UNIT – I

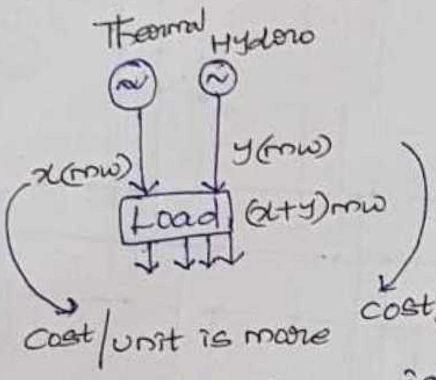
# ECONOMICAL OPERATION OF POWER SYSTEM

## Optimal operation of Generators:

The optimal system operation involves the consideration of

1. Economy of operation
2. System security.
3. Optimal releases of water at hydro generation etc.
4. Emission of fossil Fuel.

## Economy of operation:




Economy of operation is also called as economic dispatch.

The main aim of economic dispatch problem is to minimize total cost of generating real power at various stations while satisfying the load and losses in transmission lines.

Total generation = Total load + Total losses.

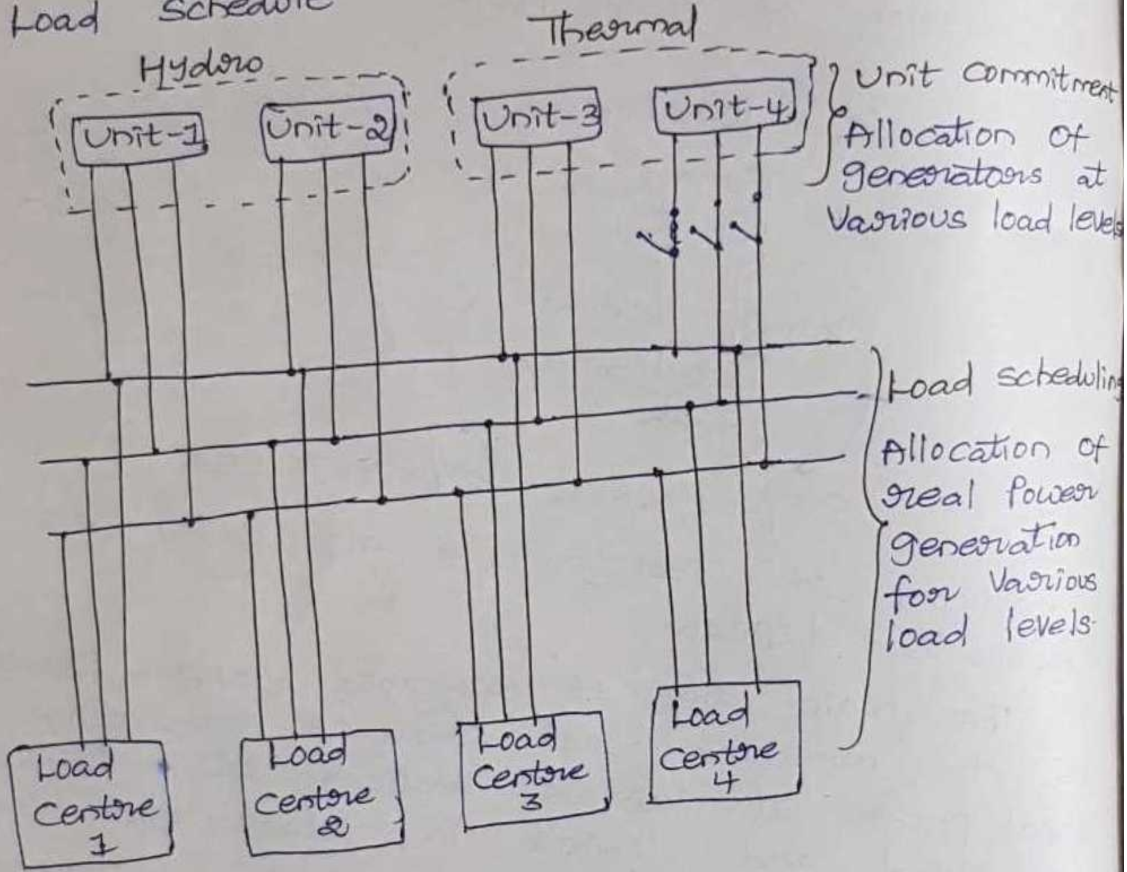
Ex: A hydro plant which operates in addition with thermal plants have negligible operation cost.

But in hydro plant, there is a limitation of availability of water over a period of time which must be used to save maximum fuel and thermal cost.

Economy of operation is predominant, in determining the allocation of generation to each station for various system load levels.

* This allocation of real power at generator buses can be partitioned into two sub problems

1. Unit commitment
2. Load schedule




Unit commitment:

optimal allocation of generators at each generating station at various load levels.

It is not economical to run all the units available all the time.

To determine the units of a plant that should operate for a particular load is the problem of unit commitment.

Load scheduling: optimal allocation of generation at each station for various system load levels to minimize the ~~total~~ cost of system.

System security: All the loads must be interconnected to all available generating stations such that if any major shut down takes place in ~~one~~ any unit, interruption must be rectified within short duration.

Note:
The major component of generator operating cost is the fuel input per hour.

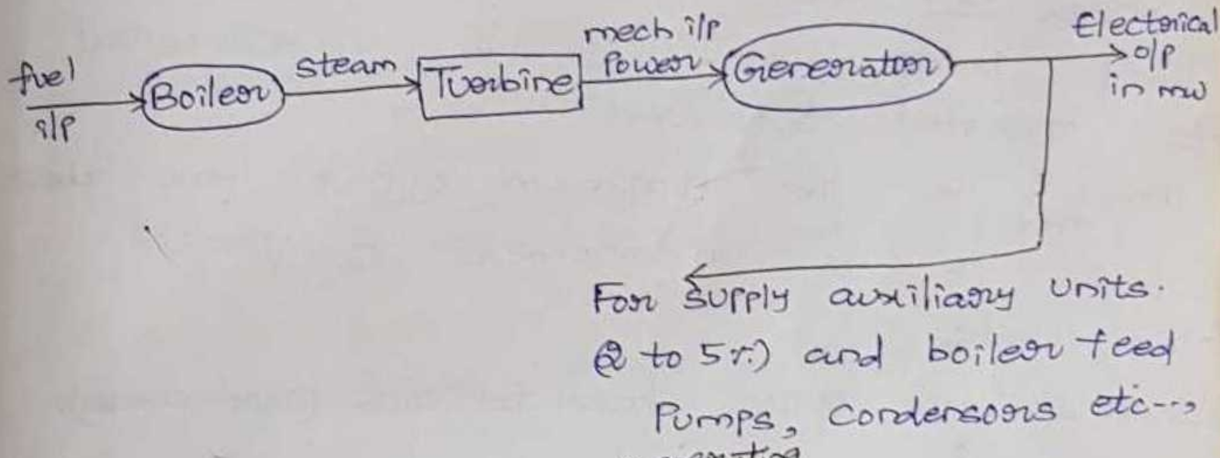
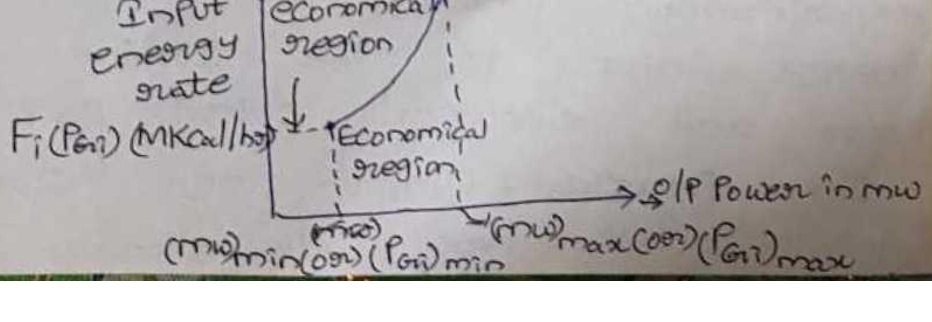



Fig. Thermal Power Generating station

The above figure represents a single unit consisting of boiler, turbine and generator.
Total generator operating cost includes fuel, labour and maintainance cost.
For simplicity, fuel cost is the only one consider to be variable (i.e for thermal and nuclear power stations).

Case I: Input - output characteristics:



The input - output curve of a generating unit specifies the input energy rate $F_i(P_{Gi})$ (mcal/hr) (or) cost of fuel used per hour. $C_i(P_{Gi})$ (Rs/hr) as a function of generator power input $P_{Gi}$.

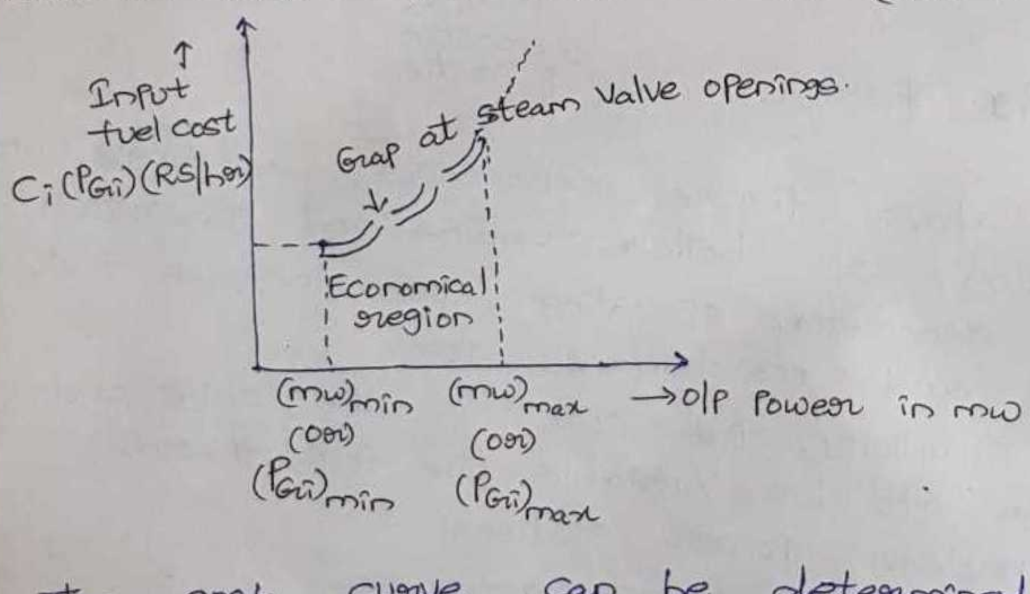
A typical input - output curve is shown in figure which is concave upwards.

Input - output curve is expressed in terms of input energy rates rather than fuel cost per hour because fuel cost can change daily (or) monthly in comparison with fuel energy used per hour.

mega watt $(mw)_{min}$ is the minimum loading limit below which it is uneconomical to operate the unit.

$(mw)_{max}$ is the maximum output limit above which it is uneconomical to operate the unit.

Case II: Input - output characteristics (cost curve):

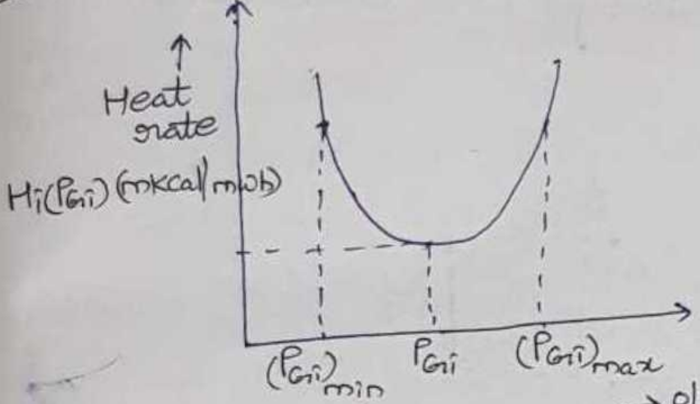



The cost curve can be determined experimentally which gives relation between energy input to the turbine in Rs per hour and electrical output power in mw.

In practical, input - output curve is not smooth but has discontinuous at steam valve openings. This is because the power output of the fossil plant is increased sequentially by opening a set of valves at inlet to its steam turbine.

The throttling losses in a valve are large when it is just open and small when it is fully open.

Case III: Heat rate curve:




It is defined as inverse of ratio of fuel input (mKcal/hr) to corresponding power output (mw)

The heat rate curve $H_i(P_{Gi})$ which is the heat energy needed to generate one unit of electrical energy.

The above figure shows the heat rate curve.

The generating unit efficiency can be defined as the ratio of electrical energy output generated to fuel energy input.

$$\text{Generator efficiency } \eta = \frac{\text{Electrical o/p Power}}{\text{Fuel i/p}}$$

$$= \frac{mw}{mKcal/hr} = \frac{mwhr}{mKcal} = \frac{1}{HRC}$$

where HRC = Heat rate curve

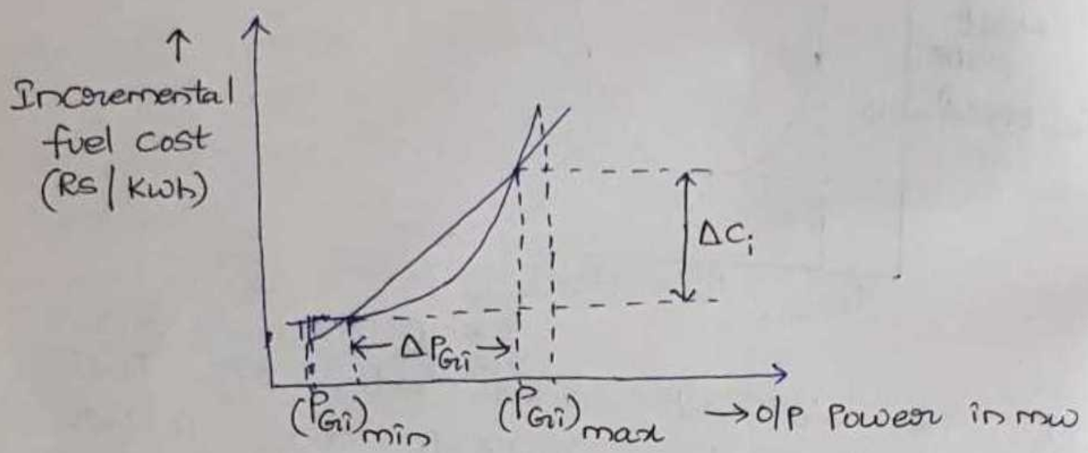
Generating unit is most efficient at minimum heat rate which corresponds to a particular $P_{Gi}$ in above figure.

