UNIT-II

Short circuit Analysis

Per-Unit System of representation - One line diagram.

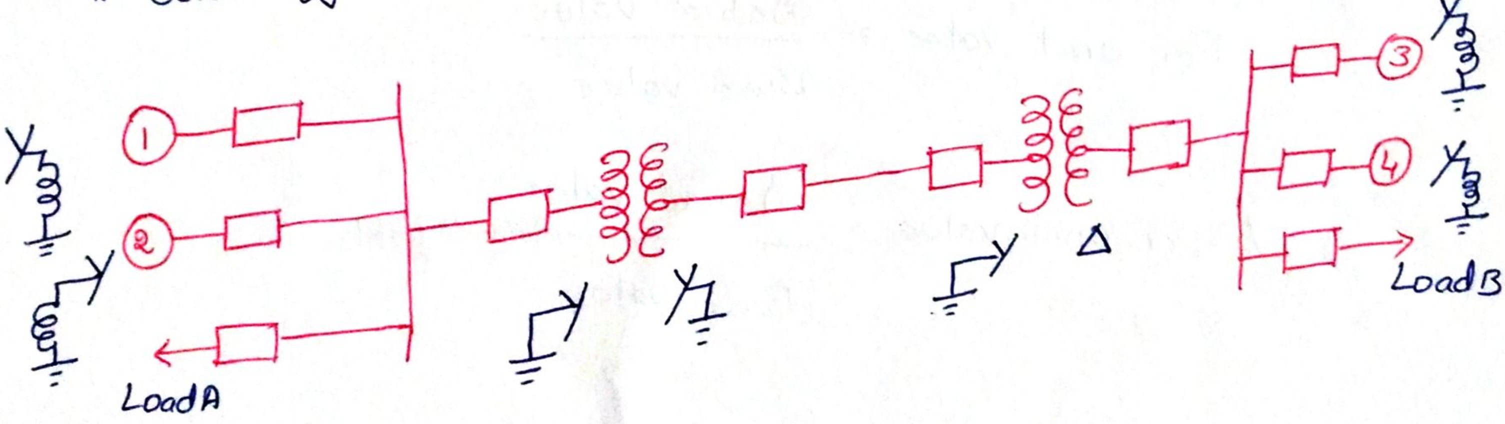
* The complete diagram of a power system representing all the three phases becomes too complicated for a system of practical size, so it is much more practical to represent a power system by means of simple symbols for each components resulting which is called one line diagram.

* One line diagram of a power system shows the main connections arrangement of components.

* The points that are followed for constructing the one line diagram are

1. Using suitable symbols, represent the generators, motors, transformers and loads.
2. Circuit breakers are represented as rectangular blocks.

* Below fig. shows the one line diagram of a simple power system.



Per-unit equivalent reactance

Per-Unit quantities-

* The components or various sections of power system may operate at different voltage and power levels.
* It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of components of power system are expressed with reference to a common value called base value.
* Hence for analysis purpose a base value is chosen for voltage, power, current and impedance.
* Then all the voltage, power, current impedance ratings of the components are expressed as a percent or per unit value of the base value.
* Per unit value of any quantity is defined as ratio to actual value to the chosen base value of same units and is expressed as a decimal.

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}}$$

$$\text{\% Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \times 100$$

Single phase system in per unit quantity

Let the base values are

Base power = Base KVA = KVA_b

Base voltage = Base voltage in kV = KV_b

Base current = Base current in Amp = I_b

Base Impedance = Base impedance in (Ω) = Z_b

Formulae related to various quantities:

Base current $I_b = \dfrac{KVA_b}{KV_b}$ in amps

Base impedance $Z_b = \dfrac{KV_b * 1000}{I_b}$ in Ω

From above equations

$$\text{Base impedance } Z_b = \frac{KV_b * 1000}{I_b}$$

$$= \frac{KV_b * 1000}{KVA_b / KV_b} = \frac{(KV_b)^2}{\frac{KVA_b}{1000}}$$

$$Z_b = \frac{(KV_b)^2}{MVA_b}$$

$$\text{PU impedance} = \frac{\text{Actual impedance } (\Omega)}{\text{Base impedance } (\Omega)}$$

3-ϕ system in PU Quantity

Base current $I_B = \frac{KVA_B}{\sqrt{3}\ KV_B}$ amp.

Base impedance $Z_B = \frac{KV_B \times 1000}{\sqrt{3} * I_B}$

$$= \frac{KV_B}{\sqrt{3}} * \frac{\sqrt{3}\ KV_B}{KVA_B} * 1000 = \frac{(KV_B)^2 * 1000}{KVA_B}\ \Omega$$

PU impedance $Z_{PU} = \frac{\text{Actual Imp}}{\text{Base Imp.}} = \frac{\text{Actual Imp} * KVA_B}{(KV_B)^2 * 1000}$

Change of base.

