bearing capacity of the tower, etc.

Mechanical Loading The mechanical loading depends on span length, type of conductor materials, area of cross-section of conductors, sag level difference between adjacent towers, etc.

7.3 LINE SUPPORTS

The line supports are the poles which support an overhead transmission line. They should be equipped to carry the load of the conductors including the additional weight due to deposition of ice and wind pressure. The various types of line supports used are:

7.3.1 Wooden Poles

The use of wooden poles as line supports is limited to low voltage power distribution and spans up to about 60 m. Wooden poles are elastic and protected by metallic caps at the top to protect it from decay. The advantage of wooden poles is that they are easily installed and cheap when compared to other varieties of line supports.

The main disadvantage of wooden poles is their tendency to rot at the ground level. But well-seasoned and preserved wood serves the purpose better; their life span is about 25–30 years.

Fig. 7.1 Tabulator steel pole

7.3.2 Tubular Steel Poles

They are used for system voltages up to 33 kV in low and high-voltage distribution systems. These are costlier than wooden and RCC poles. When compared to wooden poles, tubular steel poles have advantages like light weight, long life, and greater strength. These are used for a longer span, i.e., from 50 to 80 m. All steel supports should be well-galvanized and should have a life of at least 30 years. Their life can be increased by regularly painting and preserving the structure.

Now-a-days tubular poles are used for lighting the streets in towns and cities (as shown in Fig. 7.1).

7.3.3 RCC Poles

They are used for system voltage up to 33 kV. Generally, they are rectangular in shape at the bottom and square at the top. They are stronger and more durable and are, therefore, used for longer length spans, i.e., from 80 to

200 m. These are also used for low and high voltage distribution systems. These poles have a long life and require low maintenance but are cumbrous and heavy, therefore, adding to the transportation costs.

Now-a-days, tubular RCC poles (see Fig. 7.2) are used for 33 kV transmission lines in coastal regions.

Fig. 7.2 33 kV Tubular RCC pole

7.3.4 Latticed Steel Towers

These towers are robust in construction. They can be used for spans 300 m or above. They are used for the transmission of power above 66 kV and are more useful for valleys, railway lines, rivers, etc. They are mechanically very strong and have a long life. They are capable of withstanding the most severe climatic conditions and cannot be destroyed by forest fires. The problems caused due to lightning are minimized due to the presence of lightning conductors on each tower.

At a reasonable additional cost, double circuit transmission lines can be set on the same tower as shown in Figs. (7.3) and (7.4) thereby reducing the discontinuity of supply to a large extent. In case of breakdown of one circuit, it becomes possible to transmit the power through the other.

These towers are used for transmitting huge power at high voltage levels, i.e., 132 kV, 220 kV, 400 kV 765 kV, etc.

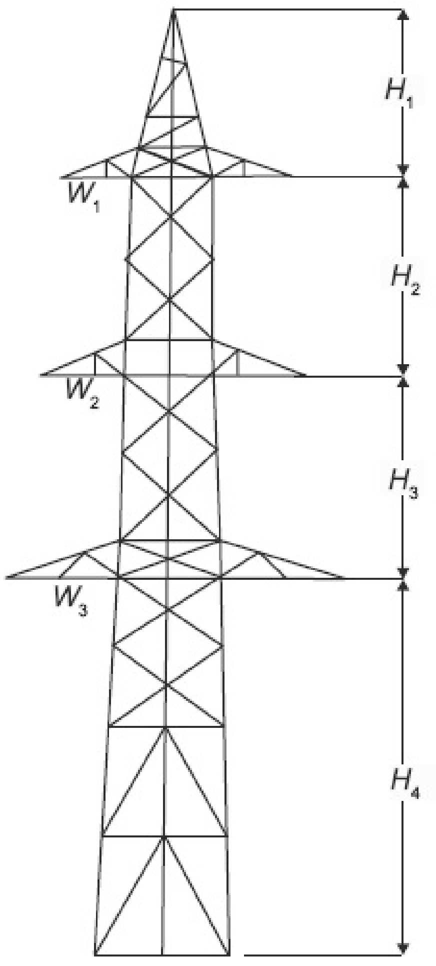

Fig. 7.3 Lattice tower

Table 7.1 From Fig. 7.3, spacing between conductors for different voltages

Voltage/ Spacing	Type of tower*	W_1 (mm)	W_2 (mm)	W_3 (mm)	H_1 (mm)	H_2 (mm)	H_3 (mm)	H_4 (mm)	Span (m)	Designed for
132 kV	0	3200	3200	3200	3900	3860	3860	11300	120	Panter
(narrow	30	3200	3200	3200	3900	3860	3860	8000	120	
base)	60	4400	4400	4400	7550	3620	3620	12300	120	
220 kV	A	4200	4400	4900	4730	5300	5300	20850	380	Zebra
(TSP)	B	4200	4350	4850	7280	5100	5100	19085	380	
	C	4900	5000	5500	8450	5200	5300	19085	380	

[*] The type of tower designed depends on the angle of deviation.

• *W_1, W_2, W_3 are the width of tower cross arm and H_1, H_2, H_3, H_4 are the height clearances between cross arms.*

Fig. 7.4 220 kV Double circuit tower

Test Yourself

1. In general, why are RCC poles preferred in coastal areas instead of iron poles for the distribution of electrical power?

7.4 SAG

When a flexible wire of uniform cross-sectional area is suspended between two supports at the same level, it experiences a tensile stress which is due to the weight of the conductor acting vertically downwards. Due to this, the conductor forms a caternary curve between the towers. The difference in level between the points of support and the lowest point on the conductor (which is caternary in shape) is known as sag.

The factors affecting the sag in an overhead line are as given below:

1. **Weight of the conductor:** The sag is directly proportional to the weight of the conductor. Sag increases with an increase in the weight of the conductor. Where there is ice formation and wind pressure upon the conductor, the weight of the conductor increases, which leads to increased sag. It is generally measured in kilogram/metre (kg/m).
2. **Length of the span:** Sag is directly proportional to the span length. With increased span length, the weight of the conductor between the supports increases. Therefore, higher the value of the span length, higher is the sag. It is measured in metre (m).
3. **Working tensile strength:** Sag is inversely proportional to the working tensile strength of the conductor at constant temperature. It is measured in kilogram (kg).
4. **Temperature:** If there is a change in temperature, there is a change in the length of the conductor. Therefore, the length of the conductor increases with the rising temperature between the fixed supports. The sag too will, therefore, increase with an increase in temperature. It is measured in (°C).

Test Yourself

1. Is sag necessary in overhead power supply system? Justify your answer.

7.4.1 Calculation of Sag at Equal Supports

When the conductor is supported by two supports P and Q as shown in Fig. 7.5, it forms a caternary curve due to the weight of the conductor, and is horizontal at O.

Let S be the length of the conductor POQ, suspended between the supports at the same level.

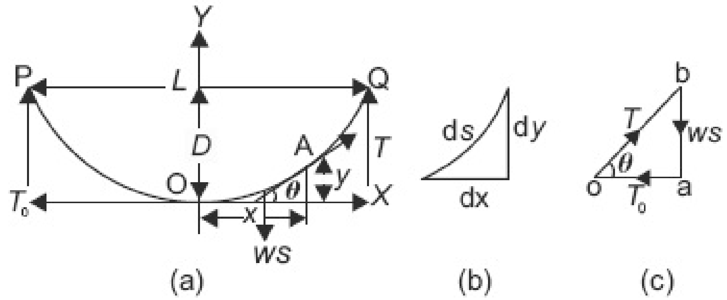

Fig. 7.5 Reference to sag calculation for equal supports

Let W =	Weight of the conductor per metre length (kg/m)
D =	Maximum sag, i.e., the level difference between the points of support and the lower point on the conductor (m)
T =	Tension at any point A of the conductor acting towards supports (kg)
T_0 =	Tension at point O of the conductor acting horizontally (kg)
s =	Length of the conductor of a small section OA (m).

Consider a small section OA of a conductor of length '*s*'. Assume that it is in equilibrium due to three forces, tension T at A, tension T_0 (H) at O and the weight of the conductor ws, acting downwards. These three forces can be represented by a triangle of forces oab as shown in Fig. 7.5(c).

From Fig. 7.5(c),

$$\tan\theta = \frac{ws}{T_0} \tag{7.1}$$

From Fig. 7.5(b), consider a small section of length ds and its horizontal and vertical sections dx and dy, respectively. From the triangle, shown in Fig. 7.5(b)

$$\tan\theta = \frac{dy}{dx} \quad (7.2)$$

$$\text{and, } \frac{dy}{dx} = \frac{T_y}{T_x} = \frac{ws}{H} \text{ as } T_0 = H \quad (7.3)$$

$$\text{also, } ds^2 = dx^2 + dy^2 \quad (7.4)$$

$$\text{or } \frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{w^2s^2}{H^2}\right)} \quad (7.5)$$

$$\text{or } dx = \frac{ds}{\sqrt{1+\left(\frac{w^2s^2}{H^2}\right)}} \quad (7.6)$$

Integrating Eq. (7.6) with respect to *s*, we get

$$x = \left(\frac{H}{w}\right)\sinh^{-1}\left(\frac{ws}{H}\right) + k \quad (7.7)$$

where, *k* is a constant of integration

From initial conditions, at $s = 0$, $x = 0$, then we get $k = 0$ (at point O)

Therefore, Eq. (7.7) becomes

$$x = \left(\frac{H}{w}\right)\sinh^{-1}\left(\frac{ws}{H}\right) \quad (7.8)$$

$$s = \left(\frac{H}{w}\right)\sinh\left(\frac{wx}{H}\right) \quad (7.9)$$

$$s = \frac{H}{w}\left[\frac{wx}{H} + \frac{w^3x^3}{6H^3} + \ldots\right] \; (\therefore \text{Neglect higher order terms}) \quad (7.10)$$

$$\cong x + \frac{w^2x^3}{6T^2} \cong x\left[1 + \frac{w^2x^2}{6T^2}\right] \quad (7.11)$$

So that length of the conductor in half-span, *s* is

$$s = l + \frac{w^2l^3}{6T^2} = l\left[1 + \frac{w^2l^2}{6T^2}\right] \quad (7.12)$$

$$\text{or } S = \frac{L}{2} + \frac{w^2L^3}{48T^2} \quad (7.13)$$

where,

l = half span length

L = span length

From Eq. (7.3)

$$\frac{dy}{dx} = \frac{ws}{H} \quad (7.14)$$

Substituting *s* from Eq. (7.9) in Eq. (7.14)

$$\frac{dy}{dx} = \frac{w}{H} \times \frac{H}{w} \sinh \frac{wx}{H} = \sinh \frac{wx}{H} \tag{7.15}$$

Integrating Eq. (7.15) with reference to *x*, we get

$$y = \frac{H}{w} \cosh \frac{wx}{H} + B \tag{7.16}$$

where, *B* is the constant of integration

From initial conditions, when $x = 0$, $y = 0$, so that

$$0 = \frac{H}{w} + B$$

$$B = -\frac{H}{w} \tag{7.17}$$

Substitute the value of *B* from Eq. (7.17) in Eq. (7.16). Then,

$$y = \frac{H}{w}\left[\cosh \frac{wx}{H} - 1\right] \tag{7.18}$$

$$y = \frac{H}{w}\left[1 + \frac{w^2 x^2}{2H^2} + \frac{w^4 x^4}{24H^4} - 1\right] \cong \frac{wx^2}{2H} \tag{7.19}$$

$$\therefore y \cong \frac{wx^2}{2T}, \text{ since } T \text{ is very nearly equal to } H$$

$$\text{Sag, } D = \frac{wl^2}{2T} \tag{7.20}$$

where, l is the half-span length

$$\text{and, Sag, } D = \frac{wL^2}{8T} \tag{7.21}$$

where, L is the span length, i.e., distance between two supports.

7.4.2 Effect of Ice Covering and Wind Pressure

The Eq. (7.21) holds good only for still air at normal temperature conditions and the weight of the conductor is acted upon alone. In actual practice, like in the case of snowfall and storm, the effective weight of the conductor is increased due to the weight of snow and wind pressure, hence increase in sag in both vertical and horizontal directions. If d is the diameter of the conductor and t is thickness of ice covering in the radial direction (see Fig. 7.6), the overall diameter of the conductor becomes

$$D = d + 2t. \tag{7.22}$$

Therefore, the sectional area of ice is

$$= \frac{\pi D^2}{4} - \frac{\pi d^2}{4} = \frac{\pi}{4}\left((d+2t)^2 - d^2\right)$$
$$= \pi t\,(d + t) \tag{7.23}$$

If d and t are in metres, then the Eq. (7.22) represents the volume of ice in cubic metre per metre length of the conductor.

If ρ be the density of ice in kg/m^3, then weight w_i of ice per metre length is

$$w_i = \rho \times \pi t (d + t) \text{ kgs} \tag{7.24}$$
$$\rho = 915 \text{ kg/m}^3$$

Therefore, the resultant weight of the conductor acting downwards is

$$w_r = w_c + w_i \text{ kgs/m} \tag{7.25}$$

where, w_c is the weight of the conductor per metre length.

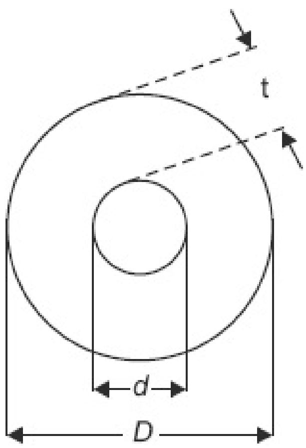

Fig. 7.6 Illustration of effect of ice covering and wind pressure

The wind pressure on the conductor is assumed to be in the horizontal direction and is taken as pressure on the projected area of the conductor. The projected surface of the conductor of length 1 m is

$$= (d + 2t) \times 1 \text{ m}^2 \tag{7.26}$$

Therefore, weight of the conductor due to wind pressure is

$$w_w = p\,(d + 2t) \text{ kg/m} \tag{7.27}$$

where, p is the wind pressure in kg/m^2.

Thus, the resultant weight w_r acting on the conductor in Fig. 7.7, is given as

$$w_r = \sqrt{(w_c + w_i)^2 + w_w^2} \tag{7.28}$$

The angle through which the line will be deflected due to the horizontal wind pressure is

$$\theta = \cos^{-1}\left(\frac{w_c + w_i}{w_r}\right) = \sin^{-1}\left(\frac{w_w}{w_r}\right) \tag{7.29}$$

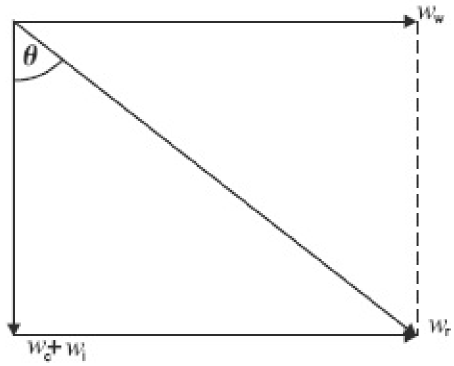

Fig. 7.7 Illustration of weights

Then the sag in vertical plane = deflection in the inclined plane × cosθ

$$= \frac{w_r l^2}{2T} \cos\theta \tag{7.30}$$

Then, the sag in horizontal plane = deflection in the inclined plane × sinθ

$$= \frac{w_r l^2}{2T} \sin\theta \tag{7.31}$$

Disadvantages of loose span:

Following are the disadvantages if the sag is more than the prescribed value:

1. When the span is loose, i.e., the length of the conductor required for a given length of line is more the sag is more too.
2. If the sag is more than the prescribed maximum, then there is a possibility that conductors might clash during windy days and hence, produce short circuits.
3. The conductors of a loose span swing irregularly and vibrate and hence, are subjected to strain at the point of their fixing. Thus, there is a possibility of them breaking.

4. To allow for the minimum clearance from the live conductor to earth under statutory rules, the heights of the structures have to be increased thus, leading to increased costs.

7.4.3 Safety Factor

It is defined as the ratio of the ultimate strength to the working stress.

EXAMPLE 7.1

An overhead transmission line has a span of 220 m, the conductor weighing 804 kg/km. Calculate the maximum sag if the ultimate tensile strength of the conductor is 5,758 kg. Assume a safety factor of 2.

Solution:

Span length, L = 220 m

Ultimate tensile strength of conductor = 5,758 kg

Safety factor = 2.

$$\text{Maximum tension, } T = \frac{\text{Ultimate strength}}{\text{Factor of safety}} = \frac{5758}{2} = 2879 \text{ kg}$$

$$\text{Weight of the conductor, } w = 804 \text{ kg/km} = 804 \times 10^{-3} \text{ kg/m}$$

$$\text{Maximum sag, } D = \frac{wL^2}{8T} = \frac{804 \times 10^{-3} \times (220)^2}{8 \times 2879} = 1.69 \text{ m.}$$

EXAMPLE 7.2

A transmission line has a span of 150 m between level supports. The line conductor has a cross-sectional area of 1.25 cm^2 and it weighs 120 kg per 100 m. If the breaking stress of the copper conductor is 4220 kg/cm^2. Calculate the maximum sag for a safety factor of 4. Assume a maximum wind pressure of 90 kg/m^2 of projected surface.

Solution:

Span length, L = 150 m

Cross-sectional area of conductor, a = 1.25 cm^2

Weight of the conductor = 120 kg/100 m = 1.2 kg/m

Breaking stress = 4220 kg/cm^2

Safety factor = 4

$$\text{Tension, } T = \frac{\text{Breaking stress} \times \text{cross sectional area}}{\text{Factor of safety}}$$

$$\text{Allowable maximum tension, } T = \frac{4220 \times 1.25}{4} = 1319 \text{ kg}$$

$$\text{Diameter of conductor, } d = \sqrt{\frac{1.25 \times 4}{\pi}} = \sqrt{\frac{1.25 \times 4}{3.14}} = 1.26 \text{ cm}$$

Weight of conductor due to wind pressure, w_W = 90 kg/m^2 × 1.26 × 10^{-2} = 1.134 kg/m

$$\text{Resultant of weight of conductor, } w_r = \sqrt{1.2^2 + 1.134^2} = 1.651 \text{ kg/m}$$

$$\therefore \text{Sag, } D = \frac{w_r L^2}{8T} = \frac{1.65 \times 150^2}{8 \times 1319} = 3.52 \text{ m.}$$

EXAMPLE 7.3

Calculate the minimum sag permissible for a 160 m span, 1.0 cm diameter copper conductor allowing a maximum tensile stress of 2000 kg/cm^2. Assume a horizontal wind pressure of 4 kg/cm^2 of projected area. Take the specific gravity of copper as 8.9 gm/cm^3.

Solution:

Span length, L = 160 m

Diameter of conductor, d = 1.0 cm

$$\text{Cross-sectional area of conductor, } a = \frac{\pi}{4} \times (1)^2 = 0.785 \text{ cm}^2$$

Weight of conductor per metre length =specific gravity × volume of 1 m conductor

$$w_C = 8.9 \text{ gm/cm}^3 \times 0.785 \text{ cm}^2 \times 100$$
$$= 698.6 \text{ gm}$$
$$= 0.6986 \text{ kg}$$

Weight of the conductor per metre length due to wind pressure, w_W = pressure × projected area in cm^2

$$= 4 \text{ kg/cm}^2 \times 0.785 \text{ cm}^2$$
$$= 3.14 \text{ kg}$$

$$\text{Resultant weight of conductor, } w_r = \sqrt{w_C^2 + w_W^2}$$
$$= \sqrt{0.6986^2 + 3.14^2} = 3.217 \text{ kg}$$

Maximum tensile stress = 200 kg/cm^2

$$= 2000 \times 0.785 \text{ kg}$$
$$= 1570 \text{ kg}$$
$$\text{Sag, } D = \frac{w_r L^2}{8T} = \frac{3.217 \times 160^2}{8 \times 1570} = 6.54 \text{ m.}$$

EXAMPLE 7.4

A transmission line has a span of 150 m between supports, the supports being at the same level. The conductor has a cross-sectional area of 2 cm^2. The ultimate strength is 5,000 kg/cm^2. The specific gravity of the material is 8.9 gm/cm^3. If the coating of ice is 1.0 cm, calculate the sag at the centre of the conductor if safety factor is 5.

Solution:

Span length, L = 150 m

The coating of ice, t = 1.0 cm

Safety factor = 5

Cross-sectional area of the conductor, a = 2 cm^2

$$\text{Diameter of conductor, } d = \sqrt{\frac{2\times 4}{\pi}}$$
$$= 1.60 \text{ cm}$$
$$\text{Ultimate strength} = 5000 \text{ kg/cm}^2$$
$$= 5000 \times 2 \text{ kg}$$
$$= 10{,}000 \text{ kg}$$
$$\text{Allowable maximum tension, } T = \frac{10{,}000}{5}$$
$$= 2000 \text{ kg}$$

Specific gravity of copper conductor = 8.9 gm/cm^3

$$\therefore \text{ Weight of conductor, } w_c = 8.9 \times 10^6 \times 2 \times 10^{-4} \text{ gm/m}$$
$$= \frac{8.9\times 10^6 \times 2 \times 10^{-4}}{1000} \text{ kg/m}$$
$$= 1.78 \text{ kg/m}$$

$$\text{Weight of conductor due to ice loading, } w_i = \text{Density of ice} \times \frac{\pi}{4}\left[(d+2t)^2 - d^2\right]$$
$$= 915 \times \frac{\pi}{4}\left[(1.6+2.0)^2 - 1.6^2\right] \times 10^{-4}$$
$$= 0.7474 \text{ kg/m}$$
$$\text{Resultant weight of conductor, } w_r = w_c + w_i$$
$$= 1.78 + 0.7474 = 2.5274 \text{ kg}$$

$$\text{Maximum allowable sag, } D = \frac{w_r L^2}{8T}$$
$$= \frac{2.5274 \times 150^2}{8 \times 2000} = 3.554 \text{ m.}$$

EXAMPLE 7.5

An overhead line has a span of 160 m of copper conductor between level supports. The conductor diameter is 1.2 cm and has a breaking stress of 35 kg/mm^2. Calculate (a) the deflecting sag (b) the vertical sag. The line is subject to a wind pressure of 40 kg/m^2 of projected area and radial ice coating of 9.53 mm thickness. The weight of ice is 913.5 kg/m^3. Allow a factor of safety of 2 and take the density of copper as 8.9 g/cm^3.

Solution:

The length of span, L = 160 m

Diameter of the conductor is, d = 1.2 cm

Breaking stress = 35 kg/mm^2

= 35×10^6 kg/m^2

Wind pressure of projected area = 40 kg/m^2

Thickness of ice, t = 9.53 mm

The weight of the ice = 913.5 kg/m^3

Safety factor = 2

Density of copper = 8.9 gm/cm^3

$$\text{Working tension} = \frac{\text{breaking stress} \times \text{area}}{\text{factor of safety}} = \frac{35 \times 10^6 \times \frac{\pi}{4} \times (0.012)^2}{2} = 1979.2 \text{ kg}$$

$$\text{Weight of the conductor per metre length, } w_c = 8.9 \times 10^3 \times \pi \times \frac{(0.012)^2}{4}$$

$$= 1.007 \text{ kg}$$

Weight of the conductor due to ice coating per metre length, $w_i = p\pi t(d+t)$

$$w_i = 913.5 \times \pi \times 9.53 \times 10^{-3} \times (0.012 + 9.53 \times 10^{-3})$$

$$= 0.5888 \text{ kg}$$

Weight of the conductor due to wind force per metre length, $w_w = p(d+2t)$

$$w_w = 40 \times (1.2 \times 10^{-2} + 2 \times 9.53 \times 10^{-3})$$

$$= 1.2424 \text{ kg}$$

$$\text{Resultant force per metre length of conductor, } w_r = \sqrt{(w_c + w_i)^2 + w_w^2}$$

$$= \sqrt{(1.007 + 0.5888)^2 + (1.2424)^2} = 2.0224 \text{ kg}$$

1. The deflecting sag (or) sag in a vertical direction is,

$$D = \frac{w_r L^2}{8T}$$

$$= \frac{2.0224 \times (160)^2}{8 \times 1979.2} = 3.27 \text{ m}$$

2. Vertical sag, $D' = D \cos \theta$

$$= D \times \frac{w_c + w_i}{w_r} = \frac{(w_c + w_i) L^2}{8T}$$

$$= \frac{(1.007 + 0.5888)(160)^2}{8 \times 1979.2} = 2.58 \text{ m.}$$

EXAMPLE 7.6

A transmission line conductor has an effective diameter of 19.5 mm and weighs 1.0 kg/m. If the maximum permissible sag with a horizontal wind pressure of 39 kg/m^2 of projected area and 12.7 mm radial ice coating is 6.3 m. Calculate the permissible span between two supports at the same level allowing a safety factor of 2. Ultimate strength of the conductors is 8,000 kg and weight of ice is 910 kg/m^3.

Solution:

Diameter of the conductor is, d = 19.5 cm

Weight of conductor, w_c = 1.0 kg/m

Wind pressurer, P = 39 kg/m^2

Radial ice coating, t = 12.7 mm

Maximum permisable sag, D = 6.3 m

Safety factor = 2

Ultimate strength = 8000 kg

Weight of ice, w_i = 910 kg/m^3

$$\text{Maximum allowable tension, } T = \frac{\text{Ultimate strength}}{\text{Safety factor}} = \frac{8000}{2} = 4000 \text{ kg}$$

$$\begin{aligned}\text{Are of ice-section } &= \pi t\,(d + t) \\ &= \pi \times 12.7(19.5 + 12.7) \\ &= 1284.723 \text{ mm}^2 \\ &= 1284 \times 10^{-6} \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Weight of ice, } w_i &= 910 \times 1284 \times 10^{-6} \\ &= 1.1691 \text{ kg/m}\end{aligned}$$

$$\begin{aligned}\text{Wind pressure, } w_w &= 39(d + 2t) \\ &= 39(19.5 + 2 \times 12.7) \times 10^{-3} \\ &= 1.751 \text{ kg/m}\end{aligned}$$

$$\text{Resultant weight, } w_r = \sqrt{w_w^2 + \left(w_c + w_i\right)^2}$$

$$= \sqrt{1.751^2 + \left(1.0 + 1.1691\right)^2} = 2.7876 \text{ kg/m}$$

$$\therefore \text{ Sag, } D = \frac{w_r L^2}{8T}$$

$$6.3 = \frac{2.7876 L^2}{8 \times 4000}$$

$$L^2 = \frac{6.3 \times 8 \times 4000}{2.7876}$$

$\therefore$ Length of span, L = 269 m.

EXAMPLE 7.7

A transmission line conductor consists of a hard drawn copper 240 mm^2 cross-section and has a span of 160 m between level supports. The conductor has an ultimate tensile stress of 42.2 kg/mm^2 and allowable tension is not to exceed 1/5th of ultimate tensile strength. Determine (i) the sag, (ii) the sag with a wind pressure of 1.35 kg/m and an ice coating of 1.0 cm thickness, (iii) the vertical sag. Take the specific gravity of hard drawn copper as 8.9 gm/cc and the weight of ice as 915 kg/m^3.

Solution:

Cross-sectional area of conductor, a = 240 mm^2

$$\text{Diameter of conductor, } d = \sqrt{\frac{240 \times 4}{\pi}} = 17.5 \text{ mm}$$

Span length, L = 160 m

$$\begin{aligned}
\text{Ultimate tensile stress} &= 42.2 \text{ kg/mm}^2 \\
&= 42.2 \times 240 \text{ kg} \\
&= 10128 \text{ kg}
\end{aligned}$$

$$\text{Allowable tension, } T = \frac{1}{5}(\text{Ultimate Tensile Stress})$$

$$= \frac{1}{5} \times 10128 = 2025.6 \text{ kg}$$

$$\begin{aligned}
\text{Weight of conductor, } w_c &= \text{Specific gravity} \times \text{cross-sectional area} \\
&= 8.9 \times 10^6 \times 240 \times 10^{-6} \text{ gm/m} \\
&= 2136 \text{ gm/m} = 2.136 \text{ kg/m}
\end{aligned}$$

Weight of conductor due to wind pressure, w_w = 1.35 kg/m

Weight of conductor due to ice loading, w_i = density of ice $\times \frac{\pi}{4}\left[(d+2t)^2 - d^2\right]$

$$= 915 \times \frac{\pi}{4}\left[(1.75+2.0)^2 - 1.75^2\right] \times 10^{-4}$$

$$= 0.79 \text{ kg/m}$$

Resultant weight of conductor, $w_r = \sqrt{(w_c + w_i)^2 + w_w^2}$

$$= \sqrt{(2.136+0.79)^2 + 1.35^2}$$

$$= 3.223 \text{ kg/m}$$

1. $\text{Sag} = \dfrac{w_c L^2}{8T} = \dfrac{2.136 \times 160^2}{8 \times 2025.6} = 3.3746 \text{ m}$
2. Sag with ice and wind effect,

$$D = \frac{w_r L^2}{8T} = \frac{3.223 \times 160^2}{8 \times 2025.6} = 5.1 \text{ m}$$

3. Vertical sag, $D_v = \left(\dfrac{w_c + w_i}{w_r}\right) \times D$

$$= 5.1 \times \left(\frac{2.136 + 0.79}{3.223}\right) = 4.63 \text{ m.}$$

EXAMPLE 7.8

An overhead line is erected across a span of 200 m on level supports. The conductor has a diameter of 1.25 cm and a dead weight of 1.09 kg/m. The line is subjected to wind pressure of 37.8 kg/m^2 of the projected area. The radial thickness of ice is 1.25 cm. Calculate the sag (i) in an inclined direction,(ii) in a vertical direction. Assume a maximum working stress of 1050 kg/cm^2. One cubic metre of ice weighs 913.5 kg. (This question is solved using MATLAB programs)

Solution:

Length of the span, L =200 m

Diameter of the conductor, d = 1.25 cm

Dead weight, w_c = 1.09 kg/m

Wind pressure, P = 37.8 kg/m^2

The radial thickness of ice, t = 1.25 cm

Maximum working stress = 2

$$\text{Working tension, } T = \rho \times \frac{\pi}{4}(d^2) = 1050 \times \frac{\pi}{4}(1.25)^2 = 1{,}288.5 \text{ kg}$$

Weight of ice coating per metre length, $w_i = 913.5 \times \pi\, t\,(d + t)$

$$= 913.5 \times \pi \times 1.25 \times 10^{-2} \times (1.25 + 1.25) \times 10^{-2}$$
$$= 0.897 \text{ kg}$$

Wind force per metre length of conductor, $w_w = P \times (d + 2t)$

$$= 37.8\,(1.25 + 2 \times 1.25) \times 10^{-2}$$
$$= 1.4175 \text{ kg/m}$$

Resultant force per metre length of conductor, $w_r = \sqrt{(w_c + w_i)^2 + w_w^{\,2}}$

$$= \sqrt{(1.09 + 0.897)^2 + 1.4175^2}$$
$$= 2.441 \text{ kg}$$

1. Sag in inclined direction, $D = \dfrac{w_r L^2}{8T} = \dfrac{2.441 \times 200^2}{8 \times 1288.5} = 9.472 \text{ m}$

2. Sag in vertical direction, $D' = D\cos\theta = D \times \dfrac{(w_c + w_i)}{W_r} = \dfrac{(w_c + w_i)}{8T} L^2$

$$= \frac{(1.09 + 0.897) \times 200^2}{8 \times 1288.5}$$
$$= 7.71 \text{ m.}$$

7.4.4 Calculation of Sag at Different Level Supports

When a transmission line is run on inclined planes, i.e., hilly areas, the supports P and Q are at unequal levels as shown in Fig. 7.8.

Let P and Q be the supports at different levels and O be the lowest point on the curve which is not at centre in this case, then

The sag with respect to lower support, $D_1 = \frac{wx_1^2}{2T}$ (7.32)

The sag with respect to upper support, $D_2 = \frac{wx_2^2}{2T}$ (7.33)

$\therefore$ The difference in level, $h = D_2 - D_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T} = \frac{w}{2T}(x_2^2 - x_1^2)$

$$= \frac{w}{2T}(x_2 + x_1)(x_2 - x_1) \quad (7.34)$$

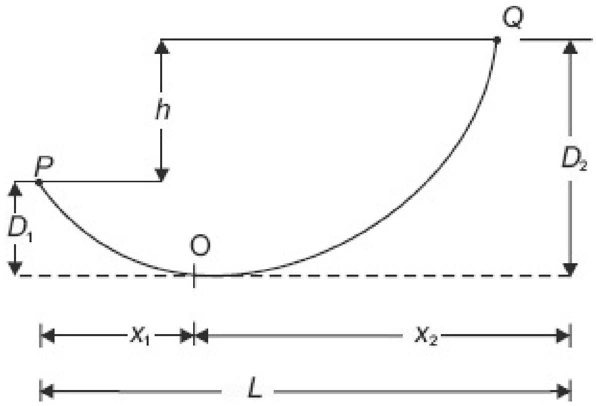

Fig. 7.8 Illustration of unequal supports

From Fig. 7.8, $x_2 + x_1 = L$ and $D_2 - D_1 = h$

$$\therefore h = \frac{wL}{2T}(x_2 - x_1) \quad (7.35)$$

$$\text{or } x_2 - x_1 = \frac{2Th}{wL} \quad (7.36)$$

$$\text{and } x_2 + x_1 = L \tag{7.37}$$

Adding Eqs. (7.35) and (7.36), we get

$$x_2 = \frac{L}{2} + \frac{Th}{wL} \tag{7.38}$$

Substituting x_2 from Eq. (7.38) in Eq. (7.36)

$$T = H;\ y = \frac{wx^2}{2T};\ D = \frac{wl^2}{2T} \text{ and } s = l + \frac{w^2 l^3}{6T^2} \tag{7.39}$$

where,

L =	span length
h =	difference in levels between supports
x_1 =	Distance between the shorter support and lower point O on conductor
x_2 =	Distance between the longer support and lower point O on conductor
T =	Tension in the conductor
w =	Weight per metre length of the conductor.

EXAMPLE 7.9

A transmission line conductor at a river crossing is supported from two towers at heights of 60 and 80 m above water level (see Fig. 7.9). The horizontal distance between the towers is 300 m. If the tension in the conductor is 2000 kg, find (i) the maximum clearance between the conductor and water, (ii) the clearance between the conductor and water at a point mid way between the towers. Weight of conductor is 0.844 kg/m. Assume that the conductor takes the shape of a parabola. (This question is solved using MATLAB programs)

Solution:

The working tension, T = 2000 kg

Span, L = 300 m

Weight of the condcutor, w = 0.844 kg/m

Now, the difference in height of tower,

$$h = 80 - 60 = 20\ \text{m}$$

$$x_1 = \frac{L}{2} - \frac{Th}{wL} = \frac{300}{2} - \frac{2000 \times 20}{0.844 \times 300} = 150 - 158 = -8\ \text{m}$$

$$x_2 = \frac{L}{2} + \frac{Th}{wL}$$

$$= \frac{300}{2} + \frac{2000 \times 20}{0.844 \times 300} = 150 + 158 = 308\ \text{m}$$

Sag with respect to the lower support,

$$D_1 = \frac{wx_1^2}{2T} = \frac{0.844 \times 8^2}{2 \times 2000} = 0.0135\ \text{m}$$

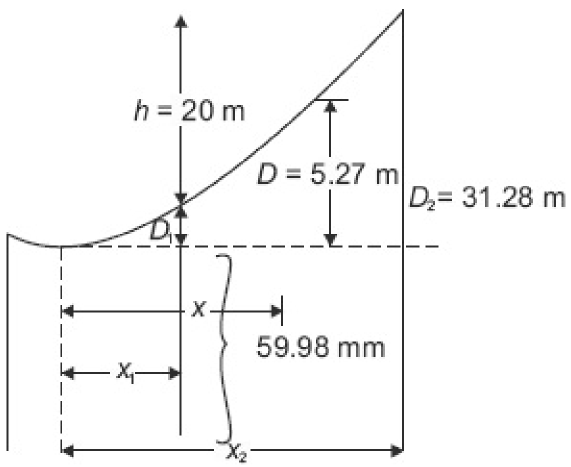

Fig. 7.9 Unequal supports

1. Clearance between the water level and lower point of the conductor = 60 – 0.0135 = 59.9865 m Distance from the midpoint between the two towers and the lower point of the conductor, x = 150 + 8 = 158 m

$$\text{Sag at the mid point of the supports, } D = \frac{wx^2}{2T} = \frac{0.844 \times 158^2}{2 \times 2000} = 5.27 \text{ m}$$

2. Clearance between the water and conductor at mid point between two conductors,

$$= 59.9865 + 5.27 = 65.2565 \text{ m.}$$

EXAMPLE 7.10

An overhead conductor consists of seven strands of silicon-bronze having a cross-sectional area of 2.0 cm^2 and ultimate strength of 10000 kg/cm^2. When connected between supports 650 m apart with a 20 m difference in level as seen in Fig. 7.10, determine the vertical sag, which must be allowed such that the factor of safety shall be 5. Assume the wire weight to be 2 kg/m, ice loading to be 1 kg/m and wind loading to be 1.75 kg/m.

Solution:

The height difference, h = 20 m

Cross sectional area, a = 2 cm^2

Span length, L = 650 m

Weight of conductor, w_c = 2 kg/m

Weight of conductor due to ice loading, w_i = 1 kg/m

Weight of conductor due to wind, w_w = 1.75 kg/m

Ultimate tensile strength = 10000 kg/cm^2

= 10000 × 2 = 20000 kg

Maximum allowable tension = 20000/5 = 4000 kg

Resultant force per metre length, $w_r = \sqrt{(w_c + w_i)^2 + w_w{}^2} = \sqrt{(2+1)^2 + (1.75)^2}$

$= 3.473$ kg/m.

Distance of lowest point of conductor from the lowest support level, $x_1 = \dfrac{L}{2} - \dfrac{T \times h}{w_r \times L}$

$$= \frac{650}{2} - \frac{4000 \times 20}{3.473 \times 650}$$

$$= 325 - 44.296$$

$$= 289.56 \text{ m}$$

Sag, $D = \dfrac{w_r (x_1)^2}{2T} = \dfrac{3.473 \times 289.56^2}{2 \times 4000} = 36.4$ m

The vertical sag, $D' = \dfrac{(w_c + w_i)(x_1)^2}{2T} = \dfrac{(2+1) \times (289.56)^2}{2 \times 4000} = 31.44$ m.

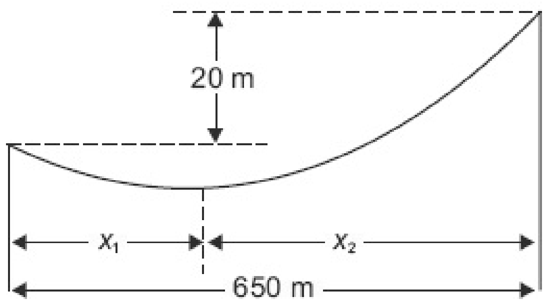

Fig. 7.10 Unequal supports

EXAMPLE 7.11

An overhead line of cross-section 2.5 cm^2 is supported at a river crossing from two towers at heights of 50 m and 100 m above the water level. The horizontal distance between the towers is 400 m. If the maximum stress in the line does not exceed 1/5th of UTS of 4000 kg/cm^2 and weight of the conductor is 0.9 kg/m, find the clearance between the conductor and water at a point mid-way between the towers.

Solution:

Cross-sectional area of conductor, a = 2.5 cm^2

Span length, L = 400 m

Ultimate tensile stress $= 4000 \text{ kg/cm}^2$

$$= 4000 \times 2.5 = 10000 \text{ kg}$$

Factor of safety $= 5$

Allowable tension, $T = \frac{1}{5}$ (UTS)

$$= \frac{1}{5} \times 10000 = 2000 \text{ kg}$$

Weight of the conductor, w_c = 0.9 kg/m

Level difference, $h = D_2 - D_1 = \frac{w}{2T}(x_2{}^2 - x_1{}^2) = 50 \text{ m}$

and, $x_1 + x_2 = L = 400$ m

$$(x_2 - x_1)(x_2 + x_1) = \frac{2 \times 2000 \times 50}{0.9} = 222222.2$$

$\therefore x_2 - x_1 = 555.6$ m

and $x_2 + x_1 = 400.00$ (given)

$$\therefore x_2 = \frac{955.5}{2} = 477.8 \text{ m}$$

And, $x_1 = 400 - x_2 = -77.8$ m

$$\therefore D_1 = \frac{wx_1^2}{2T} = \frac{0.9 \times (77.8)^2}{2 \times 2000} = 1.362 \text{ m}$$

$$D_2 = \frac{wx_2^2}{2T} = \frac{0.9 \times (477.8)^2}{2 \times 2000} = 51.367 \text{ m}$$

Clearance of water and lower point on the conductor is = 50 – 1.362 m = 48.638 m

Distance from mid-point between two supports and the lower point on the conductor = 277.78 m

Sag at mid point of the supports, $D = \frac{wx_2^2}{2T} = \frac{0.9 \times 277.78^2}{2 \times 2000} = 17.364 \text{ m}$

Clearance of water to conductor at mid way between two conductors = 48.638 + 17.364 = 66 m.

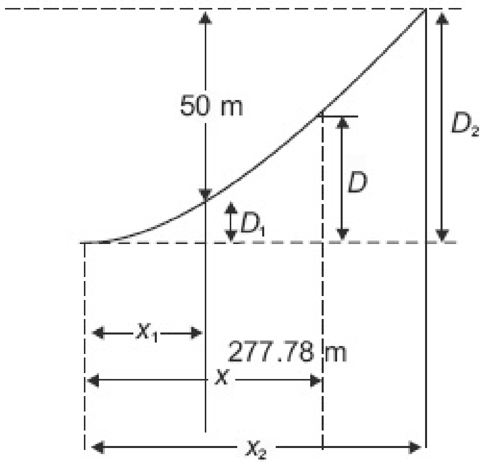

Fig. 7.11 Unequal supports

EXAMPLE 7.12

Two towers of height 40 m and 90 m, respectively support a transmission line conductor at a water crossing. The horizontal distance between the towers is 500 m. If the allowable tension in the conductor is 1600 kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.1 kg/m. Bases of the towers can be considered to be at water level.

Solution:

Level difference, $h = D_1 - D_2 = 50$ m

Span length, $L = 500$ m

$$= x_2 + x_1 = 500 \qquad (1)$$

Weight of the conductor, $w_c = 1.1$ kg/m

Tension in the conductor, $T = 1600$ kg

$$\text{Level difference, } h = D_2 - D_1 = \frac{w}{2T}(x_2^2 - x_1^2) = 50$$

$$\therefore x_2{}^2 - x_1{}^2 = \frac{50 \times 2 \times 1600}{1.1} = 145454.5$$

$$\text{or } x_2 - x_1 = \frac{145454.5}{500} = 291 \qquad (2)$$

From Eqs. (1) and (2)

$$x_2 = 395.5 \text{ and } x_1 = 104.5 \text{ m}$$

$$\therefore \text{Sag, } D_1 = \frac{wx_1^2}{2T} = \frac{1.1 \times 104.5^2}{2 \times 1600} = 3.754 \text{ m}$$

Minimum clearance from water level = 40 – 3.754 = 36.246 m

Distance between mid-point of the supports and lower point of the conductor, x = 250 – 104.5 = 145.5 m

$$\text{Sag at mid point of the supports, } D = \frac{wx_2^2}{2T} = \frac{0.9 \times 277.78^2}{2 \times 2000} = 17.364 \text{ m}$$

∴ Clearance between the mid-way between supports and ground level = 36.246 + 7.277 = 43.52 m.

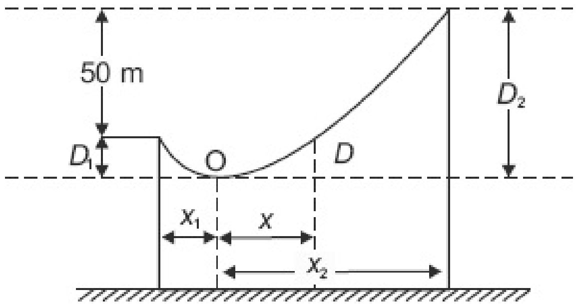

Fig. 7.12 Unequal supports

EXAMPLE 7.13

An overhead conductor having an ultimate strength of 8000 kg/cm^2 and an area of 2 cm^2 is erected between supports placed 600 m apart and having a level difference of 15 m. If the minimum ground clearance is to be 40 m, find the tower heights. The conductor is subjected to a horizontal wind pressure of 1.5 kg/m. The self-weight of the conductor is 1.75 kg/m. Assume a safety factor of 4.

Solution:

Cross-sectional area, a = 2 cm^2

Ultimate tensile stress = 8000 kg/cm^2

∴ Ultimate tensile stress = 8000 × 2 = 16000 kg

$$\text{Allowable tension, } T = \frac{16000}{4} = 4000 \text{ kg}$$

Clearance = 40 m

Span length, L = 600 m

Self weight of conductor, w_c = 1.75 kg/m

Weight of conductor due to wind, W_w= 1.5 kg/m

$$\therefore \text{Resultant weight, } w_r = \sqrt{w_c^2 + w_w^2} = \sqrt{1.75^2 + 1.5^2} = 2.305 \text{ kg}$$

$$\text{Span length } L, x_2 + x_1 = 600 \qquad (1)$$

$$\text{Level difference, } h = 15 = \frac{w_r}{2T}\left(x_2^2 - x_1^2\right)$$

$$x_2^2 - x_1^2 = \frac{15 \times 2 \times 4000}{2.305} = 52063.31$$

$$\text{or } x_2 - x_1 = \frac{52063.31}{600} = 86.77 \text{ m} \qquad (2)$$

From Eqs. (1) and (2)

$$x_2 = \frac{600 + 86.77}{2} = 343.39 \text{ m}$$

$$x_1 = 600 - 343.39 = 256.61 \text{ m}$$

$$\therefore \text{Sag with respect to lower level support, } D_1 = \frac{wx_1^2}{2T} = \frac{2.305 \times 256.61^2}{2 \times 4000} = 18.97 \text{ m}$$

The height of the lower level tower = 40 + 18.97 = 58.97 m

∴ The height of the second tower = 58.97 + 15 = 73.97 m.

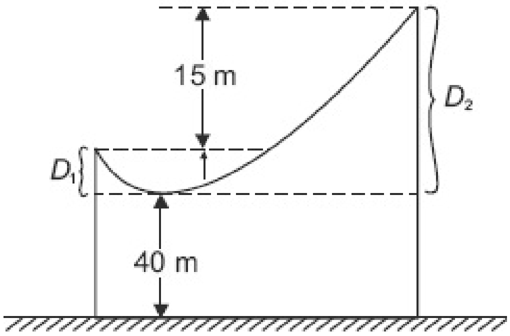

Fig. 7.13 Unequal supports

EXAMPLE 7.14

An overhead line is supported between two towers 300 m apart having a difference on their levels equal to 4 m. Calculate the sag if the wind pressure is 40 kg/m^2 of projected area and the safety factor is 2.The conductor data is nominal copper of area 11 cm^2. A stranded wire of diameter 30 + 7/2.79 cm, steel cored aluminium of weight 844 kg/km and ultimate tensile strength of 7950 kg is used.

Solution:

The diameter of stranded conductor is given = $(2n + 1)d$

where,

d is the diameter of the strand

and n is the number of the layers

Therefore, $3n^2 + 3n + 1$ = number of strands

$3n^2 + 3n + 1 = 37$

$n^2 + n - 12 = 0 \Rightarrow n = 3$

The number of strands in different layers excluding the central core are 6, 12 and 18.

Span length, $L = 300$ m

Level difference, $h = 4$ m

∴ Overall diameter of conductor, $d = (2n + 1)2.79$

$= (2 \times 3 + 1)2.79 = 19.53$ mm

Cross-sectional area of conductor, $a = \frac{\pi}{4}d^2 = 3 \text{ cm}^2$

Ultimate tensile stress = 7950 kg

$$\text{Allowable tension} = \frac{7950}{2} = 3975 \text{ kg}$$

Self-weight of conductor, W_c = 844 kg/km

Weight of conductor due to wind pressure, $W_w = 40 \times 1.953 \times 10^{-2}$
$= 0.7812$ kg/m

Result weight of conductor, $w_r = \sqrt{w_c^2 + w_w^2} = \sqrt{0.844^2 + 0.7812^2} = 1.15$ kg/m

Span length L, $x_2 + x_1 = 300$ m (1)

$$\text{Level difference, } h = D_2 - D_1 = \frac{w_r}{2T}(x_2^2 - x_1^2) = 4 \text{ m}$$

$$x_2^2 - x_1^2 = \frac{4 \times 2 \times 3975}{1.15} = 27651$$

$$\therefore x_2 - x_1 = \frac{27651}{300} = 92.17 \text{ m} \quad (2)$$

From Eqs. (1) and (2)

$$x_2 = \frac{300 + 92.17}{2} = 196.1 \text{ m}$$

$$\text{and, } x_1 = 300 - 196.1 = 103.9 \text{ m}$$

$$\text{Sag, } D_1 = \frac{wx_1^2}{2T} = \frac{1.15 \times 103.9^2}{2 \times 3975} = 1.562 \text{ m}$$

$$\text{Similarly, } D_2 = \frac{1.15 \times 196.1^2}{2 \times 3975} = 5.563 \text{ m.}$$

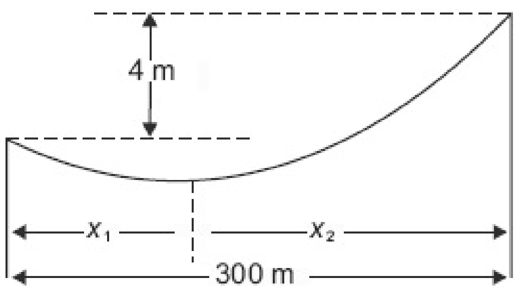

Fig. 7.14 Unequal supports

EXAMPLE 7.15

An overhead line has the following data (as illustrated in Fig. 7.15): span length 185 m, difference in levels of supports 6.5 m, conductor diameter 1.82 cm, weight per unit length of conductor 1.5 kg/m, wind pressure 39 kg/m^2 of projected area. Maximum tensile strength of the conductor is 4250 kg/cm^2, safety factor is 5. Calculate the length of the lower support.

Solution:

Span length, L = 185 m

Difference of level supports, h = 6.5 m

Conductor diameter, d = 1.82 cm

Weight per meter length of wire, w_c = 1.5 kg/m

Wind pressure of projected area = 39 kg/m^2

Weight of conductor due to wind pressure is,

$$w_w = 39 \times 1.82 \times 10^{-2}$$
$$= 0.7098 \text{ kg/m}$$

Resultant weight of conductor is

$$w_r = \sqrt{\left(w_c^2 + w_w^2\right)} = \sqrt{1.5^2 + 0.7098^2} = 1.6595 \text{ kg/m}$$

Maximum tensile strength of the conductor = 4250 kg/cm^2

Safety factor = 5

Area of cross section, $a = \frac{\pi d^2}{4} = \frac{3.14 \times (1.82)^2}{4} = 2.6 \text{ cm}^2$

Allowable tension, $T = \frac{\text{maximum stress} \times \text{area}}{\text{factor of safety}} = \frac{4250 \times 2.6}{5} = 2210 \text{ kg}$

$$x_1 = \frac{L}{2} - \frac{Th}{w_r L}$$

$$= \frac{185}{2} - \frac{2210 \times 6.5}{1.6595 \times 185}$$

$$= 45.71 \text{ m}$$

Now, $x_2 = \frac{L}{2} + \frac{Th}{w_r L}$

$$= \frac{185}{2} + \frac{2210 \times 6.5}{1.6595 \times 185}$$

$$x_2 = 139.29 \text{ m}$$

Sag with respect to lower support, $D_1 = \frac{w_r x_1^2}{2T} = \frac{1.6595 \times (45.71)^2}{2 \times 2210} = 0.7845 \text{ m}$

Sag with respect to higher support, $D_2 = \frac{w_r x_2^2}{2T} = \frac{1.6595 \times (139.29)^2}{2 \times 2210} = 7.2844 \text{ m}.$

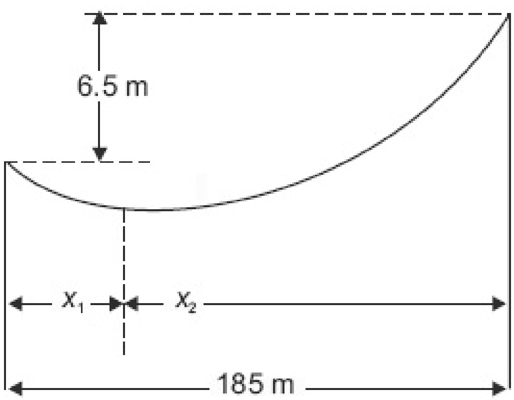

Fig. 7.15 Unequal supports

EXAMPLE 7.16

A 132 kV transmission line uses ACSR conductors whose data are: nominal copper area 110 mm^2, size 30 + 7/2.79 mm, weight 844 kg/km, ultimate strength 7950 kg. Calculate the height above ground at which the conductors with a span of 250 m should be supported, the safety factor being 2. Wind pressure 90 kg/m^2 of projected area. Ground clearance required is 7 m.

Solution:

Weight of conductor, w = 844 kg/km

Cross-sectional area, a = 110 mm^2

Ultimate strength = 7950 kg

Span length, L = 250 m

Safety factor = 2

Wind pressure = 90 kg/m^2

Ground clearance = 7 m

Diameter of conductor, $d = \sqrt{\frac{110 \times 4}{\pi}}$

$= 11.83$ mm

Weight of the conductor, w_c = 844 kg/km

= 0.844 kg/m

Allowable tension, $T = \frac{7950}{2} = 3975$ kg

Weight of the conductor due to wind pressure, $w_w = 90 \times 11.83 \times 10^{-3}$

= 1.065 kg/m

Resultant weight, $w_r = \sqrt{0.844^2 + 1.065^2} = 1.359$ kg

Sag, $D = \frac{w_r L^2}{8T} = \frac{1.359 \times 250^2}{8 \times 3975} = 2.67$ m

Height of the supports = 7 + 2.67 = 9.67 m.

7.5 STRINGING CHART

For the erection of a transmission line, the designer has to design the line so that it is able to withstand the worst probable conditions. Hence, the designer must know the sag and the tension in the overhead line to be allowed.

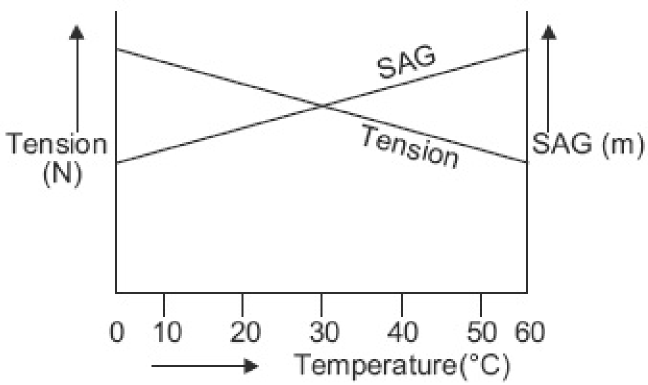

Fig. 7.16 Stringing chart

Figure 7.16 shows the effect of temperature on the sag and the tension of a given transmission line under loading conditions. This graph is called a stringing chart.

From the stringing chart, the sag and tension, can be obtained at any given temperature. The data for sag and tension to be allowed at a particular temperature too can be determined from this chart. When erecting transmission line conductors, the correct adjustment of sag and tension can, therefore, be ensured by consulting a stringing chart.

The expression for determining a stringing chart of a line is derived as below:

Let w_1 be the load per unit length, f_1 be the stress, s_1 be the conductor length in half span, D_1 be the sag and t_1 be the temperature at maximum load conditions (with the ice, wind and a low temperature of usually −5.5 °C). w_2, f_2, S_2, D_2 and t_2 will be the values under stringing conditions. a is the area of cross-section of the conductor, α is the coefficient of linear expansion and E is the modulus of elasticity.

$$T = H;\ y = \frac{wx^2}{2T};\ D = \frac{wl^2}{2T} \text{ and } s = l + \frac{w^2l^3}{6T^2} \tag{7.39}$$

$$f = \frac{T}{a}, \text{ so that } s_1 = l + \frac{w_1^2 l^3}{6f_1^2 a^2} \tag{7.40}$$

The effect of increase of temperature from t_1 to t_2 is the reduction in stress from f_1 to f_2. The decrease in length on this account is

$$\frac{f_1 - f_2}{E} s_1 = \frac{f_1 - f_2}{E} l$$

The new length s_2 is

$$s_2 = s_1 + (t_2 - t_1)\alpha l - \frac{f_2 - f_1}{E} l \tag{7.41}$$

$$\text{Byt } s_2 = l + \frac{w_2^2 l^3}{6 f_2^2 a^2} \tag{7.42}$$

Substituting s_1 and s_2 from Eqs. (7.40) and (7.42) in Eq. (7.41)

$$l + \frac{w_1^2 l^3}{6 f_1^2 a^2} + (t_2 - t_1)\alpha l - \left(\frac{f_1 - f_2}{E}\right) l = l + \frac{w_2^2 l^3}{6 f_2^2 a^2}$$

$$\text{or } \frac{w_1^2 l^2}{6 f_1^2 a^2} + (t_2 - t_1)\alpha - \left(\frac{f_1 - f_2}{E}\right) = \frac{w_2^2 l^2}{6 f_2^2 a^2}$$

$$\text{or } \frac{w_1^2 l^2 E}{6 f_1^2 a^2} + (t_2 - t_1)\alpha E + (f_2 - f_1) = \frac{w_2^2 l^2 E}{6 f_2^2 a^2}$$

$$\text{or } f_2^2 \left[f_2 - f_1 + (t_2 - t_1)\alpha E + \frac{w_1^2 l^2 E}{6 f_1^2 a^2} \right] = \frac{w_2^2 l^2 E}{6 a^2} \tag{7.43}$$

This equation is a cubic equation and can be solved graphically or analytically and knowing the value of f_2, D_2 can be determined

$$\because D_2 = \frac{w_2 l^2}{2 T_2} = \frac{w_2 l^2}{2 f_2 a} \tag{7.44}$$

Test Yourself

1. How is a stringing chart useful for electrical field engineers?

EXAMPLE 7.17

An overhead line has a conductor diameter of 1.6 cm and is erected across a span of 200 m on level supports. The radial thickness of ice under severe conditions is 1.25 cm and the dead weight of the conductor is 0.7 kg/m run. The ultimate stress of the conductor is 7000 kg, the modulus of elasticity is 7.5×10^5 kg/cm^2 and its coefficient of linear expansion is 16.5 $\times 10^{-6}$/°C. Assume a wind pressure of 39 kg/m^2 and ice covering at temperature of –5.0° C as the worst conditions, a safety factor of 2 being required under these conditions. The weight of ice is 913.5 kg/m^3. Find the sag in still air at the time of erection when the temperature is 35° C.

Solution:

Conductor diameter, d = 1.6 cm

Span length, L = 200 m

Half span length, l = 100 m

Radial thickness, t = 1.25 cm

The ultimate stress of conductor = 7000 kg

Modulus of elasticity, $E = 7.5 \times 10^5$ kg/cm^2

Coefficient of linear expansion = 16.5×10^{-6}/°C

Wind pressure, p = 39 kg/m^2

Safety factor = 2

The weight of ice, w_i = 913.5 kg/m^3

Dead weight of the conductor, w_c = 0.7 kg/m

Weight of conductor due to wind pressure, $w_w = p(d + 2t)$

$$= 39\left(\frac{1.6}{100}+\frac{2\times 1.25}{100}\right)=1.599 \text{ kg/m}$$

Weight of the conductor due to ice loading, $w_i = \rho \pi t\,(d + t)$

$$= 913.5 \times \pi \times \frac{1.25}{100}\left(\frac{1.6\ +\ 1.25}{100}\right) = 1.022 \text{ kg/m}$$

Resultant weight, $w_r = \sqrt{(w_c + w_i)^2 + w_w^2}$

$$= \sqrt{(0.7+1.022)^2 + 1.599^2} = 2.35 \text{ kg/m}$$

Area of cross-section of conductor

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (1.6)^2}{4} = 2.0106 \text{ cm}^2$$

Now we have, $t_2 = 35°$ C, $t_1 = -5°$ C, $\alpha = 16.5 \times 10^{-6}$ /°C

$E = 7.5\ 10^5$ kg/cm²,

$w_2 = w_c = 0.7$ kg/m

$$T_1 = \frac{7000}{2} = 3500 \text{ kg}$$

Let us write $f = \frac{T}{A}$ and $t = t_2 - t_1$

We know that

$$T_2^3 + T_2^2\left[AE\left(\frac{w_1^2 l^2}{6T_1^2}+\alpha t\right)-T_1\right] = \frac{w_2^2 l^2 AE}{6}$$

Substituting the given values, we get

$$T_2^3 + T_2^2\left[2.0106\times 7.5\times 10^5\left(\frac{2.35^2\times 100^2}{6\times 3500^2}+16.5\times 10^{-6}\times 40\right)-3500\right]$$
$$= \frac{0.7^2\times 100^2\times 2.0106\times 7.5\times 10^5}{6}$$

$$T_2^3 - 1371.74\ T_2^2 = 1231495200$$

Solving the above equation, we get T_2 = 1766.42, – 197.34, – 197.34 kgs

Sag at the time of erection,
$$D_2 = \frac{w_2 l^2}{2T_2}$$
$$= \frac{0.7\times 100^2}{2\times 1766.42} = 1.98 \text{ m.}$$

7.6 EFFECTS AND PREVENTION OF VIBRATIONS (VIBRATIONS AND DAMPERS)

The overhead line experiences vibrations in the vertical plane. These vibrations are of two types (i) aeoline vibrations (ii) galloping vibrations. Normal swinging of conductors due to wind force is harmless if the clearance between conductors is large.

Aeoline Vibrations Aeoline vibrations are also known as resonant vibrations. These vibrations are of high frequency and low amplitude with a loop length of 1-10 m depending upon the values of tension T, weight per unit length wand stress f. Aeoline vibrations are common to all conductors and are more or less always present in overhead lines. The harmful effects of these vibrations occur at clamps or supports due to which the conductor breaks.

Galloping Vibrations Galloping vibrations are also known as dancing vibrations. The dancing vibrations are low frequency vibrations which occur in the conductors when there are sleet storms accompanied by strong winds. The amplitude of these vibrations is about 6 m so that the conductors start to "dance". This dancing occurs horizontally and vertically, and therefore, the operation of conductors without touching one another becomes difficult. The conductors follow elliptical pathways. These vibrations occur due to the irregular coating of sleet and the stranding of conductors. There is no technique for the prevention of these vibrations. However, a horizontal conductor configuration is preferred for reducing these vibrations. The resonant vibrations which reach the conductors at the clamps or supports are prevented by using dampers.

The stock-bridge damper as shown in Figs. 7.17 and 7.18 consists of two weights attached to a piece of stranded cable, which is clamped to the line conductor. The energy of the vibrations is absorbed by the stranded cable and hence the vibration is damped out.

Another type of damper consists of a box containing a weight, resting on a spring. In this case, the spring absorbs the energy of the vibration.

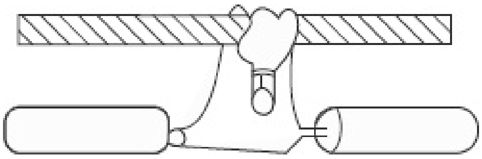

Fig. 7.17 Stock damper

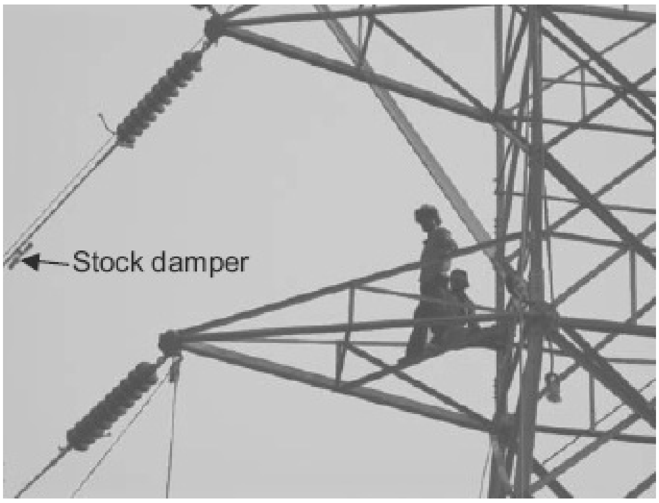

Fig. 7.18 Stock damper

Test Yourself

1. In practice, is it possible to avoid vibrations by using dampers? If yes, how?

7.7 SAG TEMPLATE

At the time of planning, sag template is necessary to decide the location of the towers along the route of transmission lines. It is prepared either on celluloid or upon tracing cloth for a particular conductor and tower height instead of calculating the sag for every span on a straight line.

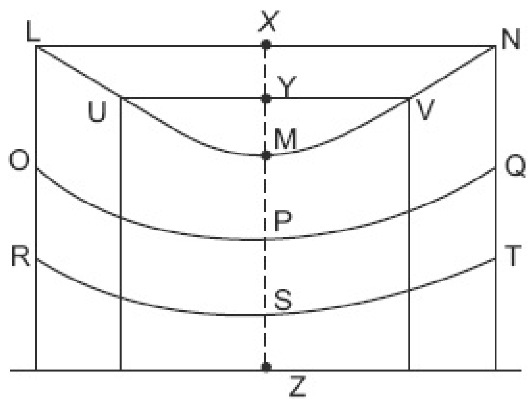

Fig. 7.19 Sag template curve

Sag template curve shown in Fig. 7.19 provides the following:

1. The horizontal and vertical distances indicate span length and sag, respectively
2. The upper curve LMN represents conductor length in the span length of LN; the maximum sag of the conductor is XM. For the span length UV, the maximum sag is YM
3. The middle line OPQ is below the upper curve LMN by uniform vertical distance which is equal to the desired minimum vertical clearance to ground
4. The lower curve RST is below the top one by uniform vertical distance equal to the height of standard tower measured to the point of support of the lowest conductor and it indicates the tower footing line.

If the location of the left tower is decided, the location of the right hand tower can be determined by adjusting the sag template.

7.8 CONDUCTOR SPACING AND GROUND CLEARANCE

The spacing of conductors is decided not only by the electrical consideration of the working voltage, but also by mechanical factors such as length of span, weight of conductors, prevalent wind direction, etc. With the increase of sag, the magnitude of the swing also increases and proper care is to be taken to maintain clearance under unfavourable condition. Figure 7.20 shows the details of spacing. For suspension insulators, a greater spacing is required to allow for the swing of the insulator string.

For HT or EHV lines, with a voltage level of 66 kV or above the conductor spacing depends upon the configuration of the conductors.

Minimum Clearance The clearance of line conductors from the ground and building at different places according to IE rules as given in Table 7.2.

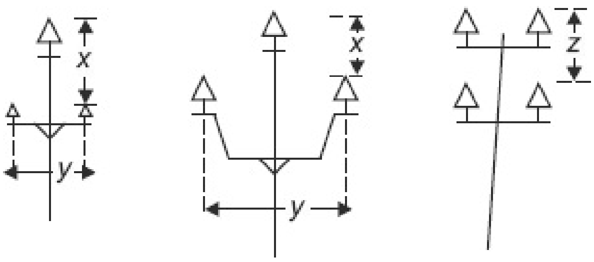

Fig. 7.20 Spacing between conductors

Table 7.2 Spacing between conductors for different voltages

Voltage/spacing	11 kV	33 kV	66 kV
x	0.914 m	1 m	1.8 m
y	1.35 m	1.5 m	2.4 m
z	0.914 m	1.05 m	1.8 m

Table 7.3 Clearance along across the road and cross country (IE Rule No. 77)

Voltage	Min. height of over line conductor across any street	Min. height of over line conductor along any street	Min. height of over line conductor erected elsewhere
LT and medium (250 V and 250 – 650 V)	5.791 m	5.486 m	4.572 m
HT (650 V to 33 kV)	6.07 m	5.79 m	5.55 m
EHV (above 33 kV)	5.18 m + For every 33 kV or part of it 0.305 m However, the min. is 6.07 m	5.18 m + For every 33 kV or part of it 0.305 m However, the min. is 6.07 m	5.18 m + For every 33 kV or part of it 0.305 m However, the min. is 6.07 m

Table 7.4 Clearances from building (IE Rule No. 79)

Voltage	Vertical clearance (m)	Horizontal clearance (m)
LT	2.438	12.218
High voltage up to 11 kV	3.658	1.21
High voltage above 33 kV	3.658	1.829
For EHV line (for every additional 33 kV)	0.305	0.305

Source: A.P. Transco Engineers' association dairy in 2007.

CHAPTER AT A GLANCE

1. **Line Supports:** The supports for an overhead transmission line that are equipped to carry the load of the conductors on the transmission line even with the additional weight caused by ice deposition and wind pressure.
2. The various types of line supports are wooden poles, tubular steel poles, RCC poles and latticed steel towers.
3. **Sag:** The difference in level between the points of supports and the lowest point on the conductor is known as sag.
4. The factors affecting the sag in an overhead line are the weight of the conductor, the length of the span, the working tensile strength of the conductor and the temperature.
5. Sag at equal supports is given by

$$D = \frac{wl^2}{2T}$$

where, l is the half-span length

also sag, $D = \frac{wL^2}{8T}$

where, L is the span length.

6. Sag at different level supports

$$x_2 = \frac{L}{2} + \frac{Th}{wL}$$
$$x_1 = \frac{L}{2} - \frac{Th}{wL}.$$

7. **Stringing charts:** The curves that represent the following relations: tension vs. temperature and sag vs. temperature are called stringing charts. Such a chart is helpful in knowing the sag and tension at any given temperature, thus helping in ensuring the correct adjustment of sag and tension when setting up transmission lines.
8. **Sag template:** It is prepared either on celluloid or upon tracing cloth for a particular conductor and tower height instead of calculating the sag for every span on a straight line.
9. **Vibrations and dampers**: The overhead line experiences vibrations of two types in the vertical plane. In addition to normal swinging of conductors due to wind force, they are (i) aeoline vibrations (ii)galloping vibrations.

   Dampers are used for reducing these conductor vibrations.

SHORT ANSWER QUESTIONS

1. What are the different types of line supports?
2. What are the main components of overhead lines?
3. What are the different types of conductor materials used for overhead lines?
4. Define line supports.
5. Define sag.
6. What are the factors affecting sag?
7. Write down the expression for working stress.
8. Write down the expression for vertical sag.

9. Write down the expression for wind force/metre length.
10. Define stringing charts.
11. Write down the expression for sag when supports are at equal levels.
12. Write down the different types of steel towers.
13. Define span.
14. What is the use of sag template?
15. How can you reduce the vibration of the conductors?
16. What are the disadvantages of loose span?

MULTIPLE CHOICE QUESTIONS

1. An overhead line may be used to ______________ electric power.
    1. transmit
    2. distribute
    3. either a or b
    4. both a and b
2. The main components of overhead lines are _______________.
    1. conductors and supports
    2. insulators
    3. cross arms
    4. all of these
3. The most commonly used conductor materials are _______________.
    1. copper and aluminium
    2. steel-cored aluminium
    3. galvanized steel
    4. all of these
4. Stranded conductor materials are used in order to ______________ the flexibility.
    1. increase
    2. decrease
    3. constant
    4. none of these
5. The purpose of a line support is to provide_______________.
    1. high mechanical strength
    2. less weight
    3. low cost
    4. all of these
6. Wooden poles, steel poles, RCC poles are the various kinds of ______________.
    1. conductors
    2. insulators
    3. line supports
    4. none of these
7. For long distance transmission at higher voltage _______________ are employed.
    1. wooden poles

2. steel poles
3. RCC poles
4. steel towers

8. The difference in level between points of supports and lowest point on the conductor is called
   1. tension
   2. sag
   3. length
   4. none of these
9. Formula for sag when supports are at equal level is ____________.
   1. $\frac{wL^2}{8T}$
   2. $\frac{wl^2}{6T}$
   3. $\frac{wl}{8T}$
   4. $\frac{wl^2}{8T}$
10. A 132 kV transmission line has a conductor weight of 680 kg/km, a length of span of 260 m, an ultimate strength of 3100 kg and a safety factor of 2. The sag value is _______________.
    1. 3 m
    2. 4 m
    3. 3.7 m
    4. 3.5 m
11. A 132 kV transmission line has a conductor weight of 680 kg/km, a length of span of 260 m, an ultimate strength of 3100 kg, a safety factor of 2, and a ground clearance of 10 m. The height of the conductor is __________________.
    1. 10 m
    2. 3.7 m
    3. 13.7 m
    4. 6.3 m
12. The formula for working tension is ____________.
    1. $\frac{\text{Safety factor}}{\text{Ultimate strength}}$
    2. $\frac{\text{Ultimate strength}}{\text{Safety factor}}$
    3. both
    4. none of these

13. A 100 kV transmission line has an ultimate strength of 3000 kg, a safety factor of 3 and a working tension of ____________________ kg.
    1. 1000
    2. 2000
    3. 3000
    4. 4000
14. If the sag of transmission line increases, tension ____________.
    1. increases
    2. decreases
    3. remains constant
    4. none of these
15. The span of transmission line between towers take the from of ____________.
    1. hyperbola
    2. parabola
    3. catanary
    4. ellipse
16. String chart is useful in ______________.
    1. finding the sag in the conductor
    2. the design of tower
    3. the design of the insulator string
    4. finding the length of conductor
17. The sag template is required for ________________.
    1. footing of towers
    2. finding the sag in the conductor
    3. design of insulator
    4. none of these
18. Formula for vertical sag
    1. $D\sin\theta$
    2. $D\cos\theta$
    3. $D\tan\theta$
    4. $D\sec\theta$
19. Sag is provided in overhead lines so that _______________.
    1. safe tension is not exceeded
    2. span length is reduced
    3. length of the conductor is decreased
    4. all the above
20. An overhead line has a weight of conductor of 1.78 kg/m and a wind force of 1.5 kg/m. The total weight per metre is then ________________ kg.
    1. 2
    2. 3
    3. 2.33
    4. 2.63
21. Spacing of conductors should provide safety against ______________.
    1. flash over
    2. light over
    3. tension
    4. voltage

22. To protect the conductors _______________ are used.
    1. springs
    2. dampers
    3. vibrators
    4. none of these
23. If an ACSR conductor has specification 48/7, it means that it has _____________ aluminium and _____________ steel strands.
    1. 7, 48
    2. 6, 48
    3. 48, 7
    4. 48, 6
24. The transmission line between towers assumes the shape of a _______________.
    1. parabola
    2. hyperbola
    3. catanary
    4. ellipse
25. In a power transmission line, the sag depends on ____________.
    1. conductor material
    2. tension in conductor
    3. transmission line span
    4. all of these

Answers:

1. c	2. d	3. b	4. a	5. d
6. c	7. d	8. b	9. a	10. c
11. c	12. b	13. a	14. b	15. b
16. a	17. a	18. b	19. a	20. c
21. a	22. b	23. c	24. c	25. d

REVIEW QUESTIONS

1. What is a sag template? Explain how this is useful for the location of towers and the stringing of power conductors.
2. What is a stringing chart? Explain clearly the procedure adopted for stringing the power conductors on the supports.
3. What are the factors affecting sag?
4. Deduce expressions for calculating sag and conductor length of an overhead line when the supports are at the same level.
5. What is sag? Derive the equation for sag when the conductor takes the form of a parabola, supported at equal levels.
6. Explain the necessity of a stringing chart for a transmission line and show how such a chart can be constructed.
7. Deduce expressions for (i) total length of conductor, and (ii) tension at the ends in terms of span length, horizontal tension, maximum sag, and weight of conductor per unit length.

8. Explain how sag is determined for an overhead line conductor taking into account the effects of wind and ice loading.
9. Show that the sag on level supported line conductor of span *L*, weight per unit length *W* kg, and minimum tension in the line conductor T_0 is given by

$$s = \frac{wL^2}{8T_0}$$

   What will be the sag if level difference is *h* metres?
10. Derive the expression for sag and tension when the supports are at unequal heights.
11. Describe the vibration of power conductors and explain the methods used to damp out these vibrations.

PROBLEMS

1. Determine the sag of an overhead line for the following data: span length 160 m, conductor diameter 0.95 cm, weight per unit length of the conductor 0.65 kg/m, ultimate stress 4250 kg/cm^2, wind pressure 40 kg/cm^2 of projected area and safety factor 5.
2. Determine the maximum sag of an overhead line conductor having a diameter of 19.5 mm and weight of 0.85 kg/m. The span length is 275 m, wind pressure is 40 kg/m^2 of projected area with ice coating of 13 mm. The ultimate strength of the conductor is 8000 kg, the safety factor is 2 and ice weighs 910 kg/m^3.
3. An overhead line has a conductor of cross-section 2.5 cm^2 and a span length of 150 m. Determine the sag which must be allowed if the tension is not to exceed one-fifth of the ultimate strength of 4175 kg/cm^2
   1. In still air
   2. With a wind pressure of 1.3 kg/m and an ice coating of 1.25 kg/m
   3. Determine also the vertical sag in the latter case
4. A transmission line conductor with diameter 14.77 mm, cross-sectional area of 120 mm^2 and weighing 1118 kg/km has a span of 200 m. The supporting structures being level, the conductor has an ultimate tensile stress of 42.2 kg/mm^2 and allowable tension not greater than 1/4th of the ultimate strength. Determine the following
   1. Sag in still air
   2. Sag with a wind pressure of 60 kg/m^2 and an ice coating of 10 mm. Also calculate the vertical sag under this condition. Assume density of ice as 0.915 gm/cc.
5. A transmission line conductor at a river crossing is supported by two towers at heights of 50 and80 m above water level. The

horizontal distance between the towers is 300 m. If the tension in the conductor is 2000 kg, determine the clearance between the conductors and water at a point midway between the towers.

6. An overhead line consisting of a copper conductor has the following data: area 0.484 cm^2, overall diameter 0.889 cm, weight 428 kg/km, breaking strength 1973 kg, safety factor 2 and spans length 200 m. Determine the maximum sag of the line for the following conditions:
   1. due to the weight of the conductor.
   2. due to additional weight of ice loading of 1 cm thickness.

8

Overhead Line Insulators

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Discuss the various types of overhead line insulators
- Understand the calculation of voltage distribution and string efficiency
- Discuss the methods to reduce string efficiency
- Discuss about testing of insulators

8.1 INTRODUCTION

Overhead line conductors are bare and not covered with any insulated coating. They are secured to the supporting structures by means of insulating fittings, called insulators. These insulators impede the flow of current from the conductors to the earth through the conductor supports. They provide insulation to the power conductors, passing through the clamps of the insulators, from the ground and are connected to the cross-arm of the supporting structure. Thus, insulators play an important role in the successful operation of overhead lines.

The insulators used must have the following requirements or properties:

1. Mechanically they must be very strong in order to withstand the weight of the conductors.
2. Dielectric strength must be very high.
3. They must have a high ratio of puncture strength to flash over voltage.
4. They should not be porous.
5. They must be free from internal impurities or flaws.
6. They must be impervious to the entry of gases or liquids into the materials.

7. They should not be affected by changes in temperature.

8.2 INSULATOR MATERIALS

The materials most commonly used for insulators of an overhead transmission line are porcelain, glass, steatite, and special composition materials that are used to a limited extent.

1. Porcelain Porcelain material is used extensively for the manufacture of insulators. It is produced by firing a mixture of 20% silica, 30% feldspar and 50% china clay at high temperature. It is mechanically stronger and costlier than glass. Dust deposits and temperature changes do not affect its surface. The dielectric strength of porcelain is 60 kV/cm of thickness. A single porcelain unit can be used up to 33 kV.

Manufacturing insulators using porcelain at low temperatures, improves its mechanical properties but at the same time the material remains porous.

2. Glass Glass insulators are used because of lower costs and high dielectric strength (140 kV/cm of thickness). The design is simple and even single piece of units can be obtained. It is very easy to detect any fault because of transparency. It can withstand high stresses when compared to porcelain and its thermal expansion is low. When properly annealed, glass has a high resistivity. The major disadvantage of glass is that moisture condenses more readily on its surface and facilitates the accumulation of dirt deposits, thus giving a high surface leakage on its surface and hence its use is limited to a voltage of about 33 kV.

3. Steatite Steatite is a magnesium silicate, which is a combination of magnesium oxide and silica in various proportions. It has a higher tensile strength and bending stress than that of porcelain. It is used, therefore, to manufacture insulators at tension towers or where the transmission line takes a sharp turn.

4. Composite Insulator Composite insulators also referred to as non-ceramic insulators (NCI) or polymer insulators are made of two components of different materials. These insulators normally employ insulator housings made of lighter weight polymeric materials such as ethylene propylene rubber (EPR), polytetrofluoroethylene (PTFE), silicon rubber, or other similar materials and used mostly in transmission systems. They generally consist of a fibreglass rod having a number of weather sheds constructed of a highly insulating polymeric material attached to the rod along its length. Composite insulators with shield layers of a synthetic material are most commonly used because of their lightweight structure and the hydrophobic property of the shield layers i.e., the insulators employed outdoors are therefore highly water repellent, which is conducive to repelling dirt and thus, to low-leakage current losses. Composite insulators are generally produced by preparing the screens individually and then fitting the required number of them onto a shank coated with extrudate and vulcanizing them with the coat, or by centrally placing a rod with a predetermined number of screens in a two-part mould and injecting all the screens at once.

Composite insulators for high voltages should satisfy certain specific electrical requirements.

They are:

- The carrier rod must be insulated electrically in its axial direction.
- There should be no electrical conduction between the insulating cover and the interior carrier rod.

The insulating cover used should have the following properties:

- It should prove resistant to the weather, UV, ozone, etc.
- It should have enough mechanical resistance to cold weather conditions.
- It should have enough electrical-tracking resistance.
- It should be flexible, halogen-free and flame retardant.

Test Yourself

1. Why are glass insulators increasingly being replaced with porcelain insulators?

8.3 TYPES OF INSULATORS

The insulators used for overhead transmission lines are:

1. Pin type
2. Suspension type
3. Strain type
4. Shackle type

8.3.1 Pin Type Insulators

A pin type insulator is small, simple in construction, and cheap. These insulators are mounted on the cross-arm of the pole. The line conductor is placed in the groove at the top of the insulator and is tied down with the binding wire of the same material as the conductor. Pin insulators do not take tension. This type of insulators is used for the transmission and distribution of electrical power up to 33 kV. Owing to higher voltages, they tend to be very heavy and more costly than suspension type insulators. The sectional view of a pin type insulators are shown in Figs. 8.1(a) and (b).

The leakage resistance of such an insulator is directly proportional to the length and inversely proportional to the width of the leakage path. Hence, after certain point, increasing the diameter of the rain sheds (petticoats) does not appreciably lead to the increase in the leakage resistance. Increase in the number of rain sheds increases the length of the path without necessarily increasing the width and therefore forms the best means of securing a high leakage resistance.

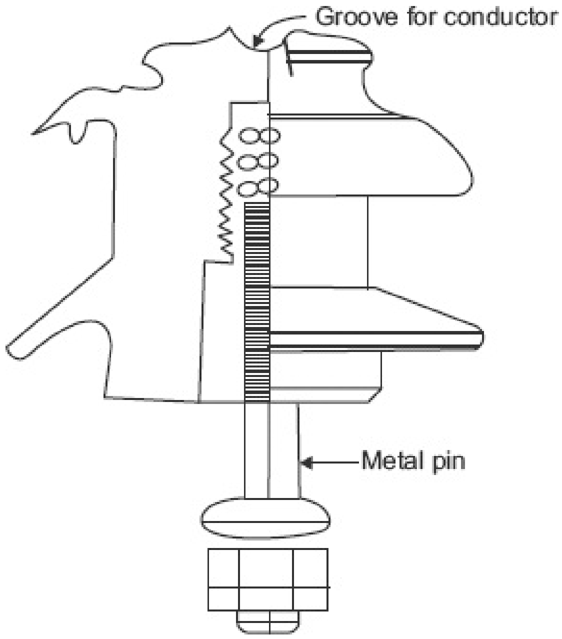

Fig. 8.1(a) Pin type insulators

Fig. 8.1(b) View of 33 kV pin insulator

The cost of the pin insulator beyond 50 kV is roughly given as

Cost α V^x, for $x > 2$

The main advantage of a pin type insulator is that, it is cheaper than the suspension type insulator for voltages below 50 kV. In practice, it is not used for more than 33 kV.

For these, the ratio of puncture strength to flash over voltage is known as safety factor. It is desirable that the value of safety factor is high so that the flash over takes place before the insulator gets punctured.

8.3.2 Suspension Type Insulator

For voltages above 33 kV, the suspension type or disc type insulator is used (Fig. 8.2). Number of separate discs are joined with each other by using metal links to form a string. The insulator string is suspended from the cross-arm of the support. The conductor is attached to the lowest unit of the string. Each unit of suspension type insulators is designed for comparatively low voltages, of about 11 kV and depending upon the operating voltage the number of units can be connected to series by links as shown in Figs. 8.3 and 8.4. For example, if the operating voltage is 132 kV, the number of units required is 12.

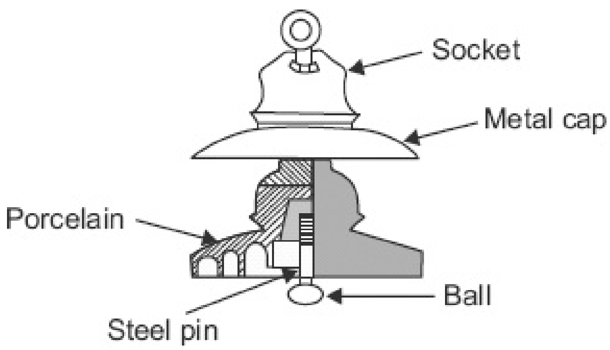

Fig. 8.2 Suspension type Insulator

There are three types of suspension insulators.

They are:

1. Hewlett or interlinking type
2. Cemented cap type
3. Core and link type

The most commonly used type is the cemented cap type of insulator as shown in Fig. 8.2.

Fig. 8.3 Two 11 kV disc insulators

Fig. 8.4 Suspension type seven discs insulator

The main advantages of suspension type over pin type insulators are:

1. The cost of suspension type insulators for operating voltages above 50 kV is less when compared to that of pin type insulators (cost increases with increased voltages).
2. Each disc in a string is designed for a comparatively lower voltage (up to 11 kV).
3. In the event of the failure of an insulator disc in a string, only that particular disc requires replacement rather the whole string.
4. In this arrangement, mechanical stresses are reduced and the line enjoys more flexibility.
5. If any additional insulation is required to increase the line voltages, they can be obtained by providing more discs in series to the string.

The main disadvantage in this type of insulators is large amplitude because suspension type insulators require larger spacing between the conductors of the string when compared to the pin type insulators.

8.3.3 Strain Insulator

This type of insulator is used for handling the mechanical stresses at angle positions of the line i.e., corner or sharp curve, end of lines, intermediate anchor towers and long river-crossings. The discs of a strain insulator are the same as that of a suspension type insulator but are arranged on a horizontal plane. In such a circumstance, for low-tension (LT) lines, shackle insulators are used as a strain insulator, but for high-tension (HT) lines, strain insulators consisting of an assembly of suspension type insulators are used. On extra long spans i.e., at river crossings, two or more strings of strain insulators are used in parallel. The arrangement of the strain type insulators is shown in the Figs. 8.5(a) and (b).

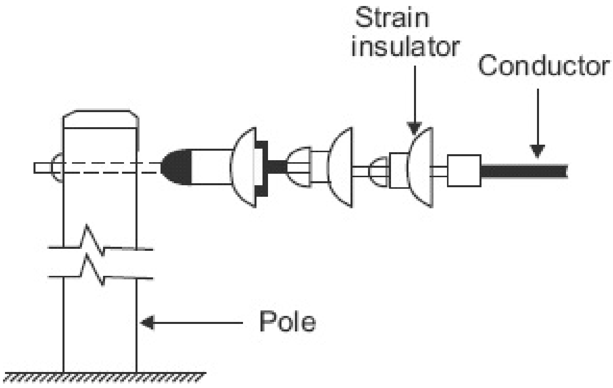

Fig. 8.5(a) Strain insulators

Fig. 8.5(b) Strain insulators

8.3.4 Shackle Type Insulator

It is used as a strain insulator and is shown in Fig. 8.6(a). These are frequently used for low-voltage distribution lines as shown in Fig. 8.6(b). The insulators are also used in section poles, end poles, on sharp curves and for service connections. These are bolted to the pole or the cross-arm. Both the low voltage conductors and the house service wires are attached to the shackle insulator.

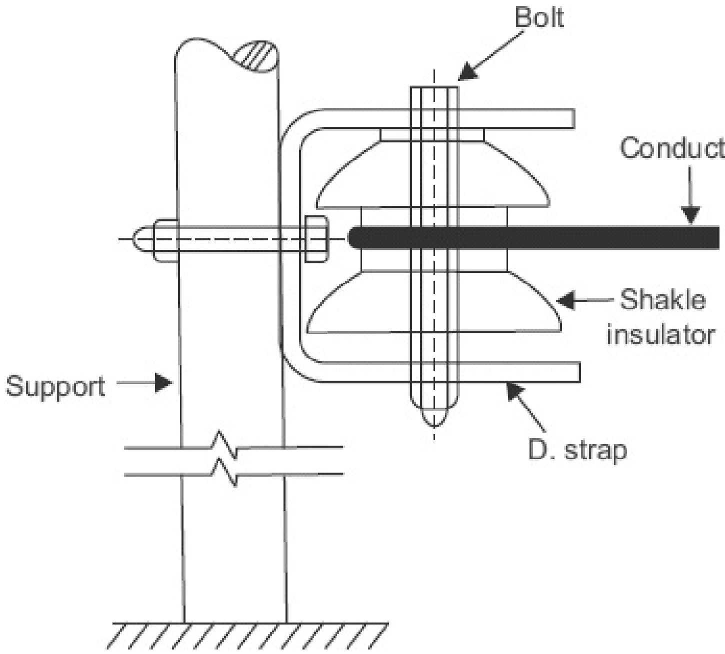

Fig. 8.6(a) Shackle type insulator

Fig. 8.6(b) View of shackle insulators

Test Yourself

1. Why are pin insulators not used for extra high voltages?

8.4 POTENTIAL DISTRIBUTION OVER A STRING OF SUSPENSION INSULATORS

The unit formed by connecting several discs in a series is known as a string of insulators. The insulators consist of metal fittings and the metal fitting of each unit has a capacitance relative to the metal fitting of the next unit. The capacitance due to two metal fittings on either side of an insulator is known as mutual capacitance. In addition, the capacitance between the metal fitting of each unit and the earth or the tower is known as shunt capacitance. In this method, the capacitance between the conductor and the metal link is neglected. Due to the shunt capacitance, the charging current in all the discs of a string is not equal. Therefore, the voltage across each unit will be different and the disc nearer to the line conductor will have the maximum voltage and minimum voltage across the top unit (near the cross arm).

8.4.1 Mathematical Expression for Voltage Distribution

Equivalent circuit of a string of suspension insulators (four units) is shown in Fig. 8.7.

$$\text{Let } m = \frac{\text{Capacitance to ground}}{\text{Mutual capacitance}} \tag{8.1}$$

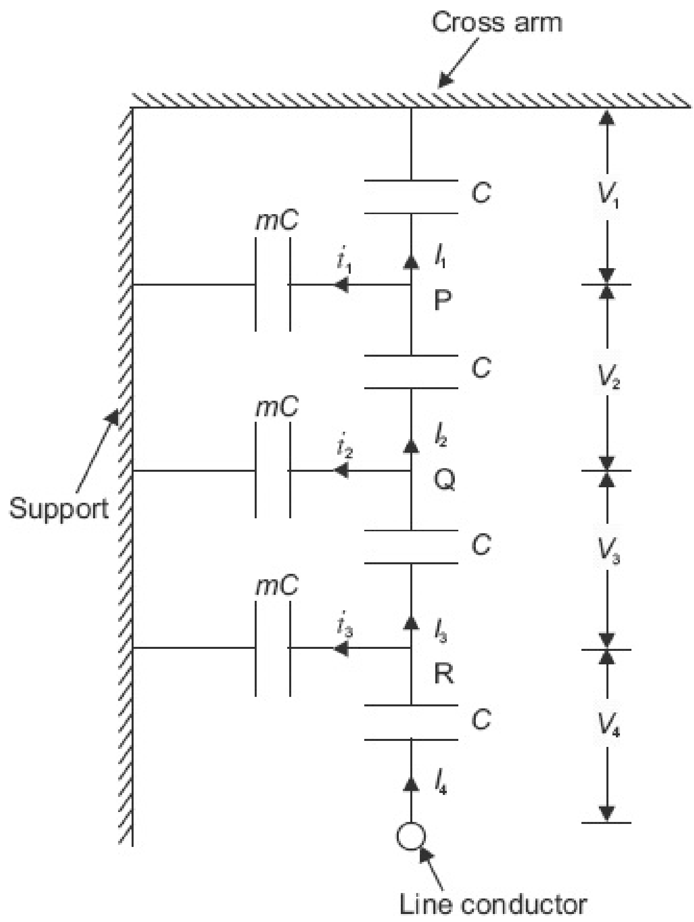

Fig. 8.7 Equivalent circuit of a string of suspension insulators

The mutual capacitance between the links be *C*; shunt capacitance between links and earth be *mC*;

The voltage across the first unit (nearest to the cross-arm) be V_1;

Voltage across the second, third, and fourth units are V_2, V_3, V_4, respectively;

and the voltage between conductor and earth (operating voltage) is *V*.

Applying Kirchhoff's Current Law (KCL) at node P

$$I_2 = I_1 + i_1 \tag{8.2}$$
$$V_2 \omega C = V_1 \omega C + V_1 m \omega C$$
$$V_2 = (1+m)V_1 \tag{8.3}$$

Applying KCL at Q,

$$I_3 = I_2 + i_2 \tag{8.4}$$
$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) m \omega C$$
$$V_3 = (1+m)V_1 + mV_1 + m(1+m)V_1$$
$$= V_1(1+m)(1+m) + mV_1$$
$$= V_1(m^2 + 3m + 1)$$
$$\therefore V_3 = V_1(1 + 3m + m^2) \tag{8.5}$$

Similarly at R,

$$I_4 = I_3 + i_3$$
$${}_4\omega C = V_3 \omega C + (V_1 + V_2 + V_3) m \omega C$$
$$= V_1(1+3m+m^2) + mV_1 + m(1+m)V_1 + V_1 m(1+3m+m^2)$$
$$= V_1(1+3m+m^2+m+m+m^2+m+3m^2+m^3)$$
$$\therefore V_4 = V_1(1+6m+5m^2+m^3) \tag{8.6}$$

An examination of Eqs. (8.3), (8.5) and (8.6) reveal that the voltage across each unit is not equal (non-uniform) and it increases towards the conductor.

If the number of units is increased, it is more difficult to calculate the potential across each unit by using the above technique. Therefore, the potential across the i^{th} unit from the bottom is given by the emperical formula as

$$V_i = \frac{V \times 2\sinh\left[\frac{1}{2}\sqrt{m}\right]\cosh\left[\left(n - i + \frac{1}{2}\right)\sqrt{m}\right]}{\sinh\left[n\sqrt{m}\right]} \quad (8.7)$$

8.5 STRING EFFICIENCY

The voltage across the unit nearer to the conductor is more than the voltage in the unit nearer to the support. This unequal potential distribution over the string is expressed in terms of string efficiency.

The efficiency of a string is defined as the ratio of voltage across the whole string to the product of the number of discs and the voltage across the disc nearer to the line conductor. Mathematically it can be expressed as,

$$\text{String efficieny, } \eta = \frac{\text{Voltage across the string}}{n \times \text{Voltage across the unit near to the power conductor}} \quad (8.8)$$

where, *n* is the number of units in the string.

Hundred percent string efficiency means that voltage across each disc will be exactly same, this value is very difficult to achieve, however, efforts may be made to approach this value.

Test Yourself

1. Why is potential distribution not uniform over a string of suspension insulators?
2. Is string efficiency in a DC system 100%? Justify your answer.

EXAMPLE 8.1

A string of suspension insulator consists of four units. The voltage between each pin and earth is $\frac{1}{10}th$ of the self-capacitance of the unit. The voltage between the line conductor and earth is 132 kV (Fig. 8.8). Find (i) the voltage distribution across each unit, and (ii) the string efficiency.

Solution:

Number of units, $n = 4$

Given that the capacitance between each pin and earth as $\frac{1}{10}th$ of the self capacitance, i.e., $m = 0.1$

The total voltage, $V = 132$ kV

Voltage across first unit = V_1 kV

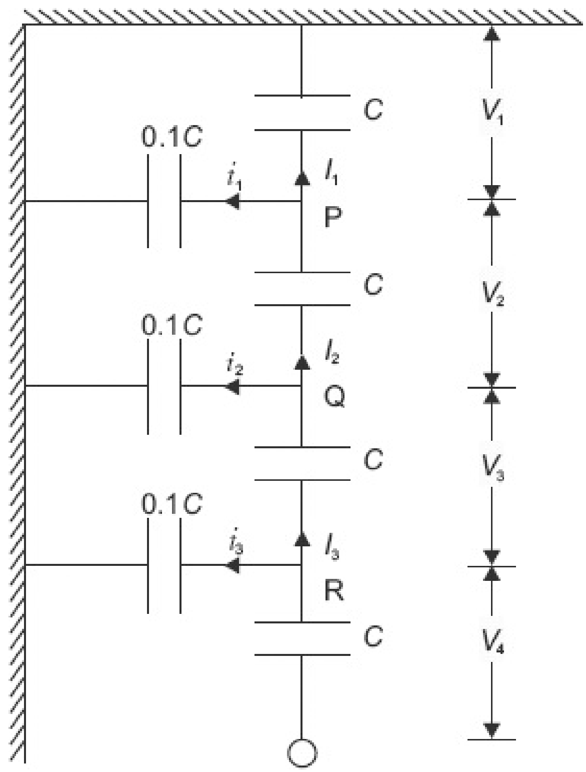

Fig. 8.8 Circuit diagram for Example 8.1

By applying KCL at point P

$$
\begin{aligned}
I_2 &= I_1 + i_1 \\
V_2\omega C &= V_1\omega C + V_1 m\omega C \\
V_2 &= V_1(1+m) \\
&= V_1(1+0.1) \\
V_2 &= 1.1V_1
\end{aligned}
$$

By applying KCL at point Q

$$
\begin{aligned}
I_3 &= I_2 + i_2 \\
V_3\omega C &= V_2\omega C + (V_1 + V_2)m\omega C \\
V_3 &= V_1 m + V_2(1+m) \\
&= V_1(1+3m+m^2) \\
&= (1+0.3+0.01)V_1 \\
V_3 &= 1.31V_1
\end{aligned}
$$

By applying KCL at point R

$$
\begin{aligned}
I_4 &= I_3 + i_3 \\
V_4\omega C &= V_3\omega C + (V_1 + V_2 + V_3)m\omega C \\
V_4 &= V_1(1+6m+5m^2+m^3) \\
&= (1+0.6+0.05+0.001)V_1 \\
V_4 &= 1.651V_1
\end{aligned}
$$

But we know the total voltage, $V = V_1 + V_2 + V_3 + V_4$

$$
\begin{aligned}
132 &= V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1 \\
5.061V_1 &= 132 \\
\therefore V_1 &= 26.082\ \text{kV}
\end{aligned}
$$

Then,

$$V_2 = 1.1 \times 26.082 = 28.69 \text{ kV}$$
$$V_3 = 1.31 \times 26.082 = 34.167 \text{ kV}$$
$$V_4 = 1.651 \times 26.082 = 43.061 \text{ kV}$$

The string efficiency, $\eta_s = \dfrac{V}{n \times V_4} \times 100 = \dfrac{132}{4 \times 43.061} \times 100 = 76.635\%.$

EXAMPLE 8.2

In a string of three units, the capacitance between each link pin to earth is 11% of the capacitance of one unit (Fig. 8.9). Calculate the voltage across each unit and the string efficiency when the voltage across the string is 33 kV.

Solution:

Number of units = 3

Capacitance between each unit = *C*

∴ Capacitance between each conductor and earth = 0.11 *C*

Ratio of shunt capacitance to mutual capacitance,

$$m = \frac{\text{Capacitance to ground}}{\text{Mutual capacitance}} = \frac{0.11C}{C} = 0.11$$

Applying Kircnhoff's laws at node P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V_2 \omega C &= V_1 \omega C + 0.11 V_1 \omega C \\ V_2 &= 1.11 V_1 \end{aligned}$$

Applying Kirchhoff's laws at node Q, we get

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V_3 \omega C &= V_2 \omega C + (V_1 + V_2)\, 0.11 \omega C \\ V_3 &= 1.11 V_2 + 0.11 V_1 \\ &= 1.2321 V_1 + 0.11 V_1 \\ &= 1.3421 V_1 \end{aligned}$$

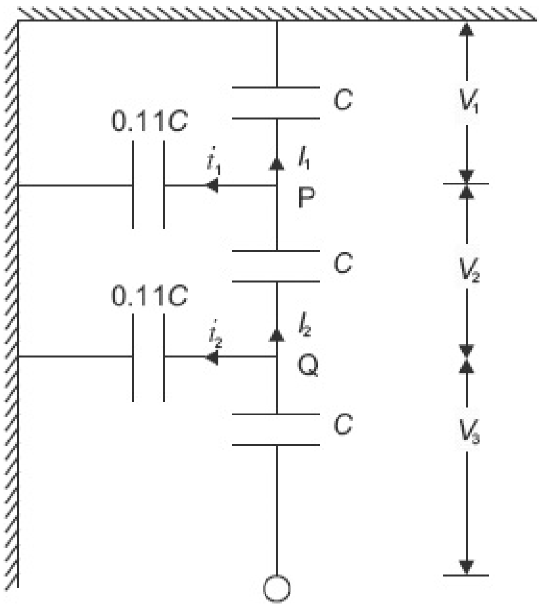

Fig. 8.9 Circuit diagram for Example 8.2

Voltage across the string is

$$\begin{aligned}\therefore V &= V_1 + V_2 + V_3 \\ &= V_1 + 1.11V_1 + 1.3421V_1 \\ &= 3.4521V_1\end{aligned}$$

The voltage across the string per phase, $V = \dfrac{33}{\sqrt{3}}$ kV

$= 19.05$ kV

$\therefore 3.4521V_1 = 19.05$

The voltage across the top disc, $V_1 = \frac{19.05}{3.4521} = 5.519$ kV/phase
$= 9.56$ kV (L–L)

The voltage across the second disc, $V_2 = 1.11V_1$
$= 1.11 \times 5.519 = 6.126$ kV/phase
$= 10.61$ kV (L–L)

The voltage across the lower disc, $V_3 = 1.3421V_1$
$= 1.3421 \times 5.519 = 7.41$ kV/phase
$= 12.829$ kV (L–L)

$$\text{String efficiency} = \frac{V}{3 \times V_3} \times 100 = \frac{19.05}{3 \times 7.41} \times 100 = 85.69\%.$$

Example 8.3

A string of suspension insulators consists of three units. The capacitance between each pin and earth is 15% of the self-capacitance of the unit (Fig. 8.10). If the maximum peak voltage per unit is not to exceed 35 kV, determine the greatest working voltage and the string efficiency.

Solution:

Number of units = 3

$m = 0.15$

Maximum voltage across an unit of string = 35 kV

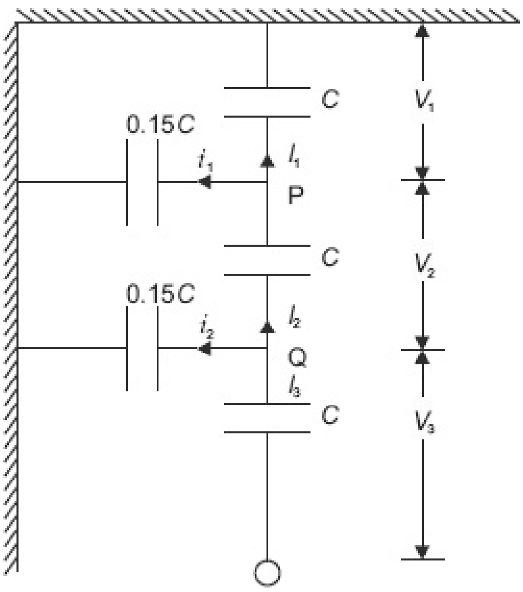

Fig. 8.10 Circuit diagram for Example 8.3

From Fig. 8.10, the voltage across the string, $V = V_1 + V_2 + V$

Apply KCL at P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V_2\omega C &= V_1\omega C + 0.15V_1\omega C \\ \therefore V_2 &= 1.15V_1 \end{aligned}$$

KCL at Q,

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V_3\omega C &= V_2\omega C + (V_1 + V_2)\,0.15\omega C \\ V_3 &= 1.15V_2 + 0.15V_1 \\ &= 1.3225V_1 + 0.15V_1 \\ &= 1.4725V_1 \end{aligned}$$

But the voltage across the lower disc, $V_3 = 35$ kV (peak)

$$= \frac{35}{\sqrt{2}} \text{ kV (r.m.s.)}$$

$$\therefore V_3 = 24.75 \text{ kV}$$

The voltage across the top unit, $V_1 = \dfrac{V_3}{1.4725} = \dfrac{24.75}{1.4725} = 16.81$ kV

The voltage across the second disc, $V_2 = 1.15V_1 = 19.33$ kV

$$\begin{aligned} \text{Maximum working voltage} &= V_1 + V_2 + V_3 \\ &= 16.81 + 19.33 + 24.75 \text{ kV} \\ &= 60.89 \text{ kV} \end{aligned}$$

String efficiency, $\eta = \dfrac{V}{3 \times V_3} \times 100 = \dfrac{60.89}{3 \times 24.75} \times 100 = 82\%.$

Example 8.4

Each conductor of a 33 kV, three-phase system is suspended by a string of three similar insulators; the capacitance of each disc is 10 times the capacitance to ground (Fig. 8.11). Calculate the voltage across each insulator and also determine the string efficiency.

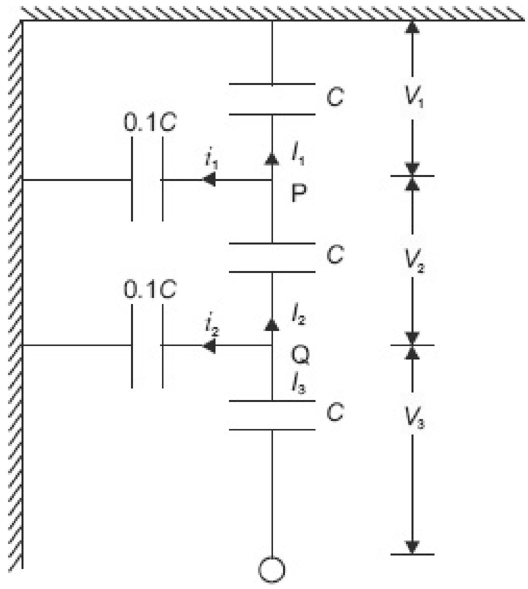

Fig. 8.11 Circuit diagram for Example 8.4

Solution:

Voltage between line conductor and earth, V = 33 kV (L–L)

$$m = \frac{\text{Shunt capacitance}}{\text{Mutual capacitance}} = \frac{1}{10} = 0.1$$

$\therefore$ Capacitance to ground = $0.1C$

Applying KCL and node P, $I_2 = I_1 + i_1$

$$\omega CV_2 = \omega CV_1 + 0.1 \times \omega CV_1$$
$$V_2 = 1.1V_1$$

At node Q, $I_3 = I_2 + i_2$

$$\omega CV_3 = \omega CV_2 + \omega C \times 0.1(V_1 + V_2)$$
$$V_3 = 1.1V_1 + 0.1(1.1\,V_1 + V_1)$$
$$V_3 = 1.31\,V_1$$
$$V = V_1 + V_2 + V_3 = V_1 + 1.1V_1 + 1.31\,V_1$$
$$V = 3.41\,V_1$$

But $V = 33\text{ kV}(\text{L}-\text{L}) = \frac{33}{\sqrt{3}} = 19.05\text{ kV/phase}$

$$\therefore 3.41V_1 = 19.05$$

Voltage across the top disc, $V_1 = \frac{19.05}{3.41} = 5.586$ kV/phase

Voltage across the second disc, $V_2 = 1.1\,V_1$

$= 1.1 \times 5.586 = 6.145$ kV/phase

$= 10.644$ kV (L–L)

Voltage across the lower disc, $V_3 = 1.31\,V_1$

$= 1.31 \times 5.586 = 7.32$ kV/phase

$= 12.674$ kV (L–L)

$$\text{String efficiency} = \frac{V}{n \times V_3} \times 100$$

$$= \frac{19.05}{3 \times 7.32} \times 100$$

$$= 86.75\%.$$

Example 8.5

A string of suspension insulators consists of four units and the capacitance to ground is 12% of its mutual capacitance (Fig. 8.12). Determine the voltage across each unit as a fraction of the operating voltage. Also determine the string efficiency.

Solution:

$$m = \frac{\text{Shunt capacitance}}{\text{Mutual capacitance}} = 0.12$$

KCL at P,

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + 0.12 V_1 \omega C$$

$$V_2 = 1.12 V_1$$

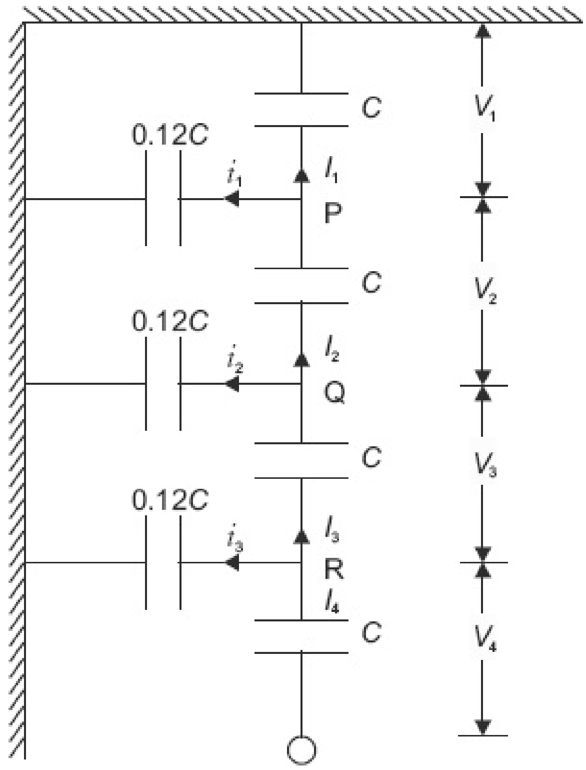

Fig. 8.12 Circuit diagram for Example 8.5

KCL at Q,

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V_3\omega C &= V_2\omega C + (V_1 + V_2)\, 0.12\omega C \\ V_3 &= 1.12V_2 + 0.12V_1 \\ &= 1.2544V_1 + 0.12V_1 \\ V_3 &= 1.3744V_1 \end{aligned}$$

Similarly at R,

$$\begin{aligned} I_4 &= I_3 + i_3 \\ V_4\omega C &= V_3\omega C + (V_1 + V_2 + V_3)\, 0.12\omega C \\ V_4 &= 1.12V_3 + 0.12V_2 + 0.12V_1 \end{aligned}$$

Substituting the values of V_3 and V_2 in above equation

$$V_4 = 1.793728V_1$$

$\therefore$ Voltage across string, $V = V_1 + V_2 + V_3 + V_4$

$$= V_1 + 1.12V_1 + 1.3744V_1 + 1.793728V_1$$

$$= 5.288V_1$$

$$\therefore V_1 = \frac{V}{5.288} \times 100 = 18.92\%V \text{ volts}$$

$$V_2 = 1.12V_1 = 1.12 \times 18.92\%V = 21.18\%V \text{ volts}$$

$$V_3 = 1.3744V_1 = 1.3744 \times 18.92\%V = 25.99\%V \text{ volts}$$

$$V_4 = 1.793728V_1 = 1.793728 \times 18.92\%V = 33.94\%V \text{ volts}$$

$$\text{String efficiency} = \frac{V}{n \times V_4} = \frac{V}{4 \times V_4}$$

$$= \frac{5.288V_1}{4 \times 1.793728V_1}$$

$$= 73.7\%.$$

Example 8.6

Each conductor of a three-phase high voltage transmission line is suspended from the cross arm of a steel tower by a string of four suspension type disc insulators (Fig. 8.13). If the voltage across the second unit is 13.2 kV and that across the third unit is 20 kV, calculate the voltage between the conductors.