The heat rate varies with output power $(P_{Gi})$

and curve indicates increase in heat rate (or) are decrease in generator efficiency, at low and high power limits.

For 100% conversion of fuel energy to electrical energy, heat rate is approximately 0.859 mKcal/mw (1 mKcal = 1.164 mwhr) is the equivalent of heat.

Case IV: (Incremental) Fuel cost curve (IFC):




Incremental fuel cost (or) Incremental fuel rate is defined as ratio of a small incremental change in input to corresponding small incremental change in output.

$$\text{IFc (or) IC (or) } \lambda = \frac{\Delta \text{ input}}{\Delta \text{ output}} = \frac{\Delta C_i}{\Delta P_{Gi}} = \frac{Rs/hr}{mw} = \frac{Rs}{mwhr}$$

where $C_i$ is fuel cost of $i^{th}$ units in Rs/hr

$P_{Gi}$ is power output of $i^{th}$ unit in MW.

$\lambda$ is Lagrange multiplier

From Graph,

slope of fuel cost curve i.e $\frac{\Delta C_i}{\Delta P_{Gi}} = \frac{\delta C_i}{\partial P_{Gi}}$ is called the incremental fuel cost (IFc) and is expressed in Rs/mwhr

Expression:

The factors influencing power generation at minimum cost are efficiency of generators, fuel cost and transmission losses.

For generators cost curves are not smooth, however the curves can be approximated using functions.

Therefore we will assume a quadratic function of fuel cost of generator in rupees/hr.

In practical, fuel cost of generator can be represented as quadratic function of real power generation.

Let the cost of the fuel be 'K' (Rs/mKcal)

The input fuel cost be $C_i(P_{Gi})$ in Rs/hr

$$C_i(P_{Gi}) = K \cdot F_i(P_{Gi}) = K \cdot P_{Gi} \cdot H_i(P_{Gi}) \text{ in Rs/hr} \rightarrow ①$$

(fuel cost) (fuel energy) (Heat rate)

The heat rate may be approximated in the form of

$$H_i(P_{Gi}) = a_i'/P_{Gi} + b_i' + c_i' P_{Gi} \text{ (mKcal/mwh)} \rightarrow ②$$

From eq① and eq②, we will get a quadratic expression for input energy rate.

$$F_i(P_{Gi}) = a_i' + b_i' \cdot P_{Gi} + c_i' \cdot P_{Gi}^2 \text{ mKcal/hr} \rightarrow ③$$

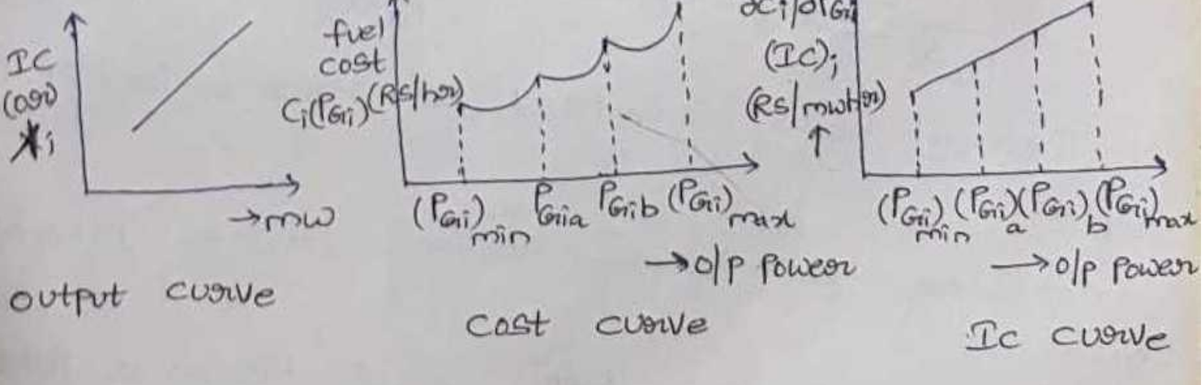
From eq② and eq③, we get a quadratic expression for fuel cost.

$$C_i(P_{Gi}) = K \cdot F_i(P_{Gi})$$
$$= Ka_i' + Kb_i' \cdot P_{Gi} + K \cdot c_i' \cdot P_{Gi}^2 \text{ Rs/hr.}$$

$$\therefore C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \text{ Rs/hr} \rightarrow ④$$

Differentiate above equation, we get

$$(IC)_i = \lambda_i = b_i + 2c_i P_{Gi} \text{ Rs/mwh.}$$




output curve

cost curve

Ic curve

Case V: Incremental production cost:

The sum of incremental fuel cost and other incremental running expenses is called incremental production cost (IPC)

The IPC of given unit consist of incremental fuel cost + incremental cost of fuel, labours, ~~labour~~, suppliers, maintainance and water etc--,.

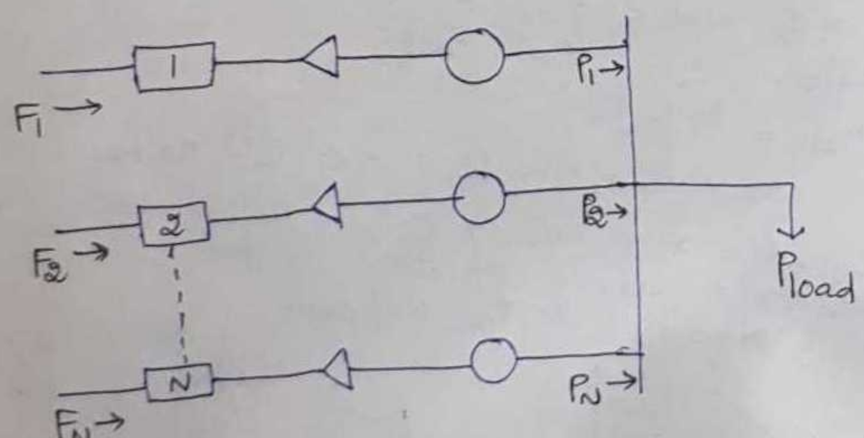
It is very difficult to express these cost as function of output and also they form a small fraction of incremental cost of fuel.

∴ For scheduling real power generation, incremental production cost is assumed to be equal to incremental fuel cost (IFC).

optimal allocation of total load among different units (when losses are neglected):

(or)

Condition for optimal allocation:




N- Thermal units committed to serve a load of $P_{load}$

step I:

Consider a power station having 'N' no. of units.

Let us assume that it is known a prior which generators are to run to meet a particular load demand on the station.

i.e, given a station with 'N' generators committed and the active Power load $P_d$, the real Power Generation $P_{Gi}$ for each generator has to be allocated. so as to minimize the total cost.

∴ The cost of production of electrical energy is given by

$$C = \sum_{i=1}^{n} C_i(P_{Gi}) \text{ Rs/hr} \rightarrow ①$$

subject to the inequality constraint,

$$P_{Gi\,(min)} \leq P_{Gi} \leq P_{Gi\,(max)}, \quad i=1,2,3 \cdots n \rightarrow ②$$

From eq①, it is assumed that the cost of real power generation $P_{Gi}$ is dependant of real Power generation and is insensitive to reactive Power generation $Q_{Gi}$

Step II:

To minimize the cost of generation, these is subjected to the equality constraints and is given by

$$\sum_{i=1}^{n} P_{Gi} - P_D = 0 \rightarrow ③$$

This is a constrained optimization problem.

Step III:

To get the solution for optimization problem, is defined by considering an objective function through eq② equality constraint eq①, with an Lagrangian multiplier (λ) as total cost, the

$$c' = C - \lambda\left[\sum_{i=1}^{n} P_{Gi} - P_D\right] \rightarrow ④$$

∴ Condition for optimality (reduction) of above equation is $\frac{\delta c'}{\partial P_{Gi}} = 0$.

$$\frac{\partial c'}{\partial P_{Gi}} = \frac{\partial c}{\partial P_{Gi}} - \frac{\partial}{\partial P_{Gi}}\left[\lambda\left|\sum_{i=1}^{n} P_{Gi} - P_D\right]\right] = 0$$

$$= \frac{\partial c}{\partial P_{Gi}} - \lambda\,(1-0) = 0$$

$$\frac{\partial c'}{\partial P_{Gi}} = \frac{\partial c}{\partial P_{Gi}} - \lambda = 0 \rightarrow (5)$$

$$\frac{\partial c}{\partial P_{G1}} - \lambda = 0$$

$$\frac{\partial c}{\partial P_{G2}} - \lambda = 0 \; - - - - - - \frac{\partial c}{\partial P_{Gn}} - \lambda = 0$$

$$\frac{\partial c}{\partial P_{G1}} = \frac{\partial c}{\partial P_{G2}} = - - - - - \frac{\partial c}{\partial P_{Gn}} = \lambda \rightarrow (6)$$

Step IV:

The expression of 'c' is Variable seperable form.

$\therefore$ overall cost = sum of cost of each generating unit which is a function of $P_G$ of that unit only.

i.e $\frac{\partial c}{\partial P_{G1}} = \frac{\partial c_1}{\partial P_{G1}}$

$$\frac{\partial c}{\partial P_{G2}} = \frac{\partial c_2}{\partial P_{G2}}$$

$$\vdots$$

$$\frac{\partial c}{\partial P_{Gn}} = \frac{\partial c_n}{\partial P_{Gn}}$$

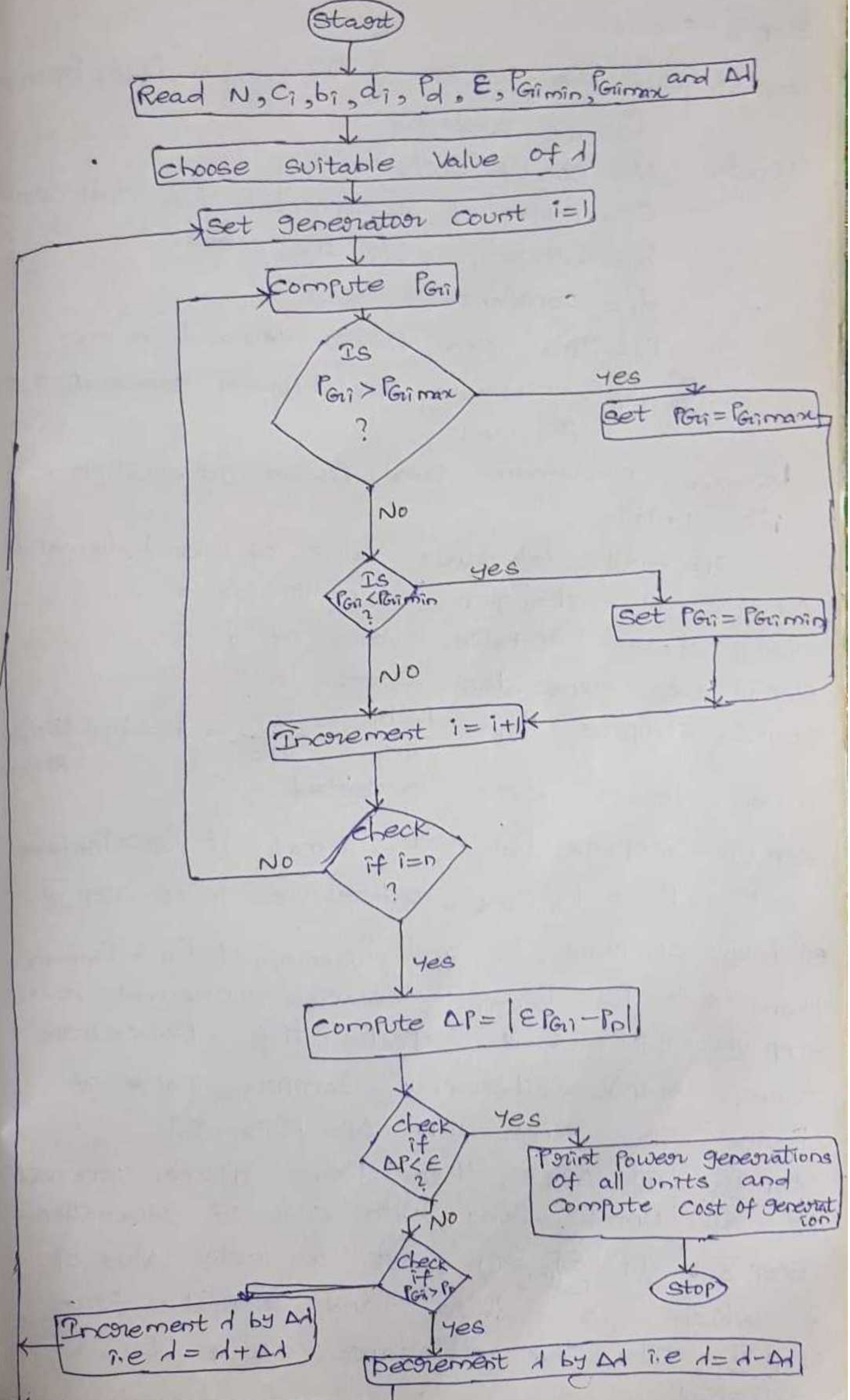
$\therefore$ condition for optimal allocation of total load among generator when losses are neglected is given by

$$\frac{\partial c_1}{\partial P_{G1}} = \frac{\partial c_2}{\partial P_{G2}} = - - - - - \frac{\partial c_n}{\partial P_{Gn}} = \lambda \rightarrow (7)$$

For $i^{th}$ unit, $\frac{\partial c_i}{\partial P_{Gi}} = \lambda \rightarrow (8)$

This represents the incremental cost of the individual units are same and are equal to '$\lambda$'. It is called co-ordination equation.