PU impedance to new base

$$Z_{punew} = Z_{puold} \times \frac{KVA_{new}}{KVA_{old}} * \frac{(KV_{old})^2}{(KV_{new})^2}$$

Pro. A 3-ϕ alternator with a rating of 10 MVA, 33 KV has its armature resistance of 15 Ω/ph and synchronous reactance of 80 Ω /ph. Determine PU impedance of the alternator.

Power = 10 MVA ; voltage = 33 KV

Choose the base values

Base power, MVA_b = 10 MVA

Base voltage, KV_b = 33 kV

Base impedance $Z_b = \frac{(KV_b)^2}{MVA_b}$

$$Z_b = \frac{(33)^2}{10} = 108.9\ \Omega$$

The actual value of impedance $Z = (15 + j80)\ \Omega$

P.U value $Z_{pu} = \frac{\text{Actual value}}{\text{Base value}}$

$$Z_{pu} = \frac{15 + j80}{109} = (0.138 + j0.735)\ PU$$

Pro. A single phase transformer of 11KV/400V, 50 Hz, 150 KVA has primary resistance and reactances are 2 Ω and 10 Ω. The secondary resistance & reactances are 0.01 Ω and 0.05 Ω respectively. Determine the P.U values of transformers.

Case(i) Referring to primary side

Base voltage KV_b = 11 KV

Base voltage MVA_b = 150 KVA = 0.15 MVA

Base impedance $Z_b = \frac{11^2}{0.15} = 806.7\ \Omega$

$$\text{Transformation ratio} = \frac{400}{11000} = 0.0364$$

Total resistance referring to primary side

$$R_{01} = r_1 + \frac{r_2}{k^2} = 2 + \frac{0.01}{(0.0364)^2} = 9.547\,\Omega$$

Total leakage reactance referring to primary side

$$X_{01} = x_1 + \frac{x_2}{k^2}$$

$$= 10 + \frac{0.05}{(0.0364)^2} = 47.74\,\Omega$$

Total impedance referring to primary side

$$Z_{01} = R_{01} + j\,X_{01} = (9.547 + j\,47.74)\,\Omega$$

Total impedance in p.u. value is

$$Z_{pu} = \frac{\text{Actual value}}{\text{Base value}} = \frac{9.547 + j\,47.74}{806.7}$$

$$= (0.0118 + j\,0.0592)\ \text{PU}$$

Case ii   Referring to secondary side

Total resistance referring to secondary side

$$R_{02} = r_2 + k^2 r_1$$

$$= 0.01 + (0.0364)^2 * 2 = 0.01265\,\Omega$$

Total leakage reactance referring to secondary side

$$X_{02} = x_2 + k^2 x_1$$

$$= 0.05 + (0.0364)^2\,10 = 0.0632\,\Omega$$

Total impedance referring to secondary side

$$Z_{02} = R_{02} + j\,X_{02} = (0.01265 + j\,0.0632)\,\Omega$$

Percentage reactance (Short circuit current & MVA calculations)

* The reactance of synchronous machines, generators, transformers, reactors etc. is usually expressed in percentage reactance to permit rapid short circuit calculation.
* The percentage reactance of a circuit is defined as.
* A reactance of X%. will have a voltage drop of IX of normal voltage when carrying full load current corresponding to normal rating or it is the percentage of total plate voltage drop in the circuit when full load current is flowing.

$$\% X = \frac{IX}{V} * 100 \quad — ①$$

where, I is the full load current

V - is the phase voltage

X is the reactance in ohms/phase.

* If X is the only reatance in the circuit, the short circuit current is given by V/X.
* Therefore, from equ. ①. short circuit current is given by

$$I_{sc} = \frac{V}{X} = I * \frac{100}{\% X} \quad — ②$$

* From equation ②, we can write

$$X = \frac{\% X V}{100 I} = \frac{(\% X) V^2}{100 VI}$$

$$= \frac{(\% X) \left[\frac{V}{1000}\right]\left[\frac{V}{1000}\right] * 1000}{100 \left[\frac{V}{1000}\right] I}$$

$$= \frac{[\% X][KV]^2 10}{KVA}$$

$$\% X = \frac{(KVA)\, X}{10\,(KV)^2} \quad \text{——— (3)}$$

The advantage of this method is that these values remain unchanged, as they are referred through transformers and this makes the procedure made simple and permits quick calculations.

Base KVA

* From equation (3), we may know that the percentage reactance of equipment depends upon its KVA rating. Since different equipments in any power system may have different KVA ratings and since the percentage reatance quoted by the manufatures as for the respective KVA ratings.
* The value of this base KVA is quite an important and may be
    (a) Equal to that of the largest plant capacity
    (b) Equal to that of the total plant capacity
    (c) Any arbitrary value.

The percentage reactance at base KVA

$$= \frac{\text{Base KVA}}{\text{Plant KVA}} * \text{Percentage reactance at plant KVA}$$

Short circuit MVA

The product of normal system voltage and short circuit current at the point of fault expressed in MVA is known as "short circuit MVA".

Let V = normal phase voltage in kV

I = Full load current in kA

$\%X$ = Percentage reactance of the system on base MVA upto the fault point.