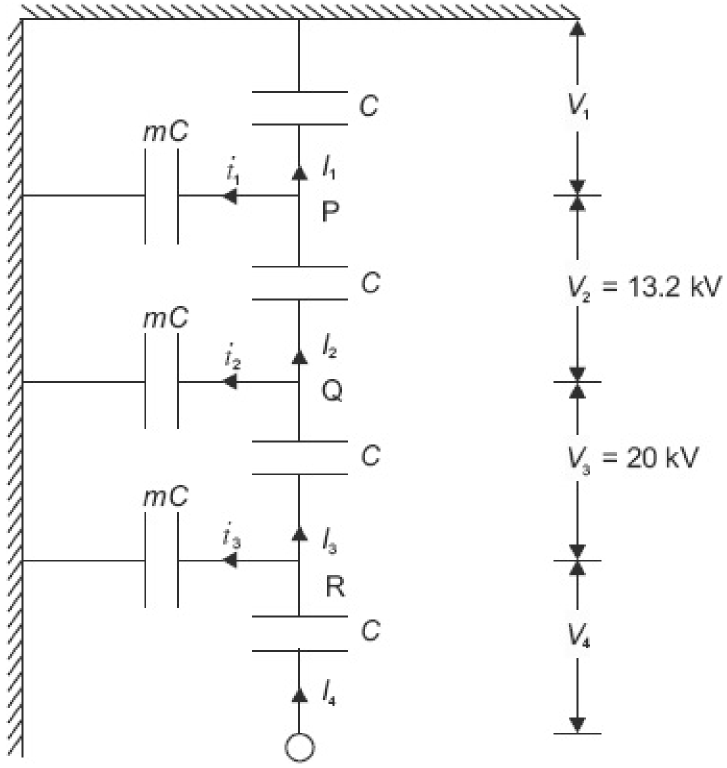

Fig. 8.13 Circuit diagram for Example 8.6

Solution:

KCL at node P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V_2\omega C &= V_1\omega C + mV_1\omega C \\ V_2 &= (1+m)V_1 \end{aligned}$$

But V_2 = 13.2 kV is given

$$(1+m)\,V_1 = 13.2\text{ kV}$$

$$\therefore V_1 = \frac{13.2\text{ kV}}{(1+m)} \qquad (1)$$

Similarly at Q,

$$
\begin{aligned}
I_3 &= I_2 + i_2 \\
V_3\omega C &= V_2\omega C + (V_1 + V_2)m\omega C \\
V_3 &= (1 + m)V_2 + mV_1 \\
&= (1 + m)^2 V_1 + mV_1 \\
&= (1 + 3m + m^2)V_1
\end{aligned}
$$

But $V_3 = 20$ kV

$\therefore (1 + 3m + m^2)V_1 = 20$ kV

$$V_1 = \frac{20 \text{ kV}}{1+3m+m^2} \qquad (2)$$

Equating Eqs. (1) and (2),

$$
\begin{aligned}
&\frac{20}{1+3m+m^2} = \frac{13.2}{(1+m)} \\
&13.2m^2 + 39.6m + 13.2 = 20 + 20m \\
&13.2m^2 + 19.6\text{m} - 6.8 = 0 \\
&m = \frac{-19.6 \pm \sqrt{19.6^2 + 4\times 6.8\times 13.2}}{2\times 13.2} \\
&\quad = \frac{-19.6 \pm \sqrt{743.2}}{26.4} \\
&\quad = 0.29 \\
&\therefore V_1 = \frac{13.2}{1+m} = \frac{13.2}{1+0.29} = 10.23 \text{ kV}
\end{aligned}
$$

Similarly at R,

$$
\begin{aligned}
I_4 &= I_3 + i_3 \\
V_4\omega C &= V_3\omega C + (V_1 + V_2 + V_3)m\omega C \\
&= (1 + 3m + m^2)V_1 + [m + (m + m^2) + \\
&\qquad (1 + 3m + m^2)m]V_1 \\
&= (1 + 6m + 5m^2 + m^3)V_1
\end{aligned}
$$

Voltage across the string is,

$$\begin{aligned} V &= V_1 + (1+m)V_1 + (1+3m+m^2)V_1 + \\ &\quad (1+6m+5m^2+m^3)V_1 \\ &= V_1(4+10m+6m^2+m^3) \\ V &= 7.43\,V_1 \\ V &= 7.43 \times 10.23 \\ &= 76\text{ kV.} \end{aligned}$$

Example 8.7

A string of six insulator units has a self-capacitance equal to 11 times the pin to earth capacitance (Fig. 8.14). Find the string efficiency.

Solution:

Given data:

No of insulators, $n = 6$

Self capacitance = 11C

Where, C is pin to earth capacitance

$$\begin{aligned} m &= \frac{\text{Shunt capacitance}}{\text{Self-capacitance}} = \frac{C}{11C} \\ &= 0.091 \end{aligned}$$

Voltage across second unit, V_2

$$\begin{aligned} V_2 &= (1+m)V_1 \\ &= 1.091V_1 \end{aligned}$$

Voltage across third unit, V_3

$$\begin{aligned} V_3 &= V_1(1+3m+m^2) \\ &= V_1(1+3(0.091)+(0.091)^2) \\ V_3 &= 1.2813V_1 \end{aligned}$$

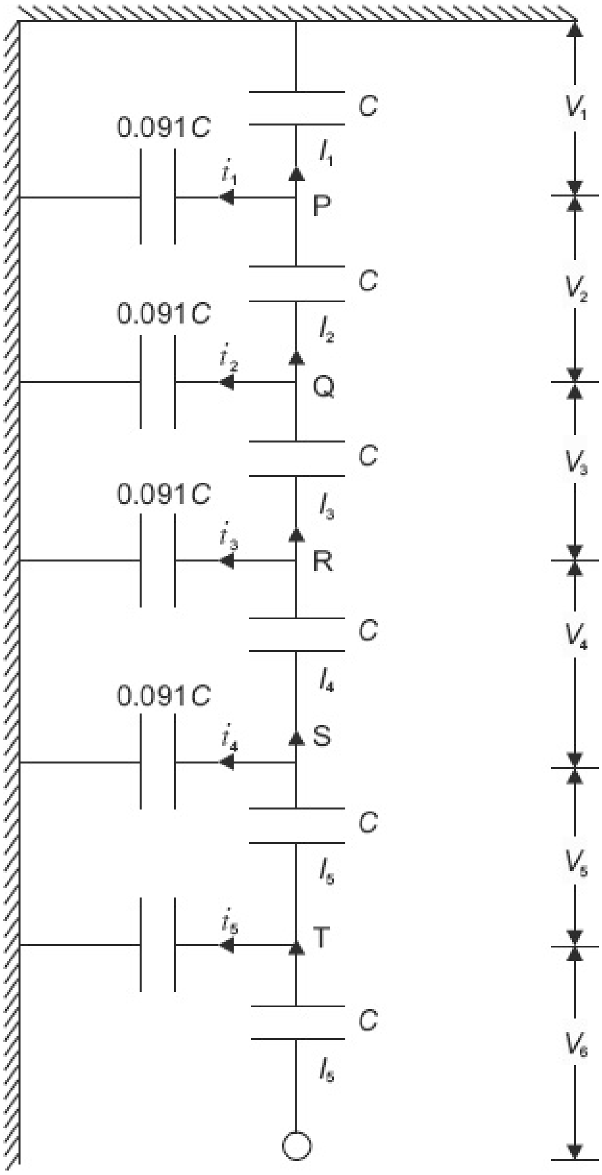

Fig. 8.14 Circuit diagram for Example 8.7

Voltage across fourth unit, V_4

$$V_4 = V_1\,(1 + 6m + 5m^2 + m^3)$$
$$V_4 = V_1\,(1 + 6\times 0.091 + 5\times 0.091^2 + 0.091^3)$$
$$= 1.588V_1$$

Voltage across fifth unit, V_5

$$= V_1 (1 + 10m + 15m^2 + 7m^3 + m^4)$$
$$= V_1 (1 + 10 \times 0.091 + 15 \times 0.091^2 + 7 \times 0.091^3 + 0.091^4)$$
$$= 2.0396V_1$$

Voltage across sixth unit, V_6

$$\omega CV_6 = \omega CV_5 + \omega C(V_1 + V_2 + V_3 + V_4 + V_5)$$
$$= V_5 + m\ (V_1 + V_2 + V_3 + V_4 + V_5)$$
$$= 2.0396V_1 + 0.091(V_1 + 1.091V_1 + 1.2813V_1 + 1.588V_1 + 2.0396V_1)$$
$$V_6 = 2.67V_1$$

$$\text{Total operating voltage, } V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$$
$$= V_1 + 1.091V_1 + 1.2813V_1 + 1.588V_1 + 2.0396V_1 + 2.67V_1$$
$$= 9.67V_1$$

$$\text{String efficiency, } \eta = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_6}$$
$$= \frac{9.67V_1}{6 \times 2.67V_1} \times 100 = 60.36\%.$$

Example 8.8

A three-phase overhead transmission line is being supported by three disc suspension insulators. The potential across the first and second insulators are 11 kV and 13.2 kV, respectively (Fig. 8.15). Calculate (i) the line voltage, and (ii) string efficiency.

Solution:

Number of units in a string = 3

Let *m* be the ratio of capacitance between pin and earth capacitance

If *C* farad is the self-capacitance of each unit, then
Capacitance between pin and earth = *mC*

Applying KCL at node P,

$$I_2 = I_1 + i_1$$
$$V_2\omega C = V_1\omega C + V_1 m\omega C$$
$$V_2 = V_1(1+m)$$
$$m = \frac{V_2 - V_1}{V_1} = \frac{13.2 - 11}{11} = 0.2$$

Applying KCL at node Q, we have

$$I_3 = I_2 + i_2$$
$$V_3\omega C = V_2\omega C + (V_1 + V_2)mV_1\omega C$$
$$V_3 = V_2 + m(V_1 + V_2)$$
$$= 13.2 + 0.2 \times (11 + 13.2)$$
$$= 18.04 \text{ kV}$$
$$\text{Voltage between line to earth} = V_1 + V_2 + V_3$$
$$= 11 + 13.2 + 18.04 = 42.24 \text{ kV}$$

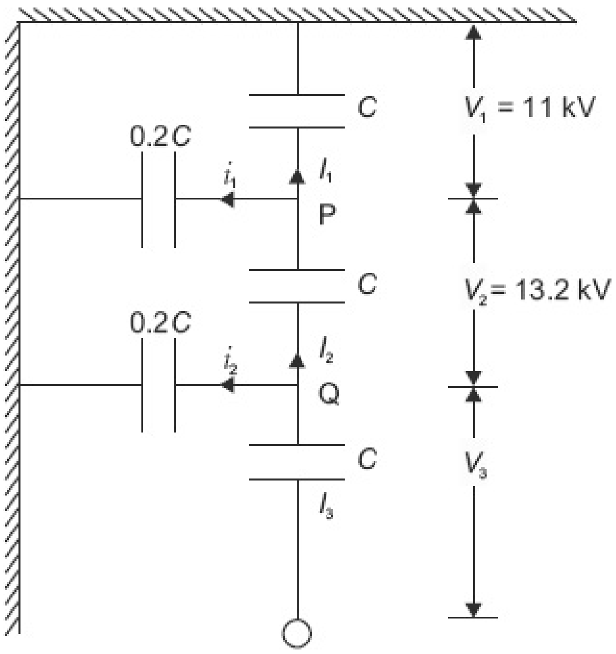

Fig. 8.15 Circuit diagram for Example 8.8

1. Line voltage $= \sqrt{3} \times 42.24 = 73.16$ kV

2. String efficiency,

$$\eta = \frac{\text{Voltage across string}}{\text{Number of units} \times V_3} \times 100$$

$$= \frac{42.24}{3 \times 18.04} \times 100 = 78.05\%.$$

EXAMPLE 8.9

A three-phase 66 kV transmission line is carried by strings of five suspension insulators. The capacity of each unit insulator to the capacity relative to earth is 4:1 (Fig. 8.16). Calculate the potential across each unit and the string efficiency. Assume that there is no leakage.

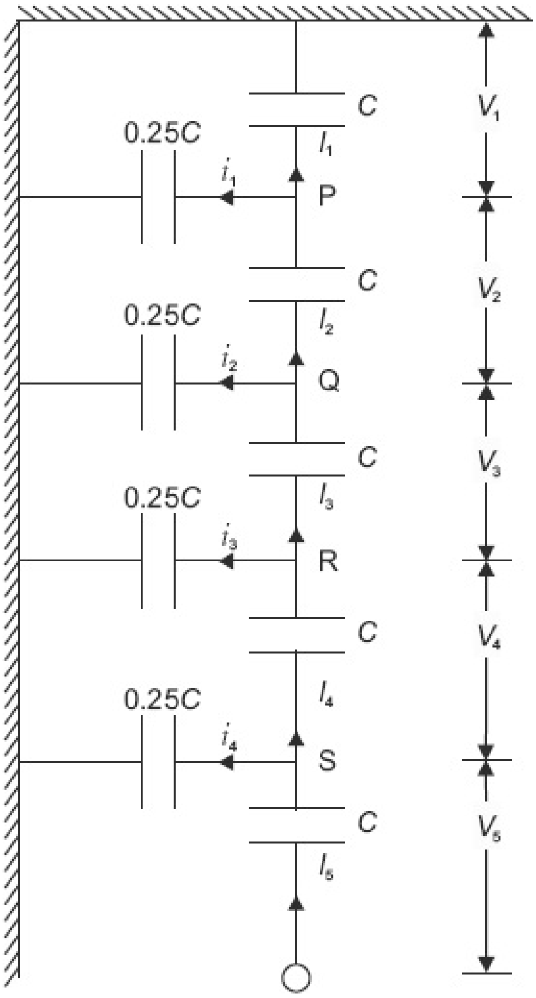

Fig. 8.16 Circuit diagram for Example 8.9

Solution:

Number of units = 5

$$m = \frac{\text{Capacitance relative to earth}}{\text{Self capacitance}} = \frac{1}{4} = 0.25$$

By applying KCL at node P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V_2\omega C &= V_2\omega C + (V_1 + V_1)\,\omega C \\ V_2 &= V_1(1 + \text{m}) = 1.25\,V_1 \end{aligned}$$

By applying KCL at node Q,

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V_3\omega C &= V_2\omega C + (V_1 + V_2)m\omega C \\ V_3 &= V_2 + V_1\text{m} + V_2\text{m} \\ &= V_2(1 + m) + \text{m}V_1 \\ V_3 &= 1.25V_2 + 0.25V_1 = 1.8125V_1 \end{aligned}$$

By applying KCL at node R,

$$\begin{aligned} I_4 &= I_3 + i_3 \\ V_4\omega C &= V_3\omega C + (V_1 + V_2 + V_3)m\omega C \\ &= V_3(1 + m) + \text{m}V_1 + \text{m}V_2 \\ &= 1.25V_3 + 0.25V_1 + 0.25V_2 \\ &= 2.828125V_1 \end{aligned}$$

By applying KCL at node S,

$$\begin{aligned} I_5 &= I_4 + i_4 \\ V_5\omega C &= V_4\omega C + (V_1 + V_2 + V_3 + V_4)m\omega C \\ &= V_4(1 + m) + \text{m}V_1 + \text{m}V_2 + \text{m}V_3 \\ &= 1.25V_4 + 0.25V_1 + 0.25V_2 + 0.25V_3 \\ &= 4.55078V_1 \end{aligned}$$

Voltage across the string is,

$$\begin{aligned} V &= V_1 + V_2 + V_3 + V_4 + V_5 \\ &= V_1 + 1.25V_1 + 1.8125V_1 + 2.83V_1 + 4.55V_1 \\ &= 11.4414V_1 \end{aligned}$$

Given voltage across string, V = 66 kV (L–L)

$$\begin{aligned} V_{ph} &= \frac{66}{\sqrt{3}} = 38.105 \text{ kV} \\ \therefore V_1 &= \frac{38.105}{11.44} = 3.33045 \text{ kV} \\ V_2 &= 1.25V_1 = 4.16 \text{ kV} \\ V_3 &= 1.8125V_1 = 6.037 \text{ kV} \\ V_4 &= 2.83V_1 = 9.418 \text{ kV} \\ \text{And} \quad V_5 &= 4.55V_1 = 15.15 \text{ kV} \end{aligned}$$

String efficiency, $\eta = \frac{V}{n \times V_5} \times 100 = \frac{38.105}{5 \times 15.15} \times 100 = 50.297\%.$

EXAMPLE 8.10

A string of suspension insulators consists of five units each having capacitance C. The capacitance between 1 each unit

and earth is $\frac{1}{8}$ of *C*. Find the voltage distribution across each

insulator in the string as a percentage of voltage of conductor to earth. If each insulator in the string is designed to withstand a voltage of 35 kV maximum, find the operating voltage of the line where five suspension insulator strings can be used.

Solution:

Total number of units = 5

The capacitance between each unit and earth, $m = \frac{\frac{1}{8}C}{C} = 0.125$

Applying KCL at Node P,

$$
\begin{aligned}
I_2 &= I_1 + i_1 \\
\omega C V_2 &= \omega C V_1 + \omega C V_1 \\
C V_2 &= C V_1 + m C V_1 \\
V_2 &= (1 + \mathrm{m}) V_1 \qquad (1)
\end{aligned}
$$

Applying KCL at node Q,

$$
\begin{aligned}
I_3 &= I_2 + i_2 \\
\omega C V_3 &= \omega C V_2 + \omega m C (V_1 + V_2) \\
V_3 &= V_2 + m(V_1 + V_2) \\
&= m V_1 + (1 + m) V_2 \\
&= m V_1 + (1 + m)(1 + m) V_1 \\
V_3 &= V_1 (1 + 3m + m^2) \qquad (2)
\end{aligned}
$$

Applying KCL at node R,

$$
\begin{aligned}
I_4 &= I_3 + i_3 \\
\omega C V_4 &= \omega C V_3 + \omega m C (V_1 + V_2 + V_3) \\
V_4 &= V_3 + m(V_1 + V_2 + V_3) \\
&= V_1 (1 + 6m + 5m^2 + m^3)
\end{aligned}
$$

Applying KCL at node S,

$$
\begin{aligned}
I_5 &= I_4 + i_4 \\
\omega\, C V_5 &= \omega\, C V_4 + \omega\, m C (V_1 + V_2 + V_3 + V_4) \\
V_5 &= V_4 + m(V_1 + V_2 + V_3 + V_4) \\
V_5 &= V_1 [1 + 10m + 15m^2 + 7m^3 + m^4]
\end{aligned}
$$

Voltage across the sting is,

$$
\begin{aligned}
V &= V_1 + V_2 + V_3 + V_4 + V_5 \\
&= V_1 (5 + 20m + 21m^2 + 8m^3 + m^4)
\end{aligned}
$$

Voltage across first insulator,

$$V_1 = \frac{V}{5 + 20m + 21m^2 + 8m^3 + m^4}$$

$V_1 = 0.1275V$ or 12.75% of V $(\because m = 0.125)$

$V_2 = (1 + m)V_1 = (1 + 0.125) \times 0.1275V$

$= 0.1306V = 14.34\%$ of V volts

$V_3 = (1 + 3m + m^2)V_1 = (1 + 3 \times 0.166$

$+ (0.166)^2) \times 0.1116V$

$= 1.525 \times 0.1116V$

$= 0.1702V = 17.73\%$ of V volts

$V_4 = (1 + 6m + 5m^2 + m^3)V_1$

$V_4 = 0.2392V$ or 23.33% of V volts

$V_5 = (1 + 10m + 15m^2 + 7m^3 + m^4)$

$V_1 = 0.3185V$ or 31.85% of V

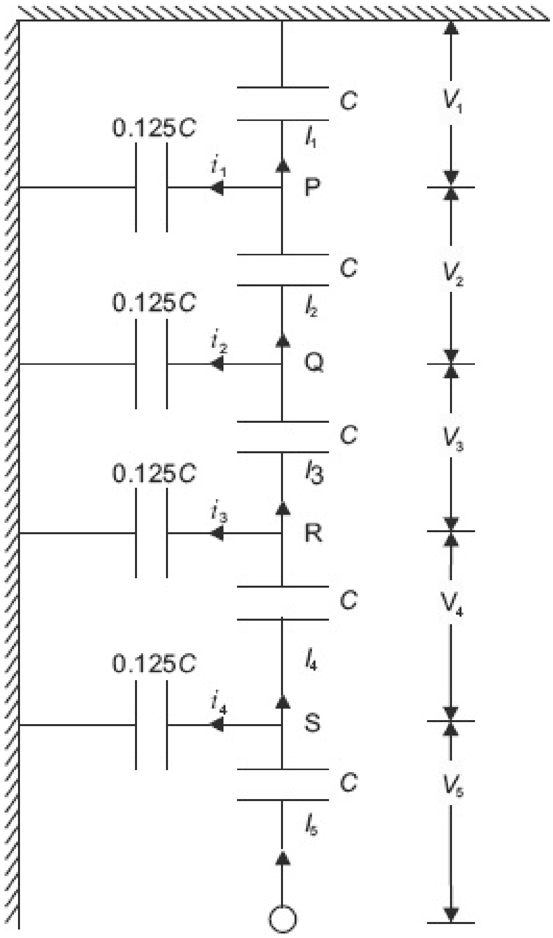

Fig. 8.17 Circuit diagram for Example 8.10

Voltage across the bottom unit, $V_5 = \frac{35}{\sqrt{2}}$ kV $= 0.3185V$

Voltage between conductor and earth, $V = \frac{V_5}{0.3185} = \frac{\frac{35}{\sqrt{2}}}{0.3185} = 77.7 \text{ kV}$

Operating line voltage

$= \sqrt{3}\ V = \sqrt{3} \times 77.7 \text{ kV} = 134.59 \text{ kV(L – L)}.$

EXAMPLE 8.11

In the following Figs. 8.18(a) and (b) which represent equivalent capacitor arrangements for strings of suspension insulators $C' = 0.125C$, the maximum voltage across any unit is not to exceed 13.2 kV. Find the maximum voltage that each arrangement can withstand.

Solution:

(i) From Fig. 8.18(a), the maximum voltage across every unit is not to exceed 13.2 kV meaning 13.2 kV is the voltage across the unit near to the conductor.

$\therefore V_2 = 13.2 \text{ kV}$

Applying KCL at point P

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V_2 \omega C &= V_1 \omega C + V_1 \omega (0.125C) \\ \therefore V_2 &= 1.125 V_1 \\ V_1 &= \frac{V_2}{1.125} = \frac{13.2}{1.125} = 11.73 \text{ kV} \end{aligned}$$

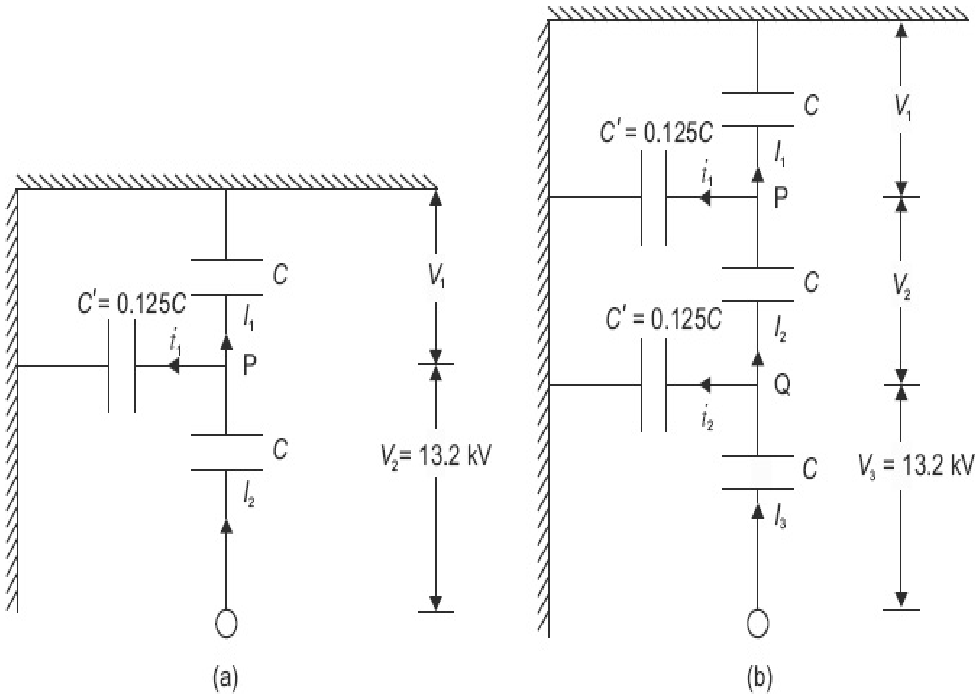

Fig. 8.18 Circuit diagram for Example 8.11

∴ The maximum voltage across the strings,

$$V = V_1 + V_2 = 11.73 + 13.2 = 24.93 \text{ kV}$$

(ii) From Fig. 8.18(b) the maximum voltage across every unit is not to exceed 13.2 kV i.e., the voltage across the unit near to the conductor, V_3 = 13.2 kV

∴ V_3 = 13.2 kV

KCL at point P,

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + V_1 \omega (0.125C)$$

$$\therefore V_2 = 1.125 V_1$$

KCL at point Q,

$$
\begin{aligned}
I_3 &= I_2 + i_2 \\
V_3\omega C &= V_2\omega C + (V_1 + V_2)0.125\omega C \\
V_3 &= 1.125V_2 + 0.125V_1 \\
&= 1.3906V_1 \\
\therefore V_1 &= \frac{V_3}{1.3906} = \frac{13.2}{1.3906} = 9.492 \text{ kV} \\
\therefore V_2 &= 1.125V_1 = 1.125 \times 9.492 = 10.68 \text{ kV}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \text{Voltage across strings, } V &= V_1 + V_2 + V_3 \\
&= 9.492 + 10.68 + 13.2 \\
&= 33.37 \text{ kV.}
\end{aligned}
$$

■

EXAMPLE 8.12

A string of suspension insulators consists of six units. If the maximum peak voltage per unit is 33 kV, calculate (i) the maximum voltage for which this string can be used, (ii) voltage across the third unit from the bottom (iii) the string efficiency. Assume capacitance between each pin and earth as 12% of the self-capacitance of each unit. ((This question is also solved using MATLAB programs in the appendix)

Solution:

(i) The potential across the i^{th} unit from the bottom is given by (empirical formula)

$$
V_i = \frac{V \times 2 \sinh\left[\frac{1}{2}\sqrt{m}\right]\cosh\left[\left(n - i + \frac{1}{2}\right)\sqrt{m}\right]}{\sinh\left[n\sqrt{m}\right]}
$$

The maximum voltage appears across the first unit from bottom, therefore $i = 1$

$$
\begin{aligned}
m &= \frac{\text{Capacitance to ground}}{\text{Mutual capacitance}} \\
&= 0.12
\end{aligned}
$$

Putting $n = 6$ and $i = 1$, we get

$$V_1 = \frac{V \times 2\sinh\left[\frac{1}{2}\sqrt{0.12}\right]\cosh\left[\left(6-1+\frac{1}{2}\right)\sqrt{0.12}\right]}{\sinh\left[6\sqrt{0.12}\right]}$$

$$= \frac{V \times 2\sinh\left[\frac{1}{2}\times 0.3464\right]\cosh\left[(5.5)0.3464\right]}{\sinh\left[6\times 0.3464\right]}$$

$$= \frac{V \times 2\sinh[0.1732]\cosh[1.905]}{\sinh[2.0785]}$$

$$= \frac{V \times 2\times 0.1741\times 3.4341}{3.9337}$$

But V_1 = 33 kV (peak),

Therefore,

$$33 = \frac{V \times 2\times 0.1741\times 3.4341}{3.9337}$$

$$\therefore V = \frac{33\times 3.9337}{2\times 0.1741\times 3.4341}$$

$$= 108.56 \text{ kV(peak)}.$$

(ii) Voltage across the third unit from bottom,

$$V_3 = \frac{V \times 2\sinh\left[\frac{1}{2}\sqrt{0.12}\right]\cosh\left[\left(6-3+\frac{1}{2}\right)\sqrt{0.12}\right]}{\sinh\left[6\sqrt{0.12}\right]}$$

$$V_3 = \frac{108.56\times 2\sinh\left[\frac{1}{2}\sqrt{0.12}\right]\cosh\left[\left(6-3+\frac{1}{2}\right)\sqrt{0.12}\right]}{\sinh\left[6\sqrt{0.12}\right]}$$

$$= \frac{108.56\times 2\sinh\left[\frac{1}{2}\times 0.3464\right]\cosh\left[(3.5)0.3464\right]}{\sinh\left[6\times 0.3464\right]}$$

$$= \frac{108.56\times 2\sinh[0.1732]\cosh[1.2124]}{\sinh[2.0784]}$$

$$= \frac{108.56\times 2\times 0.1741\times 1.8295}{3.9337} = 17.58 \text{ kV}$$

(iii)

$$\text{String efficieny, } \eta = \frac{\text{Voltage across the string}}{n \times \text{Voltage across the unit near the power conductor}}$$
$$= \frac{108.56}{6 \times 33}$$
$$= 54.83\%.$$

8.6 METHODS OF IMPROVING STRING EFFICIENCY

In order to improve the string efficiency, it is necessary to equalize the potential across the various units of the string. The various methods for equalizing the potential across units are

1. Selection of *m*,
2. Grading of insulators, and
3. Guard ring

8.6.1 Selection of m

It is observed from Eqs. (8.3), (8.5) and (8.6) that the voltage across each unit is not equal and increases towards power conductors i.e., the voltage across the unit nearer to the power conductor is more than the other units. From the expression of the string efficiency, for higher value of the voltage across the unit nearer to the power conductor, string efficiency is less. If the total voltage across the string is distributed across each unit equally, then the voltage across the power conductor is reduced. So the efficiency of the string increases. This can be obtained by reducing the value of *m*. If *m* approaches zero, the voltage across each unit will be equalized. In order to reduce the ground capacitance, the distance between the string of the insulator and the steel tower is increased or the length of the cross-arm is increased. However, there is a limit to increase in cross-arm because it is uneconomical and hence this method is not practicable.

8.6.2 Grading of Units

Unequal distribution of voltage across each disc is due to leakage of current from the insulator pin to the tower structure. This leakage current cannot be eliminated. The other possibility of equalizing the voltage across the disc is by using different capacities of discs. Therefore, for equalizing the voltage across each unit, the top unit of the string must have minimum capacitance (nearer to the cross-arm) and the bottom unit must have maximum capacitance (nearer to the power conductor). Here the capacitance between the metal pin of the disc and the power conductor is neglected because of small value.

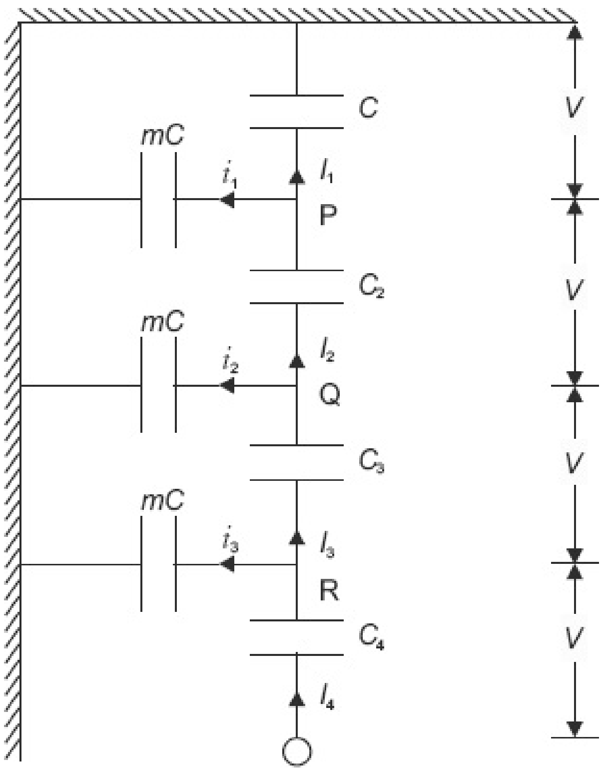

Fig. 8.19 Equivalent circuit of a string of suspension insulators

Let C be the capacitance of the top unit and the capacitance of other units are C_2, C_3,..., and C_n, respectively and voltages across each unit be V.

Applying KCL at node P in Fig. 8.19,

$$I_2 = I_1 + i_1 \tag{8.9}$$

$$V\omega C_2 = V\omega C + V\omega mC$$

$$\therefore C_2 = (1+m)C \tag{8.10}$$

Applying KCL at Q,

$$I_3 = I_2 + i_2 \tag{8.11}$$

$$V\omega C_3 = V\omega C_2 + 2Vm\omega C$$

$$C_3 = C_2 + 2mC$$

$$= (1+m)C + 2mC$$

$$\therefore C_3 = (1+3m)C \tag{8.12}$$

Similarly at R,

$$I_4 = I_3 + i_3 \tag{8.13}$$

$$V\omega C_4 = V\omega C_3 + 3Vm\omega C$$

$$C_4 = C_3 + 3mC$$

$$= (1+3m)C + 3mC$$

$$\therefore C_4 = (1+6m)C \tag{8.14}$$

Similarly the capacitance of $(n+1)^{th}$ unit is

$$\therefore C_{n+1} = C_n + nmC \tag{8.15}$$

The main disadvantages of this method are:

1. Difficulty in manufacturing different capacities of discs.
2. Difficulty in storing and handling the discs.

To overcome the above difficulties grading of insulators or guard ring is used.

Example 8.13

An insulator string containing five units has equal voltage across each unit by using discs of different capacitances (Fig. 8.20). If the top unit has a capacitance of *C* and pin to tower capacitance of all units is 20% of the mutual capacitance of top unit, calculate mutual capacitance of each disc in a string.

Solution:

Let the voltage across each unit be V

And the capacitances of the remaining four units from top are C_2, C_3, C_4 and C_5, respectively,

Applying KCL at P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V\omega C_2 &= V\omega C + V\omega \times 0.2C \\ C_2 &= C + 0.2C = 1.2C \qquad (1) \end{aligned}$$

Applying KCL at Q,

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V\omega C_3 &= V\omega C_2 + 2V\omega \times 0.2C \\ C_3 &= C_2 + 0.4C \\ C_3 &= 1.2C + 0.4C = 1.6C \qquad (2) \end{aligned}$$

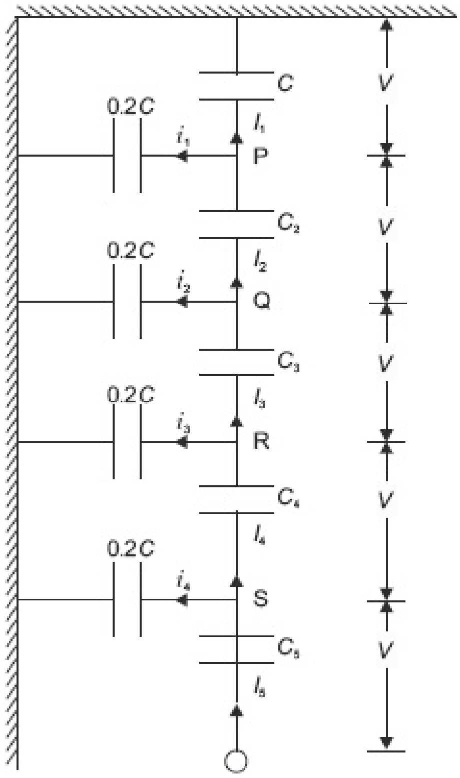

Fig. 8.20 Circuit diagram for Example 8.13

Applying KCL at R,

$$
\begin{aligned}
I_4 &= I_3 + i_3 \\
V\omega C_4 &= V\omega C_3 + 3V\omega \times 0.2C \\
C_4 &= C_3 + 0.6C \\
C_4 &= 1.6C + 0.6C = 2.2C \qquad (3)
\end{aligned}
$$

Applying KCL at S,

$$
\begin{aligned}
I_5 &= I_4 + i_4 \\
V\omega C_5 &= V\omega C_4 + 4V\omega \times 0.2C \\
C_5 &= C_4 + 0.8C \\
C_5 &= 2.2C + 0.8C = 3C. \qquad (4)
\end{aligned}
$$

Example 8.14

A string of four suspension insulators is to be graded to obtain uniform distribution of voltage across the string (Fig. 8.21). If the capacitance to ground of each unit is 10% of the capacitance of the top unit, determine the capacitance of the remaining three units.

Solution:

Let the capacitance of top unit i.e., near to the tower is *C* and remaining capacitance are C_2, C_3 and C_4.

Applying KCL at node P,

$$\begin{aligned} I_2 &= I_1 + i_1 \\ V\omega C_2 &= V\omega C + 0.1 V\omega C \\ \therefore C_2 &= 1.1C \end{aligned}$$

KCL at node Q,

$$\begin{aligned} I_3 &= I_2 + i_2 \\ V\omega C_3 &= V\omega C_2 + 0.1 \times 2V\omega C \\ C_3 &= C_2 + 0.2C \\ &= (1.1 + 0.2)C \\ &= 1.3C \end{aligned}$$

Similarly at node R,

$$\begin{aligned} I_4 &= I_3 + i_3 \\ V\omega C_4 &= V\omega C_3 + 0.1 \times 3V\omega C \\ C_4 &= 1.3C + 0.3C \\ C_4 &= 1.6C. \end{aligned}$$

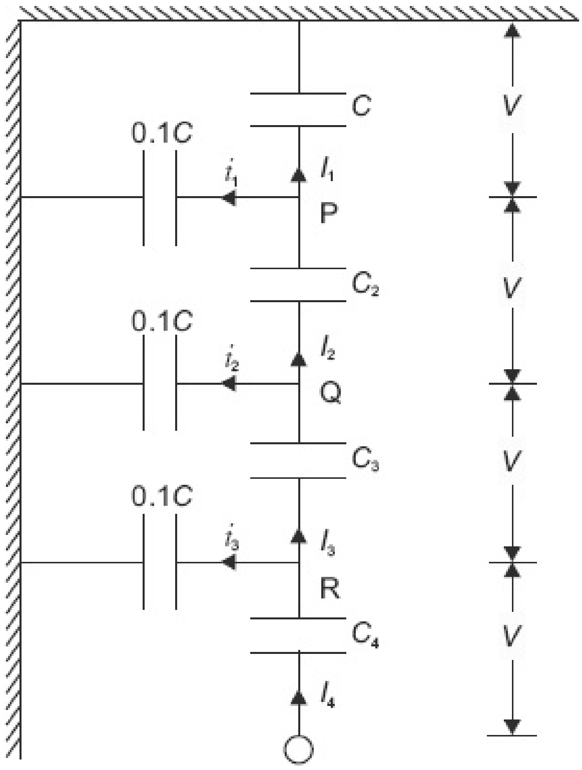

Fig. 8.21 Circuit diagram for Example 8.14

8.6.3 Guard Ring or Static Shielding

The same voltage across each disc of a string can be maintained by using a guard or grading ring. A grading ring is a large metal ring, connected to the conductor and surrounding the bottom unit that is to the line. This ring increases the capacitance between the metal fittings and the line, which was neglected in the previous two methods. Consider a string of four units as shown in Fig. 8.22.

Let the self-capacitance of each unit be C and the capacitance between each metal fitting and guard ring be C_1, C_2, C_3 and V be the voltage across each unit.

Applying KCL at node P,

$$I_2 + i_1' = I_1 + i_1 \tag{8.16}$$

Since the capacitance of each unit is same, their charging currents are equal

$$\text{i.e., } I_1 = I_2 = I_3 = I_4$$
$$i_1' = i_1$$
$$3V\omega C_1 = Vm\omega C$$
$$\therefore C_1 = \frac{mC}{3} \qquad (8.17)$$

Applying KCL at Q,

$$I_3 + i_2' = I_2 + i_2 \qquad (8.18)$$
$$i_2' = i_2$$
$$2V\omega C_2 = 2V\omega mC$$
$$\therefore C_2 = \frac{2mC}{2} = mC$$

Similarly at R

$$I_4 + i_3' = I_3 + i_3 \qquad (8.20)$$
$$i_3' = i_3$$
$$V\omega C_3 = 3V\omega mC$$
$$C_3 = 3mC \qquad (8.21)$$

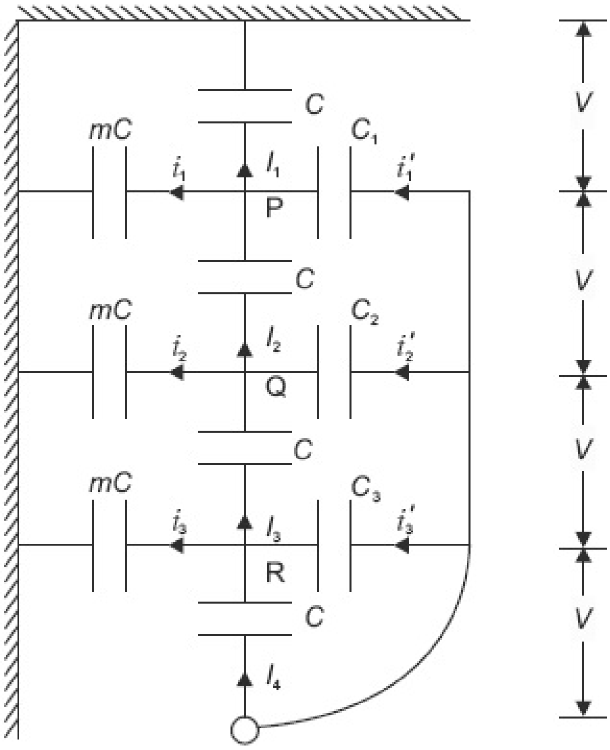

Fig. 8.22 Equivalent circuit of a string of suspension insulators with guard ring

Similarly, if we are using *k* units in a string then capacitance between guard ring and the pin of the n^{th} unit is

$$C_n = \frac{n}{k-n} mC \tag{8.22}$$

In practice, it is impossible to obtain an equal distribution of voltage across each disc of a string by providing guard ring, but considerable improvements are possible.

This guard ring serves the purpose of an arcing shield when used in combination with an arcing horn fixed at the top end of the string. Location of guard ring is shown in Fig. 8.23.

Fig. 8.23 Stock damper and Arcing horn

Example 8.15

A string of six suspension insulators is to be fitted with a guard ring (Fig. 8.24). The pin-to-earth capacitance and pin-to-pin capacitance are all equal to *C*. What should be the values of the line to pin capacitances to have uniform voltage distribution over the string.

Solution:

Voltage across unit is V

Applying KCL at node P,

$$I_2 + i_1' = I_1 + i_1$$

But the charging currents, $I_1 = I_2 = I_3 = I_4 = I_5 = I_6$

$$i_1' = i_1$$
$$5V\omega C_1 = V\omega C$$
$$\therefore C_1 = \frac{C}{5}$$

Similarly at Q,

$$i_2' = i_2$$
$$4V\omega C_2 = 2V\omega C$$
$$\therefore C_2 = 2\frac{C}{4} = \frac{C}{2}$$

At node R,

$$i_3' = i_3$$
$$3V\omega C_3 = 3V\omega C$$
$$\therefore C_3 = C$$

At node S,

$$i_4' = i_4$$
$$2V\omega C_4 = 4V\omega C;$$
$$C_4 = 2C$$

At node T,

$$i_5' = i_5$$
$$V\omega C_5 = 5V\omega C$$
$$C_5 = 5C.$$

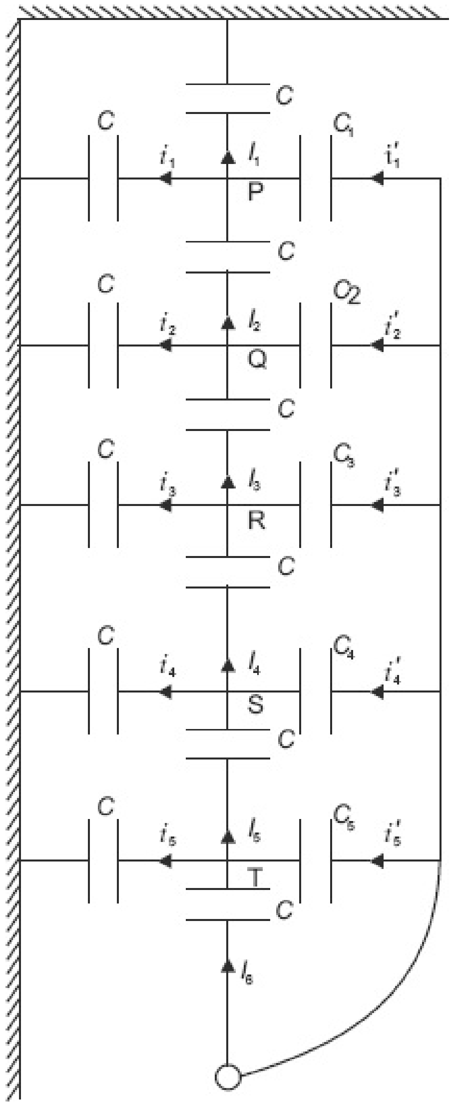

Fig. 8.24 Circuit diagram for Example 8.15

Example 8.16

Each of the three insulators forming a string has a self-capacitance of *C* F. The shunting capacitance of the connecting metalwork of each insulator is 0.2*C* to earth and 0.15*C* to the line. A guard ring increases the capacitance to the line of the metalwork of the lowest insulator to 0.3*C*. Calculate the string efficiency of this arrangement with the guard ring.

Solution:

(i) Appling KCL at node P,

$$I_2 + i_1' = I_1 + i_1$$
$$V_2\omega C + (V_2 + V_3)0.15\omega C = V_1\omega C + 0.2V_1\omega C$$
$$\therefore 1.2V_1 - 1.15V_2 - 0.15V_3 = 0 \qquad (1)$$

Similarly at Q,

$$I_3 + i_2' = I_2 + i_2$$
$$V_3\omega C + 0.15V_3\omega C = V_2\omega C + 0.2(V_1 + V_2)\omega C$$

$$1.15V_3 = 1.2V_2 + 0.2V_1$$
$$\therefore 0.2V_1 + 1.2V_2 - 1.15V_3 = 0$$

Method 1

$$1.2V_1 - 1.15\,V_2 - 0.15V_3 = 0 \qquad (1)$$
$$0.2V_1 + 1.2V_2 - 1.15V_3 = 0 \qquad (2)$$

By solving the above two equations we get,

$$V_2 = \frac{675}{835}V_3$$

Substitute the value of V_2 in equation (1)

$$1.2V_1 - 1.15\times\frac{675}{835}V_3 - 0.15V_3 = 0$$
$$\therefore V_1 = 0.8997V_3$$

Voltage across the string is

$$\therefore V = V_1 + V_2 + V_3$$
$$= 0.8997V_3 + 0.8084V_3 + V_3 = 2.7081V_3$$
$$\therefore V_3 = 36.93\%V$$
$$V_2 = 0.8084V_3 = 29.85\%V$$
$$V_1 = 0.8997V_3 = 33.22\%V$$
$$\%\text{String efficiency} = \frac{V}{3 \times V_3} = \frac{2.7081V_3}{3 \times V_3}$$
$$= 90.27\%.$$

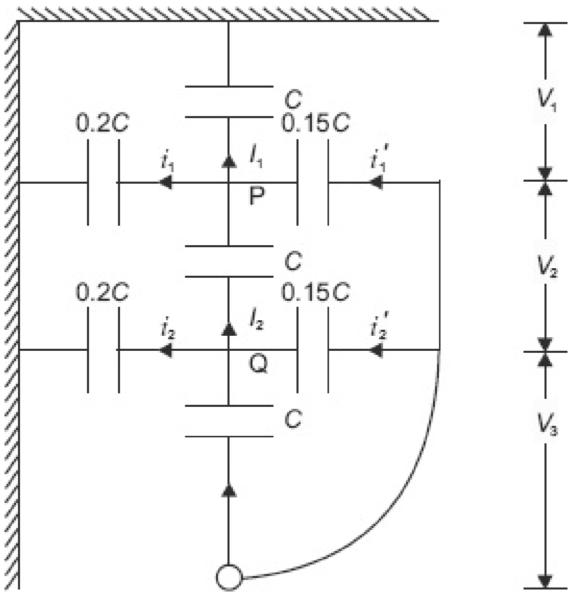

Fig. 8.25 Circuit diagram for Example 8.16

Method 2

Equations (1) & (2) are modified as

$$120\frac{V_1}{V_3} - 115\frac{V_2}{V_3} = 15, \qquad 20\frac{V_1}{V_3} + 115\frac{V_2}{V_3} = 15$$

Let $\frac{V_1}{V_3} = x$ and $\frac{V_2}{V_3} = y$

$$120x - 115y = 15; \; 20x + 120y = 115$$

Solving these two equations, we get,

$$\frac{V_2}{V_3} = y = 0.8084; \ \frac{V_1}{V_3} = x = 0.8997$$

$$V_2 = 0.8084V_3; \ V_1 = 0.8997V_3$$

Voltage across the string is

$$\begin{aligned} \therefore V &= V_1 + V_2 + V_3 \\ &= 0.8997V_3 + 0.8084V_3 + V_3 \\ &= 2.7081V_3 \\ \therefore V_3 &= 36.93\%V \\ V_2 &= 0.8084V_3 = 29.85\%V \\ V_1 &= 0.8997V_3 = 33.22\%V \end{aligned}$$

$$\%\text{String efficiency} = \frac{V}{3\times V_3} = \frac{2.7081V_3}{3\times V_3} = 90.27\%$$

(ii) If the pin to line capacitance is increased from 0.15C to 0.3C then, from Eq. (1),

$1.2V_1 - 1.15V_2 - 0.15V_3 = 0$

KCL at node Q,

$$I_3 + i_2' = I_2 + i_2$$

$$V_3\omega C + 0.3V_3\omega C = V_2\omega C + 0.2(V_1 + V_2)\omega C \qquad (\text{since } C_3 = 0.3C)$$

$$1.3V_3 = 0.2V_1 + 1.2V_2$$

$$0.2V_1 + 1.2V_2 - 1.3V_3 = 0$$

The above two equations can be written as

$$120\frac{V_1}{V_3} - 115\frac{V_2}{V_3} = 15 \quad \Rightarrow 120x - 115y = 15$$

$$2\frac{V_1}{V_3} + 12\frac{V_2}{V_3} = 13 \quad \Rightarrow 2x + 12y = 13$$

Solving these two equations, we get

$$V_2 = 0.9162\,V_3$$
$$V_1 = 1.0028\,V_3$$

Voltage across the string is

$$\begin{aligned}
\therefore V &= V_1 + V_2 + V_3 \\
&= 1.0028V_3 + 0.9162V_3 + V_3 \\
&= 2.919V_3 \\
V_3 &= 34.26\%V \\
V_2 &= 31.39\%V;\ V_1 = 34.35\%V \\
\%\text{String efficiency} &= \frac{V}{3\times V_3} = \frac{2.919V_3}{3\times V_3} \\
&= 97.3\%
\end{aligned}$$

We can observe the case (i) and case (ii), the string efficiency is improved from 90.27% to 97.3% after using the guard ring.

8.7 ARCING HORN

In case of an insulator flashover, the porcelain is often cracked or broken up by the power. As a precaution against this, arcing horns or rings are installed on a few overhead lines. One of the horns or rings is earthed and the others are electrically connected with the line conductor through the insulators as shown in Fig. 8.26. They are arranged in such a way that the arc is taken by the electrodes and held at sufficient distance from the porcelain to prevent damage by the heat of the arc. In the ring design, the advantage is that the arc can form at any point round the insulator and if formed at the windward side, it may be blown around without damaging the insulator.

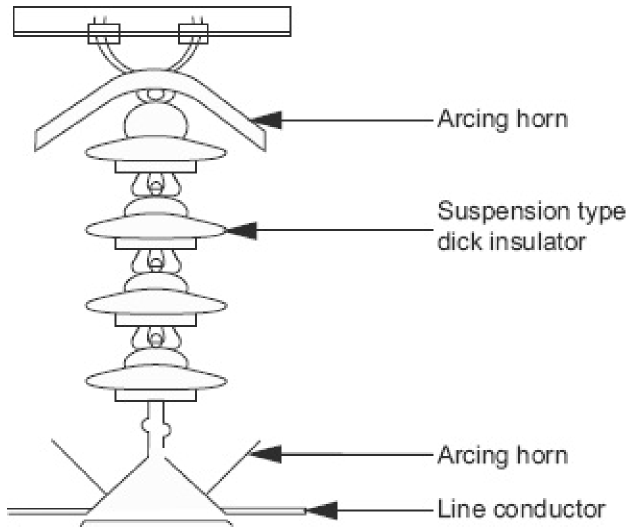

Fig. 8.26 Arcing horn

In addition, the horns at the earth-end are shaped in such a way to transfer (lead this end of) the arc upwards and outwards from the insulator string with a view to increase the arc length as quickly as possible and thereby allowing it to self extinguish. Application of these devices lowers the flashover voltage of the insulator.

8.8 TESTING OF INSULATORS

The insulator should have good mechanical and dielectric strengths to withstand the load and operating or flashover voltages, respectively. However, it should be free from pores or voids, which may damage the insulator. For this, the following three tests are performed

1. Flashover tests
2. Performance tests and
3. Routine tests

8.8.1 Flashover Tests

Three types of flashover tests are to be conducted before the insulator can be said to have passed the flashover test. They are power frequency dry flashover test, power frequency wet flashover test and impulse frequency flashover test.

a. Power Frequency Dry Flashover Test In this test, voltage is applied between the electrodes of the insulator mounted in the manner in which it is to be used. Gradually the applied voltage is increased until the surrounding air breaks down. This voltage is known as flashover voltage, and must be greater than that of the minimum specified voltage. The insulator must be capable of withstanding the minimum specified voltage for one minute.

b. Power Frequency Wet Flashover Test In this case, again the insulator is mounted in the same manner as that of dry flashover test and the voltage is applied gradually, in addition to that, the insulator is sprayed with water at an angle of 45° in such a manner that its precipitation should not be more than 5.08 mm/min. The insulator must be capable of withstanding minimum voltage for 30 sec.

c. Impulse Frequency Flashover Test In this test, a generator develops a very high voltage at a frequency of several hundred kilohertz. This voltage is applied to the insulator and spark-over voltage is noted. The ratio of impulse spark-over voltage to spark-over voltage at power frequency is called impulse ratio.

$$\text{Impulse ratio} = \frac{\text{Impulse spark-over voltage}}{\text{Spark-over voltage power frequency (50 Hz)}}.$$

8.8.2 Performance Test

Before giving the satisfactory performance of the insulators, the following tests are to be conducted.

a. Puncture Voltage Test The purpose of this test is to determine the puncture voltage. In this test, the insulator is suspended in insulating oil. The voltage is applied and increased gradually until the puncture takes place. The voltage at which the puncture starts is called puncture voltage. This voltage for suspension type insulator is 30% higher than that of the dry flash-over voltage.

b. Mechanical Strength Test This test is conducted to determine the ultimate mechanical strength of pin type insulator. The insulator is mounted on a steel pin and 250% of working load is applied for one minute.

c. Electro-mechanical Test This is conducted only for suspension type insulators. In this test, a tensile stress, which is equal to 250% of working tensile strength is applied for one minute. After this, the insulator is tested for 75% of dry spark-over voltage.

d. Porosity Test This test is conducted to determine the degree of porosity. In this, a freshly fired insulator sample is taken and broken into pieces and immersed in a 1% alcohol solution of fuchsine dye under pressure of 150 kg/cm^2. After one hour, the pieces are removed from the testing pot and are observed for the penetration of the dye. This gives the degree of porosity indication.

8.8.3 Routine Tests

Before the finally recommended insulator is used in the field, it is required to satisfy the following routine tests.

a. High Voltage Test In this test, the pin insulators are inverted and are placed in water up to the neck. The spindle hole is also filled with water, and a high voltage is applied for 5 min. After the completion of this test, the insulator should remain undamaged.

b. Proofload Test In this test, all types of testing insulators are assembled and a tensile load of 20% in excess of the working load is applied for one minute.

After completion of this test, no damage should occur to the insulator.

c. Corrosion Test In this case, the insulator with its fittings is suspended in a copper sulphate ($CuSO_4$) solution at 15.2 °C for one minute. Then, the insulator is removed, wiped, cleaned and put again in $CuSO_4$ solution. This procedure is repeated four times, which results in zero metal deposits over the insulator.

8.9 CAUSES OF FAILURE OF INSULATORS

Interruption of power supply in the overhead transmission lines is due to the failure of insulators. The following are the main causes of such failure.

1. Mechanical stress
2. Cracking
3. Porosity
4. Puncture and flashover
5. Short circuit

i. Mechanical Stress This occurs due to high compressive strength of the porcelain and it fails because of low-tension. The failure of insulator due to mechanical stress is rare because defective pieces are eliminated in routine tests.

ii. Cracking of Insulator In pin-type and cemented-cap type suspension insulators this type of failure is frequent. Generally, expansion and contraction is uneven in cement, steel, and porcelain due to seasonal change (i.e., heat, cold, dryness, and dampness). This produces high stresses and causes failure of tension. This can be reduced by using elastic cushions in between the shells.

iii. Porosity Insulators made of porcelain are porous. Hence, they absorb water from the atmosphere or from cement, which leads to reduction in insulation resistance and causes current leakage. This may result in an insulator failure. This can be minimized by glazing the insulator.

iv. Flash-over Overheating of the insulator is due to flash-over between one metal part to other. This causes unequal expansion of porcelain, thus shattering the insulator with cracking. This can be avoided by providing the arcing horns.

v. Short Circuit Sometimes arcs are generated due to short circuit between conductors or conductor to earth. This may cause failure of insulator. This can be minimized by increasing clearance between the conductors and conductors to earth.

CHAPTER AT A GLANCE

1. **Insulators:** The insulators for overhead transmission (or distribution) lines provide insulation to the power conductor from the ground. The insulators are connected to the cross-arm of the supporting structure and the power conductor passes through the clamp of the insulator. The main cause of failure of insulators is due to flash over or puncture.
2. **Insulator materials:** The most commonly used materials for insulators on overhead lines are glazed porcelain, glass, steatite, and other special composition materials.
3. **Types of insulators:** The most commonly used insulators are pin type, suspension type, strain type, and shackle type insulators.
4. **Pin type insulators:** This type of insulator is small, simple in construction and cheap. These insulators are mounted on the cross-arm of the pole. It is used for transmission and distribution of electrical power for voltages up to 33 kV.
5. **Suspension type insulators:** For voltage above 33 kV, the suspension type or disc type insulators are used. Each unit of suspension type insulators is designed for comparatively low voltage, 11 kV. There are three types of suspension insulators. They are Hewlett or interlinking, cemented cap, and core and link types

   .
6. **Strain insulators:** These type of insulators are used in transmission lines under the following circumstances – when the transmission line comes to an end, has sharp curves, intermediate anchor towers and long river crossings. The discs of a strain insulator are in a vertical plane as compared to the discs of suspension type insulator.
7. In general, potential distribution across the ith unit from the bottom is given

$$V_i = \frac{V \times 2\sinh\left[\frac{1}{2}\sqrt{m}\right]\cosh\left[\left(n - i + \frac{1}{2}\right)\sqrt{m}\right]}{\sinh\left[n\sqrt{m}\right]}$$

where, $m = \frac{\text{Capacitance to ground}}{\text{Mutual capacitance}}$, n = number of discs in the string.

8. **String efficiency:** It is the ratio of voltage across the whole string to the product of the number of discs and the voltage across the disc nearer to the line conductor.

   i.e., String efficieny, $\eta = \frac{\text{Voltage across the string}}{n \times \text{Voltage across the unit near the power conductor}}$

   In order to improve the string efficiency, it is necessary to equalize the potential across the various units of the string.
9. Various methods of improving the string efficiency are: selection of m, grading of insulators and guard ring.
10. **Testing of insulators:** These tests may be divided into three types and they are:

    design tests, performance tests and routine tests.

SHORT ANSWER QUESTIONS

1. What are the various types of insulators?
2. Give two properties of insulators.
3. Give two applications of insulators.
4. What are the common materials used for insulators?
5. Why is porcelain the most commonly used material for insulators?
6. Define string efficiency.
7. What are the causes of insulator failure?
8. What is the formula for safety factor of insulator?
9. Why are insulators are used in overhead lines?
10. What are the disadvantages of pin type insulators?
11. What are the advantages of suspension type insulators?
12. What is the disadvantage of using suspension type insulators?
13. What is a strain insulator and where is it used?
14. Write down the different methods for improving string efficiency.
15. What is meant by guard ring?
16. What is the need for grading insulators?
17. What is the need of arcing horns for insulators?

MULTIPLE CHOICE QUESTIONS

1. The insulator provides necessary_____ between line conductors and prevents any leakage current from conductors to earth.
   1. insulation
   2. conduction
   3. dielectric
   4. semiconductor
2. Types of insulators are_________
   1. pin and suspension
   2. strain
   3. shackle and stay
   4. all of these
3. Suspension type cables are used for _________ kV.
   1. 11
   2. 33
   3. more than 33
   4. more than 220
4. Up to 33 kV ________ type insulators are preferred.
   1. suspension
   2. pin
   3. strain
   4. shackle
5. The most commonly used material for insulators of overhead lines is ________ .
   1. porcelain
   2. glass
   3. mica
   4. PVC
6. The groove on the upper end of the insulator is provided for________ conductor.
   1. insulation
   2. housing
   3. conduction
   4. induction
7. The dielectric strength of a porcelain material is ________ k V/cm.
   1. 60
   2. 20
   3. 50
   4. 40
8. The failure of insulators is due to ___________.
   1. flash over
   2. puncture
   3. either a or b
   4. none of these
9. The insulator may puncture due to _______ produced by the arc.
   1. breaking
   2. loss
   3. heat

4. power

10. Safety factor of insulator is _________ .
    1. $\frac{\text{Puncture strength}}{\text{Flash over voltage}}$
    2. $\frac{\text{Puncture strength}}{\text{Flash over voltage}} \times 100$
    3. $\frac{\text{Flash over voltage}}{\text{Puncture strength}}$
    4. $\frac{\text{Flash over voltage}}{\text{Puncture strength}} \times 100$
11. Formula for string efficiency of three-disc suspension insulators is ________.
    1. $\frac{V}{3 \times V_3} \times 100$
    2. $\frac{V}{2 \times V_3} \times 100$
    3. $\frac{V}{6 \times V_3} \times 100$
    4. $\frac{V}{3 \times V_2} \times 100$
12. The shunt capacitance is increased then the string efficiency is _______ .
    1. increased
    2. decreased
    3. constant
    4. none of these
13. Purpose of guard ring is to_________ the string efficiency.
    1. increased
    2. decreased
    3. constant
    4. none of these
14. Methods of improving the string efficiency are ______ .
    1. using longer cross arms
    2. grading the insulator
    3. using guard ring
    4. all of these
15. The maximum voltage appears across the disc _____ to the

conductor.
1. near
2. away
3. undefined
4. none of these

16. Which of the following insulator will be selected for high voltage applications?
    1. pin-type
    2. suspension type
    3. strain type
    4. shackle type
17. When an insulator breaks down because of a puncture, it is________ damaged.
    1. partially
    2. permanently
    3. temporarily
    4. none of these
18. The insulator is so designed that it should fail only by_________ .
    1. flash over
    2. puncture
    3. either or
    4. none of these
19. A shorter string has ______ string efficiency than a larger one.
    1. less
    2. more
    3. no change
    4. none of these
20. When the potential across each disc is the same, then the string efficiency is_________.
    1. 50%
    2. 25%
    3. 100%
    4. 75%
21. To support the insulators________ are provided.
    1. cross arms
    2. conductors
    3. guard ring
    4. pole
22. The longer the cross arms, the_________ the string efficiency.
    1. greater
    2. less
    3. constant
    4. none of these
23. The discs of the strain insulators are used in_________.
    1. vertical plane only
    2. horizontal plane
    3. both a and b
    4. inclined
24. A three-phase overhead transmission line is being supported by three discs suspension insulators, the potential across the first

and the second insulator are 8 kV and 11 kV, respectively. The line voltage is ________ kV.

1. 8
2. 11
3. 64.28
4. 37.12

25. A three-phase overhead transmission line is being supported by three disks suspension insulators, the potential across the first and the second insulator are 8 kV and 11 kV, respectively. String efficiency is ____________.
    1. 60%
    2. 68.28%
    3. 68%
    4. 63%

Answers

1. a	2. d	3. c	4. b	5. a
6. b	7. a	8. c	9. c	10. a
11. a	12. b	13. a	14. d	15. a
16. b	17. d	18. a	19. b	20. c
21. a	22. a	23. a	24. c	25. b

REVIEW QUESTIONS

1. Explain the working of the different types of insulators used in overhead line. Give their field of application.
2. Explain why suspension insulators are preferred for high voltage transmission lines. What is a strain insulator and where is it used?
3. What are the advantages and disadvantages of suspension type insulators over pin type insulators? Sketch the sectional view of one unit of the suspension type insulator and describe its construction.
4. Define string efficiency. Why is it necessary to have high string efficiency? How can it be achieved?
5. Discuss the methods for improving the string efficiency of overhead line insulators.
6. Write a short note on insulator failure.
7. Explain the use of grading rings and arcing horns on suspension insulators.
8. What electrical and mechanical characteristics are required for a good insulator for using HV transmission lines?
9. Explain what is meant by a string efficiency of a suspension insulator consisting of a number of units. What causes the string efficiency to be less than 100%?

10. Explain why the voltage across the insulator string is not equal and describe practical methods to improve them.
11. Discuss the method of grading the string unit in insulations.
12. Show that the voltage distribution across the units of a string insulator is not uniform.

PROBLEMS

1. Determine the string efficiency of a string insulator of five units having self-capacitance equal to 10 times the pin to earth capacitance.
2. A string insulator consists of five units. The capacitance from each joint to tower is 12% of the capacitance of each unit. Determine the voltage across the lowest unit as percentage of the total voltage. Also calculate string efficiency.
3. A string of suspension insulators consists of five units each having capacitance *C*. The capacitance between each unit and earth is 1/8 of *C*. Determine the voltage distribution across each insulator in the string as a percentage of voltage of conductor to earth. If the insulators in the string are designed to withstand 36 kV maximum, calculate the operating voltage of the line where five suspension insulator strings can be used.
4. A string of four insulators has a self-capacitance of *C* Farads. The shunting capacitance of the connecting metalwork of each insulator is 0.3*C* to earth and 0.2*C* to the line. A guard ring increases the capacitance to the line of the metalwork of the lowest insulator to 0.5*C*. Determine the string efficiency of this arrangement with the guard ring.
5. A string consisting of seven suspension discs is fitted with a grading ring. Each pin to earth capacitance is *C*. If the voltage distribution is uniform, determine the values of line to pin capacitance.
6. Each conductor of a 33 kV, three-phase system is suspended by a string of three similar insulators. The capacitance between each insulator pin and earth is 13% of self capacitance of each insulator. Find (a) the distribution of voltage over three insulators and (b) string efficiency.
7. A three-phase overhead transmission line is suspended by a suspension type insulator which consists of three units. The potential across top unit and middle unit are 7 kV and 10 kV, respectively. Calculate (a) the ratio of capacitance between pin and earth to the self capacitance of the each unit, (b) the line voltage, and (c) string efficiency.

9

Underground Cables

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Understand the need for underground cables
- Provide constructional features and grading of cables
- Calculate the capacitance for three-core cables
- Provide the thermal characteristics and testing of cables

9.1 INTRODUCTION

Underground cables are used for power transmission and distribution in all such places where it is difficult to use an overhead system. In thickly populated areas such as cities and towns, the cables are laid below the ground surface. The combination of conductor and insulator over it is called a cable. Insulation is provided for the external protection of the cable against mechanical damage, moisture entry, and chemical reactions. Though the initial investment is more compared to that in overhead lines, the maintenance required is negligible compared to that of an overhead system. Underground power system usage ensures the prevention of interference with communication lines and also guarantees a more picturesque landscape un-chequered sky and no suspended wire-networks.

9.2 GENERAL CONSTRUCTION OF A CABLE

The general construction of the cable is shown in Fig. 9.1. The various parts in the cable are:

Core All cables have one or more cores at the centre, depending upon the service required (single-phase or three-phase). The material used for the core is generally copper although at times aluminium is also used.

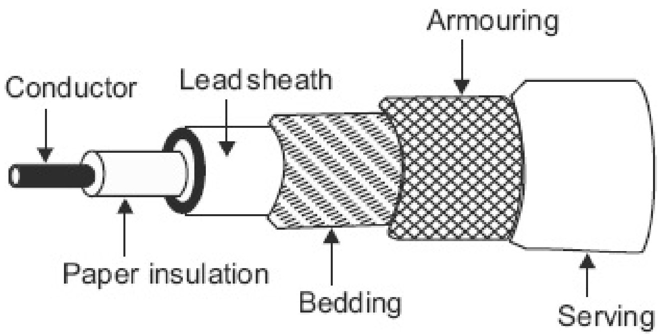

Fig. 9.1 Construction of a cable

Insulation The insulation over the core is generally provided with different layers. Thickness of each layer depends upon the operating voltage. The materials used for insulation are paper, varnished cambric, vulcanized bitumen, poly vinyl chloride (PVC), and cross-linked polyethylene (XLPE).

Metallic Sheath For protecting the insulation material from moisture, gases and any other harmful liquids in the soil, a metallic sheath is provided over the insulation. It also protects the insulation from mechanical damage. The metallic sheath is usually of lead or lead alloy. Recently, aluminium is also being used as metallic sheath because it has greater mechanical strength, low weight and cost when compared to lead sheath.

Bedding For the protection of metallic sheath against corrosion and mechanical injury, a layer of “bedding” consisting of paper tape compounded with a fibrous material like jute or Hessian tape is used. This protects the metallic sheath from corrosion and mechanical injury due to armouring.

Armouring Armouring is provided over the bedding to protect the cable from mechanical injury while laying and handling it. It consists of one or two layers of galvanized steel wires or two layers of steel tape.

Serving A layer of fibrous material is again used over the armouring which is same as bedding. This is called serving. Serving is useful to protect the armouring from atmospheric conditions.

9.3 TYPES OF CABLES

The type of cables used depends upon the voltage and service requirements. Based on the voltage, the cables are classified as:

1. Low tension cables – for operating voltage up to 1 kV
2. High tension cables – for operating voltage up to 11 kV
3. Super tension cables – for operating voltage up to 33 kV
4. Extra high tension cables – for operating voltage up to 66 kV
5. Extra super-voltage power cables – for operating voltage of 132 kV and above

9.3.1 Low Tension Cables

These are used for voltages below 1 kV. This type of cable does not require any special construction because electrostatic stresses developed are very small and the thermal conductivity is also not very important. The insulation may consist of paper impregnated with oil. Resin is used to increase viscosity and thus to prevent damage. Varnished cambric is used for insulation. Over the insulation, a lead sheath is provided to protect the cable from moisture and handling. The cables are of two types, they are single-core and multi-core cables.

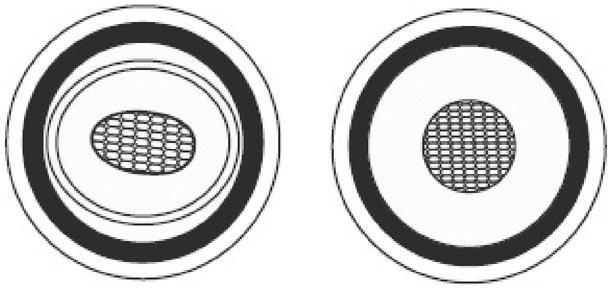

Fig. 9.2 Single-core cable

Single-core Cables A single core cable is shown in Fig. 9.2. This cable consists of a copper/aluminium stranded circular core. This conductor is insulated with paper insulation over which a lead sheath is provided. The lead sheath protects the cable against the moisture entering them. Above the lead sheath, there is a provision of a layer of compounded fibrous material or Hessian tape in order to protect the metallic sheath against corrosion. These are usually not provided with armouring in order to avoid excessive loss in the armour.

Multi-core Cables Consider a three-core cable. In this, all three cores are insulated with each other and placed. These are either circular-shaped, oval or sector-shaped conductors wrapped around by impregnated paper. The multi-core cables are of belt type or sector shaped. The shaped core improves the copper space factor. Insulating belt of paper is provided surrounding the three cores. To protect the cable from the entry of moisture, a lead sheath is enclosed over the cable. A coating of lime wash is applied outside the cable in order to prevent adhesion. View of three core LT cable is shown in Fig. 9.3(a).

9.3.2 High Tension Cables

These are used for three-phase medium and high voltage distribution. Paper is generally used for insulation. The three-core belted type cable belongs to this type. The construction is same as that of the low tension cables. The three-core belted type cable has the three insulated cores. The three cores are again insulated with impregnated paper; over this a lead sheath is provided to prevent moisture entering into the cable as shown in Fig. 9.3(b).

9.3.3 Super Tension Cables

These cables are used for operating voltages up to 33 kV and these are classified as:

- H-type or screened cables
- SL type cables
- HSL type cables

Fig. 9.3(a) View of three-core LT cable (11 kV)

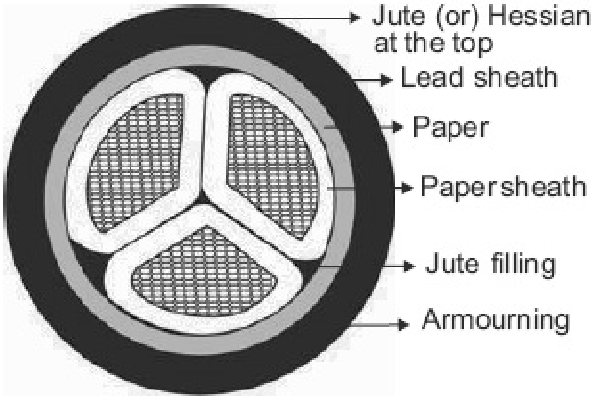

Fig. 9.3(b) Three-core high tension cable

H-type or Screened Cables These cables contain three paper insulated cores as shown in Fig. 9.4. A layer of perforated metallized paper is provided over the insulation of each conductor. These cables are laid up in the ordinary way with the metallized screens in contact with each other. Cotton tape with fine wires of copper is wound around the three cores which act as an insulator. The layers of lead sheath, bedding, armouring, and serving are provided as usual.

Advantages:

- The electric stresses are entirely radial and reduce the dielectric loss since all four screens and the lead sheath are at the earth potential.
- The metal foil increases the heat dissipating power and hence there is no sheath loss.
- There is an increase in the current carrying capacity of the cable.
- There is no possibility of forming voids and vacuous spaces.

SL Type Cables In this type of cable, each core has its own lead sheath over the insulation with an impregnated paper as shown in Fig. 9.5. In addition to the overall sheath, there is an overall layer of bedding, armouring and serving as usual.

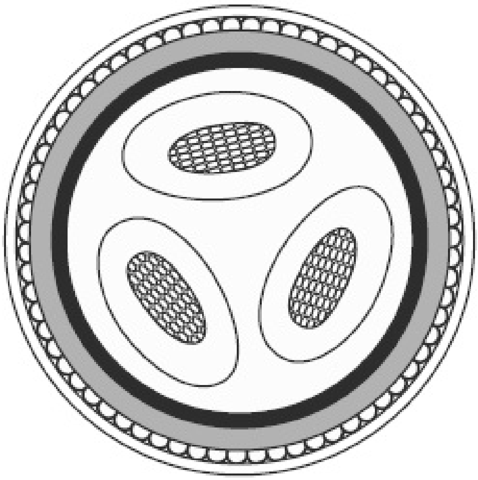

Fig. 9.4 Three-core H-type cable

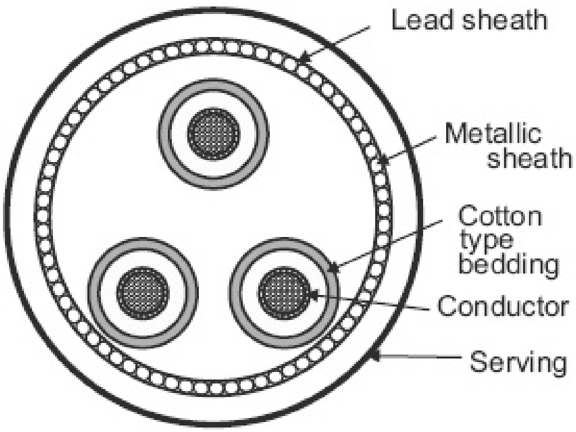

Fig. 9.5 Three-core SL type cable

Advantages:

- There is no overall lead sheath due to which the bending of the cable is possible.
- Electrical stresses are radial.
- Reduced possibility of oil drainage on hill routes.

HSL Type Cables This type cable is the combination of H type and SL type cables as shown in Fig. 9.6. In this, each core has insulation with impregnated paper and a separate lead sheath is provided. All three cores are arranged in required form and then insulated by providing the layers of bedding, armouring, and serving as usual.

The advantages of screened cables over belted type cables are:

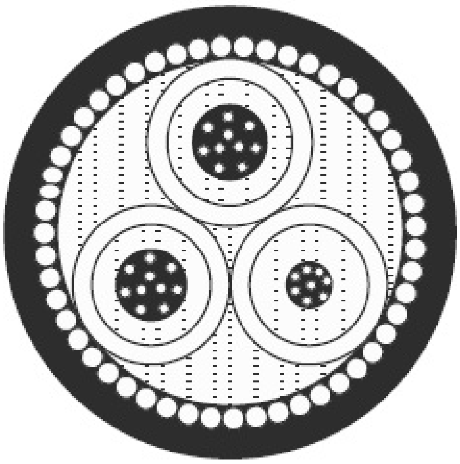

Fig. 9.6 Three-core HSL type cable

- The possibility of reducing core-to-core faults because this has greater core-to-core thickness to a given overall diameter.
- The electric stresses in the cables are uniformly radial in all sections.

9.3.4 Extra High Tension Cables

Three core cables are generally used for voltages up to 33 kV. Because of weight and size, three-core cables are not economical and practicable. To meet the increased voltage working demand, super tension cables are used. But, there are greater sheath losses in single-core cables. Due to the presence of voids in extra high tension voltage cables, there is a danger of breakdown of dielectric. Due to the failure of the dielectric, there exists ionization and allied chemical reactions, which damage the insulation.

There are two methods employed to minimize the formation of voids. They are by using:

- Thin oil under pressure (oil-filled) cables
- Gas pressure cables

Oil-filled Cables Figure 9.7 shows a single-core oil-filled cable. In this type of cable, an oil channel is formed at the centre of the core by stranding the conductor wire built around a hollow cylindrical steel spiral. This channel formed at the centre of the core is filled with low viscosity mineral oil by means of oil reservoirs and feeding tanks placed at about every 500–600 m length along the route of the cable and maintained at a constant pressure (not below the atmospheric pressure) at any point along the cable.

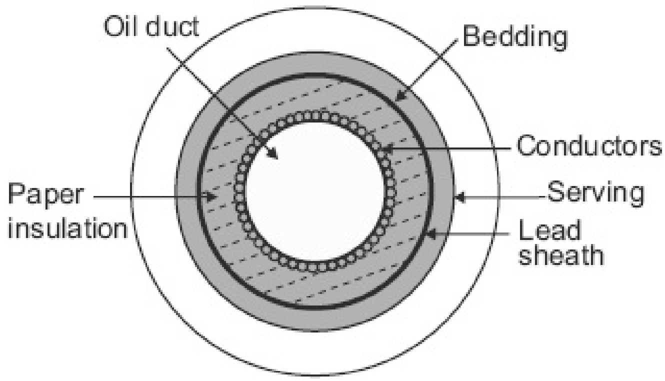

Fig. 9.7 Single-core cable

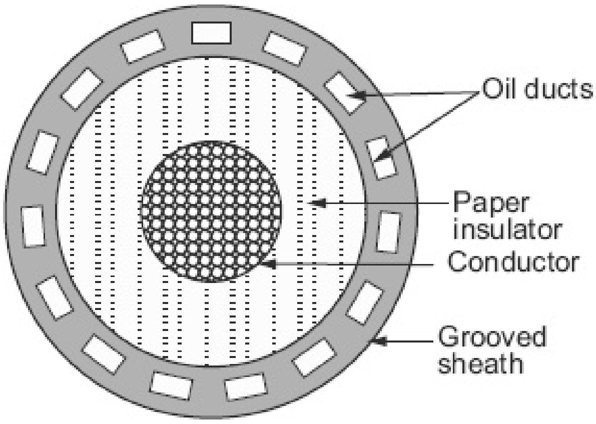

Fig. 9.8 Single-core sheath-channel cable

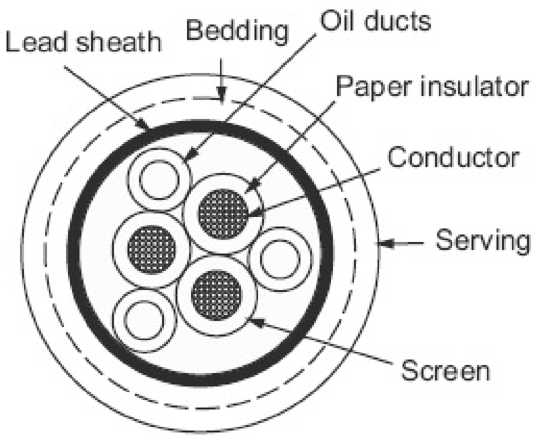

Fig. 9.9 Three-core filler-space channels

1. **Single-core Cables** Oil-filled single-core cable is shown in Fig. 9.7. These cables are so designed that when oil gets expanded due to rise in temperature of the cable, the extra oil gets collected in the external reservoir, which sends it back during contraction of oil when the temperature falls under light load conditions.
2. **Single-core Sheath-channel Cable** The conductor is solid in this type of cable like that of solid cables and is insulated with paper. The oil channel is provided by grooving the sheath and is shown in Fig. 9.8.
3. **Three-core Filler-space Channels** Three-core, oil-filled cables are shown in Fig. 9.9. The oil channels are located within the hollow filler space. The channels are of perforated metal

ribbon tubing and are at earth potential. These channels are filled with oils in the factory and are dispatched by placing them lengthwise over drums provided with tanks having oil under pressure.

The advantages of oil filled cables over the solid cables:

1. The thickness of the required dielectric decreases; as a result the overall size and weight of the cable reduces.
2. The possibility of formation of voids, oxidation and ionization is eliminated.
3. More perfect impregnation can take place.
4. Current rating of the cable increases because of reduced thermal resistance of the cable.
5. The maximum allowable stress increases.
6. Possibility of increased temperature range in service.
7. The possibility of detection of fault is easy, since as soon as the lead at any section deteriorates, the oil will start leaking.

Disadvantages:

1. High cost.
2. Installation and maintenance is complicated.

4. **Gas-filled Cables** In gas-filled cables, the paper dielectric is impregnated with petroleum jelly. The space between layers of dielectric paper is filled with dry nitrogen gas with a pressure of 1400 kN/m^2. This pressure is maintained constantly by pumps and lead sheath inside the cable. In case of single-core cables, a small clearance is left to allow the gas to flow axially. In three-core cables, the clearance is not necessary because the filler spaces and strands will allow the gas to flow.

Fig. 9.10 Three-core gas-pressure cables

Gas-pressure Cables These are mainly of two types:

1. External-pressure cables
2. Internal-pressure cables

1. External-pressure Cables This type of cable, recently introduced, has been developed for higher voltages. For this type of cable, pressure is applied externally and raised to such an extent that no ionization can take place. Due to this increased pressure, radial compression takes place which tends to close voids if any and the operating power factor of such a cable is improved. The construction of the cable is similar to that of the ordinary solid type except that it is triangular instead of a circular cross-section. The triangular section reduces its weight and has low thermal resistance.

Advantages:

- In comparison with the normal cable, these cables can carry from 1.4 to 1.6 times the load current, double the voltage and transmit from 2.4 to 3.2 times the power.
- The maximum potential gradient is 100 kV/cm and power factor is 0.5% at 15° C.
- The steel pipe provides mechanical protection to the cables and acts as an ideal method of cable laying.
- The nitrogen in the steel pipes helps in quenching any flame.
- Reservoirs are not needed.
- Maintenance cost is low.

The only disadvantage of these cables is that its initial installation cost is very high.

2. Internal-pressure Cables These cables are of three types:

1. High-pressure gas-filled cables
2. Gas-cushion cables
3. Impregnated-pressure cables

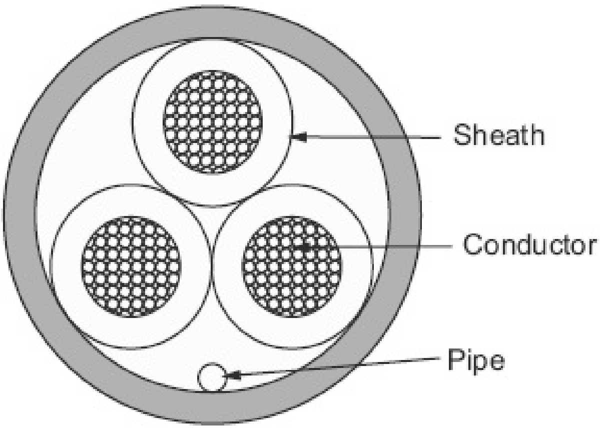

Fig. 9.11 Three-core impregnated pressure cable

1. *High-pressure gas-filled cables:* For this type of cable, the space for the gas is provided in the dielectric itself which is inert like nitrogen at a pressure of about 6 atmospheres for extra high-tension voltage cables and 12 atmospheres for super-voltage cables. This facilitates the axial flow of gas, which also passes along the un-impregnated strand. This clearance is not essential in the case of multi-core cables; the filler spaces and strands provide a low resistance path for the flow of gas.

2. *Gas-cushion cables:* For this type of cable, screened space is provided in between the lead sheath and the dielectric along the length of the cable. The screened space is sub-divided by means of barriers and the inert gas is stored at various points all along the length of the cable. The advantage of this cable is that no separate arrangement is required for the transmission of pressure to the cables from outside. Hence, the cable is a complete unit with its own armouring, without requiring the external pipe protection.

3. *Impregnate-pressure cable:* The main difference between this cable and a solid cable is that it consists of mass-impregnated paper dielectric, which is maintained under a pressure of 14 atmospheres by means of

nitrogen. To cater to the large hoop and longitudinal-stress setup, a special reinforcement is provided. The advantages of internal-pressure cables

- No external accessories required.
- The cable can be used for vertical run without any fear of leakage with suitable designs.
- With increase in pressure there is improvement in the power factor of the cable dielectric.
- The extra super-voltage power cables are used for operating voltage beyond 132 kV.

9.4 ADVANTAGES AND DISADVANTAGES OF UNDERGROUND CABLES OVER OVERHEAD LINES

Advantages:

- The underground cable is not affected by lightning, thunderstorms and other severe weather conditions. So, there are no interruptions in service to consumers.
- Accidents caused by the breaking of the conductors will be reduced.
- The maintenance cost of underground system is very low because of less chance of fault occurrence.
- Because of less spacing between the conductors in an underground system, inductance is very low as compared to overhead lines; therefore, the voltage drop is less.
- In an underground system, surge effect is smoothened down as the sheath absorbs surge energy.
- In an underground system, there is no interference to communication lines.
- The underground system of distribution or transmission is more aesthetic because no wiring is visible.

Disadvantages:

- Underground cables are more expensive than overhead lines due to the high cost of trenching, conduits, cables, manholes and other special equipment.
- As transmission voltage increases, the cost of the cable increases.
- In this system, expansion for new loads is not possible. It can be met by installation of new lines only.
- Joining of underground cables is difficult, so tapping of loads and service mains is not conveniently possible in an underground system.
- The cables have high capacitance due to less spacing between the conductors so it draws high charging current.
- Though there is only a rare chance of faults occurring in an underground system, it is very difficult to locate the fault point and its repair is difficult and expensive.

Test Yourself

1. Why is there no interference to communication lines in an underground cable system?
2. Is charging current high in cables? If yes, justify your answer.

9.5 PROPERTIES OF INSULATING MATERIALS FOR CABLES

The insulation provided to underground cables should have the following properties:

- High resistivity
- High dielectric strength
- Low thermal coefficient
- Low viscosity at working temperature
- High tensile strength and plasticity
- Low absorption of moisture
- Non-inflammable
- Chemically stable (not affected by acids or alkalies)
- High mechanical strength
- Tough and flexible

9.5.1 Insulating Materials

The various insulating materials used in cables are as follows:

Rubber It is the most commonly used insulating material. Natural rubber is produced from the latex of the rubber tree. Synthetic rubber is produced from alcohol or oil products. Its relative permittivity is between 2 and 3 and dielectric strength is in between 30 and 35 kV/mm. It absorbs moisture slightly and the maximum safe temperature as low as around 38° C.

Vulcanised India Rubber (VIR) It is useful for low-voltage power distribution systems only, because of its small size. It is prepared by mixing India rubber with mineral matter such as sulphur, zinc oxide, red lead, etc. It has a reasonable value of dielectric strength (15 kV/mm). Its use is limited because of its low melting point, low chemical resistance capability and short span of life. The advantage of using vulcanized India rubber is that when it is used for a cable, the cable becomes stronger and more durable. It can withstand high temperatures and remain more elastic than pure rubber. The drawback of using this material for insulation is that it attacks copper. Hence, before using VIR as insulation, the copper conductor must be tinned well.

Impregnated Paper This is used as an insulator in case of power cables because it has low capacitance, high dielectric strength (30 kV/mm), and is economical. The paper is manufactured with wood pulp, rags or plant fibre by a suitable chemical process. It has high resistance due to high resistivity under dry condition. The drawback of this cable is that it absorbs a small amount of moisture only, which reduces the insulation resistance. Therefore, a paper insulated cable always requires some sort of protective covering and it is impregnated in insulating oil before use.

Poly Vinyl Chloride (PVC) It is a synthetic compound material and comes as a white odourless, tasteless, chemically inert, non-inflammable, and insoluble powder. It is combined chemically with a plastic compound and is used over the conductor as an insulation cover. PVC has good dielectric strength and a dielectric constant of 5. Its maximum continuous temperature rating is 75° C. It is inert to oxygen and almost chemically stable, so it is preferred over VIR cables. PVC insulated cables are usually employed for medium and low voltage domestic, industrial lights, and power installations.

XLPC Presently, cross-linked polyethylene (XLPE) is preferably used all over the world due to its increased longevity, security and current load resistance. When these insulated conductors are laid up together, interstices may be filled with fillers, if required. These conductors consist of non-hygroscopic polypropylene fillers and a separator tape of non-hygroscopic polypropylene material wherever required. Round/flat aluminium wires and round/flat galvanized steel wires/tapes are used for single and multi-conductors respectively. Extruded PVC/special PVC compound such as flame retardant (FR), flame retardant low smoke (FRLS), and low smoke zero halogen (LSOH) can be used for the outer sheath to suit different environments and fire risk conditions. Special LSF (low smoke and fumes) compounds can be provided for installation where fire and associated problems such as emission of smoke and toxic fumes offer a serious potential threat. Operating and short circuit temperatures are 90° and 250°, respectively. This cable can be operated up to 33 kV.

The merits of this cable are:

- Light in weight
- Low dielectric constant
- Good mechanical strength
- Low overall physical dimension

Gutta-percha Similar to rubber, it is used for low-voltage distribution purposes. It becomes brittle in air and is generally used for submarine cables for insulating telephone and telegraph wires.

Silk and Cotton These types of insulation materials are used on conductors which are used for low voltage purposes. These insulated wires are usually used for instrument and motor winding.

Enamel Insulation Enamelled wires are used for instruments and motor winding. These wires are cheaper than silk or cotton insulated wires.

9.6 INSULATION RESISTANCE OF CABLES

The path of current leakage through the insulating material is radial. Resistance of a leakage path of the insulating material can be determined by considering a single core cable of conductor radius $r\left(=\frac{d}{2}\right)$, internal

sheath radius $R\left(=\frac{D}{2}\right)$, length l and resistivity of dielectric

material, ρ.

Let us consider a small elemental radial length dx at a radial distance x from the centre of the core ($x > r$).

Now, the length through which the leakage current will flow is dx and area of cross-section is $2\pi xl$.

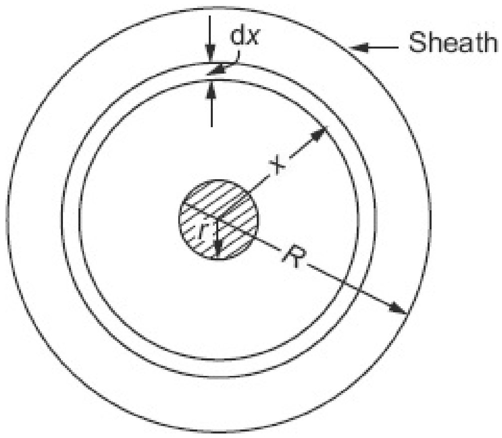

Fig. 9.12 A cable selection

Insulation resistance of a small length dx due to

leakage current $= \frac{\rho\,dx}{2\pi\,xl}$

$$\text{Insulation resistance, } R_i = \int_r^R \frac{\rho \, dx}{2\pi x l} = \frac{\rho}{2\pi l}[\ln x]_r^R$$

$$= \frac{\rho}{2\pi l} \ln \frac{R}{r} \qquad (9.1)$$

where, R_i is the resistance of the insulation

R is the radius of the cable

r is the radius of the core

From Eq. (9.1), the insulation resistance varies inversely with the length of the cable.

EXAMPLE 9.1

The insulation of a single core cable is 500 MΩ/km. If the core diameter is 2.6 cm and resistivity of insulation is 5×10^{14} Ω-cm, find the insulation thickness.

Solution:

Given data:

Length of the cable, l =1 km =1000 m

Cable insulation resistance, R_i = 500 MΩ = 500×10^6 Ω

Conductor radius, $r = \frac{2.6}{2} = 1.3$ cm

Resistivity of the insulation, $\rho = 5 \times 10^{14}$ Ω-cm $= 5 \times 10^{12}$ Ω-m

Let R be the internal sheath radius,

$$\text{Now, } R_i = \frac{\rho}{2\pi l} \ln \frac{R}{r}$$

$$\text{Or } \ln \frac{R}{r} = \frac{2\pi l R_i}{\rho} = \frac{2\pi \times 1000 \times 500 \times 10^6}{5 \times 10^{12}} = 0.6283$$

$$\text{Or } \frac{R}{r} = 1.8745$$

$$\text{Or } R = 1.8745\, r = 1.8745 \times 1.3 = 2.4368 \text{ cm}$$

$$\therefore \text{Insulation thickness} = R - r = 2.4368 - 1.3 = 1.14 \text{ cm.}$$

EXAMPLE 9.2

A single-phase core cable 6 km long has an insulation resistance of 0.45 MΩ. The core diameter is 2.2 cm and the diameter of the cable over the insulation is 5.5 cm. Calculate the resistivity of the insulation material.

Solution:

Given data:

Length, l =6 km

Insulation resistance, R_i =0.45 MΩ.

Diameter of core, d = 2.2 cm

Radius of core, r = 1.1 cm

Diameter of the insulation, D = 5.5 cm

Radius of the insulation, R = 2.75 cm

$$\text{Insulation resistance, } R_i = \frac{\rho}{2\pi l}\ln\frac{R}{r}$$

$$\therefore \text{Resistivity of the insulation material, } \rho = \frac{2\pi l R_i}{\ln\frac{R}{r}}$$

$$= \frac{2\times 3.141\times 6\times 10^3\times 0.45\times 10^6}{\ln\frac{2.75}{1.1}}$$

$$= 18.514\times 10^9\ \Omega/\text{m}.$$

9.7 CAPACITANCE OF A SINGLE-CORE CABLE

Consider a single-core cable of a conductor of radius $r\left(=\frac{d}{2}\right)$, internal sheath radius $R\left(=\frac{D}{2}\right)$ and a charge of q C/m length. Then, the electric flux density at a distance x m from the core centre is $\frac{q}{2\pi x}$ C/m^2.

The dielectric field intensity, $E = \frac{q}{2\pi\varepsilon x}$ V/m

The potential difference between conductor and sheath, $V = \int_r^R \frac{q}{2\pi\varepsilon x}\,dx$

$$= \frac{q}{2\pi\varepsilon}\int_r^R \frac{dx}{x} = \frac{q}{2\pi\varepsilon}\ln\frac{R}{r} \text{ volts}$$

$\therefore$ Capacitance of a cable per metre $= \frac{q}{V} = \frac{2\pi\varepsilon}{\ln\frac{R}{r}}$ F/m

where, $\varepsilon = \varepsilon_0\varepsilon_r = \frac{\varepsilon_r}{36\pi \times 10^9}$

where, ε_r is the relative permittivity

$\therefore$ Capacitance of a cable per metre $= \frac{\varepsilon_r \times 10^{-9}}{18\ln\frac{R}{r}}$ F/m.

EXAMPLE 9.3

Calculate the capacitance and charging current of a three-phase, single-core 33 kV, 50 Hz, 2 km long cable having a core diameter of 2 cm and a sheath diameter of 6 cm. Relative permittivity of the insulation is 3.0.

Solution:

Length, l = 2 km

Diameter of core, d = 2.0 cm

Radius of core, r = 1.0 cm

Diameter of the sheath, D = 6.0 cm

Radius of the sheath, R = 3.0 cm

Relative permittivity of insulating material, ε_r = 3.0

Operating voltage, $V_{(L\text{-}L)}$ = 33 kV

Capacitance of a cable per metre $= \frac{\varepsilon_r \times 10^{-9}}{18\ln\frac{R}{r}}$ F/m

$$= \frac{3\times10^{-9}}{18\ln\frac{3}{1}} = 151.76\times10^{-12} \text{ F/m}$$

$$= 151.76 \times 10^{-12} \times 2000 = 303.52 \times 10^{-9} \text{ F}$$

Charging current, $I = \omega C V_{ph} = 2\pi \times 50 \times 303.52 \times 10^{-9} \times \frac{33}{\sqrt{3}} \times 1000 = 1.817$ A.

9.8 DIELECTRIC STRESS IN A CABLE

The dielectric stress (or potential gradient) at any point is defined as the rate of increase of potential at that point.

Let, r be the radius of the conductor

R be the inner radius of the sheath

q be the charge per metre axial length

The potential gradient at a distance x m from the centre of the conductor within the dielectric material,

$$g = \frac{q}{2\pi\varepsilon x} \tag{9.2}$$

is also equal to the electric field intensity, E where, ε is the permittivity of the dielectric material.

The potential difference between the surface of the conductor and the inner surface of the sheath,

$$V = \int_r^R E\,dx = \int_r^R \frac{q}{2\pi\varepsilon x}\,dx$$

$$= \frac{q}{2\pi\varepsilon}\ln\left(\frac{R}{r}\right) \text{ volts} \tag{9.3}$$

From Eq. (9.3),

$$\text{Charge, } q = \frac{2\pi\varepsilon V}{\ln\left(\dfrac{R}{r}\right)}$$

Substitute the value of q in Eq. (9.2)

$$g = \frac{q}{2\pi\varepsilon x} = \frac{V}{x \ln\left(\frac{R}{r}\right)} \tag{9.4}$$

The gradient (stress) is maximum at the surface of the conductor at $x = r$ and minimum at the inner surface of the sheath i.e., at $x = R$.

From Eq. (9.4),

$$\therefore \text{The maximum potential gradient (stress), } g_{max} = \frac{V}{r \ln\left(\frac{R}{r}\right)} \tag{9.5}$$

$$\text{and minimum gradient, } g_{min} = \frac{V}{R \ln\left(\frac{R}{r}\right)} \tag{9.6}$$

Test Yourself

1. Why is potential gradient maximum at the inner core of cables?

EXAMPLE 9.4

A single-core cable has an inner diameter of 5.5 cm and core diameter of 2.0 cm. Its paper dielectric has maximum working dielectric stress of 55 kV/cm. Calculate the maximum permissible line voltage when such cables are used for three-phase power system.

Solution:

Given data:

Diameter of core, d = 2.0 cm

Radius of core, r = 1.0 cm

Diameter of the sheath, D = 5.5 cm

Radius of the sheath, R = 2.75 cm

Maximum dielectric stress, g_{max} = 55 kV/cm

$$g_{max} = \frac{V}{r \ln\left(\frac{R}{r}\right)}$$

$$\therefore 55 = \frac{V}{\ln\left(\frac{2.75}{1}\right)}$$

Phase voltage, V_{ph} = 55.64 (peak) kV = 39.342 kV(rms)

Line to line voltage, $V_L = \sqrt{3} \times 39.342 = 68.14$ kV.

9.9 ECONOMICAL CORE DIAMETER

For a given fixed value of operating voltage and overall diameter of the cable, core diameter can be determined by differentiating the Eq. (9.5) w.r.t "*r*" and equating to zero.

For minimum value of g_{max},

$$\frac{d}{dr} g_{max} = 0$$

$$\text{we know, } g_{max} = \frac{V}{r \ln\left(\frac{R}{r}\right)}$$

$$\frac{d}{dr}g_{max} = \frac{0 - V\left[r \times \frac{1}{\left(R/r\right)} \times -\frac{R}{r^2} + \ln\left(\frac{R}{r}\right)\right]}{\left(r\ln\left(\frac{R}{r}\right)\right)^2} = 0$$

$$-1 + \ln\left(\frac{R}{r}\right) = 0$$

$$\ln\left(\frac{R}{r}\right) = 1$$

$$\frac{R}{r} = e = 2.718 \tag{9.7}$$

From Eq. (9.5), the maximum gradient, $g_{max} = \frac{V}{r\ln\frac{R}{r}}$

$$= \frac{V}{r} \quad (\because R = re)$$

$$\therefore \text{Economical core radius, } r = \frac{V}{g_{max}}. \tag{9.8}$$

EXAMPLE 9.5

Determine the economical core diameter of a single-core cable working on 22 kV, single-phase system. The maximum permissible stress in the dielectric is not to exceed 33 kV/cm.

Solution:

Peak value of cable voltage, $V = 22 \times \sqrt{2}$

$= 31.11$ kV

Maximum permissible stress, $g_{max} = 33$ kV/cm

$\therefore$ Most economical conductor diameter, $d = \frac{2V}{g_{max}}$

$$= \frac{2 \times 31.11}{33} = 1.885 \text{ cm.}$$

EXAMPLE 9.6

Determine the economical-core diameter of a single-core cable working on 210 kV, three-phase system. The maximum permissible stress in the dielectric is not to exceed 230 kV/cm.

Solution:

Operating voltage, V = 210 kV(L–L)

$$\text{Peak value of operating voltage} = \frac{210}{\sqrt{3}} \times \sqrt{2} = 171.46 \text{ kV}$$

Maximum permissible stress, g_{max} = 230 kV/cm

$$\therefore \text{ Most economical conductor diameter, } d = \frac{2V}{g_{max}} = \frac{2 \times 171.46}{230} = 1.49 \text{ cm.}$$

9.10 GRADING OF CABLES

It has been observed that the voltage gradient is maximum at the surface of the conductor and minimum at the inner surface of the sheath (i.e., the stress decreases from conductor surface to sheath). This causes breakdown in the insulation. For avoiding this breakdown, it is advisable to have more uniform stress distribution throughout the dielectric. The process of achieving uniform distribution in dielectric stress is called the grading of cables. There are two methods to achieve it, they are:

1. The application of layers of different dielectric materials called "Capacitance grading".
2. Providing metallic intersheath between successive layers of the same dielectric materials and maintaining appropriate potential level at the intersheath is called "Intersheath grading".

9.10.1 Capacitance Grading

In capacitance grading, uniformity in dielectric stress is achieved by using various layers of different dielectrics in such a manner that if the permittivity ε_r of any layer is inversely proportional to its distance from the centre,

i.e., $\varepsilon_r \propto \frac{1}{x}$, then the product $\varepsilon_r x$ remains constant, i.e.,

electric stress 'g'at any distance $x = \frac{q}{2\pi\varepsilon_0\varepsilon_r x}$ or $g \propto \frac{q}{2\pi\varepsilon_0}$

which is a constant (since $\varepsilon_r x$ is constant).

Thus, if such a condition is satisfied, the value of dielectric stress at any point will be constant and independent of the distance from the centre and the grading will be an ideal one. But practically such type of dielectric is not possible since it requires infinite number of dielectrics. The practical way is to have two or three different types of dielectrics.

Consider a single-core cable having a core of radius r ($d/2$) and insulation having three layers of radii r_1, r_2, and R, the dielectric materials of each layer having permittivities, ε_1, ε_2 and ε_3, respectively with their arrangement in circular shape as shown in Fig. 9.13.

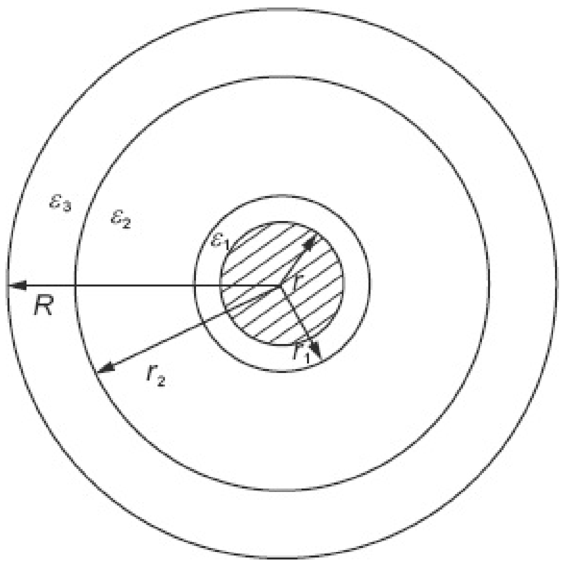

Fig. 9.13 Capacitance graded cable

The dielectric stress within the inner dielectric at a radius x_1, $g_1 = \frac{q}{2\pi\varepsilon_1 x_1}$

where,

q is the charge per metre axial length.

Dielectric stress in the middle dielectric at a radius

$$x_2,\ g_2 = \frac{q}{2\pi\varepsilon_2 x_2}$$

Dielectric stress in the outer dielectric at a radius x_3,

$g_3 = \frac{q}{2\pi\varepsilon_3 x_3}$

The potential difference between core and inner dielectric,

$$V_1 = \int_r^{r_1} \frac{q}{2\pi\varepsilon_1 x_1} dx = \frac{q}{2\pi\varepsilon_1} \ln\left(\frac{r_1}{r}\right) \quad (9.9)$$

The potential difference between inner and middle dielectrics,

$$V_2 = \int_{r_1}^{r_2} \frac{q}{2\pi\varepsilon_2 x} dx = \frac{q}{2\pi\varepsilon_2} \ln\left(\frac{r_2}{r_1}\right) \quad (9.10)$$

The potential difference between middle and sheath,

$$V_3 = \int_{r_2}^{R} \frac{q}{2\pi\varepsilon_3 x} dx = \frac{q}{2\pi\varepsilon_3} \ln\left(\frac{q}{r_2}\right) \tag{9.11}$$

The total potential difference, $V = V_1 + V_2 + V_3$

$$= \frac{q}{2\pi\varepsilon_1} \ln\left(\frac{r_1}{r}\right) + \frac{q}{2\pi\varepsilon_2} \ln\left(\frac{r_2}{r_1}\right) + \frac{q}{2\pi\varepsilon_3} \ln\left(\frac{R}{r_2}\right) \tag{9.12}$$

$$= \frac{q}{2\pi}\left[\frac{1}{\varepsilon_1} \ln\left(\frac{r_1}{r}\right) + \frac{1}{\varepsilon_2} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{\varepsilon_3} \ln\left(\frac{R}{r_2}\right)\right] \tag{9.13}$$

$$\text{and the capacitance, } C = \frac{q}{V} = \frac{2\pi}{\left[\frac{1}{\varepsilon_1} \ln\frac{r_1}{r} + \frac{1}{\varepsilon_2} \ln\frac{r_2}{r_1} + \frac{1}{\varepsilon_3} \ln\frac{R}{r_2}\right]} \tag{9.14}$$

Dielectric stress is maximum at, $x_1 = r$, $x_2 = r_1$ and $x_3 = r_2$

From Eq. (9.2) at $x_1 = r$

$$g_{1\max} = \frac{q}{2\pi\varepsilon_1 r} = \frac{q}{2\pi\varepsilon_1 r}$$

$$= \frac{1}{2\pi\varepsilon_1 r} \frac{2\pi V}{\left[\frac{1}{\varepsilon_1} \ln\left(\frac{d_1}{d}\right) + \frac{1}{\varepsilon_2} \ln\left(\frac{d_2}{d_1}\right) + \frac{1}{\varepsilon_3} \ln\left(\frac{D}{d_2}\right)\right]}$$

$$= \frac{V}{r\left[\ln\frac{r_1}{r} + \frac{\varepsilon_1}{\varepsilon_2} \ln\frac{r_2}{r_1} + \frac{\varepsilon_1}{\varepsilon_3} \ln\frac{R}{r_2}\right]} \tag{9.15}$$

Similarly at $x_2 = r_1$

$$g_{2\max} = \frac{q}{2\pi\varepsilon_2 r_1} = \frac{V}{r_1\left[\frac{\varepsilon_2}{\varepsilon_1}\ln\frac{r_1}{r} + \ln\frac{r_2}{r_1} + \frac{\varepsilon_2}{\varepsilon_3}\ln\frac{R}{r_2}\right]} \quad (9.16)$$

and at $x_3 = r_2$

$$g_{3\max} = \frac{q}{2\pi\varepsilon_3 r_2} = \frac{V}{r_2\left[\frac{\varepsilon_3}{\varepsilon_1}\ln\frac{r_1}{r} + \frac{\varepsilon_3}{\varepsilon_2}\ln\frac{r_2}{r_1} + \ln\frac{R}{r_2}\right]} \quad (9.17)$$

EXAMPLE 9.7

The inner and outer diameters of a cable are 3 cm and 8 cm, respectively. The cable is insulated with two materials having permittivity of 5 and 3.5 with corresponding stresses of 38 kV/cm and 30 kV/cm. Calculate the radial thickness of each insulating layer and the safe working voltage of the cable.

Solution:

Given data:

Diameter of core, d = 3 cm

Radius of core, r = 1.5 cm

Diameter of cable, D =8 cm

Radius of cable, R = 4 cm

Permittivity of two dielectric materials, $\varepsilon_1 = 5$, $\varepsilon_2 = 3.5$

$$g_{1\max} = \frac{q}{2\pi\varepsilon_1 r}$$

$$g_{2\max} = \frac{q}{2\pi\varepsilon_r r_1}$$

$$\frac{g_1}{g_2} = \frac{\varepsilon_2 r_1}{\varepsilon_1 r}$$

$$\frac{38}{30} = \frac{3.5 \times r_1}{5 \times 1.5}$$

∴ Radius of inner dielectric, r_1 = 2.7 cm

Radial thickness of inner dielectric = 2.7–1.5 = 1.2 cm

Radial thickness of outer dielectric = 4 – 2.7 = 1.3 cm

$$\text{Maximum voltage of the cable, } V = g_{1\max} r \ln\frac{r_1}{r} + g_{2\max} r_1 \ln\frac{R}{r_1}$$

$$= 38\times1.5\times\ln\frac{2.7}{1.5} + 30\times2.7\times\ln\frac{4}{2.7} = 65.34 \text{ kV}$$

$$\text{Working voltage (r.m.s)} = \frac{65.34}{\sqrt{2}} = 46.2 \text{ kV.}$$

EXAMPLE 9.8

A cable has been insulated with two insulating materials having permittivity of 4 and 2.5, respectively. The inner and outer diameters of the cable are 2.2 cm and 7 cm. If the dielectric stress is 40 kV/cm and 30 kV/cm, calculate the radial thickness of each insulating layer and the safe working voltage of the cable.

Solution:

Given data:

Maximum permissible stress, g_{1max} = 40 kV/cm

Maximum permissible stress, g_{2max} = 30 kV/cm

Diameter of core, d = 2.2 cm

Radius of core, r = 1.1 cm

Diameter of cable (sheath), D = 7 cm

Radius of cable, R = 3.5 cm

Permittivity of two dielectric materials, ε_1 = 4.0, ε_2 = 2.5

Let the diameter over the insulation of inner dielectric be d_1 cm.

$$\frac{g_{1\max}}{g_{2\max}} = \frac{\varepsilon_2 d_1}{\varepsilon_1 d} = \frac{2.5 d_1}{4\times2.2} = 0.2841 d_1$$

$$0.2841 d_1 = \frac{40}{30}$$

∴ Diameter of inner dielectric materials, d. = 4.7 cm

$$\text{Radial thickness of inner dielectric} = \frac{4.7 - 2.2}{2} = 1.25 \text{ cm}$$

$$\text{Radial thickness of outer dielectric} = \frac{7.0 - 4.7}{2} = 1.15 \text{ cm}$$

$$\text{Peak voltage of cable} = \frac{g_{1\max}}{2} \times d \ln\left(\frac{d_1}{d}\right) + \frac{g_{2\max}}{2} \times d_1 \ln\left(\frac{D}{d_1}\right)$$

$$= \frac{40}{2} \times 2.2 \ln\left(\frac{4.7}{2.2}\right) + \frac{30}{2} \times 4.7 \ln\left(\frac{7.0}{4.7}\right)$$

$$= 33.4 + 28.08 = 61.48 \text{ kV}$$

$$\text{Operating voltage (r.m.s)} = \frac{61.48}{\sqrt{2}} = 43.47 \text{ kV.}$$

EXAMPLE 9.9

A single core lead sheathed cable is graded by using two dielectrics of relative permittivity 3.8 (inner) and 2.8 (outer), the thickness of each being 0.75 cm. The core diameter is 1 cm; system voltage is 66 kV, three-phase. Determine the maximum stress in two dielectrics.

Solution:

Given data:

Diameter of the core, d =1 cm

Thickness, t = 0.75 cm

Diameter of inner dielectric, $d_1 = d + 2t = 2.5$ cm

Diameter of cable, $D = d_1 + 2t = 4$ cm

Permittivity of two dielectric materials, $\varepsilon_1 = 3.8$, $\varepsilon_2 = 2.8$

$$\text{Maximum voltage, } V = \frac{66}{\sqrt{3}} \times \sqrt{2} = 53.89 \text{ kV}$$

$$\text{Maximum dielectric stress, } g_{1\max} = \frac{2V}{d\left(\ln\frac{d_1}{d} + \frac{\varepsilon_1}{\varepsilon_2}\ln\frac{D}{d_1}\right)}$$

$$= \frac{2 \times 53.88}{1\left(\ln\frac{2.5}{1} + \frac{3.8}{2.8}\ln\frac{4}{2.5}\right)} = 69.35 \text{ kV/cm}$$

$$\text{Maximum dielectric stress, } g_{2\max} = \frac{2V}{d_1\left(\frac{\varepsilon_2}{\varepsilon_1}\ln\frac{d_1}{d} + \ln\frac{D}{d_1}\right)}$$

$$= \frac{2 \times 53.88}{2.5\left(\frac{2.8}{3.8}\ln\frac{2.5}{1} + \ln\frac{4}{2.5}\right)} = 37.65 \text{ kV/cm.}$$

EXAMPLE 9.10

A 66 kV concentric cable with two intersheaths has a core diameter of 2.3 cm; dielectric material of 3.5 mm thickness constitutes three zones of insulation. Determine the maximum stress in each of the three layers, if 22 kV is maintained across each of the inner two layers.

Solution:

Given data:

Operating voltage, $V = \frac{66}{\sqrt{3}} \text{kV} = 38.105 \text{ kV}$

Diameter of core, d = 2.3 cm

Radius of core, r = 1.15 cm

Thickness of the dielectric material, t = 3.5 mm

Voltage across each layer = $V_1 = V_2 = \frac{22}{\sqrt{3}} = 12.7 \text{ kV}$

Radius of first dielectric material, $r_1 = r + t$ = 1.15 + 0.35 = 1.5 cm

Radius of second dielectric material, $r_2 = r_1 + t$ = 1.6 + 0.35 = 1.95 cm

But we know that, for same maximum and minimum stresses

$$\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2}$$

$$\frac{r_1}{r} = \frac{R}{r_2}$$

$$\therefore \text{ Radius of cable, } R = \frac{r_1 \times r_2}{r} = \frac{1.5 \times 1.95}{1.15} = 2.54 \text{ cm}$$

$$\text{The maximum stress in first layer, } g_{1\max} = \frac{V_1}{r \ln\left(\frac{r_1}{r}\right)} = \frac{12.7}{1.15 \ln\left(\frac{1.5}{1.15}\right)} = 41.56 \text{ kV/cm}$$

$$\text{The maximum stress in second layer, } g_{2\max} = \frac{V_2}{r_1 \ln\left(\frac{r_2}{r_1}\right)} = \frac{12.7}{1.5 \ln\left(\frac{1.95}{1.5}\right)} = 32.27 \text{ kV/cm}$$

$$\text{The maximum stress in third layer, } g_{3\max} = \frac{V_3}{r_2 \ln\left(\frac{R}{r_2}\right)} = \frac{12.7}{1.95 \ln\left(\frac{2.54}{1.95}\right)} = 24.64 \text{ kV/cm.}$$

EXAMPLE 9.11

A single-core lead-covered cable is to be designed for 66 kV to earth. Its conductor radius is 10 mm and its three insulating materials A, B, and C have relative permittivity of 6, 5, and 4, whose corresponding maximum permissible stress are 4.0, 3.0, and 2.0 kV/mm, respectively. Find the maximum diameter of the lead sheath.