Flow chart:


Start
Read N, ci, bi, di, Pd, E, PGimin, PGimax and Δλ
choose suitable value of λ
Set generator count i=1
Compute PGi
Is PGi > PGimax ?
yes
Set PGi = PGimax
NO
Is PGi < PGimin ?
yes
Set PGi = PGimin
NO
Increment i = i+1
Check if i=n ?
NO
yes
Compute ΔP = |ΣPGi − PD|
check if ΔP<E ?
Yes
Print power generations of all units and compute cost of generation
Stop
NO
Check if PGi > PD
yes
Increment λ by Δλ i.e λ = λ+Δλ
Decrement λ by Δλ i.e λ = λ−Δλ

Algorithm:

Step I: Start

Step II: Read the values of $N, c_i, b_i, d_i, P_d, \epsilon, P_{Gi\,min}$, $P_{Gi\,max}$ and $\Delta\lambda$

where $N$ = no. of units

$c_i$ = slope of incremental fuel cost curve

$b_i$ = Intercept of IFC curves

$d_i$ = constant

$P_d$ = Total real power demand in MW

$P_{Gi\,min}$ = minimum real power generation of $i^{th}$ unit

$P_{Gi\,max}$ = maximum real power generation of $i^{th}$ unit

$\epsilon$ = specified tolerance value of load demand

$\Delta\lambda$ = small change in multiplier $\lambda$.

Step III: choose suitable value of $\lambda$

Step IV: set generator count $i=1$.

Step V: compute $P_{Gi} = \frac{\lambda - b_i}{2c_i}$, $\frac{\partial c_i}{\partial P_{Gi}} = \lambda = b_i + 2c_i P_{Gi}$ Rs/MWh

when losses are neglected.

Step VI: Compare $P_{Gi}$ and $P_{Gi\,max}$, if $P_{Gi} > P_{Gi\,max}$ set $P_{Gi} = P_{Gi\,max}$, otherwise go to step VII.

Step VII: Compare $P_{Gi}$ and $P_{Gi\,min}$, if $P_{Gi} < P_{Gi\,min}$ then set $P_{Gi} = P_{Gi\,min}$, otherwise increment $i = i+1$

Step VIII: If $i < n$, repeat the procedure from step V, otherwise compute value of change in power i.e $\Delta P = |\Sigma P_{Gi} - P_D|$

Step IX: If $\Delta P < \epsilon$, then Print power generation of all units along with cost of generation.

Step X: If $\sum_{i=1}^{n} P_{Gi} > P_d$, set a new value of $\lambda$ which is "$\lambda - \Delta\lambda$" and repeat from step IV till the tolerance value '$\epsilon$' is satisfied

Step XI: If $\sum_{i=1}^{n} P_{Gi} < P_d$, set new value of $\lambda$ which is "$\lambda + \Delta\lambda$" and repeat from step IV till the tolerance value is satisfied.

Problems:

1. The power system consist of two 125 MW units whose input costs are represented by the equations $C_1 = 0.04P_1^2 + 22P_1 + 800$ Rs/hr, $C_2 = 0.045P_2^2 + 15P_2 + 1000$ Rs/hr. If total received power is 200 MW. Determine the total load sharing between them for most economic operation.

Sol Given data,

No. of units = 2

Rated capacity of each unit = 125 MW

Cost of unit I, $C_1 = 0.04P_1^2 + 22P_1 + 800$ Rs/hr

Cost of unit II, $C_2 = 0.045P_2^2 + 15P_2 + 1000$ Rs/hr.

Incremental fuel cost of unit I is

$$\frac{\partial C_1}{\partial P_1} = 2\times 0.01P_1 + 22$$

$$= 0.08P_1 + 22 = \lambda_1 \rightarrow (1)$$

Incremental fuel cost of unit II is

$$\frac{\partial C_2}{\partial P_2} = 2\times 0.045P_2 + 15$$

$$= 0.09P_2 + 15 = \lambda_2 \rightarrow (2)$$

Total received power, $P_R = P_D = 200$ MW

$$P_1 + P_2 = 200 \text{ MW} \rightarrow (3).$$

For economic load scheduling (or) For optimal allocation,

$$\frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \lambda$$

Equating eq (1) and eq (2), we get

$$0.08P_1 + 22 = 0.09P_2 + 15$$

$$0.08P_1 - 0.09P_2 + 7 = 0$$

$$\Rightarrow 0.08P_1 - 0.09P_2 = -7 \rightarrow (4)$$

Solving eq ③ and eq ④, we get

$P_1 = 64.7$ mw , $P_2 = 135.29$ mw

Substitute $P_1$ in eq ①, we get

$\lambda = 27.176$ Rs/mwh

2. The fuel cost in Rs/hr of two units in a plant are given by $C_1 = C_1(P_1) = 1.0 + 25P_1 + 0.25P_1^2$ Rs/hr $C_2 = C_2(P_2) = 1.5 + 45P_2 + 0.2P_2^2$ Rs/hr. If the total demand on generator is 250 mw. Calculate the economic load scheduling of two units.

Given data,

sol No. of units = 2

Cost of unit I, $C_1 = C_1(P_1) = 1.0 + 25P_1 + 0.25P_1^2$ Rs/hr

Cost of unit II, $C_2 = C_2(P_2) = 1.5 + 45P_2 + 0.2P_2^2$ Rs/hr

Incremental fuel cost of unit I is

$$\frac{\partial C_1}{\partial P_1} = 0.5P_1 + 25 = \lambda_1 \rightarrow ①$$

Incremental fuel cost of unit II is

$$\frac{\partial C_2}{\partial P_2} = 0.4P_2 + 45 = \lambda_2 \rightarrow ②$$

Total received power, $P_R = P_D = 250$ mw

$$P_1 + P_2 = 250 \text{ mw} \rightarrow ③$$

For economic load scheduling (or) For optimal allocation,

$$\frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \lambda$$

Equating eq ① and eq ②, we get

$$0.5P_1 - 0.4P_2 = 20 \rightarrow ④$$

Solving eq ③ and eq ④, we get

$P_1 = 133.33$ mw , $P_2 = 116.66$ mw

Substitute $P_1$ in eq ①, we get

$\lambda = 91.665$ Rs/mwh

3. The fuel cost curve of two generators are given as $C_A(P_A) = 800 + 45P_A + 0.01 P_A^2$ Rs/hr, $C_B(P_B) = 2000 + 43P_B + 0.03 P_B^2$ Rs/hr and if the total load supplied is 700 mw, find the optimal dispatch with and without considering the generator limits have been expressed as $50\,mw \le P_A \le 200\,mw$, $50\,mw \le P_B \le 600\,mw$. Compare the systems incremental cost with and without generator limits considered.

Sol Given data,

No. of units = 2.

Cost of unit A, $C_A(P_A) = 800 + 45P_A + 0.01 P_A^2$ Rs/hr

Cost of unit B, $C_B(P_B) = 2000 + 43P_B + 0.03 P_B^2$ Rs/hr.

without considering Generator Limits:

Incremental fuel cost of unit A is

$$\frac{\partial C_A}{\partial P_A} = 2 \times 0.01 P_A + 45 = 0.02P_A + 45 = \lambda_1 \rightarrow ①$$

Incremental fuel cost of Unit B is

$$\frac{\partial C_B}{\partial P_B} = 0.006 P_B + 43 = \lambda_2 \rightarrow ②$$

Equating eq① and eq②, we get

$$0.02P_A + 45 = 0.006P_B + 43$$

$$\Rightarrow 0.02P_A - 0.006P_B = -2 \rightarrow ③$$

Total Load Supplied = 700mw

$$P_1 + P_2 = 700 \rightarrow ④$$

Solving eq③ and eq④, we get

$$P_A = 84.6\,mw, \quad P_B = 615.384\,mw$$

substitute $P_A$ in eq①, we get

$$\lambda = 46.69 \text{ Rs/mwh}.$$

with considering Generator limits:

$\therefore$ The value of power generation of unit A i.e $P_A$ is within minimum and maximum limit

The value of power generation of unit B i.e $P_B$ is violating the limits. so set $P_B$ at the uppermost limit i.e 600 mw

$\therefore P_A + P_B = 700$

$P_A + 600 = 700\,mw$

$P_A = 700 - 600 = 100\,mw$

4. Three power plants of total capacity 425 mw are scheduled for operation to supply total load of 300 mw. Find the optimum load scheduling if the plants have the following incremental cost characteristics and the generator constraints.

$$\frac{\partial C_1}{\partial P_{G_1}} = 30 + 0.15 P_{G_1} \; ; \; 25 \leq P_{G_1} \leq 125\,mw$$

$$\frac{\partial C_2}{\partial P_{G_2}} = 40 + 0.2 P_{G_2} \; ; \; 30 \leq P_{G_2} \leq 100\,mw$$

$$\frac{\partial C_3}{\partial P_{G_3}} = 15 + 0.18 P_{G_3} \; ; \; 15 \leq P_{G_3} \leq 200\,mw$$

Sol Given data,

$$\frac{\partial C_1}{\partial P_{G_1}} = 30 + 0.15 P_{G_1} \; ; \; 25 \leq P_{G_1} \leq 125\,mw \rightarrow ①$$

$$\frac{\partial C_2}{\partial P_{G_2}} = 40 + 0.2 P_{G_2} \; ; \; 30 \leq P_{G_2} \leq 100\,mw \rightarrow ②$$

$$\frac{\partial C_3}{\partial P_{G_3}} = 15 + 0.18 P_{G_3} \; ; \; 15 \leq P_{G_3} \leq 200\,mw \rightarrow ③$$

$$P_1 + P_2 + P_3 = 425\,mw \rightarrow ④$$

solving ①, ②, ③, ④, we get

$P_{G_1} = 103.22\,mw$

$P_{G_2} = 27.44\,mw$

$P_{G_3} = 169.354\,mw$

① = ② => $0.15P_{G1} - 0.2P_{G2} = 10$ → ④

① = ③ => $0.15P_{G1} - 0.18P_{G3} = -15$ → ⑤

② = ③ => $0.2P_{G2} - 0.18P_{G3} = -25$ → ⑥

$P_1 = 103.22\,mw$

$P_2 = 27.41\,mw$

$P_3 = 169.354\,mw$.

∴ The Value of Power generation of unit I and Unit III are within the minimum and maximum limits.

The Value of Power generation of unit II is Violating the limit. So set $P_{G2}$ at the lowermost limit i.e 30 mw

$$\therefore P_D = P_{G1} + P_{G2} + P_{G3} = 300\,mw$$

$\because P_{G2} = 30\,mw$.

$$P_{G1} + P_{G3} = 270\,mw$$

$$\frac{\partial C_1}{\partial P_{G1}} = \frac{\partial C_3}{\partial P_{G3}}$$

$$30 + 0.15P_{G1} = 15\cancel{0} + 0.18P_{G3}$$

$$0.15P_{G1} - 0.18P_{G3} = \cancel{120}\ -15$$

$$P_{G1} = 101.81\,mw$$

$$P_{G3} = 168.181\,mw$$

5. The Cost curves of two generators may be approximated by second degree Polynomial

$C_1 = 0.1P_{G1}^2 + 20P_{G1} + \alpha_1$

$C_2 = 0.1P_{G1}^2 + 30P_{G2} + \alpha_2$ where $\alpha_1$ and $\alpha_2$ are constants. If the total demand on generator is 200 mw. Find the optimum generator settings. How many Rs/hr would be loosing if generators were operated about 15% of optimum setting.

6. The incremental fuel cost of two units in a generating station are as follows.

$\frac{\partial c_1}{\partial P_{G_1}} = 0.15 P_{G_1} + 35$ ; $\frac{\partial c_2}{\partial P_{G_2}} = 0.2 P_{G_2} + 28$. Assuming continuous running with a total load of 150mw. Calculate the saving per hour obtained by using most economical division of load between the units as compared with loading each equally. The maximum and minimum operating loadings are same for each unit and are 125mw and 20 mw respectively.