From equation $I_{sc} = \frac{V}{X} = I * \frac{100}{\%X}$

$\therefore$ The Short circuit MVA for 3-ϕ circuit $= 3VI_{sc}$

$$= 3VI * \frac{100}{\%X}$$

$$= \text{Base MVA} * \frac{100}{\%X}$$

i.e Short circuit MVA is obtained by multiplying the base MVA by $\frac{100}{\%X}$

(i) When reactance is in percentage value

Short circuit current $I_{sc} = \frac{I*100}{\%X}$

Hence short circuit MVA is

$$= \frac{100}{\%X} * \text{base MVA}$$

$$= \left[\frac{100}{\%X} * \text{base kVA}\right] * 10^{-3}$$

Short circuit current interms of short circuit MVA and voltage is

$$= \frac{\text{Short circuit MVA} \times 10^{3}}{\text{Line to line kV} * \sqrt{3}} \text{ amp.}$$

(ii) When reactance is in ohms

$$\text{Short circuit current (rms)} = \frac{V}{\sqrt{3} \times X} \text{ amps}$$

$$\text{Short circuit MVA} = \frac{\text{Short circuit current} * V * \sqrt{3}}{10^6}$$

Fault Level calculations.

* In a power system, the maximum the fault current that can flow into a zero impedance fault is necessary to be known for switch gear solution.
* The fault level is usually expressed in MVA, with the maximum fault current value being converted using the nominal voltage rating.

$$MVA_{base} = \sqrt{3}.\ \text{Nominal voltage (KV)}.\ I_{base}\ (KA)$$

$$MVA_{fault} = \sqrt{3}.\ \text{Nominal voltage (KV)}.\ I_{sc}\ (KA)$$

Where ~~MAV~~

MVA_{fault} - Fault Level at a given point in MVA

I_{base} - Rated or base line current

I_{sc} - Short circuit line current flowing into a fault.

The pu value of the fault Level may be written as

$$\text{Fault Level} = \frac{\sqrt{3}.\ \text{Nominal voltage}.\ I_{sc}}{\sqrt{3}.\ \text{Nominal voltage}.\ I_{base}} = \frac{\sqrt{3}\ I_{sc}}{\sqrt{3}\ I_{bas}}$$

$$= I_{sc.pu} = \frac{V_{nominal.pu}}{Z_{pu}}$$

The pu voltage for nominal value is unity, so that

$$\text{Fault Level (PU)} = \frac{1}{Z_{pu}}$$

$$\text{Fault MVA} = \text{Fault Level (PU)} * MVA_{base} = \frac{MVA_{base}}{Z_{pu}}$$

Applications of series reactors-

Series reactors are mainly used to

1. Reduce fault currents and
2. Match impedance of parallel feeders.

Air core series reactors have the advantages that they

1. Cannot saturate under fault conditions
2. have low losses
3. have a long life ~~and~~ a
4. Virtually maintinance free

Series reactors require integration into the electricity network.

This requires consideration of aspects such as physical layout, protection coordination, and voltage control.

Symmetrical Component theory

Symmetrical component transformation.

The three unbalanced phasors of a 3-ϕ system as shown in fig a can be resolved into three component sets of balanced phasors.

1. Positive sequence components
2. Negative sequence components
3. Zero sequence components.

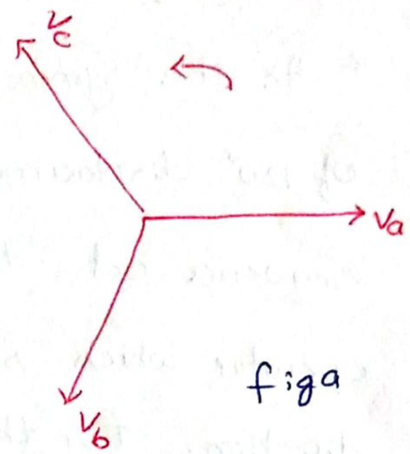


fig a

Positive sequence components

A set of three phasors equal in magnitude displaced each other by 120° in phase and having the same phase sequence as the original phasors.

These phasors can be denoted by V_{a1}, V_{b1} and V_{c1} are shown in fig b.

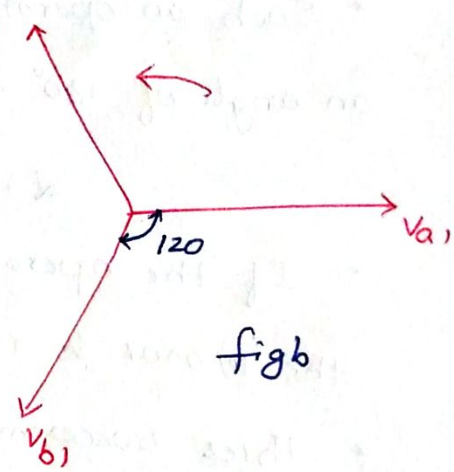


fig b

Negative sequence components

A set of three phasors equal in magnitude displaced each other by 120° in phase and having the phase sequence opposite to that of the original phasors.