Solution:

Given data:

Line voltage = 66 kV

Conductor radius, r = 10 mm

Maximum voltage, $V_{\max} = \sqrt{2} \times 66 = 93.34$ kV

$$\begin{aligned} \text{Maximum permissible stress, } g_{1\max} &= 4.0 \text{ kV/cm} \\ g_{2\max} &= 3.0 \text{ kV/cm} \\ g_{3\max} &= 2.0 \text{ kV/cm} \end{aligned}$$

Relative permittivity of insulating materials A, $\varepsilon_a = 6$

Relative permittivity of insulating materials B, $\varepsilon b = 5$

Relative permittivity of insulating materials C, $\varepsilon_c = 4$

We know that

$$\varepsilon_a r \cdot g_{1\max} = \varepsilon_b r_1 \cdot g_{2\max} = \varepsilon_c r_2 \cdot g_{3\max}$$

$$\text{Radius of dielectric material A, } r_1 = \frac{\varepsilon_a r \cdot g_{1\max}}{\varepsilon_b \cdot g_{2\max}} = \frac{6\times10\times4}{5\times3} = 16 \text{ mm}$$

$$\text{Radius of dielectric material B, } r_2 = \frac{\varepsilon_a r \cdot g_{1\max}}{\varepsilon_c \cdot g_{3\max}} = \frac{6\times10\times4}{4\times2} = 30 \text{ mm}$$

$$\text{Since, } V_{\max} = g_{1\max} r \ln\left(\frac{r_1}{r}\right) + g_{2\max} r_1 \ln\left(\frac{r_2}{r_1}\right) + g_{3\max} r_2 \ln\left(\frac{R}{r_2}\right)$$

$$93.338 = 4\times10\times\ln\left(\frac{16}{10}\right) + 3\times16\times\ln\left(\frac{30}{16}\right) + 2\times30\times\ln\left(\frac{R}{30}\right)$$

$$93.338 = 18.8 + 30.17 + 60\ln\left(\frac{R}{30}\right)$$

$$60\ln\left(\frac{R}{30}\right) = 44.367$$

$$\ln\left(\frac{R}{30}\right) = 0.73945$$

∴ Radius of lead sheath, R = 62.84 mm

The inner diameter of lead sheath, $D = 2R = 2 \times 62.84 = 125.69$ mm.

EXAMPLE 9.12

A three-phase single-core lead covered cable has a radius of core 0.5 cm and a sheath of internal diameter 10 cm. Its three insulating materials A, B, and C have relative permittivity of 4.5, 3.5, and 2.5 with maximum permissible stress of 50, 40, and 30 kV/cm, respectively. Find the operating voltage of the cable.

Solution:

Given data:

Conductor radius, r =0.5 cm

Inner radius of sheath, R = 5 cm

Maximum permissible stress $g_{1\max} = 50$ kV/cm

$g_{2\max} = 40$ kV/cm

$g_{3\max} = 30$ kV/cm

Relative permittivity of insulating materials A, ε_A = 4.5

Relative permittivity of insulating materials B, ε_B = 3.5

Relative permittivity of insulating materials C, ε_C = 2.5

We know that

$$\varepsilon_A r \cdot g_{1\max} = \varepsilon_B r_1 \cdot g_{2\max} = \varepsilon_C r_2 \cdot g_{3\max}$$

$$\text{Radius of dielectric material A, } r_1 = \frac{\varepsilon_A r \cdot g_{1\max}}{\varepsilon_B \cdot g_{2\max}} = \frac{4.5 \times 0.5 \times 50}{3.5 \times 40} = 0.8 \text{ cm}$$

$$\text{Radius of dielectric material B, } r_2 = \frac{\varepsilon_A r \cdot g_{1\max}}{\varepsilon_C \cdot g_{3\max}} = \frac{4.5 \times 0.5 \times 50}{2.5 \times 30} = 1.5 \text{ cm}$$

$$\text{Since, } V_{\max} = g_{1\max} \times r \ln \frac{r_1}{r} + g_{2\max} \times r_1 \ln \frac{r_2}{r_1} + g_{3\max} \times r_2 \ln \frac{R}{r_2}$$

$$= 50 \times 0.5 \ln \frac{0.8}{0.5} + 40 \times 0.8 \ln \frac{1.5}{0.8} + 30 \times 1.5 \ln \frac{5}{1.5}$$

$$= 11.75 + 20.12 + 54.18 = 86.05 \text{ kV}$$

$$\therefore \text{ Operating voltage, } V = \frac{86.05}{\sqrt{2}} = 60.85 \text{ kV.}$$

EXAMPLE 9.13

A single-core, lead-sheathed cable is graded by using three dielectrics of relative permittivity 6, 5, and 4, respectively. The conductor diameter is 2.5 cm and overall diameter is 7 cm. If the three dielectrics are worked at the same maximum stress of 38 kV/cm, find the safe working voltage of the cable. What will be the value of safe working voltage for an un-graded cable, assuming the same conductor and overall diameter and the maximum dielectric stress?

Solution:

Given data:

Diameter of core, d =2.5 cm

Diameter of cable, D = 7 cm

Relative permittivity of insulating materials 1, ε_1 = 6

Relative permittivity of insulating materials 2, ε_2 = 5

Relative permittivity of insulating materials 3, ε_3 = 4

Maximum gradient of each dielectric, $g_{\max}$ = 38 kV/cm

1. **Graded cable:** As the maximum stress in the three dielectrics is the same,

$$\text{i.e., } \varepsilon_1 d = \varepsilon_2 d_1 = \varepsilon_3 d_2$$
$$6 \times 2.5 = 5 \times d_1 = 4 \times d_2$$
$$d_1 = 3 \text{ cm and } d_2 = 3.75 \text{ cm}$$

∴Diameters of first and second dielectrics are 3 cm and 3.75 cm, respectively.

$$\text{Permissible peak voltage for the cable, } V = \frac{g_{\max}}{2}\left[d \ln\frac{d_1}{d} + d_1 \ln\frac{d_2}{d_1} + d_2 \ln\frac{D}{d_2}\right]$$
$$= \frac{38}{2}\left[2.5 \ln\frac{3}{2.5} + 3 \ln\frac{3.75}{3} + 3.75 \ln\frac{7}{3.75}\right]$$
$$= 65.85 \text{ kV(peak)}$$
$$\text{Safe working voltage (r.m.s) for cable } = \frac{65.85}{\sqrt{2}} = 46.56 \text{ kV(r.m.s)}$$

2. **Un-graded cable:**

$$\text{Permissible peak voltage for the cable, } V = \frac{g_{\max}}{2} d \ln\frac{D}{d}$$
$$= \frac{38}{2} \times 2.5 \ln\frac{7}{2.5} = 48.91 \text{ kV}$$
$$\text{Safe working voltage (r.m.s) for cable } = \frac{48.91}{\sqrt{2}} = 34.5824 \text{ kV.}$$

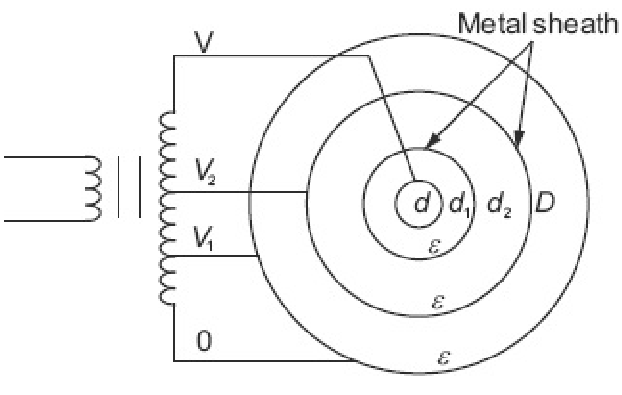

Fig. 9.14 Stress distribution without intersheath of cable

9.10.2 Intersheath Grading

This method is useful to distribute the potential gradient uniformly throughout the insulation by providing metallic intersheaths between the conductor and the lead sheath. Here, the same insulating material is separated into two or more layers by using thin metallic intersheaths. The potential of these intersheaths is maintained at certain voltages which are in between the operating and the earth potential, by connecting them to the tapping on the secondary side of the transformer as shown in Fig. 9.14.

Let the intersheaths be provided at the radius of r_1 and r_2 and their potentials kept at V_1 and V_2, respectively.

From Eq. (9.2) the gradient g_1 between the conductor and first intersheath at a distance x varies inversely proportional to x.

$$\text{i.e.,}\quad g_1 \propto \frac{1}{x} \Rightarrow g_1 = \frac{k}{x}$$

The potential difference between the conductor and the first inter-sheath is

$$\therefore g_1 = \frac{V - V_1}{x \ln \frac{r_1}{r}} \tag{9.18}$$

From Eq. (9.18), the maximum stress occurs at $x =$

$$\frac{d}{2} = r$$

$$\therefore g_{1\max} = \frac{V - V_1}{r \ln \frac{r_1}{r}} \tag{9.19}$$

With similar reasoning, the maximum stress between first and second inter-sheaths and the second and outmost sheath occurs at $x = r_1 = \frac{d_1}{2}$ and $x = r_2 = \frac{d_2}{2}$, respectively.

$$g_{2\max} = \frac{V_1 - V_2}{r_1 \ln \frac{r_2}{r_1}} \tag{9.20}$$

$$\text{and } g_{3\max} = \frac{V_2}{r_2 \ln \frac{R}{r_2}} \tag{9.21}$$

By selecting proper values of V_1 and V_2, $g_{1\max}$ can be made equal to $g_{2\max}$ and so on.

The maximum stresses $g_{1\max}$, and $g_{2\max}$ can be made equal by selecting proper values of V_1 and V_2.

The stress distribution after intersheath grading is shown in Fig. 9.15.

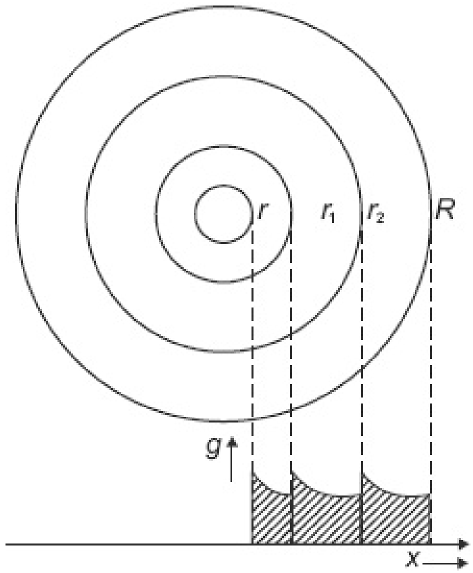

Fig. 9.15 Stress distribution with intersheath of cable

By choosing r_1 and r_2, the stress can also be made to vary between the same limits, such that

$$\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha$$

But the condition of homogeneous dielectric requires $g_{1max} = g_{2max} = g_{2max}$

$$\frac{V-V_1}{r\ln\frac{r_1}{r}}=\frac{V_1-V_2}{r_1\ln\frac{r_2}{r_1}}=\frac{V_2}{r_2\ln\frac{R}{r_2}}$$

$$\text{or } \frac{V-V_1}{r\ln\alpha}=\frac{V_1-V_2}{r_1\ln\alpha}=\frac{V_2}{r_2\ln\alpha}$$

$$\text{or } \frac{V-V_1}{r}=\frac{V_1-V_2}{r_1}=\frac{V_2}{r_2}$$

$$\text{or } V_2=\frac{r_2}{r_1}\left[V_1-V_2\right]=\alpha V_1-\alpha V_2$$

$$\text{or } V_2=\frac{\alpha V_1}{1+\alpha}$$

From the relationship, $\frac{V_1-V_2}{r_1}=\frac{V-V_1}{r}$

$$\therefore V_1-V_2=\frac{r_1}{r}\left[V-V_1\right]$$

$$V_1-V_2=\alpha V-\alpha V_1$$

$$\text{or } V_1\left(1+\alpha-\frac{\alpha}{1+\alpha}\right)=\alpha V$$

$$\therefore V_1=\frac{\alpha(1+\alpha)V}{1+\alpha+\alpha^2} \qquad (9.22)$$

$$\text{and } V_2=\frac{\alpha^2}{1+\alpha+\alpha^2}V \qquad (9.23)$$

The maximum stress, $g_{1\max}=\frac{V-V_1}{r\ln\frac{r_1}{r}}=\frac{V-V_1}{r\ln\alpha}$

$$=\frac{V-\frac{\alpha(1+\alpha)V}{1+\alpha+\alpha^2}}{r\ln\alpha}$$

$$=\frac{V}{r\ln\alpha}\times\frac{1}{1+\alpha+\alpha^2}$$

$$\therefore g_{1\max} = \frac{V}{\left(1+\alpha+\alpha^2\right) r \ln \alpha} \tag{9.24}$$

As $\frac{R}{r} = \alpha^3$ and $\ln \frac{R}{r} = \ln \alpha^3 = 3 \ln \alpha$

$$\therefore g_{1\max} = \frac{V}{\frac{1}{3}(1+\alpha+\alpha^2) r \ln \frac{R}{r}} = \frac{3V}{(1+\alpha+\alpha^2) r \ln \frac{R}{r}} \tag{9.25a}$$

$$\text{The maximum gradient without using any inter-sheath} = \frac{V}{r \ln \frac{R}{r}} \tag{9.25b}$$

From eqs. (9.25a) and (9.25b), the value of the maximum gradient when intersheaths are used is

reduced $\frac{3}{(1+\alpha+\alpha^2)}$ times the value of the maximum

gradient without using inter-sheaths.

EXAMPLE 9.14

A single-core cable for a 132 kV, three-phase system has conductor of radius 1.2 cm and inside sheath radius of 6 cm. It has two inter-sheaths. The stress varies between the maximum and minimum limits in the three layers of dielectric. Find the radii of the intersheath and their voltages.

Solution:

Given data:

Supply voltage, $V = \frac{132}{\sqrt{3}} = 76.21$ kV/phase

Radius of core, r = 1.2 cm,

Radius of sheath (inside), R = 6 cm

Since maximum and minimum stresses are the same,

$$\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha,$$

$$\therefore \frac{R}{r} = \alpha^3$$

$$\alpha = \left(\frac{6}{1.2}\right)^{1/3} = 1.71$$

The radii of intersheaths, $r_1 = \alpha r = 1.71 \times 1.2 = 2.05$ cm

$r_2 = \alpha r_1 = 1.71 \times 2.05 = 3.51$ cm

Potential of first intersheath, $V_1 = V \dfrac{\alpha(1+\alpha)}{1+\alpha+\alpha^2}$

$$= 76.21 \times \left[\frac{1.71(1+1.71)}{1+1.71+1.71^2}\right] = 76.21 \times \left[\frac{4.6341}{5.6341}\right]$$

$$= 62.68 \text{ kV/ph}$$

Potential of second intersheath, $V_2 = V \dfrac{\alpha^2}{1+\alpha+\alpha^2}$

$$= 76.21 \times \frac{1.71^2}{1+1.71+1.71^2} = 39.55 \text{ kV/phase.}$$

EXAMPLE 9.15

A single-core 66 kV cable working on three-phase system has a conductor diameter of 2 cm and a sheath of inside diameter 5.5 cm. If the two intersheaths are introduced in such a way that the variation of stress is in between the same maximum and minimum in the three layers. Determine the following

1. **Positions of intersheath**
2. **Voltage on the intersheath**
3. **Maximum and minimum stresses.**

Solution:

Diameter of the core, d = 2 cm

Internal diameter of the sheath, D = 5.5 cm

Supply voltage, $V = \dfrac{66}{\sqrt{3}} \times \sqrt{2} = 53.8$ kV (Peak)

1. **Positions of intersheath:** Assume the diameters of intersheath are

d_1 and d_2 cm, respectively and V_1 and V_2 are the potentials maintained at intersheath 1 and 2, respectively.

$$g_{1\max} = \frac{V - V_1}{\frac{d}{2}\ln\left(\frac{d_1}{d}\right)}$$

$$g_{2\max} = \frac{V_1 - V_2}{\frac{d_1}{2}\ln\left(\frac{d_2}{d_1}\right)}$$

$$g_{3\max} = \frac{V_0}{\frac{d_2}{2}\ln\left(\frac{D}{d_2}\right)}$$

For equal maximum stresses,

$$g_{1\max} = g_{2\max} = g_{3\max}$$

$$\therefore \frac{V - V_1}{\frac{d}{2}\ln\left(\frac{d_1}{d}\right)} = \frac{V_1 - V_2}{\frac{d_1}{2}\ln\left(\frac{d_2}{d_1}\right)} = \frac{V_2}{\frac{d_2}{2}\ln\left(\frac{D}{d_2}\right)}$$

In order that stress may vary between the same maximum and minimum in the three layers, we have,

$$\frac{d_1}{d} = \frac{d_2}{d_1} = \frac{D}{d_2} \qquad (1)$$

$$\text{and} \quad \frac{V - V_1}{d} = \frac{V_1 - V_2}{d_1} = \frac{V_2}{d_2} \qquad (2)$$

From Eq. (1), we get,

$$d_1^2 = d \times d_2$$
$$= 2d_2$$
$$d_2 = \frac{d_1^2}{2}$$
$$d_1 d_2 = D \times d$$
$$= 5.5 \times 2$$
$$= 11 \text{ cm}$$

$$d_1 \times \frac{d_1^2}{2} = 11 \text{ cm}$$

$$d_1 = (22)^{1/3} = 2.8 \text{ cm}$$

$$d_2 = \frac{d_1^2}{2} = \frac{2.8^2}{2} = 3.92 \text{ cm}$$

$$\therefore \alpha = \frac{d_1}{d} = \frac{2.8}{2} = 1.4 \text{ cm}$$

2. **Voltage on intersheaths**

   The maximum potential at intersheath 1,

$$V_1 = \frac{\alpha(1+\alpha)V}{1+\alpha+\alpha^2}$$
$$= \frac{1.4(1+1.4)}{1+1.4+1.4^2} \times 53.8 \text{ kV}$$
$$= 41.46 \text{ kV}$$

   The maximum potential at intersheath 2,

$$V_2 = \frac{\alpha^2}{1+\alpha+\alpha^2} V$$
$$= \frac{1.4^2}{1+1.4+1.4^2} \times 53.8$$
$$= 24.18 \text{ kV}$$

3. **Maximum and minimum stresses**

   Stresses on first dielectric:

$$\text{Maximum stress} = \frac{V - V_1}{\frac{d}{2}\ln\left(\frac{d_1}{d}\right)} = \frac{53.8 - 41.46}{1\ \ln\frac{2.8}{2}} = 36.68\ \text{kV/cm}$$

$$\text{Minimum stress} = \frac{V - V_1}{\frac{d_1}{2}\ln\left(\frac{d_1}{d}\right)} = \frac{53.8 - 41.46}{1.4\ln\frac{2.8}{2}} = 26.2\ \text{kV/cm}$$

Stresses on second dielectric:

$$\text{Maximum stress} = \frac{V_1 - V_2}{\frac{d_1}{2}\ln\left(\frac{d_2}{d_1}\right)} = \frac{41.46 - 24.18}{\frac{2.8}{2}\times\ln\left(\frac{3.92}{2.8}\right)} = 36.68\ \text{kV/cm}$$

$$\text{Minimum stress} = \frac{V_1 - V_2}{\frac{d_2}{2}\ln\left(\frac{d_2}{d_1}\right)} = \frac{41.46 - 24.18}{\frac{3.92}{2}\times\ln\left(\frac{3.92}{2.8}\right)} = 26.2\ \text{kV/cm.}$$

EXAMPLE 9.16

An 85 kV, single-core metal-sheathed cable is to be graded by means of a metallic intersheath such that the overall diameter of the cable is minimum. The insulating material can be worked at 55 kV/cm. Find the diameter of the intersheath and the voltage at which it must be maintained. Compare the conductor and outside diameters of this cable with those of an ungraded cable employing the same insulating material and working under the same conditions.

Solution:

Let the conductor, intersheath and outside diameters be d, d_1 and D, respectively and let the intersheath be maintained at a potential V_1. The conductor is at the voltage V volts.

Considering the conductor and intersheath diameters it is clear that intersheath diameter d_1 is 'e' times the conductor diameter d.

$$\therefore d_1 = 2.718\, d \qquad (\text{since } e = 2.718) \tag{1}$$

For the conductor and intersheath, maximum stress occurs at a point distant $\frac{d}{2}$ from the centre and its value is

$$g_{max} = \frac{V - V_1}{(d/2)\ln(d_1/d)} \tag{2}$$

Similarly, the maximum stress between the intersheath and metal sheath occurs at the intersheath surface i.e, $\frac{d_1}{2}$ its value being

$$g_{max} = \frac{V_1}{(d_1/2)\ln(D/d_1)} \tag{3}$$

For equal maximum stresses, g_{max} must be of the same value for both.

Therefore, from Eqs. (1) and (2)

$$g_{max} = \frac{V - V_1}{(d/2)\ln(d_1/d)} = \frac{2.718(V - V_1)}{(d_1/2)\ln(d_1/d)} \qquad (\because \text{from Eq. (1) } d = \frac{d_1}{2.718})$$

$$\text{or } d_1 = \frac{5.436(V - V_1)}{g_{max}\ln(d_1/d)} = \frac{5.436(V - V_1)}{g_{max}} \qquad (\because \ln\frac{d_1}{d} = \ln e = 1) \tag{4}$$

Substituting the value of d_1 from Eq. (4) in Eq. (3), we have

$$g_{max} = \frac{V_1}{\frac{d_1}{2}\ln\frac{D}{d_1}} = \frac{2V_1}{\frac{5.436(V - V_1)}{g_{max}}\ln\frac{Dg_{max}}{5.436(V - V_1)}} \tag{5}$$

Therefore from Eq. (5), $g_{max} = \dfrac{g_{max} V_1}{2.718(V - V_1)\ln \dfrac{Dg_{max}}{5.436(V - V_1)}}$

or $Dg_{max} = 5.436(V - V_1)e^{\left\{\frac{V_1}{2.718(V - V_1)}\right\}}$ (6)

In the above equation, g_{max} and V are constants, for minimum value of D (the overall diameter) $\dfrac{\partial D}{\partial V_1}$ should be zero. This gives

$$\frac{\partial D}{\partial V_1} = \frac{5.436}{g_{max}} \frac{\partial}{\partial V_1}\left[(V - V_1)e^{\left\{\frac{V_1}{2.718(V - V_1)}\right\}}\right] = 0$$

$$= \frac{5.436}{g_{max}}\left[(V - V_1)\left(e^{\left\{\frac{V_1}{2.718(V - V_1)}\right\}}\right)\left(\frac{2.718(V - V_1) + 2.718V_1}{2.718^2(V - V_1)^2}\right) + \left[e^{\left\{\frac{V_1}{2.718(V - V_1)}\right\}}\right](-1)\right] = 0$$

After simplification of the above equation V

$$\frac{V}{2.718(V - V_1)} = 1$$

$$\text{or} \quad V_1 = \frac{1.718}{2.718}V = 0.632V$$

$$\text{From Eq. (2), } d = \frac{2(V - V_1)}{g_{max}} = 0.736\frac{V}{g_{max}}$$

From Eq. (4), $d_1 = \dfrac{5.436(V - V_1)}{g_{max}} = \dfrac{2V}{g_{max}}$

And from Eq. (6), $D = \dfrac{5.436(V - V_1)}{g_{max}} \varepsilon^{\left\{\frac{V_1}{2.718}(V - V_1)\right\}}$

$= \dfrac{5.436 \times 0.368V}{g_{max}} \varepsilon^{V_1/V} = \dfrac{2V}{g_{max}} \varepsilon^{0.632} = 3.76 \dfrac{V}{g_{max}}$

Further the cable is ungraded, $D = ed = 2.718d$ (since e = 2.718)

and $g_{max} = \dfrac{V}{(d/2)\ln(D/d)} = \dfrac{2V}{d}$

Finally, substituting the given values

Intersheath potential V_1 =0.632 V =0.632 × 85 =53.80 kV to neutral

Core diameter, $d = 0.736 \dfrac{V}{g_{max}} = 0.736\left(\dfrac{85}{55}\right) = 1.14$ cm

Intersheath diameter, $d_1 = \dfrac{2V}{g_{max}} = \dfrac{2 \times 85}{55} = 3.10$ cm

Overall diameter, $D = \dfrac{3.76V}{g_{max}} = \dfrac{3.76 \times 85}{55} = 5.83$ cm

And in the case of an ungraded cable

Core diameter, $d = \dfrac{2V}{g_{max}} = 3.10$ cm

And overall diameter, $D = 2.718d = 2.718 \times 3.1 = 8.43$ cm.

9.10.3 Practical Aspects of Cable Grading

- Capacitance grading is difficult due to the non-availability of required permittivities of materials. The use of rubber (ε_r = 4 – 6) and impregnated paper (ε_r = 3 – 4) have been suggested. But rubber is expensive and the cost of dielectric becomes too high. Another difficulty is the possible change of permittivity with time. As a result, this may completely change the voltage gradient distribution and may even lead to complete rupture of the cable dielectric material at normal operating voltage.
- It is difficult to arrange for proper intersheath voltages in intersheath grading. There is a chance of damaging the thin intersheath during the laying of the cable in case of inter-sheaths grading. The charging currents carried by the inter-sheaths may cause overheating.

For the above reasons, grading has come to be increasingly replaced by oil-filled and gas-filled cables.

9.11 POWER FACTOR IN CABLES (DIELECTRIC POWER FACTOR)

The operating voltage = V volts, i.e., the voltage across insulation material is the same as the operating voltage.

∴ The current through the insulation, $I_i = \frac{V}{R_i}$ A and this is in phase with V.

If C is the capacitance of a single-core cable, then the charging current, $I_c = V\omega C$ (A)

And this charging current leads the voltage V by 90°. The total current I is the vector sum of I and I_c and leads the applied voltage V by angle ϕ (power factor angle). The phasor diagram is shown in Fig. 9.16(a).

$$\text{Power factor of the cable } = \cos\phi = \cos\left(\frac{\pi}{2} - \delta\right) = \sin\delta$$

$$\text{When } \delta \text{ is very small, then } \sin\delta \approx \delta = \text{power factor of the cable } = \frac{1}{\omega C R_i}$$

$$\text{Dielectric loss per phase, } P = VI\cos\phi = VI\sin\delta = V\frac{I_c}{\cos\delta}\sin\delta = V \times V\omega C\tan\delta = V^2\omega C\tan\delta \quad (9.26)$$

From the Eq. (9.26), dielectric power losses are proportional to the square of the voltage. Therefore, for low voltage cables of about 22 kV, these losses are small and hence neglected. For high voltages above 22 kV they are to be considered.

From Fig. 9.16(b), the power factor of a cable changes with the temperature. As the temperature increases, the power factor of the cable increases. Thus, there is an increase in the loss of the cable [from Eq. (9.26)]. If the cable operates in such a condition, the process is continued until the insulation is damaged.

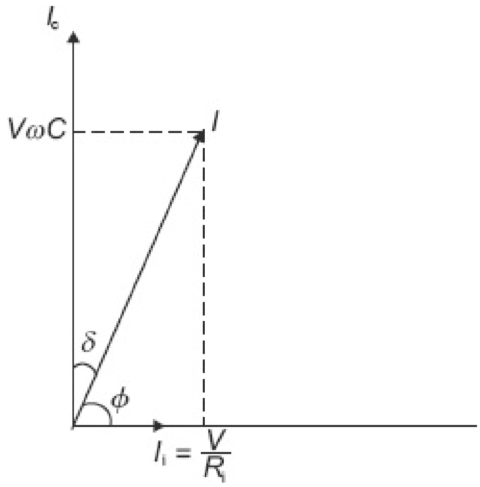

Fig. 9.16(a) Phasor diagram of a cable

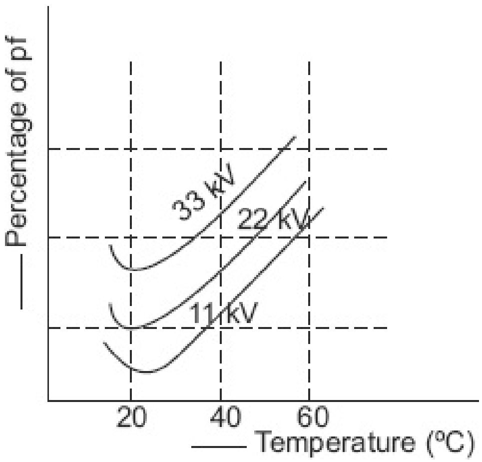

Fig. 9.16(b) Variation of power factor with temperature

EXAMPLE 9.17

A single-core cable 2 km long has a core diameter of 1.8 cm and the diameter of the inside sheath is 4 cm. The relative permittivity of the insulating material is 4. The power factor of the circuit is 0.07 and the supply voltage is 11 kV, three phase, 50 Hz. Determine (i) The capacitance of the cable, (ii) The charging current per conductor, and (iii) Dielectric loss

Solution:

Diameter of core, d = 1.8 cm

Diameter of lead sheaths, D = 4 cm

Power factor angle of the cable, $\phi = \cos^{-1} 0.07 = 86^{\circ}$

1. Capacitance of a single core 2 km long cable, $C = \dfrac{2\pi\varepsilon}{\ln\dfrac{D}{d}} \times 2 = \dfrac{10^{-9} \times 4}{18 \ln\dfrac{4}{1.8}} \times 2 = 0.5566\ \mu\text{F}$

2. Charging current, $I_{ch} = \omega CV$
$$= 2 \times \pi \times 50 \times 0.5566 \times 10^{-6} \times \frac{11}{\sqrt{3}} \times 10^{3}$$
$$= 1.11\ \text{A}$$

3. Dielectric loss $= VI_c \tan\delta = \dfrac{11}{\sqrt{3}} \times 10^{3} \times 1.11 \times \tan 4^{\circ} = 492.94$ W.

9.12 CAPACITANCE OF A THREE-CORE CABLE

The three-core cable has a capacitance between conductors and between conductor and sheath due to the existence of an electrostatic field caused by the potential difference between conductors and conductor and sheath.

Let the capacitance formed between the core and the sheath = C_s

And the capacitance formed between the two conductors = C_c

The three-core cable with capacitances formed is shown in Fig. 9.17(a). This can be represented in the other form also because the sheath is at same potential as shown in Fig. 9.17(b).

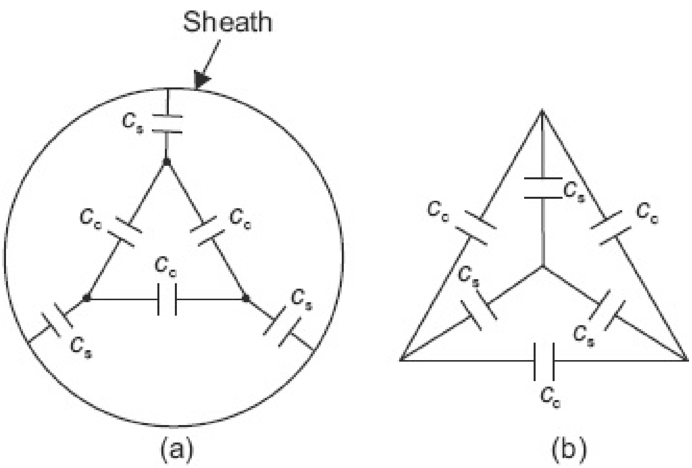

Fig. 9.17 Three-core cable

If the delta connected capacitances (between cores) are converted into an equivalent star arrangement then the equivalent capacitance value between the star point and the core is,

$C_1 = 3C_c$.

The potential of the star point may be assumed to be at sheath potential, i.e., at zero.

Then, the capacitance of each conductor to neutral, $C_N = C_s + 3C_c$ (9.27)

It is very difficult to calculate the capacitance of each conductor to neutral C_N of the cable from geometry. Therefore, the empirical formula for the capacitance between core and the neutral is,

$$C_N = \frac{0.03\varepsilon_r}{\ln\left[1 + \frac{T+t}{d}\left\{3.84 - 1.7\frac{t}{T} + 0.52\frac{t^2}{T^2}\right\}\right]} \ \mu\text{F/km} \quad (9.28)$$

where, ε_r = relative permittivity of the dielectric material

d = conductor diameter

t = belt insulation thickness

T = conductor insulation thickness

All of the above parameters are in the same units. The main uncertainty in this formula is that of the value of ε_r.

There are two unknowns C_c and C_s to find the capacitance per phase of the cable.

9.12.1 Measurement of C_C and C_S

1. Bunch the three cores and measure the capacitance between the bunched cores and the sheath. Take this as C_A.

$$C_A = 3C_S$$
$$C_S = \frac{C_A}{3}$$

2. Connect any two cores to the sheath and measure the capacitance between remaining core and sheath. Take this as C_B.

Take this as C_B.
$$C_B = C_s + 2C_c$$

From the two measurements

$$C_S = \frac{C_A}{3}$$
$$\therefore C_C = \frac{C_B - C_s}{2}$$
$$= \frac{C_B - \frac{C_A}{3}}{2}$$
$$= \frac{C_B}{2} - \frac{C_A}{6}$$

Substituting the above expressions in Eq. (9.27), we get

Capacitance/phase,

$$C_N = \frac{C_A}{3} + 3\left(\frac{C_B}{2} - \frac{C_A}{6}\right)$$

$$\therefore C_N = \frac{3C_B}{2} - \frac{C_A}{6}.$$

EXAMPLE 9.18

An H-type cable, 30 km long has a capacitance per kilometre of 0.45 μF between any two conductors. The supply voltage is three-phase, 33 kV, and 50 Hz. Determine the charging current.

Solution:

$$\text{Supply voltage, } V = 33 \text{ kV(L–L)}$$

$$= \frac{33}{\sqrt{3}} = 19.05 \text{ kV/phase}$$

Capacitance for 30 km, $C = 30 \times 0.45\ \mu F = 13.5\ \mu F$

Capacitance of each core to neutral, $C_N = 2C = 2 \times 13.5\ \mu F = 27\ \mu F$

$$\text{Charging current per phase } = V\omega C_N$$

$$= 19.05 \times 10^3 \times 2\pi \times 50 \times 27 \times 10^{-6}$$

$$= 161.59 \text{ A.}$$

EXAMPLE 9.19

The capacitance of three-core cable belted type is measured and found to be as follows:

(a)Capacitance between three cores bunched together and to the sheath is 7.5 μF

(b)Capacitance between the conductor and the other two connected together to the sheath is 4.5 μF. Calculate the capacitance to neutral and the total charging kVA when the cable is connected to a 11 kV, 50 Hz, three-phase supply.

Solution:

Let earth capacitance = C_S

Core capacitance = C_C

1. Capacitance with three cores bunched together and the sheath, C_A = 7.5 μF

$$\text{i.e., } C_A = 3C_s = 7.5\ \mu\text{F}$$
$$C_s = 2.5\ \mu\text{F}$$

2. Capacitance between the conductor and the other two cores connected together to the sheath,

$$C_B = 4.5\ \mu\text{F}$$
$$\text{i.e., } C_B = C_s + 2C_c = 4.5$$
$$= 2.5 + 2C_c = 4.5$$
$$C_c = 1.0\ \mu\text{F}$$

$$\text{Capacitance to neutral, } C_N = C_s + 3C_c$$
$$= 2.5 + 3 \times 1.0$$
$$= 5.5\ \mu\text{F}$$
$$\text{Capacitance/phase} = 5.5\ \mu\text{F}$$
$$\text{Line voltage (given) } V_L = 11\ \text{kV}$$
$$V_{ph} = \frac{11}{\sqrt{3}} = 6.35\ \text{kV}$$
$$\text{Charging current} = I_C$$
$$= (2\pi fC)V_{ph}$$
$$= 2\pi \times 50 \times 5.5 \times 10^{-6} \times 6.35 \times 10^3 = 10.97\ \text{A}$$
$$\text{Total charging} = \sqrt{3}\, V_L I_C = \sqrt{3} \times 11 \times 10.97 = 209.04\ \text{kVA.}$$

EXAMPLE 9.20

A three-core, three-phase metal-sheathed cable on testing for capacitance give the following results:

(i) Capacitance between all conductors bunched and the sheath = 0.90 μF.

(ii) Capacitance between two conductors bunched with sheath and third conductor = 0.46 μF, with the sheath insulated. Find the capacitance

(a) Between any two conductors

(b) Between any two bunched conductors and the third conductor

(c) Calculate the capacitance to neutral and charging current taken by the cable when connected to 11 kV, three-phase, 50 Hz system.

Solution:

Given that:

1. Capacitance between all the conductors bunched and sheath = 0.9 μF

$$C_A = 3\,C_s = 0.9\,\mu F$$
$$C_s = 0.3\,\mu F$$

2. Capacitance between two conductors bunched with sheath and third conductor = 0.46 μF

$$C_B = 2C_c + C_s = 0.46\,\mu F$$
$$2C_c = 0.46 - C_s$$
$$= 0.46 - 0.3 = 0.16\,\mu F$$
$$C_c = 0.08\,\mu F$$

(a) Capacitance between any two conductors $= \frac{C_s}{2} + \frac{3C_c}{2}$

$$= \frac{0.3}{2} + \frac{3\times 0.08}{2} = 0.27\,\mu F$$

(b) Between any two bunched conductors and the third conductor $= 2C_C + \frac{2C_s}{3}$

$$= 2\times 0.08 + \frac{2\times 0.3}{3} = 0.16 + 0.2 = 0.36\ \mu F$$

(c) Capacitance per phase, $C_N = C_s + 3C_c = 0.3 + 0.08 \times 3 = 0.54\,\mu F$

$$\text{Charging current} = \frac{11}{\sqrt{3}} \times 10^3 \times 2\pi \times 50 \times 0.54 \times 10^{-6}$$
$$= 1.077\ A.$$

EXAMPLE 9.21

The capacitance per km of a three-phase belted cable is 0.3 μF between the two cores with the third core connected to the lead sheath. Calculate the charging current taken by 5 km of this cable when connected to a three-phase, 50 Hz, 11 kV supply.

Solution:

Capacitance between 1 and 2 cores with third core connected to the sheath is,

$$= C_C + \frac{1}{2}C_C + \frac{1}{2}C_S = \frac{3}{2}C_C + \frac{1}{2}C_S$$
$$= \frac{1}{2}(3C_C + C_S) = 0.3\ \mu F$$

But, $3C_C + C_S = C_N$

The measure value of capacitance = $\frac{1}{2} C_N$

Capacitance per phase, $C_N = 2 \times 0.3 = 0.6\ \mu F$

$$\therefore \text{Charging current per phase per kilometre is } = \omega C_N V_{ph}$$
$$= 2\pi \times 50 \times 0.6 \times 10^{-6} \times \frac{11}{\sqrt{3}} \times 10^3$$
$$= 1.197\ A$$

Charging current for 5 km = 1.197 × 5 = 5.985 A

9.13 HEATING OF CABLES

According to the concept of Thermodynamics, the temperature of any body depends on the rate of generation and dissipation of heat by the body.

The temperature rise of a cable under working conditions depends upon the following factors:

- Heat generated inside the cable, i.e., upto its peripheral
- The rate of dissipation of heat upto peripheral of the cable
- Surrounding medium and temperature
- The current rating of the cable
- Nature of the load

9.13.1 Generation of Heat Within the Cables

The main sources of generation of heat inside the cables are:

- Copper loss
- Dielectric loss
- Sheath and armouring losses

Copper Loss Copper loss occurs mainly due to the presence of resistance in the cables. The resistance of the conductor mainly depends upon the temperature by the following relation

$$R_t = R_0(1 + \alpha t)$$

where,

R_t = resistance at temperature t

R_0 = resistance at initial temperature

α = temperature coefficient

t = change in temperature

Hence, as the temperature increases the value of the resistance of the conductor increases and hence more losses result in more heat.

Dielectric Loss Losses occurring in the dielectric of cables are called dielectric losses. These are mainly caused due to the leakage of charging currents.

As explained in the Section 9.10,

$$\text{Dielectric loss per phase, } P = VI\cos\phi = VI\sin\delta = V\frac{I_{ch}}{\cos\delta}\sin\delta$$
$$= V \times V\omega C\tan\delta = V^2\omega C\tan\delta.$$

Sheath and Armouring Losses The presence of lead sheath influences the electrical characteristics of the circuit when power is transmitted by the cable. This effect is considerably more for single-core cables than that of three-core cables. The flow of currents in the conductor produces magnetic fields around the conductor which link with the lead sheath and induce an e.m.f and current in it. This induced current depends upon the frequency, current in the conductor, sheath resistance, arrangement of cables and on the conditions that the sheath is bounded or unbounded.

The approximate sheath losses due to sheath eddy currents is given by

$$\text{Sheath loss} = I^2\left[\frac{78\omega^2}{R_s}\left(\frac{r_m}{d}\right)^2 \times 10^{-9}\right] \text{W/phase} \quad (9.30)$$

where,

I = current per conductor

$\omega = 2\pi f$

r_m = mean radius of the sheath

R_s = sheath resistance in ohms

d = Inter-axial spacing of conductors

These losses are negligible as they form only about 2% of the core losses.

9.14 THERMAL CHARACTERISTICS

Cables have the following thermal characteristics:

9.14.1 CURRENT CAPACITY

Cables should not be operated at high temperatures because of the following reasons:

1. At high temperatures, expansion of oil may cause the sheath to burst.
2. The viscosity of the oil may decrease at a higher temperature.
3. Dielectric losses increase with increased temperature, so it may cause breakdown of insulation.

The maximum operating temperature of oil impregnated cables is 60° C.

$$\text{Current, } I = \sqrt{\frac{30}{nR\,(S+G)}}\ \text{A}$$

where,

n = number of cores or phases

R = conductor resistance at 60° in ohms/metre

S = thermal resistance of the cable between the cores and the sheath in thermal ohms per metre

G = thermal resistance between the sheath and earth in thermal ohms per metre, and the dielectric and sheath losses are neglected.

Thermal resistance:

Thermal resistance of single core cable, $S = \frac{k}{2\pi l} \ln \frac{D}{d}\ \Omega$

where, k = thermal resistivity

D = cable diameter

D = core diameter

l = length.

9.15 TESTING OF CABLES

The testing of cables are carried out are as per British specifications. The tests to be carried out on paper insulation cables with lead sheaths are, therefore, as follows:

9.15.1 Acceptance Tests at Works

Conductor Resistance Test In this test, the DC resistance of the cable core is measured at room temperature and corrected to the standard reference temperature of 20° C.

Voltage Test In this case, the voltage of frequency 25–100 Hz is applied to the cable. It is gradually increased up to a rated value for 15 minutes continuously between each conductor to sheath and between the conductors. For SL or H-type cables, the lower and upper range of voltages applied between the conductor and sheath varies between 15 kV and 22 kV. This voltage test is made before and after a bending test.

9.15.2 Sample Tests at Working

In this test, after bending, voltage tests are conducted on the finished cables on certain samples in order to ascertain the declared value. Drainage tests are conducted in order to ensure that the paper does not become dry at the higher points of the cables which are vertically installed, because otherwise, the cable may be damaged at the weak spot of dried out paper.

9.15.3 Performance Tests

For this test, after the installation of the cable, voltage tests are performed but with reduced voltages. For SL-type cables, the voltage values are 20 kV.

9.15.4 Tests on Oil-Filled and Gas-Filled Cables

For these cables (pressure cables), impulse tests are conducted instead of repeated application of the normal operating voltage for stipulated durations.

Load-cycle Tests A test loop is formed out of the cable and each type of accessory, is subjected to 20 load cycles at a minimum conductor temperature of 5°C in excess of the design value. The temperature limit is increased in certain cables. The cable is energized to 150% of the normal operating voltage. The cables are tested at a minimum stipulated internal pressure.

Impulse Test In this test, a loop is formed out of the cables and each type of accessory is subjected to 10 positive and 10 negative impulses at the voltage given in Table 9.1. The values given in table are based on the empirical formula 4.5(E+10) kV where E is the phase voltage. At the higher operating voltages, the impulse-withstand value becomes relatively less significant since the latter is independent of the service voltage. For these higher voltages it is considered that the ratio (Peak impulse voltage/ Peak service voltage) can be reduced with safety.

TABLE 9.1 IMPULSE TEST RESULTS

Operating voltage (kV)	Impulse test max. operating voltage ratio	Impulse test voltage (kV)
33	194	7.2
132	640	6.0

Dielectric Thermal-resistance Test In this test, the thermal resistance of the cable is to be measured.

Mechanical Test This test is conducted for measuring mechanical strength of the metallic reinforcement. Here a specified maximum (twice) internal pressure is applied on a sample cable.

Bending Test The cable is subjected to three bending cycles round a drum of diameter 20 times the diameter of the pressure-retaining sheath. The sample is then required to withstand the routine voltage test carried out on all production lengths of cable.

Cold Power Factor/Voltage Tests The power factor of a 91.4 m length of cable is measured at 0.5, 1.0, and 2 times the operating voltage with the cable at the stipulated minimum internal pressure. The values must be within the marker’s guaranteed values.

9.15.5 Tests When Installed

A voltage test similar to the above is carried out in the same manner but with somewhat reduced voltages.

9.15.6 Tests on Pressurized Cables

In this test, an AC high-voltage (3.46 times the operating voltage) is applied for 15 minutes. This test imposes the unnecessary limitations on the dielectric stress for which the cable is designed. It has been replaced by an impulse test.

9.16 LAYING OF CABLES

The following methods are used for laying of cables:

- Direct system
- Draw in system
- Solid system

9.16.1 Direct System

In this method, the cables are laid in a trench and are closed with soil as shown in Fig. 9.18. Generally, the cables laid in a trench are covered with bricks, planks, tiles or concrete slabs. The cable is laid with a serving of bituminized paper and Hessian over the sheath. The serving is enough to protect the cable from corrosion and electrolysis.

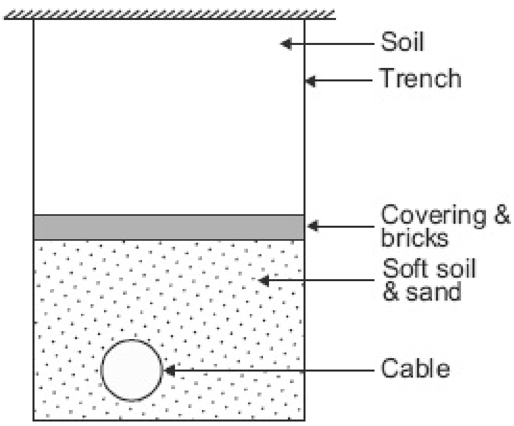

Fig. 9.18 Direct laying

Advantages

- This method is cheap and simple
- Gives the best condition for dissipating the heat generated in the cables
- Eliminates trouble due to vibrations caused by traffic

Disadvantages

- Extension of load is possible only by installation of cable in new trench
- Incurs double the cost due to new installation of cable for new loads

9.16.2 Draw-in System

This method is used in highly populated areas such as big cities, towns, etc. Ducts or pipes are laid in the ground with manholes at suitable positions and distances as shown in Fig. 9.19 and cables are drawn-in whenever needed.

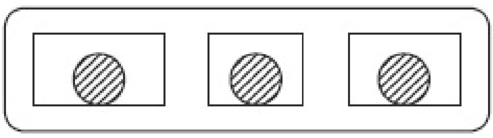

Fig. 9.19 Multiple butt ended stone-ware conduit

Advantages

- Easy to repair.
- Alterations or additions to the new loads can be made without digging the ground.
- Less installation cost for new loads.

9.16.3 Solid Systems

For this type of laying, cables are laid in wood, earthenware, cast iron or Asphalt troughs filled up with bitumen or pitch, which is poured in after being heated to fluid state as shown in Fig. 9.20. Cables laid in this manner are lead covered and provides good mechanical protection. Asphalt troughing is very suitable for a solid system because it is easy to be laid.

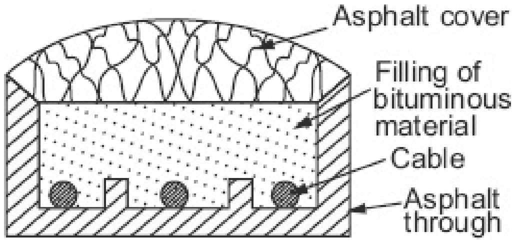

Fig. 9.20 Asphalt thronging containing cable

After filling with bitumen, a covering is provided by bricks, tiles or wood.

Advantages

- For troughing material, wood is cheaper than the others.

Disadvantages

- More expenses when compared with direct laid system.
- Facility to dissipation of heat from the cable is poor.

9.17 CABLE FAULTS

The faults which are most likely to occur are:

1. A breakdown of the insulation of the cable which allows current to flow from the core to earth or to the cable sheath (called a ground fault).
2. A cross or short-circuit fault in which case the insulation between two cables or between two cores of a multi-core cable is faulty.
3. An open circuit fault where the conductor is broken or a joint is pulled out.

9.18 DETERMINATION OF MAXIMUM CURRENT CARRYING CAPACITY

The thermal stresses on the system will be more due to frequent overloading. The temperature rise of cables beyond permissible values due to overloading may cause the following:

- The insulation voids created by unequal expansions may cause damage to insulation.
- Oil expansion causes sheath burst.
- Draining of oil from higher levels is possible as an effect of change in viscosity of the oil.
- It may cause thermal instability.

Thus, the maximum loadability of the cable depends upon the rise in temperature of the insulation and the surrounding environment.

This in-turn results in limiting the current rating of the conductor. It also depends upon

1. Purpose of layout of cables.
2. Methods employed in layouts.
3. Spacing of conductors.
4. Number of cores.
5. Thermal conductivity of the soil in which the cable is laid out.

Cable manufacturers use multiplying factors to take care of each of them. An approximate indication of the current carrying capacity for Indian conditions is given in Table 9.2.

Table 9.2 Area of cross-section and corresponding current rating of cable

Area (mm2)	Current carrying capacity (A)
33	15
10	60
16	77
25	99
35	120

70	175
95	210
185	300
300	385
500	470
630	525

Source: *A.P. Transco Employes Association dairy (2007).*

CHAPTER AT A GLANCE

1. The combination of conductor and its insulation is called cable.
2. The main parts in the cable are: core, insulation, metallic sheath, bedding, armouring, and serving.
3. The types of cable used will depend upon the voltage and service requirements and they are:
    1. Low tension cables - for operating voltage up to 1 kV
    2. High tension cables - for operating voltage up to 11 kV
    3. Super tension cables - for operating voltage up to 33 kV
    4. Extra high tension cables - for operating voltage up to 66 kV
    5. Extra super-voltage power cable - for operating voltage beyond 132 kV
4. Properties of insulating materials for cables are high resistivity, dielectric strength, tensile strength and plasticity, and mechanical strength.
5. The various insulating materials used for cables are rubber, vulcanized india rubber, impregnated paper, poly vinyl chloride, and enamel insulation.
6. Insulation resistance of cable is given by $R_i = \frac{\rho}{2\pi l} \ln \frac{R}{r}$
7. The potential gradient at any point is defined as the rate of increase of potential at that point.

$$\text{Maximum potential gradient, } g_{max} = \frac{V}{r \ln\left(\frac{D}{d}\right)} = \left(\frac{V}{r \ln\left(\frac{R}{r}\right)}\right).$$

$$\text{Minimum potential gradient, } g_{min} = \frac{V}{R \ln\left(\frac{D}{d}\right)}.$$

8. Economical core diameter 2V
9. **Grading of cable:** The process of achieving uniform distribution in dielectric stress is called grading of cables. There are two kinds of grading: capacitance grading and intersheath grading.
    1. The process of applying layers of different materials for the dielectrics is called "Capacitance Grading".
    2. The process of providing metallic intersheath between successive layers of the same dielectric materials and maintaining appropriate potential level at the intersheath is called "Intersheath Grading".
10. Power factor of the cable, $\delta = \frac{G_i}{\omega C}$.
11. The tests to be carried on paper insulation cables with lead sheaths are: Acceptance tests at works, sample tests at work, performance tests, tests on oil filled and gas filled cables.

SHORT ANSWER QUESTIONS

1. What are the different types of cables?
2. What are the advantages of underground cables?
3. What are the disadvantages of underground cables?
4. List the various parts of cables.
5. What are the different types of insulating materials used in cables?
6. Write down the different types of cables generally used for three-phase service.
7. What is meant by grading of cables?
8. What are the different methods of grading of cables?
9. What are the different types of screened and pressure cables?
10. What are the different types of laying of underground cables?

MULTIPLE CHOICE QUESTIONS

1. The advantage of the cables over overhead transmission lines is
    1. easy maintenance
    2. low cost
    3. can be used in congested areas
    4. can be used in high voltage circuits
2. Low-tension cables are generally used up to
    1. 0.2 kV
    2. 0.5 kV
    3. 0.7 kV
    4. 1 kV
3. High-tension cables are generally used up to
    1. 11 kV
    2. 33 kV
    3. 66 kV
    4. 132 kV
4. Belted cables are generally used up to ________________ .
    1. 11 kV
    2. 33 kV
    3. 66 kV
    4. 132 kV
5. ____________cables are used for 132 kV lines.
    1. high-tension
    2. super-tension
    3. extra high-tension
    4. extra super-voltage
6. Pressure cables are generally not used beyond
    1. 11 kV
    2. 33 kV
    3. 66 kV
    4. 132 kV
7. Solid-type cables are considered unreliable beyond 66 kV because
    1. insulation may melt due to higher temperature
    2. skin effect dominates the conductor
    3. of the corona loss between conductor and sheath material
    4. there is a danger of break down of insulation due to pressure of voids
8. Cables generally used beyond 66 kV are
    1. oil filled
    2. SL type
    3. belted
    4. armoured
9. The most commonly used insulation in high voltage cables is ____________
    1. impregnated paper
    2. rubber
    3. VIR
    4. cloth
10. The insulation resistance should have
    1. high resistance
    2. high mechanical strength
    3. both (a) and (b)

4. strong

11. The insulating material for the cables should
    1. be acid proof
    2. be non-inflammable
    3. be non-hygroscopic
    4. all of these
12. If a cable of homogeneous insulation has a maximum stress of 10 kV/mm, then the dielectric strength of insulation should be
    1. 5 kV/mm
    2. 10 kV/mm
    3. 15 kV/mm
    4. 30 kV/mm
13. Which of the following insulation is used in cables?
    1. varnished cambric
    2. rubber
    3. paper
    4. any of the above
14. Empire tape is
    1. varnished cambric
    2. varnished rubber
    3. impregnated paper
    4. cloth
15. The thickness of the layer of insulation on the conductor in cables depends upon
    1. reactive power
    2. power factor
    3. voltage
    4. current carrying capacity
16. Dielectric strength of rubber is around
    1. 5 kV/mm
    2. 15 kV/mm
    3. 30 kV/mm
    4. 200 kV/mm
17. If a cable is to be designed for use on 1000 kV, the insulation preferred is
    1. poly vinyl chloride
    2. vulcanized rubber
    3. impregnated paper
    4. compressed SF_6 gas
18. The most economic load on an under ground cable is____________
    1. greater than its surge loading
    2. less than its surge loading
    3. equal to its surge loading
    4. zero
19. The minimum and maximum dielectric stress in a cable is at____________
    1. lead sheath, conductor surface
    2. conductor surface, lead sheath
    3. both at the center of conductor
    4. both at lead sheath

20. Void formation in the dielectric material of an underground cable may be controlled by .
    1. using a high permittivity solid dielectric
    2. providing a strong metallic sheath outside the cable
    3. filling oil at high pressure as dielectric
    4. none of these
21. Cables in power transmission line are provided with intersheaths to_______________
    1. minimize stress
    2. minimize high voltage
    3. provide uniform stress distribution
    4. minimize charging current
22. The effect of bonding the cable is to______________
    1. increase the effective resistance but reduce inductance
    2. increase the effective resistance and inductance
    3. decrease the effective resistance and inductance
    4. decrease the effective resistance but increase the inductance
23. Which of the following protects a cable against mechanical injury
    1. bedding
    2. sheath
    3. armouring
    4. paper
24. The bedding on a cable consists of
    1. hessian cloth
    2. jute
    3. paper
    4. both b and c
25. A cable immediately above the metallic sheath_____________ is provided
    1. earthing connection
    2. bedding
    3. armouring
    4. paper
26. Conduit pipes are normally used to protect______________ cables
    1. unsheathed
    2. armoured
    3. PVC sheathed
    4. all of these
27. In capacitance grading of the cables we use a____________________ dielectric.
    1. composite
    2. porous
    3. homogeneous
    4. hygroscopic
28. The material for armouring on the cable is usually
    1. steel tape
    2. galvanized steel wire
    3. both a and b
    4. copper

29. In cables, the charging current
    1. lags the voltage by 90°
    2. leads the voltage by 90°
    3. lags the voltage by 180°
    4. leads the voltage by 180°
30. In cables, sheaths are used to
    1. prevent the moisture from entering the cable
    2. provide enough strength
    3. provide proper insulation
    4. to enter the moisture into the cable
31. The intersheaths in the cables are used to
    1. minimize the stress
    2. avoid the requirement of good insulation
    3. provide proper stress distribution
    4. provide enough strength
32. The electrostatic stress in the ungrounded cable is
    1. same as the conductor and sheath
    2. minimum at the conductor and maximum at the sheath
    3. maximum at the conductor and minimum at the sheath
    4. zero at the conductor as well as on the sheath
33. The breakdown of the insulation of the cable can be avoided economically by the use of______________________
    1. intersheaths
    2. insulating materials with different dielectric constants
    3. both a and b
    4. steel tape
34. A cable carrying alternating current has
    1. hysterisis losses only
    2. hysterisis and leakage losses only
    3. hysterisis and leakage and copper losses only
    4. hysterisis, leakage, copper losses, and friction losses
35. Capacitance grading of cable implies
    1. use of dielectrics of different permeabilities
    2. grading according to capacitance of cables per kilometre length
    3. cables using single dielectrics in different concentrations
    4. capacitance required to be introduced at different lengths to counter the effect of inductance
36. Underground cables are laid at sufficient depth
    1. to minimize temperature stresses
    2. to avoid being unearthed easily due to removal of soil
    3. to minimize the effect of shocks and vibrations due to passing vehicles, etc
    4. for all of the above reasons
37. Cables for 220 kV lines are invariably
    1. mica insulated
    2. paper insulated
    3. compressed oil or compressed gas insulated
    4. rubber insulated
38. Void formation occurs in
    1. XLPE cables

2. oil-filled cables
3. oil-impregnated paper cables
4. gas filled cables

39. Insulation resistance of a cable 20 km long is 1 MΩ. Two cable lengths, 20 km and 10 km, are connected in parallel. The insulation resistance of the parallel combination is
    1. 1 MΩ
    2. 0.5 MΩ
    3. 0.666 MΩ
    4. 1.5 MΩ
40. If the voltage applied to the core and sheath of a cable is halved, the reactive power generated by the cable will be
    1. halved
    2. 4th of the original value
    3. doubled
    4. full

Answers:

1. c	2. c	3. a	4. b	5. d
6. c	7. d	8. a	9. a	10. c
11. d	12. b	13. d	14. a	15. c
16. c	17. d	18. b	19. a	20. c
21. c	22. a	23. c	24. a	25. b
26. a	27. a	28. c	29. b	30. a
31. c	32. c	33. c	34. b	35. a
36. c	37. c	38. c	39. c	40. b

REVIEW QUESTIONS

1. What is the relation between the conductor diameter and breakdown potential of a cable while voltage of the cable and its overall diameter are fixed? Prove the same.
2. Classify underground cables according to various parameters. Give the applications of each type of cable.
3. What are the insulating materials used for underground cables? Explain in detail the different kinds of insulating materials and their respective properties.
4. What are the limitations of solid-type cables? Explain any other kind of cable to overcome the limitations.
5. Describe with a neat sketch the construction of three-core belted type cable. Discuss the limitations of such a cable.
6. Explain and distinguish between the constructional features of the (i) H-Type (ii) SL-Type (iii) HSL-Type of cables.
7. What are oil-filled cables? Describe the construction of oil-filled cables.
8. Describe the function of sheath in cables. How are sheath losses reduced in modern multi-core cables?

9. Prove that for a concentric cable of given dimension and given maximum potential gradient in the dielectric, the maximum permissible voltage between the core and the sheath is independent of the permittivity of the insulating material.
10. Derive an expression for the capacitance of a three-core cable.
11. Find expressions for capacitance, insulation resistance and dielectric stress of a single-core cable.
12. Derive expressions for electric stress of a single core cable. Where is the stress maximum? Where is it minimum and why?
13. Discuss the methods of grading cables. Why are they not used generally?
14. Describe an experiment to determine the capacitance of a three-core belted cable.
15. Derive a relation between the conductor radius and inside sheath radius of a single-core cable so that electric stress of the conductor surface may be minimum.
16. Prove that for a concentric cable of given dimension and given maximum potential gradient in the dielectric, the maximum permissible voltage between the core and the sheath is independent of the permittivity of the insulating material.
17. Derive an expression for electric stress in a single core cable.
18. Enumerate the advantages of overhead lines compared to UG cables for transmission at high voltages.

PROBLEMS

1. A single-core cable, composed of a conductor of 2 cm diameter, has an insulation thickness of 0.5 cm and an insulation resistance of 450×10^6 Ω/km. What thickness of a similar material would be needed for a 3 cm diameter conductor cable in order to have an insulation resistance of 900×10^6 Ω/km.
2. A single-core, 2 km long cable has a conductor radius of 1.2 cm and an insulation thickness of 4 mm. If the resistivity of dielectric is 7×10^{12} Ω-m, determine the insulation resistance of the cable.
3. A single-core, 3000 m long cable has a core diameter of 1.6 cm, a sheath diameter of 5 cm and an insulation resistance of 1800 MΩ. Determine the resistivity of the dielectric.
4. A single-core, lead-sheathed cable has a core diameter of 1.2 cm and is graded by using two dielectrics of relative permittivity 3.5 (inner) and 3.0 (outer). The thickness of each being 1 cm, system voltage is 66 kV, three-phase. Determine the potential gradient at the surface of the conductor and at the other points.
5. Determine the economic value of the diameter of a single-core cable working on a 66 kV, three-phase system. Also calculate the overall diameter of the insulation if the maximum permissible stress in the dielectric is not to exceed 50 kV/cm.
6. A 33 kV, three-phase, 2.5 km long feeder consists of single-core cables having a conductor radius of 12 mm and an insulation thickness of 8 mm. The dielectric has a relative permitivity of 3

and the power factor of the unloaded cable is 0.3. Determine the following:

1. capacitance per phase
2. charging current per phase
3. total charging kVAr
4. dielectric per phase and
5. maximum electric stress in the cable.

7. A 3 km long, 11 kV, three-phase, three-core, belted cable have the following data
   1. Capacitance between two conductors joined to sheath and the third conductor is 1.8 μF.
   2. Capacitance between all the three conductors joined and the sheath is 2.1 μF.

   Determine

   1. Effective capacitance of each core to neutral and
   2. Capacitance between any two cores.
8. A single-core, lead-sheathed cable is graded by using three dielectrics of relative permittivity 5, 4, and 3, respectively. The conductor diameter is 1.8 cm and the inner radius of the sheath is 3 cm. Assuming that all the three dielectrics are worked at the same maximum potential gradient of 40 kV/cm, determine the potential difference in kV between the core and earthed sheath.
9. The capacitance of three-core belted type cable has the following data
   1. Capacitance between three cores bunched together and the earthed sheath is 6.6 μF
   2. Capacitance between the conductor and the other two connected together to the sheath is 4 μF

   Determine the capacitance to neutral and the total charging current drawn by the cable, when the cable is connected to a 66 kV, 50 Hz, three-phase supply.
10. A three-core, three-phase metal-sheathed cable has the following data
    1. Capacitance between all conductors bunched and sheathed is 0.9 μF
    2. Capacitance between two conductors bunched with sheath and with a third conductor is 0.7 μF

    Determine the capacitance when the sheath is insulated for the following conditions

    1. Between any two conductors
    2. Between any two bunched conductors and the third conductor.
    3. Calculate the capacitance to neutral and charging current taken by the cable when connected to 66 kV, three-phase, 50 Hz systems.
11. In a three-phase, three-core metal-sheathed cable, the measured capacitance between any two cores is 2.1 μF. Determine the kVA taken by the cables when it is connected to 50 Hz, 13.2 kV bus

bars.

10

Power Factor Improvement

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Provide the reasons for poor power factor
- Understand the various methods of power factor improvement
- Determine the best economical power factor

10.1 INTRODUCTION

Electrical power or energy, generated at power stations and transmitted through transmission lines, is then distributed to the consumer. The quantity of power transmitted and distributed is based on the power factor (pf) of load and the parameters of the transmission lines.

Almost all the power system loads are of the inductive type and have undesirably low lagging power factor. Low power factor and increase in current results in additional losses in all the components of power system from the generating station to the consumers. A power factor close to unity is preferred for the economical and better distribution of electrical energy. In this chapter, various methods are discussed to improve the power factor of the system. Figure 10.1 shows a power factor meter.

10.2 POWER FACTOR

Power factor is the ratio of real power P to apparent power S or the cosine of angle between voltage and current in an AC circuit and is denoted by $\cos\phi$. For all types of inductive loads, the angle between voltage V and current I is negative and the cosine of this angle is called the lagging power factor. Similarly, when the angle between *V* and *I* is positive it is called leading power factor (this occurs for capacitive loads). The phasor diagram for lagging power factor is shown in Fig. 10.2(a). If all the components of Fig. 10.2(a) are multiplied by voltage *V*, the power triangle as shown in Fig. 10.2(b) is obtained.

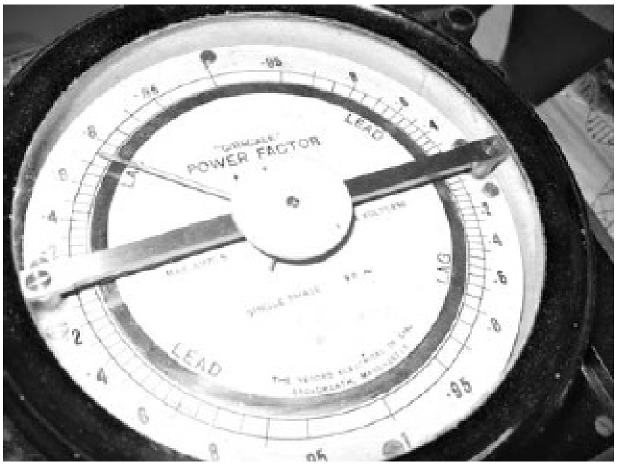

Fig. 10.1 Power factor meter

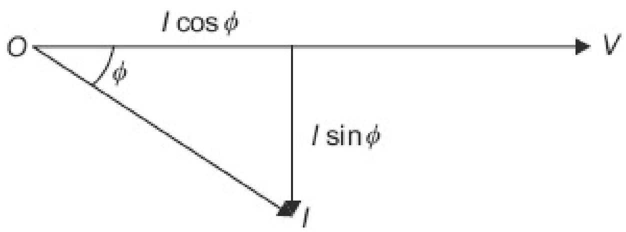

Fig. 10.2(a) Phasor diagram

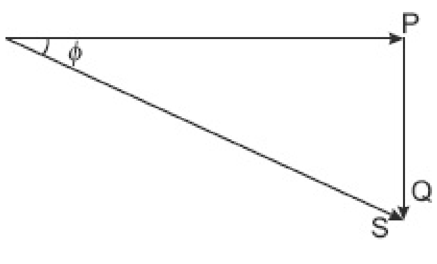

Fig. 10.2(b) Power triangle

$$\therefore \text{Power factor} = \cos\phi = \frac{P}{S}$$
$$= \frac{P}{VI} = \frac{\text{Active power}}{\text{Apparant power}}.$$

10.2.1 Causes of Low Power Factor

1. The induction motors work at a low lagging power factor at light loads and improved power factor with increased loads.
2. The transformers have a lagging power factor because they draw magnetizing current.
3. Miscellaneous equipment like arc lamps, electric discharge lamps, welding equipment etc., operates at a low power factor.
4. The industrial heating furnaces operate at a low lagging power factor.
5. The variation of load on the power system also causes low power factor.

Test Yourself

1. Why is low lagging power factor undesirable?
2. Why do power systems generally operate at a lagging power factor?

10.2.2 Effects or Disadvantages of Low Power Factor

For a given load P, with voltage V_L and power factor cosθ, the load current is given by

$$I_L = \frac{P}{\sqrt{3}V_L \cos\phi}.$$

From the above expression for a given load it is clear that, if the power factor is low, the load current will be higher. This leads to the following effects:

1. **Effect on transmission lines:** For the fixed active power to be transmitted over the line, the lower the power factor the higher will be the load current to be carried by the line. Since the maximum permissible current density of the line conductor is fixed, the cross-sectional area of the conductor is to be increased in order to carry larger current. This results in an increased volume of the conductor material which in turn increases the capital cost of transmission lines.

   Further, increase in the current causes increase in line losses with a reduction in the efficiency of the line. Also, the line voltage regulation is poor.

2. **Effect on transformers:** A reduction in the power factor causes a reduction in the kW capacity of a transformer.
3. **Effect on switchgear and bus bar:** The lower the power factor at which a given power is to be supplied, the larger is the cross-sectional area of the bus bar and the larger is the contact surface of the switch gear.
4. **Effects on generators:** With a lower power factor, the kW capacity of a generator is reduced. The power supplied by the exciter is increased. The copper losses in the generator are increased, which results in low efficiency of the generator.
5. **Effect on prime movers:** When the power factor is decreased, the alternator develops more reactive kVA, i.e., the reactive power generated is more. This requires a certain amount of power to be supplied by the prime mover. Therefore, a part of the prime mover capacity is idle and it represents dead investment. The efficiency of the prime mover is reduced..
6. **Effect on existing power systems:** For the same active power, the operation of an existing power system at a lower power factor necessitates the overloading of the equipment when a full rated load is drawn.

10.3 ADVANTAGES OF POWER FACTOR IMPROVEMENT

From the earlier discussions, it is observed that, if the power station works at a low power factor, the capital cost of generation, transmission and distribution systems is increased. Higher capital charges means higher annual fixed charges, which will increase the cost per unit supply to the consumer. Therefore, the maintenance of a high power factor (close to unity) is always advantageous for both consumers and suppliers.

Following are the advantages of power factor improvement:

- The kW capacity of the prime movers is better utilized due to decreased reactive power.

- This increases the kilowatt capacity of the alternators, transformers, and the lines.
- The efficiency of the system is increased.
- The cost per unit decreases.
- The voltage regulation of the lines is improved.
- Power losses in the system are reduced due to reduction in load current.
- Investment on generators, transformers, transmission lines and distributors per kilowatt of the load supplied is reduced.
- kVA demand charges for large consumers undergoes reduction.

10.4 METHODS OF IMPROVING POWER FACTOR

In case of inductive loads the power factor is lagging. This lagging power factor can be compensated by using devices that are called compensators. These are:

1. Static capacitors
2. Synchronous condensers, and
3. Phase advancers

10.4.1 STATIC CAPACITOR

Static capacitors are connected across the mains at the load end. This supplies a reactive component of the current to reduce the out-of-phase component of current required by an inductive load i.e., it modifies the characteristics of an inductive load by drawing a leading current which counteracts or opposes the lagging component of the inductive load current at the point of installation. So the reactive VAr's transmitted over the line is reduced, thereby the voltage across the load is maintained within the specified limits.

By the application of the shunt capacitor to a line (Fig. 10.3) the magnitude of source current can be reduced, the power factor can be improved and consequently the voltage drop between the sending and receiving ends is also reduced as shown in Fig. 10.4. However, it is important to note that it does not affect the current or the power factor beyond their point of installation. A shunt capacitor of 25 P.F is shown in Fig. 10.4(c).

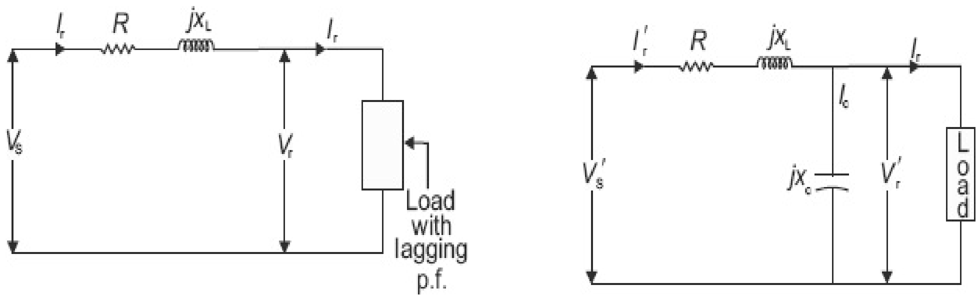

Fig. 10.3 Single line diagram without and with shunt capacitive compensation

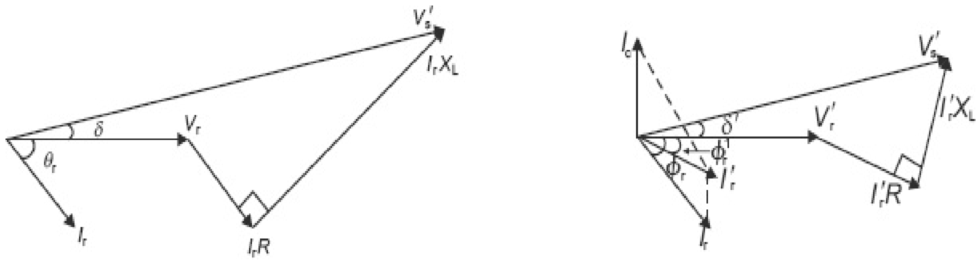

Fig. 10.4 Phasor diagram of Fig. 10.3(a) and (b),respectively

Voltage drop of the line without shunt capacitors is given by:

$$V_d = I_r R \cos\phi + I_r X_L \sin\phi \tag{10.1}$$

$$\text{With shunt capacitor, } V_d = I_r R \cos\phi + (I_r - I_c)\, X_L \sin\phi \tag{10.2}$$

where, I_c is the reactive component of current leading the supply voltage by 90°.

The voltage rise due to the location of the capacitor is the difference between the voltage drops determined by using (Eqs. 10.1) and (10.2) and is given as

Voltage rise = $I_c X_L$

and improved power factor due to placing of the shunt

capacitor of $Q_C = \frac{P}{\sqrt{P^2 + (Q_L - Q_C)^2}}$.

Fig. 10.4(c) 25 p.F. shunt capacitor

Calculation of Shunt Capacitor Rating The power factor correction can be determined from the power triangle shown in Fig. 10.5. From Fig. 10.5, the cosine of the angle $\angle OAB$ is the original power factor ($\cos\phi_1$), whereas the cosine of the angle $\angle OAC$ is the improved power factor ($\cos\phi_2$). It may be observed that the active power (OA)does not change with power factor improvement. However, the lagging kVAr of the load is reduced by the power factor correction equipment, thus improving the power factor to $\cos\phi_2$.

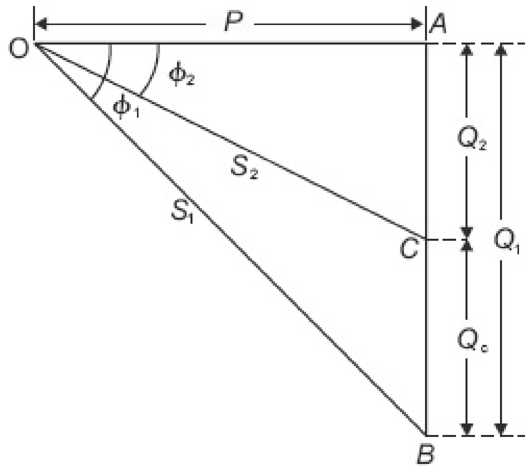

Fig. 10.5 Power triangle

Leading kVAr (Q_c) supplied by power factor correction equipment as

$$
\begin{aligned}
BC &= AB - AC \\
&= Q_1 - Q_2 \\
&= OA\ (\tan\phi_1 - \tan\phi_2) \\
&= P\ (\tan\phi_1 - \tan\phi_2) \qquad (10.3)
\end{aligned}
$$

Knowing the leading kVAr (Q_c)supplied by the power factor correction equipment, the capacitor current can be calculated.

$$
\begin{aligned}
I_c &= \frac{Q_c}{\sqrt{3}V_{L-L}} \\
X_c &= \frac{V_{ph}}{I_{cph}} \\
&= 2\pi fC \\
\therefore C &= \frac{X_c}{2\pi f}.
\end{aligned}
$$

Alternative Method Consider a single-phase load which takes lagging current I. When the capacitor is connected across the load, the current taken by the capacitor (I_c) leads the supply voltage (V) by 90°. The resultant current I' is the vector sum of I and I_c and its angle of lag is ϕ_2 which is less than ϕ_1 i.e., $\cos\phi_1$ is less than $\cos\phi_2$ as shown in Fig. 10.6. For three-phase loads the capacitors can be connected in a star or a delta arrangement.

If the power factor of the load is to be increased from $\cos\phi_1$ to $\cos\phi_2$, then the value of capacitor C required can be calculated as follows:

The active component of load current, $I_a = I\cos\phi_1$

The reactive component of load current, $I_{r1} = I\sin\phi_1$

$$I_{r1} = \frac{I_a}{\cos\phi_1} \times \sin\phi_1 = I_a \tan\phi_1$$

At fixed load the reactive component with increased power factor, $I_{r2} = I_a \tan\phi_2$.

Current taken by the capacitor, $I_C = I_a(\tan\phi_1 - \tan\phi_2)$

Applied voltage, $V = I_C X_C = I_C \dfrac{1}{2\pi f C}$ V

Value of capacitor, $C = \dfrac{I_C}{2\pi f V}$ F

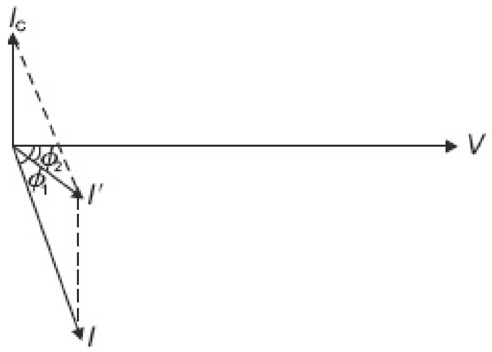

Fig. 10.6 Phasor diagram

If the power factor of the load is to be improved up to unity, the phasor diagram is shown in Fig. 10.7.

Then, $I_C = I\sin\phi_1$

And the capacitance, $C = \frac{I_C}{2\pi f V}$ F

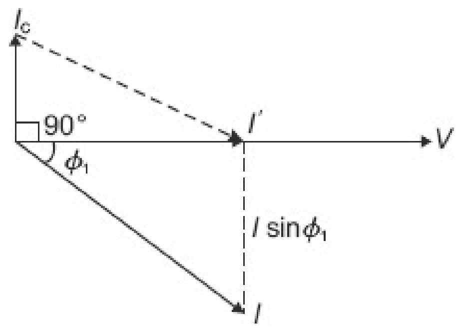

Fig. 10.7 Phasor diagram for unity power factor

Advantages of shunt capacitor:

- Losses are low.
- It requires less maintenance, because there are no rotating parts.
- Easy installation.

Disadvantages of shunt capacitor:

- Less service life.
- Easily damaged due to excess of voltage.
- Its repair is uneconomical.
- Difficult to control switching position of capacitors because of removing or adding the capacitors in the circuit for different power factors.

Test Yourself

1. How does power factor improvement reduce the rating of power system components?
2. Why is a shunt capacitor connected across the load?

10.4.2 Synchronous Condenser

The synchronous condenser is a synchronous motor running without a mechanical load. A synchronous motor takes a leading current when over-excited and, therefore behaves as a capacitor. It is connected in parallel to the supply or load as shown in Fig. 10.8(a). It generates leading current to neutralize the lagging component of the load current resulting in improved power factor.

It is already being discussed that the power factor improvement can be accomplished by the use of static capacitor bank. Various (finite) values of power factor can be achieved by the addition (or deletion) of units (capacitors) to (or from) the bank. However, by using a synchronous condenser a step less (smooth) variation of the power factor with the added advantage of ease of control can be realized.

For lower kVAr requirement static capacitor may be used. However, for a kVAr requirement in excess of 10,000, synchronous condensers are preferable and economical. View of synchronous motor is shown in Fig. 10.8(b).

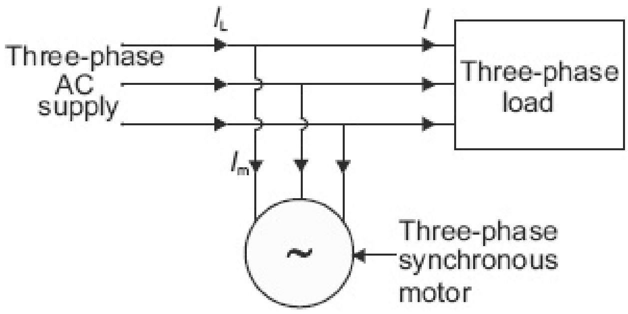

Fig. 10.8(a) Synchronous machine is connected in parallel with the supply

Fig. 10.8(b) Synchronous machine

Let P_1 be the active power of load

$\cos\phi_1$ is the power factor of load without synchronous condenser

$\cos\phi_2$ is the power factor of load after using synchronous condenser

P_S is the active power taken by the synchronous

condenser from the supply.

The rating and the power factor at which the synchronous

condenser is operating can be calculated as follows.

Reactive power of load, $Q_1 = P_1 \tan\phi_1$

When synchronous condenser is connected in the circuit,

Total load (active power) $P = P_1 + P_s$

Total reactive power, $Q_2 = P \tan\phi_2$

Reactive power supplied by the synchronous condenser, $Q = Q_1 - Q_2$.

The phasor diagram is shown in Fig. 10.9.

From Fig. 10.9, rated kVA of synchronous condenser,

$$P_{as} = \sqrt{P_s^2 + Q_r^2}$$

And power factor of synchronous condenser,

$$\cos\theta_2 = \frac{P_s}{P_{as}}.$$

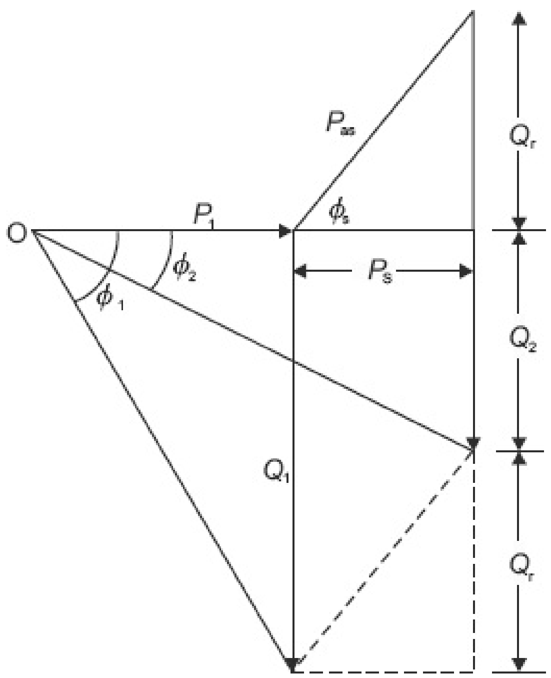

Fig. 10.9 Phasor diagram

Advantages:

- A synchronous condenser has an inherently sinusoidal wave form and the harmonic voltage does not exist.
- It can supply as well as absorb kVAr.
- The power factor can be varied smoothly.
- It allows the over-loading for short periods.
- The high inertia of the synchronous condenser reduces the effect of sudden changes in the system load and improves the stability of the system.
- It reduces the switching surges due to sudden connection or disconnection of lines in the system.

Disadvantages:

- Power loss increases.
- Uneconomical for small ratings.
- It is not possible to add or take away the units and to alter the rating of the synchronous condenser.
- The cost of maintenance is high.
- It produces noise.

10.4.3 Phase Advancers

There are special commutator machines, which are used to improve the power factor of the induction motor. When the supply is given to the stator of an induction motor, it takes a lagging current. Therefore, the induction motor has low lagging power factor. For compensating this lagging current, a phase advancer (mounted on same shaft) is used. It supplies m.m.f. to the rotor circuit at slip frequency.

Advantages:

1. The lagging kVAr drawn by the motor is reduced by compensating the stator lagging current at slip frequency.
2. Wherever the use of synchronous motors is not suitable, a phase advancer can be used.

The major drawback of this kind of a compensator is that it is not economical for motors with low ratings (less than 200 hp).

Example 10.1

A single-phase motor connected to a 230 V, 50 Hz supply takes 30 A at a power factor capacitor is shunted across the motor terminals to improve the power factor to 0.9 lag Determine the capacitance of the capacitor to be shunted across the motor terminals.

Solution:

Motor current, I_{m1} = 30 A

Active component of motor current (I_{m1}) at p.f.0.7 lag,

$$\begin{aligned} I_{a1} &= I_{m1} \cos\phi_1 \\ &= 30 \times 0.7 \\ &= 21 \text{ A} \end{aligned}$$

Reactive component of motor current (I_{m1})at p.f.0.7 lag,

$$\begin{aligned} I_{r1} &= I_{m1} \sin\phi_1 \\ &= 30 \times 0.714 \\ &= 21.42 \text{ A} \end{aligned}$$

Active component of current at improved power factor of 0.9 lag, is same as the active component of current at a power factor of 0.7 lag.

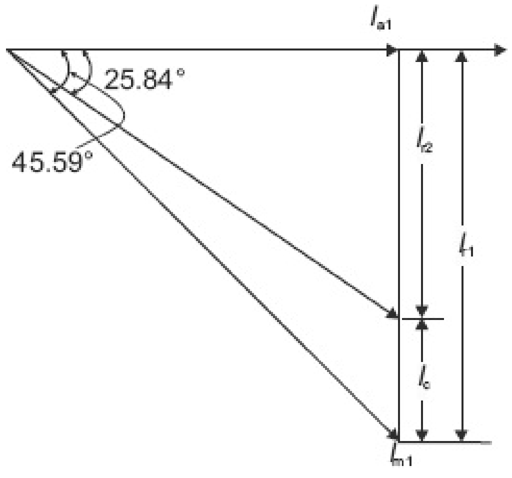

Fig. 10.10 Phasor diagram for Example 10.1

∴ The active component of motor current at a p.f.of 0.9,I_{a2} = 21.42 A

The reactive component of motor current at a p.f. of 0.9,

$$I_{r2} = \frac{21.42}{0.9} \times 0.436 = 10.374 \text{ A}$$

The reactive component of motor current to be neutralized,

$$\begin{aligned} I_c &= I_{r1} - I_{r2} \\ &= 21.42 - 10.374 = 11.046 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{It is also equal to } (I_c) &= \frac{V}{X_c} \\ &= V \times 2\pi f C \\ 11.046 &= 230 \times 2\pi \times 50 \times C \\ \text{(or)} \qquad C &= 152.87 \ \mu\text{F} \end{aligned}$$

∴ Capacity of capacitance connected across the motor terminals, C = 152.87 μF.

Example 10.2

A single-phase 400 V, 50 Hz motor takes a supply current of 50 A at power factor of 0.8 lag. The motor power factor has been improved to unity by connecting a condenser in parallel (Fig. 10.11). Calculate the capacity of the condenser required.

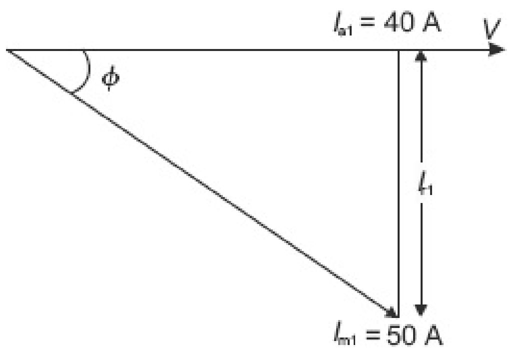

Fig. 10.11 Phasor diagram for Example 10.2

Solution:

The motor current at a p.f. 0.8 lag, I_{m1} = 50 A

Active component of motor current (I_{m1}) at p.f. 0.8 lag,

$$\begin{aligned} I_{a1} &= I_{m1} \cos\phi_1 \\ &= 50 \times 0.8 \\ &= 40 \text{ A} \end{aligned}$$

Reactive component of motor current(I_{m1}) at p.f.0.8 lag,

$$\begin{aligned} I_{r1} &= I_{m1} \sin\phi_1 \\ &= 50 \times 0.6 \\ &= 30 \text{ A} \end{aligned}$$

Active component of motor current at improved power factor of unity is same as the active component of motor current at a power factor of 0.8 lag, for a fixed load.

∴ The active component of motor current at a power factor of unity, I_{a2}

= 40 A

The reactive component of motor current to a power factor of unity,

$$, I_{r2} = \frac{40}{1} \times 0 = 0.0 \text{ A}$$

The reactive component of motor current to be neutralized,

$$\begin{aligned} I_c &= I_{r1} - I_{r2} \\ &= 30 - 0 = 30 \text{ A} \end{aligned}$$

It is equal to

$$(I_C) = \frac{V}{X_C}$$

$$\begin{aligned} &= V \times 2\pi f C \\ 30 &= 400 \times 2\pi \times 50 \times C \\ \text{(or)} \qquad C &= 238.73 \text{ μF} \end{aligned}$$

∴ Capacity of capacitance connected across the motor terminals, C = 238.73 μF.

Example 10.3

A 440 V, three phase distribution feeder has a load of 100 kW at lagging power factor with the load current of 200 A. If the power factor is to be improved, determine the

1. **uncorrected power factor and reactive load**
2. **new corrected power factor after installing a shunt capacitor of 75 kVAr.**

Solution:

1. Uncorrected power factor

$$= \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{100 \times 10^3}{\sqrt{3} \times 440 \times 200} = 0.656 \text{ lag}$$

Reactive load, $Q_L = P \tan\phi$ = 115.055 kVAr

Q_c = 75 kVAr (given)

2. Corrected power factor

$$= \frac{P}{\sqrt{(P^2 + (Q_L - Q_c)^2)}} = \frac{100}{\sqrt{(100)^2 + (115.055 - 75)^2}} = 0.928 \text{ lag.}$$

Example 10.4

A synchronous motor having a power consumption of 50 kW is connected in parallel with a load of 200 kW having a power factor of 0.8 lag. If the combined load has a power factor of 0.9, what is the value of leading reactive kVA supplied by the motor and at what power factor is it working.

Solution:

Let, power factor angle of motor = ϕ_1

Power factor angle of load = ϕ_2 = $\cos^{-1}(0.8)$ = 36.87°

Combined power factor angle (both motor and load),ϕ = $\cos^{-1}(0.9)$ = 25.84°

$\tan\phi_2$ = tan36°87' = 0.75; $\tan\phi$ = tan25°84' = 0.4842

Combined power, P = 200 + 50 =250 kW

Total kVAr of combined system =$P \tan\phi$ = 250 × 0.4842 = 121.05

Load kVAr =200 × $\tan\phi_2$ =200 × 0.75 =150

∴ Leading kVAr supplied by synchronous motor = 150 – 121.05 = 28.95

Power factor angle at which the motor is working,

$$\phi_1 = \tan^{-1}\frac{28.95}{50} = 30.07°$$

Power factor at which the motor is working = $\cos\phi_1$ = 0.865 (lead).

Example 10.5

A three phase, 5 kW induction motor has a power factor of 0.85 lag. A bank of capacitor is connected in delta across the supply terminal and power factor raised to 0.95 lag. Determine the kVAr rating of the capacitor in each phase.

Solution:

The active power of the induction motor, P = 5 kW

When the power factor is changed from 0.85 lag to 0.95 lag, by connecting a condenser bank.

The leading kVAr taken by the condenser bank

$$= P(\tan\phi_1 - \tan\phi_2)$$
$$= 5(0.6197 - 0.3287) = 1.455$$

∴ Rating of capacitor connected in each phase = $\frac{1.455}{3} = 0.485$ kVAr.

Example 10.6

A 400 V, 50 Hz, three-phase supply delivers 200 kW at a power factor of 0.7 lag. It is desired to bring the line power factor to 0.9 by installing shunt capacitors (Fig. 10.12). Calculate the capacitance if they are (a) star-connected (b) delta-connected.

Solution:

1. For star connection

   Phase voltage,

$$V_{ph} = \frac{400}{\sqrt{3}} = 230.94 \text{ V/phase}$$

Load current,

$$I = \frac{200\times10^3}{\sqrt{3}\times400\times0.7} = 412.39 \text{ A}$$

The active component of current,

$$I_a = I\cos\phi_1$$
$$= 412.39 \times 0.7 = 288.68 \text{ A}$$

Reactive component of current,

$$I_r = I\sin\theta_1 = \frac{I_a}{\cos\phi_1} \times \sin\phi_1 = I_a \tan\phi_1$$

For fixed load, let us take the reactive component of load current

$$I_{r1} = I_a \tan\phi_1$$

And the reactive component of load current with capacitor

$$I_{r2} = I_a \tan\phi_2$$

Current taken by the capacitor installed for improving power factor,

$$\begin{aligned} I_C &= I_a (\tan\phi_1 - \tan\phi_2) \\ &= 288.68(\tan(\cos^{-1} 0.7) - \tan(\cos^{-1} 0.9)) \\ &= 154.7 \text{ A} \end{aligned}$$

The value of capacitor to be connected,

$$\begin{aligned} C &= \frac{I_C}{2\pi f V_{ph}} \\ &= \frac{154.7}{2 \times \pi \times 50 \times 230.94} = 2132.26 \ \mu\text{F}. \end{aligned}$$

2. For delta connection

Phase voltage, $V_{ph} = V_L = 400$ V

Load current,

$$I = \frac{200 \times 10^3}{\sqrt{3} \times 400 \times 0.7} = 412.39 \text{ A}$$

Phase current,

$$I_{ph} = \frac{412.39}{\sqrt{3}} = 238.09 \text{ A}$$

The active component of phase current, $I_a = I\cos\phi_1 = 238.09 \times 0.7 = 166.663$ A

Current taken by the capacitor installed for improving pf,

$$\begin{aligned} I_C &= I_a(\tan\phi_1 - \tan\phi_2) \\ &= 166.663\ (\tan(\cos^{-1} 0.7) - \tan(\cos^{-1} 0.9)) \\ &= 89.312\text{ A} \end{aligned}$$

The value of capacitor to be connected,

$$\begin{aligned} C &= \frac{I_C}{2\pi f V_L} \\ &= \frac{89.312}{2\times\pi\times 50\times 400} = 710.7\ \mu\text{F}. \end{aligned}$$

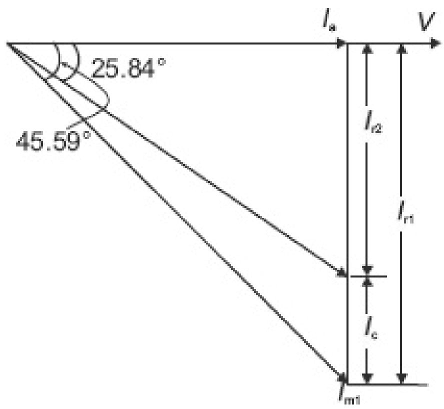

Fig. 10.12 Phasor diagram for Example 10.6

Example 10.7

A three-phase 500 hp, 50 Hz, 11 kV star-connected induction motor has a full load efficiency of 85% at a 0.75 p.f. and is connected to a feeder. If the power factor of load is desired to be corrected to 0.9 lag, determine the

1. **size of the capacitor bank in kVAr**
2. **capacitance of each unit if the capacitors are connected in Δ as well as in Y.**

Solution:

Induction motor output = 500 h_p

Efficiency, η = 85%,

$$\eta = \frac{\text{output}}{\text{input}}$$

Input of the induction motor, $P = \frac{\text{Output}}{\eta} = \frac{500}{0.85} = 588.235 \text{ hp}$
$= 588.235 \times 746 = 438.82 \text{ kW}$

Initial power factor,(cos ϕ_1) = 0.75 ⇒ tanϕ_1 = 0.88

Corrected power factor,(cosϕ_2) = 0.9 ⇒ tanϕ_2 = 0.48

Leading kVAr taken by the capacitor bank,

$$Q_c = P(\tan\phi_1 - \tan\phi_2)$$
$$= 438.82\,(0.88 - 0.48) = 175.53 \text{ kVAr}$$

Line current drawn, $I_L = \frac{Q_c}{\sqrt{3}V_{L-L}} = \frac{175.53}{\sqrt{3} \times 11} = 9.213 \text{ A}.$

Case 1: Delta connection

Charging current per phase,

$$I_c = \frac{I_L}{\sqrt{3}} = 5.319 \text{ A}$$

Reactance of capacitor bank per phase

$$= \frac{V_{L-L}}{I_c} = \frac{11 \times 10^3}{5.319} = 2.068 \text{ K}\Omega$$

$$X_c = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi f X_c}$$

Capacitance of capacitor bank, $C = \frac{1}{2\pi \times 50 \times 2.068 \times 10^3} = 1.54 \text{ μF}$

Case 2: Star connection

$I_L = I_c = 9.213$ A

Reactance of capacitor bank per phase,

$$X_c = \frac{V_{L-N}}{I_c} = \frac{11 \times 10^3}{\sqrt{3} \times 9.213} = 0.689 \text{ K}\Omega$$

Capacitance of capacitor bank, $C = \frac{1}{2\pi f X_c} = 4.619\ \mu\text{F}.$

Example 10.8

A star-connected 400 hp, 2000 V, and 50 Hz motor works at a power factor of 0.7 lag. A bank of mesh- connected condensers is used to raise the power factor to 0.9 lag (Fig. 10.13). Calculate the capacitance of each unit and total number of units required, if each unit is rated 400 V, 50 Hz. The motor efficiency is 90%.

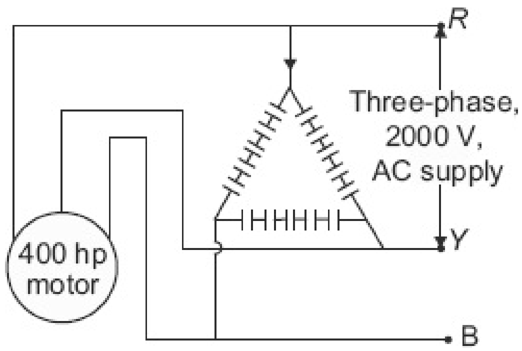

Fig. 10.13 Circuit diagram for Example 10.8

Solution:

Motor current = $400h_p$

Supply voltage = 2000 V

i.e., V = 2000 V

Power factor without condenser = 0.7 lag

Efficiency of motor, η = 0.9

$$\therefore \text{Motor current} = \frac{400 \times 746}{\sqrt{3} \times 2000 \times 0.7 \times 0.9}$$
$$= 136.73 \text{ A} \qquad (\because 1 \text{ hp} = 746 \text{ W})$$

For fixed loads, the active component of current is same for improved power factor, whereas the reactive component will be changed.

∴ The active component of current at 0.7 power factor lagging,

$$I_{a1} = I\cos\phi_1$$
$$= 136.73 \times 0.7 = 95.71 \text{ A}$$

Line current taken by the capacitor installed for improving power factor, $I_c = I_a(\tan\phi_1 - \tan\phi_2)$

$$= 95.71\ (\tan(\cos^{-1} 0.7) - \tan(\cos^{-1} 0.9))$$
$$= 51.28 \text{ A}$$

The bank of condensers used to improve the power factor is connected in delta. The voltage across each phase is 2000 V, but each unit of condenser bank is of 400 V. Therefore, each phase of the bank will have five condensers connected in series as shown in Fig. 10.13.

The current in each phase of the bank $= \frac{51.28}{\sqrt{3}} = 29.61 \text{ A}$

Let X_c be the reactance of each condenser

Then the charge current,

$$I_c = \frac{400}{X_c} = 29.61 \text{ A}$$

$$\text{or } X_c = \frac{400}{29.61} = 13.51\ \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 13.51} = 235.61\ \mu\text{F}$$

Capacitance of each phase of the bank $= \frac{235.61}{5} = 47.12\ \mu\text{F}.$

Example 10.9

A three-phase, 50 Hz, 2500 V motor develops 600 hp, the power factor being 0.8 lag and the efficiency 0.9. A capacitor bank is connected in delta across the supply terminals and the power factor is raised to unity (Fig. 10.14). Each of the capacitance units is built of five similar 500 V capacitors. Determine the capacitance of each capacitor.

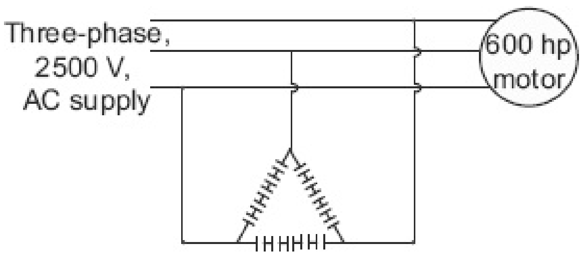

Fig. 10.14 Circuit diagram for Example 10.9

Solution:

Motor input(P)

$$= \frac{\text{output}}{\eta}$$

$$= \frac{600 \times 746}{0.9}$$

$$= 497.33\ \text{kW}$$

Leading kVAr supplied by the capacitor bank

$$= P(\tan\phi_1 - \tan\phi_2)$$
$$= 497.33\ (0.75 - 0)$$
$$= 373 \text{ kVAr}$$

Leading kVAr supplied by each sets $= \dfrac{373}{3}$
$= 124.33$ kVAr

Current per phase of capacitor bank,

$$I_c = \frac{V_{Ph}}{X_c}$$
$$= 2\pi f C V_{ph}$$
$$= 2\pi \times 50C \times 2500 \text{ A} \qquad (\because \text{For delta } V_L = V_{ph})$$

kVAr required per phase

$$= \frac{V_{Ph} I_c}{1000}$$
$$= \frac{2500 \times 2\pi \times 50 \times C \times 2500}{1000}$$
$$= 1963495.41C$$

But leading kVAr supplied by each phase = 124.33 kVAr

$$\therefore 1963495.41C = 124.33$$
$$C = \frac{124.33}{1963495.41} = 63.32\ \mu\text{F}$$

Since it is the combined capacitance of five equal capacitors joined in series.

The capacitance of each unit

$$= 5 \times 63.32\,\mu F$$
$$= 316.6\,\mu F.$$

Example 10.10

A three-phase, 50 Hz, 30 km transmission line supplies a load of 5 MW at a power factor of 0.7 lag to the receiving end where the voltage is maintained constant at 11 kV. The line resistance and inductance are 0.02 Ω and 0.84 mH per phase per kilometre, respectively. A capacitor is connected across the load to raise the power factor to 0.9 lag. Calculate (a) the value of the capacitance per phase, and (b) the voltage regulation.

Solution:

Length of line =30 km

Frequency =50 Hz

Load =5 MW at p.f.0.7 lag

Receiving-end voltage, V_r = 11 kV

Line resistance per phase =0.02 Ω/km = 0.02 × 30 = 0.6 Ω

Reactance of 30 km length per phase,

$$X = 2 \times \pi \times f \times L \times 30$$
$$= 2 \times \pi \times 50 \times 0.84 \times 10^{-3} \times 30 = 7.92\,\Omega$$

For fixed loads, the active component of power is same for improved power factor, whereas the reactive component of power will be changed.

∴ The active component of current at 0.7 p.f. lagging, *P* =5 MW.

Reactive power (MVAr) supplied by the capacitor bank

$$= P(\tan\phi_1 - \tan\phi_2)$$
$$= 5\,(1.02 - 0.484) = 2.679 \text{ MVAr}$$

Reactive power (MVAr) supplied by the capacitor bank

$$= \frac{2.679}{3}\text{MVAr}$$
$$= 0.893 \text{ MVAr} = 893 \text{ kVAr}$$

Let C be the capacitance to be connected per phase across the load.

kVAr required per phase
$$= \frac{V_{Ph} I_C}{1000} = \frac{V_{Ph}(V_{Ph}\omega C)}{1000}$$
$$= \frac{11000 \times 2\pi \times 50 \times C \times 11000}{1000}$$
$$= 38013271.11C$$

But leading kVAr supplied by each phase = 893 kVAr

$$\therefore 38013271.11C = 893$$
$$C = \frac{893}{38013271.11} = 23.49 \text{ µF}$$

Sending-end voltage with improved p.f. = $V_r + I(R\cos\phi_2 + jX\sin\phi_2)$

Receiving-end voltage, $V_r = 11 \text{ kV(L−L)} = \frac{11}{\sqrt{3}} = 6350.85 \text{ V}$

Current with improved power factor, $I = \frac{5 \times 10^6}{\sqrt{3} \times 11 \times 10^3 \times 0.9} = 291.59 \text{ A.}$

Sending-end voltage,
$$V_s = 6350.85 + 291.59\,(0.6 \times 0.9 + j7.29 \times 0.436)$$
$$= 6350.85 + 157.46 + j926.8$$
$$= 6508.31 + j926.8$$
$$= 6573.968\angle 8.1° \text{ V/ph}$$
$$= 11386.45 \text{ V(L–L)}$$

$$\% \text{ Regulation} = \frac{11386.45 - 11000}{11000} \times 100 = 3.5\%.$$

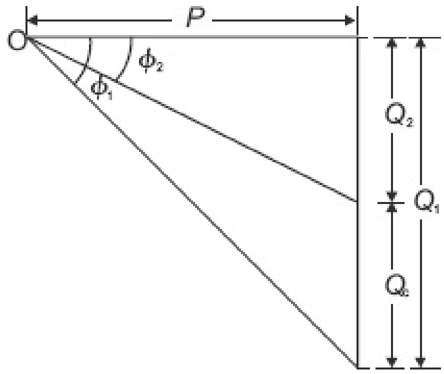

Fig. 10.15 Phasor diagram for Example 10.10

Example 10.11

A synchronous motor improves the power factor of a load of 200 kW from 0.7 lag to 0.9 lag and at the same time carries an additional load of 100 kW. Find (i) the leading kVAr supplied by the motor (ii) kVA rating of motor (iii) power factor at which the motor operates.

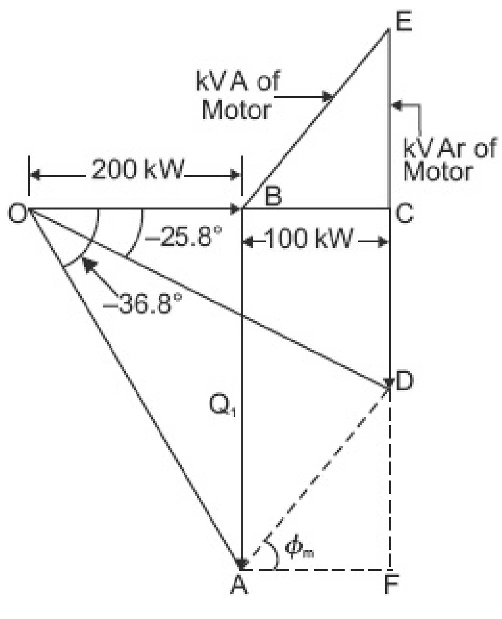

Fig. 10.16 Phasor diagram for Example 10.11

Solution:

Load, P_1 = 200 kW

Additional motor load, P_2 = 100 kW

Power factor of the load 200 kW = 0.7 lag

Power factor of the combined load (200 + 100) kW = 0.9 lag

Combined load =P_1 + P_2 =200 +100 =300 kW

Δ*OAB* is a power triangle without additional load

Δ*ODC* is the power triangle for combined load

and Δ*BEC* for the motor load

From Fig. 10.16

1. Leading kVAr taken by the motor =*CE*

$$\begin{aligned} &= CF - DC = AB - DC \\ &= 200 \tan(\cos^{-1} 0.7) - 300 \tan(\cos^{-1} 0.9) \\ &= 200 \times 1.02 - 300 \times 0.4843 \\ &= 58.71 \text{ KVAr} \end{aligned}$$

2. kVA rating of motor =*BE*

$$\begin{aligned} &= \sqrt{(BC)^2 + (EC)^2} \\ &= \sqrt{(100)^2 + (58.71)^2} \\ &= 115.96 \text{ kVA} \end{aligned}$$

3. Power factor of motor at which it operates, $\cos\phi_m = \frac{BC}{BE}$

$$\begin{aligned} &= \frac{100}{115.96} \\ &= 0.862 \text{ lead.} \end{aligned}$$

Example 10.12

A 37.3 kW induction motor has pf 0.9 and efficiency 0.9 at full load and p.f. 0.6 and efficiency 0.7 at half-load. At no-load, the current is 25% of the full-load current and pf 0.1. Capacitors are supplied to make the line pf 0.8 at half-load. With these capacitors in circuit, find the line power factor at (i) full load, and (ii) no-load.

Solution:

Full-load current,

$$I_1 = \frac{37.3 \times 10^3}{\sqrt{3} \times V_L \times 0.9 \times 0.9} = \frac{26,586}{V_L}$$

At full-load

Motor input, $P_1 = \frac{37.3}{0.9} = 41.44$ kW

Lagging kVAr drawn by the motor, $kVAr_1 = P_1 \tan\phi_1 = 41.44 \tan(\cos^{-1} 0.9) = 20.07$

At half-load

Motor input, $P_2 = \frac{0.5 \times 37.3}{0.7} = 26.64$ kW

Lagging kVAr drawn by the motor, $kVAr_2 = P_2 \tan\phi_2 = 26.64 \tan(\cos^{-1} 0.6) = 35.52$

At no-load

No load current, I_0 = 0.25 (full load current)

$$= 0.25 \times \frac{26,586}{V_L} = \frac{6646.5}{V_L}$$

Motor input at no-load,
$$P_0 = \sqrt{3} V_L I_0 \cos\phi_0$$
$$= \sqrt{3} \times \frac{6646.5}{V_L} \times V_L \times 0.1 = 1.151 \text{ kW}$$

Lagging kVAr drawn by the motor, $kVAr_0$

$$kVAr_0 = 1.151 \tan(\cos^{-1} 0.1)$$
$$= 11.452$$

Lagging kVAr drawn from the mains at half-load with capacitors,

$$kVAr_{2c} = 26.64 \tan(\cos^{-1} 0.8) = 19.98$$

kVAr supplied by capacitors, $kVAr_c = kVAr_2 - kVAr_{2c}$ = 35.52 – 19.98 = 15.54

kVAr drawn from the main at full-load with capacitors

$$kVAr_{1c} = kVAr_1 - kVAr_c$$
$$= 20.07 - 15.54 = 4.53$$

1. Line power factor at full-load

$$= \cos\left(\tan^{-1}\frac{kVAr_{1c}}{P_1}\right) = \cos\left(\tan^{-1}\frac{4.53}{41.44}\right)$$
$$= 0.994 \text{ lag}$$

2. kVAr drawn from mains at no-load with capacitors = 11.452 – 15.54 = –4.088

   Line power factor at no-load

$$= \cos\left(\tan^{-1}\frac{-4.088}{1.151}\right) = \cos(-74.27°) = 0.271 \text{ lead.}$$

Example 10.13

A single-phase system supplies the following loads

1. **Lighting load of 50 kW at unity power factor**
2. **Induction motor load of 125 kW at p.f. 0.707 lag**
3. **Synchronous motor load of 75 kW at p.f. 0.9 lead**
4. **Other miscellaneous loads of 25 kW at p.f. 0.8 lag**

Determine the total kW and kVA delivered by the system and power factor at which it works.

Solution:

Total kW of the load = 50 + 125 + 75 + 25 = 275 kW

kVAr of lighting load = 50 × 0 =0

kVAr of induction motor = $-125 \tan(\cos^{-1} 0.707)$ = –125.04

kVAr of synchronous motor = $75 \tan(\cos^{-1} 0.9)$ = 36.32

kVAr of miscellaneous loads = $25 \tan(\cos^{-1} 0.8)$ = –18.75

$\therefore$ Total kVAr of the load = 0 – 125.04 + 36.32 – 18.75 = –107.47

$$\text{Total kVA load} = \sqrt{(\text{kW})^2 + (\text{kVAr})^2} = \sqrt{(275)^2 + (107.47)^2} = 295.25$$

$$\text{Power factor} = \frac{\text{Total kW}}{\text{Total kVA}} = \frac{275}{295.25} = 0.93 \text{ lag.}$$

10.5 MOST ECONOMICAL POWER FACTOR WHEN THE KILOWATT DEMAND IS CONSTANT

For improving the power factor of load at the consumer end, the consumer must provide the equipment required for the purpose. Therefore, there is a capital investment necessary for the required correction equipment. At the same time, there is a saved amount due to the reduced demand in kVA. Therefore, the net annual savings is equal to the difference between the annual saving in maximum demand charges and the annual expenditure incurred on power factor correction equipment.

The value of a power factor at which the net annual saving is maximum is known as the most economical power factor.

Consider a peak load of PkW, which is taken by the consumer at a power factor of $\cos\phi_1$ and charge on maximum demand be Rs. X per kVA per annum.

Then the reactive component of load, $Q = P \tan\phi_1$

And kVA demand of load, $S = \dfrac{P}{\cos\phi_1}$

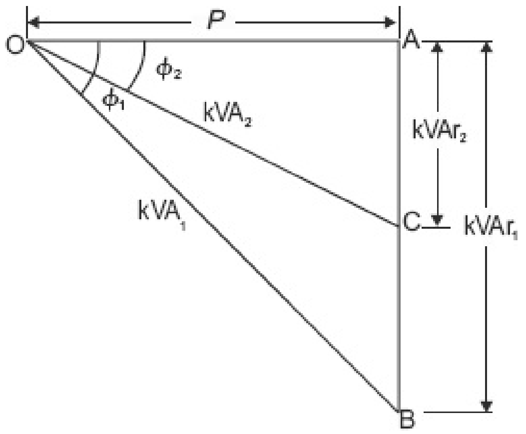

Fig. 10.17 Phasor diagram for constant kW demand

If the consumer improves the power factor to cosϕ_2 by installing power factor correction equipment. Let the expenditure incurred on the power factor correction equipment be Rs. Y per kVAr per annum. The power triangle at the original power factor cosϕ_1 is OAB and for the improved power factor cosϕ_2, it is OAC (are shown in Fig. 10.17).

kVA maximum demand at cosϕ_1, $\text{kVA}_1 = \dfrac{P}{\cos\phi_1}$

kVA maximum demand at cosϕ_2, $\text{kVA}_2 = \dfrac{P}{\cos\phi_2}$.

Because of the improvement in the power factor, the kVA maximum demand is reduced from $kVA_1$to kVA_2(since the real power remains unchanged).

