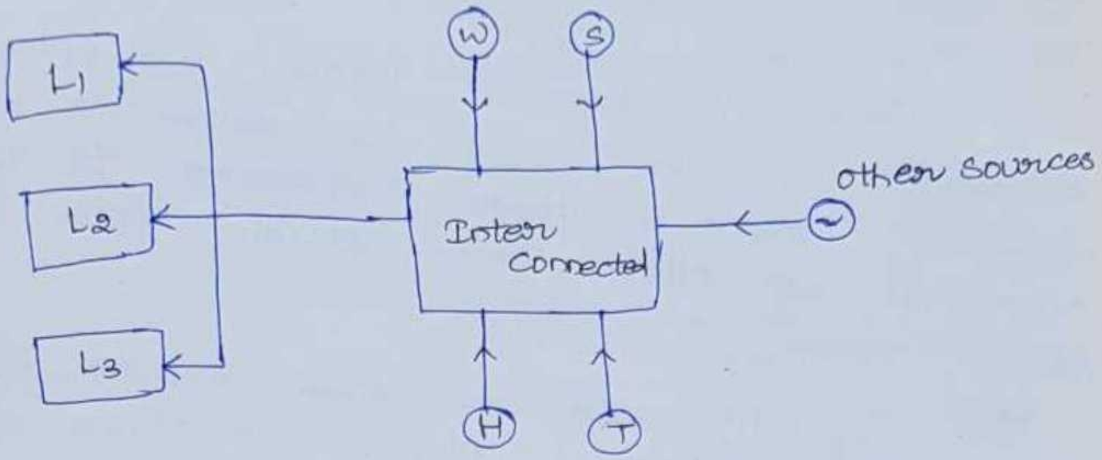
UNIT – II

HYDRO THERMAL SCHEDULING



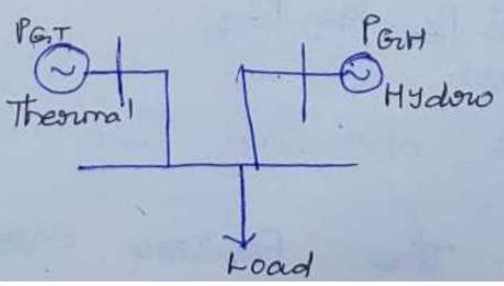
	Hydro	Thermal
Capital Cost	High	Low
Running cost	Low	high
Cost of unit	Low	High
Starting Time	0 (instant)	Time 7 to 8 hrs
Load	Peak load is allocated for HPP	Base load is allocated for TPP.

Step I: Hydro Thermal coordination:

Initially thermal plants are used to generate electrical power. Therefore there is a need for development of hydro power plant (HPP) due to the following reasons:

1. Due to increment of power in load demand.
2. Due to the high cost of fuel (coal).
3. Due to the limited range of fuel.

It is necessary to develop the hydro power plant to meet the load demand.



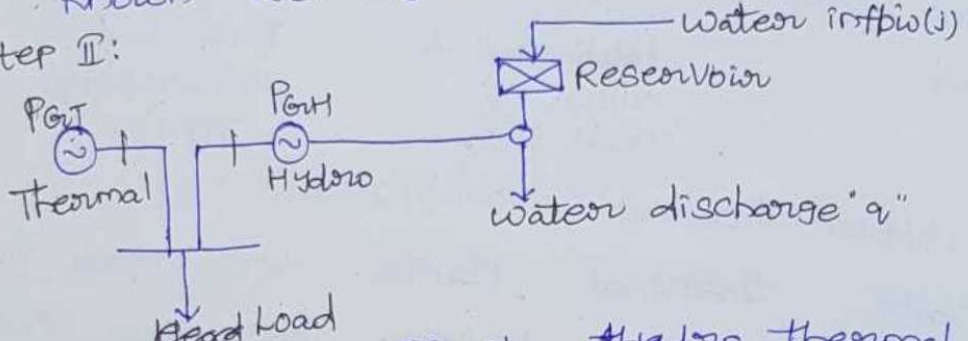
Therefore it is economical and convenient to own both thermal as well as hydro in the same grid.

In case of thermal plants, the optimal scheduling problem can be solved at any desired instant without referring to the operation at other times. It is known as static optimization problem.

The operation of a system having both hydro and thermal plants is more complex.

Hydroplants have negligible operating cost but are required to run under the availability of water for hydro generation during a given period of time. This problem is known as dynamic optimization problem.

Step II:



Consider a simple ~~hydro~~ hydro thermal system as shown in above figure which consist of one hydro and one thermal plant supplying power to the load.

For optimization problem, consider the real power generation of two plants P_{GT} and P_{GH} as control variables.

The transmission power loss in terms of B-coefficients is

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{pq} P_{Gq}$$

Advantages of operation of hydro thermal combination:

1. Flexibility : The power system reliability

can be Obtained by the Combined operation of hydro and thermal Units.

It provides the reserve ~~reset~~ Capacity to meet the random forced outages of units and unexpected load implication on system.

Hydro Plants are preferable to Operate as Peak load Plants such that their operation improves the flexibility of System operation and makes the thermal Plant operation easier.

2. Greater Economy:

In Power system, ratio of hydro power to total Power demand will result in a minimum overall cost of supply.

The run off river plants would generally meet the entire (or) Part of base loads and thermal Plants should be setup to increase the firm capacity of the system.

3. Security of supply:

water availability must depend on the season. It is high during rainy season and be reduced due to the occurance of drought during long term Plants.

Problems arise in thermal Power Plant operation i.e., due to transportation of Coal, unavailability of labour etc.,

The above facts suggested the operation of hydro thermal system to main the security of supply to consumers.

4. Better energy conservation:

During heavy run off Periods, generation of hydro Power is more which results in Conservation of fossil fuels.

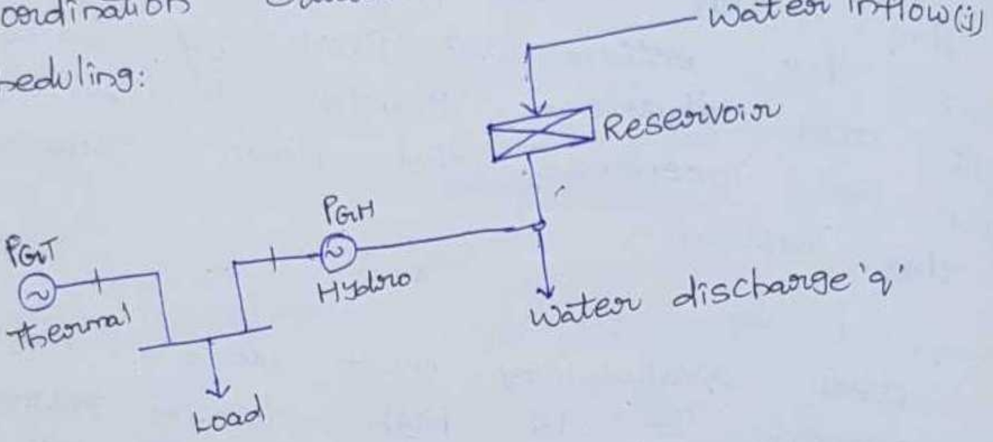
During drought, more steam power has to be generated such that the availability of water meets the minimum needs like drinking and agricultural units.

5. Reserve capacity:

For the operation of power system it is necessary that every system has some reserve capacity to meet the forced outages and unexpected load demands.

By the combined operation of hydro and thermal plants, the reserve capacity maintainance is required.

Co-ordination equation of Hydro thermal scheduling:



Step I:

Operation of a system having both hydro and thermal plants are more complex.

The hydroplants have negligible operating cost but are required to operate under constraints of availability of water in a given period of time.

Optimal scheduling problem of hydro thermal system is to minimize the operating cost of hydro thermal system.

This can be done by minimizing the fuel cost of thermal plants under the constraints of availability of water over a given Period of Operation.

Step II:

For a Period of Operation 'T', it is assumed that

1. water storage of hydro reservoir at the beginning and at the end of Period of operation 'T' are specified.
2. water inflow to reservoir and load demand on the system.

Co-ordination System:

The problem is to determine $q(t)$, water discharge as to minimize the cost of thermal generation,

$$C_T = \int_0^T c'(P_{GT}(t))\,dt \rightarrow ① \text{ [Objective function]}$$

where

C_T - cost of thermal generation.

This can be done by considering the following constraints.

Power Balanced equation:

$$P_{GT}(t) + P_{GH}(t) = P_L(t) + P_D(t)$$

$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0 \rightarrow ② \text{ (Equality constraint)}$$

where

$P_{GT}(t)$ = Thermal real Power generation

$P_{GH}(t)$ = Hydro real Power generation.

$P_L(t)$ = Total transmission losses.

$P_D(t)$ = Total Power (or) Load demand.

$$P_L(t) = B_{TT}(P_{GT}(t))^2 + B_{HH}(P_{GH}(t))^2 + 2B_{TH}P_{GT}(t) + P_{GH}(t) \rightarrow ③$$

Water Availability Equation:

Water discharge can be expressed as

$$\int_0^T q(t)\,dt = [x'(0) - x'(t)] + \int_0^T j(t)\,dt$$

$$\int_0^T q(t)\,dt - [x'(0) - x'(t)] - \int_0^T j(t)\,dt = 0 \rightarrow (4)$$

where

$x'(0)$ = Water storage in the reservoir at beginning.

$x'(t)$ = Water storage in the reservoir at 't' intervals.

$j(t)$ = Water inflow rate

$q(t)$ = water discharge rate.