These phasors can be denoted by V_{a2}, V_{b2} & V_{c2} as shown in fig c

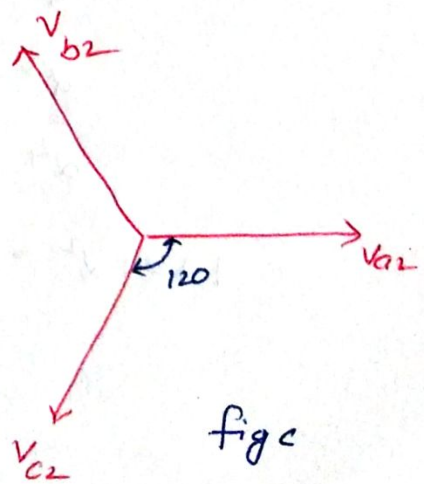
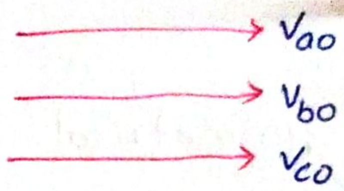


fig c

Zero sequence component

A set of three phasors equal in magnitude with zero phase displacement each other.



These phasors can be denoted by V_{ao}, V_{bo} & V_{co} as shown in fig.

Operator α

* As the symmetrical component theory involves the concept of 120° displacement in the positive sequence set and negative sequence set, therefore, it is desirable to evolve some operator which should cause 120° rotation in anticlockwise direction. For this purpose operator α is used.
* Such an operator is a complex number of unit magnitude with an angle of 120° and is defined by

$$\alpha = 1\angle 120^\circ = -0.5 + j0.866$$

* If the operator α is applied to a phasor twice in succession the phasor is rotated through 240°.
* Three successive applications of α rotate the phasor through 360°.

$$\alpha^2 = 1\angle 240 = -0.5 - j0.866$$

$$\alpha^3 = 1\angle 360 = 1\angle 0 = 1$$

$$\alpha^4 = 1\angle 480 = 1\angle 120^\circ$$

$$1 + \alpha + \alpha^2 = 1 - 0.5 + j0.866 - 0.5 - j0.866 = 0$$

Voltages & currents

* Let the three phasors are represented by a, b, c with phase sequence 'abc' then the phase negative sequence system is 'acb'. Let the subscripts 0, 1, 2 refer to zero sequence, positive sequence & negative sequences respectively.

* If V_a, V_b, V_c represent an unbalanced set of voltage phasors, the three balanced sets are written as

V_{a0}, V_{b0}, V_{c0} - Zero sequence set

V_{a1}, V_{b1}, V_{c1} = Positive sequence set

V_{a2}, V_{b2}, V_{c2} - Negative sequence set

* Each of original unbalanced phasors is the sum of its component can be written as

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \quad \text{--- (1)}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

* First consider the symmetrical component of positive sequence

$$V_{a1} = V_{a1} \angle 0$$
$$V_{b1} = \alpha^2 V_{a1} = V_{a1} \angle 240 \quad \text{--- (2)}$$
$$V_{c1} = \alpha V_{a1} = V_{a1} \angle 120$$

where $\alpha = 1\angle 120$

* And similarly negative sequence

$$V_{a2} = V_{a2} \angle 0^\circ$$
$$V_{b2} = \alpha V_{a2} = V_{a2} \angle 120$$
$$V_{c2} = \alpha^2 V_{a2} = V_{a2} \angle 240$$

--- (3)

* Substituting equation ② & ③ in ①

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$

$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

* In matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$ = Transformation matrix

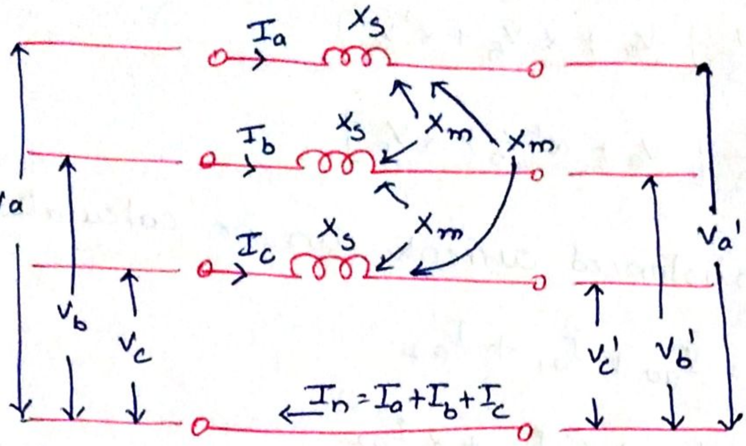
* To determine the sequence components

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = [A]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Sequence impedance



* Above fig shows 3-ϕ circuit. The lines carrying unbalanced currents I_a, I_b and I_c. The return path for I_a is sufficiently away for the manual effect to be neglected.

* Applying KVL for above fig

$$V_a - V_a' = jX_s I_a + jX_m I_b + jX_m I_c$$

$$V_b - V_b' = jX_m I_a + jX_s I_b + jX_m I_c$$

$$V_c - V_c' = jX_m I_a + jX_m I_b + jX_s I_c$$

* In matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

* Above equation can be represent in vector form

$$[V_p] - [V_p'] = [Z][I_p]$$

$$V_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad [Z] = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix}$$

where $[Z]$ = impedance.