Annual saving in maximum demand

$$= \text{Rs. } X(\text{kVA}_1 - \text{kVA}_2)$$

$$= \text{Rs. } X\left(\frac{P}{\cos\phi_1} - \frac{P}{\cos\phi_2}\right)$$

$$= \text{Rs. } XP\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right)$$

Reactive power at $\cos\phi_1$, $\text{kVAr}_1 = P\tan\phi_1$

Reactive power at $\cos\phi_2$, $\text{kVAr}_2 = P\tan\phi_2$

Leading kVAr supplied by power factor correction equipment = $P(\tan\phi_1 - \tan\phi_2)$

So annual charges towards phase advancing plant = Rs. $YP(\tan\phi_1 - \tan\phi_2)$

Net annual saving towards phase advancing circuit,

$$S = XP\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right) - YP(\tan\phi_1 - \tan\phi_2)$$

In this expression, only ϕ_2 is variable while all other quantities are fixed.

For maximum net annual saving,

$$\frac{d}{d\phi_2}(S) = 0$$

$$\frac{d}{d\phi_2}\left[XP(\sec\phi_1 - \sec\phi_2) - YP(\tan\phi_1 - \tan\phi_2)\right] = 0$$

$$\frac{d}{d\phi_2}(XP\sec\phi_1) - \frac{d}{d\phi_2}(XP\sec\phi_2) - \frac{d}{d\phi_2}(YP\tan\phi_1) + \frac{d}{d\phi_2}(YP\tan\phi_2) = 0$$

$$0 - XP\sec\phi_2\tan\phi_2 - 0 + YP\sec^2\phi_2 = 0$$

$$\text{or } -XP\tan\phi_2 + YP\sec\phi_2 = 0$$

$$\tan\phi_2 = \frac{Y}{X}\sec\phi_2$$

$$\sin\phi_2 = \frac{Y}{X}$$

Most economical power factor,

$$\cos\phi = \sqrt{1-\sin^2\phi_2}$$

$$= \sqrt{1-\left(\frac{Y}{X}\right)^2}$$

$$\text{Or } \cos\phi = \cos\left(\sin^{-1}\frac{Y}{X}\right).$$

10.6 MOST ECONOMICAL POWER FACTOR WHEN THE KVA MAXIMUM DEMAND IS CONSTANT

The necessity of calculating the most economical power factor when the kVA maximum demand is constant arises when power-supply agencies try to improve the power factor so that the kVA maximum demand on the station is reduced. Since the cost of the plant is proportional to the kVA installed, an improvement in the power factor reduces the cost of the plant. Further, the revenue returns are the function of the active power supplied.

The phasor diagram is shown in Fig. 10.18. The kVA output is constant and equal to *S*. The power factor is improved from $\cos\phi_1$to $\cos\phi_2$ due to the addition of *Q* kVAr leading. Consequently, the real power is increased from P_1 to P_2.

Let the annual interest and depreciation charges for a capacitor = *X*Rs./kVAr

Let the net return per kW of installed capacity per year= Rs. Y

From phasor diagram, leading kVAr supplied by the power factor improvement equipment is $Q = S(\sin\phi_1 - \sin\phi_2)$

∴ Annual charge on capacitor installation= Rs.

$XS(\sin\phi_1 - \sin\phi_2$.

Annual increase in revenue return because of increase in the real power

$$\begin{aligned} &= \text{Rs.} Y(P_2 - P_1) \\ &= Y(S\cos\phi_2 - S\cos\phi_1) \\ &= YS(\cos\phi_2 - \cos\phi_1) \end{aligned}$$

Net saving= $YS(\cos\phi_2 - \cos\phi_2) - XS(\sin\phi_1 - \sin\phi_2)$.

In this expression, only $\phi_2$is variable while all other quantities are fixed.

For maximum net annual saving,

$$\begin{aligned} &\frac{d}{d\phi_2}(\text{Net saving}) = 0 \\ &\frac{d}{d\phi_2}[YS(\cos\phi_2 - \cos\phi_1) - XS(\sin\phi_1 - \sin\phi_2)] \\ &\text{or } YS(-\sin\phi_2 - 0) - XS(0 - \cos\phi_2) = 0 \\ &\text{or } YS(-\sin\phi_2) = XS(-\cos\phi_2) \\ &\text{or } \tan\phi_2 = \frac{X}{Y}. \end{aligned}$$

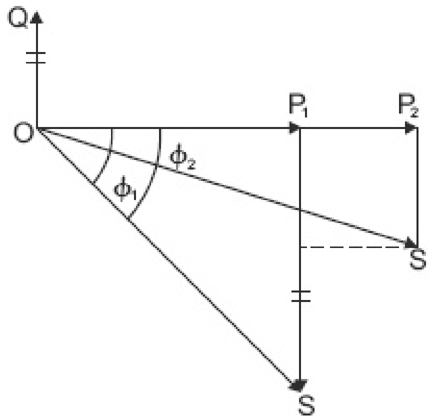

Fig. 10.18 Phasor diagram for constant kVA maximum demand

Most economical power factor, when the kVA

maximum demand is a constant $\cos\phi = \cos(\tan^{-1}\frac{X}{Y})$

It may be noted that the most economical power factor ($\cos\phi$) depends upon the relative costs of supply and power factor correction equipment but is independent of the original power factor $\cos\phi_1$.

Following are the results observed after the power factor improvement:

1. The circuit current is reduced from I to I ‘ after power factor correction current I_c.
2. The active or wattful component of current remains the same before and after power factor correction because, only the reactive component of current is reduced by the addition of a cpacitor.

$$I \cos\phi_1 = I' \cos\phi$$

3. The lagging reactive component of current is reduced after power factor improvement and is equal to the difference between lagging reactive component of load and capacitor current

$$I \sin\phi = I' \sin\phi_1; I_c$$

4. As $I\cos\phi_1 = I' \cos\phi$

$$\text{and } VI\cos\phi_1 = VI'\cos\phi$$

Therefore, active power (kW) remains unchanged due to power factor improvement.

5. As $I'\sin\phi = I\sin\phi_1 - I_c$

$$VI'\sin\phi = VI\sin\phi_1 - VI_c$$

Therefore,

Net kVAr after power factor correction = Lagging kVAr before power factor correction – Leading kVAr of equipment

Test Yourself

1. Does the system operate at unity power factor? If yes, justify.

Example 10.14

A consumer is charged at the rate of Rs. 75 per annum per kVA of maximum demand plus a flat rate per kWh. The phase advancing unit can be purchased at a rate of Rs. 70 per kVA. The rate of interest and depreciation on the capital is 12.5%. Find the most economical power factor to which it can be improved.

Solution:

Annual charges towards interest and depreciation of the phase advancing equipment

$$= \text{Rs. } 70 \times \frac{12.5}{100} = \text{Rs. } 8.75$$

Annual charge per kVA = Rs.75.00

Let ϕ_2 be the angle corresponding to the most economical power factor

$$\therefore \sin\phi_2 = \frac{8.75}{75} = 0.1166$$

The most economical power factor to which it can be improved is

$$\cos\phi_2 = \cos(\sin^{-1} 0.1166) = 0.9932 \text{ lag.}$$

Example 10.15

A generating station is working at its maximum rated capacity and is supplying the load at a power factor of 0.707 lag. An increase in demand of load can be met

1. **By setting up an extra generating plant, transmission lines etc., to meet the extra demand, or**
2. **By increasing the power factor of the system to 0.9 lag**

Determine the limiting cost per kVAr on the phase advancing plant which would justify its use. The cost of the generation plant per kVA is Rs. 150 per annum. Rate of interest and depreciation are 12%.

Solution:

Let the power supplied by the system be *S* kVA at a lagging power factor of 0.707

Load supplied = $S \times 0.707$kW

When the power factor is improved to 0.9 lag, the new load to be supplied

=$S \times 0.9$ kW (since kVA remaining constant)

∴ Extra power supplied =0.9S – 0.707S = 0.193S

For improving the power factor the reactive power is to be neutralized (partly)

Now, the reactive power at a power factor of 0.707 corresponding to a load of 0.9*S*

$$= \frac{0.9S}{0.707} \times \sin\phi_1 = \frac{0.9S}{0.707} \times \sin(\cos^{-1} 0.707) = \frac{0.9S}{0.707} \times 0.707 = 0.9S$$

The reactive power at p.f. 0.9 = $S \sin\phi_2 = S \sin(\cos^{-1} 0.9) = 0.4359S$

∴ The reactive power to be neutralized $= 0.9S - 0.4359S = 0.4641S$ kVAr

∴ Capacity of phase advancing plant $= 0.4641S$ kVAr

Let the annual cost of the phase advancing plant be Rs. X/kVAr

$$\therefore \text{Total cost of the phase advancing plant} = 0.4641SX \qquad (1)$$

Now, let the power factor remain the same at 0.707, but the extra load be supplied by installing an additional generating unit.

New power = $0.9S$

$$\text{New kVA} = \frac{0.9S}{0.707} = 1.273S$$

∴ Capacity of extra generating plant = $1.273S - S = 0.273S$

Annual rate of interest and depreciation = 12%

$$\therefore \text{Annual cost of generating plant} = 150 \times \frac{12}{100} \times 0.273S = 4.914S \qquad (2)$$

The limiting cost per kVA of the phase advancing plant can be determined from (Eqs. 1) and (2),

i.e., $0.4641SX = 4.914S$

$$\therefore X = \frac{4.914}{0.4641} = 10.588$$

For the same rate of interest and depreciation, the capital cost of the phase advancing

$$\text{Plant} = \text{Rs. } X \times \frac{100}{12} = \text{Rs. } 10.588 \times \frac{100}{12} = \text{Rs. } 88.233$$

∴ The maximum permissible cost of the phase advancing equipment = Rs.88.235/kVAr.

Example 10.16

A single-phase load of 15 MW at 66 kV and a power factor of 0.6 lag is received from a generating station, which is 100 km away. The power factor of the system is increased to 0.9 lag with the help of a phase advancing plant which costs Rs. 100 per kVA and increases the loss by 8% of its kVA rating. The cost of energy is 10 paise per unit. The rate of interest and depreciation is 10%. Assume a load factor of 50% and 0.20Ω/km is the resistance of the line (per conductor). Calculate the net saving.

Solution:

From Fig. 10.19

OA represents the load on plant, which is equal to 15MW. Its kVA rating at 0.6p.f. lag is represented by *OB*.

Load in kVA, $S_1 = \frac{15}{0.6} \times 10^3 = 25000 \text{ kVA} = OB$

New p.f. = 0.9 lag

Let *X* be the kVA rating of the phase advancing plant and is represented by *AD*. The increase in loss is 8% of its kVA rating of phase advancing plant i.e., equal to 0.08XkW and is represented by *AC*. Hence new load kVA at improved power factor is, S_2 and is represented by OD^1.

$\angle OBD'$ = 53.13° – 25.84° = 27.29°.

In $\Delta D'BC'$,

$$\cos\theta = \frac{BC'}{BD'} = \frac{0.08X}{X} = 0.08$$

or θ = 85.41°

$\angle BD'C'$ = 90° – 85.41° = 4.59°

∠ABO = 90° – 53.13° = 36.87°

In $\theta OBD'$, $\angle OD'B$ = 180° – (27.29° + 36.87° + 4.58°) = 111.26°

Hence,

$$\frac{BD'}{\sin 27.29°} = \frac{OB}{\sin 111.26°};$$
$$\frac{X}{0.4585} = \frac{25000}{0.9319}$$
$$X = 12300.14 \text{ kVA.}$$

∴ Loss in phase advancing plants =12300.14 × 0.08

=948.0 kW.

So new power taken from supply =15000 +984.0

=15984 kW

Line loop resistance =2 × 0.20 × 100 = 40Ω

Load current without phase advancer $= \frac{15000}{66 \times 0.6} = 378.79 \text{ A}$

Load current after the improvement of p.f. $= \frac{15984.0}{66 \times 0.9} = 269.1 \text{ A}$

Reduced power loss

$$= 378.79^2 \times 40 - 269.1^2 \times 40$$
$$= 40(378.78^2 - 269.08^2)$$
$$= 2842.4 \text{ kW}$$

Amount saved in line loss annually

$$= \text{Rs.} \frac{2842.4 \times 8760 \times 0.5}{100} \times 10$$
$$= \text{Rs. } 12,44,971.20$$

Amount spent annually to cover the loss in phase advancing plant

$$= \text{Rs. } \frac{984 \times 8760 \times 0.5}{100} \times 10$$
$$= \text{Rs. } 4,30,992$$

Cost of phase advancing plant = Rs.12,300.14 × 100 = Rs.12,30,014

Annual interest and depreciation of phase advancing plant

$$= \text{Rs. } 12,30,014 \times \frac{10}{100} = \text{Rs. } 1,23,001.4$$

Hence net saving = Rs(12,44,971.20 – 4,30,992 – 1,23,001.40)

= Rs. 6,90,977.80.

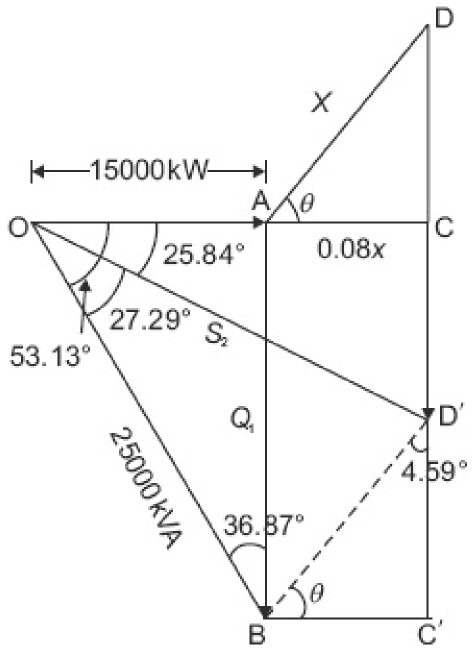

Fig. 10.19 Phasor diagram for Example 10.16

CHAPTER AT A GLANCE

1. The power factor is the ratio of kW component to the kVA.
2. Disadvantages of low power factor: If the power station works at low power factor, the capital cost of generation, transmission and distribution systems is increased.
3. Advantages of improved power factor: It improves the kW capacity of the prime movers, alternators, transformers, and lines and in voltage regulation.
4. Methods for the improvement of power factor comprise the use of static capacitors, synchronous condensers, and phase advancers.

5. **Shunt capacitor:** A shunt capacitor is connected across an inductive load to supply the reactive power of the current to reduce the out-of-phase component of current required by an inductive load.
6. **Synchronous condenser:** The synchronous condenser is a synchronous motor running without a mechanical load. A synchronous motor takes a leading current when over-excited and, therefore behaves as a capacitor.
7. **Phase advancers:** There are special commutator machines, which improve the power factor of the induction motor.
8. The value to which the power factor should be improved to have maximum net annual saving is known as the most economical power factor.

SHORT ANSWER QUESTIONS

1. Define power factor.
2. What are the disadvantages of low power factor?
3. What are the causes of low power factor of supply system?
4. What are the different methods for power factor improvement?
5. Define the most economical value of power factor.
6. What are the advantages of using a static capacitor?
7. What are the disadvantages of using a static capacitor?
8. What are the advantages of using a synchronous condenser?
9. What are the disadvantages of using a synchronous con denser?
10. What are the advantages of phase advancers?
11. What is the effect of the power factor on the real power output of an alternator?
12. How is the capital cost of a transmission line affected by a low power factor?
13. How is the power factor of an industrial consumer improved?
14. How is the power factor of a power system improved?
15. Why are static capacitors preferred for power factor improvement at higher frequencies?
16. Why is a delta-connected bank of capacitors preferred to a star-connected bank?
17. In what way does the presence of a synchronous condenser affect the rating of a circuit breaker?
18. Why is it not economical for the power factor to be raised to unity?
19. What is the condition for the power factor to be raised to unity?
20. What is the basis of determining the most economical power factor?

MULTIPLE CHOICE QUESTIONS

1. The major reason for low lagging power factor of supply system is due to the use of ____________ motors.
   1. induction
   2. synchronous
   3. DC
   4. AC
2. The maximum value of power factor can be ____________.
   1. 1
   2. 0.9
   3. 0.8
   4. 0.7
3. By improving the power factor of the system, the kilowatts delivered by generating stations are ____________.
   1. decreased
   2. increased
   3. not changed
   4. none of these
4. The most economical power factor of a consumer is generally ____________.
   1. 0.95
   2. unity
   3. 0.6 leading
   4. 0.6 lagging
5. Power factor can be improved by installing such a device in parallel with load, which takes
   1. lagging reactive power
   2. leading reactive power
   3. apparent power
   4. real power
6. The main reason for low power factor of supply system is due to the use of ____________.
   1. resistive load
   2. inductive load
   3. synchronous motor
   4. all of these
7. The only motor that can also be worked at leading power factor and can supply mechanical power ____________.
   1. synchronous induction generator
   2. synchronous motor
   3. alternator
   4. induction motor
8. An over-excited synchronous motor on no load is known as ____________.
   1. synchronous induction generator
   2. synchronous condenser
   3. alternator
   4. induction motor
9. For synchronous condensers the power factor improvement apparatus should be located at ____________.
   1. sending end

2. receiving end
3. middle
4. none of these

10. A disadvantage of synchronous condenser
    1. continues losses in motor
    2. high maintenance cost
    3. noisy
    4. all of the above
11. The smaller the lagging reactive power drawn by a circuit, its power factor will be _____________.
    1. better
    2. poorer
    3. unity
    4. none of these
12. kVAr is equal to _____________.
    1. kW tanϕ
    2. kW sinϕ
    3. kVA cosϕ
    4. kW secϕ
13. For a particular power, the current drawn by the circuit is minimum when the value of power factor is _____________.
    1. 0.8 lagging
    2. 0.8 leading
    3. unity
    4. none of these
14. Synchronous capacitors are normally _____________ cooled.
    1. air
    2. oil
    3. water
    4. paper
15. To improve the power factor of three-phase circuits, the size of each capacitor when connected in delta with respect to when connected in star is _____________.
    1. $\frac{1}{6^{th}}$
    2. $\frac{1}{4^{th}}$
    3. 3 times
    4. $\frac{1}{3^{rd}}$
16. The power factor improvement equipment is always placed _____________.
    1. at the generating station
    2. near transformer
    3. near the apparatus responsible for low power factor
    4. near the bus bar

17. A synchronous machine has higher capacity for
    1. leading power factor
    2. lagging power factor
    3. it does not depend upon the power factor of the machine
    4. none of these
18. If a synchronous machine is under excited, it takes lagging VAr's from the system when it is operated as a _____________.
    1. synchronous motor
    2. synchronous generator
    3. synchronous motor as well as generator
    4. none of these
19. A synchronous phase modifier as compared to synchronous motor used for mechanical loads has _____________.
    1. larger shaft and higher speed
    2. smaller shaft and higher speed
    3. larger shaft and smaller speed
    4. smaller shaft and smaller speed
20. The phase advancer is mounted on the main motor shaft and is connected in the _____________ motor.
    1. rotor
    2. stator
    3. core
    4. none of these
21. Industrial heating furnaces such as arc and induction furnaces operate on _____________.
    1. very low lagging power factor
    2. very low leading power factor
    3. very high leading power factor
    4. none of these
22. If a synchronous machine is over-excited. It takes lagging VAr's from the system when it is operated as
    1. synchronous motor
    2. synchronous generator
    3. synchronous motor as well as generator
    4. induction generator
23. The phase advancers are used to improve the power factor of _____________.
    1. induction motors
    2. synchronous motors
    3. induction generators
    4. none of these
24. A machine designed to operate at full load is physically heavier and is costlier if the operating power factor is
    1. lagging
    2. leading
    3. the size and cost do not depend on power factor
    4. none of these

Answers:

1. a,	2. a,	3. b,	4. a,	5. b,
6. b,	7. b,	8. b,	9. b,	10. d,
11. a,	12. a,	13. c,	14. a,	15. d,
16. c,	17. b,	18. c,	19. a,	20. b,
21. a,	22. a,	23. d,	24. a,	25. a.

REVIEW QUESTIONS

1. Explain the concept of power factor.
2. Why is the improvement of power factor equally important for both consumers and generating stations? List the various causes of low power factor and explain.
3. List and explain any four disadvantages of low power factor.
4. What is the effect of low power factor in a system?
5. Explain any three causes of low power factor of supply system.
6. What is a "static capacitor"? Clearly explain and derive the expression for the capacity of phase modifier to improve power factor of a system.
7. Explain the method of improving power factor by using synchronous condenser. Discuss the merits and demerits of the above method.
8. Explain the method of improving power factor by the method of using phase advancers.
9. Derive the equation for most economical power factor for a constant kVA type loads.
10. Clearly explain the use of static capacitors for the improvement of power factor. Derive the expression for most economical power factor for constant kW loads.
11. Compare the merits and demerits of various methods of power factor improvement methods.

PROBLEMS

1. A single-phase motor takes a current of10 A at a power factor of 0.7 lag from a 230 V, 50 Hz supply. Determine shunting capacitor value to raise the power factor to 0.9 lag.
2. An alternator is supplying a load of 300 kW at a power factor of 0.75 lag. Calculate the extra kW supplied by the alternator if the power factor is raised to 0.95 lag, for the same kVA loading?
3. A single-phase motor connected to a 240 V, 50 Hz supply takes 20 A at a power factor of 0.7 lag. A capacitor is shunted across the motor terminals to improve the power factor to 0.85 lag. Determine the capacitance of the capacitor to be used.
4. A synchronous motor improves the power factor of a load of 180 kW from 0.85 lag to unity, simultaneously the motor carries a load of 60 kW. Determine
    1. The leading kVAr supplied by the motor.
    2. The power factor at which motor operates.

5. A three-phase, 5 kW induction motor has a power factor of 0.8 lag. A bank of capacitors is connected in delta across the supply terminals and power factor is raised to 0.95 lag. Determine the kVAr rating of the capacitors connected in each phase.
6. A star-connected 400 hp, 2000 V, 50 Hz motor works at a power factor of 0.75 lag. A bank of mesh- connected condensers is used to raise the power factor to 0.98 lag. Determine the capacitance of each unit and total number of units required; if each is rated 500 V, 50 Hz. The motor efficiency is 85%.
7. A three-phase, 50 Hz, 3000 V motor develops 600 hp, the power factor being 0.75 lag and the efficiency 0.95. A bank of capacitors is connected in delta across the supply terminals and the power factor is raised to 0.98 lag. Each of the capacitance units is built of five similar 600 V capacitors. Determine capacitance of each capacitor.

11

Voltage Control

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Obtain an overview of voltage control
- Discuss the parameters or equipments causing reactive power
- Understand the methods of voltage control
- Calculate the rating of synchronous phase modifier

11.1 INTRODUCTION

A power system must be designed in such a way as to maintain the voltage variations at the consumer terminals within the specified limits. In practice, all the equipments on the power system are designed to operate satisfactorily at the rated voltages or within specified limits, at most ±6% at the consumer terminals. The main reason for variation of voltage at the consumer terminals is the variation of load on the supply power system. In case load on the supply system increases, the voltage at the consumer terminals decreases due to increase in voltage drop in power system components and vice versa when load is decreased. Most of the electronic equipment is sensitive to voltage variations; hence the voltage must be maintained at a constant. It can be maintained within the limits by providing voltage control equipment.

11.2 NECESSITY OF VOLTAGE CONTROL

The voltage at the consumer terminals changes with the variation of load on the supply system, which are undesirable due to the following reasons:

1. In case of lighting load, such as incandescent lamp, which is acutely

sensitive to voltage changes, fluctuations in voltage beyond a certain level may even decrease the life of the lamp.

2. In case of power load consisting of induction motors, voltage variations may cause variation in the torque of an induction motor as the torque is proportional to the square of the terminal voltage. If the supply voltage is low, the starting torque of the motor will be too low.
3. If the voltage variation is more than a specified value, performance of the equipment suffers and the life of the equipment is reduced.
4. The picture on a television set starts rolling if the voltage is below a certain level because the fluorescent tube refuses to glow at low voltages. Hence, voltage variations must be regulated and kept to a minimum level.

Before discussing the various methods of voltage control, it is very important to know about the various sources and sinks of reactive power in a power system.

Test Yourself

1. Why is voltage tolerance more than the frequency tolerance?

11.3 GENERATION AND ABSORPTION OF REACTIVE POWER

1. Synchronous Machine These can be used to generate or absorb reactive power. The ability to supply reactive power is determined by the short circuit ratio

$\left(\frac{1}{\text{synchronous reactance}}\right)$. An over-excited synchronous

machine generates kVAr and acts as a shunt capacitor, while an under-excited synchronous machine absorbs it and acts as a shunt reactor. The machine is the main source of supply to the system of both positive and negative VAr's.

2. Overhead Lines When fully loaded, lines absorb reactive power. With a current *IA* for a line of reactance per phase X Ω, the VAr's absorbed are I^2X per phase. On light loads the shunt capacitances of longer lines may become predominant and the lines then become VAr generators.

3. Transformers Transformers absorb reactive power. The mathematical expression for the reactive power absorbed by a transformer is $Q_T = 3|I^2|X_T$ VAr

where, X_T is the transformer reactance per phase in ohms and $|I|$ is the current flowing through in amperes.

4. Cables They act as VAr generators because they have a very small inductance and relatively a very large capacitance due to the nearness of the conductors.

5. Loads A load at 0.8 pf implies a reactive power demand of 0.75 kVAr/kW of power, which is more significant than the simple quoting of the power factor. In planning power systems, it is required to consider reactive power requirements to ascertain whether the generator is able to operate any range of power factor.

Test Yourself

1. Explain why cables qualify as VAr generators.

11.4 LOCATION OF VOLTAGE CONTROL EQUIPMENT

The consumer apparatus should operate satisfactorily. This is achieved by installing voltage-control equipment at suitable places.

The voltage-control equipment is placed in two or more than two places in a power system because:

1. The power system is a combination of wide-ranging networks and there is a voltage drop in different sections of the transmission and distribution systems.
2. The various circuits of a power system have different load characteristics.

The voltage control equipment is placed at

1. Generating stations
2. Transformer stations
3. Feeders

When power is supplied to a load through a transmission line keeping the sending-end voltage constant, the receiving-end voltage varies with magnitude of load and power factor of load. The higher the load with smaller power factor the greater is the voltage variation.

11.5 METHODS OF VOLTAGE CONTROL

The different voltage control methods are:

1. Excitation control
2. Shunt capacitors
3. Series capacitors
4. Tap changing transformers
5. Boosters
6. Synchronous condensers

11.5.1 Excitation Control

This method is used only at the generating station. Due to voltage drop in the synchronous reactance of armature, the alternator terminal voltage changes and hence the load on the supply system also undergoes a change. This can be maintained at a constant by changing the field current of the alternator. This process is called excitation control. By using an automatic or a hand-operated regulator, the excitation of the alternator can be controlled.

In modern systems automatic regulator is preferred. The two main types of automatic voltage regulators are:

1. Tirril regulator
2. Brown-Boveri regulator

1. Tirril automatic regulator Tirril regulator is a fast-acting electromagnetic regulator and gives ± 0.5% regulating deviation between no load and full load of an alternator.

Construction Tirril voltage regulator is a vibrating-type voltage regulator in which a resistance R is connected in the exciter circuit to get the required value of voltage by adjusting the proper value of resistance. Figure 11.1 shows the main parts of the Tirril voltage regulator.

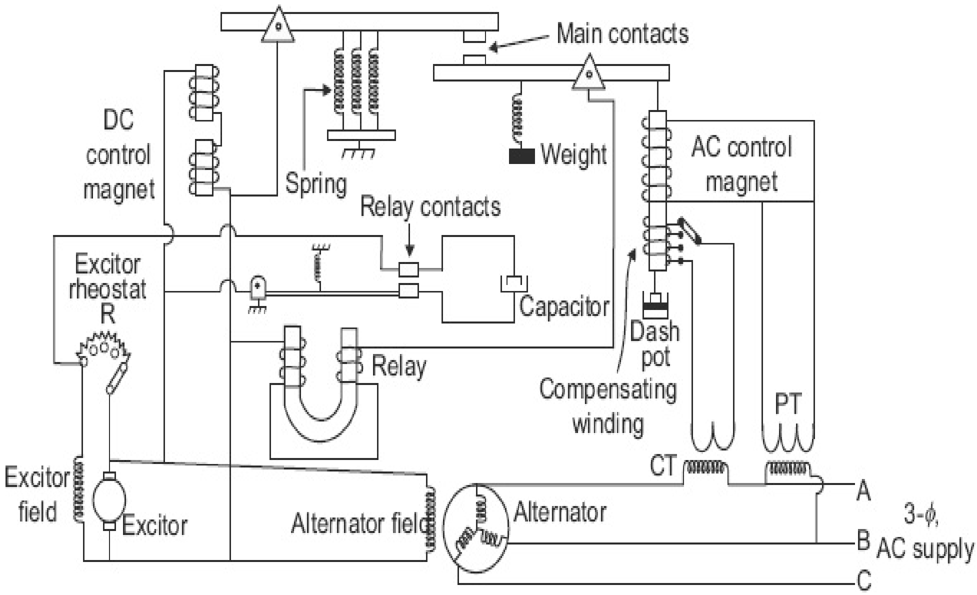

Fig. 11.1 Tirril automatic voltage regulator

Differential relay It is a 'U'-shaped (horse shoe) relay magnet. It has two identical windings on both limbs as shown in Fig. 11.1, which are connected across the armature of the exciter only when the main contacts are closed. A capacitor is connected parallel to the relay for reducing the spark when the relay contacts are opened.

Excitation system It consists of a solenoid energized by the voltage equal to the exciter terminal voltage. The counterbalance force of an excitation solenoid is provided by three springs which act in sequence and are shown in Fig. 11.1.

Main control unit It is a solenoid excited from an AC supply. The lower part of this solenoid is connected with dashpot which provides damping to the measuring unit.

Main contacts: These are attached to the levers which are operated by measuring and excitation solenoids as shown in Fig. 11.1. The lever on the left side is controlled by the DC control magnet and the lever on the right side is controlled by the AC control magnet.

Principle of operation Under normal operating conditions when the system is operating at pre-set load and voltage conditions, the main contacts are open. The field rheostat is in the circuit. If the load on the alternator increases, the terminal voltage decreases. When the pre-set excitation settings of the device is low, the m.m.f. developed by the measuring system or the solenoid is low, causing disturbance in the equilibrium and therefore main contacts are closed. These results in de-energization of differential relay and relay contacts are closed. So the resistance R in the field is short-circuited. When this is out of circuit, total field current flows through the exciter and exciter terminal voltage increases. Thus, the voltage across the alternator terminals increases due to increase in alternator field current.

Due to this increased voltage, the pull of the solenoid exceeds the spring force and so the main contacts are opened again and the resistance is inserted in the exciter field. A similar process is repeated if the terminal voltage is reduced.

2. Brown-Boveri regulator This differs from the Tirril regulator. In this, the resistance of regulator is gradually varied or varied in small steps.

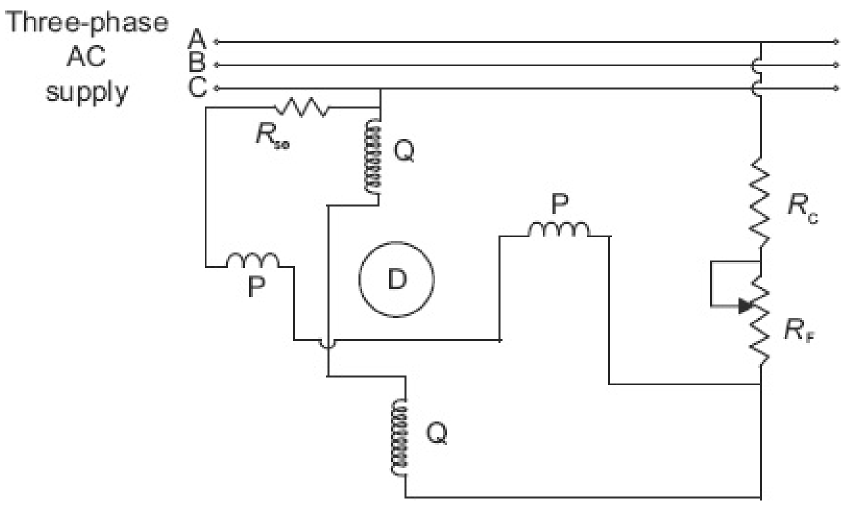

Fig. 11.2(a) Schematic diagram of Brown-Boveri regulator

Construction Brown-Boveri regulator is not a vibrating type regulator; hence wear and tear is less when compared to that of a tirril regulator. It consists of four main parts and its schematic diagram is shown in Fig. 11.2(a).

Control system: It contains two windings P and Q wound on an annular core of laminated steel sheet as shown in Fig. 11.2(a). The windings are excited from the three-phase alternator supply through the resistances R_c and R_f and resistance R_{se} is inserted in winding P. The ratio of resistance to reactance is adjusted in such a way as to get a phase angle difference between the currents in two winding. This results in the formation of a rotating magnetic field and hence develops electromagnetic torque on the aluminium drum D. This torque depends on the terminal voltage of the alternator and the resistances R_c and R_f. The torque decreases with increased values of R_f.

Operating system: It consists of two resistance sectors made up of contact blocks on the inner surface of roll contact segments as shown in Fig. 11.2(b). Contact segments and resistance sectors are made in contact by using springs. The two resistance sectors *R* and *R* are connected in series and this combination is connected in series with exciter field circuits. If the alternator voltage changes from its pre-determined value, the contact segments roll on the inside of resistance sectors and rotate clock-wise or anti-clock wise under the action of two windings P and Q.

Mechanical control torque: Mechanical torque is produced by springs (main and auxiliary) and is independent of the position of the control system. In steady deflection state, the mechanical torque is equal to electrical torque which is produced by the current in the split phase winding.

Damping torque: It consists of an aluminium disc which is rotated in between two magnets M and M and a spring S is attached to it. When there is a change in the alternator voltage, eddy currents are produced in the disc and a torque is developed, controlling the response of the moving system.

Principle of Operation Let us consider that the voltage of the alternator terminals is set to the normal value by adjusting R_c and R_f and is in position 3 on the scale. In this position, the mechanical torque is equal to the electromagnetic torque and the moving system is under equilibrium.

Let us assume that the terminal voltage of the alternator is reduced due to rise in load, and then the electromagnetic torque is reduced. At this instant, the mechanical torque is greater than the electromagnetic torque and the disc starts to rotate (assume in anti-clockwise direction). Due to this, the pointer moves to position 1. The resistance in the exciter field will be reduced which causes increase in exciter field current. So, the terminal voltage of an alternator increases.

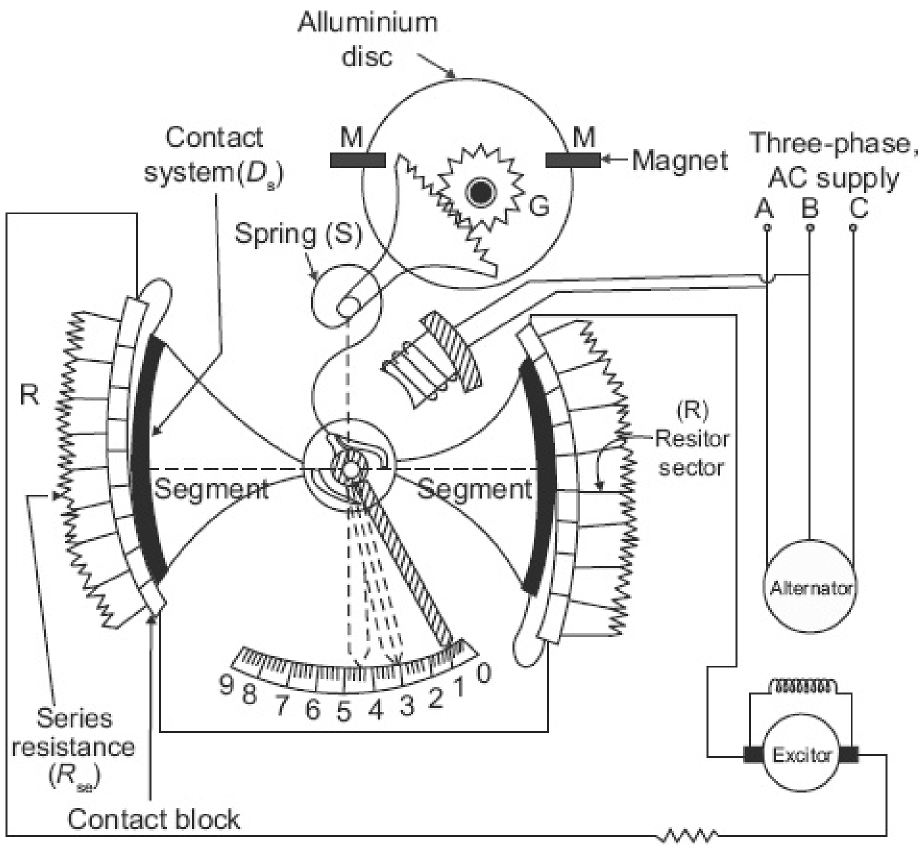

Fig. 11.2(b) Detailed diagram of Brown-Boveri regulator

11.5.2 Shunt Capacitors and Reactors

Shunt capacitors are used for lagging power factor circuits; whereas reactors are used for leading power factor circuits such as those created by lightly loaded cables. In both cases, the effect is to supply the required reactive power to maintain the values of the voltage. Apart from synchronous machines, static shunt capacitors offer the cheapest means of reactive power supply but these are not as flexible as synchronous condensers.

Capacitors are connected to a bus bar or to the tertiary winding of a main transformer. In this method, as the voltage falls, the VAr's produced by a shunt capacitor or reactor also falls. Thus, their effectiveness falls when needed. Also for light loads, when the voltage is high, the capacitor output is large and the voltage tends to become excessive. View of three-phase capacitor bank on 11 kV distribution line is shown in Fig. 11.3.

11.5.3 Series Capacitors

Capacitors are installed in series with transmission lines [shown in Fig. 11.4(b)] in order to reduce the voltage drop. The series capacitors compensate the reactance voltage drop in the line by reducing net reactance. A capacitor, in series with a transmission line, serving a lagging power factor load will cause a rise in voltage as the load increases. The power factor of the load through the series capacitor and the line must be lagging if the voltage drop is to decrease appreciably. The voltage on the load side of the series capacitor is raised above the source side, acting to improve the voltage regulation of the feeder.

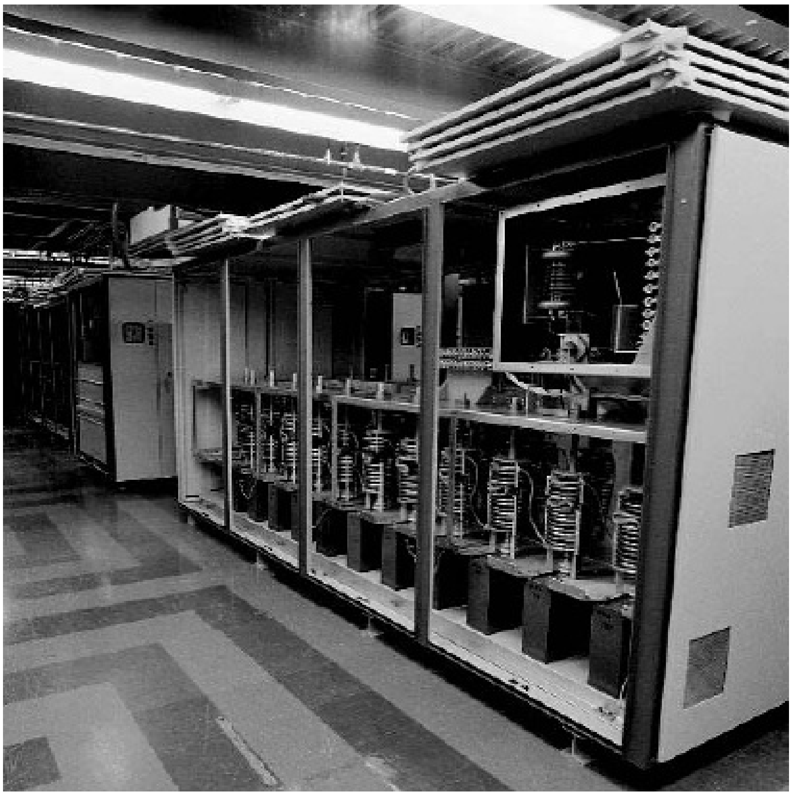

Fig. 11.3 View of three-phase capacitor bank

Since the voltage rise or drop occurs instantaneously with variations in the load, the series capacitor response as a voltage regulator is faster and smoother than the regulators.

The main drawback of this capacitor is the high voltage produced across the capacitor terminals under short circuit conditions. The drop across the capacitor is $I_f X_c$, where I_f is the fault current which is many times the full load current under certain circuit conditions. It is essential, therefore, that the capacitor is taken out of service as quickly as possible. A spark gap with a high-speed contactor can be used to protect the capacitor under these conditions.

Figures 11.4 and 11.5 show the line and its phasor diagrams without and with series compensation. The voltage drop of the line without series capacitor is approximately given by

$$V_d = IR\cos\phi + IX_L \sin\phi$$

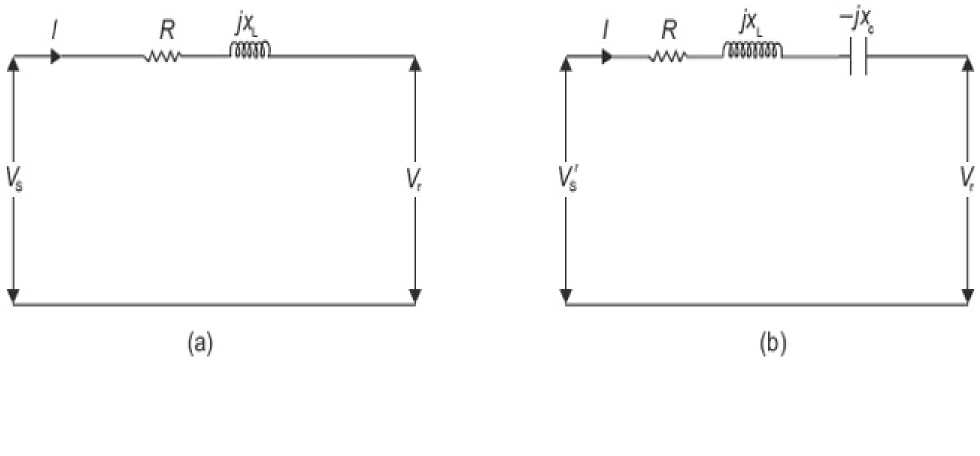

Fig. 11.4 Circuit diagram without (a) and with (b) series compensation

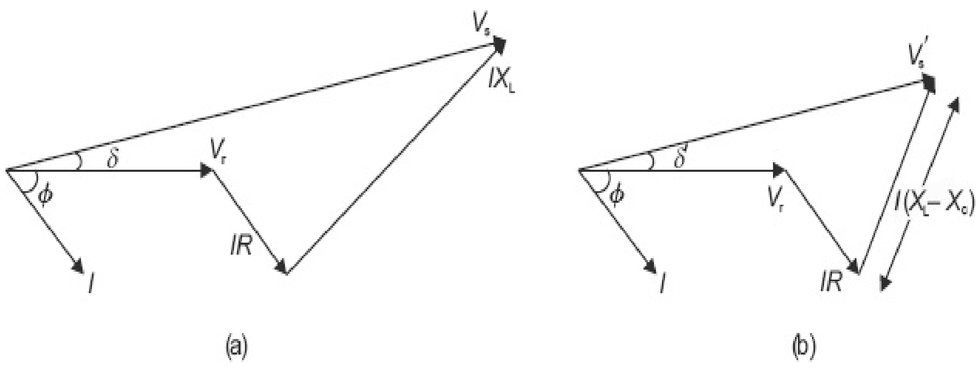

Fig. 11.5 Phasor diagrams of Fig. 11.1(a) and (b), respectively

And the voltage drop with series capacitor,

$$V_d = IR\cos\phi + I(X_L - X_c)\sin\phi$$

where,

X_c =capacitive reactance of the series capacitor.

11.5.4 Tap Changing Transformers

A tap changing transformer is a static device having a number of tap settings on its secondary side for obtaining different secondary voltages. The basic function of this device is to change the transformation ratio, whereby the voltage in the secondary circuit is varied making possible voltage control at all voltage levels at any load. The supply may not be interrupted when tap changing is done with and without load.

The types of tap changing transformers are:

1. Off-load tap changing transformer
2. On-load tap changing transformer

1. *Off-load Tap-changing Transformer* The simple tap changing arrangement of a transformer is shown in Fig.

11.6. The voltage can be varied by varying a number of tappings on the secondary side of the transformer.

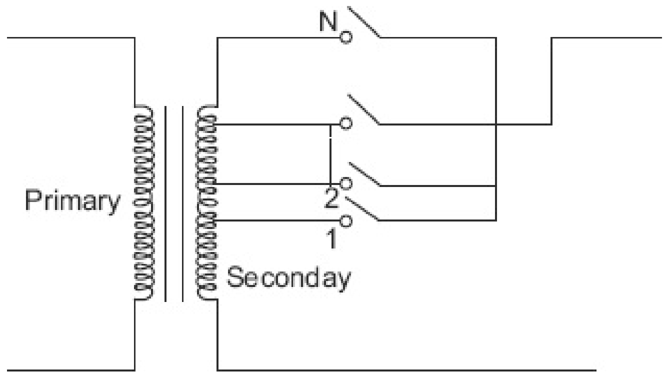

Fig. 11.6 Off-load tap changing transformers arrangement

Figure 11.6 refers to the off-load tap changing transformer which requires the disconnection of the transformer from load when the tap setting is to be changed.

The output of the secondary side of the transformer changes with the change in tap position of the secondary winding. The secondary voltage is minimum when the movable arm makes contact with stud 1; whereas it is maximum when it is in position N. When load on the transformer increases, the voltage across secondary terminals decreases. This can be increased to the desired value by adding the number of turns on the secondary terminal of the transformer by changing taps.

Thus, in the case of tap changing transformers, the main drawback is that the taps are changed only after the removal of the load. This can be overcome by using an on-load tap changing transformer with reactors.

2. *On-load Tap-changing Transformer* To supply uninterrupted power to the load (consumer), tap changing has to be performed when the system is onloaded. The secondary winding, in a tap-changing

transformer consists of two identical parallel windings with similar tappings. For example 1, 2,...N and 1′, 2′,..., N′ are the tappings on both the parallel windings of such a transformer. These two parallel windings are controlled by switches Sa and Sb as shown in Fig. 11.7(a).

In the normal operating conditions, switches S_a, S_b and tappings 1 and 1′ are closed, i.e., both the secondary windings of the transformer are connected in parallel and each winding carries half of the total load current by equal sharing. The secondary side of the transformer is at a rated voltage under no load, when the switches S_a, S_b are closed and movable arms make contact with stud 1 and 1′ whereas it is maximum (above the rated valve) under no load, when the movable arms are in position N and N′. The voltage at the secondary terminal decreases with an increase in the load. To compensate for the decreased voltages, it is required to change switches from positions 1 and 1′, to positions 2 and 2′ (number of turns on secondary is increased). For this, open any one of the switches S_a and S_b, and assume S_a is opened. At this instant, the secondary winding controlled by switch S_b carries full load current through one winding. Then, tapping is changed to position 2 on the winding of the disconnected transformer and switch S_a is closed. After this, switch S_b is opened for disconnecting its winding, the tapping position is changed from 1′ to 2′ and then switch S_b is closed. Similarly, tapping positions can be changed without interrupting the power supply to the consumers. The online tap changing transformer is shown in Fig. 11.7(b)

This method has the following disadvantages:

- It requires two winding with rated current carrying capacity instead of one winding.
- It requires two operations for change of single step.
- Complications are introduced in the design in order to obtain a high reactance between the parallel windings.

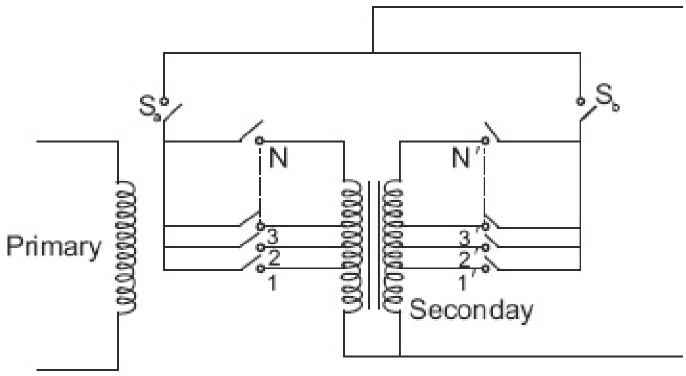

Fig. 11.7(a) On-load tap-changing transformer arrangement

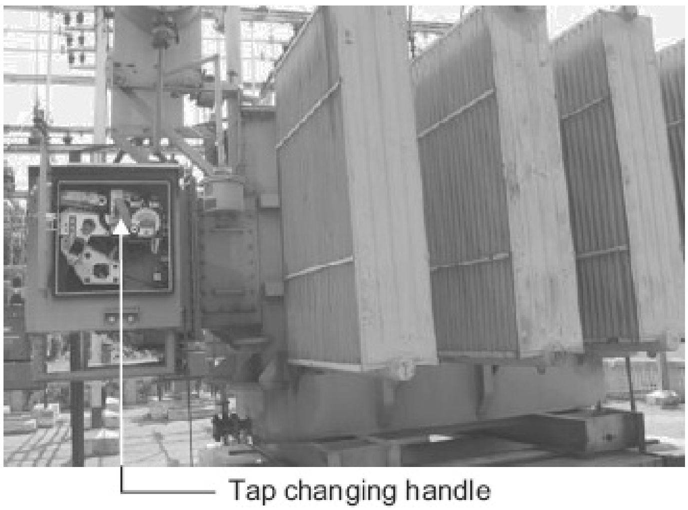

Fig. 11.7(b) View of on-load tap changing of a 2.5 MVA transformer

11.5.5 Booster Transformers

The booster transformer performs the function of boosting the voltage. It can be installed at a substation or at any intermediate point of line.

In the circuit shown in Fig 11.8(a), P and Q are the two relays. The secondary of the booster transformer is connected in series with the line whose voltage is to be controlled and the primary of the booster transformer is supplied from a regulating transformer with on-load tap changing gear. The booster can be brought in to the circuit by the closure of relay Q and the opening of relay P, and vice versa as shown in Fig. 11.8(a). The secondary of booster transformer injects a voltage in phase with the line voltages. By changing the tapping on the regulating transformer, the magnitude of V_Q can be changed and thus the feeder voltage V_F can be regulated. View of booster and distribution transformer connection (left to right) is shown in Fig. 11.8(b).

Advantages:

- It can be installed at any intermediate point in the system.
- Rating of booster transformer is about 10% of that of the main transformer (product of current and injected voltage).

Disadvantages:

When used in conjunction with main transformer:

- More expensive than a transformer with on-load tap changes.
- Less efficient due to losses in booster.
- Requires more space.

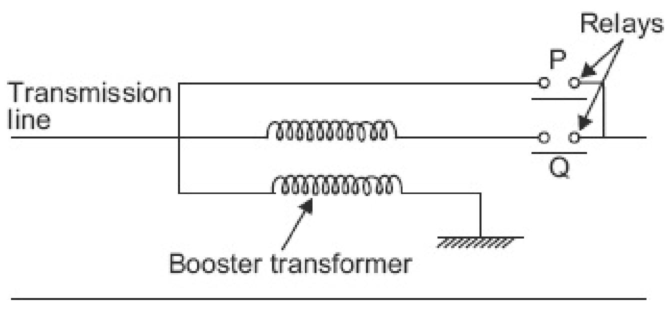

Fig. 11.8(a) Booster transformer

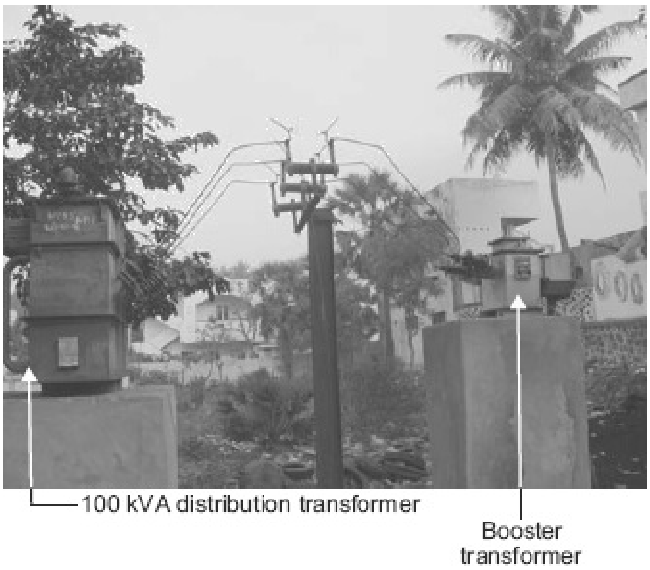

Fig. 11.8(b) View of booster and distribution transformer connection (right to left)

11.5.6 Synchronous Condensers

A synchronous condenser (synchronous phase modifier) is a synchronous motor running without mechanical load. It is connected in parallel with the load at the receiving end of the line. Depending upon its excitation, it generates or absorbs the reactive power. It takes leading current when its field is overexcited, i.e., above normal speed and takes lagging current when it is underexcited. Thus, the current drawn by a synchronous phase modifier can be varied from lagging to leading by varying its excitation. It is a very convenient device to keep the receiving-end voltage constant under any condition of load. It also improves the power factor. The output can vary smoothly.

A synchronous phase modifier has a smaller shaft and bearing, and higher speed as compared to a synchronous motor used for mechanical loads. A synchronous phase modifier has higher overall efficiency as compared with a synchronous motor.

Advantages:

- Flexibility for use in all load conditions because when the machine is under-excited it consumes reactive power.
- There is a smooth variation of reactive VAr's by synchronous capacitors.
- It can be overloaded for short periods.

Disadvantages:

- Possibility of falling out of control in case of sudden changes in voltage.
- These machines add to short-circuit capacity of the system during fault condition.

Test Yourself

1. Why are series capacitors not economical for low-voltage overhead lines?
2. Why is tap changing generally preferred on load?

Table 11.1 Comparison of shunt and series capacitors

Shunt capacitor	Series capacitor

1. Supplies fixed amount of reactive power to the system at the point where it is installed. Its effect is felt in the circuit from the location towards supply source only.
2. It reduces the reactive power flowing in the line and causes:
  1. Improvement of power factor of system
  2. Voltage profile improvement
  3. Decreases kVA loading on source, i.e., generators, transformers, and line up to location and thus provides additional capacity.
	1. The quantum of compensation is independent on load current and instantaneous changes occur. Its effect is from its location towards the load end.
	2. Its advantages are:
	  1. On tie lines, the power transfer is greater
	  2. Specifically suitable for situations when flickers due to respective load functions occur.
	3. As a thumb rule, the best location is one-third of electrical impedance from the source bus.
	4. As full-load current is to pass through, the capacity (current rating) should be more than the

3. The location has to be as near to the load point as possible. In practice, due to the high compensation required, it is found economical to provide group compensation on lines and substations. 4. As fixed kVAr is supplied, this may some times result in overcompensation in the light-load period. Switched kVAr banks are comparatively costlier than fixed kVAr and become necessary. 5. As the power factor approaches unity, larger compensation is required for the improvement of power factor. 6. Where lines are heavily loaded, compensation required will be more. 7. Cost of compensation is lower than that required for series capacitor.	load current. 5. As series capacitors carry fault current, special protection is required to protect from the fault current. 6. It causes sudden rises in voltage at the location. 7. The cost of a series capacitor is higher than that of a shunt capacitor.

Table 11.2 Comparison of synchronous condenser and static capacitors

Synchronous condenser	Static capacitors
1. Harmonics in the voltage do not exist. 2. The power factor variation is stepless (uniform). 3. It allows overloading for a short period. 4. The power loss is more. 5. It is more economical in the case of large kVAr. 6. Failure rate is less and therefore, this is more reliable.	1. Large harmonics are produced in the system. 2. The power factor varies in steps. 3. It does not allow any overloading. 4. The power loss is less. 5. It is more economical for small kVAr requirement. 6. Failure rate is more and therefore, it is less reliable.

11.6 RATING OF SYNCHRONOUS PHASE MODIFIER

The expression of the sending-end voltage in terms of transmission line constants is

$$V_s = AV_r + BI_r \tag{11.1}$$

where,

$V_S = V_S\angle\delta$ = sending-end voltage

V_r = receiving-end voltage (reference phasor)

$I_r = I_r\angle -\phi_r$ = receiving-end current

$A = A\angle\alpha$

$B = B\angle\beta$ are the line constants

The Eq. (11.1) can be written in phasor form as:

$$\begin{aligned} V_s\angle\delta &= AV_r\angle\alpha + BI_r\angle(\beta - \phi_r) \\ &= AV_r\cos\alpha + jAV_r\sin\alpha + BI_r\cos(\beta - \phi_r) + jBI_r\sin(\beta - \phi_r) \end{aligned} \tag{11.2}$$

The real part of Eq. (11.2) is

$$V_s\cos\delta = AV_r\cos\alpha + BI_r\cos(\beta - \phi_r) \tag{11.3}$$

And the imaginary part is

$$V_s\sin\delta = AV_r\sin\alpha + BI_r\sin(\beta - \phi_r) \tag{11.4}$$

Squaring and adding the Eqs. (11.3) and (11.4), we get

$$
\begin{aligned}
V_s^2 &= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r\cos\alpha\cos(\beta-\phi_r) + 2ABV_rI_r\sin\alpha\sin(\beta-\phi_r) \\
&= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r\cos(\alpha-\beta+\phi_r) \\
&= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r\left[\cos(\alpha-\beta)\cos\phi_r - \sin(\alpha-\beta)\sin\phi_r\right] \qquad (11.5)
\end{aligned}
$$

Real power at receiving end, $P_r = V_r I_r \cos\phi_r$

Reactive power at receiving end, $Q_r = V_r I_r \sin\phi_r$

and the receiving-end current can be written as

$$
\begin{aligned}
I_r &= I_r\cos\phi_r - jI_r\sin\phi_r \qquad (\because \text{lagging power factor}) \\
&= I_p - jI_q \\
\therefore I_r^2 &= I_p^2 + I_q^2 \\
\text{where, } I_p &= \frac{P_r}{V_r},\ I_q = \frac{Q_r}{V_r}
\end{aligned}
$$

Substituting the above quantities in Eq. (11.5)

$$
V_s^2 = A^2V_r^2 + B^2I_r^2 + 2ABP_r\cos(\alpha-\beta) - 2ABQ_r\sin(\alpha-\beta) \qquad (11.6)
$$

In Eq. (11.6), I_r^2 is replaced by $I_p^2$and I_q^2expressions

$$
\therefore V_s^2 = A^2V_r^2 + B^2\left[\frac{P_r^2}{V_r^2} + \frac{Q_r^2}{V_r^2}\right] + 2ABP_r\cos(\alpha-\beta) - 2ABQ_r\sin(\alpha-\beta) \qquad (11.7)
$$

The Eq. (11.7) is useful for calculating the sending-end voltage by knowing values of A, B, α, β, P_r, Q_r and V_r (or) sometimes the sending-end and receiving-end voltages are fixed and A, B, α, β, P_r, and Q_r are given. It is required to find out the rating of the phase modifier. In this case, the required quantity is Q_r, where Q_r is the net reactive power at the receiving end and not the reactive power for the load. So, if the net reactive power required to maintain certain voltages at the two ends is known, the rating of the phase modifier can be found.

Example 11.1

A three-phase overhead line has resistance and reactance per phase of 25 Ω and 90 Ω, respectively. The supply voltage is 145 kV while the load-end voltage is maintained at 132 kV for all loads by an automatically controlled synchronous phase modifier. If the kVAr rating of the modifier has the same value for zero loads as for a load of 50 MW, find the rating of the synchronous phase modifier.

Solution:

$$\therefore V_s^2 = A^2V_r^2 + B^2\left[\frac{P_r^2}{V_r^2} + \frac{Q_r^2}{V_r^2}\right] + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta) \qquad (1)$$

$$\text{We have, } V_s = AV_r + BI_r \qquad (2)$$

From given data,

Sending-end phase voltage, $V_s = \dfrac{145}{\sqrt{3}}$

Receiving-end phase voltage, $V_r = \dfrac{132}{\sqrt{3}}$

Line impedance, $Z = 25 + j90$

Assuming short line model, $V_S = V_r + I_rZ$

$$\therefore \frac{145}{\sqrt{3}} = \frac{132}{\sqrt{3}} + I_r(25 + j90)$$

$$83.71 = 76.2 + I_r \times 93.4\angle 74.47^\circ \qquad (3)$$

Comparing the Eqs. (2) and (3)

$$A = 1; \qquad \alpha = 0,$$
$$B = 93.4°, \beta = 74.47°$$
$$P_r = V_r I_r \cos\phi_r = 50 \text{ MW (given)}$$

Substituting these values in Eq. (1)

$$(83.71)^2 = 1^2(76.2)^2 + (93.4)^2\left[\frac{50^2}{(76.2)^2} + \frac{Q_r^2}{(76.2)^2}\right] + 2\times1\times93.4\times\cos(0-74.47°)\times50$$
$$-2\times1\times93.4\times Q_r \sin(0-74.47°)$$
$$1200.92\times10^6 = 3755.98\times10^6 + 1.50Q_r^2\times10^6 + 2500.7\times10^6 + 179.98Q_r\times10^6$$
$$1200.92 = 3755.98 + 1.50Q_r^2 + 2500.7 + 179.98Q_r$$
$$1.50Q_r^2 + 179.98Q_r + 5055.76 = 0$$

Solving the above equation

$$Q_r = -44.87 \text{ MVAr}$$
$$\frac{Q_r}{P_r} = \frac{V_r I_r \sin\phi_r}{V_r I_r \cos\phi_r}$$
$$= \frac{-44.87}{50}$$
$$\tan\phi_r = -0.8974$$

Power factor angle at receiving-end voltage, $\phi_r = -41.9°$

∴ The power factor is $\cos\phi_r = 0.7442$ lead

∴ The rating of the synchronous modifier = 44.87 MVAr's

EXAMPLE 11.2

A three-phase feeder, having a resistance of 3 Ω and a reactance of 10 Ω, supplies a load of 2 MW at 0.85 p.f. lagging (Fig. 11.9). The receiving-end voltage is maintained at 11 kV by means of a static condenser drawing 2.1 MVAr from the line. Calculate the sending-end voltage and power factor. What is the regulation and efficiency of the feeder?

Solution:

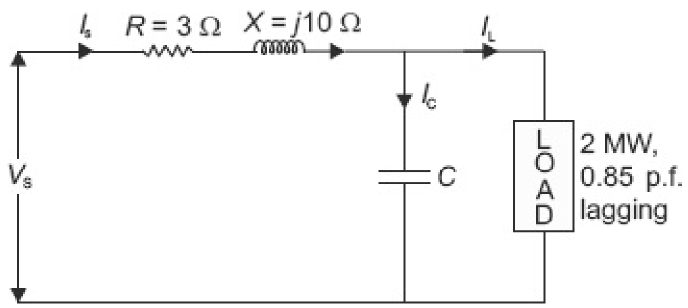

Fig. 11.9 Circuit diagram for Example 11.2

$$\text{Load current, } I_L = \frac{2000}{\sqrt{3} \times 11 \times 0.85} = 123.5 \text{ A}$$

$$\text{Shunt branch current, } I_C = \frac{2100}{\sqrt{3} \times 11} = 110.22 \text{ A}$$

$$\begin{aligned} \text{Receiving-end current, } I_S &= I_L + I_C \\ &= 123.5\angle -31.79^\circ + 110.22\angle 90^\circ \\ &= 105 - j65 + j110.22 \\ &= 105 + j45.22 \\ &= 114.3\angle 23.3^\circ \text{ A} \end{aligned}$$

The vector diagram is shown in Fig. 11.10. Here, the current is a leading current.

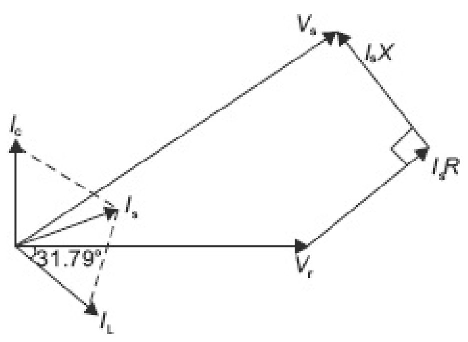

Fig. 11.10 Phasor diagram for Example 11.2

From the circuit diagram shown in Fig. 11.9,

The sending-end voltage, $V_S = V_r + I_r Z$

$$
\begin{aligned}
V_s &= V_r + I_r Z \\
&= \frac{11000}{\sqrt{3}}\angle 0^\circ + 114.3\angle 23.3^\circ \times (3 + j10) \\
&= 6350.85\angle 0^\circ + 1193.33\angle 96.6^\circ \\
&= 6350.85 + j0 - 137.16 + j1185.42 \\
&= 6213.69 + j1185.42 \\
&= 6325.74\angle 10.8^\circ \text{ V}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \text{ The sending-end voltage, } V_s &= 6325.75 \times \sqrt{3}(\text{L}-\text{L}) \\
&= 10.96 \text{ kV}
\end{aligned}
$$

From the phasor diagram shown in Fig. 11.10,

Sending-end power factor = cos (10.8° – 23.3°) = cos12.5° = 0.976 lead

$$
\begin{aligned}
\%\ \text{Regulation } &= \frac{10.96 - 11}{11} \times 100 \\
&= -0.364\%
\end{aligned}
$$

$$
\begin{aligned}
\text{Efficiency} &= \frac{\text{output}}{\text{output} + \text{losses}} \\
&= \frac{2000 \times 10^3}{2000 \times 10^3 + 3 \times (114.3)^2 \times 3} \times 100 \\
&= 94.44\%.
\end{aligned}
$$

Example 11.3

A single circuit three-phase, 220 kV, line runs at no load. Voltage at the receiving end of the line is 205 kV. Find the sending-end voltage, if the line has resistance of 21.7 Ω, reactance of 85.2 Ω and the total succeptance of 5.32×10^{-4} ℧.

The transmission line is to be represented by *π*-model.

Solution:

Sending-end voltage V_1 differs from the receiving-end voltage V_2 by the value of voltage drop due to charging current in the line impedance, as shown in Fig. 11.11. With the quadrate-axis component of voltage drop being neglected as shown in Fig. 11.12, we find $|V_1|$

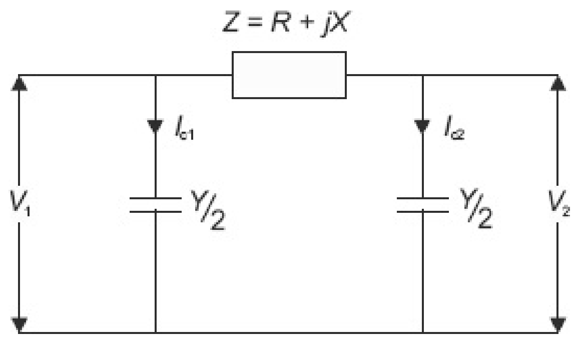

Fig. 11.11 Circuit diagram

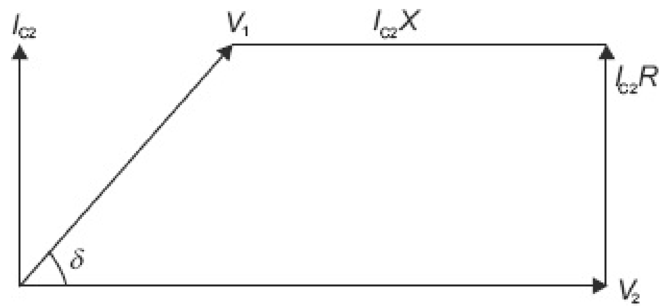

Fig. 11.12 Circuit diagram

$$|V_1| = |V_2| - \frac{Q_c}{|V_2|} X$$

We can also find out | V_1 | from expression $|V_1| = |V_2| + \dfrac{P_2R + Q_2X}{|V_2|}$

where, $Q_2 = -Q_c$ because the current is leading and $P_2 = 0$.

We are given line voltage at the receiving end, therefore, on per phase, we have

$$|V_2| = \frac{205}{\sqrt{3}} = 118.35 \text{ kV}$$

$$Q_c = |V_2|^2 \frac{B}{2} = |V_2|^2 \frac{\omega c}{2}$$

$$= (118.35)^2 \times \frac{5.32}{2} \times 10^{-4} = 3.725 \text{ MVAr}$$

X = 85.2 (given)

Hence, $$|V_1| = |V_2| - \frac{Q_c}{|V_2|} X = 118.35 - \frac{3.725}{118.35} \times 85.2$$

$$= 118.35 - 2.69 = 115.67 \text{ kV}$$

Sending-end voltage, line-to-line = $\sqrt{3}|V_1| = 200.34$ kV.

CHAPTER AT A GLANCE

1. Sources and sinks of reactive power are synchronous machine, overhead lines, transformers, cables and loads.
2. The voltage-control equipment is located at generating stations, transformer stations and feeders.
3. The various methods for voltage control are: excitation control, shunt capacitors, and series capacitors, by using tap changing transformers, boosters and synchronous condensers.
4. **Excitation control:** This method is used only at the generating station. Due to the voltage drop in the synchronous reactance of armature, whenever the load on the supply system changes, the terminal voltage of the alternator changes. This can be kept constant by changing the field current of the alternator according to the changes in load. This is known as excitation control.
5. **Shunt capacitors and reactors:** Shunt capacitors are used for lagging power factor circuits; whereas reactors are used for leading power factor circuits such as created by lightly loaded cables.
6. **Series capacitor:** It is installed in series with transmission lines to reduce the frequency of voltage drops.
7. **Tap changing transformers:** The basic operation of tap changing transformer is by changing the transformation ratio; the voltage in the secondary circuit is varied.
8. **Booster transformers:** The booster transformer performs the function of boosting the voltage. It can be installed at substation or any intermediate point of line.
9. **Synchronous condensers:** It is connected in parallel with the load at the receiving end of the line. It can generate or absorb reactive power by varying the excitation of its field winding.

SHORT ANSWER QUESTIONS

1. What are the different methods of voltage control?
2. What is meant by excitation voltage control?
3. What are the disadvantages of tap changing transformers?
4. What is the synchronous condenser?
5. What is a booster transformer?
6. How does a shunt capacitor bank control the system voltage under light loads and heavy loads?
7. Under what condition does a synchronous motor take a leading current?
8. When is the shunt inductor compensation required?

MULTIPLE CHOICE QUESTIONS

1. The voltage of the power supply at the consumers service must be held substantially _________.
    1. constant
    2. smooth variation
    3. random variation
    4. none of these
2. Low voltage reduces the_________ from incandescent lamps.
    1. power output
    2. power input
    3. light output
    4. current
3. Motors operated at below normal voltage draw abnormally_________ currents.
    1. low
    2. high
    3. medium
    4. none of these
4. Permissible voltage variation is a _________.
    1. ±10%
    2. ±20%
    3. ±50%
    4. ±5%
5. By drawing high currents at low voltages the motors get __________.
    1. over heated
    2. cooled
    3. constant heat
    4. overcooled
6. Domestic circuits supply voltage is a _________.
    1. 230 V
    2. 110 V
    3. 240 V
    4. 220 V

7. The voltage might normally vary between the limits of ________.
   1. 210 – 230 V
   2. 230 – 240 V
   3. 230 – 520V
   4. 210 – 235V
8. Above normal voltages reduces the________ of the lamps.
   1. life
   2. strength
   3. lighting
   4. colour
9. The voltage at the bus can be controlled by the injection of_____ power of the correct sign.
   1. real
   2. reactive
   3. complex
   4. both real and reactive
10. General methods of voltage control are _________.
    1. use of tap changing transformer
    2. synchronous condensers
    3. static capacitors
    4. all of these
11. Use of thyristor-controlled static compensators is_________.
    1. voltage control
    2. power control
    3. current control
    4. power factor control
12. An overexcited synchronous machine operated as generator or motor generates_________.
    1. kVA
    2. kVAr
    3. kW
    4. kI
13. Synchronous motor running at no load and overexcited is known as_________.
    1. synchronous condenser
    2. shunt capacitor
    3. series capacitor
    4. shunt reactor
14. The excitation-control method is only suitable for________ lines.
    1. short
    2. medium
    3. long
    4. all of these
15. It is_________ to maintain the same voltage at both ends of transmission lines by synchronous-condenser method.
    1. economical
    2. not economical
    3. difficult
    4. easy
16. Shunt capacitors and reactors are used across lightly loaded lines

to absorb some of the leading ________again to control the voltage.

1. VAr's
2. VA
3. V
4. I

17. Disadvantages of shunt capacitors are_______.
    1. fall of voltage
    2. reduction in VAr's
    3. reduction in effectiveness
    4. all of these
18. ________ reduces the inductive reactance between the load and the supply point.
    1. shunt capacitor
    2. shunt reactor
    3. series capacitor
    4. transformer
19. The disadvantage of a series capacitor is that it produces______ voltage across the capacitor under short circuit condition
    1. low
    2. high
    3. very low
    4. medium
20. A spark gap with a high speed contactor is the_______ used for shunt capacitor.
    1. protective device
    2. control
    3. fuse
    4. circuit breaker
21. The different types of tap changing transformers are________.
    1. off-load
    2. on-load
    3. both a & b
    4. none
22. The purposes of using booster transformers is for________ the voltage.
    1. transforming
    2. bucking
    3. boosting
    4. bucking and boosting
23. The disadvantages of the_______ transformer are, it is more expensive, less efficient and occupies more floor area.
    1. off -load tap
    2. on-load
    3. booster
    4. induction
24. If a synchronous machine is overexcited, it takes lagging VAr's from the system when it is operated as a ________.
    1. synchronous motor
    2. synchronous generator
    3. induction generator

4. synchronous phase modifier

25. For a synchronous phase modifier the load angle is_________.
    1. 0°
    2. 25°
    3. 30°
    4. 50°

Answers

1. a	2. c	3. b	4. d	5. a
6. d	7. a	8. a	9. b	10. d
11. a	12. b	13. a	14. a	15. b
16. a	17. d	18. c	19. b	20. a
21. c	22. d	23. c	24. d	25. a

REVIEW QUESTIONS

1. Why is voltage control required in a power systems? Mention the different methods of voltage control employed in a power system. Explain one method of voltage control in detail giving a neat connection diagram.
2. Why is excitation control necessary in an alternator?
3. Describe “off-load” and “on-load” tap changing transformers.
4. Explain the function of a synchronous phase modifier placed at the receiving end of the transmission line.
5. Show with the aid of a vector diagram, how the voltage at the receiving end of a transmission line can be maintained at a constant by the use of a synchronous phase modifier.

PROBLEMS

1. A three-phase, 33 kV, overhead transmission line has a resistance of 5 Ω/phase and a reactance of 18 Ω/phase. With the help of a synchronous modifier, the receiving-end voltage is kept constant at 33 kV. Calculate the kVA of the phase modifier if the load at the receiving end is 60 MW at 0.85 pf lagging. What will be the maximum load that can be transmitted?
2. If the voltage at the sending end is to be maintained at 66 kV, determine the MVA’r of the phase modifier to be installed for a three-phase overhead transmission line having an impedance of (7 + *j*19) Ω/phase, delivering a load of 80 MW at 0.85 pf lagging and with a voltage of 66 kV.
3. A three-phase induction motor delivers 450 HP at an efficiency of 95% when the operating power factor is 0.85 lag. A loaded synchronous motor with a power consumption of 110 kW is connected in parallel with the induction motor. Calculate the necessary kVA and the operating power factor of the synchronous

motor if the overall power factor is to be unity.

12

Electric Power Supply Systems

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Provide an analysis of different transmission/distribution systems
- Discuss the various choices of supply voltage and frequency
- Determine the best economic size of a conductor

12.1 INTRODUCTION

The energy generated at various generating stations is distributed to the consumers through a large network. This network can be classified into transmission and distribution systems. The transmission system transmits huge quantities of power over long distances from generating stations to load centres and large industrial consumers. Distribution systems distribute the power from substations to various consumers. Therefore, these lines constitute the infrastructure and form the major cost of the total scheme. Hence, the transmission line network should be designed in such a way that the overall transmission costs are minimized.

In order to have an economically viable transmission line network, the cost of the conductor should be as minimal as possible, which means, the conductor should amount only to the minimum possible fixed and running costs. The entire view of power systems is shown in Fig. 12.1.

12.2 COMPARISON OF CONDUCTOR EFFICIENCIES FOR VARIOUS SYSTEMS

The conductors in an overhead line insulated from each other and from the earth. The cost of insulation varies with the system. Since the conductor cost forms the bulk of the expenditure of a line, it is necessary to compare it with various systems in order to ascertain the most economical operative system for it.

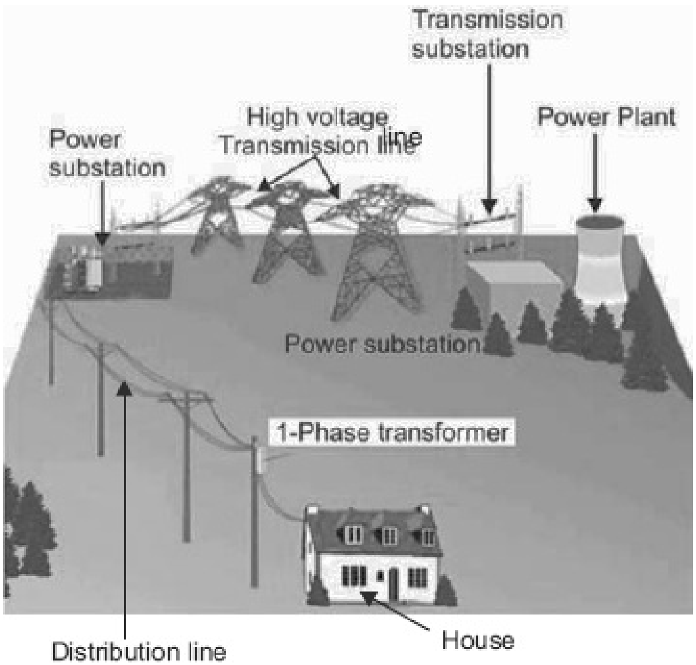

Fig. 12.1 Power systems

We will be comparing the relative amount of conductor material required for different systems of transmission and distribution for a given amount of power over a given distance, represented in terms of conductor efficiency. Conductor efficiency is the ratio of conductor material required for a given system to that required for the DC two-wire system with one wire earthed.

According to the type of supply, the transmission or distribution system is classified as DC system or AC system. If the nature of the power supply involved is DC, it is known as DC system. If the nature of the power supply involved is AC, it is known as AC system.

DC System

The DC system is classified as follows:

1. DC two-wire with one end earthed
2. DC two-wire with mid-point earthed
3. DC three-wire

Single-phase AC System

1. Single-phase, two-wire with one end earthed
2. Single-phase, two-wire with mid-point earthed
3. Single-phase, three-wire

Two-phase AC System

1. Two-phase, four-wire
2. Two-phase, three-wire

Three-phase AC System

1. Three-phase, three-wire
2. Three-phase, four-wire

From such a big list of possible transmission systems, it is difficult to ascertain which would be the best system without making comparisons. In general, the system of transmission adopted is based on economic considerations. Hence, the most suitable system for power transmission is that for which the volume of conductor material used is minimum.

Comparison of the amount of conductor material used in different transmission systems is based on the following two cases:

1. When the power is transmitted by an overhead line, a bare conductor must be insulated from the cross-arms and the supporting towers. Therefore, the maximum voltage between each conductor and earth forms the basis of comparison of volume of conductor.
2. When the power is transmitted by an underground three-core cable, conductors are insulated from each other by insulating materials. Therefore, for comparison of volume of conductor used in different systems of transmission, the maximum voltage between the conductors should be taken as the basis.

Assumptions made for comparison of conductor efficiency:

- The power transmitted over all types of systems is the same.
- The power transmitted over the given distance is the same.
- The power loss in all the systems is same.
- The maximum voltage V_m between each conductor and earth is same for overhead systems, whereas V_m is same between the conductors for underground systems.

12.2.1 Overhead Lines

In an overhead line system, a bare conductor must be insulated from the cross-arms and the supporting towers and this insulation level is determined by the maximum voltage (V_m) between conductor and earth. Let this V_m be the same for the following overhead transmission systems.

DC System

(a) Two-wire system with one-end earthed

Let V_m be the maximum voltage between conductor and earth and I be the current as shown in Fig. 12.2(a).

Power transmitted, $P = V_m I$ (12.1)

Line current, $I = \frac{P}{V_m}$

$$\text{Line loss} = 2I^2R = \frac{2P^2R}{V_m^2} \tag{12.2}$$

where, R is the resistance of each conductor.

(b) Two-wire system with mid-point earthed

Let V_m be the voltage between each conductor and earth and I_1 be the current. Assuming the middle point of the system is earthed, the line voltage is 2 V_m as shown in Fig. 12.2(b).

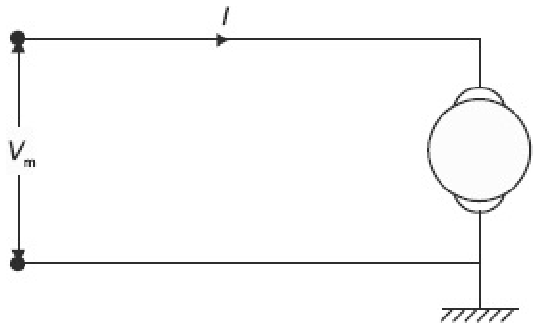

Fig. 12.2(a) DC two-wire system with one end earthed

Power transmitted, $P = 2V_m I_1$

$$\text{Line current, } I_1 = \frac{P}{2V_m} \tag{12.3}$$

$$\text{Line loss} = 2I_1^2 R_1 = \frac{2P^2 R_1}{4V_m^2} = \frac{P^2 R_1}{2V_m^2} \tag{12.4}$$

where, R_1 is the resistance of each conductor

For equal transmission loss, comparing Eqs. (12.2) and (12.4)

$$2I^2 R = 2I_1^2 R_1$$
$$\Rightarrow \frac{2P^2 R}{V_m^2} = \frac{P^2 R_1}{2V_m^2}$$
$$\therefore \frac{R}{R_1} = \frac{1}{4} = \frac{a_1}{a}.$$

Since resistance is inversely propotional to area of cross-section, $R \propto \frac{1}{a}$

$$\frac{\text{Volume of conductor material required in DC, two-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{2a_1 l}{2al} = 0.25$$

where, $2l$ is the length of two conductors of the transmission line.

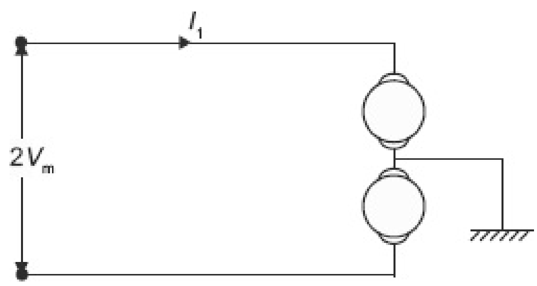

Fig. 12.2(b) DC two-wire system with mid-point earthed

Thus, the conductor material required with DC two-wire mid point earthed system is 25% of that required for DC two-wire system with one end earthed.

(c) Three-wire system

Assuming that load is balanced, the voltage between the outer wire and the central wire, which is earthed, is V_m. The voltage between the lines is $2V_m$ as shown in Fig. 12.3.

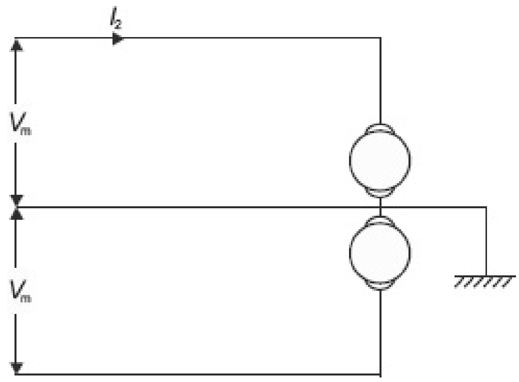

Fig. 12.3 DC three-wire system with mid-point earthed

Power transmitted, $P = 2V_m I_2$

$$\text{Line current, } I_2 = \frac{P}{2V_m} \qquad (12.5)$$

$$\text{Line loss} = 2I_2^2 R_2 = \frac{2P^2 R_2}{4V_m^2} = \frac{P^2 R_2}{2V_m^2} \qquad (12.6)$$

where, R_2 = resistance of each outer conductor.

Let the neutral conductor be half the cross-sectional area of that of outer. For a balanced system, no current is passing through the neutral wire.

For equal transmission loss, comparing Eqs. (12.2) and (12.6)

$$2I^2 R = 2I_2^2 R_2$$
$$\frac{2P^2 R}{V_m^2} = \frac{P^2 R_2}{2V_m^2}$$
$$\frac{R}{R_2} = \frac{1}{4} = \frac{a_2}{a}$$

$$\frac{\text{Volume of conductor material required in DC, three-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{a_2 \times 2.5l}{a \times 2l}$$
$$= 0.3125.$$

Thus, the conductor material required with DC three-wire system is 31.25% of that required for DC, two-wire system with one end earthed.

AC Single-phase System

(a) Single-phase, two-wire system with one end earthed

The maximum voltage between conductor and earth is V_m as shown in Fig. 12.4.

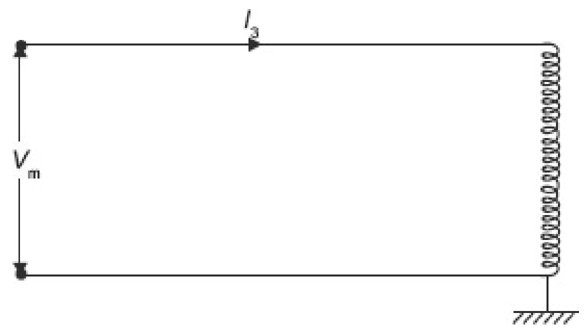

Fig. 12.4 Single-phase two-wire system with one end earthed

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Power transmitted, $P = V_{rms} I_3 \cos\phi$

$$\text{Line current, } I_3 = \frac{P}{V_{rms}\cos\phi} = \frac{\sqrt{2}P}{V_m \cos\phi} \qquad (12.7)$$

$$\text{Line losses} = 2I_3^2 R_3 = \frac{4P^2}{V_m^2 \cos^2\phi} R_3 \qquad (12.8)$$

where, R_3 is the resistance of each conductor

For equal transmission loss, comparing Eqs. (12.2) and (12.8)

$$\frac{2P^2 R}{V_m^2} = \frac{4P^2 R_3}{V_m^2 \cos^2\phi}$$

$$\therefore \frac{R}{R_3} = \frac{2}{\cos^2\phi} = \frac{a_3}{a}.$$

$$\frac{\text{Volume of conductor material required in single-phase, two-wire system with one end earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}}$$

$$= \frac{a_3 \times 2l}{a \times 2l} = \frac{2}{\cos^2\phi}.$$

(b) Single-phase, two-wire with mid-point earthed

The maximum voltage between conductor and earth is V_m as shown in Fig. 12.5.

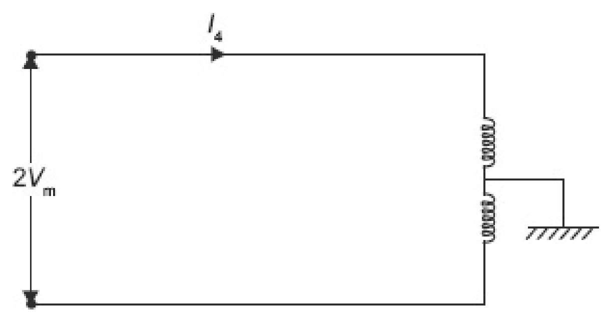

Fig. 12.5 Single-phase, two-wire with mid-point earthed

Voltage between two outer wires

$$= 2V_m \text{ (peak)}$$
$$= \frac{2V_m}{\sqrt{2}} = \sqrt{2}V_m \text{ (r.m.s)}$$

$$\text{Power transmitted, } P = \sqrt{2}V_m I_4 \cos\phi \qquad (12.9)$$
$$\text{Line current, } I_4 = \frac{P}{\sqrt{2}V_m \cos\phi}$$

$$\text{Line losses} = 2I_4^2 R_4 = \frac{P^2 R_4}{V_m^2 \cos^2\phi} \qquad (12.10)$$

where, R_4 is the resistance of each conductor

For equal transmission loss, comparing Eqs. (12.2) and (12.10)

$$\frac{2P^2 R}{V_m^2} = \frac{P^2 R_4}{V_m^2 \cos^2\phi}$$
$$\therefore \frac{R}{R_4} = \frac{1}{2\cos^2\phi} = \frac{a_4}{a}$$

$$\frac{\text{Volume of conductor material required in single-phase, two-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}}$$
$$= \frac{a_4 \times 2l}{a \times 2l}$$
$$= \frac{0.5}{\cos^2\phi}.$$

(c) Single-phase, three-wire system

Assume that load is balanced and the neutral conductor is half the area of cross-section of phase conductor (Fig. 12.6). Therefore, this type of transmission system reduces the transmission of power to a single-phase, two-wire system with mid-point earthed except that instead of 2 wires there are 2.5 wires.

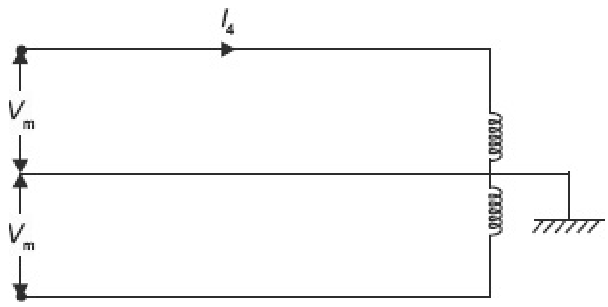

Fig. 12.6 Single-phase three-wire system with mid-point earthed

$$\frac{\text{Volume of conductor material required in single-phase, three-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}}$$

$$= \frac{a_4 \times 2.5l}{a \times 2l}$$

$$= \frac{0.625}{\cos^2\phi}.$$

AC Two-phase System

(a) Two-phase, four-wire system

As shown in Fig. 12.7, the four wires are taken from the ends of the two-phase windings and the mid points of the two windings are connected together. In this type, half of the power is transmitted by each phase, as the voltage is same as the AC single-phase, two-wire system with mid point earthed, so the current will be half of the single-phase system. If the current density remains constant, the area of cross-section will be half of the single-phase system. However, the number of wires is four; the volume of conductor remains the same as in AC single-phase, two-wire system with mid point earthed.

$$\therefore \frac{\text{Volume of conductor material required in two-phase, four-wire system}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{0.5}{\cos^2\phi}$$

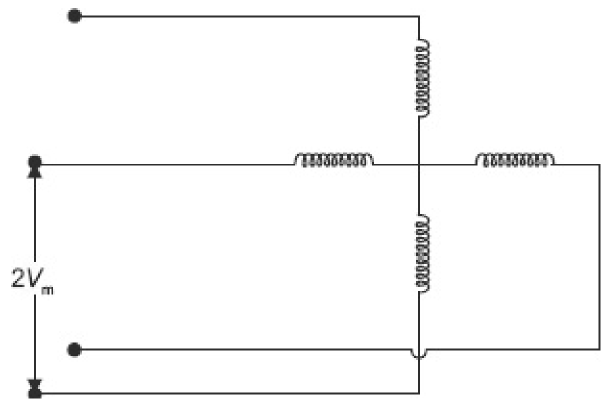

Fig. 12.7 Two-phase, four-wire system

(b) Two-phase three-wire system

The maximum voltage between conductor and earth is Vm as shown in Fig. 12.8.

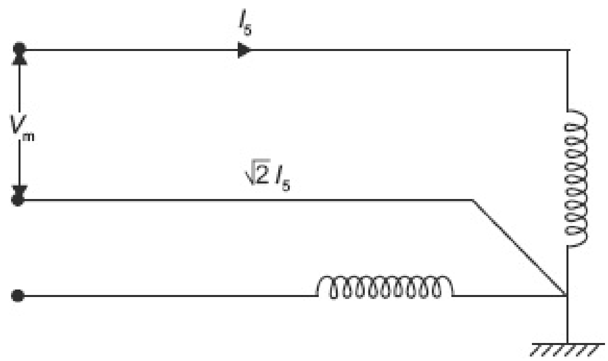

Fig. 12.8 Two-phase, three-wire system

$$V_{\text{rms}} = \frac{V_{\text{m}}}{\sqrt{2}}$$

$$\text{Power transmitted, } P = \sqrt{2} V_{\text{m}} I_5 \cos\phi \qquad (12.11)$$

$$\text{Line current, } I_5 = \frac{P}{\sqrt{2} V_{\text{m}} \cos\phi}$$

Middle wire will act as a common return and current flowing through it will be

$$\sqrt{I_5 + I_5} = \sqrt{2} I_5$$

$$\text{Line losses} = 2I_5^2 R_5 + \left(\sqrt{2} I_5\right)^2 \frac{R_5}{\sqrt{2}} = I_5^2 R_5 \left(2 + \sqrt{2}\right) = \frac{P^2}{2V_{\text{m}}^2 \cos^2\phi} R_5 \left(2 + \sqrt{2}\right) \qquad (12.12)$$

where, R_5 is the resistance of each outer conductor and resistance of middle wire will be $\frac{R_5}{\sqrt{2}}$. Since the area of cross-section of middle wire will be half of the outer wire. For equal transmission loss, comparing the Eqs. (12.2) and (12.12),

$$\frac{2P^2 R}{V_{\text{m}}^2} = \frac{P^2}{2V_{\text{m}}^2 \cos^2\phi} R_5 \left(2 + \sqrt{2}\right)$$

$$\therefore \frac{R}{R_5} = \frac{\left(2 + \sqrt{2}\right)}{4\cos^2\phi} = \frac{a_5}{a}$$

$$\frac{\text{Volume of conductor material required in two-phase, three-wire system}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{a_5 \times \left(2 + \sqrt{2}\right) l}{a \times 2l} = \frac{1.457}{\cos^2\phi}.$$

AC Three-phase Three-wire System

(a) Three-phase three-wire system

The maximum voltage between conductor and earth is Vm as shown in Fig. 12.9.

R.m.s phase voltage, $V_{rms} = \frac{V_m}{\sqrt{2}}$

Line voltage, $V_L = \frac{\sqrt{3}V_m}{\sqrt{2}}$

Power transmitted, $P = \sqrt{3}V_L I_6 \cos\phi = \sqrt{3} \times \sqrt{3}\frac{V_m}{\sqrt{2}} I_6 \cos\phi$ (12.13)

Line current, $I_6 = \frac{\sqrt{2}P}{3V_m \cos\phi}$

Line loss $= 3I_6^2 R_6 = 3 \times \frac{2P^2}{9V_m^2 \cos^2\phi} R_6 = \frac{2P^2}{3V_m^2 \cos^2\phi} R_6$ (12.14)

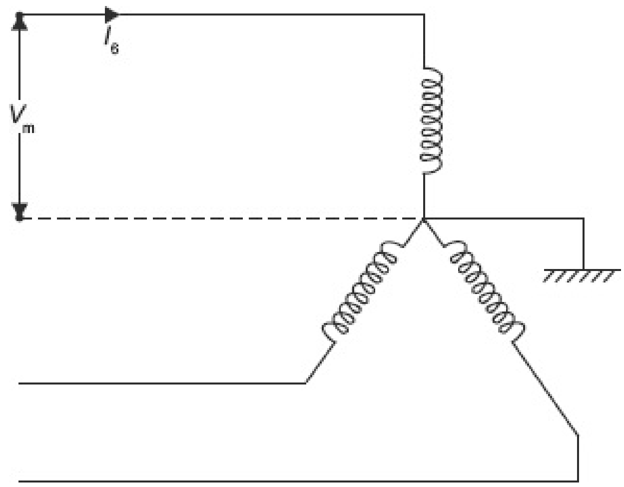

Fig. 12.9 Three-phase, three-wire system

For equal transmission loss, compare Eqs. (12.2) and (11.14)

$$\frac{2P^2R}{V_m^2} = \frac{2P^2R_6}{3V_m^2\cos^2\phi}$$

$$\therefore \frac{R}{R_6} = \frac{1}{3\cos^2\phi} = \frac{a_6}{a}$$

$$\frac{\text{Volume of conductor material required in three-phase, three-wire system}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{a_6 \times 3l}{a \times 2l} = \frac{0.5}{\cos^2\phi}.$$

(b) Three-phase four-wire system

In this case, the neutral wire is taken from the neutral point as shown in Fig. 12.10.

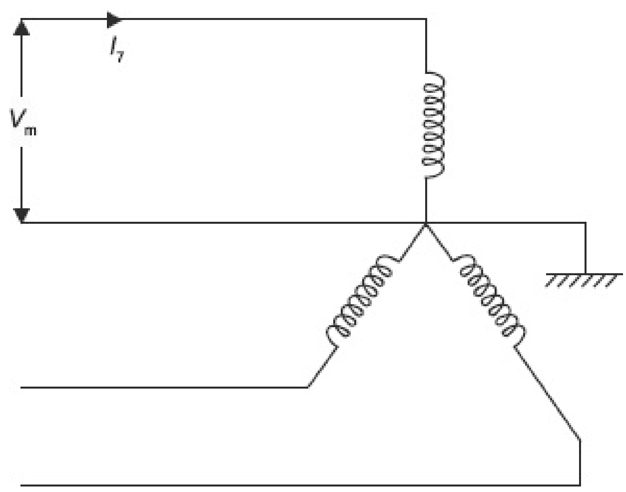

Fig. 12.10 Three-phase, four-wire system

The load will be balanced; there will be no current flowing in the neutral. The area of cross-section of the neutral wire will be half of the line wire.

$$\frac{\text{Volume of conductor material required in three-phase, four-wire system}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}} = \frac{a_6 \times 3.5l}{a \times 2l} = \frac{0.583}{\cos^2 \phi}.$$

12.2.2 Cable Systems

In the case of an underground cable system, the conductors are separated from each other by solid insulation and this insulation is determined by the maximum voltage (V_m) between conductors. Let this V_m be the same for the following underground cable transmission systems.

DC System

(a) Two-wire system with one-end earthed

Let V_m be the maximum voltage between conductors as shown in Fig. 12.11.If the current is I and the resistance of each conductor is R,

Then, power transmitted, $P = V_m I$

Line current, $I = \frac{P}{V_m}$

$$\text{Line losses} = 2I^2R = \frac{2P^2R}{V_m^2} \qquad (12.15)$$

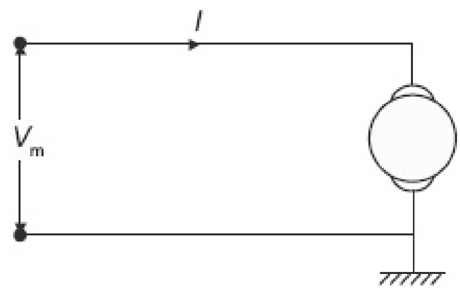

Fig. 12.11 DC two-wire system with one end earthed

(b) Two-wire system with mid-point earthed (Fig. 12.12)

This system is the same as DC two-wire system. Therefore, volume of conductor material is also the same as with one-end earthed system.

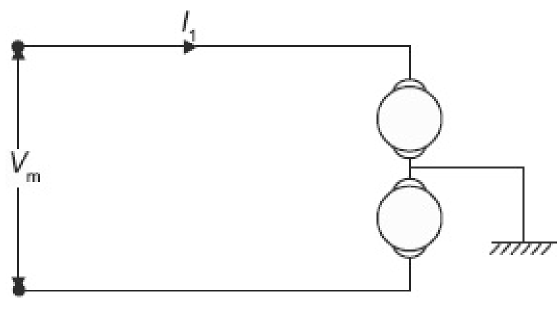

Fig. 12.12 DC two-wire system with mid-point earthed

(c) Three-wire system with mid-point earthed (Fig. 12.13)

Assuming that load is balanced. The voltage between the outer wires is V_m

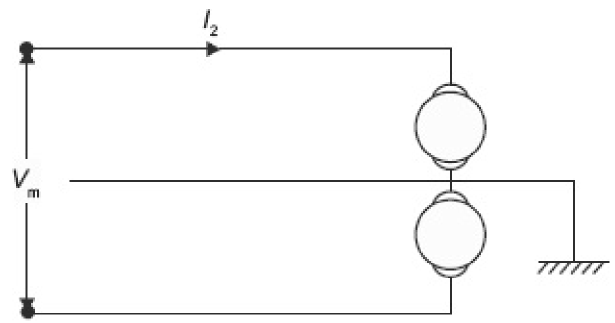

Fig. 12.13 DC three-wire system with mid-point earthed

$$\text{Power transmitted, } P = V_m I_2 \tag{12.16}$$

$$\text{Line current, } I_2 = \frac{P}{V_m}$$

$$\text{Line loss} = 2I_2^2 R_2 = \frac{2P^2 R_2}{V_m^2} \tag{12.17}$$

where, R_2 is the resistance of each outer conductor.

Let the neutral conductor be half the cross-sectional area of that of outer conductor. For a balanced system, no current is passing through the neutral wire.

For equal transmission loss, equating the Eqs. (12.15) and (12.17)

$$2I^2R = 2I_2^2R_2$$

$$\frac{2P^2R}{V_m^2} = \frac{2P^2R_2}{V_m^2}$$

$$\therefore \frac{R}{R_2} = 1 = \frac{a_2}{a}$$

$$\therefore \frac{\text{Volume of conductor material required in DC three-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC, two-wire system with one-end earthed}} = \frac{a_2 \times 2.5l}{a \times 2l}$$

$$= 1.25.$$

AC Single System

(a) Single-phase, two-wire system (Fig. 12.14)

The maximum voltage between conductors is V_m

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Power transmitted, $P = \frac{V_m}{\sqrt{2}} I_3 \cos\phi$ (12.18)

$$I_3 = \frac{P}{\frac{V_m}{\sqrt{2}}\cos\phi} = \frac{\sqrt{2}P}{V_m \cos\phi}$$

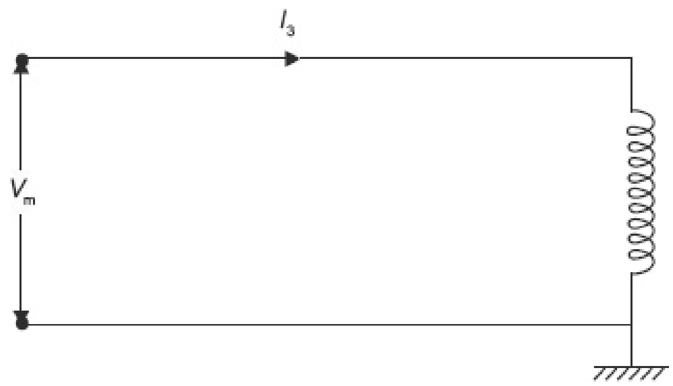

Fig. 12.14 Single-phase two-wire system

$$\text{Line losses} = 2I_3^2 R_3 = 2\times 2\frac{P^2}{V_m^2\cos^2\phi}R_3$$
$$= 4\frac{P^2}{V_m^2\cos^2\phi}R_3 \qquad (12.19)$$

For equal transmission loss, comparing the Eqs. (12.15) and (12.19)

$$2\frac{P^2}{V_m^2}R = 4\frac{P^2 R_3}{V_m^2\cos^2\phi}$$
$$\frac{R}{R_3} = \frac{a_3}{a} = \frac{2}{\cos^2\phi}.$$

$$\frac{\text{Volume of conductor material required in AC single-phase, two-wire system with one end earthed}}{\text{Volume of conductor material required in DC, two-wire system with one end earthed}}$$
$$= \frac{a_3\times 2l}{a\times 2l} = \frac{2}{\cos^2\phi}$$

(b) Single-phase, two-wire system with mid-point earthed (Fig. 12.15)

The maximum voltage between conductors is V_m.

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

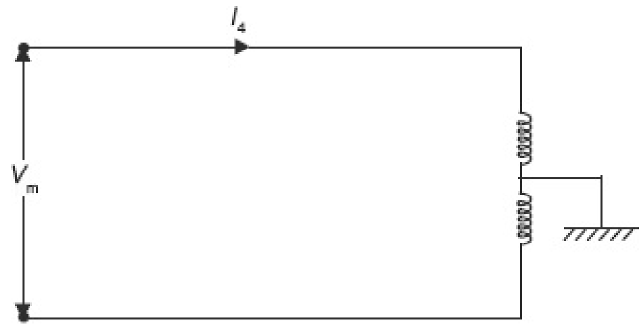

Fig. 12.15 Single-phase two-wire system with mid-point earthed

Power transmitted, $P = \frac{V_m}{\sqrt{2}} I_4 \cos\phi$ (12.20)

Line current, $I_4 = \frac{\sqrt{2}P}{V_m \cos\phi}$

Line losses $= 2I_4^2 R_4 = \frac{4P^2}{V_m^2 \cos^2\phi} R_4$ (12.21)

where, R_4 is the resistance of each conductor

For equal transmission loss, comparing Eqs. (12.15) and (12.21)

$$\frac{2P^2 R}{V_m^2} = \frac{4P^2}{V_m^2 \cos^2\phi} R_4$$

$$\therefore \frac{R}{R_4} = \frac{2}{\cos^2\phi} = \frac{a_4}{a}$$

$$\frac{\text{Volume of conductor material required in single-phase, two-wire system with mid-point earthed}}{\text{Volume of conductor material required in DC two-wire system with one end earthed}}$$

$$= \frac{a_4 \times 2l}{a \times 2l} = \frac{2}{\cos^2\phi}.$$

(c) Single-phase, three-wire system (Fig. 12.16)

Assume that the load is balanced and the neutral conductor is half the area of cross-section. Therefore, this type of transmission system reduces the transmission of power to a single-phase, two-wire system except that instead of 2 wires there are 2.5 wires.

$$\frac{\text{Volume of conductor material required in single-phase, three-wire system}}{\text{Volume of conductor material required in DC two-wire system}} = \frac{a_4 \times 2.5l}{a \times 2l} = \frac{2.5}{\cos^2\phi}.$$

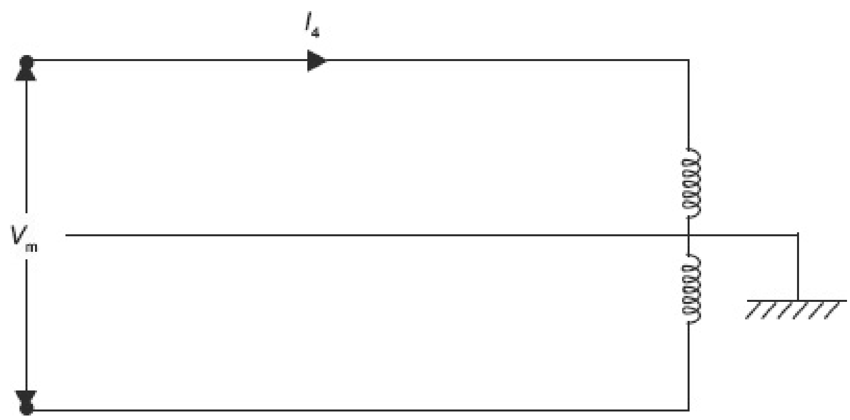

Fig. 12.16 Single-phase three-wire system

AC Two-phase System

(a) Two-phase, four-wire system

As shown in Fig. 12.17, the four wires are taken from the ends of the two-phase windings and the mid-points of the two windings are connected together. In this type, half of the power is transmitted by each phase, as the voltage is the same as the AC single-phase, two-wire system, so the current will be half of the single-phase, two-wire system. If the current density remains constant, the area of cross-section will be half of the single-phase system. However, wires are four; the volume of conductor remains the same as in AC single-phase, two-wire system.

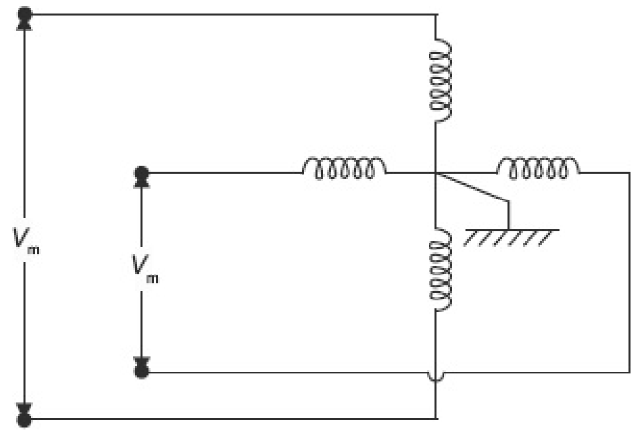

Fig. 12.17 Two-phase, four-wire system

$$\therefore \frac{\text{Volume of conductor material required in two-phase, four-wire system}}{\text{Volume of conductor material required in DC two-wire system with one end earthed}} = \frac{2}{\cos^2\phi}.$$

(b) Two-phase, three-wire systems (Fig. 12.18)

The maximum voltage between conductors = V_m.

Maximum voltage between either outer and neutral wire

$$= \frac{V_m}{\sqrt{2}}$$

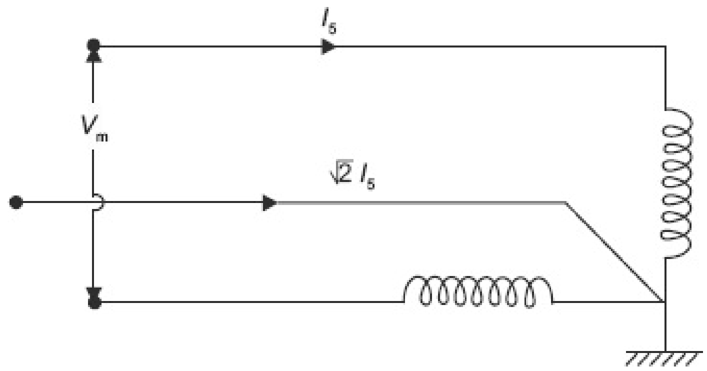

Fig. 12.18 Two-phase three-wire system

R.M.S voltage between the outer and neutral wire =

$$\frac{V_m/\sqrt{2}}{\sqrt{2}} = \frac{V_m}{2}$$

$$\text{Power transmitted, } P = 2V_{rms} I_5 \cos\phi = V_m I_5 \cos\phi \tag{12.22}$$

$$\text{Line current, } I_5 = \frac{P}{V_m \cos\phi}$$

Middle wire will act as a common return and current flowing through it will be

$$= \sqrt{I_5 + I_5} = \sqrt{2} I_5$$

$$\text{Line losses} = 2I_5^2 R_5 + \left(\sqrt{2} I_5\right)^2 \frac{R_5}{\sqrt{2}} = I_5^2 R_5 \left(2+\sqrt{2}\right) = \frac{P^2}{V_m^2 \cos^2\phi} R_5 \left(2+\sqrt{2}\right) \tag{12.23}$$

where, R_5 is the resistance of each outer conductor and resistance of middle wire will be $\frac{R_5}{\sqrt{2}}$. Since the area of cross-section of middle wire will be half of the outer wire.

For equal transmission loss, comparing the (Eqs. 12.15) and (12.23)

$$\frac{2P^2 R}{V_m^2} = \frac{P^2}{V_m^2 \cos^2\phi} R_5 \left(2+\sqrt{2}\right)$$

$$\therefore \frac{R}{R_5} = \frac{\left(2+\sqrt{2}\right)}{\cos^2\phi} = \frac{a_5}{a}$$

$$\frac{\text{Volume of conductor material required in two-phase, three-wire system}}{\text{Volume of conductor material required in DC two-wire system with one end earthed}}$$

$$= \frac{a_5 \times \left(2+\sqrt{2}\right) l}{a \times 2l} = \frac{2.914}{\cos^2\phi}.$$

AC Three-phase System

(a) Three-phase, three-wire system (Fig. 12.19)

The maximum voltage between conductors is V_{m}.

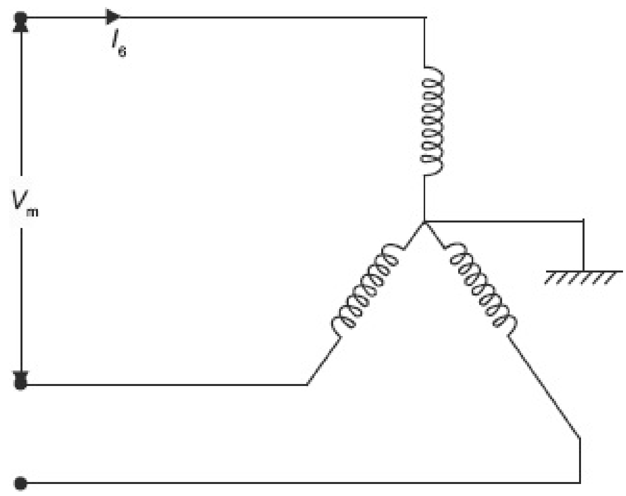

Fig. 12.19 Three-phase three-wire system

$$V_{\mathrm{rms}} = \frac{V_{\mathrm{m}}}{\sqrt{2}}$$

Line voltage, $V_{\mathrm{L}} = \frac{V_{\mathrm{m}}}{\sqrt{2}}$

Power transmitted, $P = \sqrt{3} V_{\mathrm{L}} I_6 \cos\phi = \sqrt{3} \frac{V_{\mathrm{m}}}{\sqrt{2}} I_6 \cos\phi$ (12.24)

$$\text{Line current, } I_6 = \frac{\sqrt{2}P}{\sqrt{3}V_m \cos\phi}$$

$$\text{Line loss} = 3I_6^2 R_6 = 3\frac{2P^2}{3V_m^2 \cos^2\phi} R_6$$

$$= \frac{2P^2}{V_m^2 \cos^2\phi} R_6. \tag{12.25}$$

For equal transmission loss, comparing (Eqs. 12.15) and (12.25)

$$\frac{2P^2 R}{V_m^2} = \frac{2P^2 R_6}{V_m^2 \cos^2\phi}$$

$$\therefore \frac{R}{R_6} = \frac{1}{\cos^2\phi} = \frac{a_6}{a}$$

$$\frac{\text{Volume of conductor material required in three-phase, three-wire system}}{\text{Volume of conductor material required in DC two-wire system with one end earthed}} = \frac{a_6 \times 3l}{a \times 2l} = \frac{1.5}{\cos^2\phi}.$$

(b) Three-phase four-wire system

In this case, the neutral wire is taken from the neutral point as shown in Fig. 12.20.