Hydro Power Generation:

The hydro power generation ($P_{GH}(t)$) is a function of water discharge rate and water storage.

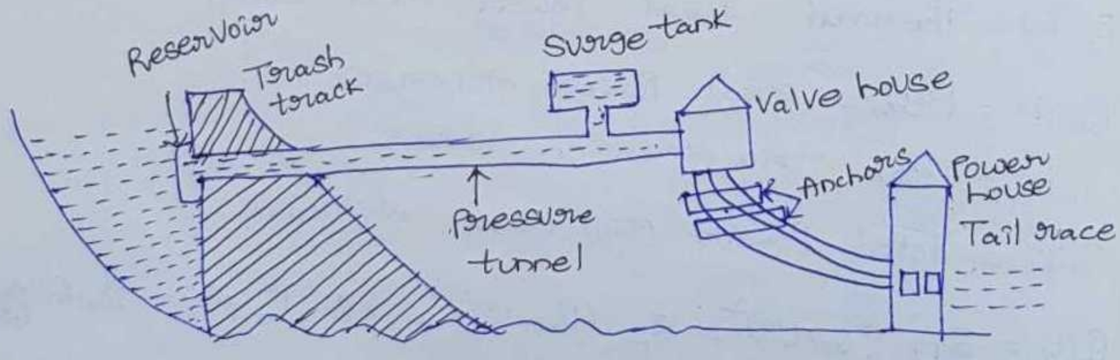
$$P_{GH}(t) = F[x'(t), q(t)] \rightarrow (5)$$

where

$x'(t)$ = water storage at time t

$q(t)$ = water discharge rate at time t.

Hydro electric Plant model:



The above figure shows the schematic diagram of Dam, hydro Plant, exit channel, tail race. The head available to turbine is slightly less than gross head due to friction in the intake penstock and the draft tube.

Let us consider water flows from the reservoir through the penstock to the inlet gates through the hydraulic turbine. The exhausted water from the turbine is carried to the tail race through the draft tube.

The efficiency of the turbine generators are in the range of 85% to 90%.

The water level at the forebay is influenced by the flow out of the reservoir including plant released and any spilling of water over the top of the dam.

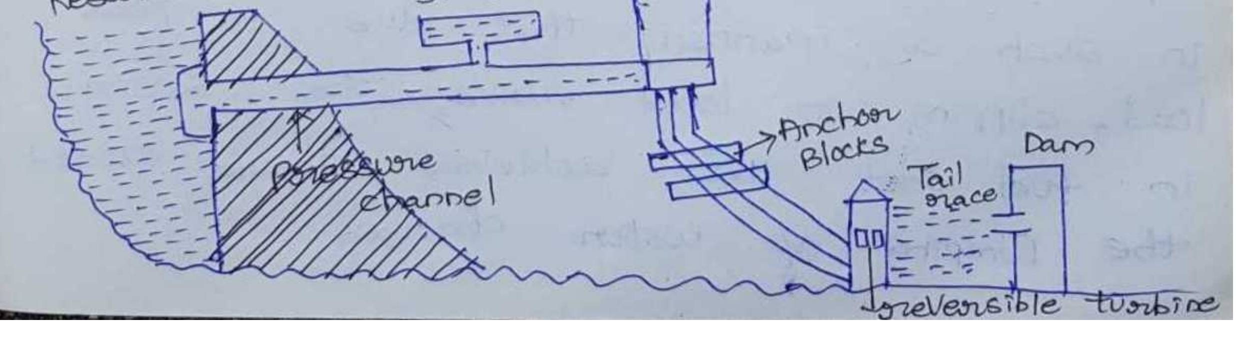
The type of turbine used in a hydro electric plant depends upon the head of the plant.

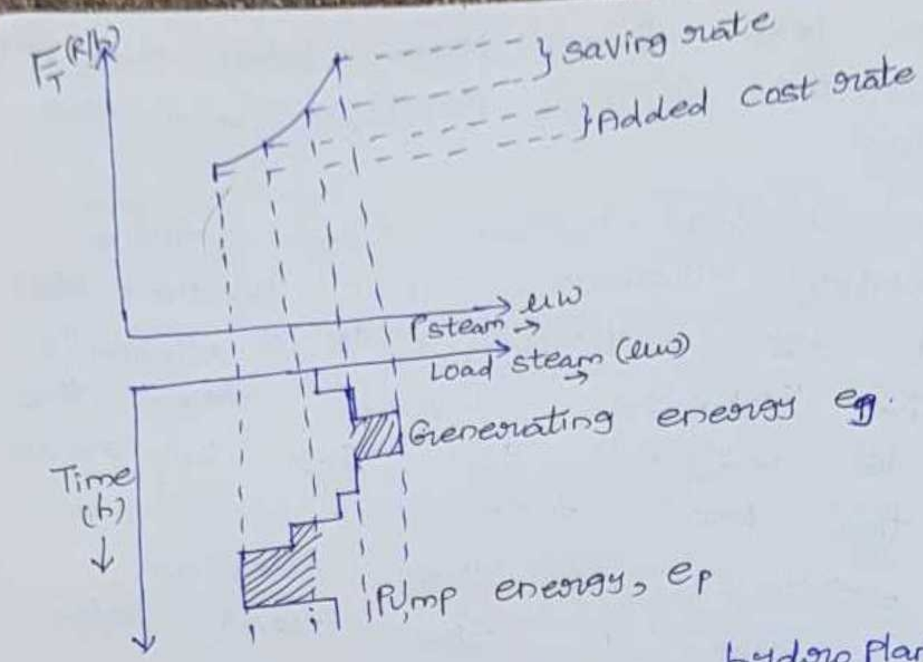
The hydroplant commonly uses the reaction type turbines.

Reaction turbines are of three types:

1. Low head - Propeller (Kaplan) - 10ft to 60ft.
2. medium head - Francis turbine - 60ft to 100ft
3. High head - Impulse or Pelton - >100ft.

Pumped storage Hydro plants:





The Pumped Storage hydro plants store water to supply Peak load demands so that fuel is saved to thermal Plants.

At light load periods, the water is Pumped to reservoir back from tail race water Pond using Power from grid.

The Powerhouse may either have both turbines and Pumps seperately (or) reversable Pump turbines.

The operating characteristics of a Pumped Storage hydro Plant is shown in above figure.

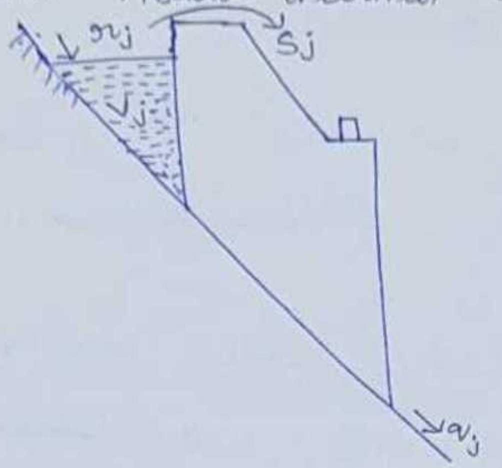
Let e_g = generation in terms of MWh
e_p = Pumping in terms of MWh } For the same amount of water

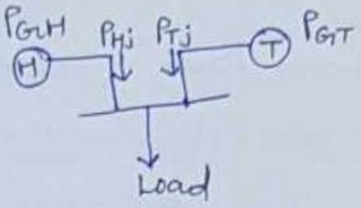
Then the cycle efficiency, $\mu = \frac{e_g}{e_p}$

Efficiency is typically about 0.67

Pumped storage Plants are to be operated in such a manner that due to Peak load, clipping on load curve, the saving in fuel cost thus achieved should exceed the Pumping of water charges.

Short term Hydro-thermal Scheduling problem:





Case I:

Let

j = Interval

r_j = water inflow rate during j^{th} interval.

V_j = Volume of water at the end of j^{th} interval

q_j = Discharge of water during j^{th} interval.

S_j = Spillage discharge during j^{th} interval.

Case II:

A short term hydro thermal scheduling plant requires that a given amount of water is used in such a way as to minimize the cost of running thermal units.

During all periods, the hydro is not sufficient to supply all load demands and that there is a maximum total volume of water that may be discharge throughout the period of T_{max} hours.

Case III:

The mathematical scheduling problem by considering 'j' no. of equal intervals with each duration n_j then the objective function is expressed as

$$\min C_t = \sum_{j=1}^{j_{max}} n_j \cdot C_j(P_{Tj}) \rightarrow (1)$$

For hydro Power plant, total discharge

$$q_{total} = \sum_{j=1}^{j_{max}} n_j \cdot q_j(P_{Hj}) \rightarrow (2)$$

$$\sum_{j=1}^{j_{max}} n_j = T_{max} \rightarrow ③$$

where n_j = length of the j^{th} interval.

For load balance i.e $j=1$ to j_{max}, equality constraints are

$P_{Dj} - P_{Tj} - P_{Hj} = 0 \rightarrow ④$ when losses are not considered

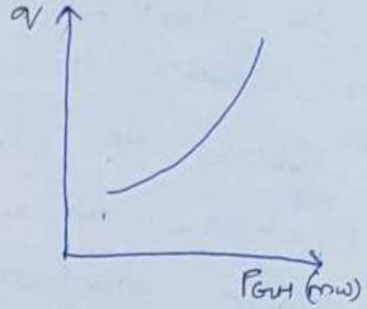
$P_{Lj} + P_{Dj} - P_{Tj} - P_{Hj} = 0 \rightarrow ⑤$ with losses

Considering other constrains for scheduling.