Sol Given $C_1 = 0.1 P_{G_1}^2 + 20 P_{G_1} + \alpha_1$

$$C_2 = 0.1 P_{G_2}^2 + 30 P_{G_2} + \alpha_2$$

Case I:

$$\lambda_1 = \frac{\partial c_1}{\partial P_{G_1}} = 0.2 P_{G_1} + 20 \rightarrow (1)$$

$$\lambda_2 = \frac{\partial c_2}{\partial P_{G_2}} = 0.2 P_{G_2} + 30 \rightarrow (2)$$

equating (1) and (2), we get

$$0.2 P_{G_1} - 0.2 P_{G_2} = 10 \rightarrow (3)$$

Given $P_D = P_{G_1} + P_{G_2} = 200 \rightarrow (4)$

$$P_{G_1} = 125^{mw}, P_{G_2} = 75 mw$$

$$\lambda_1 = 0.2(125) + 20 = 45$$

$$\lambda_2 = 0.2(75) + 30 = 45$$

Case II:

If the generator were operated about 15% of optimum setting then

$$P_{G_1} = 125 - 125 \times \frac{15}{100}$$

$$= 106.25 \text{ mw}$$

$$P_{G_2} = 75 - 75 \times \frac{15}{100}$$

$$= 63.75 \text{ mw}$$

$\therefore$ Decrease in the cost of Power generation of unit I is

change in cost I is

$$\Delta C_1 = -\int_{125}^{106.25} \left(\frac{\partial C_1}{\partial P_{G1}}\right) \partial P_{G1}$$

$$= -\int_{125}^{106.25} \partial (0.2P_{G1} + 20)\partial P_{G1}$$

$$= -\int_{125}^{106.25} (0.2\partial P_{G1}^2 + \underline{20\partial P_{G1}})$$

$$= -\int_{125}^{106.25} \partial\ (0.2P_{G1}^2 + 20P_{G1})$$

$$= -\left[0.2\left(\frac{P_{G1}^2}{2}\right) + 20 \cdot \frac{P_{G1}^2}{2}\right]_{125}^{106.25}$$

$$= 808.59 \text{ Rs/hr}$$

$$\boxed{\begin{aligned} \int x^2 &= \frac{x^3}{3} \\ \int x &= \frac{x^2}{2} \end{aligned}}$$

Decrease in cost of Power generation of unit II

$$\Delta C_2 = -\int_{75}^{63.75} \left(\frac{\partial C_2}{\partial P_{G2}}\right) \cdot \partial P_{G2}$$

$$= -\int_{75}^{63.75}$$

$\Delta C_2 = 493.593$ Rs/hr

Net save in cost $= \Delta C = \Delta C_1 - \Delta C_2 = 315$ Rs/hr.

6. Given $\dfrac{\partial C_1}{\partial P_{G1}} = 0.15P_{G1} + 35 \rightarrow (1)$

$$\frac{\partial C_2}{\partial P_{G2}} = 0.2P_{G2} + 28 \rightarrow (2)$$

equating (1) and (2), we get

$$0.15P_{G1} - 0.2P_{G2} = -7 \rightarrow (3)$$

$$P_D = P_{G1} + P_{G2} = 150 \text{ mw}$$

$P_{G1} = 65.714$ mw, $P_{G2} = 84.286$ mw

$$\lambda_1 = \frac{\partial C_1}{\partial P_{G1}} = 0.15(65.71) + 35 = 44.85$$

$$\lambda_2 = \frac{\partial C_2}{\partial P_{G2}} = 0.2(84.28) + 28 = 44.85$$

Case II: With an Equal sharing of load:

$P_{G1} = 75\,mw$ ; $P_{G2} = 75\,mw$

With an equal distribution of load, the load on Plant I is increased from 65.714 mw to 75 mw.

The increase in cost of operation of Plant I is

$$\Delta C_1 = \int_{65.714}^{75} \frac{\partial C_1}{\partial P_{G1}} \cdot \partial P_{G1}$$

$$= \int_{65.714}^{75} (0.15 P_{G1} + 35) \cdot \partial P_{G1}$$

$$= \left[ 0.15 \frac{P_{G1}^2}{2} + 35 \cdot P_{G1} \right]_{65.714}^{75}$$

$$= 423.010 \text{ Rs/hr.}$$

The decrease in cost of operation of Plant II is

$$\Delta C_2 = \int_{84.286}^{75} \frac{\partial C_2}{\partial P_{G2}} \cdot \partial P_{G2}$$

$$= \int_{84.286}^{75} (0.2 P_{G2} + 28) \partial P_{G2}$$

$$= \left[ 0.2 \frac{P_{G2}^2}{2} + 28 P_{G2} \right]_{84.286}^{75}$$

$$\Delta C_2 = 407.921 \text{ Rs/hr}$$

$\therefore$ Net Saving in cost $= \Delta C = \Delta C_1 - \Delta_2$

$$\Delta C = 423.010 - 407.921$$

$$= 15.089 \text{ Rs/hr}$$

7. Unit I: Coal Fired steam unit maximum output = 600 mw and minimum output = 150 mw. Input output curve is $H_1\left(\frac{mBtu}{h}\right) = 510.0 + 7.2P_1 + 0.00142P_1^2$

Unit II: oil Fired steam unit: maximum output is 400 mw and minimum output = 100 mw. Input output curve is $H_2\left(\frac{mBtu}{h}\right) = 310.0 + 7.8P_2 + 0.00194P_2^2$

Unit III: oil Fired steam unit: maximum output is 200 mw and minimum output = 50 mw. Input output curve is $H_3\left(\frac{mBtu}{h}\right) = 78.0 + 7.97P_3 + 0.00482P_3^2$ for Determine the economic operating point for this three units when delivering a total load of 850 mw.

Let the fuel cost be

Unit I: fuel Cost = 1.1 Rs/mBtu

Unit II: Fuel cost = 1.0 Rs/mBtu.

Unit III: Fuel Cost = 1.0 Rs/mBtu.

Sol Given Data,

No. of units, $n = 3$

$F_1(P_1) = H_1(P_1) \times$ fuel cost

$= 510.0\ \cancel{*}\ 72P_1 + 0.00142P_1^2 \times 1.1$

$= 561 + 7.92P_1 + 1.562\times10^{-3}P_1^2$

$F_2(P_2) = H_2(P_2) \times$ Fuel Cost

$= 310.0 + 7.8P_2 + 0.00194P_2^2$

$F_3(P_3) = H_3(P_3) \times$ Fuel Cost

$= 78.0 + 7.97P_3 + 0.00482P_3^2$

Now
$$\left.\begin{aligned}\frac{\partial F_1(P_1)}{\partial c_1} &= \cancel{3.124}\times10^{-3}P_1 + 7.92 = \lambda_1 \rightarrow (1)\\ \frac{\partial F_2(P_2)}{\partial c_2} &= 3.88\times10^{-3}P_2 + 7.8 = \lambda_2 \rightarrow (2)\\ \frac{\partial F_3(P_3)}{\partial c_3} &= 9.64\times10^{-3}P_3 + 7.97 = \lambda_3 \rightarrow (3)\end{aligned}\right\} \lambda_1 = \lambda_2 = \lambda_3 = \lambda$$

Total Load, $P_1 + P_2 + P_3 = 850$ mw $\rightarrow$ (4)

Equating ① and ②, we get

$$\frac{\partial F_1(P_1)}{\partial C_1} = \frac{\partial F_2(P_2)}{C_2}$$

$$0.003124P_1 + 7.92 = 0.00388P_2 + 7.8$$

$$\Rightarrow 0.003124P_1 + 7.92 - 0.00388P_2 - 7.8 = 0$$

$$\Rightarrow 0.003124P_1 - 0.00388P_2 = -0.12 \rightarrow ⓐ$$

Equating ① and ③, we get

$$\frac{\partial F_1(P_1)}{\partial C_1} = \frac{\partial F_3(P_3)}{C_3}$$

$$\Rightarrow 0.003124P_1 - 0.00964P_3 = 0.05 \rightarrow ⓑ$$

Equating ② and ③, we get

$$\frac{\partial F_2(P_2)}{\partial C_2} = \frac{\partial F_3(P_3)}{C_3}$$

$$\Rightarrow 0.00388P_2 - 0.00964P_3 = 0.17 \rightarrow ⓒ$$

$$P_1 + P_2 + P_3 = 850 \rightarrow ④$$

$$\Rightarrow P_3 = 850 - (P_1 + P_2)$$

Equating ⓐ and ⓑ, we get

$$0.003124P_1 - 0.00388P_2 + 0.12 = 0.003124P_1 - 0.00964P_3 - 0.05$$

$P_1 = 393.2464$ mw

$P_2 = 334.665$ mw

$P_3 = 122.084$ mw

$\lambda = 9.1485$

$$\Rightarrow -0.00388P_2 + 0.12 + 0.00964P_3 + 0.05 = 0$$

$$\Rightarrow -0.00388P_2 + 0.17 + 0.0964[850 - (P_1 + P_2)] = 0$$

$$\Rightarrow -0.00386P_2 + 0.17 + 0.0964 \times 850 - 0.00964P_1 - 0.00964P_2 = 0$$

$$-0.00388P_2 + 0.17 + 8.194 - 0.00964P_1 - 0.00964P_2 = 0$$

$$\Rightarrow -0.00964P_1 - 0.01352P_2 + 8.364 = 0$$

$$\Rightarrow 0.00964P_1 + 0.01352P_2 = 8.364 \rightarrow ①$$

Equating ⓑ and ⓒ, we get

$$0.003124P_1 - 0.00964P_3 - 0.05 = 0.00388P_2 - 0.00964P_3 - 0.17$$

$$\Rightarrow 0.003124P_1 - 0.00964[850 - (P_1 + P_2)] - 0.05 - 0.00388P_2 + 0.00964P_3 - 0.17$$

$$\Rightarrow 0.003124P_1 - 8.194 - 0.00964P_1 - 0.00964P_2 - 0.05 - 0.0038$$

(or)

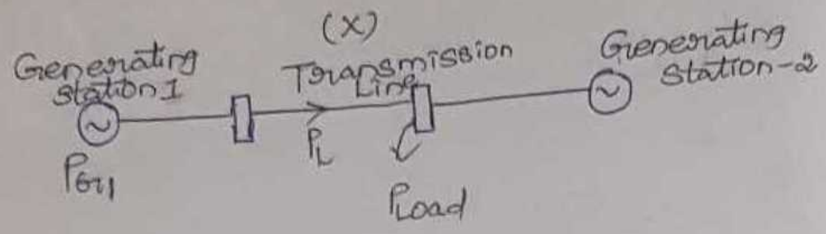
$$\lambda = \frac{P_D + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3}}{\frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3}}$$

$$\lambda = \frac{850 + \frac{7.92}{0.003124} + \frac{7.8}{0.00388} + \frac{7.97}{0.00964}}{\frac{1}{0.003124} + \frac{1}{0.00388} + \frac{1}{0.00964}}$$

$$= \frac{6222.284}{681.568}$$

$$= 9.129$$

Economic operation of Power systems (when losses are considered):



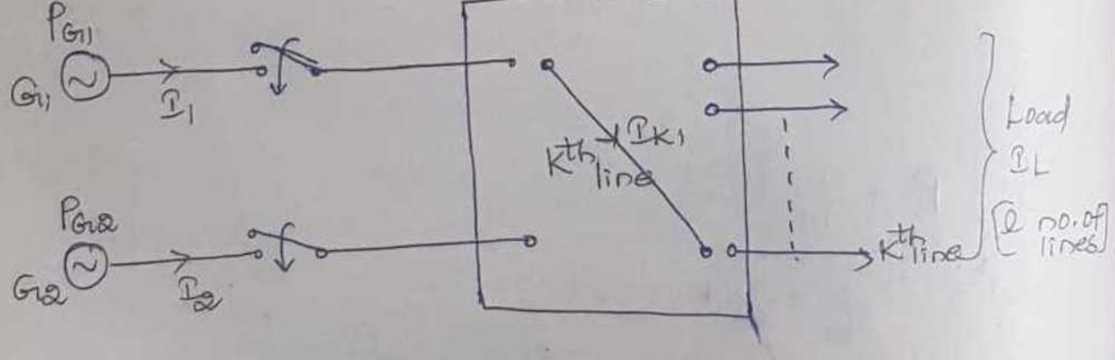

Transmission Loss expression in terms of loss coefficient (B-coefficients):

Step I:

Transmission Loss $P_L$ is expressed as a function of real power generation through loss coefficients (or) B-coefficients. i.e.,

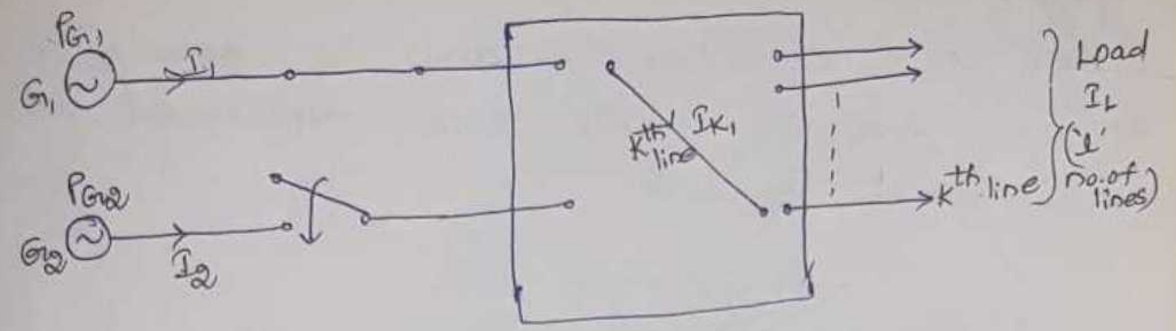
$$P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j}$$

The expression for transmission power loss is derived by using Kron's method of reducing a system ~~plan~~ to an ~~equation~~ equivalent system. The power loss of a system can be derived by considering two generating stations, supplying an arbitary no. of loads through a transmission network as shown in figure




Determination of current in $K^{th}$ line:

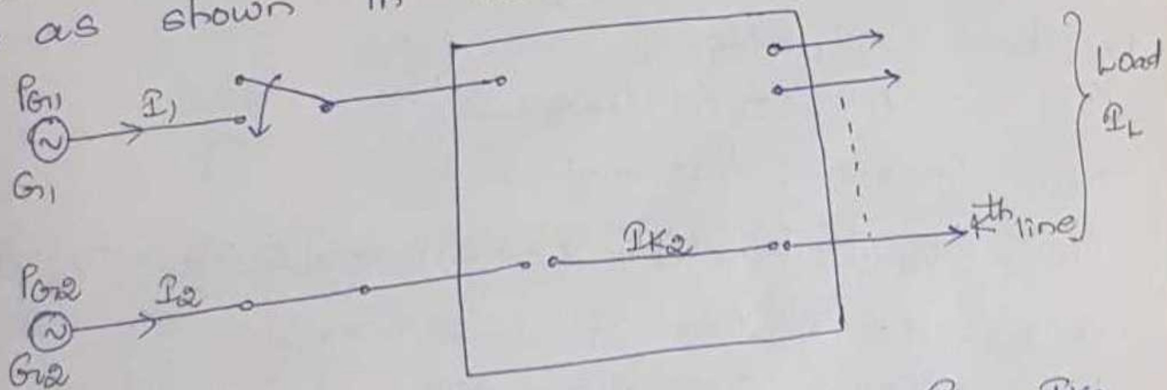
Let us assume that the entire load current is supplied by generating station 1 as shown in figure 2




current distribution factor, $N_{K_1} = \frac{I_{K_1}}{I_L} = \frac{I_{K_1}}{I_1}$

current in the $K^{th}$ line is $I_{K_1} = N_{K_1} \cdot I_1 \rightarrow (1)$

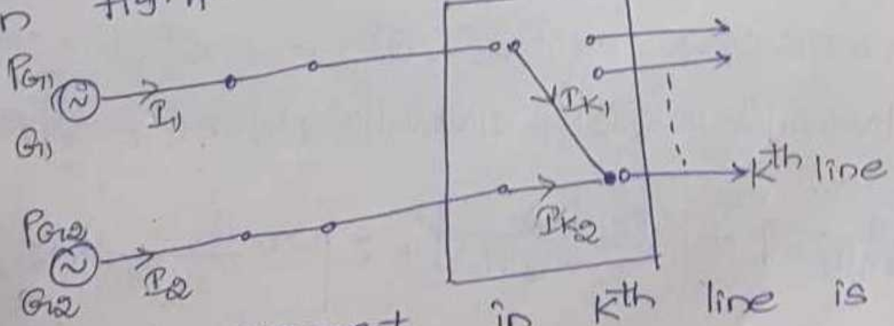
Let us assume that the entire load current is supplied by generating station 2 as shown in figure 3




$\therefore$ current distribution factor, $N_{K_2} = \frac{I_{K_2}}{I_L} = \frac{I_{K_2}}{I_2}$

$\therefore$ current in $K^{th}$ line $I_{K_2} = N_{K_2} \cdot I_2 \rightarrow (2)$

Let us assume that the total load is supplied by both the stations as shown in fig. 4.




$\therefore$ Total current in $K^{th}$ line is $I_K = I_{K_1} + I_{K_2}$

$I_K = N_{K_1} I_1 + N_{K_2} I_2 \rightarrow (3)$

Step IV:

Total Power loss through loss coefficients:

If there are '$l$' no. of lines in the system, total Power loss in system is.

$$P_L = \sum_{K=1}^{l} 3 \cdot I_K^2 \cdot R_K \rightarrow (9)$$

$$\therefore P_L = \frac{P_{G1}^2}{V_1^2 (Pf_1)^2} \sum_{K=1}^{l} NK_1^2 \cdot R_K + \frac{P_{G2}^2}{V_2^2 (Pf_2)^2} \sum_{K=1}^{l} NK_2^2 \cdot R_K +$$

$$\frac{2 \cdot P_{G1} P_{G2}}{V_1 V_2 (Pf_1)(Pf_2)} \sum_{K=1}^{l} NK_1 \cdot NK_2 \cdot R_K \cos(\sigma_2 - \sigma_1) \rightarrow (10)$$

The above expression can be written as

$$P_L = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 + 2 B_{12} P_{G1} P_{G2} \rightarrow (11)$$ [for only 2 units]

where $B_{11} = \frac{1}{V_1^2 (Pf_1)^2} \cdot \sum_{K=1}^{l} NK_1^2 \cdot R_K$

$B_{22} = \frac{1}{V_2^2 (Pf_2)^2} \sum_{K=1}^{l} NK_2^2 \cdot R_K$.

$B_{12} = \frac{1}{V_1 V_2 (Pf_1)(Pf_2)} \cdot \sum_{K=1}^{l} NK_1 \cdot NK_2 \cdot R_K \cdot \cos(\sigma_2 - \sigma_1)$

eq (11) represents the total Power loss of a transmission line as a function of real Power generation through loss coefficients.

For three units,

$$P_L = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 + B_{33} P_{G3}^2 + 2 B_{12} P_{G1} P_{G2} + 2 B_{23} P_{G2} P_{G3} + 2 B_{31} P_{G3} P_{G1}.$$

In general if the system has 'n' no. of stations supplying the total load through transmission lines, the transmission line loss is given by

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{pq} P_{Gq} \quad (\text{or}) \quad P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{Gi} B_{ij} P_{Gj}$$

Assumptions:

All the lines in the system have the Same $\frac{X}{R}$ ratio.

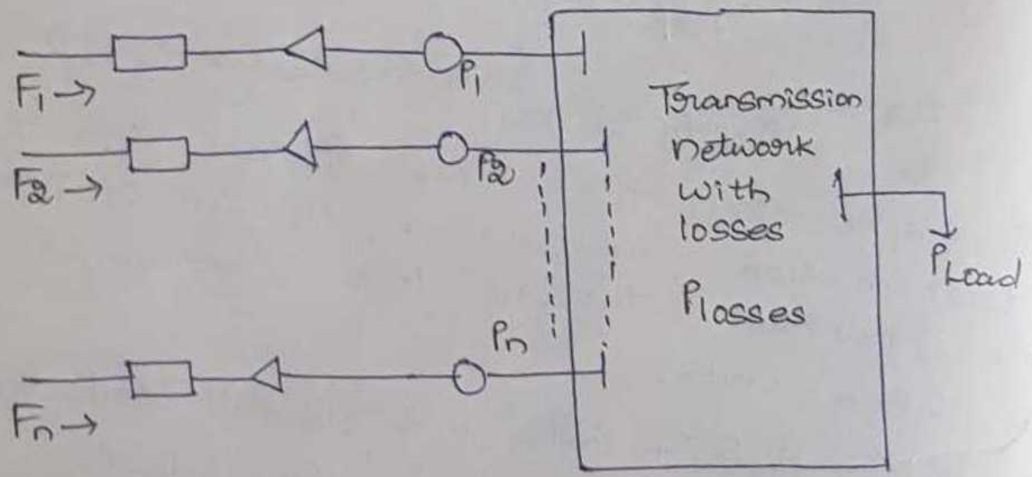
. All the load currents have the same Phase angle.

. All the load currents maintain a constant ratio to the total current.

The magnitude and Phase angle of bus Voltages at each station remains constant.

Power factor at each station bus remains constant

Optimal allocation of total load among generators when transmission losses are considered (or) Condition for optimal allocation:




Step I:

Consider a Power station having 'n' no. of Units.

$\therefore$ The cost of Production of electrical energy i.e., the objective function is given by

$$C = \sum_{i=1}^{n} C_i(P_{Gi}) \rightarrow (1)$$

where $C_i$ is cost function of $i^{th}$ unit.

Step II:

This cost is to be minimized, subjected to the equality constraint is given by

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_L(P_{Gi}) = 0 \rightarrow ②$$

This is a constrainted optimization problem.

Step III:

To get the solution for optimization problem, an objective function is defined by argumenting (increasing) with equality constrains using Lagrangian multiplier ($\lambda$) and is given by

$$C' = C - \lambda \left[P_{Gi} - P_D - P_L(P_{Gi})\right] \rightarrow ③$$

This argumented objective function is called constrained objective function.

Step IV: Condition for optimality:

The condition for optimality such as argumented function is

$$\frac{\partial c'}{\partial P_{Gi}} = 0 \text{ ; where } i = 1, 2, \ldots n$$

$$\frac{\partial c'}{\partial P_{Gi}} = \frac{\partial C_i}{\partial P_{Gi}} - \lambda \left[1 - 0 - \frac{\partial P_L}{\partial P_{Gi}}\right] = 0$$

$$\frac{\partial C_i}{\partial P_{Gi}} = \lambda \left[1 - \frac{\partial P_L}{\partial P_{Gi}}\right]$$

$$\frac{\partial C_i}{\partial P_{Gi}} \Big/ \left[1 - \frac{\partial P_L}{\partial P_{Gi}}\right] = \lambda$$

$$\frac{\frac{\partial C_i}{\partial P_{Gi}}}{\left[1 - (ITL)_i\right]} = \lambda \quad \left[\text{where } ITL = \frac{\partial P_L}{\partial P_{Gi}}\right]$$

$$\frac{(IFC)_i}{\left[1 - (ITL)_i\right]} = \lambda$$

$$\lambda = L_i \frac{\partial C_i}{\partial P_{Gi}} \rightarrow ④$$

where $L_i = \frac{1}{1 - (ITL)_i} = \frac{1}{\left[1 - \frac{\partial P_L}{\partial P_{Gi}}\right]}$ ; $L_i$ = Penality factor where ITL = Incremental transmission Losses.

Step V:

The condition for optimality when transmission losses are considered is that the incremental fuel cost of each Plant multiplied by its Penality factor must be same for all the Plants.

$$L_1 \cdot \frac{\partial C_1}{\partial P_{G_1}} = L_2 \cdot \frac{\partial C_2}{\partial P_{G_2}} = \text{-----------} = L_n \cdot \frac{\partial C_n}{\partial P_{G_n}}$$

Step VI: Determination of Incremental Transmission Losses:

Consider a system with three generating ~~stat~~ units ($n=3$)

$\therefore$ Transmission power Loss,

$$P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + B_{33} P_{G_3}^2 + 2B_{12} P_{G_1} P_{G_2} + 2B_{23} P_{G_2} P_{G_3} + 2B_{31} P_{G_3} P_{G_1}.$$

Incremental transmission loss of generator I is

$$(ITL)_1 = \frac{\partial P_L}{\partial P_{G_1}}$$

$$(ITL)_2 = \frac{\partial P_L}{\partial P_{G_2}}$$

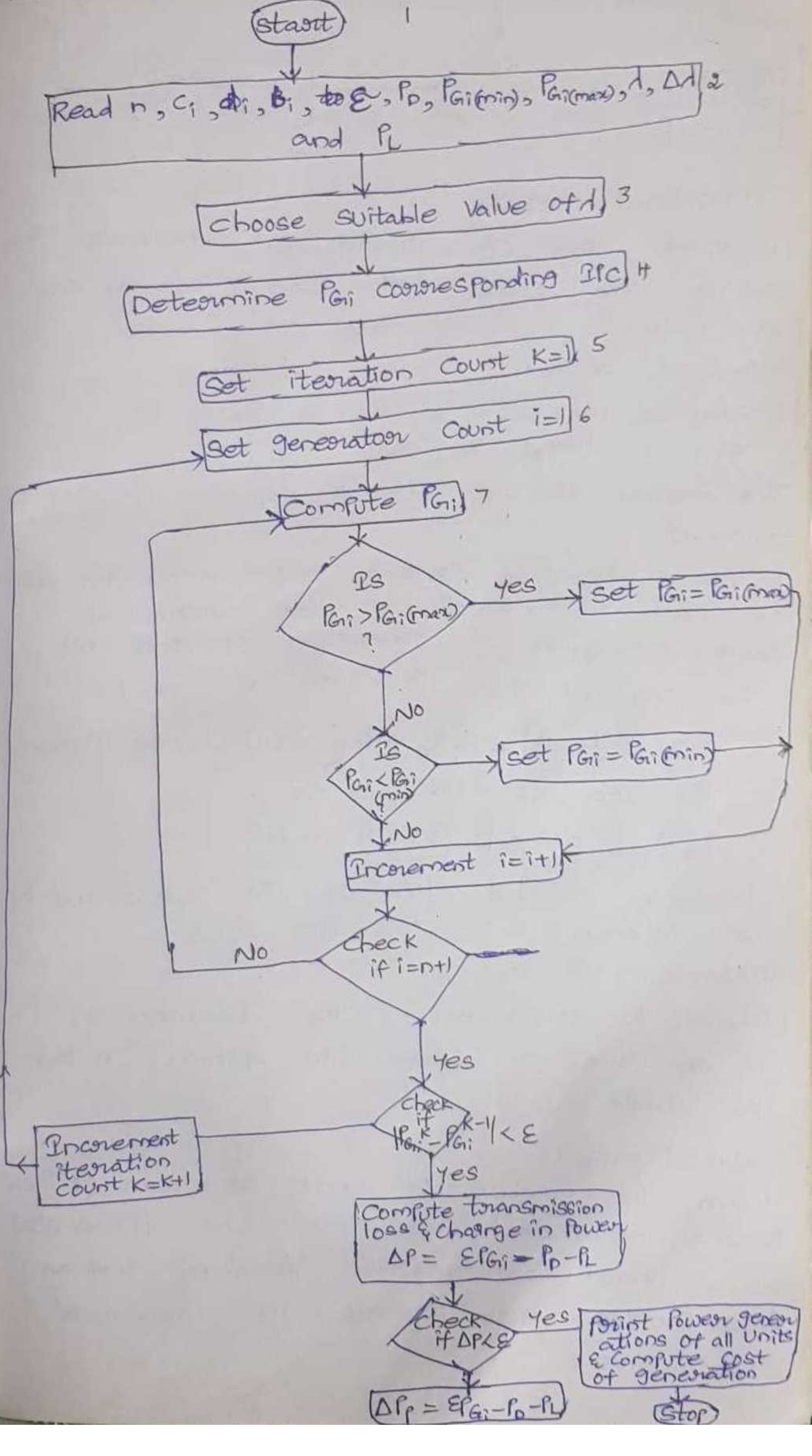
$$\vdots$$

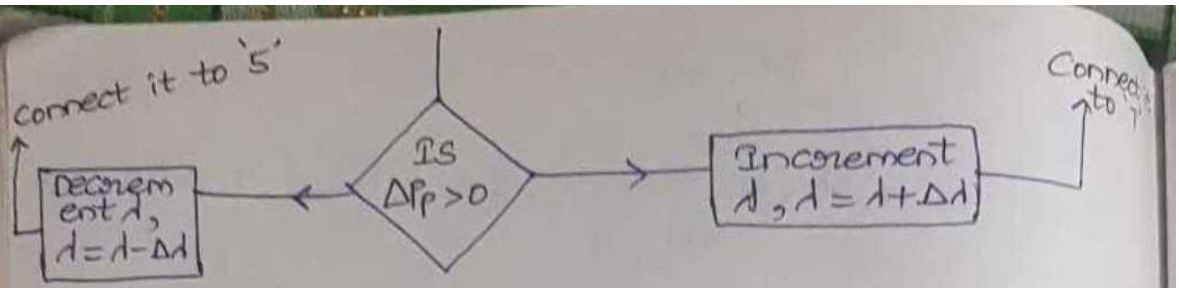
$$(ITL)_n = \frac{\partial P_L}{\partial P_{G_n}}$$

$$\therefore P_L = 2(B_{11} P_{G_1} + B_{22} P_{G_2} + B_{33} P_{G_3})$$

In general, $\dfrac{\partial P_L}{\partial P_{G_i}} = \sum_{j=1}^{n} 2 B_{ij} P_{G_i}$