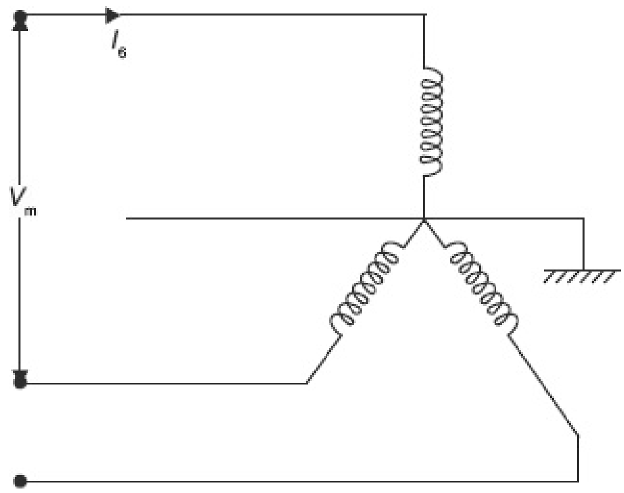

Fig. 12.20 Three-phase four-wire system

The load will be balanced; there will be no current flowing in the neutral. The area of cross-section of the neutral wire will be half of the line wire.

$$\frac{\text{Volume of conductor material required in three-phase, four-wire system}}{\text{Volume of conductor material required in DC two-wire system with one end earthed}}$$
$$= \frac{a_6 \times 3.5l}{a \times 2l} = \frac{1.75}{\cos^2\phi}.$$

Results obtained above are summarized in Table 12.1.

Table 12.1 Comparison of conductor efficiencies of different transmission systems

Sl. no	Type of system	Overhead system	Underground cable
1.	DC two-wire system with one end earthed	1	1
2.	DC two-wire system with mid-point earthed	0.25	1
3.	DC three-wire system with mid-point earthed	0.3125	1.25
4.	Single-phase, two-wire system with one end earthed	$\frac{2}{\cos^2\phi}$	$\frac{2}{\cos^2\phi}$
5.	Single-phase, two-wire system with mid-point earthed	$\frac{0.5}{\cos^2\phi}$	$\frac{2}{\cos^2\phi}$
6.	Single-phase, three-wire system with mid-point earthed	$\frac{0.625}{\cos^2\phi}$	$\frac{2.5}{\cos^2\phi}$
7.	Two-phase, four-wire system	$\frac{0.5}{\cos^2\phi}$	$\frac{2}{\cos^2\phi}$
8.	Two-phase, three-wire system	$\frac{1.457}{\cos^2\phi}$	$\frac{2.914}{\cos^2\phi}$
9.	Three-phase, three-wire system	$\frac{0.5}{\cos^2\phi}$	$\frac{1.5}{\cos^2\phi}$
10.	Three-phase, four-wire system	$\frac{0.583}{\cos^2\phi}$	$\frac{1.75}{\cos^2\phi}$

From Table 12.1, if DC system is used for transmission of power, there is greater saving in conductor material over AC system (since $\cos\phi$ is less than one). However, DC system is not used for electrical power transmission because of some technical problems (discussed in Chapter 15).

Among the AC systems, the conductor material required in three-phase, three-wire system is less compared with single-phase, two-wire system on the basis of equal voltage between conductor and earth and on the basis of voltage between the conductors, respectively.

Due to the greater convenience and efficiency of poly-phase plant, power is always generated at three-phase and hence a three-phase, three-wire system is adopted universally for transmission.

Test Yourself

1. Why is the DC system not used for distribution?
2. Is the efficiency of poly-phase systems more? If yes, why?

Example 12.1

Compare the conductor efficiencies for an AC, three-phase, four-wire and a DC, three-wire system on the bases of equal maximum potential difference between any two conductors (other than neutral). Assume equal power transmitted over equal length of line with equal power loss in the line and the neutral has half of the cross-sectional area of the outers.

Solution:

(a) DC, three-wire (Fig. 12.21)

Let, the voltage between the outer conductors be V_{m}

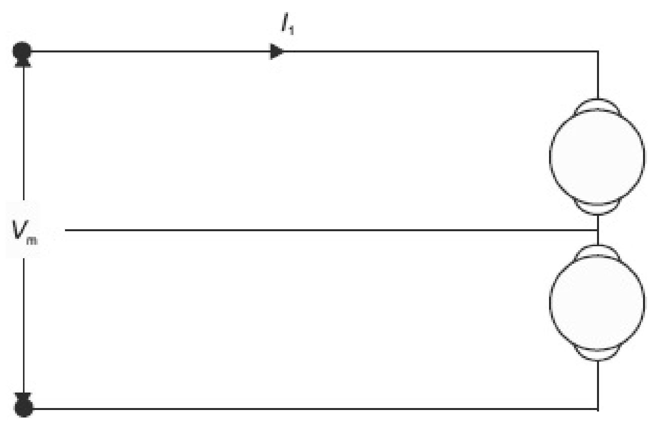

Fig. 12.21 DC three-wire system

Assuming balanced load system

$$\text{Power transmitted, } P = V_m I_1 \tag{12.26}$$

$$\text{The line current, } I_1 = \frac{P}{V_m}$$

$$\text{Power loss} = 2I_1^2 R_1 = 2\frac{P^2}{V_m^2}R_1 \tag{12.27}$$

where, R_1 is the resistance of conductor

Let a_1 be the cross-sectional area of the outer conductor

The volume of copper used in three-wire, DC system, $u_1 = 2.5a_1l$ (12.28)

(b) AC, three-phase, four-wire system (Fig. 12.22)

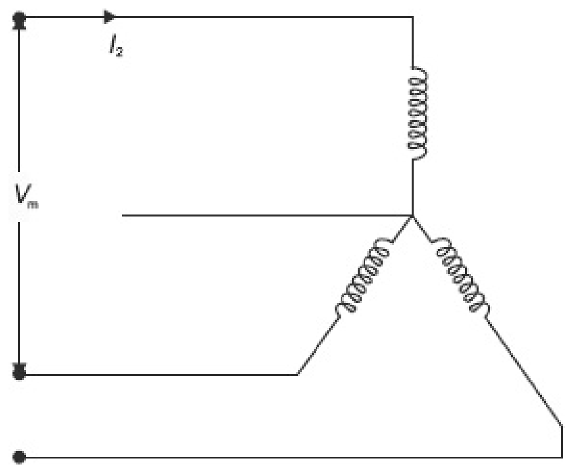

Fig. 12.22 AC three-phase, four-wire system

$$\text{Power}, P = \sqrt{3}\frac{V_m}{\sqrt{2}}I_2\cos\phi \quad (12.29)$$

$$\text{Line current}, I_2 = \frac{\sqrt{2}P}{\sqrt{3}V_m\cos\phi}$$

$$\text{Power loss} = 3I_2^2R_2$$

$$= 3\times\frac{2P^2}{3V_m^2\cos^2\phi}R_2 = \frac{2P^2R_2}{V_m^2\cos^2\phi} \quad (12.30)$$

Volume of copper used in three-phase, four-wire system, $u_2 = 3.5a_2l$ (12.31)

For equal transmission loss, comparing Eqs. (12.27) and (12.30)

$$\frac{2P^2}{V_m^2}R_1 = \frac{2P^2R_2}{V_m^2\cos^2\phi}$$

$$\frac{R_1}{R_2} = \frac{1}{\cos^2\phi} = \frac{a_2}{a_1} \tag{12.32}$$

From Eqs. (12.28) and (12.31)

$$u_1 : u_2 = 2.5a_1l : 3.5a_2l$$
$$= 2.5a_1l : 3.5 \times \frac{a_1}{\cos^2\phi}l$$
$$\therefore u_1 : u_2 = 1 : \frac{1.4}{\cos^2\phi}.$$

Example 12.2

Show that in a cable transmission scheme the ratio of volumes of conductor in DC two-wire, AC, single-phase and AC, three-phase are given by

$$u_1 : u_2 : u_3 = 1 : \frac{2}{\cos^2\phi} : \frac{1.5}{\cos^2\phi}$$

where, cosϕ is the power factor of load. Assume equal power transmitted over equal length with equal losses and maximum voltage between conductors is the same in all cases.

Solution:

(a) DC, two-wire system (Fig. 12.23)

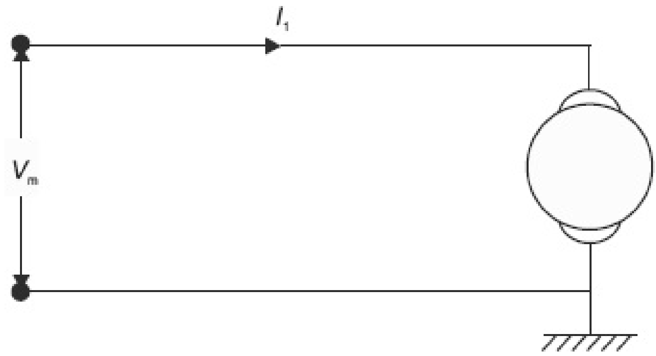

Fig. 12.23 DC two-wire system

Let, V_m be the voltage between the lines
Assuming balanced load system

Power transmitted, $P = V_m I_1$ (12.33)

The line current, $I_1 = \frac{P}{V_m}$

Power loss $= 2I_1^2 R_1 = 2\frac{P^2}{V_m^2} R_1$ (12.34)

where, R_1 is the resistance of conductor.
Let a_1 be the cross-sectional area of the outer conductor

The volume of copper used in three-phase, DC system, $u_1 = 2a_1 l$ (12.35)

(b) Single-phase, AC, two-wire system (Fig. 12.24)

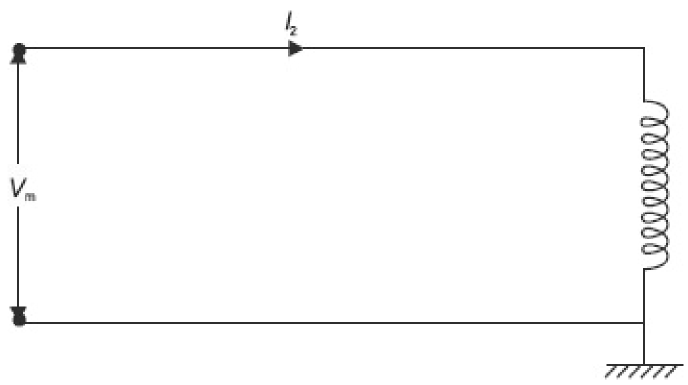

Fig. 12.24 Single-phase, AC, two-wire system

Let, V_m be the maximum voltage between lines,

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\text{Power transmitted, } P = \frac{V_m}{\sqrt{2}} I_2 \cos\phi \qquad (12.36)$$

$$\text{Line current, } I_2 = \frac{P}{\frac{V_m}{\sqrt{2}}\cos\phi} = \frac{\sqrt{2}P}{V_m \cos\phi}$$

$$\text{Power loss} = 2I_2^{\,2}R_2 = 2\times\frac{2P^2}{V_m^2\cos^2\phi}R_2 \qquad (12.37)$$

$$\text{Volume of copper used in three-phase, two-wire system, } u_2 = 2a_2l \qquad (12.38)$$

For equal transmission loss, comparing the Eqs. (12.34) and (12.37)

$$\frac{2P^2R_1}{V_m^2} = 2\times\frac{2P^2}{V_m^2\cos^2\phi}R_2$$

$$\frac{R_1}{R_2} = \frac{2}{\cos^2\phi} = \frac{a_2}{a_1} \qquad (12.39)$$

(c) AC, three-phase, three-wire system (Fig. 12.25)

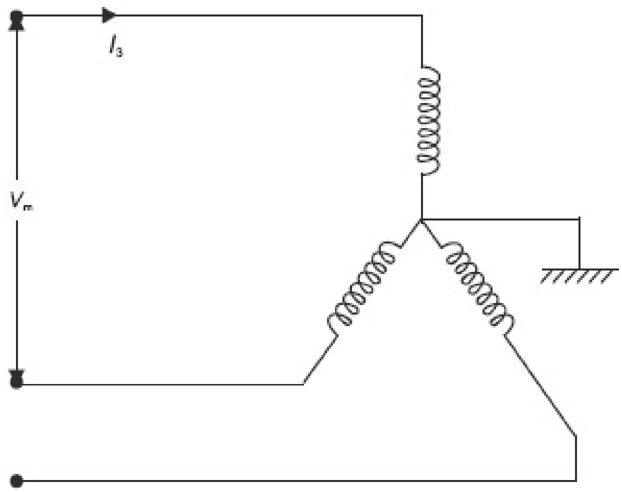

Fig. 12.25 AC, three-phase, three-wire system

$$\text{Power transmitted, } P = \sqrt{3}\frac{V_m}{\sqrt{2}} I_3 \cos\phi \tag{12.40}$$

$$\text{Line current, } I_3 = \frac{\sqrt{2}P}{\sqrt{3}V_m \cos\phi}$$

$$\text{Power loss} = 3I_3^2 R_3 = 3\frac{2P^2}{3V_m^2 \cos^2\phi} R_3 = \frac{2P^2 R_3}{V_m^2 \cos^2\phi} \tag{12.41}$$

$$\text{Volume of copper used in three-phase, three-wire system, } u_3 = 3\,a_2 l \tag{12.42}$$

For equal transmission loss, comparing Eqs. (12.34) and (12.41)

$$\frac{2P^2}{V_m^2} R_1 = \frac{2P^2 R_3}{V_m^2 \cos^2\phi}$$

$$\frac{R_1}{R_3} = \frac{1}{\cos^2\phi} = \frac{a_3}{a_1} \tag{12.43}$$

From Eqs. (12.35), (12.38) and (12.42)

$$u_1 : u_2 : u_3 = 2a_1 l : 2a_2 l : 3a_3 l$$
$$= 2a_1 l : 2\times\frac{2a_1 l}{\cos^2\phi} : 3\times\frac{a_1}{\cos^2\phi} l$$
$$\therefore u_1 : u_2 : u_3 = 1 : \frac{2}{\cos^2\phi} : \frac{1.5}{\cos^2\phi}.$$

Example 12.3

Compare the conductor weighing for (i) three-phase, four-wire (ii) DC two-wire system for the same line voltages and same losses in both the cases. Assume a balanced load and control wire to be half the cross-section of outer one.

Solution:

Let V be the voltage between two lines.

(i) AC, three-phase, four-wire system (Fig. 12.26)

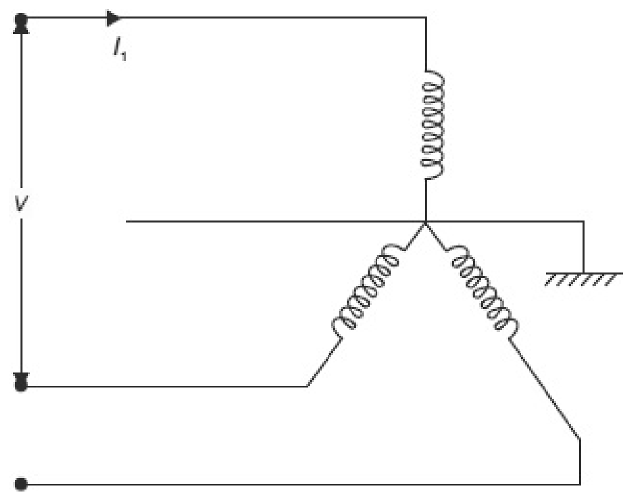

Fig. 12.26 AC, three-phase, four-wire system

Power transmitted, $P = \sqrt{3}VI_1\cos\phi$

Line current, $I_1 = \dfrac{P}{\sqrt{3}V\cos\phi}$

Power loss = $3I_1^2R_1$ (For balanced system, power loss in neutral is zero)

$$= \frac{3 \times P^2 R_1}{3V^2\cos^2\phi} = \frac{P^2R_1}{V^2\cos^2\phi} \qquad (12.44)$$

Volume of copper used, $u_1 = 3.5a_1l$

(Since cross-section of neutral conductor is $\frac{1}{2}$ of the outer conductor)

(ii) DC two-wire system (Fig. 12.27)

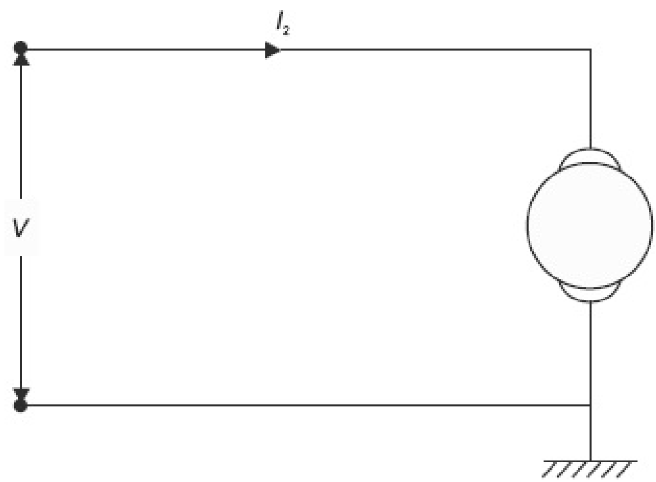

Fig. 12.27 DC two-wire system

Power transmitted, $P = VI_2$

Line current, $I_2 = \dfrac{P}{V}$

$$\text{Power loss} = 2I_2^2R_2 = 2\frac{P^2}{V^2}R_2 \qquad (12.45)$$

Volume of copper used, $u_2 = 2a_2l$

For equal transmission loss, compare Eqs. (12.44) and (12.45)

$$\frac{P^2}{V^2\cos^2\phi}R_1 = 2\frac{P^2}{V^2}R_2$$

$$\frac{R_1}{R_2} = 2\cos^2\phi = \frac{a_2}{a_1}$$

$$\therefore \frac{\text{Volume of copper used in DC two-wire system}}{\text{Volume of copper used in three-phase, four-wire system}} = \frac{2a_2l}{3.5a_1l} = 2\cos^2\phi \times \frac{2}{3.5} = \frac{4\cos\phi}{3.5}.$$

Example 12.4

Discuss the basis for comparison between overhead system and multi-core belted-type of cable system. Derive the conductor efficiencies of a three-phase, three-wire system for both the cases.

Solution:

For comparing the amount of conductor used in different systems are:

1. When the transmission is being carried on overhead lines, great care is taken to insulate the conductors from the cross-arms and supporting towers since there is a minimum disruptive stress between them. As the towers are earthed, the maximum voltage between each conductor and earth forms the basis of comparison of the volume of the conductor.
2. When the transmission is through underground three-phase cables, the maximum disruptive stress is between the two conductors of the cable. Therefore, for the comparison of the volume of the conductor used in different systems of transmission, the maximum voltage between the conductors should be the base.

(a) Three-phase, three-wire for overhead system (Fig. 12.28)

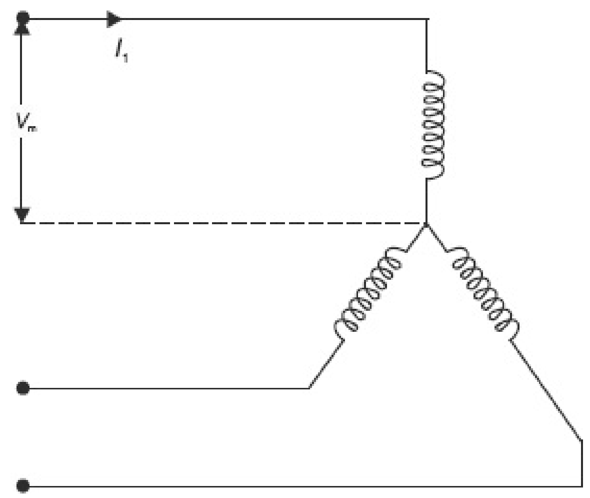

Fig. 12.28 Three-phase, three-wire overhead system

Let V_m be the maximum voltage between line and ground point.

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\text{Power transmitted, } P = 3\frac{V_m}{\sqrt{2}} I_1 \cos\phi$$

$$\text{Line current, } I_1 = \frac{\sqrt{2}P}{3V_m \cos\phi}$$

$$\text{Power loss} = 3I_1^2 R_1 = 3 \times \frac{2P^2}{9V_m^2 \cos^2\phi} \times R_1$$

$$= \frac{2P^2}{3V_m^2 \cos^2\phi} \times R_1 \qquad (12.46)$$

Volume of conductor used in three-phase, three-wire overhead system, $u_1 = 3a_1 l$

where, a_1 is the cross-sectional area of the conductor.

(b) Three-phase, three-wire for cable system (Fig. 12.29)

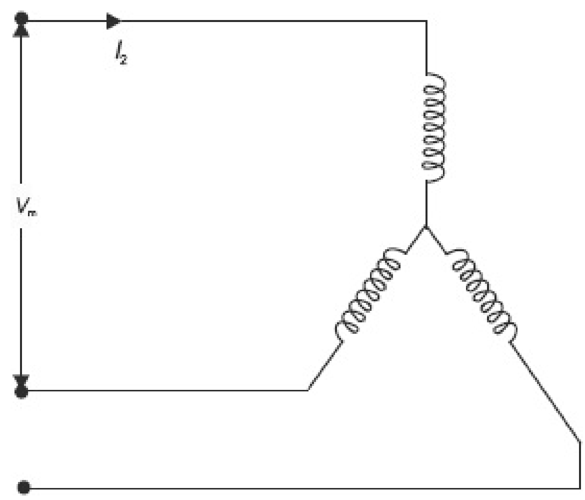

Fig. 12.29 Three-phase, three-wire for cable system

Let V_m be the maximum voltage between the lines.

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\text{Power transmitted, } P = \sqrt{3}\,\frac{V_m}{\sqrt{2}} I_2 \cos\phi$$

$$\text{Line current, } I_2 = \frac{P}{\sqrt{3} \times \frac{V_m}{\sqrt{2}} \cos\phi} = \frac{\sqrt{2}P}{\sqrt{3}\,V_m \cos\phi}$$

$$\text{Power loss} = 3I_2^2 R_2 = 3 \times \frac{2P^2}{3V_m^2 \cos^2\phi} \times R_2$$

$$= \frac{2P^2}{V_m^2 \cos^2\phi} \times R_2 \qquad (12.47)$$

Volume of conductor used in three-phase, three-wire cable system, $u_2 = 3a_2 l$

where, a_2 is the cross-sectional area of the conductor

For equal transmission loss, comparing Eqs. (12.46) and (12.47),

$$\frac{2P^2}{3V_m^2\cos^2\phi}R_1 = \frac{2P^2}{V_m^2\cos^2\phi}R_2$$

$$\frac{R_1}{R_2} = 3 = \frac{a_2}{a_1}$$

$$u_1 : u_2 = 3a_1 l : 3a_2 l$$
$$= 3a_1 l : 3 \times 3a_1 l$$

$$u_1 : u_2 = 1 : 3$$

$$\therefore \frac{\text{Volume of copper used in three-phase, three-wire overhead system}}{\text{Volume of copper used in three-phase, three-wire cable system}} = \frac{u_1}{u_2} = \frac{1}{3}.$$

Example 12.5

Power is distributed to the consumers either by a three-wire DC system (Fig. 12.30(a)) or by an AC three-phase, four-wire (Fig. 12.30(b)) system. Compare the amount of conductors required in the two systems. Assume the same voltage over consumer's terminals, same percentage loss, balanced load, and unity power factor. The middle wires are of the same cross-sectional area as that of outers.

Solution:

DC three-wire system

Let V be the voltage at consumer's terminals

DC power supplied to consumers, $P = 2VI_1$

$$\text{Line current, } I_1 = \frac{P}{2V}$$

$$\text{Power losses} = 2I_1^2 R_1 = 2 \times \frac{P^2 R_1}{4V^2} = \frac{P^2 R_1}{2V^2}$$

$$\%\text{ losses} = \frac{\frac{P^2 R_1}{2V^2}}{P} = \frac{PR_1}{2V^2} \qquad (12.48)$$

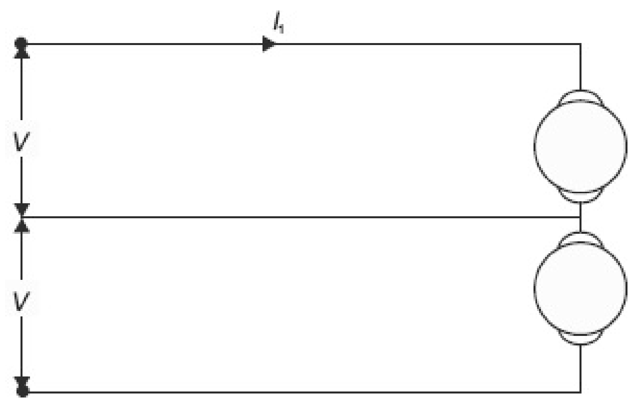

Fig. 12.30(a) DC three-wire system

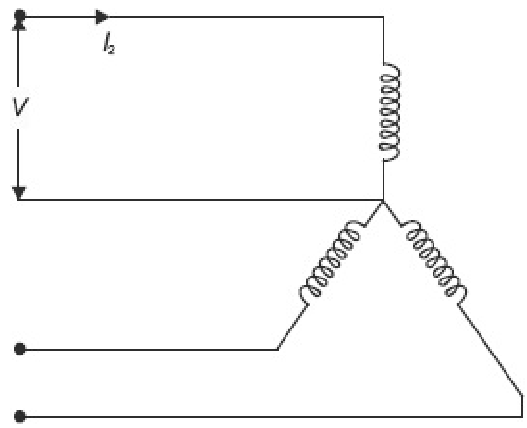

Fig. 12.30(b) AC three-phase, four-wire system

AC power supplied to consumers, $P = 3VI_2 \cos\phi = 3VI_2 \qquad [\because \cos\phi = 1]$

Line current, $I_2 = \dfrac{P}{3V}$

Power losses $= 3I_2^2 R_2 = 3 \times \dfrac{P^2 R_2}{9V^2} = \dfrac{P^2 R_2}{3V^2}$

$$\text{For same percentage losses} = \frac{\dfrac{P^2 R_2}{3V^2}}{P} = \frac{PR_2}{3V^2} \qquad (12.49)$$

For equal % losses, compare Eqs. (12.48) and (12.49)

$$\frac{PR_1}{2V^2} = \frac{PR_2}{3V^2}$$

$$\frac{R_2}{R_1} = \frac{3}{2} = \frac{a_1}{a_2}$$

$$u_1 : u_2 = 3\,a_1 l : 4a_2 l = 3\,a_1 l : 4\frac{2a_1}{3} l = 1 : \frac{8}{9}.$$

Example 12.6

An existing DC network consisting of two overhead wires is to be converted into a three-phase, three-wire system by the addition of a third conductor with the same cross-section of the existing two conductors. Calculate the percentage additional load which can now be transmitted if the voltage between wires, percentage loss, and the line remain unchanged. Assume a balanced load of unity power factor.

Solution:

Let us assume the voltage between the wires to be equal to V

Power supply by DC, two-wire system (Fig. 12.31(a)), $P = VI_1$

Line current, $I_1 = \dfrac{P}{V}$

Line losses = $2I_1^2R$

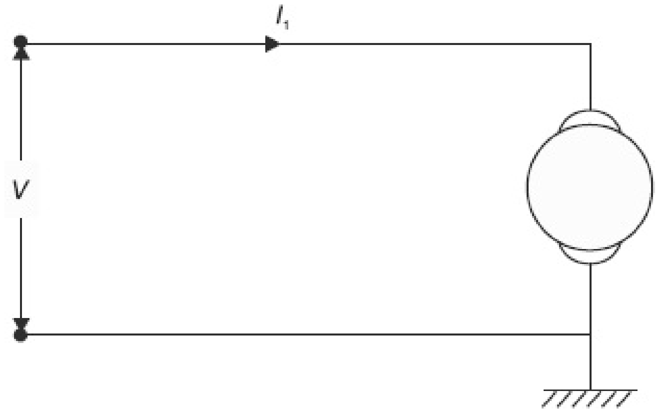

Fig. 12.31(a) DC two-wire system

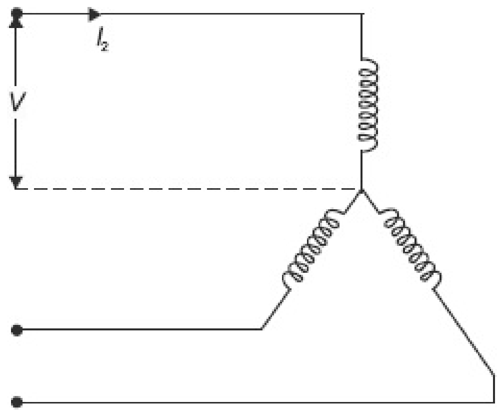

Fig. 12.31(b) AC three-phase, three-wire system

$$\% \text{ losses} = \frac{2I_1^2 R}{V I_1} = \frac{2I_1 R}{V} \tag{12.50}$$

Power supplied by the AC three-phase, three-wire system (Fig. 12.31(b)) is

$$= \sqrt{3} V I_2 \cos\phi$$
$$= \sqrt{3} V I_2 \qquad [\because \text{at u.p.f}] \tag{12.51}$$
$$\text{losses} = 3I_2^2 R$$
$$\% \text{losses} = \frac{3I_2^2 R}{\sqrt{3} V I_2} = \frac{\sqrt{3} I_2 R}{V}$$

Compare Eqs. (12.50) and (12.52)

$$\frac{2I_1 R}{V} = \frac{\sqrt{3} I_2 R}{V}$$
$$\therefore \frac{I_2}{I_1} = \frac{2}{\sqrt{3}}$$

Substitute the value of I_2 in Eq. (12.51)

$\therefore$ Power supply = $\sqrt{3} \times V \times 2\frac{I_1}{\sqrt{3}} = 2\,VI_1$

$\therefore$ Extra power transmitted = $2VI_1 - VI_1 = VI_1$

$\therefore$ Percentage of extra power transmitted = $\frac{VI_1}{VI_1} \times 100 = 100\%$.

Example 12.7

A power of 20 kW is to be distributed in cities by a two-wire, DC system or by a three-wire DC system. The consumer's terminal voltage is 200 V. Compare the weight of conductor on the basis of equal efficiencies. Assume balanced loads and cross-sectional area of the middle wire is half of that of outers.

Solution:

(a) Two-wire DC system

Power distributed, $P = VI_1 = 20$ kW

$$\text{Current, } I_1 = \frac{20 \times 1000}{200} = 100\,\text{A}$$

Let R be the resistance of conductor.

$$\text{Line drop} = 2\,I_1 R_1$$

$$\begin{aligned}\text{Line efficiency} &= \frac{VI_1}{VI_1 + 2I_1^2 R_1} \\ &= \frac{V}{V + 2I_1 R_1} \\ &= \frac{200}{200 + 2 \times 100 \times R_1} \qquad (12.53)\end{aligned}$$

(b) Three-wire DC system

Power distributed, $P = 2VI_2 = 20$ kW

$$\text{Current, } I_2 = \frac{20\times1000}{2\times200} = 50\,\text{A}$$

There is no current in the two middle wires, since load is balanced. Let R_2 be the resistance/conductor;

total line drop = $2I_2R_2$

$$\text{Line efficiency} = \frac{2VI_2}{2VI_2 + 2I_2^2R_2}$$

$$= \frac{V}{V + I_2R_2}$$

$$= \frac{200}{200 + 50R_2} \tag{12.54}$$

For equal efficiency, compare Eqs. (12.53) and (12.54)

$$\frac{200}{200 + 200R_1} = \frac{200}{200 + 50R_2}$$

$$200 + 50R_2 = 200 + 200R_1$$

$$\frac{R_2}{R_1} = \frac{200}{50} = 4 = \frac{a_1}{a_2} \qquad \left(\because \frac{a_2}{a_1} = \frac{1}{4}\right)$$

$$\therefore \frac{\text{Volume of copper used in DC, two-wire system}}{\text{Volume of copper used in DC, three-wire system}} = \frac{u_1}{u_2} = \frac{2a_1l}{2.5a_2l} = \frac{2\times4a_2l}{2.5a_2l} = 3.2.$$

Example 12.8

A three-phase, four-wire, AC distribution system with 230 V between each line and neutral has a balanced load of 500 kW at 0.8 p.f. lagging (Fig. 12.32). An additional lamp load of ratings 75 kW, 150 kW, and 225 kW are connected between each line and neutral, respectively. Calculate the neutral current.

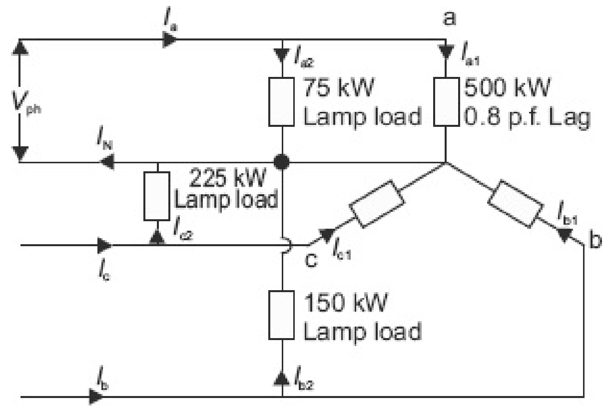

Fig. 12.32 Circuit diagram for Example 12.8

Solution:

Load, $P = 3V_{ph}I_{ph}\cos\phi = 500\,\text{kW}$

Line current due to a balanced load, $I_{a1} = \dfrac{500 \times 1000}{3 \times 230 \times 0.8} = 905.8\angle -36.87^\circ\,\text{A}$

Line current due to a balanced load, $I_{b1} = 905.8\angle -156.87^\circ\ \text{A}$

Line current due to a balanced load, $I_{c1} = 905.8\angle -276.87^\circ\ \text{A}$

Current due to an additional load on phase "a", $I_{a2} = \dfrac{75 \times 1000}{3 \times 230} = 108.7\,\text{A}.$

$\therefore$ Current due to an additional load on phase "a", $I_a = I_{a1} + I_{a2}$

$$= 905.8\angle -36.87^\circ + 108.7$$
$$= 724.64 - j543.48 + 108.7$$
$$= (833.34 - j543.48)\,\text{A}.$$

Current due to an additional load on phase "b", $I_{b2} = \dfrac{150 \times 1000}{3 \times 230} = 217.4\,\text{A}.$

$\therefore$ Current due to an additional load on phase "b", $I_b = I_{b1} + I_{b2}$

$$= 905.8\angle -156.87^\circ + 217.4$$
$$= -833 - j355.83 + 217.4$$
$$= (-615.6 - j355.83)\,\text{A}.$$

Current due to an additional load on phase "c", $I_{c1} = \dfrac{225 \times 1000}{3 \times 230} = 326.09\,\text{A}.$

$\therefore$ Current due to an additional load on phase "c", $I_c = I_{c1} + I_{c2}$

$$= 905.8\angle -276.87^\circ + 326.09$$
$$= 108.35 + j899.3 + 326.09$$
$$= (434.44 + j899.3)\,\text{A}.$$

$\therefore$ Neutral current, $I_N = I_a + I_b + I_c$

$$= (833.34 - j543.48 - 615.6 - j355.83 + 434.44 + j899.3)\,\text{A}$$
$$= 652.18 - j0.01 = 652.18\angle -8.78^\circ\,\text{A}.$$

12.3 CHOICE OF SYSTEM FREQUENCY

Frequency of a system must be fixed for maintaining constant speed of AC motors. In all interconnected power systems, the frequency is kept constant because this is useful for the operation of synchronous clocks directly from the power mains. In many countries, one common supply frequency is adopted to facilitate the interchange of energy between different organizations.

Electrical lights do not operate satisfactorily at lower frequencies because lower frequencies cause a noticeable and undesirable flicker in the light. Hence, a frequency of 50 Hz or 60 Hz is maintained. Higher frequencies also have the advantage of providing a much greater range of speed variation for synchronous and induction motors. For example, synchronous and induction motors with frequency of 50 Hz may be designed for nominal speed of 3000, 1500 or 1000 rpm, etc. While motors with frequency of 25 Hz are confined to 1500, 750 or 500 rpm. In case of transformers, at lower frequencies, the number of flux linkages required is more. Therefore, the number of turns required is more, and hence, more cost is involved.

The inductive reactance of lines and machinery also vary with frequency. The reactance at frequency 25 Hz is double the reactance at 50 Hz frequency. The high reactance of the line tends to produce very poor voltage regulation.

12.4 CHOICE OF SYSTEM VOLTAGE

In both AC and DC systems, the actual power transmitted for a given current is proportional to the system voltage. For a given power, the current decreases as the voltage increases. This possibility of a reduction in current for an increase in voltage has an important economic aspect of power transmission. In the case of a transmission system, the load which the conductors can carry will depend on the heating effects of the current.

Hence, if the current can be reduced by using a high voltage, the resistance may be increased without incurring additional losses and causing a greater temperature rise. Therefore, we can use smaller conductors, thus, saving cost. Alternatively, with the same conductor, the losses and voltage drops are reduced and the efficiency of transmission is increased.

From the above arguments, we conclude that high voltages are an essential requirement for an efficient long-distance transmission system. In addition, in case of an AC system, high power is desirable, provided the insulation levels are increased.

12.5 ADVANTAGES OF HIGH-VOLTAGE TRANSMISSION

1. If increase in transmission voltage reduces the size of the conductor, the cost of the conductor and supporting structure materials is reduced.
2. If increase in transmission voltage reduces the line current, this results in a line loss reduction.
3. With the increase in transmission voltage and reduction in line losses, the line efficiency sees an increase.
4. Due to low current at high-transmission voltage, the voltage drop in the line is low. This leads to improved voltage regulation.
5. There is flexibility for future system growth.
6. Increase in transmission capacity is possible.
7. There is an increase in surge-impedance loading.

12.6 EFFECT OF SUPPLY VOLTAGE

If the voltage is raised n times, the current in the line required to transmit the same amount of power is reduced by n times. Therefore, the line loss I^2R is reduced n^2 times. Hence, the line efficiency is increased.

Case I: Consider a DC two-wire transmission as shown in Fig. 12.33(a).

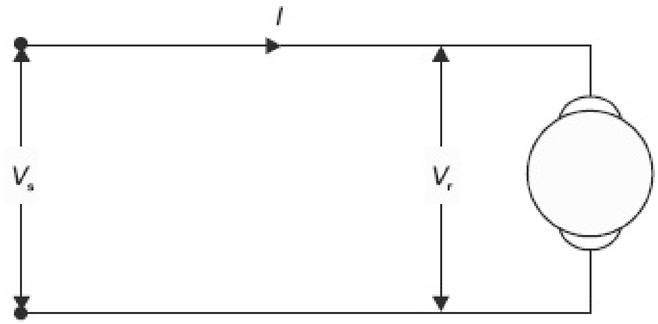

Fig. 12.33(a) DC two-wire transmission system

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}} = \frac{V_r I}{V_s I} = \frac{V_r}{V_s}$$

We know $V_r = V_s - IR$

where,

R = resistance of both go and return conductors

V_s = sending-end voltage

V_r =receiving-end voltage

I = current delivering

$$\eta\% = \left(\frac{V_s - IR}{V_s}\right) \times 100 = 100 - \frac{IR}{V_s} \times 100$$

where, $\frac{IR}{V_s} \times 100$, is the voltage drop in both the

conductors and expressed as a percentage of the sending-end voltage. It is called the percentage voltage drop. Therefore, as the voltage is increased, the efficiency is increased for a given voltage drop.

Due to the increase of voltage by n times, the current in the line is reduced n times, and hence, the line loss is reduced n^2 times. Hence, to transmit with the same loss as before, the resistance of the line can be increased n^2 times, i.e., amount of conductor material can be decreased n^2 times. Therefore, for a given loss, amount of power to be delivered, there is a great saving in conductor material.

Case II: Consider an AC, three-phase system as shown in Fig. 12.33(b).

Let the power P watt be delivered by a three-phase line at a line voltage of V and a power factor of $\cos \phi$.

The line current $I = \frac{P}{\sqrt{3}V \cos\phi}$

Let l = length of the conductor

σ = current density

ρ = specific resistance of conductor material

a = cross-section of conductor.

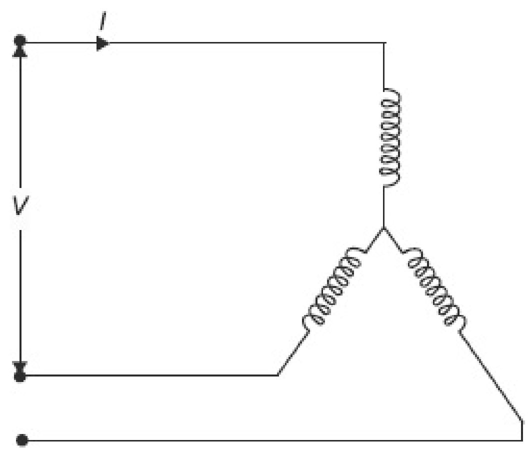

Fig. 12.33(b) AC, three-phase system

$$\text{Then, } a = \frac{I}{\sigma} = \frac{P}{\sqrt{3}\, V\sigma \cos\phi}$$

$$\text{Now } R = \frac{\rho l}{a} = \frac{\sqrt{3}\, V\sigma\rho l \cos\phi}{P}$$

Line loss = 3(loss per conductor)

$$= 3I^2R$$

$$= 3\frac{P^2}{3\,V^2\cos^2\phi} \times \frac{\sqrt{3}V\rho\sigma l\cos\phi}{P} = \frac{\sqrt{3}\sigma\rho lP}{V\cos\phi} \tag{12.55}$$

$$\text{Line input} = \text{Output} + \text{Losses} = P + \frac{\sqrt{3}\sigma\rho lP}{V\cos\phi} = P\left(1 + \frac{\sqrt{3}\sigma\rho l}{V\cos\phi}\right)$$

$$\text{Efficiency of line} = \frac{\text{Output}}{\text{Input}} = \frac{P}{P\left(1 + \frac{\sqrt{3}\sigma\rho l}{V\cos\phi}\right)} \cong \left(1 - \frac{\sqrt{3}\sigma\rho l}{V\cos\phi}\right) \tag{12.56}$$

$$\text{Voltage drop per line} = IR = \frac{\sqrt{3}\sigma\rho lV\cos\phi}{P} \times \frac{P}{\sqrt{3}V\cos\phi} = \sigma\rho l \tag{12.57}$$

$$\text{Volume of conductor} = 3la = \frac{3Pl}{\sqrt{3}\sigma V\cos\phi} = \frac{\sqrt{3}Pl}{\sigma V\cos\phi} \tag{12.58}$$

Assume transmitted power *P*, line length *l*, current density *σ* and specific resistance *ρ* are the constants of the given conductor material, the effect of supply voltage on distribution can be seen as follows:

- From Eq. (12.55), line loss is inversely proportional to *V*. It is also inversely proportional to power factor, cos*ϕ*.
- Line efficiency increases with voltage of transmission and power factor as seen from Eq. (12.56).
- As seen from Eq. (12.57) for a given current density, the resistance drop per line is constant (since *ρ* and *l* have been assumed fixed in the present case). Hence, percentage drop is decreased when *V* is increased.
- The volume of conductor required for a distribution line is inversely proportional to the voltage and the power factor is seen from Eq. (12.58).

It is clear from the above discussion that for long distance distribution of AC power, high voltage and high power factors are essential. Economical upper limit of voltage is reached when the saving in cost of conductor is offset by the increased cost of insulation and increased cost of transformers and high-voltage switches.

Test Yourself

1. Why is generation voltage restricted to 25 kV?

12.7 ECONOMIC SIZE OF CONDUCTOR (KELVIN'S LAW)

The line conductor for an electrical-power distribution system is designed in such a way that the potential drop at the far end of the feeder should be within the specified limits. But the conductors for the transmission lines must be designed on economic principles because the cost of the conductor material is the major component of the total cost of the transmission line.

In addition, if the cross-sectional area of the conductor is decreased to reduce the conductor cost the resistance increases. The annual expenditure incurred due to energy lost in the line conductors increases because of increased resistance. If the cross-section area of the conductor is decreased to decrease the initial investment on the conductor, the annual recurring expenses are increased, i.e., the initial investment and the annual charges on investment are increased, and therefore, there will be some optimum cross-sectional area for which the annual cost of the line is minimum.

The financial loss/annual expenditure occurring in a current-carrying conductor is made up of two parts:

1. The interest on the capital cost of purchase and installation of the conductors plus an allowance for depreciation.
2. The cost of energy wasted due to
   1. Loss in resistance (I^2R loss).
   2. Losses in the metallic sheaths of sheathed cables.

3. In the case of high-tension insulated cables, losses in the insulated materials are used.

For a given length of route, the cost of the conductor material is proportional to the cross-sectional area of conductor. Hence, the annual value of the interest and depreciation is also proportional to the cross-sectional area of the conductor. It can be written as:

Annual value of interest and depreciation, $C_1 \propto$ a

$$C_1 = Pa \qquad (12.59)$$

where, P is the proportionality constant

a = area of cross section

The resistance of the conductor is proportional to $\frac{1}{a}$,

and hence, for a given load condition, the energy loss in the conductor will be proportional to the resistance, i.e.,

$\frac{1}{a}$ It can be written as:

a

Annual value of the cost of energy waste, $C_2 \propto \frac{1}{a}$.

$$\therefore C_2 = \frac{Q}{a} \qquad (12.60)$$

where, Q is the proportionality constant.

∴ The total financial loss in a feeder conductor system is

$$s = Pa + \frac{Q}{a} \tag{12.61}$$

For the total annual financial loss is minimum, the $\frac{ds}{da}$ must be equal to zero.

$$\frac{ds}{da} = P - \frac{Q}{a^2} = 0$$

$$\therefore a = \sqrt{\frac{Q}{P}} \tag{12.62a}$$

$$\text{The annual cost of the capital invested } (C_1) = Pa$$
$$= \sqrt{PQ}$$
$$\text{Cost of the energy losses, } C_2 = \frac{Q}{a} = \frac{Q}{\sqrt{\frac{Q}{P}}} = \sqrt{PQ}$$

Therefore, from the above equations

$$C_1 = C_2$$

or

$$Pa = \frac{Q}{a} \tag{12.62b}$$

The most economical cross-sectional area of the conductor is the one which makes the annual interest and depreciation on capital cost of transmission line equal to the annual cost of energy wasted. This is called as Kelvin’s law.

Statements of Kelvin’s Law:

“The most economical size of the conductor for the transmission of electric energy can be determined by comparing the annual interest on capital cost of conductor plus depreciation with the cost of the energy wasted annually”.

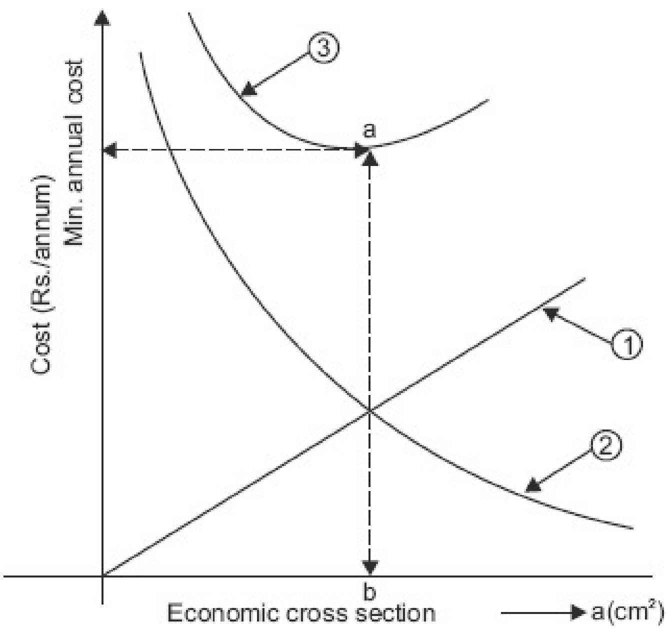

Fig. 12.34 Graphical representation of Kelvin’s Law

The relation given in Eq. (12.62b) can also be obtained with the help of a graph shown in Fig. 12.34 also. Graph (1) represents the cost of the conductor, graph (2) represents the annual cost of energy wasted and graph (3) shows the total cost. The horizontal line ‘ab’ represents the minimum annual cost and the corresponding point ‘b’ on x-axis gives the economical cross-section, i.e., the intersection of graphs (1) and (2).

12.7.1 Modification Of Kelvin’s Law

As per Kelvin’s law, the cost of poles or towers and their foundation, insulation and erection are all independent of the conductor area. However, practically it is not true because the cost of the insulation and the labour required for erection also changes with the changed cross-sectional area of the conductor (other independent items are cost of tower site, cost of location, inspection of towers, etc.).

Therefore, annual interest and depreciation can be written as (insulation, labour, and tower)

$$C_1 = (Pa + R) \tag{12.63}$$

where,

R is the part of the capital, which is constant and independent of the size of the conductors.

Therefore, now total cost, $s = \left(Pa + R + \frac{Q}{a} \right)$

$$\frac{ds}{da} = P - \frac{Q}{a^2} = 0$$

$$Pa = \frac{Q}{a}$$

$$a = \sqrt{\frac{Q}{P}} \tag{12.64}$$

Substituting Eq. (12.64) in Eq. (12.63) and Eq. (12.60)

$$C_1 = P\sqrt{\frac{Q}{P}} + R$$

$$= \sqrt{PQ} + R$$

$$\text{and } C_2 = \frac{Q}{a} = \frac{Q}{\sqrt{\frac{Q}{P}}} = \sqrt{PQ} \tag{12.65}$$

$$\therefore C_1 = C_2 + R \qquad (12.66)$$

Thus, the interest and depreciation on the initial cost of the feeder is greater than the annual cost of the energy loss.

Therefore, the most economical size of a conductor can be determined by comparing the annual cost of energy loss and the annual interest and depreciation on the portion of initial expenditure on the line, which can be considered proportional to the weight of conductor material used.

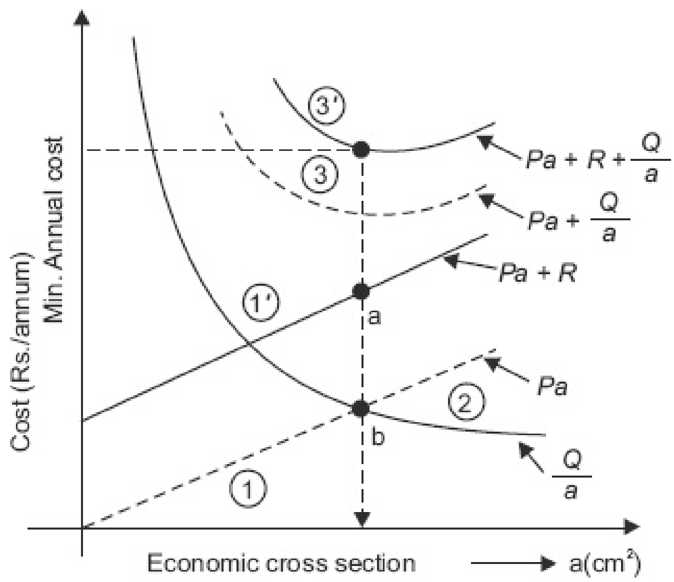

Fig. 12.35 Graphical representation of modified Kelvin's Law

The above results can be obtained with the help of a graph shown in Fig. 12.35. The ordinate of graph (1′) is greater than that of graph (1) by the distance '*ab*', i.e., the amount of independent portion of the capital expenditure.

Test Yourself

1. Why is it important to use only the most economically sized conductor?

12.7.2 Practical Limitations to the Application of Kelvin's Law

Though theoretically Kelvin's law is true, in practice there are limitations due to the following:

1. The size of the conductor determined may be of small area of cross-section, causing high voltage drop or high voltage regulation in the line.
2. The size of the conductor determined may be weak or strong from a mechanical point of view.
3. For smaller cross-sectional area, the current density in the conductor is high which may give rise to excessive heating. The only remedy is to increase the cross-sectional area of the conductor.
4. The diameter of the conductor may be small causing high corona loss.
5. The rate of interest and depreciation may vary from time to time and from place to place. Thus, the other data remaining the same, the conductor of economically designed lines will have different cross-sectional areas at different times and in different countries.
6. It is difficult to estimate the energy loss in the line without actual load curves, which are definitely not available at the time of estimation.

Example 12.9

Determine the most economical cross-section of a conductor for a three-phase, 8 km long line to supply a load at a constant voltage of 33 kV. During a 24-hour period, the load is 2500 kW for 8 hour, 1500 kW for 10 hours, and 1000 kW for 6 hours at u.p.f. The capital cost per km of the line is Rs. (16,250 + 5000*a*) where *a* is in cm^2. Interest and depreciation charges are 8% and cost of energy is 5 paise per unit. The resistance per km of a conductor of 1 cm^2 cross-section is 0.176 Ω.

Solution:

Resistance per kilometre of each conductor, $R = \frac{0.176}{a}\,\Omega$.

Line voltage = 33 kV = 33×10^3 V

The load currents at various load are;

$$\text{Current at a load of 2500 kW, } I_1 = \frac{2500 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 1} = 43.74 \text{ A.}$$

$$\text{Current at a load of 1500 kW, } I_2 = \frac{1500 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 1} = 26.24 \text{ A.}$$

$$\text{Current at a load of 1500 kW, } I_3 = \frac{1000 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 1} = 17.49 \text{ A.}$$

$$\text{Energy loss per day in three-phase line} = 3 \times \frac{0.176}{a} \times \frac{365}{1000} \left[(43.74)^2 \times 8 + (26.24)^2 \times 10 + (17.49)^2 \times 6 \right]$$

$$= \frac{0.528}{a} [8769.6] = \frac{4630.344}{a} \text{ kWh.}$$

$$\text{Annual cost of energy loss} = \text{Rs.} \frac{5}{100} \times \frac{4630.344}{a} = \frac{231.517}{a}.$$

$$\text{Variable annual charge} = 8\% \text{ of capital (variable) of line.}$$

$$= \text{Rs. } 0.08 \times 5000a = \text{Rs } 400a$$

According to Kelvin's law, for most economical cross section conductor.

Variable annual charge = Annual cost of energy lost

$$400a = \frac{231.517}{a}.$$

$$\therefore \text{Cross-sectional area of conductor, } a = \sqrt{\frac{231.517}{400}} = 0.76 \text{ cm}^2.$$

Example 12.10

A two-conductor cable, one km in length, is required to supply a constant load of 200 A throughout the year. The cost of the cable is Rs. (50*a* + 25)/m where, a is the area of the conductor in cm^2. Determine the most economical cross-section of the conductor if the cost of energy is 5 paise/kWh and interest and depreciation charges amount to 10%. Specific resistivity of the conductor is 1.85 μΩ-cm.

Solution:

Resistance of each conductor, $R = \frac{\rho L}{a} = \frac{1.85 \times 10^{-6} \times 1000 \times 100}{a} = \frac{0.185}{a}$

Annual energy loss $= \frac{2I^2 R \times 8760}{1000}$ kWh

$$= 2 \times 200^2 \times \frac{0.185}{a} \times \frac{8760}{1000} = \frac{129648}{a}$$

Annual cost of energy loss $= \frac{5}{100} \times \frac{129648}{a} = \text{Rs.} \frac{6482.4}{a}$

Capital cost of the cable/km $=$ Rs. $50a \times 1000 = 50000a$

Interest and depreciation on capital cost $= \frac{10}{100} \times 50000a = \text{Rs.} 5000a$

According to Kelvin's law

$$5000a = \frac{6482.4}{a}$$

More economic cross section of conductor, a =1.1386 cm^2.

Example 12.11

The daily load cycle of a three-phase transmission line, 20 km long, is equivalent to a current of 100 A for 8 hours, 75 A for 7 hours, and 20 A for 9 hours. The capital cost per kilometre of line per conductor is Rs. (9500 + 24000*a*) where a is the cross-section in cm^2 of the conductor.

Find the most economical size for the conductor, assuming interest and depreciation of 10% per annum and energy cost to be 5 paise per kWh. Resistance per km of a conductor of 1 cm^2 cross-section is 0.19 Ω.

Solution:

Resistance of each conductor per kilometre length,

$$R = \frac{0.19}{a} \Omega$$

Load current for 8 hours, I_1 = 100 A

Load current for 7 hours, I_2 = 75 A

Load current for 9 hours, I_3= 20 A

Energy lost per day in three-phase line

$$\text{Energy wasted per day of a three-phase system} = 3 \times \frac{0.19}{a} \times \frac{1}{1000}\left[(100)^2 \times 8 + (75)^2 \times 7 + (20)^2 \times 9\right]$$

$$= \frac{0.57}{1000a} \times [80000 + 39375 + 3600]$$

$$= \frac{70.1}{a} \text{kWh.}$$

$$\text{Cost of annual energy wasted} = \text{Rs.} \frac{5}{100} \times \frac{70.1}{a} \times 365.$$

$$= \text{Rs.} \frac{1279.25}{a}.$$

Variable annual charge = 10% of capital cost

$= 0.1 \times 24000a$

$= 2400a.$

According to Kelvin's law

$$\frac{1279.25}{a} = 2400a$$

$a^2 = 0.533$

The most economical size of conductor, $a = 0.73$ cm^2.

Example 12.12

Find the most economical size for a conductor required to transmit a maximum loading of 5000 kVA over a three-phase overhead line operated at 33 kV, the annual load factor being 50%, allow 10% for interest and depreciation. Cost per kilometre length of line is Rs. (24000*a* + 1000) where *a* is cross-sectional area of the conductor in cm^2, cost of energy waste is 10 paise/kWh and $K = I_{rms}//I_{av}$ = 1.2 for a load factor of 50%. Resistance per kilometre of a conductor of 1 cm^2 cross-section of 0.173 Ω.

Solution:

$$\text{Maximum current transmitted, } I_{max} = \frac{5000}{\sqrt{3} \times 33} = 87.48 \text{ A}$$

$$\text{Load factor} = \frac{I_{av}}{I_{max}} = 0.5$$

$$\therefore I_{av} = 0.5 I_{max} = 0.5 \times 87.48 = 43.74 \text{ A}$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = 1.2$$

$$\therefore I_{rms} = 1.2 I_{av} = 1.2 \times 43.74 = 52.486 \text{ A}$$

$$\text{Power loss} = 3 I_{rms}^2 R = 3 \times (52.486)^2 \times \frac{0.173}{a} = \frac{1429.75}{a} \text{ W}$$

$$\text{Annual energy wasted} = \frac{1429.75}{a} \times \frac{8760}{1000} = \frac{12524.63}{a} \text{ kWh}$$

$$\text{Cost of energy wasted} = \text{Rs.} \frac{12524.63}{a} \times 0.1 = \text{Rs.} \frac{1252.463}{a}$$

Interest and depreciation on capital cost = (24000a + 1000) × 0.1 = 2400a + 100

According to Kelvin's law

$$2400a = \frac{1252.463}{a}$$

$$\therefore a^2 = \frac{1252.463}{2400} = 0.522$$

∴ Most economic cross section of conductor, a =0.722 cm^2.

Example 12.13

Determine the most economical cross-section of a three-core cable 300 m long supplying a load of 150 kW at 400 V and 0.85 p.f. lagging for 5000 hours in a year and no load for the rest of the year. The cost of the cable including its installation is Rs. (200*a* + 40)/m where *a* is in square cm. Interest and depreciation on total cost is 15% and unit cost of energy is 15 paise. Assume resistance of 1 km length and 1 cm^2 section of single core conductors as 1/5.8 Ω.

Solution:

Current corresponding to 150 kW load at 400 V and 0.85 p.f. lagging

$$I = \frac{150 \times 10^3}{\sqrt{3} \times 400 \times 0.85} = 254.71 \text{ A.}$$

$$\text{Energy wasted per annum} = \left[3 \times (254.71)^2 \times \frac{0.173}{a} \times 300 \times 10^{-3} \times 5000\right] \times 10^{-3} \qquad \left(\because R = \frac{1}{5.8} \simeq 0.173\right)$$

$$= \frac{50503}{a} \text{ kWh.}$$

$$\text{Annual cost of energy wasted as losses} = \frac{15}{100} \times \frac{50503}{a}$$

$$= \text{Rs.} \frac{7575.44}{a}$$

Capital cost of cable = Rs. $(200a + 40) \times 300$

$$\text{Interest depreciation on capital cost of cable} = (200a + 40) \times 300 \times \frac{15}{100} = \text{Rs.} (9000a + 1800)$$

According to Kelvin's law

$$9000a = \frac{7575.44}{a}.$$

Economic cross-section conductor, a = 0.9174 cm^2.

CHAPTER AT A GLANCE

1. For the transmission line to be economical, the cost of the conductor should be as small as possible.
2. Different types of transmission systems are DC system, single-phase AC system, two-phase AC system and three-phase AC system.
3. When overhead lines transmit power, the maximum voltage between each conductor and the earth forms the basis of comparison of volume of the conductor.
4. When an underground three-core cable transmits the power, maximum voltage between conductors should be taken as a base for comparison of volume of conductor.
5. Effect of supply voltage: If the voltage is raised n times, the current in the line required to transmit the same amount of power is reduced by n times. Therefore, the line loss I^2R is reduced n^2 times. Hence, the line efficiency is increased.
6. Kelvin's law states that the most economical size of the conductor for the transmission of electric energy can be determined by comparing the annual interest on capital cost

of conductor plus depreciation with the cost of the energy wasted annually.

SHORT ANSWER QUESTIONS

1. What is meant by an electric-supply system?
2. What are the principle components of an electric-supply system?
3. How many parts can the large network of conductors be broadly divided into between the power station and consumers?
4. What are the advantages of DC transmission?
5. What are the disadvantages of DC transmission?
6. What are the advantages of AC transmission?
7. What are the disadvantages of AC transmission?
8. What are the advantages of high transmission voltage?
9. What are the limitations of high transmission voltage?
10. What are the various systems of power transmission?
11. State any two limitations of Kelvin's law.
12. Define economical transmission voltage.

MULTIPLE CHOICE QUESTIONS

1. In a three-wire DC system, the current in the positive outer is 100 A and in the negative outer is 50 A, the current in the neutral is
    1. 0 A
    2. 50 A
    3. 150 A
    4. 100 A
2. In a transmission system, the weight of conductor used is proportional to
    1. V^2
    2. V
    3. $\frac{1}{V^2}$
    4. $\frac{1}{V}$
3. DC higher voltages cannot be generated because of
    1. commutating and insulating problems
    2. commutating problems
    3. insulating problems
    4. brushes problem
4. The economic size of a conductor is determined by
    1. Kelvin's law
    2. Kirchhoff's law
    3. Faraday's law
    4. Newton's law

5. Conductor material required for AC three-phase, three-wire system, on the basis of maximum potential difference between any two conductors, over DC two-wire system of transmission for the similar conductors is
    1. equal
    2. 1.25 times
    3. $\frac{2}{\cos^2\phi}$
    4. $\frac{1.5}{\cos^2\phi}$
6. Transmission of electrical power is normally done by
    1. two-wire DC
    2. three-wire DC
    3. three-phase, three-wire AC
    4. single-phase AC
7. The AC system of transmission is more advantageous than DC system as it makes use of
    1. alternators
    2. transformers
    3. reactors
    4. bus bars
8. For comparing relative amounts of conductor material of different systems of transmission, the conditions assumed are
    1. equal power
    2. equal length of line
    3. equal line loss
    4. equal power, length of line and line loss
9. On the basis of maximum potential difference between any conductor and earth, the conductor material required for AC single-phase, two-wire system over DC two-wire system is
    1. unity
    2. 1.25 units
    3. $\frac{2}{\cos^2\phi}$
    4. $\frac{1.5}{\cos^2\phi}$
10. The standard supply frequency adopted in our country is
    1. 60 Hz
    2. 50 Hz
    3. 25 Hz
    4. 40 Hz
11. At 50 Hz frequency, the reactance of lines and machinery is more than at 25 Hz frequency by
    1. 3 times

2. 2.5 times
3. 2 times
4. 1.5 times

12. Amount of conductor material required in a supply system for a given power, length of line and line loss is
    1. inversely proportional to the square of the voltage
    2. directly proportional to the square of the voltage
    3. inversely proportional to the voltage
    4. directly proportional to the voltage
13. Amount of conductor material required in an AC supply system is
    1. directly proportional to the power factor
    2. directly proportional to the square of the power factor
    3. inversely proportional to the power factor
    4. inversely proportional to the square of the power factor
14. Increase in supply voltage
    1. increases the efficiency of line
    2. decreases the efficiency of line
    3. increases the volume of conductor material
    4. decreases the cost of supports
15. When the voltage of a supply system is raised by m times, then the line loss is reduced by
    1. m times
    2. m^2times
    3. m^3
    4. $\sqrt{m}$ times
16. Annual cost of the combined interest that is proportional to the depreciation is
    1. (cross-section of the conductor)^2
    2. cross-section of the conductor
    3. $\frac{1}{(\text{cross section of the conductor})^2}$
    4. $\frac{1}{\text{cross section of the conductor}}$
17. Annual cost of energy loss due to resistance of conductor is
    1. inversely proportional to the cross-section of conductor
    2. proportional to the cross-section of conductor
    3. proportional to the square of the conductor cross-section
    4. inversely proportional to the square of the conductor cross-section
18. Kelvin's law is used to determine
    1. economical voltage of the line
    2. economical current density
    3. the cost of annual power loss
    4. the annual interest and depreciation charges
19. Kelvin's law is not applicable for
    1. overhead lines

2. high voltage transmission lines
3. low voltage lines
4. underground cable system

20. Conductor material should have
    1. low specific resistance
    2. high specific resistance
    3. low conductivity
    4. low mechanical strength
21. The conductor efficiency of overhead lines is calculated, based on the maximum voltage between conductor and
    1. conductor
    2. earth
    3. line
    4. phase
22. As per IE rules, voltage at consumer terminals must not vary by ________ % of declared voltage
    1. 5
    2. 10
    3. 15
    4. 12.5
23. As the temperature increases, the temperature co-efficient of resistance.
    1. increases
    2. does not change
    3. doubles
    4. decreases
24. Resistivity depends upon the
    1. length of the conductor
    2. area of cross-section of the conductor
    3. material of the conductor
    4. temperature of the conductor
25. Electrical lights do not operate satisfactorily at ________ frequencies.
    1. higher
    2. double
    3. lower
    4. none

Answers:

1. c	2. c	3. a	4. a	5. d
6. c	7. b	8. d	9. c	10. b
11. c	12. a	13. d	14. a	15. b
16. b	17. a	18. d	19. d	20. a
21. b	22. a	23. d	24. d	25. c

REVIEW QUESTIONS

1. Enumerate the difference between systems of transmitting electric power. Explain how comparison is made between them from the point of view of the conductor used in each system.
2. State and prove Kelvin's law for size of conductors for transmission. Discuss its limitations.
3. Explain the following with neat diagrams.
    1. AC three-phase, three-wire distribution system.
    2. AC three-phase, four-wire system.
4. State Kelvin's law. Explain its significance and limitations.
5. Explain the choice of frequency.
6. Discuss the choice of voltage.
7. What are the advantages of high-voltage transmission?
8. Explain the effect of supply voltage on volume of conductor material and power factor of the system.
9. State and explain the Kelvin's law for size of conductors for transmission.
10. Explain the modified Kelvin's law for size of conductors for transmission.

PROBLEMS

1. Compare the cost of materials for a three-phase AC and three-wire DC for transmission of a given amount of power over the same distance and with the same efficiency and voltage to earth.
2. A three-wire DC system is converted to a three-phase, four-wire AC system by the addition of another wire which has one section equal to one of the outers. For the same effective voltage between outers and the neutral at the customer terminals and the same percentage loss, determine the additional percentage load that can be supplied. Assume a balanced load and that the AC system has a power factor of unity.
3. In a two-wire DC system, a feeder is working on a supply voltage of 220 V for supplying a load. Determine the percentage saving in the conductor material if the supply voltage is increased to 440 V without changing the load.
4. Compare the weights of copper on the basis of equal efficiency for a two-wire DC system and three-wire DC system if a power of 15 kW is to be distributed by either of the above systems. The consumer's terminal voltage is 220 V. Assume that the load is balanced and the middle wire has half the cross-sectional area of that of the outer.
5. A two-core, 500 V, 5 kilometre long feeder supplies a maximum current of 225 A for six months. The cost of the cable including the installation charges is Rs. (22a + 5.6) per metre and the interest and depreciation charges are 8%. The cost of energy is 6 paise/kWh. The resistance of the conductor which is 1 kilometre long and has a cross-section of 1 cm is 0.173 Ω. Determine the most economical cross- section of the conductor.
6. A single-phase two-wire 1.5 kilometre long cable supplies a

constant load of 225 A throughout the year. Determine the cross-sectional area if the cost of the cable is Rs. (200*a* + 30) per metre, where '*a*' is the area of cross-section of the conductor in cm^2. The cost of energy is 8 paise/kWh and the interest and depreciation charges amount to 12%. Specific resistivity of copper is 1.73 μΩ-cm.

13

Substations

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Provide an overview of substations
- Describe the location and constructional features of substations
- Classify bus bar arrangements
- Discuss the various methods of grounding

13.1 INTRODUCTION

The substation may be defined as an assembly of apparatus installed to perform switching, voltage transformation, power factor correction, power and frequency-converting operations.

The purpose of a substation is to take power at high voltages from the transmission or subtransmission level, reduce its voltage and supply it to a number of primary voltage feeders for distribution in the area surrounding it. In addition, it performs operational and emergency switching and protection duties at both the transmission and feeder level. It is also used as a local site for communication, storage of tools, etc. The sectional view of a 33/11 kV substation is shown in Fig. 13.1.

13.2 FACTORS GOVERNING THE SELECTION OF SITE

Voltage levels, voltage regulation, the cost of subtransmission, substation, primary feeder mains and distribution transformers dictate the location of a substation. However, the following rules are to be considered for the selection of an ideal location for a substation:

- The substation should be located nearer the load centre of its service areas, so that its distance from the substation is minimum.
- Proper voltage regulation should be possible without taking extensive measures.
- There should be proper access for incoming sub-transmission lines and outgoing primary feeders.
- It should provide enough space for future expansion.
- It should help minimize the number of customers affected by any service interruption.

Fig. 13.1 View of 33/11 kV Substation

13.3 CLASSIFICATION OF SUBSTATION

Substations are classified based on the service, location, function and type of apparatus they use.

13.3.1 According to Service

Substations are classified according to voltage levels, switching operations, power factor correction, change in frequency and conversion of AC to DC as follows:

Transformer Substations These substations transform power from one voltage to another as per requirement. These are:

1. Transmission or primary substations These substations receive power from local generating stations (11 kV or 33 kV) and step up the voltage (220 kV or 400 kV) for primary transmission so that huge amounts of power can be transmitted over long distances to the load centres economically.

2. Subtransmission or secondary substation These substations receive the power from primary transmission substations at high voltages (above 132 kV) and step down the voltage to 33 kV or 11 kV for secondary transmission or primary distribution.

3. Step down or distribution substations These substations receive the power from sub-transmission substations or directly from power stations and step down the voltage for secondary distribution, i.e. 400 V for three phases or 230 V for single phase for household consumers.

Industrial Substations Some industrial consumers require huge amounts of power, it is advisable for such consumers to install individual substations. These substations are called industrial substations.

Switching Substations These substations are used for switching operations of power lines without the transformation of voltage. In this substation, different connections are made between different transmission lines.

Synchronous Substations At these substations, synchronous phase modifiers are installed for the improvement of the power factor of the system.

Frequency Change Substations These substations are used for converting normal frequency to other useful frequency and are supplied to industries which require high or low frequency.

Converting Substations These substations are used for converting AC into DC. This is useful for special purposes such as electric traction, electric welding, battery charging, etc.

13.3.2 According to Design

The main components of substation equipment are insulators, bus bars, circuit breakers, transformers, switches, relays, etc., which are properly protected for continuity and quality of supply.

According to design, the substations are classified as indoor and outdoor substations.

Indoor Substations Indoor substations are those whose apparatus are installed within a building. These substations are generally used up to 11 kV voltage only. Generally these types of substations are installed where the atmosphere is contaminated with impurities such as metal-corroding gases and fumes, conductive dust, etc.

Outdoor Substations Outdoor substations are of two types. They are:

1. Pole-mounted substations These are used for distribution purposes only and are usually mounted on double or four-pole structures with suitable platforms.

2. Foundation-mounted substations These are also called plinth-mounted substations.

These are used for high-rating transformers due to the heavy weight of the transformer.

13.4 MERITS AND DEMERITS OF INDOOR AND OUTDOOR SUBSTATIONS

Outdoor substations have the following merits over indoor substations:

1. All the equipment is visible. So the identification of fault is easier.
2. Expansion of the substation is easier.
3. Takes less erection time.

4. There is no necessity of building. So it requires less building material.
5. The construction work required is comparatively smaller, and hence, the cost of the switchgear installation is low.
6. The spacing between the apparatus is more, so less damage occurs due to faults.

The demerits of outdoor substations over indoor substations are:

1. Switching operations, the supervision and maintenance of apparatus are to be performed in the open air during all kinds of weather.
2. Requires more space for arranging apparatus in the substation.
3. The apparatus is exposed to the sun. It requires special design, therefore, for withstanding high temperatures.
4. The apparatus requires more maintenance due to dust and dirt deposition on the outdoor substation equipment.
5. These are prone to lightning strokes.

The choice of the particular arrangement depends upon the relative importance of safety, reliability, flexibility of operation, initial cost, easy maintenance, availability of good area, location of connecting lines and provision for expansions and appearance.

13.5 SUBSTATION EQUIPMENT

The various equipments required in a substation depend upon the type of substation, service requirement and protection importance. However, the following main equipments are generally used in most of the substations.

Bus Bars A bus bar term is used for a conductor carrying current to which many connections are made. These are generally used in substations where the number of incoming lines and outgoing lines take place at the same voltage. The sectional view of a 33/11 kV bus bar is shown in Fig. 13.2.

Bus bars used in substations are of copper or aluminium and they are bare rectangular cross-section bars or round solid bus bars, but the former is more commonly used since it is more economical as compared to the latter. According to the type of the construction of the bus bar, it can be classified into two kinds. These are:

1. Copper or copper-clad steel tubes or aluminium tubes supported on post insulators.
2. Standard copper or ACSR wires or cellular hollow wires strung between strain insulators. They are of 5–6 m in length.

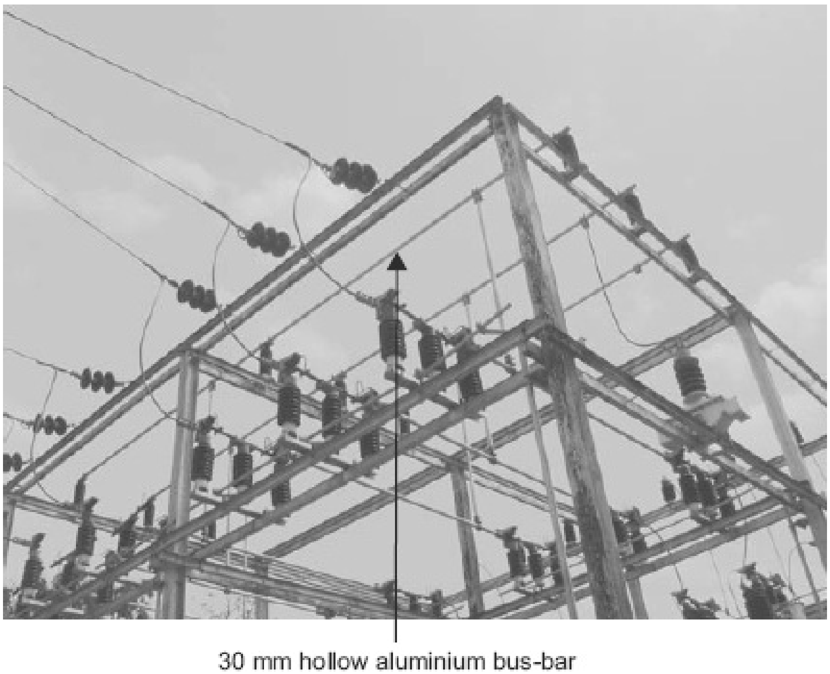

Fig. 13.2 View of 33/11 kV bus bars

Bus-bar arrangements: The selection of bus bars depends upon the following:

1. Amount of flexibility required in operation.
2. Immunity from total shut down
3. Initial cost of installation.
4. Load handled by the bus bar.

In addition, the bus bar arrangement in substation design includes the decision about whether

- Rigid bus or strain bus will be used.
- Aluminium bus or copper bus will be used.
- Shield wire should be used.
- Disconnecting switches should be with two insulator stacks or with three insulators.
- The ground switch is to be used.

- Horizontal or vertical switches are to be used.

Insulators Porcelain insulators are used in substations to support and insulate the live conductors and bus bars.

Switchgear Under normal working conditions, any equipment will carry the rated load current of the circuit. Under abnormal conditions such as short circuits or lightning strokes, the equipment will be called upon to carry heavy currents. The equipment may also be subjected to high voltages. Hence, equipment of all different voltage ratings, whether they are in the generating stations or substations, are invariably provided with switching apparatus necessary both, under normal and abnormal operating conditions. Switching apparatus are (i) isolators, (ii) air-break switches, (iii) low-voltage knife switches, (iv) circuit breakers and (v) fuses.

Isolators Isolators are used for isolating the circuit when the current has already been interrupted. They allow currents into the circuit until circuit health is restored. Isolators are used only for connecting and disconnecting parts of electrical installations after de-energizing them by opening their circuits with the respective circuit breakers.

Transformers A transformer is a static device used to transform power from one voltage level to another voltage level without changing the frequency. Transformers can be classified into two kinds: power transformers and instrument transformers.

1. Power transformers Step-up power transformers are used at generating stations to step up the voltage for transmission, whereas step-down transformers are used at receiving-end stations to step down the voltage for secondary transmission, and primary and secondary distribution.

In case of transformers of large KVA rating, a single three-phase unit or three single-phase units are used.

Single three-phase units are economical and occupy less space. The connections are also relatively simpler. But three single-phase units are easier to handle. They are, however, more expensive and require more space.

2. Instrument transformers Instrument transformers are used (a) to protect personnel and apparatus from high voltage, and (b) to permit the use of reasonable insulation levels and current-carrying capacity in relays and motors.

Indicating and Metering Instruments Ammeters, voltmeters, PF meters, watt meters, energy meters, kVAr meters are installed in substations for control and measurement purposes. In case of all substations (including 33 kV/11 kV substations), it is necessary to provide recording voltmeters.

Protective Relays These are installed for the protection of equipment against faults or overloads.

Lightning Arresters All the equipment in the outdoor stations should be protected against direct lightning strokes and travelling waves reaching the station over the transmission lines. By shielding the station equipment, equipment can be protected against the direct strokes.

Fire-fighting Equipment Fire-fighting equipment such as carbon-dioxide cylinders on wheeled trucks fitted with suitable metal hoses, nozzles, and sand and water buckets of adequate capacity should be provided in all substations to deal with fires likely to be encountered. All fire-fighting equipment should be located at convenient places.

Substation Auxiliary Supply In any substation, it is a practice to connect two transformers to the 11 kV main bus bars for the supply of auxiliaries at a voltage of 400 V/230 V. Electric supply is required for auxiliaries like lighting circuits, air blast fans of power transformers, battery-charging sets, oil-servicing facilities, compressor units, ventilating fans of the station buildings, water supply and heating system equipment, etc.

13.6 TYPES OF BUS BAR ARRANGEMENTS

The different types of bus bar arrangements are:

1. Single bus bar
2. Single bus bar system with sectionalization
3. Double bus bar with single breaker
4. Double bus bar with two circuit breakers
5. Breakers and a half with two main buses
6. Main and transfer bus bar
7. Double bus bar with bypass isolator
8. Ring bus

13.6.1 Single Bus Bar

It consists of a single bus bar and all the incoming and outgoing lines are connected to the same bus bar as shown in Fig. 13.3. Here, the 11 kV incoming lines are connected to the bus bar through isolators and circuit breakers. Three-phase, 400 V and single-phase, 230 V outgoing lines are connected through isolator, circuit breaker, and step-down transformer from the bus bar. This type of arrangement is suitable for DC stations and small AC stations. The major drawback of this system is that, if the fault occurs on any section of the bus bar, the entire bus bar is to be de-energized for carrying out the repair work. So, this results in a loss of continuity of service of all feeders. Similarly, the periodical maintenance work on bus bars can also be carried out only by disconnecting the whole supply.

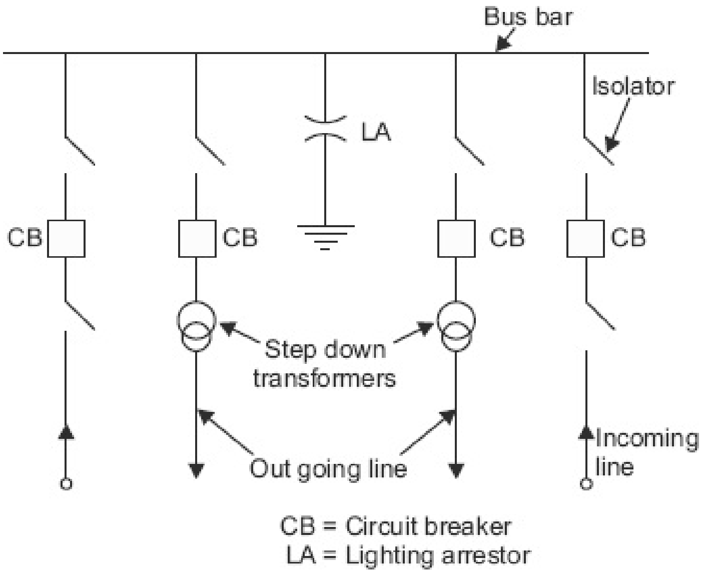

Fig. 13.3 Layout diagram of single bus bar

The equipment connections are very simple, and hence, the system is very convenient to operate. This arrangement is not popular for voltages above 33 kV. The indoor 11 kV substations often use single bus bar arrangements.

Merits:

- Each of the outgoing circuits requires a single-circuit breaker. So, this type of arrangement is the cheapest one.
- The relaying system is simple.
- The maintenance cost is low.
- The bus bar potential can be used for the line relays.

Demerits:

- Maintenance without interruption of supply is not possible.
- Expansion of the substation without shutdown is not possible.

13.6.2 Single-Bus Bar System with Sectionalization

The sectionalization of the bus bar ensures continuity of supply on the other feeders, during the time of maintenance or repair of one side of the bus bar. The whole of the supply need not be shut down. The number of sections of a bus bar is usually 2 or 3 is a substation as shown in Fig. 13.4, but actually it is limited by the short-circuit current to be handled. Another advantage of sectionalization is that the circuit breaks of low breaking capacity can be used on the sections as compared to the previous case. In case of duplicate feeders, they are connected to different sections of the bus bars so that in the event of a fault on one of the bus bar sections, the feeders connected to it are immediately transferred to the healthy-bus bar section and the faulty section is isolated.

An important point to note is that the sections should be synchronized before the bus coupled is closed for sharing the load.

Advantages:

- The operation of this system is simple as in case of the single bus bar.
- The maintenance cost of this system is comparable with the single bus bar.
- For maintenance or repair of the bus bar, only one half of the bus bar is required to be de-energized. So complete shut down of the bus bar is avoided.
- It is possible to utilize the bus bar potential for line relays.

Disadvantages:

- In case of a fault on the bus bar, one half of the section will be switched off.
- For regular maintenance also, one of the bus bars is required to be de-energized.
- For maintaining or repairing a circuit breaker, it is required to be isolated from the bus bar.

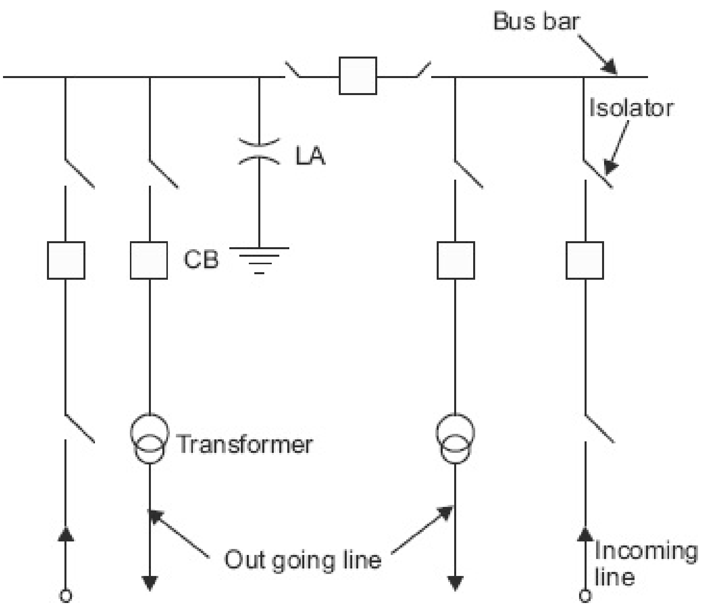

Fig. 13.4 Layout diagram of single-bus bar system with sectionalization

13.6.3 Double Bus Bar with Single Breaker

This system is shown in Fig. 13.5. It consists of two identical bus bars, one is the main bus bar and another one is spare bus bar. Each bus bar has the capacity to take up the entire substation load. Each load may be fed from either bus bar. The infeed and load circuits can be further divided into two separate groups based on operational considerations (maintenance or repair). Any bus bar may be taken out for maintenance and cleaning of insulators.

With the help of bus coupler, the incoming and/or outgoing lines are connected to any bus bar through isolator and circuit breaker. This system is adopted when the voltage is greater than 33 kV. This arrangement does not permit breaker maintenance without causing interruption in supply.

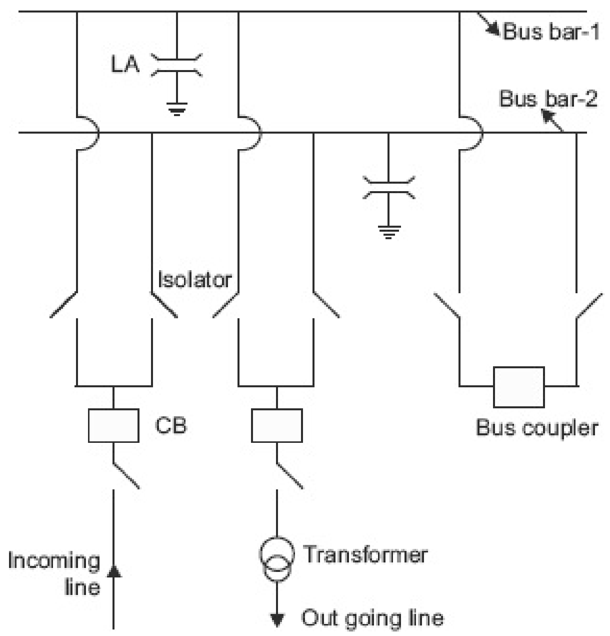

Fig. 13.5 Layout diagram of double bus bar with single breaker

Advantages:

- Permits some flexibility with two operating buses.
- Any main bus may be isolated for maintenance.
- The circuit can be transferred readily from one bus to another by using bus-coupler and bus- selector disconnect switches.

Disadvantages:

- One extra breaker is required for the bus coupler.
- Three switches are required per circuit.
- High exposure to bus faults.
- If bus coupler fails, the entire substation runs out of service.

13.6.4 Double Bus Bar with Two Circuit Breakers

Figure 13.6 shows the schematic diagram of double bus bar arrangement with two breakers per circuit. This is a simple and flexible arrangement. It is expensive, and hence, is rarely used. When it is used, it is used in large generating stations which require a high-security connection. It provides the best maintenance facilities for maintenance to be carried out on the circuit breakers. Thus, when one circuit breaker is opened for maintenance or repair works, the load can be transferred on to the other circuit breaker very easily.

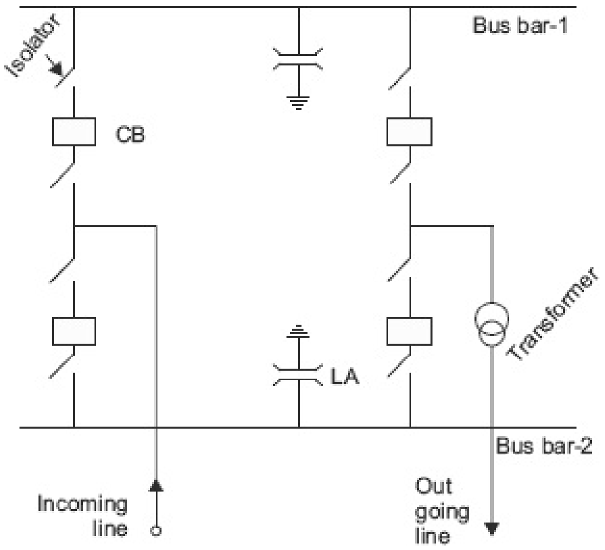

Fig. 13.6 Layout diagram of double bus bar with two circuit breakers

Advantages:

- Two circuit breakers in each circuit.
- Has flexibility to connect the feeder circuits to any bus.
- For service maintenance any breaker can be taken out.
- High reliability.

Disadvantages:

- More expensive.
- If circuits are not connected to both buses, the bus bar loses half the circuit for breaker failure and interprets supplies.

13.6.5 Breakers and a Half with Two Main Buses

The schematic diagram of this arrangement is shown in Fig. 13.7. This method is an improved version of double bus bar with two circuit breakers and uses lesser number of circuit breakers. In this method, one spare breaker is provided for every two circuits. When the breaker (own) is taken out for maintenance, the protection is complicated since it must associate the central breaker with the feeder.

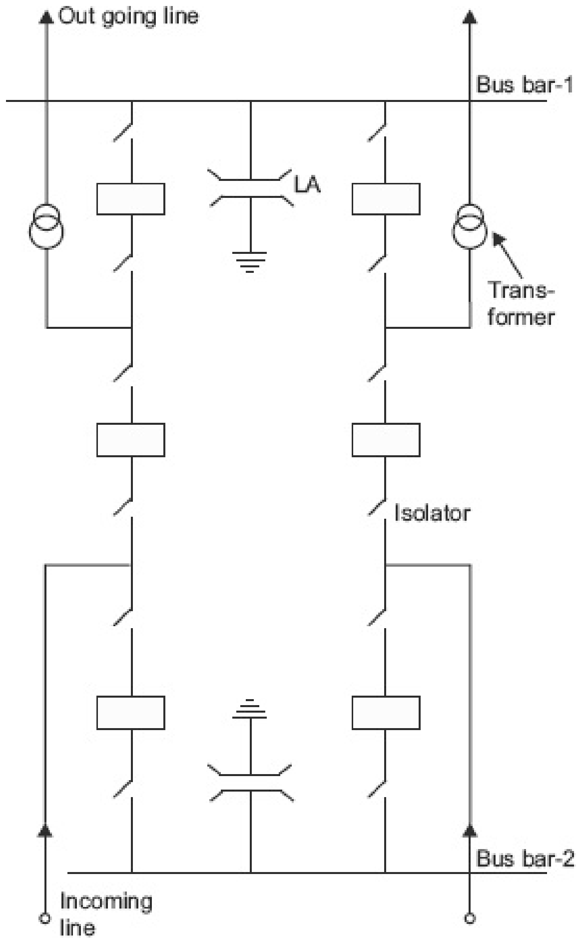

Fig. 13.7 Layout diagram of breakers and a half with two main buses

Advantages:

- This system is more economical as compared to a double-bus double-breaker arrangement.
- A fault in a breaker or in a bus will not interrupt the supply.
- Addition of circuits to the system is possible.
- High reliability.
- Any main bus can be taken out of service at any time for maintenance.

Disadvantages:

- 1½ breaker per circuit.
- The relaying becomes more complicated as compared to that in a single-bus arrangement.

- The maintenance cost is higher.

13.6.6 Main and Transfer Bus Bar

The schematic diagram of this commonly used arrangement is shown in Fig. 13.8. This arrangement is an alternative to the double bus bar scheme. In this arrangement any line circuit breaker can be taken out for maintenance and repair without affecting the supply. This is done by closed transfer circuit breaker and changing the load to transfer bus bar and then removing the line breaker from service. Only one breaker at a time can be removed from service and the transfer breaker takes its place when it is out of service.

In a substation, to work on a bus bar, it is often necessary to remove it from service. This is possible only by transferring the load to the other bus bar. This is not possible in this scheme. Hence, the absence of this facility to remove any bus bar from service is the only drawback.

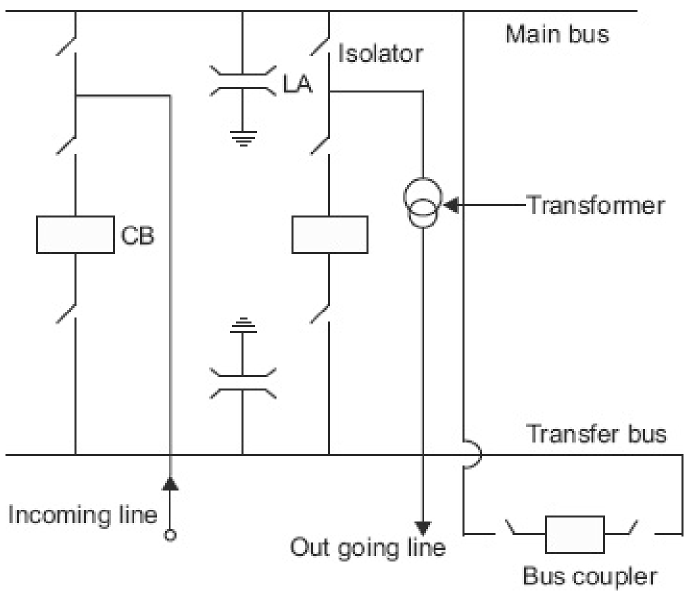

Fig. 13.8 Layout diagram of main and transfer bus bar

Advantages:

- It ensures supply in case of bus fault. In case of any fault in a bus, the circuit can be transferred to the transformer bus.
- It is easy to connect the circuit from any bus.
- The maintenance cost of substation decreases.
- The bus potential can be used for relays.

Disadvantages:

- Requires one extra breaker for the bus tie.
- Switching is somewhat complicated while maintaining a breaker.
- Failure of bus or any circuit breaker results in shutdown of entire substation.

13.6.7 Double Bus Bar with Bypass Isolator

This is a commonly used arrangement also known as sectionalized double bus bar arrangement and is shown in Fig. 13.9. This is a combination of a double-bus and main transfer-bus scheme. Any of the bus bars can act as a main bus and another bus is used as the transfer bus. The advantage of this method is that any circuit breaker or any bus bar can be taken out for service without affecting the supply. In substations, it is frequently necessary to take bus bar or the circuit breaker out of service for maintenance or repair. So this scheme is the recommended one both because it is simple and economical.

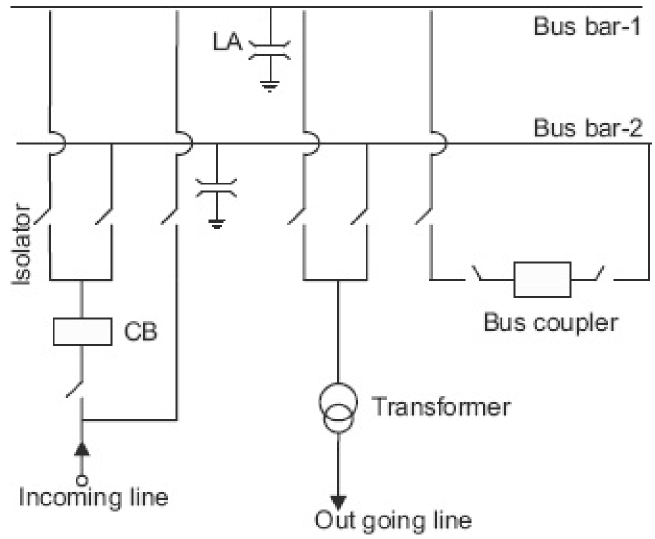

Fig. 13.9 Layout diagram of double bus bar with bypass isolator

13.6.8 Ring Bus

This is an extension of the sectionalized bus bar arrangement. By using two bus couplers, as shown in Fig. 13.10, the ends of the bus bars are returned upon themselves to form a ring. The sectionalizing and bus coupler are in series. There is a greater flexibility of operation. This is not a commonly used arrangement at present.

Different types of ring or mesh buses utilized are:

1. Simple ring.
2. Rectangular ring.
3. Circulating ring.
4. Zigzag ring.

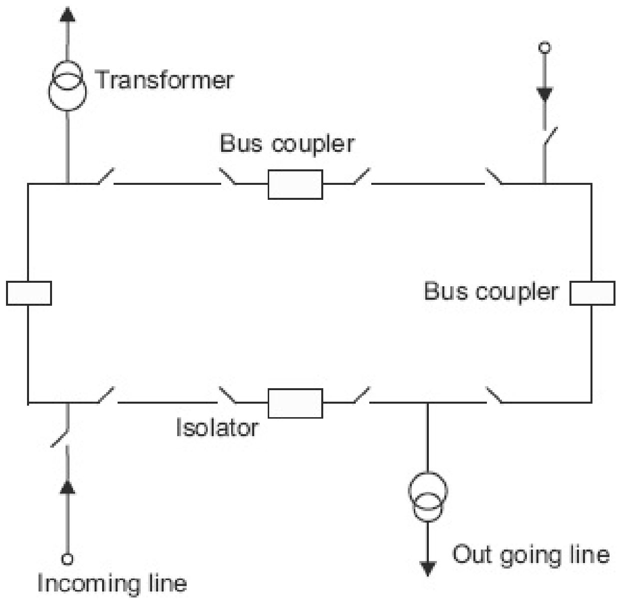

Fig. 13.10 Layout diagram of ring bus

Advantages:

- Low initial and ultimate cost.
- Flexible operation for breaker maintenance.
- Any breaker can be removed for maintenance without interrupting load.
- Required only one breaker per circuit.
- Does not use main bus.

Disadvantages:

- It is necessary to trip two circular breakers to isolate a faulted line, which makes the relaying quite complex.
- It is necessary to supply potential to relays separately to each of the circuits.
- It is difficult to add any new circuit to the ring.

13.7 POLE AND PLINTH-MOUNTED TRANSFORMER SUBSTATIONS

These types of stations are very small in size. The cost of these substations is less as they require less space and no building, as such, to function. All the equipment used is of outdoor type. The layout diagram of a pole-mounted transformer substation is shown in Fig. 13.11(a). When these types of substations are used for local distribution, the low-tension (LT) line originates at the same support, whereas, when it is used for a single consumer, connections may be given through underground cables. For small-capacity Transformers, i.e., up to 200 kVA, the transformer is mounted on a platform erected on the structure as shown in Fig. 13.11(c). Beyond 200 kVA, Transformers become heavy, and hence, are mounted on plinth-mounted platforms.

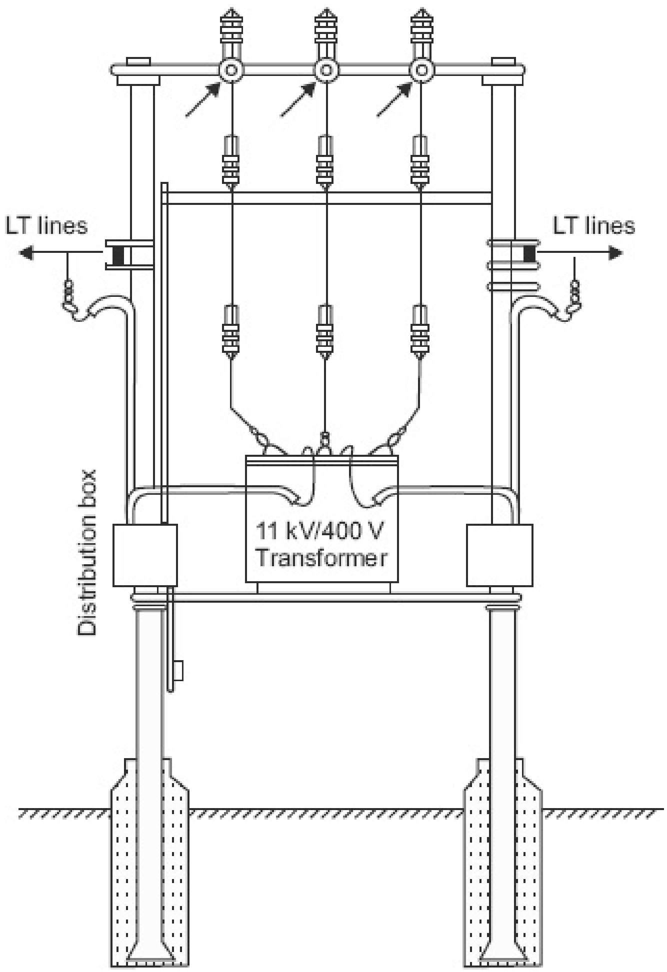

Fig. 13.11(a) Schematic diagram of pole mounted substation

The layout diagram as shown in Fig. 13.11 (b), has all the accessories named. These substations are used for secondary distribution, and hence, are seen in any locality of a town or a village.

The HT side of the transformer is protected by a Horn-gap fuse and switching is done by an air-break switch operated from the ground. On the LT side, the fuse gear is enclosed in a sheet-metal cube mounted on the pole itself.

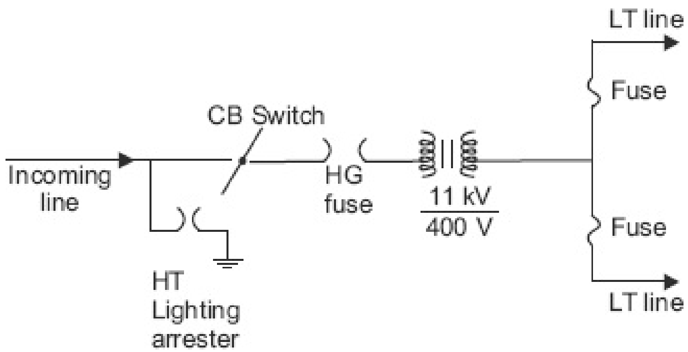

Fig. 13.11(b) Layout diagram of Fig. 11.11(a)

Fig. 13.11(c) View of 11 kV/400 V plinth mounted transformer

13.8 OPTIMAL SUBSTATION LOCATION

Every consumer in a utility system should be supplied from the nearest substation. Supplying each consumer from the nearest substation, i.e., distribution-delivery distance must be as short as possible, which reduces the feeder cost, electric-power loss costs and service-interruption exposure.

To serve every consumer from the nearest substation and to locate substations as close as possible to as many consumers as possible, it is necessary to optimize site, size and service area. For optimal sub-station location, the following method is used.

13.8.1 Perpendicular Bisector Rule

It is a graphical method of obtaining optimum location of substation for serving the consumers from the nearest substation. It is also useful to determine the location of a new substation to maximize its closeness to as many consumer loads as possible. This rule is a very useful concept. Application of this rule to a service-area map consists of several steps:

1. Draw a straight line between a proposed substation site and each of its neighbours.
2. Perpendicularly bisect each of those lines, i.e., divide it in two equal parts.
3. The set of all the perpendicular bisectors around a substation defines its service area.
4. The target load for this substation is the sum of all loads in its service area.

This process is illustrated in Fig. 13.12. Step (2) of this process determines a set of lines that are equidistant between the substation and each of its neighbours. The set of all such lines around a proposed substation site encloses the area that is closer to it than any other substation. As a starting point in the planning process, this should be considered as its preferred service area. The sum of all loads inside this set of lines defines the required peak demand to be served by the substation.

The impact on the loading of the nearby substation can be determined in a similar manner, by using the perpendicular bisector method to identify how their service area boundaries change, what area they give up to the new substation and how much their load is reduced by the new substation taking a part of their service area away from them.

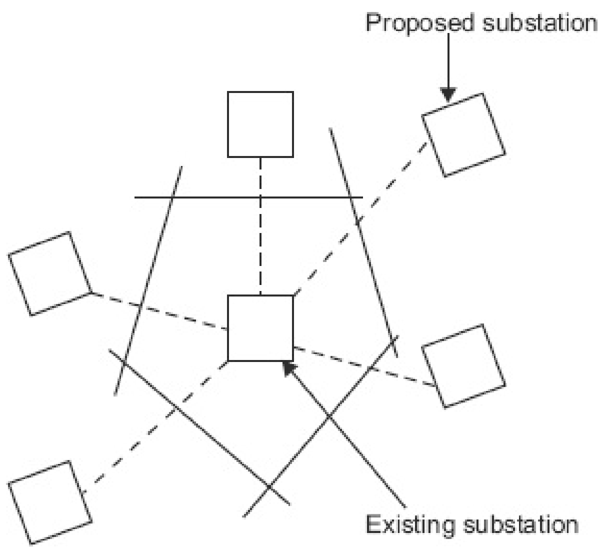

Fig. 13.12 Illustration of perpendicular bisector rule

13.9 BASIC TERMS OF EARTHING

Earth An object is said to be earthed when it is electrically connected to an earth electrode. A conductor is said to be solidly earthed when it is electrically connected to an earth electrode without intentional addition of resistance or impedance in the earth connection.

Earth Electrode A metal plate, pipe, any other conductor or an array of conductors electrically connected to the general mass of the earth.

Earthing Lead The conductor by which the earth electrode is connected to neutral is called earthing lead.

13.10 GROUNDING OR NEUTRAL EARTHING

The word “earthing” and “grounding” have the same meaning. But equipment earthing is different from neutral-point earthing. Equipment earthing refers to connecting the non-current carrying metallic parts to earth available in the neighbourhood of electrical circuits. For example: motor body, switch gear, Transformer tank, etc.

Thus, the purpose of neutral earthing and that of equipment earthing are distinctly different. Equipment earthing ensures safety while neutral earthing is done mostly to ensure that the stator short-circuit current is limited and for stability reasons.

The very purpose of earthing is to safeguard the equipment against possible damage due to electric shock, fire, etc. It is very important to have good and effective earthing or grounding. Today, the neutral grounding is an important aspect of power-system designs because the performance of the system, in terms of short circuits, stability, protection, etc., is greatly affected by the state of the neutral. In most of the modern high-voltage systems, the neutral of the system is solidly grounded, i.e., the neutral is connected directly to the ground.

Earthing is provided with the following objectives:

- For safety of personnel from electric shock.
- For safety of equipment and personnel against lightning and voltage surges.
- For reducing the voltage stress on the lines and the equipment with respect to earth under various operating and fault conditions and also for controlling the earth fault currents for protective relays.

Disadvantages of ungrounded system:

The disadvantages of the ungrounded system are:

- In ungrounded systems, arcing grounds are established due to line-to-ground fault.
- The voltage of two healthy lines is increased by V3 times and the capacitive currents are also increased by V3 times. This causes stress on the insulation of all the machines and equipment connected to the system.

- In ungrounded systems, earth faults cannot be easily sensed and the earth fault relaying becomes complicated.
- The over voltages due to induced static charges is not conducted to earth in ungrounded systems.
- The voltages due to lightning surges do not find the path of earth.
- Neutral point is shifted.

Advantages of neutral grounding:

The advantages of neutral grounding are:

- Surge voltages due to the arcing grounds are reduced or eliminated.
- The voltages of phases are limited to the line-to-ground voltages.
- The life of insulation is long due to prevention of voltage surges or sustained over voltages, thereby reducing maintenance cost, repairs, and breakdowns with improved continuity.
- Stable neutral point.
- The earth fault relaying is relatively simple. A useful amount of earth fault is available to operate earth-fault relay.
- The over voltages due to lightning are discharged to earth.
- By connecting resistance or reactance in earth connection, the earth fault current can be controlled.
- Improved service reliability is achieved due to limitation of arcing grounds and prevention of unnecessary tripping of circuit breakers.
- Greater safety to personnel and equipment due to operation of fuses or relays on earth fault and limitation of voltages.
- Life of equipments, machines and installation is improved due to limitation of voltage.

Test Yourself

1. What are the differences between neutral earthing and equipment earthing?

13.11 EARTHING OF SUBSTATIONS

Earthing systems are provided in a substation for the following main reasons:

- To provide a discharge path for lightning arrests, gaps, etc.
- To provide proper grounding for substation equipments.
- To keep the non-current carrying metal parts, such as transformer tank, structures, etc., safe at earth potential even though the insulation fails.

The fault current through the neutral of the transformer to earth is to be distributed over a large area; otherwise the potential gradient of the earth around the earth connection may be dangerous. Earth mat consisting of heavy gauge-bonded conductors or cost iron grid buried at a depth of not less than 30 cm and connected to the earth electrode, serves best to distribute the heavy fault current over a large earth area. For small stations, more number of earthing stations are provided at different places and are interconnected which helps to reduce the potential gradient.

The resistance of the earth connections cannot be fixed to a definite value because it depends on the kVA rating and the voltage of the system. For high value of the kVA rating and lower values of system voltages, the station earth resistance should be low.

For pole-mounted and plinth-mounted transformer substations, three earthings are provided, one for lightning arrestors, one for transformer neutral and the other for all other points.

13.12 METHODS OF NEUTRAL GROUNDING

The various methods of grounding the neutral of the system are:

1. Solid grounding
2. Resistance grounding
3. Reactance grounding
4. Peterson-coi1 grounding (or resonant grounding)
5. Voltage transformer grounding
6. Earthed transformer

13.12.1 Solid Grounding or Effective Grounding

The term effective grounding is the same as solid grounding. The neutral is directly connected to ground without any impedance between neutral and ground as shown in Fig. 13.13.

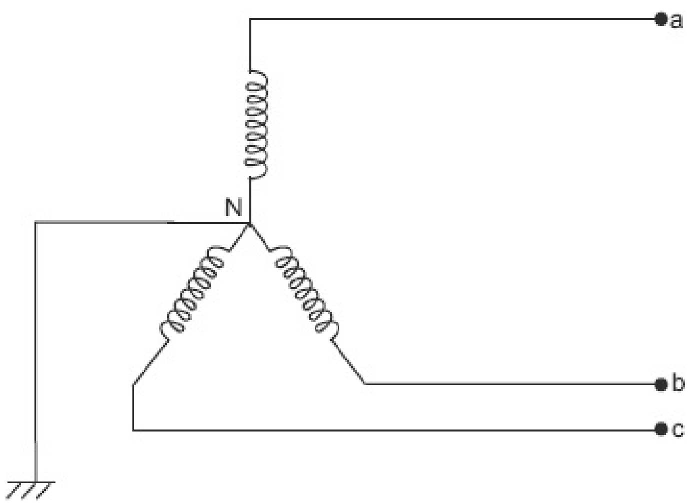

Fig. 13.13 Solidly grounded system

Consider a line to ground fault on any one phase, say phase ‘a’ at a point F as shown in Fig. 13.14. As a result of this fault, the line to neutral voltage of phase ‘a’ becomes zero, but the phase to the earth voltage of the remaining two healthy phases will be approximately constant, as shown in Fig. 13.14.

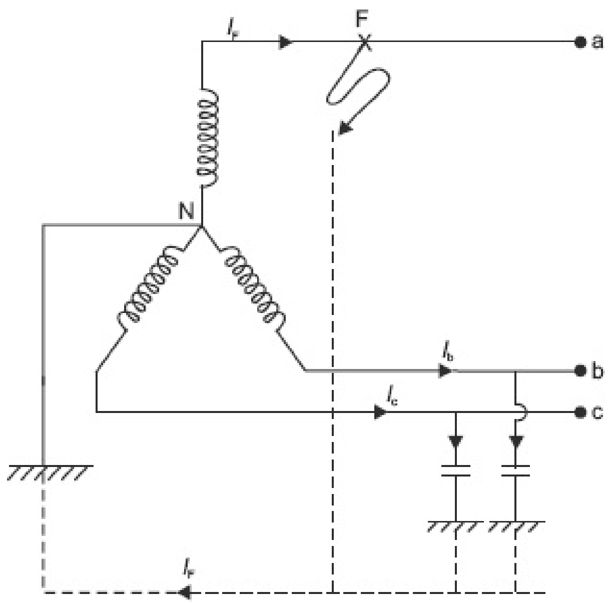

Fig. 13.14 Solidly grounded system with fault at phase ‘a’

The capacitive currents flowing in the healthy phases are I_b and I_c. The resultant capacitive current I_{cf} will be the vector sum of healthy phase currents I_b and I_c. It should be noted that in this system, in addition to charging currents, the power source also feeds the fault current I_f. This fault current will go to the fault point through the faulty phase and then return back to power source through the earth and the neutral conductor. Generally, the generator or transformer has its own reactance in series. When the generator reactance is too low, solid grounding of the generator without any external impedance may cause the single line to ground fault current from the generator to exceed the maximum three-phase fault current which the generator can deliver. This may exceed the short-circuit current for which its windings are braced. If the reactance of the generator or transformer is very large, then also the purpose of the grounding is defeated. The resistance of the earth path is negligible, therefore, the magnitude of the fault current If is only dependent upon the zero-sequence impedance of the power source and that of the phase conductor up to the point of fault.

The resistive component of the zero-sequence impedance is usually negligible, so high-magnitude fault current is opposite to the total capacitive current. This capacitive current lags the faulty phase voltage by approximately 90°. As shown in Fig. 13.15, I_f will have an opposite phase relationship to that of I_{cf}. Due to this, the effect of the capacitive current will be neutralized by high faulty current, thereby eliminating the chances of arcing grounds and over-voltages up to a greater extent.

As explained above, a high fault current will be formed. This is a disadvantage, as it causes the system to become unstable. Thus, to keep the system stable, this method is usually employed so that the circuit impedance is sufficiently high to keep the fault current within limits.

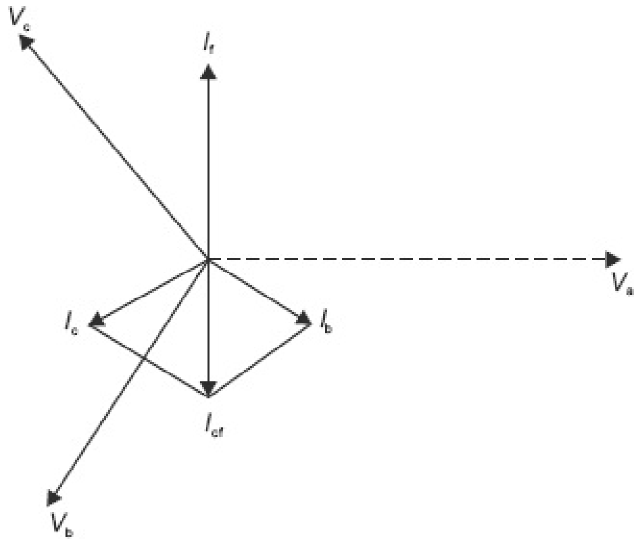

Fig. 13.15 Phasor diagram of solidly grounded system

Advantages:

The size and cost of the transformer is reduced, as less insulation is required when the neutral is earthed. Thus, progressively less insulation is used on a transformer winding as we approach the neutral point. For high-voltage systems, below 33 kV with total capacity not exceeding 5,000 kVA, it is economical to operate with a solid earthed neutral point.

Disadvantages:

- Heavy earth-fault current results in greater interference to neighbouring communication circuits.
- The major drawback is that it is difficult to handle the circuit breakers due to high fault current. For this reason, heavy contacts in circuit breakers are to be provided. Thus, the application of this system beyond 33 kV is uneconomical, and hence, not used.