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}[V_a + \alpha V_b + \alpha^2 V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + \alpha^2 V_b + \alpha V_c]$$

Similory unbalanced current can be calculated as

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$

In matrix form

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

the symmetrical component are

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c).$$

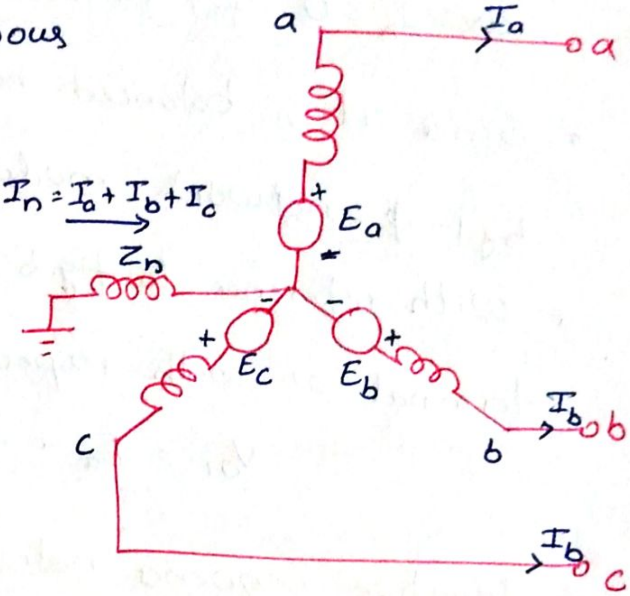
In matrix form.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Sequence Networks

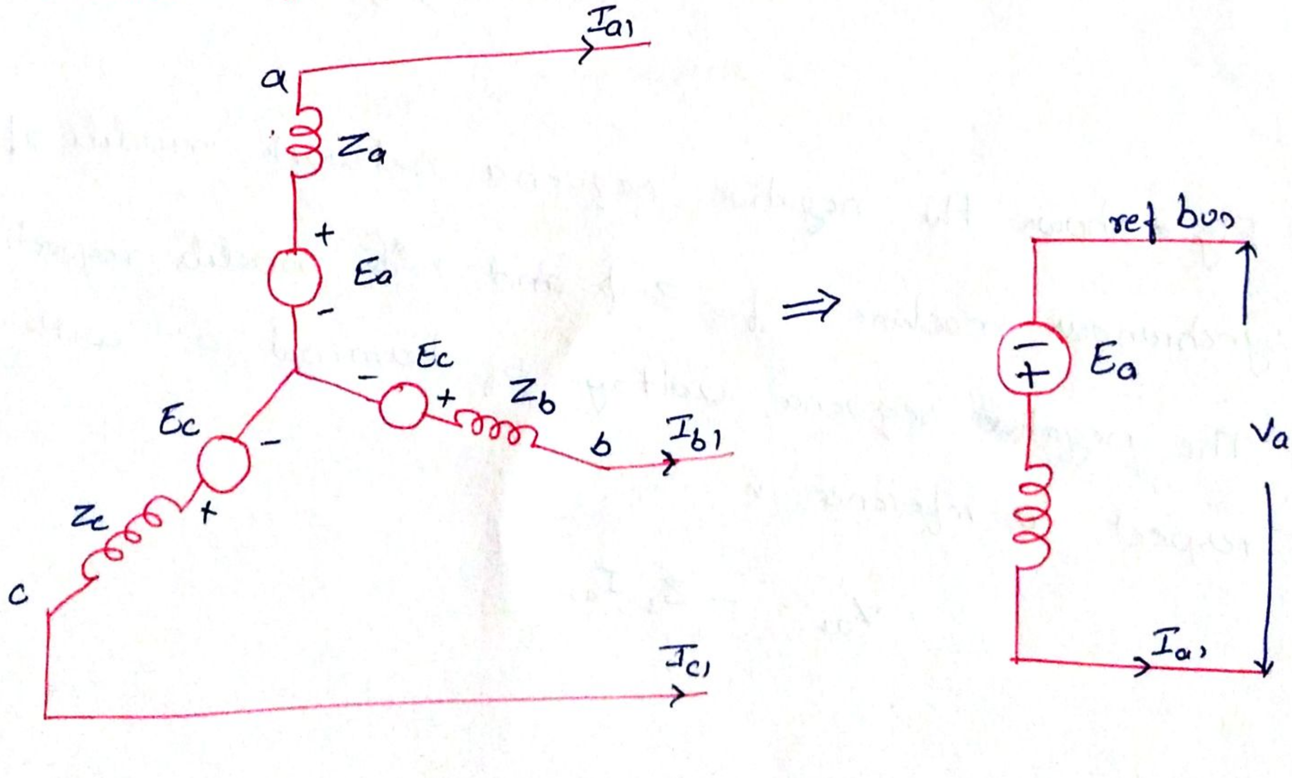
Sequence impedances and networks of a Synchronous generator

* Fig. shows an unloaded synchronous generator grounded through impedance Z_n.