Flow chart:


start
Read n, $c_i$, $a_i$, $b_i$, $\varepsilon$, $P_D$, $P_{Gi(min)}$, $P_{Gi(max)}$, $\lambda$, $\Delta\lambda$ and $P_L$
Choose suitable value of $\lambda$
Determine $P_{Gi}$ corresponding IIC
Set iteration count K=1
Set generator count i=1
Compute $P_{Gi}$
Is $P_{Gi} > P_{Gi(max)}$ ?
yes
Set $P_{Gi} = P_{Gi(max)}$
NO
Is $P_{Gi} < P_{Gi(min)}$ ?
set $P_{Gi} = P_{Gi(min)}$
No
Increment i=i+1
Check if i=n+1
No
yes
Check if $|P_{Gi}^{K} - P_{Gi}^{K-1}| < \varepsilon$
Increment iteration count K=K+1
yes
Compute transmission loss & change in power $\Delta P = \Sigma P_{Gi} - P_D - P_L$
Check if $\Delta P < \varepsilon$
Yes
Print power generations of all units & compute cost of generation
Stop
$\Delta P_f = \Sigma P_{Gi} - P_D - P_L$
1
2
3
4
5
6
7




Computational methods:

Different types of computational methods for solving the above optimization problem are as follows.

1. Analytical method
2. Graphical method
3. Using a digital computer.

The method to be adopted depends on the following:

1. The mathematical equation representing the IFC of each unit, which can be determined from the cost of generation of that unit.

The cost of the $i^{th}$ unit is given by.

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i \text{ (second-degree Polynomial)}$$

$\therefore$ The IFC of $i^{th}$ unit is

$$(IC)_i = a_i P_{Gi} + b_i \text{ (Linear model)}$$

Where $a_i$ is the slope of IFC curve and $b_i$ the intercept of the IFC curve.

(ii) Number of units (N).

(iii) Need to represents the discontinuities (if any due to Steam Valve opening) in the IFC curve.

Analytical method:

When the number of units are small (either 2 or 3), incremental cost curves are approximated as a linear or quadratic variation and no discontinuities are present in incremental cost curves.

we Know that the IFC of $i^{th}$ unit

$$(IC)_i = \frac{\partial C_i}{\partial P_{Gi}} = a_i P_{Gi} + b_i$$

For an optimal solution, the IFC of all the units must be the same (neglecting the transmission losses).

$$\text{i.e.,} \quad \frac{\partial C_1}{\partial P_{G1}} = \frac{\partial C_2}{\partial P_{G2}} = \cdots = \frac{\partial C_n}{\partial P_{Gn}} = \lambda$$

The analytical method consist of the following steps:

1. choose a particular Value of $\lambda$.
   i.e., $\lambda = a_i P_{Gi} + b_i$
2. Compute $P_{Gi} = \frac{\lambda - b_i}{a_i}$, for the $i^{th}$ unit.
3. Find total real Power generation
   $= \sum_{i=1}^{n} P_{Gi}$ for all $i = 1, 2, ---, n$
4. Repeat the procedure from step (ii) for different Values of $\lambda$.
5. Plot a graph between total Power generation and $\lambda$.
6. For a given Power demand ($P_D$), estimate the Value of $\lambda$ from figure shown below. Value of $\lambda$ will be optimal solution for That Value of $\lambda$ optimization Problem.

Graphical method:

For obtaining the solution in this method, the following procedure is required.

1. Consider the incremental cost curves of all Units: i.e., $(IC)_i = a_i P_{Gi} + b_i$ for all $i = 1, 2, ---, n$.

and total load demand $P_D$ is given

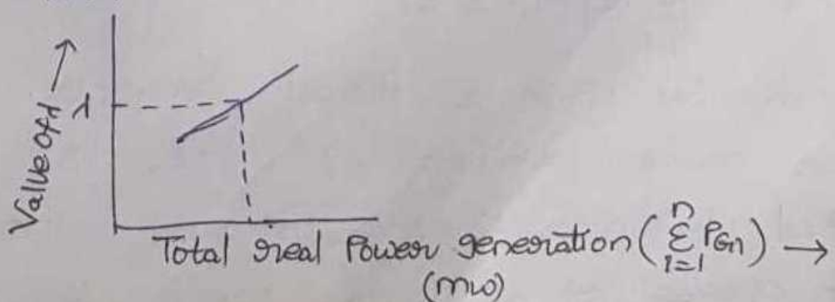
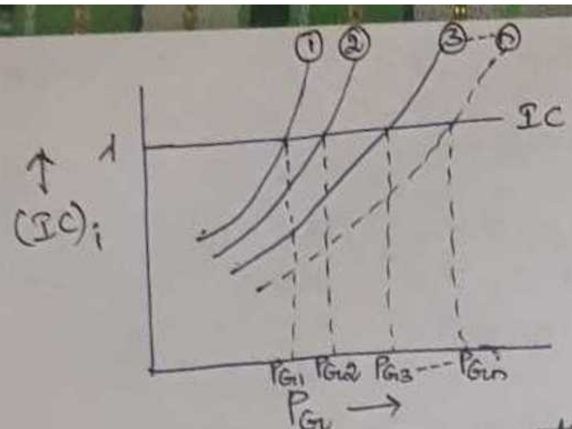



Fig. Estimation of optimum Value $\lambda$ – analytical method.



(Real Power generation in mw)

Fig. Graphical method.

2. For each unit, draw a graph between $P_G$ and (IC) as shown in above figure.
3. choose a particular value of $\lambda$ and $\Delta\lambda$
4. Determine the corresponding real-power generation of all units:

   i.e., $P_{G1}, P_{G2}$ ---- $P_{Gn}$.
5. Compute the total real power generation $= \sum_{i=1}^{n} P_{Gi}$
6. check the real-power balance of eq② as follows:

(a) If $\sum_{i=1}^{n} P_{Gi} - P_D = 0$, then $\lambda$ choosen will be optimal solution and incremental costs of all units become equal.

(b) If $\sum_{i=1}^{n} P_{Gi} - P_D < 0$, increase $\lambda$ by $\Delta\lambda$ and repeat the procedure from step 4.

(c) If $\sum_{i=1}^{n} P_{Gi} - P_D > 0$, decrease $\lambda$ by $\Delta\lambda$ and repeat the procedure from step 4.

7. This process is repeated until $\sum_{i=1}^{n} P_{Gi} - P_D$ is within a specified tolerance ($\varepsilon$), say 1 mw.

   i.e., $\sum_{i=1}^{n} P_{Gi} - P_D \leq \varepsilon$.

Solution by using a digital computer:

For more number of units, the $\lambda$-iterative method is more accurate and incremental cost curves of all units are to be stored in memory.

Information about IFC curves is given for all units i.e., $\lambda = (IC)_i = a_i P_{Gi} + b_i$.

(or) $P_{Gi} = \frac{\lambda - b_i}{a_i} = \frac{\lambda}{a_i} - \frac{b_i}{a_i}$ [when losses are neglected]

Let $\alpha_i = \frac{b_i}{a_i}$

$\beta_i = \frac{1}{a_i}$ and so on.

$\therefore P_{Gi} = \alpha_i + \beta_i (IC)_i + \gamma_i (IC_i)^2 + \cdots \rightarrow (1)$

For $i = 1, 2, \ldots, n$.

The number of terms included depends on degree of accuracy required and coefficients $\alpha_i, \beta_i$ and $\gamma_i$ are to be taken as input.

Algorithm for $\lambda$-iterative method:

1. Guess the initial value of $\lambda^0$ with the use of cost-curve equations.
2. Calculate $P^0_{Gi}$ according to eq (1) i.e.,
$$P^0_{Gi} = \alpha_i + \beta_i (\lambda^0)_i + \gamma_i (\lambda^0)^2_i + \cdots$$
3. Calculate $\sum_{i=1}^{n} P^0_{Gi}$.
4. check whether $\sum_{i=1}^{n} P^0_{Gi} = P_D$:
$$\left[\sum_{i=1}^{n} P^0_{Gi} - P_D \le \varepsilon \text{ (a tolerance value)}\right]$$
5. If $\sum_{i=1}^{} P^0_{Gi} < P_D$, set a new value for $\lambda$, i.e., $\lambda' = \lambda^0 + \Delta\lambda$ and repeat from step (ii) till the tolerance value is satisfied.
6. If $\sum_{i=1}^{n} P^0_{Gi} > P_D$, set a new value for $\lambda$, i.e., $\lambda' = \lambda^0 - \Delta\lambda$ and repeat from step (ii) till the tolerance value is satisfied.
7. stop.

Algorithm:

step I: Start

step II: Read the values of $n, c_i, d_i, b_i, \varepsilon, P_D, P_{Gi(min)}$, $P_{Gi(max)}$, $\lambda$, $\Delta\lambda$ and $P_L$

where $n$ = no. of units

$C_i$ = slope of incremental fuel cost curve.
$b_i$ = Intercept of incremental fuel cost curve
$d_i$ = Constant.
$P_D$ = Total real Power demand in mw
$P_{Gi(min)}$ = minimum Power generation in mw
$P_{Gi(max)}$ = maximum Power generation in mw
$\lambda$ = Lagrangian multiplier
$\Delta\lambda$ = change in Value of $\lambda$.
$P_L$ = Power loss in transmission line.
$\varepsilon$ = tolerance (allowable error).

Step III: Choose suitable Value of $\lambda$

Step IV: Determine real Power generation of $i$th unit corresponding to $\lambda_{PC}$.

Step V: Set iteration count $K=1$.

Step VI: Set generator count $i=1$.

Step VII: Compute $P_{Gi}$.

Step VIII: Compare $P_{Gi}$ and $P_{Gi(max)}$. If $P_{Gi} > P_{Gi(max)}$ then set $P_{Gi} = P_{Gi(max)}$ otherwise go to next step.

Step IX: Compare $P_{Gi}$ and $P_{Gi(min)}$. If $P_{Gi} < P_{Gi(min)}$ then set $P_{Gi} = P_{Gi(min)}$ otherwise increment $i = i+1$.

Step X: If $i \leq n+1$ repeat the procedure from step VII.

Step XI: If $i=n+1$, check the condition $|P_{Gi}^{K} - P_{Gi}^{K-1}| < \varepsilon$ and the compute transmission loss and change in Power $\Delta P = \Sigma P_{Gi} - P_D - P_L$

Step XII: If $|P_{Gi}^{K} - P_{Gi}^{K-1}| \geq \varepsilon$ then increment iteration count $K=K+1$ then repeat from step VI.

Step XIII: check if $\Delta P < \varepsilon$, if Yes, stop doing further calculations and start calculating the cost of generation otherwise go to next step.

step $\overline{XIV}$: Calculate value of $P_P = \Sigma P_{G_i} - P_D - P_L$

step $\overline{XV}$: check if $\Delta P_P > 0$, if yes, decrement the value of $\lambda$ and then $\lambda = \lambda - \Delta\lambda$ and repeat from step $\overline{V}$, else increment $\lambda = \lambda + \Delta\lambda$ and repeat from step $\overline{VII}$.

problems:

1. The incremental fuel cost for two plants are $\frac{\partial C_1}{\partial P_{G_1}} = 0.075 P_{G_1} + 18$ Rs/mwh, $\frac{\partial C_2}{\partial P_{G_2}} = 0.08 P_{G_2} + 16$ Rs/mwh. The loss coefficients are given as $B_{11} = 0.0015$/mw, $B_{12} = -0.0004$/mw, $B_{22} = 0.0032$/mw. For $\lambda = 25$ Rs/mwh, find the real power generations, total load demand and the transmission power loss.

Sol Given data,

No. of units, $n=2$

$$\left.\begin{aligned}\frac{\partial C_1}{\partial P_{G_1}} &= 0.075 P_{G_1} + 18 \text{ Rs/mwh}\\ \frac{\partial C_2}{\partial P_{G_2}} &= 0.08 P_{G_2} + 16 \text{ Rs/mwh}\end{aligned}\right\} \rightarrow (a)$$

Loss coefficients,

$B_{11} = 0.0015$/mw

$B_{12} = -0.0004$/mw

$B_{22} = 0.0032$/mw

$\lambda = 25$ Rs/mwh.