13.12.2 Resistance Grounding

When it becomes necessary to limit the high earth-fault current, a current-limiting device is introduced between neutral and earth. As shown in Fig. 3.16, a pure resistance is introduced in the neutral and the earth as the current-limiting device. The value of the resistance commonly used is quite high as compared with the system reactance. The resistor may comprise of a metallic resistance unit or a liquid resistor. It is more usual to use liquid resistors if the voltage is 6.6 kV or more.

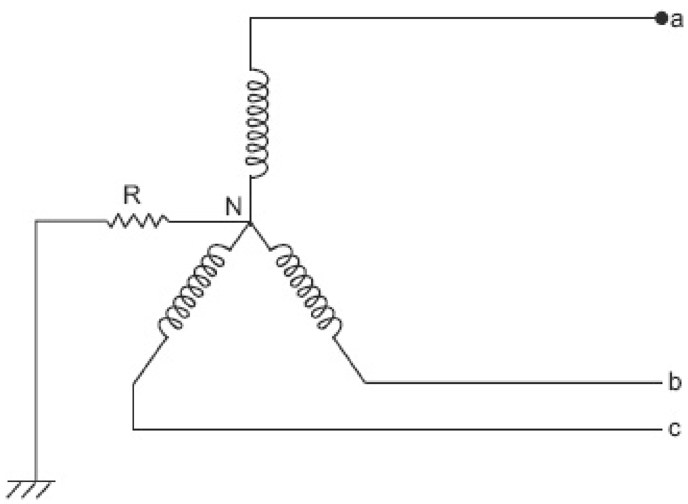

Fig. 13.16 Resistance grounding system

With the increase in operating voltage, value of resistance required for grounding also increases to limit the short-circuit current during line-to-ground faults. Resistance grounding reduces the arcing-ground hazards and permits ready relaying of ground faults. This system can be operated for obtaining the voltage characteristics similar to that of the isolated, earthed system.

Let us assume that an earth fault occurs in phase ‘a’ of a resistance-grounded neutral system as shown in Fig. 13.17. The phasor diagram is shown in Fig. 13.18. Let I_b and I_c be the capacitive currents flowing in the healthy ‘b’ and 'c' phases. In this case, the fault current depends on line resistance upon the point of fault, upon the value of resistance inserted in the earth circuit, and in addition depends on zero-sequence impedance of power source.

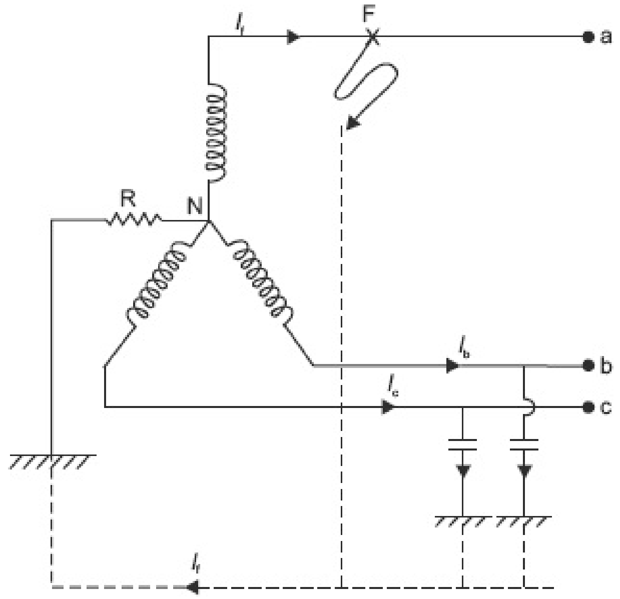

Fig. 13.17 Resistance grounding system with fault at phase 'a'

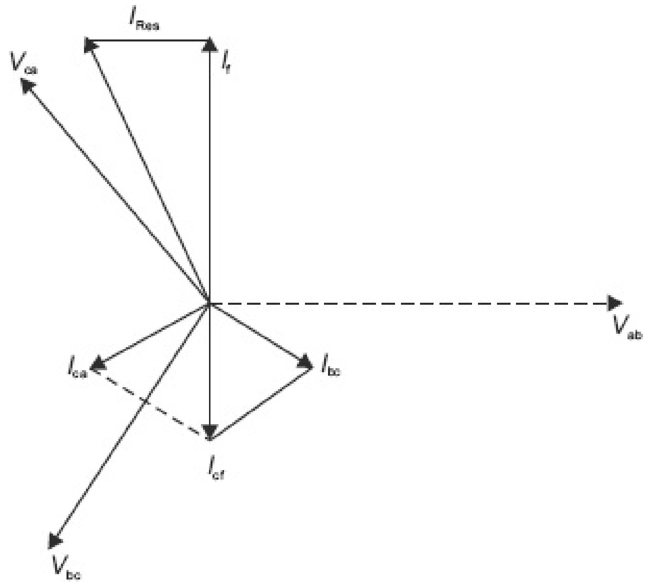

Fig. 13.18 Phasor diagram of resistance-grounded system

Fault current has two components, one is in phase with the faulty-phase voltage and the other lagging the faulty phase voltage by 90°. The lagging component of this current will be in phase opposition to capacitive current and its value changes with changing earth resistance.

This type of resistance earthing is commonly employed for the system operating at voltages between 2.2 kV and 33 kV and the power source capacity exceeds 5,000 kVA.

The advantages of resistance grounding:

- It permits the use of discriminative productive gear.
- Transient ground faults are converted into controlled current faults.
- It minimizes the hazards of arcing grounds.
- In general, a resistance-grounding system will have ground fault current materially lower than that of the effectively grounded system, and therefore, the inductive interference to the neighbouring communication circuits is also lesser than that in an effectively grounded system.
- Power dissipation in the grounding resistance may improve the system stability because it reduces the accelerating power.

Disadvantages of resistance grounding:

- The system is costlier than the solidly-grounded system.
- There is enormous energy loss in neutral ground resistance due to dissipation of fault energy.

13.12.3 Reactance Grounding

In this system, a highly inductive reactance coil is inserted between the neutral and earth to limit the fault current as shown in Fig. 13.19. If resistance is used, fault current is limited and system reactance provides the necessary phase opposition between capacitive ground current and fault current. Reactance grounding provides additional reactance, thereby, neutralising the capacitive currents. Hence, for circuits where high charging currents are involved such as transmission lines, underground cables, etc., reactance grounding is provided.

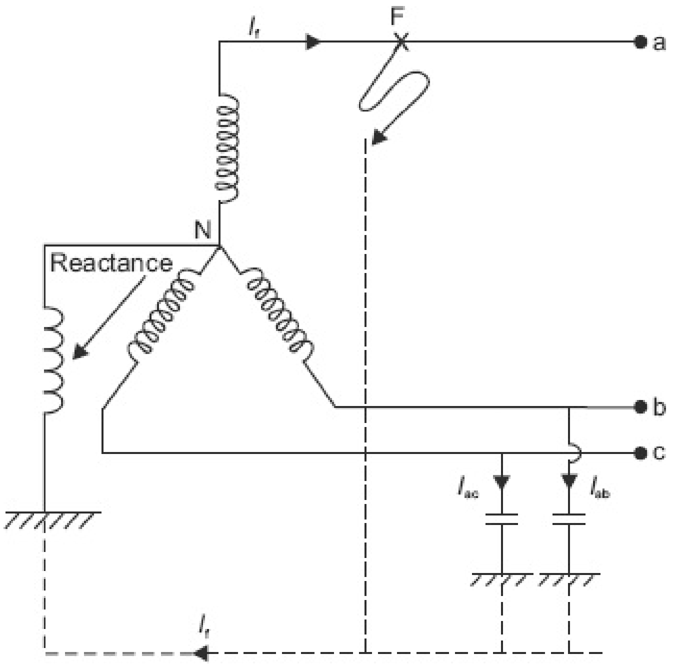

Fig. 13.19 Reactance-grounding system

For reactance earthing, it is essential that the magnitude of the earthing fault current should be at least 25% of the three-phase fault current. This is higher than that required in a resistance-earthing system. In addition to zero-sequence impedance of the power source and faulty phase up to fault point, the fault current is dependent upon the reactance. Thus, by varying the reactance in the earth circuit, the magnitude of the fault current can be changed. Its characteristics are similar to the solid-earth system.

Reactance-grounding system lies between effective grounding and resonant grounding.

The main characteristics of reactance grounding:

1. Ground fault current is reduced, but it is much larger than capacitive fault current.

2. The voltage across a healthy phase is between 0.8 and 1 per unit of line-to-line voltage.
3. Arcing grounds are avoided.
4. Transient ground faults are converted into controlled-current faults.
5. Ground faults relaying is simple and satisfactory.
6. Since the ground fault current is reduced, the inductive interference with the parallel communication circuits is also reduced.

Reactance grounding may be used for grounding the neutral of circuits where high- charging currents are involved such as transmission lines, underground cables, synchronous motors, synchronous capacitors, etc. This system ensures satisfactory relaying, partial grading of apparatus insulation, reduced interference with communication circuits as compared to solid-grounded systems.

Advantages:

- Satisfactory relaying.
- Partial grading of the apparatus insulation.
- Relatively less interference to neighbouring communication circuits as compared to a solidly-earthed system.

Disadvantage:

- The generation of high transient over-voltages, prevents the reactance-earthing system from being commonly used.

13.12.4 Peterson-coil Grounding

The Peterson coil, also known as arc-suppression coil or ground-fault neutralizer, is reactor provided with an iron core connected in the neutral-grounding circuit. It is provided with tapping so that the value of reactance can be changed to neutralize the capacitance of the system. The reactance is selected so that the current through the reactor is equal to the line-charging current which would flow into the line-to- ground fault if the system were operated with the neutral ungrounded. The reactance balances the system ground capacitance so that the resultant ground-fault current is practically zero. This method is also known as resonant grounding.

Consider a line to ground fault in line ‘a’ at point F as shown in Fig. 13.20. The phasor diagram is shown in Fig. 13.21. Consequently, the line-to-ground voltage of phase ‘a’ becomes zero and $I_b = I_c = 0$. The voltages of the remaining two healthy phases ‘b’ and ‘c’ increase from phase values to line values. The currents through the capacitances C_b and C_c become I_{ba} and I_{ca}. The current I_{ba} leads V_{ba} by 90°.

The resultant of I_{ba} and I_{ca} is I_{cf}. The phasor diagram is shown in Fig. 13.21.

$$I_{ba} = \frac{V_{ba}}{X_{cb}} = \frac{\sqrt{3}V_p}{X_c} \tag{13.1}$$

$$I_{ca} = \frac{V_{ca}}{X_{cc}} = \frac{\sqrt{3}V_p}{X_c} \tag{13.2}$$

$$I_{ba} = I_{ca} \tag{13.3}$$

$$I_c = I_{ba} + I_{ca} \tag{13.4}$$

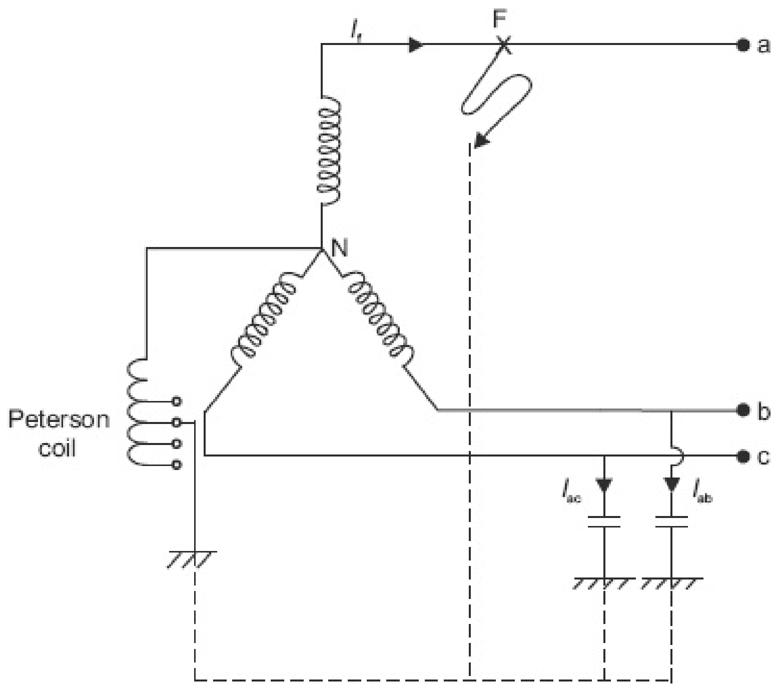

Fig. 13.20 LG fault on phase 'a'

From the phasor diagram 13.20,

$$I_{cf} = \sqrt{3}I_{ba} = \sqrt{3}I_{ca} = \frac{\sqrt{3}\times\sqrt{3}V_p}{X_c} = \frac{3V_p}{X_c} \qquad (13.5)$$

The current I_L through the inductances L of the Peterson coil is given by

$$|I_L| = \left|\frac{V_{na}}{\omega L}\right| = \frac{V_p}{\omega L} \qquad (13.6)$$

But V_{na} is equal to $-V_a$. The current I_L lags behind V_{na} by 90°. The current I_{cf} is in phase opposition to I_L. Hence, I_L is equal to I_{cf} and there will be no current passing through the ground and no tendency of formation of arcing grounds. With the use of a Peterson coil, the arc current is reduced to such a small value that it is usually self-extinguishing. Therefore, Peterson coils are also known as ground-fault neutralizers or arc-suppression coils.

The inductance value for the Peterson-coil can be calculated as follows:

For no current through the ground fault,

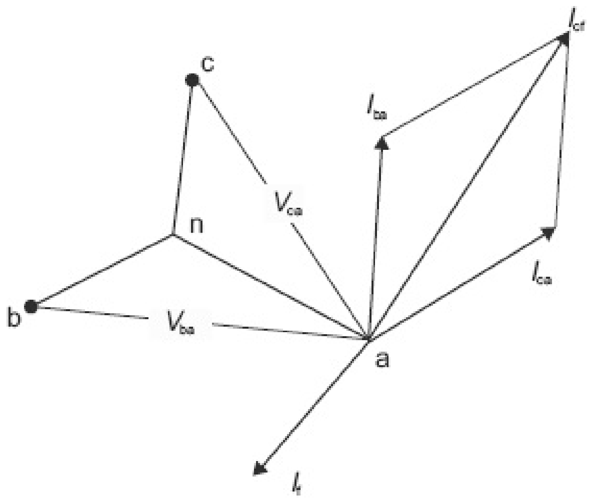

Fig. 13.21 Phasor diagram with LG fault on phase 'a'

$$|I_L| = |I_{cf}|$$

$$\frac{V_p}{\omega L} = \frac{3V_p}{X_c} = 3V_p \omega C \qquad (13.7)$$

$$L = \frac{1}{3\omega^2 C} \qquad (13.8)$$

Eqn. (13.8) can also be written as

$$\omega = \frac{1}{\sqrt{(3C)L}} \qquad (13.9)$$

The above relationship is also a condition for resonance at the supply frequency ω rad/sec.

In other words, this condition states that the inductance must be "tuned" to the capacitance if the Peterson coil is to fulfill its function.

Example 13.1

A 33 kV, 50 Hz network has capacitance to neutral of 1.2 μF per phase. Calculate the reactance of an arc-suppression coil suitable for the system to avoid the adverse effect of the arcing ground.

Solution:

Supply frequency, f =50 Hz

Capacitor of each conductor to earth, $C = 1.2\ \mu F$

$= 1.2 \times 10^{-6}$ F

Necessary reactance of arc-suppression coil, $\omega L = \frac{1}{3\omega C}$

$= \frac{1}{3 \times 2\pi \times 50 \times 1.2 \times 10^{-6}}$

$= 884.2\ \Omega.$

Example 13.2

A 230 kV, three-phase, 50 Hz, 200 km transmission line has a capacitance to earth of 0.015 μP/km/phase. Calculate the inductance and kVA rating of the Peterson coil used for earthing the above system.

Solution:

Capacitor of each line conductor to earth, $C = 200 \times 0.015\ \mu F$

$= 3.0 \times 10^{-6}$ F

Necessary inductance of Peterson coil, $L = \frac{1}{3\omega^2 C}$

$$= \frac{1}{3 \times (2\pi \times 50)^2 \times 3 \times 10^{-6}}$$

$= 1.126$ H

Current through Peterson coil, $I_L = \frac{V_P}{\omega L}$

$$= \frac{230 \times 1000}{\sqrt{3} \times 2\pi \times 50 \times 1.126}$$

$= 375.387$ A

Rating of the Peterson coil $= \frac{230 \times 1000}{\sqrt{3}} \times 375.387 \times \frac{1}{1000}$

$= 49847.85$ kVA.

Example 13.3

A 132 kV, three-phase, 50 Hz transmission line 200 km long, consists of three conductors of effective diameter 18 cm, arranged in a vertical plane with 3.5 m spacing and regularly transposed. Find the inductance and MVA rating of the arc-suppression coil in the system.

Solution:

The radius of conductor, d =18 cm

The radius of conductor, r = 9 cm

Conductors are arranged in a vertical plane and the arrangement is shown in Fig. 13.22.

Therefore GMD $= \sqrt[3]{3.5\times3.5\times7} = 4.41$ m geometrical

The capacitance per phase of a line, $C = \dfrac{2\pi\varepsilon_0}{\ln\dfrac{\text{GMD}}{r}}$ F/m

$$\therefore C = \frac{10^{-9}}{18\ln\dfrac{\text{GMD}}{r}} = \frac{10^{-9}}{18\ln\left(\dfrac{\sqrt[3]{3.5\times3.5\times7}}{9\times10^{-3}}\right)}$$

$$= 8.969\times10^{-12} \text{ F/m}$$

$$= 8.969\times10^{-12}\times200\times10^{3}$$

$$= 1.7937\ \mu\text{F}.$$

Now, $\omega L = \dfrac{1}{3\omega C}$

$$L = \frac{1}{3\omega^2 C} = \frac{10^6}{3\times314^2\times1.7937} = 1.883 \text{ H}$$

$\therefore$ MVA rating of the suppressor coil $= \dfrac{V^2}{3\omega L} = \dfrac{132\times132}{3\times314\times1.883} = 9.918$ MVA per coil.

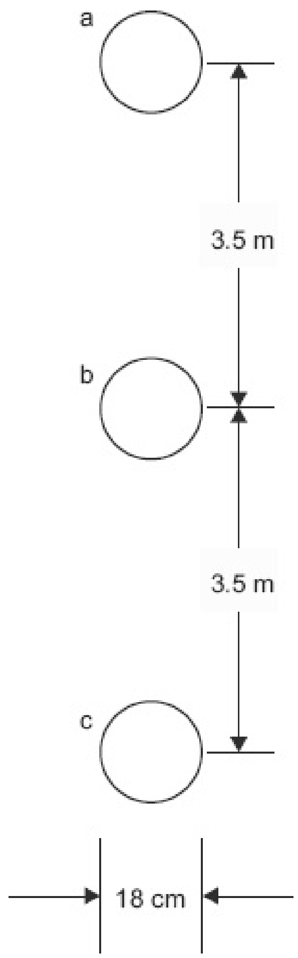

Fig. 13.22 Configuration of conductors

Example 13.4

Determine the value of reactance to be connected in the neutral connection to neutralize the capacitance current, of an overload line having line-to-ground capacitance of each line equal to 0.01 μF. Frequency = 50 Hz.

Solution:

Inductance, $L = \frac{1}{3\omega^2 C}$ H

L = inductance of coil connected in neutral to ground circuit (Henries)
$\omega = 2\pi f$, f = frequency in Hz
C = capacitance to earth of each phase
Here, we have to determine L,

$$L = \frac{1}{3 \times (314)^2 \times 0.01 \times 10^{-6}}$$
$$= 338.08 \text{ H.}$$

Example 13.5

A 50 Hz overhead line has line-to-earth capacitance of 1.25 μF. It is used as an earth-fault neutralizer. Calculate the reactance connected to neutralize the capacitance of:

1. **100% of the length of the line.**
2. **90% of the length of line.**
3. **85% of the length of line.**

Solution:

1. Capacitance of 100% length of the line, $C = 1.25 \times 10^{-6}$ F

$$\omega L = \frac{1}{3\omega C}$$
$$L = \frac{1}{3\omega^2 C}$$
$$L = \frac{1}{3 \times (314)^2 \times 1.25 \times 10^{-6}}$$
$$= 2.705 \text{ H.}$$

To neutralize capacitance of 100% of the line reactance required is 2.705 H.

2. Capacitance of 90% length of line = 1.25 μF × 0.9 = 1.125 μF

$$\therefore L = \frac{1}{3 \times \omega^2 \times C} = \frac{1}{3 \times (314)^2 \times 1.125 \times 10^{-6}} = 3.0 \text{ H}$$

3. Capacitance of 85% length of line = 1.25 μF × 0.85 = 1.0625 μF

$$\therefore L = \frac{1}{3 \times \omega^2 \times C} = \frac{1}{3 \times (314)^2 \times 1.0625 \times 10^{-6}} = 3.182 \text{ H.}$$

Example 13.6

A 33 kV, three-phase, 50 HZ, overhead line of 100 km long has a capacitance to earth of each line equal to 0.02 μF per km. Determine the inductance and kVA rating of the arc suppression coil.

Solution:

$$\text{Inductive coil for suppression coil, } L = \frac{1}{3\omega^2 C}$$

$$= \frac{1}{3 \times (314)^2 \times 0.02 \times 100 \times 10^{-6}} = 1.69 \text{ H}$$

For ground fault, the current in neutral is given by

$$I_{ph} = \frac{V_{ph}}{\omega L} = \frac{33 \times 1000}{\sqrt{3} \times 314 \times 1.69} = 35.904 \text{ A}$$

$$\text{kVA rating of suppression coil} = V_{ph} \times I_{ph}$$

$$= 35.904 \times \frac{33}{\sqrt{3}} = 684.05 \text{ kVA.}$$

13.12.5 Grounding Transformer

If a neutral point is required or not available in case of delta connections and bus bar points, a zig-zag transformer is used. Earthed transformers are used for providing the neutral point for such cases. It is a core-type transformer having three limbs built-up in the same manner as that of a power transformer. Each limb accommodates two equally-spaced windings and the way they are connected is shown in Fig. 13.23. It will be seen that the current in the two halves of the winding on each limb acts in opposite directions. These currents do not allow undeserving harmonics to prevail in the circuit, and thereby, the stresses on the insulation of the transformer are considerably reduced.

The impedance of the earthing transformers is quite low, and therefore, the fault current will be quite high. The magnitude of the fault current is limited by inserting resistance either in the neutral circuit as shown in Fig. 13.24 or in the windings of the earthing transformer. Components of various currents flowing under the conditions are also shown therein.

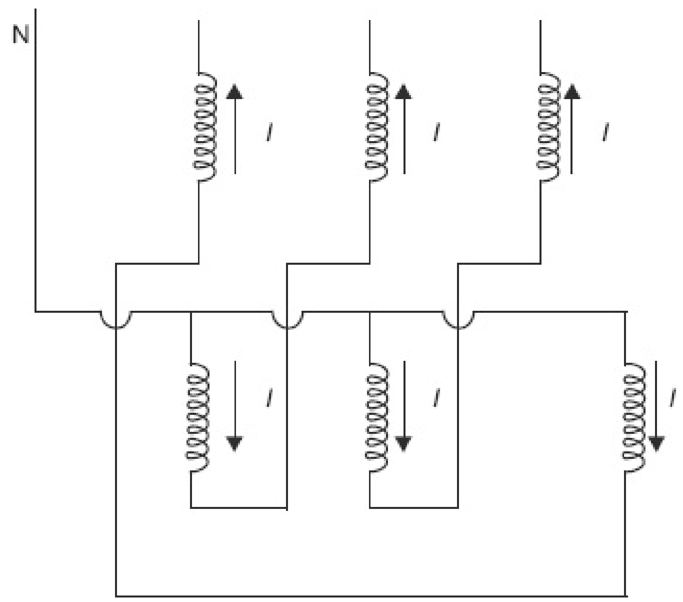

Fig. 13.23 Representation of an earthing transformer

The terminals of the earthing transformers are soldered to the power transformer for obtaining a solid connection between them. The capacity of the earthing transformer is denoted by the fault current it is capable of handling. Under normal operating conditions, it is only iron losses that are continuously present; copper losses are present only when the fault occurs. These copper losses are present only for short periods due to the short duration of fault (in the order of a few seconds).

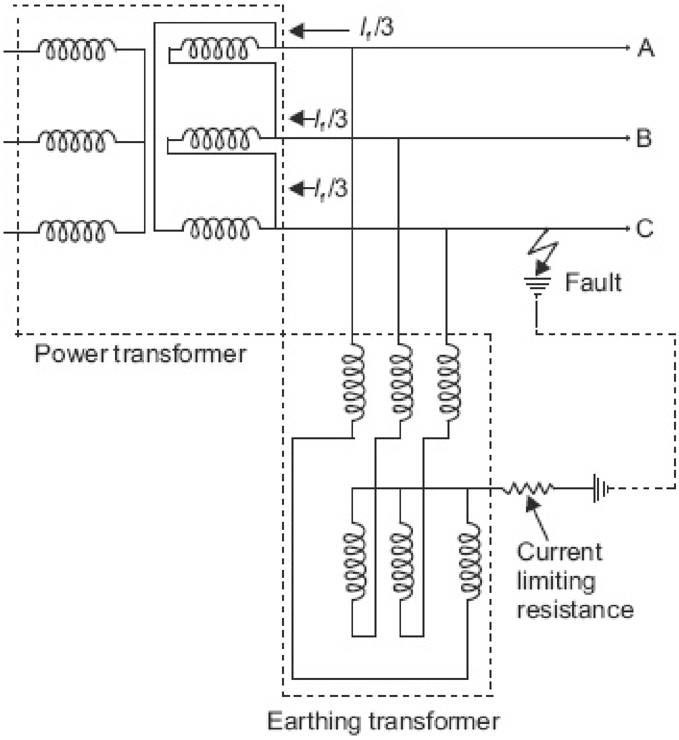

Fig. 13.24 Insertion of resistance in the neutral circuit

13.13 GROUNDING GRID

Grounding grids are used where there is a chance of flow of large fault currents through the system. A grounding grid is composed of a number of rods that are joined together through copper conductors to reduce the overall grounding resistance. They are also known as earthing mats or earthing grids. This arrangement should be made near the ground surface as it limits the potential gradient. Driving rods deep into the soil, however, becomes an expensive process, especially in the case of hard soil or soils having low conductivity. The performance of a grounding grid system suffers when the rods are removed from the mesh.

The following factors have to be taken into account while designing a grounding grid:

- Voltage between the ground surface and the conductor should be kept at a minimum in case of a fault.
- The voltage at the time of fault should not cause any harm to the persons or the non- current carrying equipment near the grounding system.
- There should be proper co-ordination between the fault current that may flow into the ground through grounding grids and the protective relays.

CHAPTER AT A GLANCE

1. A substation receives power at high voltage from the transmission or subtransmission system and reduces its voltage to consumer level.
2. Substations are classified according to the service, mounting, function and type of apparatus used.
3. The various equipments in a substation are to be connected by the conductors. These conductors are called bus bars.
4. The various kinds of bus bar arrangements are: Single bus bar, single bus bar system with sectionalization, double bus bar with single breaker, double bus bar with two circuit breakers, breakers and a half with two main buses, main and transfer bus bar, double bus bar with bypass isolator and ring bus.
5. **Single bus bar:** It consists of a single bus bar and all the incoming and outgoing lines are connected. It is suitable for DC stations and small AC stations.
6. Double bus bar with single breaker: It consists of two bus bars; one is the main bus bar and the other, the spare bus bar. This arrangement has been generally adopted where the continuity of supply is required.
7. Main and transfer bus bar: This is an alternative arrangement to the double bus bar scheme. In this arrangement, any line circuit breaker can be taken out for cleaning and repair without affecting the supply.
8. Double bus bar with bypass isolator: This is a combination of a double-bus and main transfer-bus scheme. Anyone of the bus bars can act as a main bus and the other bus be used as the transfer bus.

SHORT ANSWER QUESTIONS

1. Define substation.

2. What is meant by transmission substation?
3. What is meant by subtransmission substation?
4. What is meant by distribution substation?
5. What is meant by switching substation?
6. What is meant by indoor substation?
7. What are the advantages of outdoor substation?
8. What is the function of an isolator?
9. What is the need of an instrument transformer?
10. What is the need of lightning arresters?
11. How are substations classified on the basis of mounting?
12. What is the essential difference between a circuit breaker and an isolator?
13. Why are isolators provided on either side of a circuit breaker?
14. What is the effect of sectionalization on the rating of a circuit breaker?
15. What parameter limits the number of sections in a sectionalized bus bar?
16. When is group switching adopted?
17. What is the important advantage of a main and transfer bus bar arrangement?
18. What is the major drawback of a main and transfer bus bar arrangement from the point of view of protection?
19. What is meant by earth electrode?
20. What are the methods of earthing?
21. What are the objectives of providing earth?

MULTIPLE CHOICE QUESTIONS

1. The insulation coordination for UHV lines is done based on
    1. lightning surges
    2. lightning surges and switching surges
    3. switching surges
    4. faults
2. In comparison to line insulation, the insulation level of the station equipment is
    1. less
    2. more
    3. equal
    4. not related directly
3. When a wave reaches an open circuit the_________ at the termination are double the incident values
    1. voltage
    2. current
    3. both the voltage and current
    4. impedance
4. When a wave reaches a short circuit the__________ at the termination are zero
    1. voltage
    2. current

3. both the voltage and current
4. impedance

5. A backward wave means a
   1. negative voltage wave
   2. negative current wave
   3. a wave travelling in the negative direction
   4. a wave travelling in the positive direction
6. It is fatal to touch a live wire as
   1. the voltage may cause burns to the skin
   2. it may cause flow of current through the human body
   3. the current may cause burns to the skin or inside body
   4. it may cause damage to heart and nervous system
7. Earthing of the voltage is necessary for protection against
   1. overloading
   2. voltage fluctuation
   3. danger of electric shock
   4. high conductor temperature
8. The earth wire should be
   1. good conductor of electricity
   2. mechanically strong
   3. both a and b
   4. poor conductor of electricity
9. Earth wires are made of
   1. copper
   2. aluminium
   3. iron
   4. galvanized standard steel
10. The size of the earth wire is determined on the basis of
    1. voltage of the service line
    2. current carrying capacity of the service line
    3. atmospheric conditions
    4. power rating
11. The earth wire should not be of size smaller than
    1. 10 SWG copper
    2. 8 SWG copper
    3. 6 SWG copper
    4. 4 SWG copper
12. Neutral earthing is provided for
    1. the safety of people from electric shock
    2. the safety of equipment and personnel against lightning and voltage surges
    3. reducing voltage stress on the lines and equipment with respect to earth under various operating and fault conditions
    4. controlling the earth fault currents for protective relaying
    5. both c and d
13. Isolated neutral system is disadvantaged by
    1. voltage oscillations
    2. difficult earth fault relaying
    3. persistent arcing ground
    4. all of these

14. The advantage of neutral earthing are
    1. safety of personnel
    2. reduction of earth fault current
    3. elimination of arcing ground
    4. safety of machines
15. The neutral of the power system may be connected to earth
    1. directly
    2. through a resistor
    3. through a reactor
    4. any of these
16. Grounding is done generally at
    1. receiving end
    2. supply end
    3. either/or
    4. mid point
17. Solid earthing is done for voltages below
    1. 400 V
    2. 600 V
    3. 33 kV
    4. 66 kV
18. Earthing of transformer neutral through reactance will improve its
    1. transient stability
    2. steady state stability
    3. a and b
    4. dynamic stability
19. Resistance earthing is done for voltages between
    1. 3.3 and 11 kV
    2. 11 and 33 kV
    3. 33 and 66 kV
    4. 66 kV and 132 kV
20. Peterson coil is used for
    1. grounding of system neutral
    2. reducing the fault of system
    3. connecting two interconnected systems
    4. reducing the impedance
21. During arcing ground conditions, the phase voltage of the system rises to ________ times its normal value.
    1. 20
    2. 15
    3. 5 or 6
    4. $\sqrt{3}$
22. Arcing on transmission lines is prevented by connecting an__________ in neutral.
    1. inductor
    2. resistor
    3. capacitor
    4. protective layer
23. The positive, negative and zero-sequence impedance of a solidly-

grounded system under steady-state condition always follow relations

1. $Z_0 < Z_1 < Z_2$
2. $Z_1 > Z_2 > Z_0$
3. $Z_0 > Z_1 > Z_2$
4. $Z_2 < Z_0 < Z_1$

24. A grounding transformer is a system usually connected to
    1. delta/delta
    2. star/star
    3. zig-zag/delta
    4. delta/star
25. The method of neutral grounding affects the
    1. positive sequence network
    2. negative sequence network
    3. zero sequence network
    4. positive and negative sequence network

Answers

1. c	2. d	3. a	4. a	5. c
6. b	7. c	8. c	9. d	10. b
11. a	12. e	13. d	14. c	15. d
16. c	17. c	18. a	19. a	20. a
21. c	22. a	23. b	24. b	25. c

REVIEW QUESTIONS

1. What are the factors considered when selecting a location for a substation?
2. What are the merits and demerits of indoor substations over outdoor substations?
3. What are the factors to be considered for selecting bus bars?
4. Briefly discuss the classification of substations.
5. Explain the single bus bar arrangement, its merits and demerits.
6. Briefly discuss the equipments of substations.
7. Explain the single bus bar system with sectionalization and list their merits and demerits.
8. Explain the main and transfer bus bar system with a circuit diagram.
9. Explain the key features of a substation with a suitable diagram.
10. Discuss the merits of earthing if (a) solid (b) through a resistance.
11. Explain the advantages of grounding power-system neutrals.
12. Explain with the help of circuit and phasor diagrams the function of a Peterson coil in a three-phase system.
13. Derive an expression for the inductance of a Peterson coil in terms of the capacitance of the protected line.
14. Explain the concept of neutral grounding for generator protection.

15. What are the disadvantages of an ungrounded system?

PROBLEMS

1. A 33 kV, three-phase, 50 Hz, 50 km overhead line has a capacitance of earth of each line equal to 0.015 mF/km. Determine the inductance and kVA rating of the arc-separation coil.
2. Determine the reactance of a Peterson coil suitable for a 33 kV, three-phase transmission system of which the capacitance to earth of each conductor is 3.5 μF.
3. A 50 Hz transmission line has a capacitance of 0.15 μF per phase. Determine the inductance of the Peterson coil to neutralize the effect of capacitance for the following cases:
   1. Complete length of line.
   2. 95% of length of line.
   3. 85% of the length of the line.
4. A 132 kV, 50 HZ, three-phase, 60 km long transmission line has a capacitance of 0.12 μF/km per phase. Determine the inductive reactance and MVA rating of the arc-suppression coil suitable for the line to eliminate the arcing-ground phenomenon.
5. A 132 kV, three-phase, 50 HZ overhead line of 100 km length has a capacitance to earth of 0.012 μF/km. Determine the inductance and MVA rating of the arc-suspension coil suitable for this line.

14

Distribution Systems

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Obtain an overview of electrical power distribution
- Provide classification of distribution systems
- Discuss design considerations
- Provide an analysis of AC and DC distribution systems

14.1 INTRODUCTION

The power generated is usually three-phase 50 Hz and 11 kV. The generated voltage is stepped up to 220 kV or 400 kV by means of step-up transformers. Then by means of three-phase transmission systems, the generated power is distributed to the various consumption points. Here, the voltage is stepped down to 132 kV or 33 kV and further carried through sub-transmission systems where the voltage is further stepped down to 11 kV. This voltage is again stepped down to 415 V or 230 V, either as three-phase or single-phase, respectively, for small consumption purposes.

The distribution system is a part of the power system, existing between distribution sub-stations and the consumers. The distribution system is further classified on the basis of voltage as primary and secondary distribution systems (11 kV, 415 V/230 V). Distribution system is shown in Fig. 14.1.

14.2 PRIMARY AND SECONDARY DISTRIBUTION

14.2.1 Primary Distribution

The part of the electrical-supply system existing between the distribution substations and the distribution transformers is called the primary system. It is made of circuits, known as primary feeders or primary distribution feeders. The most commonly used nominal primary voltage is 11 kV.

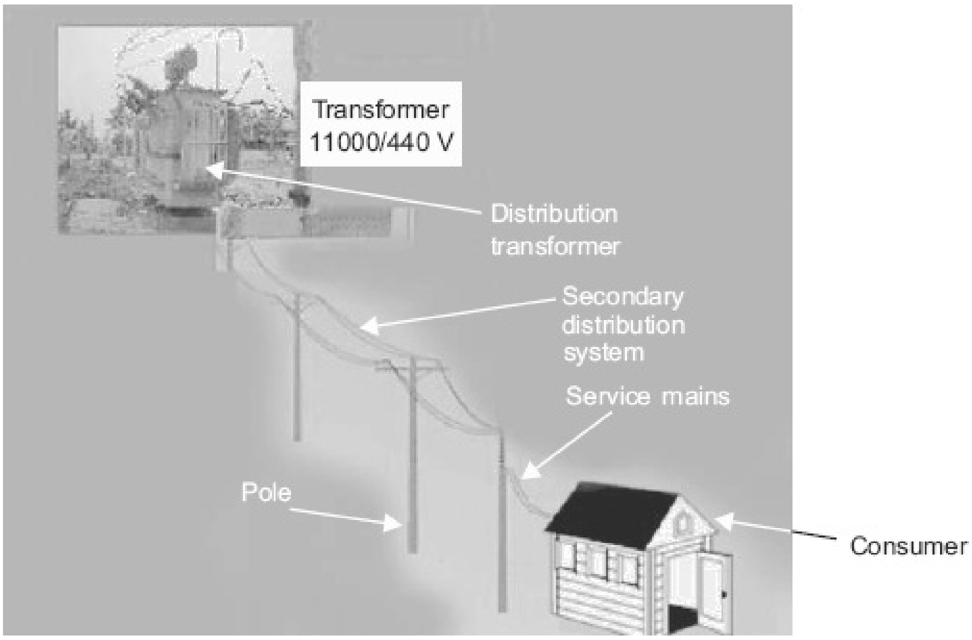

Fig. 14.1 View of distribution system

14.2.2 Secondary Distribution

The secondary distribution system receives power from the secondary side of distribution transformers at low voltage and supplies power to various connected loads via service lines. The secondary distribution system is the final sub-system of the power system.

The secondary distribution systems are generally of the radial type except for some specific service areas such as hospitals, business centres and military installations, which require highly reliable service, and therefore, may be of a grid or mesh type.

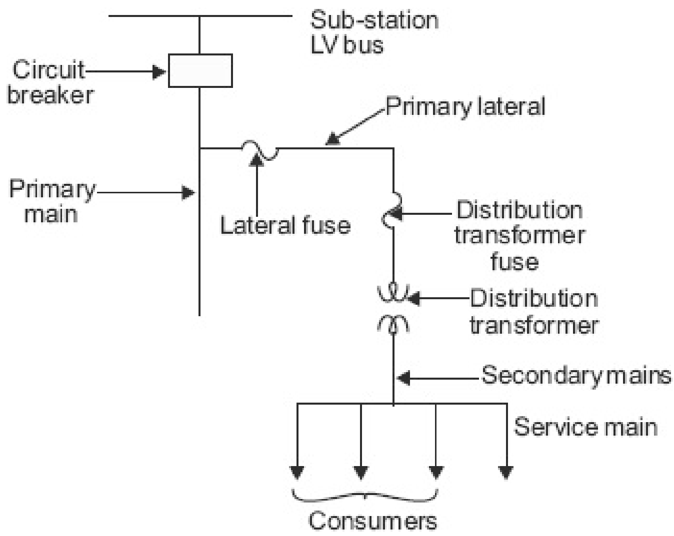

Fig. 14.2 Radial secondary distribution system configuration

As shown in Fig. 14.2, the secondary transformer in a radial secondary circuit are located near the load centres. The primary-distribution transformer receives power from primary laterals via a fuse cut-out or a fuse switch, whereas the secondary side of the distribution transformer (DT) supplies power through secondary mains, service conductor to service meter. This includes the following:

- A separate service system for each customer with separate DT and secondary connection (e.g., single phase DT).
- The radial system with a common secondary main which is supplied by one DT and feeding a group of customers (e.g., three-phase DT).
- The parallel connection system with a common secondary main that is supplied by several DT's that are all fed like common primary feeders (e.g., secondary banking of DT's).

Test Yourself

1. Why are LVDS systems being increasingly replaced by HVDS systems?

14.3 DESIGN CONSIDERATIONS IN A DISTRIBUTION SYSTEM

Good voltage regulation is the most important factor in a distribution system for delivering good service to the consumer. For this purpose, careful consideration is required for the design of feeders and distributor networks.

Feeders are the conductors that connect substations to consumer ports and have large current- carrying capacity. The current loading of a feeder is uniform along the whole of its length since no tappings are taken from it. The design of a feeder is based mainly on the current that is to be carried.

Distributors are the conductors, which run along a street or an area to supply power to consumers. These can be easily recognized by the number of tappings, which are taken from them for the supply to various consumer terminals. The current loading of a distributor is not uniform and it varies along the length while its design is largely influenced by the voltage drop along it.

Service Mein and Sub Main The service mains are the conductors forming connecting links between distributors and metering points of the consumer's terminal. Figure 14.1 shows the layout of a distribution system. The term sub main refers to the several connections given to consumers from one service main. The area of cross-section of a sub-main conductor is greater than that of the service mains.

14.4 DISTRIBUTION SYSTEM LOSSES

It has been established that 70% of the total losses occur in the primary and secondary distribution system, while transmission and sub-transmission lines account for only 30% of the total losses. Distribution losses amount to 15.5% of the generation capacity and the target level is to bring it down to 7.5%. Therefore, the primary and secondary distribution systems must be planned with care to ensure losses within acceptability limits.[1]

14.4.1 Factors Effecting Distribution-System Losses

Factors contributing to the increase in line losses in the primary and secondary distribution systems are:

Inadequate Size of Conductor As stated above, the rural load is usually scattered and fed by radial feeders. The conductor size of the feeders must be adequate. The size of the conductor should be selected on the basis of km-kVA capacity of the stranded conductors.

Feeder Length In practice, 11 kV and 415 V lines in rural areas are hurriedly extended radially over long distances to feed loads scattered over large areas. This results in high line resistance, low voltage, and high current, and therefore, leads to high PR losses in the line.

Location of Distribution Transformers Often the distribution transformers are not located centrally in relation to the intended customer. Consequently, the farthest customers obtain an extremely low voltage even though a reasonably good voltage level is maintained at the secondary transformer. This again leads to higher line losses.

Therefore, in order to reduce the voltage drop in the line transmitting power to consumers located farthest from the line, the DT should be located at the load center to keep the voltage drop within permissible limits.

Low Voltage Whenever the voltage applied to an induction motor deviates from rated voltage, its performance is adversely affected. A reduced voltage, in case of an induction motor, results in higher currents drawn for the same output leading to higher losses. This can be overcome by adjusting the tap changer at power transformer and at distribution transformer, if available.

Use of Over-rated Distribution Transformers Studies on 11 kV feeders have revealed that often the rating of distribution transformers (DTs) is much higher than the maximum kVA demand on the LT feeder. An overrated transformer produces an unnecessarily high iron loss.

From the above, it is clear that the rating should be judiciously selected to keep the losses within the permissible limits.

Low Power Factor In most of the LT distribution systems, it is found that the power factor varies from as worse as 0.65 to 0.75. A low power factor contributes towards high distribution losses. For a given load, if the power factor is low, the current drawn is high, consequently the losses proportional to a square of the current, will be more.

Thus, line losses owing to the poor power factor can be reduced by improving the power factor by using shunt capacitors for the purpose.

14.4.2 Methods for the Reduction of Line Losses

As discussed in the previous section, the distribution-system losses are on the higher side in the Indian power system. The Government of India has decided to reduce the line losses and set a target for the reduction of transmission and distribution losses by 1% per annum in order to realize an overall reduction of 5% in the national average by the end of the 8th five-year plan.[2]

The following methods are adopted for the reduction of distribution-system losses:

1. HV distribution system
2. Feeder reconfiguration
3. Reinforcement of the feeder
4. Grading of conductor
5. Construction of new substation
6. Reactive-power compensation

HV Distribution System The low-voltage distribution system contributes a majority of the total distribution losses because of poor voltage regulation. The low tension distribution system, which is based on the European practice, where loads are concentrated in small areas with high-load densities in addition to high power factor. The load factor is most ill suited to cater to the scattered and highly inductive load with very low load densities, low power factor, and low load factor in developing countries. The present situation is that LV lines are extended, irrespective of voltage drops, up to the full capacity of the DT and sometimes over and above the transformer capacity. Hence, no purpose will be served by prescribing low kVA-km loading limits for LV lines. The only practice and feasible solution is to eliminate or minimize LV lines by switching over to single-phase high voltage distribution. By adopting HV distribution, the losses in the LV distribution can be minimized.

Feeder Reconfiguration It is defined as the process of altering the topological structure of distribution feeders by changing the open/closed status of the sectionalising and tie switches. Feeder reconfiguration allows the transfer of loads from heavy loaded feeders to moderately heavy loaded feeders. Such transfers are effective not only in terms of altering the levels of loads on the feeders being switched, but also in improving the voltage profile along the feeders and effecting reduction in the overall system power losses.

Reinforcement of the Feeder There are more losses in the distribution system due to the fact that the conductor size used at the time of erection of the feeders is no more optimal with reference to the increased total load. The total cost is the sum of the fixed cost of investment on the line and the variable cost of energy losses in the conductor due to the power flow.

Addition of new load on an existing feeder is limited by its current-carrying capacity. Therefore, if the existing feeder gets overloaded, the alternative to catering to the extra load is only the reinforcement of the feeder. This method is considered to be good for short-term planning measures.

Reinforcement of conductors is considered necessary as the smaller-sized conductors encounter high losses due to unplanned use. However, at the time of reinforcement much supply interruptions will take place, which leads to loss of revenue.

Grading of Conductor In normal practice, the conductor used for radial distribution feeders is of uniform cross-sectional area. However, the load magnitude at the substation is high and it reduces as we proceed to the tail-end of the feeder. This indicates that the use of a higher-sized conductor, which is capable of supplying load from the source point, is not necessary at the tail-end point. Similarly, the use of different conductor cross-sections for intermediate sections will lead to a minimum capital investment cost and line loss.

The use of a larger number of conductors of different cross-sectional areas will result in increased costs of inventory. A best choice can, however, be made in selecting the size of cross-sectional area for optimal design.

Tie lines are the most economical method to reduce losses, but in practice, it is uncommon in rural India. Constructing new tie lines for small excess loads leads to unnecessary increase in capital investment.

Construction of New Substation If a new substation is to be constructed and connected to an existing network, several possible solutions are to be studied. These solutions may include various connection schemes of the substation and several feasible locations, while the principal connection scheme is defined by a limited number of possibilities. The number of possible sites of the newly constructed HT (33 kV) line and its location determines the cost of their construction and operation. Due to the large number of possible sites, an economical comparison may overlook the optimum technical solution. The final decision is usually influenced by additional factors such as topography, land ownership, environment considerations, etc. The optimum site for a substation is defined as that location which will result in minimum cost for construction and minimum losses. These include both the investments for the 11 kV and 33 kV voltage systems and the cost of operating the system.

Therefore, by constructing a new substation at load centres, the line losses will be reduced due to an improvement in the voltage profile and a reduction in the length of the lines. However, for an excess small quantum of load, the decision for the construction of new substations cannot be made as the capital investment is high and the substations run under load condition for a long time resulting in poor return on the capital. So, in such situations, alternative arrangements can be attempted.

Reactive Power Compensation It is universally acknowledged that the voltage-reactive power control function has a pivotal role to play in the distribution automation. The problem of reactive power compensation can be attempted by providing static capacitors.

The method presently used to compensate the reactive power component is to increase the reactive power by increasing the terminal voltage of the generator (or), by increasing the field current of the synchronous machine in condenser mode at the generating stations. This procedure is not effective because the power-system losses will be further increased due to the increase of reactive power in the transmission system. An alternative method for compensating the reactive power is the use of capacitors in distribution systems at customer points.

Shunt capacitors supply the amount of reactive power to the system at the point where they are connected. Capacitors are mainly used to develop reactive power near the point of consumption. By capacitor compensation at load, the user reaps the same advantage as the power utility for higher power factor on a small scale. In addition, if each load is compensated, the power factor remains relatively constant since in plants, loads are switched on and off and the dangers of over-consumption do not exist. If, however, power factor has been corrected only at the service entry, system power can make relatively wide swings, as heavy loads are frequently switched on and off. Suitable capacitor banks at grid or main substations are desirable to feed reactive power of lines, transformers and domestic consumers, etc. who have no capacitors at terminals.

There are two methods of capacitor compensation viz.

1. Series compensation (capacitors are placed in series with line)
2. Shunt compensation (capacitors are placed in parallel with load)

The fundamental function of capacitors, whether they are series or shunt in a power system is to generate reactive power to improve power factor and voltage, thereby enhancing the system capacity and reducing losses. In series capacitors, the reactive power is proportional to the square of the load current, whereas in shunt capacitors it is proportional to the square of the voltage.

Test Yourself

1. Why are distribution systems losses more when compared to transmission losses?

14.5 CLASSIFICATION OF DISTRIBUTION SYSTEM

Distribution systems can be classified as follows:

14.5.1 Type of Current

Distribution system can be classified into two according to the type of current used. These are:

1. AC distribution system, and
2. DC distribution system

AC distribution system The electrical power is always generated, transmitted, and distributed in the form of AC. The main reason for adopting the AC system over a DC system for the generation, transmission, and distribution of electrical power is that the alternating voltage can be conveniently changed to any desired value with the help of transformers, since a transformer is a device used for the step-up or step-down of the voltage to the required levels. Alternating voltage can be increased to the economical value required for transmission (high voltage) and can be reduced to a safe value for distribution (low voltage) of electrical power. The AC distribution is classified as primary and secondary distribution systems.

DC distribution system Though electrical power is completely generated, transmitted and distributed as AC, certain applications like electro-chemical works, variable speed operation of DC motors, etc., absolutely need DC. Hence, for these applications, AC is converted into DC at the substations and distributed either by a two-wire or a three-wire system.

In general, AC distribution is adopted because it is simpler and cheaper than the DC distribution system.

14.5.2 Type of Construction

Distribution systems can be classified into two according to the type of construction procedure used. These are:

1. Overhead system
2. Underground system

The overhead system is employed for distribution because it is cheaper than the underground system and the underground system is employed only when the overhead system is restricted by local laws.

14.5.3 Type of Service

Distribution systems can be classified into four according to the services they best cater to. These are:

1. General lighting and power
2. Industrial power
3. Railway
4. Streetlight etc.

14.5.4 Number of Wires

Distribution systems can be classified into three according to the number of wires used. These are:

1. Two wire
2. Three wire
3. Four wire

For distribution purposes, overhead AC systems are universally employed. Three-phase, three-wire systems are used for power loads and three-phase, four-wire systems are used for both lighting and power loads. In case of DC supply systems, a three-wire distribution is usually preferred due to its advantages over a two-wire system.

14.5.5 Scheme of Connection

Distribution systems can be classified into three according to their schemes of connection. These are:

1. Radial distribution system
2. Ring or loop distribution system
3. Interconnected distribution system

Test Yourself

1. Why are AC distribution systems preferred over DC distribution systems?

14.6 RADIAL DISTRIBUTION SYSTEM

Most distribution systems are designed to be radial distribution systems as shown in Fig. 14.3. In a radial distribution system, only one path is connected between each customer and the substations. The electrical power flows from the substation to the customer along a single path. This, if interrupted, results in complete loss of power to the customer. In India, 99% of distribution of power is by radial distribution system only.

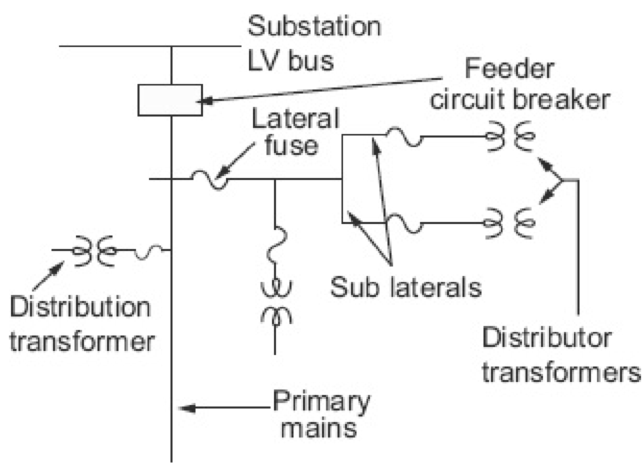

Fig. 14.3 Radial primary feeder configuration

Advantages:

- Its initial cost is minimum.
- Simple in planning, design and operation.

Disadvantages:

- Low reliability factor.
- Distributor nearer to the feeding end is heavily loaded.
- The consumers at the far end of the feeder would be subjected to series voltage fluctuations with the variations in load.

14.7 RING OR LOOP DISTRIBUTION SYSTEM

Figure 14.4 shows the loop distribution system. This distribution system consists of two or more paths between the power sources (substations) and the customer. It is selected to carry its normal load plus the load of the other half of the loop also. Therefore, the size of the feeder conductor in a loop distribution system is the same throughout the loop.

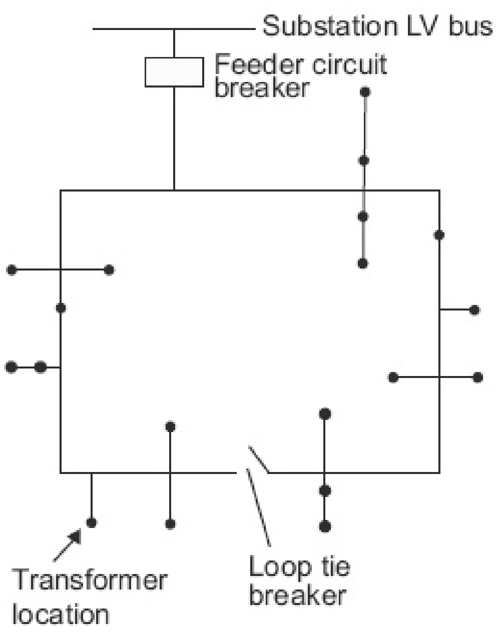

Fig. 14.4 Loop primary feeder configuration

Advantages:

- Less conductor material is required as each part of the ring carries less current than in the radial system.
- Less voltage fluctuations.
- It is more reliable.

Disadvantage:

- It is difficult to design when compared to the designing of a radial system.

14.8 INTERCONNECTED DISTRIBUTION SYSTEM

Figure 14.5 shows an interconnected feeder which is supplied by a number of feeders. The radial primary feeders can be tapped off from the interconnecting tie feeders. They can also serve directly from the sub-stations. Each tie feeder has two associated circuit breakers at each end in order to have less load interruptions due to a tie feeder fault.

The reliability and the quality of the service of the interconnected-type distribution arrangement are much higher than the radial and loop arrangements. However, it is more difficult to design and operate than the radial or loop type systems.

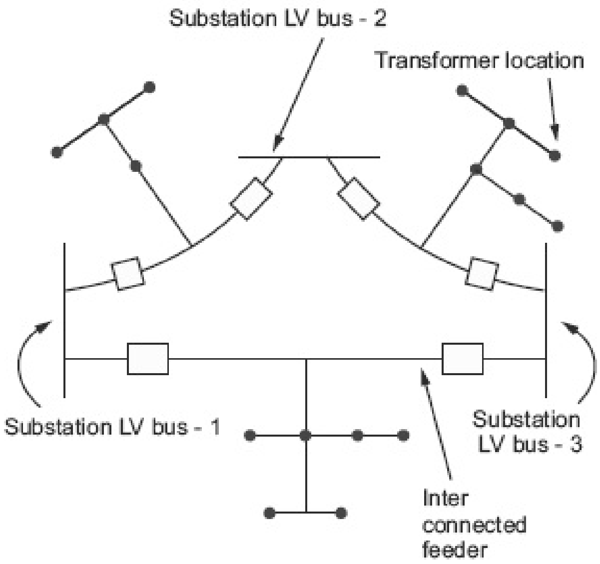

Fig. 14.5 Interconnected-type primary feeder configuration

Advantages:

- It increases the reliability of supply.
- Losses are less and efficiency is more.
- Quality of service is improved.

Disadvantages:

- Its initial cost is more.
- Difficult in planning, design and operation.

The voltage drop along the distributor is considered as a main factor while designing a distributor. This drop depends upon the nature of load of the distributor and also on the feeding, whether it is fed at one end only or at both ends.

According to loading, a distributor may be classified as:

1. Distributor with concentrated loading
2. Uniformly-loaded distributor

According to feeding, a distributor may be classified as:

1. Feed at one end
2. Feed at both ends
    1. With equal voltages
    2. With unequal voltages

14.9.1 Distributor Fed at One End with Concentrated Loads

In this type of feeding, the distributor is connected to the supply at one end and loads are taken from different points along the length of distributor as shown in Fig. 14.6.

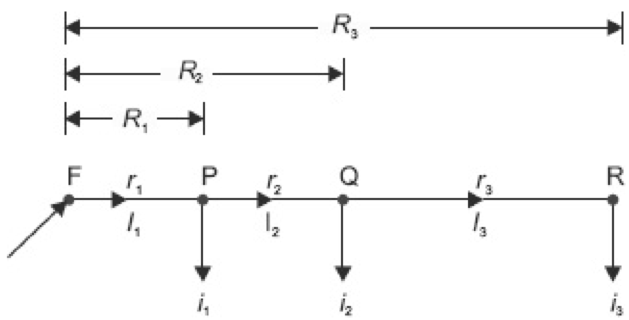

Fig. 14.6 Distributor fed at one end

Let i_1 i_2, i_3 be the currents tapped off from the distributor. I_1, $I_2$and I_3 are the currents passing in various branches (sections). r_1 r_2, r_3 and R_1 R_2, R_3 are the resistances of various sections and total resistances from feeding point F to the successive tapping points, respectively.

The voltage drop from F to R is

$$e = I_1 r_1 + I_2 r_2 + I_3 r_3 \quad (14.1)$$

But $I_1 = i_1 + i_2 + i_3$

$I_2 = i_2 + i_3$

and $I_3 = i_3$

Substituting the above currents in Eq. (14.1)

$$\begin{aligned} e &= (i_1 + i_2 + i_3)r_1 + (i_2 + i_3)r_2 + i_3 r_3 \\ &= i_1 r_1 + i_2(r_1 + r_2) + i_3(r_1 + r_2 + r_3) \\ &= i_1 R_1 + i_2 R_2 + i_3 R_3 \quad (14.2) \end{aligned}$$

Where, $R_1 = r_1$

$R_2 = r_1 + r_2$

$R_3 = r_1 + r_2 + r_3$.

Thus, the drop at the far end of the distributor with concentrated loads fed at one end is equal to the sum of the moments of various currents tapped off about the feeding point.

Voltage drop at any intermediate point “Q”

Voltage drop between the points F and Q is given by

$$\begin{aligned} V_{FQ} &= I_1 r_1 + I_2 r_2 \\ &= (i_1 + i_2 + i_3)r_1 + (i_2 + i_3)r_2 \\ &= i_1 R_1 + i_2 R_2 + i_3 R_3 \quad (14.3) \end{aligned}$$

Thus, the drop at any intermediate point is equal to the sum of the moments of the currents up to that point plus the moment of all the currents beyond that point assumed to be acting at that point.

Example 14.1

A DC two-wire distributor, 500 m long and fed at one end is shown in Fig. 14.7. The total resistance of the distributor is 0.02 Ω. Determine the voltage at the fed end F when the voltage at the far end R is 220 V.

Solution:

Total resistance of the conductors = 0.02 Ω

Resistance of the conductors per metre = $= \frac{0.02}{500} = 4 \times 10^{-5}$ Ω/m.

Voltage drop up to the far end R from fed end F,

$$V_{FR} = 40 \times 100 \times 4 \times 10^{-5} + 80 \times 300 \times 4 \times 10^{-5} + 60 \times 500 \times 4 \times 10^{-5}$$
$$= 4 \times 10^{-5} (40 \times 100 + 80 \times 300 + 60 \times 500)$$
$$= 4 \times 10^{-5} (4000 + 24000 + 30000) = 4 \times 10^{-5} \times 58000 = 2.32 \text{ V}$$

$\therefore$ Voltage at fed end F, V_F = Voltage at R + V_{FR} = 220 + 2.32 = 222.32 V.

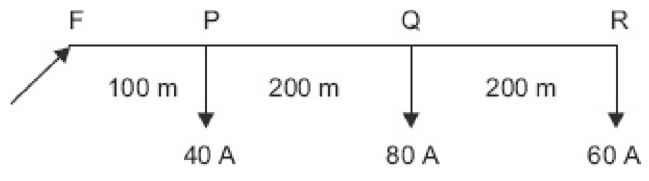

Fig. 14.7 DC two-wire distributor

Example 14.2

A two wire 400 m long distributor is loaded as follows:

Metres from feeding point :	100	200	275	325	400
Load in ampere :	25	10	30	50	20

If the resistivity of conductor is 1.5 X10^{-8} Ω-m, what must be the cross-section of each conductor, in order that the voltage drop may not exceed 10 V.

Solution:

Resistivity, $\Omega = 1.5 \times 10^{-8}$ Ω-m

Maximum voltage drop = 10 V

Let, the cross-section of the wire = a sq. m

l_1, l_2,l_3, l_4 and l_5 are the length of each node point from feeding end respectively.

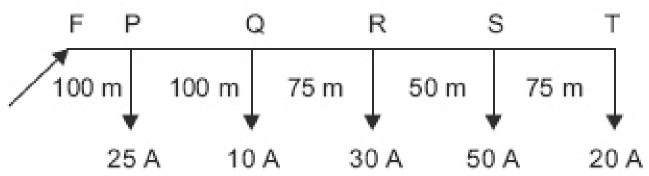

Fig. 14.8 Two wire distributor

Total voltage drop in the distributor

$$= 2\left\{\frac{25\rho l_1}{a} + 10\frac{\rho l_2}{a} + 30\frac{\rho l_3}{a} + 50\frac{\rho l_4}{a} + 20\frac{\rho l_5}{a}\right\}$$

(since go and return conductors)

$$= \frac{2\rho}{a}(25l_1 + 10l_2 + 30l_3 + 50l_4 + 20l_5)$$

$$= \frac{2\times1.5\times10^{-8}}{a}(25\times100 + 10\times200 + 30\times275 + 50\times325 + 20\times400)$$

$$= \frac{2\times1.5\times10^{-8}}{a}\times37000 = \frac{11.1\times10^{-4}}{a}$$

Equating to the drop given, we get

$$\frac{11.1\times10^{-4}}{a}=10$$

$$\therefore \text{Cross sectional area, } a=\frac{11.1\times10^{-4}}{10}=1.11\times10^{-4}\text{ sq.m}$$

Therefore, area of cross-section of each conductor is 1.11 cm^2.

14.9.2 Distributor Fed at Both Ends with Concentrated Loads

When voltage is fed at one end for a long distributor, the voltage drop would be high. In order to reduce voltage drop the distributor is fed at both ends, either at equal or unequal voltages. Alternatively, for a given drop of voltage, a conductor of smaller size can be used. Hence, a distributor fed at both ends is economical.

When voltage is fed at both the ends, the minimum potential point occurs in between the feeding points and it varies with load on different sections of the distributor.

Advantages:

1. Voltage drop is reduced or for a given voltage drop a smaller size conductor is required.
2. Continuity of supply is maintained from the other feeding point, in case any fault occurs in the feeders.
3. Maintains the continuity of supply to the remaining sections, in case any branch of the distributor is isolated due to a fault.

Voltage Drop Calculations Consider a distributor fed at F_1 and F_2 with voltages V_1 and V_2, respectively as shown in Fig. 14.9. Let i_1, i_2 and i_3 be the currents tapped off from it, r_1, r_2, r_3 and $r_4$are the resistances of various sections. I_1 and I_2 are the currents supplied from F_1 and F_2, respectively. Then,

$$I_1+I_2=i_1+i_2+i_3 \qquad (14.4)$$

Also, the sum of the voltage drops in different sections from F1,while F2is equal to the difference of feeding voltages at F1and F2.

$$\text{i.e., } V_1 - V_2 = I_1 r_1 + (I_1 - i_1) r_2 + (I_1 - i_1 - i_2) r_3 + (I_1 - i_1 - i_2 - i_3) r_4 \quad (14.5)$$

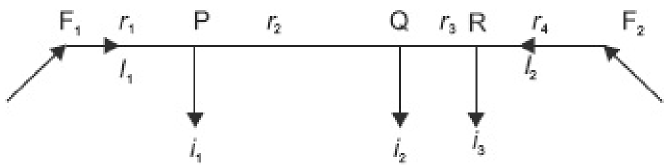

Fig. 14.9 Distributor fed at both ends

From Eqs. (14.4) and (14.5), I_1 and I_2can be determined. Thus the currents in various branches can be determined. Calculating the currents in various sections, the voltage drop can be calculated from any load point to a feeding point.

Example 14.3

A DC two-wire distributor F_1F_2 is fed at both ends at the same voltage of 220 V. The length of the distributor is 250 m and the loads tapped off from the end F_1 are:

Distance in metre	**:**	**50**	**75**	**100**	**150**
Load in amps	**:**	**10**	**40**	**30**	**25**

The resistance per kilometre of both distributor is 0.2 Ω. Find (i) the current in each section and (ii) the voltage at each load point.

Solution:

Resistance per kilometre of the conductor (go and return) = 0.2 Ω.

∴ Resistance per metre for go and return = $\frac{0.2}{1000}$ = 2×10^{-4} Ω/m.

Let, I_1 and I_2 be the currents supplied from the ends F_1 F_2, respectively. Then

$$I_1 + I_2 = 10 + 40 + 30 + 25 = 105\text{ A} \quad (1)$$

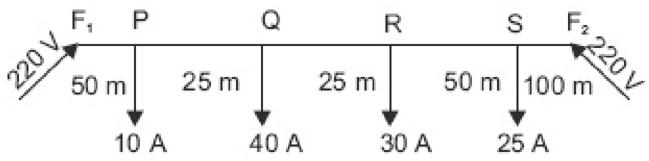

Fig. 14.10 DC two wire distributor fed at both ends

The voltage drop in the distributor from F_1 to F_2 is

$$2 \times 10^{-4}\{I_1 \times 50 + (I_1 - 10)25 + (I_1 - 50)25 + (I_1 - 80)50 + (I_1 - 105) \times 100\} = V_{F1} - V_{F2}$$
$$50I_1 + 25I_1 + 25I_1 + 50I_1 + 100I_1 - 250 - 1250 - 4000 - 10500 = 220 - 220 = 0$$
$$250I_1 - 16000 = 0$$
$$250I_1 = 16000$$
$$\therefore I_1 = \frac{16000}{250} = 64\text{ A.}$$

Substituting I_1 in Eq. (1), we have

$I_2 = 105 - 64 = 41$ A.

1. Current in various sections of the distributor

$$I_{F_1P} = I_1 = 64\text{ A}$$
$$I_{PQ} = I_1 - 10 = 54\text{ A}$$
$$I_{QR} = 54 - 40 = 14\text{ A}$$
$$I_{F_2S} = I_2 = 41\text{ A}$$
$$I_{SR} = I_2 - 25 = 16\text{ A}$$

2. The voltage at each load point

$$
\begin{aligned}
V_P &= V_{F1} - \text{drop in section } F_1P \\
&= 220 - 64 \times 50 \times 2 \times 10^{-4} \\
&= 219.36 \text{ V}
\end{aligned}
$$

$$
\begin{aligned}
V_Q &= V_P - \text{drop in PQ} \\
&= 219.36 - 54 \times 25 \times 2 \times 10^{-4} \\
&= 219.09 \text{ V} \\
V_R &= V_Q - \text{drop in QR} \\
&= 219.09 - 14 \times 25 \times 2 \times 10^{-4} \\
&= 219.02 \text{ V} \\
V_S &= V_{F2} - \text{drop in } F_2S \\
&= 220 - 41 \times 100 \times 2 \times 10^{-4} \\
&= 219.18 \text{ V.}
\end{aligned}
$$

$$
\begin{aligned}
V_Q &= V_P - \text{drop in PQ} \\
&= 219.36 - 54 \times 25 \times 2 \times 10^{-4} \\
&= 219.09 \text{ V} \\
V_R &= V_Q - \text{drop in QR} \\
&= 219.09 - 14 \times 25 \times 2 \times 10^{-4} \\
&= 219.02 \text{ V} \\
V_S &= V_{F2} - \text{drop in } F_2S \\
&= 220 - 41 \times 100 \times 2 \times 10^{-4} \\
&= 219.18 \text{ V.}
\end{aligned}
$$

Example 14.4

A two-wire distributor is fed at F_1 and F_2 at 230 V and 220 V, respectively. Loads of 150 A and 100 A are taken at points P and Q. Resistance of both the conductors between F_1P is 0.03 Ω, between PQ is 0.05 Ω and between QF_2 is 0.02 Ω. Determine the current in each section of the distributor and voltage at each load point.

Solution:

From the Fig. 14.11, let the current supplied from feed ends F_1 and F_2 be I_1 and I_2, respectively,

$$I_1 + I_2 = 150 + 100 = 250\text{ A} \qquad (1)$$

Voltage drop in the distributor from F_1 to F_2 = $V_{F1} - V_{F2}$ = 230 – 220 = 10 V.

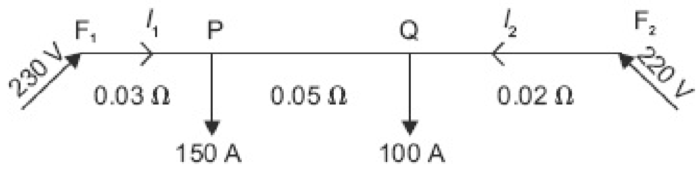

Fig. 14.11 DC two wire distributor fed at both ends

$$\therefore I_1 \times 0.03 + (I_1 - 150) \times 0.05 + (I_1 - 250) \times 0.02 = 10$$
$$\Rightarrow 0.03I_1 + 0.05I_1 + 0.02I_1 - 7.5 - 5 = 10$$
$$\Rightarrow 0.1I_1 - 12.5 = 10$$
$$0.1I_1 = 10 + 12.5 = 22.5$$
$$\therefore I_1 = \frac{22.5}{0.1} = 225\text{ A.}$$

Substituting I_1 in Eq. (1), we get

I_2 = 250 – 11 = 250 – 225 = 25 A

∴ Current in various sections

$$I_{F1P} = I_1 = 225\text{ A}$$
$$I_{PQ} = I_1 - 150 = 75\text{ A}$$
$$I_{F2Q} = I_2 = 25\text{ A}$$

The voltage at each load point

$$\begin{aligned} V_P &= V_{F1} - \text{drop in } F_1P \\ &= 230 - 225 \times 0.03 \\ &= 223.25 \text{ V} \\ V_Q &= V_{F2} - \text{drop in } F_2Q \\ &= 220 - 25 \times 0.02 \\ &= 219.5 \text{ V.} \end{aligned}$$

Example 14.5

A DC two-wire distributor is fed at F_1 and F_2 at 220 V and 225 V, respectively. The total length of the distributor is 225 m. The loads tapped off from fed end F_1

Load in ampere	**:**	**20**	**40**	**25**	**35**
Distance in metre	**:**	**50**	**75**	**100**	**125**

The resistance per kilometre of one conductor is 0.3 Ω. Determine the current in various sections of the distributor and the voltage at the point of minimum potential.

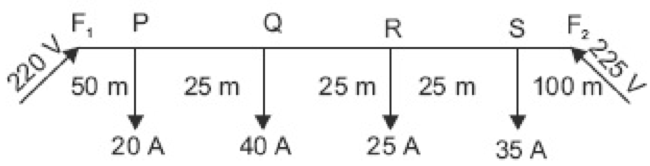

Fig. 14.12 DC two wire distributor fed at both ends

Solution:

Resistance per kilometre per conductor = 0.3 Ω

∴ Resistance per kilometre for both the conductors (go and return) =

2 x 0.3 = 0.6 Ω

$$\text{Resistance per metre for both the conductors} = \frac{0.6}{1000} = 6 \times 10^{-4}\ \Omega/\text{m}.$$

Let the current supplied from fed ends F_1 and F_2 be I_1 and I_2, respectively,

$$I_1 + I_1 = 20 + 40 + 25 + 35 = 120\ \text{A}. \qquad (1)$$

The voltage drop in the distributor from F_1 and F_2 is

$$6 \times 10^{-4}\,[I_1 \times 50 + (I_1 - 20)25 + (I_1 - 60)25 + (I_1 - 85) \times 25 + (I_1 - 120)100] = V_{F1} - V_{F2}$$

$$\text{or } 6 \times 10^{-4}\,(50I_1 + 25I_1 + 25I_1 + 25I_1 + 100I_1 - 500 - 1500 - 2125 - 12000) = 220 - 225 = -5$$

$$\text{or } 6 \times 10^{-4}\,(225I_1 - 16125) = -5$$

$$225I_1 - 16125 = \frac{-5}{6\times10^{-4}} = -8333.33$$

$$225I_1 = -8333.33 + 16125 = 7791.67$$

$$\therefore I_1 = \frac{7791.67}{225} = 34.63\ \text{A}.$$

Substituting I_1 in Eq. (1), we get

$$I_2 = 120 - 34.63 = 85.37\ \text{A}.$$

$\therefore$ Current in various sections are:

$$I_{F_1P} = I_1 = 34.63\ \text{A}$$
$$I_{PQ} = I_1 - 20 = 14.63\ \text{A}$$
$$I_{F_2S} = I_2 = 85.37\ \text{A}$$
$$I_{SR} = I_2 - 35 = 50.37\ \text{A}$$
$$I_{RQ} = I_{SR} - 25 = 25.37\ \text{A}.$$

The load at the point of minimum potential is supplied from both the ends. Looking at the current distribution in various sections, the load 40 A tapped at the point Q is the point of minimum potential.

The voltage at the point of minimum potential

$= V_{F1}$ – drop in F_1P – drop in PQ

$= 220 - 6 \times 10^{-4} (34.63 \times 50 + 14.63 \times 25)$

$= 220 - 1.25835$

$= 218.74$ V.

14.9.3 Uniformly Loaded Distributor Fed at One End

The single-line diagram of a two-wire DC distributor "FA" fed at one end F and loaded uniformly with i amps per metre length is shown in Fig. 14.13(a).

Let i = current tapped off per metre length

r = resistance per metre length for go and return

l = total length of distributor

Consider a point P on the distributor at a distance of x metres from the feeding point F as shown in Fig. 14.13(b). Then the current at point P is,

$$= il - ix = i(l - x) \text{ A}$$

Consider a small section dx at a distance x from the sending-end.

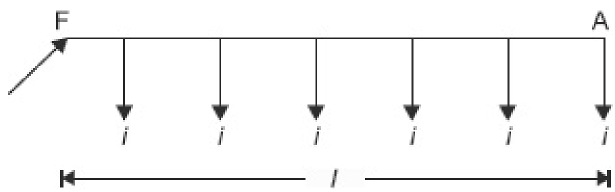

Fig. 14.13(a) Single line diagram of uniformly distributed load

Voltage drop in small section dx is

$$dV = i(l - x)r\,dx$$

Then the voltage drop at any point *x* from feed point F is

$$V_{Fx} = \int_0^x dV = \int_0^x i(l - x)r\,dx = ir\left(lx - \frac{x^2}{2}\right) \quad (14.6)$$

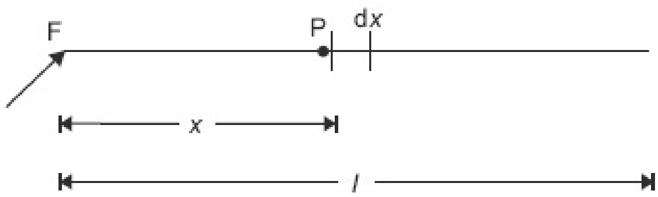

Fig. 14.13(b) Small equivalent section of Fig. 14.13(a)

The voltage drop over the whole distributor can be obtained by substituting x = lin Eq. (14.6).

The voltage drop over the distributor

$$\begin{aligned} V_{FA} &= ir\left(l^2 - \frac{l^2}{2}\right) \\ &= ir\left(\frac{l^2}{2}\right) \\ &= \frac{1}{2}(il)(rl) \\ &= \frac{1}{2}IR \qquad (14.7) \end{aligned}$$

Where, *I* = *il* = total current feeding at point F

R = *rl* = total resistance of the distributor.

Thus a uniformly distributed feed at one end gives a total voltage drop equal to that produced by the whole of load assumed to be concentrated at the middle point.

Power loss over the length, dx = (current in length dx)^2 × (resistance of length dx)

$$= \left[i\left(l-x\right)\right]^2 r\mathrm{d}x.$$

Total power loss over the whole distributor is

$$\begin{aligned} P_{\text{loss}} &= \int_0^l \left[i\left(l-x\right)\right]^2 r\mathrm{d}x \\ &= \int_0^l i^2\left(l^2+x^2-2lx\right) r\mathrm{d}x \\ &= i^2 r\left(l^3+\frac{l^3}{3}-l^3\right) \\ &= \frac{i^2 r l^3}{3}. \end{aligned}$$

$$\therefore P_{\text{loss}} = \frac{i^2 r l^3}{3} = \frac{I^2 R}{3} \tag{14.8}$$

Example 14.6

A DC two-wire distributor of length 1000 m is loaded uniformly at 2 A/m run. The distributor is fed at one end at 220 V. Determine (i) the voltage drop at a distance 250 m from the feeding station and (ii) voltage drop at the far end. The loop resistance is 3 × 10^{-5} Ω/m.

Solution:

The distributor is as shown in Fig. 14.14

Uniformly distributed current, i = 2 A/m

Loop resistance, r = 3 × 10^{-5} Ω/m

Length of the distributor, l = 1000 m

Voltage drop at distance, x = 250 m

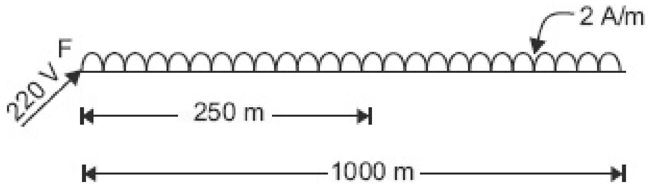

Fig. 14.14 Uniformly loaded DC two wire distributor

We know, the drop at any point, $x = ir\left(lx - \frac{x^2}{2}\right)$ V

1. When x = 250 m

$$\text{Drop} = 2 \times 3 \times 10^{-5} \left(1000 \times 250 - \frac{250^2}{2}\right)$$
$$= 6 \times 10^{-5} (250000 - 31250)$$
$$= 13.125 \text{ V.}$$

2.

$$\text{At the far end drop} = \frac{irl^2}{2}$$
$$= \frac{2 \times 3 \times 10^{-5} \times 1000^2}{2} = 30 \text{ V.}$$

14.9.4 Uniformly Distributed Load Fed at Both Ends at the Same Voltage

When the voltage at both ends is the same, evidently, the middle point becomes the point of minimum potential. Thus, the distributor can be imagined to be cut into two at the middle point, giving rise to two uniformly-loaded distributors each fed at one end as shown in Fig. 14.15.

From Eq. (14.6) the drop at any point 'x' is

$$ir\left(lx - \frac{x^2}{2}\right) \tag{14.9}$$

The drop at the middle point is maximum i.e., at

$l = \frac{l}{2}$ and $x = \frac{l}{2}$.

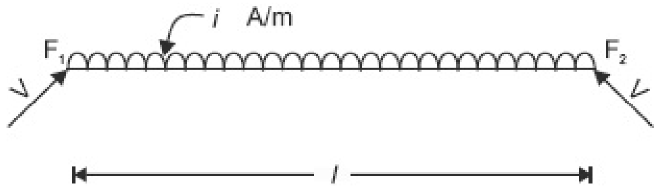

Fig. 14.15 Uniformly-distributed load fed at both ends

Substituting the above quality in Eq. (14.9), we

$$\text{maximum drop} = ir\left(\frac{l}{2}\times\frac{l}{2} - \frac{\left(\frac{l}{2}\right)^2}{2}\right)$$

$$= ir\left(\frac{l^2}{4} - \frac{l^2}{8}\right)$$

$$= \frac{irl^2}{8}$$

$$= \frac{IR}{8} \tag{14.10}$$

$$\text{Minimum voltage} = V - \frac{IR}{8}.$$

Example 14.7

A uniformly-loaded DC two-wire distributor 500 m long is loaded at 3 A/m run. Resistance of the loop is 0.01 Ω/km. Determine the maximum voltage drop if the distributor is fed at both ends at the same voltage.

Solution:

The distributor is as shown in Fig. 14.16

Uniformly distributed current, $i = 3$ A/m

Loop resistance, $r = 0.01\ \Omega/\text{km}$

$= 0.01 \times 10^{-3}\ \Omega/\text{m}$

Length of the distributor, $l = 500$ m

$$\text{Maximum voltage drop} = \frac{irl^2}{8}\ \text{V}$$

$$= \frac{3 \times 0.01 \times 10^{-3} \times 500^2}{8}$$

$$= 0.9375\ \text{V.}$$

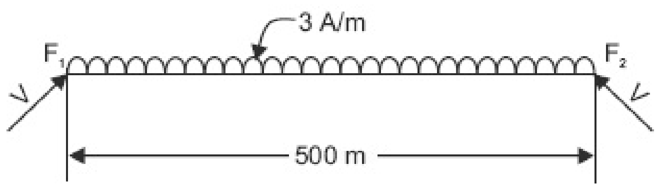

Fig. 14.16 Uniformly-loaded DC two-wire distributor

Example 14.8

A DC two-wire distributor F_1F_2 is 400 m long is shown in Fig. 14.17 and 220 V is fed at both ends. The resistance of each conductor is 0.02 Ω/m. Calculate the point of minimum potential and it's voltage.

Solution:

Let Q be the point of minimum potential and *r* be the resistance per metre of both the conductors.

Let *x* be the current supplied from F_1 to load point, Q.

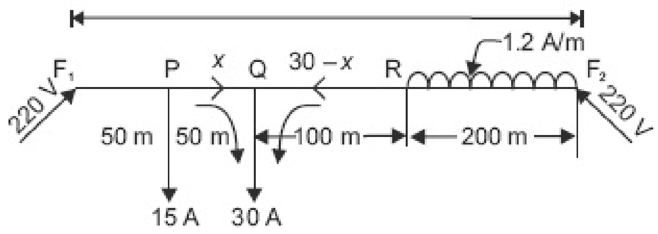

Fig. 14.17 DC two-wire distributor

Voltage drop in section

$$F_1Q = [(15 + x)50 \times r + x\, r \times 50]\ \text{V} \qquad (1)$$

Voltage drop in section F2Q = Drop of voltage in F2R due to concentrated load + Drop of volts in RQ.

$$= \frac{1.2 \times r \times 200^2}{2} + (30 - x) r \times 100 \qquad (2)$$

Since voltage at feed points F_1 and F_2 is same. i.e., from Eqs. (1) and (2)

$$\left[(15 + x)50 \times r + x\, r \times 50\right] = (30 - x) r \times 100 + \frac{1.2 \times r \times 200^2}{2}$$

$$750 + 50x + 50x = 3000 - 100x + 24000$$

$$200x = 27000 - 750$$

$$200x = 26250$$

$$x = \frac{49675}{200} = 131.25\ \text{A.}$$

The current required at point Q = 30 A, but 131.25 A is the current that passes from P to Q.

The remaining current (131.25 – 30 = 101.25 A) is supplied to the distributed load.

Hence, the point of minimum potential occurs at $= 50 + 50 + 100 + \dfrac{101.25}{1.2}$

$= 200 + 84.375 = 284.375$ m, from fed end F_1.

The distance of minimum potential from end $F_2 = 400 - 284.375$

$= 115.625$ m

$$\text{voltage drop} = \frac{irl^2}{2} = \frac{1.2 \times 0.04 \times (115.625)^2}{1000 \times 2} = 0.321 \text{ V}$$

Hence the minimum potential = 220 – 0.321 = 219.678 V.

14.9.5 Uniformly Distributed Load Fed at Both Ends at Different Voltages

Consider a distributor F_1 F_2 of length l metres as shown in Fig. 14.18.

Let i = current rating of the distributor F_1F_2 (A/m)

r = resistance go and return (Ω/m)

V_1 and V_2 = voltages at the feed points F_1 and F_2, respectively.

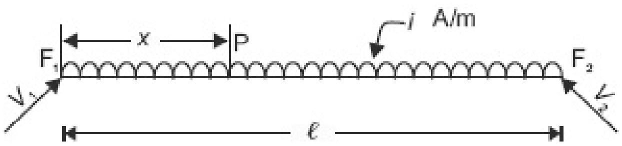

Fig. 14.18 DC two-wire distributor

Suppose the point of minimum potential P is situated at a distance x metres from the feeding point Fig. 14.18 DC two-wire distributor F_1, then current feeding from feed point is ix.

$$\therefore \text{Voltage drop in section } F_1P = ir\frac{x^2}{2} \text{ V} \tag{14.11}$$

$$\text{Similarly voltage drop in section } F_2P = ir\frac{(l-x)^2}{2} \text{ V}$$

$$\text{Voltage at point } P, V_p = V_1 - \text{drop in section } F_1P$$

$$= V_1 - ir\frac{x^2}{2}.$$

$$\text{Also, voltage at point P, } V_P = V_2 - \text{drop in section } F_2P$$

$$= V_2 - ir\frac{(l-x)^2}{2} \tag{14.12}$$

Equating Eqs. (14.11) and (14.12)

$$\therefore V_1 - ir\frac{x^2}{2} = V_2 - ir\frac{(l-x)^2}{2}$$

$$V_1 - V_2 = ir\frac{x^2}{2} - ir\frac{(l-x)^2}{2}$$

$$= \frac{irl}{2}(2x - l)$$

$$\therefore x = \frac{l}{2} + \frac{(V_1 - V_2)}{irl} \tag{4.13}$$

Thus knowing x, drop in sections F_1P and F_2P can be calculated. Hence the voltage at the point of minimum potential can be determined.

Example 14.9

A 300 m distributor fed from both ends F_t and F_2 is loaded uniformly at the rate of 2 A/m run. The resistance of loop is 0.2 Ω/km. Find the minimum voltage and the point where it occurs, if the feeding points F_1 and F_2 are maintained at 225 V and 220 V, respectively. Also find the currents supplied from the feeding points F_1 and F_2.

Solution:

The distributor is as shown in Fig. 14.19.

Uniformly distributed current, i = 2 A/m

$$\text{Loop resistance, } r = 0.2\ \Omega/\text{km} = 2 \times 10^{-4}\ \Omega/\text{m}$$

Length of the distributor, l = 300 m

Voltage at F_1, V_{F1} = 225 V and

Voltage at F_2, V_{F2} = 220 V.

Let P be the point of minimum potential and has a distance x from the feeding point F_1. We know

$$x = \frac{l}{2} - \frac{(V_{F2} - V_{F1})}{irl}$$

$$= \frac{300}{2} - \frac{(220 - 225)}{2 \times 2 \times 10^{-4} \times 300}$$

$$= 150 + \frac{5}{1200 \times 10^{-4}} = 191.67 \text{ m.}$$

$$\therefore \text{Voltage drop in section } F_1P = \frac{irx^2}{2}$$

$$= \frac{2 \times 2 \times 10^{-4} \times 191.67^2}{2}$$

$$= 7.347 \text{ V.}$$

$$\therefore \text{Voltage at the point of minimum potential} = V_{F1} - \text{drop in section } F_1P$$

$$= 225 - 7.347$$

$$= 217.65 \text{ V.}$$

$$\text{Current supplied from feed point } F_1,\ I_{F1} = ix$$

$$= 2 \times 191.67 = 383.34 \text{ A}$$

$$\text{Current supplied from feed point } F_2,\ I_{F2} = i(l - x)$$

$$= 2(300 - 191.67)$$

$$= 216.66 \text{ A.}$$

14.10 RING DISTRIBUTION

It is a distributor which is arranged in the form of a closed circuit and which can be fed at one or more points. For the purpose of calculating voltage distribution, a ring distributor fed at one point can be treated as an equivalent to a straight distributor fed at both ends at the same voltage using a ring distributor. Voltage drop is reduced or for a given drop of voltage, a reduced size of conductor can be used.

In a ring distributor any two load points are joined by means of a connector. This connector is also known as an interconnector. The purpose of the inter-connector is to reduce voltage drop in various sections. The current distributor in such a network may be easily obtained either by applying Thevenin's theorem or Kirchhoff's laws.

14.10.1 Advantages of Using Interconnector

When there is an interconnection of any two load points in the ring distributor, there will be reduction in the voltage drop in various sections, and hence, the power loss in the system is reduced. Such an interconnected system will give continuous service even if one of station is shutdown. The advantages of interconnection are as follows:

- Increased security of service.
- Reduction in number of standby plants.
- The total capital cost and running cost can be reduced by dividing the total load.

Example 14.10

A 250 m ring distribution has loads as shown in Fig. 14.20(a). The resistance of both the distributor is 0.2 Ω/km. If the distributor is fed at 220 V at P, find the voltage at Q, R and S.

Solution:

Resistance of both conductors per metre $= \frac{0.2}{1000} = 2 \times 10^{-4}\ \Omega/\text{m}$

Let the current in section PQ = *IA*

And the current in various sections are shown in Fig. 14.20(b).

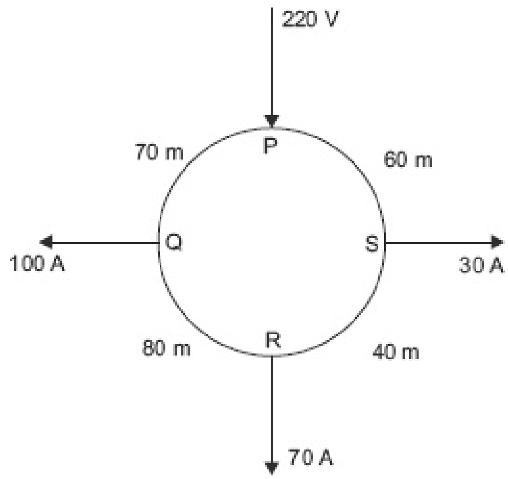

Fig. 14.20(a) Ring distribution

Applying KVL around the ring

i.e., drop in PQ + drop in QR + drop in RS + drop in SP = 0

$$2 \times 10^{-4}(I \times 70 + (I - 100) \times 80$$
$$+(I - 170) \times 40 + (I - 200) \times 60) = 0$$
$$\text{(or) } I \times 70 + (I - 100) \times 80$$
$$+(I - 170) \times 40 + (I - 200)60 = 0$$

$$70I + 80I + 40I + 60I = 8000 + 6800 + 12000$$
$$\therefore I = \frac{2680}{250} = 107.2 \text{ A.}$$

Then current in different sections are

$I_{PQ} = 107.2$ A
$I_{QR} = I - 100 = 7.2$ A
$I_{RS} = I - 170 = -62.8$ A
$I_{SR} = 170 - I = 62.8$ A
$I_{SP} = I - 200 = -92.8$ A
$I_{PS} = 200 - I = 92.8$ A.

Voltage at different load point

$V_Q = V_P$ – drop in section PQ
$= 220 - 107.2 \times 70 \times 2 \times 10^{-4} = 218.499$ V

$V_R = V_Q$ – drop in section QR
$= 218.499 - 7.2 \times 80 \times 2 \times 10^{-4} = 218.384$ V

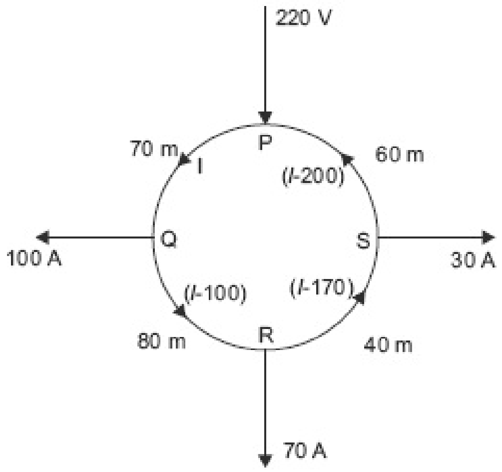

Fig. 14.20(b) Current in various sections of Fig. 14.20(a)

$V_S = V_R$ – drop in section RS
$= 218.384 - (-62.8) \times 40 \times 2 \times 10^{-4}$
$= 218.886$ V

(or)

$V_S = V_p$ – drop in section PS
$= 220 - 92.8 \times 60 \times 6 \times 10^{-4}$
$= 218.886$ V.

Example 14.11

Find the currents fed at M and P and also in the various sections of the ring main distributors as shown in Fig. 14.21.

The voltage at feed points M and P is 220 V. Also find the potential difference at each load point. Resistances given are the loop resistances.

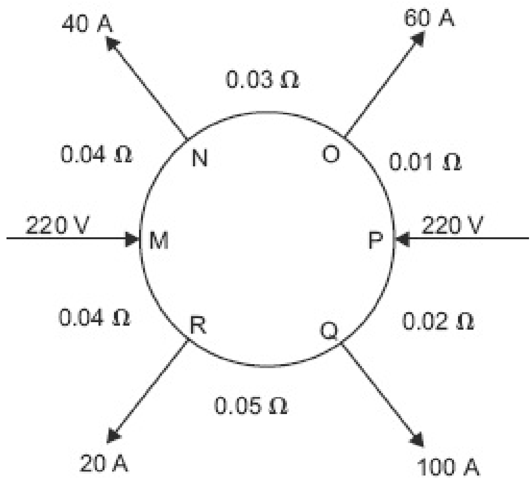

Fig. 14.21 Ring-main distributor

Solution:

Let the current in section MN = I_1

Voltage drops in various sections are

Drop MN + drop NO + drop OP = $V_m - V_p$

$$\therefore I_1 \times 0.04 + (I_1 - 40) \times 0.03 + (I_1 - 100) \times 0.01 = 220 - 220 = 0$$

$$0.04I_1 + 0.03I_1 + 0.01I_1 - 1.2 - 1 = 0$$

$$0.08I_1 - 2.2 = 0$$

$$I_1 = \frac{2.2}{0.08} = 27.5 \text{ A}$$

Current in section NO $= I_1 - 40 = 27.5 - 40 = -12.5$ A

(or) Current in section ON $= 12.5$ A

Current in section OP $= (I_1 - 100) = 27.5 - 100 = -72.5$ A

(or) Current in section PO $= 72.5$A

Current in section MR $= I_2$

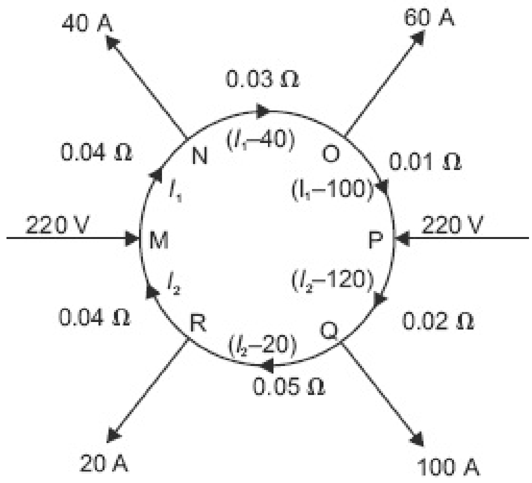

Fig. 14.22(a) Current in various sections of Fig. 14.21

Current in various sections as shown in Fig. 14.22(a) drop MR + drop RQ + drop QP = $V_m - V_p$

$$\therefore I_2 \times 0.04 + (I_2 - 20) \times 0.05 + (I_2 - 120) \times 0.02 = 220 - 220 = 0$$

$$\Rightarrow 0.04I_2 + 0.05I_2 + 0.02I_2 - 1 - 2.4 = 0$$

(or)

$$0.11I_2 - 3.4 = 0$$

$$\text{(or) } I_2 = \frac{3.4}{0.11} = 30.909 \text{ A.}$$

Current in section RQ $= I_2 - 20 = 30.909 - 20 = 10.909$ A

Current in section QP $= (I_2 - 120) = 30.909 - 120 = -89.091$ A

(or) current in section PQ = 89.091 A.

current in section PQ = 89.091 A. Current direction in various sections are shown in Fig. 14.22(b)

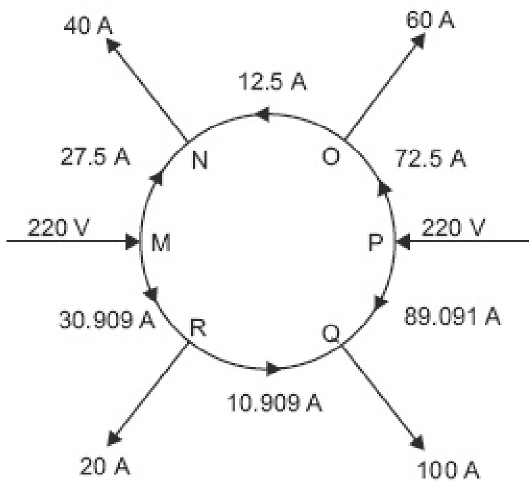

Fig. 14.22(b) Current direction in various sections of Fig. 14.21

From the Fig. 14.22(b), it is clear that

The current fed at M = 27.5 + 30.909 = 58.409 A

The current fed at P = 72.5 + 89.091 = 161.591 A

Voltage at various load points

$$V_N - V_M = \text{drop in MN} = 220 - 27.5 \times 0.04 = 218.9\ \text{V}$$
$$V_0 - V_P = \text{drop in PO} = 220 - 72.5 \times 0.01 = 219.275\ \text{V}$$
$$V_R = V_M - \text{drop in MR} = 220 - 30.909 \times 0.04 = 218.763\ \text{V}$$
$$V_Q = V_P - \text{drop in PQ} = 220 - 89.091 \times 0.02 = 218.218\ \text{V.}$$

Example 14.12

Find the current in various sections and voltage at various load points of the ring distributor as shown in Fig. 14.23.

Solution:

The total current fed at M, I = 60 + 70 + 100 + 30 = 260 A Let the current in section MN = I_1

Also let the current in section NP = current in inter- connector = I_2.

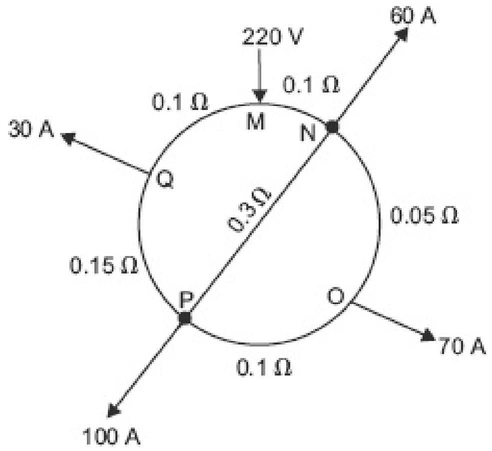

Fig. 14.23 Ring distributor

Then, the current in various sections is shown in Fig. 14.24 Considering the closed loop MNPQM and applying KVL,

$$\text{i.e. } 0.1I_1 + 0.3I_2 - 0.15(230 - I_1) - 0.1(260 - I_1) = 0$$
$$\text{(or) } 0.35I_1 + 0.3I_2 + 60.5 = 0 \quad (1)$$

Similarly for loop NOPN,

$$0.05(I_1 - I_2 - 60) + 0.1(I_1 - I_2 - 130) - 0.3I_2 = 0$$
$$\text{(Or) } 0.15I_1 - 0.45I_2 - 16 = 0.$$
$$\text{Or } I_1 = 106.66 + 3I_2 \quad (2)$$

Substituting I_1 from Eq. (2) in Eq. (1) and by solving we get

$$\therefore I_1 = 158.147 \text{ A}$$
$$I_2 = 17.162 \text{ A}$$

Therefore, currents in various sections are

I_{MN} = 158.147 A

I_{NO} = 80.985 A

I_{OP} = 10.985 A

I_{MQ} = 101.853 A

I_{QP} = 71.853 A

I_{NP} = 17.162 A

Voltage at various load points is

$V_N = V$M – drop in MN

= 220 – 0.1 × 158.147 = 204.185 V.

similarly

V_O = 204.185 – 0.05 × 80.985 = 200.136 V

V_P = 200.136 – 0.1 × 10.985 = 199.037 V

V_Q = 220 – 0.1 × 101.853 = 209.815 V.

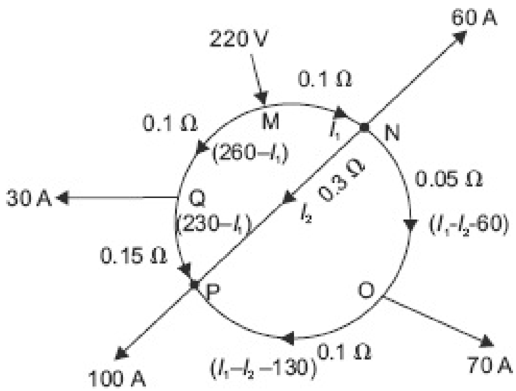

Fig. 14.24 Current in various sections of Fig. 14.23

Example 14.13

The ring-main distributor is fed at point O and loaded at points P and Q as shown in Fig. 14.25. The resistances of various sections (both go and return) are shown in the figure. Find the current in PQ and voltage drops in various sections.

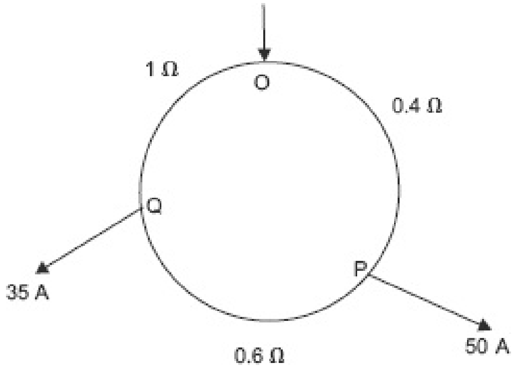

Fig. 14.25 Ring-main distributor

Solution:

This problem can be solved using Thevenin's theorem.

Imagine feeder PQ is removed (see Fig. 14.26)

Voltage drops in OP section = 50 × 0.4 = 20 V

Voltage drops in OQ section = 1 x 35 = 35 V

By comparing the voltage drops in OP and OQ sections, the potential at P is more than Q.

Voltage between P and Q = V_{PQ} = 15 V

∴ Current flows from P to Q. R_{Th} between P and Q = 1 + 0.4 = 1.4 Ω

$$\therefore I_{PQ} = \frac{15}{0.6+1.4} = \frac{15}{2} = 7.5 \text{ A from P to Q}$$

I_{OP} = 50 + 7.5 = 57.5 A

V_{OP} = 57.5 × 0.4 = 23 V

V_{PQ} = 7.5 × 0.6 = 4.5 V

I_{OQ} = 35 – 7.5 = 27.5 A

V_{OQ} = 27.5 × 1 = 27.5 V.

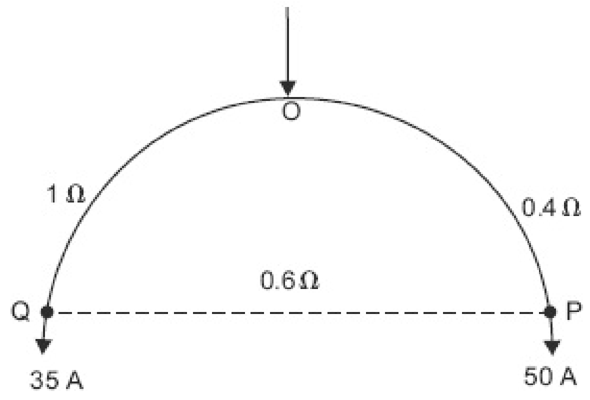

Fig. 14.26 Thevenin's equivalent of Fig. 14.25

14.11 STEPPED DISTRIBUTOR

It is also known as tapered distributor. It is necessary to design each distributor with the minimum volume of conductor material by satisfying the voltage limits.

Loads when tapped from any distributor changes the current along the length of the distributor. For example, a uniformly loaded distributor supplying voltage at one end carries a current that changes from maximum value at supply-end to the minimum at the other end. If the uniform cross-section conductor is used throughout the distributor, it affects savings. If the conductor cross-section is determined based on the current to be carried i.e., if the conductor is stepped, the required conductor material is minimum. When the conductor is tapered then the cross-section of the conductor at any point is proportional to the square root of the distance from the far end of the distributor. Practically, it is not possible. The stepped distributor is shown in Fig. 14.27.

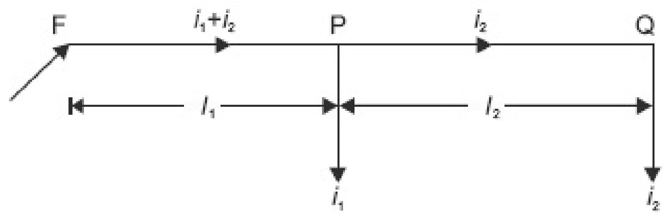

Fig. 14.27 Stepped distributor

Consider distribution FQ with loads i_1 and i_2 tapped off at the points P and Q, respectively. Let l_1, a_1 and l_2, a_2 be the lengths and cross-section areas of sections FP and PQ, respectively.

Resistance of section FP (go and return wire), $r_1 = \frac{2\rho l_1}{a_1}$

Similarly for section PQ, $r_2 = \frac{2\rho l_2}{a_2}$

Voltage drop in section FP, $V_1 = (i_1 + i_2) r_1 = \frac{2\rho l_1}{a_1}(i_1 + i_2)$ (14.14)

(or) $a_1 = \frac{2\rho L_1}{V_1}(i_1 + i_2)$ (14.15)

Drop in section PQ,

V_2 = total permissible drop along the distribution – drop in section PQ

i.e., $r_2 i_2 = V - V_1$

(or) $\frac{2\rho l_2}{a_2} i_2 = V - V_1$ (14.16)

(or) $a_2 = \frac{2\rho l_2 i_2}{V - V_1}$ (14.17)

Volume of conductor material in the distributor (go and return) is

$$u = 2l_1 a_1 + 2l_2 a_2$$
$$u = \frac{4\rho l_1^2}{V_1}(i_1 + i_2) + \frac{4\rho l_2^2 i_2}{V - V_1} \tag{14.18}$$

Differentiating the Eq. (14.18) with respect to V_1 and equating to zero for the minimum volume of conductor material

$$\frac{\mathrm{d}u}{\mathrm{d}V_1} = \frac{-4\rho l_1^2}{(V_1)^2}(i_1 + i_2) + \frac{4\rho l_2^2 i_2}{(V - V_1)^2} = 0$$

Substituting the expression of V_1 and $V - V_1$ from Eqs. (14.14) and (14.16) in above expression

$$0 = 4\rho\left(\frac{l_2^2 i_2 a_2^2}{4\rho^2 l_2^2 i_2^2} - \frac{l_1^2 (i_1 + i_2) a_1^2}{4\rho^2 l_1^2 (i_1 + i_2)^2}\right)$$
$$\therefore \frac{a_1}{a_2} = \sqrt{\frac{i_1 + i_2}{i_2}} \tag{14.19}$$

From Eq. (14.19), we see that various sections of a stepped distributor should be proportional to the square root of the currents in order to make the cross-sectional area of the conductor more economical.

The disadvantages of this distributor are as follows:

- This method requires more jointing than the scheme of uniform cross-section.
- In future, the addition of branches to any system may change the current distribution entirely in different branches of the distributor.

Test Yourself

1. Why are stepped-type distributors not commonly used?

14.12 AC DISTRIBUTION

All the methods used for solution of DC distributors will apply equally well for the solution of AC distributors. The resistances have to be replaced by impedance, the currents in various sections of the distributor and voltages at load points are the vector sum and not simply arithmetic sums. Further, the loads tapped off from a given distribution will be at different power factors. The phase angles at the load points may be referred to the supply or it may be referred to the voltage, which is taken as a reference vector or it may be referred to the voltage at the load points itself.

To illustrate the procedure, we will consider an AC distribution with two concentrated loads:

1. Power factor is specified with respect to receiving-end voltage.
2. Power factor refers to the respective load points.

14.12.1 Power Factor Referred to the Receiving-end

Consider the distributor FQ with concentrate loads of I_1 and I_2 tapped off with power factors of $\cos\phi_1$ and $\cos\phi_2$ at points Q and P, respectively are shown on Fig. 14.28(a). Let $(R_2 + jX_2)$and $(R + jX_1)$ be the impedances of sections FP and PQ.

The current in section PQ is = $I_1(\cos\phi_1 - j\sin\phi_1)$

Drop in section PQ,

$V = I_1(\cos\phi_1 - j\sin\phi_1)\ (R_1 + jX_1)$.

Current in section FP, $I = I_1 + I_2$

$$= I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)$$
$$= I(\cos\phi - j\sin\phi).$$

Voltage drop in section FP,

$$V_2 = I(\cos\phi - j\sin\phi)(R_2 + jX_2)$$

$\therefore$ Voltage at point F,

$$V_F = V_Q + V_1 + V_2$$
$$= \text{Voltage at the sending-end} = |V_F|\angle\phi_s$$

where, ϕ_s = sending-end phase angle.

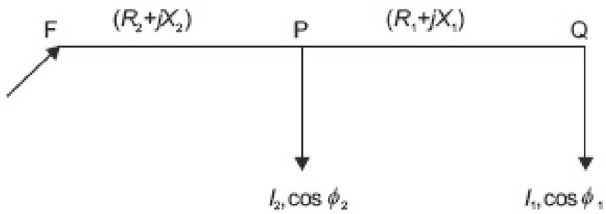

Fig. 14.28(a) AC distributor

The phase diagram of AC distributor is shown in Fig. 14.28(b). Here, the receiving, end voltage V_Q is taken as reference vector. For this case power factors of the loads are referred to V_Q, hence I_1 and I_2 lags behind V_Q by an angle ϕ_1 and ϕ_2, respectively.

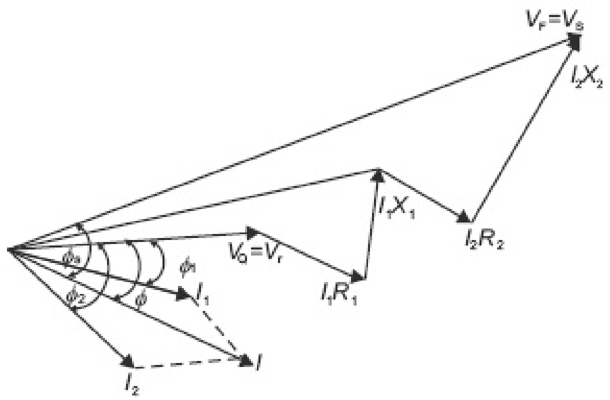

Fig. 14.28(b) Phasor diagram

Example 14.14

A two-wire distributor 1200 m long is loaded as shown in Fig. 14.29. The power factors at the two load points refer to the voltage at R. The impedance of each line is (0.15 + j0.2) Ω. Calculate the sending-end voltage, current, and power factor. The voltage at point R is 230 V.

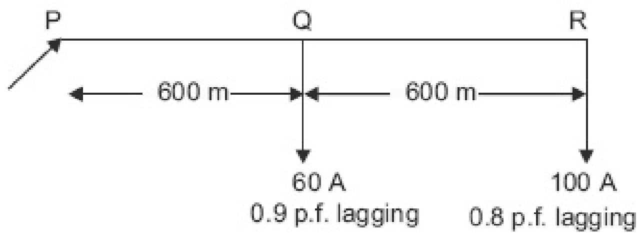

Fig. 14.29 AC two-wire distributor

Solution:

Total length of distributor from P to R = 1200 m

As Q is midpoint, the length is

$$PQ = QP = \frac{PR}{2} = 600 \text{ m}.$$

The impedance of each line = $Z_{PQ} = Z_{QR} = (0.15 + j0.2)\ \Omega$

Let V_R be the voltage at point R and is taken as reference vector.
Then, $V_R = 230 + j0$ V.

Load current at point R, $I_R = 100\angle -\cos^{-1} 0.8$ A

$I_R = 100\angle -36.869^\circ$ A

$I_R = 80 - j60 \text{ A} = I_{QR}$

Load current at point Q, $I_Q = 60\angle -\cos^{-1} 0.9$ A

$= 60\angle -25.842^\circ$ A

$I_Q = (54 - j26.16)$ A

Current in section PQ, $I_{PQ} = I_Q + I_R = 54 - j26.16 + 80 - j60$

$= 134 - j86.16 = 159.3\angle -32.74^\circ$ A.

Voltage drop in section QR, $V_{QR} = I_{QR} Z_{QR}$

$= 100\angle -36.869^\circ \times (0.15 + j0.2)$

$= 25\angle 16.26^\circ = (24 + j7)$ V.

And drop in section PQ, $V_{PQ} = I_{PQ} Z_{PQ}$

$= 159.3\angle -32.74^\circ \times 0.25\angle 53.13^\circ$

$= 39.825\angle 20.39^\circ$

$V_{PQ} = (37.329 + j13.875)$ V.

Sending-end voltage, $V_P = V_R + V_{PQ} + V_{QR}$

$= 230 + j0 + 37.329 + j13.875 + 24 + j7$

$= 291.329 + j20.875$ V

$= 292.1\angle 4.244^\circ$ V.

$\therefore$ Sending-end voltage, $V_P = 292.1$ V.

Sending-end current, $I_{PQ} = 159.3\angle -32.74^\circ$ A.

And power factor at sending-end, $\cos(36.84) = 0.8$ lag.

Example 14.15

A single-phase distributor 2 km long supplies a load of 120 A at 0.8 pf lagging at its far end and a load of 80 A at 0.9 pf lagging at its midpoint. Both power factors are referred to the voltage at the far end. The impedance per kilometre (go and return) is (0.05 + *j*0.1) Ω. If the voltage at the far end is maintained at 230 V, determine the following:

1. **Voltage at the sending-end, and**
2. **Phase angle between the voltages at both the ends.**

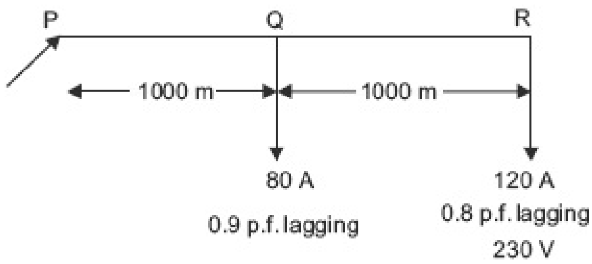

Fig. 14.30 AC two-wire distributor

Solution:

The distributor PR with Q as the midpoint is shown in Fig. 14.30.