$V_j|_{j=0} = V_S$ [starting Volume]

$V_j|_{j=j_{max}} = V_E$ [Ending Volume]

$q_{min} \leq q \leq q_{max}$



Case IV: solution for optimal Condition:

without Losses:

The Lagrangian function,

$$L = \left[\sum_{j=1}^{j_{max}} n_j c_j(P_{Tj}) + \lambda_j \left[P_{Dj} - P_{Tj} - P_{Hj}\right]\right] + \nu\left[\sum_{j=1}^{j_{max}} n_j q_j(P_{Hj}) - q_{total}\right] \rightarrow ⑥$$

For a specific interval, rewrite $j=k$

$$L = n_K c_K(P_{TK}) + \lambda_K (P_{DK} - P_{TK} - P_{HK}) + \nu\left[n_K q_K(P_{HK}) - q_{total}\right] \rightarrow ⑦$$

Let us assume that thermal Plant is supplying maximum load then condition for scheduling is

$$\frac{\partial L}{\partial P_{TK}} = 0$$

$$\lambda_K = n_K \cdot \frac{\partial c_K(P_{TK})}{\partial P_{TK}} \rightarrow ⑧$$

Similarly

Let us assume that hydro plant is supplying maximum load then condition for scheduling is $\frac{\partial L}{\partial P_{HK}} = 0$

$$\lambda_K = V_{nK} \cdot \frac{\partial q_K(P_{HK})}{\partial P_{HK}} \rightarrow 9$$

Case V: with Losses:

The Lagrangian Function,

$$L = \left[\sum_{j=1}^{j_{max}} n_j c_j(P_{Tj}) + \lambda_j \left[P_{Lj} + P_{Dj} - P_{Tj} - P_{Hj}\right] \right] + V \cdot \left[\sum_{j=1}^{j_{max}} n_j q_j (P_{Hj}) - q_{total} \right] \rightarrow 10$$

For a specific interval, rewrite $j = K$.

$$L = \sum_{j} \left[n_K c_K (P_{TK}) + \lambda_K \left(P_{LK} + P_{DK} - P_{TK} - P_{HK}\right)\right] + V\left(n_K q_K (P_{HK}) - q_{total}\right)$$

Let us assume that thermal plant is supplying maximum load then condition for scheduling is $\frac{\partial L}{\partial P_{TK}} = 0$

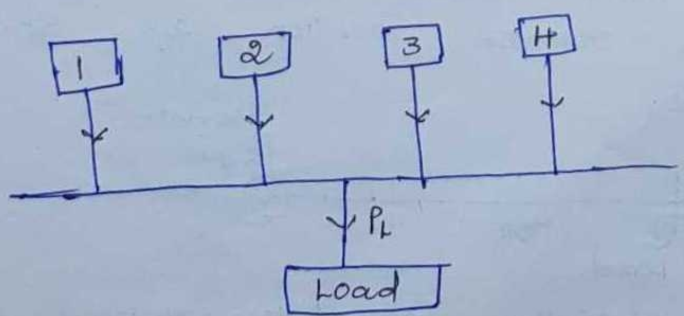
$$\lambda_K = n_K \cdot \frac{\partial c_K(\partial P_K)}{\partial P_{TK}} + \lambda_K \frac{\partial P_{LK}(P_{TK})}{\partial P_{TK}} \rightarrow 10$$

Let us assume that hydro plant is supplying maximum load then condition for scheduling is $\frac{\partial L}{\partial P_{HK}} = 0$

$$\lambda_K = \cancel{q_K} \cdot V \cdot n_K \frac{\partial q_K(P_{HK})}{\partial P_{HK}} + \lambda_K \frac{\partial P_{LK}(P_{HK})}{\partial P_{HK}} \rightarrow 11$$

Spinning Reserve:

Case I:



Spinning Reserve, S.R = Total amount of generation available from all units + Total load demands and total losses being supplied

(Or)

S.R = Total Capacity of the Synchronized units + Total load and losses at a specified time

The additional Capacity of generating units in order to lead the load requirements and losses is called as spinning reserve.

If "N" no. of generators are operating in parallel in a generating station and if any one of generators fails to operate then load must be shared by "N-1" generators but total demand cannot be meet by these "N-1" generators.

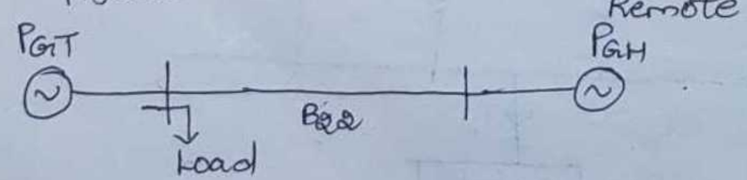
Hence in order to meet the load demand either a stand by unit (or) static reserve capacity must be maintained.

But in thermal power plants, starting a standby units takes a duration of 7-8 hours due to the above reason continuity is affected but also cost is increased.

Therefore it is economical to maintain spinning reserves in thermal plants as well as in hydro plants due to minimum starting time.

Problems:

1. A two plant system has a thermal station near the load centre and a hydro power station at a remote location as shown in below figure.



The characteristics of both stations are

$C_1 = 26 P_{GT} + 0.045 P_{GT}^2$ Rs/hr, $w_2 = 7P_{GH} + 0.004 P_{GH}^2$ m³/sec and $\gamma_2 = 4\times10^{-4}$ Rs/m³. The transmission loss coefficient $B_{22} = 0.0025$ mw^{-1}. Determine the Power generation at each station and the Power received by load when $\lambda = 65$ Rs/mwh.

Sol Given data,

No. of units, $n=2$

cost of thermal unit, $C_1 = 26P_{GT} + 0.045P_{GT}^2$ Rs/hr.

$$\frac{\partial C_1}{\partial P_{GT}} = 26 + 2\times0.045 P_{GT} = 26 + 0.09 P_{GT} \text{ Rs/Mwh.}$$

$$w_2 = 7P_{GH} + 0.004 P_{GH}^2 \text{ m}^3/\text{sec} \Rightarrow \frac{\partial w_2}{\partial P_{GH}} = 7 + 0.008 P_{GH} \text{ m}^3/\text{sec}$$

step I: optimal condition in thermal:

$$\frac{\partial C_1}{\partial P_{GT}} \cdot L_1 = \frac{\partial C_2}{\partial P_{GT}} \cdot L_2 \text{ ---------} = \lambda$$

Consider $\frac{\partial C_1}{\partial P_{GT}} \cdot L_1 = \lambda \rightarrow ①$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{GT}}}$$

$$P_L = B_{TT} P_{GT}^2 + B_{HH} P_{GH}^2 + 2B_{TH} P_{GT} P_{GH}$$

$= \cancel{0.0025\times}$ Now $\frac{\partial P_L}{\partial P_{GT}} = 2B_{TT} P_{GT} + 2B_{TH} P_{GH}$.

Assume $P_{GT} = B_{TH} = 0$ then $\frac{\partial P_L}{\partial P_{GT}} = 0$.

$$\therefore L_1 = \frac{1}{1-0} = 1$$

substitute L_1 in eq ①, we get

$$26 + 0.09 P_{GT} = 65 \Rightarrow 0.09 P_{GT} = 65 - 26$$

$$P_{GT} = 433.33 \text{ mw}$$

step II: optimal condition for hydro Power Plant:

$$\gamma_1 \frac{\partial C_1}{\partial P_{GH}} \cdot L_1 = \frac{\partial C_2}{\partial P_{GH}} \cdot L_2 \gamma_2 = \text{ --------- } \lambda$$

Consider $\gamma_2 \cdot \frac{\partial w_2}{\partial P_{GH}} \cdot L_2 = \lambda \rightarrow ②$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{GH}}}$$

w.K.T, $P_L = B_{TT} P_{GT}^2 + B_{HH} P_{GH}^2 + 2 B_{TH} P_{TG} P_{GH}$

$$\frac{\partial P_L}{\partial P_{GH}} = 2 B_{HH} P_{GH} + 2 B_{TH} P_{GT}$$

$$B_{22} = B_{HH} = 0.0025$$

$$\frac{\partial P_L}{\partial P_{GH}} = 0.005 P_{GH}$$

$$L_2 = \frac{1}{1 - 0.005 P_{GH}}$$

Substitute L_2 in eq ②, we get

$$4 \times 10^{-4} \times 7 + 0.008 P_{GH} \cdot \frac{1}{1 - 0.005 P_{GH}} = 65$$

$$\Rightarrow (4 \times 10^{-4} \times 7 + 0.008 P_{GH})(1 - 0.005 P_{GH}) = 65$$

$$P_{GH} = 199.99 \text{ mw}$$

$$P_L = B_{TT} P_{GT}^2 + B_{HH} P_{GH}^2 + 2 B_{HT} P_{GT} P_{GH}.$$

$B_{HT} = B_{TT} = 0$ then

$$P_L = 0.0025 \times 199.99^2 = 99.99 \text{ mw}$$

we Know that

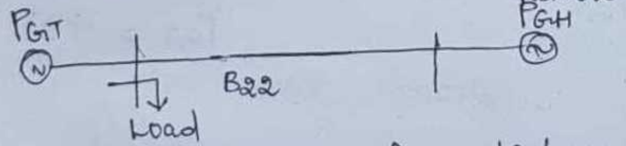
$$P_D + P_L = P_{GT} + P_{GH}$$

$$\Rightarrow P_D = P_L - P_{GT} - P_{GH}$$

$$P_D = 99.99 - 433.33 - 199.99$$

$$= 533.33 \text{ mw}$$

2. A two plant system having a steam plant near load centre and hydro plant at remote location as shown in below figure



The load is 500mw for 16 hours a day and 350mw for 8 hours a day. The characteristics of units are $C_1 = 120 + 45 P_{GT} + 0.075 P_{GT}^2$, $W_2 = 0.6 P_{GH} + 0.00283 P_{GH}^2$ m³/sec, Loss coefficient, $B_{22} = 0.001 \text{ mw}^{-1}$. Find the generation schedule, daily water used by hydroplant, daily operating cost of thermal Power Plant for $\gamma_j = 85.5$ Rs/m³.hr.