* E_a, E_b and E_c are the induced Emf's of the three phase.
* If any fault occurs at the terminals, the currents I_a, I_b & I_c flow in the lines, whenever fault involves, ground current $I_n = I_a + I_b + I_c$ flows through ground impedance 'Z_n'.
* Unbalanced currents can be resolved into their symmetrical components I_{a1}, I_{a2} and I_{a0}.

Positive sequence impedance



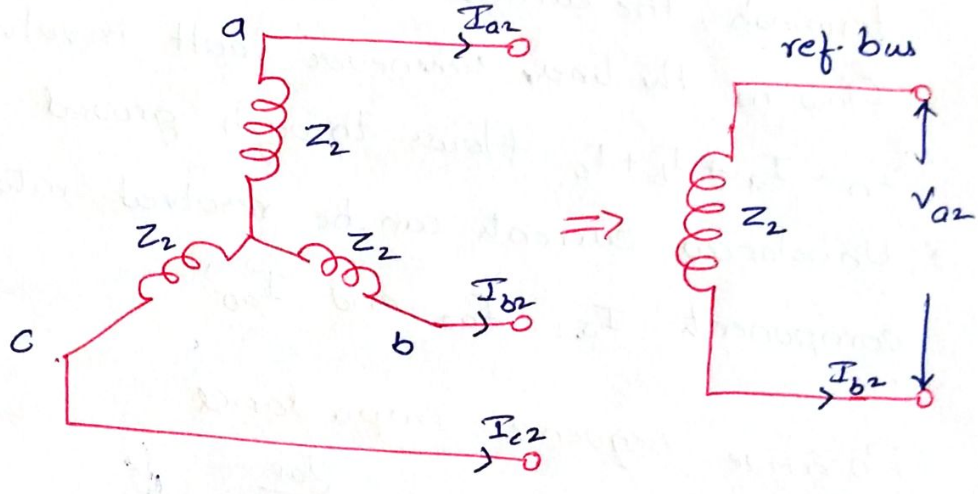
* fig(a) shows the 3-ϕ positive sequence network model of synchronous machine, Z_n does not appear in the model as $I_n = 0$ for positive sequence currents.

* Since it is balanced network this can be represented by 1-ϕ network model as shown in fig b.

* With reference to fig b, the positive sequence voltage of terminal 'a' with respect to reference bus is

$$V_{a1} = E_a - Z_1 I_{a1}$$

Negative sequence network

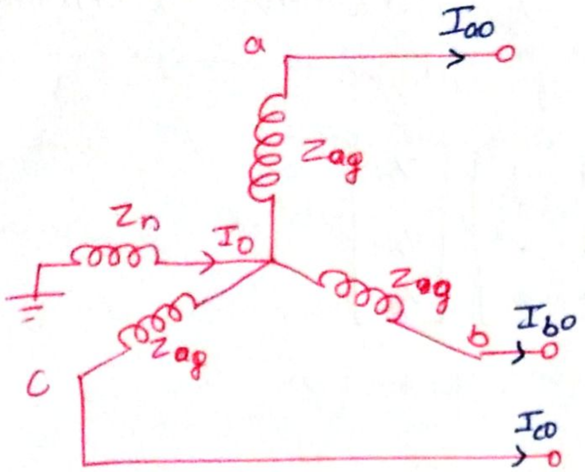


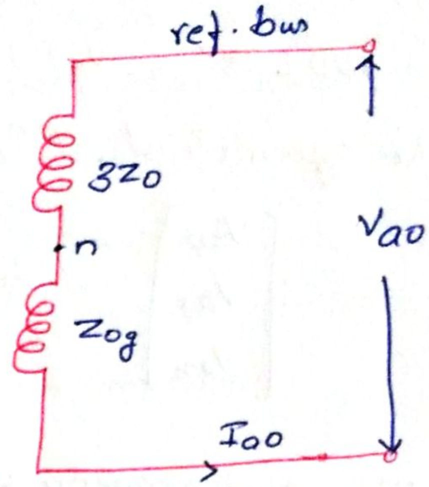
* Fig. shows the negative sequence network models of a synchronous machine of 3-ϕ and 1-ϕ models respectively.

* The negative sequence voltage of terminal 'a' with respect to reference is

$$V_{a2} = -Z_2 I_{a2}$$

Zero sequence impedance





* Above fig. shows the zero sequence network models of 3-ϕ and 1-ϕ of synchronous generator respectively.

* The current flowing in the impedance Z_n is $I_n = 3I_{ao}$.