For optimal condition,

$$L_1 \cdot \frac{\partial C_1}{\partial P_{G_1}} = L_2 \cdot \frac{\partial C_2}{\partial P_{G_2}} = \lambda.$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G_1}}} \; ; \; L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G_2}}}$$

For $n=2$, $P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + 2 B_{12} P_{G_1} P_{G_2}$.

$$\frac{\partial P_L}{\partial P_{G_1}} = 2 P_{G_1} \cdot B_{11} + 2 B_{12} \cdot P_{G_2}.$$

$$\frac{\partial P_L}{\partial P_{G2}} = 2B_{22}P_{G2} + 2B_{12}P_{G1}$$

$$\left.\begin{aligned}\frac{\partial P_L}{\partial P_{G1}} &= 2\times0.0015\,P_{G1} - 0.0004\,P_{G2} \\ \frac{\partial P_L}{\partial P_{G2}} &= 2\times0.0032\,P_{G2} - 2\times0.0004\,P_{G1}\end{aligned}\right\} \rightarrow ①$$

$$P_{G} + P_D - P_L = 0$$

$$\Rightarrow P_{G1} + P_{G2} - P_D - P_L = 0.$$

$$P_{G1} + P_{G2} = P_D + P_L \rightarrow ②$$

$$\left.\begin{aligned}L_1 \cdot \frac{\partial C_1}{\partial P_{G1}} &= \lambda \\ L_2 \cdot \frac{\partial C_2}{\partial P_{G2}} &= \lambda\end{aligned}\right\} \rightarrow ③$$

Now substitute eq ① and eq ② in eq ③, we get

$$\therefore\ L_1 \cdot \frac{\partial C_1}{\partial P_{G1}} = \lambda \Rightarrow$$

$$\Rightarrow \frac{1}{[1 - 2\times0.0015P_{G1} - 2\times0.0004P_{G2}]}[0.075P_{G1} + 18] = 25$$

$$\Rightarrow 0.075P_{G1} + 18 = 25[1 - 2\times0.0015P_{G1} - 2\times0.0004P_{G2}]$$

$$\Rightarrow 0.075P_{G1} + 18 = 25 - 0.075P_{G1} - 0.02P_{G2}$$

$$\Rightarrow 0.15P_{G1} - 0.02P_{G2} - 7 = 0 \rightarrow ④$$

similary

$$L_2 \cdot \frac{\partial C_2}{\partial P_{G2}} = \lambda \Rightarrow$$

$$\Rightarrow \frac{1}{(1 - 2\times0.0032P_{G2} - 2\times0.0004P_{G1})}[0.08P_{G2} + 16] = 25$$

$$\Rightarrow 0.08P_{G2} + 16 = 25(1 - 2\times0.0032P_{G2} - 2\times0.0004P_{G1})$$

$$\Rightarrow 0.08P_{G2} + 16 = 25 - 0.16P_{G2} - 0.02P_{G1}$$

$$\Rightarrow -0.02P_{G1} + 0.24P_{G2} - 9 = 0 \rightarrow ⑤$$

By solving eq ④ and eq ⑤, we get

$P_{G1} = 52.24$ mw , $P_{G2} = 41.85$ mw we Know that,

Power loss, $P_L = B_{11}P_{G1}^2 + B_{22}P_{G2}^2 + 2B_{12}P_{G1}P_{G2}$

$\therefore P_L = 0.0015 \times 52.24^2 + 0.0032 \times 41.85^2 + 2 \times -0.0004 \times 52.24 \times 41.85$

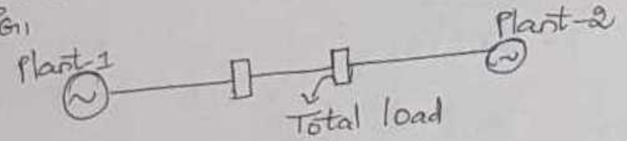
$P_L = 7.95$ mw

$P_D = P_{G1} + P_{G2} - P_L = 52.24 + 41.85 - 7.95$

$P_D = 86.14$ mw

2. A system consists of 2 power plants connected by a transmission line. ~~Determine~~ The total load, located at plant 2 is shown in figure. Data of evaluating loss coefficients consists of information that a power transfer of 100mw from station 1 to station 2 results in a total loss of 8mw. Find the required generation at each station and the power received by the load when 'λ' of the system is 100 Rs/mwh. Their IFC's of two plants are given by

$\frac{\partial C_1}{\partial P_{G1}} = 0.12P_{G1} + 65$ Rs/mwh, $\frac{\partial C_2}{\partial P_{G2}} = 0.25P_{G2} + 75$ Rs/mwh.




sol Given data,

No. of units, $n = 2$

$$\left.\begin{aligned} \frac{\partial C_1}{\partial P_{G1}} &= 0.12P_{G1} + 65 \text{ Rs/mwh} \\ \frac{\partial C_2}{\partial P_{G2}} &= 0.25P_{G2} + 75 \text{ Rs/mwh} \end{aligned}\right\} \rightarrow (a)$$

we Know that,

For 'n' no. of stations supplying the total load through transmission lines, transmission line loss is given by.

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{pq} P_{Gq}.$$

$\because n = 2$, $P_L = \sum_{p=1}^{2} \sum_{q=1}^{2} P_{Gp} B_{pq} P_{Gq}$

wh $\therefore P_L = P_{G1}^2 B_{11} + 2B_{12}P_{G1}P_{G2} + P_{G2}^2 B_{22}$

Since Power transfer of 100mw from Plant-1 to Plant-2 i.e $P_{G_1} = 100mw$.

Consider $P_{G2}, B_{21}, B_{22} = 0$.

$$\therefore P_L = B_{11} P_{G1}^2$$

Given $P_L = 8mw$

$$\Rightarrow 8 = B_{11} \times 100^2$$

$$B_{11} = \frac{8}{100^2} = 8\times10^{-4} mw.$$

$$\therefore P_L = 8\times10^{-4} \cdot P_{G1}^2$$

$$\frac{\partial P_L}{\partial P_{G1}} = 2B_{11}P_{G1} + 2B_{22}P_{G2}$$

$$= 2P_{G1} \times 8\times10^{-4}$$

$$\left[\because B_{22}, P_{G2} = 0 \quad 2B_{22}P_{G2} = 0\right]$$

$$\left.\begin{aligned} \frac{\partial P_L}{\partial P_{G1}} &= 16\times10^{-4} P_{G1} \\ \frac{\partial P_L}{\partial P_{G2}} &= 0 \quad [\because P_{G2} = 0] \end{aligned}\right\} \rightarrow ①$$

Now,

$$\left.\begin{aligned} &\text{Penality factor for Plant-I is} \\ &L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{1}{1 - 16\times10^{-4} P_{G1}} \\ &\text{Penality factor for Plant II is} \\ &L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1-0} = 1 \end{aligned}\right\} \rightarrow ②$$

Condition for optimality is

$$\frac{\partial C_1}{\partial P_{G1}} L_1 = \frac{\partial C_2}{\partial P_{G2}} L_2 = \lambda \quad \text{where } \lambda = 100 \text{ Rs/mwh}$$

Substitute eq ② and eq ① in eq ②, we get.

$$\Rightarrow (0.12P_{G1} + 65)\left(\frac{1}{1 - 16\times10^{-4} P_{G1}}\right) = (0.25P_{G2} + 75) \times 1 = 100$$

$\Rightarrow$ Consider

$$0.12P_{G1} + 65 \times \left(\frac{1}{1 - 16\times10^{-4} P_{G1}}\right) = 100$$

$$\Rightarrow 0.12P_{G1} + 65 = 100(1 - 16\times10^{-4} P_{G1})$$

$$\Rightarrow 0.12P_{G1} + 65 = 100 - 100\times16\times10^{-4} P_{G1}$$

$$\Rightarrow 0.12P_{G1} + 65 = 100 - 0.16P_{G1}$$

$$\Rightarrow 0.28P_{G1} - 35 = 0$$

$0.28P_{G1} = 35$

$\Rightarrow P_{G1} = \frac{35}{0.28}$

$P_{G1} = 125\,mw$

Now consider,

$0.25P_{G2} + 75 = 100$

$\Rightarrow 0.25P_{G2} - 25 = 0$

$\Rightarrow P_{G2} = \frac{25}{0.25} = 100\,mw$

Now Power received by load when $\lambda = 100\,Rs/mwh$ is

$P_D = (P_{G1} + P_{G2}) - \text{losses}$

$= (125 + 100) - P_L$

$= (125 + 100) - 8\times10^{-4} \cdot P_{G1}^2$

$= (125 + 100) - 8\times10^{-4} \times 125^2$

$P_D = 212.5\,mw$

2. The incremental production cost of 2 units are given by $(IPC)_1 = (IC)_1 = 0.07P_1 + 16\,Rs/mwh$ $(IPC)_2 = (IC)_2 = 0.08P_2 + 12\,Rs/mwh$. The loss coefficients of system are given by $B_{11} = 0.001$, $B_{12} = B_{21} = -0.005$, $B_{22} = 0.0024$. The total load to be met is 150 mw, determine the economic operation schedule if the transmission line losses are co-ordinated and the losses are included but not co-ordinated.

Sol Given data,

No. of units, $n = 2$.

$(IPC)_1 = (IC)_1 = \frac{\partial C_1}{\partial P_{G1}} = 0.07P_1 + 16\,Rs/mwh.$

$(IPC)_2 = (IC)_2 = \frac{\partial C_2}{\partial P_{G2}} = 0.08P_2 + 12\,Rs/mwh.$

Loss Coefficients,

$B_{11} = 0.001$, $B_{21} = B_{12} = -0.005$, $B_{22} = 0.0024$.

$P_D = 150\,mw$ = Power demand.

Finding Terms:

(i) Economic Operating schedule $\left.\begin{array}{l}P_1 = ?\\ P_2 = ?\end{array}\right\}$ when losses are Co-ordinated

$\Sigma P_{G_1} - P_D - P_L = 0$

(ii) $\left.\begin{array}{l} P_1 = ? \\ P_2 = ? \end{array}\right\} \rightarrow$ when losses are not Co-ordinated considered:

when losses are considered:

Penality factor, $L_1 = \dfrac{1}{1 - \frac{\partial P_L}{\partial P_{G_1}}}$

$P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + 2 B_{12} P_{G_1} P_{G_2}$

$\dfrac{\partial P_L}{\partial P_{G_1}} = 2B_{11} P_{G_1} + 2B_{12} P_{G_2} = 2 \times 0.001 P_{G_1} + 2 \times -0.005 P_{G_2}$

$\dfrac{\partial P_L}{\partial P_{G_1}} = 0.002 P_{G_1} + 0.01 P_{G_2}$

$L_1 \cdot \dfrac{\partial C_1}{\partial P_{G_1}} = \lambda \Rightarrow \dfrac{1}{1 - (0.002 P_{G_1} + 0.01 P_{G_2})} (0.07 P_{G_1} + 16) = \lambda$

$\Rightarrow 0.07 P_{G_1} + 16 = \lambda (1 - 0.002 P_{G_1} + 0.01 P_{G_2})$

$0.07 P_{G_1} + 16 = \lambda - 0.002 \lambda P_{G_1} + 0.01 P_{G_2} \lambda \rightarrow (1)$

$\dfrac{\partial P_L}{\partial P_{G_2}} = 2B_{22} P_{G_2} + 2B_{12} P_{G_1} = 2 \times 0.0024 P_{G_2} + 2 \times -0.005 P_{G_1}$

$L_2 \cdot \dfrac{\partial C_2}{\partial P_{G_2}} = \lambda \Rightarrow \dfrac{1}{1 - (0.0048 P_{G_2} - 0.01 P_{G_1})} (0.08 P_{G_2} + 12) = \lambda$

$\Rightarrow 0.08 P_{G_2} + 12 = \lambda - 0.0048 P_{G_2} \lambda + 0.01 P_{G_1} \lambda$

$\Rightarrow 0.08 P_2 + 12 - \lambda + 0.0048 P_{G_2} \lambda + 0.01 P_{G_1} \lambda = 0$

$\Rightarrow -0.01 P_{G_1} \lambda + 0.0048 P_{G_2} \lambda + 0.08 P_2 + 12 - \lambda = 0 \rightarrow (2)$

Here there is no relation for generation and demand.

when losses are not considered:

$\dfrac{\partial C_1}{\partial P_1} = 0.07 P_1 + 16 \text{ Rs/mwh} \rightarrow (1); \quad \dfrac{\partial C_2}{\partial P_2} = 0.08 P_2 + 12 \text{ Rs/mwh} \rightarrow (2)$

$P_D = P_1 + P_2 = 150 \text{ mw} \rightarrow (3)$

For optimal allocation, $\dfrac{\partial C_1}{\partial P_1} = \dfrac{\partial C_2}{\partial P_2} = \lambda$

Equate (1) and (2), we get,

$0.07 P_1 - 0.08 P_2 + 4 = 0 \rightarrow (4)$

solving (3) and (4), we get

$P_1 = 53.33 \text{ mw}, \; P_2 = 96.667 \text{ mw}$

substitute $P_1$ in eq(1), we get

$\lambda = 19.733 \text{ Rs/mwh}$

3. The system consisting of two generating plants with fuel cost of $C_1 = 0.03 P_{G1}^2 + 15P_{G1} + 1$ Rs/hr, $C_2 = 0.04P_{G2}^2 + 21P_{G2} + 1.4$ Rs/hr. The system is operating on economical dispatch with 100 MW of power generation by each plant. The incremental transmission loss of plant 2 is 0.25. Find the penalty factor of plant 1

Sol Given data,

$$C_1 = 0.03P_{G1}^2 + 15P_{G1} + 1 \text{ Rs/hr}$$

$$C_2 = 0.04P_{G2}^2 + 21P_{G2} + 1.4 \text{ Rs/hr}$$

$$P_{G1} = P_{G2} = 100 \text{ MW}$$

$$\frac{\partial P_L}{\partial P_{G2}} = (ITL)_2 \text{ of plant 2} = 0.25$$

we know that,

Penalty factor of plant-2 is

$$L_2 = \frac{1}{1-(ITL)_2} = \frac{1}{1-\frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1-0.25} = \frac{1}{0.75} = 1.333$$

Incremental fuel cost of plant-1 is

$$\frac{\partial C_1}{\partial P_{G1}} = 2\times 0.03P_{G1} + 15 = 0.06P_{G1} + 15$$

Incremental fuel cost of plant-2 is

$$\frac{\partial C_2}{\partial P_{G2}} = 2\times 0.04P_{G2} + 21 = 0.08P_{G2} + 21$$

condition for optimality is

$$L_1 \cdot \frac{\partial C_1}{\partial P_{G2}} = L_2 \cdot \frac{\partial C_2}{\partial P_{G2}} = \lambda$$

$$\Rightarrow L_1(0.06P_{G1} + 15) = 1.333 \times (0.08P_{G2} + 21)$$

$$L_1(0.06\times 100 + 15) = 1.333\times(0.08\times 100 + 21)$$

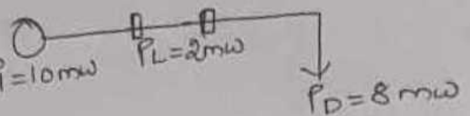
$$21 L_1 = 1.333\times 0.08\times 100 + 21\times 1.333$$

$$21 L_1 = 38.657$$

$$\Rightarrow L_1 = \frac{38.657}{21} = 1.84$$

$\therefore$ Penalty factor of plant 1 is $L_1 = 1.84$.