Sol Given data,

No. of units = 2.

$C_1 = 120 + 45P_{GT} + 0.0075P_{GT}^2$

$\Rightarrow \frac{\partial C_1}{\partial P_{GT}} = 45 + 0.015P_{GT}$

$w_2 = 0.6P_{GH} + 0.00283P_{GH}^2$

$\Rightarrow \frac{\partial w_2}{\partial P_{GT}} = 0.6 + 0.00566P_{GH}$

step I:

optimal condition in thermal is

$$\frac{\partial w_1}{\partial P_{GH}} \cdot L_1 = \frac{\partial w_2}{\partial P_{GH}} L_2 = ----- = \lambda$$

consider $\gamma_1 \frac{\partial w_1}{\partial P_{GH}} = \lambda_1 \rightarrow ①$

$\Rightarrow$ ~~$45 + 0.015P_{GT} = \lambda$~~ $\cancel{0.6P_{GH}}^{0.6} + 0.00566P_{GH} = \lambda$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{GT}}} = 1.$$

$P_L = B_{TT}P_{GT}^2 + B_{HH}P_{GH}^2 + 2B_{TH}P_{GT}P_{GH}$

$P_L = B_{HH}P_{GH}^2 = 0.001P_{GH}^2$

$\frac{\partial P_L}{\partial P_{GH}} = 0.002P_{GH}.$

$$L_1 = \frac{1}{1 - 0.002P_{GH}}$$

substitute L_2 in eq ①, we get

$\Rightarrow$ Now

Power Balance equation is

$P_{GT} + P_{GH} = P_D + P_L \rightarrow ②$

$$\frac{\partial C_1}{\partial P_{GT}} \cdot L_1 = \gamma_j \frac{\partial w_1}{\partial P_{GH}} \left(\frac{1}{1 - \frac{\partial P_L}{\partial P_{GH}}} \right) = \lambda$$

when $P_D = 500$ mw:

$$0.15P_{GT} + 45 = 85.5 \; 0.6 + 0.00566P_{GH} \times \frac{1}{1 - 0.002P_{GH}}$$

Sol Given data,

$$C_1 = 120 + 45P_{GT} + 0.075P_{GT}^2 \Rightarrow \frac{\partial C_1}{\partial P_{GT}} = 45 + 0.15P_{GT} \rightarrow ①$$

$$\omega_2 = 0.6P_{GH} + 0.00283P_{GH}^2 \rightarrow \frac{\partial \omega_2}{\partial P_{GH}} = 0.6 + 0.00566P_{GH} \rightarrow ②$$

$$B_{22} = B_{HH} = 0.001$$

$$\frac{\partial C_1}{\partial P_{GT}} \cdot L_T = \lambda \Rightarrow L_T = \frac{1}{1 - \frac{\partial P_L}{\partial P_{GT}}} = \frac{1}{1-0} = 1$$

$$P_L = B_{TT}P_{GT}^2 + B_{HH}P_{GH}^2 + 2B_{HT}P_{GT}P_{GH}$$

$$\frac{\partial P_L}{\partial P_{GT}} = 2B_{TT}P_{GT} + 2B_{HT}P_{GT} = 0$$

optimal Condition for ~~thermal~~ Hydro is

$$\gamma_j \frac{\partial \omega_2}{\partial P_{GH}} \cdot L_H = \lambda$$

$$\frac{\partial C_1}{\partial P_{GT}} \cdot L_T = \gamma_j \frac{\partial \omega_2}{\partial P_{GH}} \cdot L_H \;;\; \frac{\partial P_L}{\partial P_{GH}} = 2 \times 0.001P_{GH} = 0.002P_{GH}$$

$$(45 + 0.15P_{GT}) = 85.5\,(0.6 + 0.00566P_{GH}) \times \frac{1}{(1 - 0.002P_{GH})}$$

$$\Rightarrow (45 + 0.15P_{GT})(1 - 0.002P_{GH}) = 85.5(0.6 + 0.00566P_{GH})$$

$$\Rightarrow 45 - 0.09P_{GH} + 0.15P_{GT} - 0.0003P_{GT}P_{GH} = 51.3 + 0.48393P_{GH}$$

$$\Rightarrow -0.15P_{GT} + 0.57393P_{GH} + 0.0003P_{GT}P_{GH} + 6.3 = 0 \rightarrow ③$$

Case I: $P_D = 500\,mw$

$$P_{GT} + P_{GH} = P_D + P_L$$

$$\Rightarrow P_{GT} + P_{GH} = 500 + 0.001P_{GH}^2 = 0$$

$P_{GT} = 500 + 0.001 P_{GH}^2 - P_{GH} \rightarrow (4)$

Substitute eq (4) in eq (3), we get

$-0.15(500 + 0.001P_{GH}^2 - P_{GH}) + 0.57393 P_{GH} - 0.0003(500 + 0.001 P_{GH}^2 - P_{GH}) P_{GH} + 6.3 = 0.$

$-75 + 1.5\times10^{-4} P_{GH}^2 + 0.15 P_{GH} + 0.57393 P_{GH} + 0.15 P_{GH} + 3\times10^{-7} P_{GH}^3 + 0.0003 P_{GH}^2 + 6.3 = 0$

$\Rightarrow 0.000000 P_{GH}^3 - 0.00045 P_{GH}^2 + 0.87393 P_{GH} - 68.7 = 0$

$\Rightarrow P_{GH} = 81.87$ mw

Substitute P_{GH} in eq (4), we get

$P_{GT} = 500 + 0.001\times 81.87^2 - 81.87 = 424.83$ mw

$\therefore P_L = P_{GT} + P_{GH} - P_D$

$= 424.83 + 81.87 - 500$

$= 6.70$ mw.

Case II: $P_D = 350$ mw

$P_{GT} + P_{GH} = P_D + P_L$

$P_{GT} + P_{GH} = 350 - 0.001 P_{GH}^2$

$P_{GT} = 350 - 0.001 P_{GH}^2 - P_{GH} \rightarrow (a)$

Substitute eq (a) in eq (3), we get

$\Rightarrow -0.15(350 - 0.001 P_{GH}^2 - P_{GH}) + 0.57393 P_{GH} + 0.0003(350 - 0.001 P_{GH}^2 - P_{GH}) P_{GH} + 6.3 = 0$

$\Rightarrow -52.5 + 0.00015 P_{GH}^2 + 0.15 P_{GH} + 0.57393 P_{GH} + 0.105 P_{GH} - 0.0000003 P_{GH}^3 - 0.0003 P_{GH}^2 + 68 = 0$

$\Rightarrow -0.000000 P_{GH}^3 - 0.00015 P_{GH}^2 + 0.82893 P_{GH} - 46.2 = 0$

$\Rightarrow P_{GH} = 58.585$ mw

Substitute P_{GH} in eq (a), we get

$P_{GT} = 350 - 0.001\times(58.586)^2 - 58.586.$

$P_{GT} = 294.847$ mw

$\therefore P_L = P_{GT} + P_{GH} - P_D = 294.847 + 58.586 - 350$

$P_L = 3.4321$ mw

Daily water used by the hydro plant

$w_2 = 0.6 P_{GH} + 0.00283 P_{GH}^2 \; m^3/sec$

Daily water quantity used for a 500mw load

Daily water quantity used for 16 hours + daily water quantity used for 8 hours.

for a 350mw load

$$w_2 = \left\{ \left[0.6(81.876) + 0.00283(81.876)^2\right] \times 16 + \left[0.6(58.586) + 0.00283(58.586)^2\right] \times 8 \right] \times 3600$$

$$w_2 = [68.09 \times 16 + 44.86 \times 8] \times 3600$$

$$w_2 = 1448.32 \times 3600$$

$$= 5.21 \times 10^6 \; m^3$$

Daily operating cost of thermal plant is

Daily water used

Now, $C_1 = 120 + 45 P_{GT} + 0.0075 P_{GT}^2$

$\therefore C_1 = [120 + 45 P_{GT} + 0.0075 P_{GT}^2]$ for both 500 mw and 350 mw

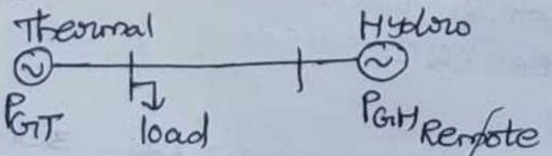
$$\Rightarrow C_1 = \left[\left(120 + 45 \times 424.83 + 0.0075 \times (424.83)^2\right) \times 16 + \left(120 + 45 \times 294.847 + 0.0075 \times (294.847)^2 \times 8\right) \right] \times 3600$$

$$= [329455.2635 + 18604.2002] \times 3600$$

$$= \cancel{1253014069} = 6,83,589.96 \text{ Rs per day.}$$

$\cancel{\le 1253.01}$

3. A load is feeded by two plants, one is thermal and other is hydro plant. A load is located near thermal power plant as shown in below figure.