* The zero sequence voltage of terminal 'a' with respect to ground is

$$V_{ao} = -3Z_n I_{ao} - Z_{og} I_{ao}$$

$$= -(3Z_n + Z_{og}) I_{ao}$$

$$V_{ao} = -Z_0 I_{ao}$$

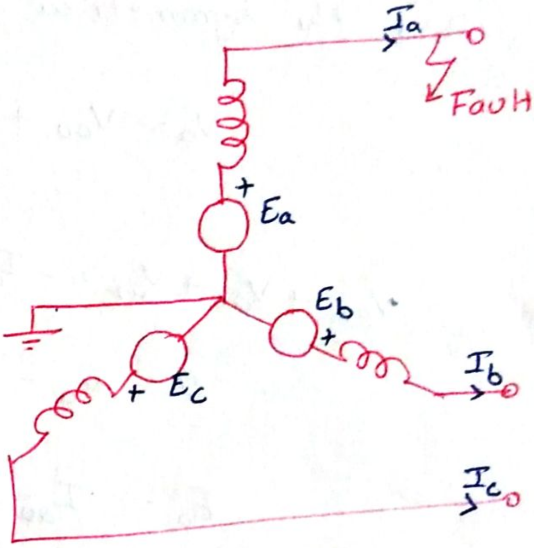
The zero sequence impedance is

$$Z_0 = 3Z_n + Z_{og}$$

Unsymmetrical fault analysis

* Most of the faults that occur on the power system are unsymmetrical faults. The faults due to open of one or more conductors through breaks or the action of fuses is called series faults.
* And the other faults that may consist of short circuit occur as LG, LL, LLG may or may not contain impedance.

Single line to ground fault (L-G)



* Fig. shows a single line to ground fault occurs at phase 'a'.
* The fault current flows through phase 'a' and the remaining phase currents are zero because the machine is under no load condition.
* The voltage current relations are

$$V_a = 0 \; ; \quad I_b = 0 \; ; \quad I_c = 0 \; ; \quad I_a = I_f$$

* The sequence network equations are

$$V_{a0} = -I_{a0} Z_0 , \quad V_{a1} = E_a - I_{a1} Z_1 \; ; \quad V_{a2} = -I_{a2} Z_2$$

* From the symmetrical component relations, the symmetrical currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

From above equations

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_0$$

And the symmetrical components for voltage is

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0 \qquad [\because V_a = 0]$$

$$V_{a0} + V_{a1} + V_{a2} = -I_{a0} Z_0 + E_0 - I_{a1} Z_1 - I_{a2} Z_2 = 0$$

$$0 = E_a - (I_{a0} Z_0 + I_{a1} Z_1 + I_{a2} Z_2)$$

$$E_a = I_{a0} Z_0 + I_{a1} Z_1 + I_{a2} Z_2$$

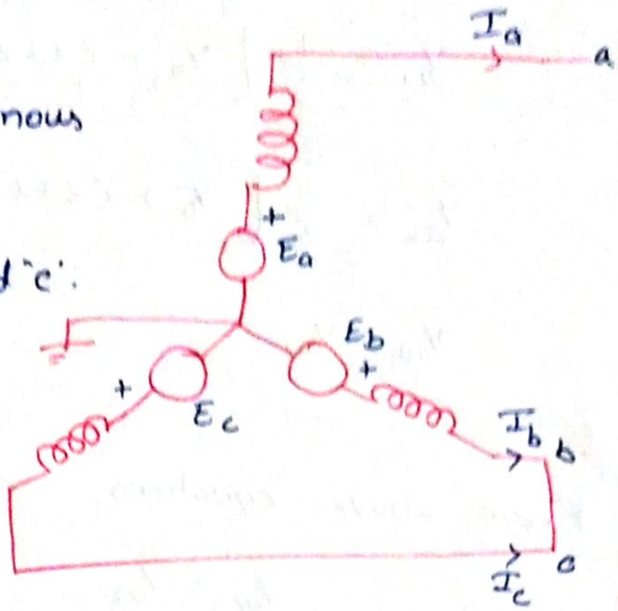
$$E_a = \frac{1}{3} I_a Z_0 + \frac{1}{3} I_a Z_1 + \frac{1}{3} I_a Z_2 = \frac{1}{3} I_a (Z_0 + Z_1 + Z_2)$$

$$I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2}$$

$$I_a = 3I_{a1} = 3I_{a2} = 3I_{a0} = 3 \frac{E_a}{Z_0 + Z_1 + Z_2}$$

This is fault current for L-G fault.

Line to line fault (LL)



* Fig a shows an unloaded synchronous generator and line to line fault occurs between the phases 'b' and 'c'.

* The fault current and voltages can be calculated from fig a.