4. Determine the incremental cost of received Power and Penality factor of Plant shown in figure, if the incremental cost of Production is $\frac{\partial C_1}{\partial P_1} = 0.1P_1 + 3$ Rs/mwh.




Sol Given data,

$$\frac{\partial C_1}{\partial P_1} = 0.1P_1 + 3 \text{ Rs/mwh}$$

$P_1 = 10mw$

$P_L = 2mw$

$P_D = 8mw$

we know that

Incremental cost of received Power is given by $\frac{\partial C_1}{\partial P_{G1}} \cdot L_1$

where $L_1$ is Penality factor

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{1}{1 - \frac{2}{10}} = \frac{10}{8} = 1.25$$

$$\therefore \quad \frac{\partial C_1}{\partial P_{G1}} \cdot L_1 = (0.1P_1 + 3) \times \frac{10}{8} = \lambda$$

$$\lambda = 0.1 \times 10 \times 1.25 + 3 \times 1.25$$

$$= 5 \text{ Rs/mwh.}$$

5. Two thermal Plants are interconnected and following are the incremental Production Cost of Plants in Rs/mwh is $\frac{\partial C_1}{\partial P_1} = 20 + 10P_1$; $\frac{\partial C_2}{\partial P_2} = 15 + 10P_2$. where $P_1$ and $P_2$ are Plant Powers expressed in Per Unit in 100 mVA Base. Transmission loss is given by $P_L = 0.1P_1^2 + 0.2P_2^2 + 0.1P_1P_2$ in Per Unit. If the Incremental cost (IC) of received Power $\lambda = 50$ Rs/mwh. Find optimal generation.

Sol Given data,

$$\frac{\partial C_1}{\partial P_1} = 20 + 10P_1 \text{ Rs/mwh} \Big\} \rightarrow (a)$$

$$\frac{\partial C_2}{\partial P_2} = 15 + 10P_2 \cdot \text{ Rs/mwhr} \rightarrow ⓐ$$

$$P_L = 0.1P_1^2 + 0.2P_2^2 + 0.1P_1P_2 \text{ P.u} \rightarrow ①$$

$$\lambda = 50 \text{ Rs/mwh}$$


we know that,

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2 \rightarrow ②$$

Compare ① and ②, we get

$$B_{11} = 0.1, \quad B_{22} = 0.2, \quad 2B_{12} = 0.1 \Rightarrow B_{12} = 0.05$$

$$\lambda = 50 \text{ Rs/mwh}$$

Condition for optimality when transmission losses are considered is given by

$$\frac{\partial C_1}{\partial P_1} \cdot L_1 = \frac{\partial C_2}{\partial P_2} L_2 = \lambda \rightarrow ③$$

Incremental transmission losses of Plant-1 is

$$(ITL)_1 = \frac{\partial P_L}{\partial P_{G1}} = 2B_{11}P_{G1} + 2B_{12}P_{G2}$$

$$= 2 \times 0.1 P_{G1} + 2 \times 0.05 P_{G2} = 0.2P_{G1} + 0.1P_{G2}$$

similarly $(ITL)_2 = \frac{\partial P_L}{\partial P_{G2}} = 2B_{22}P_2 + 2B_{12}P_1$

$$= 2 \times 0.2P_2 + 2 \times 0.05P_1$$

$$= 0.4P_2 + 0.1P_1.$$

Penalty factor for Plant-1 is

$$L_1 = \frac{1}{1-(ITL)_1} = \frac{1}{1-\left(\frac{\partial P_L}{\partial P_1}\right)} = \frac{1}{1-(0.2P_1+0.1P_2)}$$

Penalty factor for Plant-2 is

$$L_2 = \frac{1}{1-(ITL)_2} = \frac{1}{1-\left(\frac{\partial P_L}{\partial P_2}\right)} = \frac{1}{1-(0.4P_2+0.1P_1)}$$

$\rightarrow ④$

substitute eq ⓐ and eq ④ in eq ③, we get

$$\frac{\partial C_1}{\partial P_1} \cdot L_1 = \lambda$$

$$\Rightarrow (20+10P_1) \cdot \frac{1}{1-(0.2P_1+0.1P_2)} = 50$$

$$\Rightarrow 20 + 10P_1 = 50(1 - 0.2P_1 - 0.1P_2)$$

$$20 + 10P_1 = 50 - 10P_1 - 5P_2$$

$\Rightarrow 20P_1 + 5P_2 - 30 = 0 \rightarrow (5)$

$\frac{\partial C_2}{\partial P_2} L_2 = \lambda$

$\Rightarrow 15 + 10P_2 \cdot \frac{1}{1-(0.4P_2+0.1P_1)} = 50$

$\Rightarrow 15 + 10P_2 = 50\,(1 - 0.4P_2 - 0.1P_1)$

$\Rightarrow 15 + 10P_2 = 50 - 20P_2 - 5P_1$

$\Rightarrow 5P_1 + 30P_2 - 35 = 0 \rightarrow (6)$

solving (5) and (6), we get.

$P_1 = 1.2608$ Pu , $P_2 = 0.9565$ Pu.

$P_1 = 1.2608 \times 100 = 126.08$ mw , $P_2 = 0.9565 \times 100 = 95.65$ mw

Substitute $P_1$ and $P_2$ Values in (1), we get

$P_L = 0.1P_1^2 + 0.2P_2^2 + 0.1P_1P_2$

$= 0.1 \times 1.2608^2 + 0.2 \times 0.9565^2 + 0.1 \times 1.2608 \times 0.9565$

$= 0.4625$ Pu

$= 0.4625 \times 100$ mw

$= 46.25$ mw

Optimal Generation,

$P_1 = P_1 \times$ base mVA

$= 1.2608$ Pu $\times 100$

$= 1.2608 \times 100$

$= 126.08$ mw

$P_2 = P_2 \times$ base mVA

$= 0.9565 \times 100$

$= 95.65$ mw

6. The Cost curve of two Plants are $C_1 = 0.05P_{G_1}^2 + 20P_{G_1} + 150$ Rs/hr ; $C_2 = 0.05P_{G_2}^2 + 15P_{G_2} + 180$ Rs/hr. The loss coefficient for above system is given as $B_{11} = 0.0015$/mw , $B_{12} = B_{21} = -0.0004$/mw, $B_{22} = 0.0032$/mw. Determine the economical generation scheduling corresponding to $\lambda = 30$ Rs/mwh. and the Corresponding system load that can be met with. If total load Connected to system is 120 mw by taking 4% change in Value of $\lambda$, what should be Value of $\lambda$ in next iteration.

Sol Given data,

$C_1 = 0.05P_{G1}^2 + 20P_{G1} + 150$ Rs/hr

$C_2 = 0.05P_{G2}^2 + 15P_{G2} + 180$ Rs/hr

$B_{11} = 0.0015/\text{MW}$, $B_{12} = B_{21} = -0.0004/\text{MW}$, $B_{22} = 0.0032/\text{MW}$

$\lambda = 30$ Rs/MWh

Now

$$\left.\begin{aligned} \frac{\partial C_1}{\partial P_{G1}} &= 2\times 0.05P_{G1} + 20 = 0.1P_{G1} + 20 \text{ Rs/hr} \\ \frac{\partial C_2}{\partial P_{G2}} &= 2\times 0.05P_{G2} + 15 = 0.1P_{G2} + 15 \text{ Rs/hr} \end{aligned}\right\} \rightarrow (1)$$

Transmission Loss, $P_L = \sum_{p=1}^{n}\sum_{q=1}^{n} P_{Gp}B_{pq}P_{Gq} \rightarrow (a)$

Given $n = 2$, $\therefore P_L = \sum_{p=1}^{2}\sum_{q=1}^{2} P_{Gp}B_{pq}P_{Gq}$

$$P_L = B_{11}P_{G1}^2 + B_{22}P_{G2}^2 + 2B_{12}P_{G1}P_{G2} \rightarrow (a)$$

$$= 0.0015P_{G1}^2 + 0.0032P_{G2}^2 - 2\times 0.0004P_{G1}P_{G2}$$

$$= 0.0015P_{G1}^2 + 0.0032P_{G2}^2 - 0.0008P_{G1}P_{G2}$$

Now

$$(ITL)_1 = \frac{\partial P_L}{\partial P_{G1}} = 2B_{11}P_{G1} + 2B_{12}P_{G2}$$

$$= 2\times 0.0015P_{G1} + 2\times -0.0004P_{G2}$$

$$= 0.003P_{G1} - 0.0008P_{G2}$$

$$(ITL)_2 = \frac{\partial P_L}{\partial P_{G2}} = 2B_{12}P_{G1} + 2B_{22}P_{G2}$$

$$= 2\times -0.0004P_{G1} + 2\times 0.0032P_{G2}$$

$$= -0.0008P_{G1} + 0.0064P_{G2}$$

Penality factor for Plant - 1 is

$$L_1 = \frac{1}{1-(ITL)_1} = \frac{1}{1 - 0.003P_{G1} + 0.0008P_{G2}}$$

Penality factor for Plant - 2 is

$$L_2 = \frac{1}{1-(ITL)_2} = \frac{1}{1 + 0.0008P_{G1} - 0.0064P_{G2}}$$

$\rightarrow (2)$

Condition for optimality is

$$\frac{\partial C_1}{\partial P_{G1}}L_1 = \frac{\partial C_2}{\partial P_{G2}}L_2 = \lambda \;;\; \lambda = 30 \text{ Rs/MWh} \rightarrow (3)$$

Substitute (1) and (2) in eq (3), we get

$$\frac{\partial C_1}{\partial P_{G1}} \cdot L_1 = \lambda \Rightarrow 0.1P_{G1}+20 \times \left(\frac{1}{1-0.003P_{G1}+0.0008P_{G2}}\right) = 30$$

$$\Rightarrow 0.1P_{G1}+20 = 30(1-0.003P_{G1}+0.0008P_{G2})$$

$$0.1P_{G1}+20 = 30-0.09P_{G1}+0.024P_{G2}$$

$$\Rightarrow 0.19P_{G1}-0.024P_{G2}-10=0 \rightarrow (4)$$

$$\frac{\partial C_2}{\partial P_{G2}} L_2=\lambda \Rightarrow (0.1P_{G2}+15)\times\left(\frac{1}{(1+0.0008P_{G1}-0.0064P_{G2})}\right)=30$$

$$0.1P_{G2}+15 = 30+30\times0.0008P_{G1}-30\times0.0064P_{G2}$$

$$\Rightarrow 0.1P_{G2}+15 = 30+0.024P_{G1}-0.192P_{G2}$$

$$\Rightarrow 0.1P_{G2}+15=30+0.024P_{G1}-0.192P_{G2}$$

$$\Rightarrow 0.292P_{G2}-0.024P_{G1}=15=0 \rightarrow (5)$$

Solving (4) and (5), we get.

$P_{G1} = 59.74$ mw , $P_{G2} = 56.28$ mw

Substitute $P_{G1}$, $P_{G2}$ values in eq(2), we get

$$P_L = 0.0015\times59.74^2 - 0.0008\times56.28\times59.74+0.0032\times56.28^2$$

$$P_L = 12.799 \text{ mw}$$

Load, $P_D = P_{G1}+P_{G2}-P_L$

$= 59.74+56.28-12.799$

$= 103.221$ mw

Case II:

Given $\lambda = 30$ Rs/mwh

4% change in $\lambda = 30\times\frac{4}{100} = 1.2$ Rs/mwh.

change in Power demand, $\Delta P_D = 120-103.22$

$\Delta P_D = 16.78 \text{ mw} > 0$

Decrement '$\lambda$' by $\lambda-\Delta\lambda = 30-1.2$

$= 28.8$ Rs/mwh.