The characteristics of two plants are as follows

$$C_T = 0.04 P_{GT}^2 + 30 P_{GT} + 20 \; Rs/hr$$

$$w_H = 0.0012 P_{GH}^2 + 7.5 P_{GH} \; m^3/sec$$

$\gamma_H = 2.5\times10^{-5}$ Rs/m^3. The transmission loss coefficients $B_{22} = 0.0015\ mw^{-1}$. Determine the power generation of both thermal and hydro plants, the load connected when $\lambda = 45$ Rs/mw. Also determine the daily water used by hydro plant and daily cost of totally operating plant with the load connected for 24 hours.

Sol) Given data,

$$C_T = 0.04P_{GT}^2 + 30P_{GT} + 20 \text{ Rs/hr}$$

$$\Rightarrow \frac{\partial C_T}{\partial P_{GT}} = 0.08P_{GT} + 30 \rightarrow (1)$$

$$\omega_H = 0.0012P_{GH}^2 + 7.5P_{GH}\ m^3/sec$$

$$\Rightarrow \frac{\partial \omega_H}{\partial P_{GH}} = 0.0024P_{GH} + 7.5 \rightarrow (2)$$

$$B_{22} = B_{HH} = 0.0015\ mw^{-1}$$

$$\frac{\partial C_T}{\partial P_{GT}} \cdot L_T = \lambda \Rightarrow \boxed{L_T = \frac{1}{1-\frac{\partial C_T}{\partial P_{GT}}} = \frac{1}{1-\frac{\partial C_T}{\partial P_{GT}}}}$$

$$\Rightarrow (0.08P_{GT}+30) \cdot L_T = \lambda$$

$$P_L = B_{TT}P_{GT}^2 + B_{HH}P_{GH}^2 + 2B_{HT}P_{GT}P_{GH}$$

$$P_L = 0.0015P_{GH}^2 \Rightarrow \frac{\partial P_L}{\partial P_{GT}} = 0$$

$$L_T = 1$$

$$\Rightarrow 0.08P_{GT} + 30 = 45 \Rightarrow 0.08P_{GT} = 15$$

$$P_{GT} = \frac{15}{0.08} = 187.5$$

Now $\gamma_H \cdot \frac{\partial \omega_2}{\partial P_{GH}} \cdot L_H = \lambda$

$$\frac{\partial P_L}{\partial P_{GH}} = 2B_{HH}P_{GH} + 2B_{HT}P_{GT} = 2\times0.0015P_{GH} + 0$$

$$\frac{\partial P_L}{\partial P_{GH}} = 0.003P_{GH}$$

$$L_H = \frac{1}{1-\frac{\partial P_L}{\partial P_{GH}}} = \frac{1}{1-0.003P_{GH}} = 1$$

$$\therefore \gamma_H \cdot \frac{\partial \omega_2}{\partial P_{GH}} \cdot \frac{1}{1-0.003P_{GH}} = 45$$

$$\Rightarrow 2.5\times10^{-5} \cdot (0.0024P_{GH} + 7.5)\frac{1}{1-0.003P_{GH}} = 45$$

$$\Rightarrow 6\times10^{-8}P_{GH} + 1.875\times10^{-4} = 45(1-0.003P_{GH})$$

$$\Rightarrow 6\times10^{-8} P_{GH} + 1.875\times10^{-4} = 45 - 1.875\times10^{-4}$$

$$6\times10^{-8} P_{GH} = 44.998$$

$$P_{GH} = \frac{44.998}{6\times10^{-8}} = 333.33$$

$$\therefore P_L = B_{TT} P_{GT}^2 + B_{HH} P_{GH}^2 + 2B_{HT} P_{GT} P_{HG}$$

$$= B_{HH} P_{GH}^2$$

$$= 0.0015\times 333.33^2$$

$$= 166.66 \text{ mw}$$

$$P_L = P_{GT} + P_{GH} - P_D \Rightarrow P_L + P_D - P_{GT} - P_{GH} = 0$$

$$\Rightarrow P_D = -\left[P_D - P_{GT} - P_{GH}\right]$$

$$\Rightarrow P_D = -P_D + P_{GT} + P_{GH}$$

$$P_D = P_{GT} + P_{GH} - P_L$$

$$= 187.5 + 333.33 - 166.66$$

$$= 354.17 \text{ mw}$$

$$\therefore C_T = \left[0.04 P_{GT}^2 + 30 P_{GT} + 20\right]\times 24$$

$$= \left[0.04\times 187.5^2 + 30\times 187.5 + 20\right]\times 24$$

$$= 169230 \text{ Rs/day}$$

$$w_H = \left[0.0012 P_{GH}^2 + 7.5 P_{GH}\right]\times 3600\times 24$$

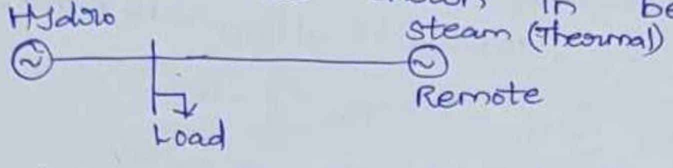
$$= \left[0.0012\times 333.33^2 + 7.5\times 333.33\right]\times 24\times 3600$$

$$= 22.751\times10^7$$

4. A thermal station and a hydro station supplying an area joined. The hydro station is run 16 hours daily and thermal is run through 24 hours. The incremental fuel cost of thermal plant are $C_T = 6 + 12P_{GT} + 0.04 P_{GT}^2$ Rs/hr. If the load on the thermal station when both plants are in operation is 350mw, the incremental water rate of hydro power plant is $28 + 0.03P_{GH}$ m³/mwsec. The total quantity of water utilized during a 16 hour operation of hydro plant is 450 million m³.

Find the generation of hydro Plant at cost of water used. Assume that total load on hydro Plant is constant for the 16 hour Period.

5. A two plant system that has hydro plant near load centre and steam plant at the remote location as shown in below figure.



The load is 400 mw for 14 hour a day and 200 mw for 10 hour a day. The characteristics of units are $C_T = 150 + 60P_{GT} + 0.1 P_{GT}^2$ Rs/hr,
$\omega_H = 0.8P_{GH} + 0.000333 P_{GH}^2$ m³/sec. The loss coefficient $B_{22} = 0.001$ mw^{-1}. Find the generation schedule, daily water used by hydro plant and daily operating cost of thermal plant for $\gamma_j = 77.5$ Rs/m³hr

sol

Hsol Given data,

$C_T = 6 + 12P_{GT} + 0.04 P_{GT}^2$ Rs/hr $\Rightarrow \dfrac{\partial C_T}{\partial P_{GT}} = 12 + 0.08P_{GT} \rightarrow ①$

$\dfrac{\partial \omega_H}{\partial P_{GH}} = 28 + 0.03P_{GH}$ m³/sec $\Rightarrow \dfrac{\partial \omega_H}{\partial P_{GH}} = 0.03 \rightarrow ②$

we know that

$$\frac{\partial C_T}{\partial P_{GT}} \cdot L_1 = \lambda$$

where $L_1 = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_{GT}}}$

$$P_L = B_{TT}P_{GT}^2 + B_{HH}P_{GH}^2 + 2B_{HT}P_{GH}P_{GT}$$

$$\frac{\partial P_L}{\partial P_{GT}} = 2B_{TT}P_{GT} + 2B_{HH}P_{GH} + 2B_{HT}P_{GH} = 0$$

$$\therefore L_1 = \frac{1}{1-1} = 0$$

$$\Rightarrow \frac{\partial C_T}{\partial P_{GT}} = \lambda$$

$\Rightarrow 0.08 \times 350 + 12 = \lambda \Rightarrow \lambda = 40$ Rs/mwh

Given $\dfrac{\partial \omega_H}{\partial P_{GH}} = 28 + 0.03P_{GH}$ m³/sec $\rightarrow ②$

w.k.T, $\gamma_H \cdot \dfrac{\partial \omega_H}{\partial P_{GH}} \cdot L_H = \lambda$

$\Rightarrow \gamma_H (28+0.03P_{GH}) 1 = 40$

Daily water:

Total Quantity of water used during 16 hours of operation is 450 million m^3

$$450\times10^6 = (28+0.03P_{GH})P_{GH} \times 16\times3600$$

$$\Rightarrow 450\times10^6 = [28P_{GH}+0.03P_{GH}^2] \times 16\times3600$$

$$\Rightarrow 450\times10^6 = (28P_{GH}+0.03P_{GH}^2)\times 57600$$

$$\Rightarrow 450\times10^6 = 1612800P_{GH} + 1728P_{GH}^2$$

$$\Rightarrow 1728P_{GH}^2 + 1612800P_{GH} - 450\times10^6 = 0$$

$$P_{GH} = 224.84 \text{ mw}$$

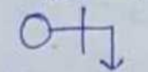
$$\therefore \gamma_H (28+0.03P_{GH})1 = 40 \Rightarrow \gamma_H(28+0.03\times224.84) = 40$$

$$\Rightarrow \gamma_H = \frac{40}{28+0.03\times224.84}$$

$$= 1.1522 \text{ Rs/m}^3\text{ hr}$$

5sol Given data,

Thermal



$B_{22} = B_{TT}$

$B_{11} = B_{HH}$

$B_{12} = B_{TH}$.