The impedance of distributor kilometre = $(0.05 + j0.1)\ \Omega$

$$\therefore \text{Impedance of section PQ, } Z_{PQ} = (0.05 + j0.1) \times \frac{1000}{1000} = 0.1118\angle 63.43^\circ\ \Omega$$

$$\therefore \text{Impedance of section QR, } Z_{QR} = (0.05 + j0.1) \times \frac{1000}{1000} = 0.1118\angle 63.43^\circ\ \Omega$$

Let the voltage V_R at point R be taken as a reference vector.

$\therefore V_R = 230 + j0$ V.

1. Load current at point R, $I_R = 120(0.8 - j0.6) = (96 - j72)$A

   Load current at point Q, $I_Q = 80(0.9 - j0.436) = (72 - j34.88)$A

   Current in section QR, $I_{QR} = I_R = (96 - j72)$ A

   Current in section PQ, $I_{PQ} = I_Q + I_R = 72 - j34.88 + 96 - j72 = 168 - j106.88$A

Voltage drop in section QR, $V_{QR} = I_{QR} Z_{QR}$

$$= (92 - j72)(0.05 + j0.1)$$
$$= 120\angle -36.869^\circ \times 0.1118\angle 63.435^\circ$$
$$= 13.42\angle 26.746^\circ \text{ V}$$
$$= (12 + j6) \text{ V.}$$

Drop in section PQ, $V_{PQ} = I_{PQ} Z_{PQ}$

$$= (168 - j106.88)(0.05 + j0.1)$$
$$= 199.12\angle -32.464^\circ \times 0.1118\angle 63.435^\circ$$
$$= 22.3\angle 30.971^\circ \text{ V}$$
$$= (19.12 + j11.48) \text{ V.}$$

∴ Sending-end voltage, $V_P = V_R + V_{QR} + V_{PQ}$

$$= 230 + j0 + 12 + j6 + 19.12 + j11.48$$
$$= (261.12 + j17.48) \text{ V}$$
$$= 261.70\angle 3.83^\circ \text{ V}$$

2. The phase angle difference between V_P and $V_R = 3.83°$.

Example 14.16

Determine the total voltage drop of a single-phase distributor as shown in Fig. 14.31. The impedance is (0.25 + *j*0.125) per kilometre run (go and return).

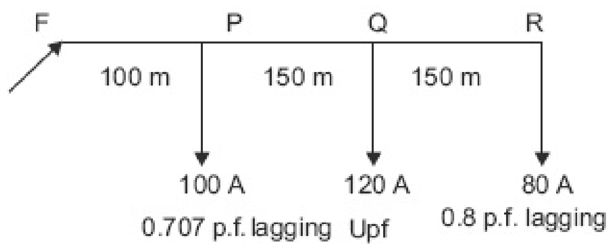

Fig. 14.31 AC two-wire distributor

Solution:

$$\text{Impedance of section FP, } Z_{FP} = (0.25 + j0.125)\,\frac{100}{1000}$$
$$= (25 + j12.5)\,10^{-3}\,\Omega.$$
$$\text{Impedance of section PQ, } Z_{PQ} = (0.25 + j0.125)\,\frac{150}{1000}$$
$$= (37.5 + j18.75)10^{-3}\,\Omega.$$
$$\text{Impedance of section QR, } Z_{QR} = (0.25 + j0.125)\,\frac{150}{1000}$$
$$= (37.5 + j18.75)10^{-3}\,\Omega$$
$$\text{Current in section FP} = I_P + I_Q + I_R$$
$$= 100(0.707 - j0.707) + 120(1 + j0) + 80(0.8 - j0.6)$$
$$= (254.7 - j118.7)\text{ A.}$$
$$\text{Current in section PQ} = I_Q + I_R$$
$$= 120(1 + j0) + 80(0.8 - j0.6)$$
$$= (184 - j48)\text{ A.}$$
$$\text{Current in section QR} = I_R$$
$$= 80(0.8 - j0.6) = (64 - j48)\text{ A.}$$
$$\text{Volt drop in section FP} = \text{Current in FP} \times Z_{FP}$$
$$= (254.7 - j118.7)\,(25 + j12.5)\,10^{-3}$$
$$= (7.85 + j0.216)\text{ V.}$$
$$\text{Volt drop in section PQ} = \text{Current in PQ} \times Z_{PQ}$$
$$= (184 - j48)\,(37.5 + j18.75)\,10^{-3}$$
$$= (7.8 + j1.65)\text{ V.}$$

$$\text{Volt drop in section QR} = \text{Current in QR} \times Z_{QR}$$
$$= (64 - j48)\,(37.5 + j18.75)\,10^{-3}$$
$$= (3.3 - j0.6)\text{ V.}$$
$$\therefore \text{Total volt drop} = \text{drop in FP} + \text{drop in PQ} + \text{drop in QR}$$
$$= (7.85 + j0.216) + (7.8 + j1.65) + (3.3 - j0.6)\text{ V}$$
$$= 18.95 + j1.266$$
$$= 18.99\angle 3.82^\circ\text{ V}$$
$$= 18.99\text{ V.}$$

Example 14.17

A single-phase distributor PQR fed at P is as shown in Fig. 14.32. The power factors are lagging with respect to the voltage at the far end. The impedances between the sections PQ and QR is (0.1 + *j*0.15) Ω. If the voltage at the far end is 230 V, calculate the voltage at the supply-end and also its phase angle with respect to the far end.

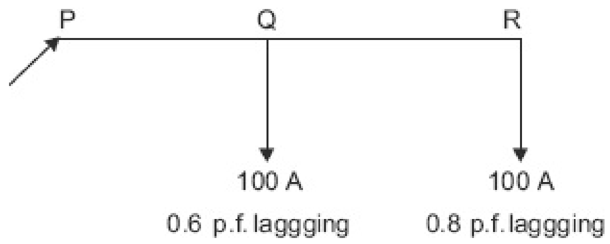

Fig. 14.32 AC single-phase distributor

Solution:

Voltage at point Q, $V_Q = V_R + \text{drop in section QR} = V_R + I_R Z_{QR}$ (since $I_{QR} = I_R$)

$$= 230 + 100\,(0.8 - j0.6)\,(0.1 + j0.15)$$
$$= (247 + j6)\text{ V}$$

Current in section PQ, $I_{PQ} = I_Q + I_R$

$$= 100\,(0.6 - j0.8) + 100(0.8 - j0.6)$$
$$= (140 - j140)\text{ A}$$

$\therefore\ V_P = V_Q + \text{drop in PQ} = V_Q + I_{PQ} Z_{PQ}$

$$= 247 + j6 + (140 - j140)\,(0.1 + j0.15)$$
$$= 282 + j13\text{ V.}$$

$\therefore$ (i) Voltage at the supply-end, $V_P = 282.3\angle 2.64^\circ$ V

(ii) The phase angle difference betwee V_P and V_R = 2.64ΰ.

14.12.2 Power Factor Referred to Respective Load Voltages

If the power factors are referred to the respective load point voltage, the current I_2lags behind voltage V_P by an angle ϕ_2, whereas, current I_1lags behind the voltage V_Q by an angle ϕ_1(not V_Q as in above case). For Fig. 14.28(a), the phasor diagram in this case will be shown in Fig. 14.33. V_1 and V_2 are the voltage drops in section PQ and FP, respectively. Sending-end voltage, current, and power factor can be determined similar to the earlier section.

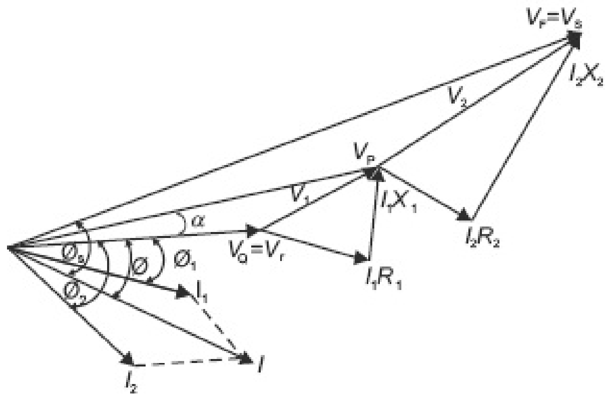

Fig. 14.33 AC distributor

Voltage drop in section PQ,

$$V_1 = I_1(\cos\phi_1 - j\sin\phi_1)(R_1 + jX_1)$$

$$\text{Voltage at point P} = V_Q + \text{Voltage drop in section PQ}$$
$$= V_Q + V_1 = V_P\angle\alpha$$

Now $I_2 = I_2\angle -\phi_2$ with respect to voltage V_P

$$\therefore I_2 = I_2\angle -(\phi_2 - \alpha)$$

$$I_{FP} = I_2 + I_1 = I_2(\cos(\phi_2 - \alpha) - j\sin(\phi_2 - \alpha)) + (I_1(\cos\phi_1 - j\sin\phi_1))$$

Voltage drop in section FP, $V_2 = I_{FP}(R_2 + jX_2)$

Voltage at point F, $V_F = V_P + V_2$.

Test Yourself

1. Why are AC distribution systems preferred to DC distribution systems?

Example 14.18

A single-phase two-wire AC feeder is loaded as shown in Fig. 14.34. The power factors are lagging and are referred to the voltage at the respective load points. The impedances of

section PQ and QP are (0.03 + *j*0.05) and (0.05 + *j*0.08) Ω, respectively. Determine the voltage magnitude and phase angle at the supply end, if the voltage at the far end is 230 V.

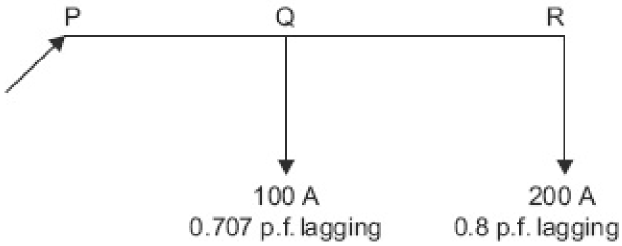

Fig. 14.34 AC single-phase distributor for Example 14.18

Solution:

Voltage at Q, $V_Q = V_R$ + drop in section QR

$$= 230 + I_R Z_{QR} = 230 + I_R(\cos\phi_R - j\sin\phi_R)(R_{QR} + jX_{QR})$$
$$= 230 + 200(0.8 - j0.6)(0.05 + j0.08)$$
$$= 247.6 + j6.8 = 247.69\angle 1.573^\circ \text{ V}$$

V_Q leads V_R by angle $\alpha = 1.573^\circ$

The current I_Q lags the reference vector by $(\phi_1 - \alpha) = (45^\circ - 1.573) = 43.427^\circ$

$I_{PQ} = I_R + I_Q$

$$= 200(0.8 - j0.6) + 100(\cos 43.427 - j\sin 43.427)$$
$$= 200(0.8 - j0.6) + 100(0.727 - j0.6867)$$
$$= (232.7 - j188.67) \text{ A}$$

$\therefore V_P = V_Q$ + drop in section PQ $= V_Q + I_{PQ} Z_{PQ}$

$$= 247.6 + j6.8 + (232.7 - j188.67)(0.03 + j0.05)$$
$$= (264.01 + j12.77) \text{ V.}$$

$\therefore$ Supply-end voltage, $V_P = 264.32\angle 2.77^\circ$ V

Phase angle between V_R and V_P, $\delta = 2.77^\circ$.

Example 14.19

A single-phase distributor has loop resistance of 0.3 Ω and a reactance of 0.4 Ω. The far end of the distributor has a load current of 100 A and a power factor 0.8 lag at 220 V. The midpoint Q of the distributor has a load current of 50 A at power factor 0.9 lag with reference to voltage Q. Determine the sending-end voltage and power factor.

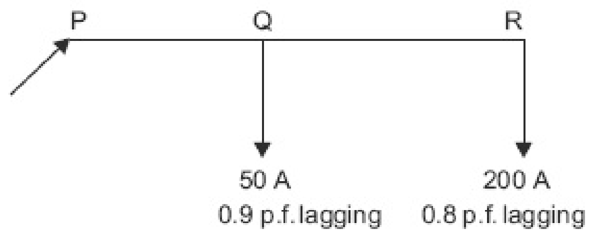

Fig. 14.35 AC single-phase distributor for Example 14.19

Solution:

Loop impedance of the single phase distributor, $Z_{PR} = (0.3 + j0.4)\ \Omega$

$$\text{Impedance of section PQ} = \frac{Z_{PR}}{2}$$
$$= \frac{0.3 + j0.4}{2} = (0.15 + 0.2)\ \Omega$$
$$(\because \text{ Q is midpoint of the line})$$

Let the voltage V_R at far end point R be taken as the reference vector.

$\therefore V_R = 220 + j0$

Load current at point R, $I_R = 100(0.8 - j0.6) = (80 - j0.6)$ A

Current in section QR, $I_{QR} = I_R = (80 - j60)$ A

$$\text{Drop in section QR, } V_{QR} = I_{QR}Z_{QR}$$
$$= (80 - j60)(0.15 - j0.2)$$
$$= (24 + j7)\text{V}.$$
$$\text{Voltage at point Q, } V_Q = V_R + V_{QR}$$
$$= (220 + j0) + (24 + j7)$$
$$= (244 + j7)\text{V} = 244\angle 1.64^\circ \text{ V}.$$

Phase angle between V_Q and V_R, $\alpha = 1.64°$

The load current I_Q has a power factor of 0.9 lag with respect to V_Q.

$\therefore I_Q$ behind V_Q by an angle $\phi_Q = \cos^{-1} 0.9 = 25.84°$

∴ Phase angle between I_Q and V_R, $\phi'_R = \phi_R - \alpha$

$$= 25.84 - 1.64$$
$$= 24.2°.$$

Load current at Q, $I_Q = I_Q(\cos\phi'_Q - j\sin\phi'_Q)$

$$= 50(\cos 24.2 - j\sin 24.2)$$
$$= 50(0.9121 - j0.4099)$$
$$= (45.6 - j20.49)\text{ A.}$$

Current in section PQ, $I_{PQ} = I_Q + I_R = (45.6 - j20.49) + (80 - j60)$

$$= (125.6 - j80.49)\text{ A.}$$

Drop in section PQ, $V_{PQ} = I_{PQ}Z_{PQ} = (125.6 - j80.49)(0.15 + j0.2)$

$$= 34.938 + j13.3046\text{ V.}$$

Sending-end voltage, $V_P = V_Q + V_{PQ} = 244 + j7 + 34.938 + j13.3046$

$$= 278.938 + j20.046 = 279.66\angle 4.11°\text{ V} = 279.66\text{ V.}$$

Its phase angle θ between V_P and $V_R = 4.11°$:

Supply current $I_{PQ} = (125.6 - j80.49)$

$$= 149.18\angle -32.65°\text{ A}$$

∴ Supply power factor $= \cos(4.11° + 32.65°)$

$$= \cos(36.76°) = 0.801\text{ lag.}$$

Example 14.20

A single-phase ring distributor PQR is fed at P. The loads at Q and R are 40 A and 60 A, both at power factor of 0.8 lag and expressed relative to voltage at P. The total impedance of the three sections PQ, QR and RP are (2 + *j*1), (2 + *j*3) and (1 + *j*2) Ω, respectively. Find the current in each section with respect to the supply voltage at P.

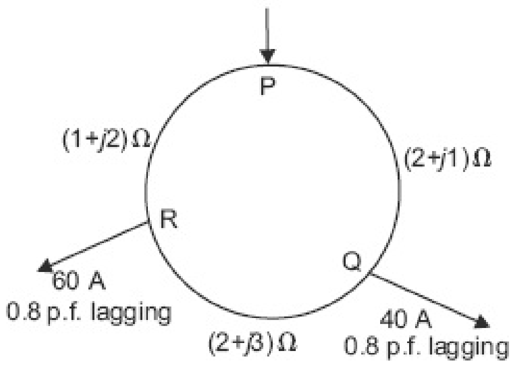

Fig. 14.36(a) A single-phase ring distributor for Example 14.20

Solution:

Thevenin's theorem will be used to solve this problem. The ring distributor is shown in Fig. 14.36(a).

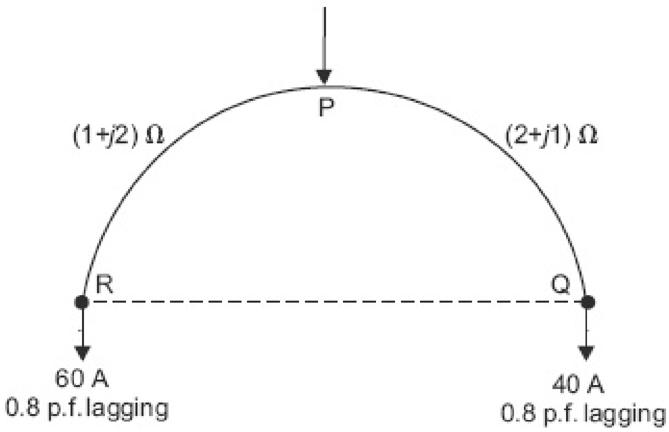

Fig. 14.36(b) Equivalent of Fig. 14.36 when feeder QR is removed

Imagine feeder QR is removed as shown in Fig. 14.36(b)

$$\begin{aligned}
\text{Current in PQ} &= 40(0.8 - j0.6) \\
&= (32 - j24)\text{ A} \\
\text{Current in PR section} &= 60(0.8 - j0.6) \\
&= (48 - j36)\text{ A.} \\
\text{Voltage drop in section PQ} &= (32 - j24)(2 + j1) \\
&= (88 - j16)\text{ V.} \\
\text{Voltage drop in section PR} &= (48 - j36)(1 + j2) \\
&= (120 + j60)\text{ V.}
\end{aligned}$$

Point R is at a lower potential as compared to point Q, since drop in sections PR is more.

Potential difference between Q and R

$= (120 + j60)–(88–j16) = (32 + j76)$ V.

Impedance of the network as looked into from points Q and R = $2+j1 + 1 + j2 = 3 + j3$.

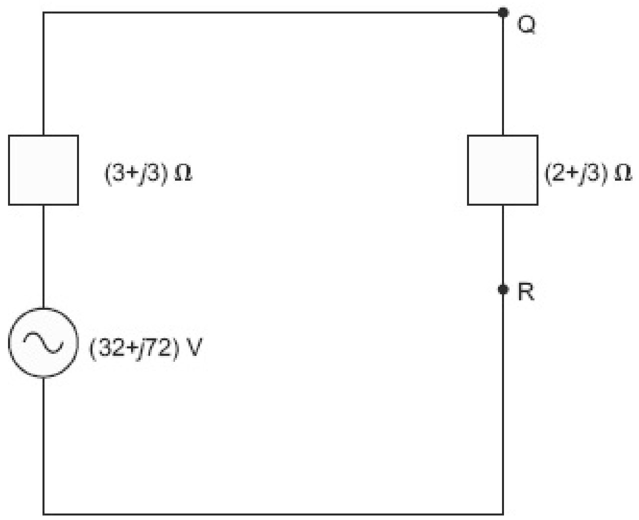

Fig. 14.36(c) Thevenin's equivalent of Fig. 14.36

The equivalent Thevenin's source is shown in Fig. 14.36(c) with feeder QR connected across it.

$$\therefore \text{Current in QR} = \frac{32 + j76}{(3 + j3) + (2 + j3)}$$

$$= \frac{32 + j76}{5 + j6} = \frac{82.462\angle 67.17^\circ}{7.81\angle 50.2^\circ}$$

$$= 10.56\angle 16.97$$

$$= 10.1 + j3.08 \text{ A.}$$

$$\text{Current in PQ} = (32 - j24) + 10.1 + j3.08$$

$$= 42.1 - j20.92$$

$$= 47.01\angle -26.42^\circ \text{ A.}$$

$$\text{Current in PR} = (48 - j36) - 10.1 - j3.08$$

$$= 37.9 - j39.08$$

$$\text{Total current fed at P} = (42.1 - j20.92) + (37.9 - j39.08) = 80 - j60 \text{ A.}$$

14.13 AC THREE-PHASE DISTRIBUTION

In three-phase systems, all the calculations should be carried out on a single-phase basis and the final results can be given in line values if required. Further, in three-phase four-wire distribution systems, motor loads and three-phase balance loads are connected to the three-phase lines and single-phase loads are distributed between each phase end to neutral. As far as possible, the connections are usually arranged to balance the loads and the current passing through the neutral is minimum. When the load is unbalanced (i.e., each phase impedance and/or powers), the current passing through the neutral is maximum.

Example 14.21

A three-phase distribution system is shown in Fig. 14.37. Power is supplied at 11 kV (line voltage) and balanced load of 50 A/phase at power factor 0.8 lag and 70 A at power factor 0.9 lag are taken at Q and R, respectively. The impedance of the feeders are PQ = (5 + *j*9) Ω, QR = (6 + *j*10)Ω and RP = (4 + *j*8) Ω. Calculate the voltage at Q and R and the current in each branch. Power factors are assumed with respect to voltage at P.

Solution:

Thevenin's theorem will be used to solve this problem.

Imagine feeder QR is removed as shown in Fig. 14.38.

$$\text{Voltage drop in PQ section} = 50\angle -36.87^\circ \times 10.30\angle 60.94^\circ = 515\angle 24.07^\circ \text{ V.}$$

$$\text{Supplied voltage} = \frac{11\times 10^3}{\sqrt{3}} = 6350.85 \text{ V}$$

$$\therefore \text{Voltage at Q} = (6350.85 + j0) - 470.22 - j210.04 = 5880.633 - j210.04 = 5884.38\angle -2.06^\circ \text{ V.}$$

$$\text{Voltage drop in PR section} = 70\angle -25.84^\circ \times (4 + j8) = 70\angle -25.84^\circ \times 8.94\angle 63.44 = 626.1\angle 37.6 = (496.05 + j382.01)\text{ V.}$$

$$\text{Voltage at R} = 6350.85 - 496.05 - j382.01 = 5854.8 - j382.01 = 5867.75\ \angle -3.733^\circ \text{ V}$$

Here the potential at Q is higher than at R.

$$\therefore V_{QR} = 5880.633 - j210.04 - 5854.8 + j382.01$$
$$= 25.833 + j171.97 = 173.9\angle 81.46^\circ$$
$$Z_{\text{Thevenin's}} = 5 + j9 + 4 + j8 = 9 + j17$$
$$I_{QR} = \frac{173.9\angle 81.46}{9 + j17 + 6 + j10} = \frac{173.9\angle 81.46}{30.89\angle 60.94}$$
$$= 5.63\angle 20.52^\circ \text{ A.}$$

$$I_{PQ} = 50\angle -36.81^\circ + 5.63\angle 20.54^\circ$$
$$= 40 - j30 + 5.263 + j1.972$$
$$= 45.263 - j28.028 = 53.24\angle -31.7^\circ \text{A.}$$

$$I_{PR} = 70\angle -25.84 - 5.263 - j1.972$$
$$= 63 - j30.51 - 5.263 - j1.972$$
$$= 57.737 - j32.482$$
$$= 66.25\angle -29.36^\circ \text{A.}$$

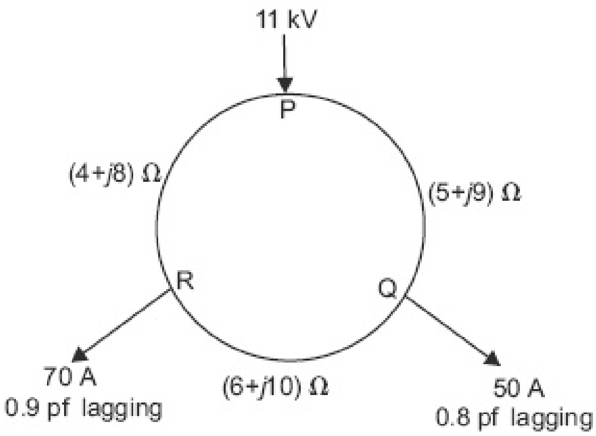

Fig. 14.37 Three-phase distribution

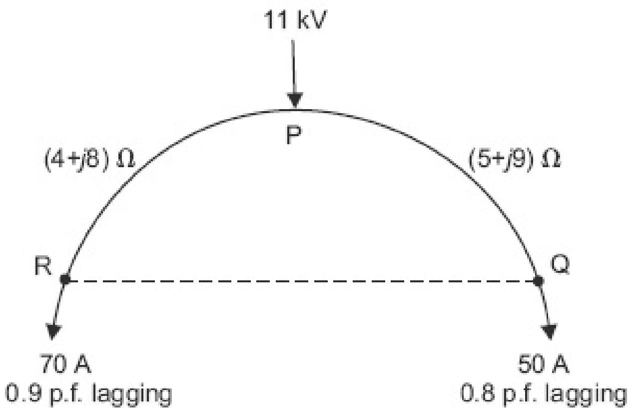

Fig. 14.38 Equivalent of Fig. 14.37 when feeder QR is removed

Example 14.22

A three-phase ring main PQRS fed at P of 11 kV, supplies balanced loads of 50 A at power factor 0.8 lag at Q, 120 A at unity power factor at R and 70 A at 0.866 lag at S, the resistances being referred to the various sections are: section PQ = (1 + *j*0.6) Ω; section QR = (1.2 + *j*0.9) Ω; section RS = (0.8 + *j*0.5) Ω; section SP = (3 + *j*2) Ω (Fig. 14.39). Determine the currents in various sections and station bus-bar voltages at Q, R and S.

Solution:

Let the current in section PQ is $(x + jy)$

$\therefore$ Current in section QR, $I_{QR} = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)$

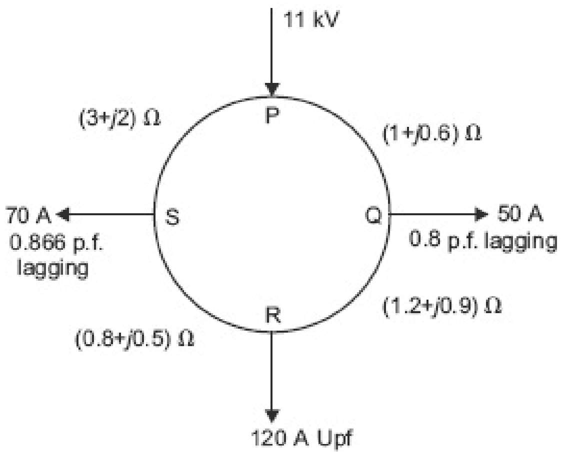

Fig. 14.39 Three-phase ring main distributor

$\therefore$ Current in section RS, $I_{RS} = (x - 40) + j(y + 30) - (120 + j0)$
$= (x - 160) + j(y + 30).$

$\therefore$ Current in section SP, $I_{SP} = (x - 160) + j(y + 30) - 70(0.866 - j0.5)$
$= (x - 220.6) + j(y + 65).$

Drop in section PQ, $I_{PQ}Z_{PQ} = (x + jy)(1 + j0.6)$
$= (x - 0.6y) + j(y + 0.6x).$

Drop in section QR, $I_{QR}Z_{QR} = ((x - 40) + j(y + 30))(1.2 + j0.9)$
$= (1.2x - 0.9y - 75) + j(0.9x + 1.2y).$

Drop in section RS, $I_{RS}Z_{RS} = ((x - 160) + j(y + 30))(0.8 + j0.5)$
$= (0.8x - 0.5y - 143) + j(0.5x + 0.8y - 56).$

Drop in section SP, $I_{SP}Z_{SP} = ((x - 220.6) + j(y + 65))(3 + j2)$
$= (3x - 2y - 791.8) + j(2x + 3y - 246.2).$

Applying KVL to mesh PQRSP, we have

Drop in PQ + Drop in QR + Drop in RS + Drop in SP = 0

$$(x-0.6y)+j(y+0.6x)+(1.2x-0.9y-75)+j(0.9x+1.2y)+$$
$$(0.8x-0.5y-143)+j(0.5x+0.8y-56)+(3x-2y-791.8)+j(2x+3y-246.2)=0$$
$$(6x-4y-1009.8)+j(4x+6y-302.2)=0.$$

As the real and imaginary parts have to be separately zero

$(6x - 4y - 1009.8) = 0$

and $(4x + 6y - 302.2) = 0$

Solving the above two expressions

$$\begin{aligned} 12x - 8y - 2019.6 &= 0 \\ 12x + 18y - 906.6 &= 0 \\ \hline -26y - 1113.0 &= 0 \end{aligned}$$

$$y = \frac{-1113.0}{26} = -42.8 \text{ A}$$
$$4x = 302.2 + 6 \times 42.8 = 302.2 + 256.8 = 559$$
$$x = \frac{559}{4} = 139.75 \text{ A.}$$

∴ Current in section PQ = (139.75–j42.8) A

Current in section QR = (99.75–j12.8) A

Current in section RS = (-20.25–j12.8) A

Current in scction SP = (-80.85 + j 22.2) A

$$\text{Voltage at supply-end } P, V_P = \frac{11000}{\sqrt{3}} = 6351 \text{ V/ph}$$
$$\therefore \text{Voltage at station Q}, V_Q = V_P - I_{PQ}Z_{PQ} = (6351 + j0) - (139.75 - j42.8)(1 + j0.6)$$
$$= (6185.62 - j41.02) \text{ V/ph.}$$
$$\text{Voltage at station R}, V_R = V_Q - I_{QR}Z_{QR}$$
$$= [(6185.62) - j41.02] - (99.75 - j12.8)(1.2 + j0.9)$$
$$= (6054.08 - j115.39) \text{ V/ph.}$$
$$\text{Voltage at station S}, V_S = V_R - I_{RS}Z_{RS}$$
$$= (6054.08 - j115.39) - (-20.25 - j12.8)(0.8 + j0.5)$$
$$= (6063.88 - j95.025) \text{ V/ph.}$$

CHAPTER AT A GLANCE

1. **Distribution system:** It is the part of the power system that lies between the distribution sub-station and the consumers.

2. **Primary distribution:** Power received by major substations at high voltages through transmission lines are stepped down to voltages somewhat higher than general utilization levels. This process is called primary distribution.
3. **Secondary distribution:** It receives power from the secondary side of the distribution transformer at low voltage and supplies power to various connected loads.
4. **Feeders:** The feeders are the conductors which connect the substations to the areas served by these stations.
5. **Distributor:** The distributors are the conductors which run along a street or an area to supply power to consumers.
6. **Service main:** The service mains are the connecting links between distributor and consumer terminals.
7. Distribution systems are classified according to type of current, construction and service, number of wires, and scheme of connection.
    1. According to type of current: AC and DC distribution systems.
    2. According to the type of construction: Overhead and underground systems.
    3. According to the type of service: General lighting and power, industrial power, railway and streetlight etc.
    4. According to the number of wires: Two, three and four-wire systems.
    5. According to the scheme of connection: Radial, ring or loop and inter- connected distribution systems.
8. **Radial distribution system:** The radial system is characterized by having only one path between each customer and a substation.
9. **Loop distribution system:** It consists of two paths between the power sources and every customer.
10. **Interconnected distribution system:** It consists of multiple paths between all points in the network.
11. **According to loading, a DC distributor may be classified as** Concentrated loading and uniformly distributed load.
12. **According to feeding, a DC distributor may be classified as** fed at one end only and fed at both ends.
13. **Distributor fed at one end with concentrated loads:** In this type of feeding, the distributor is connected to the supply at one end and loads are taken at different points along the length of distributor.
14. **Distributor fed at both ends with concentrated loads:** In order to reduce voltage drop the distributor is fed at both ends, either at equal or unequal voltages.

15. Uniformly loaded distributor fed at one end gives a total voltage drop equal to that produced by the whole of load assumed concentrated at the middle point.
16. Uniformly distributed load fed at both ends at the same voltage, evidently the middle point becomes the point of minimum potential.

SHORT ANSWER QUESTIONS

1. Define distribution system.
2. What are the parts of a distribution system?
3. Define the terms: feeder, distributors and service mains.
4. How is distribution systems classified?
5. Why is AC transmission systems used more when compared to DC transmission systems?
6. What are the uses of a distribution system?
7. What are the drawbacks of the radial system?
8. What are the advantages of the ring-main system?

MULTIPLE CHOICES

1. Cross-section of the conductor is obtained, on the basis of current loading for
    1. feeders
    2. distributors
    3. service mains
    4. line
2. Cross-section of the conductor is obtained, on the basis of volt drop for
    1. service mains
    2. distributors
    3. feeders
    4. line
3. Conductors which carry bulk loads from a generating station or a substation are called
    1. bus bars
    2. distributors
    3. feeders
    4. service mains
4. The number of circuits through which, a consumer is fed in a ring-main system is
    1. 1
    2. 2
    3. 3
    4. 4
5. Low-voltage, three-phase AC distribution is done at
    1. 11 kV
    2. 6.6 kV

3. 400 V
4. 230 V

6. The total voltage drop is equal to that produced by the whole of the load assumed concentrated at the mid point, in DC two-wire distributor
   1. fed at one end with concentrated loads
   2. fed at both ends with concentrated loads
   3. fed at one end with uniform load
   4. fed at both ends with uniform load
7. The point of minimum potential for a uniform distributor, fed at one end is at
   1. the middle
   2. the far end
   3. a point between the far end and the middle
   4. a point between the feeding end and the middle
8. The voltage drop at the far end of a uniformly-loaded distributor of 50 m length of resistance 0.01 Ω/m (both wires) with a load of 2 A/m length is
   1. 230 V
   2. 25 V
   3. 1 V
   4. 0.02 V
9. An interconnected system provides
   1. the use of higher voltages
   2. reduction in length of lines
   3. the easier use of protective devices
   4. greater security of service
10. Thevenin's theorem is used to solve
    1. DC networks
    2. AC networks
    3. either DC or AC networks
    4. none
11. The voltage most commonly used for the primary distribution is
    1. 11 kV
    2. 400 V
    3. 230 V
    4. 132 kV
12. Advantage of radial system of distribution over other systems is its________ initial cost.
    1. less
    2. high
    3. moderate
    4. very high
13. In a distributor fed at both ends with concentrated loads, the point of minimum potential always occurs at a point.
    1. receiving-end
    2. mid
    3. sending-end
    4. none of these
14. In solving AC networks and distributors, the resistances have to be replaced by

1. admittance
2. conductance
3. impedance
4. succeptance

15. A uniformly-loaded distributor fed at one end only has power loss of "x" watts, if the distributor is now fed from both ends
    1. $x/2$
    2. $x/4$
    3. $x/8$
    4. $x/6$
16. Floating neutral in a three-phase supply is considered undesirable as it causes
    1. high voltage across the load
    2. low voltage across the load
    3. unequal line voltages across the load
    4. equal line voltages across the load
17. A line which connects a consumer to the distributor is called a
    1. feeder
    2. service main
    3. distributor
    4. line
18. A line which connects a distributor to the substation is called
    1. feeder
    2. distributor
    3. service main
    4. line
19. Which of the following power distribution system gives the better reliability?
    1. ring-main system
    2. radial system
    3. DC three-wire system
    4. tapered distribution
20. Three-phase, four-wire AC system of distribution is used for
    1. unbalanced load
    2. balanced load
    3. all types of load
    4. none of these
21. The fundamental difference between a transmission line and a feeder is that
    1. transmission line is shorter and operates at less voltage than feeder
    2. transmission line is shorter and operates at more voltage than feeder
    3. transmission line is longer and operates at more voltage than feeder
    4. transmission line is longer and operates at less voltage than feeder
22. A distribution transformer usually is a
    1. star-star transformer
    2. delta-delta transformer
    3. star-delta transformer

4. delta-star transformer
23. Electrical power is transmitted from generating stations, to the areas to be served, by means of
    1. belt transmission
    2. water conduits
    3. conductor system
    4. air
24. Which of the following distribution systems is the simplest and has lowest installation cost?
    1. inter-connected system
    2. radial system
    3. ring system
    4. tapered system
25. A uniformly-loaded DC two-wire distributor 300 m long is loaded at 2 A/m run. Resistance of the loop is 0.05 Ω/km. Calculate the maximum voltage drop if the distributor is fed at both ends at the same voltage.
    1. 0.0125 V
    2. 1.125 V
    3. 4.5 V
    4. 3 V

Answers:

1. a	2. b	3. c	4. b	5. c
6. c	7. b	8. c	9. d	10. c
11. a	12. a	13. b	14. c	15. b
16. c	17. b	18. a	19. a	20. a
21. c	22. b	23. a	24. b	25. b

REVIEW QUESTIONS

1. A distributor of length L metres has a distributed load of IA/m. Prove that maximum voltage drop when fed at both ends with equal voltages is one-fourth of that when fed at one end.
2. What general considerations govern the design of distribution system? Bring out their relative importance.
3. Prove that the total voltage drop in a uniformly-loaded distributor fed at one end is equal to the drop produced by the whole load assumed contracted at the midpoint of the distributor.
4. What are the important requirements for a good distribution system?
5. What are the different types of distributor?
6. Prove that the voltage drop of two-wire DC distributor fed at one end is equal to the sum of moment of load currents about the feeding point.
7. Find out the voltage-drop expression of a two-wire DC distribution with a uniform load being fed at one end.

8. Find out the voltage-drop expression of a two-wire DC distribution with a uniformly-distributed load being fed at both ends.
9. What is a ring distributor? How many types of ring distributors do we have? What are the advantages of providing interconnectors in the ring distributor?
10. Derive an expression for the power loss in a uniformly-loaded distributor fed at one end.
11. Define and explain the following terms (i) feeder, (ii) distributor, and (iii) service mains.
12. Discuss briefly the design considerations in a distribution system.
13. Discuss the relative merits and demerits of DC distribution and AC distribution systems.
14. Compare (i) radial distribution system, (ii) ring distribution system, and (iii) network distribution system. State applications.
15. Explain: (i) primary distribution system and (ii) secondary distribution system.
16. Draw a single-line diagram for a radial primary feeder and mention the factors that influence the selection of primary feeders.
17. Explain the basic design practice of secondary distribution systems.
18. What is an inter-connector? Discuss the advantages of using it in a distribution system.

PROBLEMS

1. A DC distributor is fed at both ends with the same voltage of 230 V. The total length of the feeder is 300 m and the loads are tapped as follows: 50 A at 60 m from P, 40 A at 100 m from P, 35 A at 120 m from P and 25 A at 200 m from P. The resistance per kilometre of the conductor for go and return is 0.8 Ω. Calculate the following:
    1. The current in each section
    2. The point of minimum potential
    3. The voltage at minimum potential.
2. A DC 350 m long distributor is uniformly loaded at a rate of 0.9 A/m. Determine the voltage drop at a distance of 250 m:
    1. When it is fed from one end at 230 V
    2. When it is fed from both ends at 230 V

    The resistance of each conductor for both go and return per metre is 0.0001Ω. Also, find the power loss in the above two cases.
3. A two-wire DC distributor PQ is 1.5 kilometre long and supplies loads of 75 A, 100 A, 150 A and 40 A to points situated at 400 m, 750 m, 1200 m, and 1500 m from the feeding point P. Each conductor has a resistance of 0.02 Ω/kilometre. If a supply voltage 250 V is maintained at point P, determine the voltage at each load.

4. A two-wire DC distributor PQ of total length of 500 m is loaded as follows:

Distance from P (m)	: 125	250	325	425
Loads (A)	: 75	150	200	275

   The feeding point P is maintained at 420 V and Q is maintained at 400 V. If each conductor has a resistance of 0.012 Ω/100 m, determine the following:

   1. The current supplied from P to Q
   2. The power dispatched in the distributor.
5. A two-wire DC distributor cable PQ, 1.5 kilometre long has a total resistance of 0.5 Ω. The ends P and Q are fed at 230 V. The cable is uniformly loaded at 1 A/m length and with concentrated loads of 100 A, 75 A, 120 A, and 30 A at a distance of300 m, 500 m, 800 m and 1200 m, respectively, from the end P. Determine the following:
   1. The point of minimum potential
   2. Currents supplied from ends P and Q
   3. The value of minimum potential
6. A single-phase line 3 kilometre long distributor PQR has a resistance and reactance (go and return) of 0.05 and 0.2 Ω /kilometre respectively. "P" is the feeding point, Q is the mid-point of the distributor taking a load of 50 A at 0.707 p.f. lagging and R is the far end taking a load of 100 A at UPF. The voltage at the far end is 230 V. Calculate the voltage at the sending end and the phase angle difference between the voltages of two ends if
   1. Power factors of the loads are with reference to far-end voltage
   2. Power factors of the loads are with reference to the voltages at the load points.
7. A two-wire feeder PQR has a load of 100 A at R and 50 A at Q, both at p.f. 0.707 lagging. The impedance of PQ is (0.03 + j0.05) Ω and that of QR is (0.07 + j0.015) Ω. If the voltage at the far end R is to be maintained at 415 V, calculate the voltage at Q and P.
8. Consider a ring distributor PQR, the impedances of the sections PQ, QR, and RP are (2 + j2), (1 + j1.5), and (3 + j4) Ω, respectively. The supply is fed at point P, while at Q and R, loads of 40 A at 0.707 p.f. lagging and 75 A at 0.85 p.f. lagging are drawn. The power factors are with respect to voltage at P. Determine the current in each section.
9. A single-phase two-wire feeder 1 kilometre long supplies a load of 50 A at 0.707 p.f. lagging, 35 A at 0.8 p.f. lagging and 60 A at 0.9 p.f. lagging at distance of 250 m, 500 m and 1000 m, respectively from the feeding end. The resistance and reactance of the feeder per kilometre length are 0.05 and 0.2, respectively. If the voltage at the far end is to be maintained at 230 V, determine the voltage

at the receiving end?

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15

EHV and HVDC Transmission Lines

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Obtain an overview of EHV and HVDC transmission lines
- Classify the HVDC transmission system
- Explain the concepts of conversion and inversion

15.1 INTRODUCTION

All developed and developing countries depend upon electrical energy for industrial, commercial, agricultural, domestic and social purposes. Therefore, the basic infrastructure, that is, generating stations and transmission and distribution lines have become a crucial part of modern socio-industrial landscape. Hydropower sites, however, are located far away from load centres while thermal plants are located near coal mines, for economic reasons and also to avoid pollution. Hence, the energy produced at the generating stations has to be transmitted over long distances to load centres through transmission systems. This transmission can be done more economically by using extra high-voltage (EHV) or high-voltage direct current (HVDC) transmission lines.

Nominal-system voltage levels recognized in India are 132, 220, 400, 500 and 750 kV. At present, the highest AC transmission voltage is 750 kV (under planning), as opposed to just 400 kV in the 1990s. EHV transmission lines are now commonly used for transmitting bulk power in various national grids. In future, we may go for ultra high voltage (UHV) transmission voltages in the range of 1000, 1100, 1200 kV and above.

The problems associated with the transmission of AC power at high voltage, over long distances have again brought focus on the development of high voltage DC systems (HVDC). India has been a pioneer developer of HVDC since 1990 when the 1000 MW Rihand-Dadri line was commissioned in UP. Since then, many 500 MW lines have come up. The 2000 MW Talcher-Kolar link is the biggest so far and spans

four states: Orissa, Andhra Pradesh, Tamil Nadu and Karnataka. The highest HVDC transmission voltage in India is ±500 kV.

15.2 NEED OF EHV TRANSMISSION LINES

EHV lines are preferred for the following reasons:

- Improves the performance of transmission lines i.e., efficiency and regulation increases with increase in transmission voltage.
- Reduces the requirement of conductor material due to increased transmission voltage as the volume of the conductor is inversely proportional to the transmission line voltage.
- The installation cost of transmission line per kilometre decreases as the volume of conductor decreases, and hence, the cost of line supports reduces.
- There is an increase in the transmission capacity of the line as the power-transfer capability is directly proportional to the product of sending and receiving-end voltages.
- Flexibility for future growth of the system; with an increase in transmission voltage, there is a possibility to supply more power.
- There is an increase in surge impedance loading as it is directly proportional to the square of the transmission voltage.
- There is a possibility of interconnecting power systems for the economical operation of power systems.
- With increase in operating voltage, the number of circuits and the requirement of land are reduced considerably.

15.3 ADVANTAGES AND DISADVANTAGES OF EHV LINES

ADVANTAGES:

- Voltage can be stepped up or stepped down to the required level using transformers.
- Operation is simple as the generated power can directly be sent to the

receiving end without the need of converters.
- Equipments are simple and reliable.
- The line can be easily tapped or extended as the power need not be converted.
- Control of power flow in the lines is simple and natural because of the use of compensating devices.
- Generation and distribution of power is in AC only.

DISADVANTAGES:

- Current density increases as voltage increases with charging current.
- Bundled conductors are required for reducing corona effect.
- Surface-voltage gradient on conductors becomes higher as the voltage increases.
- Corona problems increase in the voltage gradient.
- High electrostatic field under the overhead line increases the charging current.
- Over-voltage problem due to switching surges.
- Insulation coordination based upon switching impulse level.

Test Yourself

1. Why has the EHV transmission voltage remained restricted to 400 kV so far?
2. Why is insulation coordination based upon the switching-impulse level?

15.4 METHODS OF INCREASING TRANSMISSION CAPABILITY OF EHV LINES

The power-transfer capability of EHV transmission lines can be increased by maintaining the load-end voltage within specified limits. The following methods are used for maintaining the voltage within limits:

1. Shunt capacitor banks
2. Shunt reactors
3. Synchronous condensers
4. Static VAR compensators at heavy loads
5. Series compensation

Shunt capacitor banks: Shunt capacitor banks are operated to inject the reactive power for maintaining the voltage within limits at heavy loads. These are installed near the load terminals i.e., factory substations and switching substations.

Shunt reactors: Shunt reactors are used to absorb the reactive power from the line to control the voltage under lightly-loaded and no-load conditions. These are provided at both ends of all transmission lines.

Synchronous condensers: Synchronous condenser is the synchronous motor running under no load. It generates and absorbs the reactive power depending upon the excitation. Under low loads, it works as a VAR absorber. Under heavy loads, it works as a VAR generator. So, by adjusting the excitation of the synchronous machine, the receiving-end voltage can be maintained within the limits.

Static VAR compensators: The functioning of a static VAR compensator is the same as that of a synchronous condenser. The only difference is that there are no moving parts. The important features of this compensator are higher speed of response, better reliability factor, low maintenance cost and reduced power system oscillation. It also improves the power transfer capacity.

Series compensation: In series compensation, a capacitor is connected in series with the line. So, the net reactance of the line will be reduced, and due to the reduced reactance, the net voltage drop is also reduced. The performance of the transmission line is thus improved. These various compensation methods have been discussed in detail in Chapters 10 and 16.

15.5 HVDC TRANSMISSION SYSTEM

Power generating plants are located far away from the load centres because of pollution hazards and the location also depends upon the availability of hydropower sites. Therefore, the process of transmission of power generated at great distances from the target load centres is the major factor that affects the cost of energy. The problem of transmission of AC power, especially from long distances, can be overcome by DC transmission.

The first 20 MW, 100 kV HVDC link in the world was built in 1954 between the mainland of Sweden and the island of Gotland. At present, the biggest HVDC link has reached 6300 MW, 300 kV (bipolar). The top view of a 400 kV HVDC substation is given in Fig. 15.1.

Fig. 15.1 View of 400 kV HVDC substation

15.6 COMPARISON BETWEEN AC AND DC TRANSMISSION SYSTEMS

The two main factors to be considered while comparing the two kinds of systems are:

1. Economic advantages
2. Technical advantages

15.6.1 Economic Advantages

1. The HVDC transmission system requires only one conductor when compared to the AC transmission system which requires several conductors. So, the cost of an HVDC transmission line is less when compared to that of an AC transmission system.
2. The supporting structure required for an HVDC transmission system is narrow, whereas an AC transmission system requires a lattice structure. So, the cost of the supporting tower is less when compared to that of an AC system.
3. Line losses in HVDC transmission are less when compared to the

AC transmission for the same power transfer capacity. So, the energy cost in HVDC transmission lines is less when compared to that in AC transmission systems.

4. The HVDC lines can be built in two stages. The second stage can be built whenever the extra power transfer capability is needed. The first stage can be operated as a monopolar line with one conductor and ground as a return, whereas, the second stage can be operated as bipolar with two conductors without ground return. The investment on the second stage can, thus, be postponed.

15.6.2 Technical Advantages

1. **Reactive power requirement:** When the load impedance equals the surge impedance of the line, the reactive power generated by the line capacitance equals the reactive power absorbed by the line inductance. The line cannot always be operated at its natural load since the loads vary with time. In DC transmission, no reactive power is transmitted over the line. The reactive power is independent of line length but varies with the transmitted power.
2. **System stability:** In order to maintain stability, the length of an uncompensated AC line must be less than 500 km, whereas, when series compensation is used, the length may be longer than 500 km. A DC transmission system has no such stability problems.
3. **Short-circuit current:** If two AC systems are interconnected by an AC line, then the short-circuit current in the system increases. Therefore, the existing circuit breakers (CBs) have to be replaced with new CBs of high ratings. However, in DC lines, the contribution of short-circuit current is the same as the rated current-carrying capacity of a DC line.
4. **Independent control of AC systems:** The AC systems which are interconnected by a DC line can be controlled independently. They are independent with reference to frequency, system control, short-circuit rating, future extension etc.
5. **Fast change of energy flow:** The transmitted power is proportional to the difference in terminal voltages. So, the direction of energy flow can be changed by changing the values of DC voltages.
6. **Less corona loss and radio interference:** The corona loss of a transmission system is proportional to (f + 25), the frequency of a DC system is zero. So, corona losses in a DC system are less when compared to an AC system of the same conductor diameter and voltage.
7. **Greater reliability:** If a fault occurs in one conductor of a bipolar system, the other conductor continues to operate with ground return. So, a two-conductor bipolar DC line is more reliable than a three-conductor three-phase AC line.

Test Yourself

1. Why is corona effect less in HVDC transmission lines?

15.7 ADVANTAGES AND DISADVANTAGES OF HVDC SYSTEMS

The advantages of HVDC transmission systems are:

1. Only one or two conductors are required for HVDC transmission, whereas, three conductors are required for an AC transmission. Hence, the cost involved is comparatively less.
2. It can carry more power at the same operating voltage.
3. There is a considerable economy in insulation. For the same operating voltage, the insulation between the tower and the conductors and the clearance above the earth required is small when compared to AC lines.
4. Due to the absence of charging current and stability limits, there is no limitation of distance for power transfer.
5. Dielectric power loss in cables is less in DC than in AC. Hence, the current-carrying capacity of cables can be increased considerably.
6. There is no skin effect. So, smaller size conductors can be used.
7. By the asynchronous link provided between two DC systems, the problem of voltage control, system stability, and frequency control is simplified.
8. Corona loss and radio interference problems are less in DC.
9. In long HVDC transmission lines, the receiving-end voltage does not rise above the sending-end voltage. So, there is no Ferranti effect.
10. Detection and clearance of fault is faster.
11. The short circuit current required is low when compared to AC systems.
12. By having a DC link in parallel with an AC link, the stability limit of the existing AC system can be increased. So, the existing AC lines can be used for high capacity transmission.
13. No reactive compensation is required.
14. There is a possibility of using the ground as return path.

The disadvantages of HVDC transmission systems are:

1. Reactive power cannot be transmitted through a DC line.
2. The terminal equipment for conversion from AC to DC and back to AC is expensive.
3. The protection scheme of a DC system is expensive.
4. Ripples and harmonics are generated when AC is converted into DC (both on the AC and the DC side) necessitating installation of filters for their elimination.
5. Voltage cannot be stepped up.

15.8 HVDC TRANSMISSION SYSTEM

A DC link is the DC power transmission network which consists of transformers, converter units, and

conductors.

The classification of HVDC systems depends upon the arrangement of the pole and the earth return. They are as follows:

1. Monopolar link
2. Bipolar link
3. Homopolar link

15.8.1 MONOPOLAR LINK

A monopolar link has only one conductor, with ground or sea as a return path. Generally, the conductor has a negative polarity with respect to the earth. See Fig. 15.2.

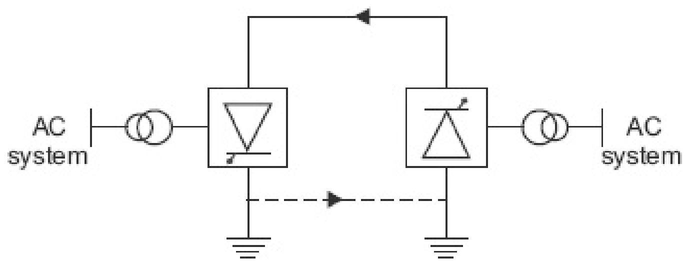

Fig. 15.2 Monopolar link

ADVANTAGES:

- The corona effect in a DC line is less because of the negative polarity of the line conductor as compared to the positive polarity in AC.
- Less conductor material is required as ground is used as the return path.
- Negative polarity is preferred because of less radio interference.
- Less insulation cost.

DISADVANTAGES:

- Ground return path is objectionable as it leads to corrosion of buried metallic structures such as pipes, cables, fences, etc.
- Causes disturbances in underground communication systems.
- Power transmitting capacity is less as compared to Bipolar and Homopolar systems.
- Power interruption problems due to faults are frequent.

15.8.2 Bipolar Link

Bipolar links have two conductors, one at the positive potential and the other at the negative (same magnitude) with respect to the ground. At each terminal, two identical sets of converters are connected in series, on the DC side. The junction between the two sets of converters is grounded at one or both ends as shown in Fig. 15.3. Normally, both conductors operate at equal potentials. So, the current through the ground is zero.

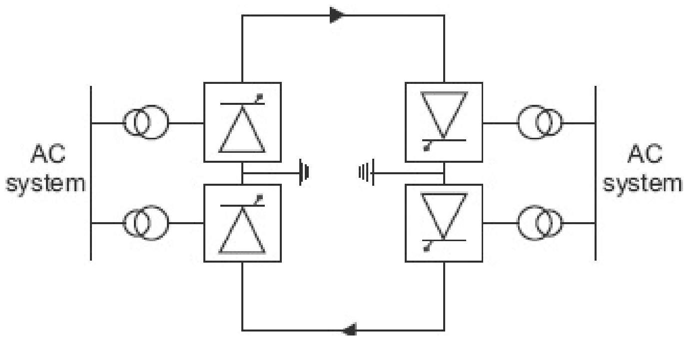

Fig. 15.3 Bipolar link

ADVANTAGES:

- Power transmitting capacity is doubled when compared to monopolar link.
- If a fault occurs in one conductor, power can be transmitted through ground therefore, power interruptions can be avoided.
- Reliability is high when compared to monopolar link.

DISADVANTAGES:

- Terminal equipment cost is high.
- More conductor material is required.
- Radio interference is high.
- Corona loss is high.

15.8.3 Homopolar Link

Homopolar link has two or more conductors, all having the same polarity, and ground is used as the return path.

Usually, negative polarity (with respect to the earth conductors) is used as shown in Fig. 15.4.

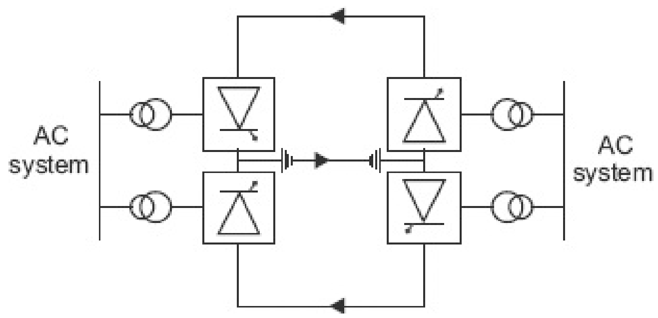

Fig. 15.4 Homopolar link

ADVANTAGES:

- Corona effect is less in negative polarity conductors.
- Less conductor material is required because ground is used as the return path.
- Negative polarity conductor has less radio interference.
- We can avoid power interruption due to faults by transmitting power through other conductors.
- Reliability is high.
- Insulation costs are low.

DISADVANTAGES:

- Ground return path causes corrosion of buried metallic structures.
- Causes disturbance in underground communication cables.

Test Yourself

1. Why is the corona effect less in negative polarity conductors?

15.9 RECTIFICATION

Generally, the valves of the rectifier operate only in one direction i.e., from anode to cathode. Transformers are connected to give three-phase, six-phase and twelve-phase supply to the rectifier valves.

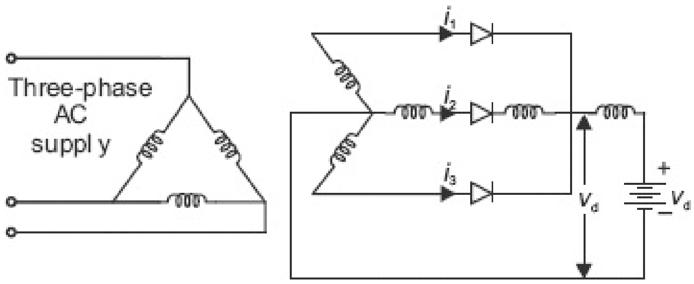

Fig. 15.5 Three-pulse converter

Consider an n-phase system for general analysis of a rectifier circuit. For simple analysis, consider a circuit diagram of a three-phase, three-pulse bridge converter shown in Fig. 15.5.

From the voltage wave forms of three phases shown in Fig. 15.6(a), consider a point 'O' as reference,

the valves start conducting at 30° and continues up to

150° i.e., $\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ to $\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$. In general, for an

n-phase system, the valve starts conducting at $\left(\frac{\pi}{2}+\frac{\pi}{n}\right)$

and continues up to $\left(\frac{\pi}{2}+\frac{\pi}{n}\right)$.

From the above discussion, the conduction takes place only in the positive half cycle of the voltage, the average value of DC voltage will be,

$$V_0 = \frac{1}{\frac{2\pi}{n}} \int_{\left(\frac{\pi}{2}-\frac{\pi}{n}\right)}^{\left(\frac{\pi}{2}+\frac{\pi}{n}\right)} (V_m \sin\theta \, d\theta)$$

$$= \frac{1}{\frac{2\pi}{n}} \int_{\left(\frac{\pi}{2}-\frac{\pi}{n}\right)}^{\left(\frac{\pi}{2}+\frac{\pi}{n}\right)} (V_m \sin\theta \, d\theta)$$

$$= \frac{-nV_m}{2\pi} [\cos\theta]_{\frac{\pi}{2}-\frac{\pi}{n}}^{\frac{\pi}{2}+\frac{\pi}{n}}$$

$$= V_m \frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}.$$

For three phase system, $n = 3$

$$V_0 = V_m \frac{\sin\frac{\pi}{3}}{\frac{\pi}{3}} = 0.827 V_m.$$

The wave form of an anode current is a rectangular pulse of magnitude I_d and length 120°. So, its average value is $\frac{I_d}{\sqrt{3}} = 0.577 I_d$.

The secondary current of a transformer is equal to the anode current and it cannot be reduced to zero or a finite value instantly because of the inductance. Conduction of an anode can be controlled by applying suitable pulse at a suitable instant to the gate terminal. Once conduction starts, the grid loses its control over the conduction period. Figure 15.6(c) shows the wave form of the voltage with grid control angle. Let a positive pulse be applied to the grid in such a way that the conduction is delayed by an angle α.

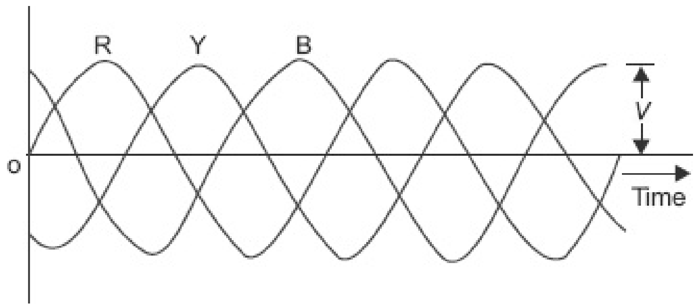

Fig. 15.6(a) a three-phase input voltage

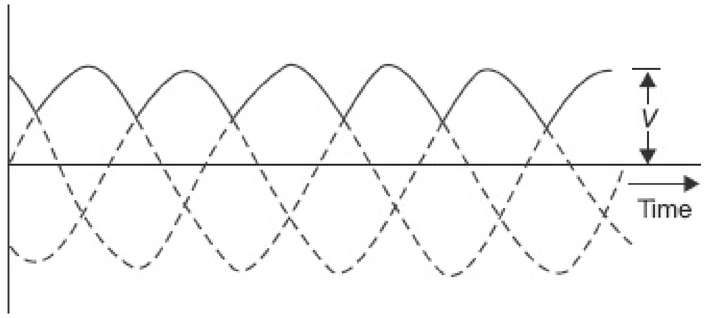

Fig. 15.6(b) Wave form of an anode voltage

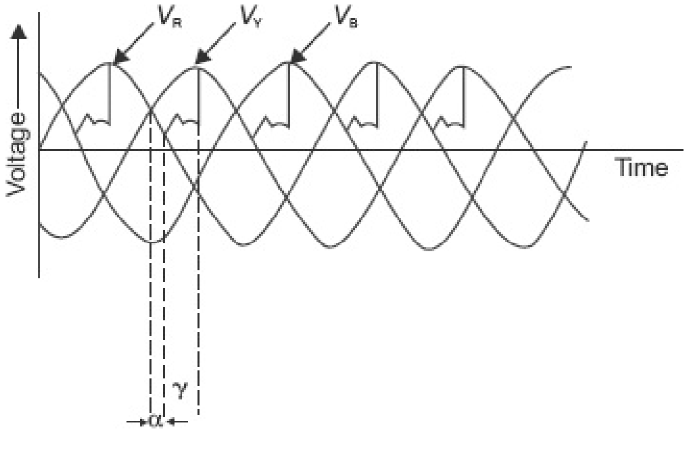

Fig. 15.6(c) Wave form of voltage with grid control angle

The average output voltage of *n*-phase system with delay angle α is

$$V_0' = \frac{1}{\frac{2\pi}{n}} \int_{\left(\frac{\pi}{2}-\frac{\pi}{n}\right)+\alpha}^{\left(\frac{\pi}{2}+\frac{\pi}{n}\right)+\alpha} \left(V_m \sin\theta \, d\theta\right)$$

$$= \frac{-nV_m}{2\pi}\left[\cos\theta\right]_{\frac{\pi}{2}-\frac{\pi}{n}-\alpha}^{\frac{\pi}{2}+\frac{\pi}{n}+\alpha}$$

$$= \frac{nV_m}{2\pi}\left[\sin\left(\frac{\pi}{n}+\alpha\right)+\sin\left(\frac{\pi}{n}-\alpha\right)\right]$$

$$= \frac{nV_m}{2\pi}\left[\sin\left(\frac{\pi}{n}\right)\cos(\alpha)\right]$$

$$= V_0 \cos\alpha.$$

15.10 THREE-PHASE BRIDGE CONVERTER

Three-phase bridge converters are used at converter stations for converting AC to DC and vice-versa in case of HVDC transmission systems. The connection of a bridge circuit (also called Graetz circuit) is shown in Fig. 15.7(a). This is a six-pulse converter configuration. A twelve-pulse configuration can be obtained by connecting two six-pulse converters in series.

Let the instantaneous line-to-phase voltages be

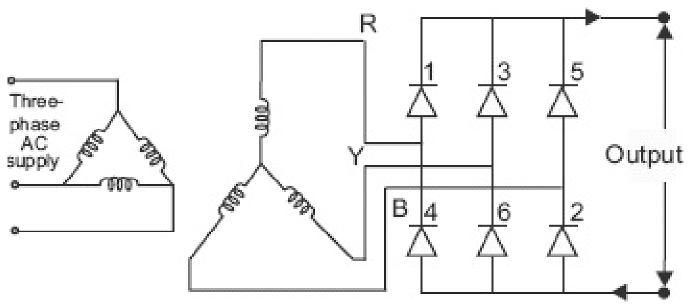

Fig. 15.7(a) Three-phase bridge converter

Let the instantaneous line-to-phase voltages be,

$$V_R = V_{max} \sin \omega t \quad (15.1)$$
$$V_Y = V_{max} \sin (\omega t - 120^o) \quad (15.2)$$
$$V_B = V_{max} \sin (\omega t - 240^o) \quad (15.3)$$

and its wave forms are shown in Fig. 15.7(b).

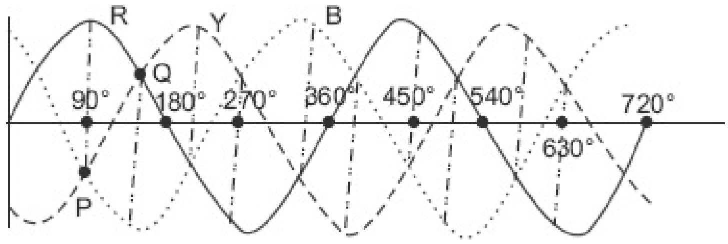

Fig. 15.7(b) Wave forms of a three-phase bridge converter

The operation of a bridge circuit can be explained as follows:

Corresponding to the point 'P', V_R is more positive and V_B is more negative. Therefore, conduction takes place between phases 'R' and 'B'. Thus, the valves 1 and 2 will conduct. This will continue up to point 'Q'. At point 'Q', V_Y is more positive and V_B is more negative. Therefore, conduction takes place between phases 'Y' and 'B'. Thus, the valves 2 and 3 will conduct. The complete sequence of valves conducting is, therefore, (1,2), (2,3), (3,4), (4,4), (5,6) and (6,1). It should be noted that each valve conducts for 120° and the interval between two consecutive firing pulses is 60° in steady state. The output voltage for an *n*-phase circuit is,

$$V_0 = V_m \frac{\sin \frac{\pi}{n}}{\frac{\pi}{n}}.$$

For a three-phase system, (n = 3)

$$V_0 = V_m \frac{\sin\frac{\pi}{3}}{\frac{\pi}{3}}$$

$$= \frac{3V_m}{\pi} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi} V_m.$$

So, the output voltage of a three-phase circuit is

$$V_0 = \frac{3\sqrt{3}}{2\pi} V_m.$$

Test Yourself

1. Why are twelve-pulse converters commonly used for conversion for HVDC?

15.11 INVERSION

In case of an inverter operation, the direct current is transferred to an alternating-current system. This can be obtained by the reversal of the average direct voltage. The voltages across the bridge-rectifier output terminals depend on the firing angle or the delay angle.

The converter operates in the rectifier mode when the delay angle α is less than 90°. The average value of output voltage is positive. Therefore, power will be transferred from AC to DC. If α is more than 90°, the output voltage across the bridge circuit terminals is negative, thus, power will be transferred from DC to AC. The circuit diagram for an inverter is shown in Fig. 15.8.

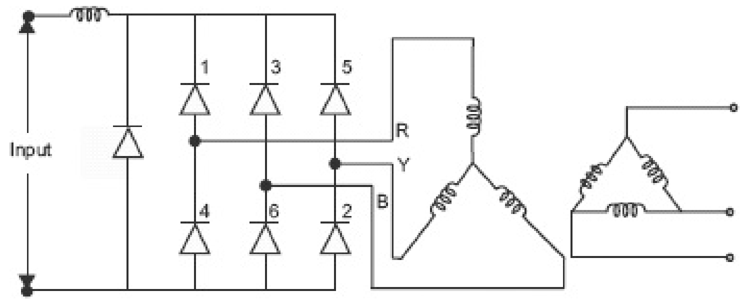

Fig. 15.8 Inverter

15.12 COMPONENTS OF HVDC TRANSMISSION SYSTEM

The typical diagram showing the important components of the HVDC transmission system is shown in Fig. 15.9. Converter stations comprise of the most important component of HVDC transmission systems. These are the two types:

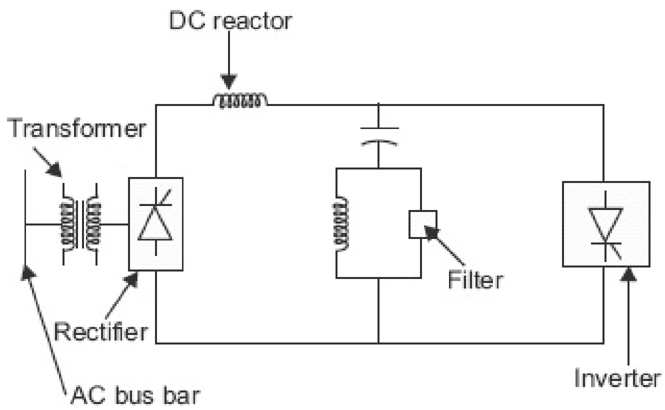

Fig. 15.9 Overview of HVDC transmission

1. Rectifier station
2. Inverter station

Rectification is the process of conversion from AC to DC. Here, the supply taken from the AC bus bar is fed to the rectifying unit as input and the output obtained from this is DC. The DC supply, thus available, is used for transmission. These stations are located at the starting points of the HVDC transmission system.

Inversion is the process of conversion DC to AC. Here, the HVDC supply is taken as the input to the inverter unit and the output obtained is the AC. These stations are located at the end point of the HVDC transmission system.

In modern times, three-phase, 12-pulse converters are used for conversion processes. The other major components are converter transformers, DC reactor and filters.

1. **Converter transformer:** It is connected between the converter valves and the AC bus. This is a specially designed transformer that can withstand the direct voltage stress and harmonic losses due to harmonic currents, unlike a power transformer. They are either a single three-phase unit or three one-phase units.
2. **DC reactor:** It is connected in series with the rectifier. It is necessary for converter operation and for smoothing the DC current i.e., to eliminate the ripple content in the DC components. It also safeguards the rectifier under short-circuit conditions.
3. **Filter:** During the conversion process, harmonics are generated. Filters are used to eliminate generated harmonics.

15.13 HARMONIC FILTERS

Filters are used to filter the unwanted signals (i.e., harmonics). There are two types of filters used in HVDC transmission systems. They are:

1. AC filters
2. DC filters

1. **AC filter:** It is used to reduce the harmonic components in the AC power flow. AC filters are connected in parallel between the AC bus bars and the earth. They offer high impedance to power frequency and low impedance to harmonic frequencies, since harmonic frequencies are more than the power frequencies. Thereby, harmonic frequencies are eliminated from AC networks by passing them to ground.
2. **DC filters:** These are used to reduce the ripple content in the voltage or current on DC line. These are used in conjunction with the DC reactors and DC surge capacitors. These are located between the pole bus and the neutral bus. These filters offer low

impedance to certain DC harmonics, which are eliminated from the DC line by passing them to ground.

15.14 APPLICATION OF HVDC TRANSMISSION SYSTEM

For generation, transmission and distribution of electrical power, three-phase AC systems are used and they have a definite superiority over HVDC.

However, HVDC transmission is a strong alternative to EHV AC transmission for the following applications:

- High power transmission by overhead lines for long distances.
- Medium and long high-power submarine or underground cables.
- Multi-terminal HVDC system for interconnecting three or more than three-phase systems.
- Frequency conversion.
- Electric traction.

CHAPTER AT A GLANCE

1. **Extra high-voltage transmission:** Transmission which involves voltages between 330 and 1000 kV.
2. **Ultra high-voltage transmission:** Transmission which involves voltages in the range of 1000, 1100, 1200 kV and above.
3. **Shunt reactor:** A device used to absorb the reactive power from the line to control the voltage under lightly-loaded conditions.
4. **Series compensation:** A device used for increasing the power transfer capacity.
5. **Monopolar link:** A monopolar link has only one conductor, with ground or sea as a return path.
6. **Bipolar link:** A bipolar link is an HVDC transmission link that has two conductors: one at the positive potential and the other at negative (same magnitude) with respect to the ground.
7. **Homopolar link:** A homopolar link is an HVDC transmission link that has two or more conductors, all having the same polarity. The ground is used as return the path.
8. **Rectification:** The process of conversion from AC to DC.
9. **Inversion:** The process of conversion from DC to AC.
10. **Filters:** A filter is a device used to filter unwanted signals called harmonics, which are generated during the conversion process.

11. **AC filter:** A filter is used to reduce the harmonic components in the AC power flow.
12. **DC filter:** A filter is used to reduce the ripple content in the voltage or current on DC line.

SHORT ANSWER QUESTIONS

1. What is the range of voltage in extra high-voltage transmission?
2. What are the voltages under ultra high-voltage transmission?
3. What are the uses of a shunt reactor?
4. What is meant by a monopolar link?
5. What is meant by a bipolar link?
6. What is meant by a homopolar link?
7. What do you mean by rectification?
8. What do you mean by inversion?
9. What is the function of a filter?
10. How do you define an HVDC-system link?
11. What are the major components of an HVDC transmission system?

MULTIPLE CHOICE QUESTIONS

1. Which one of the following is the ideal voltage for an EHV transmission system?
    1. 110 kV
    2. 132 kV
    3. 400 kV
    4. 11 kV
2. What is the highest AC transmission voltage in India?
    1. 220 kV
    2. 400 kV
    3. 750 kV
    4. 132 kV
3. What is the highest DC transmission voltage in India?
    1. 1000 kV
    2. 300 kV
    3. 600 kV
    4. 132 kV
4. What is the need of an EHV transmission system?
    1. reduction in conductor material
    2. increased surge-impedance loading
    3. increase in transmission capacity
    4. all of these
5. What is the drawback of an EHV transmission system?
    1. corona problems
    2. over-voltage problem
    3. simple in operation
    4. both a and b

6. What is the function of a shunt reactor?
   1. absorb the reactive power
   2. generate the reactive power
   3. absorb or generate the reactive power
   4. absorb the active power
7. Which DC transmission network consists of only one conductor, with ground as a return path?
   1. monopolar link
   2. bipolar link
   3. homopolar link
   4. none of these
8. The cost of a transmission line includes the
   1. investment
   2. operational cost
   3. both a and b
   4. none of these
9. The investment includes
   1. right-of-way
   2. transmission towers
   3. conductors, isolators
   4. all of these
10. The corona effects on DC system is __________ significant than on AC
    1. less
    2. more
    3. no
    4. none of these
11. Disadvantages of a DC transmission system:
    1. inability to use transformers for DC
    2. high cost of conversion equipment
    3. complexity
    4. all of these
12. Application of DC transmission system is more reliable for
    1. long distance bulk power transmission
    2. underground or underwater cable
    3. asynchronous interconnection AC systems
    4. all of these
13. How many types of DC links are presented in an HVDC system?
    1. 1
    2. 2
    3. 3
    4. 4
14. How many conductors are available in a monopolar link?
    1. 1
    2. 2
    3. 3
    4. 4
15. What is the function of a smoothing reactor?
    1. to smooth AC voltage
    2. to smooth DC current
    3. improving power

4. all of these

16. For long-distance bulk power transmission, the voltage level is chosen to minimize the
    1. power
    2. cost
    3. current
    4. all of these
17. Ripple content will be lessened by a _________ converter
    1. six-pulse
    2. three-pulse
    3. two-pulse
    4. twelve-pulse
18. What is the advantage of three-phase converters over single-phase converters?
    1. less output
    2. more output
    3. high ripple
    4. all of these
19. Skin effect is absent for_________ transmission
    1. AC
    2. DC
    3. underground cable
    4. all of these
20. Rectification means _________ conversion
    1. AC to DC
    2. DC to AC
    3. AC to variable AC
    4. DC to variable DC

Answers:

1. c	2. c	3. c	4. d	5. d
6. a	7. a	8. c	9. d	10. a
11. d	12. d	13. c	14. a	15. b
16. b	17. d	18. b	19. b	20. a

REVIEW QUESTIONS

1. What is the need of EHV transmission lines?
2. What are the advantages and disadvantages of EHV transmission lines?
3. Explain the methods of increasing transmission capability of EHV lines.
4. Discuss the economic advantages of an HVDC transmission system.
5. Explain the technical advantages of an HVDC transmission system.
6. What are the advantages and disadvantages of an HVDC

transmission over EHV transmission systems?

7. What are the different types of HVDC transmission systems? Describe them.
8. What are the merits and demerits of the different types of HVDC transmission systems?
9. Explain the three-phase bridge converter and derive the expression for output voltage.
10. What are the major components of an HVDC transmission system? Explain each component.
11. Explain the uses of the different types of harmonic filters.

16

Flexible AC Transmission Systems

CHAPTER OBJECTIVES

After reading this chapter, you should be able to:

- Understand the purpose of FACTS (Flexible AC Transmission Systems) devices
- Classify various FACTS devices
- Describe various FACTS circuits
- Understand voltage stability

16.1 INTRODUCTION

The development and growth in the power industry requires the power-transmission systems to be highly stable with maximum thermal limits and the quality of power is given utmost importance.

For a profitable operation of generation, transmission and distribution systems, the incorporation of advanced technology and increased efficiency in utilization and control of the existing power-transmission system infrastructure are mandatory.

Power-electronic-based technologies, called FACTS technology, allows us to achieve quality and reliability of the transmission systems with minimum investment, not only by avoiding new transmission lines but also by improving operational features.

16.2 FACTS

FACTS is an acronym for flexible AC transmission system. It is an evolving technology-based solution envisioned to help the utility industry to deal with changes in the power-delivery business.

Terms and Definitions

The definitions presented here are divided into basic definitions and definition of controllers that serve specific function(s). The given categorization is the result of extensive discussions.

Flexibility of electric power transmission: The ability to accommodate changes in the electrical power-transmission systems or operating conditions while maintaining sufficient steady state and transient limits.

Flexible AC transmission system (FACTS): AC transmission systems incorporating power-electronic-based and other static controllers to enhance controllability and increase power-transfer capability.

FACTS controller: A power-electronic-based system and other static equipment that provide control of one or more AC transmission system parameters.

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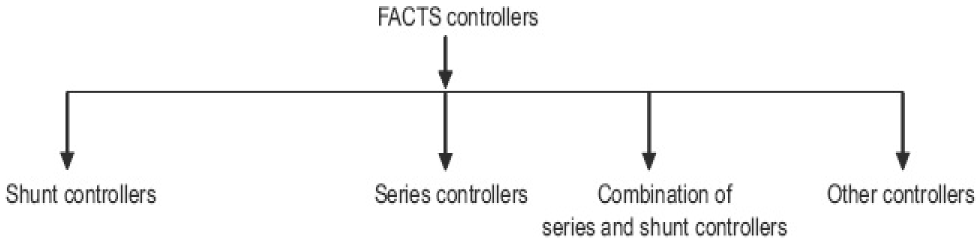
16.3 FACTS CONTROLLERS

FACTS controllers are classified as:



Some of the definitions presented in the following are organized according to their respective connection to the controlled AC transmission system.

16.3.1 Shunt–Connected Controllers

A shunt controller may be a variable impedance or a variable source frequency or it may be a combination of both. The function of the shunt controller is to inject current into the transmission line at the point of installation. The current obtained by connecting a variable admittance to the line voltage is the same as that of the current injected into the overhead line. When the injected current is inphase quadrature with the overhead line voltage, this controller controls only supplies or consumes the variable reactive power. However, if injected current is not inphase quadrature with the line voltage, it controls the active power also.

Types of Shunt Controllers

Static synchronous compensator (SSC or STATCOM):A static synchronous generator operated as a shunt-connected static VAr compensator whose capacitive or inductive output current can be controlled independently of the AC system voltage.

Static VAr compensator (SVC): It is a static VAr generator or absorber whose output is adjusted to exchange capacitive or inductive current so as to control or maintain specific parameters of the power system.

Thyristor-switched capacitor (TSC): A shunt connected thyristor-switched capacitor whose effective reactance is varied in a stepwise manner by a full or a zero operation conduction of the thyristor valve.

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16.3.2 Series-Connected Controllers

Series controllers may be a variable impedance or a variable source frequency. The function of the series controller is to inject voltage in series with the transmission line. Even though the current that flows through the transmission line gets multiplied with the variable impedance, the voltage obtained represents an injected series voltage in the line. When the injected voltage is inphase quadrature with the overhead line current, this controller controls only the variable reactive power or if injected voltage is not inphase quadrature with the line current, it controls active power also.

Types of Series Controllers

Static synchronous series compensator (SSSC): A static synchronous generator operated without an external electric-energy source as a series compensator whose output voltage is in quadrature with, and controllable independently of, the line current, for the purpose of increasing or decreasing the overall reactive voltage drop across the line, and thereby, controlling the transmitted electrical power. The SSSC includes energy-storage or energy-absorbing devices during transient periods to enhance the dynamic behaviour of the power system by additional temporary active-power compensation. The overall resistive voltage drop across the line may increase or decrease momentarily.

Thyristor-controlled series capacitor (TCSC): TCSC consists of a series capacitor bank shunted by thyristor-controlled reactor in order to provide a smoothly-variable series capacitive reactance.

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16.3.3 Combined Shunt and Series-Connected Controllers

By combining individual shunt and series-connected controllers in a co-ordinated manner, a new type of controller can be obtained. The main function of this controller is to inject the current into the system (principle of shunt controller) and to inject voltage in series with the line (principle of series controller). Exchange of active power between the series and parallel controllers through power link can be achieved by using this type of controllers. The most important kind of shunt and series controllers is unified power flow controller (UPFC).

Unified Power Flow Controller (UPFC)

A combination of a STATCOM and an SSSC coupled via a common DC link, to allow bi-directional flow of active power between the series-output terminals of the SSSC and the shunt output terminals of the STATCOM, is controlled to provide concurrent active and reactive series line compensation without an external electric-energy source. The UPFC can control transmission-line voltage, impedance or impedance angle (i.e., active and reactive power flows) simultaneously or independently.

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16.4 CONTROL OF POWER SYSTEMS

In theory, a transmission line can carry power up to its thermal loading limits. But in practice, to reach the thermal limit, the system meets the following constraints:

- Transmission stability limits
- Voltage limits
- Loop flows

The transmission stability limits refer to the limits of transmittable power with which a transmission system can ride through major faults in the system with its power-transmission capability intact. With voltage limits of the system already defined, voltage can be kept within permitted deviations from nominal, usual limit is ±5%. The voltage is governed by a quantity named reactive power. Longer the line and/or the heavier the flow of active power, the stronger will be the flow of reactive power, as a consequence of which the voltage will drop, until at some critical level, the voltage collapses altogether. Loop flows can be a problem as they are governed by the laws of nature which may not be coincident with the interests of humans. This means that power which is to be sent from point 1 to point 2 in a grid will not necessarily take the shortest, direct route, but will go uncontrolled and fan out to take unwanted paths available in the grid, thereby, generating additional losses and possibly also overloading sections of neighbouring power systems.

FACTS is designed to remove the above constraints and to meet planners, investors and operators' goals without their having to undertake major system additions. It enables different ways of attaining an increase of power transmission capacity at optimum conditions, i.e., at maximum availability, minimum transmission losses and minimum environmental impact at minimum investment cost and time expenditure.

Power-quality improvement and protecting equipment in transmission and distribution is another major reason for the implementation of the FACTS technology. This addition may cause more benefits in terms of employment and GDP growth but if the supplying grid is weak or inefficient, the addition of the FACTS technology may be a nuisance due to the pollution of the grid. This pollution may cause impediment to industry and sometimes leads to a complaint.

FACTS will offer remedy in such cases, by enabling confinement or neutralizing of electrical disturbances such as voltage sags and fluctuations, harmonic distortion, and phase unbalance in three-phase systems. As a useful value-add, improved economy of the process or processes in system will also be usually achieved.

16.4.1 Facts Devices

FACTS means application of power-electronic-devices to transmission.

Depending on the nature of this equipment, the FACTS technology is used for the following functions (and combination):

1. Rapid dynamic response.
2. Frequent variations in output.
3. Smoothly adjustable output.

A few important devices are SVC, STATCOM and TCSC while UPFC, IPC, TCPST and DVR (dynamic voltage restorers) are some of the others.

Table 16.1 Examples of conventional equipment for enhancing power-system control

Equipment	Controlling parameter
Series capacitor	Controls impedance.
Switched shunt capacitor and reactor, Load tap changing transformer and synchronous phase modifier	Controls voltage.
Phase-shifting transformer	Controls angle.
Special stability controls	Typically focuses on voltage control but can often include direct control of power.

Table 16.2 Example of FACTS controllers for enhancing power-system control

FACTS controller	Control parameters
Static VAr compensator (SVC)	Voltage control, VAr compensation, damping oscillations, transient and dynamic stability, voltage stability.
Static synchronous compensator (STATCOM without storage)	Voltage control, VAr compensation, damping oscillations, voltage stability.
Static synchronous compensator (STATCOM with storage)	Voltage control, VAr compensation, damping oscillations, transient and dynamic stability, voltage stability, AGC.
Thyristor-controlled braking resistor (TCBR), Static synchronous series compensator (SSSC without storage)	Damping oscillations, transient and dynamic stability. Current control, damping oscillations, transient and dynamic stability, voltage stability, fault current limiting.

Static synchronous series compensator (SSSC with storage)	Current control, damping oscillations, transient and dynamic stability, voltage stability.
Thyristor-controlled series capacitor (TCSC)	Current control, damping oscillations, transient and dynamic stability, voltage stability, fault current limiting.
Thyristor-controlled series reactor (TCSR)	Current control, damping oscillations, transient and dynamic stability, voltage stability, fault current limiting.
Thyristor-controlled voltage regulator (TCVR)	Reactive power control, voltage control, damping oscillations, transient and dynamic stability, voltage stability.
Thyristor-controlled phase shifting transformer (TCPST or TCPR)	Active power control, damping oscillations, transient and dynamic stability, voltage stability.
Unified power flow (UPFC)	Active and reactive power control, voltage control, VAr compensation, damping oscillations, transient and dynamic stability, voltage stability, fault current limiting.
Super-conducting magnetic energy storage (SMES)	Controls voltage and power.

16.4.2 Benefits Of Control Of Power Systems

When FACTS controllers are used, the following improvements in the power system may be obtained:

- Stability.
- Loading capability.
- Security and reliability.
- Added flexibility in setting new generation.
- No need for additional transmission lines.

These advantages are important for the design and operation of power system. But cost-benefit analysis is also crucial for constraining implementation of FACTS solutions to conventional ones.

These benefits depend on following criteria:[1]

- Voltage-stability; e.g., P–V voltage or power criteria with minimum limits and Q–V reactive power criteria with minimum limits.
- Dynamic-voltage; e.g., Avoiding voltage collapse and minimum transient voltage dip/sag criteria.
- Transient stability.
- Power-system oscillation damping; e.g., Minimum damping ratio.

All the above benefits can be measured from the following:

- Power transfer through transmission line.
- Power plant output.
- Regional load level.

16.4.3 FACTS Technology: Opportunities

The following opportunities create the FACTS technology:

- The desired amount of power flows through the prescribed routes by the control of power.
- Secure loading of transmission lines near their steady-state, short-time and dynamic limits.
- Enhancing and securing transmission interconnections for emergency power with neighbouring utilities reduces the generation reserve margins.
- Contain cascading outages by limiting the impact of multiple faults.
- Undertake and effectively utilize upgrading of transmission lines by increasing voltage and/or current capacity.

These are the main opportunities for enhancing transmission productivity and under the new circumstances, FACTS could represent the vehicle to meet the transmission wheeling requirements for a competitive generation industry. In this respect, FACTS technology represents a revolution in power-transmission systems.

16.5 BASIC RELATIONSHIP FOR POWER-FLOW CONTROL

The basic concept of control of power transmission in real time assumes rapid change in those parameters of the power system which determine the power flow. Considering the possibilities of power-flow control, power relationships for the simple two-machine model is depicted in Figs. 16.1(a) and (b).

Figure 16.1(b) shows the sending and receiving-end generators with voltage phasors V_s and V_r, inductive transmission-line impedance (X_L) in two sections $\frac{X_L}{2}$ and generalized power-flow controller operated (for convenience) at the middle of the line. The power-flow controller consists of two controllable elements, i.e., a voltage source (V_{xy}) and a current source (I_x) are connected in series and shunt, respectively with the line at the midpoint. Both the magnitude and the angle of the voltage V_{xy} are freely variable, whereas, only the magnitude of current I_x is variable; its phase angle is fixed at 90° with respect to the reference phasor of mid-point voltage V_m. The basic power-flow relation is shown in Fig. 16.1 by using FACTS controller in a normal transmission system.

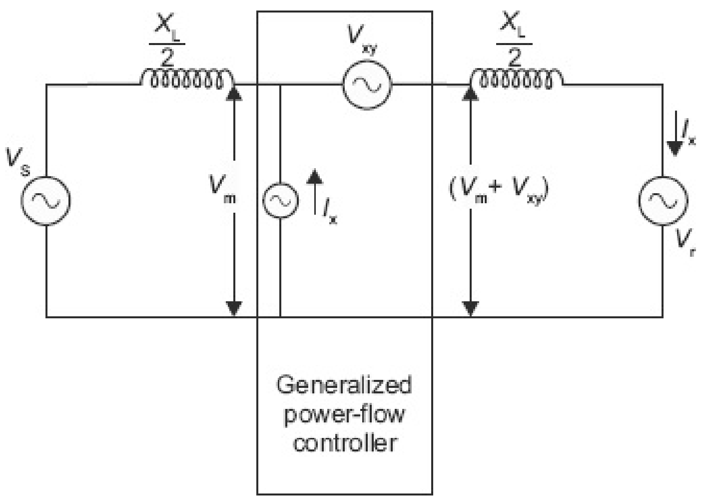

Fig. 16.1(a) Simple two-machine power system with a generalized power-flow controller

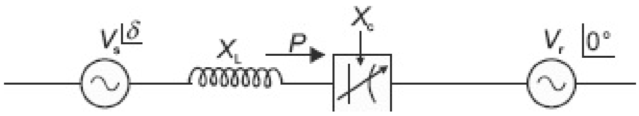

Fig. 16.1(b) Power-flow relation

The four classical cases of power transmission are:

1. Without line compensation
2. With series capacitive compensation
3. With shunt compensation
4. With phase angle control, can be obtained by appropriately specifying V_{xy} and I_x in the generalized power-flow controller

Without line compensation Consider that the power-flow controller is off, i.e., both V_{xy} and I_x are zero. Then, the power transmitted between the sending and receiving-end generators can be expressed by the well-known formula,

$$P_{(1)} = \frac{V^2}{X}\sin\delta \quad (\text{assume } V_r = V_s = V)$$

where δ is the angle between the sending and receiving-end voltage phasors. Power $P_{(1)}$ is plotted against angle δ as in Fig. 16.2.

With series capacitive compensation Assume a parallel current source, $I_x = 0$ and a series voltage source, $V_{xy} = -jnX_L I$, i.e., the voltage inserted in series with the line lags the line current by 90° with an amplitude proportional to the magnitude of the line current and that of the line impedance. In other words, the voltage source acts at the fundamental frequency precisely as a series-compensating capacitor. The degree of series compensating is defined by coefficient n (i.e., $0 \le n \le 1$). The relation of P against δ can be written as

$$P_{(2)} = \frac{V^2}{X_L(1-n)} \sin\delta .$$

With shunt compensation Consider the series voltage source V_{xy} = 0 and parallel current source

$I_x = -j\frac{4V}{X_L}\left(1-\cos\frac{\delta}{2}\right)$, i.e., the current source I_x draws just

enough capacitive current to make the magnitude of the mid-point voltage, V_m equal to V. In other words, the reactive current source acts like an ideal shunt compensator which segments the transmission line into

two independent parts, each with an impedance of $\frac{X_L}{2}$,

by generating the reactive power necessary to keep the mid-point voltage constant, independent of angle δ. For this case of ideal mid-point compensation, the relation of P against δ can be written as

$$P_{(3)} = \frac{2V^2}{X_L} \sin\frac{\delta}{2}.$$

With phase angle control Assume that I_x = 0 and V_{xy} = $\pm jV_m \tan\alpha$, i.e., a voltage (V_{xy}) with the amplitude $\pm jV_m \tan\alpha$, is added in quadrature to the mid-point voltage (V_m) to produce the desired α phase shift. The basic idea behind phase shifter is to keep the transmitted power at a desired level independent of angle δ in a predetermined operating range. Thus, for example, the power can be kept at its peak value after angle δ is $\pi/2$ by controlling the amplitude of the quadrature voltage V_{xy} so that the effective phase angle ($\delta - \propto$) between the sending and receiving-end voltages stays at $\pi/2$. In this way, the actual transmitted power may be increased significantly, even though the phase shifter does not increase the steady-state power transmission limit. Considering, ($\delta - \propto$) as the effective phase angle between the sending-end and receiving-end voltage, the transmitted power can be expressed as

$$P_{(4)} = \frac{V^2}{X_L}\sin(\delta - \alpha).$$

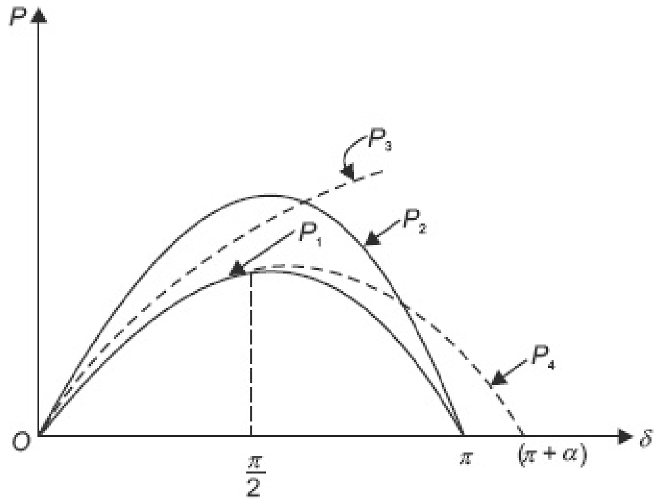

Fig. 16.2 The basic power-transmission characteristics for four different cases

From Fig. 16.2, it can be seen that the power flow in a transmission system without compensating is less as shown in $P_{(1)}$ curve. Power is increased by using series-capacitor compensation shown in $P_{(2)}$ curve. Power-angle curve with shunt compensator is shown in $P_{(3)}$ curve; in this case, power is increased and it seems that voltage is also increased. The concept of phase-angle control is shown in $P_{(4)}$ curve, by shifting the curve to ($\pi + \alpha$), higher power can be obtained.

16.5.1 Shunt Compensator

A shunt-connected static VAr compensator, composed of thyristor-switched capacitors (TSCs) and thyristor-controlled reactors (TCRs), is shown Fig. 16.3. With proper coordination of the capacitor switching and reactor control, the VAr output can be varied continuously between the capacitive and inductive rating of the equipment. The compensator is normally operated to regulate the voltage of the transmission system at a selected terminal, often with an appropriate modulation option to provide damping if power oscillation is detected.

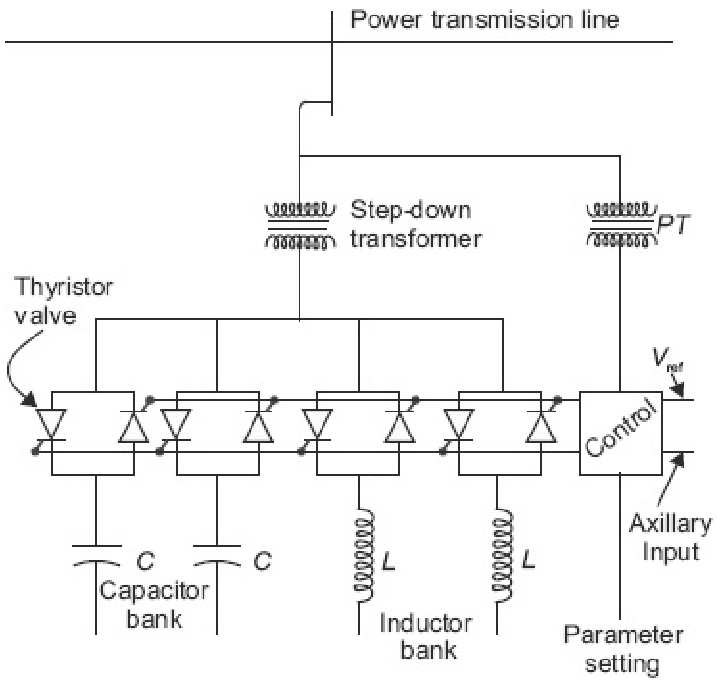

Fig. 16.3 Static VAr compensator employing thyristor-switched capacitors and thyristor controlled reactor

16.5.2 Thyristor-Controlled Reactor (TCR)

With the increase in the size and complexity of the power system, fast reactive power compensation has become necessary in order to maintain the stability of the system.

The thyristor-controlled shunt reactors have made it possible to reduce the response time to a few milliseconds. Thus, the reactive power compensator utilizing the thyristor-controlled shunt reactors have become popular. An elementary single-phase thyristor-controlled reactor is shown in Fig. 16.4.

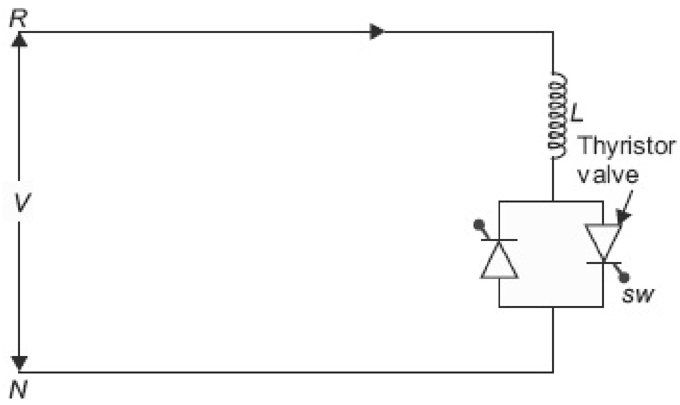

Fig. 16.4 Thyristor-controlled reactor

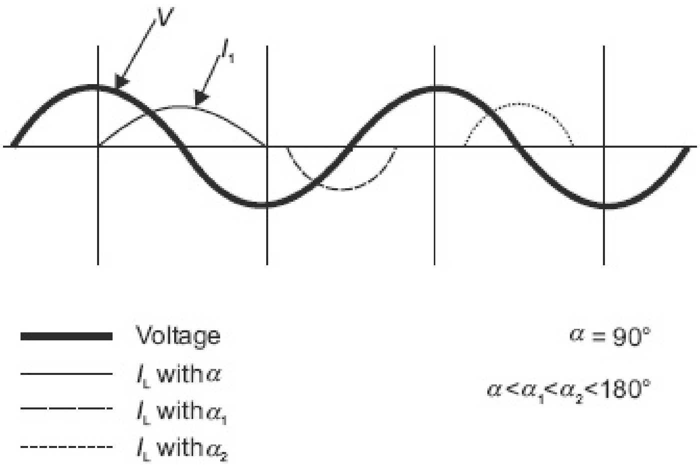

Fig. 16.5 Thyristor-controlled reactor waveform

It consists of a fixed reactor of inductance *L* and a bi-directional thyristor valve. The thyristor valve can be brought into conduction by the application of a gate pulse to thyristor and the valve will be automatically blocked immediately after the AC current crosses zero. The current in the reactor can be controlled from maximum to zero by the method of firing-angle control. Partial conduction is obtained with higher value of firing-angle delay. The effect of increasing the gating angle is to reduce the fundamental component of the current. This is equivalent to an increase in the inductance of the reactor, reducing its current. So far as the fundamental component of the current is concerned, the TCR is a controllable susceptance, and can therefore, be used as a static compensator.

The current in this circuit is essentially reactive, lagging the voltage by 90° and this is continuously controlled by the phase control of the thyristors. The conduction angle control results in a non-sinusoidal current wave form in the reactor. In other words, the thyristor-controlled reactor generates harmonics. For identical positive and negative current (half-cycle time), only odd harmonics are generated, as shown in Fig. 16.5. Using filters can reduce the magnitude of harmonics.

The characteristics of TCR are:

- It controls continuously.
- No transients are produced.
- Generates only harmonics.

16.5.3 Thyristor-Switched Capacitor (Tsc)

A shunt-connected thyristor-switched capacitor shows that effective reactance is varied in a stepwise manner by full- or zero-conduction operation of the thyristor valve.

Thyristor switched capacitor (TSC) is also a subset of SVC in which thyristor-based AC switches are used to switch in and out shunt capacitor units, in order to achieve the required step change in the reactive power supplied to the system. Unlike shunt reactors, shunt capacitors cannot be switched continuously with variable firing-angle control.

Depending on the total VAr requirement, a number of capacitors are used which can be switched into or out of the system individually. The control is done continuously by sensing the load VArs. A single-phase thyristor-switched capacitor is shown in Fig. 16.6.

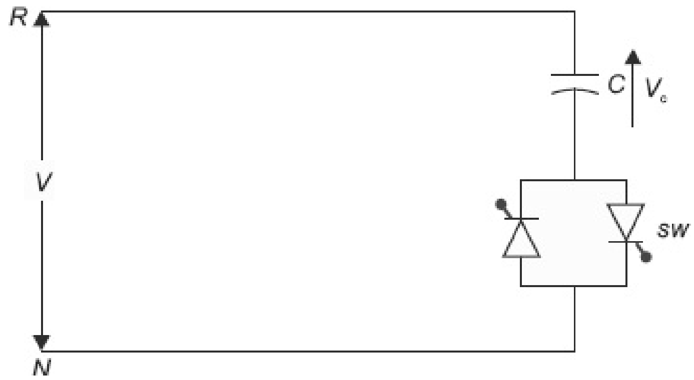

Fig. 16.6 Thyristor-switched capacitor

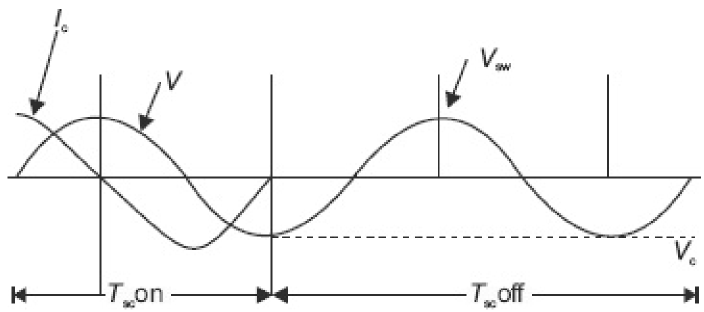

Fig. 16.7 Thyristor-switched capacitor wave forms

It consists of a capacitor, a bi-directional thyristor valve and a relatively small surge current in the thyristor valve under abnormal operating conditions (e.g., control malfunction causing capacitor switching at a "wrong time"); it may also be used to avoid resonance with system impendence at particular frequencies. The problem of achieving transient-free switching of the capacitors is overcome by keeping the capacitors charged to the positive or negative peak value of the fundamental frequency-network voltage at all times when they are in the standby state. The switching-on transient is then selected at the time when the same polarity exists in the capacitor voltage. This ensures that switching on takes place at the natural zero passage of the capacitor current. The switching, thus, takes place with practically no transients. This is called zero-current switching.

Switching off a capacitor is accomplished by suppression offering pulses to the anti-parallel thyristors so that it will block as soon as the current becomes zero. In principle, the capacitor will then remain charged to the positive or negative peak voltage and be prepared for a new transient-free switching-on as shown in Fig. 16.7.

The characteristics of TSC are:

- It is a steeped control.
- No transients are produced.
- No harmonics are generated.
- Losses are less.
- More flexible.

16.5.4 Series Compensator

In the thyristor-switched capacitor scheme, increasing the number of capacitor banks in series controls the degree of series compensation. To accomplish this, each capacitor bank is controlled by a thyristor- bypass switch or value. The operation of the thyristor switches is coordinated with voltage and current zero-crossing; the thyristor switch can be turned on to bypass the capacitor bank when the applied AC voltage crosses zero, and its turn-off has to be initiated prior to a current zero at which it can recover its voltage-blocking capability to activate the capacitor bank. Initially, the capacitor is charged to some voltage, while switching the SCRs, they may get damaged because of this initial voltage. In order to protect the SCRs from this kind of a damage, a resistor is connected in series with capacitor as shown in Fig. 16.8.

In the fixed capacitor, thyristor-controlled reactor scheme as shown in Figs. 16.9 and 16.10, the degree of series compensation in the capacitive operating region is increased (or decreased) by increasing (or decreasing) the current in the TCR. Minimum series compensation is reached when the TCR is off. The TCR may be designed for a substantially higher maximum admittance at full thyristor conduction than that of the fixed shunt-connected capacitor. In this case, the TCR time with an appropriate surge-current rating can be used essentially as a bypass switch to limit the voltage across the capacitor during faults and the system contingencies of similar effect.

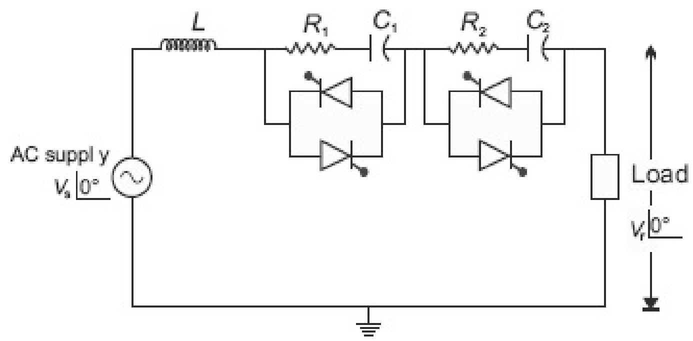

Fig. 16.8 Series compensator

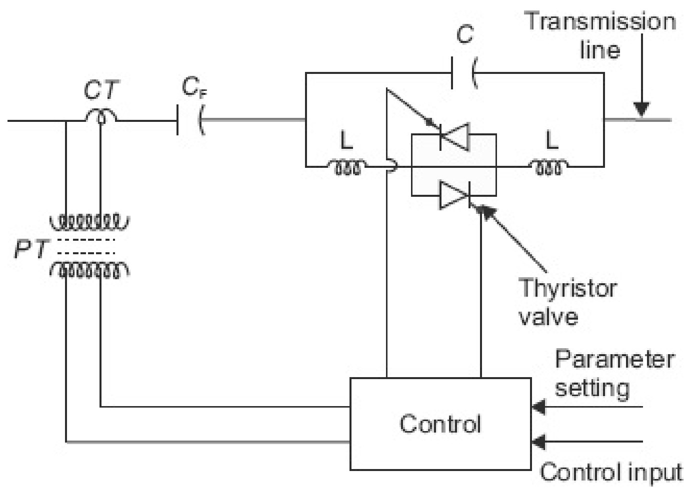

Fig. 16.9 Thyristor-controlled capacitor

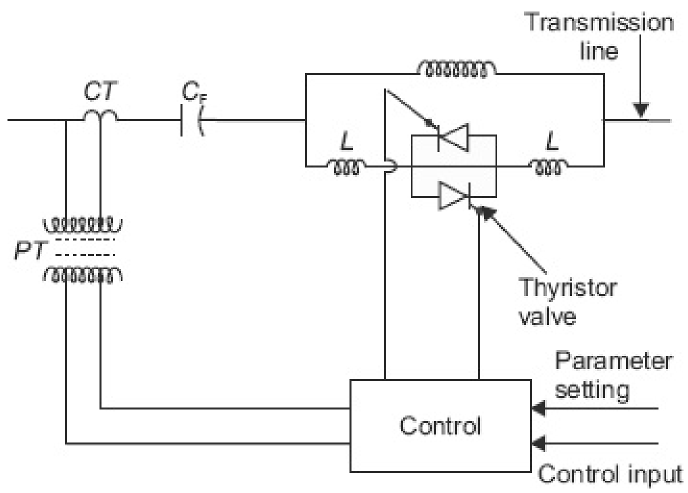

Fig. 16.10 Thyristor-controlled reactor

Controllable series compensation can be highly effective in damping power oscillation and preventing loop flows of power.

The expression for power transferred is given by

$$P = \frac{V_s V_r}{X} \sin \delta,$$

where,

V_s – Sending-end voltage

V_r – Receiving-end voltage

δ – Angle between V_s and V_r

$X = X_L - X_C$.

In inter-connected power systems, the actual transfer of power from one region to another might take unintended routes depending on impedances of transmission lines connecting the areas. Controlled series compensation is a useful means for optimizing power flow between regions for varying loading and network configurations. It becomes possible to control power flows in order to achieve a number of goals:

- Minimizing system losses.
- Reduction of loop flows.
- Elimination of line overloads.
- Optimizing load sharing between parallel circuits.
- Directing power flows along contractual paths.

16.5.5 Unified Power-Flow Controller (UPFC)

In the unified power-flow controller, an AC voltage generated by a thyristor-based inverter is injected in series with the phase voltage. In Fig. 16.11, converter 2 performs the main function of the UPFC by injecting, through a series transformer, an AC voltage with controllable magnitude and phase angle in series with the transmission line. The basic function of converter 1 is to supply or absorb the real power demanded by converter 2 at the common DC link. It can also generate or absorb controllable reactive power and provide independent shunt reactive compensation for the line. Converter 2 supplies or absorbs the required reactive power locally and exchanges the active power as a result of the series-injection voltage.

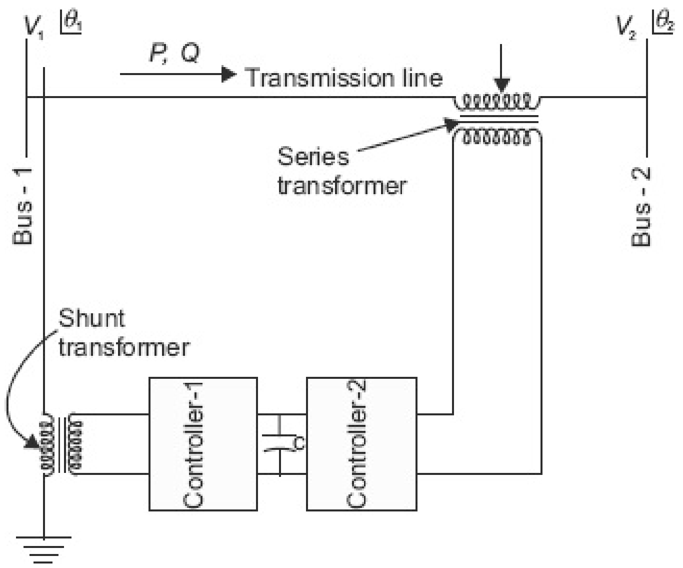

Fig. 16.11 Unified power-flow controller

Generally, the impedance control would cost less and be more effective than the phase-angle control, except where the phase angle is very small or very large or varies widely.

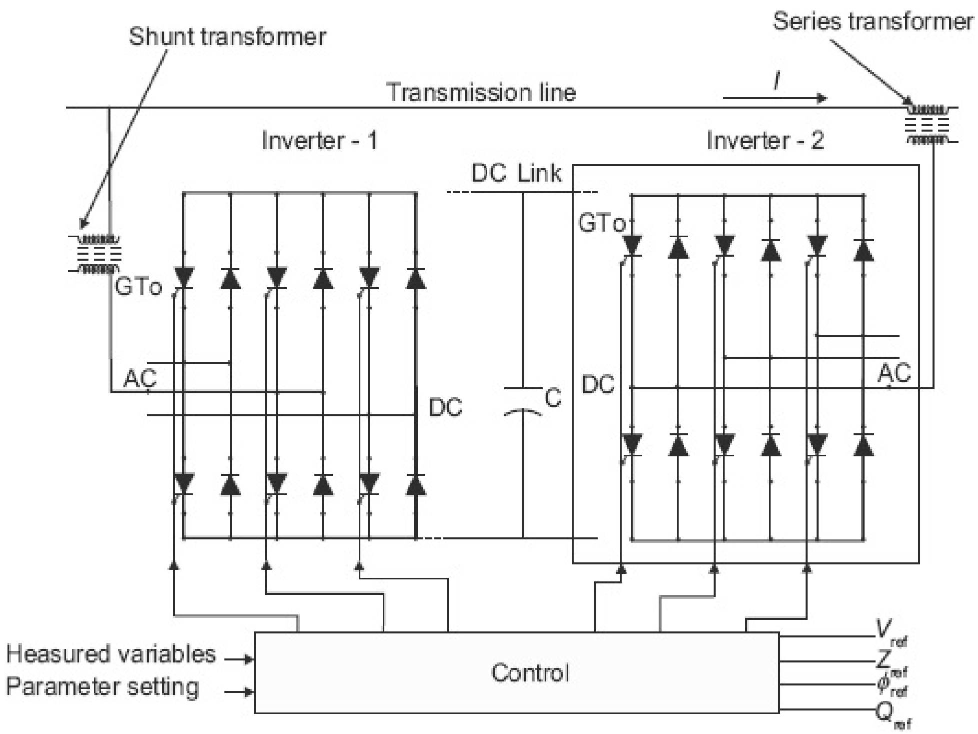

Fig. 16.12 Implementation of the UPFC using two voltage source inverters with a direct voltage link simultaneously.

In general, it has three control variables and can be operated in different modes. The shunt-connected converter regulates the voltage bus '*i*' in Fig. 16.12, and the series-connected converter regulates the active and reactive power or active power and the voltage at the series-connected node. In principle, a UPFC is able to perform the functions of the other FACTS devices, which have been described, namely voltage support, power-flow control and improved stability.

Test Yourself

1. Why is UPFC more useful than other FACTS devices?

16.6 "FACTS" FOR MINIMIZING GRID INVESTMENTS

The important incentive for considering FACTS in grid planning is that it is an economically and environmentally-attractive alternative for investments in more transmission lines. For example, it can be shown that the cost of installing series capacitors as means for improving the power-transmission capacity of existing lines amounts to only a fraction of the cost for installing one or several new lines. This is valid for all existing transmission voltages and for all transmission distances where series compensation comes into consideration.

By considering series compensation from the very beginning, power transmission between regions can be planned with a minimum of transmission circuits, thereby, minimizing costs as well as environmental impact from the start.

Test Yourself

1. How do FACTS devices minimize the grid investment?

16.7 VOLTAGE STABILITY

Voltage has always been considered as an integral part of the power-system response and is an important aspect of system stability and security. Their voltage instability and collapse cannot be separated from the general problem of system stability. There are several factors which contribute to voltage collapse -increased loading on transmission lines, reactive-power constraints, on-load tap changer (OLTC) dynamics and load characteristics.

In contrast to voltage stability, the problem of loss of synchronism due to uncontrolled generator rotor swings is termed as angle stability. Loss of synchronism may also be accompanied by voltage instability.

16.7.1 Voltage Stability – What is it?

Voltage instability does not refer to the problem of low voltage in a steady-state condition. As a matter of fact, it is possible that the voltage collapse may precipitate even if the initial operating voltages are at acceptable levels.

Voltage collapse may be fast or slow. Fast voltage collapse is due to induction-motor loads or HVDC converter stations and slow voltage collapse is due to on-load tap changer and generator-excitation limiters.

Voltage stability is sometimes also termed as load stability. The terms voltage instability and voltage collapse are often used interchangeably.

It is to be understood that the voltage stability is a subset of overall power-system stability and is a dynamic problem. Voltage instability generally results in monotonically (or aperiodically) – decreasing voltages. Sometimes voltage instability may manifest as undamped (or negatively-damped) voltage oscillations prior to voltage collapse.

Definition: A power system at a given operating state and subjected to a given disturbance is voltage stable if voltages near loads approach post-disturbance equilibrium values.

The concept of voltage stability is related to transient stability of a power system.

Voltage collapse: Following voltage instability, a power system undergoes voltage collapse if the post-disturbance equilibrium voltages near the load are below acceptable limits. The voltage collapse may be total or partial.