* Current & voltage relations are

$$I_a = 0 \; ; \quad I_b + I_c = 0 \quad ; \quad V_b = V_c$$

* The sequence currents can be calculated by using the equation

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a1} = \frac{1}{3}(\alpha - \alpha^2) I_b$$

$$I_{a2} = \frac{1}{3}(\alpha^2 - \alpha) I_b$$

$$I_{a1} = -I_{a2}$$

$$I_{a0} = I_b = -I_b$$

* The symmetrical components of voltages are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a1} = \frac{1}{3}\left[V_a + (\alpha + \alpha^2) V_a \right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + (\alpha + \alpha^2) V_b \right]$$

$$V_{a0} = V_{a0}$$

From above equations

$$V_{a1} = V_{a2}$$

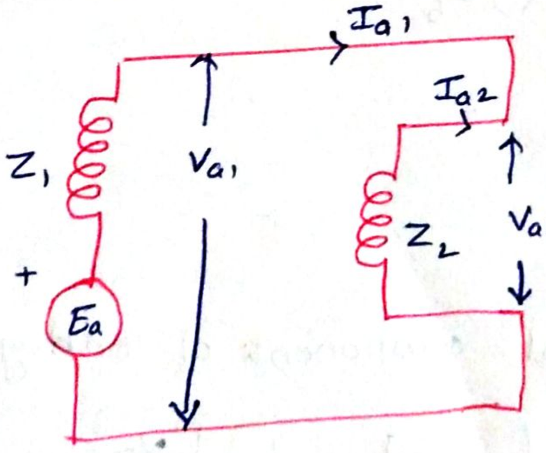
$$E_a - I_{a1} Z_1 = - I_{a2} Z_2$$

$$E_a = I_{a1} Z_1 - I_{a2} Z_2$$

$$= (Z_1 + Z_2) I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} = - I_{a2}$$

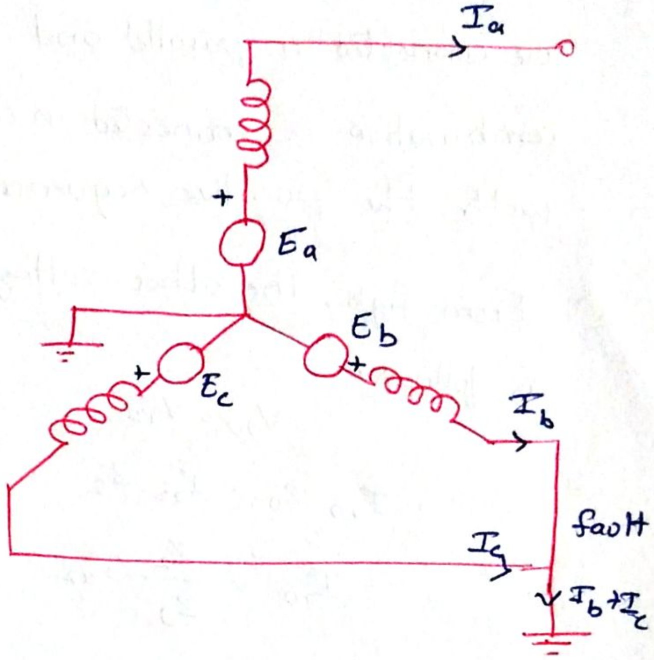
By using the above equation the equivalent circuit of a double line faulted synchronous generator as shown in fig.



Double line to ground fault (LLG)

* Fig a shown double line to ground fault at phases 'b' and 'c'
* The fault current is passed to ground.

Fig a the terminal conditions are



$I_a = 0$; $V_b = 0$; $V_c = 0$

* From equation

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

* The symetrical components of equation are

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] = \frac{1}{3}V_a$$

$$V_{a1} = \frac{1}{3}[V_a + \alpha V_b + \alpha^2 V_c] = \frac{1}{3}V_a$$

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c) = \frac{1}{3}V_a$$

$$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3}$$

- The symmetrical components of currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

* Fault current $I_f = I_b + I_c = 2I_{a0} + (\alpha^2+\alpha)I_a + (\alpha+\alpha^2)I_{a1}$

$$= 2I_{a0} + (\alpha^2+\alpha)(I_{a1}+I_{a2})$$

$$= 2I_{a0} + (\alpha^2+\alpha)(-I_{a0})$$

$$I_b + I_c = 3I_{a0}$$

* From above relations the equivalent circuit for double line to ground fault is shown in fig b.

* The negative and zero sequence networks are connected in parallel and this combination is connected in series with the positive sequence network

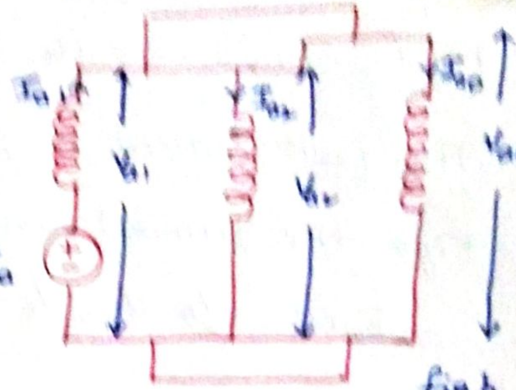


fig b

* From fig b, the other voltages and currents can be calculated as follows:

$$V_{a0} = V_{a2}$$

$$I_{a0} Z_0 = I_{a2} Z_2$$

$$I_{a0} = \frac{Z_2}{Z_0} I_{a2}$$

and $$V_{a1} = V_{a2}$$

$$E_a - I_{a1} Z_1 = - I_{a2} Z_2$$

$$I_{a2} = \frac{-[E_a - I_{a1} Z_1]}{Z_2}$$

* From sequence network diagram of fig b we can write

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$$