$$C_T = 150 + 60P_{GT} + 0.1P_{GT}^2 \text{ Rs/hr} \Rightarrow \frac{\partial C_T}{\partial P_{GT}} = 60+0.2P_{GT} \rightarrow ①$$

$$w_H = 0.8P_{GH} + 0.000333P_{GH}^2 \text{ m}^3/\text{sec} \Rightarrow \frac{\partial w_H}{\partial P_{GH}} = 0.8+0.000666P_{GH} \rightarrow ②$$

$$B_{22} = B_{TT} = 0.001 \text{ mw}^{-1}$$

$$P_L = B_{TT}P_{GT}^2 + B_{HH}P_{GH}^2 + 2B_{HT}P_{GT}P_{GH}$$

$$\frac{\partial P_L}{\partial P_{GT}} = 2B_{TT}P_{GT} + 2B_{HH}P_{GH} + 2B_{HT}P_{GH}$$

$$= 2\times0.001 = 0.002P_{GT}$$

$$L_T = \frac{1}{1-\frac{\partial P_L}{\partial P_{GT}}} = \frac{1}{1-0.002P_{GT}} = 1.002$$

W.K.T, $\gamma_j \frac{\partial C_T}{\partial P_{GT}} = L_T = \lambda$

$\Rightarrow_{max} (60+0.2P_{GT}) \cdot \dfrac{1}{1-0.002P_{GT}} = \lambda$

$\Rightarrow (60+0.2P_{GT})(1-0.002P_{GT}) \times 77.5 = \lambda$

$\Rightarrow [60 - 0.12P_{GT} + 0.2P_{GT} - 4\times10^{-4}P_{GT}] \times 77.5 = \lambda$

$\Rightarrow 450 + 6.2P_{GT} - 0.031P_{GT} = \lambda$

$\Rightarrow 450 + 6.169P_{GT} = \lambda \rightarrow (3)$

w.K.T, $P_{GT} + P_{GH} = P_D + P_L$

For $P_D = 400\,mw$, $P_L = 0.0001P_{GP}^2$

$\Rightarrow P_{GT} + P_{GH} = 400 + 0.0001P_{GT}^2$

$\Rightarrow P_{GH} = 0.0001P_{GT}^2 - P_{GH} + 400 \rightarrow (2)$

Now $\dfrac{\partial w_H}{\partial P_{GH}} \cdot L_H = \lambda$

$\Rightarrow 0.8 + 0.000666P_{GH} = \lambda \rightarrow (4)$

eq (3) = eq (4) $\Rightarrow 450 + 6.169P_{GT} = 0.8 + 0.000666P_{GH}$

$\Rightarrow 450 + 6.169P_{GT} - 0.8 - 0.000666P_{GH} = 0$

$\Rightarrow 449.2 + 6.169P_{GT} - 0.000666(0.0001P_{GT}^2 - P_{GH} + 400) = 0$

$\Rightarrow 449.2 + 6.169P_{GT} - 6.66\times10^{-8}P_{GT}^2 + 0.000666P_{GH} - 0.2664 = 0$

$\Rightarrow -6.66\times10^{-8}P_{GT}^2 + 6.169P_{GT} + 0.000666P_{GH} +$

$P_{GT} = 55. -\,mw$ $\quad P_{GH} = 347.97\,mw$, $P_L = 3.03$

For 200mw:

$P_{GT} = 31.26\,mw$ $\quad P_{GH} = 169.6928\,mw$, $P_L = 0.9788$

$C_T = 73843.2688$ Rs/hr

$w_H = 21.294705\times10^6\,m^3$

$P_{GH} = 400 + 0.001(55.37)^2 - 55.37 = 347.69\,mw.$

$P_L = 0.001(55.37)^2 = 3.06\,mw$

For 200 mw, $\dfrac{\left[\dfrac{600+0.2P_{GT}}{1-0.002P_{GT}} - 62\right]}{0.0515} = 200 + 0.01P_{GT}^2 - P_{GT}$

$= 31.45\,mw$

$$P_{GH} = 200 + 0.01\,(31.45)^2 - 31.45 = 169.54\ \text{mw}$$

$$P_L = 0.001\,(31.45)^2 = 0.9891\ \text{mw}.$$

$$C_T = \left[150 + 60(55.37) + 0.1\,(55.37)^2\right] \times 14 + \left[\left(150 + 60(31.45) + 0.1\,(31.45)^2\right) \times 8\right]$$

$$= 74262.07\ \text{Rs/day}.$$

$$W_H = \begin{pmatrix} 0.8 \times 347.69 + 0.00033 \times 47.69^2 \times 14 + \\ 0.8 \times 169.54 + 0.000383 \times 169.54^2 \times 8 \end{pmatrix} \times 3600$$

$$= 20.22 \times 10^6\ \text{m}^3/\text{day}$$

Unit Commitment:

The total load of Power system is not Constant but Varies throughout the day and reaches a different Peak Value from one day to another day. It follows a Particular hourly load cycle over a day. There will be different discrete load levels at each Period as shown in below figure.

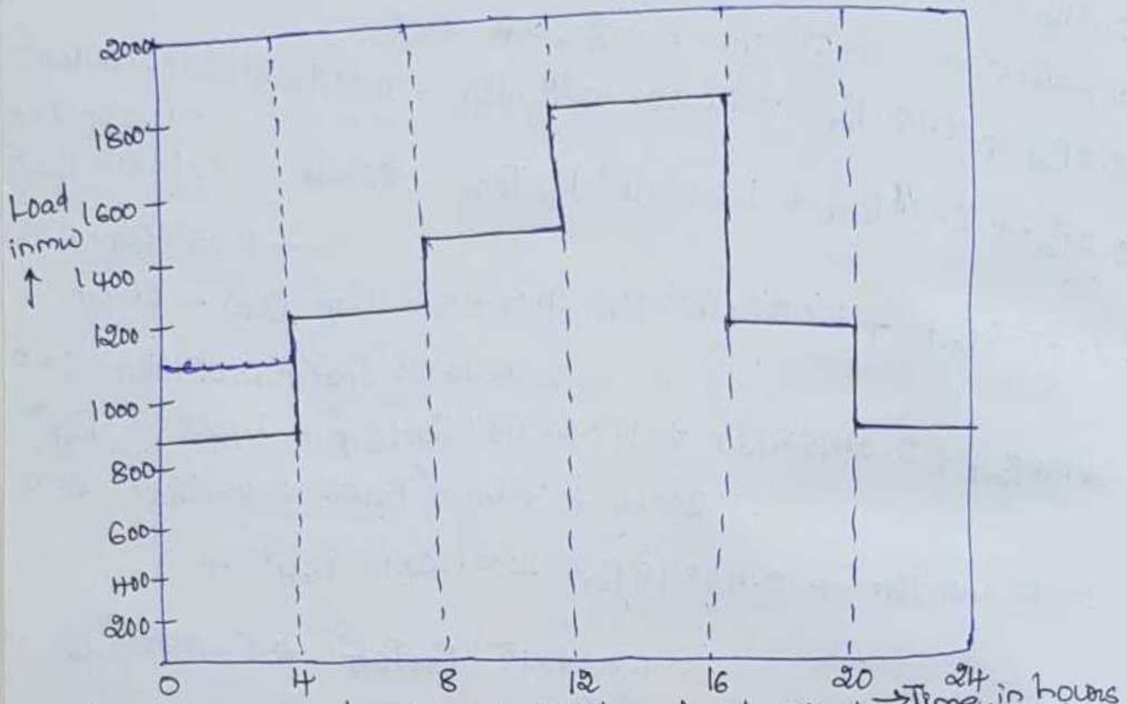


Fig. Discrete levels of system load of daily load cycle.

Due to the above reason it is not advisable to run all available Units all the time, and it is necessary to decide in advance which generators are to start up, when to connect them to network, the sequence in which the operating units should be shut down, and for how long, the computational procedure for making such decisions is called as U.C. (Unit commitment). and a unit when scheduled and for connection to the system is said to be committed.

The problem of U.C. is nothing but to determine the units that should operate for particular load to "Commit" a generating unit is to "turn it ON" i.e., to bring it upto a speed, synchronize it to the system, and connect it, so that it can deliver power to network.

Unit commitment operation of simple peak-Valley load Patterns: Shut down (load) rule:

Let us assume that load follows a simple Peak Valley Pattern as shown in below figure

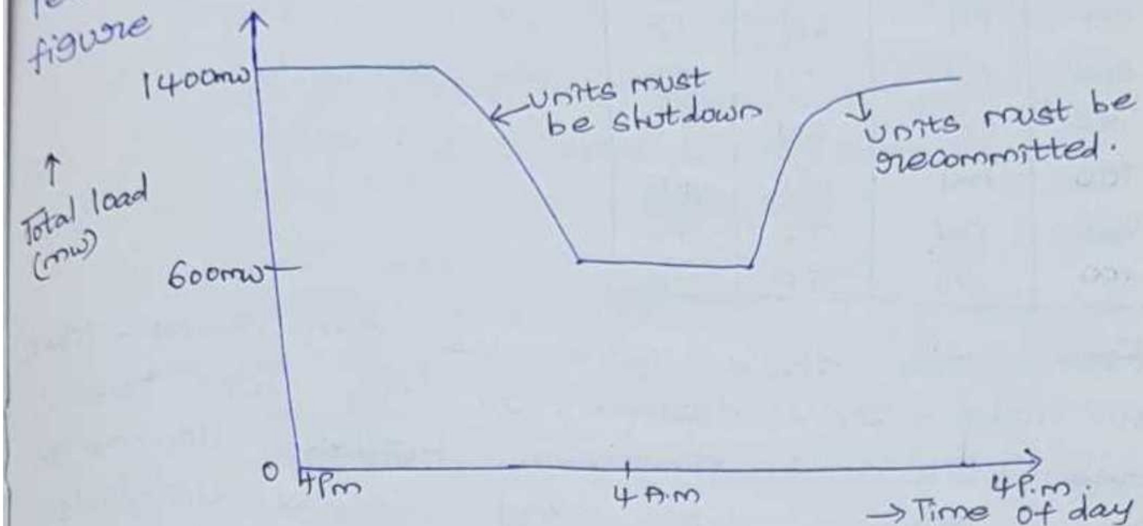


Fig. Simple Peak Valley load Pattern.