The absence of voltage stability leads to voltage instability and results in progressive decrease of voltages.

16.7.2 Derivation Of Voltage Stability Index

Consider a typical branch consisting of sending and receiving-end buses as shown in Fig. 16.13.

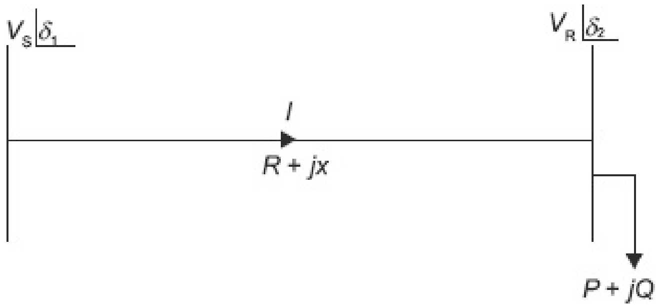

Fig. 16.13 Single-line model of a typical branch.

Current flowing through the branch, $I = \dfrac{V_s\angle\delta_1 - V_r\angle\delta_2}{R + jX}$

$$P - jQ = V_r^* I$$

$$\therefore P - iQ = \frac{V_s V_r \angle(\delta_1 - \delta_2) - V_r^2}{R + jX}$$

$$\Rightarrow (P - jQ)\,(R + jX) = V_s V_r \angle(\delta_1 - \delta_2) - V_r^2$$

$$\Rightarrow (RP + XQ) + j(XP - RQ) = V_s V_r \angle(\delta_1 - \delta_2) - V_r^2$$

$$V_r^2 + (RP + XQ) + j(XP - RQ) = V_s V_r \angle\delta_1 - \delta_2$$

The real term of above equation is

$$V_s\, V_r \cos(\delta_1 - \delta_2) = V_r^2 + (RP + XQ)$$

And imaginary part is

$$V_s\, V_r \sin(\delta_1 - \delta_2) = XP - RQ$$

Squaring and adding the above two terms, we get

$$V_r^4 + R^2P^2 + X^2Q^2 + 2R \times PQ + 2V_r^2 (RP + XQ) + X^2P^2 + R^2Q^2 - 2R \times PQ = V_s^2V_r^2$$
$$\Rightarrow V_r^4 + P^2(R^2 + X^2) + Q^2(R^2 + X^2) + 2V_r^2(RP + XQ) - V_s^2V_r^2 = 0$$
$$\Rightarrow V_r^4 + V_r^2(2RP + 2XQ - V_s^2) + (P^2 + Q^2)(R^2 + X^2) = 0$$

The above equation is a quadratic equation of V^2_r. The system is to be stable

if $V_r^2 \geq 0$.

It is possible when

$$b^2 - 4ac \geq 0$$
$$\text{i.e., } [2(RP + XQ) - V_s^2)]^2 - 4(P^2 + Q^2)(R^2 + X^2) \geq 0$$
$$\text{or } 4R^2P^2 + 4X^2Q^2 + 4R \times PQ + V_s^4 - 4V_s^2(RP + XQ) - 4R^2P^2 - 4X^2P^2 - 4R^2Q^2 - 4X^2Q^2 \geq 0$$

Simplifying the above equation, we get

$$V_s^4 - 4V_s^2(RP + XQ) - 4(PX - RQ)^2 \geq 0$$
$$\text{or } 4(PX - RQ)^2 + 4V_s^2(RP + XQ)) \leq V_s^4$$

Dividing both sides by V^4_s

$$4\left(\frac{PX - RQ}{V_s^2}\right)^2 + 4\left(\frac{RP + XQ}{V_s^2}\right) \leq 1$$
$$\therefore L_{(\text{p.u.})} = 4\left\{\left(\frac{PX - RQ}{V_s^2}\right)^2 + \left(\frac{RP + XQ}{V_s^2}\right)\right\},$$

where L = stability index.

For stable systems, $L \leq 1$.

Test Yourself

1. What happens if the voltage stability index is greater than one?

CHAPTER AT A GLANCE

1. **Flexible AC transmission system:** Alternating current transmission systems incorporating power-electronic-based and other static controllers enhance controllability and increase power-transfer capability.
2. **FACTS controller:** A power-electronic-based system and other static equipment that provides control of one or more AC transmission system parameters.
3. **Static synchronous compensator (SSC or STATCOM):** A static synchronous generator operated as a shunt-connected static VAr compensator.
4. **Static VAr compensator (SVC):** A static VAr generator or absorber whose output is adjusted to the exchange capacitive or inductive current so as to control or maintain specific parameters of the power system.
5. **Thyristor-switched capacitor (TSC):** A shunt-connected thyristor-switched capacitor whose effective reactance is varied in a stepwise manner by full or zero operation conduction of the thyristor valve.
6. **Thyristor-switched reactor (TSR):** A shunt-connected thyristor-switched inductor whose effective reactance is varied in a stepwise manner by full or zero-conduction operation of the thyristor valve.
7. **Static synchronous-series compensator (SSSC):** A static, synchronous generator operated without an external electric-energy source as a series compensator.
8. **Thyristor-controlled series capacitor (TCSC):** A series capacitor bank shunted by thyristor-controlled reactor in order to provide a smoothly variable series capacitive reactance.
9. **Unified power-flow controller (UPFC):** A combination of a STATCOM and an SSSC which are coupled through a common DC link, to allow the bi-directional flow of active power.

SHORT ANSWER QUESTIONS

1. What is meant by flexible AC transmission system (FACTS)?
2. Why should electric power transmission be flexible?
3. What is meant by static synchronous compensator (SSC or STATCOM)?

4. Define the static VAr compensator (SVC).
5. What do you mean by static synchronous series compensator (SSSC)?
6. What is the purpose of a thyristor-controlled series capacitor (TCSC)?
7. What do you mean by unified power-flow controller (UPFC)?
8. Define voltage stability.
9. Define voltage collapse.
10. What is the function of a shunt controller?
11. What is the function of a series controller?

MULTIPLE CHOICE QUESTIONS

1. What are the advantages of FACTS technology?
    1. quality improvement
    2. reliability improvement
    3. quality and reliability improvement
    4. voltage improvement
2. Power-electronics-based technology is known as
    1. FACTS technology
    2. electronics technology
    3. VLSI technology
    4. nano technology
3. What are the types of FACTS controllers?
    1. shunt controllers
    2. series controllers
    3. combination of series & shunt controllers
    4. all of these
4. What are the benefits of controlling power systems?
    1. stability improvement
    2. loading capability improvement
    3. security and reliability improvement
    4. all of these
5. What is the function of series controller?
    1. inject voltage
    2. inject current
    3. inject real power
    4. inject reactive power
6. What is the function of shunt controller?
    1. inject voltage
    2. inject current
    3. inject real power
    4. inject reactive power
7. For stable system, the stability index will be
    1. greater than unity
    2. less than unity
    3. unity
    4. none of these
8. Stability index depends on

    1. line parameters
    2. load parameters
    3. line and load parameters
    4. power factor
9. Which of the following is not a FACTS device?
    1. SVC
    2. STATCOM
    3. SCR
    4. SVR
10. UPFC is a combination of
    1. STATCOM and an SSSC
    2. STATACOM and an SVC
    3. SSSC and an SVC
    4. SVC and an SCR

Answers:

1. c	2. a	3. d	4. d	5. a
6. b	7. b	8. c	9. c	10. a

REVIEW QUESTIONS

1. Define shunt-connected controllers.
2. Define series-connected controllers.
3. What are the uses of series controller and shunt controller?
4. Explain the benefits of controlling power systems.
5. What are the opportunities of using the FACTS technology?
6. Describe the basic relationship of power-flow control with diagrams.
7. Explain thyristor-controlled reactor.
8. Explain series compensator.
9. Describe the unified power-flow controller with diagram.
10. State and explain the concept of voltage stability.

REFERENCES

1. Paserba, John J. (2003). “How FACTS Controllers Benefit AC Transmission System”, *IEEE Conference*, 949–956.

Appendix 1

DATASHEETS

Table A.1 Electrical parameters of three-phase overhead lines

Rated voltage (kV)	400	220			132			66			33		11				
Code	Moose	Goat	Zebra	Moose	Wolf	Panther	Goat	Dog	Wolf	Panther	Dog	Wolf	Squirrel	Weasle	Ferret	Rabbit	Racoon
Area of crosssection (cm^2)	5.97	1.85	2.6	3.25	0.95	1.3	1.85	0.65	0.95	1.3	0.65	0.95	0.13	0.2	0.25	0.3	0.48
Diameter (cm)	2.74	2.597	2.862	3.177	1.813	2.1	2.597	1.416	1.813	2.1	1.416	1.813	0.0633	0.777	0.90	1.005	1.227
Resistance (Ω/km)	0.07	0.089897	0.0680058	055174	0.18441	0.13751	0.089897	0.27452	0.18441	0.13751	0.27452	0.184415	1.3741	0.91167	0.679558	0.544946	0.365631
Inductance (mH/km)	0.952	1.2405	1.2221	1.2002	1.297	1.268	1.225	1.21	1.16	1.131	1.071	1.0218	1.132	1.091	1.062	1.040	1
Capacitance (μF/km)	0.013	0.00887	0.009	0.00916	0.00848	0.00868	0.00898	0.00909	0.00948	0.00973	0.01027	0.01077	–	–	–	–	–
Surge impedance (Ω)	295	374	368	362	391	382	369	–	350	–	323	–	–	–	–	–	–
Surge impedance loading (MW)	490	129.41	131.5	133.702	44.563	45.613	47.22	–	12.446	–	3.372	–	–	–	–	–	–

Source: A.P. Transco Engineers' association dairy in 2007.

Table A.2 Transmission lines clearance

				Insulators		Clearance	
Voltage (kV)	Span (mts)	Conductor	Earth wire	Suspension (kN)	Tension (kN)	Ground (mts)	Vertical (mts)
132	320	30 + 7/300 Panther ACSR	7/3.15 HTGS	70	120	6.1	3.9
220	350	54 + 7/3.53 Moose ACSR	7/3.15 HTGS	70	120	7.0	4.9
	370	54 + 7/3.18 Zebra ACSR					
400	400	54 + 7/3.53 Moose ACSR	7/3.66	120	160	8.84	8.0

Voltage (kV)	Horizontal (mts)	P&T Crossing	Rly. Crossing	Building		River	Right of way	Max. Sag
				Vertical	Horizontal			
132	6.8	2.75	14.6	4.58	2.75	4.30	27	6.500
220	8.4	3.05	15.4	5.49	3.66	5.10	35	9.675
400	11.0	4.48	17.9	7.32	5.49	6.40	52	12.865

Courtesy: A.P.S.E.B employs association dairy in 2007.

Table A.3 Standard Bay/Bus width in metres

Description	400 kV	220 kV	132 kV	33 kV	11 kV
Bay width	27.0	17.0	12.2	5.0	3.5
Bus width	27.0	17.0	11.0	4.7	–
Phase to phase	7.0	4.5	3.0	0.73	–
Phase to earth	6.5	4.0	3.1	0.6	–

Courtesy: A.P.S.E.B employs association dairy in 2007.

Table A.4 Capacitors to be installed for motors

Rating of the motor (HP)	kVAr rating of capacitor various r.p.m of the motors			
	750 (rpm)	1000 (rpm)	1500 (rpm)	3000 (rpm)
3	1	1	1	1
5	2	2	2	2
7.5	3	3	3	3
10	4	4	4	3
15	6	5	5	4
20	8	7	6	5
25	9	8	7	6
30	10	9	8	7
40	13	11	10	9
50	15	15	12	10
60	20	20	16	14
75	24	23	19	16

Courtesy: A.P.S.E.B employs association dairy in 2007.

Table A.5 Power factors of some common types of loads

Load	Power factor
Incandescent lamps	1.00
Arc lamps used in cinemas	0.3 to 0.7
Fluorescent lamps	0.6 to 0.8
Resistance heaters	1.0
Induction heaters	0.85
Arc furnaces	0.85
Induction furnaces	0.60
Arc welders	0.3 to 0.4
Resistance welders	0.65
Induction motors	0.8

Courtesy: A.P.S.E.B employs association dairy in 2007.

Table A.6 Conversion table (Amps to MVA)

Amps	11 kV	33 kV	66 kV	132 kV	220 kV	400 kV
1	0.019	0.057	0.114	0.229	0.381	0.69
2	0.038	0.114	0.229	0.457	0.762	1.39
3	0.057	0.171	0.343	0.686	1.143	2.08
4	0.076	0.228	0.457	0.914	1.524	2.77
5	0.095	0.286	0.572	1.143	1.905	3.46
6	0.114	0.343	0.686	1.372	2.286	4.16
7	0.133	0.400	0.800	1.600	2.667	4.85
8	0.152	0.457	0.915	1.829	3.048	5.54
9	0.171	0.514	1.028	2.058	3.429	6.23
10	0.191	0.571	1.143	2.286	3.810	6.93
11	0.210	0.629	1.257	2.515	4.192	7.62
12	0.229	0.686	1.372	2.744	4.473	8.31
13	0.248	0.743	1.486	2.972	4.954	9.01
14	0.267	0.800	1.600	3.201	5.334	9.70
15	0.286	0.857	1.715	3.429	5.416	10.39
16	0.305	0.915	1.829	3.658	6.097	11.08
17	0.324	0.972	1.943	3.887	6.478	11.78
18	0.343	1.029	2.058	4.115	6.859	12.47
19	0.362	1.086	2.172	4.344	7.240	13.16
20	0.381	1.143	2.286	4.573	7.621	13.86
25	0.476	1.429	2.858	5.716	9.526	17.32
30	0.572	1.715	3.429	6.859	11.432	20.78
35	0.667	2.000	4.001	8.002	13.337	24.25
40	0.726	2.286	4.73	9.145	15.242	27.71
45	0.857	2.572	5.144	10.288	17.147	31.18
50	0.953	2.858	5.716	11.432	19.053	36.64
60	1.143	3.429	6.8559	13.718	22.863	41.57
70	1.334	4.001	8.002	16.004	26.674	48.50
80	1.524	4.573	9.145	18.290	30.484	55.42
90	1.715	5.144	10.288	20.557	34.295	62.35

Courtesy: A.P.S.E.B employs association dairy in 2007.

Table A.7 Conversion table (MVA to Amps)

MVA	11 kV	33 kV	66 kV	132 kV	220 kV	400 kV
1	52.5	17	8.7	4.3	2.2	1.4
2	105.5	35	17.5	8.7	4.4	2.9
3	157.5	52	26.2	13.0	6.6	4.8
4	209.9	70	35.0	17.5	8.7	5.7
5	262.4	87	43.7	21.9	10.9	7.2
6	314.9	105	52.5	26.2	13.1	8.7
7	367.4	122	61.2	30.6	15.3	10.8
8	419.9	140	70.0	35.0	17.5	11.5
9	472.4	157	78.7	39.4	19.7	13.0
10	524.9	175	87.0	43.7	21.9	14.4
11	577.4	192	96.0	48.1	24.0	15.9
12	629.8	210	105.0	52.5	26.2	17.3
13	682.3	227	114.0	56.9	28.4	18.8
14	734.8	245	122.0	61.2	30.6	20.2
15	787.3	262	131.0	65.6	32.8	21.6
16	839.8	280	140.0	70.0	35.0	23.8
17	892.3	297	149.0	74.3	37.1	24.5
18	944.8	315	157.0	78.7	39.4	26.0
19	997.0	332	166.0	83.1	41.5	27.47
20	1050.0	350	175.0	87.5	43.7	28.9
25	1312.0	437	219.0	109.0	54.7	36.8
30	1575.0	525	262.0	131.0	65.6	43.3
35	1837.0	612	306.0	153.0	76.5	50.5
40	2099.0	700	350.0	175.0	87.5	57.7
45	2362.0	787	394.0	197.0	98.4	65.0
50	2624.0	875	437.0	218.0	109.3	72.2
60	3149.0	1050	525.0	262.0	131.2	86.6
70	3674.0	1225	612.0	306.0	153.0	101.0
80	4199.0	1400	700.0	350.0	175.0	115.0
90	4724.0	1575	787.0	394.0	197.0	130.0

Courtesy: A.P.S.E.B employs association dairy in 2007.

Appendix 2

ANSWERS TO PROBLEMS

CHAPTER 2

1. 1.24 mH/km
2. 1.32 mH/km
3. 1.19 mH/km, 0.3755 Ω/km
4. 1.159 mH/km
5. L_a = 1.3012 – *j*0.055 mH/km, L_b = 1.2693 + *j*0 mH/km,

   L_c = 1.3012 + *j*0.0552 mH/km
6. 0.5586 mH/km
7. 4.527 μF
8. 10.4021 pp/km
9. 2.9775 μF
10. 0.0184 μF/km
11. 0.0173 μF/km
12. 8.7917 pF/km

CHAPTER 3

1.
    1. 31.92 kV,
    2. 74.09 A,
    3. 97.7%
2.
    1. 2.1%,
    2. 0.895 lagging,
    3. 98.6%
3.
    1. 23.85 kV,
    2. 7.56%,
    3. 95%
4. 4.99%, 94.7%
5. 34.485 kV, 1760.03
6.
    1. *A* = D = (0.99475 + *j*0.00225), *B* = (15 + *j*35) Ω, *C* = (–0.0000003 + *j*0.000299) S
    2. 122.796 kV,

3. 12.22%,
4. 93.93%

CHAPTER 4

1. At full load: (i) 275.65 kV, (ii) 498.72 A, (iii) 30%

   At no load: (i) 212 kV, (ii)438.78 A,(iii)–3.4%
2. 186.849 kV, 177.82 A, 0.04013 lead
3. 269.636 kV, 29.44%
4. $A = D = (0.9754 + j0.0014605)$, $B = (3.688542 + j62.31818)\ \Omega$,
   $C = (-0.3836 \times 10^{-6} + j0.0007789)$ S
5.
   1. $A = D = 0.4715\angle 46.57°$, $B = 526.57\angle 68.31°\ \Omega$, $C = 1.964 \times 10^{-3}\angle 99.07°$ S
   2. $A = D = 0.5204\angle 43.4°$, $B = 518.01\angle 67.09°\ \Omega$, $C = 1.934 \times 10^{-3}\angle 98.11°$ S
6. 280.592 kV, 243.35 A

CHAPTER 5

1.
   1. 10^4 A,
   2. 14000 A,
   3. 85.714 Ω,
   4. –800 kV, –0.4, 0.6
2.
   1. 90.91 kV, 2.2727 A,
   2. –409.09 kV, 1.022 A
3.
   1. 127.66 kV, 255.32 A, 425.533 A;
   2. –72.34 kV, 204.233 A
4. 15.39 kV, 0.535 kA
5. 32.834 kV
6. 485.22 kV

CHAPTER 6

1. 150.39 cm
2. 70.26 kV/ph
3. 93.78 kW/ph, 142.91 kV/ph, 134.5 kV/ph
4. 97.24 kV/ph, 100.4 kV/ph, 121.17 kV/ph
5. 6.92 kW/km
6.
   1. 94.27 kV(rms),
   2. 124.29 kV,
   3. 141.08 kV,
   4. 4741.4 kW,

    5. 42.51 kW
7. 29.9 kW/ph

CHAPTER 7

1. 3.766 m
2. 6.5 m
3.
    1. 3 m,
    2. 5 m,
    3. 4.68 m
4.
    1. 4.41 m,
    2. 10.96 m, 7.23 m
5. 60.22 m
6.
    1. 2.17 m,
    2. 3.472 m

CHAPTER 8

1. 66.92%
2. 31.47% V, 63.57%
3. V_1 = 12.75% V, V_2 = 14.34% V, V_3 = 17.73% V, V_4 = 23.33% V, V_5 = 31.85% V and V = 113.03 kV
4. 73.25%, 91.5%
5. C_1 = 0.166 C, $C2$ = 0.4 C, $C3$ = 0.75 C, C_4 = 1.333 C, C_5 = 2.5 C, C_6 = 6 C
6.
    1. V_1 = 5.386 kV, V_2 = 6.08 kV, V_3 = 7.58 kV
    2. 83.94%
7.
    1. 0.428,
    2. 59.38 kV,
    3. 66.1%

CHAPTER 9

1. 3.6 cm
2. 159.87 MΩ
3. 29.7×10^{12} Ω – m
4. g_{1max}= 71.15 kV/cm, g_{2max} = 83 kV/cm
5. 10.14 cm
6.
    1. 818.08×10^{-9} F/phase,
    2. 4.9 A,
    3. 280.07 kVAr,
    4. 28.005 kW,

    5. 31.13 kV/cm
7.
    1. 2.35 μF,
    2. 1.175 μF
8. 62.56 kV
9. 4.9 μF, 58.629 A
10.
    1. 0.45 μF,
    2. 0.6 μF,
    3. 10.768 A
11. 229.78 kVA

CHAPTER 10

1. 51.92 μF
2. 80 kW
3. 103 μF
4.
    1. 82.49 kVAr,
    2. 0.588 lag
5. 0.702 kVAr
6. 47.414 μF, 4
7. 143.22 μF

CHAPTER 11

1. 93.41 MVAr, 42.68 MW
2. 101.15 MVAr
3. 245 kVA, 0.44 leading

CHAPTER 12

1. $\frac{4}{3\cos^2\phi}$
2. 50%
3. 50%
4. 0.3125
5. 1.617 cm^2
6. 0.715 cm^2

CHAPTER 13

1. 251.68 kVA
2. 303.15 Ω
3.
    1. 22.52 H,

2. 23.7 H,
3. 0.03775 H

4. 147.36 Ω, 39.41 MVA per coil
5. 0.28 H, 66 MVA per coil

CHAPTER14

1.
   1. I_{PR} = 96 A, I_{RS}= 46 A, I_{ST} = 6 A, I_{UT} = 29 A, I_{QU}=54 A
   2. Minimum potential will occur at point T
   3. V_T = 223.82
2.
   1. 11.25 V, 729 W
   2. 1.125 V, 156.09 W
3. V_R = 244.16 V, V_S = 240.10 V, V_T = 236.68 V, V_Q= 236.2 V
4.
   1. Current from *P* = 409.16 A and *Q* = 290.84 A
   2. 10.507 kW
5.
   1. Point of minimum potential is T
   2. 1080.25 A
   3. 173.83 V
6.
   1. 267.05 V, 14.74°
   2. 239.387 V, 7.2°
7. $V_Q = 430.59\angle 0.752$ V, $V_P = 439.106\angle 1.014°$ V
8. Current in section $QR = 28.42\angle -81.319°$ A, Current in

   $PR = 48.052\angle 60.54$
9. 242.93 V

Appendix 3

ANSWERS TO ODD QUESTIONS

CHAPTER 2

1. It is the opposition of line conductors to the current flow. The resistance is distributed uniformly along the whole length of the line.
3. The capacitance of a capacitor is defined as the charge per unit potential difference. capacitance, $C = \frac{q}{V}$ F.
5. Capacitance, $C_N = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}$ F/m.
7. The alternating magnetic flux in a conductor caused by the current flowing in a neighbouring conductor gives rise to circulating currents, which cause an apparent increase in the resistance of a conductor. This phenomenon is called proximity effect.
9. The value of inductance, which is due to flux linkages, caused by current in both conductors (one of which is the return path for the current in the other) is called the inductance per loop length. Loop inductance is double the inductance per conductor in single-phase line.
11. An increase in transmission voltage results in reduction in losses, increase in transmission efficiency, improvement of voltage regulation and reduction in conductor material requirement.
13. The use of more than one three-phase circuit on the same tower requires greater reliability and a higher transmission capacity. If such circuits are so widely separated that there is negligible mutual inductance between them, the inductive reactance of the equivalent single circuit would be half of the individual circuit considered alone.

CHAPTER 3

1. The different types of overhead lines are (i) Short transmission lines, (ii) Medium transmission lines, and (iii) Long transmission lines.

3. The efficiency of a transmission line is defined as the ratio of receiving-end power to the sending-end power.

5. (i) End capacitor method, (ii) Nominal-T method, and (iii) Nominal-σ method.

7. A lumped parameter is an element in which physically separate resistors, inductors and capacitors can be represented.

9. All overhead lines having lengths up to 100 km are generally categorized as short lines. Lines having length ranging from 100 km to 250 km are termed as medium lines. Lines having length greater than 250 km are termed as long lines.

11. The capacitance for medium lines is considered as lumped parameter.

13. The order of error is nearly 10%.

15. The capacitance of each conductor is concentrated in the middle while the series impedance is split into two equal parts.

CHAPTER 4

1. $V_S = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l$.

3. Surge impedance of a line is defined as the square root of

$$\frac{L}{C}, \text{ i.e., } Z_C = \sqrt{\frac{L}{C}}.$$

5. The expression for sending current for a long transmission line by the

$$\bar{I}_S = \bar{V}_r \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ} + \bar{I}_r \cosh \sqrt{YZ}.$$

7. The advantages of constant-voltage transmission are:

- Same voltage is maintained at both ends of the line for all loads.
- For long distance more power transmission, it is possible to carry increased power for a given size of conductor.
- Improved power factor is affected at times of heavy loads.

9. 1/3 of the total power in a system is lost due to charging current.

11. The rigorous solution method is required in order to obtain a fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line.

13. The following two methods are generally used to compute the hyperbolic functions. They are

1. Convergent real angle methods, and
2. Convergent complex angle method.

15. For a lossless line, resistance *R* and conductance *G* are neglected.

17. In a long transmission line, if the sending-end voltage is numerically equal to the receiving-end voltage and sending-end current is numerically equal to the receiving-end current, then the line is said to be a tuned line.

CHAPTER 5

1. The effect of a lightning discharge or a sudden switching in or out in a power system is to impress electrical energy, which moves along the line at nearly the speed of light, in a transmission line. This is called a travelling wave.
3. The characteristic impedance of transmission lines range between 200 Ω to 500 Ω. For cables, it is in the order of 30 Ω to 50 Ω and the transformers have several thousand ohms as their impedance.
5. The equation of a travelling wave is in the form $V = V_0\,(e^{-\alpha t} - e^{\beta t})$ where, V_0 represents a factor that depends on the peak value, and, α and β are constants which control the wave front and wave tail times, respectively.
7. When a line is terminated by an inductance

   Reflected voltage,

   $$v(t) = 2v' e^{-\frac{Z_0}{L}t}$$

   and current $i(t) = \dfrac{2v'}{Z_0}\left(1 - e^{-\frac{Z_0}{L}t}\right)$

   When a line is terminated by a capacitance

   Reflected voltage, $v(t) = 2v'\left(1 - e^{-\frac{Z_0}{C}t}\right)$

   and current $i(t) = 2\dfrac{v'}{Z_0}\left(1 - e^{-\frac{Z_0}{C}t}\right)$

9. The crest of a wave is its maximum amplitude and is expressed in kV or kA.

11. The coefficient of refraction $= \dfrac{2R}{Z_0 + R}$

    and, the coefficient of reflection $= \dfrac{R - Z_0}{Z_0 + R}$

13. With the use of Bewley's Lattice diagram, the position and direction of the motion of every incident, reflected and refracted wave on the system at all instants can be known at a glance.

CHAPTER 6

1. The phenomenon whereby the conductors on the transmission line are ionized, a violet glow is formed, a hissing noise is emitted and ozone gas is produced, is known as Corona.

3. It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.

5. (i) Electrical factors, (ii) atmospheric factors, and (iii) factors connected with the conductors.

7.

- Due to corona formation, the air surrounding the conductor becomes conducting and hence, the virtual diameter of the conductor is increased.
- Corona reduces the effects of the transients produced by surges.

9. $P_C = \frac{244(f+25)}{\delta}\sqrt{\frac{r}{D}}(V - V_0)^2 \times 10^{-5}$ kW.

11.

- A faint violet glow is observed around the conductor.
- A hissing sound is produced.
- It produces ozone, which can be detected by its characteristic odour.
- The glow is maximum over rough and dirty surfaces of the conductor.
- Corona is accompanied by a power loss.
- The charging current under corona condition increases because the corona introduces harmonic currents.
- When corona is present, the effective capacitance of the conductor increases.
- It works as a safety value for surges.

CHAPTER 7

1. The following are the types of line supports that are most commonly used:

- Wooden poles
- Reinforced concrete poles
- Tubular steel poles
- Latticed steel towers.

3. The most commonly used conductor materials for overhead lines are copper, aluminium, galvanized steel, steel-cored aluminium and cadmium copper.

5. The difference in level between points of supports and the lowest point on the conductor is called sag.

7. Working stress = $\frac{\text{Ultimate strength}}{\text{Safety factor}}$.

9. Windforce/metre length = pressure × projected area in square metre^2

11. Sag, $D = \frac{WL^2}{8T}$.

13. The distance between adjacent supporting towers is called the span.

15. The vibration of the conductors can be reduced by using stock bridge dampers.

CHAPTER 8

1. The types of insulators are:

- Pin type.
- Suspension type.
- Strain type.
- Shackle type.

3.

- Provides insulation between conductor and its support.
- Prevents flowing of any leakage current from conductor to earth.

5. It is stronger, mechanically, than glass, gives less trouble from leakage and is less affected by changes of temperature.

7. The main causes of an insulator failure are abnormal stress, cracks, flash over and ageing.

9.

- To provide insulation between the conductor and the supporting tower.
- To prevent leakage current to flow from conductor to earth.

11.

- Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.
- If any one disc is damaged the whole string does not become useless because the damaged disc can be replaced by the sound one.

13. When the line encounters a dead end or a sharp curve, the line is subjected to great tension. In order to relieve the line of this excessive tension strain insulators are used.

15. Guard ring is a large metal ring surrounding the bottom unit and connected to the metal work at the bottom of this unit i.e., to the line.

17. In case of an insulator flashover, the porcelain is often cracked or broken up by the power. To protect the insulator from this, arcing horns are installed on overhead lines.

CHAPTER 9

1.

- Low tension cables-upto 1000 V.
- High tension cables-upto 11000 V.
- Super tension cables-from 22 kV to 33 kV.
- Extra high tension cables-from 33 kV to 66 kV.
- Extra super-voltage cables-beyond 132 kV.

3.

- More expensive.
- Insulation problems at high voltage levels as compared with the overhead system.

5. The different types of insulating materials used in cables are:

- Rubber.
- Vulcanized India rubber.
- Impregnated paper.
- Varnished cambric.
- Polyvinyl chloride (PVC).

7. Grading of cables refers to the distribution of dielectric material such that the difference between the maximum and minimum gradient is reduced, so that a cable of the same size may be operated at higher voltages or for the same operating voltage a relatively smaller size cable could be used.

9. The screened cables are:

- H-type cables.
- S-L type cables.

The pressure cables are:

- Oil-filled cables.
- Gas pressure cables.

CHAPTER 10

1. The power factor is defined as the ratio of real power to the apparent power. The cosine of angle between voltage and current in an AC circuit is also known as power factor.

3.

- Most of the AC motors are of induction type, which have low lagging power factor.
- Arc lamps, electric discharge lamps and industrial heating furnaces operate at low lagging power factor.
- The load on the power system is varying; it is high during mornings and evenings and low at other times. During the low load period, supply voltage is increased which increases magnetization current. This results in the decreased power factor.

5. The value to which the power factor should be improved to have maximum net annual saving is known as the most economical power factor.

7.

- They have a short service life ranging from 8 to 10 years.
- They are easily damaged if the voltage exceeds the rated value.
- Once the capacitors are damaged, their repair is uneconomical.

9.

- There are considerable losses in the motor.
- The maintenance cost is high.
- It produces noise.
- Except in size above 500 kVA the cost is greater than that of static capacitors of the same rating.
- As a synchronous motor has no self-starting torque, therefore, auxiliary equipment has to be provided for this purpose.

11. The higher the power factor, the higher is the real power generation.

13. By the use of shunt capacitors.

15. Because the required capacitance is inversely proportional to the frequency.

17. The rating of the circuit breakers is increased in the presence of a synchronous condenser.

19. The lagging kVAr must be completely neutralized by the leading kVAr supplied by the phase advancing unit.

CHAPTER 11

1. The following methods are generally employed for controlling the receiving-end voltage:

    1. By excitation control.
    2. By using tap changing transformer.
    3. Auto-transformer tap changing.
    4. Booster transformer.
    5. Induction regulators.
    6. By synchronous condensers.

3.

1. During switching, the impedance of the transformer is increased and there will be a voltage surge.
2. There are twice as many tappings as the voltage steps.

5. The transformer which is used to control the voltage of the transmission line at a point far away from the main transformer, is known as Booster transformer.

7. The synchronous motor takes a leading current when its field is overexcited under high-load conditions.

CHAPTER 12

1. The conveyance of electric power from a power station to the consumer premises is known as electric supply system.

3. The large network of conductors between the power station and consumers are broadly divided into two parts and they are:

1. Transmission system.
2. Distribution system.

5.

- Electric power cannot be generated at high DC voltage due to commutation problems.
- The DC voltage cannot be stepped up for transmission of power at high voltages.

7.

- An AC line requires more conductors than a DC line.
- The construction of an AC transmission line is more complicated than a DC transmission line.

9.

- The increased cost of insulating the conductors.
- The increased cost of transformers, switchgear and such other terminal apparatus.

11.

- It is not easy to estimate the energy loss in the line without actual load curves, which are not available at the time of estimation.
- Interest and depreciation on the capital outlay cannot be determined accurately.

CHAPTER 13

1. It is defined as an assembly of apparatus installed to perform switching, voltage transformation, power factor correction, power and frequency-converting operations.

3. A subtransmission substation receives the power from primary transmission substations at high voltages (above 132 kV) and steps down the voltage to 33 kV or 11 kV for secondary transmission or primary distribution.

5. A switching substation is used for switching operations of power lines without the transformation of voltage. In this substation different connections are made between different transmission lines.

7.

1. All the equipment is visible. So, the identification of fault is easier.
2. Expansion of the substation is easier.
3. Takes less erection time.
4. There is no necessity of building. So it requires less building material.

9. Instrument transformers are used to:

1. Protect personnel and apparatus from high voltage.
2. Permit the use of reasonable insulation levels and current-carrying capacity in relays and motors.

11. Substations can be classified, on the basis of mounting, into the following categories: Indoor, outdoor and pole-mounted substations.

13. To facilitate repair and maintenance work to be carried out on the circuit breaker.

15. Short-circuit current determines the number of sections in a sectionalized bus bar.

17. Commissioning of new feeders without causing interruption to the load.

19. An earth electrode is a metal plate, pipe, or any other conductor, or an array of conductors electrically connected to the general mass of the earth.

21.

- For safety of personnel from electric shock.
- For safety of equipment and personnel against lightning and voltage surges.
- For reducing the voltage stress on the lines and the equipment with respect to earth under various operating and fault conditions and also for controlling the earth fault currents for protective relays.

CHAPTER 14

1. The part of a power system which distributes electric power for local use is known as distribution system.

3.

- Feeders: A feeder is a conductor which connects the substation to the area where power is to be distributed.
- Distributors: A distributor is a conductor from which tapings are taken for supply to the consumers.
- Service mains: A service main is generally a small cable which connects the distributor to the consumer's terminals.

5. AC systems are universally adopted and preferred to DC transmission systems because these are simpler and more economical.

7.

- The end nearest to the feeding-point end of the distributor will be heavily loaded.
- The consumers are dependent on a single feeder and single distributor. Therefore, any fault on the feeder or distributor cuts off supply to the consumer who is on the side of the fault away from the substation.
- The consumers at the distant end of the distributor would be subjected to serious voltage fluctuations when the load on the distributor changes.

CHAPTER 15

1. The range of voltage in extra high-voltage transmission is between 330 kV and 1000 kV.

3. Shunt reactors are used to absorb the reactive power from the line to control the voltage under lightly-loaded conditions.

5. A bipolar link is an HVDC transmission link that has two conductors: one at the positive potential and the other at negative (same magnitude) with respect to the ground.

7. Rectification is the process of conversion from AC to DC.

9. A filter is a device used to filter unwanted signals called harmonics, which are generated during the conversion process.

11. The major components of an HVDC transmission are rectifier stations, inverter stations, converter transformers, DC reactor and filters.

CHAPTER 16

1. FACTS is an alternating current transmission systems incorporating power-electronic-based and other static controllers to enhance controllability and increase power-transfer capability.

3. The static synchronous compensator refers to a static synchronous generator operated as a shunt- connected static VAr compensator.

5. A static synchronous series compensator is a static, synchronous generator operated without an external electric-energy source as a series compensator.

7. A unified power-flow controller refers to a combination of a STATCOM and an SSSC which are coupled via., a common DC link, to allow the bi-directional flow of active power.
9. A power system undergoes voltage collapse if the post-disturbance equilibrium voltages near the load are below acceptable limits.
11. The function of series controller is to inject voltage in series with the transmission line.

Appendix 4

SOLUTIONS USING MATLAB PROGRAMS

CHAPTER 2

EXAMPLE 2.7

Calculate the inductance of a conductor (line-to-neutral) of a three-phase system as shown in Fig. 2.17, which has 1.2 cm diameter and conductors are placed at the corner of an equilateral triangle of sides 1.5 m.

MATLAB programme for Example 2.7

```
% TO FIND INDUCTANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

s = input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

r = d/2;

effR = 0.7788*r;

L = 2*10^(-7)*log(s*100/effR)*1000
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:1.2

d=

  1.200
```

```
ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:1.5

s=

 1.500

L (H/km) =

 0.00115429238467
```

EXAMPLE 2.9

Calculate the inductance per phase of a three-phase transmission line as shown in Fig. 2.19. The radius of the conductor is 0.5 cm. The lines are untransposed.

MATLAB Programme for Example 2.9

```
% TO FIND INDUCTANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:)

D12=input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

D23=input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

D31 = input('ENTER THE LENGTH OF THE SIDE
OF THE TRIANGLE in metres:')

r = d/2;

effR = 0.7788*r*0.01;

La = 2*10^(-7)*((log(sqrt(D12*D31)/effR)-
j*((sqrt(3)/2)*log(D31/D12))))

Lb = 2*10^(-7)*((log(sqrt(D12*D23)/effR)-
j*((sqrt(3)/2)*log(D23/D12))))

Lc = 2*10^(-7)*((log(sqrt(D31*D23)/effR)-
j*((sqrt(3)/2)*log(D23/D31))))
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:1

d =

1

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:3.5

D12 = 3.5000

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:3.5

D23 = 3.5000

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:7

D31 = 7

La(H/km) =1.429530986161494e-006
-1.200566133852944e-007i

Lb(H/km) =1.360216268105499e-006

Lc(H/km) =1.429530986161494e-006
+1.200566133852944e-007i
```

EXAMPLE 2.11

Calculate the inductance and reactance of each phase of a three-phase 50 Hz overhead high-tension line (HTL) which has conductors of 2.5 cm diameter. The distance between the three-phases are (i) 5 cm between A and B, (ii) 4 m between B and C and (iii) 3 m between C and A as shown in Fig. 2.23. Assume that the phase conductors are transposed regularly.

MATLAB programme for Example 2.11

```
% TO FIND INDUCTANCE

clc;

clear all;

d=input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')
```

```
D12=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

D23=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

D31=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

f = input('ENTER FREQUENCY OF THE SYSTEM:')

r = d/2;

effR = 0.7788*r;

Deq = (D12*D23*D31)^(1/3);

L = 2*10^(-7)*log(Deq/(effR*0.01))*1000

X = 2*pi*f*L
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN cm:2.5

d = 2.500

ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:3

D12 = 3

ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:4

D23 = 4

ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:5

D31 = 5

ENTER FREQUENCY OF THE SYSTEM:50

f = 50

L(H/km) =0.00119936183218

X(Ohm/km) =0.37679063209718
```

EXAMPLE 2.13

A 300 kV, three-phase bundle conductor line with two sub-conductor per phase has a horizontal configuration as shown in Fig. 2.24. Find the inductance per phase, if the radius of each sub conductor is 1.2 cm.

MATLAB programme for Example 2.13

```
% TO FIND INDUCTANCE

clc;

clear all

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

r = d/2;

effR = 0.7788*r;

Daa1 = input('ENTER Daa1in metres:')

Da1a = input('ENTER Da1a in metres:')

Dab = input('ENTER Dab in metres:')

Dab1 = input('ENTER Dab1 in metres:')

Dac = input('ENTER Dac in metres:')

Dac1 = input('ENTER Dac1 in metres:')

Da1b = input('ENTER Da1b in metres:')

Da1b1 = input('ENTER Da1b1 in metres:')

Da1c = input('ENTER Da1c in metres:')

Da1c1 = input('ENTER Da1c1 in metres:')

Dbc = input('ENTER Dbc in metres:')

Db1c = input('ENTER Db1c in metres:')

Dbc1 = input('ENTER Dbc1 in metres:')

Db1c1 = input('ENTER Db1c1 in metres:')

Dm1 =
nthroot((Dab*Dab1*Dac*Dac1*Da1b*Da1b1*Da1c
*Da1c1),8);
```

```
Dm2 =
nthroot((Dab*Dab1*Da1b*Da1b1*Dbc*Db1c*Dbc1
*Db1c1),8);

Dm3= Dm1;

Dm = nthroot((Dm1*Dm2*Dm3),3);

Ds = sqrt(effR*0.01*Daa1);

L = 2*10^(-7)*log(Dm/Ds);
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:2.4

d =2.400

ENTER Daa1in metres:0.4

Daa1 = 0.400

ENTER Da1a in metres:0.4

Da1a= 0.4000

ENTER Dab in metres:10

Dab = 10

ENTER Dab1 in metres:10.4

Dab1 = 10.4000

ENTER Dac in metres:20

Dac = 20

ENTER Dac1 in metres:20.4

Dac1 = 20.40000

ENTER Da1b in metres:9.6

Da1b= 9.60000000000000

ENTER Da1b1 in metres:10

Da1b1=10

ENTER Da1c in metres:19.6
```

```
Da1c=19.6000000000000
ENTER Da1c1 in metres:20
Da1c1 = 20
ENTER Dbc in metres:10
Dbc = 10
ENTER Db1c in metres:9.6
Db1c = 9.60000000000000
ENTER Dbc1 in metres:10.4
Dbc1=10.4000000000000
ENTER Db1c1 in metres:10
Db1c1=10
L(H) =1.065580823245456e-006
```

EXAMPLE 2.14

Calculate the inductance per phase of a three-phase double circuit line as shown in Fig. 2.27, if the conductors are spaced at the vertices of a hexagon of side 2 m each. The diameter of each conductor is 2.0 cm.

MATLAB programme for Example 2.14

```
% TO FIND INDUCTANCE
clc;
clear all;
d = input('ENTER THE DIAMETER OF THE CONDUCTOR IN cm:')
r = d/2;
effR = 0.7788*r*0.01;
l=input('ENTER THE LENGTH OF THE SIDE OF THE HEXAGON in metres:')
L = 10^(-7)*log((sqrt(3)/2)*(l/effR))
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN cm:2

d = 2

ENTER THE LENGTH OF THE SIDE OF THE HEXAGON in metres:2

l = 2

L(H/km) =5.404477335806238e-007
```

EXAMPLE 2.16

Calculate the inductance per phase of a three-phase, double circuit line as shown in Fig. 2.30. The diameter of each conductor is 1.5 cm.

MATLAB programme for Example 2.16

```
% TO FIND INDUCTANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE CONDUCTOR IN cm:')

r = d/2;

effR = 0.7788*r*0.01;

H=input('ENTER VALUE OF HORIZONTAL DISTANCE IN metres:')

V=input('ENTER VALUE OF VERTICAL DISTANCE IN metres:')

m = sqrt((H^2) + (V^2));

n = sqrt((H^2) + ((2*V)^2));

L = 2*10^(-7)*log((2^(1/6))*((V/effR)^(0.5))*((m/n)^(1/3)))
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN cm:1.5

d = 1.5000
```

```
ENTER VALUE OF HORIZONTAL DISTANCE IN
metres:6

H=6

ENTER VALUE OF VERTICAL DISTANCE IN
metres:2.5

V = 2.5000

L(H/km) =6.167769885861392e-007
```

EXAMPLE 2.22

Calculate the capacitance of a conductor per phase of a three-phase 400 km long line, with the conductors spaced at the corners of an equilateral triangle of side 4 m and the diameter of each conductor being 2.5 cm.

MATLAB programme for Example 2.22

```
% TO FIND CAPACITANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

s = input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

l = input('ENTER LENGTH OF THE LINE')

r = d/2*0.01;

C = (10^(-9))/(18*log(s/r))*l*1000
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:2.5

d = 2.5000

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:4

s = 4

ENTER LENGTH OF THE LINE400
```

```
l = 400

C(F) =3.852459361818897e-006
```

EXAMPLE 2.24

Calculate the capacitance per phase of a three-phase three-wire transposed system as shown in Fig. 2.38 when the conductors are arranged at the corners of a triangle with sides measuring 1.0 m, 1.5 m, and 2.0 m. Diameter of each conductor is 1.2 cm.

MATLAB programme for Example 2.24

```
% TO FIND CAPACITANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE CONDUCTOR IN cm:')

D12=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

D23=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

D31=input('ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:')

r = d/2*0.01;

Deq = (D12*D23*D31)^(1/3);

C = (10^(-9))/(18*log(Deq/r))*1000;
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN cm:1.2

d = 1.2000

ENTER THE LENGTH OF THE SIDE OF THE TRIANGLE in metres:1.5

D12=1.5000
```

```
ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:2

D23 = 2

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:2.5

D31 = 2.5000000

C(F) =9.599016197705905e-009
```

EXAMPLE 2.28

Calculate the capacitance (phase-to-neutral) of a three-phase 100 km long double circuit line shown in Fig. 2.43 with conductors of diameter 2.0 cm each arranged at the corners of an hexagon with sides measuring 2.1 m.

MATLAB Programme for Example 2.28

```
% TO FIND CAPACITANCE

clc;

clear all;

d=input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

r = d/2*0.01;

l=input('ENTER THE LENGTH OF THE SIDE OF
THE HEXAGON in metres:')

L = input('ENTER LENGTH OF THE LINE')

C = (10^(-9))/(18*
(log(sqrt(sqrt(3)*l/(2*r)))))*10^5
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:2

d = 2

ENTER THE LENGTH OF THE SIDE OF THE
HEXAGON in metres:2.1

l = 2.1000
```

```
ENTER LENGTH OF THE LINE100

L = 100

C(F) =2.135410731484511e-006
```

EXAMPLE 2.29

Calculate the capacitance per phase of a three-phase, double circuit line as shown in Fig. 2.45. The diameter of each conductor is 1.5 cm.

MATLAB programme for Example 2.29

```
% TO FIND CAPACITANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE CONDUCTOR IN cm:')

r = d/2*0.01;

H=input('ENTER VALUE OF HORIZONTAL DISTANCE IN metres:')

V=input('ENTER VALUE OF VERTICAL DISTANCE IN metres:')

m = sqrt((H^2) + (V^2));

n = sqrt((H^2) + ((2*V)^2));

C= 1/(9*log((2^(1/3))*(V/r)*((m/n)^(2/3))))
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN cm:1.5

d = 1.5000

ENTER VALUE OF HORIZONTAL DISTANCE IN metres:6

H=6

ENTER VALUE OF VERTICAL DISTANCE IN metres:2.5
```

```
V = 2.5000

C(μF/km) = 0.0188
```

EXAMPLE 2.30

Calculate the capacitance per phase of a three-phase double circuit line as shown in Fig. 2.46. The diameter of the conductor is 2 cm. Assume that the line is completely transposed.

MATLAB programme for Examples 2.17 and 2.30

```
% TO FIND INDUCTANCE & CAPACITANCE

clc;

clear all;

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

r = d/2;

effR = 0.7788*r*0.01; Dab=input('ENTER
Dabin metres:')

Dab1 = input('ENTER Dab1in metres:')

Daa1 = input('ENTER Daa1in metres:')

Dac = input('ENTER Dac in metres:')

Dac1 = input('ENTER Dac1 in metres:')

Da1b = input('ENTER Da1b in metres:')

Da1c = input('ENTER Da1c in metres:')

Da1b1 = input('ENTER Da1b1 in metres:')

Da1c1 = input('ENTER Da1c1 in metres:')

Dbb1 = input('ENTER Dbb1 in metres:')

Dbc = input('ENTER Dbc in metres:')

Db1c = input('ENTER Db1c in metres:')

Dbc1 = input('ENTER Dbc1 in metres:')

Db1c1 = input('ENTER Db1c1 in metres:')
```

```
Dm1 =
nthroot((Dab*Dab1*Dac*Dac1*Da1b*Da1b1*Da1c
*Da1c1),8);

Dm2 =
nthroot((Dab*Dab1*Da1b*Da1b1*Dbc*Db1c*Dbc1
*Db1c1),8);

Dm3= Dm1;

Dm = nthroot((Dm1*Dm2*Dm3),3);

Ds1 = nthroot((effR*effR*Daa1*Daa1),4);

Ds2 = nthroot((effR*effR*Dbb1*Dbb1),4);

Ds3= Ds1;

Ds = nthroot((Ds1*Ds2*Ds3),3);

L = 0.2*log((Dm/Ds))

C= 1/(18*log(Dm/Ds))
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:2

d=2

ENTER Dabin metres:3.64

Dab = 3.64000000000000

ENTER Dab1in metres:7.826

Dab1 = 7.82600000000000

ENTER Daa1in metres:9.22

Daa1 = 9.22000000000000

ENTER Dac in metres:7

Dac = 7

ENTER Dac1 in metres:6

Dac1 = 6

ENTER Da1b in metres:7.826
```

```
Da1b = 7.82600000000000
ENTER Da1c in metres:6
Da1c = 6
ENTER Da1b1 in metres:3.64
Da1b1=3.64 000000000000
ENTER Da1c1 in metres:7
Da1c1 = 7
ENTER Dbb1 in metres:8
Dbb1 = 8
ENTER Dbc in metres:3.64
Dbc = 3.64000000000000
ENTER Db1c in metres:7.826
Db1c = 7.82600000000000
ENTER Dbc1 in metres:7.826
Dbc1 = 7.82600000000000
ENTER Db1c1 in metres:3.64
Db1c1=3.64 000000000000
L(mH/km) =0.61599539955856
C(µF/km) =0.01803765274720
```

EXAMPLE 2.33

Calculate the capacitance per phase of a three-phase, three-wire system by considering earth effect, when the conductors are arranged in a horizontal plane with spacing $D_{12} = D_{23}$ = 3.5 m, and D_{31} = 7 m as shown in the Fig. 2.52. The conductors are transposed and each has a diameter of 2.0 cm. Assume that the transmission line is 4 m above the ground level.

MATLAB programme for Example 2.33

```
% TO FIND CAPACITANCE
clc;
```

```
clear all;

d = input('ENTER THE DIAMETER OF THE
CONDUCTOR IN cm:')

D12=input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

D23=input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

D31=input('ENTER THE LENGTH OF THE SIDE OF
THE TRIANGLE in metres:')

r = d/2*0.01;

h=input('ENTER DISTANCE OF THE
TRANSMISSION LINE ABOVE THE GROUND IN
metres:')

H1 = 2*h;

H2 = sqrt(((2*h)^2) + (D12^2));

H3 = sqrt(((2*h)^2) + (D31^2));

Deq = (D12*D23*D31)^(1/3);

Heq = H1/(((H2^2)*H3)^(1/3));

C = (10^(-9))/(18*log((Deq/r)*Heq))*1000
```

Result:

```
ENTER THE DIAMETER OF THE CONDUCTOR IN
cm:2

d = 2

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:3.5

D12 = 3.5000

ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:3.5

D23 = 3.5000
```

```
ENTER THE LENGTH OF THE SIDE OF THE
TRIANGLE in metres:7

D31 = 7

ENTER DISTANCE OF THE TRANSMISSION LINE
ABOVE THE GROUND IN metres:4

h = 4

C (F) =9.359320827157641e-009
```

CHAPTER 3

MATLAB programme for medium line model

Programme-1: To find the sending-end quantities

A three-phase transmission line is 140 km long. The resistance per phase is 0.04 ohms per kilometre and the inductance per phase is 0.95 mH per kilometre. The shunt capacitance is 0.0105 uF per km. The receiving-end load is 90 MVA with 0.85 pf lagging at 110 kV. Determine the voltage, powers at the sending end, voltage regulation and efficiency by using medium line models.

```
% MEDIUM LINE END CAPACITANCE, NOMINAL-T,
NOMINAL-PI METHODS COMPARISIONS

clc;

clear all;

format long;

R=input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L=input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct=input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G=input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
µSeimens/Km:')
```

```
l=input('ENTER THE VALUE OF LENGTH OF THE
LINE IN Km:')

f=input('ENTER THE FREQUENCY OF THE SYSTEM
IN Hz:')

R=R*l;

L=L*l*0.001;

Ct=Ct*l* 0.000001;

G=G*l;

z=R+j*2*pi*f*L;

y=G+j*2*pi*f*Ct;

m=menu('SPECIFY THE TYPE OF NETWORK','END
CAPACITANCE','NOMINAL T','NOMINAL PI')

switch m

  case {1}

 A=1+y*z;

 B=z;

 C=y;

 D=1;

  case {2}

 A=1+(y*z/2);

 B=z*(1+y*z/4);

 C=y;

 D=1+(y*z/2);

  otherwise

 A=1+((z*y)/2);

 B=z;

 C=y*(1+((z*y) /4));

 D=A;
```

```
end
TM=[A B;C D];
VRPh=input('ENTER THE RECEIVING END PHASE
VOLTAGE IN KV:')
VRL=VRPh*(sqrt(3));
SR=input('ENTER THE RECEIVING END LOAD IN
MVA:')
Pf=input('ENTER THE RECEIVING END LOAD
POWER FACTOR:')
h=acos(Pf);
SR=SR*(cos(h)+j*sin(h));
PR=real(SR);
IR=conj(SR)/(3*conj(VRL));
SM=TM*[VRL;IR];
VS = SM(1,1);
IS = SM(2,1);
Pfs=cos(angle(VS)-angle(IS));
SS = 3*VS*conj(IS);
VS=sqrt(3)*abs(VS);
IS=abs(IS)*1000;
VREG=((VS/(abs(TM(1,1)))-VRPh)/VRPh)*100;
PS=real(SS);
QS=imag(SS);
eff=PR/PS*100;
z
y
TM
fprintf('SENDING END VOLTAGE(L-L KV) =
%g\n', VS);
```

```
fprintf('SENDING END CURRENT (A) =
%g\n',IS);

fprintf('SENDING END POWER FACTOR = %g\n',
Pfs);

fprintf('SENDING END REAL POWER (KW) =
%g\n ', PS);

fprintf('SENDING END REACTIVE POWER (KVAR)
= %g\n ', QS)

fprintf('PERCENTAGE VOLTAGE REGULATION =
%g\n ', VREG);

fprintf('PERCENTAGE EFFICIENCY: %g',eff)
```

Result:

```
ENTER THE ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN
Ohms/Km:0.04

R = 0.04000000000000

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.95

L = 0.95000000000000

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.0105

Ct = 0.01050000000000

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:140

l = 140

ENTER THE FREQUENCY OF THE SYSTEM IN Hz:50

f = 50

m = 1
```

ENTER THE RECEIVING END PHASE VOLTAGE IN kV:110

VRPh = 11

ENTER THE RECEIVING END LOAD IN MVA:90

SR = 90

ENTER THE RECEIVING END LOAD POWER FACTOR :0.85

pf=0.85000000000000

z = 5.60000000000000 +41.78318229274425i

y = 0 +4.618141200776996e-004i

Case-I: End Capacitance Method

TM = 0.98070393643543 + 0.00258615907244i 5.60000000000000 +41.78318229274425i

0 + 0.00046181412008i 1.00000000000000

SENDING END VOLTAGE(L-L KV) = 132.545

SENDING END CURRENT (A) = 457.607

SENDING END POWER FACTOR = 0.761677

SENDING END REAL POWER (KW) = 80.018

SENDING END REACTIVE POWER (KVAR) = 68.0712

PERCENTAGE VOLTAGE REGULATION = 22.866

PERCENTAGE EFFICIENCY: 95.6035

Case-II: Nominal - T Method:

TM = 0.99035196821772 + 0.00129307953622i 5.54597102201920 +41.58524018008296i

0 + 0.00046181412008i 0.99035196821772 + 0.00129307953622i

SENDING END VOLTAGE (L-L KV) = 133.413

SENDING END CURRENT (A) = 453.09

```
SENDING END POWER FACTOR = 0.76504

SENDING END REAL POWER (KW) = 80.0988

 SENDING END REACTIVE POWER (KVAR) =
67.4243

 PERCENTAGE VOLTAGE REGULATION = 22.4656

 PERCENTAGE EFFICIENCY: 95.507
```

Case-III: Nominal-PI Method:

```
TM = 0.99035196821772 + 0.00129307953622i
5.60000000000000 +41.78318229274425i

 -0.00000029858119 + 0.00045958632142i
0.99035196821772 + 0.00129307953622i

SENDING END VOLTAGE(L-L KV) = 133.556

SENDING END CURRENT (A) = 453.141

SENDING END POWER FACTOR = 0.764428

SENDING END REAL POWER (KW) = 80.1298

 SENDING END REACTIVE POWER (KVAR) =
67.5804

 PERCENTAGE VOLTAGE REGULATION = 22.5971

 PERCENTAGE EFFICIENCY: 95.4701
```

Programme-2: To find the receiving-end quantities

A three-phase transmission line is 135 km long. The series impedance is $Z = 0.04 + j0.95$ ohms per phase per kilometre, and shunt admittance is $Y = j5.1$ μ-mhos per phase per km. The sending-end voltage is 132 kV, and the sending-end current is 154 A at 0.9 power factor lagging. Determine the voltage, current and power at the receiving end and the voltage regulation using medium line-π model.

```
% TO FIND RECEIVING END QUANTITIES FOR
MEDIUM LINES USING NOMINAL PI METHOD

clc;

clear all;
```

```
R=input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L=input('ENTER THE VALUE OF INDUCTIVE
REACTANCE/PHASE OF THE LINE IN Ohms/Km:')

Ct=input('ENTER THE VALUE OF CAPACITIVE
REACTANCE/PHASE OF THE LINE IN
SEIMENS/Km:')

G=input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE :')

l=input('ENTER THE VALUE OF LENGTH OF THE
LINE IN Km:')

f=input('ENTER THE FREQUENCY OF THE SYSTEM
IN Hz')

R=R*l;

L=L*l;

Ct=Ct*l;

G=G*l;

z=R+j*L;

y=G+j*Ct;

A=1+((z*y)/2);

B=z;

C=y*(1+((z*y)/4));

D=A;

TM=[A B;C D];

VSPh=input('ENTER THE SENDING END PHASE
VOLTAGE')

VSL=VSPh/(sqrt(3));

IS=input('ENTER THE SENDING END CURRENT IN
AMPERES')

IS = IS*0.001;
```

```
Pf=input('ENTER THE SENDING END LOAD POWER
FACTOR')

h=-acos(Pf);

IS=IS*(cos(h)+j*sin(h));

RM=inv(TM)*[VSL;IS];

VR=RM(1,1);

IR=RM(2,1);

Pfr=cos(angle(VR)-angle(IR));

SR=3*VR*conj(IR);

VR=sqrt(3)*abs(VR);

IR=abs(IR)*1000;

VREG=((VSPh/(abs(TM(1,1)))-VR)/VR)*100;

PR=real(SR);

QR=imag(SR);

z

y

TM

fprintf('RECEIVING END VOLTAGE(L-L KV) =
%g\n', VR);

fprintf('RECEIVING END CURRENT (A) =
%g\n',IR);

fprintf('RECEIVING END POWER FACTOR =
%g\n', Pfr);

fprintf('RECEIVING END REAL POWER(KW) =
%g\n ', PR);

fprintf('RECEIVING END REACTIVE
POWER(KVAR) = %g\n ', QR);

fprintf('PERCENTAGE VOLTAGE REGULATION =
%g\n ', VREG);
```

Result:

ENTER THE VALUE OF RESISTANCE/PHASE OF THE LINE IN Ohms/Km:0.04

R = 0.04000000000000

ENTER THE VALUE OF INDUCTIVE REACTANCE/PHASE OF THE LINE IN Ohms/Km:0.95

L = 0.95000000000000

ENTER THE VALUE OF CAPACITIVE REACTANCE/PHASE OF THE LINE IN SEIMENS/Km:5.1*0.000001

Ct = 5.100000000000000e-006

ENTER THE VALUE OF CONDUCTANCE/PHASE OF THE LINE :0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN Km:135

l = 135

ENTER THE FREQUENCY OF THE SYSTEM IN Hz:50

f = 50

ENTER THE SENDING END PHASE VOLTAGE In kV:132

VSPh = 132

ENTER THE SENDING END CURRENT IN AMPERES:154

IS = 154

ENTER THE SENDING END LOAD POWER FACTOR:0.9

Pf = 0.90000000000000

z = 5.400000000000000e+000 +1.282500000000000e+002i

y = 0 +6.884999999999999e-004i

```
TM = 0.955849937500 + 0.001858950000i
5.400000000000 + 128.250000000000i

 -0.000000639944 + 0.000673301341i
0.955849937500 + 0.001858950000i

RECEIVING END VOLTAGE (L-L KV) = 113.961

RECEIVING END CURRENT (A) = 175.705

RECEIVING END POWER FACTOR = 0.900639

RECEIVING END REAL POWER = 31.2358

RECEIVING END REACTIVE POWER (KW) =
15.0716

PERCENTAGE VOLTAGE REGULATION (KVAR) =
21.179
```

CHAPTER 4

MATLAB Programmes for long line models

Programme-1: To find the sending-end quantities

A three-phase, 50 Hz, 240 kV transmission line is 200 km long. The line parameters are *r* = 0.017 ohms/phase/ km, *L* = 0.94 mH/phase/ km, *C* = 0.0111 µF/phase/km. Calculate the line performance when load at the receiving-end is 500 MW, 0.9 pf lagging at 220 kV.

```
% COMPARISION OF NOMINAL-PI AND NOMINAL-T
METHODS FOR LONG LINES

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')
```

```
G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')
l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')
f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')
z = (R+j*2*pi*f*L*0.001);
y = (G+j*2*pi*f*Ct* 0.000001);
gm = sqrt(z*y);
zc = sqrt(z/y);
m = menu('specify the type of
network','EQUALENT PI','EQUQLENT T')
switch m
case {1}
z1 = z*sinh(gm*l) / (gm*l);
y1 = y*tanh(gm*l/2)/ (gm*l/2);
otherwise
y1 = y*sinh(gm*l)/(gm*l);
z1 = z*tanh(gm*l/2)/(gm*l/2);
end
A = 1+(z1*y1/2);
B = z1;
C = y1*(1+(z1*y1/4));
D = A;
TM = [A B;C D];
Z = B;
Y = (2*tanh(gm*l/2))/zc;
```

```
VRL = input('ENTER THE RECEIVING END LINE
VOLTAGE')

VRP = VRL/(sqrt(3));

PR = input('ENTER THE RECEIVING END LOAD
IN MW')

Pf = input('ENTER THE RECEIVING END LOAD
POWER FACTOR')

h = acos(Pf);

SR = PR/Pf;

SR = SR*(cos(h)+j*sin(h));

QR = imag(SR);

IR = conj(SR)/(3*conj(VRP));

SM = TM*[VRP;IR];

VS = SM(1,1);

IS = SM(2,1);

Pfs = cos(angle(VS)-angle(IS));

SS = 3*VS*conj(IS);

VSA= angle(VS)*(180/pi);

ISA = angle(IS)*(180/pi);

VS = sqrt(3)*abs(VS);

IS = abs(IS)*1000;

VREG = ((VS/(abs(TM(1,1)))-VRL)/VRL)*100;

PS = real(SS);

QS = imag(SS);

eff = PR/PS*100;

PL = PS-PR;

QL = QS-QR;

z
```

```
y

zc

Z

Y

TM

fprintf('SENDING END VOLTAGE(L-L KV) = %g
at %g degrees \n', VS,VSA);

fprintf('SENDING END CURRENT (A) = %g at
%g degrees \n',IS,ISA);

fprintf('SENDING END POWER FACTOR = %g\n',
Pfs);

fprintf('SENDING END REAL POWER = %g\n
PS);

fprintf('SENDING END REACTIVE POWER (KW)
=%g\n QS);

fprintf('PERCENTAGE VOLTAGE REGULATION
(KVAR) =%g\n VREG);

fprintf('REAL POWER LOSS (MW) =%g\n',PL);

fprintf('REACTIVE POWER LOSS (MVAR)
=%g\n',QL);

fprintf('EFFICIENCY = %g', eff);
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.017

R = 0.0170

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.94

L = 0.9400

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.0111
```

Ct = 0.0111

ENTER THE VALUE OF CONDUCTANCE/PHASE OF THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN Km:200

l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz:50

f = 50

m = 1

ENTER THE RECEIVING END LINE VOLTAGE In kV:220

VRL = 220

ENTER THE RECEIVING END LOAD IN MW:50 0

PR = 500

ENTER THE RECEIVING END LOAD POWER FACTOR:0.9

Pf = 0.9000

z = 0.01700000000000 + 0.29530970943744i

y=0 +3.487167845484671e-006i

zc = 2.911267529483590e + 002 -8.372669679985799e+000i

Case-I: Nominal PI method

Z = 0.0168 + 0.2 933i

Y=1.3896e-007 +6.9984e-004i

TM = 1.0000 - 0.0000i 0.0168 + 0.2933i

-0.0000 + 0.0000i 1.0000 - 0.0000i

SENDING END VOLTAGE(L-L KV) = 220.362 at 0.168516 degrees

SENDING END CURRENT (A) = 1457.96 at -25.8419 degrees

SENDING END POWER FACTOR=0.898714

SENDING END REAL POWER = 500.107

SENDING END REACTIVE POWER (KW) =244.031

PERCENTAGE VOLTAGE REGULATION (KVAR) = 0.164499

REAL POWER LOSS (MW) = 0.106864

REACTIVE POWER LOSS (MVAR) = 1.8703

PERCENTAGE OF EFFICIENCY = 99.9786

Case-II: Nominal-T method

Z = 0.0171 + 0.2963i

Y=1.3896e-007 +6.9984e-004i

TM= 0.99999948686194 + 0.00000002943785i 0.01711743438890 + 0.29632419574163i

-0.00000000137255 + 0.00000346327561i 0.99999948686194 + 0.00000002943785i

SENDING END VOLTAGE (L-L KV) = 220.366 at 0.170206 degrees

SENDING END CURRENT (A) = 1457.76 at -25.8264 degrees

SENDING END POWER FACTOR=0.89882

SENDING END REAL POWER=500.109

SENDING END REACTIVE POWER (KW) =243.883

PERCENTAGE VOLTAGE REGULATION (KVAR) =0.166386

REAL POWER LOSS (MW) =0.109075

REACTIVE POWER LOSS (MVAR) =1.72148

PERCENTAGE OF EFFICIENCY = 99.9782

Comment: From the results of the above two cases, the results obtained from equivalent-PI method are similar to practical values. Hence, the equivalent-PI method is used for solving the following problems.

Programme-2: To find the sending-end quantities

A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μF/phase/km. Calculate the receiving-end quantities and the performance of the line using equivalent w-model, when 300 MW and 250 MVAr are being transmitted at 220 kV from the sending-end.

```
%TO FIND RECEIVING END QUANTITIES FOR LONG
LINES USING NOMINAL-PI METHOD

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')

f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')

z = (R+j*2*pi*f*L*0.001);

y = (G+j*2*pi*f*Ct* 0.000001);

gm = sqrt(z*y);

zc = sqrt(z/y);

A = cosh(gm*l);
```

```
B = zc*sinh(gm*l);

C = sinh(gm*l)/zc;

D = cosh(gm*l);

TM = [A B;C D];

Z = B;

Y = (2*tanh(gm*l/2))/zc;

VSL= input('ENTER THE SENDING END LINE
VOLTAGE') VSP = VSL/(sqrt(3));

PS = input('ENTER THE SENDING END REAL
POWER IN MW') QS = input('ENTER THE
SENDING END REACTIVE POWER IN MVAR') h =
atan(QS/PS); SS = PS + j*QS;

IS = conj(SS)/(3*conj(VSL));

RM = inv(TM)*[VSP;IS]; VR = RM(1,1); IR =
RM(2,1);

Pfr = cos(angle(VR)-angle(IR));

VRA = angle(VR)*(180/pi);

IRA = angle(IR)*(180/pi);

SR = 3*VR*conj(IR);

VR = sqrt(3)*abs(VR);

IR = abs(IR)* 1000;

VREG = ((VSPh/(abs(TM(1,1)))-VR)/VR)*100;

PR = real(SR);

QR = imag(SR);

eff = PR/PS*100;

PL = PS-PR;

QL = QS-QR;

z

y
```

```
zc

Z

Y

TM

fprintf('RECEIVING END VOLTAGE(L-L KV) =
%g at %g degrees \n', VR,VRA);

fprintf('RECEIVING END CURRENT (A) = %g at
%g degrees \n',IR,IRA);

fprintf('RECEIVING END POWER FACTOR = %g\n
', Pfr);

fprintf('RECEIVING END REAL POWER = %g\n
', PR);

fprintf(‘RECEIVING END REACTIVE POWER (KW)
= %g\n ‘, QR);

fprintf(‘PERCENTAGE VOLTAGE REGULATION
(KVAR) = %g\n ‘, VREG);

fprintf(‘REAL POWER LOSS (MW) = %g\n’,PL);

fprintf(‘REACTIVE POWER LOSS (MVAR) =
%g\n’,QL);

fprintf(‘EFFICIENCY = %g’, eff);
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02

Ct = 0.0200
```

```
ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:200

l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz:50

f = 50

ENTER THE SENDING END LINE VOLTAGE IN kV:
220 VS

Ph = 220

ENTER THE SENDING END REAL POWER IN MW:30
0

PS = 300

ENTER THE SENDING END REACTIVE POWER IN
MVAR:250

QS = 250

z = 0.01500000000000 + 0.31101767270539i

y = 0 +6.283185307179586e-006i

zc = 2.225505958391533e + 002
-5.363553706465585e + 000i

Z = 2.92228950396776 +61.3981788015 6411i

Y = 0.00000040102997 + 0.00126488706125i

TM = 0.96116970498754 + 0.00186049434616i
2.92228950396776 +61.39817880156411i

-0.00000078341370 + 0.00124032946544i
0.96116970498754 + 0.00186049434616i

RECEIVING END VOLTAGE (L-L KV) =159.251 at
-30.1535 degrees
```

```
RECEIVING END CURRENT (A) = 1092.47 at
-46.0621 degrees

RECEIVING END POWER FACTOR = 0.9617

RECEIVING END REAL POWER = 289.797

RECEIVING END REACTIVE POWER (KW) =
82.5979

PERCENTAGE VOLTAGE REGULATION (KVAR) =
43.7273

REAL POWER LOSS (MW) = 10.2027

REACTIVE POWER LOSS (MVAR) = 167.402

PERCENTAGE OF EFFICIENCY = 96.5991
```

Programme-3: To find the sending-end quantities when load impedance is given

Note: For this case, the receiving current can be calculated using the formula given below

$$I_R = \frac{V_{Rph}}{Z_L}$$
$$S_R = 3 \times V_{Rph} \times I_R^*$$

Where, Z_L = complex load impedance

A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μF/phase/km. Calculate the sending-end quantities and the line performance when the receiving-end load impedance is 200 ohms at 220 kV.

```
% TO FIND SENDING END QUANTITIES USING
LOAD IMPEDANCE FOR LONG LINES

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')
```

```
Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')

f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')

z = (R+j*2*pi*f*L*0.001);

y = (G+j*2*pi*f*Ct* 0.000001);

gm = sqrt(z*y)

zc = sqrt(z/y)

A = cosh(gm*l)

B = zc*sinh(gm*l);

C = sinh(gm*l)/zc;

D = cosh(gm*l);

TM = [A B;C D];

Z = B;

Y = (2*tanh(gm*l/2))/zc;

VRL = input('ENTER RECEIVING END VOLTAGE
IN KV:')

VRP = VRL/(sqrt(3));

VRPhA = input('ENTER RECEIVING END VOLTAGE
PHASE ANGLE IN DEGREES:')

VRP = VRP(cos(VRPhA)+j*sin(VRPhA));

ZL = input('ENTER RECEIVING END COMPLEX
LOAD IMPEDENCE:')

IR = VRP/ZL;

SR = 3*VRL*IR;
```

```
PR = real(SR);
QR = imag(SR);
SM = TM*[VRP;IR];
VS = SM(1,1)
IS = SM(2,1)
Pfs = cos(angle(VS)-angle(IS)
SS = 3*VS*conj(IS);
VSA= angle(VS)*(180/pi);
ISA =angle(IS)*(180/pi);
VS = sqrt(3)*abs(VS);
IS = abs(IS)*1000;
VREG = ((VS/(abs(TM(1,1)))-VRL)/VRL)*100;
PS = real(SS);
QS = imag(SS);
eff = PR/PS*100;
PL = PS-PR;
QL = QS-QR;
z
y
zc
Z
Y
TM
fprintf('SENDING END VOLTAGE(L-L KV) = %g
at %g degrees \n', VS,VSA
fprintf('SENDING END CURRENT (A) = %g at
%g degrees \n',IS,ISA);
```

```
fprintf('SENDING END POWER FACTOR = %g\n',
Pfs);

fprintf('SENDING END REAL POWER = %g\n
PS);

fprintf('SENDING END REACTIVE POWER (KW)
=%g\n QS);

fprintf('PERCENTAGE VOLTAGE REGULATION
(KVAR) =%g\n VREG);

fprintf('REAL POWER LOSS (MW) =%g\n',PL);

fprintf('REACTIVE POWER LOSS (MVAR) =%g\n
',QL);

fprintf('EFFICIENCY = %g', eff);

ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R=0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02

Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:200 l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz50

f = 50

ENTER RECEIVING END VOLTAGE IN KV:220

VRPh = 220
```

```
ENTER RECEIVING END VOLTAGE PHASE ANGLE IN
DEGREES:0

VRPhA = 0

ENTER RECEIVING END COMPLEX LOAD
IMPEDENCE:200

ZL = 200

z = 0.0150 + 0.3110i

y = 0 +6.2832e-006i

zc = 2.2255e+002 -5.3636e+000i

Z = 2.9223 +61.3982i

Y= 0.0000 + 0.0013i

TM = 0.9612 + 0.0019i 2.9223 +61.3982i

-0.0000 + 0.0012i 0.9612 + 0.0019i

SENDING END VOLTAGE (L-L KV) = 225.168 at
17.5635 degrees

SENDING END CURRENT (A) = 630.627 at
14.5777 degrees

SENDING END POWER FACTOR = 0.998643

SENDING END REAL POWER = 245.613

SENDING END REACTIVE POWER (KW) = 12.8107

PERCENTAGE VOLTAGE REGULATION (KVAR) =
6.48392

REAL POWER LOSS (MW) = 3.6127

REACTIVE POWER LOSS (MVAR) = 12.8107

EFFICIENCY = 98.5291
```

Programme-4: To find the shunt reactor rating under open-circuit condition

A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μP/phase/km. Calculate the receiving-end voltage when the line is terminated in an open-

circuit and is energized with 220 kV at the sending end. Also find the reactance and the MVAr of a three- phase shunt reactor to be installed at the receiving-end in order to limit the no-load receiving-end voltage to 220 kV.

```
% TO FIND SHUNT REACTOR RATING FOR OPEN
CIRCUIT ON RECEIVING END SIDE

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')

f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')

z = (R+j*2*pi*f*L*0.001);

y = (G+j*2*pi*f*Ct* 0.000001);

gm = sqrt(z*y);

zc = sqrt(z/y);

VSL = input('ENTER SENDING END VOLTAGE:')

VSph = VSL/sqrt(3);

VRph = VSph/(cosh(gm*l));

VRL = VRph*sqrt(3);

VRA = angle(VRL)*(18 0/pi);

IS = (sinh(gm*l)*VRph)/zc* 1000;
```

```
ISA =angle(IS)*(180/pi);

pfs = cos(ISA*(pi/180));

XL = abs(sinh(gm*l)*zc/(1-cosh(gm*l)));

Q = (VSL^2)/XL;

fprintf('RECEIVING END VOLTAGE = %g KV AT
(in DEGREES) %g\n', VRL,VRA)

fprintf('SENDING END CURRENT=%g AMPS AT
(in DEGREES) % g\n ',abs(IS), ISA)

fprintf('SENDING END POWER FACTOR = %g\n
',pfs)

fprintf('SHUNT REACTOR REACTANCE (OHMS) =%
g\n',XL)

fprintf('SHUNT REACTOR RATING (MVar) =%g
',Q)
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km: 0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km: 0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km :0.02

Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km: 0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km: 200

l = 200
```

```
ENTER THE FREQUENCY OF THE SYSTEM IN Hz50

f = 50

ENTER SENDING END VOLTAGE: 220

VSL = 220

RECEIVING END VOLTAGE = 228.887 KV AT (in
DEGREES) -0.110905

SENDING END CURRENT = 163.907 AMPS AT (in
DEGREES) 89.9253

SENDING END POWER FACTOR = 0.00130404 LEAD

SHUNT REACTOR REACTANCE (OHMS) = 1581.17

SHUNT REACTOR RATING (MVAR) =30.6103
```

Programme-5: To find the sending-end and receiving-end currents under short circuit at the receiving-end

A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μF/phase/km. Calculate the receiving-end and the sending-end currents when the line is short-circuited at the receiving-end, if the sending-end line-to-line voltage is 220 kV.

```
% TO FIND IR & IS FOR SHORT CIRCUIT ON
RECEIVING END

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')
```

```
l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')

f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')

z = (R+j*2*pi*f*L*0.001);

y = (G+j*2*pi*f*Ct* 0.000001);

gm = sqrt(z*y); zc = sqrt(z/y);

VSL = input('ENTER SENDING END VOLTAGE:')

VSph= VSL/sqrt(3);

IR = VSph/(zc*sinh(gm*l))* 1000;

IRA = angle(IR)*(180/pi);

IS = cosh(gm*l)*IR;

ISA = angle(IS)*(180/pi);

fprintf('RECEIVING END CURRENT = % g AT
(in DEGREES) %g\n ',abs(IR),IRA)

fprintf('SENDING END CURRENT = % g AT (in
DEGREES) %g\n ',abs(IS),ISA)
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02

Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0
```

```
ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:200

l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz50

f = 50

ENTER SENDING END VOLTAGE:22 0

VSL = 220

RECEIVING END CURRENT (A) = 2066.4 AT (in
DEGREES) -87.275

SENDING END CURRENT (A) = 1986.17 AT (in
DEGREES) -87.1641
```

Programme-6: To find the shunt capacitor rating

> **A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μF/phase/km. The receiving-end load is 400 MVA at 0.85 pf lagging. Calculate the MVAr and the capacitance of the shunt capacitors to be installed at the receiving-end to keep the receiving-end voltage at 220 kV when the line is energized with 220 kV at the sending-end.**

```
% TO FIND SHUNT CAPACITOR RATING

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct=input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')
```

```
f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')
z = (R+j*2*pi*f*L*0.001);
y = (G+j*2*pi*f*Ct* 0.000001);
gm = sqrt(z*y);
zc = sqrt(z/y);
A = cosh(gm*l);
B = zc*sinh(gm*l);
C = sinh(gm*l)/zc;
D = cosh(gm*l);
TM = [A B;C D];
Xn = zc*sinh(gm*l);
anA = angle(A);
anB = angle(B);
VSL = input('ENTER SENDING END LINE TO
LINE VOLTAGE:')
VRL = input('ENTER DESIRED RECEIVING END
LINE TO LINE VOLTAGE:')
SR = input('ENTER RECEIVING END LOAD IN
MVA:')
pfr = input('ENTER RECEIVING END POWER
FACTOR:')
h = acos(pfr);
SR = SR*(cos(h)+j*sin(h));
IRb = conj(SR)/(sqrt(3)*conj(VRL));
SMb = TM*[VRL/sqrt(3);IRb];
VSb = SMb(1,1)*sqrt(3);
ISb = SMb(2,1);
```

```
VREGb = (abs(VSb)/(abs(A))-
(abs(VRL)))/(abs(VRL))*100;

pfsb = cos(angle(VSb)-angle(ISb));

PR = real(SR);

QR = imag(SR);

delta = anB-acos ((PR + (abs (A)* (((abs
(VRL) A2)*cos (a n B - anA))/abs(B))))*
(abs(B)/(abs(VSL)*abs(VRL))));

QRn= ((abs(VSL)*abs(VRL)/abs(B))*sin(anB-
delta))-(abs(A)*(((abs(VRL)A2)*sin(anB-
anA))/abs(B)));

QC = (QRn-QR)*j;

XC = (abs(VRL)A2)/conj(QC);

Cn = (10A6)/(2*pi*f*abs(XC));

IC = VRL/(sqrt(3)*XC);

ICA =angle(IC)*(180/pi);

SR = PR+j*QRn;

IR = conj(SR)/(sqrt(3)*conj(VRL));

IRA = angle(IR); pfr = cos(IRA);

VS = (A*(VRL/sqrt(3))+B*IR)*sqrt(3);

VSA = angle(VS);

IS = ((sinh(gm*l)*VRL/sqrt(3))/zc) +
(cosh(gm*l)*IR);

ISA = angle(IS);

pfs = cos(VSA-ISA);

SS = sqrt(3)*VS*conj(IS);

PS = real(SS);

QS = imag(SS);

PL = PS-PR;
```

```
QL = QS-QRn;

VREG = (abs(VS)/(abs(A))-
(abs(VRL)))/(abs(VRL))*100;

eff = PR/PS*10 0;

fprintf('VOLTAGE REGULATION IN PERCENTAGE
BEFORE : %g\n',VREGb)

fprintf('POWER FACTOR BEFORE: %g',pfsb)

fprintf('SENDING END VOLTAGE (L-L KV) = %g
AT (in DEGREES) %g\n',abs(VS),VSA*
(180/pi))

fprintf('RECEIVING END VOLTAGE (L-L KV)
=%g\n',VRL)

fprintf('ACTIVE LOAD POWER = %g AND
REACTIVE LOAD POWER = %g\n',PR,QR)

fprintf('SHUNT CAPACITOR RATING %g OHMS,
%g uF, (inMVAR)%g\n',abs(XC),Cn,abs(QC))

fprintf('SHUNT CAPACITOR CURRENT = %g at
(in degrees) %g\n',abs(IC)* 1000,ICA)

fprintf('Pr = %g\n Qr = %g\n Ir = %g at in
degrees %g\n Is = %g at in degrees %g\n
pfr = %g\n

pfs = %g\n        PS = %g\n        QS =
%g\n       PL = %g\n       QL = %g\
',PR,QRn,abs(IR)*1000,IRA*
(180/pi),abs(IS)*1000,ISA*
(180/pi),pfr,pfs,PS,QS,PL,QL)

fprintf('VOLTAGE REGULATION IN PERCENTAGE
: %g\n',VREG)

fprintf('EFFICIENCY OF TRANSMISSION LINE :
%g',eff)
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02

Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:200

l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz:
50

f = 50

ENTER SENDING END LINE TO LINE VOLTAGE IN
kV:220

VSL = 220

ENTER DESIRED RECEIVING END LINE TO LINE
VOLTAGE IN kV:220

VRL = 220

ENTER RECEIVING END LOAD IN MVA:400

SR = 400

ENTER RECEIVING END POWER FACTOR:0.85

pfr = 0.8500

VOLTAGE REGULATION IN PERCENTAGE BEFORE :
37.1106
```

```
SENDING END POWER FACTOR BEFORE: 0.742582

EFFICIENCY IN PERCENTAGE BEFORE : 72.61

SENDING END VOLTAGE (L-L KV) = 220 AT (in
DEGREES) 25.9181

SHUNT CAPACITOR RATING = 175.634
OHMS,18.1235 mF, 275.573 MVAR

SHUNT CAPACITOR CURRENT(A) = 723.192 at
(in degrees) 90

VOLTAGE REGULATION IN PERCENTAGE AFTER:
4.03971

EFFICIENCY OF TRANSMISSION LINE AFTER :
97.8277
```

Programme-7: To construct locus for P_R verses Q_R

A three-phase, 50 Hz, transmission line is 200 km long. The line parameters are r = 0.015 ohms/phase/km, L = 0.99 mH/phase/km, C = 0.02 μF/phase/km. Construct locus for P_R versus Q_R.

```
% TO FIND RECEIVING END POWER CIRCLE
DIAGRAM

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

l = input('ENTER THE VALUE OF LENGTH OF
THE LINE IN Km:')
```

```
f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')

z = (R+j*2*pi*f*L*0.001);

y = (G+j*2*pi*f*Ct* 0.000001);

gm = sqrt(z*y);

zc = sqrt(z/y);

A = cosh(gm*l);

B = zc*sinh(gm*l);

C = sinh(gm*l)/zc;

D = cosh(gm*l);

TM = [A B;C D];

VR = input('ENTER RECEIVING END LINE
VOLTAGE : ')

VS = input('ENTER SENDING END LINE
VOLTAGE:')

anA = angle(A);

anB = angle(B);

for VS = VR:0.05*VR:1.2*VR

for delta =1:38

PR (delta) = ((abs (VS)*abs (VR) /abs (B))
*cos (anB-delta* (pi/180))) - (abs(A)*
(((abs(VR)^2)*cos(anB-anA))/abs(B)));

QR (delta) =
((abs(VS)*abs(VR)/abs(B))*sin(anB-delta*
(pi/180)))- (abs(A)*(((abs(VR)^2)*sin(anB-
anA))/abs(B)));

end

xlabel('Pr (KW)')

ylabel('Qr (KVAR)')

plot(PR,QR)
```

```
hold on
end
```

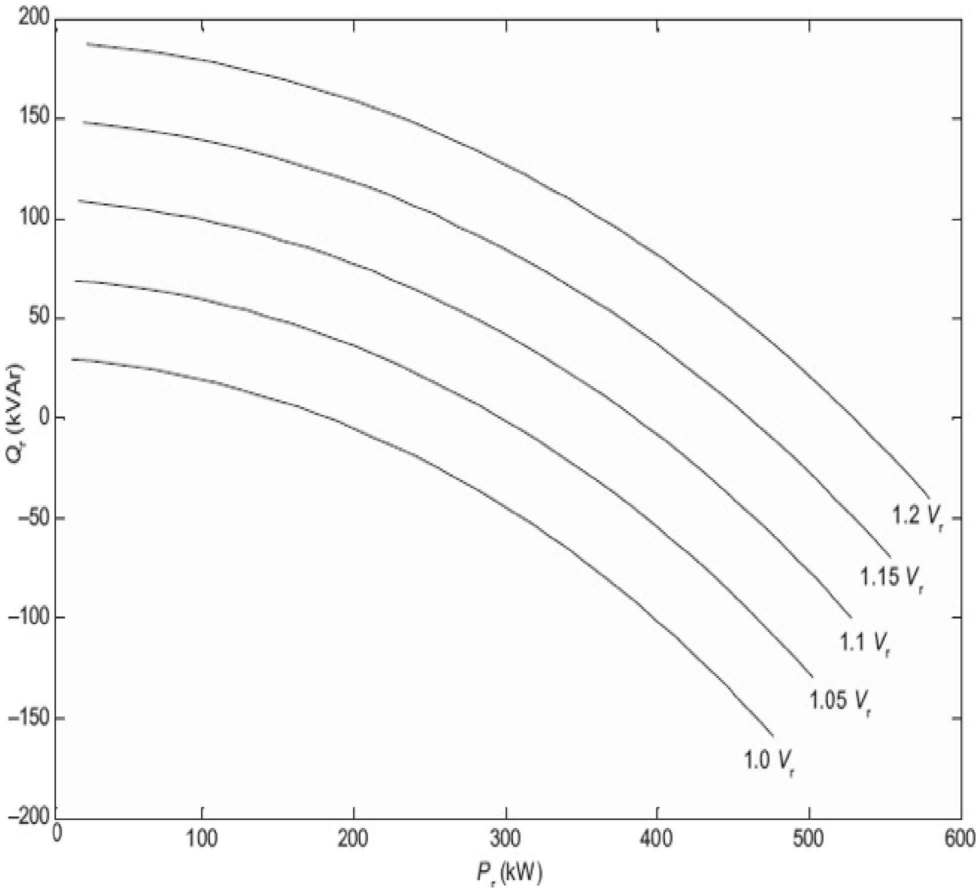

Fig. Plot for P_r verses Q_r

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02
```

```
Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE VALUE OF LENGTH OF THE LINE IN
Km:200 l = 200

ENTER THE FREQUENCY OF THE SYSTEM IN Hz50

f = 50

ENTER RECEIVING END LINE VOLTAGE : 220

VR = 220

ENTER SENDING END LINE VOLTAGE:220

VS = 220
```

Programme-8: To draw the no-load voltage profile with respect to line length

A three-phase, 50 Hz, 500 kV, transmission line is 200 km long. The line parameters are *r* = 0.015 ohms/phase/km, *L* = 0.99 mH/phase/km, *C* = 0.02 μP/phase/km. Draw the no-load voltage profile with respect to length of the line.

```
% TO FIND NOLOAD VOLTAGE PROFILE

clc;

clear all;

R = input('ENTER THE VALUE OF
RESISTANCE/PHASE OF THE LINE IN Ohms/Km:')

L = input('ENTER THE VALUE OF
INDUCTANCE/PHASE OF THE LINE IN mH/Km:')

Ct = input('ENTER THE VALUE OF
CAPACITANCE/PHASE OF THE LINE IN uF/Km:')

G = input('ENTER THE VALUE OF
CONDUCTANCE/PHASE OF THE LINE IN
mSeimens/Km:')

f = input('ENTER THE FREQUENCY OF THE
SYSTEM IN Hz')
```

```
z = (R+j*2*pi*f*L*0.001);
y = (G+j*2*pi*f*Ct* 0.000001);
gm = sqrt(z*y);
zc = sqrt(z/y);
VS = 500;
for l = 1:200
VR(l) = VS/cosh(gm*l);
end
xlabel('length(KM)')
ylabel('Vr (kv)')
plot(1:200,abs(VR))
```

Result:

```
ENTER THE VALUE OF RESISTANCE/PHASE OF THE
LINE IN Ohms/Km:0.015

R = 0.0150

ENTER THE VALUE OF INDUCTANCE/PHASE OF THE
LINE IN mH/Km:0.99

L = 0.9900

ENTER THE VALUE OF CAPACITANCE/PHASE OF
THE LINE IN uF/Km:0.02

Ct = 0.0200

ENTER THE VALUE OF CONDUCTANCE/PHASE OF
THE LINE IN mSeimens/Km:0

G = 0

ENTER THE FREQUENCY OF THE SYSTEM IN Hz50

f = 50
```

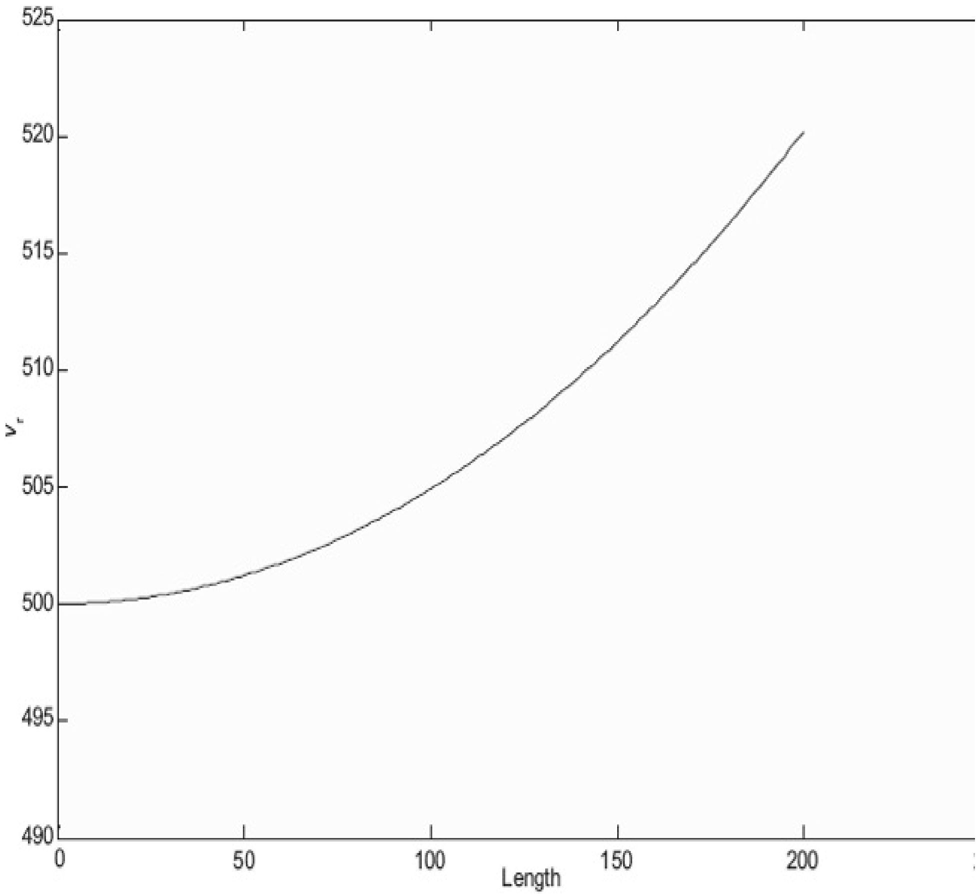

Fig. No load voltage verses Line length

CHAPTER 7

EXAMPLE 7.8

An overhead line is erected across a span of 200 m on level supports. The conductor has a diameter of 1.25 cm and a dead weight of 1.09 kg/m. The line is subjected to wind pressure of 37.8 kg/sq. m of the projected area. The radial thickness of ice is 1.25 cm. Calculate the sag, (i) in an inclined direction, (ii) in a vertical direction. Assume a maximum working stress of 1,050 kg/sq. cm. One cubic meter of ice weighs 913.5 kg.

MATLAB programme for Example 7.8

```
clc;

clear all;

L=input('enter span length in metres:')
```

```
d=input('enter the diametre of the
conductor in cm:')

w=input('enter weight of the conductor:')

wp=input('enter wind pressure in kg/sq.cm:
')

t=input('enter thickness of ice:')

ws=input('enter working stress:')

wc=input('enter weight of ice:')

T=ws*(pi/4)*(d^2);

wi=wc*pi*t*(d+t)* 0.0001;

ww=wp*(d+2*t)*0.01;

wr=sqrt((w+wi)^2+ww^2);

D=wr*L^2/(8*T)

D1 = D*(w+wi)/wr
```

Result:

```
enter span length in metres:200

L = 200

enter the diametre of the conductor in
cm:1.25

d = 1.2500

enter weight of the conductor:1.0 9

w = 1.0900

enter wind pressure in kg/sq.cm:37.8

wp = 37.8000

enter thickness of ice:1.25

t = 1.2500

enter working stress:1050

ws = 1050
```

```
enter weight of ice:913.5
wc = 913.5000
D(m) = 9.4706
D1(m) = 7.7096
```

EXAMPLE 7.9

A transmission line conductor at a river crossing is supported from two towers at heights of 60 and 80 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductor is 2000 kg, find (i) the maximum clearance between the conductor and water, (ii) the clearance between the conductor and water at a point mid way between the towers. Weight of conductor is 0.844 kg/m. Assume that the conductor takes the shape of a parabola.

MATLAB programme for Example 7.9

```
% TO FIND SAG of unequal supports
clc;
clear all;
H1=input('ENTER THE HEIGHT OF FIRST TOWER IN metres:')
H2 = input('ENTER THE HEIGHT OF THE SECOND TOWER IN metres:')
h = abs(H1-H2);
L=input('ENTER THE SPACING BETWEEN TWO TOWERS IN metres:')
T=input('ENTER THE TENSION IN THE CONDUCTOR:')
W = input('ENTER WEIGHT OF THE CONDUCTOR IN kg:')
x1=(L/2)-(T*h/(W*L));
x2=(L/2) + (T*h/(W*L));
D1=W*x1^2/(2*T);
mc=H1-D1;
```

```
x=(L/2)-x1;

D=W*x^2/(2*T); mpc=59.9865+D;

fprintf('MINIMUM CLEARENCE BETWEEN THE
WATER metres: %g\n',mc)

fprintf('CLEARENCE BETWEEN THE WATER LEVEL
CONDUCTOR in metres: %g\n',mpc)
```

Result:

```
ENTER THE HEIGHT OF FIRST TOWER IN
metres:60

H1 = 60

ENTER THE HEIGHT OF THE SECOND TOWER IN
metres:80

H2 = 80

ENTER THE SPACING BETWEEN TWO TOWERS IN
metres:300

L = 300

ENTER THE TENSION IN THE CONDUCTOR:2000

T = 2000

ENTER WEIGHT OF THE CONDUCTOR IN kg:0.844

W = 0.8440

MINIMUM CLEARENCE BETWEEN THE WATER LEVEL
AND LOWER POINT OF THE CONDUCTOR in
metres: 59.9866

CLEARENCE BETWEEN THE WATER LEVEL AND
CONDUCTOR AT THE MID POINT OF THE
CONDUCTOR in : 65.2524
```

EXAMPLE 7.10

An overhead conductor consists of seven strands of silicon-bronze having cross-sectional area of 2.0 cm^2 and ultimate strength of 10000 kg/cm^2. When connected between supports 650 m apart and having a 20 m difference in level, determine the vertical sag, which must be allowed such that

the factor of safety shall be 5. Assume the wire weight 2 kg/m, ice loading is 1 kg/m and wind loading is 1.75 kg/m.

MATLAB programme for Example 7.10

```
% TO FIND VERTICAL SAG

clc;

clear all;

a=input('ENTER CROSS-SECTIONAL AREA OF THE
CONDUCTOR IN sqcm:')

S=input('ENTER STRENGTH OF THE CONDUCTOR
IN Kg/sqcm:')

D=input('ENTER THE SPACING BETWEEN TOWERS
in metres:')

h=input('ENTER THE DIFFERENCE BETWEEN THE
CONDUCTOR ENDS in metres:')

SF=input('ENTER SAFETY FACTOR:')

W=input('ENTER WEIGHT OF THE WIRES IN
Kg/m:')

IL=input('ENTER ICE LOADING IN Kg/m:')

WL=input('ENTER WIND LOADING IN Kg/m:')

TS = S*a;

MT=TS/SF;

F = sqrt((W+IL)^2+WL^2);

x1 = D/2-(MT*h/(F*D));

D1 = F*x1^2/(2*MT);

VS=(IL+W)*(x1A2)/(2*MT);

fprintf('SLANT SAG in metres: %g\n',D1)

fprintf('VERTICAL SAG in metres: %g',VS)
```

Result:

```
ENTER CROSS-SECTIONAL AREA OF THE
CONDUCTOR IN sqcm:2
```

```
a = 2

ENTER STRENGTH OF THE CONDUCTOR IN
Kg/sqcm:10000

S = 10000

ENTER THE SPACING BETWEEN TOWERS in
metres:650

D = 650

ENTER THE DIFFERENCE BETWEEN THE CONDUCTOR
ENDS in metres:20

h = 20

ENTER SAFETY FACTOR:5

SF = 5

ENTER WEIGHT OF THE WIRES IN Kg/m:2

W = 2

ENTER ICE LOADING IN Kg/m:1

IL = 1

ENTER WIND LOADING IN Kg/m:1.75

WL = 1.7500

SLANT SAG in metres: 36.4011

VERTICAL SAG in metres: 31.4425
```

CHAPTER 8

EXAMPLE 8.13

A string of suspension insulators consists of 6 units. If the maximum peak voltage per unit is 33 kV, calculate (i) the maximum voltage for which this string can be used, (ii) Voltage across the third unit from the bottom (iii) the string efficiency. Assume capacitance between each pin and earth as 12% of the self-capacitance of each unit.

MATLAB programme for Example 8.13

```
% TO FIND STRING EFFICIENCY
```

```
clc;
clear all;
n=input('ENTER THE NUMBER OF INSULATER
UNITS:')
v=input('ENTER MAXIMUM PEAK VOLTAGE PER
UNIT IN KV:')
m=input('ENTER CAPACITANCE BETWEEN EACH
PIN AND EARTH AS A PERCENT OF THE SELF
CAPACITANCE OF EACH UNIT:')
m=m/10 0;
b=1;
Vmax=(v*
(sinh(n*sqrt(m))))/(2*sinh(sqrt(m)/2)*cosh
((n-b+0.5)*sqrt(m)));
b=input('ENTER TO WHICH UNIT FROM THE
BOTTOM VOLTAGE IS REQUIRED:')
V=(Vmax*2*sinh(sqrt(m)/2)*cosh((n-
b+0.5)*sqrt(m)))/(sinh(n*sqrt(m)));
Streff=Vmax/(n*v)*10 0;
fprintf('Vmax (in volts) = %g\n',Vmax)
fprintf('VOLTAGE at %g UNIT FROM THE
BOTTOM (in volts) = %g\n',b,V)
fprintf(' PERCENTAGE STRING EFFICIENCY =
%g \n',Streff)
```

Result:

```
ENTER THE NUMBER OF INSULATER UNITS:6
n=6
ENTER MAXIMUM PEAK VOLTAGE PER UNIT IN KV:
33
v = 33
```

ENTER CAPACITANCE BETWEEN EACH PIN AND EARTH AS A PERCENT OF THE SELF CAPACITANCE OF EACH UNIT: 12

m = 12

ENTER TO WHICH UNIT FROM THE BOTTOM VOLTAGE IS REQUIRED: 3

b=3

Vmax (kV) = 10 8.546

VOLTAGE at 3 UNIT FROM THE BOTTOM (kV) = 17.5769

PERCENTAGE OF STRING EFFICIENCY = 54.8212

Glossary

AC filter Filter used to reduce the harmonic components in the AC power flow.

Annual load factor It is the ratio of average demand to maximum demand in a particular year.

Audio noise Noise having frequencies upto 20 kHz.

Bipolar link HVDC transmission link that has two conductors, one at positive potential and the other negative (same magnitude) with respect to the ground.

Booster transformer Transformer which boosts either the voltage or the current.

Brown-Boveri regulator Regulator in which resistance is gradually varied either continuously or in small steps.

Bundled conductors Aconductor which is formed by two or more than two sub- conductors in each phase.

Bus bar A conductor carrying current to which many connections are made.

Cable The combination of conductor and the insulation over the conductor.

Capacitance grading The process of achieving uniform distribution in dielectric stress by applying layers of different dielectric materials for the dielectrics.

Capacitor It is defined as the charge per unit potential difference.

Charging current in lines Current produced by the line capacitance.

Circuit breaker Device that makes or breaks the electrical circuit under all conditions.

Converting substation Substation that is used for converting AC into DC or vice versa.

Copper loss Losses that occur mainly due to the presence of resistance in the cables.

Corona The phenomenon of the ionization of air surrounding the power conductors

Critical disruptive voltage The minimum voltage at which the ionization just takes place.

Current transformer Transformers that change the current from higher values to the measuring level current.

DC filter Filter used to reduce the ripple content in the voltage or current on DC line.

Delta connected load If starting end of one load is connected to finishing end of other load in the form of loop. Then it is called as delta connected load.

Dielectric loss The losses occurring in the dielectric of cables which are mainly due to leakage of charging currents.

Distributed parameters An element in which resistors, inductors, and capacitors cannot be electrically separated and individually isolated as separate elements.

Distribution substation Substation which step-down the voltage from the distribution voltage to the voltage needed by consumers.

Distribution system It is a part of power system, which is in between distribution sub- station and the consumers.

Distributor Conductors which run along a street or an area to supply the power to consumers.

Earth electrode A metal plate or pipe or other conductor or an array of conductors electrically connected to the general mass of the earth.

Earthing lead The conductor by which the electrode is connected to earth.

Economical transmission voltage The transmission voltage for which the cost of conductors and cost of insulators, transformers, switchgear and other terminal apparatus is minimum.

Efficiency of a transmission line The ratio of receiving-end power to the sending-end power.

External pressure cables Cables that are developed for higher voltages for which pressure is applied externally and raised to such an extent that no ionization can take place **in the cable**.

Extra high voltage transmission Transmission which involves voltages in between 330 kV and 1000 kV.

FACTS controller A power electronic-based system and other static equipment that provide control of one or more AC transmission system parameters.

Feeders Conductors which connect the substations to the areas to be served.

Ferranti effect The phenomenon of rise in voltage at the receiving end **due to** open circuited or lightly loaded line (**condition**).

Filters Device used to filter the unwanted signals.

Flexibility of electric power transmission The ability to accommodate changes in the electrical power transmission system or operating conditions while maintaining sufficient steady state and transient limits.

Flexible AC transmission system (FACTS) Alternating current transmission systems incorporating power electronic-based and other static controllers to enhance controllability and increase power transfer capability.

Frequency change substation Substation that is used for converting normal frequency to required frequency.

Gas cushions cables Cables in which the screened space is provided in between the lead sheath and the dielectric along the length of the cable for the flow of gas.

Gas filled cables Cable which consists of a special duct for the flow of dry nitrogen gas through the cable, the pressure of gas being 1400 kN/sq. m.

Grading of cables The process of achieving uniform distribution in dielectric stress.

Grounding grid The number of rods joined together through copper conductors which reduces the overall grounding resistance.

Grounding transformer A core type of transformer having three limbs built up in the same manner as that of power transformer used if a neutral point is required or it is not available in case of delta connection and bus-bar points.

Guard ring Device with which voltage across each disc of a string can be maintained constant.

High tension cables Cables used for power flow for voltages upto 11 kV.

Homo polar link HVDC transmission link that has two or more conductors all having the same polarity and ground is used as return path.

Impregnated paper Insulator in case of power cables having low capacitance and high dielectric strength, manufactured with wood pulp rags or plant fiber by a suitable chemical process.

Impregnated pressure cable Cable which consists of mass-impregnated paper as dielectric which is maintained under a pressure of 14 atmospheres by means of nitrogen.

Indoor substation Substation in which apparatus of a substation are installed with in a building.

Inductance The property of the conductor by virtue of which it opposes any change in direction or magnitude of current flowing through itself.

Industrial substation Substation that is used to supply power only for industrial consumers.

Instrument transformers Transformers are used to protect personnel and apparatus from high voltage and to permit the use of reasonable voltage level and current carrying capacity in relays and motors.

Insulator Material which does not allow the current to flow through it.

Interphase power controller (IPC) A series connected controller of active and reactive power consisting, in each phase, of inductive and capacitive branches subjected to separately phase-shifted voltages.

Intersheath grading The process of achieving uniform distribution in dielectric stress by providing metallic intersheath between successive layers of the same dielectric materials and maintaining appropriate potential level at the intersheath.

Inversion The process of conversion of DC to AC.

Isolators Device used for isolating the electrical circuits under no load.

Kelvin law The most economical size of the copper conductor for the transmission of electric energy can be determined by comparing the annual interest on capital cost of conductor plus depreciation with the cost of the energy wasted annually.

Lightning arresters Device which bypasses the high voltage surges to the ground.

Load duration curve It is a plot of the load demand arranged in a descending order of magnitude on the *y*-axis and the time in hours on the *x*-axis.

Load end capacitance method Method in which the entire line capacitance is assumed to be concentrated at the receiving end.

Load shedding It is by which parts of an electric power system are disconnected in an attempt to prevent failure of the entire system due to overloading.

Long lines The transmission line having length above 160 km with an operating voltage above 100 kV.

Low tension cables Cables used for power flow for voltages below 1 kV.

Medium transmission line The transmission line having length between 80 km and 160 km with an operating voltage from 20 kV to 100 kV.

Metallic sheath Layer provided over the insulation which protects the insulation material from moisture, gases and any other harmful liquids which might persist underground.

Monopolar link HVDC transmission link that has only one conductor with ground or sea as a return path.

Most economical power factor The value to which the power factor should be improved so as to have maximum net annual saving.

Nominal T-method Method in which the capacitance of each conductor is concentrated in the middle of the line while the series impedance is split into equal parts.

Nominal-π method It is the method in which the capacitance of each conductor is divided into two halves, one half being shunted across receiving end and the other half across the sending end of the line and the total impedance is placed in between them.

Oil filled cables Cables containing a special duct for the flow of mineral oil through them and these are used to carry voltages from 33 kV to 400 kV.

Outdoor substations Substation in which apparatus of a substation are installed in an open place.

Penstock It is a conduit which supplies water to the turbines from the water reservoir.

Phase advancers Special commutator machines, which are used to improve the power factor of the induction motor.

Poly vinyl chloride (PVC) A synthetic compound material which acts as an insulator and comes as white odorless, tasteless, chemically inert, non-inflammable and insoluble powder.

Potential transformer Transformers that change the Voltages from higher values to the measuring level voltage.

Power factor The ratio of kW (real power) component to the kVA (apparent power) component or the cosine of angle between voltage and current in an AC circuit.

Power transformers Transformer used to step up the voltage from the available voltage.

Primary distribution Electrical supply system which is in between the distribution substations and the distribution transformers.

Primary substation Substation which step down the voltage from transmission voltage to sub-transmission voltage.

Radio interference The radio interference is a noise type that occurs in the AM radio reception.

Rectification The process of conversion of AC to DC.

Regulation The change in voltage at the receiving end when full load is thrown off; with constant sending end voltage.

Relay Device that detects the fault and initiate the operation of the circuit breaker.

Resistance Voltage per unit current at constant temperature.

Safety factor The ratio of puncture strength to flash over voltage.

Sag The difference in level between the points of supports and the lower point on the conductor.

Secondary distribution Electrical supply system which is in between the secondary side of distribution transformer and supplies power to various connected loads.

Secondary substation Substation which step down the voltage from sub transmission voltage to distribution voltage.

Service mains Conductors connecting links between distributor and metering point of the consumer's terminal.

Short transmission line The transmission line having length less than 80 km with an operating voltage upto 20 kV.

Shunt reactor Device used to absorb the reactive power from the line to control the voltage under lightly loaded condition.

Skin effect The tendency of AC current to concentrate near the surface of a conductor.

Solid earth A conductor is said to be solidly earthed when it is electrically connected to an earth electrode without intentional addition of resistance or impedance in the earth connection.

Span The distance between adjacent supporting towers is called the span.

Spinning reserve It is the generating capacity which is connected to bus bar and is ready to take load.

Star connected load If all the finishing ends of three loads are connected at one common point, it is called as star-connected load.

Static capacitor Capacitors that are connected across the mains at the load end for improving the power factor.

Static switchable shunt capacitor bank Device which injects the capacitive power at suitable location of the line for maintaining the bus voltage within the limits.

Static synchronous compensator (SSC or STATCOM) A static synchronous generator operated as a shunt-connected static VAr compensator.

Static synchronous series compensator (SSSC) A static, synchronous generator operated without an external electric energy source as a series compensator.

Static VAr compensator (SVC) A static VAr generator or absorber whose output is adjusted to exchange capacitive or inductive current so as to control or maintain specific parameters of the power system.

Static VAr generator or absorber (SVG) A static electrical device, equipment, or system which is capable of drawing controlled inductive and/or capacitive current from an electrical power system.

Static VAr system (SVS) A combination of different static and mechanically switched VAr compensators whose outputs are coordinated.

String efficiency The ratio of voltage across the whole string to the product of the number of discs and the voltage across the disc nearer to the line conductor.

Stringing chart Graph that represents the relation between temperature and sag and temperature and tension at a fixed span of a given transmission line under loading conditions.

Sub main Several connections of consumers taken from one service mains

Substation It is an assembly of apparatus installed to perform switching, voltage transformation, power factor correction, power and frequency converting operations.

Super tension cables Cables used for power flow for voltages upto 33 kV

Superconducting magnetic energy storage (SMES) Electronic converter that rapidly injects and/or absorbs real and/or reactive power or dynamically controls power flow in an AC system.

Surge impedance loading (SIL) The MW loading at which the line's MVAr usage is equal to the line's MVAr production.

Switching substation substation that is used for switching operations of power lines.

Switchgear Device used for switching, controlling and protecting the electrical circuits and equipments.

Synchronous condenser Synchronous motor running without a mechanical load.

Synchronous substation Substation in which synchronous phase modifiers are installed for improvement of power factor of the system.

Tap changing transformer Transformer having tap setting arrangement in its secondary circuit.

Thyristor controlled braking resistor (TCBR) A shunt connected thyristor-switched resistor, which is controlled to aid stabilization of a power system or to minimize power acceleration of a generating unit during a disturbance.

Thyristor controlled phase shifting transformer (TCPST) A phase shifting transformer, adjusted by thyristor switches to provide a rapidly variable phase angle.

Thyristor controlled reactor (TCR) A shunt connected thyristor-controlled inductor whose effective reactance is varied in a continuous manner by partial-conduction control of the thyristor valve.

Thyristor controlled series capacitor (TCSC) A series capacitor bank shunted by thyristor controlled reactor in order to provide a smooth variable series capacitive reactance.

Thyristor controlled series compensation An impedance compensator which is applied in series on an AC transmission system to provide smooth control of series reactance.

Thyristor controlled series reactor (TCSR) A series inductive reactor shunted by a thyristor controlled reactor in order to provide a smooth variable series inductive reactance.

Thyristor controlled voltage limiter (TCVL) A thyristor-switched Metal-Oxide Varistor (MOV) used to limit the voltage across its terminals during transient conditions.

Thyristor switched capacitor (TSC) A shunt connected thyristor-switched capacitor whose effective reactance is varied in a stepwise manner by full or zero operation conduction of the thyristor valve.

Thyristor switched reactor (TSR) A shunt connected thyristor-switched inductor whose effective reactance is varied in a stepwise manner by full or zero conduction operation of the thyristor valve.

Thyristor switched series capacitor (TSSC) A capacitive reactance compensator which consists of a series capacitor bank shunted by a thyristor switched reactor to provide a step-wise control of series capacitive reactance.

Thyristor switched series compensation An impedance compensator which is applied in series with an AC transmission system to provide a step-wise control of series reactance.

Thyristor switched series reactor (TSSR) An inductive reactance compensator which consists of series reactor shunted by thyristor switched reactor in order to provide a step- wise control of series inductive reactance.

Tirril regulator A vibrating type voltage regulator, in which a fixed resistance is cut in and cutout in the exciter field circuit of the alternator.

Transformer substation Substation which transform power from one voltage to another voltage.

Transformer Device used to change the voltage from one level to another level.

Tuned transmission lines If the sending end voltage and current line is numerically equal to the receiving end voltage and current **of the line** respectively.

Ultra high voltage transmission Transmission which involves voltages in the range of 1000 kV, 1100 kV, 1200 kV and above.

Unified power flow controller (UPFC) A combination of a STATCOM and a SSSC which are coupled via, a common DC link, to allow bi-directional flow of active power.

VAr compensating system (VCS) A combination of different static and rotating VAr compensators whose outputs are coordinated.

Visual critical voltage The minimum voltage at which the corona just becomes visible.

Voltage stability A power system at a given operating state and subjected to a given disturbance is voltage stable if voltages near loads approach post-disturbance equilibrium values.

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