To optimize the system operation, some units must be shut down as the load decreases and is then recommitted (Put into service) as it goes back up.

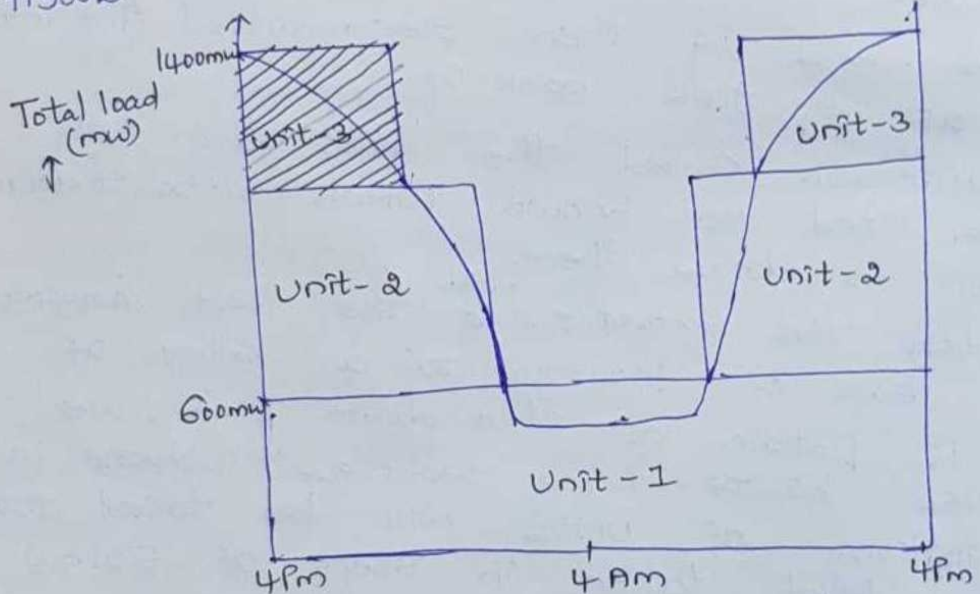
one approach called the "shut down rule" must be used to known which units to draw and when to draw them.

Consider the example, with the load Varying from a Peak of 1400mw to a Valley of 600mw. To obtain a shut down rule, we simply use "brute-force technique" where in all combinations of units will be tried for each load level taken in steps of 50mw

Load	Optimum Generation		
	Unit-I	Unit-II	Unit-III
1400	ON	ON	ON
1350	ON	ON	ON
1300	ON	ON	ON
1250	ON	ON	ON
1200	ON	ON	ON
1150	ON	ON	ON

Load	Unit commitment U-I	U-II	U-III
1100	ON	ON	OFF
1050	ON	ON	OFF
1000	ON	ON	OFF
950	ON	ON	OFF
900	ON	ON	OFF
850	ON	ON	OFF
800	ON	ON	OFF
750	ON	ON	OFF
700	ON	OFF	OFF
650	ON	OFF	OFF
600	ON	OFF	OFF

From the above table, we can observe that for load above 1100 mw, running all the three units is economical, between 1100 mw to 750 mw running first and second units is economical, below of 700 mw running of only one unit is economical as shown in below figure.



Comparison with economic load dispatch:

Economic dispatch economically distributes the actual system load as it rises to the various units that are already on line. However the unit commitment problems plans for best set of units to be available to supply the predicted or forecasted load of the system over a future time period.

Need for unit Commitment:

The plant commitment and unit ordering schedules extend the period of optimization from a few minutes to several hours. weekly pattern can be developed from daily schedules, likewise ~~mostly~~ monthly, seasonal and annual schedules can be prepared by taking into consideration repititive nature of load demand and seasonal variations.

A great deal of money can be saved by turning off the units when they are not needed for the time. If operation of system is to be optimized, the U.C. schedules are required for economically committing units in the plants to serve with the time at which individual units should be taken out from are return to the service.

This problem is of importance for scheduling thermal units in thermal plants, as for other types of generation such as hydro their aggregate costs (such as start up cost, operating fuel cost & shut down cost are negligible) so that their ON/OFF status is not important

constraints in unit commitment:

Spinning Reserve:

It is the term used to describe total amount of generation available from all synchronised units on system minus (−) the present load and losses being supplied. Here the synchronised units on system may be named as units spinning on the system.

$$\text{Spinning reserve} = \begin{bmatrix}\text{Total generation}\\ \text{output of all}\\ \text{synchronized}\\ \text{units at a}\\ \text{particular time}\end{bmatrix} - \begin{bmatrix}\text{load at}\\ \text{that time} +\\ \text{losses at}\\ \text{that time}\end{bmatrix}$$

i.e $P_{GSR} = \sum_{i=1}^{n} P_{Gi} - (P_D + P_L)$

where

P_{GSR} = Spinning reserve

P_{Gi} = Power generation of ith synchronized.

P_D = Total load on system.

P_L = Power loss of system.

Spinning reserve must be maintained so that the failure of one or more units does not cause too far a drop in system frequency. Simply if one unit fails, there must be an ample reserve on other units to make up for the loss in a specified time period.

The spinning reserve must be a given percentage of forecasted peak load demand (or) it must be capable of taking up the loss of most heavily loaded unit in a given period of time.

The reserves must be properly allocated among fast responding units and slow responding units such that this allows the automatic generation control system. to restore frequency and quickly interchange the time of outage of a generating unit.

Beyond the spinning reserve, the unit commitment problem may consider various classes of scheduled reserves or offline reserves this include quick start diesel or gas turbine units as well as most hydro units and pumped storage units that can be brought online, synchronised and brought upto maximum capacity quickly. As such this units can be counted in overall reserve assessment as long as their time to come up to maximum capacity is taken into consideration.

Reserves should be spread well around entire power system to avoid transmission system limitations and to allow different

Parts of system to run as Islands, should they become electrically disconnected.

Thermal Unit Constraints:

A thermal unit can undergo only gradual temperature changes and this translates into time Period (of some hours) require to bring the unit on the line. Due to such limitations in operation of a thermal Plant, the following Constraints are to be considered

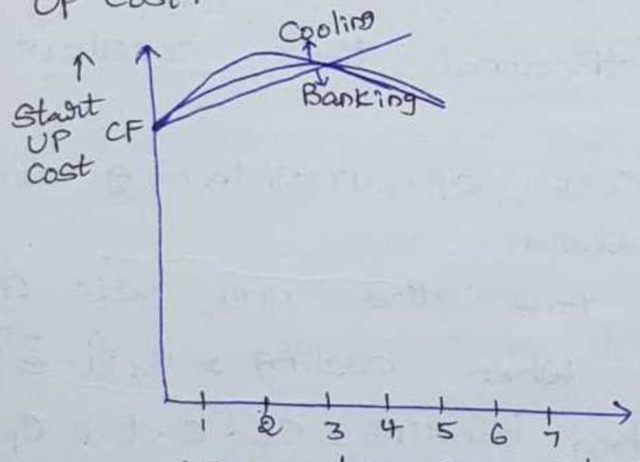
(i) minimum UP Time: During minimum up time, once unit is operating, it should not be turned off immediately.

(ii) minimum down Time: The minimum down time is the minimum time during which the unit is in down state i.e., once unit is decommitted, there is a minimum time before it can be recommitted.

(iii) crew Constraints:

If a Plant consist of two or more units, they cannot be turned on at same time since there are not enough crew members to attempt the both units while starting up.

Start UP cost:



Time dependant Start UP Cost.

Fig (a).

In addition to above constraints, because the temperature and Pressure of the thermal unit must be moved slowly, a certain amount of energy must be expended to bring the Unit on line. and is brought into Unit Commitment Problem as a Start up cost.

The start up cost may vary from a maximum "cold start" value to a very small value if the unit was only turned off recently and it is still relatively close to operating temperature.

. Two approaches to treating a thermal unit during its down ~~steam~~ state.

The first approach (Cooling) allows the unit's boiler to cool down and then heat back upto a operating temperature in time for a scheduled turn ON.

The second approach (Banking) requires that sufficient energy be given as input to the boiler to just maintain the operating temperature.

The best approach can be chosen by comparing the cost for the above two approaches.

Let "C_c" be the cold start cost, 'c' be the fuel cost, "C_f" be the fixed cost (

'α' be the thermal time constant for the unit.

"C_t" be the cost of maintaining unit at a operating temperature.

't' be the time the unit was cooled (hrs)

Start up cost when cooling $= C_c(1-e^{-t/\alpha})c+C_f$

start up cost when banking $= C_t \cdot c \cdot t + C_F$

Up to a certain number of hours, the cost of banking is less than cost of cooling as shown in fig (a).

The capacity limits of thermal units may change frequently due to maintainance or unscheduled outages of various equipment in plant and this must also be taken into consideration in the unit commitment problem.

Hydro Constraints:

The hydro thermal scheduling will be explained as seperated from unit commitment problem. Operation of a system having both hydro and thermal plants is more complex, as hydro plants have negligible operating costs, but are required to operate under constraints of water available for hydro generation in given period of time.

The problem of minimising the operating cost of thermal system can be viewed as one of minimizing the fuel cost of thermal plants under constraints of water availability for hydro generation over a given period of operation.

Must Run:

It is necessary to give a must-run recognization to some units of plant during certain events of the year, to yield the voltage support on transmission network or for such purpose as supply of steam for uses outside the steam plant itself.

Fuel constraints:

A system in which some units have limited fuel or else have constraints that require them to burn a specified amount of fuel in a given time presents a most challenging unit commitment problem.

Cost Function Formulation:

Let

"F_i" be the cost of operation of i^{th} unit.

"P_{Gi}" is the output of the i^{th} unit.

"C_i" is the running cost of i^{th} unit

$$\therefore \quad F_i = C_i \cdot P_{Gi}$$

"C_i" may vary depending on the load condition

Let C_{ij} be the variable cost coefficient for the ith unit when operating at jth load for which the corresponding active power is P_{Gij}

$\therefore$ $F_i = C_{ij} P_{Gij}$

Since the level of operation is a function of time, the cost efficiency may be described with another index to denote the time of operation, so that it becomes C_{ijt} for the sub interval "t" corresponding to a power output P_{Gijt}

If each unit is capable of operation at "t" discrete levels then the running cost "F_{it}" of the ith unit in the time interval "t" is given by

$$F_{it} = \sum_{i=1}^{K} C_{ijt} P_{Gijt}$$

If there are "n" number of units available in time interval "t" then the total running cost of "n" unit during time interval "t" is

$$F_{int} = \sum_{i=1}^{n} \sum_{j=1}^{K} C_{ijt} P_{Gijt}$$

For the entire time period of optimization having "T" sub intervals of time, the overall running cost may become

$$F_T = \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{j=1}^{K} C_{ijt} P_{Gijt}$$

Start up Cost Consideration:

Suppose that for a plant to be brought into service, an additional expenditure C_{sci} has to be added in addition to the running cost i.e., (C_{sci} = start up cost at ith unit), the cost of starting "x" no. of units during any sub interval "t" is given by

$$F_{sc} = \sum_{i=1}^{x} C_{sci} \delta_{it}$$

where $S_{it} = 1$; if the i^{th} unit is started in sub interval "t" and otherwise $S_{it} = 0$.

over the complete scheduling the "x" sub-interval, the overall cost is

$$F_{SCT} = \sum_{t=1}^{T} \sum_{i=1}^{x} C_{sci} S_{it}$$

Shut down cost consideration:

Similarly, if the plant is taken out of service during scheduling period, it is necessary to consider shut down cost, if "y" number of units to be shut down during sub interval "t", the shut down cost becomes

$$F_{sd} = \sum_{i=1}^{y} C_{sdi} \sigma_{it}$$

where

σ_{it} = if the i^{th} unit is thrown out of service in sub interval "t" and otherwise $\sigma_{it} = 0$

over the complete scheduling the "T" sub-interval the shut down cost is

$$F_{sdT} = \sum_{t=1}^{T} \sum_{i=1}^{y} C_{sdit} \sigma_{it}$$

Now the total expression for the cost function including the running cost, the start up cost and the shut down cost is written as

$$F_T = \sum_{t=1}^{T} \left| \sum_{q=1}^{n} \sum_{j=1}^{t} C_{ijt} P_{Gijt} + \sum_{i=1}^{x} C_{scit} S_{it} + \sum_{i=1}^{y} C_{sdit} \sigma_{it} \right.$$

Unit Commitment- Solution methods:

The most important techniques for the solution of a unit commitment problem are

1. Priority - list schemes

2. Dynamic programming (DP) method.

3. Lagrange's relaxation (LR) method.

A simple shut-down rule or priority list scheme could be obtained after an exhaustive enumeration of all unit

Combinations at each load level.

Enumeration Scheme:

A straight forward but highly time consuming way of finding most economical combination of units to meet a particular load demand is to try all possible combinations of units that can supply this load. This load is divided optimally among units of each combination by the use of co-ordination equations so as to find most economical operating cost of combination. Then the combination that has the least operating cost among all these is determined.

Some combinations will be infeasible if the sum of all maximum MW for units committed is less than load (or) if the sum of all MW for units committed is greater than the load.

Ex: Let us consider a plant having three units. The cost characteristics and maximum and minimum limits of power generation (MW) of each unit are as follows.

Unit-1:

$$C_1 = 0.002842\,P_{G_1}^2 + 8.46\,P_{G_1} + 600 \text{ Rs/hr}, \quad 200 \le P_{G_1} \le 650$$

Unit-2:

$$C_2 = 0.002936\,P_{G_2}^2 + 8.32\,P_{G_2} + 420 \text{ Rs/hr}, \quad 150 \le P_{G_2} \le 450$$

Unit-3:

$$C_3 = 0.006449\,P_{G_3}^2 + 9.884\,P_{G_3} + 110 \text{ Rs/hr}, \quad 100 \le P_{G_3} \le 300$$

To supply a total load of 600 MW most economically, the combinations of units and their generation status are tabulated as shown in following table.

No. of combinations $= 2^n = 2^3 = 8$.

Combination	Status of Units: Unit-1	Unit-2	Unit-3	P_{Gmax}	P_{Gmin}	P_{G1}	P_{G2}	P_{G3}	C_1	C_2	C_3	Total generation cost ($C_1+C_2+C_3$) (in Rs)
1.	OFF	OFF	OFF	0	0	Infeasible	Infeasible	Infeasible	–	–	–	–
2.	OFF	OFF	ON	300	100	"	"	"	–	–	–	–
3.	OFF	ON	OFF	450	150	"	"	"	–	–	–	–
4.	OFF	ON	ON	750	250	0	450	150	–	4,758,54	1,882,805	6,641,345
5.	ON	OFF	OFF	650	200	600	0	0	6,6984	–	–	6,698,400
6.	ON	OFF	ON	950	300	500	0	100	5,540,5	–	1,162,89	6,703,390
7.	ON	ON	OFF	1100	350	292.77	307.23	0	3,320,43	3,252,28	–	6,573,71
8.	ON	ON	ON	1400	450	241.95	258.05	100	2,813,267	881,877	1,162,89	4,858,035

Table 1. Combination of units and their status for dispatch of 600 MW load

Unit	Rs/mwh	P_{Gmax}	P_{Gmin}
2	9.834	450	150
1	9.838	650	200
3	11.1738	300	100

Table 2. Priority ordering of units.

Combination of Units	For Combination P_{Gmin}	For Combination P_{Gmax}
2, 1 and 3	450	1,400
2 and 1	350	1,100
2	150	450

Table 3. Priority list for supply of 1400 MW

Priority- list method:

A simple but sub-optimal approach to the problem is to impose priority ordering, where in the most efficient unit is loaded first to be followed by less efficient units in order as load increases.

From table 1, we construct priority list as follows,

First the full load average production cost will be calculated.

The full load average production cost of unit1 = 9.838 Rs/mwh

Unit 2 = 9.834 Rs/mwh

Unit 3 = 11.173 Rs/mwh

2. Dynamic Programming:

It is based on Principle of optimality explained by Bellman in 1957 which states that an optimal Policy has the property, that whatever the initial state and initial decisions are done, the remaining decisions must constitute an optimal Policy with regard to state resulting from first decision.

This method Consist of a distinct number of stages. However it is suitable only when the decision at later stages do not effect the operation at earlier stages

solution of an optimal Unit Commitment problem with DP method:

Dynamic programming has many advantages, the main advantage is reduction in the size of the problem.

The imposition (drawback) of priority list arranged in order of full load average cost rate would result in a correct dispatch and commitment only if

1. No load costs are zero.
2. Unit input and output characteristics are linear between zero output and full load.
3. There are no other limitations.
4. Start up costs are a fixed amount.

In the DP approach, we assume that

1. A state consists of an array(n) of units with specified operating units and the rest are at offline.

The start up cost of a unit is independent of time if it has been offline.

There are no costs for shutting down a unit.

There is a strict priority order and in each interval specified minimum amount of

Capacity must be operating.

A feasable state is one at which the Committed units can supply required load and that meets the minimum amount of Capac in each period.

Practically, a UC table is to be made for complete load cycle. The DP method is more efficient for preparing UC table if the available load demand is assumed to increase in small but finite size steps.

In DP it is not necessary to solve Co-ordinate equations, while at the same time the unit combinations are to be tried

The total no. of units available, their individual cost characteristics and load cycle on station are assumed to be known a prior

Procedure for preparing the unit commitment table using DP approach:

Step I: Start arbitarily with consideration of any two units

Step II: Arrange the combined output of the two units in the form of discrete load levels.

Step III: Determine the most economical combination of two units for all load levels. It is to be observed that at each load level, economic operation may be to run either unit or both units with certain load sharing between two units.

Step IV: obtain the most economical cost curve in discrete form for two units and that can be treated as cost curve of single equivalent unit.

Step V: Add the third unit and repeat the procedure to find cost curve of three combined units. It may be noted that, by this procedure, the operating combinations of third and first, third and second units are not required to be worked out resulting in considerable saving in computation.

step VI: Repeat the process till all available units are exhausted.

The main advantage of this DP method is that having obtained the optimal way of loading "K" units, it is quite easy to determine the optimal way of loading "K+1" units.

mathematical Expression:

Let $F_N(x)$ = minimum cost in Rs/hr of generating of "x" MW by "N" no. of units

$f_N(y)$ = cost of generation of "y" MW by the "N" units

$f_{N-1}(x-y)$ = minimum cost of generation of "x-y" MW by "N-1" units

$$\therefore F_N(x) = \min_y \{f_N(y) + F_{N-1}(x-y)\}$$